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Industrial statistics

Lecture Notes
for SSTA403

Contents

Chapter 1

CPM and PERT

1.1 Network analysis

Large and complex projects of any organization involve a number of inter-related activities with limited resources such as labour, machines, material, money and time. It is not possible for the management to find and execute an optimum solution just by intuition and work experience.

The main objective of project management can hence be described in terms of successful completion of a project within time, budgeted cost and t the technical specifications/ It also becomes essential to incorporate any change in the initial plan and immediately know the effect of the change from time to time.

Definition 1.1 (Network analysis)**.** *The statistical tool develop to help in planning, managing and controlling the project is known as* ***network analysis.***

Thus network analysis deals with dividing the given project into smaller well-defined tasks and studying the interdependencies of these tasks. When the interdependencies of these tasks are best represented, they form a **network**; and hence the name **network analysis.** It involves three main steps:

1. It defines the jobs to be done.

2. It integrates the jobs in a logical time sequence.

3. It controls the progress of the project plan.

When all the tasks are accomplished, the project is said to be completed.

Project scheduling consists of four main steps:

1. **Planning:** The project is divided into a number of well defined tasks / activities. The time estimation for the completion of each activities and the sequential order in which the activities should be executed is indicated.

2. **Scheduling:** The objective of this phase is to construct a time chart showing the start and finish time for each activity by applying forward and backward pass techniques and identify the critical path to indicate the critical activities; which require special attention if the project is to be completed on time; along with the slack and float for the non-critical paths.

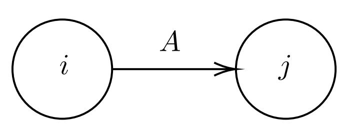
3. **Allocation of resources:** A resource is a physical variable required such as labour, equipment, space, etc. In this phase, the resources are allocated to difference activities to achieve the desired objective.

4. **Controlling:** It refers to analysing and evaluating the actual progress against the plan. In this final phase of project management, a financial and technical control is obtained over the project, by having progress reports from time to time and updating the networking continuously. Reallocation of resources, crashing and review of project with periodical reports are carried out in this phase of the project.

1.1.1 Required definitions:

1. **Activity:** An individual operation which consumes resources, has a beginning and an end, is called an activity.

   An arrow is used to represent an activity, with its head indicating the direction of the progress of the project.

   Where A is the activity, i and j are events.

2. **Event:** An event represents a point in time satisfying the completion of some activities and the beginning of the new ones.

   It is also called as a **node** or **connector** and is usually represented by a circle in the network.

3. **Tail event:** It is the event from which the activity begins, i.e. the event from which the arrow emerges.

4. **Head event:** It is the event into which the activity ends, ie/ te event into which the arrow enters.

5. **Dummy activity:** An activity which does not consume any resources but only shows the technological dependence of activities is called a dummy activity. It is represented by a dotted line in the network.

6. **Network:** When all the activities and events are connected logically and sequentially, they form a network.

7. **Start event:** An event which does not have a preceding activity si called the start event of the network.

   Thus, the start event will have no arrows entering into it, but will only have arrows emerging out of it.

8. **End event:** An event which does not have a succeeding activity is called the end event of the network.

9. **Merge event:** Merge event is the vent into which more than one activities end, i.e. the event into which many arrows enter.

10. **Burst event:** Burst event is the event from which more than one activities start, i.e. the event from which many arrows emerge

11. **Predecessor activity:** Activity which must be completed before a particular activity starts is know na as a predecessor activity.

12. **Successor activity:** Activity which must follow a particular activity is known as a successor activity.

13. **Critical activity:** An activity which cannot be delayed if the project is to be completed on time is known as a critical activity.

    Total float for every critical activity is zero.

14. **Critical event:** An event which cannot be delayed if the project is to completed is known as a critical event.

    Slack of every critical event is zero

15. **Path:** A path is connected sequence of activities from start event to end event.

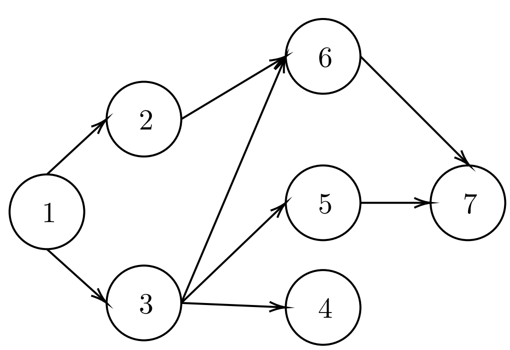
16. **Critical path:** A path connecting the start event and the end event through the critical events or critical activities is called the critical path.

