PES UNIVERSITY
ಪಿಇಎಸ್ ವಿಶ್ವವಿದ್ಯಾಲಯ

MODULE 1

CONVENTIONS AND BASIC ANALYSIS

1.1 Introduction

Today we live in a predominantly electrical world. Electrical technology is a driving force in the changes that are occurring in every engineering discipline. For example, surveying is now done using lasers and electronic range finders.

Circuit analysis is the foundation for electrical technology. An indepth knowledge of circuit analysis provides an understanding of such things as cause and effect, feedback and control and, stability and oscillations. Moreover, the critical importance is the fact that the concepts of electrical circuit can also be applied to economic and social systems. Thus, the applications and ramifications of circuit analysis are immense.

In this chapter, we shall introduce some of the basic quantities that will be used throughout the text. *An electric circuit or electric network is an interconnection of electrical elements linked together in a closed path so that an electric current may continuously flow.* Alternatively, an electric circuit is essentially a pipe-line that facilitates the transfer of charge from one point to another.

1.2 Current, voltage, power and energy

The most elementary quantity in the analysis of electric circuits is the electric charge. Our interest in electric charge is centered around its motion results in an energy transfer. Charge is the intrinsic property of matter responsible for electrical phenomena. The quantity of charge q can be expressed in terms of the charge on one electron. which is -1.602×10^{-19} coulombs. Thus, -1 coulomb is the charge on 6.24×10^{18} electrons. The current flows through a specified area A and is defined by the electric charge passing through that area per unit time. Thus we define q as the charge expressed in coulombs.

Charge is the quantity of electricity responsible for electric phenomena.

The time rate of change constitutes an electric current. Mathemetically, this relation is expressed as

$$i(t) = \frac{dq(t)}{dt} \tag{1.1}$$

or

$$q(t) = \int_{-\infty}^{t} i(x)dx \tag{1.2}$$

The unit of current is ampere(A); an ampere is 1 coulomb per second.

Current is the time rate of flow of electric charge past a given point.

The basic variables in electric circuits are current and voltage. If a current flows into terminal a of the element shown in Fig. 1.1, then a voltage or potential difference exists between the two terminals a and b. Normally, we say that a voltage exists across the element.

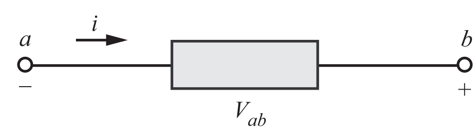


Figure 1.1 Voltage across an element

The voltage across an element is the work done in moving a positive charge of 1 coulomb from first terminal through the element to second terminal. The unit of voltage is volt, V or Joules per coulomb.

We have defined voltage in Joules per coulomb as the energy required to move a positive charge of 1 coulomb through an element. If we assume that we are dealing with a differential amount of charge and energy,

then

$$v = \frac{dw}{dq} \tag{1.3}$$

Multiplying both the sides of equation (1.3) by the current in the element gives

$$vi = \frac{dw}{dq}\left(\frac{dq}{dt}\right) \quad \Rightarrow \quad \frac{dw}{dt} = p \tag{1.4}$$

which is the time rate of change of energy or power measured in Joules per second or watts (W).

p could be either positive or negative. Hence it is imperative to give sign convention for power. If we use the signs as shown in Fig. 1.2., the current flows out of the terminal indicated by x, which shows the positive sign for the voltage. In this case, the element is said to provide energy to the charge as it moves through. Power is then provided by the element.

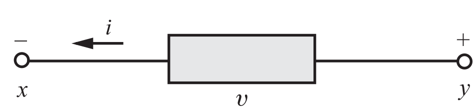


Figure 1.2 An element with the current leaving from the terminal with a positive voltage sign

Conversely, power absorbed by an element is $p = vi$, when i is entering through the positive voltage terminal.

Energy is the capacity to perform work. Energy and power are related to each other by the following equation:

$$Energy = w = \int_{-\infty}^{t} p\ dt$$

EXAMPLE 1.1

Consider the circuit shown in Fig. 1.3 with $v = 8e^{-}t$ V and $i = 20e^{-t}$ A for $t \geq 0$. Find the power absorbed and the energy supplied by the element over the first second of operation. we assume that v and i are zero for $t < 0$.

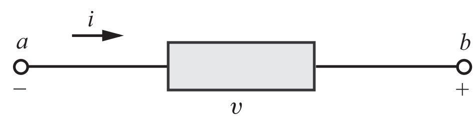


Figure 1.3

SOLUTION

The power supplied is

$$\begin{aligned} p &= vi = (8e^{-t})(20e^{-t}) \\ &= 160e^{-2t}\ \text{W} \end{aligned}$$

The element is providing energy to the charge flowing through it.

The energy supplied during the first seond is

$$\begin{aligned} w &= \int_0^1 p\ dt = \int_0^1 160e^{-2t} dt \\ &= 80(1 - e^{-2}) = \mathbf{69.17\ Joules} \end{aligned}$$

1.3 Linear, active and passive elements

A linear element is one that satisfies the principle of superposition and homogeneity.
In order to understand the concept of superposition and homogeneity, let us consider the element shown in Fig. 1.4.

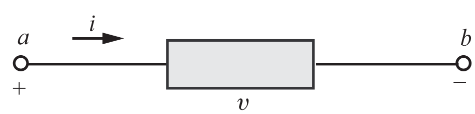


Figure 1.4 An element with excitation i and response v

The excitation is the current, i and the response is the voltage, v. When the element is subjected to a current i_1, it provides a response v_1. Furthermore, when the element is subjected to a current i_2, it provides a response v_2. If the principle of superposition is true, then the excitation $i_1 + i_2$ must produce a response $v_1 + v_2$.

Also, it is necessary that the magnitude scale factor be preserved for a linear element. If the element is subjected to an excitation βi where β is a constant multiplier, then if principle of homogencity is true, the response of the element must be βv.

We may classify the elements of a circuir into categories, passive and active, depending upon whether they absorb energy or supply energy.

1.3.1 Passive Circuit Elements

An element is said to be passive if the total energy delivered to it from the rest of the circuit is either zero or positive.

Then for a passive element, with the current flowing into the positive (+) terminal as shown in Fig. 1.4 this means that

$$w = \int_{-\infty}^{t} vi \ dt \geq 0$$

Examples of passive elements are resistors, capacitors and inductors.

1.3.1.A Resistors

Resistance is the physical property of an element or device that impedes the flow of current; it is represented by the symbol R. Resistance of a wire element is calculated using the relation:

Figure 1.5 Symbol for a resistor R

$$R = \frac{\rho l}{A} \tag{1.5}$$

where A is the cross-sectional area, ρ the resistivity, and l the length of the wire. The practical unit of resistance is ohm and represented by the symbol Ω.

An element is said to have a resistance of 1 ohm, if it permits 1A of current to flow through it when 1V is impressed across its terminals.

Ohm's law, which is related to voltage and current, was published in 1827 as

$$v = Ri \tag{1.6}$$

or

$$R = \frac{v}{i}$$

where v is the potential across the resistive element, i the current through it, and R the resistance of the element.

The power absorbed by a resistor is given by

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{1.7}$$

Alternatively,

$$p = vi = (iR)i = i^2 R \tag{1.8}$$

Hence, the power is a nonlinear function of current i through the resistor or of the voltage v across it.

The equation for energy absorbed by or delivered to a resistor is

$$w = \int_{-\infty}^{t} p d\tau = \int_{-\infty}^{t} i^2 R \ d\tau \tag{1.9}$$

Since i^2 is always positive, the energy is always positive and the resistor is a passive element.

1.3.1.B Inductors

Whenever a time-changing current is passed through a coil or wire, the voltage across it is proportional to the rate of change of current through the coil. This proportional relationship may be expressed by the equation

$$v = L\frac{di}{dt} \tag{1.10}$$

Where L is the constant of proportionality known as inductance and is measured in Henrys (H). Remember v and i are both funtions of time.
Let us assume that the coil shown in Fig. 1.6 has N turns and the core material has a high permeability so that the magnetic fluk ϕ is connected within the area A. The changing flux creates an induced voltage in each turn equal to the derivative of the flux ϕ, so the total voltage v across N turns is

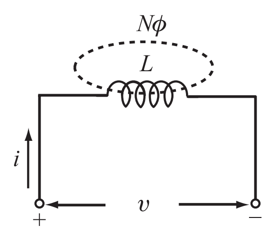


Figure 1.6 Model of the inductor

$$v = N\frac{d\phi}{dt} \tag{1.11}$$

Since the total flux $N\phi$ is proportional to current in the coil, we have

$$N\phi = Li \tag{1.12}$$

Where L is the constant of proportionality. Substituting equation (1.12) into equation(1.11), we get

$$v = L\frac{di}{dt}$$

The power in an inductor is

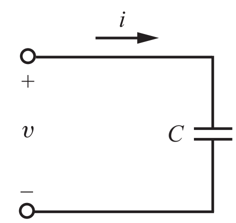
$$p = vi = L\left(\frac{di}{dt}\right)i$$

The energy stored in the inductor is

$$\begin{aligned} w &= \int_{-\infty}^{t} p\ d\tau \\ &= L\int_{i(-\infty)}^{i(t)} i\ di = \frac{1}{2}Li^2 \text{ Joules} \end{aligned} \tag{1.13}$$

Note that when $t = -\infty, i(-\infty) = 0$. Also note that $w(t) \geq 0$ for all $i(t)$, so the inductor is a passive element. The inductor does not generate energy, but only stores energy.

1.3.1.C Capacitors



1.7 Circuit symbol for a capacitor

A capacitor is a two-terminal element that is a model of a device consisting of two conducting plates seperated by a dielectric material. Capacitance is a measure of the ability of a deivce to store energy in the form of an electric field. ***Capacitance is defined as the ratio of the charge stored to the voltage difference between the two conducting plates or wires,***

$$C = \frac{q}{v}$$

The current through the capacitor is given by

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \tag{1.14}$$

The energy stored in a capacitor is

$$w = \int_{-\infty}^{t} vi \; d\tau$$

Remember that v and i are both functions of time and could be written as $v(t)$ and $i(t)$.

Since
$$i = C\frac{dv}{dt}$$

we have
$$\begin{aligned} w &= \int_{-\infty}^{t} v \; C\frac{dv}{d\tau} \; d\tau \\ &= C \int_{v(-\infty)}^{v(t)} v \; dv = \frac{1}{2}Cv^2 \Big|_{v(-\infty)}^{v(t)} \end{aligned}$$

Since the capacitor was uncharged at $t = -\infty$, $v(-\infty) = 0$.

Hence
$$\begin{aligned} w &= w(t) \\ &= \frac{1}{2}Cv^2(t) \text{ Joules} \end{aligned} \tag{1.15}$$

Since $q = Cv$, we may write

$$w(t) = \frac{1}{2C}q^2(t) \text{ Joules} \tag{1.16}$$

Note that since $w(t) \geq 0$ for all values of $v(t)$, the element is said to be a passive element.

1.3.2 Active Circuit Elements (Energy Sources)

An active two-terminal element that supplies energy to a circuit is a source of energy. An ideal voltage source is a circuit element that maintains a prescribed voltage across the terminals regardless of the current flowing in those terminals. Similarly, an ideal current source is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

These circuit elements do not exist as practical devices, they are only idealized models of actual voltage and current sources.

Ideal voltage and current sources can be further described as either independent sources or dependent sources. An independent source establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone. In contrast, a dependent source establishes a voltage or current whose value depends on the value of the voltage or current elsewhere in the circuit. We cannot specify the value of a dependent source, unless you know the value of the voltage or current on which it depends.

The circuit symbols for ideal independent sources are shown in Fig. 1.8.(a) and (b). Note that a circle is used to represent an independent source. The circuit symbols for dependent sources are shown in Fig. 1.8.(c), (d), (e) and (f). A diamond symbol is used to represent a dependent source.

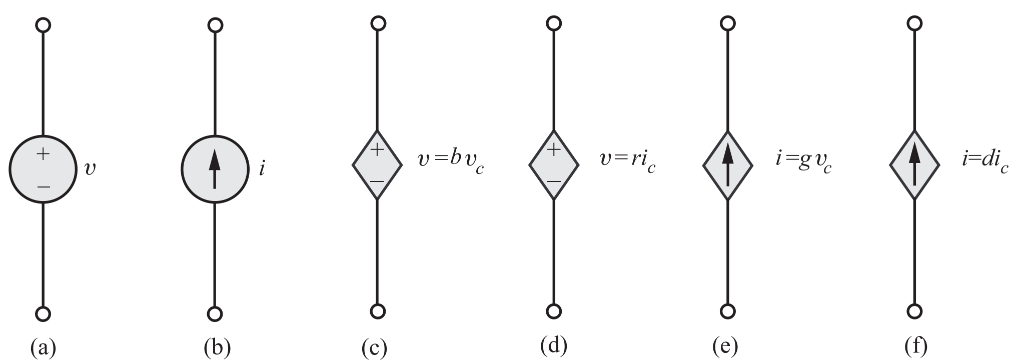


Figure 1.8 (a) An ideal independent voltage source
(b) An ideal independent current source
(c) voltage controlled voltage source
(d) current controlled voltage source
(e) voltage controlled current source
(f) current controlled current source

1.4 Unilateral and bilateral networks

A Unilateral network is one whose properties or characteristics change with the direction. An example of unilateral network is the semiconductor diode, which conducts only in one direction.

A bilateral network is one whose properties or characteristics are same in either direction. For example, a transmission line is a bilateral network, because it can be made to perform the function equally well in either direction.

1.5 Network simplification techniques

In this section, we shall give the formula for reducing the networks consisting of resistors connected in series or parallel.

1.5.1 Resistors in Series

When a number of resistors are connected in series, the equivalent resistance of the combination is given by

$$R = R_1 + R_2 + \cdots + R_n \tag{1.17}$$

Thus the total resistance is the algebraic sum of individual resistances.

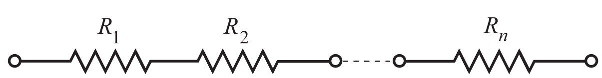


Figure 1.9 Resistors in series

1.5.2 Resistors in Parallel

When a number of resistors are connected in parallel as shown in Fig. 1.10, then the equivalent resistance of the combination is computed as follows:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} \tag{1.18}$$

Thus, the reciprocal of a equivalent resistance of a parallel combination is the sum of the reciprocal of the individual resistances. Reciprocal of resistance is conductance and denoted by G. Consequently the equivalent conductance,

$$G = G_1 + G_2 + \cdots + G_n$$

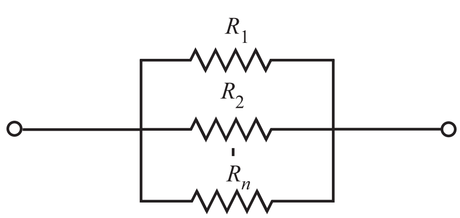


Figure 1.10 Resistors in parallel

1.5.3 Division of Current in a Parallel Circuit

Consider a two branch parallel circuit as shown in Fig. 1.11. The branch currents I_1 and I_2 can be evaluated in terms of total current I as follows:

$$I_1 = \frac{IR_2}{R_1 + R_2} = \frac{IG_1}{G_1 + G_2} \tag{1.19}$$

$$I_2 = \frac{IR_1}{R_1 + R_2} = \frac{IG_2}{G_1 + G_2} \tag{1.20}$$

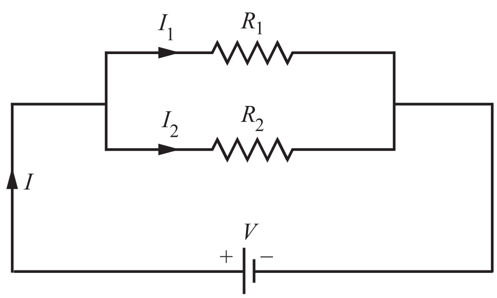


Figure 1.11 Current division in a parallel circuit

That is, current in one branch equals the total current multiplied by the resistance of the other branch and then divided by the sum of the resistances.

EXAMPLE 1.2

The current in the 6Ω resistor of the network shown in Fig. 1.12 is 2A. Determine the current in all branches and the applied voltage.

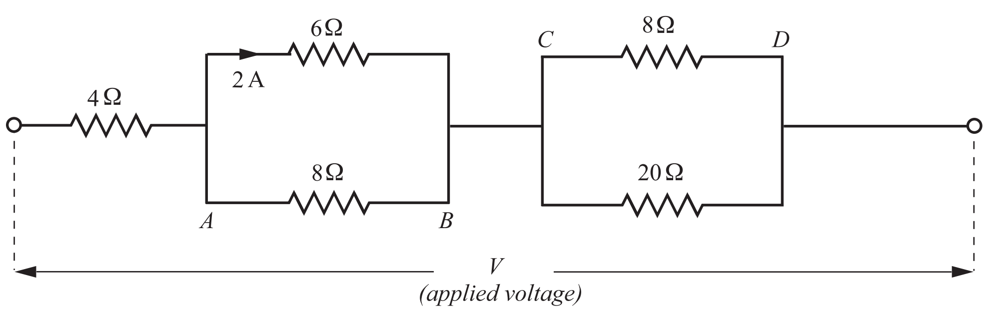


Figure 1.12

SOLUTION

Voltage across $\quad 6\Omega = 6 \times 2$

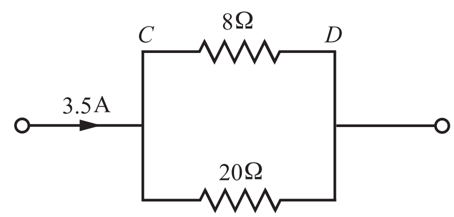
$\qquad = 12$ volts

Since 6Ω and 8Ω are connected in parallel, voltage across $8\Omega = 12$ volts.

$$\left.\begin{array}{r}\text{Therefore, the current through}\\ 8\Omega\,\text{(between A and B)}\end{array}\right\} = \frac{12}{8} = \mathbf{1.5\ A}$$

Total current in the circuit $= 2 + 1.5 = \mathbf{3.5\ A}$

Current in the 4Ω branch $= 3.5$ A



$$\begin{aligned}
\text{Current through } 8\Omega \text{ (betwen C and D)} &= 3.5 \times \frac{20}{20+8}\\
&= \mathbf{2.5\ A}\\
\text{Therefore, current through}\qquad 20\Omega &= 3.5 - 2.5\\
&= \mathbf{1A}\\
\text{Total resistance of the circuit} &= 4 + \frac{6\times 8}{6+8} + \frac{8\times 20}{8+20}\\
&= 13.143\Omega\\
\text{Therefore applied voltage,}\qquad V &= 3.5 \times 13.143 \quad (\because\ V = IR)\\
&= \mathbf{46\ Volts}
\end{aligned}$$

EXAMPLE 1.3

Find the value of R in the circuit shown in Fig. 1.13.

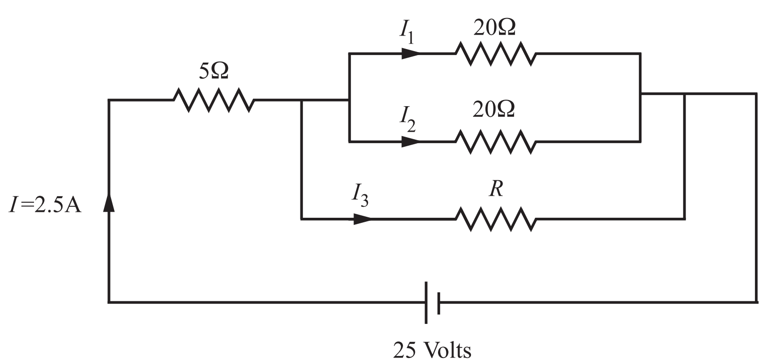


Figure 1.13

SOLUTION

Voltage across $5\Omega = 2.5 \times 5 = 12.5$ volts

Hence the voltage across the parallel circuit $= 25 - 12.5 = 12.5$ volts

$$\begin{aligned}
\text{Current through}\qquad 20\Omega &= I_1 \text{ or } I_2\\
&= \frac{12.5}{20} = 0.625\text{A}
\end{aligned}$$

Therefore, current through $R = I_3 = I - I_1 - I_2$

$$= 2.5 - 0.625 - 0.625$$

$$= 1.25 \text{ Amps}$$

Hence, $$\mathrm{R} = \frac{12.5}{1.25} = 10\Omega$$

1.6 Kirchhoff's laws

In the preceeding section, we have seen how simple resistive networks can be solved for current, resistance, potential etc using the concept of Ohm's law. But as the network becomes complex, application of Ohm's law for solving the networks becomes tedious and hence time consuming. For solving such complex networks, we make use of Kirchhoff's laws. Gustav Kirchhoff (1824-1887), an eminent German physicist, did a considerable amount of work on the principles governing the behaviour of eletric circuits. He gave his findings in a set of two laws: (i) current law and (ii) voltage law, which together are known as Kirchhoff's laws. Before proceeding to the statement of these two laws let us familarize ourselves with the following definitions encountered very often in the world of electrical circuits:

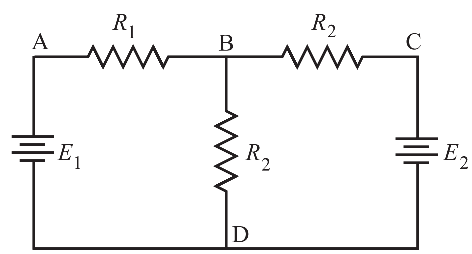


Figure 1.14 A simple resistive network for difining various circuit terminologies

(i) *Node*: A node of a network is an equi-potential surface at which two or more circuit elements are joined. Referring to Fig. 1.14, we find that A,B,C and D qualify as nodes in respect of the above definition.

(ii) *Junction*: A junction is that point in a network, where three or more circuit elements are joined. In Fig. 1.14, we find that B and D are the junctions.

(iii) *Branch*: A branch is that part of a network which lies between two junction points. In Fig. 1.14, BAD,BCD and BD qualify as branches.

(iv) *Loop*: A loop is any closed path of a network. Thus, in Fig. 1.14, ABDA,BCDB and ABCDA are the loops.

(v) *Mesh*: A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 1.14, ABDA and BCDB are the examples of mesh. Once ABDA and BCDB are taken as meshes, the loop ABCDA does not qualify as a mesh, because it contains loops ABDA and BCDB.

1.6.1 Kirchhoff's Current Law

The first law is Kirchhoff's current law(KCL), which states that the algebraic sum of currents entering any node is zero.

Let us consider the node shown in Fig. 1.15. The sum of the currents entering the node is

$$-i_a + i_b - i_c + i_d = 0$$

Note that we have $-i_a$ since the current i_a is leaving the node. If we multiply the foregoing equation by -1, we obtain the expression

$$i_a - i_b + i_c - i_d = 0$$

which simply states that the algebraic sum of currents leaving a node is zero. Alternately, we can write the equation as

$$i_b + i_d = i_a + i_c$$

which states that the sum of currents entering a node is equal to the sum of currents leaving the node. If the sum of the currents entering a node were not equal to zero, then the charge would be accumulating at a node. However, a node is a perfect conductor and cannot accumulate or store charge. Thus, the sum of currents entering a node is equal to zero.

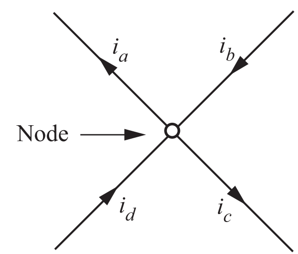


Figure 1.15 Currents at a node

1.6.2 Kirchhoff's Voltage Law

Kirchhoff's voltage law(KVL) states that the algebraic sum of voltages around any closed path in a circuit is zero.

In general, the mathematical representation of Kirchhoff's voltage law is

$$\sum_{j=1}^{N} v_j(t) = 0$$

where $v_j(t)$ is the voltage across the j^{th} branch (with proper reference direction) in a loop containing N voltages.

In Kirchhoff's voltage law, the algebraic sign is used to keep track of the voltage polarity. In other words, as we traverse the circuit, it is necessary to sum the increases and decreases in voltages to zero. Therefore, it is important to keep track of whether the voltage is increasing or decreasing as we go through each element. We will adopt a policy of considering the increase in voltage as *negative* and a decrease in voltage as *positive*.

