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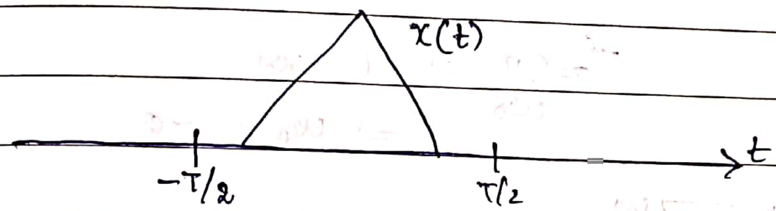
Chapter-4 : Continuous-Time Fourier Trasform

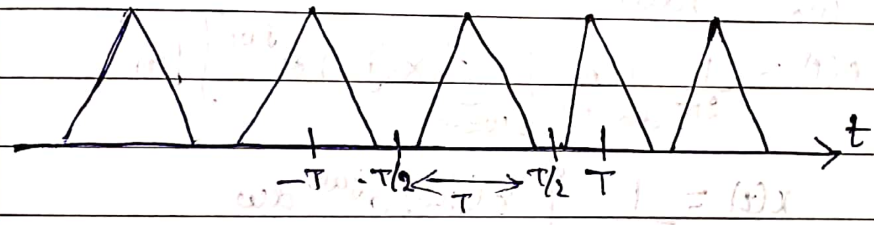
Periodic signal:

$$x_T(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Aperiodic signal representation by fourier Integral

Let $x_T(t)$ be a periodic signal with time period T and $x(t) = \lim_{T\to\infty} x_T(t)$, where $x(t)$ is an aperiodic signal





the fourier series representation of $x_T(t)$,

$$x_T(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{—— (1)}$$

where $a_k = \frac{1}{T}\int_{-T/2}^{T/2} x_T(t) e^{-jk\omega_0 t} dt$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt$$

$$a_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

Lets define $\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt}$

$$a_k = \frac{1}{T} X(jk\omega_0) \quad \text{—— (2)}$$

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② $\longrightarrow$ ①

$$x_T(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X(jk\omega_0) e^{jk\omega_0 t}$$

$$T = \frac{2\pi}{\omega_0}$$

$$x_T(t) = \frac{1}{2\pi} \sum_{k=-\infty} X(jk\omega_0) e^{jk\omega_0 t} \omega_0$$

$$x(t) = \lim_{T\to\infty} x_T(t) = \frac{1}{2\pi} \lim_{T\to\infty} \left[\sum_{k=-\infty}^{\infty} X(jk\omega_0) e^{jk\omega_0 t} \omega_0 \right]$$

$$T = \frac{2\pi}{\omega_0} \Rightarrow T \longrightarrow \infty$$

$$\Rightarrow \omega_0 \longrightarrow 0$$

$k\omega_0 \longrightarrow \omega$

$\omega_0 \longrightarrow d\omega$

$$x(t) = \frac{1}{2\pi} \lim_{d\omega\to 0} \left[\sum_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} \right] d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$F[x(t)] = X(j\omega)$

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt}$$

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega}$$ //

Existene of fourier transform

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

conditions for fourier series

i) $x(t) \rightarrow$ absolutely integrated

2) $x(t) \rightarrow$ find the no. of maximas and minimas

3) $x(t) \rightarrow$ find the no. of discontinuities and each of those discontinuities must be finite

EX: obtain the FT of the following signals

1) $x(t) = e^{-at} u(t)$; $a>0$

2) $x(t) = e^{-a(t)}$; $a>0$

3) $x(t) = \delta(t)$

sketch the spectrum

Ans 1) $x(t) = e^{-at} u(t)$; $a>0$

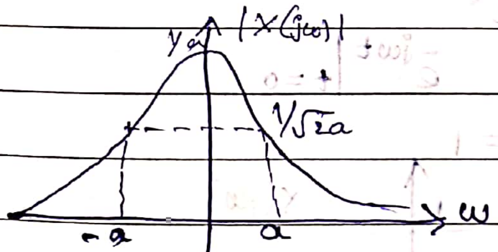
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

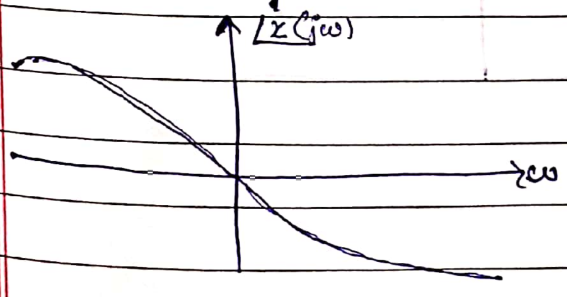
$$= \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$X(j\omega) = \frac{1}{a+j\omega} \; ; \; a>0$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$





