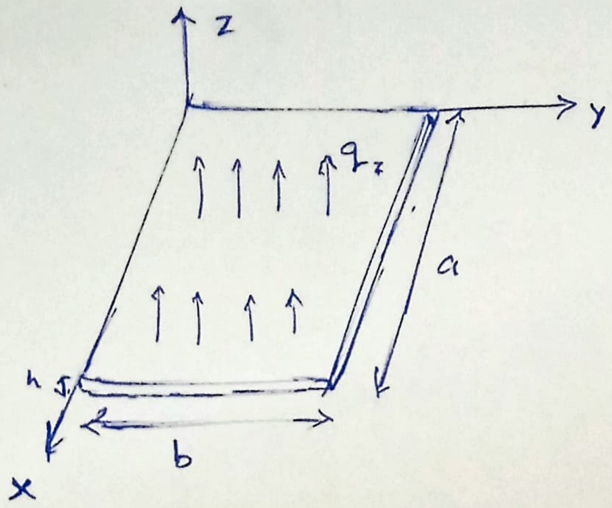
1.



$a = b = 0.5m$

$h = 4 \times 10^{-3} m$

$E = 200 GPa$

$\nu = 0.3$

Governing eq is $\nabla^4 w = \frac{q_z}{D}$

$$D = \frac{Eh^3}{12(1-\nu^2)} = 1172.16 Nm$$

~~x=0, y=0~~ Boundary conditions

$w(0,y) = 0, \quad w(a,y) = 0$

$w(x,0) = 0, \quad w(x,b) = 0$

2. Assume a double fourier series

$$w_n = \sum_{m=1,3,5}^{n} \sum_{n=1,3,5}^{n} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Solving it $A_{mn} = \frac{4}{ab\pi^4 D} \dfrac{\int_{x=0}^{a}\int_{y=0}^{b} q_z \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) dx\, dy}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$

$$A_{mn} = \frac{4q_z}{\pi^6 D m^3 n^3} \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^{-2}$$

$$w_n = \frac{4q_z}{\pi^6 D} \sum_{m=1,3,5..}^{n} \sum_{n=1,3,5}^{n} \frac{\sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{b}\right)}{m^3 n^3\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

4. $\sigma_{xx}(z) = \frac{12z}{h^3} M_x$ $\quad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$

$\sigma_{yy}(z) = \frac{12z}{h^3} M_y$ $\quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$

$\sigma_{xy}(z) = \frac{12z}{h^3} M_{xy}$ $\quad M_{xy} = 0$

The ~~plot~~ point of highest deflection occurs at $\left(\frac{a}{2}, \frac{b}{2}\right)$

$w_{max} = w\left(\frac{a}{2}, \frac{b}{2}\right)$

$w_{2_{max}} = 14.2\,\mu m$

$w_{4_{max}} = 13.17\,\mu m$

$w_{6_{max}} = 13.39\,\mu m$

5. $\sigma_{yield} = 450\,MPa$

$\sigma_{VM} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy}}$

$= \sqrt{\left(\frac{6M_x}{h^2}\right)^2 + \left(\frac{6M_y}{h^2}\right)^2 - \frac{6M_x}{h^2} \cdot \frac{6M_y}{h^2}}$

