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Investigating Damped Oscillatory Motion

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1. Introduction

The overarching purpose is to fit different damping models to data and inspect which model fits the data the best. For the completion of this goal, data will be generated, filtered, analyzed, & fitted. Futhermore, an explanation would be provided for the chosen model.

2. Chosen Phenomenon and Data Source

The phenomenon selected is Simple harmonic oscillations in a lab environment. Data collected by a device called 'iOLab' in a lab environment is opted as the source of this investigation. The data set consists of Time (s), Force (N), Displacenment (m), Velocity (ms^{-1}) and, Acceleration (ms^{-2}). *(The file used will be available in repository)*

3. Equations To Fit Data

There are 2 special damping models:

1. Velocity damped friction where the damping factor depends on the velocity of the oscillator
2. Constant friction model where the damping value is independent of the velocity, and rather depends on the coefficient of friction (μ) on the surface in contact (which is basically how much friction is provided by the surface the oscillator is oscillating on).

3.1. Velocity-Dependant Friction Model

The case is modelled by the following differential equation:

$$\ddot{y}(t) + 2\beta\dot{y}(t) + (\omega_0)^2 y(t) = 0 \tag{1}$$

Where, β is a constant of proportionality and is equal to $\frac{b}{2m}$ v is the velocity m is the mass of the iOLab Friction force = bv The solution of the aforementioned equation follows, which is used to model line of best fit:

$$y(t) = Ae(-\beta t)\cos(\omega t + \Phi) \tag{2}$$

3.2. Constant Friction Model

In this the damping factor is constant and the following differential equation is used to model the line of best fit:

$$\ddot{y}(t) + (\omega_0)^2 y(t) \pm \frac{f}{m} = 0 \tag{3}$$

where, f is the constant friction.

The $\pm$ sign depends on the direction of motion as one direction is taken as positive and the reverse is negative and friction will oppose the direction of motion. Consider that the spring starts at y = $-A_0$ and ends at y = A_1

The solution of the aforementioned equation follows:

$$A_n - A_{n+1} = \frac{2f}{k} \tag{4}$$

For further comparison of the oscillation models, an undamped model was also added cllaed the simple harmonic model.

3.3. Simple Harmonic Oscillation Model

The case is modelled by the following differential equation:

$$\ddot{y}(t) + (\omega_0)^2 y(t) = 0 \tag{5}$$

whose solution is:

$$y(t) = A\cos(\omega_0 t + \phi), \tag{6}$$

for some amplitude A and phase Φ. The amplitude remains constant throughout, as this is an undamped case.

4. Data Generation For Testing

To ensure the validity of the models: they will be applied to random data generated by Numpy. Moreover, they will be applied to a known damped case and be assessed based on error and the perfection of the fit.

During this process, it was realized that simple random data would not be possible to analyze adequately. To further expand, the random data fits none of the three models describred in the equations to fit as none of them have a sinusoidal pattern. Therefore, data based on the simple harmonic model was generated and noise was applied at 20% at random, uniformly.

What follows is the data generated's plot:

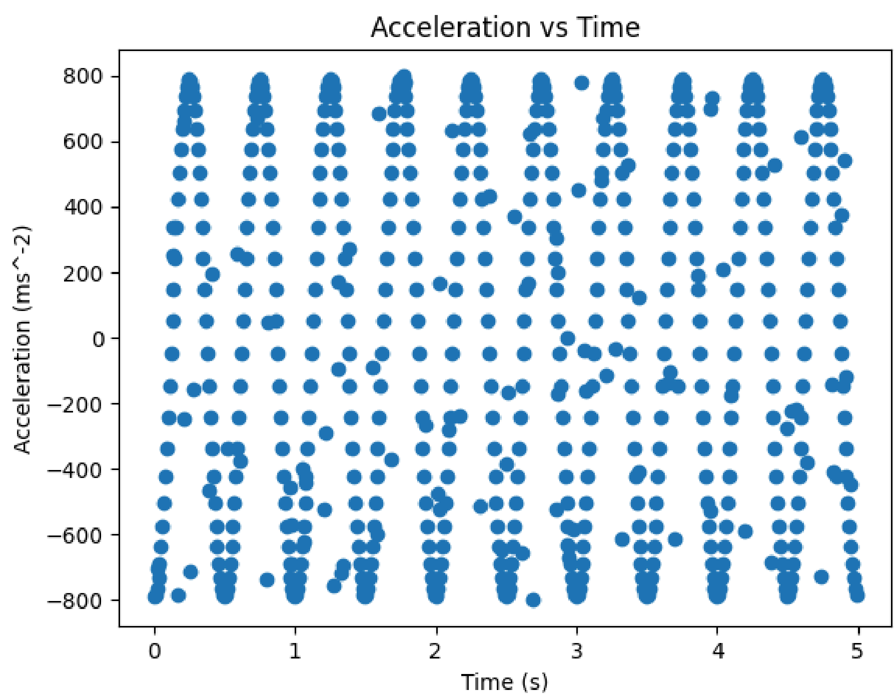

Figure 1: plot of acceleration vs time.