2) $x(t) = e^{-a|t|}$; $a>0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

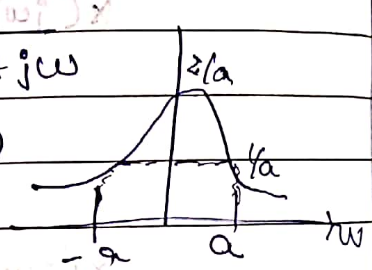
$$e^{-a|t|} = \begin{cases} e^{at} & , t<0 \\ e^{-at} & , t \geq 0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{t(a-\omega j)} + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$X(j\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2} \quad \text{(real)}$$



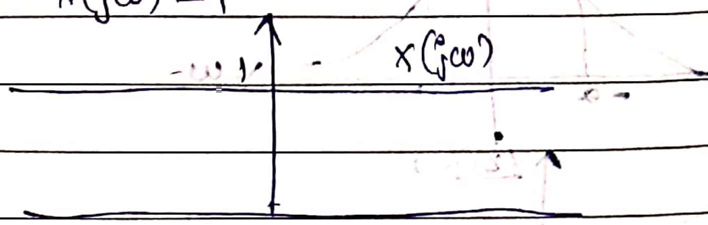
3) $x(t) = \delta(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

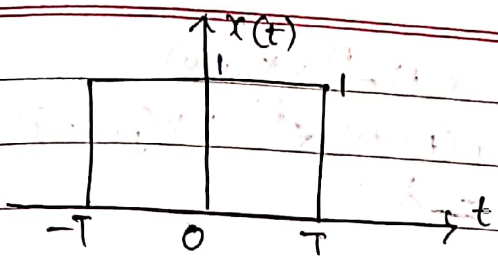
$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$= e^{-j\omega t}\Big|_{t=0}$$

$$X(j\omega) = 1$$



4)



$$X(j\omega) = \int_{-T}^{T} 1.\, e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-T}^{T}$$

$$X(j\omega) = \frac{e^{-j\omega T}}{-j\omega} - \frac{e^{-j\omega(-T)}}{-j\omega}$$

$$= \frac{e^{-j\omega T}}{-j\omega} + \frac{e^{j\omega T}}{+j\omega}$$

$$X(j\omega) = \frac{2\sin\omega T}{\omega}$$

Properties of fourier Transform

1) **linearity:** $x(t) \xrightarrow{FT} X(j\omega)$

$$y(t) \xrightarrow{FT} Y(j\omega)$$

$$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$$

proof:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$a\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t}\, d\omega\right) + \frac{b}{2\pi}\int_{-\infty}^{\infty} Y(j\omega) e^{j\omega t}\, d\omega$$

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2) Time shift: if $x(t) \xleftrightarrow{FT} X(j\omega)$

then $y(t) = x(t-t_0) \xleftrightarrow{FT} Y(j\omega) = e^{-j\omega t_0} X(j\omega)$

λ

Proof: $X(j\omega)$

Duality

1) $\delta(t) = x(t) \xleftrightarrow{FT} X(j\omega) = 1$

$1 = X(jt) \xleftrightarrow{FT} 2\pi x(-\omega) = 2\pi\delta(-\omega)$

$= 2\pi\,\delta(\omega)$

2) $y(t) = e^{-a(t)} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2} = Y(j\omega)$

$X(jt) = \frac{2a}{a^2+t^2} \xleftrightarrow{FT} 2\pi e^{-a|-\omega|} = 2\pi y(-\omega)$

3) $x(t) = e^{-t-1} u(t)$

$= e^{-t} e^{-1} u(t)$

$e^{-at} u(t) \longrightarrow \frac{1}{a+j\omega}$

$e^{-t} u(t) \longleftrightarrow \frac{1}{1+j\omega}$

$$\boxed{e^{-1} e^{-t} u(t) \xleftrightarrow{FT} \frac{e^{-1}}{1+j\omega} = X(j\omega)}$$

$X^*(j\omega) = \frac{e^{-1}}{1-j\omega} = X(-j\omega)$

$X^*(j\omega) = X(-j\omega)$

$x(t) \longrightarrow$ conjugate symmetry

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$e^{-at} \; e^{at}$

$\frac{e^{at} - e^{-at}}{2j}$

4) $e^{-t} e^{-j\pi t} u(t) = x(t)$

$x(t) = e^{-j\pi t} y(t)$

$y(t) = e^{-t} u(t) \xleftrightarrow{FT} Y(j\omega) = \frac{1}{1+j\omega}$

$$\boxed{x(t) = e^{-j\pi t} y(t) \xleftrightarrow{FT} Y(j(\omega+\pi)) = \frac{1}{1+j(\omega+\pi)} = X(j\omega)}$$