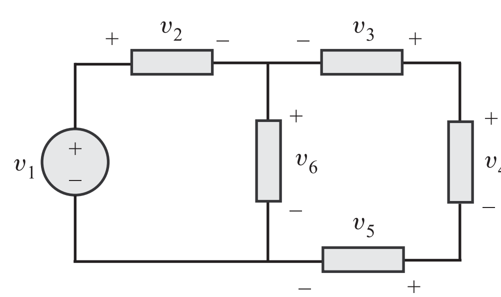


Figure 1.16 Circuit with three closed paths

Consider the circuit shown in Fig. 1.16, where the voltage for each element is identified with its sign. The ideal wire used for connecting the components has zero resistance, and thus the voltage across it is equal to zero. The sum of voltages around the loop incorporating v_6, v_3, v_4 and v_5 is

$$-v_6 - v_3 + v_4 + v_5 = 0$$

The sum of voltages around a loop is equal to zero. A circuit loop is a conservative system, meaning that the work required to move a unit charge around any loop is zero.

However, it is important to note that not all electrical systems are conservative. Example of a nonconservative system is a radio wave broadcasting system.

EXAMPLE 1.4

Consider the circuit shown in Fig. 1.17. Find each branch current and voltage across each branch when $R_1 = 8\Omega, v_2 = -10$ volts $i_3 = 2$A and $R_3 = 1\Omega$. Also find R_2.

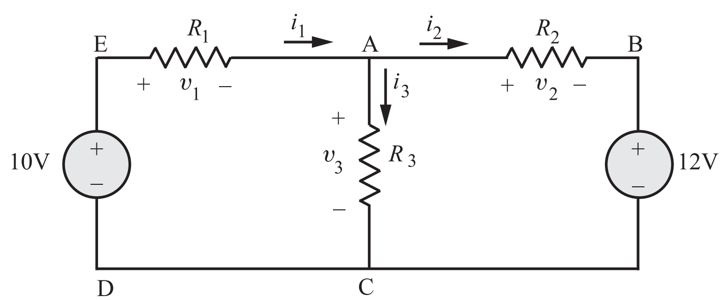


Figure 1.17

SOLUTION

Applying KCL (Kirchhoff's Current Law) at node A, we get

$$i_1 = i_2 + i_3$$

and using Ohm's law for R_3, we get

$$v_3 = R_3 i_3 = 1(2) = \mathbf{2V}$$

Applying KVL (Kirchhoff's Voltage Law) for the loop EACDE, we get

$$-10 + v_1 + v_3 = 0$$

$$\Rightarrow \quad v_1 = 10 - v_3 = \mathbf{8V}$$

Ohm's law for R_1 is

$$v_1 = i_1 R_1$$

$$\Rightarrow \quad i_1 = \frac{v_1}{R_1} = \mathbf{1A}$$

Hence, $$i_2 = i_1 - i_3$$
$$= 1 - 2 = -\mathbf{1A}$$

From the circuit, $$v_2 = R_2 i_2$$

$$\Rightarrow \quad R_2 = \frac{v_2}{i_2} = \frac{-10}{-1} = \mathbf{10\Omega}$$

EXAMPLE 1.5

Referring to Fig. 1.18, find the following:

(a) i_x if $i_y = 2\text{A}$ and $i_z = 0\text{A}$

(b) i_y if $i_x = 2\text{A}$ and $i_z = 2i_y$

(c) i_z if $i_x = i_y = i_z$

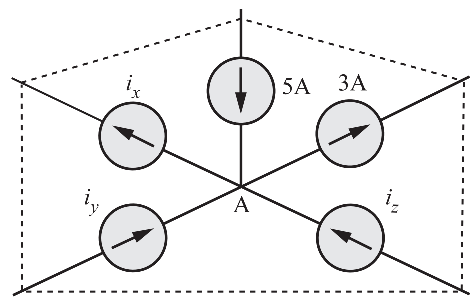


Figure 1.18

SOLUTION

Applying KCL at node A, we get

$$5 + i_y + i_z = i_x + 3$$

(a) $i_x = 2 + i_y + i_z$

$= 2 + 2 + 0 = \mathbf{4A}$

(b) $i_y = 3 + i_x - 5 - i_z$

$= -2 + 2 - 2i_y$

$\Rightarrow \quad i_y = \mathbf{0A}$

(c) This situation is not possible, since i_x and i_z are in opposite directions. The only possibility is $i_z = 0$, and this cannot be allowed, as KCL will not be satisfied ($5 \neq 3$).

EXAMPLE 1.6

Refer the Fig. 1.19.

(a) Calculate v_y if $i_z = -3\text{A}$

(b) What voltage would you need to replace 5 V source to obtain $v_y = -6$ V if $i_z = 0.5\text{A}$?

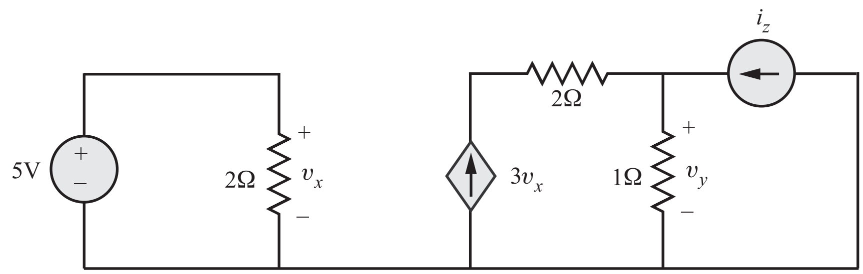


Figure 1.19

SOLUTION

(a) $v_y = 1\ (3\ v_x + i_z\ )$

Since $\quad v_x = 5\text{V}$ and $i_z = -3\text{A}$,

we get $\quad v_y = 3(5) - 3 = \mathbf{12V}$

(b)
$$v_y = 1\ (3\ v_x + i_z) = -6$$
$$= 3\ v_x + 0.5$$
$$\Rightarrow \qquad 3\ v_x = -6.5$$

Hence, $\quad v_x = \mathbf{-2.167\ volts}$

EXAMPLE 1.7

For the circuit shown in Fig. 1.20, find i_1 and v_1, given $R_3 = 6\Omega$.

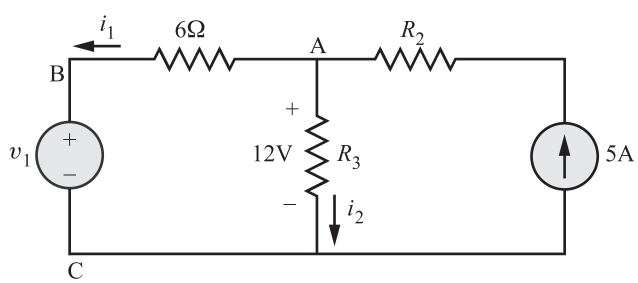


Figure 1.20

SOLUTION

Applying KCL at node A, we get

$$-i_1 - i_2 + 5 = 0$$

From Ohm's law, $\quad 12 = i_2 R_3$

$$\Rightarrow \qquad i_2 = \frac{12}{R_3} = \frac{12}{6} = 2\text{A}$$

Hence, $\quad i_1 = 5 - i_2 = \mathbf{3A}$

Applying KVL clockwise to the loop CBAC, we get

$$-v_1 - 6i_1 + 12 = 0$$
$$\Rightarrow \qquad v_1 = 12 - 6i_1$$
$$= 12 - 6(3) = -\mathbf{6volts}$$

EXAMPLE 1.8

Use Ohm's law and Kirchhoff's law to evaluate (a) v_x, (b) i_{in}, (c) I_s and (d) the power provided by the dependent source in Fig 1.21.

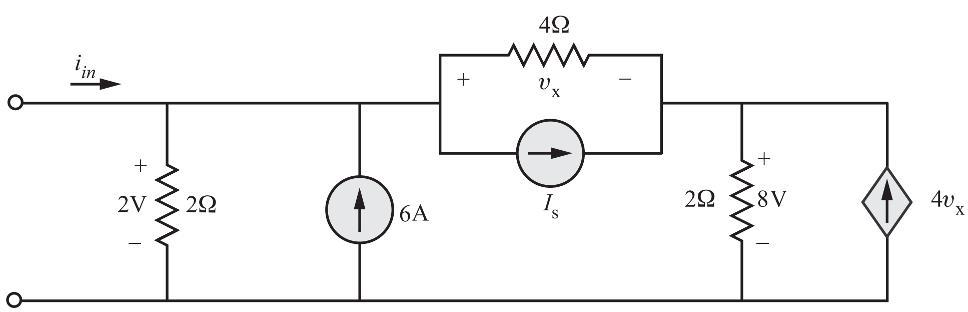


Figure 1.21

SOLUTION

(a) *Applying KVL*, (Referring Fig. 1.21 (a)) we get

$$-2 + v_x + 8 = 0$$
$$\Rightarrow \qquad v_x = -\mathbf{6V}$$

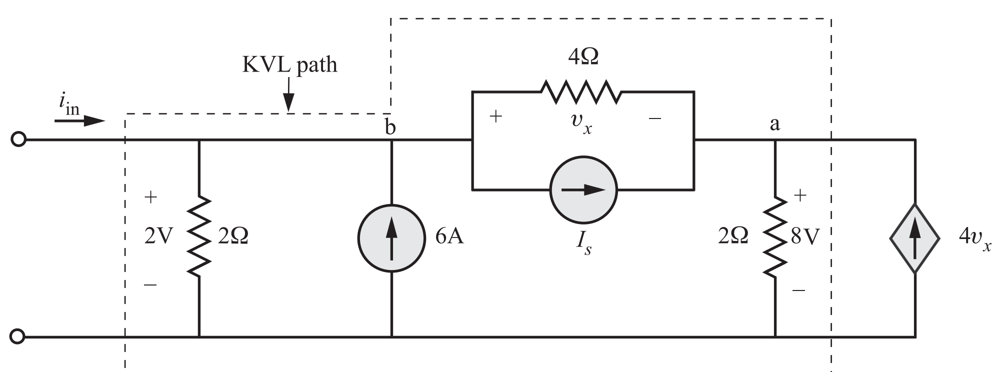


Figure 1.21(a)

(b) *Applying KCL at node a*, we get

$$I_s + 4v_x + \frac{v_x}{4} = \frac{8}{2}$$
$$\Rightarrow \quad I_s + 4(-6) - \frac{6}{4} = 4$$
$$\Rightarrow \quad I_s - 24 - 1.5 = 4$$
$$\Rightarrow \quad I_s = \mathbf{29.5A}$$

(c) *Applying KCL at node b*, we get

$$i_{in} = \frac{2}{2} + I_s + \frac{v_x}{4} - 6$$
$$\Rightarrow \quad i_{in} = 1 + 29.5 - \frac{6}{4} - 6 = \mathbf{23A}$$

(d) The power supplied by the dependent current source = 8 $(4v_x) = 8 \times 4 \times -6 = \mathbf{-192W}$

EXAMPLE 1.9

Find the current i_2 and voltage v for the circuit shown in Fig. 1.22.

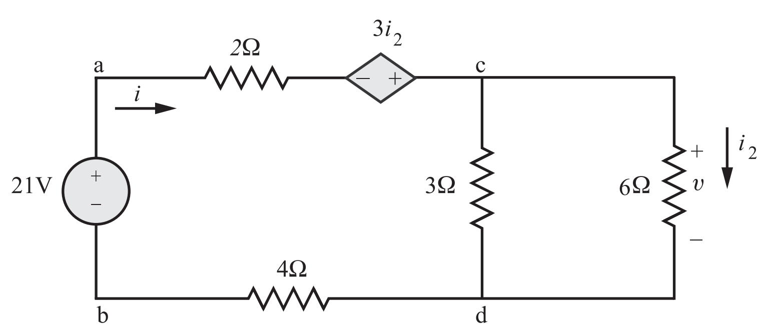


Figure 1.22

SOLUTION

From the network shown in Fig. 1.22, $i_2 = \dfrac{v}{6}$

The two parallel resistors may be reduced to

$$R_p = \frac{3 \times 6}{3 + 6} = 2\Omega$$

Hence, the total series resistance around the loop is

$$R_s = 2 + R_p + 4$$
$$= 8\Omega$$

Applying KVL around the loop, we have

$$-21 + 8i - 3i_2 = 0 \tag{1.21}$$

Using the principle of current division,

$$\begin{aligned} i_2 &= \frac{iR_2}{R_1 + R_2} = \frac{i \times 3}{3+6} \\ &= \frac{3i}{9} = \frac{i}{3} \\ \Rightarrow \qquad i &= 3i_2 \end{aligned} \tag{1.22}$$

Substituting equation (1.22) in equation (1.21), we get

$$-21 + 8(3i_2) - 3i_2 = 0$$

Hence, $$i_2 = \mathbf{1A}$$

and $$v = 6i_2 = \mathbf{6V}$$

EXAMPLE 1.10

Find the current i_2 and voltage v for resistor R in Fig. 1.23 when $R = 16\Omega$.

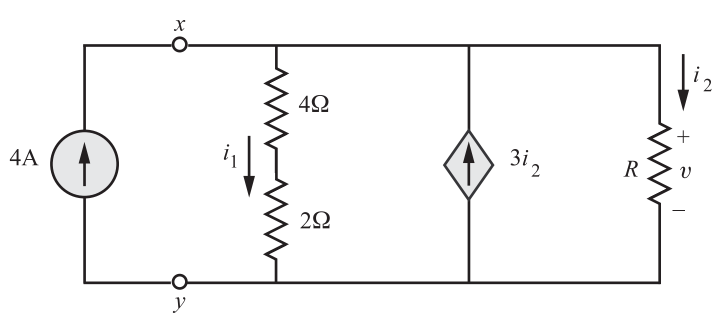


Figure 1.23

SOLUTION

Applying KCL at node x, we get

$$4 - i_1 + 3i_2 - i_2 = 0$$

Also, $$i_1 = \frac{v}{4+2} = \frac{v}{6}$$

$$i_2 = \frac{v}{R} = \frac{v}{16}$$

Hence, $$4 - \frac{v}{6} + 3 \times \frac{v}{16} - \frac{v}{16} = 0$$

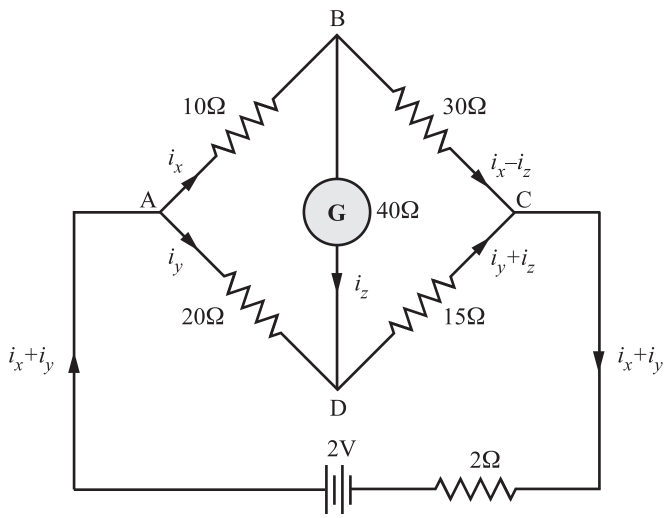
$$\Rightarrow \qquad v = \mathbf{96 volts}$$

and $$i_2 = \frac{v}{6} = \frac{96}{16} = \mathbf{6A}$$

EXAMPLE 1.11

A wheatstone bridge ABCD is arranged as follows: AB = 10Ω, BC = 30Ω, CD = 15Ω and DA = 20Ω. A 2V battery of internal resistance 2Ω is connected between points A and C with A being positive. A galvanometer of resistance 40Ω is connected between B and D. Find the magnitude and direction of the galvanometer current.

SOLUTION



Applying KVL clockwise to the loop ABDA, we get

$$10i_x + 40i_z - 20i_y = 0$$
$$\Rightarrow \quad 10i_x - 20i_y + 40i_z = 0 \tag{1.23}$$

Applying KVL clockwise to the loop BCDB, we get

$$30(i_x - i_z) - 15(i_y + i_z) - 40i_z = 0$$
$$\Rightarrow \quad 30i_x - 15i_y - 85i_z = 0 \tag{1.24}$$

Finally, *applying KVL clockwise to the loop ADCA*, we get

$$20i_y + 15(i_y + i_z) + 2(i_x + i_y) - 2 = 0$$
$$\Rightarrow \quad 2i_x + 37i_y + 15i_z = 2 \tag{1.25}$$

Putting equations (1.23),(1.24) and (1.25) in matrix form, we get

$$\begin{bmatrix} 10 & -20 & 40 \\ 30 & -15 & -85 \\ 2 & 37 & 15 \end{bmatrix} \begin{bmatrix} i_x \\ i_y \\ i_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Using Cramer's rule, we find that

$$i_z = \mathbf{0.01\ A\ (Flows\ from\ B\ to\ D)}$$

1.7 Multiple current source networks

Let us now learn how to reduce a network having multiple current sources and a number of resistors in parallel. Consider the circuit shown in Fig. 1.24. We have assumed that the upper node is $v(t)$ volts positive with respect to the lower node. Applying KCL to upper node yields

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

$$\Rightarrow \quad i_1(t) - i_3(t) + i_4(t) - i_6(t) = i_2(t) + i_5(t) \tag{1.26}$$

$$\Rightarrow \quad i_o(t) = i_2(t) + i_5(t) \tag{1.27}$$

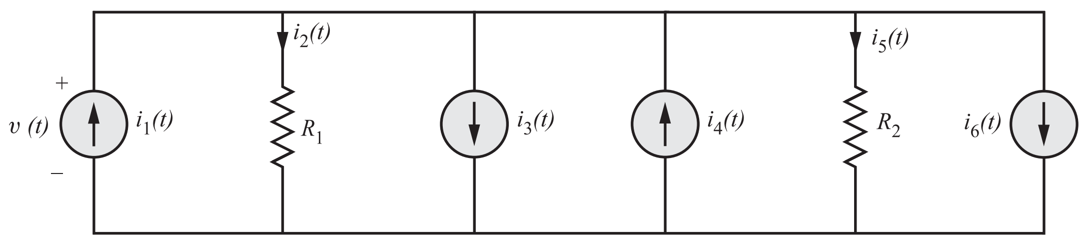


Figure 1.24 Multiple current source network

where $i_o(t) = i_1(t) - i_3(t) + i_4(t) - i_6(t)$ is the algebraic sum of all current sources present in the multiple source network shown in Fig. 1.24. As a consequence of equation (1.27), the network of Fig. 1.24 is effectively reduced to that shown in Fig. 1.25. Using Ohm's law, the currents on the right side of equation (1.27) can be expressed in terms of the voltage and individual resistance so that KCL equation reduces to

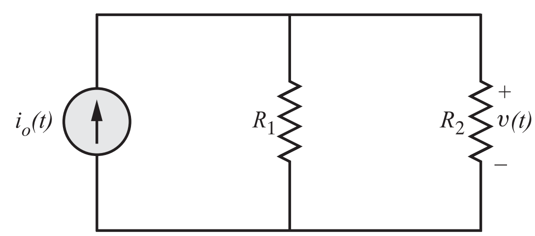


Figure 1.25 Equivalent circuit

$$i_o(t) = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v(t)$$

Thus, we can reduce a multiple current source network into a network having only one current source.

1.8 Source transformations

Source transformation is a procedure which transforms one source into another while retaining the terminal characteristics of the original source.

Source transformation is based on the concept of equivalence. An equivalent circuit is one whose terminal characteristics remain identical to those of the original circuit. The term equivalence as applied to circuits means an identical effect at the terminals, but not within the equivalent circuits themselves.

We are interested in transforming the circuit shown in Fig. 1.26 to a one shown in Fig. 1.27.

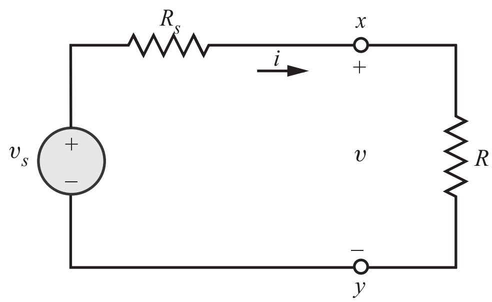


Figure 1.26 Voltage source connected to an external resistance R

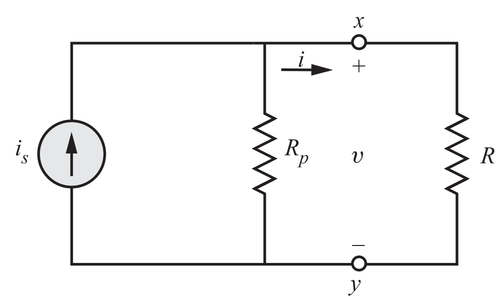


Figure 1.27 Current source connected to an external resistance R

We require both the circuits to have the equivalence or same characteristics between the terminals x and y for all values of external resistance R. We will try for equivanlence of the two circuits between terminals x and y for two limiting values of R namely $R = 0$ and $R = \infty$. When $R = 0$, we have a short circuit across the terminals x and y. It is obligatory for the short circuit to be same for each circuit. The short circuit current of Fig. 1.26 is

$$i_s = \frac{v_s}{R_s} \tag{1.28}$$

The short circuit current of Fig. 1.27 is i_s. This enforces,

$$i_s = \frac{v_s}{R_s} \tag{1.29}$$

When $R = \infty$, from Fig. 1.26 we have $v_{xy} = v_s$ and from Fig. 1.27 we have $v_{xy} = i_s R_p$. Thus, for equivalence, we require that

$$v_s = i_s R_p \tag{1.30}$$

Also from equation (1.29), we require $i_s = \dfrac{v_s}{R_s}$. Therefore, we must have

$$v_s = \left(\frac{v_s}{R_s}\right) R_p$$

$$\Rightarrow \qquad R_s = R_p \tag{1.31}$$

Equations(1.29) and (1.31) must be true simulaneously for both the circuits for the two sources to be equivalent. We have derived the conditions for equivalence of two circuits shown in Figs. 1.26 and 1.27 only for two extreme values of R, namely $R = 0$ and $R = \infty$. However, the equality relationship holds good for all R as explained below.

Applying KVL to Fig. 1.26, we get

$$v_s = iR_s + v$$

Dividing by R_s gives

$$\frac{v_s}{R_s} = i + \frac{v}{R_s} \tag{1.32}$$

If we use KCL for Fig. 1.27, we get

$$i_s = i + \frac{v}{R_p} \tag{1.33}$$

Thus two circuits are equal when

$$i_s = \frac{v_s}{R_s} \text{ and } R_s = R_p$$

Transformation procedure: If we have embedded within a network, a current source i in parallel with a resistor R can be replaced with a voltage source of value $v = iR$ in series with the resistor R.

The reverse is also true; that is, a voltage source v in series with a resistor R can be replaced with a current source of value $i = \dfrac{v}{R}$ in parallel with the resistor R. Parameters within the circuit are unchanged under these transformation.

EXAMPLE 1.12

A circuit is shown in Fig. 1.28. Find the current i by reducing the circuit to the right of the terminals $x - y$ to its simplest form using source transformations.

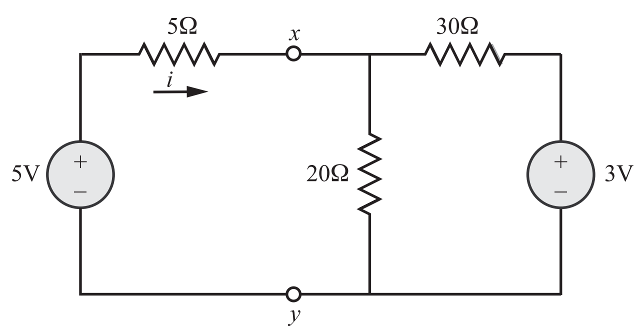
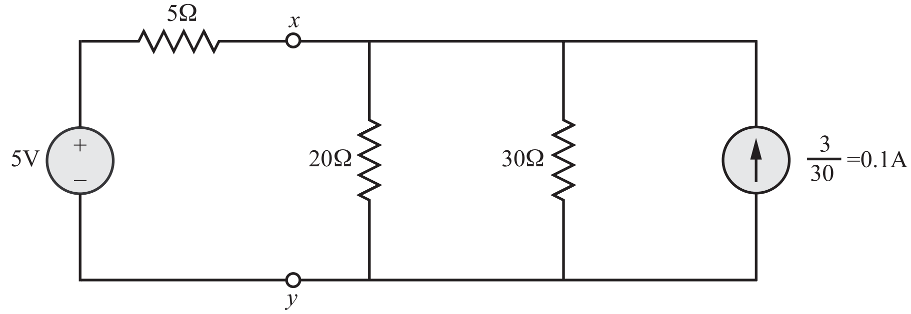


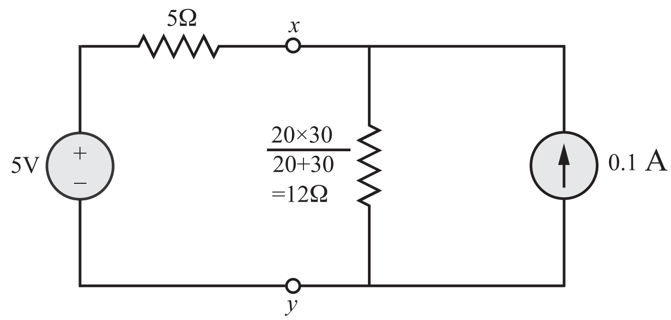
Figure 1.28

SOLUTION

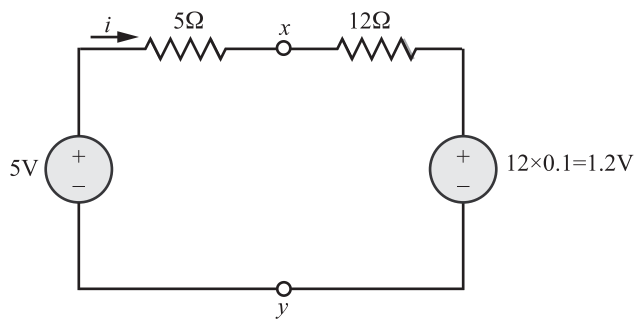
The first step in the analysis is to transform 30 ohm resistor in series with a 3 V source into a current source with a parallel resistance and we get:



Reducing the two parallel resistances, we get:



The parallel resistance of 12Ω and the current source of 0.1A can be transformed into a voltage source in series with a 12 ohm resistor.



