Types

1 byte. = 8 bits

2^8 256/2 = 128

0 - 256

-128 - 127:

{unsigned variable}

A Type Is A Set

```
int n;
```

- When you declare that a variable has a certain type, you are saying that the values the variable can have are elements of a certain set
- *A type is a set of values*
  - plus a low-level representation
  - plus a collection of operations that can be applied to those values

Today: A Tour Of Types

- There are too many to cover them all
- Instead, a short tour of the type menagerie
- Most ways you can construct a set in mathematics are also ways to construct a type in some programming language
- We will organize the tour around that connection

Outline

- Type Menagerie
  - Primitive types
  - Constructed types
- Uses For Types
  - Type annotations and type inference
  - Type checking
  - Type equivalence issues

Primitive vs. Constructed Types

- Any type that a program can use but cannot define for itself is a *primitive type* in the language
- Any type that a program can define for itself (using the primitive types) is a *constructed type*
- Some primitive types in ML: **`int`**, **`real`**, **`char`**
  - An ML program cannot define a type named **`int`** that works like the predefined **`int`**
- A constructed type: **`int list`**
  - Defined using the primitive type **`int`** and the **`list`** type constructor

Primitive Types

- The definition of a language says what the primitive types are
- Some languages define the primitive types more strictly than others:
  - Some define the primitive types exactly (Java)
  - Others leave some wiggle room—the primitive types may be different sets in different implementations of the language (C, ML)

Comparing Integral Types

C:

consider inf.

`char`
`unsigned char`
`short int`
`unsigned short int`
`int`
`unsigned int`
`long int`
`unsigned long int`

No standard implementation, but longer sizes must provide at least as much range as shorter sizes.

Java:

`byte` (1-byte signed)
`char` (2-byte unsigned)
`short` (2-byte signed)
`int` (4-byte signed)
`long` (8-byte signed)

Scheme:

`integer`

Integers of unbounded range

Issues

- What sets do the primitive types signify?
  - How much is part of the language specification, how much left up to the implementation?
  - If necessary, how can a program find out? (**`INT_MAX`** in C, **`Int.maxInt`** in ML, etc.)
- What operations are supported?
  - Detailed definitions: rounding, exceptions, etc.
- The choice of representation is a critical part of these decisions

Outline

- Type Menagerie
  - Primitive types
  - Constructed types
- Uses For Types
  - Type annotations and type inference
  - Type checking
  - Type equivalence issues

Constructed Types

- Additional types defined using the language
- Today: enumerations, tuples, arrays, strings, lists, unions, subtypes, and function types
- For each one, there is connection between how *sets* are defined mathematically, and how *types* are defined in programming languages

Making Sets by Enumeration

- Mathematically, we can construct sets by just listing all the elements:

$$S = \{a, b, c\}$$

Making Types by Enumeration

- Many languages support *enumerated types*:

C:
```
enum coin {penny, nickel, dime, quarter};
```
Ada:
```
type GENDER is (MALE, FEMALE);
```
Pascal:
```
type primaryColors = (red, green, blue);
```
ML:
```
datatype day = M | Tu | W | Th | F | Sa | Su;
```

- These define a new type (= set)
- They also define a collection of named constants of that type (= elements)

Representing Enumeration Values

- A common representation is to treat the values of an enumeration as small integers
- This may even be exposed to the programmer, as it is in C:

```
enum coin { penny = 1, nickel = 5, dime = 10, quarter = 25 };

enum escapes { BELL = '\a', BACKSPACE = '\b', TAB = '\t',
               NEWLINE = '\n', VTAB = '\v', RETURN = '\r' };
```

Operations on Enumeration Values

- Equality test:

```
fun isWeekend x = (x = Sa orelse x = Su);
```

- If the integer nature of the representation is exposed, a language will allow some or all integer operations:

Pascal:
```
for C := red to blue do P(C)
```

C:
```
int x = penny + nickel + dime;
```

Making Sets by Tupling

- The Cartesian product of two or more sets defines sets of tuples:

$$S = X \times Y$$

$$= \{(x, y) \mid x \in X \wedge y \in Y\}$$

Making Types by Tupling

- Some languages support pure tuples:

```
fun get1 (x : real * real) = #1 x;
```

- Many others support record types, which are just tuples with named fields:

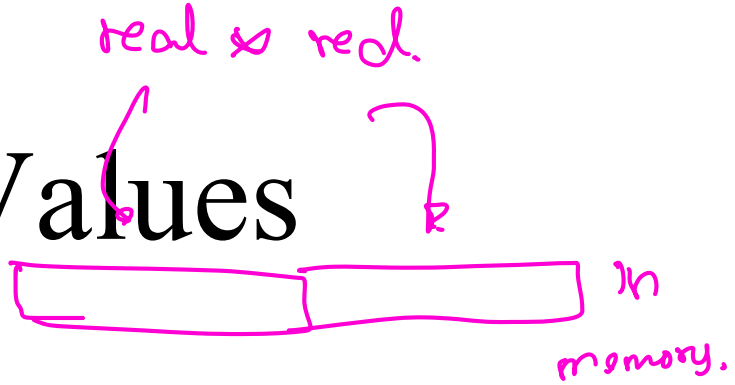
C:

```
struct complex {
  double rp;
  double ip;
};
```

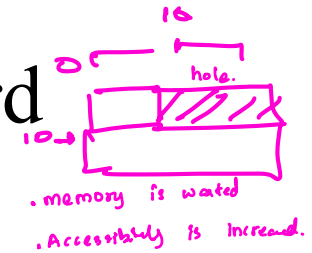
ML:

```
type complex = {
  rp:real,
  ip:real
};
fun getip (x : complex) = #ip x;
```

Representing Tuple Values



- A common representation is to just place the elements side-by-side in memory
- But there are lots of details:
  - in what order? (typical → According to the order?
  - with "holes" to align elements (e.g. on word boundaries) in memory?
  - is any or all of this visible to the programmer?



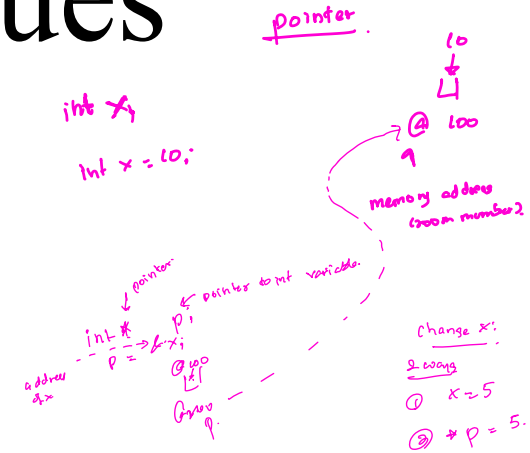
Example: ANSI C

The members of a structure have addresses increasing in the order of their declarations. A non-field member of a structure is aligned at an addressing boundary depending on its type; therefore, there may be unnamed holes in a structure. If a pointer to a structure is cast to the type of a pointer to its first member, the result refers to the first member…

Adjacent field members of structures are packed into implementation-dependent storage units in an implementation-dependent direction...

The C Programming Language, 2nd ed.
Brian W. Kernighan and Dennis M. Ritchie

Operations on Tuple Values



- Selection, of course:

  C: `x.ip`

  ML: `#ip x`

- Other operations depending on how much of the representation is exposed:

C:
```
double y = *((double *) &x);
struct person {
  char *firstname;
  char *lastname;
} p1 = {"marcia","brady"};
```

Sets Of Vectors

- Fixed-size vectors:

$$S = X^n$$
$$= \{(x_1, \ldots, x_n) \mid \forall i \,.\, x_i \in X\}$$

- Arbitrary-size vectors:

$$S = X^*$$
$$= \bigcup_i X^i$$

Types Related To Vectors

- Arrays, strings and lists
- Like tuples, but with many variations
- One example: indexes
  - What are the index values?
  - Is the array size fixed at compile time?

Index Values

- Java, C, C++:
  - First element of an array **`a`** is **`a[0]`**
  - Indexes are always integers starting from 0
- Pascal is more flexible:
  - Various index types are possible: integers, characters, enumerations, subranges
  - Starting index chosen by the programmer
  - Ending index too: size is fixed at compile time

Pascal Array Example

```
type
  LetterCount = array['a'..'z'] of Integer;
var
  Counts: LetterCount;

begin
  Counts['a'] = 1
  etc.
```

Types Related To Vectors

- Many variations on vector-related types:
  - What are the index values?
  - Is array size fixed at compile time (part of static type)?
  - What operations are supported?
  - Is redimensioning possible at runtime? (change array size)
  - Are multiple dimensions allowed?
  - Is a higher-dimensional array the same as an array of arrays?
  - What is the order of elements in memory?
  - Is there a separate type for strings (not just array of characters)?
  - Is there a separate type for lists?