5) $x(t) = \sin \pi t \; e^{-2t} u(t)$

$= e^{-2t}\left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right) u(t)$

$= \frac{1}{2j} e^{-2t} e^{j\pi t} u(t) - \frac{1}{2j} e^{-2t} e^{-j\pi t} u(t)$

to find $X(j\omega)$

$e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}$

$x_1(t) \Rightarrow \frac{1}{2j} e^{+j\pi t} e^{-2t} u(t) \xleftrightarrow{FT} \frac{1}{2j} \frac{1}{2+j(\omega-\pi)} = X_1(j\omega)$

$x_2(t) \Rightarrow \frac{-1}{2j} e^{-j\pi t} e^{-2t} u(t) \xleftrightarrow{FT} \frac{-1}{2j} \frac{1}{2+j(\omega+\pi)} = X_2(j\omega)$

$X(j\omega) = X_1(j\omega) + X_2(j\omega)$

6) $x(t) = e^{-3|t-2|}$

Ans

$= e^{-3|t|} \Rightarrow \frac{2(3)}{9+\omega^2} = \frac{6}{9+\omega^2}$

$e^{-3|t-2|} \Rightarrow \frac{6}{9+\omega^2} e^{-j\omega 2}$ //

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7) $x(t) = \frac{d}{dt}\left[t e^{-2t} \sin t \, u(t)\right]$

\# using frequency distribution

$$x(t) \longleftrightarrow X(j\omega)$$

$$(-jt)\, x(t) \longleftrightarrow \frac{d}{d\omega} X(j\omega)$$

$$t\, x(t) \longleftrightarrow j \frac{d}{d\omega} X(j\omega)$$

In general

$$t^n x(t) \longleftrightarrow (j)^n \frac{d^n}{d\omega^n} X(j\omega)$$

$$e^{-2t} u(t) \longleftrightarrow \frac{1}{2+j\omega}$$

$$t e^{-2t} u(t) \longleftrightarrow j \frac{d}{d\omega}\left(\frac{1}{2+j\omega}\right)$$

$$\frac{1}{2j} e^{jt} t e^{-2t} u(t) \longleftrightarrow \frac{1}{2j}\left(\frac{1}{2+j(\omega-1)}\right)^2$$

$$\frac{-1}{2j} e^{-jt} t e^{-2t} u(t) \longleftrightarrow \frac{-1}{2j} \frac{1}{(2+j(\omega+1))^2}$$

$$y(t) \longleftrightarrow Y(j\omega)$$

$$x(t) = \frac{dy(t)}{dt} \longleftrightarrow X(j\omega) = j\omega Y(j\omega)$$

Fourier transform of periodic signal

$$F[x_T(t)] = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$$

proof: find the ~~FT~~ FT of an impulse train

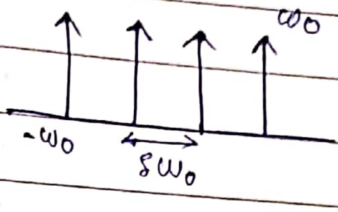
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$a_k = \frac{1}{T}$$

$$F[x(t)] = 2\pi \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - k\omega_0)$$

$$= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - k\omega_0)$$

$$= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$



$$\cancel{F[x_T(t)] = \left[\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right]}$$

prob

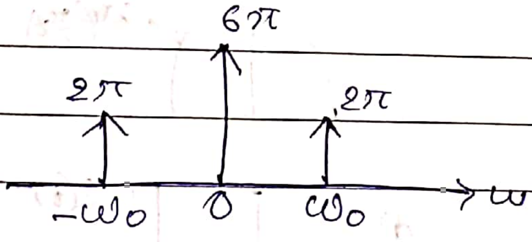
$$x(t) = 3 + 2\cos(10\pi t)$$

$$a_0 = 3 \qquad a_{-1} = 1 \qquad a_1 = 1$$

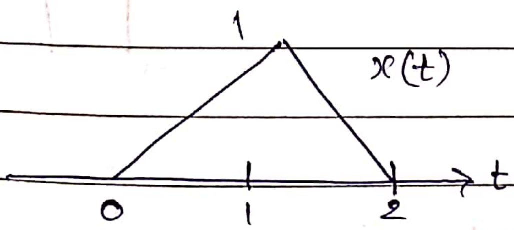
$$F[x(t)] = 2\pi \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - k\omega_0)$$

$$= 2\pi a_{-1} \delta(\omega + \omega_0) + 2\pi a_0 \delta(\omega) + 2\pi a_1 \delta(\omega - \omega_0)$$

