Signals And Systems (UE17EC204)

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Introduction to the course

Course Objectives

- To familiarize students with different types of Signals and Systems, typically encountered in Communication Engineering.
- To expose students to different transformation techniques to apply and analyze different real-life periodic and aperiodic signals to systems (typically Linear and Time-Invariant systems).
- To provide valuable insights of complex systems/signals analyzed through different techniques learnt.
- To provide sufficient understanding of different types of signals and systems and transformation techniques for future courses in Signal Processing, Image Processing, etc.

Course Outcomes

On successful completion of this course, students would be able to:

1. Understand and represent signals and perform basic operations on signals.

2. Determine Fourier representations for Continuous-time and Discrete-time signals.

3. Understand LTI systems.

4. Analyse and design signals and systems using transformation techniques.

5. Use the unilateral Z transform.

6. Apply Fourier representation properties and Z transform properties to solve problems.

Textbooks and Reference Books

Text book:

V. Oppenheim and A. S. Willsky with S. H. Nawab , *Signals and Systems*, 2nd Edition, Pearson Education, 2013.

Reference books:

1. **B. P. Lathi**, *Signal Processing and Linear Systems*, 1st Indian Edition, Oxford University Press, 2006.

2. **Simon Haykin and Barry Van Veen**, *Signals and Systems*, 2nd Edition, Wiley India, 2004.

3. **Ashok Ambardar**, *Analog and Digital Signal Processing*, 2nd Edition, Thomas Learning, 1999.

Unit I
Introduction to Signals and Systems

(Chapter 1 of prescribed Textbook – Sections 1.1 – 1.6)

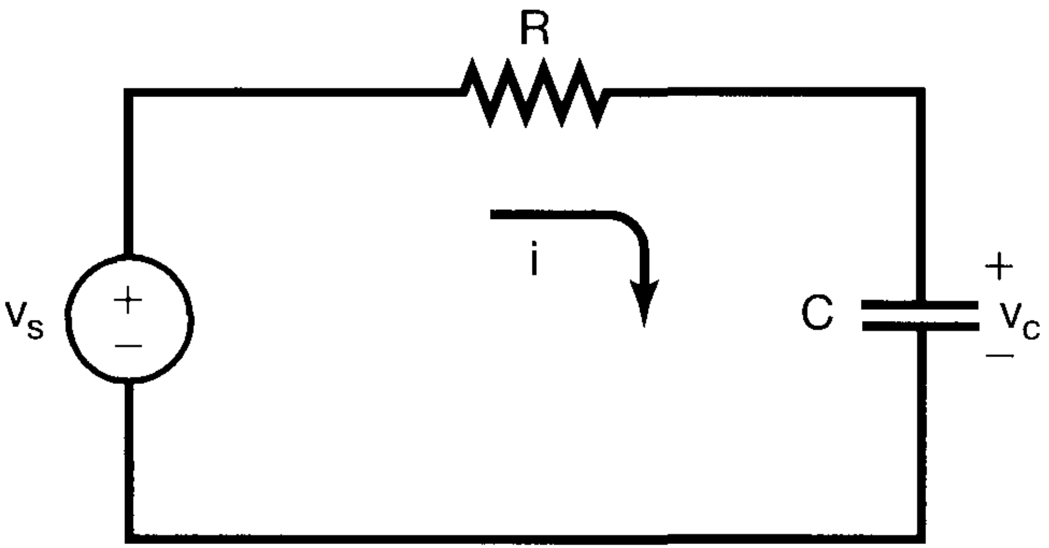
Continuous-time and Discrete-time signals

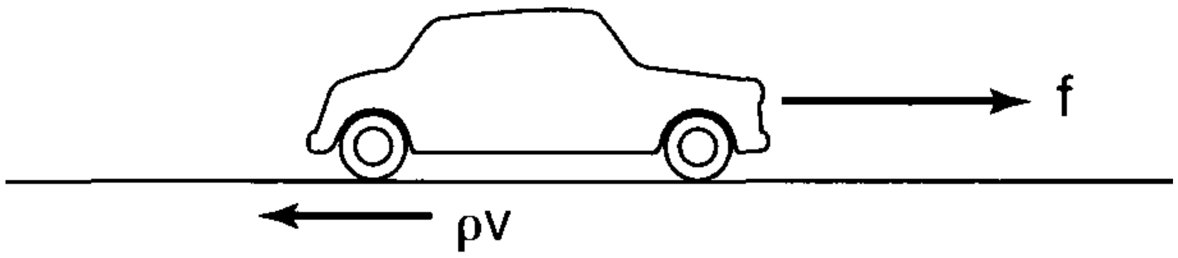
Examples and Mathematical Representation

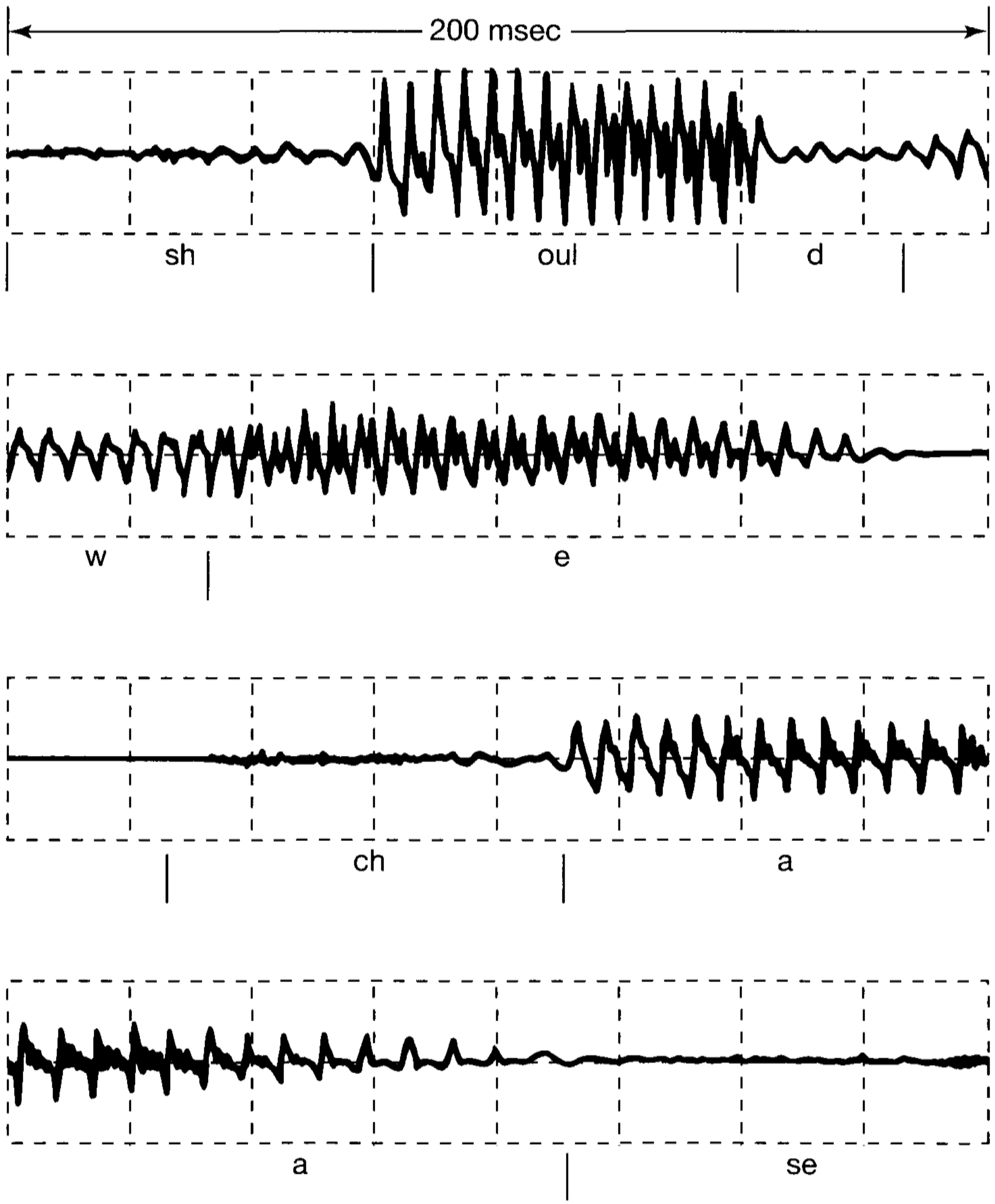
- Signals are **information bearing quantities** describing a wide variety of phenomena (Examples: speech signals, image signals, electrical signals, biomedical signals, etc.).
- Mathematically represented as **functions of one or more independent variables** (time is the independent variable referred to here).
- **Two** basic types of signals:

1. Continuous-time
2. Discrete-time

Examples of Signals







Continuous-time and discrete-time signals

Continuous-time signals (CT)

