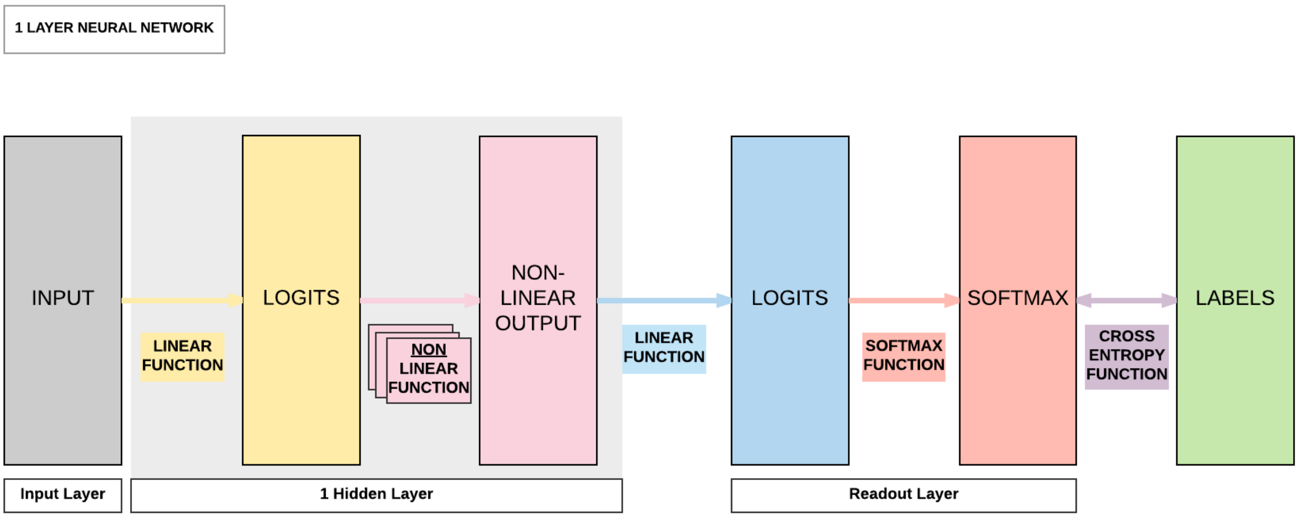
4. Optimizers

Introduction to Gradient-descent Optimizers

Model: 1 Hidden Layer Feedforward Neural Network (ReLU Activation) ¶



In this assignment, we are going to train a MLP model (developed using Pytorch) using different Optimization algorithms that have been already discussed in class.