5. Data Filtering

To make certain that the data set being fit is of good quality, the data will be filtered by removing any obvious outliers and duplicate data.

To check for duplicates the len(list) to the len(set(list)), the list being the x values (not y values as they can be similar at times), to confirm whether filtering is necessary. This is because the list in itself can have duplicates, but the set does not contain any duplicates. In this case they matched. Therefore, no further filtering was required (it is known that the random data could generate duplicates but the seed 73 in place, it turned out that no duplicates occured.)

Furthermore, it was realized in the limited dataset the outliers were not extreme and removing the tiny outliers would simply nullify the noise added during generation.

6. Data Fitting With Error

To analyze which model is the best fit, the residuals of each model will be plotted which will be assessed on distribution of the points and proximity to x-axis. further more the value of the period and damping factor and be calculated by python with covariance to assess the best fit.

6.1. Lab Data

For the **lab data** the following plots were generated along with their residuals.

6.1.1. Simple harmonic oscillation fit

This model makes a fit assuming that the oscillations are undamped. This causes the data to not correspond accurately with the line of best fit as in real life energy is lost to the environment which causes the oscillation to be damped. Not a great fit.

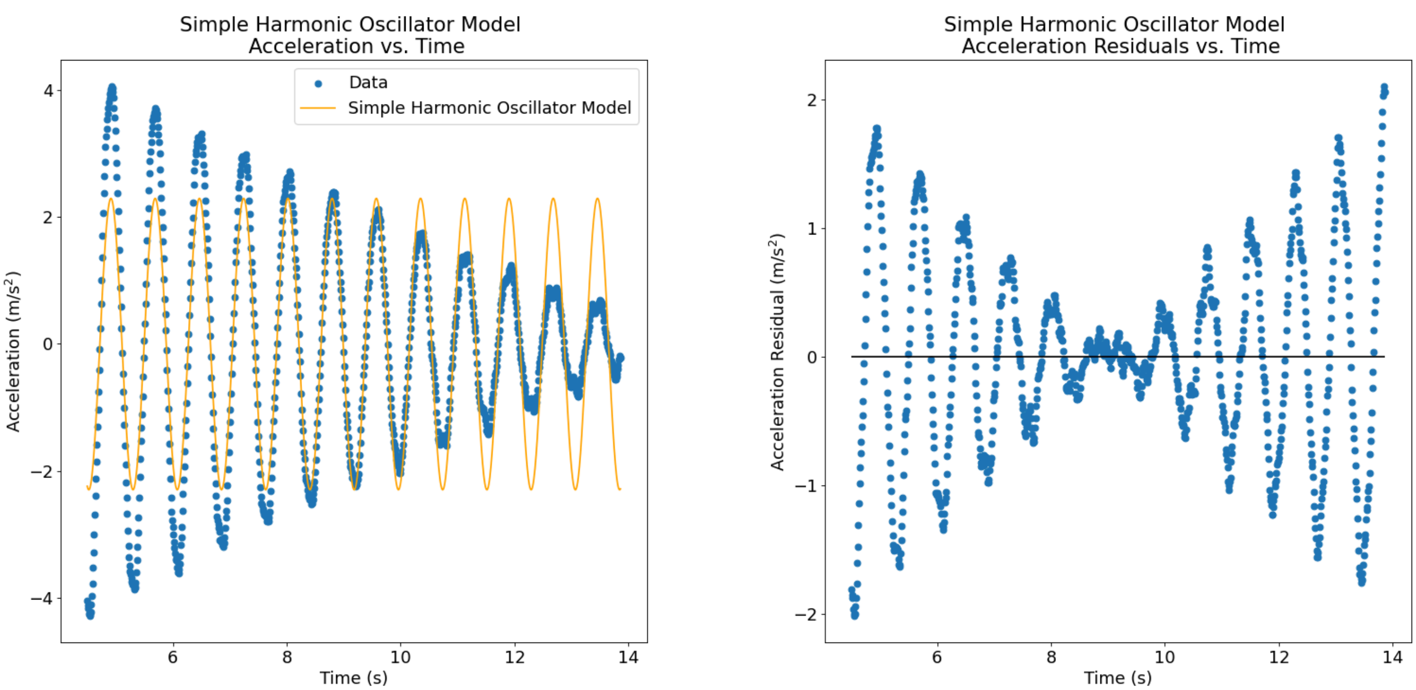

Figure 2: simple harmonic motion fit, plot of acceleration vs time and residuals vs time