    Duration-wise it is the longest path on the network. It is represented using **bolder** lines on the network.

1.1.2 Rules for drawing the network

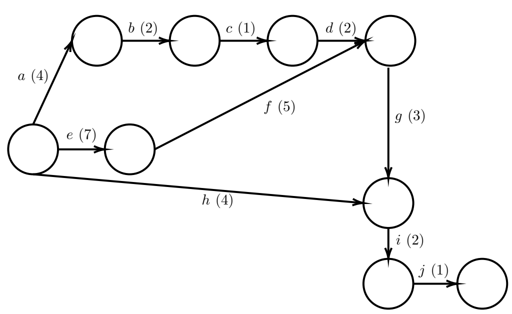
1. Each activity is represented by only one arrow in the network.
2. The arrows should be drawn as straight lines.
3. The arrows should not cross each other.
4. The length of an arrow is of no significance.
5. Time flows from left to right.
6. Arrows pointing in opposite direction are to be avoided.
7. Angles between arrows should be as large as possible.
8. Every network should have a unique start event.
9. Every network should have a unique end event.
10. Before an activity can be undertaken, all activities preceding it must be completed.
11. There should be no duplication in event numbers.
12. No two activities can have same start event and end event..
13. No loops allowed.

Example 1.2. *Event*$-2; 3; 4; 5; 6; 7$, *preceded by* $1; 1; 3; 3, 4; 2, 3, 5; 5, 6$



Example 1.3. *Make a diagram based on the following table*

Sr. No.	Activity	Duration (hrs)
a)	Preparing a pattern for casting item 'A'	4
b)	Preparing a mould	2
c)	Casting and cleaning item 'A'	1
d)	Heat treatment for 'A'	2
e)	Obtaining and installing machine 'M'	7
f)	Preparing item 'B' by 'M'	5
g)	Assembling 'A' and 'B'	3
h)	Preparing test rig	4
i)	Testing the assembly	2
j)	Packing	1



1.2 Fulkerson's Rule for Numbering the events

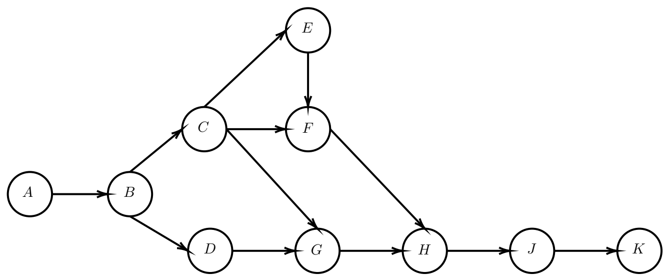
1. A start event is an event is an event which has only emerging arrows, and no entering arrows. The start event is numbered as 1.

2. Strike out all the arrows that emerge out of event 1. This will create at least one event that has no entering arrows. Number these events as $2, 3, 4, \ldots$.

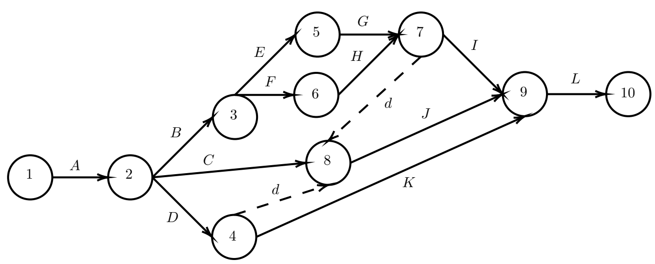
3. Continue this procedure till all the events are numbered.

Note: Events can be numbered as $1, 2, 3, \ldots$ or $10, 20, 30, \ldots$.

Example 1.4. *Draw a network for the following project and number the events according to Fulkerson's rule.*
'A' is a start event and, 'K' is end event. 'A' precedes the event 'B', 'J' is the successor event to 'F', 'C' and 'D' are successor events to 'B', 'D' is preceding event to 'G', 'E' and 'F' occurs after event 'C', 'E' precedes 'F', 'C' restrains the occurrence of 'G' and 'G' precedes 'F'.
'H' precedes 'J' and 'K' succeeds 'J', 'F' restrains the occurrence of 'H'.