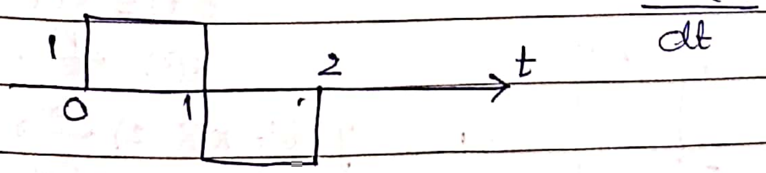
$$= 2\pi \, \delta(\omega + \omega_0) + 6\pi \delta(\omega) + 2\pi \delta(\omega - \omega_0)$$

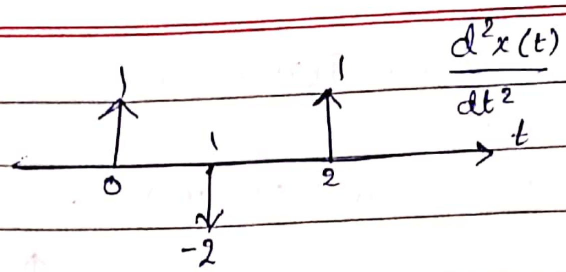


prob



successive differentiation





$$\frac{d^2x(t)}{dt^2} = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$\Downarrow FT \qquad\qquad \Downarrow FT$$

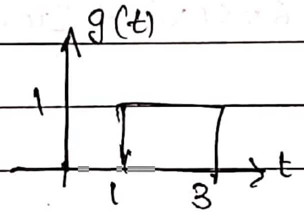
$$(j\omega)^2 X(j\omega) = 1 - 2e^{-j\omega} + e^{-j2\omega}$$

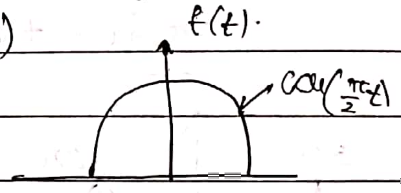
$$X(j\omega) = \frac{1 - 2e^{-j\omega} + e^{-j2\omega}}{(j\omega)^2}$$

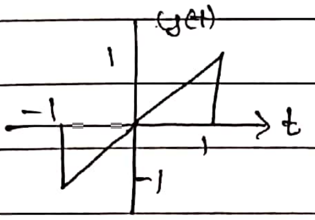
prob Given FT pair,

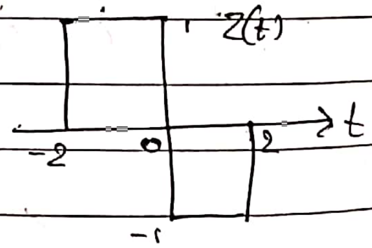
$$x(t) = \begin{cases} 1, & |t|<1 \\ 0, & |t|>1 \end{cases} \xleftrightarrow{FT} X(j\omega) = \frac{2\sin\omega}{\omega}$$

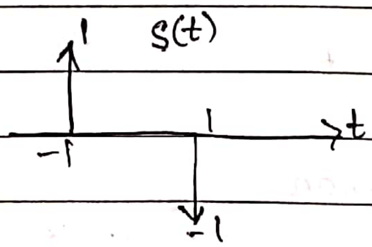
Find the FT of the foll signals

a) 

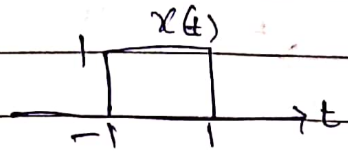
b) 

c) 

d) 

e) 

Ans



$g(t) = x(t-2)$

$x(t) \xleftrightarrow{FT} X(j\omega)$

$y(t) = x(t-2) \xleftrightarrow{FT} e^{-j2\omega} X(j\omega) =$

$e^{-2j\omega} \frac{2\sin\omega}{\omega} = G(j\omega)$

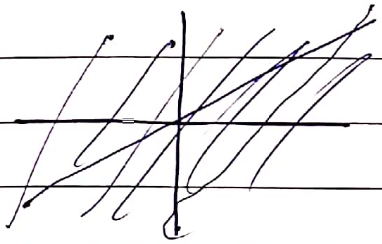
b) $f(t) = x(t)\cos\left(\frac{\pi t}{2}\right)$

$$f(t) = \frac{1}{2} e^{+j\frac{\pi}{2}t} x(t) + \frac{1}{2} e^{-j\frac{\pi}{2}t} x(t)$$ //