Calculated Parameters:

- Amplitude = 0.03501293630582916 $\pm$ 3.1060801835705897e-07 m
- Offset = $-$0.0023171345854919173 $\pm$ 0.0006634338548552344 m/s2

- Angular Frequency = 8.091310918011459 ± 3.512113194023326e-05 Hz
- Phase = −0.22281318786958898 ± 0.001024073237456813

6.1.2. Velocity-Dependant Friction Model

In this model work done by a velocity dependent friction is accounted for in a system. This model is relatively better than the former as it does damp the system, however, it fails to account for the rolling resistance, thus is not precise.

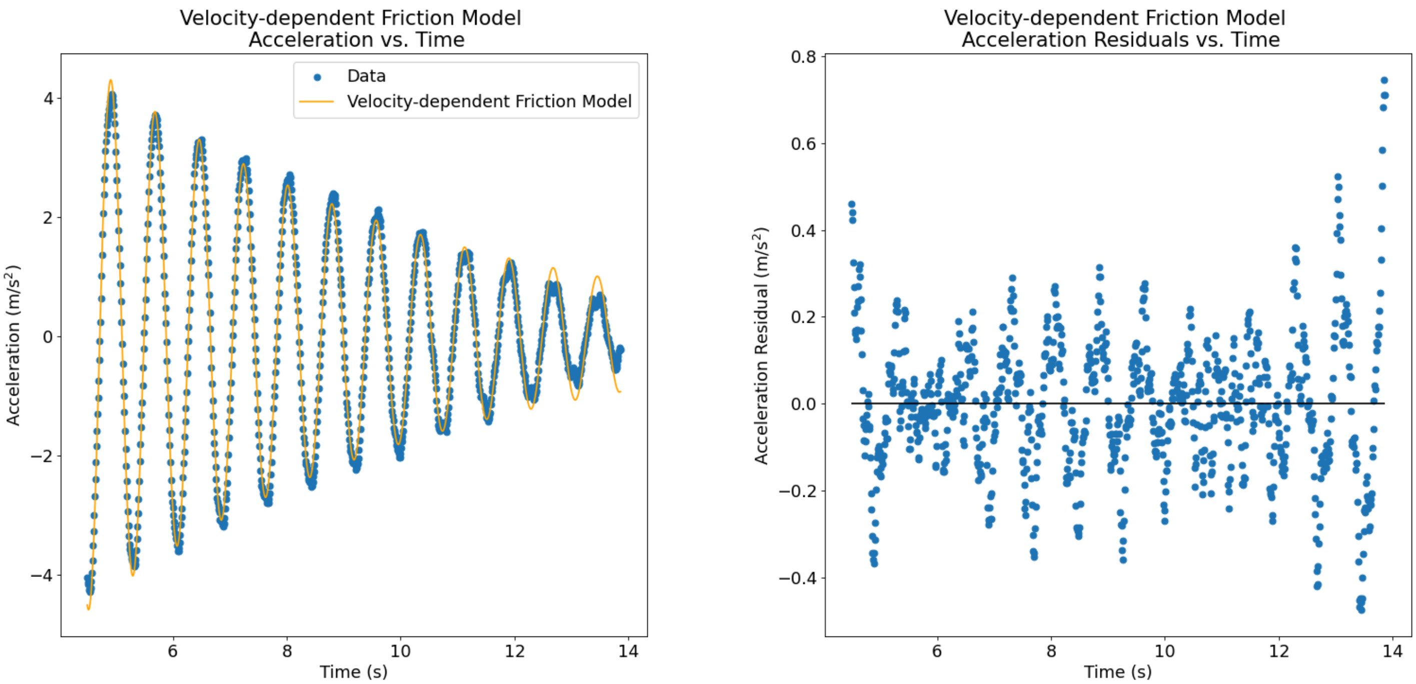

Figure 3: velocity dependant damped fit, plot of acceleration vs time and residuals vs time

Calculated Parameters:

- Amplitude = 0.07038512547285165 ± 9.949857421575286e-08 m
- Offset = 0.0030306717240111388 ± 2.7783275270267563e-05 m/s
- Omega = 8.089650011865984 ± 1.8871504492066312e-06
- Phi = −0.2507652218368846 ± 2.1128779643073925e-05
- Damping = 0.17025049996072586 ± 1.8108848553240495e-06

6.1.3. Constant Friction Model

In this model the friction is constant and the work done by friction is accounted for in the system.

