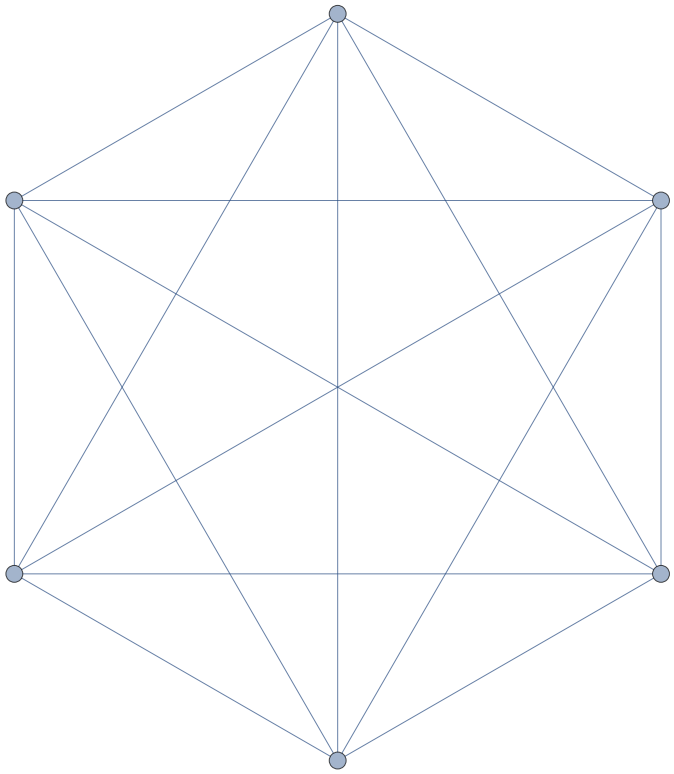
Reciprocal Multifactorial Constants

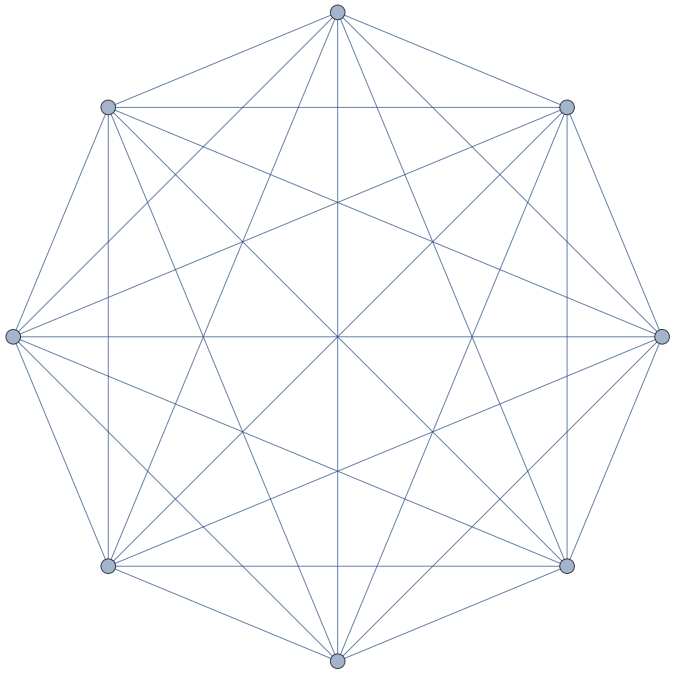
Perfect matchings for K_ 6 and K_8

In[ ]:=
```
k6 = CompleteGraph[6]
k8 = CompleteGraph[8]
```

Out[ ]=



Out[ ]=



In[ ]:=
```
l6 = Length[FindIndependentEdgeSet[k6]]
l8 = Length[FindIndependentEdgeSet[k8]]
```

Out[ ]= 3

Out[ ]= 4

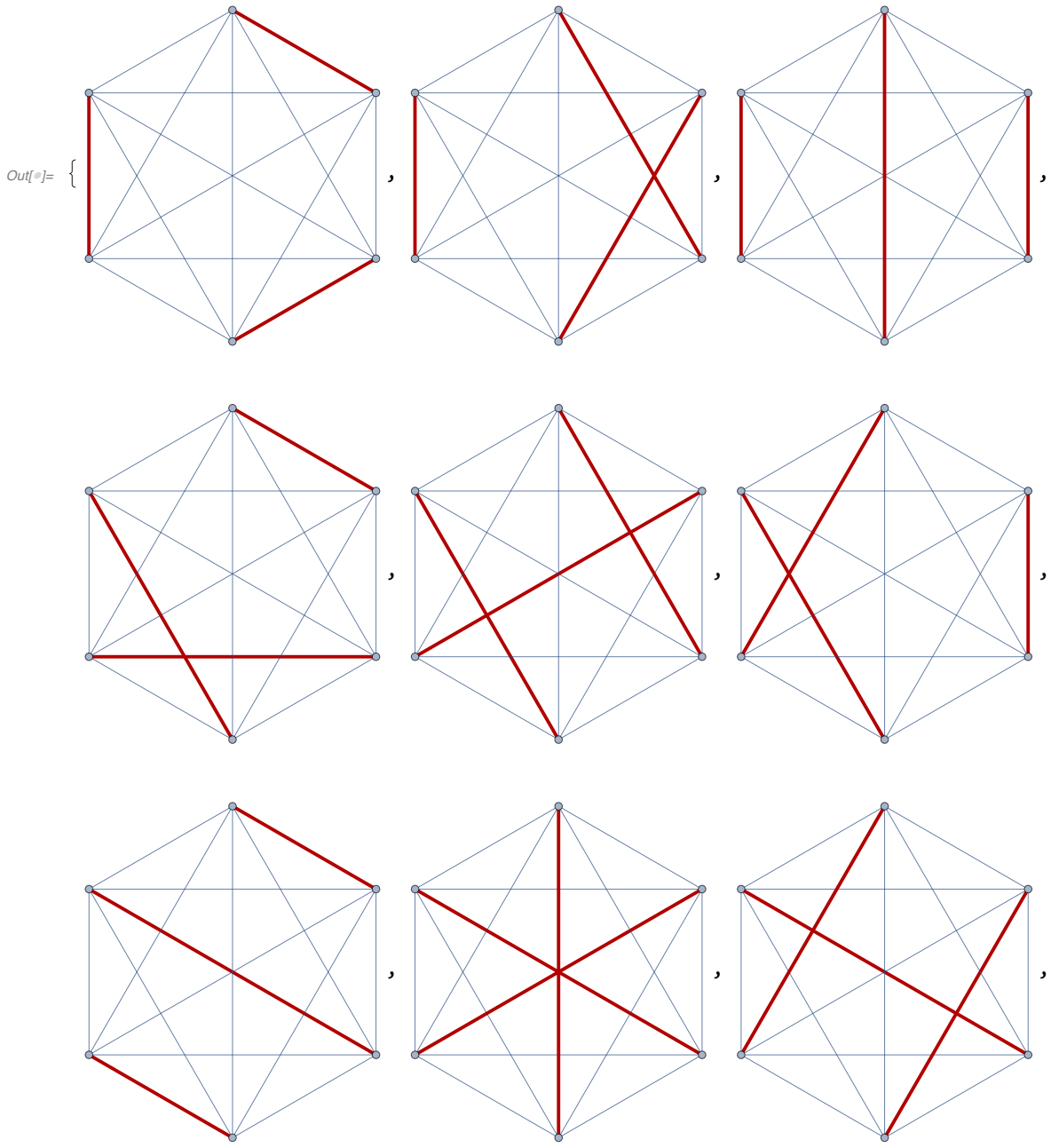
```
In[]:= esl6 = Select[Subsets[EdgeList[k6], {l6}], IndependentEdgeSetQ[k6, #] &]
       esl8 = Select[Subsets[EdgeList[k8], {l8}], IndependentEdgeSetQ[k8, #] &]
```