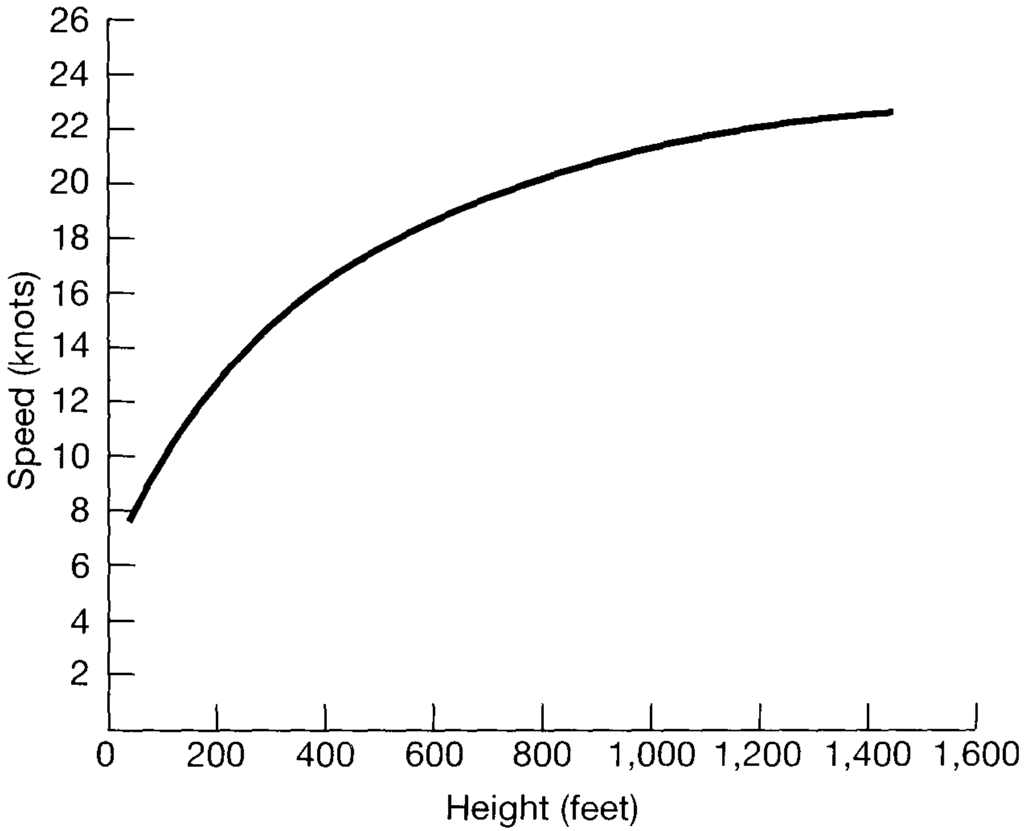
- Defined for a **continuum of values** of a **continuous independent variable**.
- **Examples:** speech signal as fn. of time, atmospheric pressure as fn. of altitude.
- Independent variable enclosed in parentheses (.)

Discrete-time signals (DT)

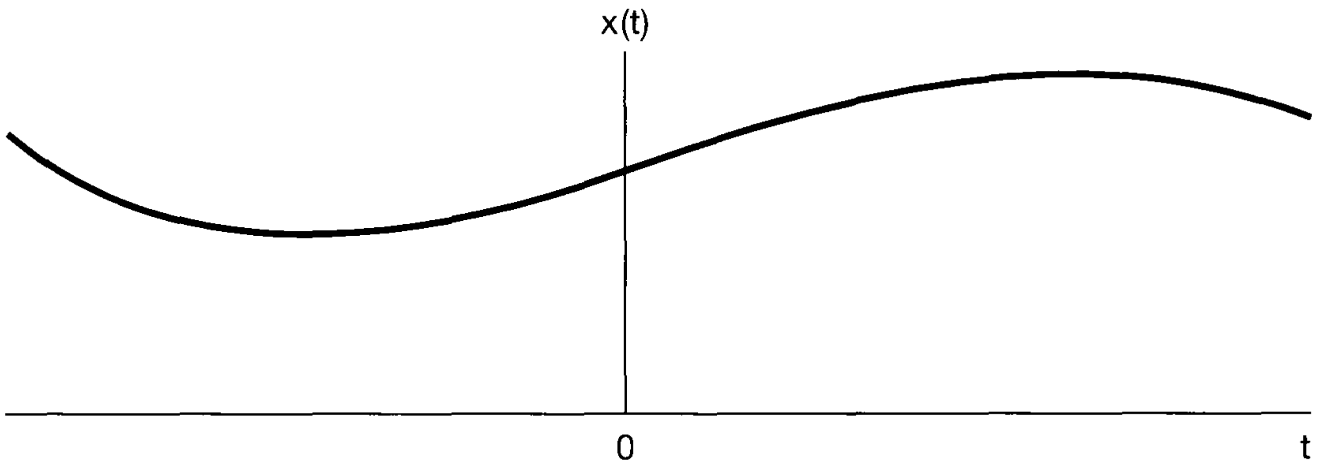
- Defined for **discrete times**, independent variable takes on **discrete set of values**.
- **Examples:** demographic study plots (average budget against family size, crime rate against total population)
- Independent variable enclosed in brackets [.]

Note: defined only for **integer** values

Continuous-time signals

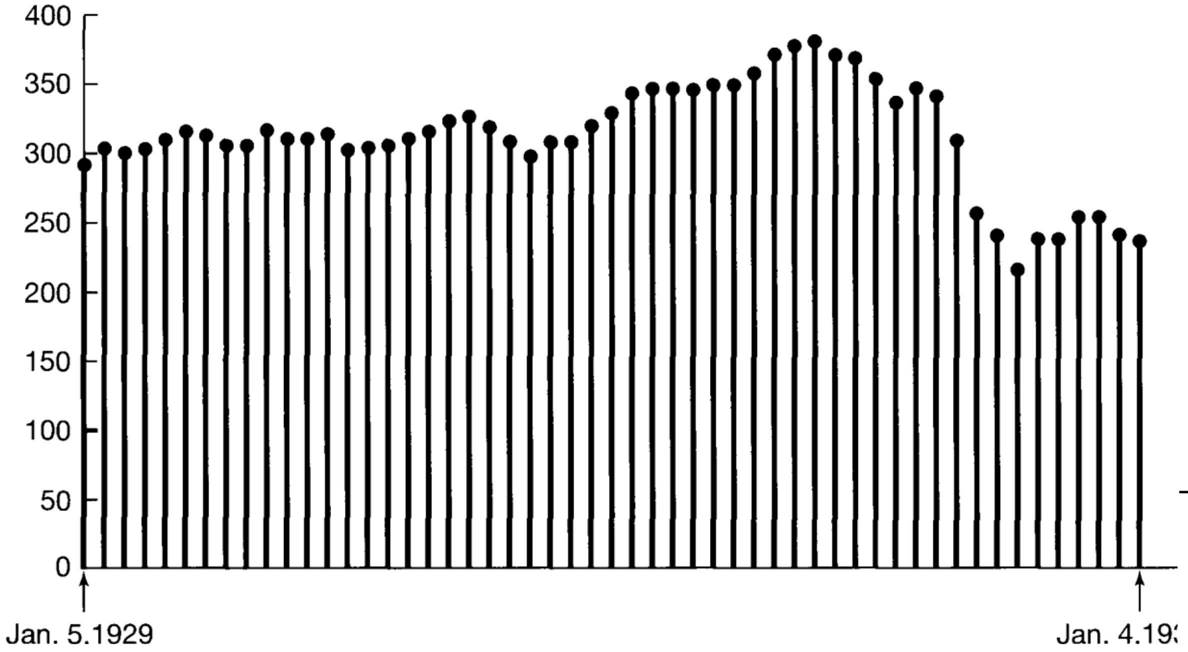


Typical Annual Vertical Wind Profile (Crawford and Hudson, August 1970)

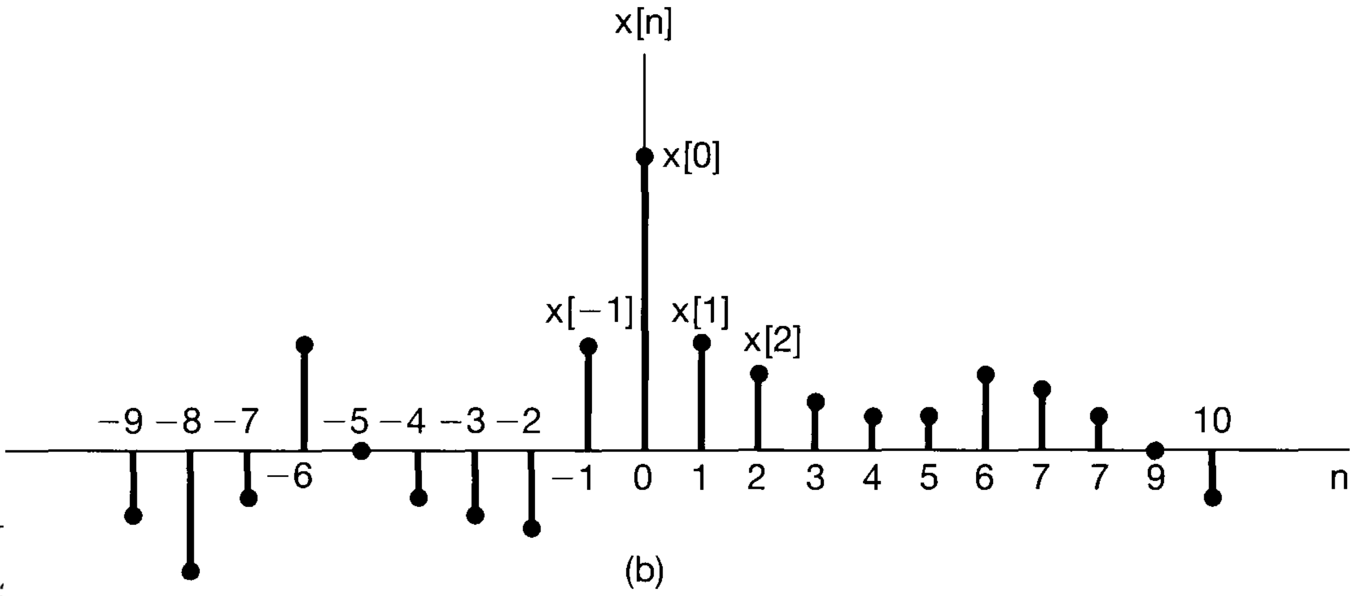


Graphical representation of a continuous-time signal

Discrete-time signals



Weekly Dow-Jones stock market index
(January 1929 – January 1930)



Graphical representation of a discrete-time signal

Continuous-time and Discrete-time signals

Signal Energy and Power

Total energy over a time interval $t1 \leq t \leq t2$ in a continuous-time signal $x(t)$ is defined as:

$$\int_{t1}^{t2} |x(t)|^2 dt$$

where $|x|$ denotes **magnitude of (possibly complex) number $\boldsymbol{x}$.**

Time averaged power: divide above equation by length $t2 - t1$

Total energy in a discrete time signal $x[n]$ over the time interval $n1 \leq n \leq n2$ is defined as:

$$\sum_{n=n1}^{n2} |x[n]|^2$$

Average power: divide above equation by $n2 - n1 + 1$

Continuous-time and Discrete-time signals

Signal Energy and Power

Over **infinite time interval**

$-\infty \leq t \leq \infty$ and $-\infty \leq n \leq \infty$,

$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$; for continuous-time signals

$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$; for discrete-time signals

Note: If $x(t)$ or $x[n]$ equals a **nonzero constant** for all time, the integral or sum above does not converge. Such signals have infinite energy, while signals with $E_\infty < \infty$ have finite energy.