$\sigma_{2_{VM}}$ ~~1050MPa~~ At $q = 1\,Pa$

$\sigma_{2_{VM}} = 1050\,MPa$

$\sigma_{4_{VM}} = 1200\,MPa$

$\sigma_{6_{VM}} = 1237.5\,MPa$

$$q_{yield} = q \times \frac{\sigma_{yield}}{\sigma_{vM}}$$

$q_{2\,yield} = 0.43\,Pa$

$q_{4\,yield} = 0.38\,Pa$

$q_{6\,yield} = 0.36\,Pa$

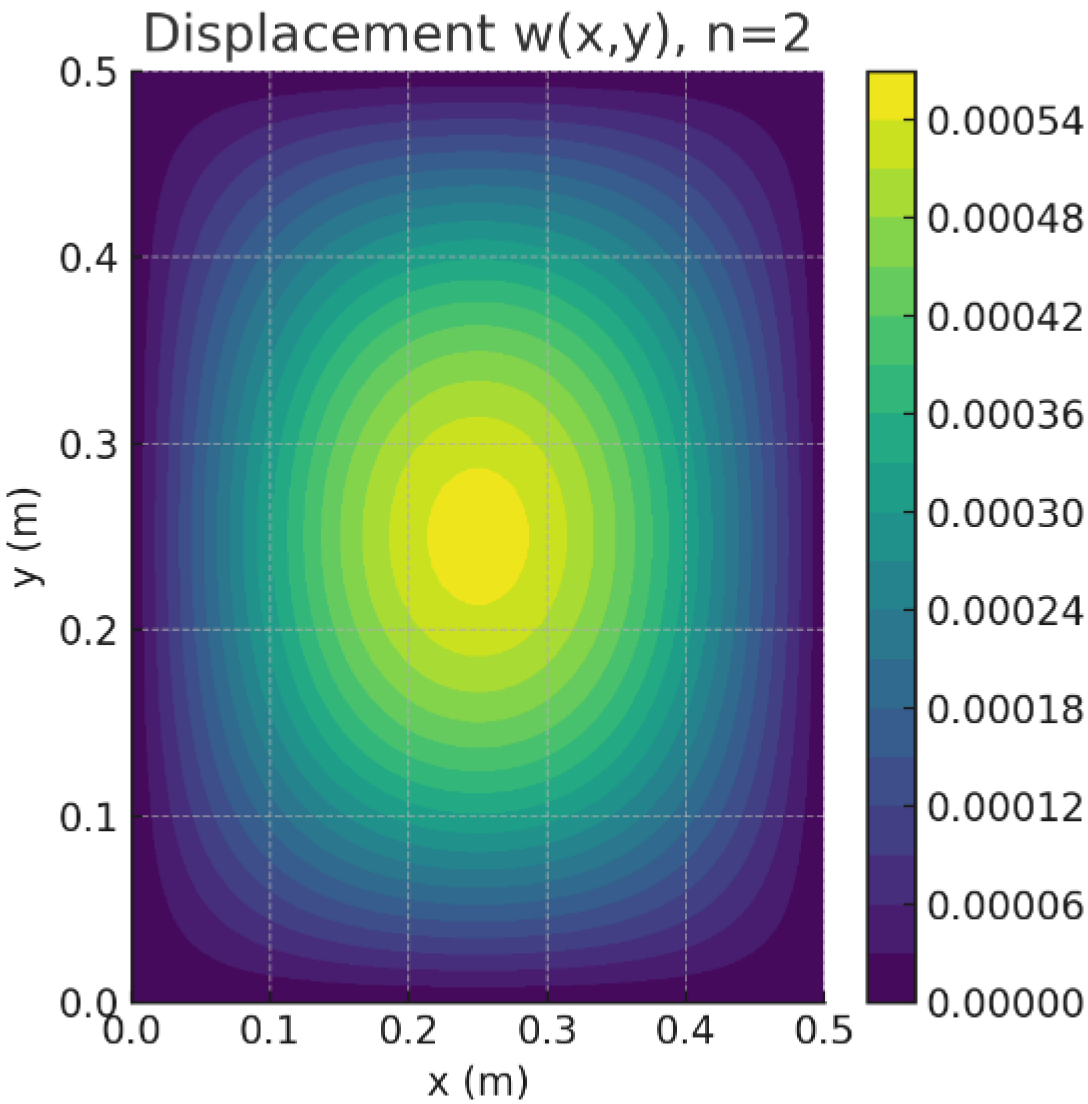
6. Strain energy

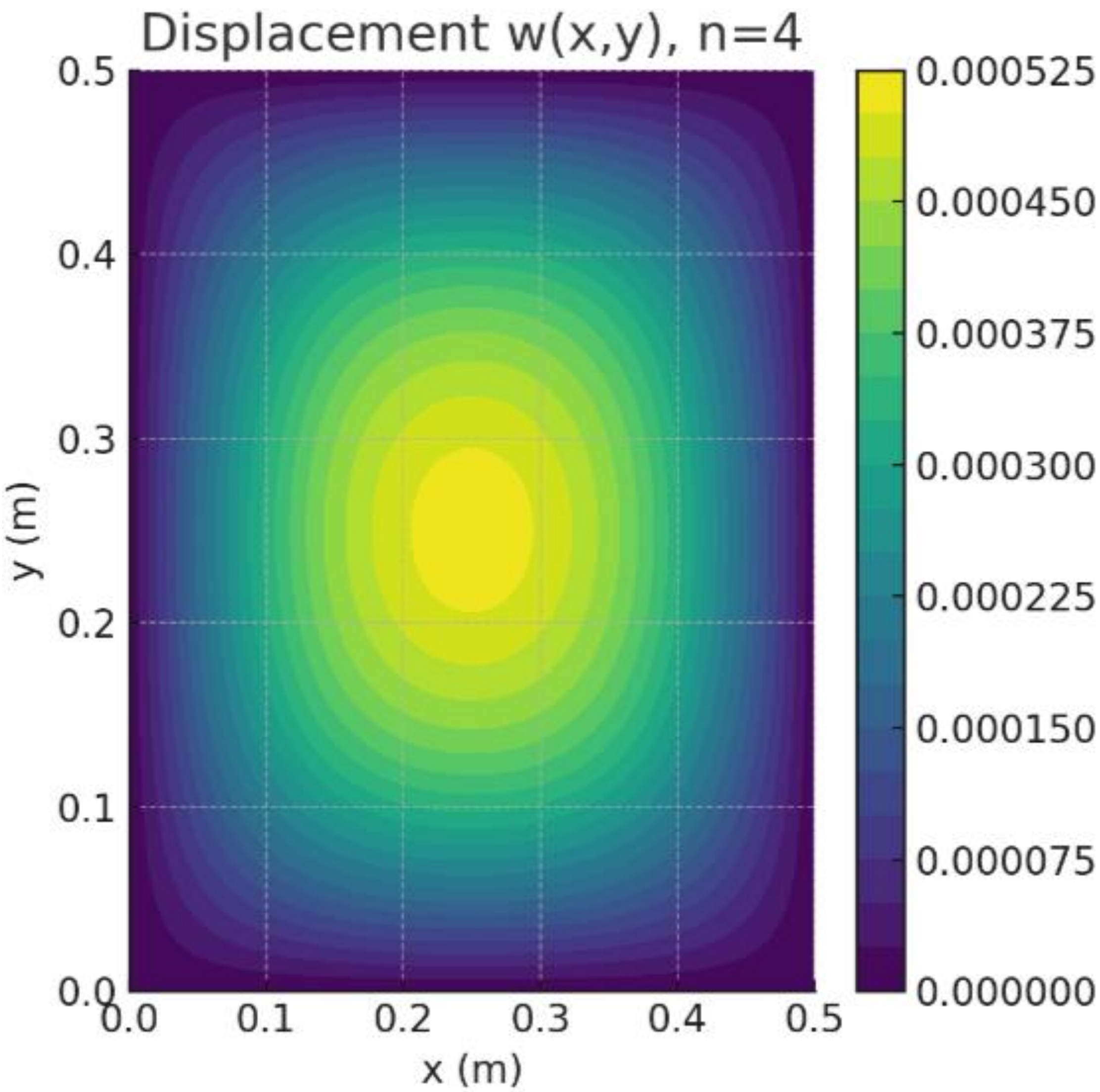
$$U = \frac{1}{2} \int_0^a \int_0^b D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dx\,dy$$