$$F(j\omega) = \frac{1}{2}X\left(j\left(\omega-\frac{\pi}{2}\right)\right) + \frac{1}{2}X\left(j\left(\omega+\frac{\pi}{2}\right)\right)$$

$$F(j\omega) = \frac{\sin(\omega-\pi/2)}{(\omega-\pi/2)} + \frac{\sin(\omega+\pi/2)}{(\omega+\pi/2)}$$

c) $\frac{dy(t)}{dt} = i(t)$



$$x(t) \longleftrightarrow X(j\omega)$$

$$t\,x(t) \longleftrightarrow j\frac{d}{d\omega}X(j\omega)$$

$$y(t) \longleftrightarrow j\frac{d}{d\omega}\left(\frac{2\sin\omega}{\omega}\right)$$ //

d) $z(t) = x(t+1) - x(t-1)$

$$Z(j\omega) = e^{j\omega}X(j\omega) - e^{-j\omega}X(j\omega)$$

$$= e^{j\omega}\frac{2\sin\omega}{\omega} - e^{-j\omega}\frac{2\sin\omega}{\omega}$$

e) $s(t) = \frac{d}{dt}x(t)$

$$S(j\omega) = j\omega X(j\omega) = j\omega\frac{2\sin\omega}{\omega}$$

Discrete time fourier Transform

$$F[x(n)] = X(e^{j\omega})$$

$$\cancel{X(e^{j\omega}) = \int x(n) e^{-j\omega n} d\omega}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

1) find the discrete time fourier transform of signals

a) $x(n) = \delta(n)$

b) $x(n) = a^n u(n)$, $|a| < 1$

c) $x(n) = (0.5)^{n+2} u(n)$

d) $x(n) = n (0.5)^{2n} u(n)$

e) $x(n) = -a^n u(-n-1)$

f) $x(n) = \{1, 2, 3, 2, 1\}$

Ans a) $F[x(n)] = X(e^{j\omega})$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n}$$

$$= e^{-j\omega n}\Big|_{n=0}$$

$$= 1$$

b) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} a^n e^{-j\omega n} \cancel{\Big]_{n=0}}$$

$$X(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}$$

***

$$\boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a|<1}$$

c) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (0.5)^{n+2} u(n)\, e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} (0.5)^{n+2} e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} (0.5)^2 (0.5)^n e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} (0.5)^2 (0.5 e^{-j\omega})^n$

$= (0.5)^2 \dfrac{1}{1-0.5e^{-j\omega}} = \dfrac{0.25}{1-0.5e^{-j\omega}}$

d) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} n(0.5)^{2n} u(n)$

$$\boxed{\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2}}$$

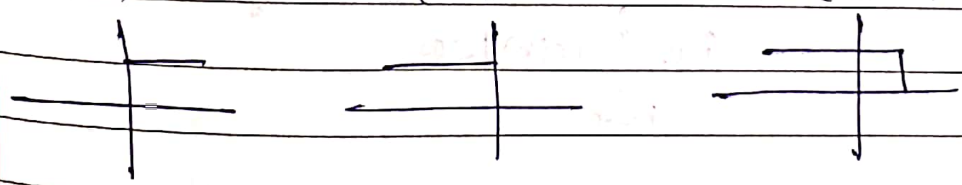
$X(e^{j\omega}) = \sum_{n=0}^{\infty} n(0.25e^{-j\omega})^n \cancel{e^{-j\omega n}}$

$= \dfrac{0.25e^{-j\omega}}{(1-0.25e^{-j\omega})^2}$

e) $x(n) = -a^n u(-n-1)$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} -a^n u(-n-1) e^{-j\omega n}$

$u(n)$ $\quad$ $u(-n)$ $\quad$ $u(-n-1)$



$= \sum_{n=-\infty}^{-1} (-a^n e^{-j\omega n})$

$n = -m$

$$= -\sum_{m=1}^{\infty} (a^{-1} e^{j\omega})^m$$

let $l = m-1 \Rightarrow m = l+1$

$m = 1 \Rightarrow l = 0$

$m = \infty \Rightarrow l = \infty$

$$= -\sum_{l=0}^{\infty} (a^{-1} e^{j\omega})^{l+1}$$

$$= -a^{-1} e^{j\omega} \sum_{l=0}^{\infty} (a^{-1} e^{j\omega})^{l}$$

$$= \frac{-a^{-1} e^{j\omega}}{1 - a^{-1} e^{j\omega}}$$

f) $$x(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{4} x(n) e^{-j\omega n}$$

$$= \cancel{e^{-j\omega n}}\, 1 + 2e^{-j\omega} + 3e^{-j2\omega} + 2e^{-j3\omega} + e^{-j4\omega}$$

