Lab -3

Bhrugu Dave (201801401)[*] and Sarthak Patel (201801435)[†]
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
CS-302, Modeling and Simulation

In this lab, we understood and analyzed the logistic equation and its variations by implementing various models to simulate the growth of the population of a species under different circumstances. We also learnt about the effect of constant and dynamic harvesting on growth of the population in both, presence and absence of deaths due to isolation.

I. INTRODUCTION

We took a glance at the population model with adding some extra features along the way in the logistic equation. We first implemented the logistic model with constant harvesting and then dynamic harvesting. In the second problem, we also considered the death due to isolation and then implemented the both type of harvesting for this scenario.

II. MODEL

Population P is modeled as,

$$\frac{dP}{dt} = r(1 - \frac{P}{K})P \tag{1}$$

Where K is carrying capacity of the population and r is the growth rate of the system.

Now, we make the above logistic equation dimensionless.

$$\frac{dn}{d\tau} = n(1 - n) \tag{2}$$

where $n = \frac{P}{K}$ and τ = t·r

Case 1: Constant Harvesting with rate h

As, we move to dimensionless equation, harvesting rate ($\widetilde{h}$) will be h/(K·r). In this case the equation becomes,

$$\frac{dn}{d\tau} = n(1 - n) - \widetilde{h} \tag{3}$$

Case 2: Dynamic Harvesting

In this case, amount harvesting will be directly proportional to the population present. So, we take proportionality constant ϵ. In this case the equation is modeled as,

$$\frac{dn}{d\tau} = n(1 - n) - \epsilon n \tag{4}$$

Death due to Isolation:

Due to this factor we consider that a species will only grow if there are enough population is present in the system. If a certain population is not present then the species will go instinct. To model this we need to add another factor in the logistic equation.

$$\frac{dn}{d\tau} = n(1 - n)(n - n_{thresh}) \tag{5}$$

Here, n_{thresh} is minimal population that should be alive so that a species can grow.

$$n_{thresh} = \frac{A}{K}$$

Where, A is threshold population and K is the carrying capacity of the system.

Case 3: Death due to Isolation with Constant Harvesting

Considering constant harvesting along with death due to isolation in Eq. 5, we get the following model:

$$\frac{dn}{d\tau} = n(1 - n)(n - n_{thresh}) - \widetilde{h} \tag{6}$$

*Electronic address: 201801401@daiict.ac.in
†Electronic address: 201801435@daiict.ac.in

Case 4: Death due to Isolation with Dynamic Harvesting

Considering the dynamic harvesting with the proportionality ϵ in the Eq. 5. We get the following model,

$$\frac{dn}{d\tau} = n(1-n)(n - n_{thresh}) - \epsilon n \quad (7)$$

III. RESULTS

For the following discussions:
p_0 = initial population,
r = growth rate,
K = carrying capacity
h = harvesting rate
A = threshold population

- Starting with simulation of first model, i.e. with constant harvesting rate, we observe linear growth in the very initial phase, after which it observes exponential growth, and once total population reaches carrying capacity, the growth terminates and total population becomes constant with value equal to carrying capacity.

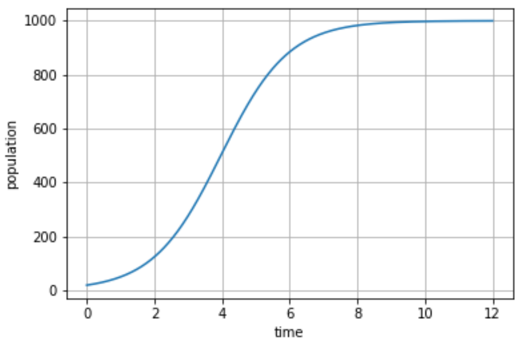

FIG. 1: Population growth with h=1, K=1000, p_0=20, r=1, and without death in isolation.

- In above discussed model, on considering harvesting rate to be dynamic, whose value is directly proportional to the value of instantaneous population, we observe that as the value of dynamic harvesting rate increases, the amount of population at saturation level decreases and recedes farther from the carrying capacity. This behaviour is due to the fact that after a certain amount of population(less than carrying capacity) is achieved, the addition in population due to growth rate equals the deaths due to harvesting, and hence carrying capacity is never reached.

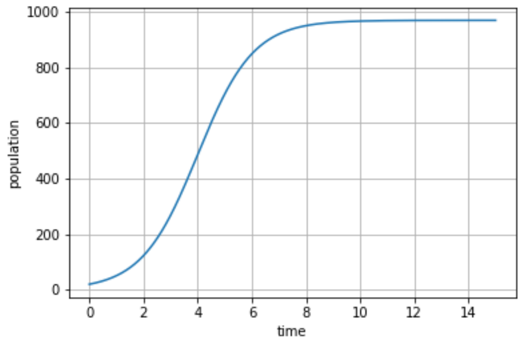

FIG. 2: Population growth with dynamic rate of harvesting, h=0.03, K=1000, p_0=20, r=1, and without death in isolation.