In [1]:

```
import torch
import torch.nn as nn
import torchvision.transforms as transforms
import torchvision.datasets as dsets

# Set seed
torch.manual_seed(0)


'''
STEP 1: LOADING DATASET
'''

train_dataset = dsets.MNIST(root='./data', train=True, transform=transforms.ToTe
nsor(), download=True)
test_dataset = dsets.MNIST(root='./data', train=False, transform=transforms.ToTe
nsor())

'''
STEP 2: MAKING DATASET ITERABLE
'''

batch_size = 100
n_iters = 3000
num_epochs = n_iters / (len(train_dataset) / batch_size)
num_epochs = int(num_epochs)

train_loader = torch.utils.data.DataLoader(dataset=train_dataset, batch_size=bat
ch_size, shuffle=True)
test_loader = torch.utils.data.DataLoader(dataset=test_dataset, batch_size=batch
_size, shuffle=False)

'''
STEP 3: CREATE MODEL CLASS
'''
class FeedforwardNeuralNetModel(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim):
        super(FeedforwardNeuralNetModel, self).__init__()
        # Linear function
        self.fc1 = nn.Linear(input_dim, hidden_dim)
        # Non-linearity
        self.relu = nn.ReLU()
        # Linear function (readout)
        self.fc2 = nn.Linear(hidden_dim, output_dim)

    def forward(self, x):
        ### START CODE HERE ###
        # Linear function
        out = self.fc1(x)
        # Non-linearity
        out = self.relu(out)
        # Linear function (readout)
        out = self.fc2(out)
        ### END CODE HERE ###
        return out
'''
STEP 4: INSTANTIATE MODEL CLASS
'''
input_dim = 28*28
```

```
hidden_dim = 100
output_dim = 10

model = FeedforwardNeuralNetModel(input_dim, hidden_dim, output_dim)

'''
STEP 5: INSTANTIATE LOSS CLASS
'''
criterion = nn.CrossEntropyLoss()


'''
STEP 6: INSTANTIATE OPTIMIZER CLASS
'''
learning_rate = 0.1

### START CODE HERE ###
optimizer = torch.optim.SGD(model.parameters(), lr=learning_rate)
### END CODE HERE ###

'''
STEP 7: TRAIN THE MODEL
'''
iter = 0
for epoch in range(num_epochs):
    for i, (images, labels) in enumerate(train_loader):
        # Load images as Variable
        images = images.view(-1, 28*28).requires_grad_()

        # Clear gradients w.r.t. parameters
        optimizer.zero_grad()

        # Forward pass to get output/logits
        outputs = model(images)

        # Calculate Loss: softmax --> cross entropy loss
        loss = criterion(outputs, labels)

        # Getting gradients w.r.t. parameters
        loss.backward()

        # Updating parameters
        optimizer.step()

        iter += 1

        if iter % 500 == 0:
            # Calculate Accuracy
            correct = 0
            total = 0
            # Iterate through test dataset
            for images, labels in test_loader:
                # Load images to a Torch Variable
                images = images.view(-1, 28*28)

                # Forward pass only to get logits/output
                outputs = model(images)

                # Get predictions from the maximum value
                _, predicted = torch.max(outputs.data, 1)
```

```
                # Total number of labels
                total += labels.size(0)

                # Total correct predictions
                correct += (predicted == labels).sum()

            accuracy = 100 * correct / total

            # Print Loss
            print('Iteration: {}. Loss: {}. Accuracy: {}'.format(iter, loss.item
(), accuracy))
```

```
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyt
e.gz
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyt
e.gz to ./data/MNIST/raw/train-images-idx3-ubyte.gz


Extracting ./data/MNIST/raw/train-images-idx3-ubyte.gz to ./data/MNI
ST/raw

Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyt
e.gz
Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyt
e.gz to ./data/MNIST/raw/train-labels-idx1-ubyte.gz


Extracting ./data/MNIST/raw/train-labels-idx1-ubyte.gz to ./data/MNI
ST/raw

Downloading http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.
gz
Downloading http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.
gz to ./data/MNIST/raw/t10k-images-idx3-ubyte.gz


Extracting ./data/MNIST/raw/t10k-images-idx3-ubyte.gz to ./data/MNIS
T/raw

Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.
gz
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.
gz to ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz


Extracting ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz to ./data/MNIS
T/raw

Iteration: 500. Loss: 0.3460221588611603. Accuracy: 91.5
Iteration: 1000. Loss: 0.21451084315776825. Accuracy: 92.47000122070
312
Iteration: 1500. Loss: 0.1919996440410614. Accuracy: 93.870002746582
03
Iteration: 2000. Loss: 0.17153751850128174. Accuracy: 94.47000122070
312
Iteration: 2500. Loss: 0.1125108525156974 8. Accuracy: 95.16999816894
531
Iteration: 3000. Loss: 0.1736811101436615. Accuracy: 95.599998474121
1
```

Optimization Process

```
parameters = parameters - learning_rate * parameters_gradients
```

Mathematical Interpretation of Gradient Descent

- Model's parameters: $\theta \in \mathbb{R}^d$
- Loss function: $J(\theta)$
- Gradient w.r.t. parameters: $\nabla J(\theta)$
- Learning rate: η
- Batch Gradient descent: $\theta = \theta - \eta \cdot \nabla J(\theta)$