Properties ~~of DTFT~~:

$$|x(e^{j\omega})| = \sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

DTFT exist if $x(n)$ is absolutely summable

i.e. $\sum_{n=-\infty}^{\infty} |x(n)| < \infty$

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Properties of DTFT:

1) **Linearity**

$$a x(n) + b y(n) \xleftrightarrow{DTFT} a X(e^{j\omega}) + b X(e^{j\omega})$$

2) **Time-shift**

$$x(n-n_0) \xleftrightarrow{DTFT} e^{-j\omega n_0} X(e^{j\omega})$$

3) **Frequency shift**

$$e^{j\beta n} x(n) \xleftrightarrow{DTFT} X(e^{j(\omega-\beta)})$$

4) **Scaling**

$$x_a(n) = \begin{cases} x\left(\frac{n}{a}\right) = x(k) & \text{if } n = ka \\ 0 & \text{if } n \neq ka \end{cases}$$

then,

$$x_a(n) \xleftrightarrow{DTFT} X_a(e^{j\omega}) = X(e^{ja\omega})$$

5) **Frequency - differentiation**

$$-jn\, x(n) \xleftrightarrow{DTFT} \frac{d}{d\omega} X(e^{j\omega})$$

6) **Summation**

$$\sum_{k=-\infty}^{n} x(k) \xleftrightarrow{DTFT} \pi X(e^{j0}) \delta(e^{j\omega}) + \frac{1}{1-e^{-j\omega}} X(e^{j\omega})$$

7) **Convolution**

$$x(n) * y(n) \xleftrightarrow{DTFT} X(e^{j\omega}) Y(e^{j\omega})$$

8) **Multiplication**

$$x(n) y(n) \longleftrightarrow \frac{1}{2\pi}\left[X(e^{j\omega}) * Y(e^{j\omega})\right]$$

Periodic convolution

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9) <u>Parsevel's theorem</u>

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

10) <u>conjugation</u>

$$x^*(n) \longleftrightarrow X^*(e^{-j\omega})$$

11) <u>Time Reversal</u>

$$x(-n) \longleftrightarrow X(e^{-j\omega})$$

12) <u>Symmetry property</u>

$x(n)$. real or even $\longleftrightarrow$ $X(e^{j\omega})$ purely real

$x(n)$, real or odd $\longleftrightarrow$ $X(e^{j\omega})$ purely imaginary

<u>problem</u>. find $X(e^{j\omega})$

1) $x(n) = \left(\frac{1}{2}\right)^n u(n-4)$

2) $x(n) = a^{|n|}, \; |a|<1$

3) $x(n) = u(n+1) - u(n-2)$

4) $x(n) = \alpha^n \cos(\omega_0 n + \phi)\, u(n)$

5) $x(n) = n\,\alpha^n u(n), \; |\alpha|<1$

<u>Ans</u> 1) $X(e^{j\omega}) = \sum_{n=4}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n}$

$m = n-4, \quad n = m+4$

$n=4 \;\; m=0$

$n=\infty \;\; m=\infty$

$$X(e^{j\omega}) = \frac{1}{1-\left(\frac{1}{2}e^{-j\omega}\right)}$$

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$$X(e^{j\omega}) = \sum_{n=4}^{\infty} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{n-4} u(n-4)$$

$$\boxed{a^n u(n) \longleftrightarrow \frac{1}{1-ae^{-j\omega}}}$$

$$X(e^{j\omega}) = \cancel{\left(\frac{1}{2}\right)^4} \frac{1}{1-\left(\frac{1}{2}\right)e^{-j\omega}} \left(\frac{1}{2}e^{-j\omega}\right)^4$$

~~using property~~

$$\cancel{x(n) = \left(\frac{1}{2}\right)^{n-4+4} u(n-4)}$$

$$\cancel{= \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)'}$$

$$\left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$\left(\frac{1}{2}\right)^{n-4} u(n-4) \longleftrightarrow \frac{e^{-j\omega 4}}{1-\frac{1}{2}e^{-j\omega}}$$

$$\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{n-4} u(n-4) \longleftrightarrow \frac{\left(\frac{1}{2}e^{-j\omega}\right)^4}{1-\frac{1}{2}e^{-j\omega}}$$