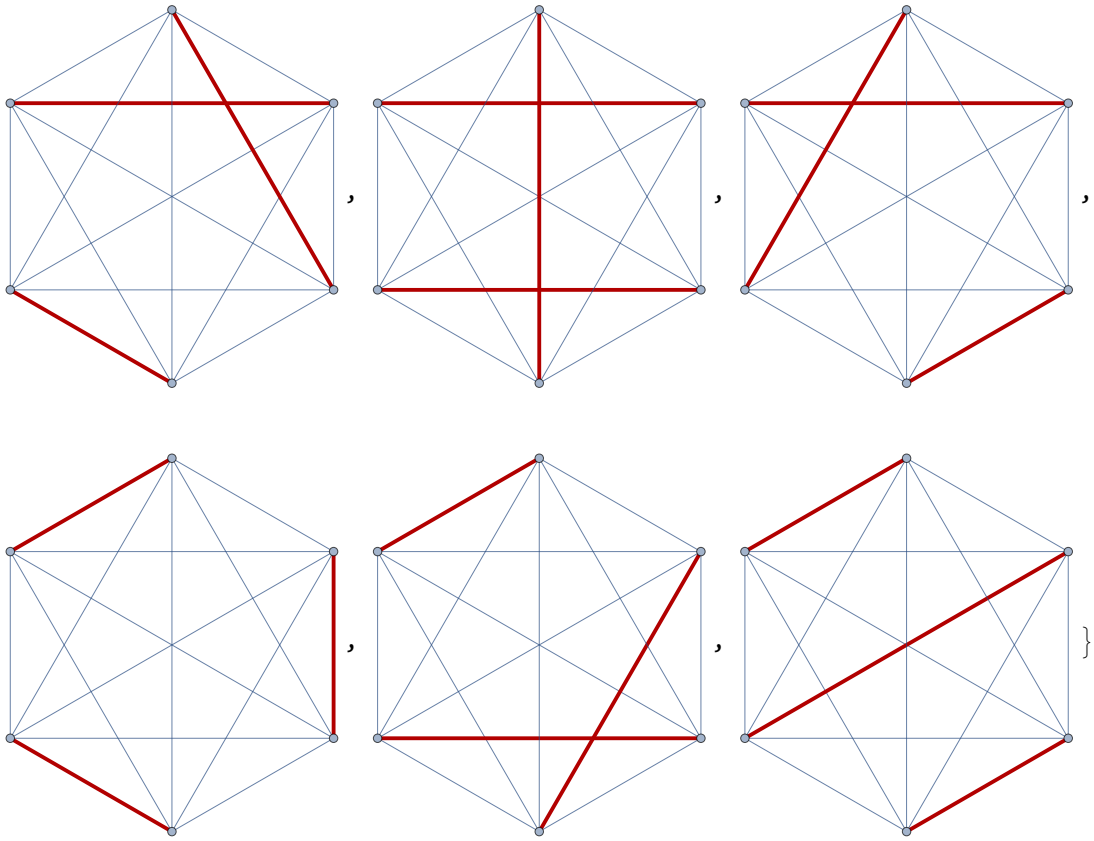
```
Out[]= {{1 ↔ 2, 3 ↔ 4, 5 ↔ 6}, {1 ↔ 2, 3 ↔ 5, 4 ↔ 6}, {1 ↔ 2, 3 ↔ 6, 4 ↔ 5}, {1 ↔ 3, 2 ↔ 4, 5 ↔ 6},
        {1 ↔ 3, 2 ↔ 5, 4 ↔ 6}, {1 ↔ 3, 2 ↔ 6, 4 ↔ 5}, {1 ↔ 4, 2 ↔ 3, 5 ↔ 6}, {1 ↔ 4, 2 ↔ 5, 3 ↔ 6},
        {1 ↔ 4, 2 ↔ 6, 3 ↔ 5}, {1 ↔ 5, 2 ↔ 3, 4 ↔ 6}, {1 ↔ 5, 2 ↔ 4, 3 ↔ 6}, {1 ↔ 5, 2 ↔ 6, 3 ↔ 4},
        {1 ↔ 6, 2 ↔ 3, 4 ↔ 5}, {1 ↔ 6, 2 ↔ 4, 3 ↔ 5}, {1 ↔ 6, 2 ↔ 5, 3 ↔ 4}}
```

```
Out[]= {{1 ↔ 2, 3 ↔ 4, 5 ↔ 6, 7 ↔ 8}, {1 ↔ 2, 3 ↔ 4, 5 ↔ 7, 6 ↔ 8}, {1 ↔ 2, 3 ↔ 4, 5 ↔ 8, 6 ↔ 7},
        {1 ↔ 2, 3 ↔ 5, 4 ↔ 6, 7 ↔ 8}, {1 ↔ 2, 3 ↔ 5, 4 ↔ 7, 6 ↔ 8}, {1 ↔ 2, 3 ↔ 5, 4 ↔ 8, 6 ↔ 7},
        {1 ↔ 2, 3 ↔ 6, 4 ↔ 5, 7 ↔ 8}, {1 ↔ 2, 3 ↔ 6, 4 ↔ 7, 5 ↔ 8}, {1 ↔ 2, 3 ↔ 6, 4 ↔ 8, 5 ↔ 7},
        {1 ↔ 2, 3 ↔ 7, 4 ↔ 5, 6 ↔ 8}, {1 ↔ 2, 3 ↔ 7, 4 ↔ 6, 5 ↔ 8}, {1 ↔ 2, 3 ↔ 7, 4 ↔ 8, 5 ↔ 6},
        {1 ↔ 2, 3 ↔ 8, 4 ↔ 5, 6 ↔ 7}, {1 ↔ 2, 3 ↔ 8, 4 ↔ 6, 5 ↔ 7}, {1 ↔ 2, 3 ↔ 8, 4 ↔ 7, 5 ↔ 6},
        {1 ↔ 3, 2 ↔ 4, 5 ↔ 6, 7 ↔ 8}, {1 ↔ 3, 2 ↔ 4, 5 ↔ 7, 6 ↔ 8}, {1 ↔ 3, 2 ↔ 4, 5 ↔ 8, 6 ↔ 7},
        {1 ↔ 3, 2 ↔ 5, 4 ↔ 6, 7 ↔ 8}, {1 ↔ 3, 2 ↔ 5, 4 ↔ 7, 6 ↔ 8}, {1 ↔ 3, 2 ↔ 5, 4 ↔ 8, 6 ↔ 7},
        {1 ↔ 3, 2 ↔ 6, 4 ↔ 5, 7 ↔ 8}, {1 ↔ 3, 2 ↔ 6, 4 ↔ 7, 5 ↔ 8}, {1 ↔ 3, 2 ↔ 6, 4 ↔ 8, 5 ↔ 7},
        {1 ↔ 3, 2 ↔ 7, 4 ↔ 5, 6 ↔ 8}, {1 ↔ 3, 2 ↔ 7, 4 ↔ 6, 5 ↔ 8}, {1 ↔ 3, 2 ↔ 7, 4 ↔ 8, 5 ↔ 6},
        {1 ↔ 3, 2 ↔ 8, 4 ↔ 5, 6 ↔ 7}, {1 ↔ 3, 2 ↔ 8, 4 ↔ 6, 5 ↔ 7}, {1 ↔ 3, 2 ↔ 8, 4 ↔ 7, 5 ↔ 6},
        {1 ↔ 4, 2 ↔ 3, 5 ↔ 6, 7 ↔ 8}, {1 ↔ 4, 2 ↔ 3, 5 ↔ 7, 6 ↔ 8}, {1 ↔ 4, 2 ↔ 3, 5 ↔ 8, 6 ↔ 7},
        {1 ↔ 4, 2 ↔ 5, 3 ↔ 6, 7 ↔ 8}, {1 ↔ 4, 2 ↔ 5, 3 ↔ 7, 6 ↔ 8}, {1 ↔ 4, 2 ↔ 5, 3 ↔ 8, 6 ↔ 7},
        {1 ↔ 4, 2 ↔ 6, 3 ↔ 5, 7 ↔ 8}, {1 ↔ 4, 2 ↔ 6, 3 ↔ 7, 5 ↔ 8}, {1 ↔ 4, 2 ↔ 6, 3 ↔ 8, 5 ↔ 7},
        {1 ↔ 4, 2 ↔ 7, 3 ↔ 5, 6 ↔ 8}, {1 ↔ 4, 2 ↔ 7, 3 ↔ 6, 5 ↔ 8}, {1 ↔ 4, 2 ↔ 7, 3 ↔ 8, 5 ↔ 6},
        {1 ↔ 4, 2 ↔ 8, 3 ↔ 5, 6 ↔ 7}, {1 ↔ 4, 2 ↔ 8, 3 ↔ 6, 5 ↔ 7}, {1 ↔ 4, 2 ↔ 8, 3 ↔ 7, 5 ↔ 6},
