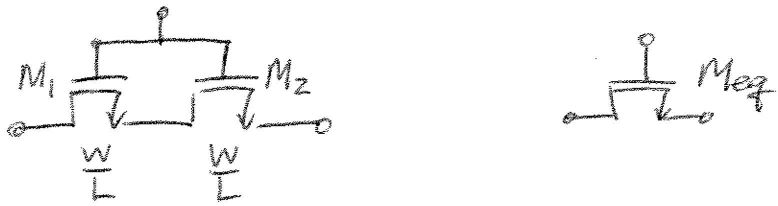
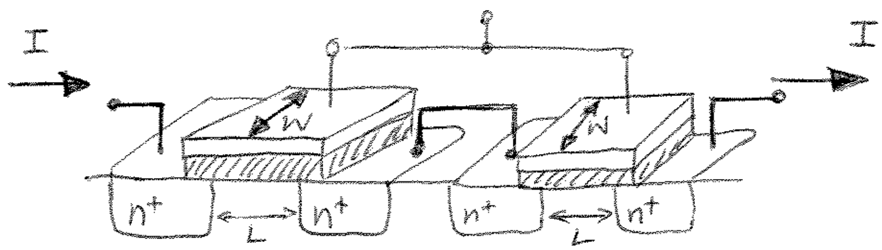
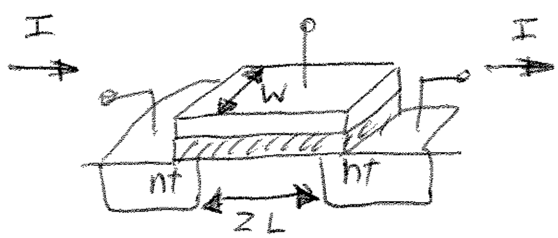
1.



Intuitively, this is similar to having twice of the original channel length:



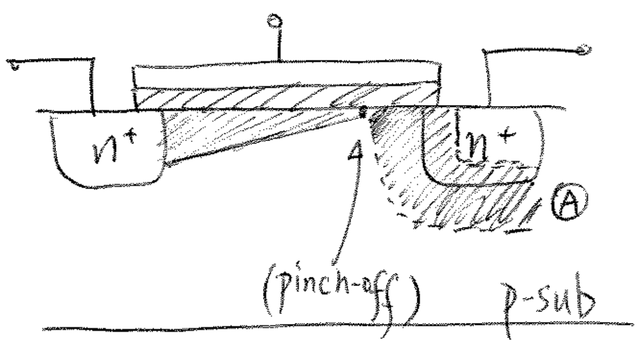
Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:



This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that v is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:



The shaded region, Ⓐ, represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which $\neq 0$.

Pinch-off implies that the depletion region created no longer has free carriers. The depletion still sweeps all electrons from inversion channel to drain.

3. Given: $C_{ox} = 10\ fF/\mu m^2$ $\quad W = 5\mu m$ $\quad L = 0.1\mu m$

$V_{GS} - V_{TH} = 1V$ $\quad V_{DS} = 0$

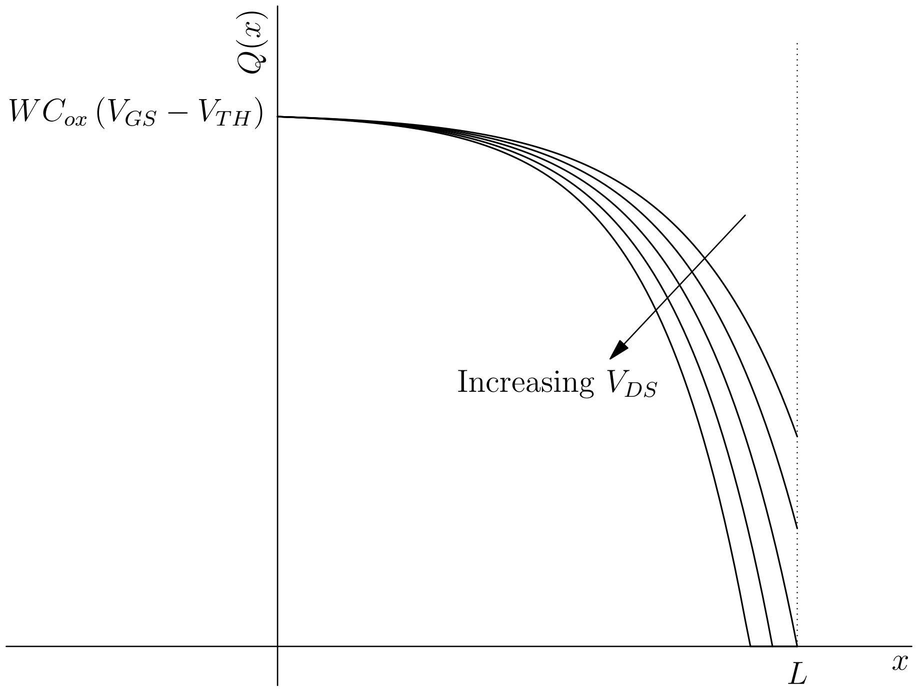
Find: total charge stored in channel, Q_{tot}

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

$$= (5\mu m)(10\ fF/\mu m^2)(1V)(0.1\mu m) = 5\ fC$$

6.4 (a)

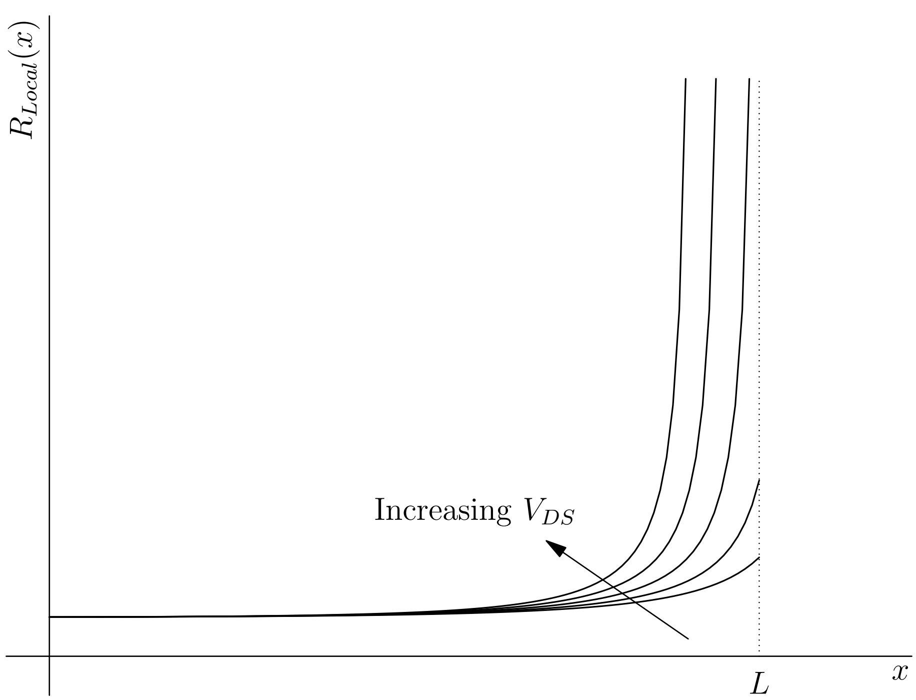
$$\begin{aligned} Q(x) &= WC_{ox}\left(V_{GS} - V(x) - V_{TH}\right) \\ &= WC_{ox}\left(V_{GS} - V_{TH}\right) - WC_{ox}V(x) \end{aligned}$$



The curve that intersects the axis at $x = L$ (i.e., the curve for which the channel begins to pinch off) corresponds to $V_{DS} = V_{GS} - V_{TH}$.

(b)

$$R_{Local}(x) \propto \frac{1}{\mu Q(x)}$$



Note that R_{Local} diverges at $x = L$ when $V_{DS} = V_{GS} - V_{TH}$.

