Calculating mathematial constants using Monte Carlo simulations

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Section 1 : Pi

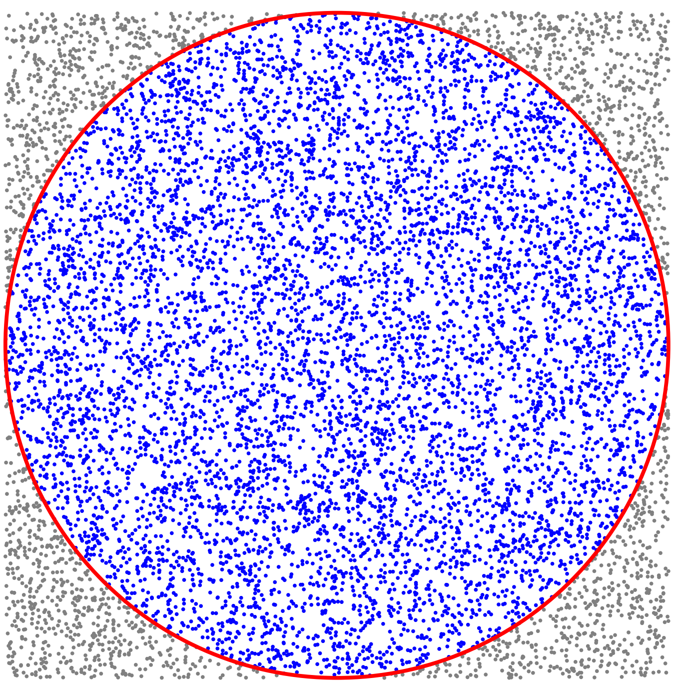
Elementary Method

In[ ]:=
```
pairs = RandomReal[{-1, 1}, {10000, 2}];
4 Count[Map[Norm, pairs], _? (# ≤ 1 &)] / 10000.
```

Out[ ]=
```
3.1252
```

In[ ]:=
```
Graphics[{PointSize[Small], Blue, Point@Select[pairs, Norm[#] ≤ 1 &], Gray,
  Point@Select[pairs, Norm[#] > 1 &], Red, Thick, Circle[]}, AspectRatio → 1]
```

Out[ ]=



In[ ]:=
```
approxPi[n_] := 4. Count[Map[Norm, RandomReal[{-1, 1}, {n, 2}]], _? (# ≤ 1 &)] / n
```

In[ ]:=
```
RepeatedExperiment = Table[approxPi[10^6], {10^3}];
```

```
Mean[RepeatedExperiment]
```