        {1 ↔ 5, 2 ↔ 3, 4 ↔ 6, 7 ↔ 8}, {1 ↔ 5, 2 ↔ 3, 4 ↔ 7, 6 ↔ 8}, {1 ↔ 5, 2 ↔ 3, 4 ↔ 8, 6 ↔ 7},
        {1 ↔ 5, 2 ↔ 4, 3 ↔ 6, 7 ↔ 8}, {1 ↔ 5, 2 ↔ 4, 3 ↔ 7, 6 ↔ 8}, {1 ↔ 5, 2 ↔ 4, 3 ↔ 8, 6 ↔ 7},
        {1 ↔ 5, 2 ↔ 6, 3 ↔ 4, 7 ↔ 8}, {1 ↔ 5, 2 ↔ 6, 3 ↔ 7, 4 ↔ 8}, {1 ↔ 5, 2 ↔ 6, 3 ↔ 8, 4 ↔ 7},
        {1 ↔ 5, 2 ↔ 7, 3 ↔ 4, 6 ↔ 8}, {1 ↔ 5, 2 ↔ 7, 3 ↔ 6, 4 ↔ 8}, {1 ↔ 5, 2 ↔ 7, 3 ↔ 8, 4 ↔ 6},
        {1 ↔ 5, 2 ↔ 8, 3 ↔ 4, 6 ↔ 7}, {1 ↔ 5, 2 ↔ 8, 3 ↔ 6, 4 ↔ 7}, {1 ↔ 5, 2 ↔ 8, 3 ↔ 7, 4 ↔ 6},
        {1 ↔ 6, 2 ↔ 3, 4 ↔ 5, 7 ↔ 8}, {1 ↔ 6, 2 ↔ 3, 4 ↔ 7, 5 ↔ 8}, {1 ↔ 6, 2 ↔ 3, 4 ↔ 8, 5 ↔ 7},
        {1 ↔ 6, 2 ↔ 4, 3 ↔ 5, 7 ↔ 8}, {1 ↔ 6, 2 ↔ 4, 3 ↔ 7, 5 ↔ 8}, {1 ↔ 6, 2 ↔ 4, 3 ↔ 8, 5 ↔ 7},
        {1 ↔ 6, 2 ↔ 5, 3 ↔ 4, 7 ↔ 8}, {1 ↔ 6, 2 ↔ 5, 3 ↔ 7, 4 ↔ 8}, {1 ↔ 6, 2 ↔ 5, 3 ↔ 8, 4 ↔ 7},
        {1 ↔ 6, 2 ↔ 7, 3 ↔ 4, 5 ↔ 8}, {1 ↔ 6, 2 ↔ 7, 3 ↔ 5, 4 ↔ 8}, {1 ↔ 6, 2 ↔ 7, 3 ↔ 8, 4 ↔ 5},
        {1 ↔ 6, 2 ↔ 8, 3 ↔ 4, 5 ↔ 7}, {1 ↔ 6, 2 ↔ 8, 3 ↔ 5, 4 ↔ 7}, {1 ↔ 6, 2 ↔ 8, 3 ↔ 7, 4 ↔ 5},
        {1 ↔ 7, 2 ↔ 3, 4 ↔ 5, 6 ↔ 8}, {1 ↔ 7, 2 ↔ 3, 4 ↔ 6, 5 ↔ 8}, {1 ↔ 7, 2 ↔ 3, 4 ↔ 8, 5 ↔ 6},
        {1 ↔ 7, 2 ↔ 4, 3 ↔ 5, 6 ↔ 8}, {1 ↔ 7, 2 ↔ 4, 3 ↔ 6, 5 ↔ 8}, {1 ↔ 7, 2 ↔ 4, 3 ↔ 8, 5 ↔ 6},
        {1 ↔ 7, 2 ↔ 5, 3 ↔ 4, 6 ↔ 8}, {1 ↔ 7, 2 ↔ 5, 3 ↔ 6, 4 ↔ 8}, {1 ↔ 7, 2 ↔ 5, 3 ↔ 8, 4 ↔ 6},
        {1 ↔ 7, 2 ↔ 6, 3 ↔ 4, 5 ↔ 8}, {1 ↔ 7, 2 ↔ 6, 3 ↔ 5, 4 ↔ 8}, {1 ↔ 7, 2 ↔ 6, 3 ↔ 8, 4 ↔ 5},
        {1 ↔ 7, 2 ↔ 8, 3 ↔ 4, 5 ↔ 6}, {1 ↔ 7, 2 ↔ 8, 3 ↔ 5, 4 ↔ 6}, {1 ↔ 7, 2 ↔ 8, 3 ↔ 6, 4 ↔ 5},
        {1 ↔ 8, 2 ↔ 3, 4 ↔ 5, 6 ↔ 7}, {1 ↔ 8, 2 ↔ 3, 4 ↔ 6, 5 ↔ 7}, {1 ↔ 8, 2 ↔ 3, 4 ↔ 7, 5 ↔ 6},
        {1 ↔ 8, 2 ↔ 4, 3 ↔ 5, 6 ↔ 7}, {1 ↔ 8, 2 ↔ 4, 3 ↔ 6, 5 ↔ 7}, {1 ↔ 8, 2 ↔ 4, 3 ↔ 7, 5 ↔ 6},
        {1 ↔ 8, 2 ↔ 5, 3 ↔ 4, 6 ↔ 7}, {1 ↔ 8, 2 ↔ 5, 3 ↔ 6, 4 ↔ 7}, {1 ↔ 8, 2 ↔ 5, 3 ↔ 7, 4 ↔ 6},
        {1 ↔ 8, 2 ↔ 6, 3 ↔ 4, 5 ↔ 7}, {1 ↔ 8, 2 ↔ 6, 3 ↔ 5, 4 ↔ 7}, {1 ↔ 8, 2 ↔ 6, 3 ↔ 7, 4 ↔ 5},
        {1 ↔ 8, 2 ↔ 7, 3 ↔ 4, 5 ↔ 6}, {1 ↔ 8, 2 ↔ 7, 3 ↔ 5, 4 ↔ 6}, {1 ↔ 8, 2 ↔ 7, 3 ↔ 6, 4 ↔ 5}}
```

```
In[]:=
```