5. $I_D = WC_{ox}\left[V_{GS} - V(x) - V_{TH}\right]\mu_n \dfrac{dV(x)}{dx}$

Define : $A = \dfrac{I_D}{WC_{ox}\mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \dfrac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

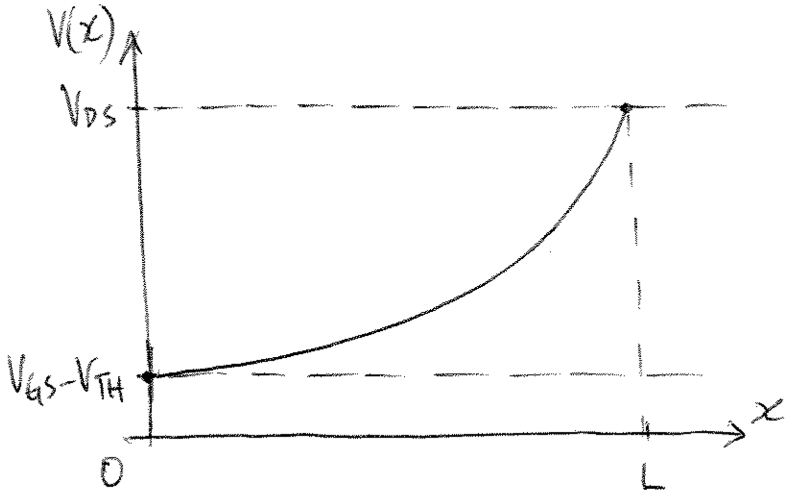
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4\cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\tfrac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2\cdot\frac{I_D}{WC_{ox}\mu_n(V_{GS} - V_{TH})^2}\right]x}\right\}$$

We know that $0 \le V(x) \le V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is > 0. Therefore, we take V_- as the solution.

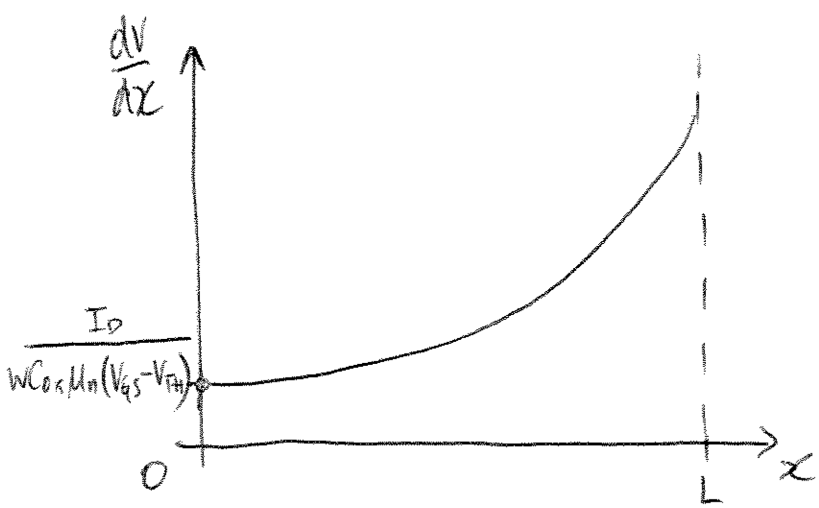
i.e. $V(x) = (V_{GS}-V_{TH})\left\{1-\sqrt{1-\left[\dfrac{2I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]x}\right\}$



$\because$ $I_D \propto W$

$\Rightarrow V(x)$ is independent of W.

$$\frac{dV}{dx} = \frac{I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})}\cdot\left[1-\frac{2I_D\cdot x}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS}-V_{TH}$ & V_{DS}, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given : NMOS $\quad I_D = 1mA \quad V_{GS}-V_{TH} = 0.6V$

$I_D = 1.6mA \quad V_{GS}-V_{TH} = 0.8V$

(triode region) $\quad \mu_n C_{ox} = 200 \frac{\mu A}{V^2}$

Find V_{DS} & W/L.

$$1mA = \mu_n C_{ox} \frac{W}{L} \left[(0.6) V_{DS} - V_{DS}^2/2\right] \quad \text{——} \quad ①$$

$$1.6mA = \mu_n C_{ox} \frac{W}{L} \left[(0.8) V_{DS} - V_{DS}^2/2\right] \quad \text{——} \quad ②$$

$$② \div ① : \quad 1.6 = \frac{0.8 V_{DS} - V_{DS}^2/2}{0.6 V_{DS} - V_{DS}^2/2} = \frac{1.6 - V_{DS}}{1.2 - V_{DS}}$$

$$\Rightarrow V_{DS} = \frac{1.6(0.2)}{0.6} \approx 0.533 \ V$$

$$\Rightarrow \quad \frac{W}{L} = \frac{I_D}{\mu_n C_{ox}\left[(V_{GS}-V_{TH})V_{DS} - V_{DS}^2/2\right]}$$

$$= \frac{1mA}{200 \frac{\mu A}{V^2} \left[(0.6V)(0.533V) - (0.533V)^2/2\right]}$$

$$\approx 28.$$

8. $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} [2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2]$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \cdot 2V_{DS} = \mu C_{ox} \frac{W}{L} V_{DS}$$

$$g_m \big|_{V_{DS}=0} = 0.$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no I_D ($\because V_{DS}=0$), it does not matter if there is an incremental change in V_{GS} (i.e. ∂V_{gs}). Since varying V_{GS} gives no change in I_D, $g_m|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $\quad \frac{W}{L} = 20$ $\quad \mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

$V_{TH} = 0.4\ V$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)\ (1.8 - 0.4)V} = 179.\ \Omega$$

10. $$500 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$$

$$400 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$$\Rightarrow \quad 500(1 - V_{TH}) \overset{?}{=} 400(1.5 - V_{TH})$$
$$500(0.6) \neq 400(1.1)$$

$\therefore$ This is not possible.

11. $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \left[\frac{\partial}{\partial V_{DS}}\left\{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]\right\}\right]^{-1}$$

$$= \left[\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) - \mu C_{ox}\frac{W}{L}V_{DS}\right]^{-1}$$

$$= \frac{1}{\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}-V_{DS})}$$

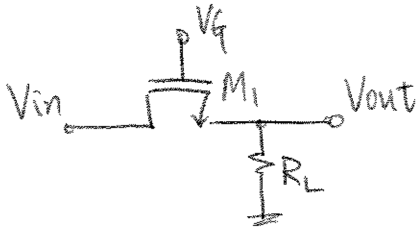
12. When MOS operates as a resistor,

$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{W L C_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,
1) use minimum channel length, and
2) maximize overdrive voltage.

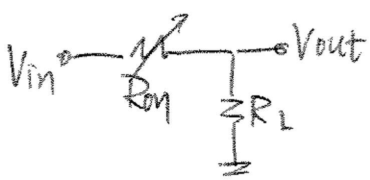
13.



Given $V_{in} \approx 0$
$V_G = 1.8\ V$
$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate M_1 as a linear resistance controlled by V_G. Therefore, the equivalent circuit becomes a resistive divider:



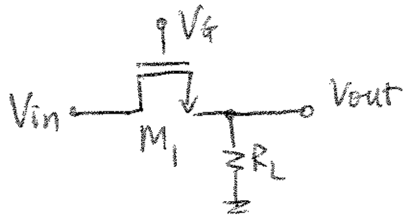
$$V_{out} = 0.95\, V_{in} = \frac{R_L}{R_{on} + R_L} V_{in}$$

$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox} (V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200 \frac{\mu A}{V^2} (1.8 - 0.4)(5.3\Omega)}$$

$$= 674.$$

14.



$V_0 \sim$ few mV.

(a) $V_{in} = V_0 \cos\omega t$ $\qquad$ $V_{out} = 0.95(V_0 \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON}+R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON}+R_L} = 0.95 V_0$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_0}{1-0.95V_0}\right)} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_0/(1-0.95V_0)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\, V_{in} = 0.95\,(V_0 \cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

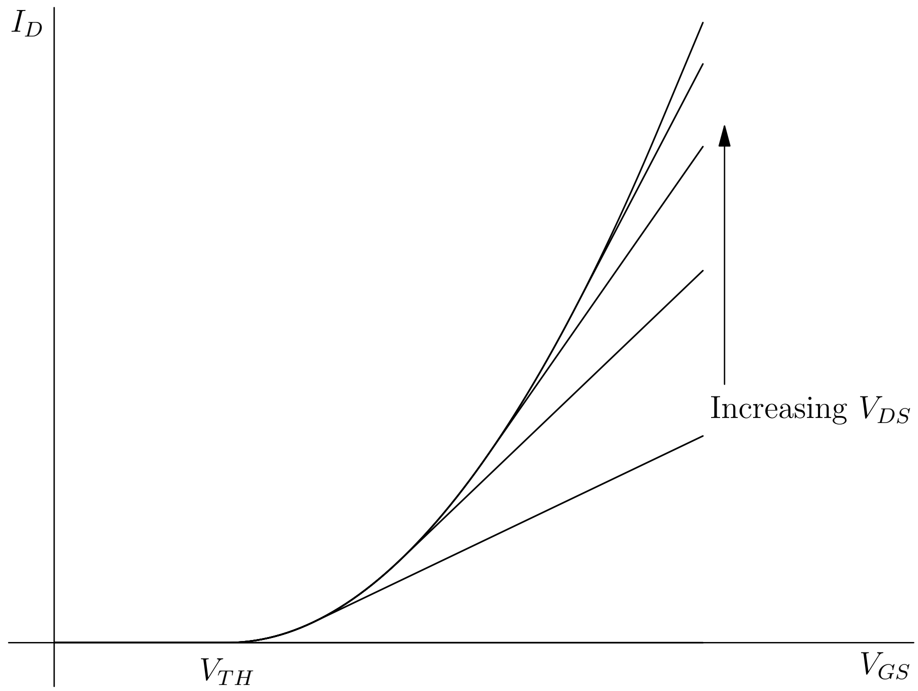
($\because$ V_0 is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the R_{ON} varies with changing sinewave. With a DC bias voltage, R_{ON} becomes more stable (independent of V_O).

