MTL 390 (Statistical Methods)
Major Examination Assignment 2 Report

Name: Bhumika Chopra Entry Number: 2018MT10748

1. Testing of Hypothesis

   It is known that Human beings and Chimpanzees evolved from the same species of ancient 4 limbed vertebrates. We'd like to compare and test the IQ of Chimpanzees alongside humans. Consider that IQ of a Chimpanzee follows a normal distribution, $X_i \sim N(\mu, 10^2)$. IQ of 10 Chimpanzees are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. Assume that the average IQ of human beings is 100, construct suitable null and alternate hypothesis to test this claim.

   Solution We construct the null and alternate hypothesis as,

   $$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100.$$

   Let $\bar{X}$ be a test statistic and $R_1 = (-\infty, 90]$ to be a rejection region. Let's compute the probability of making Type I error based on this testing procedure. Under the assumption H_0 is true,

   $$X_i \sim N(100, 10^2).$$

   Under this condition, $\bar{X} \sim N(100, 10)$ and

   $$\alpha = P(\bar{X} \leq 90) = 0.0007827011$$

   By using this test procedure, it is highly unlikely to make Type I error. Let's see what happens when we change the rejection region.
   When $R_2 = (-\infty, 95]$, $\alpha = P(\bar{X} \leq 95) = 0.05692315$

   When $R_3 = (-\infty, 99]$, $\alpha = P(\bar{X} \leq 99) = 0.3759148$

   Depending on the level of significance that we set ($\alpha = 0.05$) we will decide whether to reject the null hypothesis or not. Let the level of significance is 0.05 then we will reject the null hypothesis for rejection regions R_2, R_3.

2. Testing of Hypothesis

   The data given indicates the number of flying bomb hits recorded in each of 576 regions in Hiroshima during World War II.

Number of hits per region	0	1	2	3	4	5	>= 6
Number of regions	229	211	93	35	7	1	0

   Theorists later on claimed that the nuclear launcher made by the US was far ahead of its time and that it could actually be used to aim accurately. Others claimed that the above theory was false and the hits were actually randomly distributed over the area following a Poisson distribution. Check the validity of the claims by performing an appropriate test.

Solution We first calculate the MLE of the Poisson parameter, λ. The likelihood function for a sample of size n is

$$L(\lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

so that the log-likelihood is

$$l(\lambda) = \sum_{i=1}^{n} x_i log\lambda - n\lambda - \sum_{i=1}^{n} \log x_i!$$

Calculating the derivative and equating to 0 gives the MLE of λ,

$$\frac{\delta l}{\delta \lambda} = \frac{\sum_{i=1}^{n} x_i}{\lambda} - n = 0$$
$$\widehat{\lambda} = \overline{X} = \frac{535}{576} = 0.928.$$

where $\overline{X}$ is the sample mean. Using the Poisson probability mass function with $\lambda = 0.929$ we therefore obtain

i	0	1	2	3	4	5	>= 6
$P_i(\widehat{\lambda})$	0.3949	0.3669	0.1704	0.0528	0.0123	0.0023	0.0004

and therefore

$$D = 2\sum_{i=1}^{k} n_i \log(\frac{n_i}{nP_i(\widehat{\lambda})}) = 1.4995.$$

This is tested against χ^2 distribution with 5 degrees of freedom. This gives $P(D <= 1.4995) = 0.913$. This is a very high probability and therefore we have proved that the hits actually follow a Poisson distribution.

3. Analysis of correlation and regression

4. Analysis of correlation and regression

5. Time Series Analysis

6. Time Series Analysis