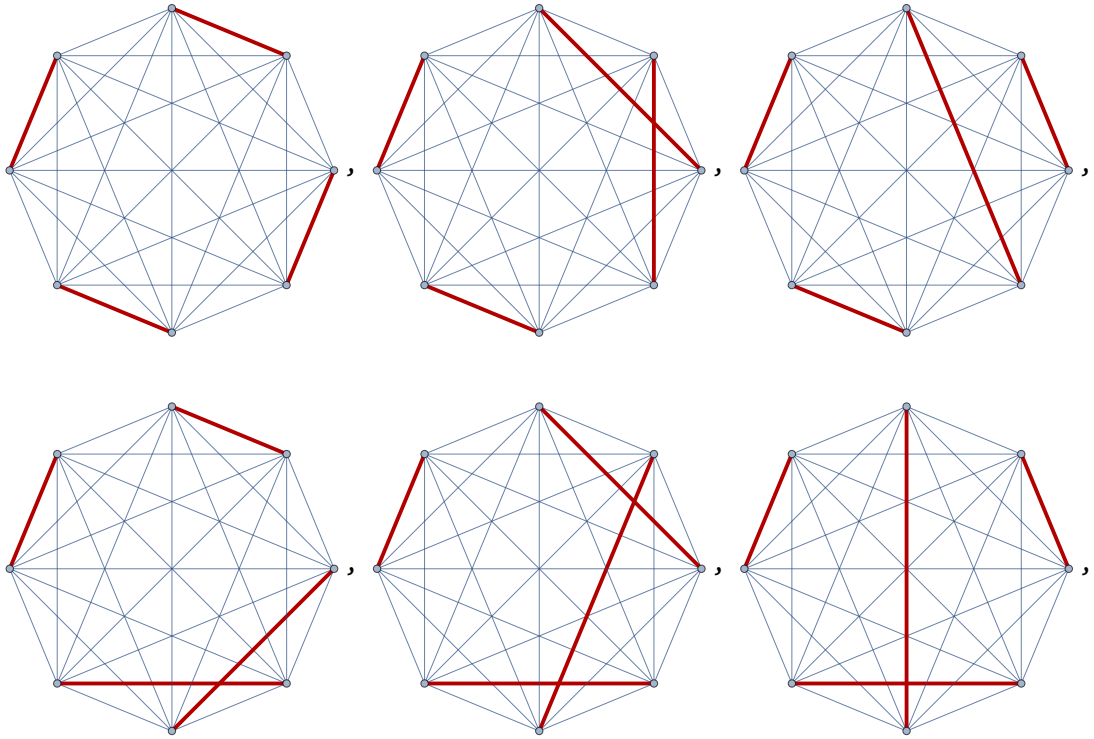
```
In[]:= Table[HighlightGraph[k6, h, GraphHighlightStyle → "Thick"], {h, esl6}]
```