Example 1.5. *B,C,D preceeding A; E,F preceeding B; G preceeding E; H preceeding F; I preceeding G,H; J preceeding G,H,C,D; K preceeding D; L preceeding I,J,K*



Proof. ☐

1.3 Management tools for scheduling a project

1. Project evaluation and review technique (PERT)
2. Critical path method (CRT)

They help in reducing project execution time.

1.4 Time estimates for the exceution of each activity

Activities which can be performed but no data is available for their time estimates such activities are called variable time or probabilistic activity.

On the other hand, there are activities for which the time estimates can be accurately determined.

Such activities are called fixed type or deterministic activities.

The projects with probabilistic activities employ PERT, whereas the projects which involve deterministic actives are handled by CPM techniques.

PERT, developed during 1950's was first used in conjunction with planning and designing of the Polaris missile project.

CPM was developed by DuPont company and applied first to the construction projects in the chemical industry.

1.5 PERT

This network analysis technique gives more emphasis on events. Since the time estimates are not known, three different types of time estimates are defiend for each activity.

1. **Optimistic time estimate (t_0)**
   This is the shortest time within which an activity can be completed.

2. **Pessimistic time estimate (t_p)**
   This is the maximum time required for the completion of an activity.

3. **Most likely time estimate (t_m)**
   This is the estimate of time, which the activity will require under normal circumstances. The frequency curve of the activity time estimates resembles the beta distribution curve.
   The mean time required for the completion of the activity (t^{ij}) is given by

$$t^{ij} = \frac{t_0 + 4t_m + t_p}{6}$$

$$\text{Variance} = \left(\frac{t_p - t_0}{6}\right)^2$$

t0	tm	tp	expected time	variance
4	5	8	5.333333333	0.4444444444
5	7	10	7.166666667	0.6944444444
2	3	7	3.5	0.6944444444
8	11	12	10.66666667	0.4444444444
4	7	10	7	1
6	8	15	8.833333333	2.25
8	12	16	12	1.777777778
5		9	2.333333333	0.4444444444
3		7	1.666666667	0.4444444444
3		11	2.333333333	1.777777778
6		13	3.166666667	1.361111111

1.5.1 Calculation of the critical path

Definition 1.6 (Earliest occurrence time). *Earliest occurrence time (T_E) is the earliest time at which an event can occur.*

Definition 1.7 (Latest Occurrence time). *Latest occurrence time (T_L) is the latest time at which an event can occur.*

Definition 1.8 (Forward pass). *The process of calculating T_E from start event to end event. Initially T_E is taken as zero. Then it is calculated by adding the previous T_E and duration of the activity. If there are more than one activity preceding then the earliest time for each event is taken as the maximum of T_E.*

Definition 1.9 (Backward pass). *content...*

Definition 1.10 (Slack). *content...*

Definition 1.11 (Critical path). *content...*

Example 1.12.

1.6 CPM

1. **Earliest start time (EST)** The EST of an activity is the earliest time of the event from which it emerges. For activity $i - j, (EST)_{ij} = T_E^i$
2. **Earliest finish time (EFT)** The EFT of an activity is defined as the sum of the earliest time at which the activity starts and the duration of the activity. For activity $i - j, (EFT)_{ij} = T_E^i + t^{ij}$

3. **Latest finish time (LFT)** The latest time by which an activity should be completed is LFT, i.e. the latest time at which the event indicating the end of the activity should occur. For activity $i-j; (LFT)_{ij} = T_L^j$

4. **Latest start time (LST)** The LST for an activity is defined as teh difference between LFT and its duration. For activity $i-j; (EST)_{ij} = T_L^j - t^{ij}$

Definition 1.13 (Float)**.** *It indicates the time by which the activity can be delayed without affecting the schedule of the project. Float for the activity has the same significance as slack for an event.*

Definition 1.14 (Total float)**.** *It is defined as the difference between the maximum time avalible to perform an activity and the actual duration of an activity.*

$$\begin{aligned}(TF)_{ij} &= (T_L^j - T_E^i) - t^{ij} \\ &= T_L^j - (T_E^i + t^{ij}) \\ &= (LFT)_{ij} - (EFT)_{ij}\end{aligned}$$

For a particular activity, if the total float is zero, it indicates that it is not affordable to delay the activity.