2) $x(n) = a^{|n|}, \quad |a|<1$ $\quad \begin{cases} -n & n<0 \\ n & n>0 \end{cases}$

$$\cancel{x(n) =} \int_{-\infty}^{-1} a^{-n} + \int_{0}^{\infty} a^{+n}$$

$$a^{|n|} = a^{-n} u(-n-1) + a^n u(n)$$

$$= \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \frac{ae^{j\omega}}{1-ae^{j\omega}} + \frac{1}{1-ae^{-j\omega}}$$

1

3) $x(n) = u(n+1) - u(n-2)$

$$\sum_{n=-1}^{2} x(n) e^{-j\omega n} = e^{j\omega n} + 1 + e \Big|_{n=-1}^{2}$$

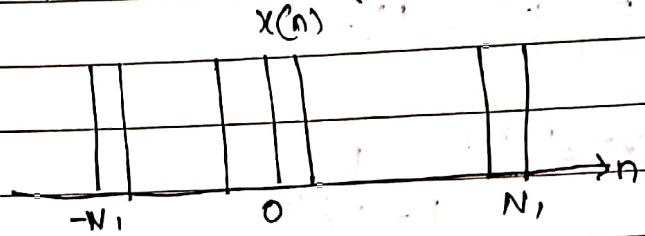
$$= e^{2j\omega} + \cancel{1} + \cancel{e} - (e^{-j\omega} + \cancel{1} + \cancel{e})$$

$$= e^{2j\omega} - e^{-j\omega}$$

4) $x(n) = \alpha^n$

find DTFT

1)



$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}, \quad a \neq 1$$

$$X(e^{j\omega}) = \sum_{n=-N_1}^{N_1} e^{-j\omega n}$$

let $m = n + N_1 \Rightarrow n = m - N_1$

$n = -N_1 \Rightarrow m = 0$

$n = N_1 \Rightarrow m = 2N_1$

$$X(e^{j\omega}) = \sum_{m=0}^{2N_1} e^{-j\omega m} e^{j\omega N_1}$$

$$= e^{j\omega N_1}\left[\frac{1 - e^{-j\omega(2N_1+1)}}{1 - e^{-j\omega}}\right]$$

$$= \frac{e^{j\omega N_1} e^{-j\frac{\omega}{2}(2N_1+1)} \left[e^{j\omega/2(2N_1+1)} - e^{-j\omega/2(2N_1+1)}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

$$\boxed{\begin{aligned} X(e^{j\omega}) &= \frac{\sin \omega/2(2N_1+1)}{\sin(\omega/2)}, \quad \omega \neq 0, \pm\pi, \pm 2\pi, \ldots \\ &= 2N_1 + 1, \quad \omega = 0, \pm\pi, \pm 2\pi, \ldots \end{aligned}}$$

2) $x(n) = \delta(6-3n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta(6-3n)\, e^{-j\omega n}$$

$$= e^{-j\omega n}\Big|_{n=2}$$

$$= e^{-j2\omega}$$

3) find the inverse DTFT of $X(e^{j\omega}) = 2\pi\,\delta(e^{j(\omega-\omega_0)})$

Ans $$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\,\delta(e^{j(\omega-\omega_0)})\, e^{j\omega n}\, d\omega$$

$$= e^{j\omega n}\Big|_{\omega=\omega_0} = e^{j\omega_0 n}$$

$$e^{j\omega_0 n} \xleftrightarrow{DTFT} 2\pi\,\delta(e^{j(\omega-\omega_0)})$$

$\omega_0 = 0$

$$1 \xleftrightarrow{DTFT} 2\pi\,\delta(e^{j\omega})$$

$$1 \xleftrightarrow{DTFT} 2\pi\,\delta(\omega)$$

4) $x(n) = \cos\left(0.2n\pi + \frac{\pi}{4}\right)$

Ans $$\frac{1}{2} e^{j0.2n\pi}\, e^{j\pi/4} + \frac{1}{2} e^{-j\pi/4}\, e^{j0.2\pi n}$$

$$1 \xleftrightarrow{DTFT} 2\pi\,\delta(e^{j\omega})$$

$$\frac{1}{2} e^{j\pi/4} \xleftrightarrow{DTFT} 2\pi\,\frac{1}{2}\, e^{j\pi/4}\,\delta(e^{j\omega})$$

$$\frac{1}{2} e^{j\pi/4}\, e^{j0.2\pi n} \longleftrightarrow \pi e^{j\pi/4}\,\delta(e^{j(\omega-0.2\pi)}) + (e^{j\omega})$$

$$\frac{1}{2} e^{-j\pi/4}\, e^{-j0.2n\pi} \longleftrightarrow \pi e^{-j\pi/4}\,\delta(e^{j(\omega+0.2\pi)})$$