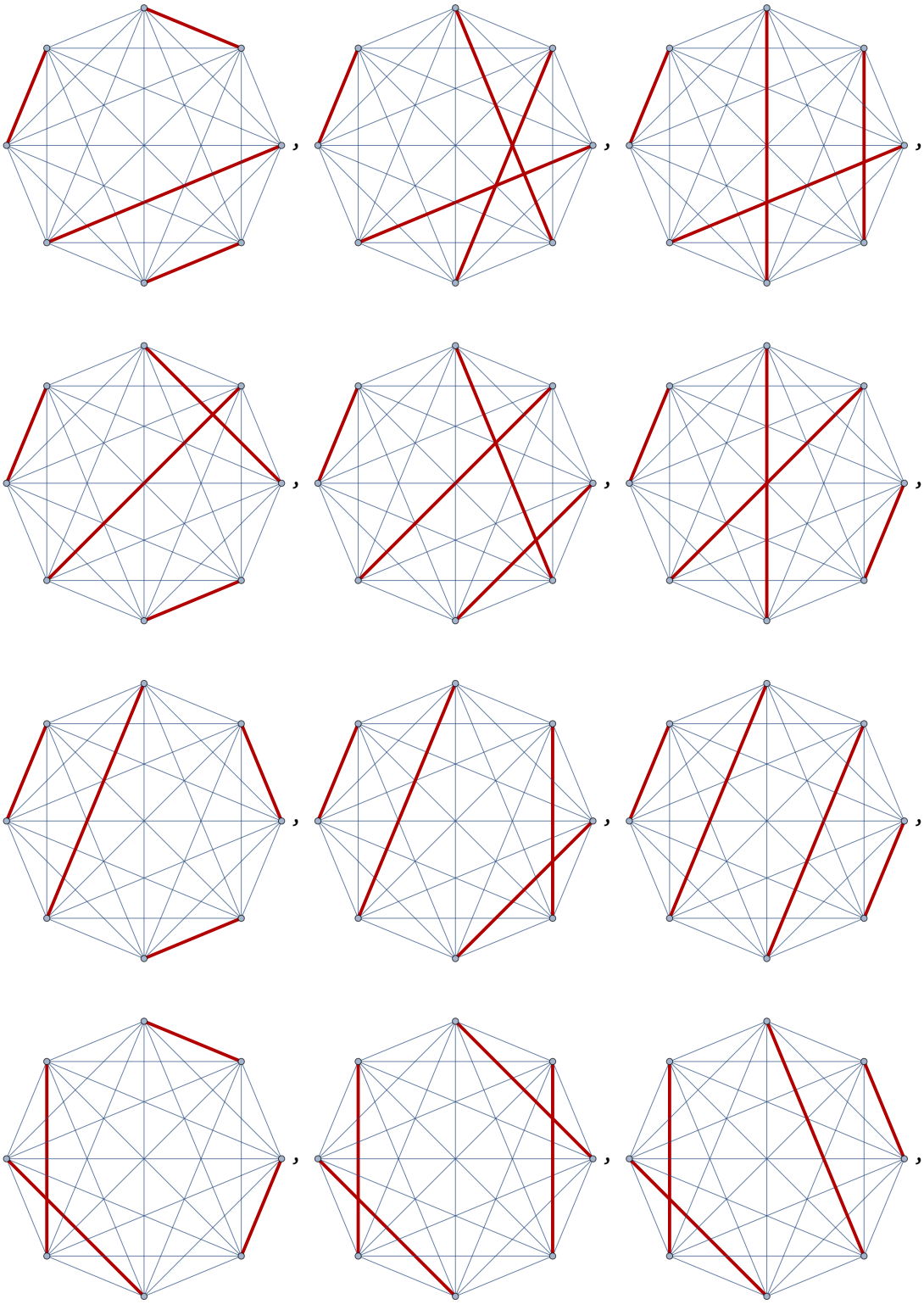
Out[ ]= 

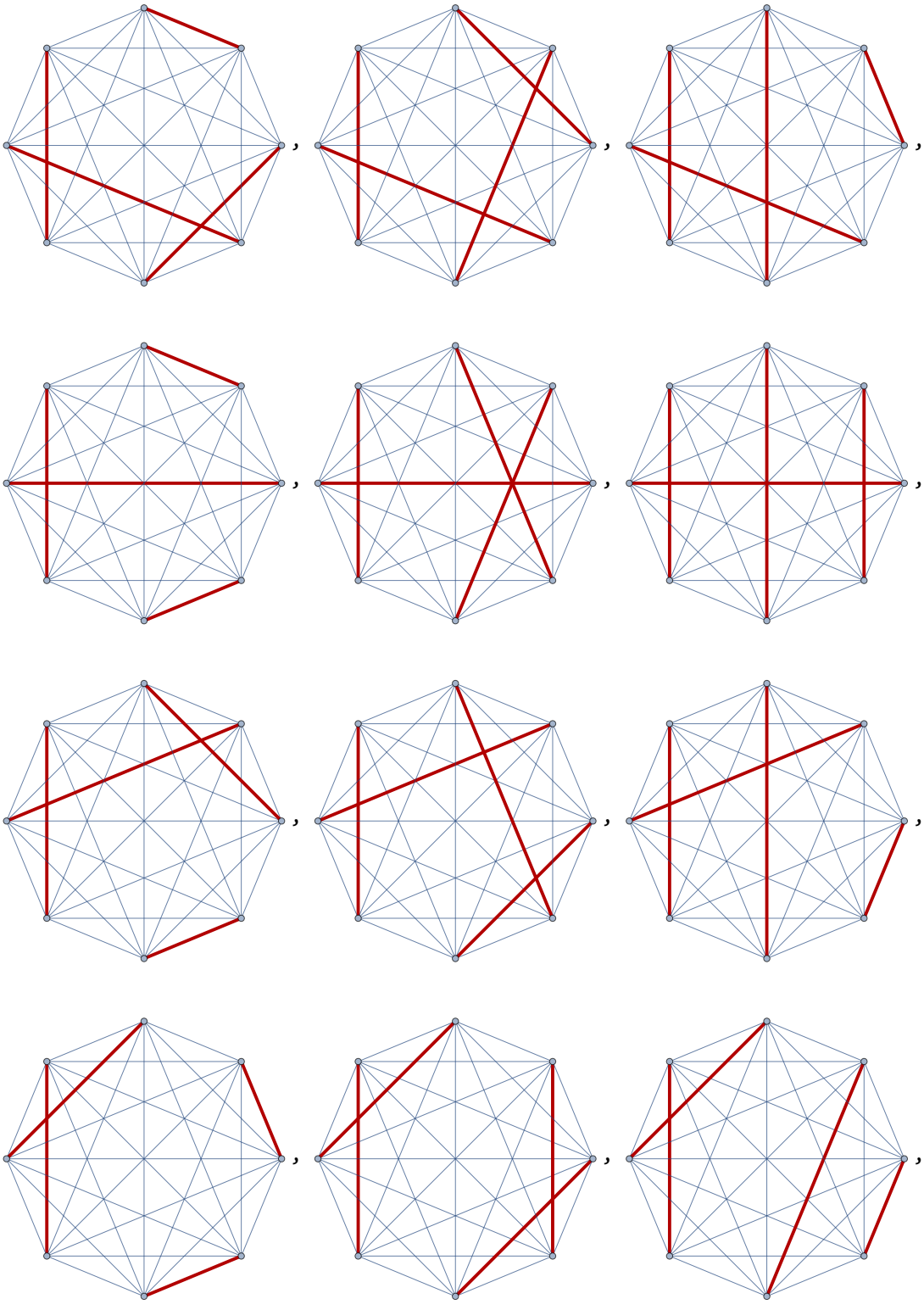


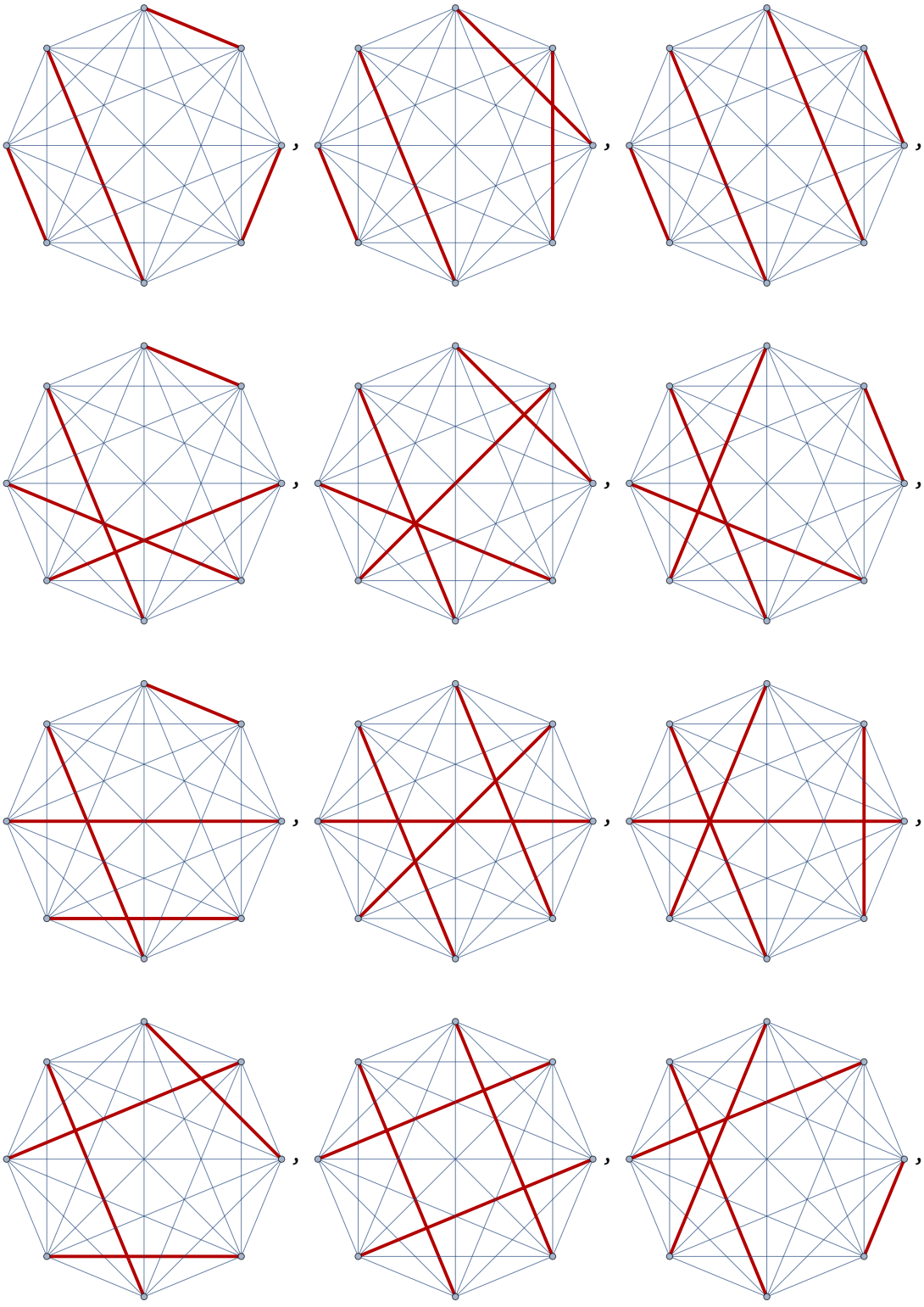
In[ ]:= `Table[HighlightGraph[k8, h, GraphHighlightStyle → "Thick"], {h, esl8}]`