6.15



Initially, when V_{GS} is small, the transistor is in cutoff and no current flows. Once V_{GS} increases beyond V_{TH}, the curves start following the square-law characteristic as the transistor enters saturation. However, once V_{GS} increases past $V_{DS} + V_{TH}$ (i.e., when $V_{DS} < V_{GS} - V_{TH}$), the transistor goes into triode and the curves become linear. As we increase V_{DS}, the transistor stays in saturation up to larger values of V_{GS}, as expected.

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) I_D cannot be increased further by increasing V_{DS}. Since this curve must be continuous, the peak I_D must originate from the peak of the parabola.

6.17

$$
\begin{aligned}
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha},\ \alpha < 2 \\
g_m &\triangleq \frac{\partial I_D}{\partial V_{GS}} \\
&= \frac{\alpha}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha-1} \\
&= \boxed{\frac{\alpha I_D}{V_{GS}-V_{TH}}}
\end{aligned}
$$

18. $I_D = W C_{ox} (V_{GS} - V_{TH}) \upsilon_{SAT}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} \upsilon_{SAT}$$

19. (a) OFF $\because V_{GS} = 0$

(b) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER : MOSFET is symmetric)

(e) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because V_{GS} = 0$

(g) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

20. (a) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(b) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &

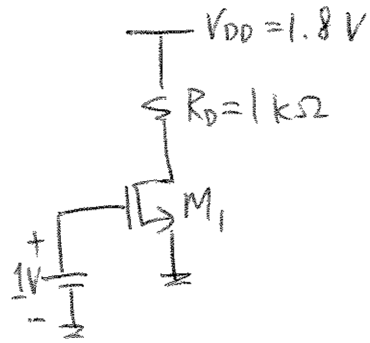
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

6.21 Since they're being used as current sources, assume M_1 and M_2 are in saturation for this problem. To find the maximum allowable value of λ, we should evaluate λ when $0.99 I_{D2} = I_{D1}$ and $1.01 I_{D2} = I_{D1}$, i.e., at the limits of the allowable values for the currents. However, note that for any valid λ (remember, λ should be non-negative), we know that $I_{D2} > I_{D1}$ (since $V_{DS2} > V_{DS1}$), so the case where $1.01 I_{D2} = I_{D1}$ (which implies $I_{D2} < I_{D1}$) will produce an invalid value for λ (you can check this yourself). Thus, we need only consider the case when $0.99 I_{D2} = I_{D1}$.

$$
\begin{aligned}
0.99 I_{D2} &= 0.99 \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_B - V_{TH}\right)^2 \left(1 + \lambda V_{DS2}\right) \\
&= I_{D1} \\
&= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_B - V_{TH}\right)^2 \left(1 + \lambda V_{DS1}\right) \\
0.99 \left(1 + \lambda V_{DS2}\right) &= 1 + \lambda V_{DS1} \\
\lambda &= \boxed{0.02\ \mathrm{V}^{-1}}
\end{aligned}
$$

22.



$\lambda = 0$, $V_{TH} = 0.4\,V$
$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

M_1 sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$$\Rightarrow V_{DS,\,edge} = (1 - 0.4)\,V = 0.6\,V$$

By KCL, $I_{D_1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$$\therefore I_{D_1} = 1.2\,mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$\Rightarrow \frac{W}{L} = \frac{2 I_{D_1}}{\mu_n C_{ox}(V_{GS} - V_{TH})^2} = \frac{2(1.2mA)}{\left(200\frac{\mu A}{V^2}\right)(1 - 0.4)^2 V^2}$$

$$\cong 33.$$

23. If gate oxide thickness, t_{ox}, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow$ $\mu_n C_{ox}$ is also halved

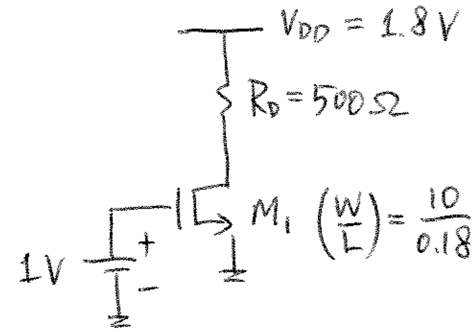
$\Rightarrow$ I_{D_1} is halved $\Rightarrow$ V_{DS} increases

$\Rightarrow$ M_1 stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2\,mA}{2} = 0.6\,mA$$

$$\Rightarrow V_{DS} = (1.8\,V) - (0.6\,mA)(1\,k\Omega) = 1.2\,V$$

24.



$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow V_{DS} \geq 1 - 0.4 = 0.6\,V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$

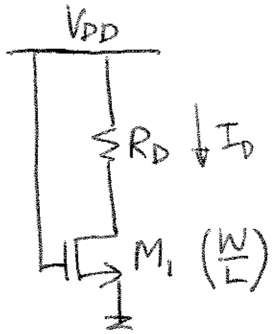
$$= \frac{1}{2}\left(200\,\frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\,mA$$

By KCL, $\frac{V_{DD} - V_{DS}}{R_D} = 2\,mA$

$\therefore V_{DD} = (2mA)(500\Omega) + 0.6\,V = 1.6\,V$

Minimum $V_{DD} = 1.6\,V$

25.



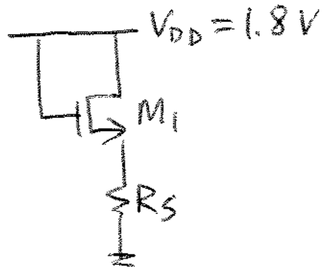
$\lambda = 0$

When M_1 operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2}_{I_D}$$

26.



$\lambda = 0$

Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

Since $V_{DS} = V_{GS}$ for M_1, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{DD} - I_1 R_S - V_{TH}\right)^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2 I_1}{\mu_n C_{ox} \left(V_{DD} - I_1 R_S - V_{TH}\right)^2}$$

5.27

$$\begin{aligned}
V_{DD} - I_D R_D = V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} \\
\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}} = \left(V_{DD} - V_{TH} - I_D R_D\right)^2 \\
I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\left(V_{DD} - V_{TH}\right)^2 - 2I_D R_D\left(V_{DD} - V_{TH}\right) + I_D^2 R_D^2\right]
\end{aligned}$$

We can rearrange this to the standard quadratic form as follows:

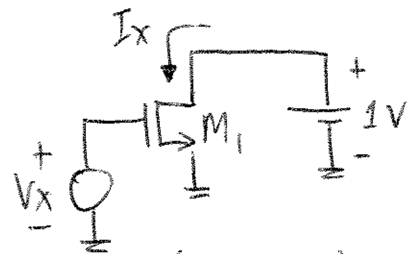
$$\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D^2\right)I_D^2 - \left(\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1\right)I_D + \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - V_{TH}\right)^2 = 0$$

Applying the quadratic formula, we have:

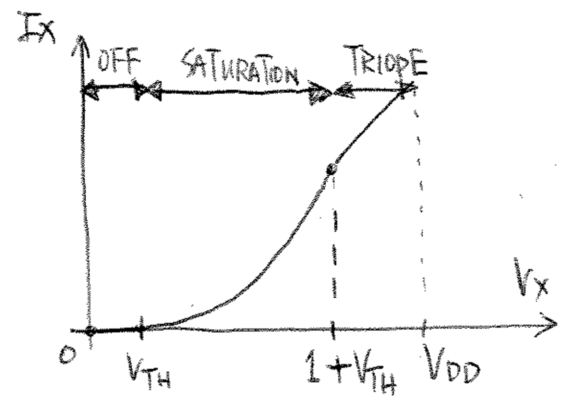
$$\begin{aligned}
I_D &= \frac{\left(\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1\right) \pm \sqrt{\left(\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1\right)^2 - 4\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right)\right)^2}}{2\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D^2\right)} \\
&= \frac{\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1 \pm \sqrt{\left(\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1\right)^2 - \left(\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right)\right)^2}}{\mu_n C_{ox}\frac{W}{L}R_D^2} \\
&= \boxed{\frac{\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right) + 1 \pm \sqrt{1 + 2\mu_n C_{ox}\frac{W}{L}R_D\left(V_{DD} - V_{TH}\right)}}{\mu_n C_{ox}\frac{W}{L}R_D^2}}
\end{aligned}$$

Note that mathematically, there are two possible solutions for I_D. However, since M_1 is diode-connected, we know it will either be in saturation or cutoff. Thus, we must reject the value of I_D that does not match these conditions (for example, a negative value of I_D would not match cutoff or saturation, so it would be rejected in favor of a positive value).

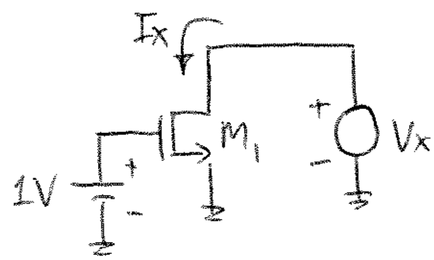
28. (a)

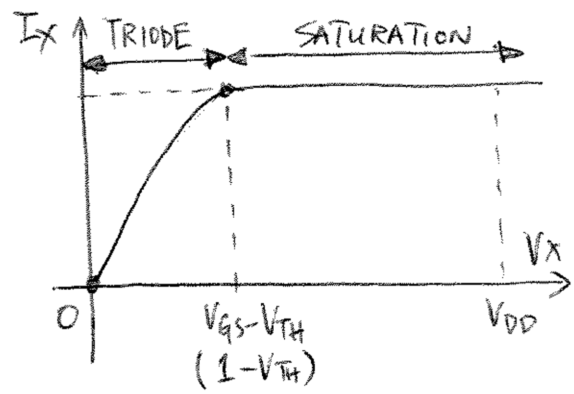


($V_{TH} = 0.4$)

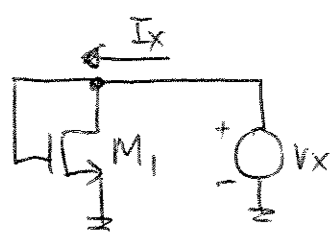


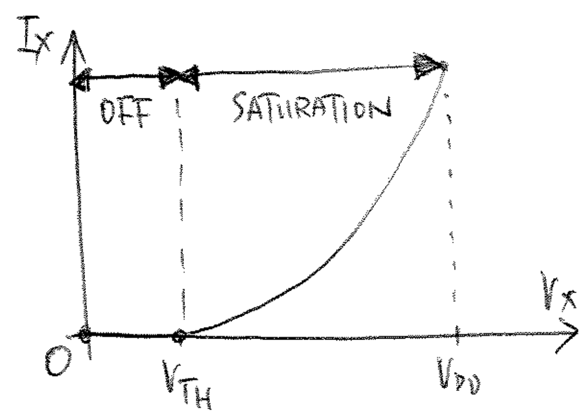
(b)



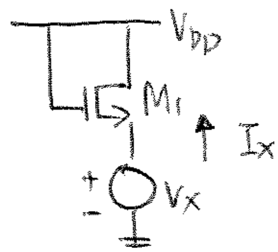


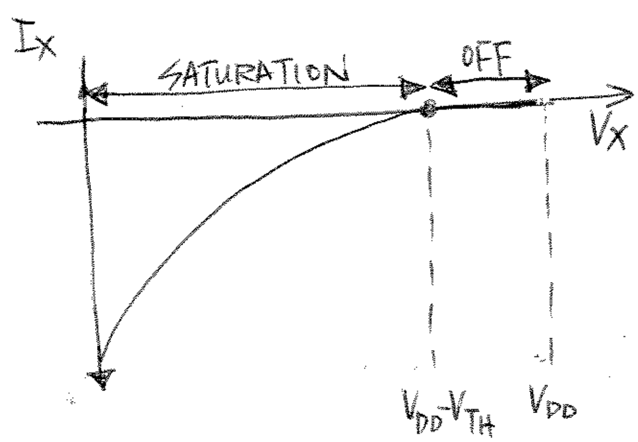
(c)



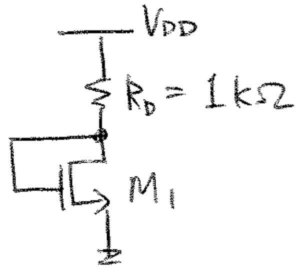


(d)





29.



$\left(\frac{W}{L}\right) = \frac{10}{0.18}$, $\lambda = 0.1\ V^{-1}$

Find I_{D1}

Since M_1 is diode-connected, it operates in saturation.

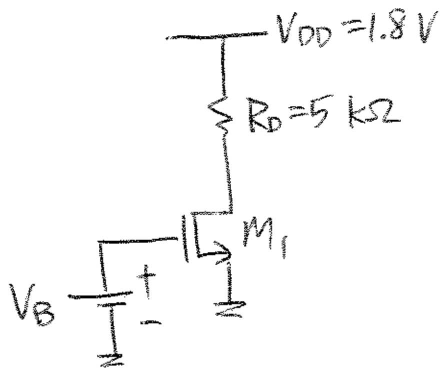
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_G - V_{TH})^2 (1 + \lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding V_G.

Using any method gives $V_G \approx 0.807\ V$

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1\ mA$

30.



$\frac{W}{L} = \frac{20}{0.18}$, $\lambda = 0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1} = \frac{V_{DD} - (V_B - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda(V_B - V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\,mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50\ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5\ mA$, find W/L.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(1/50\ \frac{1}{\Omega}\right)^2}{2\left(200\ \frac{\mu A}{V^2}\right)(0.5mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5V$, find W/L.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{GS} - V_{TH})} = \frac{\left(1/50\ \frac{1}{\Omega}\right)}{\left(200\ \frac{\mu A}{V^2}\right)(0.5\ V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5V$, find I_D.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(1/50\ \frac{1}{\Omega}\right)(0.5V)}{2} \approx 5mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$ (I_D constant)

Doubling (W/L) implies a $\sqrt{2}$ times increase in g_m: $g_{m\,NEW} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$ (I_D constant)

Doubling $(V_{GS} - V_{TH})$ decreases g_m by half:

$g_{m\,NEW} = \frac{2 I_D}{2(V_{GS} - V_{TH})} = \frac{1}{2} g_m$

(c) $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$ (W/L constant)

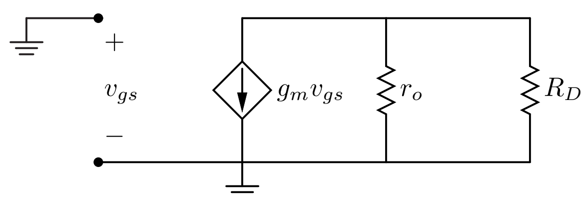
Doubling I_D increases g_m by $\sqrt{2}$ times.

(d) $g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$ ($V_{GS} - V_{TH}$ constant)

Doubling I_D increases g_m by 2 times.