Continuous-time and Discrete-time signals

Signal Energy and Power

Over **infinite time interval**

$-\infty \leq t \leq \infty$ and $-\infty \leq n \leq \infty$,

Time averaged power:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

And

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

For continuous time and discrete time, respectively

Continuous-time and Discrete-time signals

Signal Energy and Power

Three important classes of signals:

- Class of signals with finite total energy, i.e., $E_\infty < \infty$, and zero average power
- Class of signals with finite average power $P_\infty > 0$ and $E_\infty = \infty$
- Class of signals where $E_\infty = P_\infty = \infty$

Transformations of the independent variable

Examples of transformations of the independent variable

- **Time shift** – displacement or shifting of signals by t_0 and n_0 (applicable in both continuous time and discrete time). Used in: radar, sonar, seismic signal processing
- **Time reversal** – reversal or reflection of signals about the vertical axis
- **Time scaling** – linear scale change in independent variable. Example: $x(t)$ is a tape recording, then $x(2t)$ is the recording at twice the speed, and $x(t/2)$ is the recording at half the speed

Transformations of the independent variable

Periodic Signals

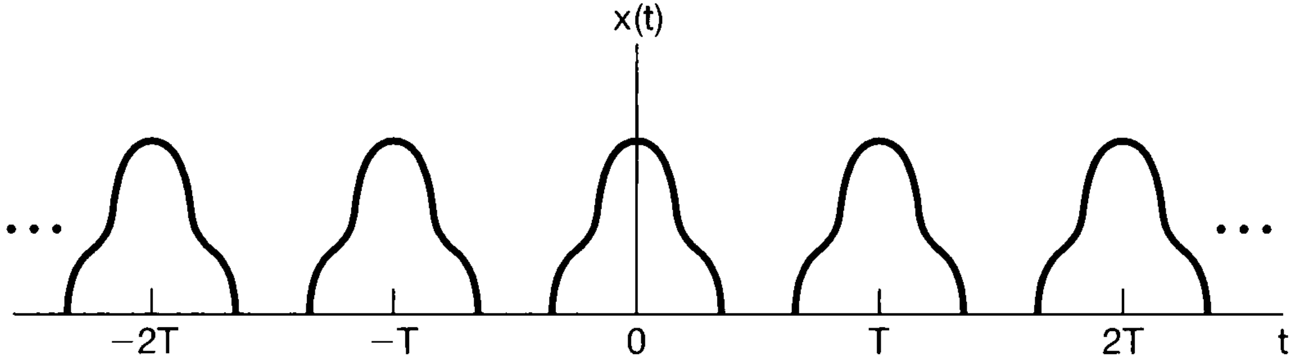
A **periodic CT signal** $x(t)$ has the property that there is a positive value of T such that

$$x(t) = x(t + T) \text{ for all values of } t$$

No change by a time shift of T

Example: natural response of systems like ideal LC circuits without resistive energy dissipation.

Fundamental period T_0: smallest positive value of T for which the above equation holds.



$x(t) = x(t + mT)$ for all values of t

Transformations of the independent variable

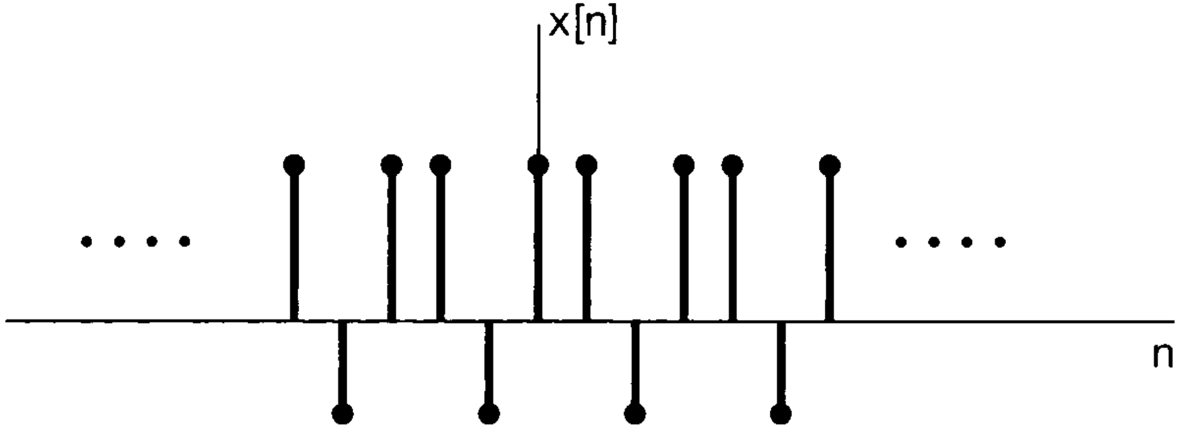
Periodic Signals

A **periodic DT signal** $x[n]$ has the property that there is a positive value of N such that

$$x[n] = x[n + N] \text{ for all values of } n$$

No change by a time shift of N

Fundamental period N_0: smallest positive value of N for which the above equation holds.



DT periodic signal with $N_0 = 3$

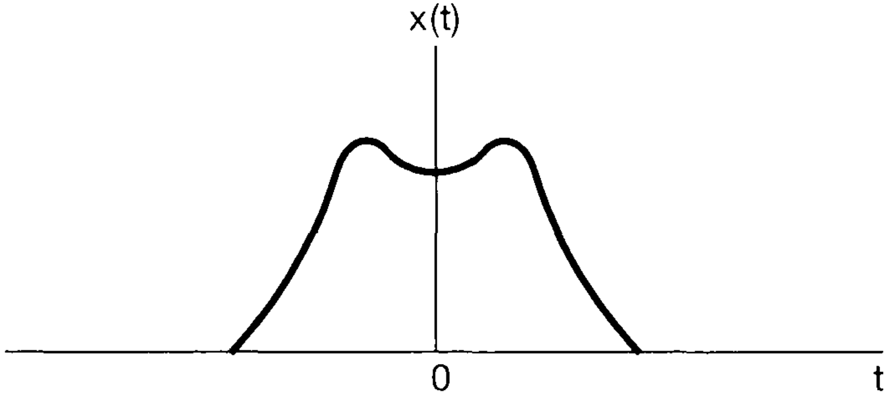
Transformations of the independent variable

Even and odd signals

A signal $x(t)$ or $x[n]$ is **even** if

$$x(-t) = x(t) \text{ or } x[-n] = x[n]$$

for CT and DT signals respectively

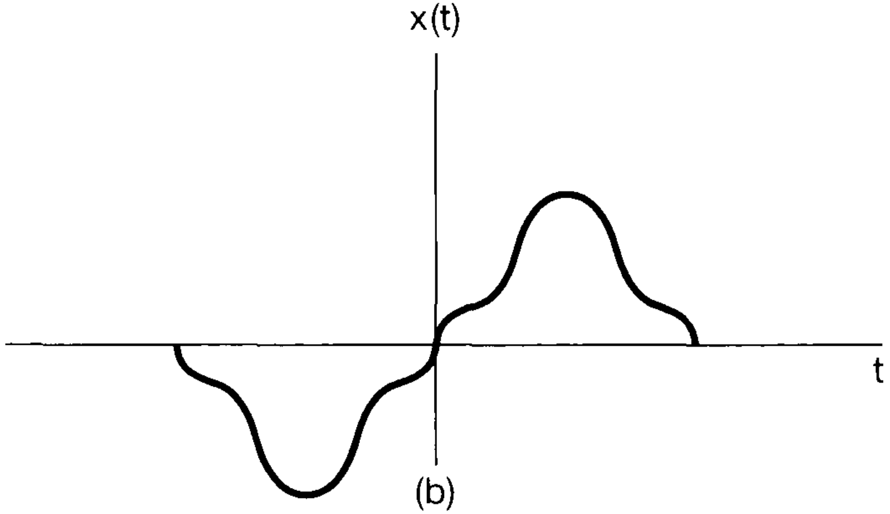


(a)

A signal $x(t)$ or $x[n]$ is **odd** if

$$x(-t) = -x(t) \text{ or } x[-n] = -x[n]$$

for CT and DT signals respectively



(b)

Transformations of the independent variable

Even and odd signals

Any signal can be broken into a **sum of two signals**, one even part and one odd part.

Even part of $x(t)$: $Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$

Odd part of $x(t)$: $Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$

Exactly analogous definitions in DT case

Exponential and Sinusoidal Signals

CT complex exponential and sinusoidal signals

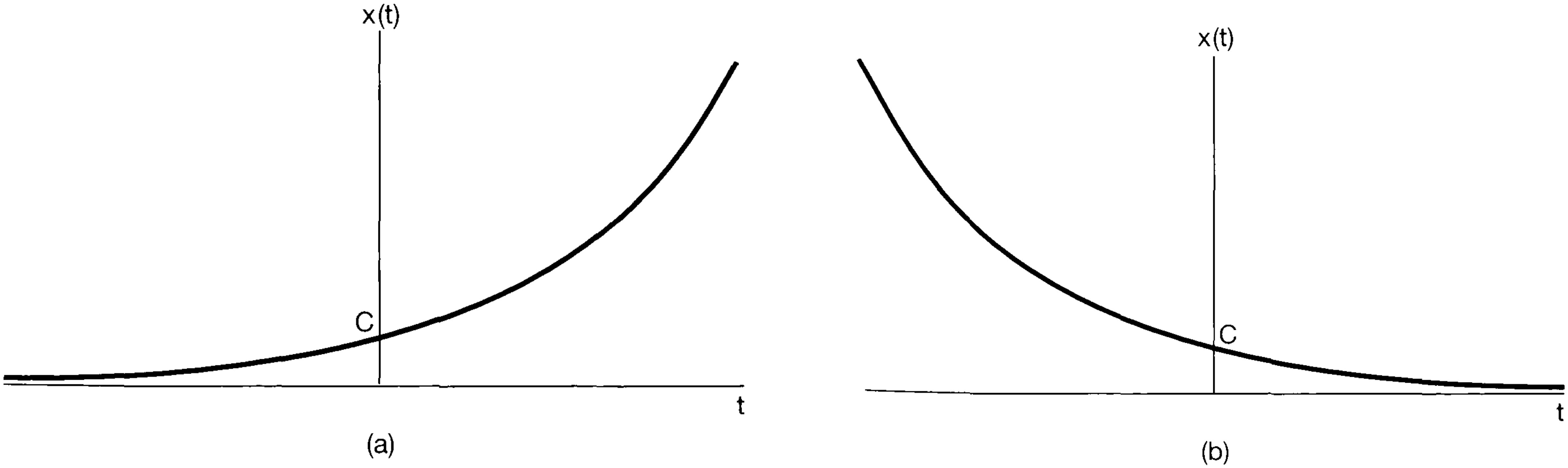
CT complex exponential signal is of the form
$x(t) = Ce^{at}$

Where C and a are complex numbers.