Maxima line of best fit: y = −0.07186222x + 1.05897853

Minima line of best fit: y = 0.07066866x + −1.03597433

The line of best fit has been calculated according to the Constant friction model. The phase and the period of the 'line of best fit' correspond to the data, and the model fits as well.

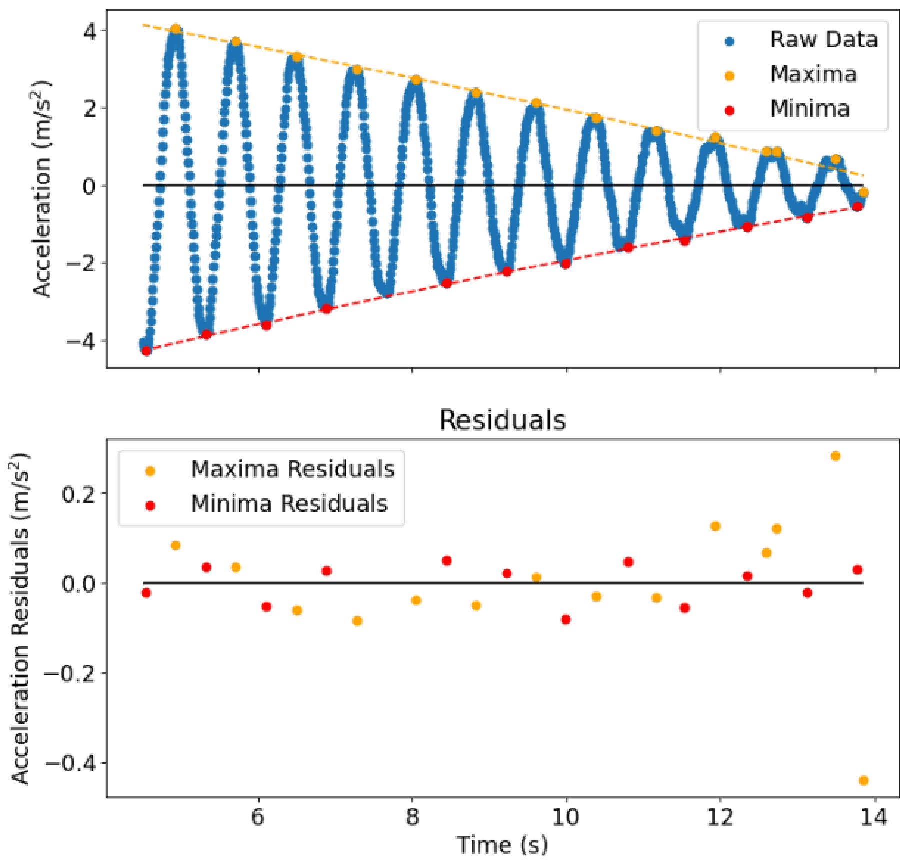

Figure 4: constricted friction damped fit, plot of acceleration vs time and residuals vs time.

6.2. Random Data

The **random data** clearly did not have damping as the amplitude did not reduce overtime. Therefore, it was visually fit to the simple harmonic oscillation. The fit did not seem visually apt which could be due to the outliers not being removed. Despite, the data being interpolated it seems as the plot didnot improve significanly.