Optimization Algorithm 1: Batch Gradient Descent

- What we've covered so far: batch gradient descent
  - $\theta = \theta - \eta \cdot \nabla J(\theta)$
- Characteristics
  - Compute the gradient of the lost function w.r.t. parameters for the entire training data, $\nabla J(\theta)$
  - Use this to update our parameters at every iteration
- Problems
  - Unable to fit whole datasets in memory
  - Computationally slow as we attempt to compute a large Jacobian matrix → first order derivative, $\nabla J(\theta)$

Optimization Algorithm 2: Stochastic Gradient Descent

- Modification of batch gradient descent
  - $\theta = \theta - \eta \cdot \nabla J(\theta, x^i, y^i)$
- Characteristics
  - Compute the gradient of the lost function w.r.t. parameters for the **one set of training sample (1 input and 1 label)**, $\nabla J(\theta, x^i, y^i)$
  - Use this to update our parameters at every iteration

Optimization Algorithm 3: Mini-batch Gradient Descent

- Combination of batch gradient descent & stochastic gradient descent
  - $\theta = \theta - \eta \cdot \nabla J(\theta, x^{i:i+n}, y^{i:i+n})$
- Characteristics
  - Compute the gradient of the lost function w.r.t. parameters for **n sets of training sample (n input and n label)**, $\nabla J(\theta, x^{i:i+n}, y^{i:i+n})$
  - Use this to update our parameters at every iteration
- This is often called SGD in deep learning frameworks

In [2]:

```
import torch
import torch.nn as nn
import torchvision.transforms as transforms
import torchvision.datasets as dsets

# Set seed
torch.manual_seed(0)

'''
STEP 1: LOADING DATASET
'''

train_dataset = dsets.MNIST(root='./data', train=True, transform=transforms.ToTe
nsor(), download=True)
test_dataset = dsets.MNIST(root='./data', train=False, transform=transforms.ToTe
nsor())

'''
STEP 2: MAKING DATASET ITERABLE
'''

batch_size = 100
n_iters = 3000
num_epochs = n_iters / (len(train_dataset) / batch_size)
num_epochs = int(num_epochs)

train_loader = torch.utils.data.DataLoader(dataset=train_dataset, batch_size=bat
ch_size, shuffle=True)
test_loader = torch.utils.data.DataLoader(dataset=test_dataset, batch_size=batch
_size, shuffle=False)

'''
STEP 3: CREATE MODEL CLASS
'''
class FeedforwardNeuralNetModel(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim):
        super(FeedforwardNeuralNetModel, self).__init__()
        # Linear function
        self.fc1 = nn.Linear(input_dim, hidden_dim)
        # Non-linearity
        self.relu = nn.ReLU()
        # Linear function (readout)
        self.fc2 = nn.Linear(hidden_dim, output_dim)

    def forward(self, x):
        ### START CODE HERE ###
        # Linear function
        out = self.fc1(x)
        # Non-linearity
        out = self.relu(out)
        # Linear function (readout)
        out = self.fc2(out)
        ### END CODE HERE ###
        return out
'''
STEP 4: INSTANTIATE MODEL CLASS
'''
input_dim = 28*28
hidden_dim = 100
```

```
output_dim = 10

model = FeedforwardNeuralNetModel(input_dim, hidden_dim, output_dim)

'''
STEP 5: INSTANTIATE LOSS CLASS
'''
criterion = nn.CrossEntropyLoss()


'''
STEP 6: INSTANTIATE OPTIMIZER CLASS
'''
learning_rate = 0.1

### START CODE HERE ###
optimizer = torch.optim.SGD(model.parameters(), learning_rate)
### END CODE HERE ###

'''
STEP 7: TRAIN THE MODEL
'''
iter = 0
for epoch in range(num_epochs):
    for i, (images, labels) in enumerate(train_loader):
        # Load images as Variable
        images = images.view(-1, 28*28).requires_grad_()

        # Clear gradients w.r.t. parameters
        optimizer.zero_grad()

        # Forward pass to get output/logits
        outputs = model(images)

        # Calculate Loss: softmax --> cross entropy loss
        loss = criterion(outputs, labels)

        # Getting gradients w.r.t. parameters
        loss.backward()

        # Updating parameters
        optimizer.step()

        iter += 1

        if iter % 500 == 0:
            # Calculate Accuracy
            correct = 0
            total = 0
            # Iterate through test dataset
            for images, labels in test_loader:
                # Load images to a Torch Variable
                images = images.view(-1, 28*28).requires_grad_()

                # Forward pass only to get logits/output
                outputs = model(images)

                # Get predictions from the maximum value
                _, predicted = torch.max(outputs.data, 1)

                # Total number of labels
```

```
                total += labels.size(0)

                # Total correct predictions
                correct += (predicted == labels).sum()

            accuracy = 100 * correct / total

            # Print Loss
            print('Iteration: {}. Loss: {}. Accuracy: {}'.format(iter, loss.item
(), accuracy))
```

```
Iteration: 500. Loss: 0.3460221588611603. Accuracy: 91.5
Iteration: 1000. Loss: 0.21451084315776825. Accuracy: 92.47000122070
312
Iteration: 1500. Loss: 0.1919996440410614. Accuracy: 93.870002746582
03
Iteration: 2000. Loss: 0.17153751850128174. Accuracy: 94.47000122070
312
Iteration: 2500. Loss: 0.11251085251569748. Accuracy: 95.16999816894
531
Iteration: 3000. Loss: 0.1736811101436615. Accuracy: 95.599998474121
1
```