Out[ ]= 3.14151

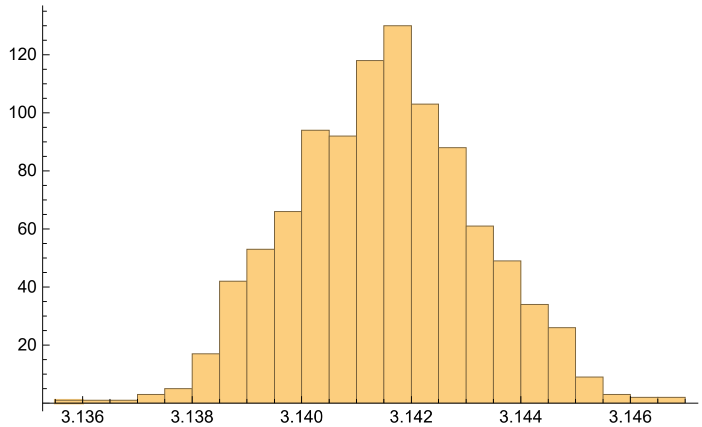
In[ ]:=
```
NumberForm[3.14151, 15]
```

Out[ ]//NumberForm=

3.141510656

In[ ]:=
```
Histogram[RepeatedExperiment]
```

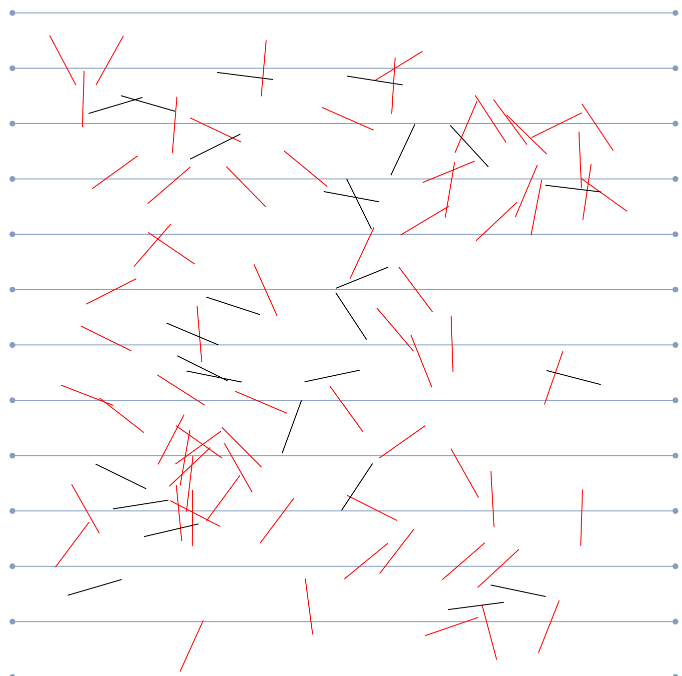
Out[ ]=



Buffon' s Needle

In[ ]:=
```
d = 0.2; n = 100;
lines = MeshRegion[Join @@ Table[{{-1 - d, y}, {1 + d, y}}, {y, -1 - d, 1 + d, d}],
   Line[Partition[Range[2 Floor[2 / d + 3]], 2]]];
needles = Table[Line[{pt, RandomPoint[Circle[pt, d]]}],
   {pt, RandomReal[{-1, 1}, {n, 2}]}];
overlap = Select[needles, ! RegionDisjoint[lines, #] &];
Show[lines, Graphics[{Red, overlap, Black, Complement[needles, overlap]}]]
N[(n) / Length[overlap]]
```

Out[ ]=



Out[ ]= 1.35135

Out[ ]= ComplexInfinity

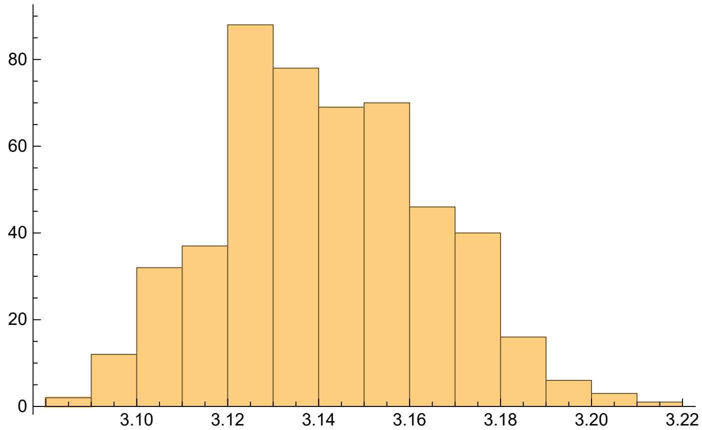
In[ ]:=
```
RepeatedBuffon = Table[
   d = 0.2; n = 10 000;
   lines = MeshRegion[Join @@ Table[{{-1 - d, y}, {1 + d, y}}, {y, -1 - d, 1 + d, d}],
     Line[Partition[Range[2 Floor[2 / d + 3]], 2]]];
   needles = Table[Line[{pt, RandomPoint[Circle[pt, d]]}],
     {pt, RandomReal[{-1, 1}, {n, 2}]}];
   overlap = Select[needles, ! RegionDisjoint[lines, #] &];
   N[(2 n) / Length[overlap]], {500}];
```