Applying KVL, we get

$$5i + 12i + 1.2 - 5 = 0$$

$$\Rightarrow \quad 17i = 3.8$$

$$\Rightarrow \quad \boldsymbol{i = 0.224\text{A}}$$

EXAMPLE 1.13

Find current i_1 using source transformation for the circuit shown Fig. 1.29.

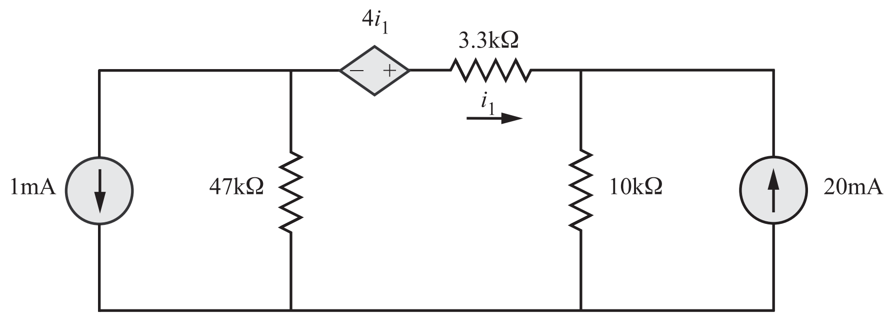


Figure 1.29

SOLUTION

Converting 1 mA current source in parallel with 47kΩ resistor and 20 mA current source in parallel with 10kΩ resistor into equivalent voltage sources, the circuit of Fig. 1.29 becomes the circuit shown in Fig. 1.29(a).

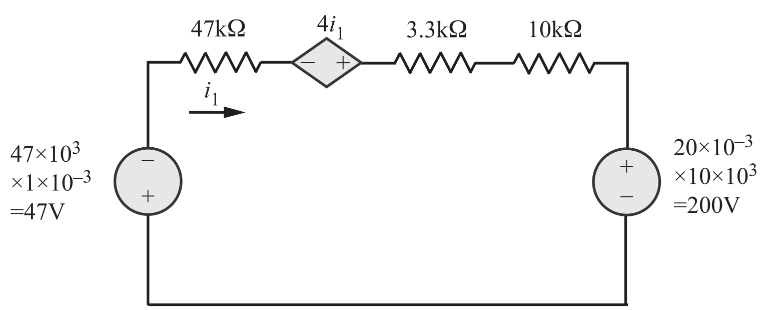


Figure 1.29(a)

Please note that for each voltage source, "+" corresponds to its corresponding current source's arrow head.

Using KVL to the above circuit,

$$47 + 47 \times 10^3 i_1 - 4i_1 + 13.3 \times 10^3 i_1 + 200 = 0$$

Solving, we find that

$$i_1 = \mathbf{-4.096\ mA}$$

EXAMPLE 1.14

Use source transformation to convert the circuit in Fig. 1.30 to a single current source in parallel with a single resistor.

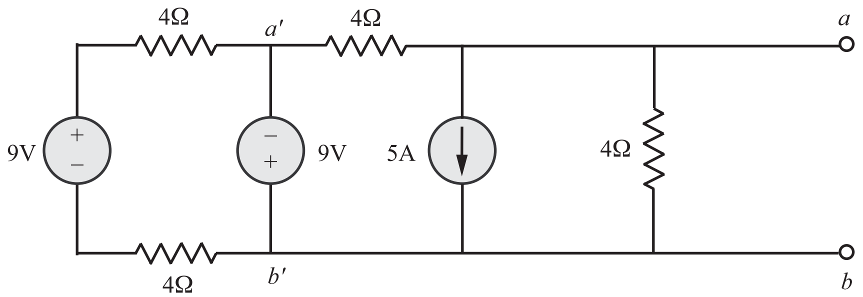
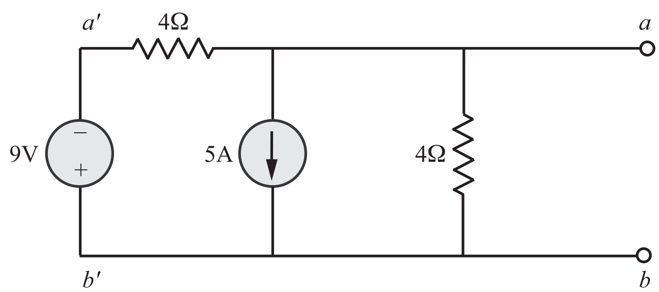


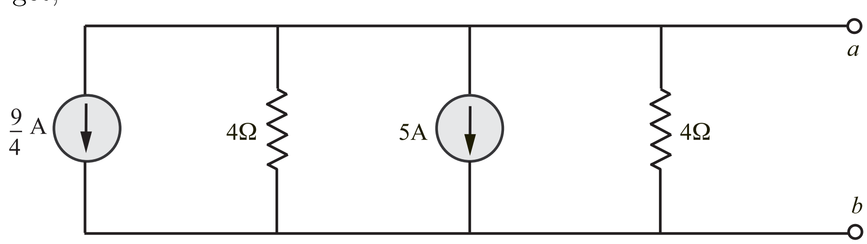
Figure 1.30

SOLUTION

The 9V source across the terminals a' and b' will force the voltage across these two terminals to be 9V regardless the value of the other 9V source and 8Ω resistor to its left. Hence, these two components may be removed from the terminals, a' and b' without affecting the circuit condition. Accordingly, the above circuit reduces to,



Converting the voltage source in series with 4Ω resistor into an equivalent current source, we get,



Adding the current sources in parallel and reducing the two 4 ohm resistors in parallel, we get the circuit shown in Fig. 1.30 (a):

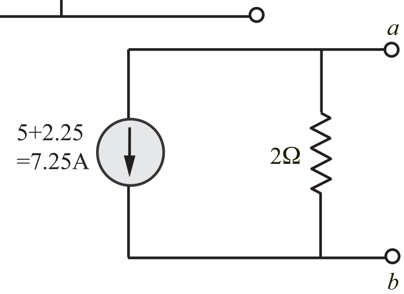


Figure 1.30 (a)

1.8.1 Source Shift

The source transformation is possible only in the case of practical sources. ie $R_s \neq \infty$ and $R_p \neq 0$, where R_s and R_p are internal resistances of voltage and current sources respectively. Transformation is not possible for ideal sources and source shifting methods are used for such cases.

Voltage source shift (E−shift):

Consider a part of the network shown in Fig. 1.31(a) that contains an ideal voltage source.

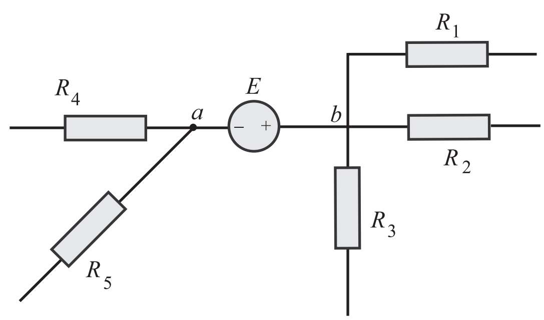


Figure 1.31(a) Basic network

Since node b is at a potential E with respect to node a, the network can be redrawn equivalently as in Fig. 1.31(b) or (c) depend on the requirements.

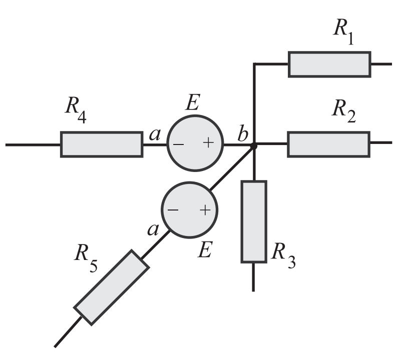


Figure 1.31(b) Networks after E-shift

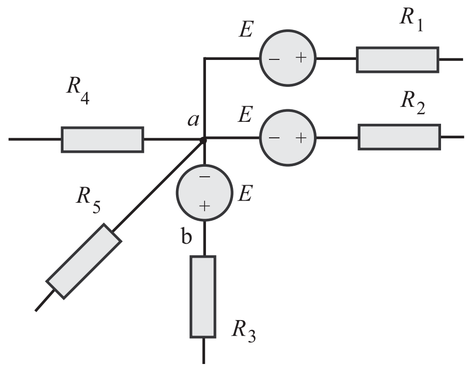


Figure 1.31(c) Network after the E-shift

Current source shift (I−shift)

In a similar manner, current sources also can be shifted. This can be explained with an example. Consider the network shown in Fig. 1.32(a), which contains an ideal current source between nodes a and c. The circuit shown in Figs. 1.32(b) and (c) illustrates the equivalent circuit after the I - shift.

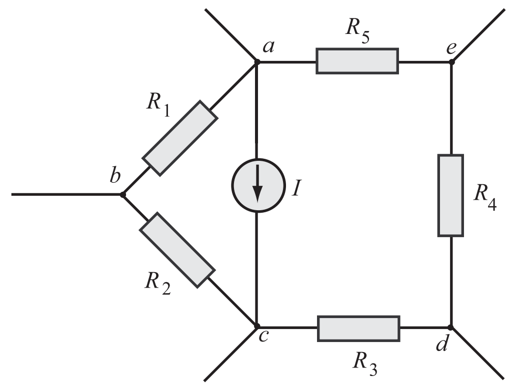


Figure 1.32(a) basic network

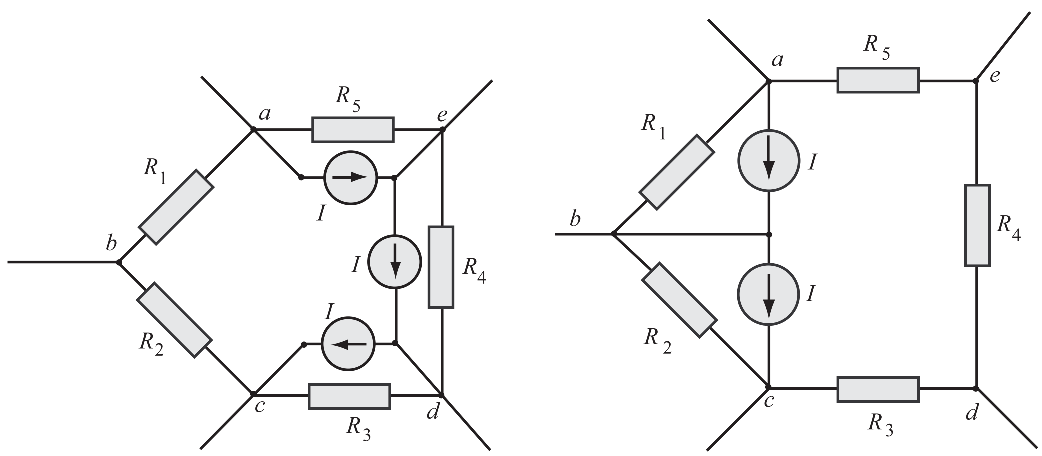


Figure 1.32(b) and (c) Networks after I--shift

EXAMPLE 1.15

Use source shifting and transformation techiniques to find voltage across 2Ω resistor shown in Fig. 1.33(a). All resistor values are in ohms.

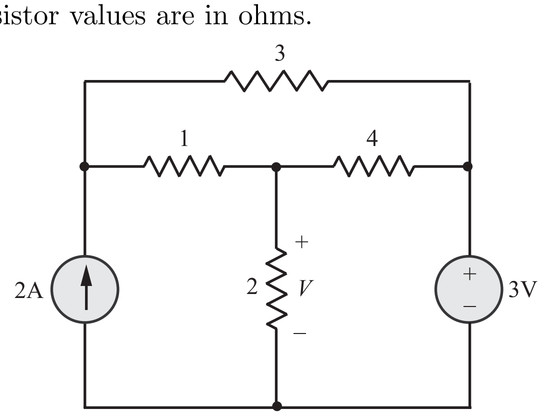
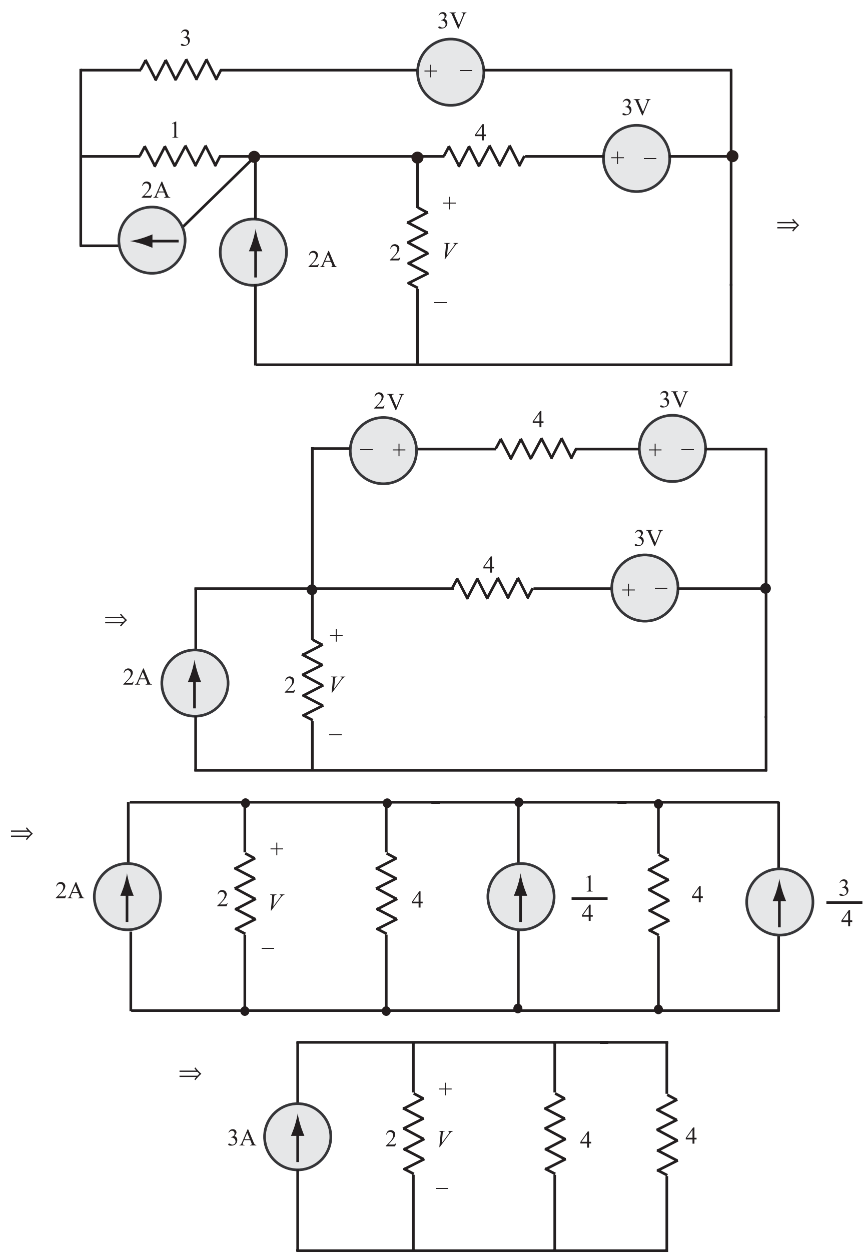


Figure 1.33(a)

SOLUTION

The circuit is redrawn by shifting 2A current source and 3V voltage source and further simplified as shown below.



Thus the voltage across 2Ω resistor is

$$V = 3 \times \frac{1}{2^{-1} + 4^{-1} + 4^{-1}} = 3\ \mathbf{V}$$

EXAMPLE 1.16

Use source mobility to calculate v_{ab} in the circuits shown in Fig. 1.34 (a) and (b). All resistor values are in ohms.

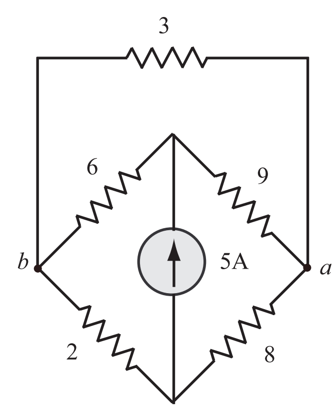


Figure 1.34(a)

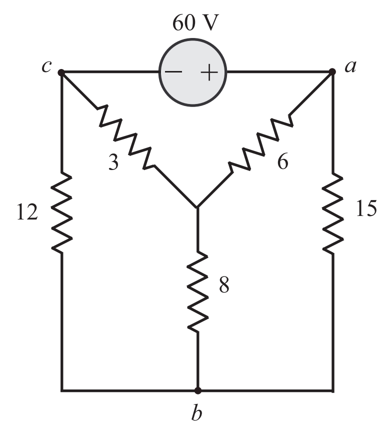
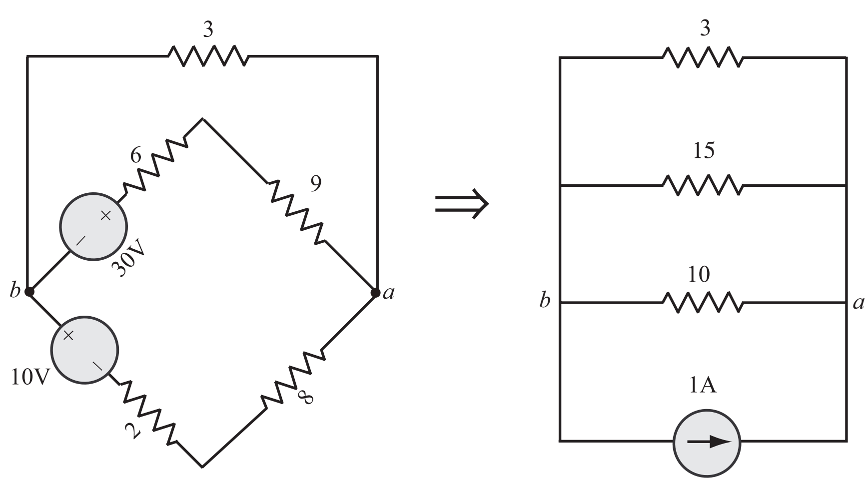


Figure 1.34(b)

SOLUTION

(a) The circuit shown in Fig. 1.34(a) is simplified using source mobility technique, as shown below and the voltage across the nodes a and b is calculated.



Voltage across a and b is

$$V_{ab} = \frac{1}{3^{-1} + 10^{-1} + 15^{-1}} = 2\ \mathbf{V}$$

(b) The circuit shown in Fig. 1.34 (b) is reduced as follows.

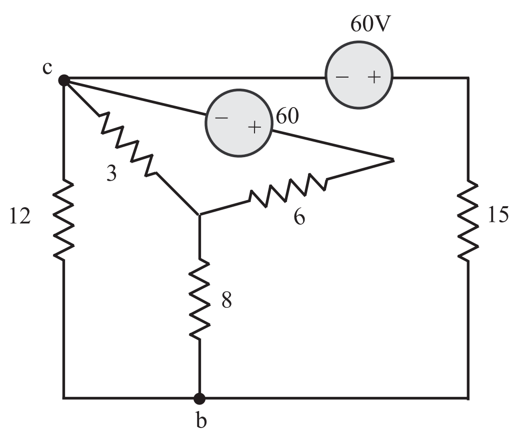


Figure 1.34(c)

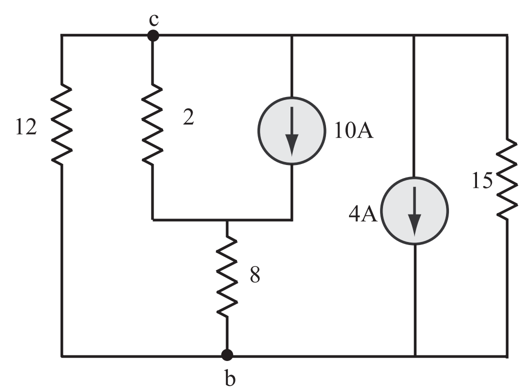


Figure 1.34(d)

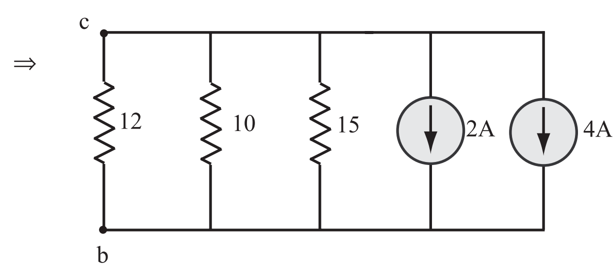


Figure 1.34(e)

From Fig. 1.34(e),

$$V_{bc} = \frac{12^{-1} \times 6}{12^{-1} + 10^{-1} + 15^{-1}} \times 12 = 24 \text{ V}$$

Applying this result in Fig. 1.34(b), we get

$$\begin{aligned} v_{ab} &= v_{ac} - v_{bc} \\ &= 60 - 24 = 36 \text{ V} \end{aligned}$$

EXAMPLE 1.17

Use mobility and reduction techniques to solve the node voltages of the network shown in Fig. 1.35(a). All resistors are in ohms.

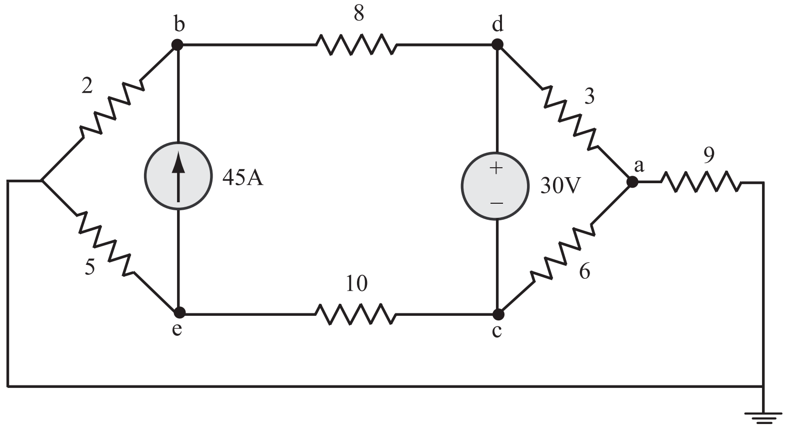


Figure 1.35(a)

SOLUTION

The circuit shown in Fig. 1.35(a) can be reduced by using desired techniques as shown in Fig. 1.35(b) to 1.35(e).