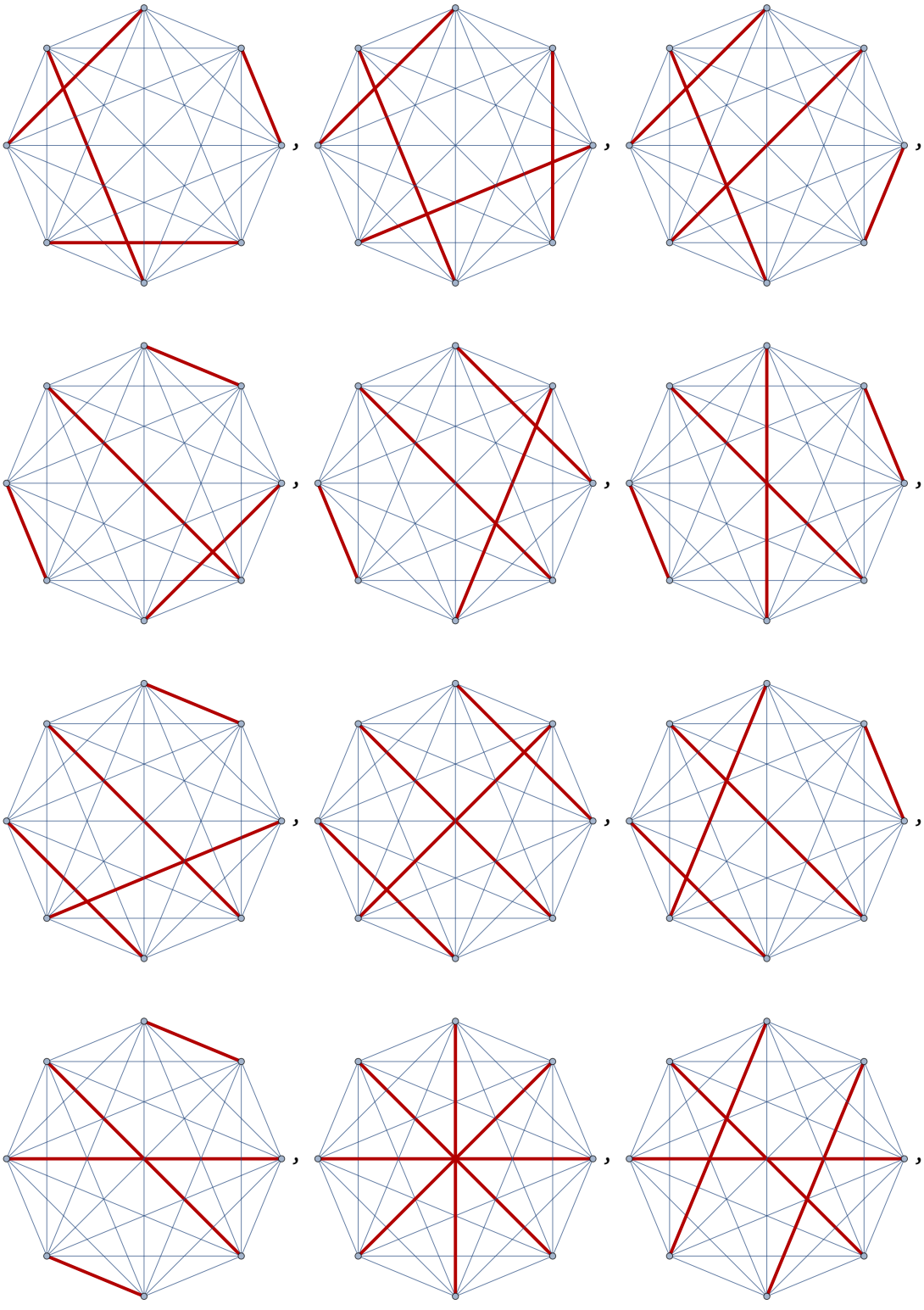
Out[ ]=

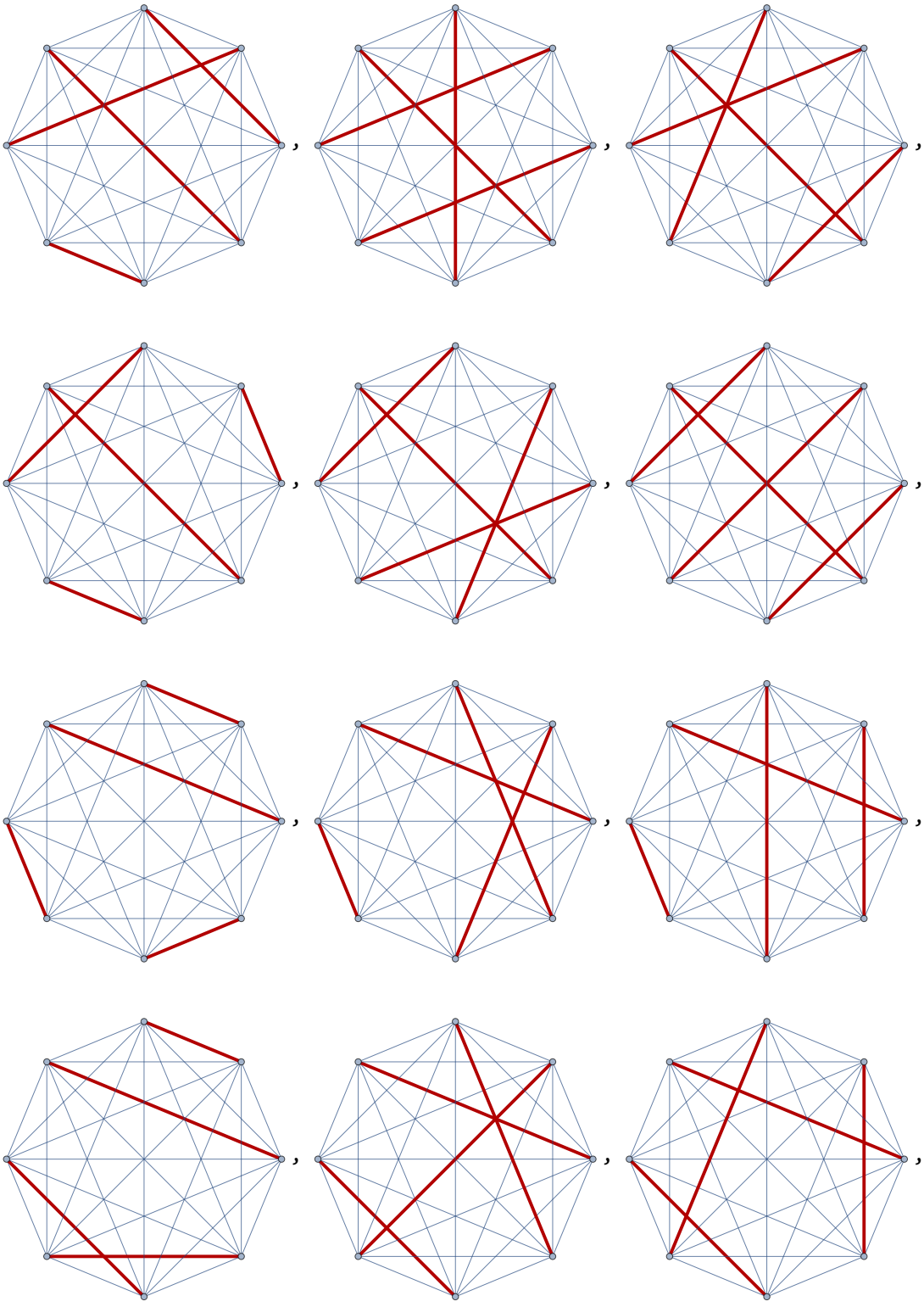


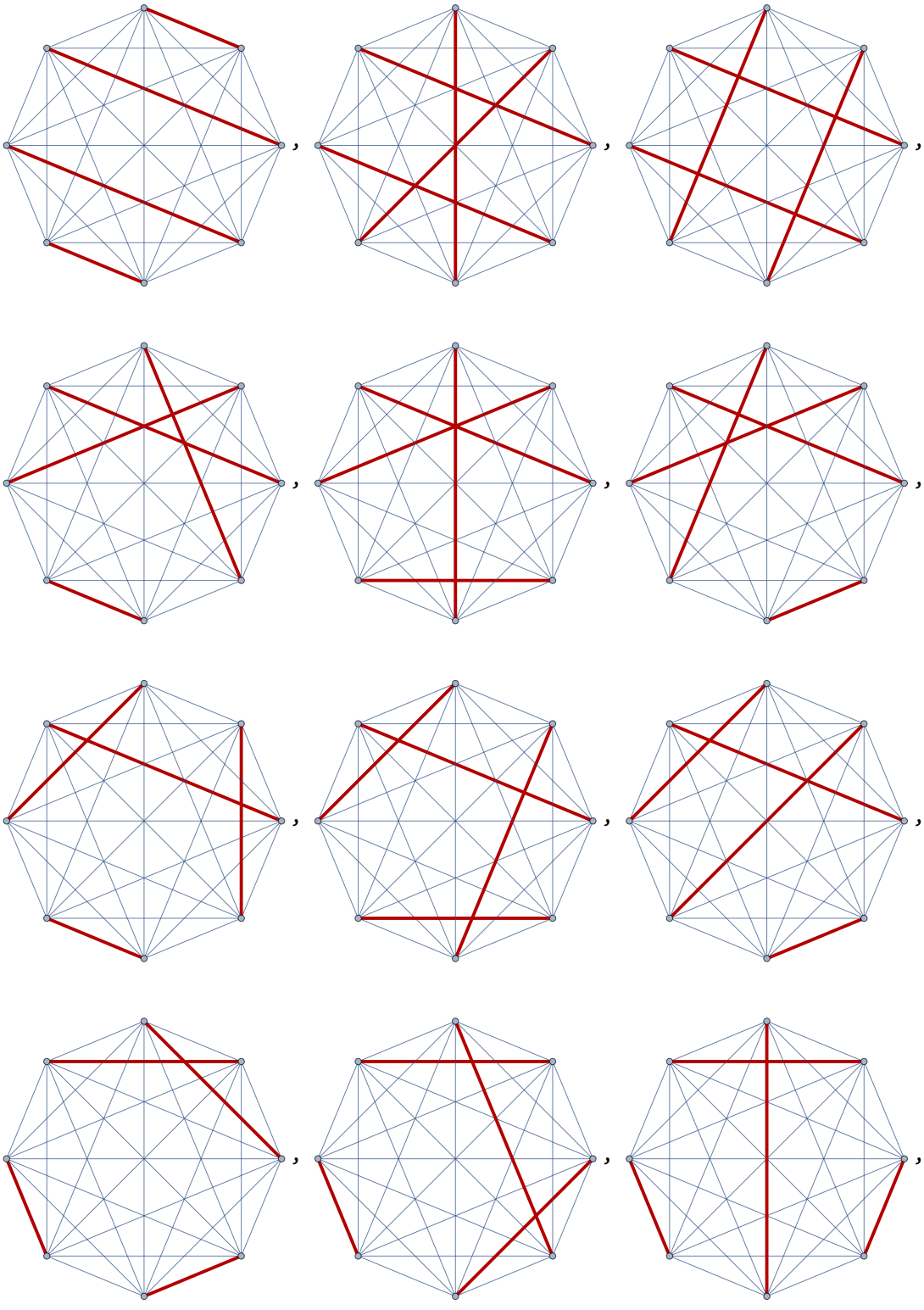


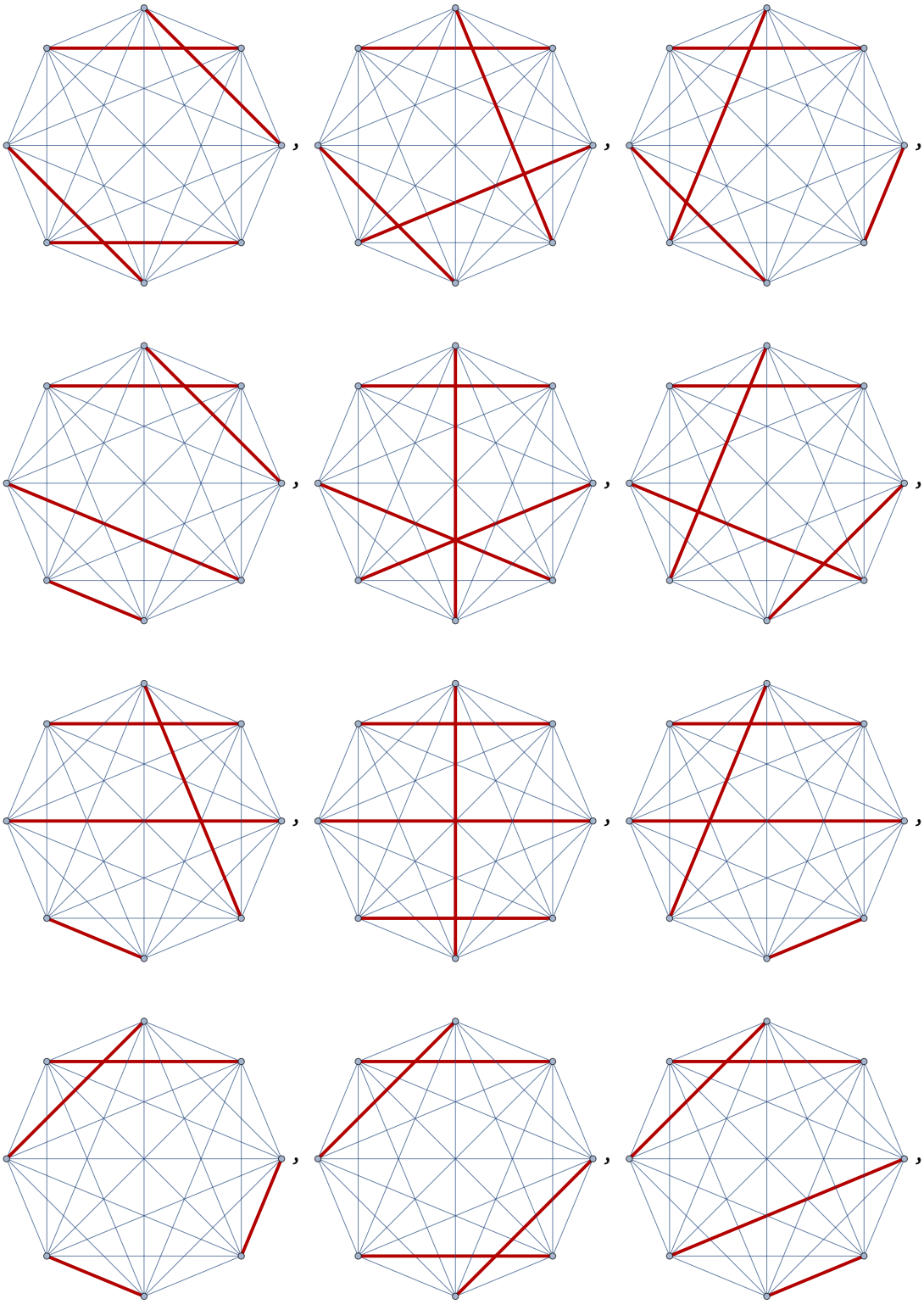


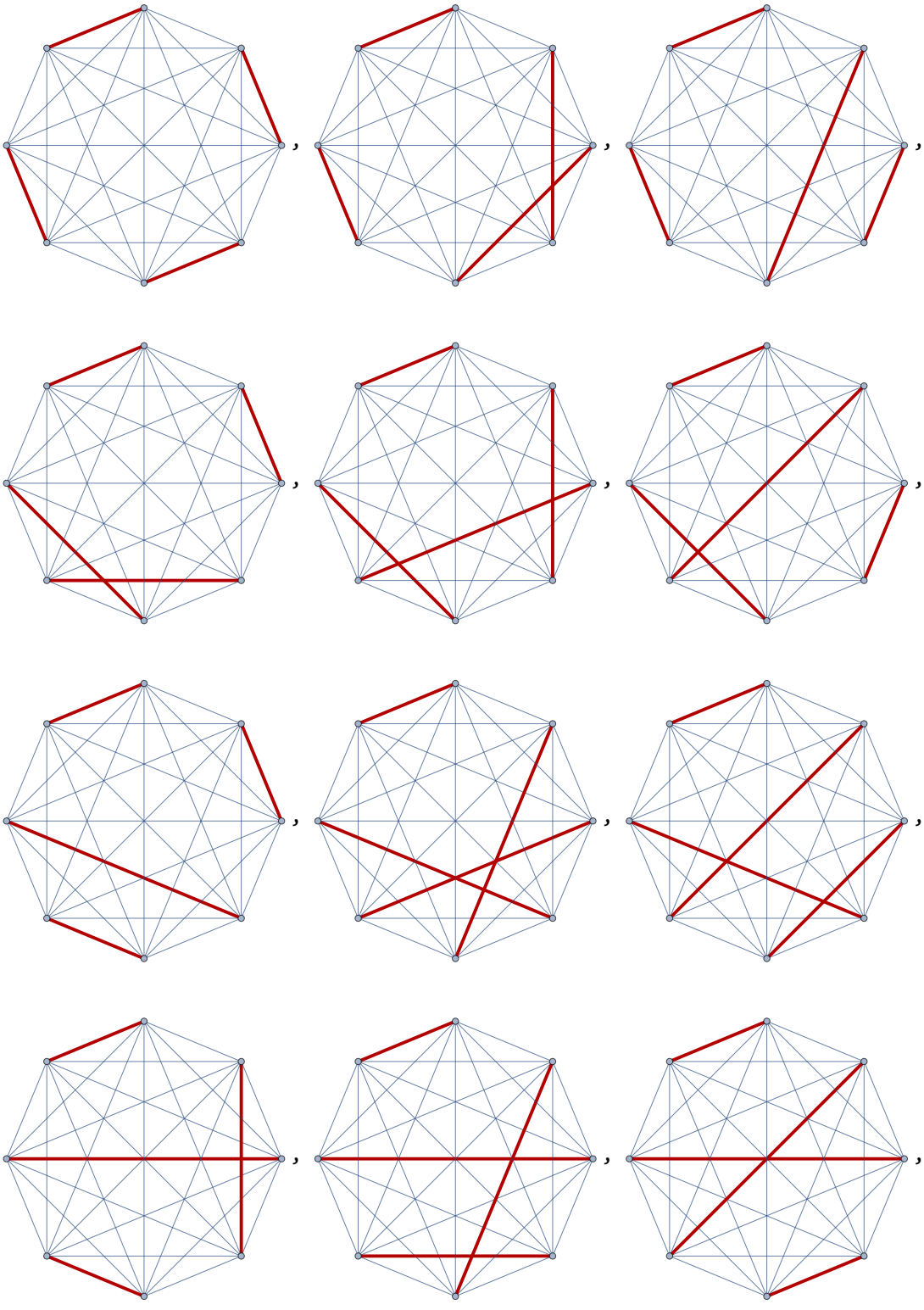


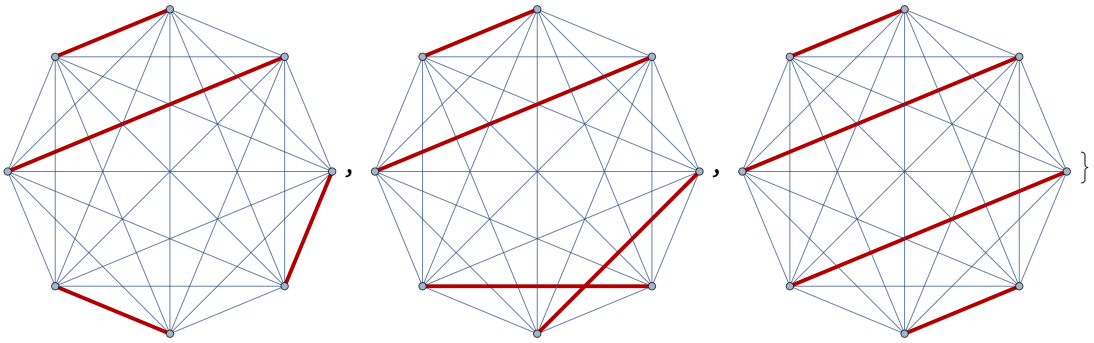












Stirling Permutations

```
In[ ]:= stirlingPermutation[k_] := (list = Riffle[Table[i, {i, 1, k}], Table[i, {i, 1, k}]];
          check[x_List] := (done = {};
            n = 1;
            While[n ≤ Length[x], If[MemberQ[done, x[[n]]], p = Position[done, x[[n]]][[1]][[1]];
              If[Min[done[[p ;;]]] ≠ x[[n]], Return[False]]];
             AppendTo[done, x[[n]]];
             n++;];
            Return[True];);
          Multicolumn[Select[Permutations[list], check[#] &], k])
```

```
In[ ]:= stirlingPermutation[4]
```

```
Out[ ]= {1, 1, 2, 2, 3, 3, 4, 4}  {1, 2, 4, 4, 2, 1, 3, 3}  {2, 2, 1, 4, 4, 1, 3, 3}  {3, 3, 2, 2, 1, 4, 4, 1}
        {1, 1, 2, 2, 3, 4, 4, 3}  {1, 2, 4, 4, 2, 3, 3, 1}  {2, 2, 1, 4, 4, 3, 3, 1}  {3, 3, 2, 2, 4, 4, 1, 1}
        {1, 1, 2, 2, 4, 4, 3, 3}  {1, 2, 4, 4, 3, 3, 2, 1}  {2, 2, 3, 3, 1, 1, 4, 4}  {3, 3, 2, 4, 4, 2, 1, 1}
        {1, 1, 2, 3, 3, 2, 4, 4}  {1, 3, 3, 1, 2, 2, 4, 4}  {2, 2, 3, 3, 1, 4, 4, 1}  {3, 3, 4, 4, 1, 1, 2, 2}
        {1, 1, 2, 3, 3, 4, 4, 2}  {1, 3, 3, 1, 2, 4, 4, 2}  {2, 2, 3, 3, 4, 4, 1, 1}  {3, 3, 4, 4, 1, 2, 2, 1}
        {1, 1, 2, 3, 4, 4, 3, 2}  {1, 3, 3, 1, 4, 4, 2, 2}  {2, 2, 3, 4, 4, 3, 1, 1}  {3, 3, 4, 4, 2, 2, 1, 1}
        {1, 1, 2, 4, 4, 2, 3, 3}  {1, 3, 3, 2, 2, 1, 4, 4}  {2, 2, 4, 4, 1, 1, 3, 3}  {3, 4, 4, 3, 1, 1, 2, 2}
        {1, 1, 2, 4, 4, 3, 3, 2}  {1, 3, 3, 2, 2, 4, 4, 1}  {2, 2, 4, 4, 1, 3, 3, 1}  {3, 4, 4, 3, 1, 2, 2, 1}