Making Sets by Union

- We can make a new set by taking the union of existing sets:

$$S = X \bigcup Y$$

Making Types by Union

- Many languages support union types:

C:

```
union element {
  int i;
  float f;
};
```

ML:

```
datatype element =
  I of int |
  F of real;
```

Representing Union Values

- You can have the two representations overlap each other in memory

```
union element {
  int i;
  char *p;
} u; /* sizeof(u) ==
        max(sizeof(u.i),sizeof(u.p)) */
```

- This representation may or may not be exposed to the programmer

Strictly Typed Unions

- In ML, all you can do with a union is extract the contents
- And you have to say what to do with each type of value in the union:

```
datatype element =
  I of int |
  F of real;

fun getReal (F x) = x
  | getReal (I x) = real x;
```

Loosely Typed Unions

- Some languages expose the details of union implementation
- Programs can take advantage of the fact that the specific type of a value is lost:

```
union element {
   int i;
   float f;
};

union element e;
e.i = 100;
float x = e.f;
```

What ANSI C Says About This

A union may be thought of as a structure all of whose members begin at offset 0 and whose size is sufficient to contain any of its members. At most one of the members can be stored in a union at any time. If a pointer to a union is cast to the type of a pointer to a member, the result refers to that member.

In general, a member of a union may not be inspected unless the value of the union as been assigned using that same member.

The C Programming Language, 2nd ed.
Brian W. Kernighan and Dennis M. Ritchie

A Middle Way: Variant Records

- Union where specific type is linked to the value of a field ("*discriminated union*")
- A variety of languages including Ada and Modula-2

Ada Variant Record Example

```
type DEVICE is (PRINTER, DISK);

type PERIPHERAL(Unit: DEVICE) is
  record
    HoursWorking: INTEGER;
    case Unit is
      when PRINTER =>
        Line_count: INTEGER;
      when DISK =>
        Cylinder: INTEGER;
        Track: INTEGER;
    end case;
  end record;
```

Same with Java inheritance

Making Subsets

- We can define the subset selected by any predicate P:

$$S = \{x \in X \mid P(x)\}$$

Making Subtypes

- Some languages support subtypes, with more or less generality
  - Less general: Pascal subranges

```
type digit = 0..9;
```

  - More general: Ada subtypes

```
subtype DIGIT is INTEGER range 0..9;
subtype WEEKDAY is DAY range MON..FRI;
```

  - Most general: Lisp types with predicates

Example: Ada Subtypes

```
type DEVICE is (PRINTER, DISK);

type PERIPHERAL(Unit: DEVICE) is
  record
    HoursWorking: INTEGER;
    case Unit is
      when PRINTER =>
        Line_count: INTEGER;
      when DISK =>
        Cylinder: INTEGER;
        Track: INTEGER;
    end case;
  end record;

subtype DISK_UNIT is PERIPHERAL(DISK);
```

Example: Lisp Types with Predicates

```
(declare (type integer x))

(declare (type (or null cons) x))

(declare (type (and number (not integer)) x))

(declare (type (and integer (satisfies evenp)) x))
```

Representing Subtype Values

- Usually, we just use the same representation for the subtype as for the supertype
- Questions:
  - Do you try to shorten it if you can? Does `X: 1..9` take the same space as `X: Integer`?
  - Do you enforce the subtyping? Is `X := 10` legal? What about `X := X + 1`?

Operations on Subtype Values

- Usually, supports all the same operations that are supported on the supertype
- And perhaps additional operations that would not make sense on the supertype:

```
function toDigit(X: Digit): Char;
```

- Important meditation:

> A subtype is a subset of values, but it can support a superset of operations.