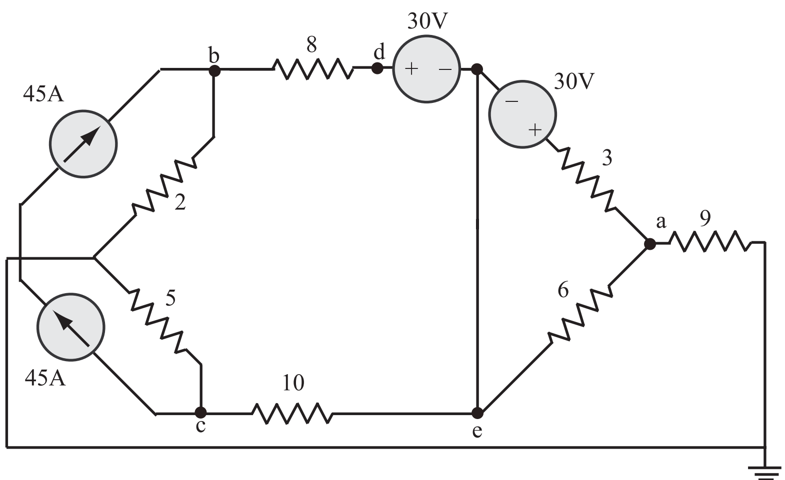


Figure 1.35(b)

From Fig. 1.35(e)

$$i = \frac{34}{17} = 2\ \text{A}$$

Using this value of i in Fig. 1.35(e),

$$V_a = -9 \times 2 = -18\ \text{V}$$

and $$V_e = V_a - 2 \times 2 - 20 = -42V$$

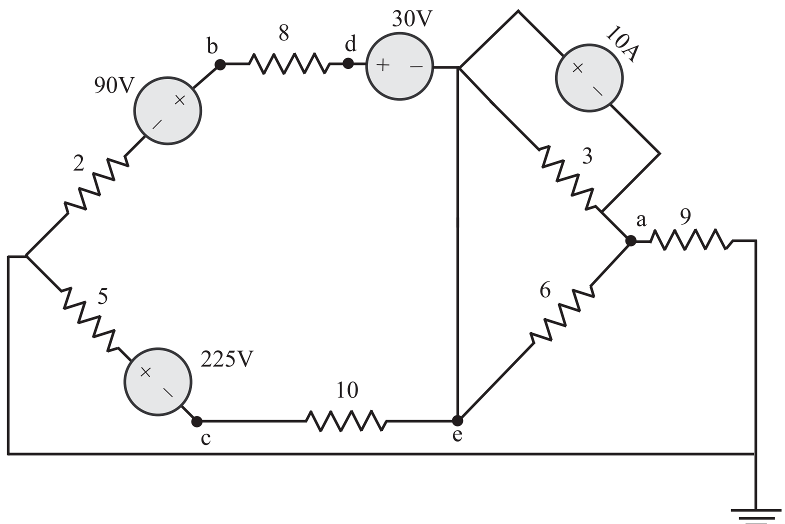


Figure 1.35(c)

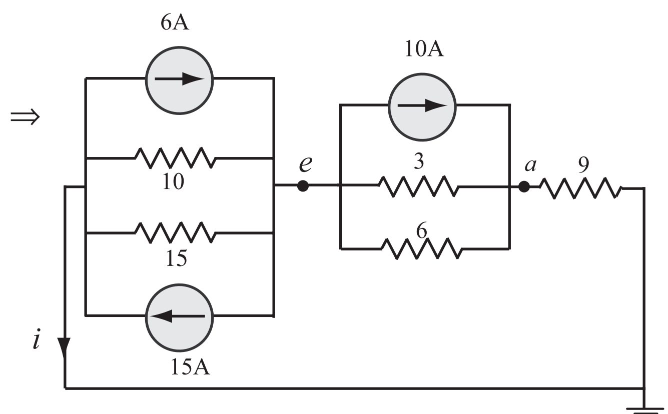


Figure 1.35(d)

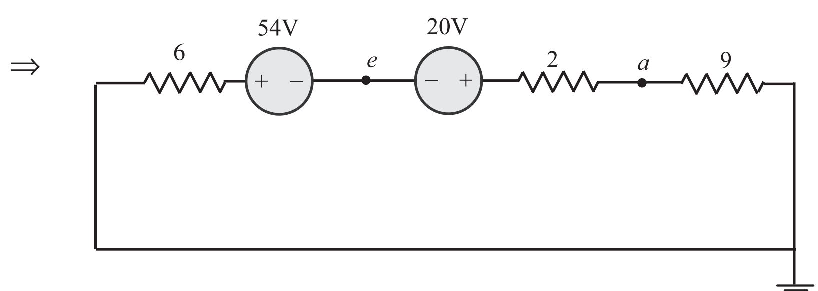


Figure 1.35(e)

From Fig 1.35(a)

$$V_d = V_e + 30 = -42 + 30 = -12\text{V}$$

Then at node b in Fig. 1.35(b),

$$\frac{V_b}{2} - 45 + \frac{V_b - V_d}{8} = 0$$

Using the value of V_d in the above equation and rearranging, we get,

$$V_b\left(\frac{1}{2} + \frac{1}{8}\right) = 45 - \frac{12}{8}$$

$$\Rightarrow \qquad V_b = 69.6 \text{ V}$$

At node c of Fig. 1.35(b)

$$\frac{V_c}{5} + 45 + \frac{V_c - V_e}{10} = 0$$

$$V_c\left(\frac{1}{5} + \frac{1}{10}\right) = -45 - \frac{42}{10}$$

$$\Rightarrow \qquad V_c = -164 \text{ V}$$

EXAMPLE 1.18

Use source mobility to reduce the network shown in Fig. 1.36(a) and find the value of V_x. All resistors are in ohms.

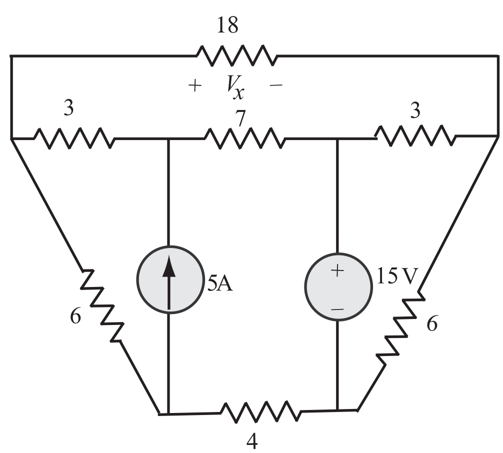


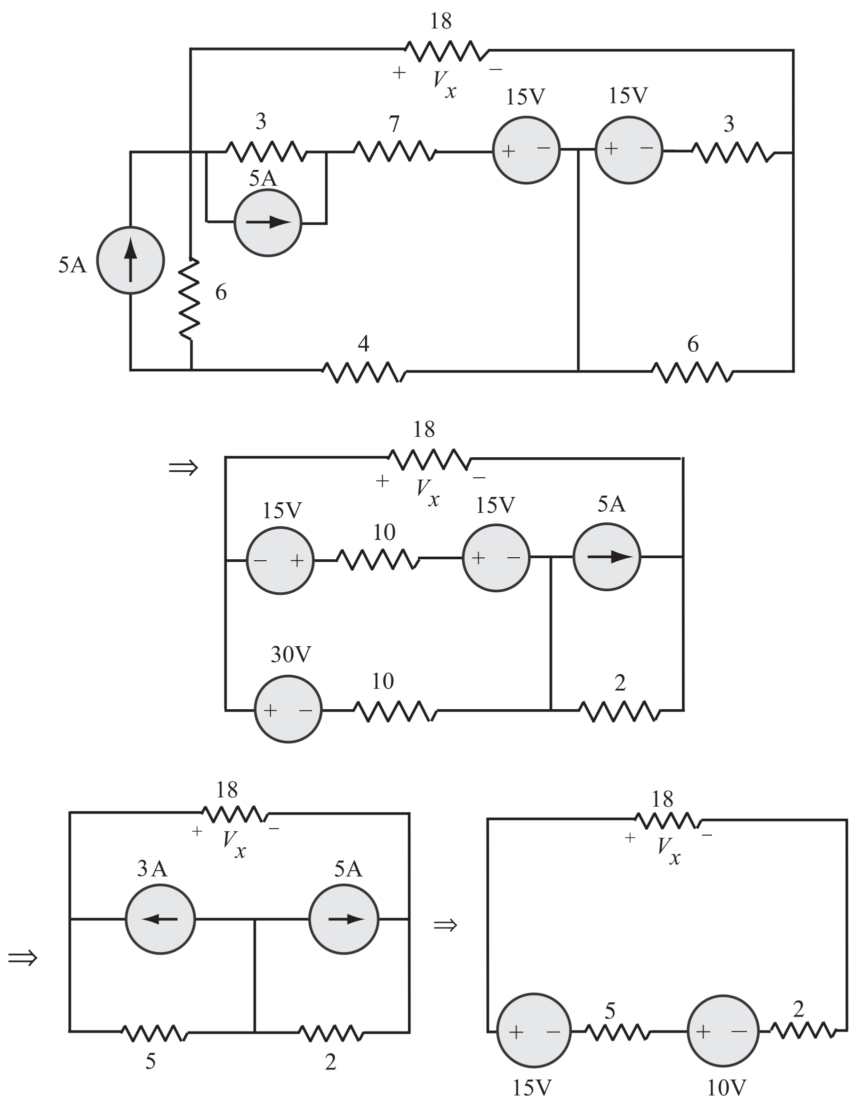
Figure 1.36(a)

SOLUTION

The circuit shown in Fig. 1.36(a) can be reduced as follows and V_x is calculated.

Thus

$$V_x = \frac{5}{25} \times 18 = 3.6\text{V}$$



1.9 Mesh analysis with independent voltage sources

Before starting the concept of mesh analysis, we want to reiterate that a closed path or a loop is drawn starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once. A mesh is a special case of a loop. A mesh is a loop that does not contain any other loops within it. The network shown in Fig. 1.37(a) has four meshes and they are identified as M_i, where $i = 1, 2, 3, 4$.

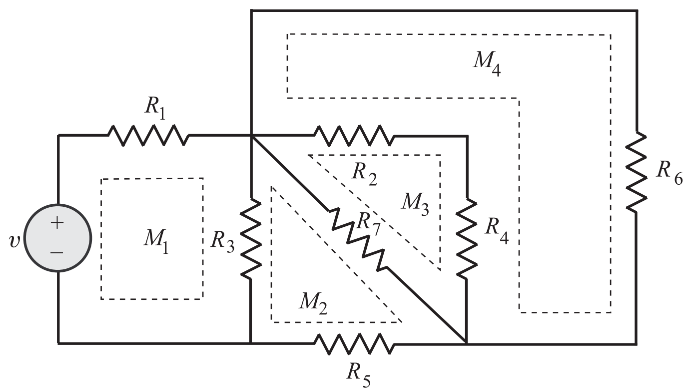


Figure 1.37(a) A circuit with four meshes. Each mesh is identified by a circuit

The current flowing in a mesh is defined as mesh current. As a matter of convention, the mesh currents are assumed to flow in a mesh in the clockwise direction.

Let us consider the two mesh circuit of Fig. 1.37(b).

We cannot choose the outer loop, $v \to R_1 \to R_2 \to v$ as one mesh, since it would contain the loop $v \to R_1 \to R_3 \to v$ within it. Let us choose two mesh currents i_1 and i_2 as shown in the figure.

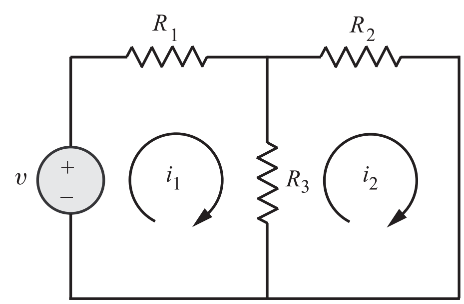


Figure 1.37(b) A circuit with two meshes

We may employ KVL around each mesh. We will travel around each mesh in the clockwise direction and sum the voltage rises and drops encountered in that particular mesh. We will adpot a convention of taking voltage drops to be *positive* and voltage rises to be *negative* . Thus, for the network shown in Fig. 1.37(b) we have

$$Mesh\ 1: -v + i_1R_1 + (i_1 - i_2)R_3 = 0 \tag{1.34}$$

$$Mesh\ 2: \qquad R_3(i_2 - i_1) + R_2i_2 = 0 \tag{1.35}$$

Note that when writing voltage across R_3 in mesh 1, the current in R_3 is taken as $i_1 - i_2$. Note that the mesh current i_1 is taken as '+ve' since we traverse in clockwise direction in mesh 1, On the other hand, the voltage across R_3 in mesh 2 is written as $R_3(i_2 - i_1)$. The current i_2 is taken as +ve since we are traversing in clockwise direction in this case too.

Solving equations (1.34) and (1.35), we can find the mesh currents i_1 and i_2.

Once the mesh currents are known, the branch currents are evaluated in terms of mesh currents and then all the branch voltages are found using Ohms's law. If we have N meshes with N mesh currents, we can obtain N independent mesh equations. This set of N equations are independent, and thus guarantees a solution for the N mesh currents.

EXAMPLE 1.19

For the electrical network shown in Fig. 1.38, determine the loop currents and all branch currents.

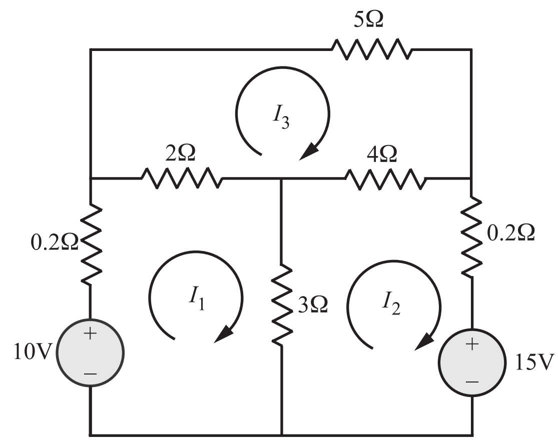


Figure 1.38

SOLUTION

Applying KVL for the meshes shown in Fig. 1.38, we have

$$Mesh\ 1: \quad 0.2I_1 + 2(I_1 - I_3) + 3(I_1 - I_2) - 10 = 0$$

$$\Rightarrow \quad 5.2I_1 - 3I_2 - 2I_3 = 10 \tag{1.36}$$

$$Mesh\ 2: \quad 3(I_2 - I_1) + 4(I_2 - I_3) + 0.2I_2 + 15 = 0$$

$$\Rightarrow \quad -3I_1 + 7.2I_2 - 4I_3 = -15 \tag{1.37}$$

$$Mesh\ 3: \quad 5I_3 + 2(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$\Rightarrow \quad -2I_1 - 4I_2 + 11I_3 = 0 \tag{1.38}$$

Putting the equations (1.36) through (1.38) in matrix form, we have

$$\begin{bmatrix} 5.2 & -3 & -2 \\ -3 & 7.2 & -4 \\ -2 & -4 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -15 \\ 0 \end{bmatrix}$$

Using Cramer's rule, we get

$$I_1 = \mathbf{0.11A}$$

$$I_2 = -\mathbf{2.53A}$$

and

$$I_3 = -\mathbf{0.9A}$$

The various branch currents are now calculated as follows:

Current through 10V battery $= I_1 = 0.11\text{A}$

Current through 2Ω resistor $= I_1 - I_3 = 1.01\text{A}$

Current through 3Ω resistor $= I_1 - I_2 = 2.64\text{A}$

Current through 4Ω resistor $= I_2 - I_3 = -1.63\text{A}$

Current through 5Ω resistor $= I_3 = -0.9\text{A}$

Current through 15V battery $= I_2 = -2.53\text{A}$

The negative sign for I_2 and I_3 indicates that the actual directions of these currents are opposite to the assumed directions.

1.10 Mesh analysis with independent current sources

Let us consider an electrical circuit source having an independent current source as shown Fig. 1.39(a).

We find that the second mesh current $i_2 = -i_s$ and thus we need only to determine the first mesh current i_1, *Applying KVL to the first mesh*, we obtain

$$(R_1 + R_2)i_1 - R_2 i_2 = v$$

Since $\qquad i_2 = -i_s,$

we get $\qquad (R_1 + R_2)i_1 + i_s R_2 = v$

$$\Rightarrow \qquad i_1 = \frac{v - i_s R_2}{R_1 + R_2}$$

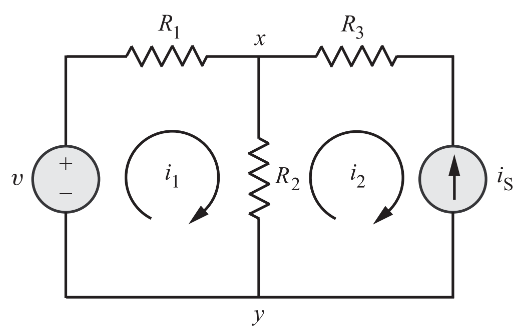


Figure 1.39(a) Circuit containing both independent voltage and current sources

As a second example, let us take an electrical circuit in which the current source i_s is common to both the meshes. This situation is shown in Fig. 1.39(b).

By *applying KCL at node x*, we recognize that, $i_2 - i_1 = i_s$

The two mesh equations (*using KVL*) are

Mesh 1 : $\qquad R_1 i_1 + v_{xy} - v = 0$

Mesh 2 : $\qquad (R_2 + R_3)i_2 - v_{xy} = 0$

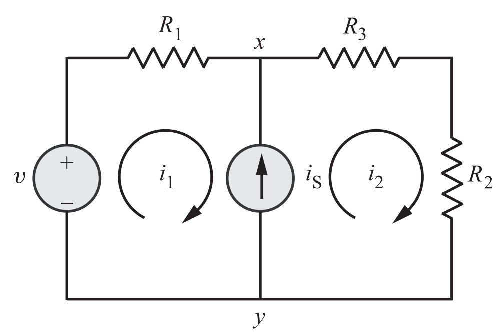


Figure 1.39(b) Circuit containing an independent current source common to both meshes

Adding the above two equations, we get

$$R_1 i_1 + (R_2 + R_3) i_2 = v$$

Substituting $i_2 = i_1 + i_s$ in the above equation, we find that

$$R_1 i_1 + (R_2 + R_3)(i_1 + i_s) = v$$

$$\Rightarrow \qquad i_1 = \frac{v - (R_2 + R_3) i_s}{R_1 + R_2 + R_3}$$

In this manner, we can handle independent current sources by recording the relationship between the mesh currents and the current source. The equation relating the mesh current and the current source is recorded as the *constraint equation*.

EXAMPLE 1.20

Find the voltage V_o in the circuit shown in Fig. 1.40.

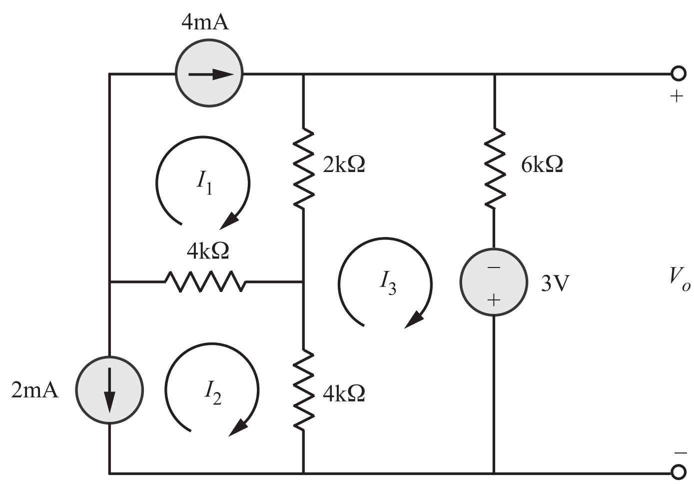


Figure 1.40

SOLUTION

Constraint equations:

$$I_1 = 4 \times 10^{-3} \text{ A}$$
$$I_2 = -2 \times 10^{-3} \text{ A}$$

Applying KVL for the mesh 3, we get

$$4 \times 10^3 [I_3 - I_2] + 2 \times 10^3 [I_3 - I_1] + 6 \times 10^3 I_3 - 3 = 0$$

Substituting the values of I_1 and I_2, we obtain

$$I_3 = 0.25 \text{ mA}$$

Hence,
$$\begin{aligned} V_o &= 6 \times 10^3 I_3 - 3 \\ &= 6 \times 10^3 (0.25 \times 10^{-3}) - 3 \\ &= \mathbf{-1.5\ V} \end{aligned}$$

1.11 Supermesh

A more general technique for mesh analysis method, when a current source is common to two meshes, involves the concept of a supermesh. A supermesh is created from two meshes that have a current source as a common element; the current source is in the interior of a supermesh. We thus reduce the number of meshes by one for each current source present. Figure 1.41 shows a supermesh created from the two meshes that have a current source in common.

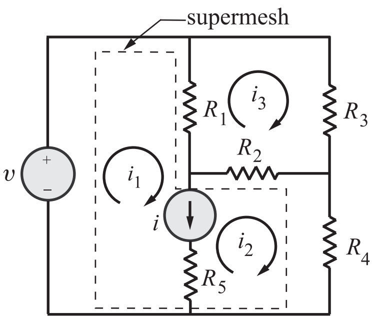


Figure 1.41 Circuit with a supermesh shown by the dashed line

EXAMPLE 1.21

Find the current i_o in the circuit shown in Fig. 1.42(a).

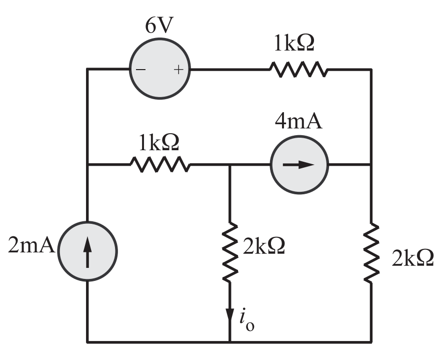


Figure 1.42(a)

SOLUTION

This problem is first solved by the techique explained in Section 1.10. Three mesh currents are specified as shown in Fig. 1.42(b). The mesh currents constrained by the current sources are

$$i = 2 \times 10^{-3} \text{ A}$$
$$i_2 - i_3 = 4 \times 10^{-3} \text{ A}$$

The KVL equations for meshes 2 and 3 respetively are

$$2 \times 10^3 i_2 + 2 \times 10^3 (i_2 - i_1) - v_{xy} = 0$$
$$-6 + 1 \times 10^3 i_3 + v_{xy} + 1 \times 10^3 (i_3 - i_1) = 0$$

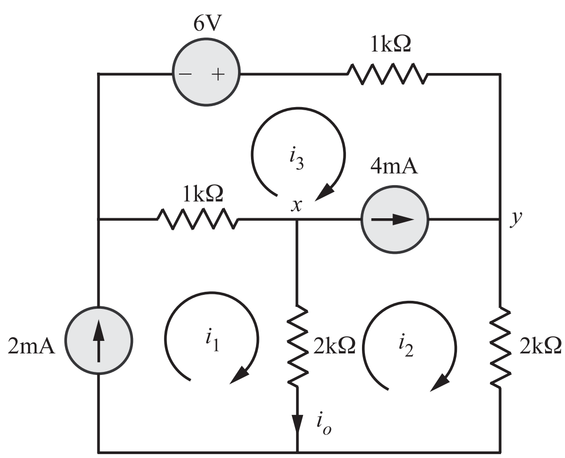


Figure 1.42(b)

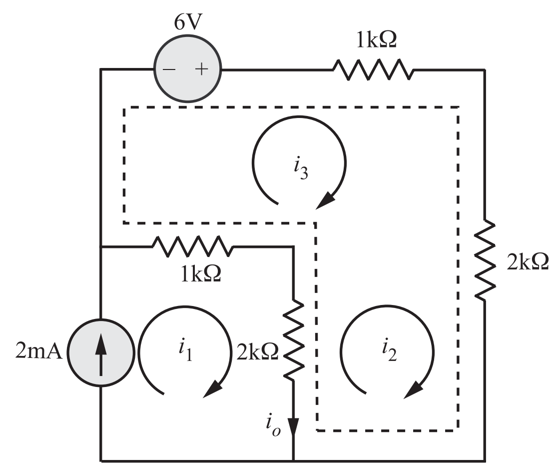


Figure 1.42(c)

Adding last two equations, we get

$$-6 + 1 \times 10^3 i_3 + 2 \times 10^3 i_2 + 2 \times 10^3 (i_2 - i_1) + 1 \times 10^3 (i_3 - i_1) = 0 \tag{1.39}$$

Substituting $i_1 = 2 \times 10^{-3}$A and $i_3 = i_2 - 4 \times 10^{-3}$A in the above equation, we get

$$-6 + 1 \times 10^3 \left[i_2 - 4 \times 10^{-3}\right] + 2 \times 10^3 i_2 + 2 \times 10^3 \left[i_2 - 2 \times 10^{-3}\right] + 1 \times 10^3 \left[i_2 - 4 \times 10^{-3} - 2 \times 10^{-3}\right] = 0$$

Solving we get

$$i_2 = \frac{10}{3} \text{ mA}$$

Thus,

$$\begin{aligned} i_o &= i_1 - i_2 \\ &= 2 - \frac{10}{3} \\ &= \frac{-4}{3} \text{ mA} \end{aligned}$$

The purpose of supermesh approach is to avoid introducing the unknown voltage v_{xy}. The supermesh is created by mentally removing the 4 mA current source as shown in Fig. 1.42(c). Then applying KVL equation around the dotted path, which defines the supermesh, using the orginal mesh currents as shown in Fig. 1.42(b), we get

$$-6 + 1 \times 10^3 i_3 + 2 \times 10^3 i_2 + 2 \times 10^3 (i_2 - i_1) + 1 \times 10^3 (i_3 - i_1) = 0$$

Note that the supermesh equation is same as equation 1.39 obtained earlier by introducing v_{xy}, the remaining procedure of finding i_o is same as before.