- Now, we consider the model which only has deaths due to isolation. By death due to isolation, we mean that only if the initial population is greater than a certain value, will the population be able to survive. For initial population less than threshold, the population will go extinct. To simulate such behavior, we need a model that has positive growth rate for population greater than threshold and negative growth rate if population is less than the threshold. Here, the idea behind the change is shown in the figure below. Which shows that after a certain point growth is positive and below that growth is negative.

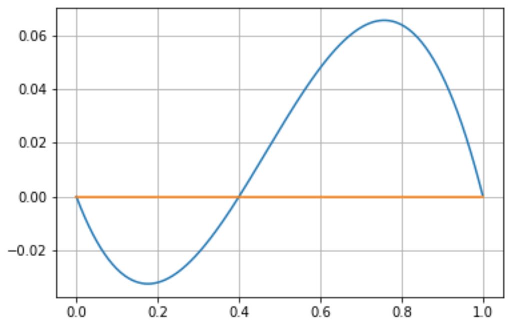

FIG. 3: Death due to isolation model

- Now, we simulate the above discussed model with varying values of initial population. In the first case, we start with an initial population which is less than the threshold population. In the second case, we start with an initial population that is greater than the threshold. The results of both are shown below in Fig. 4 and Fig. 5 respectively.

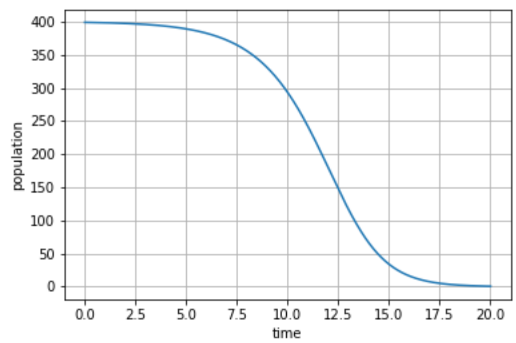

FIG. 4: Death due to isolation with K=1000, p_0=399, A=400, r=1

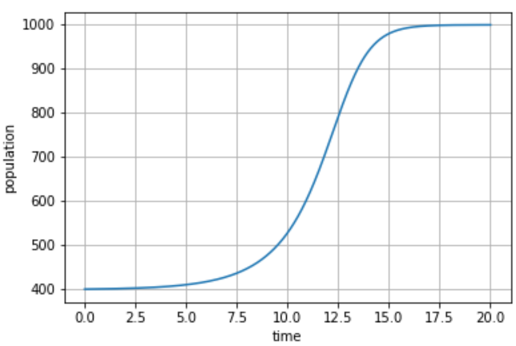

FIG. 5: Death due to isolation with K=1000, p_0=401, A=400, r=1

- Now, we also take deaths due to harvesting into consideration along with deaths due to isolation in the previous model. In this scenario, harvesting constant $\widetilde{h}$ will also affect the growth of population. If the value of $\widetilde{h}$ is greater than the growth, i.e. deaths due to harvesting are more than growth due to new births, than the population will start decreasing and eventually vanish. If the value of $\widetilde{h}$ is less than the growth, then the species will grow.

In Fig. 6, even though initial population is greater than threshold population, the species goes extinct since the growth is less than $\widetilde{h}$.

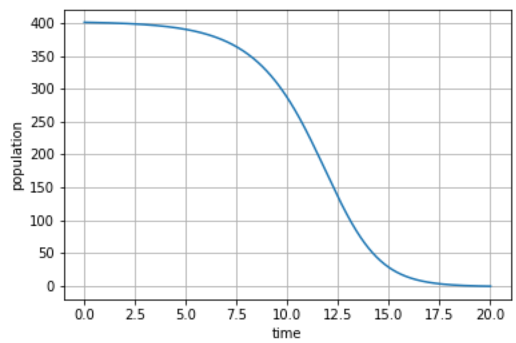

FIG. 6: Death due to isolation and constant harvesting with K=1000, p_0=401, A=400, r=2, h=1

In Fig. 7, as initial population is less than threshold population, the growth will be negative. And hence, the species will go extinct.