Optimization Algorithm 4: SGD Momentum

- Modification of SGD
  - $v_t = \gamma v_{t-1} + \eta \cdot \nabla J(\theta, x^{i:i+n}, y^{i:i+n})$
  - $\theta = \theta - v_t$
- Characteristics
  - Compute the gradient of the lost function w.r.t. parameters for **n sets of training sample (n input and n label)**, $\nabla J(\theta, x^{i:i+n}, y^{i:i+n})$
  - Use this to add to the previous update vector v_{t-1}
  - Momentum, usually set to $\gamma = 0.9$
  - Parameters updated with update vector, v_t that incorporates previous update vector
    - γv_t increases if gradient same sign/direction as v_{t-1}
      - Gives SGD the push when it is going in the right direction (minimizing loss)
      - Accelerated convergence
    - γv_t decreases if gradient different sign/direction as v_{t-1}
      - Dampens SGD when it is going in a different direction
      - Lower variation in loss minimization

In [4]:

```
import torch
import torch.nn as nn
import torchvision.transforms as transforms
import torchvision.datasets as dsets

# Set seed
torch.manual_seed(0)

'''
STEP 1: LOADING DATASET
'''

train_dataset = dsets.MNIST(root='./data', train=True, transform=transforms.ToTe
nsor(), download=True)
test_dataset = dsets.MNIST(root='./data', train=False, transform=transforms.ToTe
nsor())

'''
STEP 2: MAKING DATASET ITERABLE
'''

batch_size = 100
n_iters = 3000
num_epochs = n_iters / (len(train_dataset) / batch_size)
num_epochs = int(num_epochs)

train_loader = torch.utils.data.DataLoader(dataset=train_dataset, batch_size=bat
ch_size, shuffle=True)
test_loader = torch.utils.data.DataLoader(dataset=test_dataset, batch_size=batch
_size, shuffle=False)

'''
STEP 3: CREATE MODEL CLASS
'''
class FeedforwardNeuralNetModel(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim):
        super(FeedforwardNeuralNetModel, self).__init__()
        # Linear function
        self.fc1 = nn.Linear(input_dim, hidden_dim)
        # Non-linearity
        self.relu = nn.ReLU()
        # Linear function (readout)
        self.fc2 = nn.Linear(hidden_dim, output_dim)

    def forward(self, x):
        ### START CODE HERE ###
        # Linear function
        out = self.fc1(x)
        # Non-linearity
        out = self.relu(out)
        # Linear function (readout)
        out = self.fc2(out)
        ### END CODE HERE ###
        return out
'''
STEP 4: INSTANTIATE MODEL CLASS
'''
input_dim = 28*28
hidden_dim = 100
```

```
output_dim = 10

model = FeedforwardNeuralNetModel(input_dim, hidden_dim, output_dim)

'''
STEP 5: INSTANTIATE LOSS CLASS
'''
criterion = nn.CrossEntropyLoss()


'''
STEP 6: INSTANTIATE OPTIMIZER CLASS
'''
learning_rate = 0.1

### START CODE HERE ###
optimizer = torch.optim.SGD(model.parameters(),lr=learning_rate, momentum=0.9)
### END CODE HERE ###

'''
STEP 7: TRAIN THE MODEL
'''
iter = 0