EXAMPLE 1.22

For the network shown in Fig. 1.43(a), find the mesh currents i_1, i_2 and i_3.

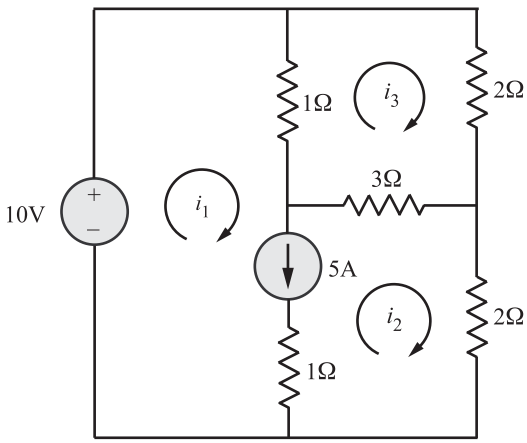


Figure 1.43(a)

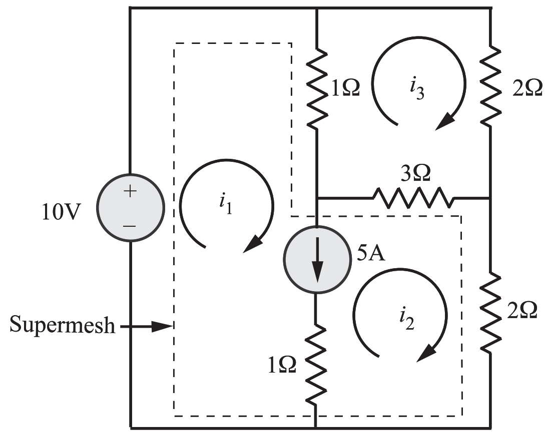


Figure 1.43(b)

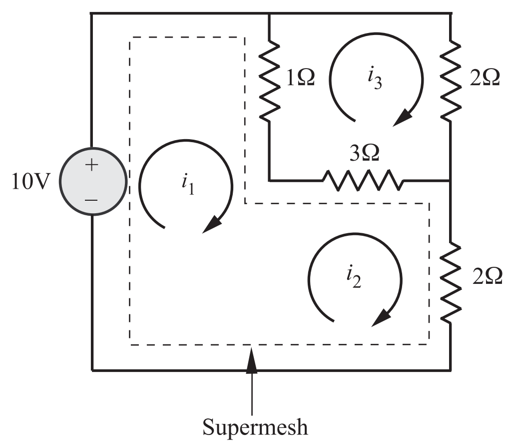


Figure 1.43(c)

SOLUTION

The 5A current source is in the common boundary of two meshes. The supermesh is shown as dotted lines in Figs.1.43(b) and 1.43(c), the branch having the 5A current source is removed from the circuit diagram. Then applying KVL around the dotted path, which defines the supermesh, using the original mesh currents as shown in Fig. 1.43(c), we find that

$$-10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0$$

For mesh 3, we have

$$1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0$$

Finally, the *constraint equation is*

$$i_1 - i_2 = 5$$

Then the above three eqations may be reduced to

$Supemesh$: $1i_1 + 5i_2 - 4i_3 = 10$

Mesh 3 : $-1i_1 - 3i_2 + 6i_3 = 0$

$current\ source$: $i_1 - i_2 = 5$

Solving the above simultaneous equations, we find that,

$$\boldsymbol{i_1 = 7.5\text{A}, i_2 = 2.5\text{A}, \text{ and } i_3 = 2.5\text{A}}$$

EXAMPLE 1.23

Find the mesh currents i_1, i_2 and i_3 for the network shown in Fig. 1.44.

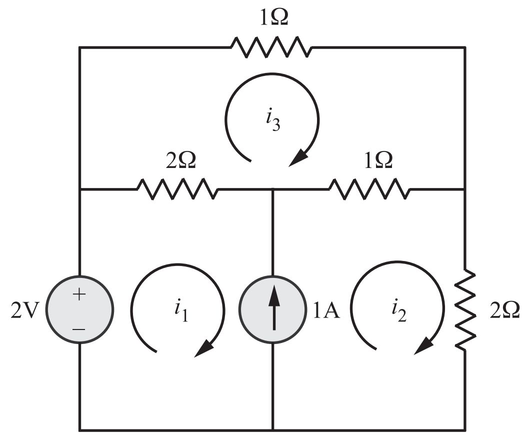


Figure 1.44

SOLUTION

Here we note that 1A independent current source is in the common boundary of two meshes. Mesh currents i_1, i_2 and i_3, are marked in the clockwise direction. The supermesh is shown as dotted lines in Figs. 1.45(a) and 1.45(b). In Fig. 1.45(b), the 1A current source is removed from the circuit diagram, then applying the KVL around the dotted path, which defines the supermesh, using original mesh currents as shown in Fig. 1.45(b), we find that

$$-2 + 2(i_1 - i_3) + 1(i_2 - i_3) + 2i_2 = 0$$

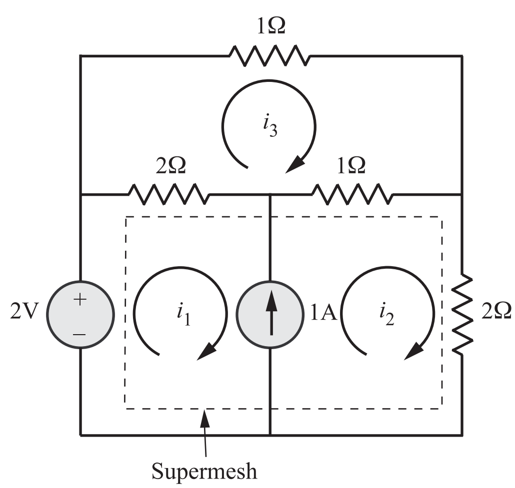


Figure 1.45(a)

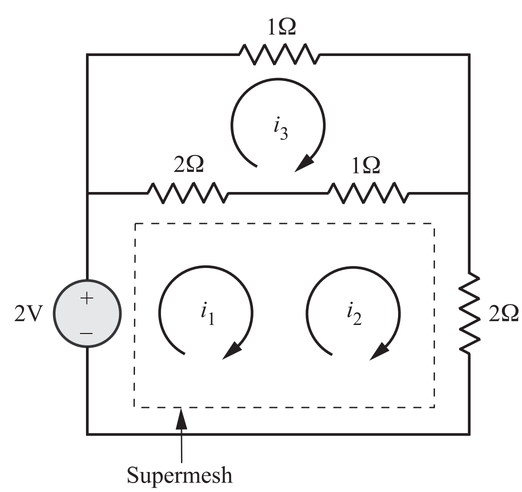


Figure 1.45(b).

For mesh 3, the KVL *equation is*

$$2(i_3 - i_1) + 1i_3 + 1(i_3 - i_2) = 0$$

Finally, the *constraint equation is*

$$i_1 - i_2 = 1$$

Then the above three equations may be reduced to

Supermesh : $2i_1 + 3i_2 - 3i_3 = 2$

Mesh 3 : $2i_1 + i_2 - 4i_3 = 0$

Current source: $i_1 - i_2 = 1$

Solving the above simultaneous equations, we find that

$\boldsymbol{i_1 = 1.55\text{A}, i_2 = 0.55\text{A}, i_3 = 0.91\text{A}}$

1.12 Mesh analysis for the circuits involving dependent sources

The persence of one or more dependent sources merely requires each of these source quantites and the variable on which it depends to be expressed in terms of assigned mesh currents. That is, to begin with, we treat the dependent source as though it were an independent source while writing the *KVL* equations. Then we write the *controlling equation* for the dependent source. The following examples illustrate the point.

EXAMPLE 1.24

(a) Use the mesh current method to solve for i_a in the circuit shown in Fig. 1.46.

(b) Find the power delivered by the independent current source.

(c) Find the power delivered by the dependent voltage source.

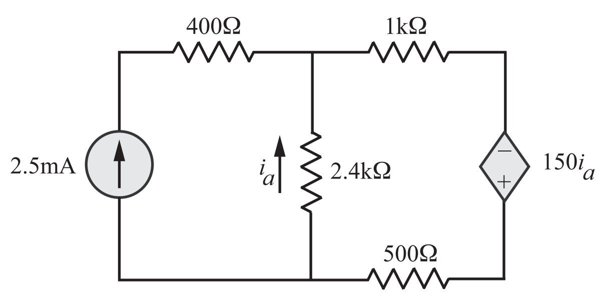


Figure 1.46

SOLUTION

(a) We mark two mesh currents i_1 and i_2 as shown in Fig. 1.47. We find that $i = 2.5$mA. *Applying KVL to mesh* 2, we find that

$$2400(i_2 - 0.0025) + 1500i_2 - 150(i_2 - 0.0025) = 0 \quad (\because i_a = i_2 - 2.5 \text{ mA})$$

$$\Rightarrow \quad 3750i_2 = 6 - 0.375$$

$$= 5.625$$

$$\Rightarrow \quad i_2 = \mathbf{1.5\ mA}$$

$$i_a = i_2 - 2.5 = \mathbf{-1.0mA}$$

(b) *Applying KVL to mesh* 1, we get

$-v_o + 2.5(0.4) - 2.4i_a = 0$

$\Rightarrow \quad v_o = 2.5(0.4) - 2.4(-1.0) = 3.4\text{V}$

$$P_{\text{ind.source}} = 3.4 \times 2.5 \times 10^{-3}$$
$$= \mathbf{8.5\ mW}(\text{delivered})$$

(c) $P_{\text{dep.source}} = 150i_a(i_2)$
$$= 150(-1.0 \times 10^{-3})(1.5 \times 10^{-3})$$
$$= -\mathbf{0.225\ mW}(\text{absorbed})$$

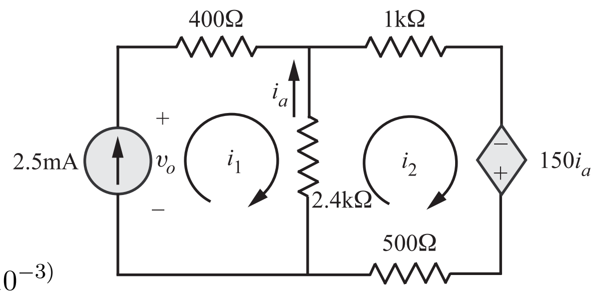


Figure 1.47

EXAMPLE 1.25

Find the total power delivered in the circuit using mesh-current method.

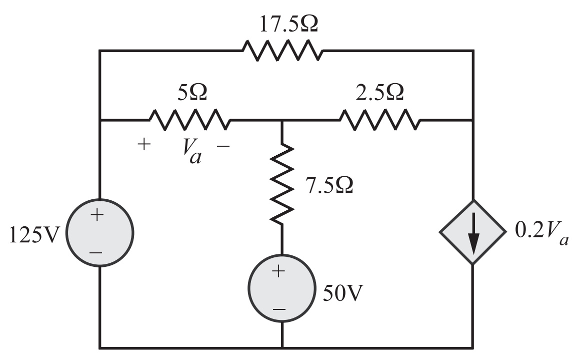


Figure 1.48

SOLUTION

Let us mark three mesh currents i_1, i_2 and i_3 as shown in Fig. 1.49.

KVL equations:

Mesh 1: $\quad 17.5i_1 + 2.5(i_1 - i_3)$
$$+5(i_1 - i_2) = 0$$
$\Rightarrow \quad 25i_1 - 5i_2 - 2.5i_3 = 0$

Mesh 2: $\quad -125 + 5(i_2 - i_1)$
$$+7.5(i_2 - i_3) + 50 = 0$$
$\Rightarrow \quad -5i_1 + 12.5i_2 - 7.5i_3 = 75$

Constraint equations:

$$i_3 = 0.2V_a$$
$$V_a = 5(i_2 - i_1)$$

Thus, $\quad i_3 = 0.2 \times 5(i_2 - i_1) = i_2 - i_1.$

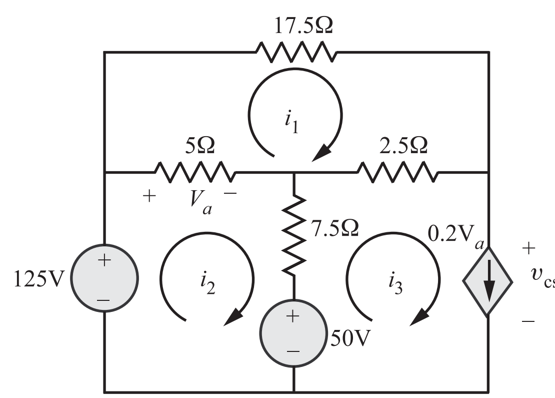


Figure 1.49

Making use of i_3 in the mesh equations, we get

$$\begin{aligned} Mesh\ 1: &\quad 25i_1 - 5i_2 - 2.5(i_2 - i_1) = 0 \\ \Rightarrow &\quad 27.5i_1 - 7.5i_2 = 0 \\ Mesh\ 2: &\quad -5i_1 + 12.5i_2 - 7.5(i_2 - i_1) = 75 \\ \Rightarrow &\quad 2.5i_1 + 5i_2 = 75 \end{aligned}$$

Solving the above two equations, we get

$$i_1 = 3.6\ \text{A}, i_2 = 13.2\ \text{A}$$

and $$i_3 = i_2 - i_1 = 9.6\ \text{A}$$

Applying KVL through the path having $5\Omega \to 2.5\Omega \to v_{cs} \to 125$V source, we get,

$$\begin{aligned} 5(i_2 - i_1) + 2.5(i_3 - i_1) + v_{cs} - 125 &= 0 \\ \Rightarrow \quad v_{cs} &= 125 - 5(i_2 - i_1) - 2.5(i_3 - i_1) \\ &= 125 - 48 - 2.5(9.6 - 3.6) = 62\ \text{V} \\ P_{vcs} &= 62(9.6) = 595.2\text{W (absorbed)} \\ P_{50\text{V}} &= 50(i_2 - i_3) = 50(13.2 - 9.6) = 180\text{W (absorbed)} \\ P_{125\text{V}} &= 125i_2 = \mathbf{1650W\ (delivered)} \end{aligned}$$

EXAMPLE 1.26

Use the mesh-current method to find the power delivered by the dependent voltage source in the circuit shown in Fig. 1.50.

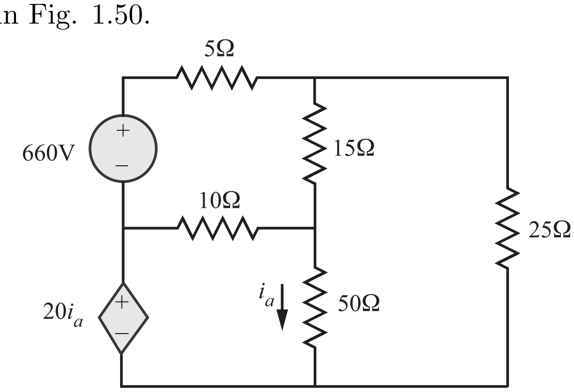


Figure 1.50

SOLUTION

Applying KVL to the meshes 1, 2 and 3 shown in Fig 1.51, we have

$$\begin{aligned} Mesh\ 1: &\quad 5i_1 + 15(i_1 - i_3) + 10(i_1 - i_2) - 660 = 0 \\ \Rightarrow &\quad 30i_1 - 10i_2 - 15i_3 = 660 \end{aligned}$$

$$\begin{aligned}
Mesh\ 2: &\qquad -20i_a + 10(i_2 - i_1) + 50(i_2 - i_3) = 0 \\
\Rightarrow &\qquad 10(i_2 - i_1) + 50(i_2 - i_3) = 20i_a \\
\Rightarrow &\qquad -10i_1 + 60i_2 - 50i_3 = 20i_a \\
Mesh\ 3: &\qquad 15(i_3 - i_1) + 25i_3 + 50(i_3 - i_2) = 0 \\
\Rightarrow &\qquad -15i_1 - 50i_2 + 90i_3 = 0
\end{aligned}$$

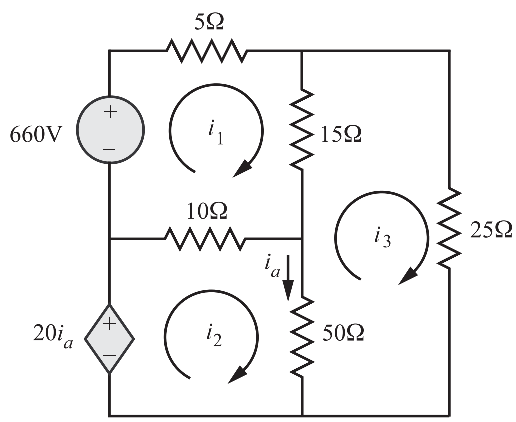


Figure 1.51

Also $\quad i_a = i_2 - i_3$

Solving, $\boldsymbol{i_1 = 42\text{A}, i_2 = 27\text{A}, i_3 = 22\text{A}, i_a = 5\text{A}}$.

Power delivered by the dependent voltage source $= P_{20i_a} = (20i_a)i_2$

$= \mathbf{2700W}$ (delivered)

1.13 Node voltage anlysis

In the nodal analysis, Kirchhoff's current law is used to write the equilibrium equations. A node is defined as a junction of two or more branches. If we define one node of the network as a reference node (a point of zero potential or ground), the remaining nodes of the network will have a fixed potential relative to this reference. Equations relating to all nodes except for the reference node can be written by applying KCL.

Refering to the circuit shown in Fig.1.52, we can arbitrarily choose any node as the reference node. However, it is convenient to choose the node with most connected branches. Hence, node 3 is chosen as the reference node here. It is seen from the network of Fig. 1.52 that there are three nodes.

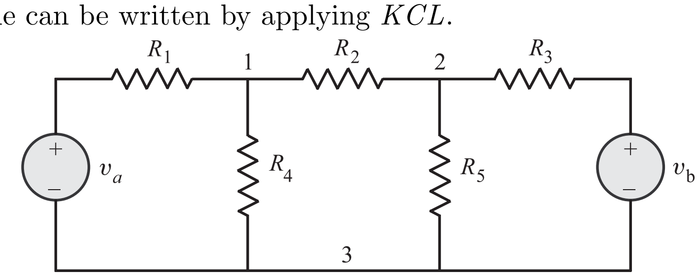


Figure 1.52 Circuit with three nodes where the lower node 3 is the reference node

Hence, number of equations based on KCL will be total number of nodes minus one. That is, in the present context, we will have only two KCL equations referred to as node equations. For applying KCL at node 1 and node 2, we assume that all the currents leave these nodes as shown in Figs. 1.53 and 1.54.

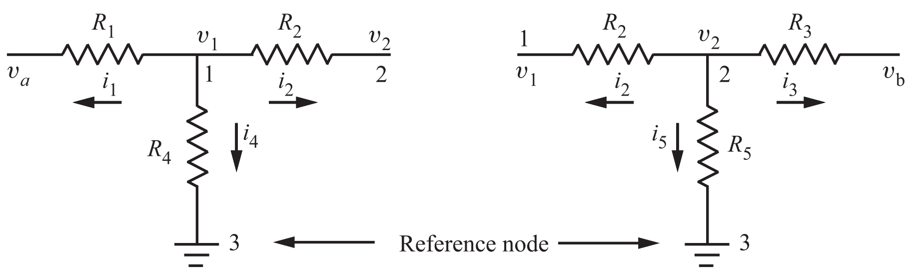


Figure 1.53 Simplified circuit for applying KCL at node 1

Figure 1.54 Simplified circuit for applying KCL at node 2

Applying KCL at node 1 *and* 2, we find that

(i) At node 1: $$i_1 + i_2 + i_4 = 0$$

$$\Rightarrow \quad \frac{v_1 - v_a}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - 0}{R_4} = 0$$

$$\Rightarrow \quad v_1 \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right] - v_2 \frac{1}{R_2} = \frac{v_a}{R_1} \tag{1.40}$$

(ii) At node 2: $$i_2 + i_3 + i_5 = 0$$

$$\Rightarrow \quad \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_b}{R_3} + \frac{v_2}{R_5} = 0$$

$$\Rightarrow \quad - v_1 \left[\frac{1}{R_2} \right] + v_2 \left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right] = \frac{v_b}{R_3} \tag{1.41}$$

Putting equations (1.40) and (1.41) in matrix form, we get

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{v_a}{R_1} \\ \frac{v_b}{R_3} \end{bmatrix}$$

The above matrix equation can be solved for node voltages v_1 and v_2 using Cramer's rule of determinants. Once v_1 and v_2 are obtainted, then by using Ohm's law, we can find all the branch currents and hence the solution of the network is obtained.

EXAMPLE 1.27

Refer the circuit shown in Fig. 1.55. Find the three node voltages v_a, v_b and v_c, when all the conductances are equal to 1S.

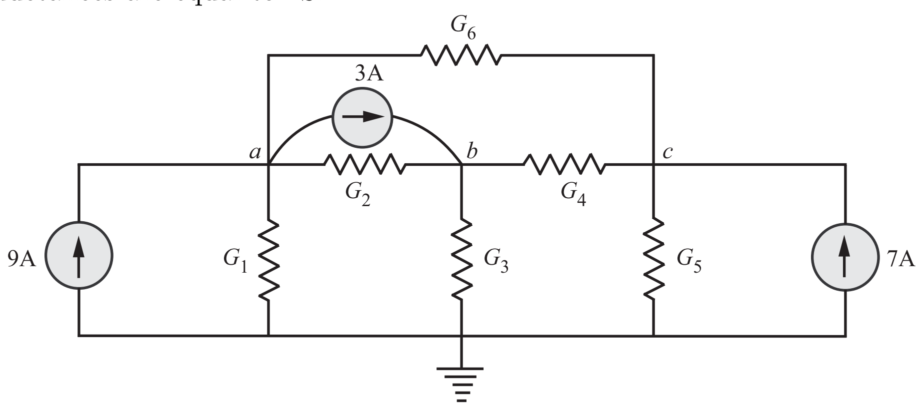


Figure 1.55

SOLUTION

At node **a**: $(G_1 + G_2 + G_6)v_a - G_2 v_b - G_6 v_c = 9 - 3$

At node **b**: $-G_2 v_a + (G_4 + G_2 + G_3)v_b - G_4 v_c = 3$

At node **c**: $-G_6 v_a - G_4 v_b + (G_4 + G_5 + G_6)v_c = 7$

Substituting the values of various conductances, we find that

$$3v_a - v_b - v_c = 6$$
$$-v_a + 3v_b - v_c = 3$$
$$-v_a - v_b + 3v_c = 7$$

Putting the above equations in matrix form, we see that

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 7 \end{bmatrix}$$

Solving the matrix equation using cramer's rule, we get

$$v_a = 5.5\text{V}, \quad v_b = 4.75\text{V}, \quad v_c = 5.75\text{V}$$

The determinant Δ used for computing v_a, v_b and v_c in general form is given by

$$G = \begin{vmatrix} \sum_a G & -G_{ab} & -G_{ac} \\ -G_{ab} & \sum_b G & -G_{bc} \\ -G_{ac} & -G_{bc} & \sum_c G \end{vmatrix}$$

where $\sum_i G$ is the sum of the conductances at node i, and G_{ij} is the sum of conductances conecting nodes i and j.

The node voltage matrix equation for a circuit with k unknown node voltages is

$$\mathbf{Gv} = \mathbf{i_s},$$

where,
$$\mathbf{v} = \begin{bmatrix} v_a \\ v_b \\ \vdots \\ v_k \end{bmatrix}$$

is the vector consisting of k unknown node voltages.

The matrix
$$\mathbf{i_a} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ \vdots \\ i_{sk} \end{bmatrix}$$

is the vector consisting of k current sources and i_{sk} is the sum of all the source currents entering the node k. If the k^{th} current source is not present, then $i_{sk} = 0$.

EXAMPLE 1.28

Use the node voltage method to find how much power the 2A source extracts from the circuit shown in Fig. 1.56.