Definition 1.15 (Critical activity)**.** *The activity for which float is zero.*

Definition 1.16 (Critical path)**.** *A path connecting all the critical activities.*

Definition 1.17 (Free float (FF))**.** *The time by which an activity can be delayed under the assumption that all events in the network occur at their eraliest is known as the Free Float.*

$$\begin{aligned}(FF)_{ij} &= (T_E^j - T_E^i) - t^{ij} \\ &= (T_E^j - T_E^i) - t^{ij} + T_L^j - T_L^j \\ &= \{(T_L^j - T_e^i) - t^{ij}\} - (T_L^j - T_E^j) \\ &= (TF)_{ij} - \textit{slack of the head event}\end{aligned}$$

Definition 1.18 (Independent float (IF))**.** *The time by which an activity can be delayed without affecting the preceding and the succeeding activities, is known as the independent float.*

$$\begin{aligned}(IF)_{ij} &= (T_E^j - T_L^i) - t^{ij} \\ &= (T_E^j - T_L^i) - t^{ij} + T_E^i - T_E^i \\ &= (FF)_{ij} - \textit{slack of the tail event}\end{aligned}$$

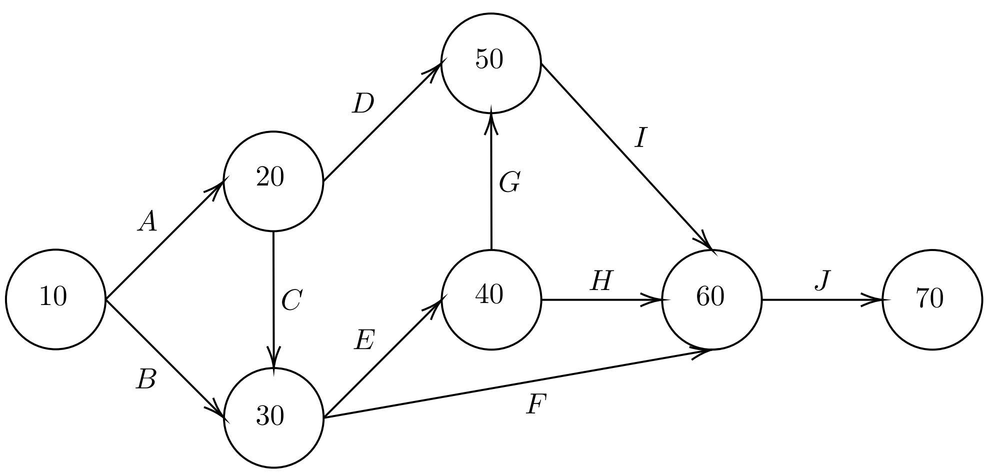
Independent float can be negative, if the duration of the activity is greater than the minimum time requred to complete the activity. If negative, indepednet float is considered as zero.

1.6.1 Example of CPM

Activity	*Duration*	*PreceedingActivity*
A	15	$-$
B	15	$-$
C	3	A
D	5	A
E	8	B, C
F	12	B, C
G	1	E
H	4	E
I	3	D, G
J	14	F, H, I

Draw the network for above project and find EST, LST, EFT, LFT and the three types of float. Hence, determine the critical path.

Proof. First draw the network diagram as follows,



$T_E = 54$

Events	t^{ij}	$EST = T_E^i$	$LFT = T_L^j$	$EFT = T_E^i + t_{ij}$	$LST = T_L^j - t^{ij}$	$TF = LFT - EFT$	$FF = TF - (T_L^j - T_E^j)$
10-20	15	0	15	15	0	0	0-(15-15)=0
10-30	15	0	18	15	3	3	3-(0)=3
20-30	3	15	18	18	15	0	0-(18-18)=0
20-50	5	15	37	20	32	17	7
30-40	6	18	26	24	20	0	0
30-60	12	18	40	30	28	10	10
40-50	1	26	37	27	36	10	0
40-60	14	26	40	40	26	0	0
50-60	3	27	40	30	37	10	10
60-70	14	40	54	54	40	0	0

Now we find total float and free float.
Total float=$LFT - EFT$. If total float is zero it implies its a critical activity.
Free float = $TF-$ Head slack
Independent float = FF - Tail slack
Head slack = $T_L^j - T_E^j$
Tail slack = $T^i - T_E^i$ □