- Signals with **real $\boldsymbol{C}$ and $\boldsymbol{a}$** – real exponential signal

1. a positive, $x(t)$ is a growing exponential
2. a negative, $x(t)$ is a decaying exponential
3. $a = 0$, $x(t)$ is constant

CT real exponential signal



CT real exponential $x(t) = Ce^{at}$ (a) a positive; (b) a negative

Exponential and Sinusoidal Signals

CT complex exponential and sinusoidal signals

- Signals with **purely imaginary $\boldsymbol{a}$** – complex exponential signal. Consider

$$x(t) = e^{j\omega_0 t}$$

Where ω_0 = fundamental frequency

This is a periodic signal; $e^{j\omega_0 t}$ = $e^{j\omega_0 (t+T)}$

If $\omega_0 = 0$, $x(t) = 1$

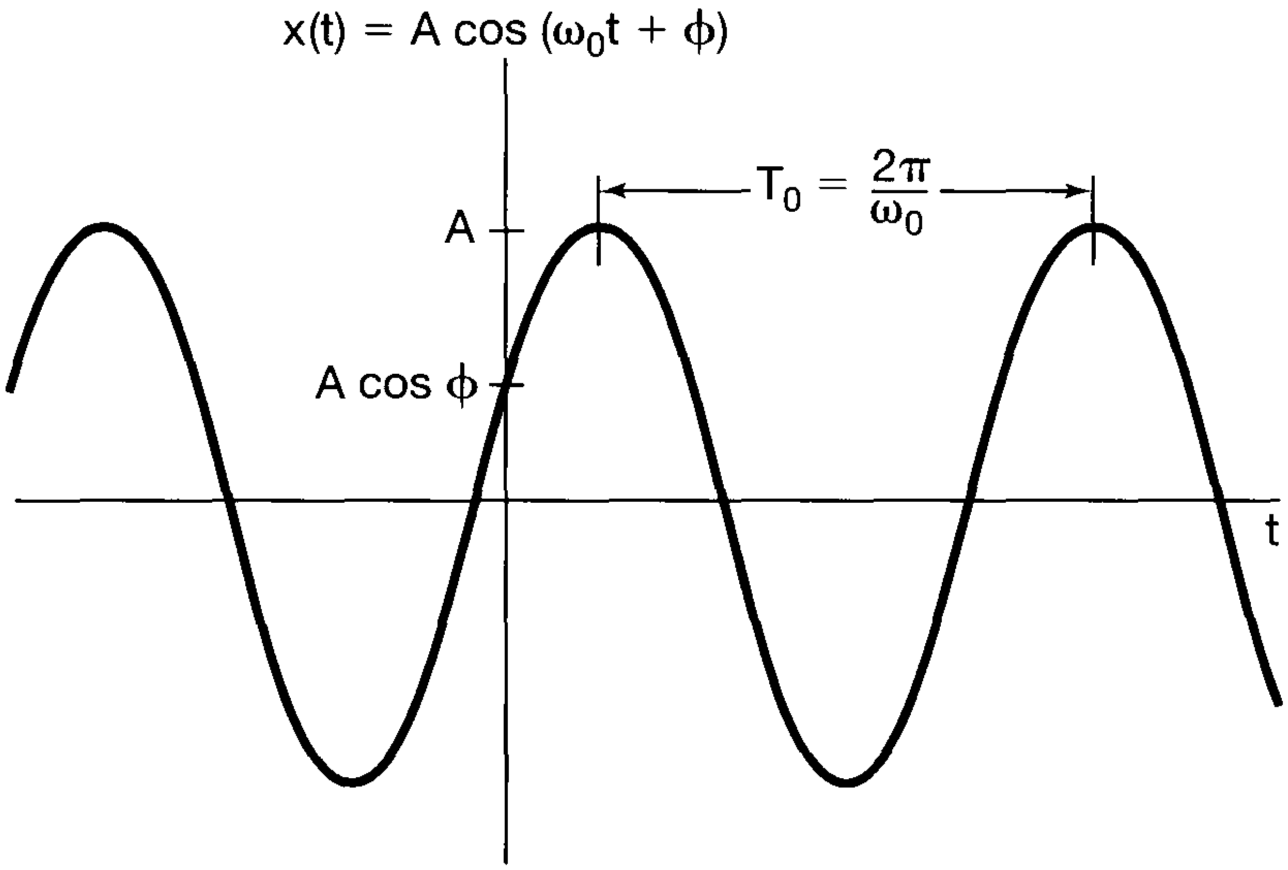
If $\omega_0 \neq 0$, fundamental period $T_0 = \frac{2\pi}{|\omega_0|}$

We know that $T_0 \propto \frac{1}{|\omega_0|}$

As ω_0 decreases, rate of oscillation slows down, period increases; opposite effects if ω_0 increases.

The fundamental period of a constant signal is undefined. Constant signals have zero period of oscillation.

Sinusoidal signal



Signal closely related to periodic complex exponential – Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

ω_0 in radians per second

ϕ in radians

Exponential and Sinusoidal Signals

CT complex exponential and sinusoidal signals

Considering a periodic exponential signal, total energy over one period

$$E_{period} = \int_0^{T0} \left|e^{j\omega_0 t}\right|^2 dt = \int_0^{T0} 1^2 dt = T_0$$

$$P_{period} = E_{period}/T_0 = 1$$

Infinite number of periods as t ranges from -∞ to ∞, therefore, total energy integrated over all time = ∞

Finite average power $P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|e^{j\omega_0 t}\right|^2 dt = 1$

Exponential and Sinusoidal Signals

CT complex exponential and sinusoidal signals

Considering general complex exponential Ce^{at} where

$C = |C|e^{j\theta}$ and $a = r + j\omega_0$

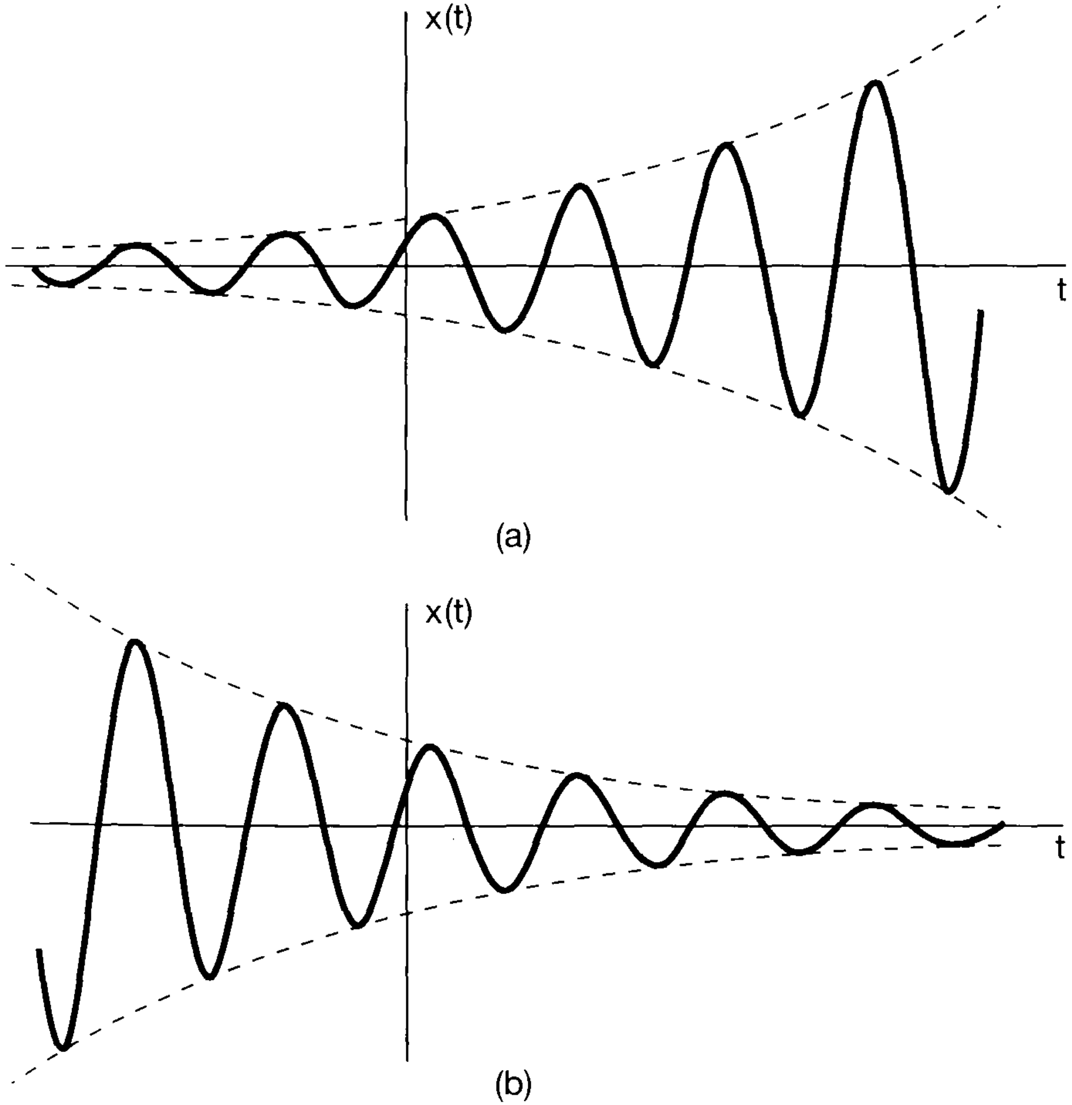
Then $Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0 t+\theta)}.$