A Word About Classes

After midterm

- This is a key idea of object-oriented programming
- In class-based object-oriented languages, a *class* can be a type: data and operations on that data, bundled together
- A *subclass* is a subtype: it includes a subset of the objects, but supports a superset of the operations
- More about this in Chapter 13

Making Sets of Functions

- We can define the set of functions with a given domain and range:

$$S = D \rightarrow R$$

$$= \{f \mid \text{dom } f = D \wedge \text{ran } f = R\}$$

$S = \underbrace{\text{int} \times \text{int}}_{D} \rightarrow \underbrace{\text{int}}_{R}$

$\{(x, y) \mid y \neq 0\}$

Making Types of Functions

- Most languages have some notion of the type of a function:

C:

```
int f(char a, char b) {
  return a==b;
}
```

ML:

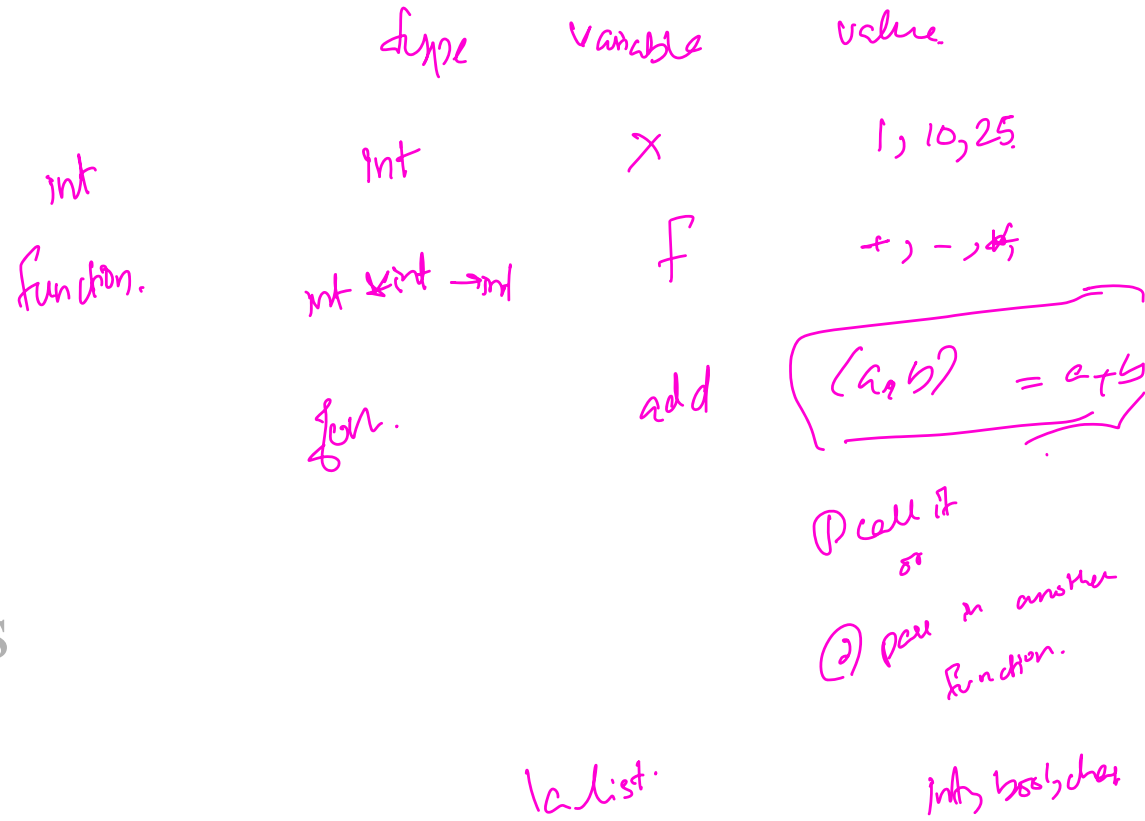
```
fun f(a:char, b:char) = (a = b);
```

Operations on Function Values

- Of course, we need to *call* functions
- We have taken it for granted that other types of values could be passed as parameters, bound to variables, and so on
- Can't take that for granted with function values: many languages support nothing beyond function call
- We will see more operations in ML

Outline

- Type Menagerie
  - Primitive types
  - Constructed types
- Uses For Types
  - Type annotations and type inference
  - Type checking
  - Type equivalence issues



Type Annotations

- Many languages require, or at least allow, type annotations on variables, functions, …
- The programmer uses them to supply static type information to the language system
- They are also a form of documentation, and make programs easier for people to read
- Part of the language is syntax for describing types (think of `*`, `->` and `list` in ML)

Intrinsic Types

- Some languages use naming conventions to declare the types of variables
  - Dialects of BASIC: `S$` is a string
  - Dialects of Fortran: `I` is an integer
- Like explicit annotations, these supply static type information to the language system and the human reader

Extreme Type Inference

- ML takes type inference to extremes
- Infers a static type for every expression and for every function
- Usually requires no annotations