        {1, 1, 3, 3, 2, 2, 4, 4}  {1, 3, 3, 2, 4, 4, 2, 1}  {2, 2, 4, 4, 3, 3, 1, 1}  {3, 4, 4, 3, 2, 2, 1, 1}
        {1, 1, 3, 3, 2, 4, 4, 2}  {1, 3, 3, 4, 4, 1, 2, 2}  {2, 3, 3, 2, 1, 1, 4, 4}  {4, 4, 1, 1, 2, 2, 3, 3}
        {1, 1, 3, 3, 4, 4, 2, 2}  {1, 3, 3, 4, 4, 2, 2, 1}  {2, 3, 3, 2, 1, 4, 4, 1}  {4, 4, 1, 1, 2, 3, 3, 2}
        {1, 1, 3, 4, 4, 3, 2, 2}  {1, 3, 4, 4, 3, 1, 2, 2}  {2, 3, 3, 2, 4, 4, 1, 1}  {4, 4, 1, 1, 3, 3, 2, 2}
        {1, 1, 4, 4, 2, 2, 3, 3}  {1, 3, 4, 4, 3, 2, 2, 1}  {2, 3, 3, 4, 4, 2, 1, 1}  {4, 4, 1, 2, 2, 1, 3, 3}
        {1, 1, 4, 4, 2, 3, 3, 2}  {1, 4, 4, 1, 2, 2, 3, 3}  {2, 3, 4, 4, 3, 2, 1, 1}  {4, 4, 1, 2, 2, 3, 3, 1}
        {1, 1, 4, 4, 3, 3, 2, 2}  {1, 4, 4, 1, 2, 3, 3, 2}  {2, 4, 4, 2, 1, 1, 3, 3}  {4, 4, 1, 2, 3, 3, 2, 1}
        {1, 2, 2, 1, 3, 3, 4, 4}  {1, 4, 4, 1, 3, 3, 2, 2}  {2, 4, 4, 2, 1, 3, 3, 1}  {4, 4, 1, 3, 3, 1, 2, 2}
        {1, 2, 2, 1, 3, 4, 4, 3}  {1, 4, 4, 2, 2, 1, 3, 3}  {2, 4, 4, 2, 3, 3, 1, 1}  {4, 4, 1, 3, 3, 2, 2, 1}
        {1, 2, 2, 1, 4, 4, 3, 3}  {1, 4, 4, 2, 2, 3, 3, 1}  {2, 4, 4, 3, 3, 2, 1, 1}  {4, 4, 2, 2, 1, 1, 3, 3}
        {1, 2, 2, 3, 3, 1, 4, 4}  {1, 4, 4, 2, 3, 3, 2, 1}  {3, 3, 1, 1, 2, 2, 4, 4}  {4, 4, 2, 2, 1, 3, 3, 1}
        {1, 2, 2, 3, 3, 4, 4, 1}  {1, 4, 4, 3, 3, 1, 2, 2}  {3, 3, 1, 1, 2, 4, 4, 2}  {4, 4, 2, 2, 3, 3, 1, 1}
        {1, 2, 2, 3, 4, 4, 3, 1}  {1, 4, 4, 3, 3, 2, 2, 1}  {3, 3, 1, 1, 4, 4, 2, 2}  {4, 4, 2, 3, 3, 2, 1, 1}
        {1, 2, 2, 4, 4, 1, 3, 3}  {2, 2, 1, 1, 3, 3, 4, 4}  {3, 3, 1, 2, 2, 1, 4, 4}  {4, 4, 3, 3, 1, 1, 2, 2}
        {1, 2, 2, 4, 4, 3, 3, 1}  {2, 2, 1, 1, 3, 4, 4, 3}  {3, 3, 1, 2, 2, 4, 4, 1}  {4, 4, 3, 3, 1, 2, 2, 1}