Using Euler's relation,

$$Ce^{at} = |C|e^{rt}\cos(\omega_0 t + \theta) + j|C|e^{rt}\sin(\omega_0 t + \theta).$$

- $r = 0$, real and imaginary parts of a complex exponential are sinusoidal
- $r < 0$, real and imaginary parts are sinusoids multiplied by a decaying exponential
- $r > 0$, real and imaginary parts are sinusoids multiplied by a growing exponential

Growing and decaying CT sinusoidal signals



(a)

(b)

Sinusoidal signal

$$x(t) = Ce^{rt}$$

$(a)\, r > 0$

$(b)\, r < 0$

Exponential and Sinusoidal Signals

DT complex exponential and sinusoidal signals

In DT, the complex exponential signal is defined by

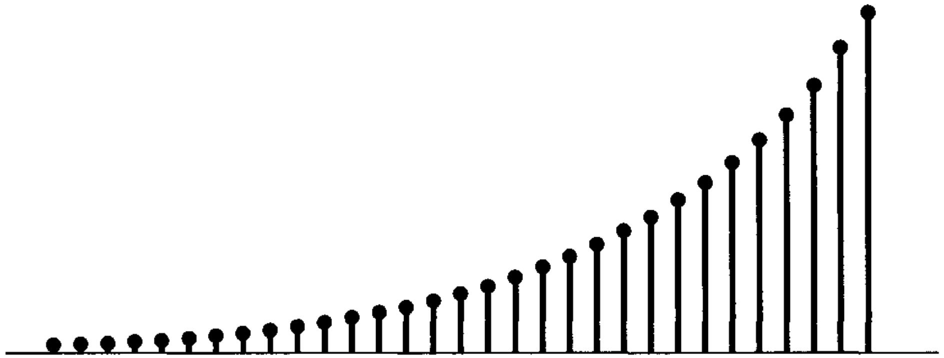
$$x[n] = C\alpha^n,$$

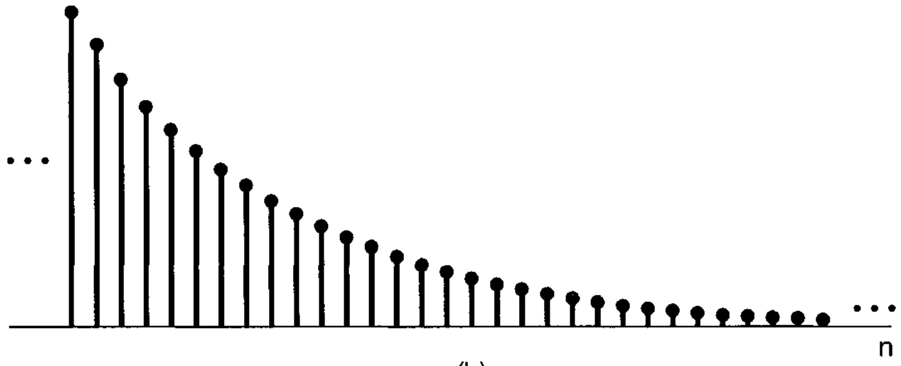
Where C and α are generally complex numbers

If C and α are real:

1. $|\alpha| > 1$, magnitude of signal grows exponentially with n
2. α is negative, sign of $x[n]$ alternates
3. $\alpha = 1$, $x[n]$ is constant
4. $\alpha = -1$, $x[n]$ alternates between C and -C

DT real exponential signals





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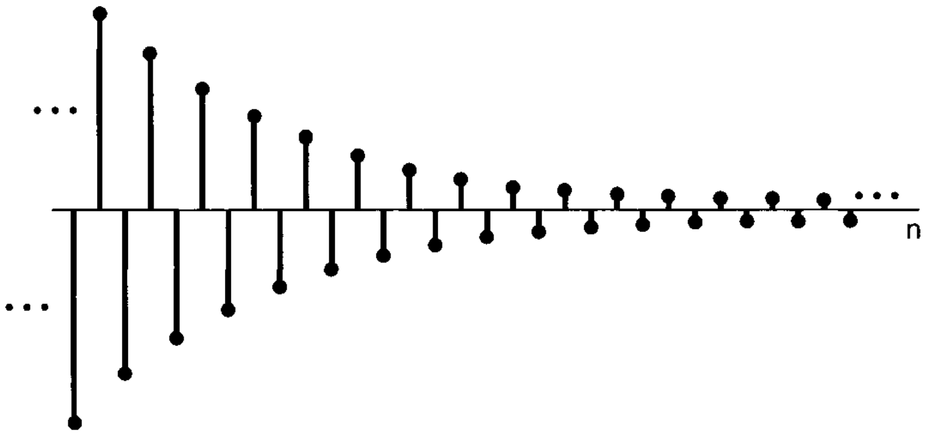

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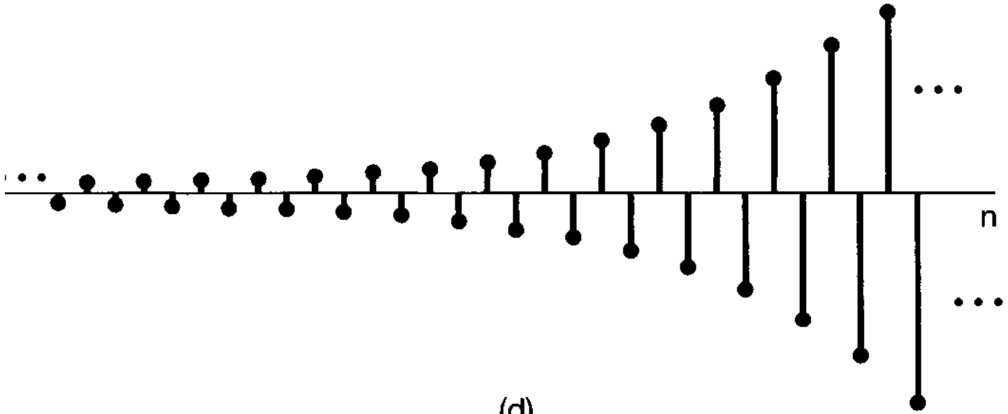


(d)

Real exponential signal $x[n] = C\alpha^t$ (a) $\alpha > 1$ (b) $0 < \alpha < 1$ (c) $-1 < \alpha < 0$ (d) $\alpha < -1$

Exponential and Sinusoidal Signals

DT complex exponential and sinusoidal signals

Considering $x[n] = e^{j\omega_0 n}$

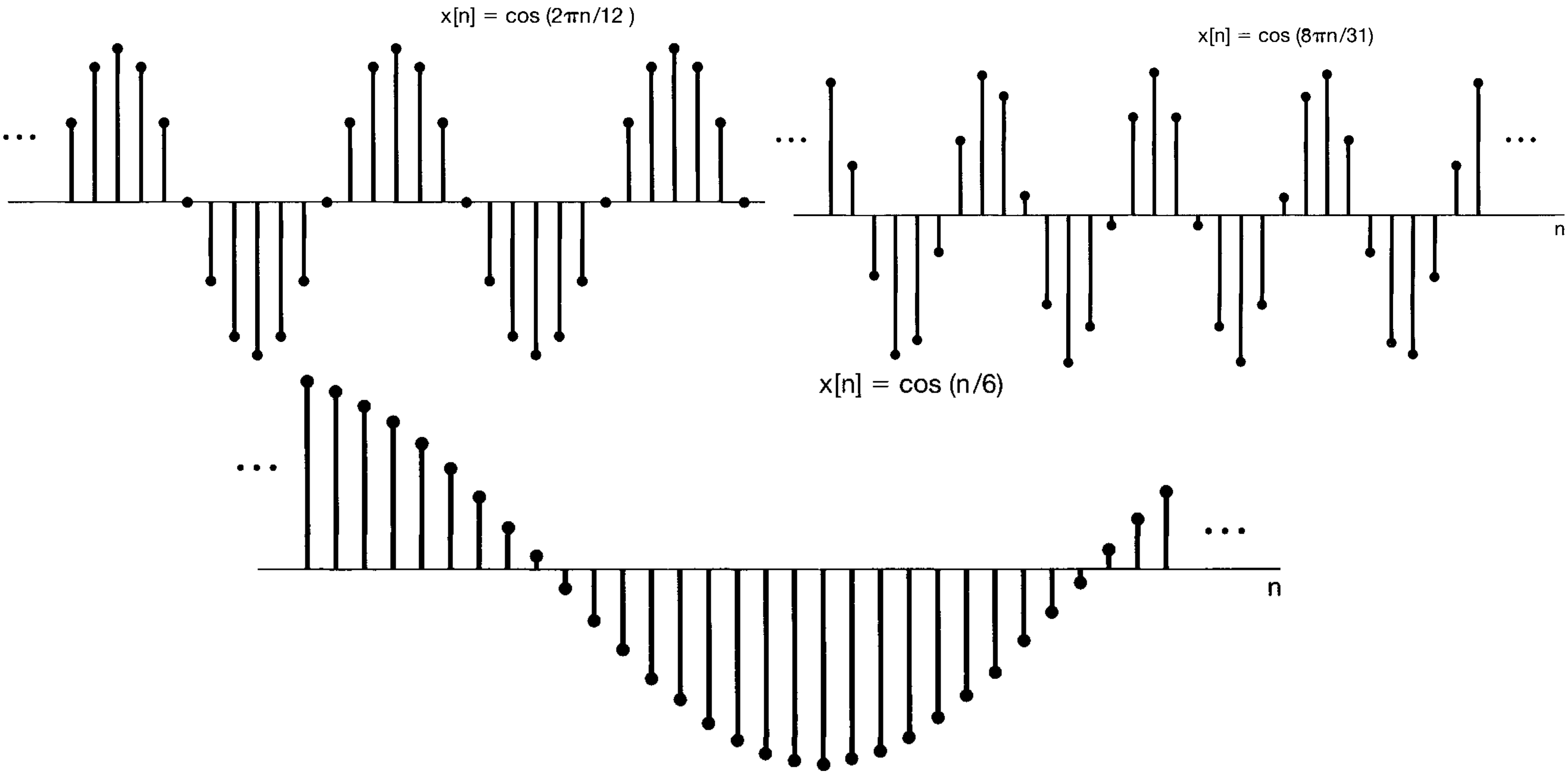
This signal is related to sinusoidal $x[n] = A\cos(\omega_0 n + \phi)$