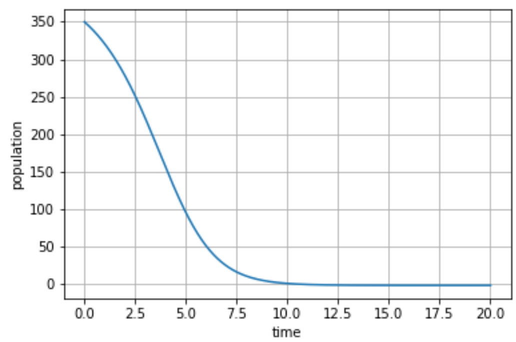

FIG. 7: Death due to isolation and constant harvesting with K=1000, p_0=350, A=400, r=2, h=1

In Fig. 8, initial population is greater than threshold population and also the growth is greater than $\widetilde{h}$. So, the species will grow and reach the carrying capacity eventually.

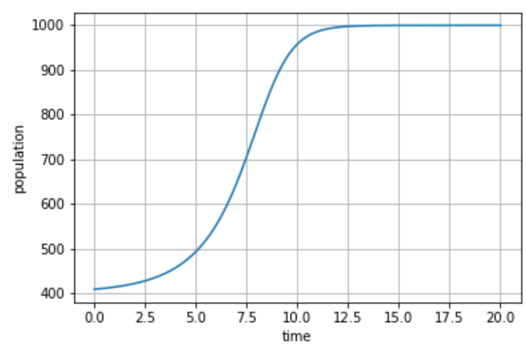

FIG. 8: Death due to isolation and constant harvesting with K=1000, p_0=410, A=400, r=2, h=1

- Now, we add the dynamic harvesting into the death due to isolation model and here too we observe similar behavior as seen in the previous case. If growth is less than the harvesting, then the species will go extinct. Else it will grow and reach the carrying capacity.

  In Fig. 9, even though initial population is greater than threshold population, the species goes extinct since the growth is less than $\widetilde{h}$.

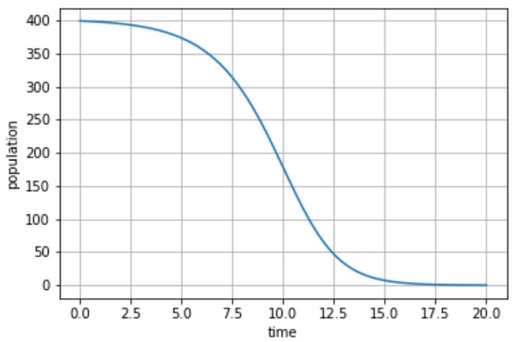

FIG. 9: Death due to isolation and dynamic harvesting with K=1000, p_0=401, A=400, r=2, ϵ=0.01

In Fig. 10, as initial population is less than threshold population, the growth will be negative. And hence, the species will go extinct.

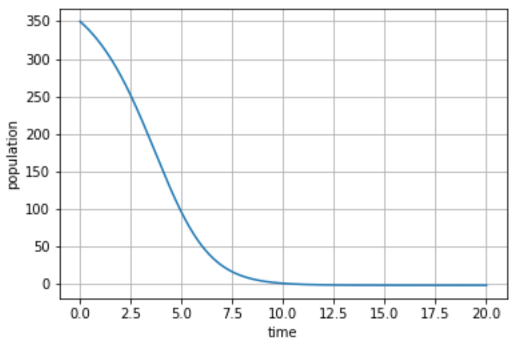

FIG. 10: Death due to isolation and dynamic harvesting with K=1000, p_0=350, A=400, r=2, ϵ=0.001

In Fig. 11, initial population is greater than threshold population and also the growth is greater than $\widetilde{h}$. So, the species will grow and reach the carrying capacity eventually.

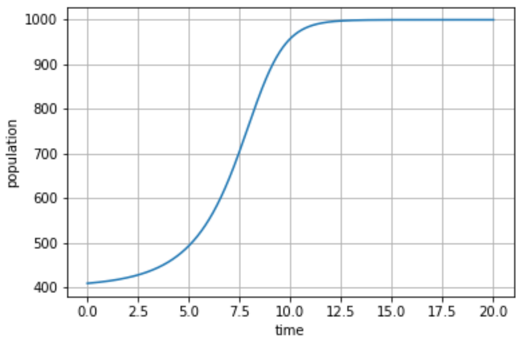

FIG. 11: Death due to isolation and dynamic harvesting with K=1000, p_0=410, A=400, r=2, ϵ=0.001

IV. CONCLUSIONS

In the case of harvesting, irrespective of the consideration of death by isolation, we observed that the species will go extinct if the deaths due to harvesting is greater than the growth due to births. Otherwise, the system will grow to the carrying capacity or the steady state population depending on the scenario simulated. By including an additional factor into the logistic model, we can inculcate the effect of death by isolation too.