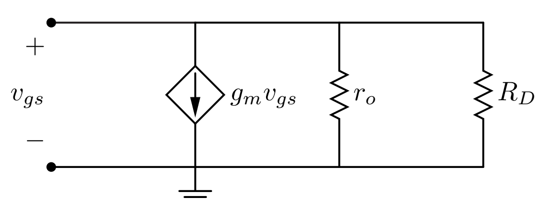
6.33 (a) Assume M_1 is operating in saturation.

$$V_{GS} = 1\text{ V}$$
$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right) R_D$$
$$V_{DS} = 1.35\text{ V} > V_{GS} - V_{TH}\text{, which verifies our assumption}$$
$$I_D = 4.54\text{ mA}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{13.333\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{2.203\text{ k}\Omega}$$



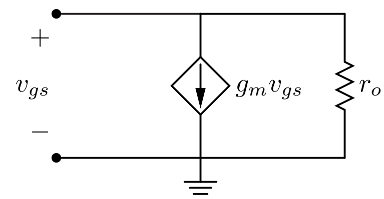
(b) Since M_1 is diode-connected, we know it is operating in saturation.

$$V_{GS} = V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right) R_D$$
$$V_{GS} = V_{DS} = 0.546\text{ V}$$
$$I_D = 251\ \mu\text{A}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{3.251\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{39.881\text{ k}\Omega}$$



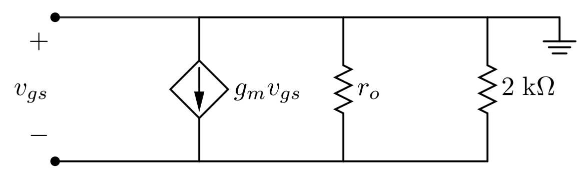
(c) Since M_1 is diode-connected, we know it is operating in saturation.

$$I_D = 1\text{ mA}$$
$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \boxed{6.667\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{10\text{ k}\Omega}$$



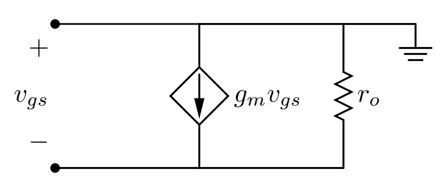
(d) Since M_1 is diode-connected, we know it is operating in saturation.

$$\begin{aligned}
V_{GS} &= V_{DS} \\
V_{DD} - V_{GS} = I_D(2\text{ k}\Omega) &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right)(2\text{ k}\Omega) \\
V_{GS} &= V_{DS} = 0.623\text{ V} \\
I_D &= 588\ \mu\text{A} \\
g_m &= \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{4.961\text{ mS}} \\
r_o &= \frac{1}{\lambda I_D} = \boxed{16.996\text{ k}\Omega}
\end{aligned}$$



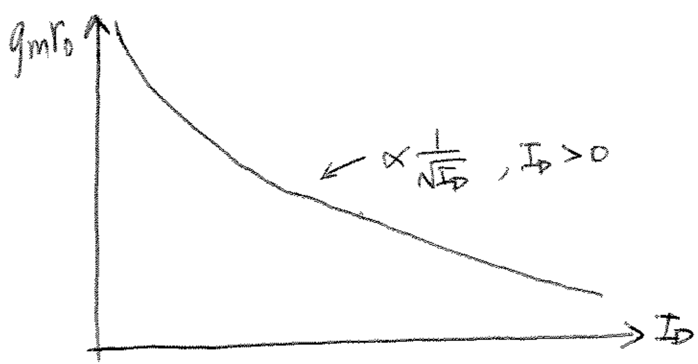
(e) Since M_1 is diode-connected, we know it is operating in saturation.

$$\begin{aligned}
I_D &= 0.5\text{ mA} \\
g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \boxed{4.714\text{ mS}} \\
r_o &= \frac{1}{\lambda I_D} = \boxed{20\text{ k}\Omega}
\end{aligned}$$



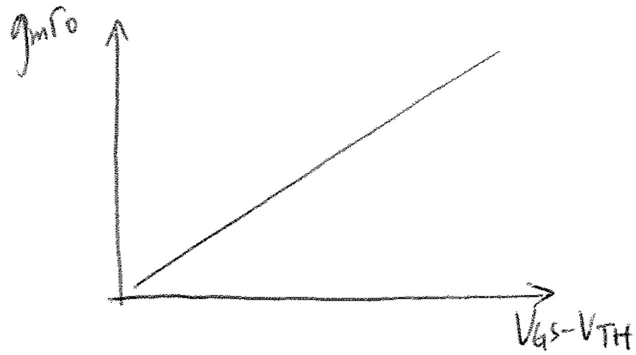
34. $g_m = \sqrt{2\mu C_{ox}\frac{W}{L} I_D}$ $\quad r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}(W/L) I_D}}{\lambda I_D} = \frac{1}{\lambda}\sqrt{\frac{2\mu C_{ox}(W/L)}{I_D}}$$

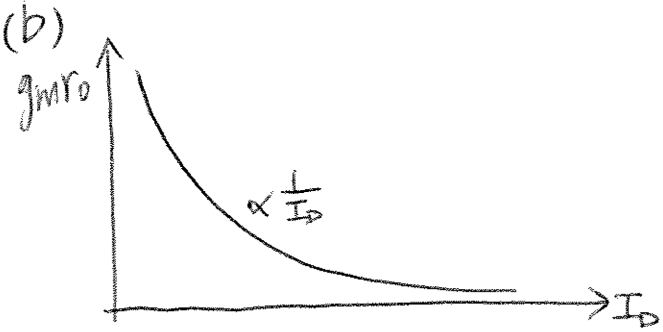


35 (a) $g_m = \mu C_{ox} \frac{W}{L}(V_{GS}-V_{TH})$ $r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox}(W/L)(V_{GS}-V_{TH})}{\lambda I_D}$$



(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$, $V_{DS} = 1.5\ V$

determine W/L if $I_D = 0.5\ mA$.

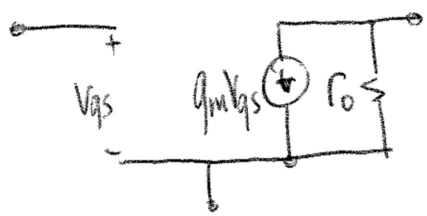
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5\ mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\ k\Omega} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\ k\Omega}\right)^2 \frac{1}{2 \mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\ k\Omega}\right)^2 \frac{1}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 5.$$

37.



Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

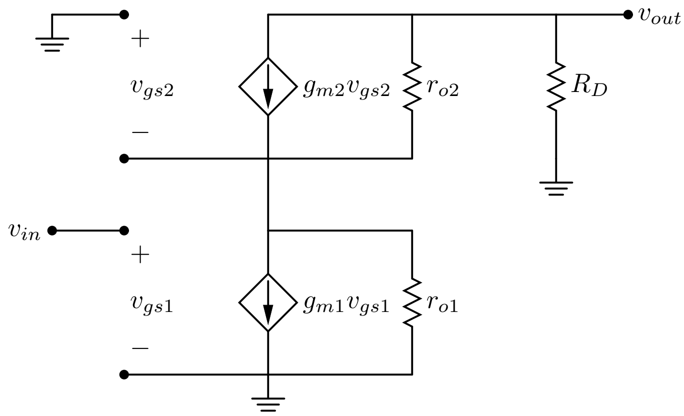
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2V^{-1})(0.5\,mA)$$
$$= 0.002\ \tfrac{1}{\Omega}$$

$$\Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

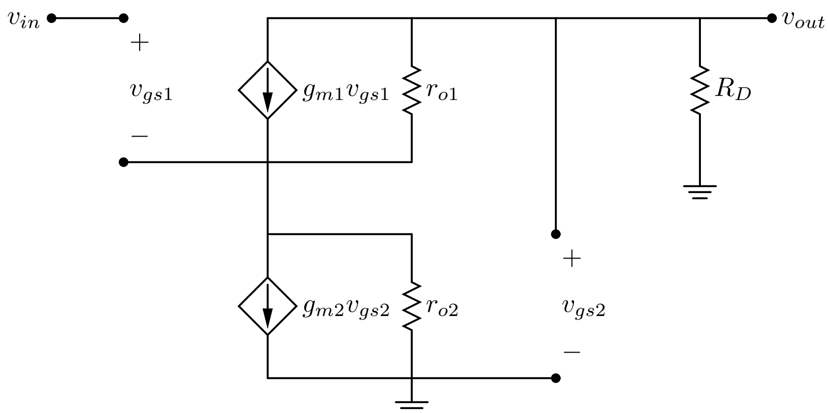
$$\therefore \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{(0.0002\ \tfrac{1}{\Omega})^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\,mA)} = 20$$

6.38 (a)



