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Calculus IV

Lecture Notes
for SMAT401

Contents

Chapter 1

Tutorial 1

Show that the limits as the function approaches (0,0) dont exist

1.1 Question 1

$$\frac{x^2 - y^2}{x^2 + y^2}$$

use x and y axis

1.2 Question 2

$$\frac{x^3 y}{x^6 + y^2}$$

consider $y = 0$ then we have limit equal 0. Consider now $y = x^3$ so now limit of $\frac{x^6}{2x^6} = \frac{1}{2}$.

1.3 Question 3

$$\frac{\sin(x^2 + y)}{x + y}$$

along the x axis (y=0) we get $\sin(x^2)/x$ and the lim is 0. But for y axis (x=0) we get $\sin(y)/y$ and the lim is 1.

1.4 Question 4

$$\frac{x^3 + y^3}{x - y}$$

take the line $y = 0$ we get $\frac{x^3+m^3x^3}{x-mx} = \frac{(1+m)x^3}{x(1-m)}$ is 0 but with $y = x - x^3$ is equal to 2.

Try with $y = x - x^2$ we get

$$\begin{aligned}\lim \frac{x^3 + (x - x^3)^3}{x - (x - x^3)} &= \lim \frac{x^3}{x^3} + \frac{()x - x^3)^3}{x^3} \\ &= 1 + \lim \frac{(x - x^3)^3}{x^3} = 1 + 1 = 2\end{aligned}$$

1.5 Question 5

$$\frac{x^2y^2}{x^2y^2 + (x - y)^2}$$

consider the line $x = 0$ then the limit is obviously 0. Now consider $y = x$ then the limit of $\frac{x^4}{x^4} = 1$.

1.6 Question 6

$$\frac{2xy^2}{x^3 + y^3}$$

take y=0 and x=y.

Tutorial 2

1.7 Question 1

$$\lim_{(x,y)\to(0,0)} xy \sin\left(\frac{1}{x^2+y^2}\right) = 0$$