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Investigating Damped Oscillatory Motion

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1. Introduction

The overarching purpose is to fit different damping models to data and inspect which model fits the data the best. For the completion of this goal, data will be generated, filtered, analyzed, & fitted. Futhermore, an explanation would be provided for the chosen model.

2. Chosen Phenomenon and Data Source

The phenomenon selected is Simple harmonic oscillations in a lab environment. Data collected by a device called 'iOLab' in a lab environment is opted as the source of this investigation. The data set consists of Time (s), Force (N), Displacenment (m), Velocity (ms^{-1}) and, Acceleration (ms^{-2}). *(The file used will be available in repository)*

3. Equations To Fit Data

There are 2 special damping models:

1. Velocity damped friction where the damping factor depends on the velocity of the oscillator
2. Constant friction model where the damping value is independent of the velocity, and rather depends on the coefficient of friction (μ) on the surface in contact (which is basically how much friction is provided by the surface the oscillator is oscillating on).

3.1. Velocity-Dependant Friction Model

The case is modelled by the following differential equation:

$$\ddot{y}(t) + 2\beta\dot{y}(t) + (\omega_0)^2 y(t) = 0 \tag{1}$$

Where, β is a constant of proportionality and is equal to $\frac{b}{2m}$ v is the velocity m is the mass of the iOLab Friction force = bv The solution of the aforementioned equation follows, which is used to model line of best fit:

$$y(t) = Ae(-\beta t)\cos(\omega t + \varphi) \tag{2}$$

3.2. Constant Friction Model

In this the damping factor is constant and the following differential equation is used to model the line of best fit:

$$\ddot{y}(t) + (\omega_0)^2 y(t) \pm \frac{f}{m} = 0 \tag{3}$$

where, f is the constant friction.

The $\pm$ sign depends on the direction of motion as one direction is taken as positive and the reverse is negative and friction will oppose the direction of motion. Consider that the spring starts at y = $-A_0$ and ends at y = A_1

The solution of the aforementioned equation follows:

$$A_n - A_{n+1} = \frac{2f}{k} \tag{4}$$

4. Data Generation For Testing

To ensure the validity of the models: they will be applied to random data generated by Numpy. Moreover, they will be applied to a known damped case and be assessed based on error and the perfection of the fit.

5. Data Filtering

To make certain that the data set being fit is of good quality, the data will be filtered by removing any obvious outliers and duplicate data.

6. Data Fitting With Error

To analyze which model is the best fit, the residuals of each model will be plotted which will be assessed on distribution of the points and proximity to x-axis. further more the value of the period and damping factor and be calculated by python with covariance to assess the best fit.

7. Explanation of Model Fit

After the completion of the fits, an explanation of the selected model would be shown along with evaluation of parameters calculated. This would provide a deeper understanding of Damped oscillatory systems.