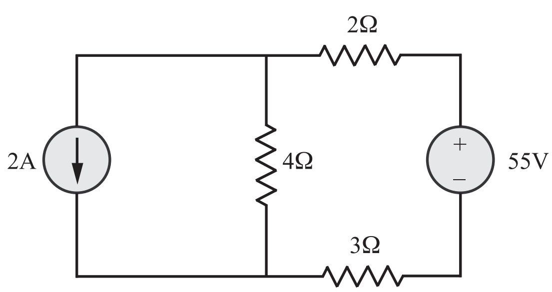


Figure 1.56

SOLUTION

Applying KCL at node a, we get

$$2 + \frac{v_a}{4} + \frac{v_a - 55}{5} = 0$$

$$\Rightarrow \quad v_a = 20\text{V}$$

$$P_{2\text{A source}} = 20(2) = 40\text{W (absorbing)}$$

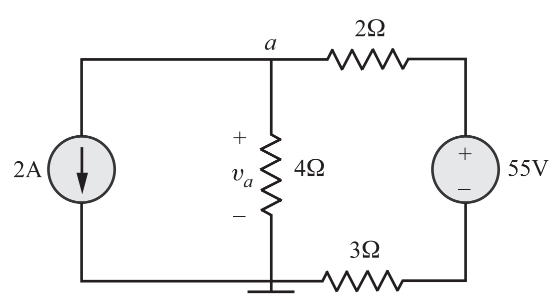


Figure 1.57

EXAMPLE 1.29

Refer the circuit shown in Fig. 1.58(a).

(a) Use the node voltage method to find the branch currents i_1 to i_6.

(b) Test your solution for the branch currents by showing the total power dissipated equals the power developed.

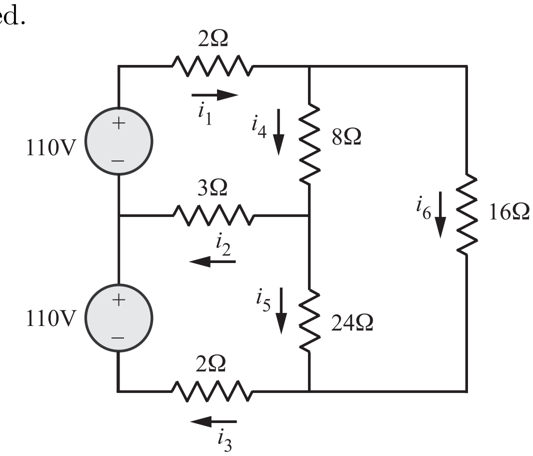


Figure 1.58(a)

SOLUTION

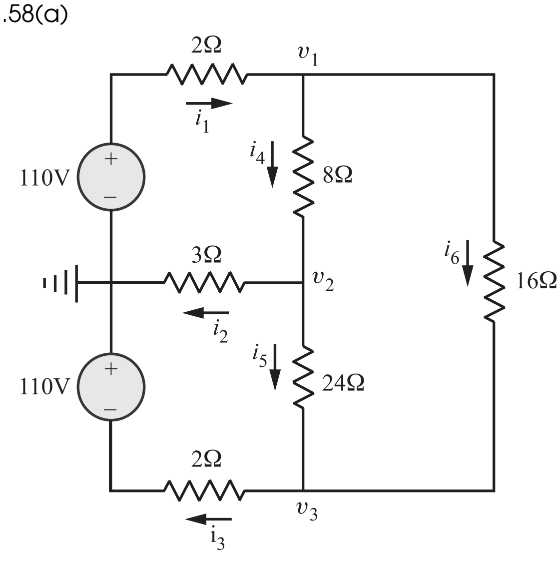


Figure 1.58(b)

(a) At node v_1:

$$\frac{v_1 - 110}{2} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{16} = 0$$
$$\Rightarrow \quad 11v_1 - 2v_2 - v_3 = 880$$

At node v_2:

$$\frac{v_2 - v_1}{8} + \frac{v_2}{3} + \frac{v_2 - v_3}{24} = 0$$
$$\Rightarrow \quad -3v_1 + 12v_2 - v_3 = 0$$

At node v_3:

$$\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0$$
$$\Rightarrow \quad -3v_1 - 2v_2 + 29v_3 = -2640$$

Solving the above nodal equations,we get

$$v_1 = 74.64\text{V}, \ v_2 = 11.79\text{V}, \ v_3 = -82.5\text{V}$$

Hence,

$$i_1 = \frac{110 - v_1}{2} = \mathbf{17.68A}$$
$$i_2 = \frac{v_2}{3} = \mathbf{3.93A}$$

$$i_3 = \frac{v_3 + 110}{2} = \mathbf{13.75A}$$
$$i_4 = \frac{v_1 - v_2}{8} = \mathbf{7.86A}$$
$$i_5 = \frac{v_2 - v_3}{24} = \mathbf{3.93A}$$
$$i_6 = \frac{v_1 - v_3}{16} = \mathbf{9.82A}$$

(b) Total power delivered $= 110i_1 + 110i_3 = \mathbf{3457.3W}$

Total power dissipated $= i_1^2 \times 2 + i_2^2 \times 3 + i_3^2 \times 2 + i_4^2 \times 8 + i_5^2 \times 24 + i_6^2 \times 16$

$= \mathbf{3457.3\ W}$

EXAMPLE 1.30

(a)Use the node voltage method to show that the output volatage v_o in the circuit of Fig 1.59(a) is equal to the average value of the source voltages.

(b) Find v_o if $v_1 = 150$V, $v_2 = 200$V and $v_3 = -50$V.

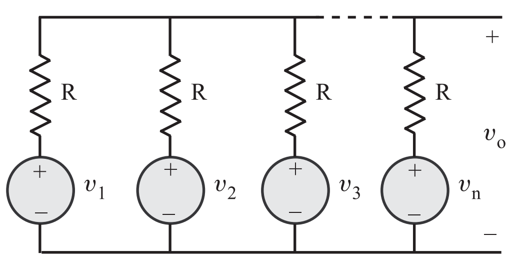


Figure 1.59(a)

SOLUTION

Applying KCL at node a, we get

$$\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$$

$$\Rightarrow\ nv_o = v_1 + v_2 + \cdots + v_n$$

$$\text{Hence, } v_o = \frac{1}{n}\left[v_1 + v_2 + \cdots + v_n\right]$$
$$= \frac{1}{n}\sum_{k=1}^{n} v_k$$

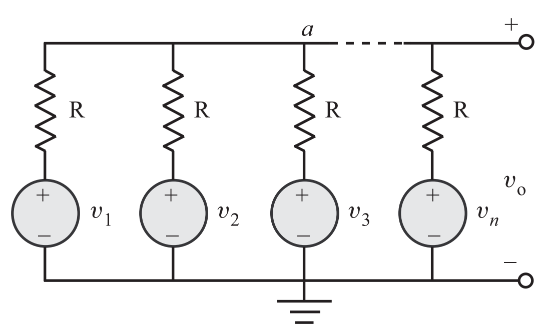


Figure 1.59(b)

(b) $$v_o = \frac{1}{3}(150 + 200 - 50) = \mathbf{100V}$$

EXAMPLE 1.31

Use nodal analysis to find v_o in the circuit of Fig. 1.60.

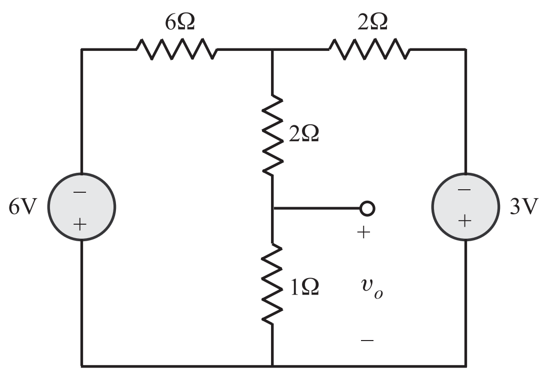


Figure 1.60

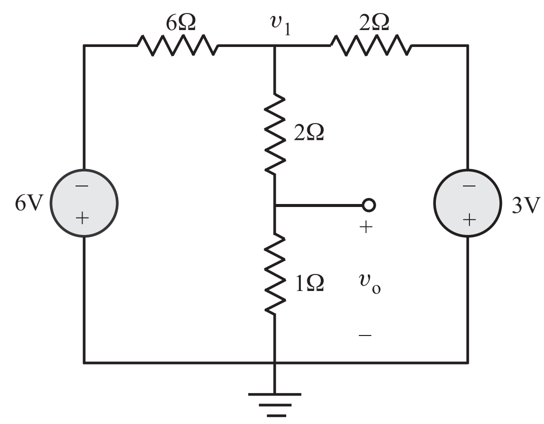


Figure 1.61

SOLUTION

Referring Fig 1.61, at node v_1:

$$\frac{v_1+6}{6}+\frac{v_1}{3}+\frac{v_1+3}{2}=0$$

$$\Rightarrow \quad \frac{v_1}{6}+\frac{v_1}{3}+\frac{v_1}{2}=-2.5$$

$$\Rightarrow \quad v_1=-2.5 \text{ V}$$

$$v_o=\left[\frac{v_1}{2+1}\right]\times 1$$

$$=\frac{-2.5}{3}\times 1$$

$$=\mathbf{-0.83volts}$$

EXAMPLE 1.32

Refer to the network shown in Fig. 1.62. Find the power delivered by 1A current source.

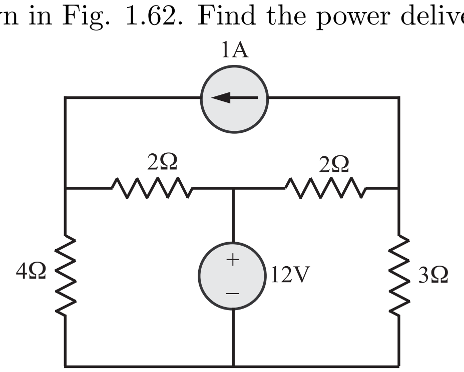


Figure 1.62

SOLUTION

Referring to Fig. 1.63, *applying KVL to the path* $v_a \rightarrow 4\Omega \rightarrow 3\Omega$, we get

$$v_a = v_1 - v_3$$

$$v_2 = 12\text{V}$$

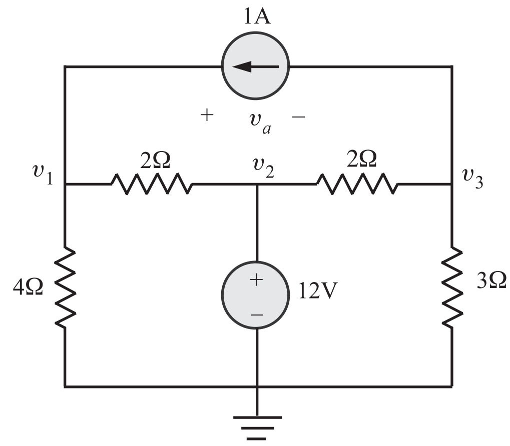


Figure 1.63

At node v_1 : $\dfrac{v_1}{4} + \dfrac{v_1 - v_2}{2} - 1 = 0$

$$\Rightarrow \quad \frac{v_1}{4} + \frac{v_1 - 12}{2} - 1 = 0$$

$$v_1 = 9.33 \text{ V}$$

At node v_2: $\dfrac{v_3}{3} + \dfrac{v_3 - v_2}{2} + 1 = 0$

$$\Rightarrow \quad \frac{v_3}{3} + \frac{v_3 - 12}{2} + 1 = 0$$

$$\Rightarrow \quad v_3 = 6\text{V}$$

Hence, $\quad v_a = 9.33 - 6 = 3.33$ volts

$$P_{1\text{A source}} = v_a \times 1$$

$$= 3.33 \times 1 = \mathbf{3.33W} \ (\mathbf{delivering})$$

1.14 Supernode

Inorder to understand the concept of a supernode, let us consider an electrical circuit as shown in Fig. 1.64.

Applying KVL clockwise to the loop containing R_1, voltage source and R_2, we get

$$v_a = v_s + v_b$$

$$\Rightarrow \quad v_a - v_b = v_s (Constraint\ equation) \tag{1.42}$$

To account for the fact that the source voltage is known, we consider both v_a and v_b as part of one larger node represented by the dotted ellipse as shown in Fig. 1.64. We need a larger node because v_a and v_b are dependent (see equation 1.42). This larger node is called the supernode.

Applying KCL at nodes a and b, we get

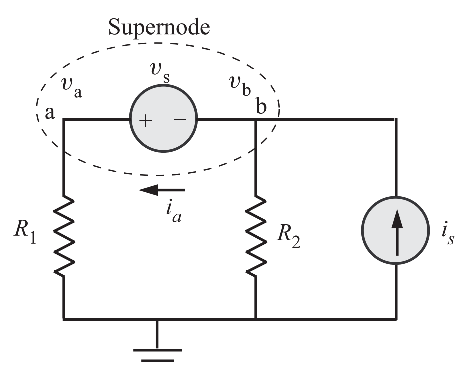


Figure 1.64 Circuit with a supernode incorporating v_a and v_b.

$$\frac{v_a}{R_1} - i_a = 0$$

and $$\frac{v_b}{R_2} + i_a = i_s$$

Adding the above two equations, we find that

$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s$$

$$\Rightarrow \qquad v_a G_1 + v_b G_2 = i_s \tag{1.43}$$

Solving equations (1.42) and (1.43), we can find the values of v_a and v_b.

When we apply KCL at the supernode, mentally imagine that the voltage source v_s is removed from the the circuit of Fig. 1.63, but the voltage at nodes a and b are held at v_a and v_b respectively. In other words, by applying KCL at supernode, we obtain

$$v_a G_1 + v_a G_2 = i_s$$

The equation is the same equation (1.43). As in supermesh, the KCL for supernode eliminates the problem of dealing with a current through a voltage source.

Procedure for using supernode:

1. Use it when a branch between non-reference nodes is connected by an independent or a dependent voltage source.
2. Enclose the voltage source and the two connecting nodes inside a dotted ellipse to form the supernode.
3. Write the constraint equation that defines the voltage relationship between the two non-reference node as a result of the presence of the voltage source.
4. Write the *KCL equation* at the supernode.
5. If the voltage source is dependent, then the *constraint equation* for the dependent source is also needed.

EXAMPLE 1.33

Refer the electrical circuit shown in Fig. 1.65 and find v_a.

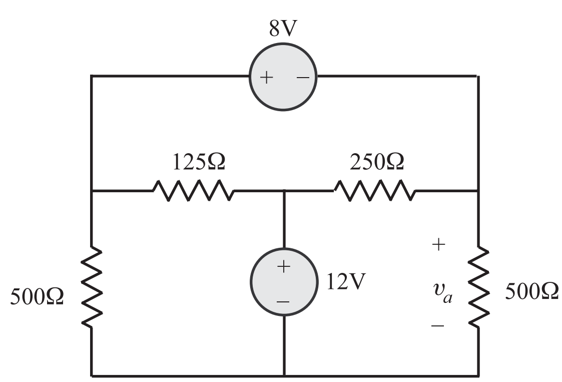


Figure 1.65

SOLUTION

The constraint equation is,

$$v_b - v_a = 8$$

$$\Rightarrow \quad v_b = v_a + 8$$

The KCL equation at the supernode is then,

$$\frac{v_a+8}{500} + \frac{(v_a+8)-12}{125} + \frac{v_a-12}{250} + \frac{v_a}{500} = 0$$

Therefore, $\boldsymbol{v_a = 4\mathrm{V}}$

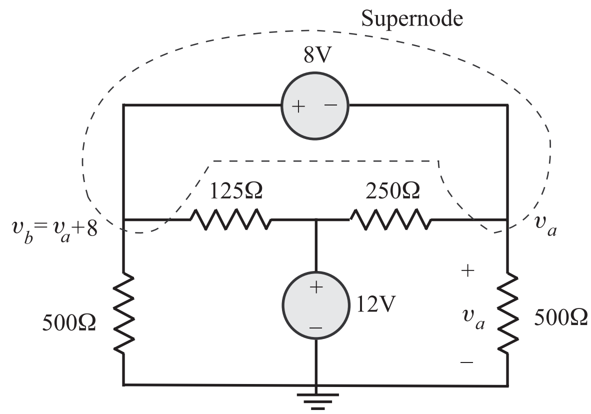


Figure 1.66

EXAMPLE 1.34

Use the nodal analysis to find v_o in the network of Fig. 1.67.

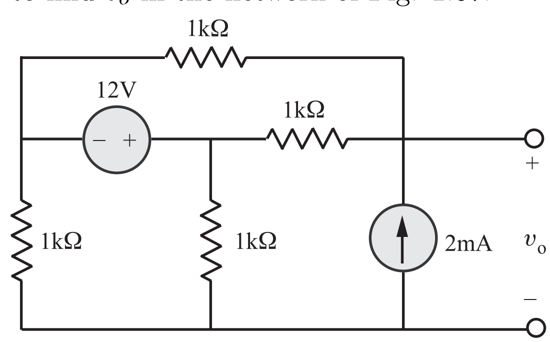


Figure 1.67

SOLUTION

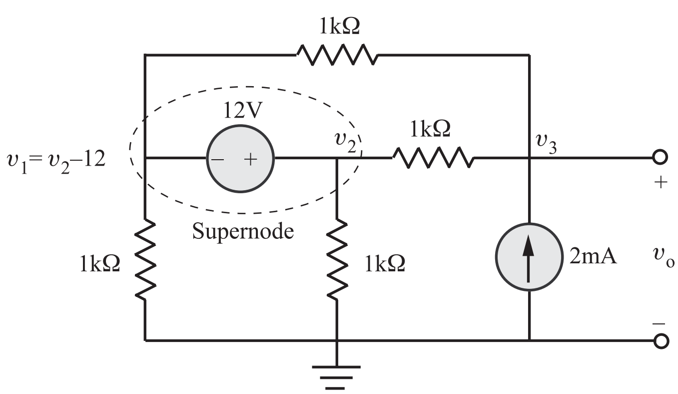


Figure 1.68

The constraint equation is,

$$v_2 - v_1 = 12$$

$$\Rightarrow \qquad v_1 = v_2 - 12$$

KCL at supernode:

$$\frac{v_2 - 12}{1 \times 10^3} + \frac{(v_2 - 12) - v_3}{1 \times 10^3} + \frac{v_2}{1 \times 10^3} + \frac{v_2 - v_3}{1 \times 10^3} = 0$$

$$\Rightarrow \qquad 4 \times 10^{-3} v_2 - 2 \times 10^{-3} v_3 = 24 \times 10^{-3}$$

$$\Rightarrow \qquad 4v_2 - 2v_3 = 24$$

At node v_3:

$$\frac{v_3 - v_2}{1 \times 10^3} + \frac{v_3 - (v_2 - 12)}{1 \times 10^3} = 2 \times 10^{-3}$$

$$\Rightarrow \qquad -2 \times 10^{-3} v_2 + 2 \times 10^{-3} v_3 = -10 \times 10^{-3}$$

$$-2v_2 + 2v_3 = -10$$

Solving we get $\quad v_2 = 7\text{V}$

$v_3 = 2\text{V}$

Hence, $\quad v_o = v_3 = \mathbf{2V}$

EXAMPLE 1.35

Refer the network shown in Fig. 1.69. Find the current I_o.

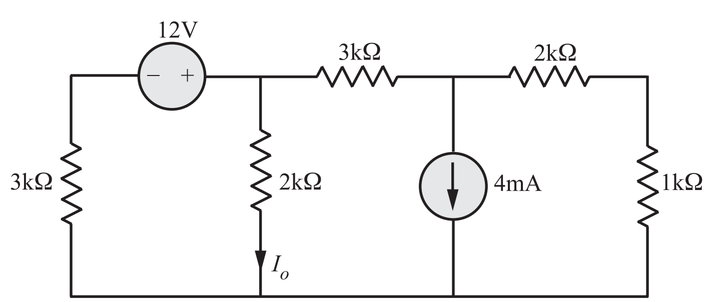


Figure 1.69

SOLUTION

Constriant equation:

$$v_3 = v_1 - 12$$

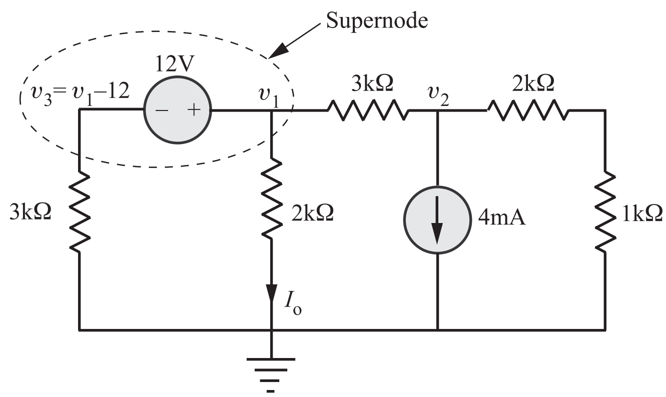


Figure 1.70

KCL at supernode:

$$\frac{v_1 - 12}{3 \times 10^3} + \frac{v_1}{2 \times 10^3} + \frac{v_1 - v_2}{3 \times 10^3} = 0$$

$$\Rightarrow \quad \frac{7}{6} \times 10^{-3} v_1 - \frac{1}{3} \times 10^{-3} v_2 = 4 \times 10^{-3}$$

$$\Rightarrow \quad \frac{7}{6} v_1 - \frac{1}{3} v_2 = 4$$

KCL at node 2:

$$\frac{v_2 - v_1}{3 \times 10^3} + \frac{v_2}{3 \times 10^3} + 4 \times 10^{-3} = 0$$

$$\Rightarrow \quad -\frac{1}{3} \times 10^{-3} v_1 + \frac{2}{3} \times 10^{-3} v_2 = -4 \times 10^{-3}$$

$$\Rightarrow \quad -\frac{1}{3} v_1 + \frac{2}{3} v_2 = -4$$

Putting the above two nodal equations in matrix form, we get

$$\begin{bmatrix} \frac{7}{6} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

Solving the above two matrix equations using Cramer's rule, we get

$$v_1 = 2\text{V}$$

$$\Rightarrow \quad I_o = \frac{v_1}{2 \times 10^3} = \frac{2}{2 \times 10^3} = \mathbf{1mA}$$

EXAMPLE 1.36

Refer the network shown in Fig. 1.71. Find the power delivered by the dependent voltage source in the network.

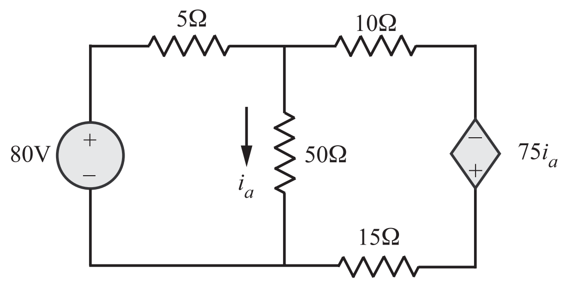


Figure 1.71

SOLUTION

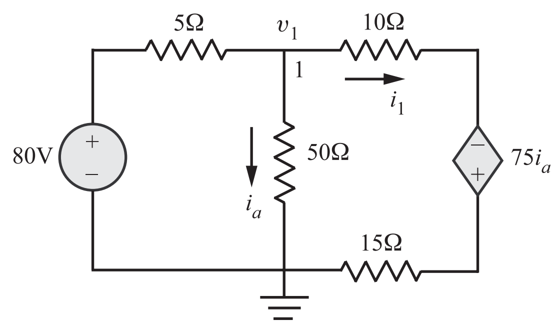


Figure 1.72

Refer Fig. 1.72, *KCL at node* 1:

$$\frac{v_1 - 80}{5} + \frac{v_1}{50} + \frac{v_1 + 75i_a}{25} = 0$$

where $i_a = \dfrac{v_1}{50}$

$$\Rightarrow \quad \frac{v_1 - 80}{5} + \frac{v_1}{50} + \frac{v_1 + 75\left(\frac{v_1}{50}\right)}{25} = 0$$

Solving we get $\quad v_1 = \mathbf{50V}$

$$\Rightarrow \qquad i_a = \frac{v_1}{50} = \frac{50}{50} = 1\text{A}$$

Also,

$$\begin{aligned} i_1 &= \frac{v_1 - (-75i_a)}{(10+15)} \\ &= \frac{v_1 + 75i_a}{(10+15)} \\ &= \frac{50 + 75 \times 1}{(10+15)} = 5\text{A} \\ P_{75ia} &= (75i_a)i_1 \\ &= 75 \times 1 \times 5 \\ &= \mathbf{375W} \text{ (delivered)} \end{aligned}$$

EXAMPLE 1.37

Use the node-voltage method to find the power developed by the 20 V source in the circuit shown in Fig. 1.73.