1.7 Project crashing

Two types

Definition 1.19 (Project crashing)**.** *Process of shortening the critical path to achieve completion of the project earlier is called project crashing.*

There are two types of cost,

1. **Direct cost:** It includes cost of materials, machinery, man hours etc. When we estimate the duration of various activities in the project it is assumed that various activities in the project it is assumed that the normal amount of labour and machine required to complete these activities. But when we want to complete the project in the shorter period, than the critical path we will need to employ more resources. Hence direct cost will increase. **yeah this makes no sense dw**

2. **Indirect cost:** It includes rent, overheads, administrative costs. Indirect costs vary with time. They are expressed on per day or per week basis

Total cost = Direct cost + indirect cost

Normal time: Under normal circumstances. **Crash time:** Minimum possible time in which activity can be completed. **Crash cost:** The direct cost associated with crash time. When the activity is crashed its direct cost will increase.

Crash time ¡ Normal time
Crash cost ¿ Normal cost

Definition 1.20 (Crash slope/Cost slope)**.** *The increase in direct cost per day. It is Crash cost - normal cost divided by normal time - crash time*

$$\frac{Crash\ cost - Normal\ cost}{Normal\ time - Crash\ time}$$

1.7.1 Procedure of crashing

1. Find the normal critical path and identify the critical activities.
2. Calculate the cost slope for the critical activities.
3. Rank the critical activities in the ascending order of their cost slopes.
4. Crash critical activities as per the ranking (i.e. the activity with the least cost slope first) to its maximum possible extent. Calculate the revised cost by adding the cost of crashing and subtracting the indirect cost; both proportion to the number of days crashed.
5. As the critical path duration gets reduced in step 4, some other path may also become critical, i.e. there may be parallel critical paths. This implies that the project duration now has to be reduced simultaneously by crashing the activities on the parallel critical paths.
6. After crashing the activities in the above manner, a point is reached where further crashing is either not possible or it does not result in reduction of project duration or cost. For different project or durations, project cost should be calculated.

Activity	t^{ij}	Activity	t^{ij}
1-2	2	4-6	3
1-3	2	5-8	1
1-4	1	6-9	5
2-5	4	7-8	4
3-6	8	8-9	3
3-7	5		

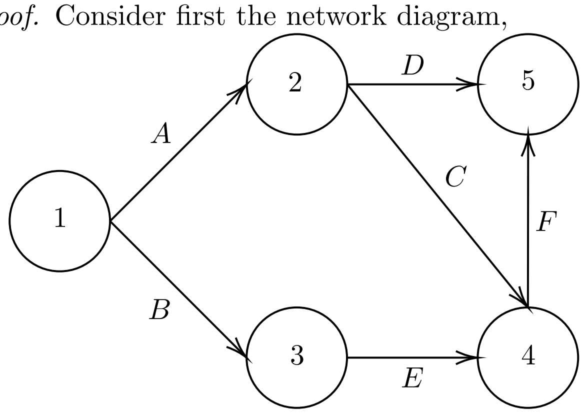
Example 1.21. *Draw a network for this project. Find total float for each activity. Hence determine critical path for network.*

Proof. $T_E = T_L = 15$ □

Example 1.22. *Indirect cost is Rs.* 70 *per day.*

Activity	Normal time	Crash time	Normal cost	Crash cost
1-2	8	6	100	200
1-3	4	2	150	350
2-4	2	1	50	90
2-5	10	5	100	400
3-4	5	1	100	200
4-5	3	1	80	100

Proof. Consider first the network diagram,



Critical path is $1-2-5 : A-D = 18$days.
Other paths are $1-3-4-5 : B-E-F = 12$days
$1-2-4-5 : A-C-F = 13$days.
Total normal cost is $= DC + IDC$

$$= DC + IDC$$
$$= 580 + 70 * 8 \qquad = 1140$$

Now see the cost slope

$$\text{Cost slope} = \frac{\Delta\text{time}}{\Delta\text{cost}}$$

Minimum total cost = optimum project cost = 1760

□

Activity	Maximum crashable days	Cost Slope / day	Rank
A(1-2)	8-6=2	50	4
B(1-3)	4-2=2	100	6
C(2-4)	2-1=1	40	3
D(2-5)	10-5=5	60	5
E(3-4)	5-1=4	25	2
F(4-5)	3-1=2	10	1

