Linear Programming
Problem Set 3 – CS 6515/4540 (Fall 2025)

Answers to problem set 3, question 10.

10 LPs

1. Let $x_A \geq 0$ and $x_B \geq 0$ denote the numbers of bundle A and bundle B Alice buys respectively. Bundle A: cost \$3, contains 1 orange and 2 bananas. Bundle B: cost \$2, contains 3 oranges and 1 banana.

   We want to minimize total cost while getting at least 8 oranges and at least 20 bananas. The LP is

$$\begin{aligned} \text{minimize} \quad & 3x_A + 2x_B \\ \text{subject to} \quad & x_A + 3x_B \geq 8 \qquad \text{(oranges)} \\ & 2x_A + x_B \geq 20 \qquad \text{(bananas)} \\ & x_A \geq 0, \quad x_B \geq 0. \end{aligned}$$

   The first two inequalities enforce the required minimum counts of oranges and bananas respectively. The objective equals the total dollars spent which we have to minimise. Non-negativity ensures that Alice cannot buy negative bundles.

2. Alice can buy $x_A = 10$ units of the first bundle and $x_B = 0$ units of the second bundle. Doing so, she ends up with 10 oranges and 20 bananas which satisfies the constraints and she ends up paying \$30. Therefore, Alice at most needs to spend \$30 unless we can find a more optimal solution.

3. We prove the lower bound $3x_A + 2x_B \geq 30$ for every feasible (x_A, x_B) by taking a nonnegative linear combination of the constraints.

   Multiply the "banana" constraint $2x_A + x_B \geq 20$ by 1.5. This yields

$$1.5(2x_A + x_B) \; = \; 3x_A + 1.5x_B \; \geq \; 1.5 \cdot 20 = 30.$$

   Since $x_B \geq 0$ we have $2x_B \geq 1.5x_B$. Therefore

$$3x_A + 2x_B \; \geq \; 3x_A + 1.5x_B \; \geq \; 30,$$

   for every feasible (x_A, x_B). Thus every feasible solution has cost at least 30.

   Combining this with the possible solution we got in (2), we conclude that the minimum possible cost is exactly 30 and the solution $(x_A = 10, x_B = 0$ is an optimal solution.