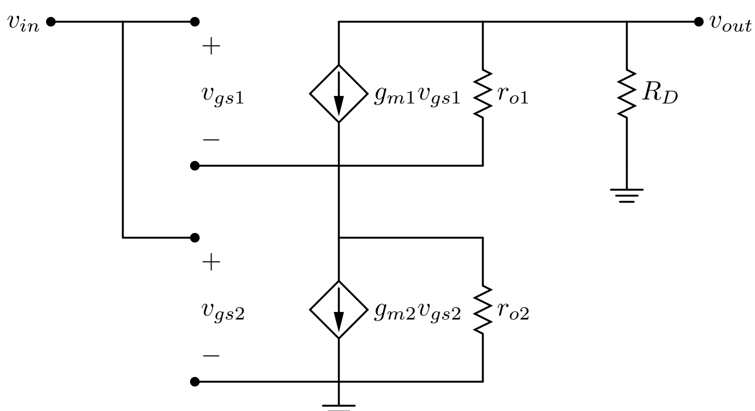
+
v_{gs2}
−
$g_{m2}v_{gs2}$
r_{o2}
R_D
v_{out}
v_{in}
+
v_{gs1}
−
$g_{m1}v_{gs1}$
r_{o1}

(b)



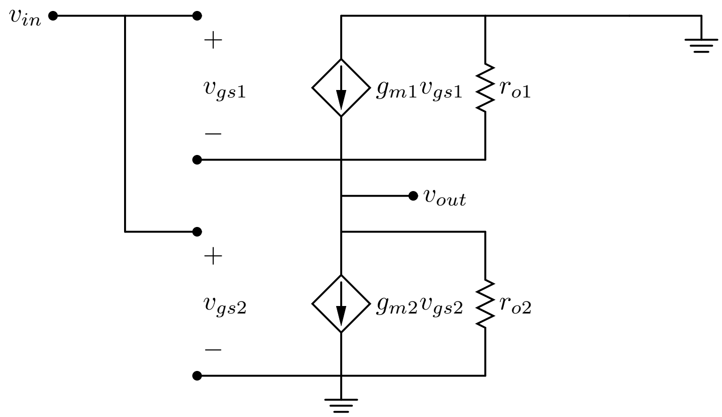
v_{in}
+
v_{gs1}
−
$g_{m1}v_{gs1}$
r_{o1}
R_D
v_{out}
$g_{m2}v_{gs2}$
r_{o2}
+
v_{gs2}
−

(c)



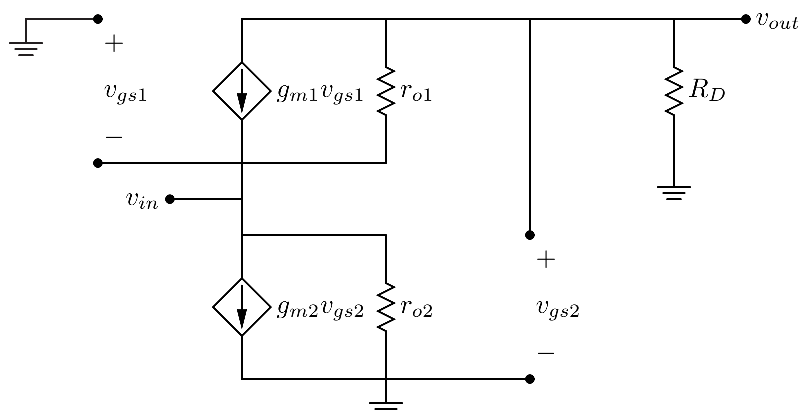
v_{in}
+
v_{gs1}
−
$g_{m1}v_{gs1}$
r_{o1}
R_D
v_{out}
+
v_{gs2}
−
$g_{m2}v_{gs2}$
r_{o2}

(d)



v_{in}
$+$
v_{gs1}
$-$
$g_{m1}v_{gs1}$
r_{o1}
v_{out}
$+$
v_{gs2}
$-$
$g_{m2}v_{gs2}$
r_{o2}

(e)



$+$
v_{gs1}
$-$
$g_{m1}v_{gs1}$
r_{o1}
v_{out}
R_D
v_{in}
$+$
v_{gs2}
$-$
$g_{m2}v_{gs2}$
r_{o2}

39. (a) OFF $\because |V_{SG}| = 0$

(b) OFF $\because |V_{SG}| < |V_{TH}| = 0.4V$

(c) SATURATION $\because |V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because V_{SG} < |V_{TH}|$

40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

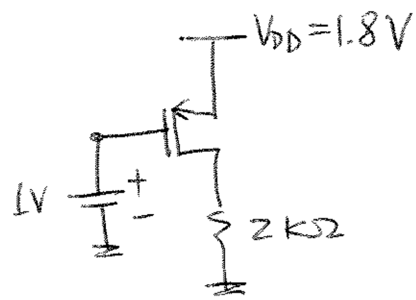
(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} < V_{SG} - |V_{TH}|$

41.



$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$
$\Rightarrow V_D = 1.4\ V.$

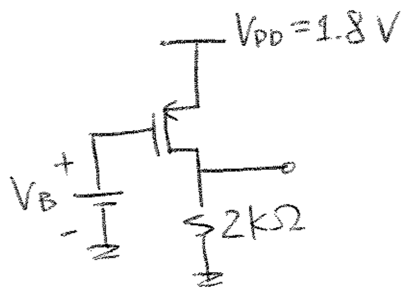
By KCL, $I_{D_1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2 = \frac{V_D}{2k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2k\Omega}\cdot\frac{2}{\mu_p C_{ox}(V_{SG} - |V_{TH}|)^2}$$

$$= \frac{1.4\ V}{2k\Omega}\cdot\frac{2}{100\ \frac{\mu A}{V^2}\ (0.8\ V - 0.4V)^2} \approx 87.5$$

42.



$\lambda = 0$

When $V_B = 1V$, $W/L = 87.5$

When $V_B = 0.8V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6\,mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2k\Omega} = \frac{(1.8 - 0.18)V}{2k\Omega} \approx 0.81\ mA$$

6.43 (a) Assume M_1 is operating in triode (since $|V_{GS}| = 1.8$ V is large).

$$\begin{aligned}
|V_{GS}| &= \boxed{1.8 \text{ V}} \\
V_{DD} - |V_{DS}| = |I_D| \, (500 \ \Omega) &= \frac{1}{2}\mu_p C_{ox} \frac{W}{L} \left[2 \left(|V_{GS}| - |V_{TH}| \right) |V_{DS}| - |V_{DS}|^2 \right] (500 \ \Omega) \\
|V_{DS}| &= \boxed{0.418 \text{ V}} < |V_{GS}| - |V_{TH}| \text{, which verifies our assumption} \\
|I_D| &= \boxed{2.764 \text{ mA}}
\end{aligned}$$

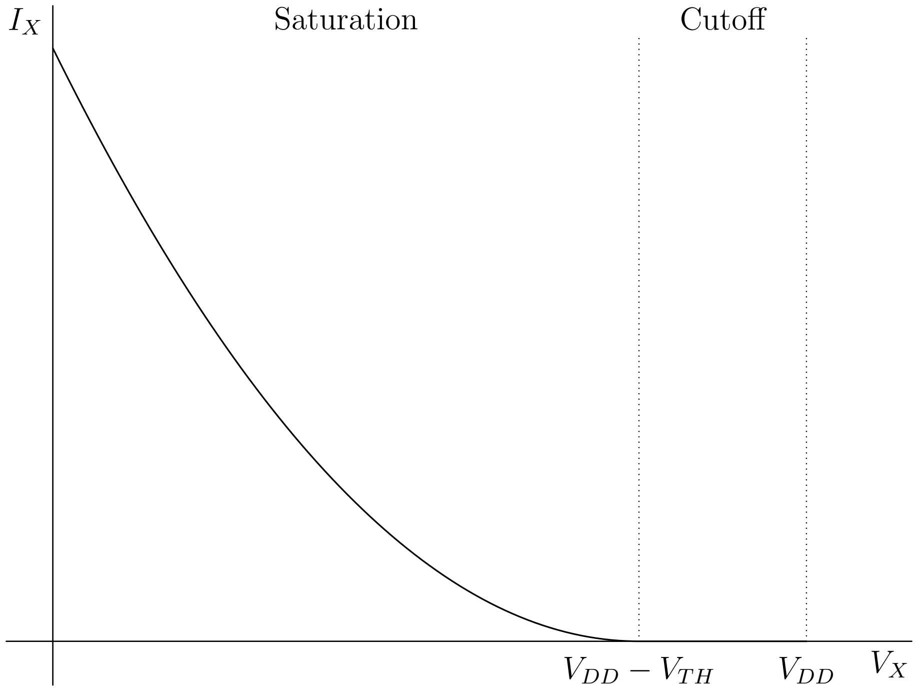
(b) Since M_1 is diode-connected, we know it is operating in saturation.

$$\begin{aligned}
|V_{GS}| &= |V_{DS}| \\
V_{DD} - |V_{GS}| = |I_D| (1 \text{ k}\Omega) &= \frac{1}{2}\mu_p C_{ox} \frac{W}{L} \left(|V_{GS}| - |V_{TH}| \right)^2 (1 \text{ k}\Omega) \\
|V_{GS}| = |V_{DS}| &= \boxed{0.952 \text{ V}} \\
|I_D| &= \boxed{848 \ \mu\text{A}}
\end{aligned}$$

(c) Since M_1 is diode-connected, we know it is operating in saturation.