Crashing	Paths			Cost (Rs.)		
	A-D	A-C-F	B-E-F	D.C.	I.D.C	T.C.
Before Crashing	18	13	12	580	1266	1840
1st Crashing A by 1	18-1= 17	13-1= 12	12	580+(50*1) =630	17*70 =1190	1820
2nd Crashing A by 2	17-1= 16	12-1= 11	12	630+(50) =680	16*70 =1120	1800
3rd Crashing D by 1	16-1= 15	11	12	680+60 =740	15*70 =1050	1790
4th Crashing D by 1	15-1= 14	11	12	740+60 =800	14*70 =980	1780
5h Crashing D by 1	13	11	12	860	910	1770
6th Crashing D by 1	12	11	12	920	840	1760
7th Crashing D by 1 and F by 1	11	10	11	920+60+10 =990	11*70 =770	1760

1.8 something

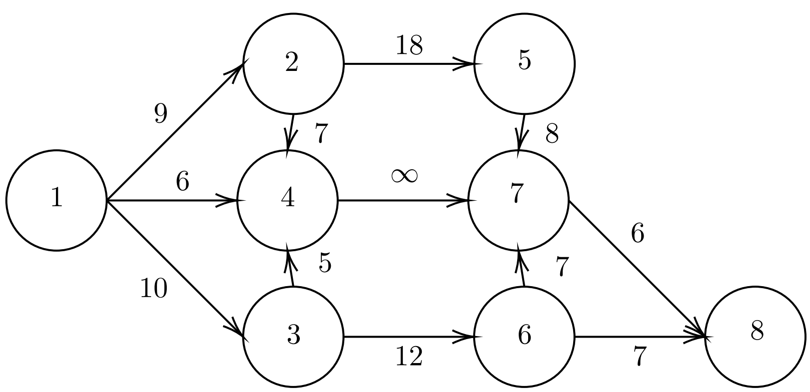
...which have occurred due to re planning or re scheduling of ...
Updating can be done in two ways,

1. Use the revised time estimates of incomplete activities and calculate from the completion of time of each event in the usual manner to know the project completion time. and represent all the activities already finished by elapsed time. Events in the revised network diagram are re numbered. And the completion of the completed activities (?????) are taken as the revised time.

Example 1.23. *After 15 days working the following progress is noted of an erection job.*

1. *Activity 1-2,1-3 and, 1-4 completed as per the original schedule.*
2. *Act 2-4, is in progress and will be completed in 3 more days*
3. *Act 3-6 is in progress and require 18 days more.*
4. *Act 6-7 appears to present some problem and its new estimated time is 12 days.*
5. *Act 6-8 can be completed in 5 days instead of 7 days.*

Proof. Consider the network diagram

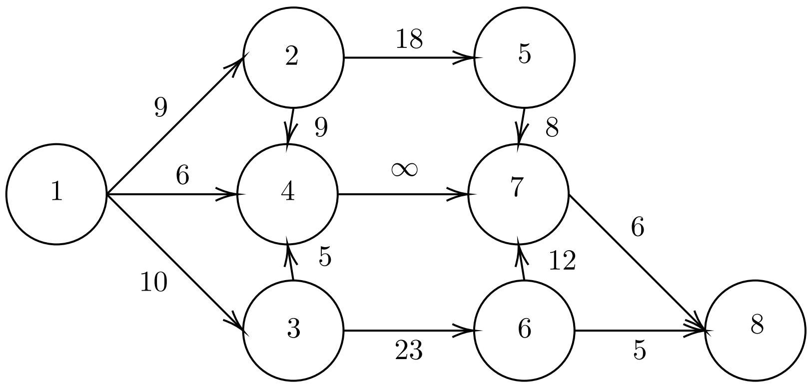


$T_E = 42, T_L = 42$ new duration of $2 - 4$ is $15 + 3 - 9 = 9$

1. Act 2-4 needs 15+3-9=9 days instead of 7.
2. Act 3-6 needs 15+18-10 = 23 days instead of 12.

3. Act 6-7=12 days instead of 7 days.
4. Act 6-8=5 days instead of 7
5. Act 2-5, 4-7, 5-7, 7-8, 3-4 remain as it is.

New network,



New $T_E, T_L = 51$ with critical path given as $1 - 3 - 6 - 7 - 8 = 51$

$\square$