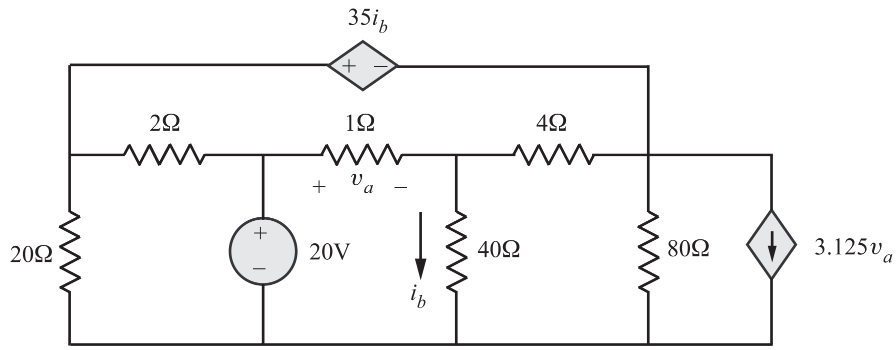


Figure 1.73

SOLUTION

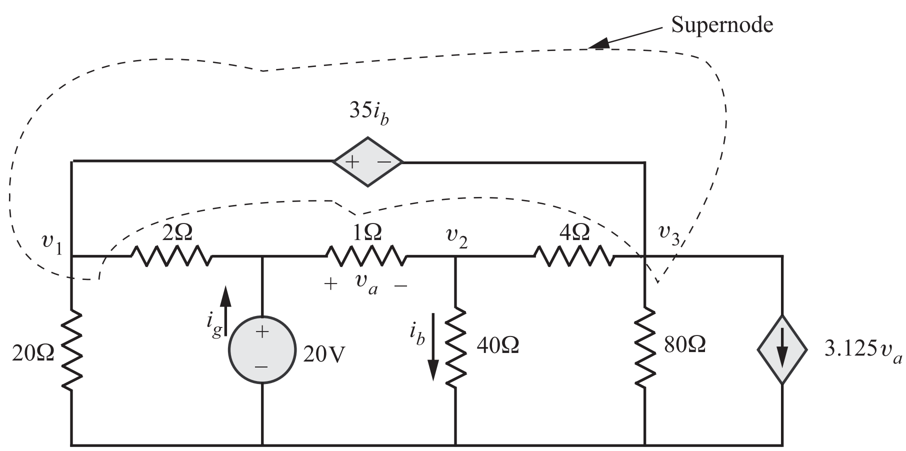


Figure 1.74

Constraint equations:

$$v_a = 20 - v_2$$
$$v_1 - 31i_b = v_3$$
$$i_b = \frac{v_2}{40}$$

Node equations:

(i) Supernode:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_a = 0$$

$$\Rightarrow \quad \frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{(v_1 - 35i_b) - v_2}{4} + \frac{(v_1 - 35i_b)}{80} + 3.125(20 - v_2) = 0$$

$$\Rightarrow \quad \frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{\left(v_1 - 35\frac{v_2}{40}\right) - v_2}{4} + \frac{\left(v_1 - 35\frac{v_2}{40}\right)}{80} + 3.125(20 - v_2) = 0$$

(ii) *At node* v_2:

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

$$\Rightarrow \qquad \frac{v_2}{40} + \frac{v_2 - (v_1 - 35i_b)}{4} + \frac{v_2 - 20}{1} = 0$$

$$\Rightarrow \qquad \frac{v_2}{40} + \frac{v_2 - \left(v_1 - 35\frac{v_2}{40}\right)}{4} + \frac{v_2 - 20}{1} = 0$$

Solving the above two nodal equations, we get

$$v_1 = -20.25\text{V}, \quad v_2 = 10\text{V}$$

Then
$$\begin{aligned} v_3 &= v_1 - 35i_b \\ &= v_1 - 35\frac{v_2}{40} \\ &= \mathbf{-29V} \end{aligned}$$

Also,
$$\begin{aligned} i_g &= \frac{20 - v_1}{2} + \frac{20 - v_2}{1} \\ &= \frac{20 + 20.25}{2} + \frac{(20 - 10)}{1} \\ &= \mathbf{30.125\ \ A} \end{aligned}$$

$$\begin{aligned} P_{20\text{V}} &= 20i_g = 20(30.125) \\ &= \mathbf{602.5\ \ W\ \ (delivered)} \end{aligned}$$

EXAMPLE 1.38

Refer the circuit shown in Fig. 1.75(a). Determine the current i_1.

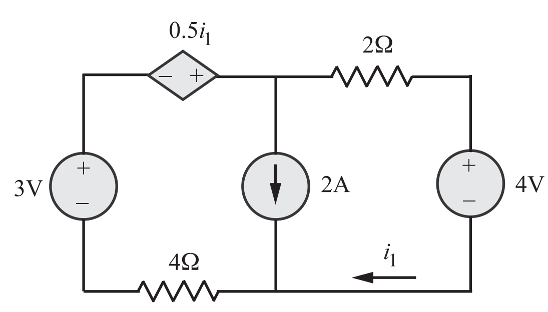


Figure 1.75(a)

SOLUTION

Constraint equation:

Applying KVL clockwise to the loop containing 3V source, dependent voltage source, 2A current source and 4Ω resitor, we get

$$-v_1 - 3 - 0.5i_1 + v_2 = 0$$
$$\Rightarrow \qquad v_1 - v_2 = -3 - 0.5i_1$$

Substituting $i_1 = \dfrac{v_2 - 4}{2}$, the above equation becomes

$$4v_1 - 3v_2 = -8$$

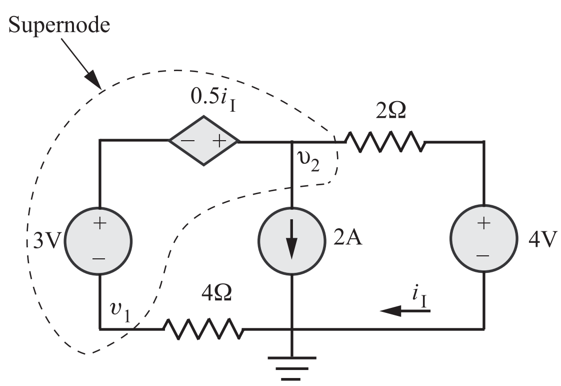


Figure 1.75(b)

KCL equation at supernode:

$$\frac{v_1}{4} + \frac{v_2 - 4}{2} = -2 \quad \Rightarrow \quad v_1 + 2v_2 = 0$$

Solving the constraint equation and the KCL equation at supernode simultaneously, we find that,

$$v_2 = 727.3 \text{ mV}$$
$$v_1 = -2v_2$$
$$= -1454.6 \text{ mV}$$

Then,
$$i_1 = \frac{v_2 - 4}{2}$$
$$= \mathbf{-1.636A}$$

EXAMPLE 1.39

Refer the network shown in Fig. 1.76(a). Find the node voltages v_d and v_c.

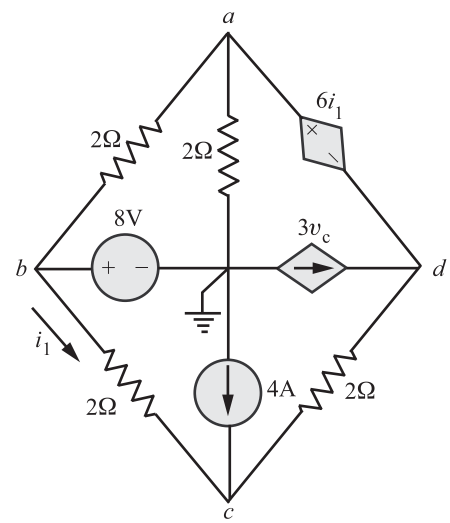


Figure 1.76(a)

SOLUTION

From the network, shown in Fig. 1.76 (b), by inspection, $v_b = 8$ V, $i_1 = \dfrac{v_b - v_c}{2}$

Constraint equation: $v_a = 6i_1 + v_d$

KCL at supernode: $\dfrac{v_a - v_b}{2} + \dfrac{v_a}{2} + \dfrac{v_d - v_c}{2} = 3v_c$

$$\Rightarrow \quad v_a\left[\frac{1}{2} + \frac{1}{2}\right] - \frac{1}{2}v_b + \frac{1}{2}\left[v_d - v_c\right] = 3v_c \tag{1.44}$$

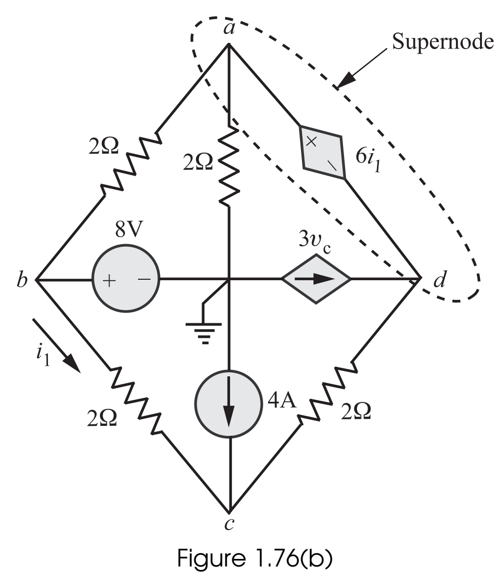


Figure 1.76(b)

Substituting $v_b = 8$ V in the constrained equation, we get

$$v_a = 6\frac{(v_b - v_c)}{2} + v_d$$
$$= 3(v_b - v_c) + v_d$$
$$= 3(8 - v_c) + v_d \tag{1.45}$$

Substituting equation (1.45) into equation (1.44), we get

$$[3(8 - v_c) + v_d] - \frac{1}{2}(8) + \frac{1}{2}[v_d - v_c] = 3v_c$$
$$\Rightarrow \qquad 24 - 3v_c + v_d - 4 + \frac{1}{2}v_d - \frac{1}{2}v_c = 3v_c$$
$$\Rightarrow \qquad -6.5v_c + 1.5v_d = -20 \tag{1.46}$$

KCL at node c: $$\frac{v_c - v_b}{2} + \frac{v_c - v_d}{2} = 4$$

Substituting $v_b = 8$V, we have $$\frac{v_c - 8}{2} + \frac{v_c - v_d}{2} = 4$$

$$\Rightarrow \qquad v_c - 8 + v_c - v_d = 8$$
$$\Rightarrow \qquad 2v_c - v_d = 16$$
$$\Rightarrow \qquad v_c - 0.5v_d = 8 \tag{1.47}$$

Solving equations (1.46) and (1.47), we get

$$v_c = \mathbf{-1.14V}$$
$$v_d = \mathbf{-18.3V}$$

EXAMPLE 1.40

For the circuit shown in Fig. 1.77(a), determine all the node voltages.

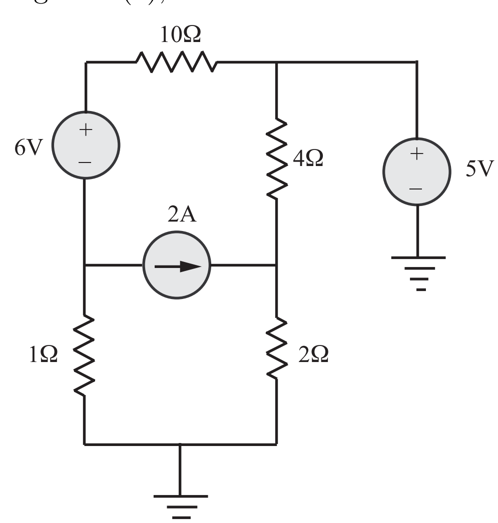


Figure 1.77(a)

SOLUTION

Refer Fig 1.77(b), by inspection, $v_2 = 5$V

Nodes 1 and 3 form a supernode.

Constraint equation:

$$v_1 - v_3 = 6$$

KCL at super node:

$$\frac{v_1 - v_2}{10} + \frac{v_3}{1} + 2 = 0$$

Substituting $v_2 = 5$V, we get

$$\frac{v_1 - 5}{10} + \frac{v_3}{1} = -2$$

$$\Rightarrow \quad v_1 - 5 + 10v_3 = -20$$

$$\Rightarrow \quad v_1 + 10v_3 = -15$$

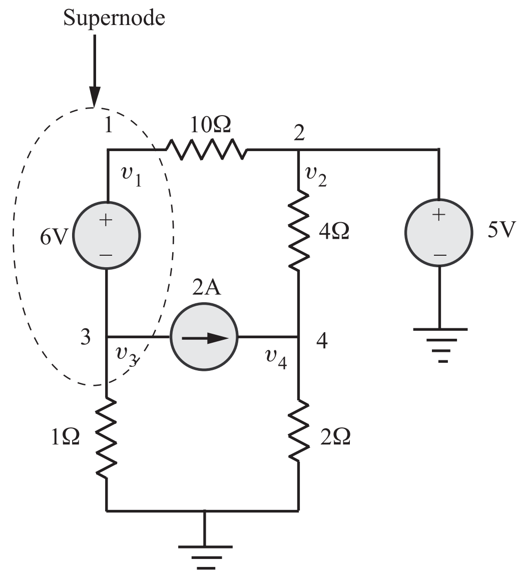


Figure 1.77(b)

Solving the constraint and the *KCL* equations at supernode simultaneously, we get

$$\boldsymbol{v_1 = 4.091\text{V}}$$
$$\boldsymbol{v_3 = -1.909\text{V}}$$

KCL at node 4 :

$$\frac{v_4}{2} + \frac{v_4 - v_2}{4} - 2 = 0$$

Substituting $v_2 = 5$V, we get

$$\frac{v_4}{2} + \frac{v_4 - 5}{4} - 2 = 0$$

Solving we get, $\quad v_4 = \mathbf{4.333V}$.

1.15 Brief review of impedance and admittance

Let us consider a general circuit with two accessible terminals, as shown in Fig. 1.78. If the time domain voltage and current at the terminals are given by

$$v = v_m \sin(\omega t + \phi_v)$$
$$i = i_m \sin(\omega t + \phi_i)$$

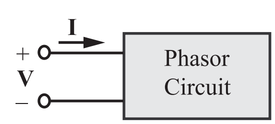


Figure 1.78 General phasor circuit

then the phasor quantities at the terminals are

$$\mathbf{V} = V_m \underline{/\phi_v}$$
$$\mathbf{I} = I_m \underline{/\phi_i}$$

We define the ratio of **V** to **I** as the impedence of the circuit, which is denoted as **Z**. That is,

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

It is very important to note that impedance **Z** is a complex quantity, being the ratio of two complex quantities, but **it is not a phasor**. That is, it has no corresponding sinusoidal time-domain function, as current and voltage phasors do. Impedence is a complex constant that scales one phasor to produce another.

The impedence **Z** is written in rectangular form as

$$\mathbf{Z} = R + jX$$

where $R = \text{Real}[\mathbf{Z}]$ is the resistance and $X = \text{Im}[\mathbf{Z}]$ is the reactance. Both R and X, like **Z**, are measured in ohms.
The magnitude of **Z** is written as $|\mathbf{Z}| = \sqrt{R^2 + X^2}$ and the angle of **Z** is denoted as $\phi_Z = \tan^{-1}\left[\dfrac{X}{R}\right]$. The relationships are shown graphically in Fig. 1.79. The table below gives the various forms of **Z** for different combinations of R, L and C.

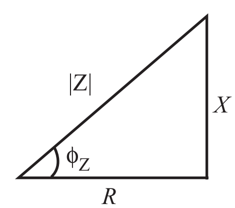


Figure 1.79 Graphical representation of impedance

	Type of the circuit	Impedance Z
1.	Purely resistive	$\mathbf{Z} = R$
2.	Purely inductive	$\mathbf{Z} = j\omega L = jX_L$
3.	Purely capactive	$\mathbf{Z} = \dfrac{-j}{\omega C} = -jX_C$
4.	RL	$\mathbf{Z} = R + j\omega L = R + jX_L$
5.	RC	$\mathbf{Z} = R - \dfrac{j}{\omega C} = R - jX_C$
6.	RLC	$\mathbf{Z} = R + j\omega L - \dfrac{j}{\omega C} = R + j(X_L - X_C)$

The reciprocal of impendance is denoted by

$$\mathbf{Y} = \frac{\mathbf{1}}{\mathbf{Z}}$$

is called admittance and is analogous to conductance in resistive circuits. Evidently, since $\mathbf{Z}$ is a complex number, so is $\mathbf{Y}$. The standard representation of admittance is

$$\mathbf{Y} = G + jB$$

The quantities $G = \text{Re}[\mathbf{Y}]$ and $B = \text{Im}[\mathbf{Y}]$ are respectively called conductance and suspectence. The units of $\mathbf{Y}$, G and B are all siemens.

1.16 Kirchhoff's Laws: Applied to alternating circuits

If a complex excitation, say $v_m e^{j(\omega t+\theta)}$, is applied to a circuit, then complex voltages, such as $v_1 e^{j(\omega t+\theta_1)}, v_2 e^{j(\omega t+\theta_2)}$ and so on, appear across the elements in the circuit. Kirchhoff's voltage law applied around a typical loop results in an equation such as

$$v_1\ e^{j(\omega t+\theta_1)} + v_2\ e^{j(\omega t+\theta_2)} + \ldots + v_N e^{j(\omega t+\theta_N)} = 0$$

Dividing by $e^{j\omega t}$, we get

$$v_1 e^{j\theta_1} + v_2 e^{j\theta_2} + \ldots + v_N e^{j\theta_N} = 0$$

$$\Rightarrow \qquad \mathbf{V}_1 + \mathbf{V}_2 + \ldots + \mathbf{V}_N = 0$$

where $$\mathbf{V}_i = V_i \underline{/\theta_i}\ , i = 1, 2, \cdots N$$

are the phasor voltage around the loop.

Thus KVL holds good for phasors also. A similar approach will establish KCL also. At any node having N connected branches,

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = 0$$

where $$\mathbf{I}_i = I_i \underline{/\theta_i}\ , i = 1, 2 \cdots N$$

Thus, KCL holds good for phasors also.

EXAMPLE 1.41

Determine $\mathbf{V}_1$ and $\mathbf{V}_2$, the node voltage phasors using nodal technique for the circuit shown in Fig. 1.80.

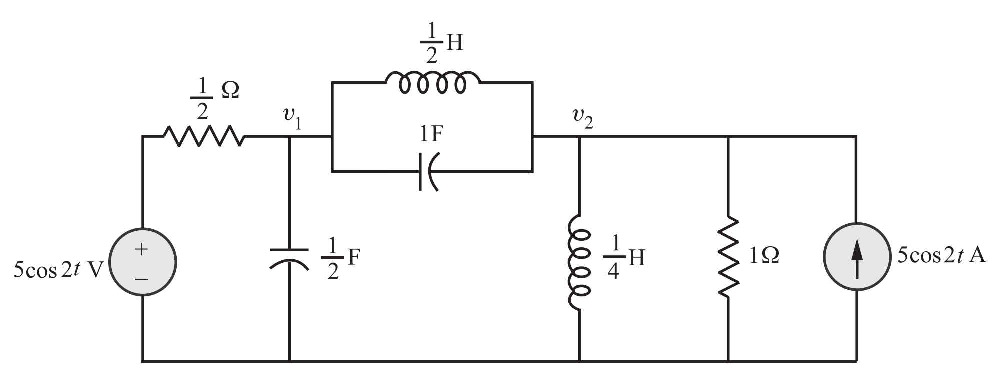


Figure 1.80

SOLUTION

First step in the analysis is to convert the circuit of Fig. 1.80 into its phasor version (frequency domain representation).

$$5\cos 2t \quad \Rightarrow \quad 5\underline{/0^\circ}\ , \omega = 2 \text{ rad/s}$$
$$\frac{1}{4}\text{ H} \quad \Rightarrow \quad j\omega L = j\frac{1}{2}\Omega$$
$$\frac{1}{2}\text{ F} \quad \Rightarrow \quad \frac{-j}{\omega C} = -j1\Omega, \qquad 1\text{F} \quad \Rightarrow \quad \frac{-j}{\omega C} = -j\frac{1}{2}\Omega$$

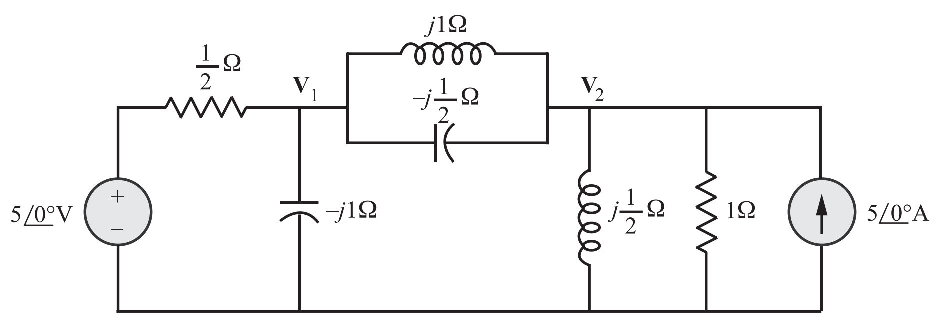


Figure 1.80(a)

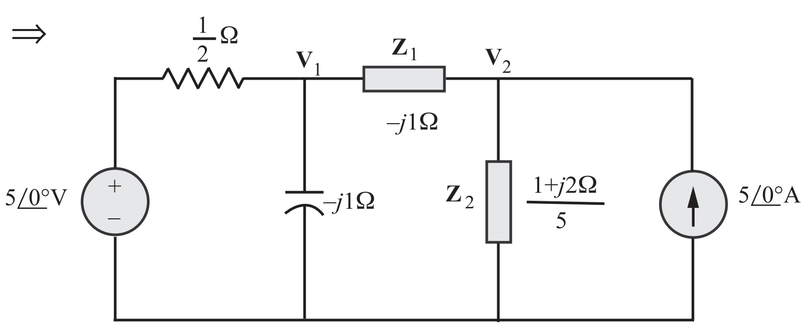


Figure 1.80(b)

Fig. 1.80(a) and (b) are the two versions of the phasor circuit of Fig. 1.80.

$$\mathbf{Z}_1 = j1\Omega || \left(-j\frac{1}{2}\Omega\right)$$
$$= \frac{j1\left(-j\frac{1}{2}\right)}{j1 - j\frac{1}{2}} = -j1\Omega$$

$$\mathbf{Z}_2 = j\frac{1}{2}\Omega||1\Omega$$
$$= \frac{\left(j\frac{1}{2}\right)(1)}{\left(j\frac{1}{2}+1\right)} = \frac{1+j2}{5}\Omega$$

KCL at node $\mathbf{V}_1$:

$$2\left(\mathbf{V}_1 - 5\,\underline{/0^\circ}\,\right) + \frac{\mathbf{V}_1}{-j1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j1} = 0$$
$$\Rightarrow \qquad (2+j2)\mathbf{V}_1 - j1\mathbf{V}_2 = 10$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j1} + \frac{\mathbf{V}_2}{\frac{1+j2}{5}} = 5\angle 0^\circ$$
$$\Rightarrow \qquad j\mathbf{V}_2 - j\mathbf{V}_1 + \mathbf{V}_2 - 2j\mathbf{V}_2 = 5$$
$$\Rightarrow \qquad -\,j1\mathbf{V}_{1+}(1-j1)\mathbf{V}_2 = 5$$

Putting the above equations in a matrix form, we get

$$\begin{bmatrix} 2+j2 & -j1 \\ -j1 & 1-j1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Solving $\mathbf{V}_1$ and $\mathbf{V}_2$ by Cramer's rule, we get

$$\mathbf{V}_1 = 2 - j1 \text{ V}$$
$$\mathbf{V}_2 = 2 + j4 \text{ V}$$

In polar form,

$$\mathbf{V}_1 = \sqrt{5}\,\underline{/-26.6^\circ}\ \text{ V}$$
$$\mathbf{V}_2 = 2\sqrt{5}\,\underline{/63.4^\circ}\ \text{ V}$$

In time domain,

$$\boldsymbol{v_1 = \sqrt{5}\cos(2t - 26.6^\circ)\ \mathrm{V}}$$
$$\boldsymbol{v_2 = 2\sqrt{5}\cos(2t + 63.4^\circ)\ \mathrm{V}}$$

EXAMPLE 1.42

Find the source voltage $\mathbf{V}_s$ shown in Fig. 1.81 using nodal technique. Take $\mathbf{I} = 3\,\underline{/45^\circ}$ A.