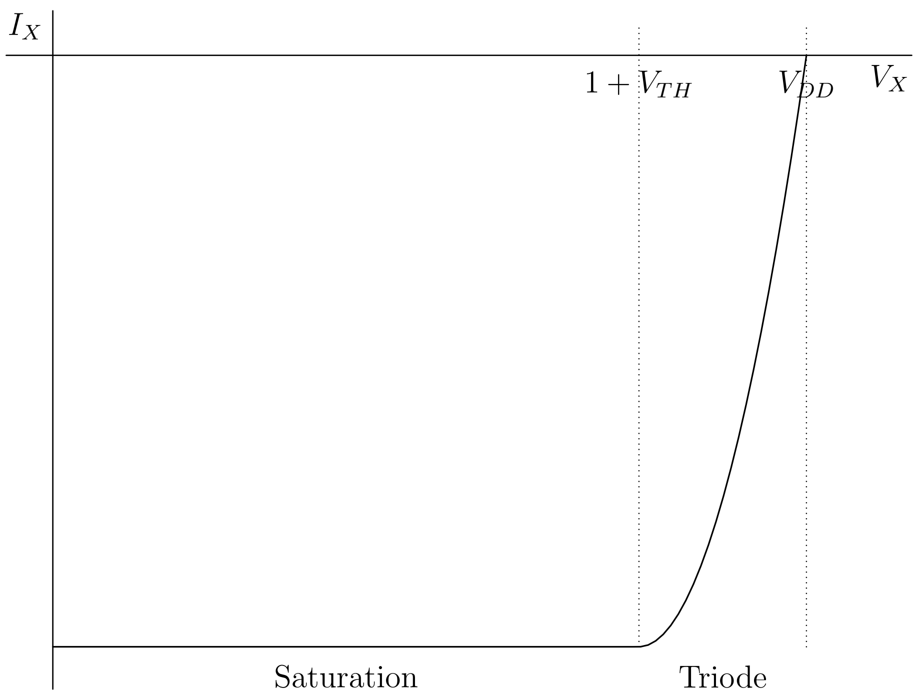
$$\begin{aligned}
|V_{GS}| &= |V_{DS}| \\
|V_{GS}| &= V_{DD} - |I_D| (1 \text{ k}\Omega) = V_{DD} - |I_D| (1 \text{ k}\Omega) = \frac{1}{2}\mu_p C_{ox} \frac{W}{L} \left(|V_{GS}| - |V_{TH}| \right)^2 (1 \text{ k}\Omega) \\
|V_{GS}| &= |V_{GS}| = \boxed{0.952 \text{ V}} \\
|I_D| &= \boxed{848 \ \mu\text{A}}
\end{aligned}$$

6.44 (a)



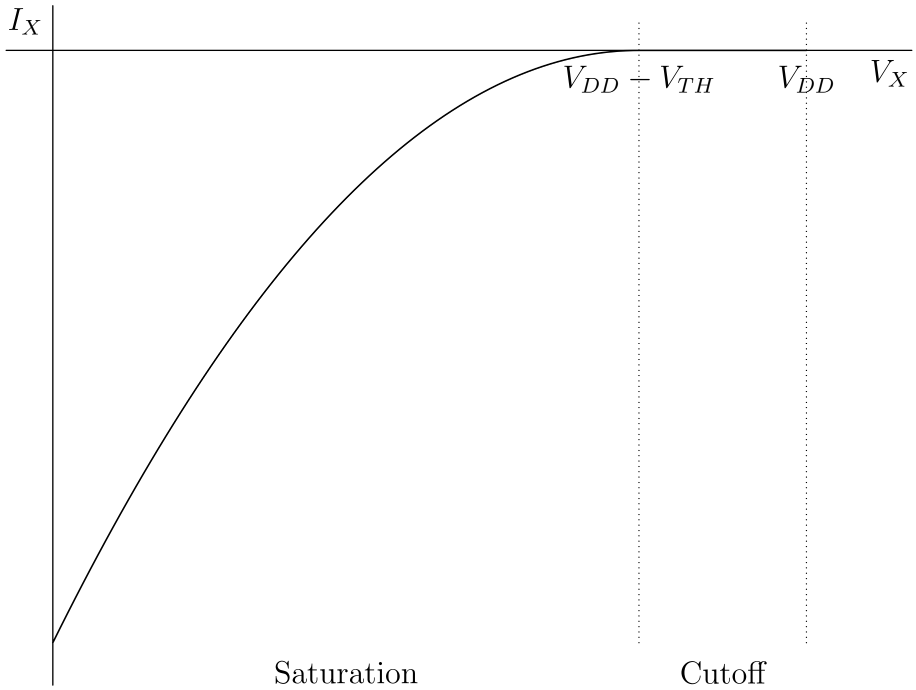
M_1 goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4$ V.

(b)



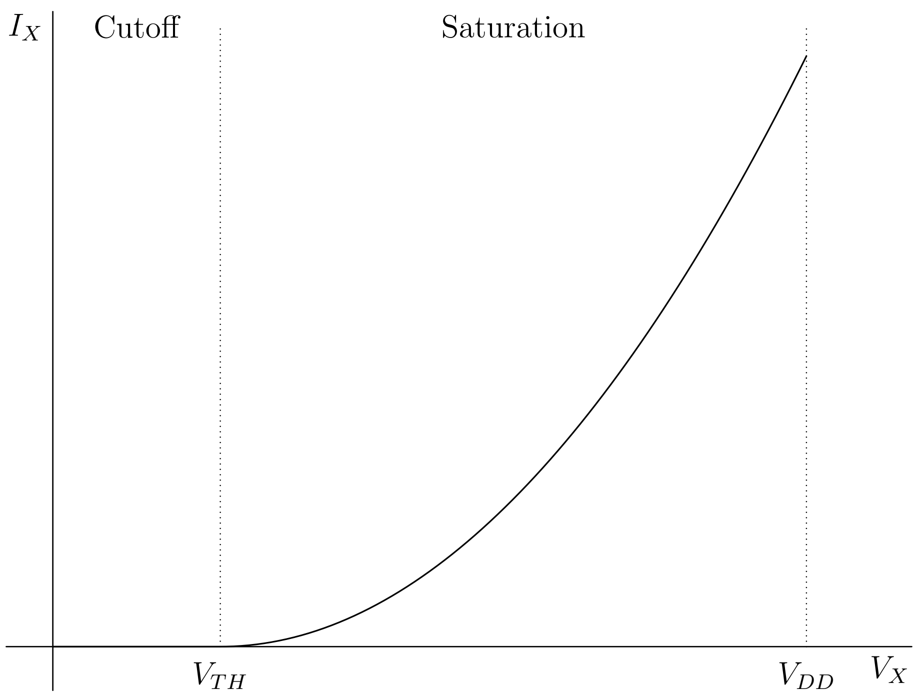
M_1 goes from saturation to triode when $V_X = 1 + V_{TH} = 1.4$ V.

(c)



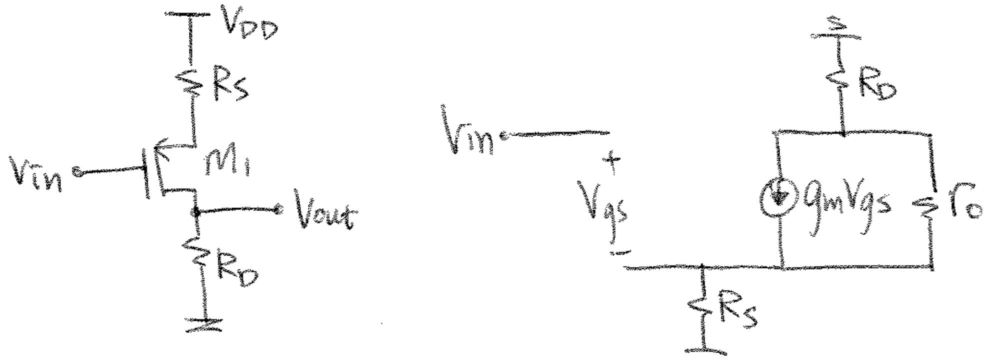
M_1 goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4$ V.