In[ ]:= `Mean[RepeatedBuffon]`

Out[ ]= 3.14169

In[•]:=
```
Histogram[RepeatedBuffon]
```

Out[•]=



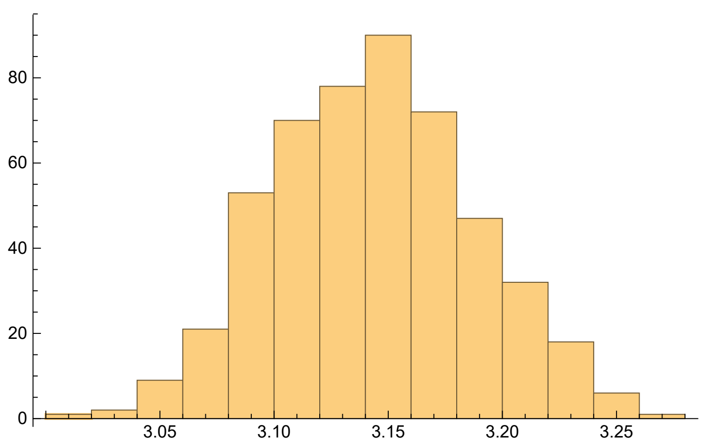
In[•]:=
```
NewRepeatedBuffon = Table[d = 0.2; n = 10000;
   lines = MeshRegion[Join @@ Table[{{-1 - d, y}, {1 + d, y}}, {y, -1 - d, 1 + d, d}],
     Line[Partition[Range[2 Floor[2 / d + 3]], 2]]];
   needles = Table[Line[{pt, RandomPoint[Circle[pt, 0.5 d]]}],
     {pt, RandomReal[{-1, 1}, {n, 2}]}];
   overlap = Select[needles, ! RegionDisjoint[lines, #] &];
   N[(n) / Length[overlap]], {500}];
```

In[•]:=
```
Mean[NewRepeatedBuffon]
```

Out[•]= 3.14413

In[•]:=
```
Histogram[NewRepeatedBuffon]
```

Out[•]=



Euler Mascheroni Constant

Gumbel Distribution

```
approxgamma[n_] := Mean[-Log[-Log[RandomReal[{0, 1}, {n}]]]]
```

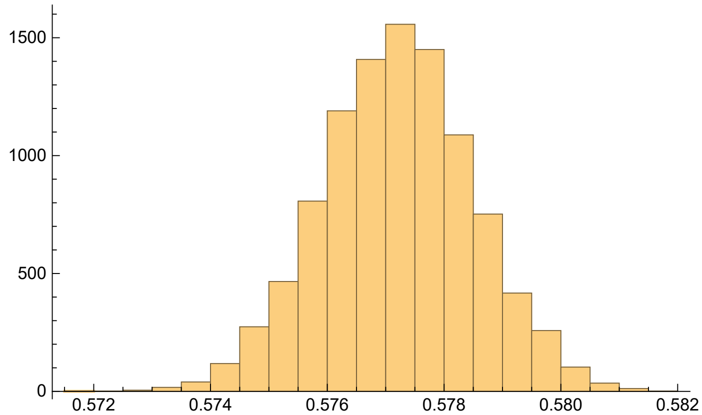
In[•]:=
```
approxgamma[10^6]
```
Out[•]=
```
0.576802
```

In[•]:=
```
RepeatedGammaExp = Table[approxgamma[10^6], {10^4}];
```

In[•]:=
```
NumberForm[Mean[RepeatedGammaExp], 15]
```
Out[•]//NumberForm=
```
0.577210180043852
```

In[•]:=
```
N[EulerGamma, 15]
```
Out[•]=
```
0.577215664901533
```

In[•]:=
```
Histogram[RepeatedGammaExp]
```
Out[•]=



Exponential

In[•]:=
```
N[Mean[Table[Module[{u = Random[], t = 1}, While[u < 1, u = Random[] + u;
     t++];
    t], {10^6}]]]
```
Out[•]=
```
2.71786
```

