Practice Problems (Exam 2) – CS 6515/4540 (Fall 2025)

Collection of problems for students to practice

1 Short Answer Questions

Give the following prompt to an LLM (like chatGPT, Gemini, CoPilot) to test your basic understanding of class topics such as Farkas' lemma, LPs, LP Duality, Vertex/Set Cover, Facility Location, basic probability theory:

"Can you give me simple short-answer/multiple-choice questions to test my knowledge of ..."

2 LPs

1. Think about how to use LP to solve following questions.

   (a) Minimize the Maximum (Minimax Problem): Minimize the largest value among a set of linear functions f_i:
   $$\min_x \max_i f_i(x)$$

   (b) Maximize the Minimum (Maximin Problem): Maximize the smallest value among a set of linear functions f_i:
   $$\max_x \min_i f_i(x)$$

   (c) Handle Absolute Value Constraints: Manage constraints that contain absolute value expressions:
   $$|c^T x| \leq d$$

   (d) Minimize L1 Distance: Minimize the L1 norm (sum of absolute residuals) in a linear system:
   $$\min_x \|Ax - b\|_1$$

2. Erickson's notes (scroll to bottom for problems)[1] (not limited to) problems 2, 4, 6

3 Approximation Algorithms

Erickson's notes (scroll to bottom for problems)[2] (not limited to) problems 1, 2, 3, 5, 6, 7, 9

4 Randomized Algorithms

1. Let $G = (V, E)$ be an undirected graph with $m = |E|$ edges. Each vertex is independently colored with one of c colors, each color equally likely.

   (a) For a fixed edge $e = \{u, v\}$, compute the probability that both endpoints have the same color.

   (b) Use linearity of expectation to find the expected number of monochromatic edges.

   (c) Show that there exists a c-coloring of G with at most $\frac{m}{c}$ monochromatic edges.

   Hint: The probability that both endpoints share the same color is $1/c$.

[1] Use this embedded hyperlink or check Canvas homepage, Extended Dance Remix section H

[2] If using the link from the canvas homepage, it's extended Dance Remix section J

2. You have black-box access to a univariate polynomial $p(x)$ of degree at most d. You may evaluate p at any integer value of your choice, but you cannot see its coefficients.

   Consider the following randomized test:

   > Pick a random integer r uniformly from $\{1, 2, \ldots, M\}$ and compute $p(r)$. If the value is 0, output "Zero"; otherwise output "Nonzero".

   (a) Explain why this algorithm never makes a mistake when $p(x)$ is identically zero.

   (b) Argue that if p is not identically zero, then the probability that the algorithm incorrectly outputs "Zero" is at most $\dfrac{d}{M}$.

   (c) If you want the error probability to be at most δ, how large should M be in terms of d and δ?

3. You are given three $n \times n$ matrices A, B, C over the integers. You need to verify whether $AB = C$ without computing the full product AB explicitly.

   Consider the following randomized algorithm (*Freivalds', 1977*):

   > Pick a random vector $r \in \{0, 1\}^n$ uniformly at random. Compute $A(Br)$ and Cr. If $A(Br) = Cr$, output "Equal", otherwise output "Not equal".

   (a) Show that if $AB = C$, the algorithm always outputs "Equal".

   (b) Show that if $AB \neq C$, then Pr[algorithm outputs "Equal"] $\leq \frac{1}{2}$.

   (c) How many repetitions are needed to make the error probability at most δ?

   (d) What is the running time of one trial, and how does it compare to explicitly computing AB?

   Hint: For (b), note that $AB \neq C$ means there exists a nonzero vector $v = (AB - C)r$; argue that for a random $r \in \{0, 1\}^n$, $\Pr[v = 0] \leq 1/2$.

4. (Advance Problem on LP-Based Randomized Rounding for Approx.) Let $G = (V, E)$ be an undirected graph and let $S = \{s_1, \ldots, s_k\} \subseteq V$ be a set of k terminals. A *multiway cut* is a set of edges $F \subseteq E$ whose removal disconnects every pair of distinct terminals, i.e., in $(V, E \setminus F)$ there is no path between s_i and s_j for any $i \neq j$.

   Let $P_{u,v}$ denote the set of all u–v paths in G. Consider the LP relaxation:

$$
\begin{aligned}
\min \quad & \sum_{e \in E} x_e & & \\
\text{s.t.} \quad & \sum_{e \in p} x_e \;\geq\; 1 & & \forall\, i \neq j,\ \forall\, p \in P_{s_i, s_j}, \\
& x_e \;\geq\; 0 & & \forall\, e \in E.
\end{aligned}
$$

   (a) Show that the LP optimum value is at most the size of a minimum multiway cut.

   (b) Consider the following randomized rounding algorithm:

      i. Solve the LP and let $x_e^\star$ be an optimal solution.

      ii. Choose $r \sim \mathrm{Unif}[0, \frac{1}{2}]$.

      iii. For each terminal $s \in S$ (in an arbitrary order), let $B_r(s) = \{v \in V : \; d_{x^\star}(s, v) \leq r\}$, where $d_{x^\star}$ is the shortest-path metric with edge lengths $x_e^\star$. Add all edges in the cut $\big(B_r(s),\, V \setminus B_r(s)\big)$ to F.

      Prove:

      (a) The algorithm always outputs a feasible multiway cut.

      (b) For any edge $e \in E$, $\Pr[e \in F] \leq 2x_e^\star$.