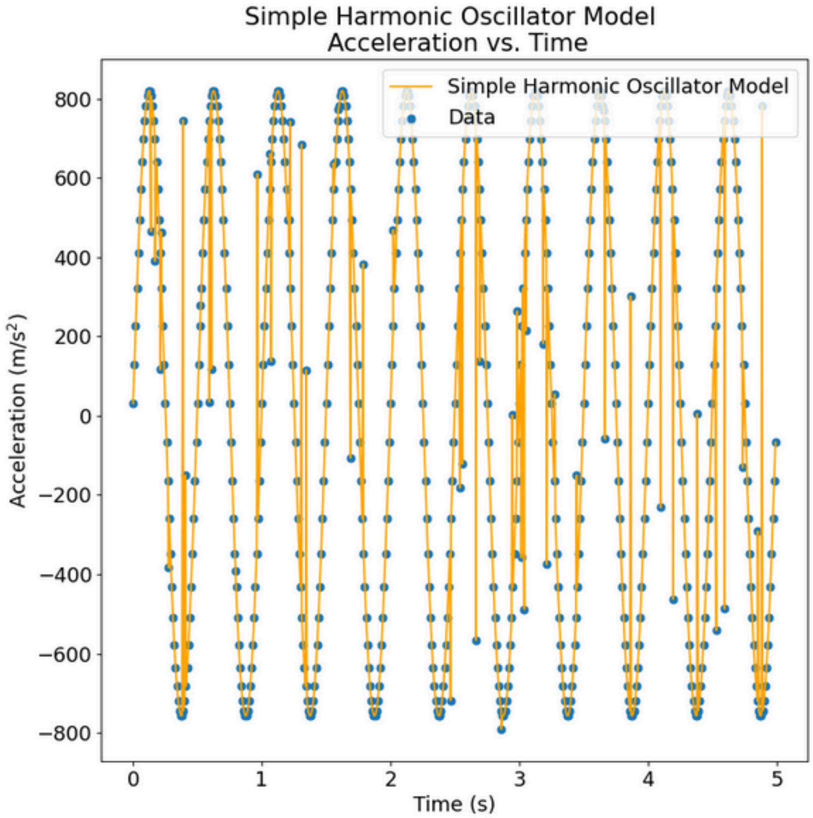

Figure 5: constricted friction damped fit, plot of acceleration vs time and residuals vs time.

7. Explanation of Model Fit

In conclusion, the constant friction model is the best at illustrating the dynamical behavior. The system is evidently damped as the oscillations do come to a stop, thus the undamped simple harmonic model fails to explain the behavior. However, the damped velocity dependent friction model is relatively better than the former as it does have a velocity dependent damping factor of 0.170 ± 1.81e-06. It's line of best fit fails to fit the data precisely and the residuals show an evident sinusoidal pattern. Additionally, the 2nd model is better than the first one due the damping being present is well illustrated by a plot of both of their residuals on the same graph. Furthermore, the covariance of the parameters decrease significantly, when switched from the 1st to the 2nd model However, they both show a clear sinusoidal trend which causes the third model to be ultimately the best model for the fit of lab data. Additionally, the residuals for the 3rd model are the best due to the close proximity to y = 0 and the randomess as shown in the graph below:

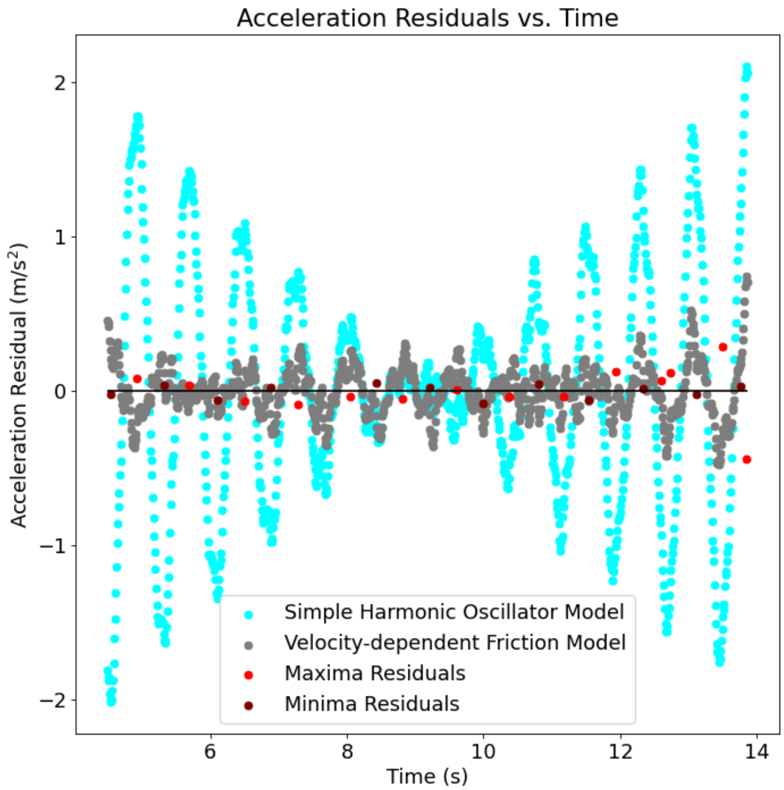

Figure 6: all residuals against time.

This matches the case as the osscilations were on a wooden surface that provided a constant friction.

For the random data it is clear that the Simple harmonic fit was the best as it not damped, therefore, the 2 damped models cant be applied to it for good results.