for epoch in range(num_epochs):
    for i, (images, labels) in enumerate(train_loader):
        # Load images as Variable
        images = images.view(-1, 28*28).requires_grad_()

        # Clear gradients w.r.t. parameters
        optimizer.zero_grad()

        # Forward pass to get output/logits
        outputs = model(images)

        # Calculate Loss: softmax --> cross entropy loss
        loss = criterion(outputs, labels)

        # Getting gradients w.r.t. parameters
        loss.backward()

        # Updating parameters
        optimizer.step()

        iter += 1

        if iter % 500 == 0:
            # Calculate Accuracy
            correct = 0
            total = 0
            # Iterate through test dataset
            for images, labels in test_loader:
                # Load images to a Torch Variable
                images = images.view(-1, 28*28)

                # Forward pass only to get logits/output
                outputs = model(images)

                # Get predictions from the maximum value
                _, predicted = torch.max(outputs.data, 1)

                # Total number of labels
```

```
                total += labels.size(0)

                # Total correct predictions
                correct += (predicted == labels).sum()

            accuracy = 100 * correct / total

            # Print Loss
            print('Iteration: {}. Loss: {}. Accuracy: {}'.format(iter, loss.item
(), accuracy))
```

```
Iteration: 500. Loss: 0.10879121720790863. Accuracy: 96.010002136230
47
Iteration: 1000. Loss: 0.12940317392349243. Accuracy: 96.23999786376
953
Iteration: 1500. Loss: 0.1231849417090416. Accuracy: 96.440002441406
25
Iteration: 2000. Loss: 0.04057228937745094. Accuracy: 97.52999877929
688
Iteration: 2500. Loss: 0.04051990807056427. Accuracy: 97.47000122070
312
Iteration: 3000. Loss: 0.18660661578178406. Accuracy: 97.62999725341
797
```

Optimization Algorithm 4: Adam

- Adaptive Learning Rates
  - $m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$
    - Keeping track of decaying gradient
    - Estimate of the mean of gradients
  - $v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$
    - Keeping track of decaying squared gradient
    - Estimate of the variance of gradients
  - When m_t, v_t initializes as 0, $m_t, v_t \rightarrow 0$ initially when decay rates small, $\beta_1, \beta_2 \rightarrow 1$
    - Need to correct this with:
    - $\hat{m}_t = \frac{m_t}{1-\beta_1}$
    - $\hat{v}_t = \frac{v_t}{1-\beta_2}$
  - $\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t + \epsilon}} \hat{m}_t$
  - Default recommended values
    - $\beta_1 = 0.9$
    - $\beta_2 = 0.999$
    - $\epsilon = 10^{-8}$
- Instead of learning rate $\rightarrow$ equations account for estimates of mean/variance of gradients to determine the next learning rate