        {1, 2, 3, 3, 2, 1, 4, 4}  {2, 2, 1, 1, 4, 4, 3, 3}  {3, 3, 1, 2, 4, 4, 2, 1}  {4, 4, 3, 3, 2, 2, 1, 1}
        {1, 2, 3, 3, 2, 4, 4, 1}  {2, 2, 1, 3, 3, 1, 4, 4}  {3, 3, 1, 4, 4, 1, 2, 2}
        {1, 2, 3, 3, 4, 4, 2, 1}  {2, 2, 1, 3, 3, 4, 4, 1}  {3, 3, 1, 4, 4, 2, 2, 1}
        {1, 2, 3, 4, 4, 3, 2, 1}  {2, 2, 1, 3, 4, 4, 3, 1}  {3, 3, 2, 2, 1, 1, 4, 4}
```

Limits for Ratio test in m(2)

In[1]:=
```
case1 = (2^(1 + 2 n) n! (1 + n)!) / (2 (1 + n))!
case2 = (2^(-1 - 2 n) (2 n)!) / (n! (1 + n)!)
```

Out[1]=
$$\frac{2^{1+2n}\, n!\, (1+n)!}{(2\,(1+n))!}$$

Out[2]=
$$\frac{2^{-1-2n}\, (2n)!}{n!\, (1+n)!}$$

In[3]:=
```
limitm2case1 = Limit[case1, n → Infinity]
limitm2case2 = Limit[case2, n → Infinity]
```

Out[3]= 0

Out[4]= 0

Computing m(1) to m(10) from n=0 to n=2000

```
Multifactorial[n_, k_] := Abs[Apply[Times, Range[-n, -1, k]]]
```

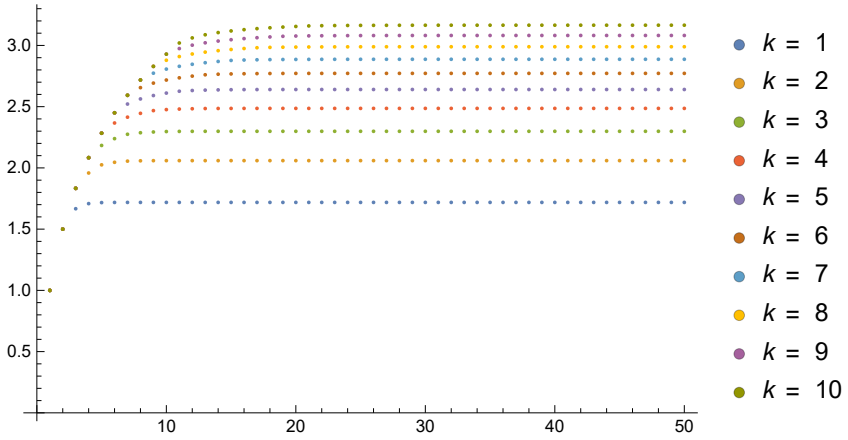
In[•]:=
```
For[i = 1, i < 11, i++, Print[N[Sum[1 / Multifactorial[n, i], {n, 0, 2000}], 10]]]
```

```
2.718281828
3.059407405
3.298913538
3.485944977
3.640224468
3.771902396
3.886959654
3.989241213
4.081375520
4.165243766
```

Plot of m (1) to m (1) superimposed on each other from n = 0 to n = 2000

```
ListPlot[Table[Sum[1 / Multifactorial[n, j], {n, 1, i}], {j, 1, 10}, {i, 1, 20}],
 PlotLegends → PointLegend[Automatic,
   PromptForm[k, #] & /@ Range[10], LegendMarkers → {Graphics[Disk[]], 6}]]
```

Out[•]=




```
ListPlot[Table[Sum[1 / Multifactorial[n, j], {n, 1, i}], {j, 1, 10}, {i, 1, 50}],
 PlotLegends → PointLegend[Automatic,
   PromptForm[k, #] & /@ Range[10], LegendMarkers → {Graphics[Disk[]], 6}]]
```

Out[•]=