The signals above are DT signals with infinite total energy but finite average power. Average power per time point = 1

By Euler's relation,

$$A\cos(\omega_0 n + \phi) = \frac{A}{2}e^{j\phi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 n}.$$

DT sinusoidal signals



Exponential and Sinusoidal Signals

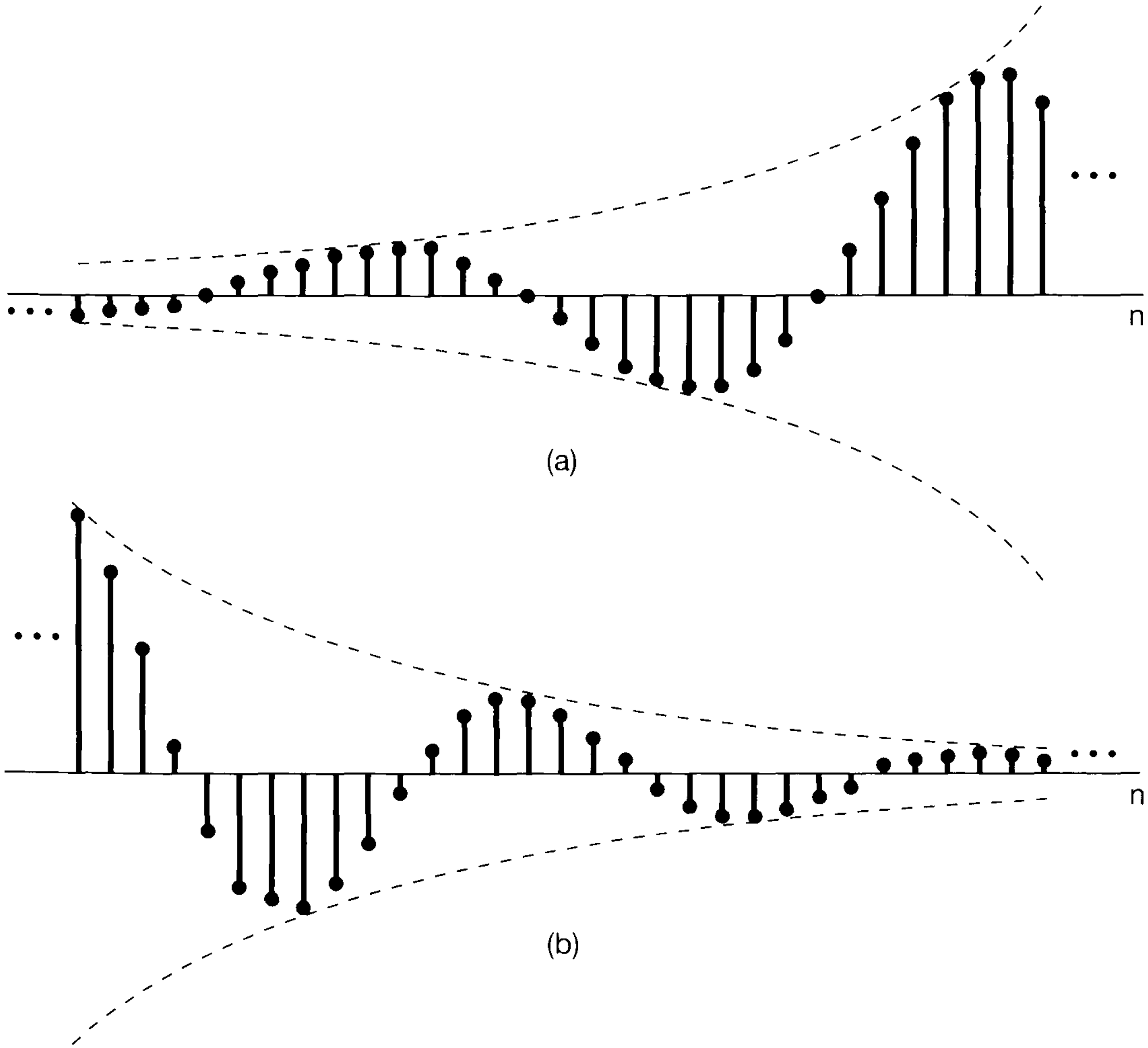
DT complex exponential and sinusoidal signals

If C and α are written in polar form, where $C = |C|e^{j\theta}$ and $\alpha = |\alpha|e^{j\omega_0}$

Then, $C\alpha^n = |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta).$

1. $|\alpha| = 1$, the real and imaginary parts of the sequence are sinusoidal
2. $|\alpha| < 1$, real and imaginary parts are sinusoids multiplied by a decaying exponential
3. $|\alpha| > 1$, real and imaginary parts are sinusoids multiplied by a growing exponential

Growing and decaying DT sinusoidal signals



(a) Growing DT sinusoidal signal

(b) Decaying DT sinusoidal signal

Exponential and Sinusoidal Signals

Periodicity property of DT complex exponentials

- For a CT exponential $e^{j\omega_0 t}$, larger ω_0 means higher rate of oscillation; $e^{j\omega_0 t}$ is periodic for all ω_0; the signal is distinct for distinct values of ω_0
- But the DT exponential $e^{j\omega_0 n}$ at frequencies $\omega_0 \pm 2\pi$, $\omega_0 \pm 4\pi$ and so on, is the same as that at ω_0.

$$e^{j(\omega_0+2\pi)n} = e^{j2\pi n} e^{j\omega_0 n} = e^{j\omega_0 n}.$$

- We only need to consider a frequency interval of length 2π to choose ω_0.
- $e^{j\omega_0 n}$ does not have a continually increasing rate of oscillation as ω_0 is increased.
- Low frequency DT exponentials have values of ω_0 near $0, 2\pi$ and other even multiples of π, whereas high frequencies are located near $\omega_0 \pm \pi$ and other odd multiples

Exponential and Sinusoidal Signals

Periodicity property of DT complex exponentials

For periodicity, $e^{j\omega_0 n}$=$e^{j\omega_0 (n+N)}$ $or\ e^{j\omega_0 N} = 1$

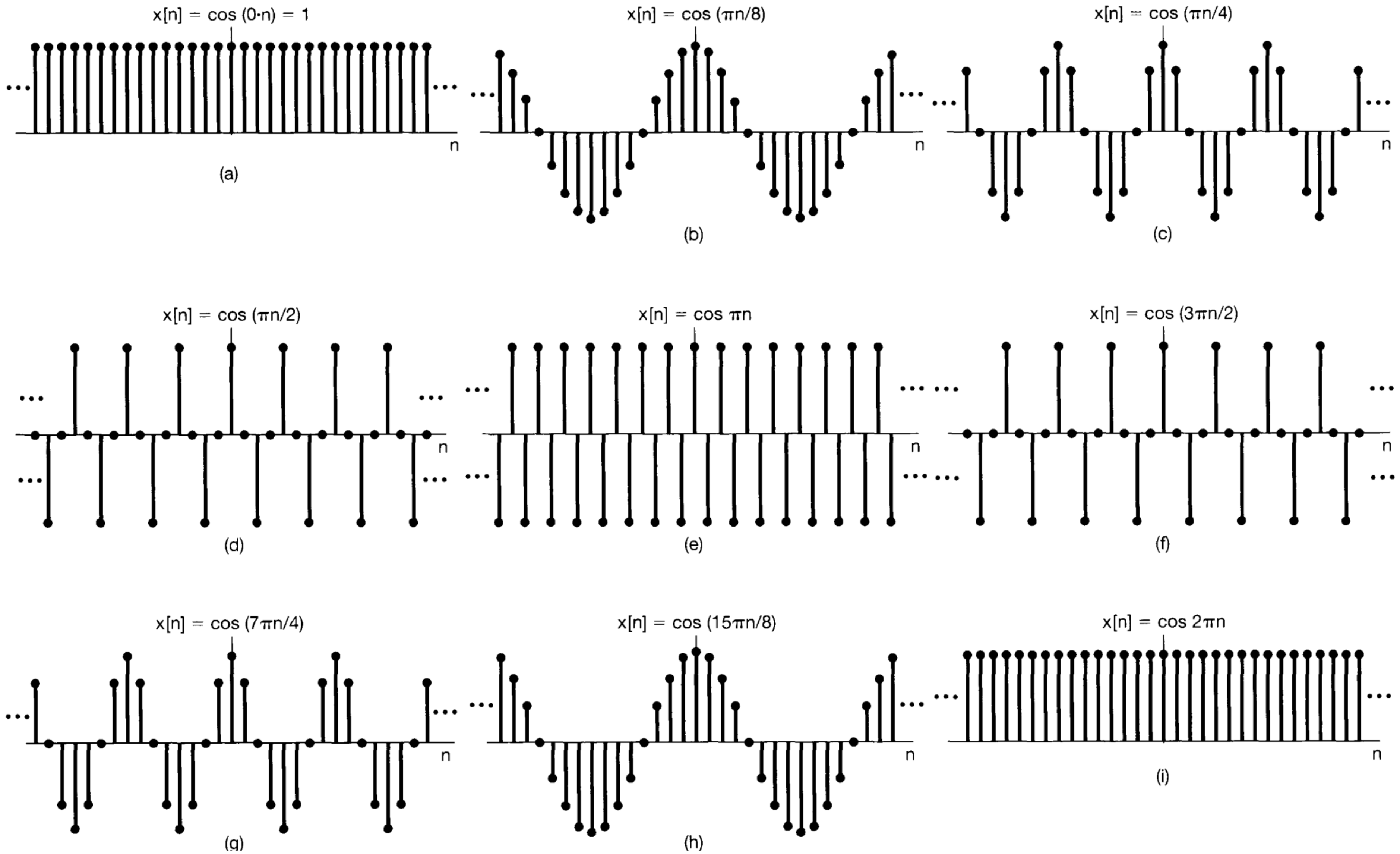
Where $N > 0$

Therefore, $\omega_0 N$ = $2\pi\mathrm{m}$ or $\frac{\omega_0}{2\pi} = \frac{m}{N}$