In [5]:

```
import torch
import torch.nn as nn
import torchvision.transforms as transforms
import torchvision.datasets as dsets

# Set seed
torch.manual_seed(0)

'''
STEP 1: LOADING DATASET
'''

train_dataset = dsets.MNIST(root='./data', train=True, transform=transforms.ToTe
nsor(), download=True)
test_dataset = dsets.MNIST(root='./data', train=False, transform=transforms.ToTe
nsor())

'''
STEP 2: MAKING DATASET ITERABLE
'''

batch_size = 100
n_iters = 3000
num_epochs = n_iters / (len(train_dataset) / batch_size)
num_epochs = int(num_epochs)

train_loader = torch.utils.data.DataLoader(dataset=train_dataset, batch_size=bat
ch_size, shuffle=True)
test_loader = torch.utils.data.DataLoader(dataset=test_dataset, batch_size=batch
_size, shuffle=False)

'''
STEP 3: CREATE MODEL CLASS
'''
class FeedforwardNeuralNetModel(nn.Module):
    def __init__(self, input_dim, hidden_dim, output_dim):
        super(FeedforwardNeuralNetModel, self).__init__()
        # Linear function
        self.fc1 = nn.Linear(input_dim, hidden_dim)
        # Non-linearity
        self.relu = nn.ReLU()
        # Linear function (readout)
        self.fc2 = nn.Linear(hidden_dim, output_dim)

    def forward(self, x):
        ### START CODE HERE ###
        # Linear function
        out = self.fc1(x)
        # Non-linearity
        out = self.relu(out)
        # Linear function (readout)
        out = self.fc2(out)
        ### END CODE HERE ###
        return out
'''
STEP 4: INSTANTIATE MODEL CLASS
'''
input_dim = 28*28
hidden_dim = 100
```

```
output_dim = 10

model = FeedforwardNeuralNetModel(input_dim, hidden_dim, output_dim)

'''
STEP 5: INSTANTIATE LOSS CLASS
'''
criterion = nn.CrossEntropyLoss()


'''
STEP 6: INSTANTIATE OPTIMIZER CLASS
'''
# learning_rate = 0.001

### START CODE HERE ###
optimizer = torch.optim.Adam(model.parameters(), lr=0.001,betas=(0.9,0.999),eps=
1e-8)
### END CODE HERE ###

'''
STEP 7: TRAIN THE MODEL
'''
iter = 0
for epoch in range(num_epochs):
    for i, (images, labels) in enumerate(train_loader):
        # Load images as Variable
        images = images.view(-1, 28*28).requires_grad_()

        # Clear gradients w.r.t. parameters
        optimizer.zero_grad()

        # Forward pass to get output/logits
        outputs = model(images)

        # Calculate Loss: softmax --> cross entropy loss
        loss = criterion(outputs, labels)

        # Getting gradients w.r.t. parameters
        loss.backward()

        # Updating parameters
        optimizer.step()

        iter += 1

        if iter % 500 == 0:
            # Calculate Accuracy
            correct = 0
            total = 0
            # Iterate through test dataset
            for images, labels in test_loader:
                # Load images to a Torch Variable
                images = images.view(-1, 28*28)

                # Forward pass only to get logits/output
                outputs = model(images)

                # Get predictions from the maximum value
                _, predicted = torch.max(outputs.data, 1)
```

```
                # Total number of labels
                total += labels.size(0)

                # Total correct predictions
                correct += (predicted == labels).sum()

            accuracy = 100 * correct / total

            # Print Loss
            print('Iteration: {}. Loss: {}. Accuracy: {}'.format(iter, loss.item
(), accuracy))
```

```
Iteration: 500. Loss: 0.2257964313030243. Accuracy: 93.3099975585937
5
Iteration: 1000. Loss: 0.17263264954090118. Accuracy: 94.72000122070
312
Iteration: 1500. Loss: 0.136827290058136. Accuracy: 95.5400009155273
4
Iteration: 2000. Loss: 0.07791975140571594. Accuracy: 96.41000366210
938
Iteration: 2500. Loss: 0.07298330217599869. Accuracy: 96.90000152587
89
Iteration: 3000. Loss: 0.1346699446439743. Accuracy: 97.209999084472
66
```

Other Adaptive Algorithms

- Other adaptive algorithms (like Adam, adapting learning rates)
  - Adagrad
  - Adadelta
  - Adamax
  - RMSProp

In [ ]:

```
!pip install nbconvert
!sudo apt-get install texlive-xetex texlive-fonts-recommended texlive-plain-gene
ric
!jupyter nbconvert --to html "/content/drive/MyDrive/Colab Notebooks/filename.ip
ynb"
```