Simple Type Inference

- Most languages require some simple kinds of type inference
- Constants usually have static types
  - Java: `10` has type `int`, `10L` has type `long`
- Expressions may have static types, inferred from operators and types of operands
  - Java: if `a` is `double`, `a*0` is `double` (`0.0`)

Outline

- Type Menagerie
  - Primitive types
  - Constructed types
- Uses For Types
  - Type annotations and type inference
  - Type checking
  - Type equivalence issues

Static Type Checking

- Static type checking determines a type for everything before running the program: variables, functions, expressions, everything
- Compile-time error messages when static types are not consistent
  - Operators: `1+"abc"`
  - Functions: `round("abc")`
  - Statements: `if "abc" then` …
- Most modern languages are statically typed

Dynamic Typing

- In some languages, programs are not statically type-checked before being run
- They are usually still *dynamically* type-checked
- At runtime, the language system checks that operands are of suitable types for operators

Example: Lisp

- This Lisp function adds two numbers:

```
(defun f (a b) (+ a b))
```

- It won't work if **a** or **b** is not a number
- An improper call, like **(f nil nil)**, is not caught at compile time
- It is caught at runtime – that is dynamic typing

It Still Uses Types

- Although dynamic typing does not type everything at compile time, it still uses types
- In a way, it uses them even more than static typing
- It needs to have types to check at runtime
- So the language system must store type information with values in memory

Static And Dynamic Typing

- Not quite a black-and-white picture
- Statically typed languages often use some dynamic typing
  - Subtypes can cause this
  - Everything is typed at compile time, but compile-time type may have subtypes
  - At runtime, it may be necessary to check a value's membership in a subtype
  - This problem arises in object-oriented languages especially – more in Chapter 13

Static And Dynamic Typing

- Dynamically typed languages often use some static typing
  - Static types can be inferred for parts of Lisp programs, using constant types and declarations
  - Lisp compilers can use static type information to generate better code, eliminating runtime type checks

Explicit Runtime Type Tests

- Some languages allow explicit runtime type tests:
  - Java: test object type with **`instanceof`** operator
  - Modula-3: branch on object type with **`typecase`** statement
- These require type information to be present at runtime, even when the language is mostly statically typed

Strong Typing, Weak Typing

- The purpose of type-checking is to prevent the application of operations to incorrect types of operands
- In some languages, like ML and Java, the type-checking is thorough enough to guarantee this—that's *strong typing*
- Many languages (like C) fall short of this: there are holes in the type system that add flexibility but weaken the guarantee

Outline

- Type Menagerie
  - Primitive types
  - Constructed types
- Uses For Types
  - Type declarations and inference
  - Static and dynamic typing
  - Type equivalence issues

Type Equivalence

- When are two types the same?
- An important question for static and dynamic type checking
- For instance, a language might permit **`a:=b`** if **`b`** has "the same" type as **`a`**
- Different languages decide type equivalence in different ways

Type Equivalence

- *Name equivalence*: types are the same if and only if they have the same name
- *Structural equivalence*: types are the same if and only if they are built from the same primitive types using the same type constructors in the same order
- Not the only two ways to decide equivalence, just the two easiest to explain
- Languages often use odd variations or combinations

Type Equivalence Example

```
type irpair1 = int * real;
type irpair2 = int * real;
fun f(x:irpair1) = #1 x;
```

- What happens if you try to pass **`f`** a parameter of type **`irpair2`**?
  - Name equivalence does not permit this: **`irpair2`** and **`irpair1`** are different names
  - Structural equivalence does permit this, since the types are constructed identically
- ML does permit it

Type Equivalence Example

```
var
   Counts1: array['a'..'z'] of Integer;
   Counts2: array['a'..'z'] of Integer;
```

- What happens if you try to assign **`Counts1`** to **`Counts2`**?
  - Name equivalence does not permit this: the types of **`Counts1`** and **`Counts2`** are unnamed
  - Structural equivalence does permit this, since the types are constructed identically
- Most Pascal systems do not permit it

Conclusion

- A key question for type systems: how much of the representation is exposed?
- Some programmers prefer languages like C that expose many implementation details
  - They offer the power to cut through type abstractions, when it is useful or efficient or fun to do so
- Others prefer languages like ML that hide all implementation details (*abstract types*)
  - Clean, mathematical interfaces make it easier to write correct programs, and to prove them correct