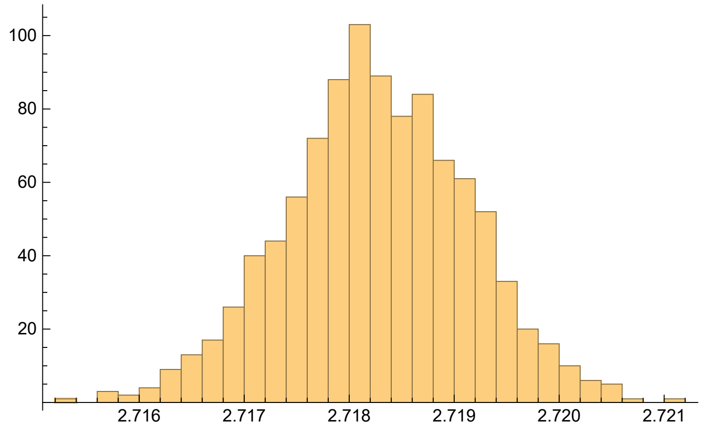
In[•]:=
```
RepeatedExp[n_, repeat_] :=
 Table[Mean[Table[Module[{u = Random[], t = 1}, While[u < 1, u = Random[] + u;
      t++];
     t], {n}]], {repeat}]
```

In[•]:=
```
RepeatedData = RepeatedExp[10^6, 1000];
```

In[•]:=
```
N[Mean[RepeatedData], 10]
```
Out[•]=
```
2.718262161
```

In[•]:=
```
Histogram[RepeatedData]
```

Out[•]=



Numerical Integration

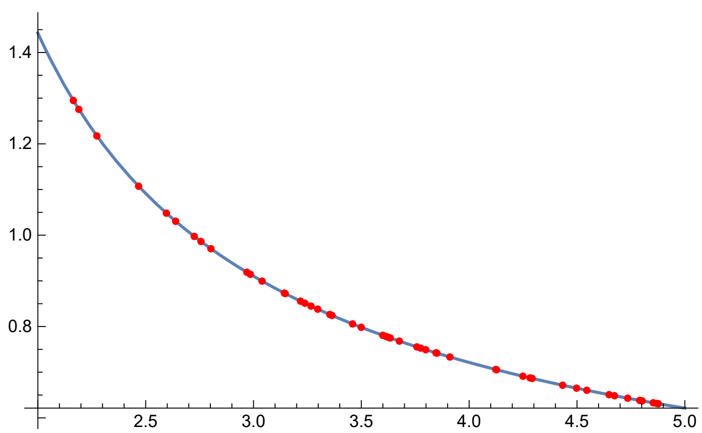
In[•]:=
```
g[x_] = 1/Log[x];
```

$$g[x\_] = \frac{1}{\text{Log}[x]};$$

In[•]:=
```
a = 2; b = 5;
```

In[•]:=
```
ExampleRand = RandomReal[{2, 5}, {50}];
ExamplePoints = g[ExampleRand];
Show[Plot[g[x], {x, 2, 5}],
 ListPlot[Transpose[{ExampleRand, ExamplePoints}], PlotStyle → {Red}]]
```

Out[•]=



In[•]:=
```
RepeatedIntegral[points_, repeat_] :=
 Table[N[(b - a)/(points - 1) Total[g[RandomReal[{a, b}, {points}]]]], {repeat}]
```

In[•]:=
```
IntegralData := RepeatedIntegral[10^6, 1000]
```

In[•]:=
```
Mean[IntegralData]
```

Out[•]= 2.58942

```
NIntegrate[g[x], {x, 2, 5}]
```

Out[ ]= 2.58942

```
In[ ]:= Histogram[IntegralData]
```

Out[ ]=