(d)



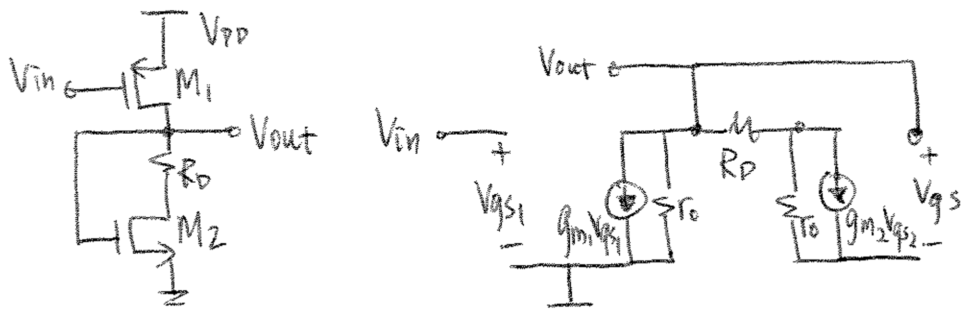
M_1 goes from cutoff to saturation when $V_X = V_{TH} = 0.4$ V.

45. (a)



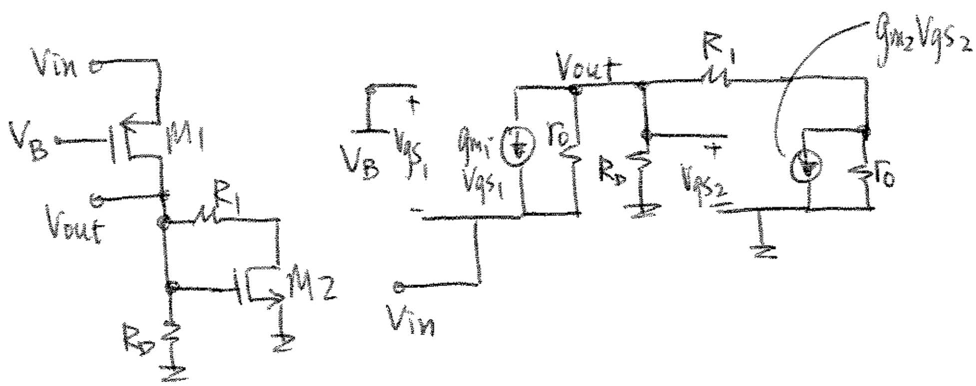
VDD
Rs
M1
Vin
Vout
RD
Vin
+
Vgs
−
gmVgs
ro
RD
Rs

(b)



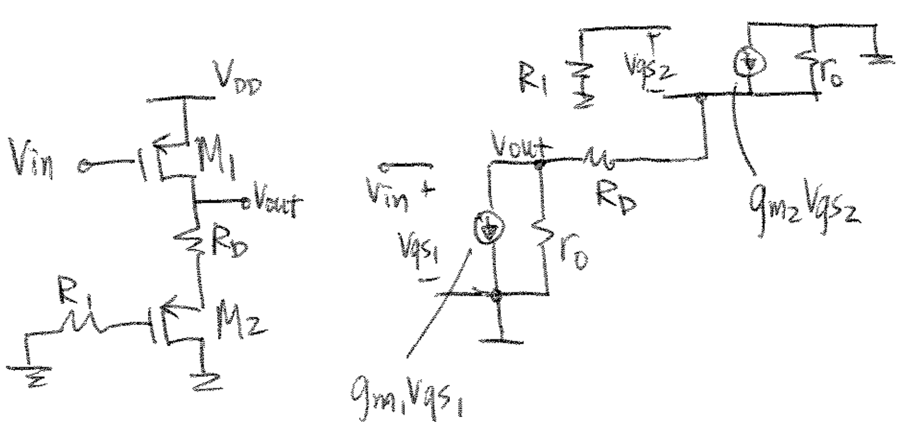
VDD
Vin
M1
Vout
RD
M2
Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

(c)



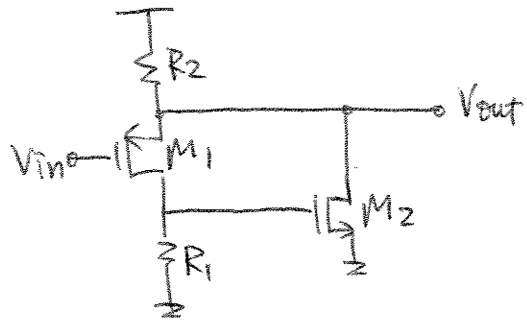
Vin
VB
M1
Vout
R1
M2
RD
Vout
R1
gm2Vgs2
VB
+
Vgs1
−
gm1Vgs1
ro
RD
+
Vgs2
−
ro
Vin

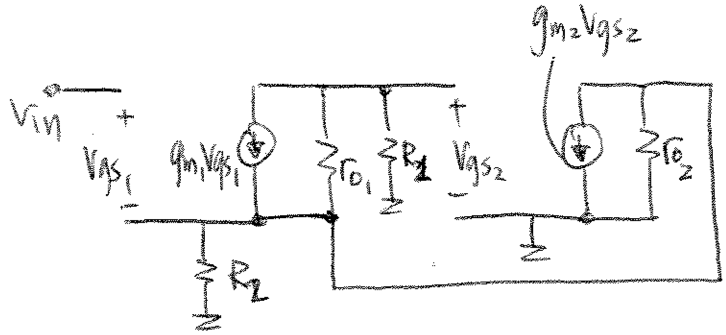
(d)



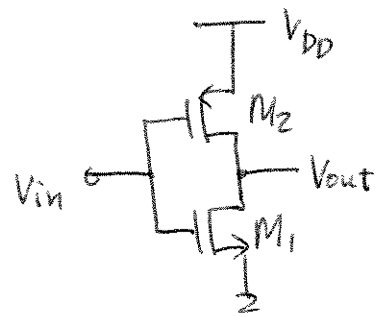
VDD
Vin
M1
Vout
RD
R1
M2
R1
+
Vgs2
−
ro
Vout
RD
gm2Vgs2
Vin
+
Vgs1
−
ro
gm1Vgs1

(e)


R2
Vout
Vin
M1
M2
R1

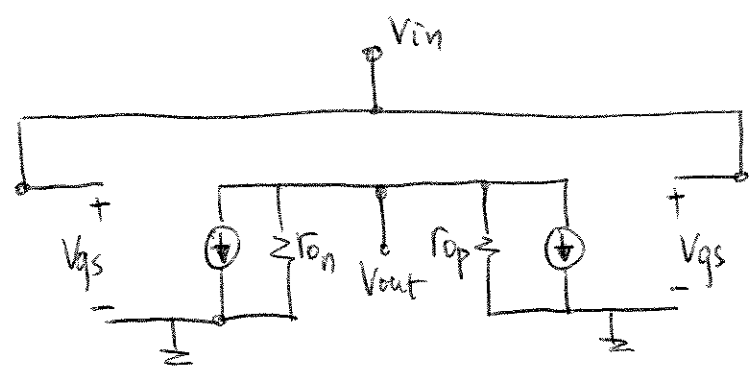

gm2Vgs2
Vin
+
Vgs1
-
gm1Vgs1
ro1
R1
+
Vgs2
-
ro2
R2

46.



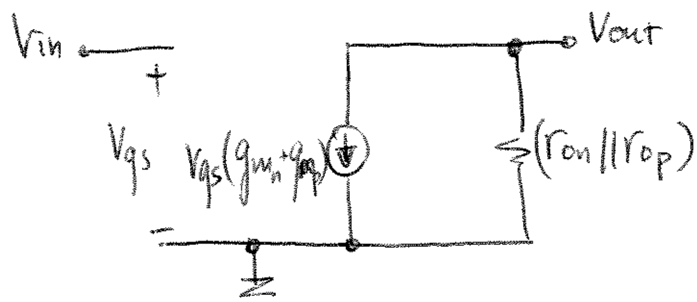
Assume λ_n & λ_p.

(a)



They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both M_1 & M_2 are in saturation, we can combine r_o's & g_m's :



$$\therefore \quad \frac{V_{out}}{V_{in}} = -(g_{mn} + g_{mp})(r_{on} \parallel r_{op})$$