Periodicity guaranteed only when $\frac{\omega_0}{2\pi}$ is rational

Fundamental period $\mathrm{N} = \mathrm{m}(\frac{\omega_0}{2\pi})$

DT sinusoidal signals for different frequencies



Comparison of the signals $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$

$e^{j\omega_0 t}$

- Distinct values for distinct values of $\boldsymbol{\omega_0}$
- Periodic for any choice of $\boldsymbol{\omega_0}$
- Fundamental frequency $\boldsymbol{\omega_0}$
- Fundamental period if $\boldsymbol{\omega_0} = 0$ is undefined, else $\frac{2\pi}{\omega_0}$

$e^{j\omega_0 n}$

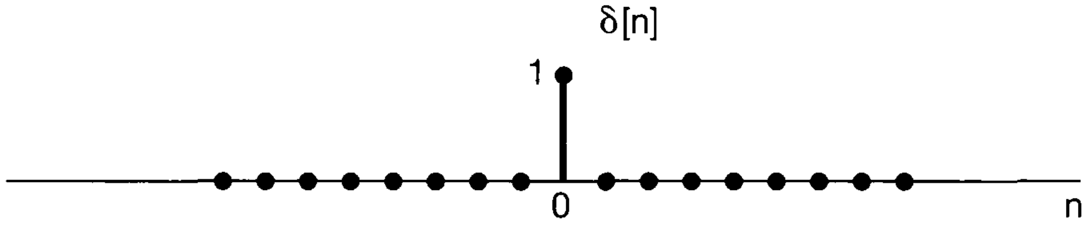
- Identical signals for values of $\boldsymbol{\omega_0}$ separated by multiples of 2π.
- Periodic only when $\boldsymbol{\omega_0} = \frac{2\pi m}{N}$ for $N > 0$ and m
- Fundamental frequency $\frac{\omega_0}{m}$
- Fundamental period if $\boldsymbol{\omega_0} = 0$ is undefined, else $m(\frac{2\pi}{\omega_0})$

The Unit Impulse and Unit Step Functions

The DT Unit Impulse and Unit Step Sequences

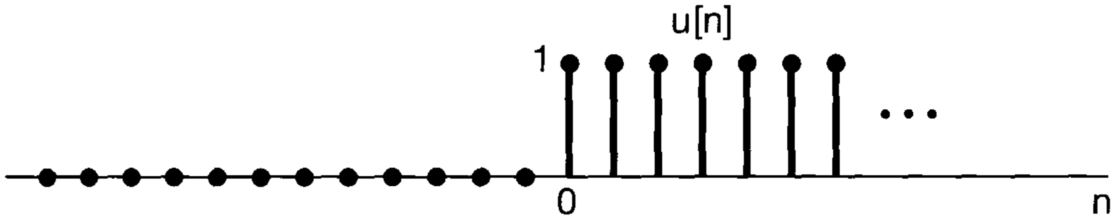
- The DT unit impulse/unit sample is defined by:

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



- The DT unit step is defined by:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



The Unit Impulse and Unit Step Functions

The DT Unit Impulse and Unit Step Sequences

- The DT unit impulse is the first difference of the discrete time step:

$$\delta[n] = u[n] - u[n-1]$$

- The DT unit step is the running sum of the unit sample:

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

or

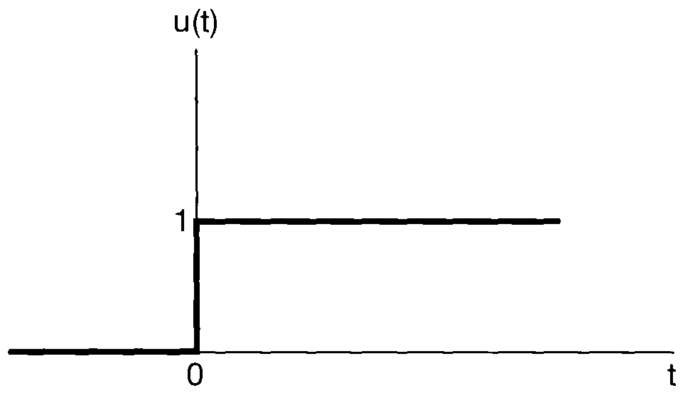
$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$ (superposition of delayed impulses)

The Unit Impulse and Unit Step Functions

The CT Unit Step and Unit Impulse Functions

- The CT unit step is defined by:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



- The CT unit impulse function $\delta(t)$ is related to the unit step in an analogous manner to relation between DT unit step and impulse functions.

The Unit Impulse and Unit Step Functions

The CT Unit Step and Unit Impulse Functions

- The CT unit step is the running integral of the CT unit impulse

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \int_{0}^{\infty} \delta(t-\sigma)d\sigma$$

- The CT unit impulse is the first derivative of the CT unit step

$$\delta(t) = \frac{du(t)}{dt}$$

But $u(t)$ is discontinuous at t $= 0$

Therefore, approximate the unit step and determine its derivative.

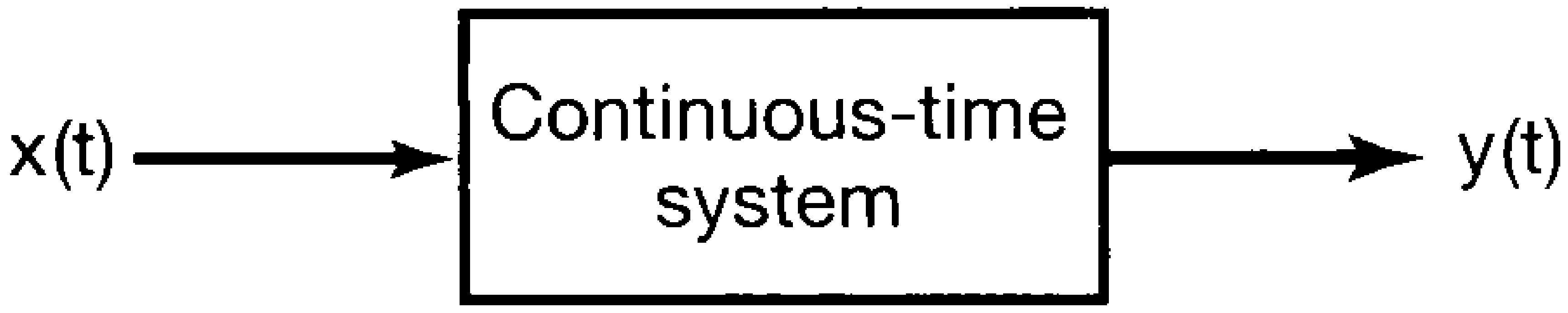
- $\delta(t)$ has no duration but unit area, scaled impulse k$\delta(t)$ has an area of k

Continuous-time and Discrete-time Systems

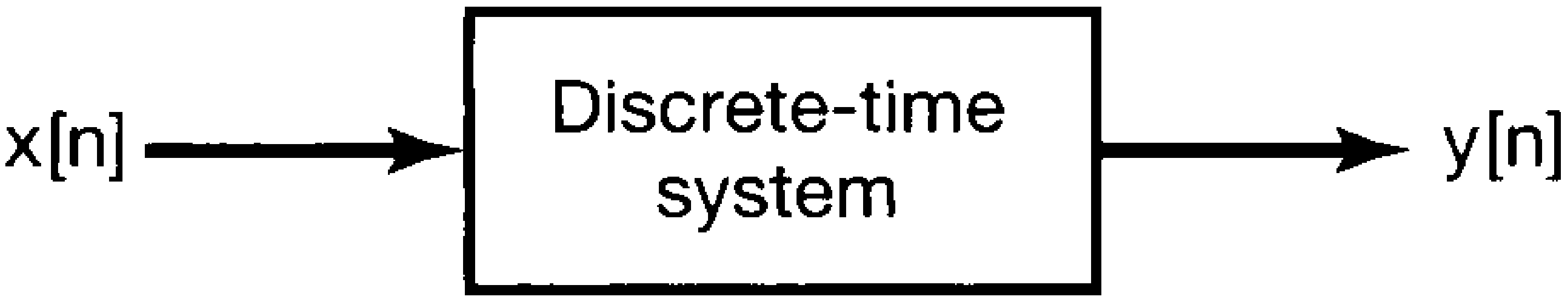
- Physical systems are an **interconnection** of components, devices or subsystems.
- The process involves **transformation of input signals** by the system or causing the system to **respond** in a particular way.
- **Continuous-time system** – system in which continuous-time input signals are applied and continuous-time signals are outputs.
- **Discrete-time system** – system that transforms discrete-time inputs into discrete-time outputs.