③

DTFT of a periodic signal

$$x(n) = \sum_{n=0}^{N-1} a_k e^{jk\omega_0 n}$$

$$a_k \xleftrightarrow{DTFT} 2\pi a_k \delta(e^{j\omega})$$

$$e^{-jk\omega_0 n} a_k \longleftrightarrow 2\pi a_k \delta(e^{j(\omega_0 - k\omega_0)})$$

$$\sum_{n=0}^{N-1} e^{jk\omega_0 n} a_k \longleftrightarrow 2\pi \sum_{k} a_k \delta(e^{j(\omega - k\omega_0)})$$

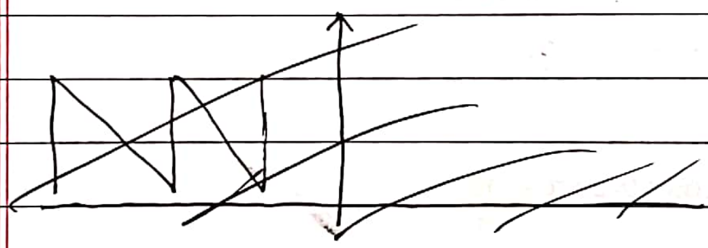
5) $y(n) = (n+1)a^n u(n)$

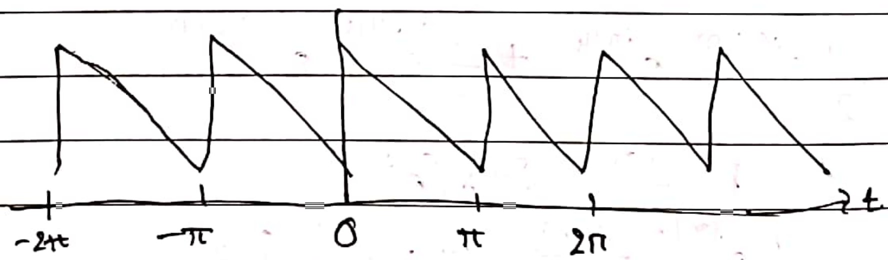
<u>Ans</u>

$$x(n) = a^n u(n)$$

$$n\, x(n) = \frac{d}{d\omega} X(e^{j\omega})$$

1) find the spectrum for gain signal





$$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$a_k = \frac{1}{T}\int x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{\pi}\int_0^{\pi} e^{-t/2} e^{-jk2t}\, dt$$

$$a_k = \frac{1}{\pi}\int_0^{\pi} e^{-t\left(\frac{1}{2}+2kj\right)}\, dt = \frac{1}{\pi}\left[\frac{e^{-t\left(\frac{1}{2}+2kj\right)}}{-\left(\frac{1}{2}+2kj\right)}\right]_0^{\pi}$$

$$= \frac{-1}{\pi\left(t\frac{1}{2}+2Kj\right)} \left. e^{-t\left(\frac{1}{2}+2Kj\right)} \right]_0^{\pi}$$

$$= \frac{-1}{\pi\left(t\frac{1}{2}+2Kj\right)} \left[e^{-\pi\left(\frac{1}{2}+2Kj\right)} - 1 \right]$$

$= \cancel{e^{-\pi\left(\frac{1}{2}+2Kj\right)}}$ when $K=1$ $\qquad K = 0.$

$$a_1 = \frac{-1}{\pi\left(\frac{1}{2}+2j\right)} \left[e^{-\pi\left(\frac{1}{2}+2j\right)} - 1 \right] \qquad a_0 = 0.5042)$$

when

2) Let $x(t)$ be a periodic signals whose fourier series coeffecients are

$$a_K = \begin{cases} 2 & ; K = 0 \\ j\left(\frac{1}{2}\right)^{|K|} & ; \text{otherwise} \end{cases}$$

use fourier series properties to answer the questions

1) is $x(t)$ real

2) is $x(t)$ even

3) is the derivative of $x(t)$ even

Ans if $x(t) \xleftrightarrow{FS} a_K$

1) if $x(t)$ is real $\longleftrightarrow a_K = a_{-K}^*$ conjugate

since $a_K \neq a_{-K}^* \Rightarrow x(t)$ is not real

2) If $x(t)$ is even $\longleftrightarrow a_K = a_{-K}$.

since $a_K = a_{-K} \Rightarrow x(t)$ is even

3) $\frac{d}{dt} x(t) \longleftrightarrow jK\omega_0 a_K$ $\qquad jK\omega_0 a_K \neq jK\omega_0 a_{-K}$

not even.

3) find the exponential series & sketch the corresponding fourier spectrum for a full wave rectified sine wave.

Ans