$U_2 = 0.014\,J$

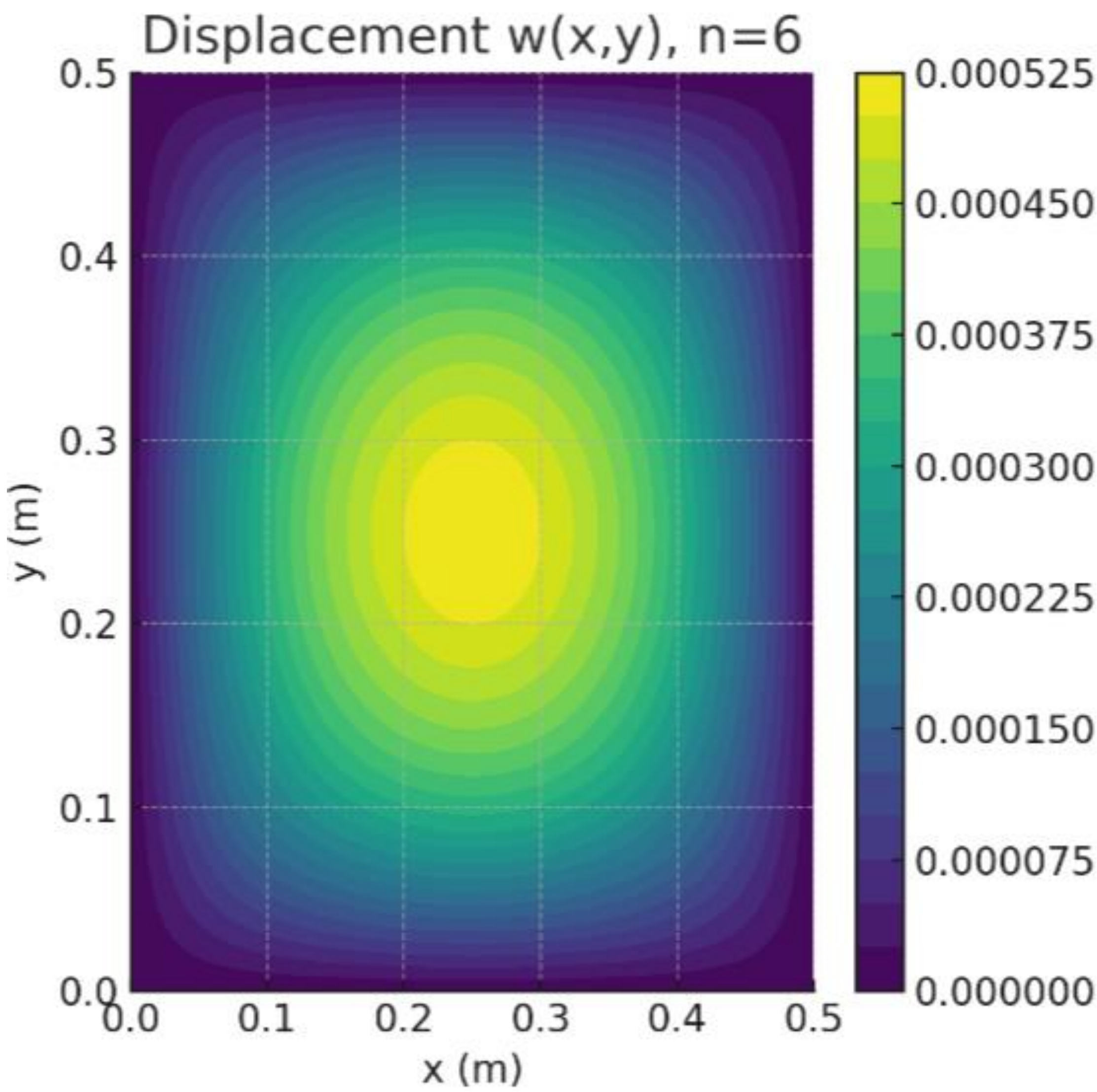
$U_4 = 0.0128\,J$

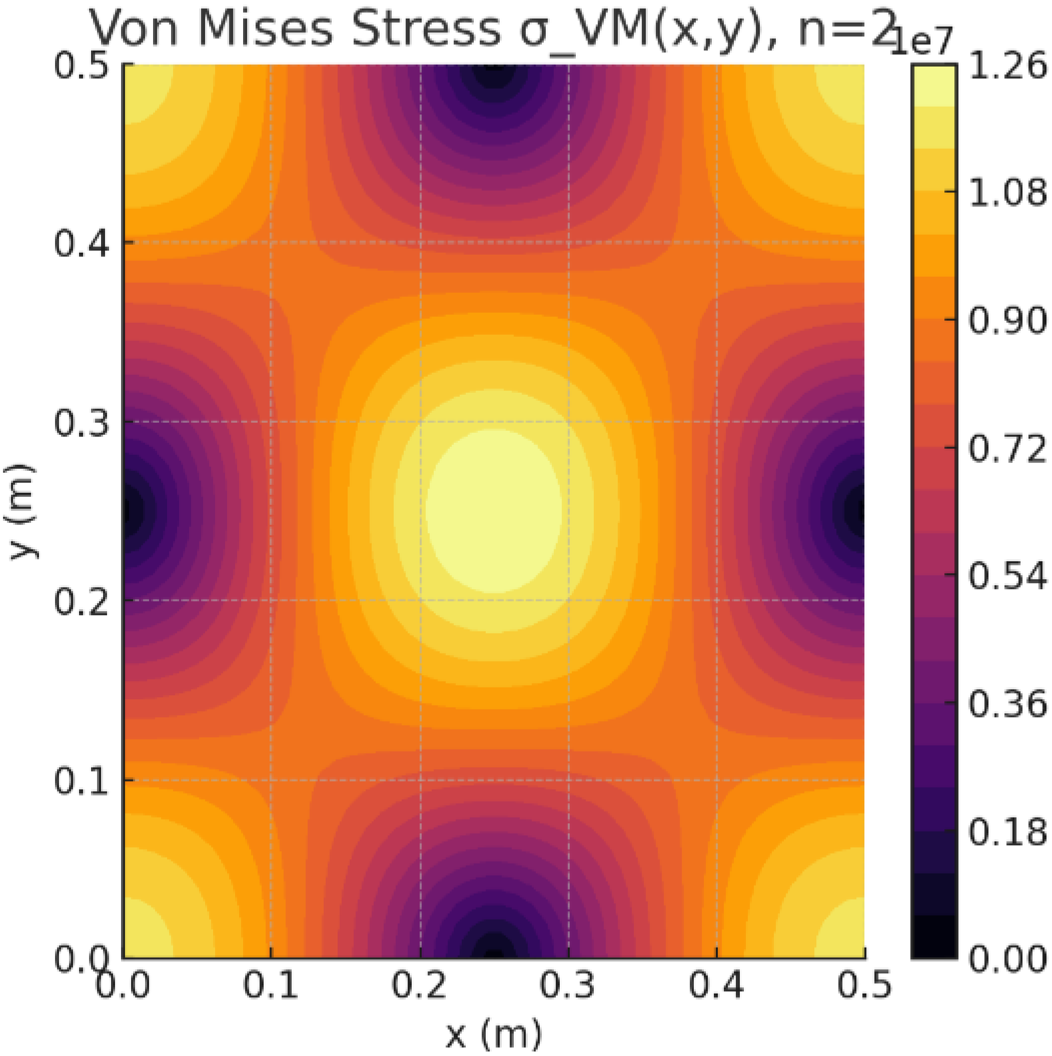
$U_6 = 0.0131\,J$


Displacement w(x,y), n=2
0.5
0.4
0.3
0.2
0.1
0.0
y (m)
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
0.00054
0.00048
0.00042
0.00036
0.00030
0.00024
0.00018
0.00012
0.00006
0.00000