CT and DT systems

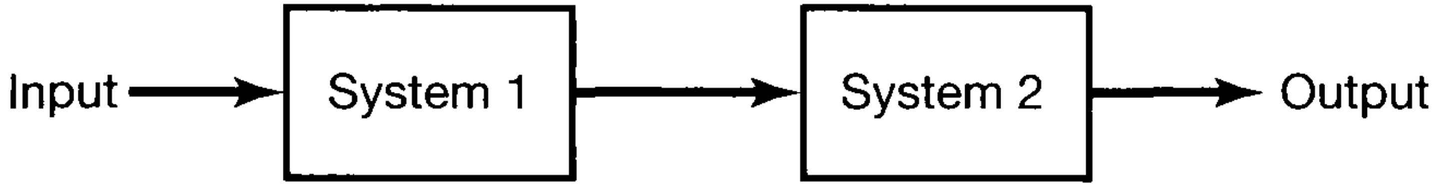
$x(t) \rightarrow y(t).$



$x[n] \rightarrow y[n].$



Interconnection of systems



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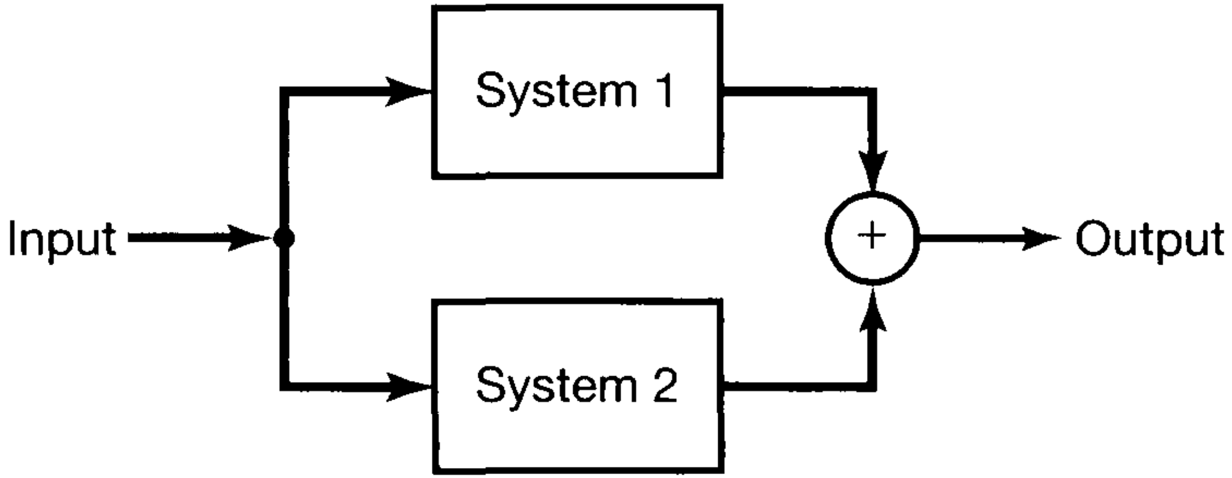
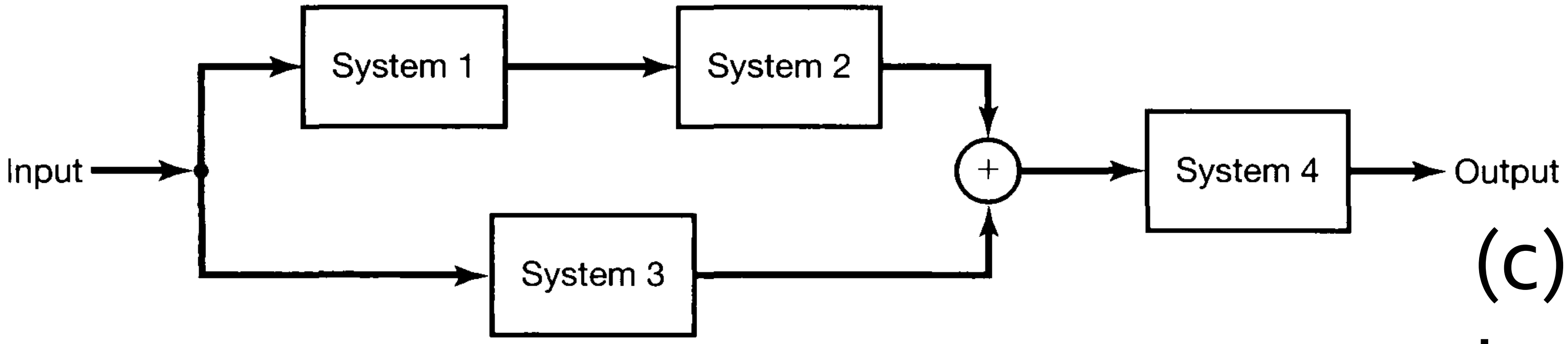


(b)

(a) Series/cascade interconnection

(b) Parallel interconnection

Interconnection of systems



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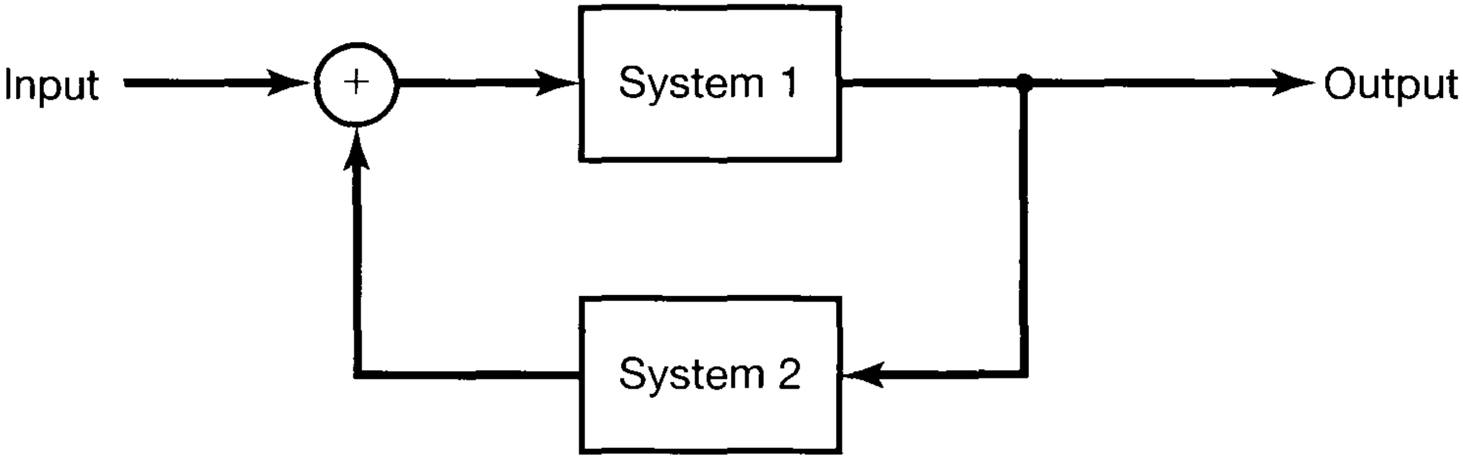


(c) Series/parallel interconnection

(d) Feedback interconnection

Basic System Properties

Systems with/without memory

- A system is said to be **memoryless** if **its output for each value** of the independent variable at a given time is dependent **only on the input at that same time**.

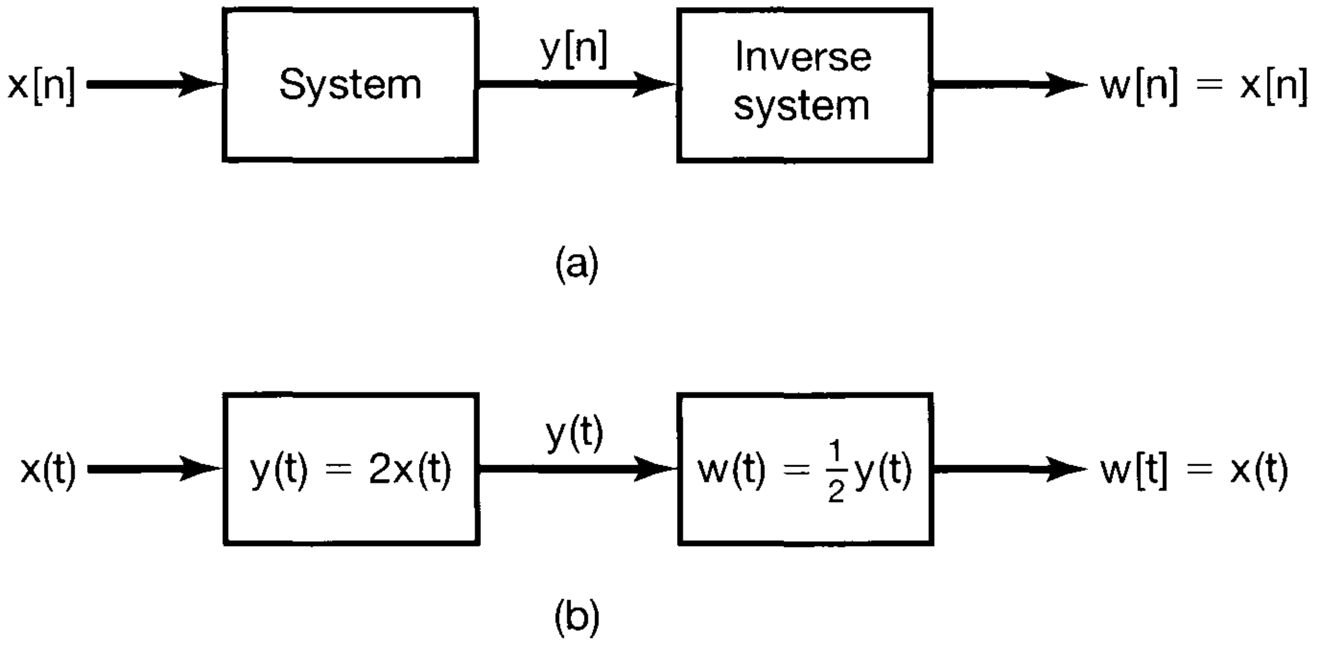
$$y[n] = (2x[n] - x^2[n])^2$$

- Examples of memoryless systems: Resistor, identity system
- A system is said to **contain memory** if current output is dependent on **past or future values** of the input and output.
- Examples of systems with memory: accumulator/summer, delay, capacitor

Basic System Properties

Invertibility and Inverse Systems

- A system is said to be **invertible** if distinct inputs lead to distinct outputs.
- If a system is invertible, **an inverse system** exists which, when cascaded with the original system, **produces an output equal to the input to the first system**.



Basic System Properties

Causality

- A system is **causal/non-anticipative** if the output at any time **depends only on present and past values** of input.
- Examples: Series RC circuit, automobile motion
- All memoryless systems are causal

Basic System Properties

Stability

- A system is **stable** when small inputs lead to non-diverging responses. If the input to a stable system is bounded, its output must also be bounded.
- Examples of stable systems: pendulum, simple RC circuit
- Examples of non-stable systems: inverted pendulum, model for a bank account balance

Basic System Properties

Time Invariance

- A system is said to be **time invariant** if the system behavior and characteristics are fixed over time.
- An RC circuit is time invariant if R and C values constant at all times.
- In terms of signals, time shift in the input signal must result in time shift in the output signal.

Basic System Properties

Linearity

- A system is said to be **linear** if it satisfies the property of superposition
- Properties of additivity and homogeneity must be satisfied:

1. Additivity: $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$
2. Homogeneity: $ax(t) \rightarrow ay(t)$, a is a complex constant.