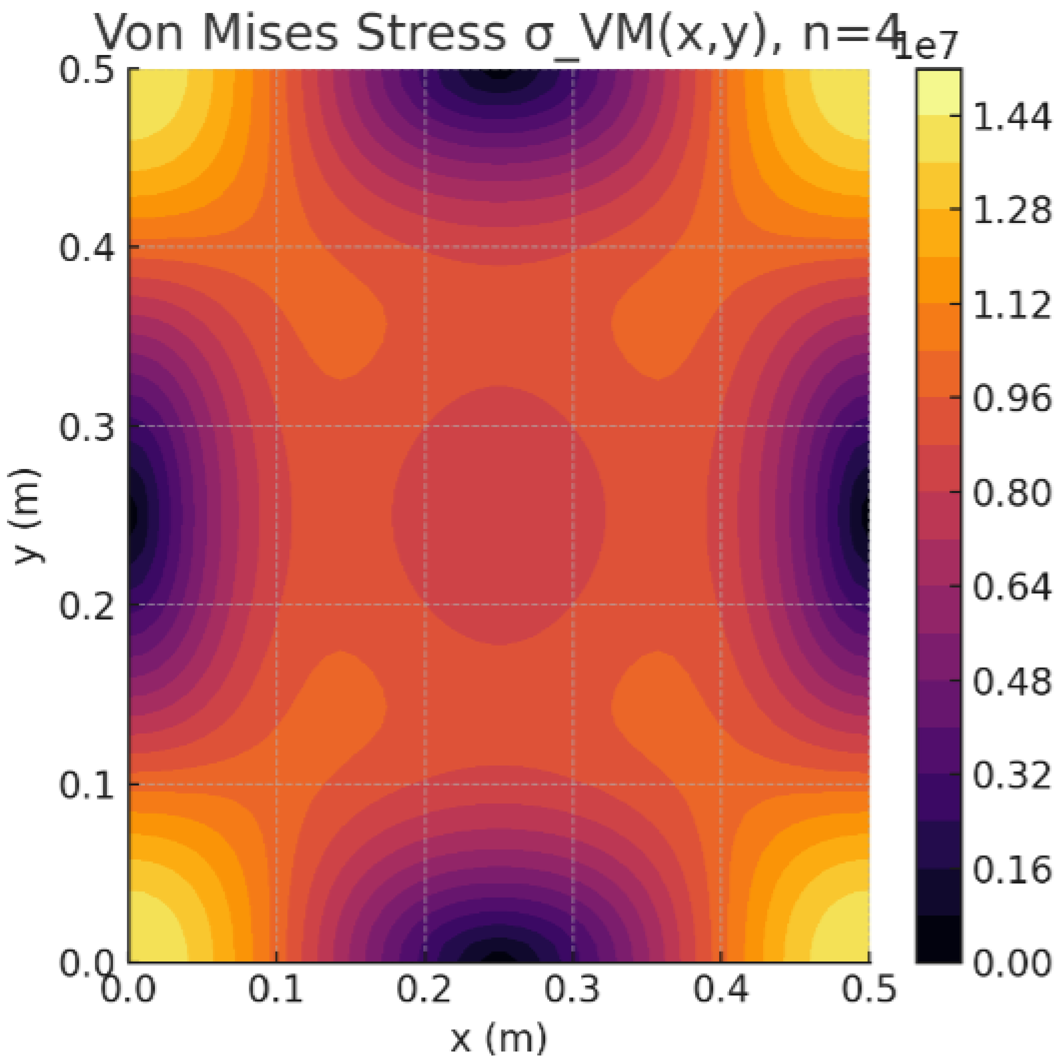


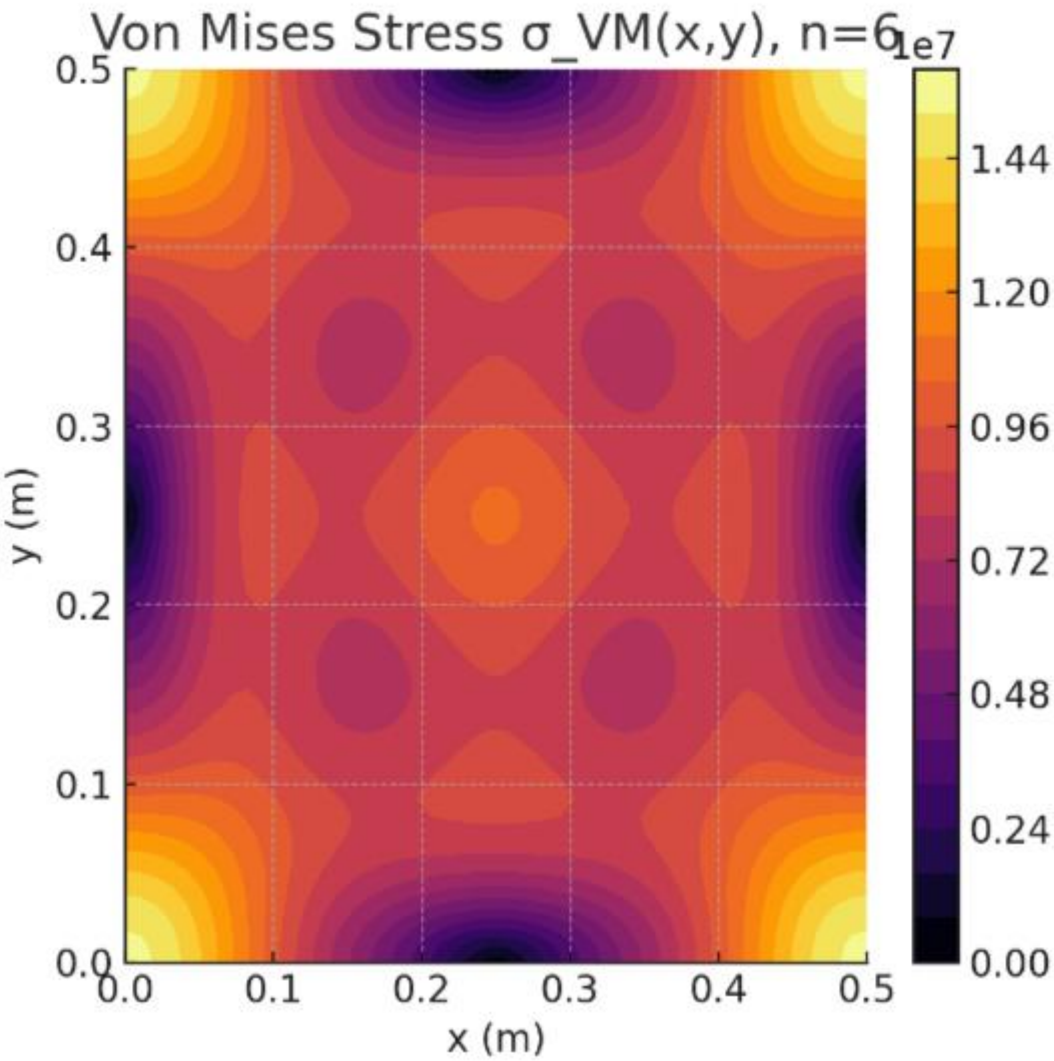
Displacement w(x,y), n=4
0.5
0.4
0.3
0.2
0.1
0.0
y (m)
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
0.000525
0.000450
0.000375
0.000300
0.000225
0.000150
0.000075
0.000000


Displacement w(x,y), n=6
0.5
0.4
0.3
0.2
0.1
0.0
y (m)
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
0.000525
0.000450
0.000375
0.000300
0.000225
0.000150
0.000075
0.000000



Von Mises Stress σ_VM(x,y), n=2
1e7
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
y (m)
1.26
1.08
0.90
0.72
0.54
0.36
0.18
0.00


Von Mises Stress σ_VM(x,y), n=4
1e7
0.5
0.4
0.3
0.2
0.1
0.0
y (m)
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
1.44
1.28
1.12
0.96
0.80
0.64
0.48
0.32
0.16
0.00


Von Mises Stress σ_VM(x,y), n=6
1e7
0.0
0.1
0.2
0.3
0.4
0.5
x (m)
y (m)
0.00
0.24
0.48
0.72
0.96
1.20
1.44