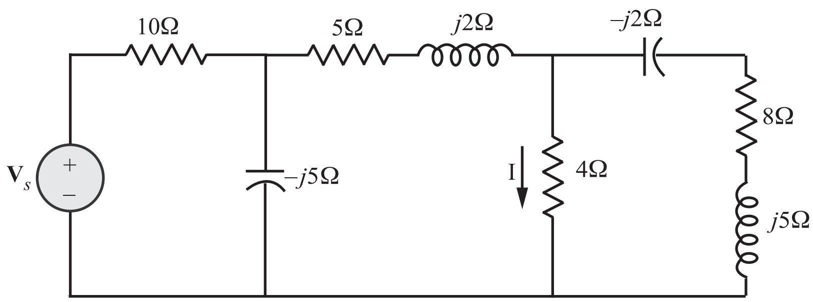


Figure 1.81

SOLUTION

Refer to Fig. 1.81(a).

KCL at node 1:

$$\frac{\mathbf{V}_1 - \mathbf{V}_s}{10} + \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{5 + j2} = 0$$

$$\Rightarrow \quad (11 + j12)\mathbf{V}_1 - (5 + j2)\mathbf{V}_s = 10\mathbf{V}_2 \tag{1.48}$$

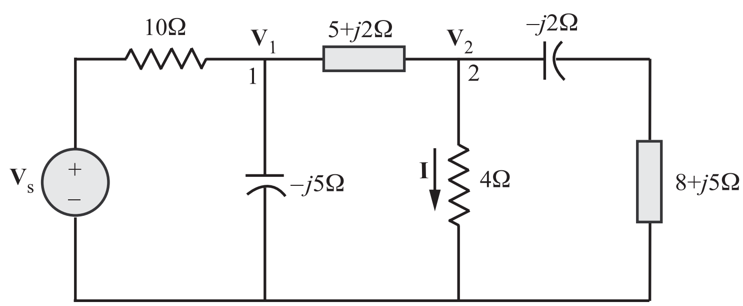


Figure 1.81(a)

KCL at node 2:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{5 + j2} + \mathbf{I} + \frac{\mathbf{V}_2}{8 + j3} = 0$$

$$\Rightarrow \quad (8 + j3)\mathbf{V}_1 = (13 + j5)\mathbf{V}_2 + (34 + j31)\mathbf{I} \tag{1.49}$$

Also,

$$\begin{aligned} \mathbf{V}_2 = 4\mathbf{I} = 4(3\,\underline{/45^\circ}) &= 12\,\underline{/45^\circ} \\ &= 6\sqrt{2} + j6\sqrt{2} \end{aligned} \tag{1.50}$$

Substituting equations (1.48) and (1.50) in equation (1.49), we get

$$(8+j3)\mathbf{V}_1 = 74.24 + j290.62$$

$$\Rightarrow \qquad \mathbf{V}_1 = \frac{300\,\underline{/75.7^\circ}}{8.54\,\underline{/20.6^\circ}}$$

$$= 35.1\,\underline{/55.1^\circ}$$

$$= 20.1 + j28.8 \text{ V}$$

Substituting $\mathbf{V}_1$ and $\mathbf{V}_2$ in equation (1.48) yields

$$(5+j2)\mathbf{V}_s = -209.4 + j473.1$$

Therefore,

$$\mathbf{V}_s = \frac{517.4\,\underline{/113.9^\circ}}{5.38\,\underline{/21.8^\circ}} = \mathbf{96.1\,\underline{/92.1^\circ}\ V}$$

EXAMPLE 1.43

Find the voltage $v(t)$ in the network shown in Fig. 1.82 using nodal technique.

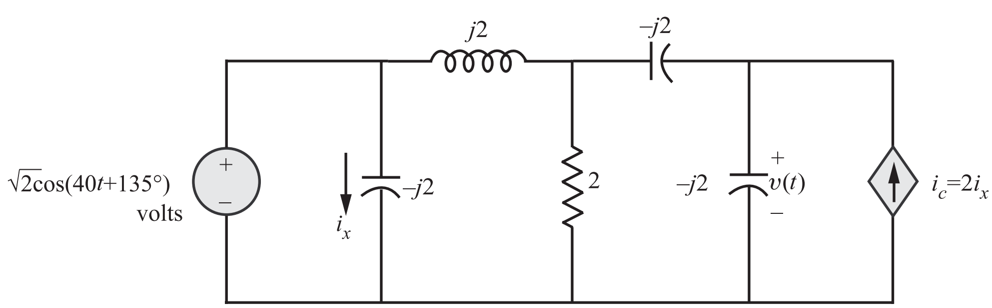


Figure 1.82

SOLUTION

Converting the circuit diagram shown in Fig. 1.82 into a phasor circuit diagram, we get

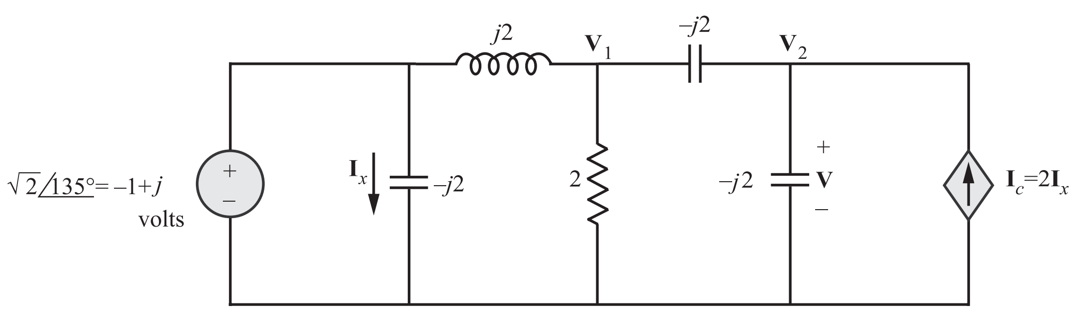


Figure 1.83

At node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1 - (-1 + j)}{j2} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j2} = 0$$

$$\Rightarrow \quad \mathbf{V}_1 - j\mathbf{V}_2 = 1 + j \tag{1.51}$$

At node$\mathbf{V}_2$: $\quad \dfrac{\mathbf{V}_2 - \mathbf{V}_1}{-j2} + \dfrac{\mathbf{V}_2}{-j2} - \mathbf{I}_c = 0$

Also $\quad \mathbf{I}_c = 2\mathbf{I}_x = \dfrac{2(-1 + j)}{-j2} = -1 - j$

Hence, $\quad \dfrac{\mathbf{V}_2 - \mathbf{V}_1}{-j2} + \dfrac{\mathbf{V}_2}{-j2} = -1 - j$

$$\Rightarrow \quad -j\mathbf{V}_1 + j2\mathbf{V}_2 = -2 - j2 \tag{1.52}$$

Solving equations (1.51) and (1.52) using Cramer's rule we get

$$\mathbf{V}_2 = \sqrt{2}\,\underline{/135^\circ}\ \mathrm{V}$$

Therefore, $\quad \boldsymbol{v(t) = v_2(t) = \sqrt{2}\cos(4t + 135^\circ)\ \mathrm{V}}$

EXAMPLE 1.44

Refer to the circuit of Fig. 1.84. Using nodal technique, find the current i.

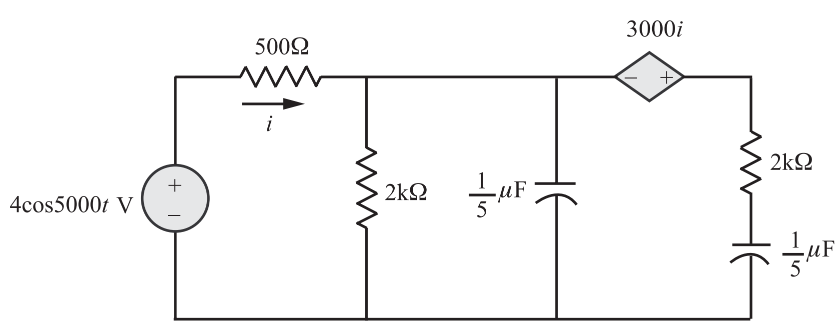


Figure 1.84

SOLUTION

Reactance of $\dfrac{1}{5}\mu$F capacitor $= \dfrac{1}{j\omega C} = \dfrac{1}{j5000 \times \dfrac{1}{5} \times 10^{-6}} = -j1\mathrm{k}\Omega$

The parallel combinations of 2kΩ and $-j1$kΩ is

$$\mathbf{Z}_p = \frac{2 \times 10^3(-j10^3)}{2 \times 10^3 - j10^3} = \frac{2}{5}(1 - j2)\mathrm{k}\Omega$$

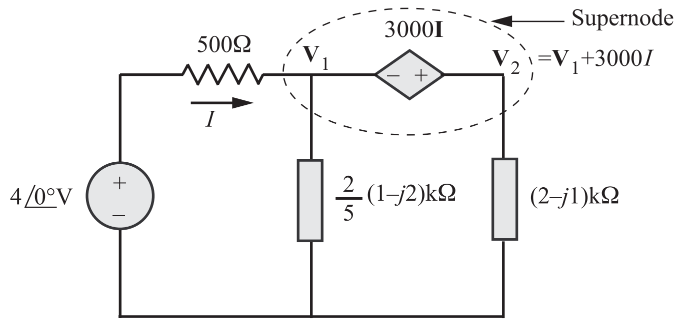


Figure 1.85

The phasor circuit of Fig. 1.84 is as shown in Fig. 1.85.
Constraint equation :

$$\mathbf{V_2 = V_1 + 3000I}$$

KCL at supernode :

$$\frac{\mathbf{V}_1 - 4\,\underline{/0^\circ}}{500} + \frac{\mathbf{V}_1}{\frac{2}{5}(1-j2)\times 10^3} + \frac{\mathbf{V}_2}{(2-j1)\times 10^3} = 0$$

Substituting $\mathbf{V}_2 = \mathbf{V}_1 + 3000\mathbf{I}$ in the above equation, we get

$$\frac{\mathbf{V}_1 - 4\,\underline{/0^\circ}}{500} + \frac{\mathbf{V}_1}{\frac{2}{5}(1-j2)\times 10^3} + \frac{\mathbf{V}_1 + 3000\mathbf{I}}{(2-j1)\times 10^3} = 0$$

Also,

$$\mathbf{I} = \frac{4\,\underline{/0^\circ} - V_1}{500} \tag{1.53}$$

Hence,

$$\frac{\mathbf{V}_1 - 4\angle 0^\circ}{500} + \frac{\mathbf{V}_1}{\frac{2}{5}(1-j2)\times 10^3} + \frac{\mathbf{V}_1 + 3000\left(\frac{4-\mathbf{V}_1}{500}\right)}{(2-j1)\times 10^3} = 0$$

Solving for $\mathbf{V}_1$ and substituting the same in equation (1.53), we get $\mathbf{I} = 24\,\underline{/53.1^\circ}$ mA
Hence, in time-domain, we have

$$\boldsymbol{i = 24\cos(5000t + 53.1^\circ)\mathrm{mA}}$$

EXAMPLE 1.45

Use nodal analysis to find $\mathbf{V}_o$ in the circuit shown in Fig. 1.86.

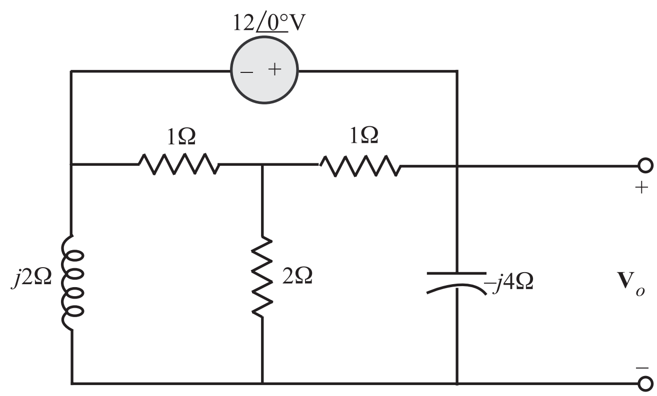


Figure 1.86

SOLUTION

The voltage source and its two connecting nodes form the supernode as shown in Fig. 1.87.

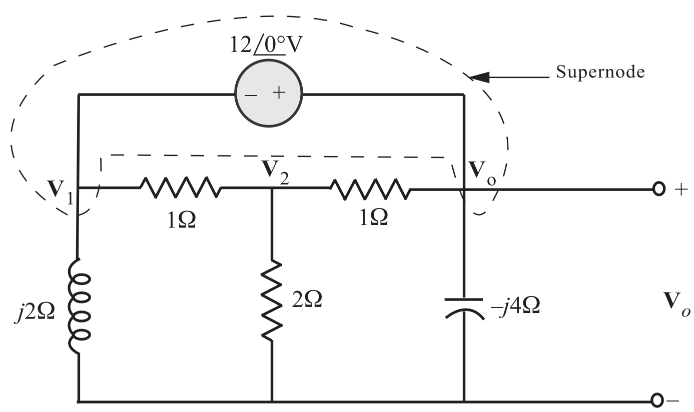


Figure 1.87

Constraint equation:

Applying KVL clockwise to the loop formed by $12\,\underline{/0^\circ}$ source, $j2\Omega$ and $-j4\Omega$ we get

$$-12\,\underline{/0^\circ} + \mathbf{V}_o - \mathbf{V}_1 = 0$$

$$\Rightarrow \qquad \mathbf{V}_1 = \mathbf{V}_o - 12\,\underline{/0^\circ}$$

KCL at supernode:

$$\frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} + \frac{\mathbf{V}_o - \mathbf{V}_2}{1} + \frac{\mathbf{V}_o}{-j4} = 0$$

Substituting $\mathbf{V}_1 = \mathbf{V}_o - 12$ in the above equation

we get, $$\frac{-j}{2}(\mathbf{V}_o - 12) + (\mathbf{V}_o - 12 - \mathbf{V}_2) + \mathbf{V}_o - \mathbf{V}_2 + \frac{j}{4}\mathbf{V}_o = 0$$

$$\Rightarrow \qquad \mathbf{V}_o\left(\frac{-j}{2} + 1 + 1 + \frac{j}{4}\right) + \mathbf{V}_2(-1 - 1) = 12 - j6$$

$$\Rightarrow \qquad \mathbf{V}_o\left(2 - \frac{1}{4}j\right) - 2\mathbf{V}_2 = 12 - j6$$

KCL at $\mathbf{V}_2$:
$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{1} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} = 0$$

Substituting $\mathbf{V}_1 = \mathbf{V}_o - 12\,\underline{/0^\circ}$ in the above equation

we get, $$\mathbf{V}_2 - (\mathbf{V}_o - 12\,\underline{/0^\circ}) + \frac{1}{2}\mathbf{V}_2 + \mathbf{V}_2 - \mathbf{V}_o = 0$$

$$\Rightarrow \qquad -2\mathbf{V}_o + \frac{5}{2}\mathbf{V}_2 = -12\,\underline{/0^\circ}$$

Solving the two nodal equations,we get

$$\mathbf{V_o = 11.056 - \mathit{j}8.09 = 13.7\,\underline{/-36.2^\circ}\ V}$$

EXAMPLE 1.46

Find i_1 in the circuit of Fig. 1.88 using nodal analysis.

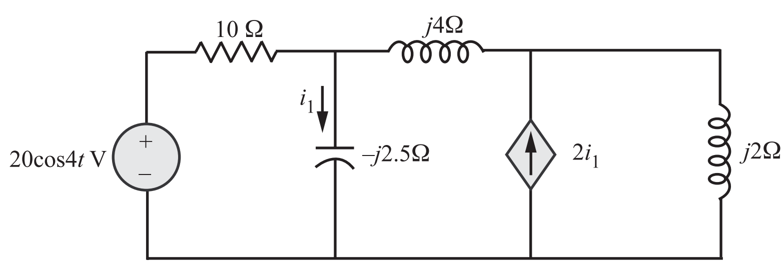


Figure 1.88

SOLUTION

The phasor equivalent circuit is as shown in Fig. 1.88(a).

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1 - 20\,\underline{/0^\circ}}{10} + \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = 0$$

$$\Rightarrow \qquad (1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{j2} = 2\mathbf{I}_1$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{-j2.5}$$

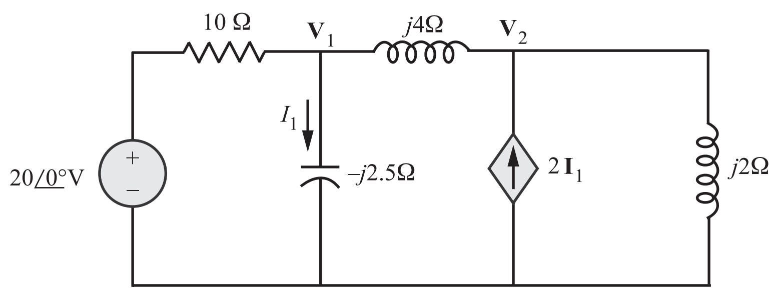


Figure 1.88(a)

Hence, $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{j2} = \frac{2\mathbf{V}_1}{-j2.5}$$

$$\Rightarrow \quad -j0.55\mathbf{V}_1 - j0.75\mathbf{V}_2 = 0$$

Multiplying throughout by $j20$, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

Putting the two nodal equations in matrix form, we get

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

Solving the matrix equation, we get

$$\mathbf{V}_1 = 18.97\,\underline{/18.43^\circ}\ \text{V}$$
$$\mathbf{V}_2 = 13.91\,\underline{/-161.56^\circ}\ \text{V}$$

The current $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{-j2.5} = 7.59\,\underline{/108.4^\circ}\ \text{A}$$

Transforming this to the time-domain, we get

$$\boldsymbol{i_1 = 7.59\cos(4t + 108.4^\circ)\,\mathrm{A}}$$

EXAMPLE 1.47

Use the node-voltage method to find the steady-state expression for $v_o(t)$ in the circuit shown in Fig. 1.89 if

$$v_{g1} = 10\cos(5000t + 53.13^\circ)\text{V}$$
$$v_{g2} = 8\sin 5000t\ \text{V}$$

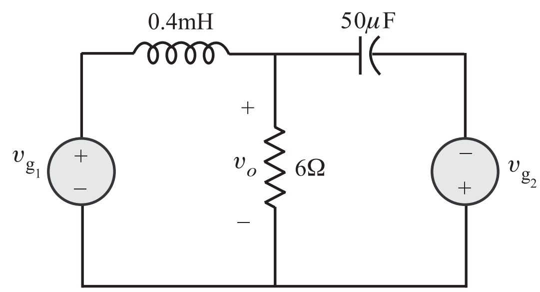


Figure 1.89

SOLUTION

The first step is to convert the circuit of Fig. 1.89 into a phasor circuit.

$$\begin{aligned} 10\cos(5000t+53.13^\circ)\text{V},\ \omega=5000\text{rad/sec} \quad &\Rightarrow \quad 10\,\underline{/53.13^\circ}=6+j8\text{V} \\ 8\sin 5000t = 8\cos(5000t-90^\circ)\text{V} \quad &\Rightarrow \quad 8\,\underline{/-90^\circ}=-j8\text{V} \\ L=0.4\text{ mH} \quad &\Rightarrow \quad j\omega L=j2\Omega \\ C=50\mu F \quad &\Rightarrow \quad \frac{1}{j\omega C}=-j4\Omega \end{aligned}$$

The phasor circuit is shown in Fig. 1.89(a).

KCL at node 1:

$$\frac{\mathbf{V}_o-(6+j8)}{j2}+\frac{\mathbf{V}_o}{6}+\frac{\mathbf{V}_o-(-j8)}{-j4}=0$$

Solving we get $\mathbf{V}_o=12\,\underline{/0^\circ}$ V

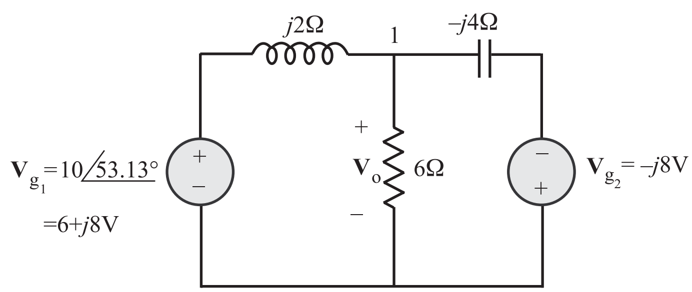


Figure 1.89(a)

Hence, the steady-state expression is

$$v_o(t)=12\cos 5000\text{t}$$

EXAMPLE 1.48

Solve the example (1.47) using mesh-current method.

SOLUTION

Refer Fig. 1.90.

$$\begin{aligned} &KVL\ to\ mesh\ 1: &&[6+j2]\mathbf{I}_1-6\mathbf{I}_2=10\,\underline{/53.13^\circ} \\ &KVL\ to\ mesh\ 2: &&-6\mathbf{I}_1+(6-j4)\mathbf{I}_2=8\,\underline{/-90^\circ} \end{aligned}$$

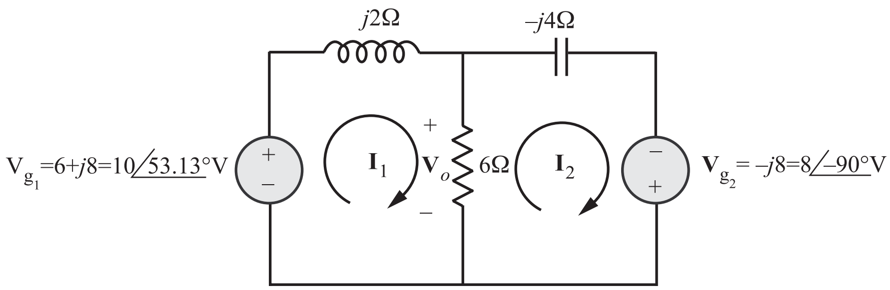


Figure 1.90

Putting the above equations in matrix form, we get

$$\begin{bmatrix} 6+j2 & -6 \\ -6 & 6-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\,\underline{/53.13°} \\ 8\,\underline{/-90°} \end{bmatrix}$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$, we get

$$\mathbf{I}_1 = 4 + j3$$
$$\mathbf{I}_2 = 2 + j3$$

Now, $$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6 = 12$$
$$= 12\,\underline{/0°}\ \ \text{V}$$

Hence in time domain, $$\boldsymbol{v_o} = \mathbf{12\cos 5000}\boldsymbol{t}\ \mathbf{Volts}$$

EXAMPLE 1.49

Determine the current $\mathbf{I}_o$ in the circuit of Fig. 1.91 using mesh analysis.

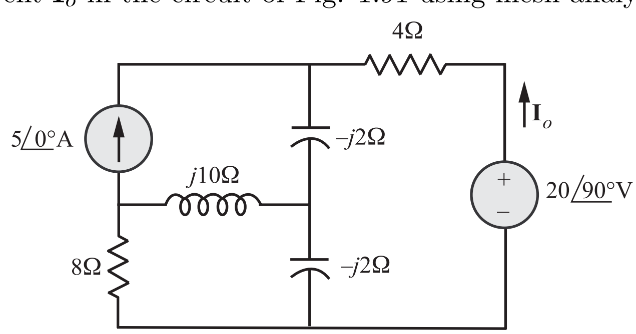


Figure 1.91

SOLUTION

Refer Fig 1.92

KVL for mesh 1 :

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

$$\Rightarrow \quad (8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j10\mathbf{I}_3 \tag{1.54}$$

KVL for mesh 2 :

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\,\underline{/90^\circ} = 0$$

$$\Rightarrow \quad j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 + j2\mathbf{I}_3 = -j20 \tag{1.55}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{1.56}$$

Sustituting the value of $\mathbf{I}_3$ in the equations (1.54) and (1.55), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$
$$= -j30$$

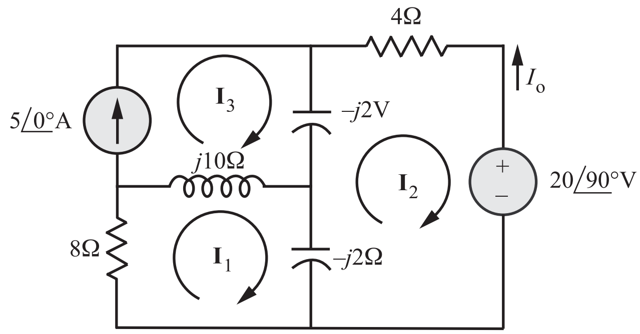


Figure 1.92

Putting the above equations in matrix form,we get

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

Using Cramer's rule,we get

$$\mathbf{I}_2 = 6.12\,\underline{/-35.22^\circ} \text{ A}$$

The required current:

$$\mathbf{I}_o = -\mathbf{I}_2$$
$$= \mathbf{6.12\,\underline{/144.78^\circ}\ A}$$

EXAMPLE 1.50

Find $\mathbf{V}_{oc}$ using mesh technique.

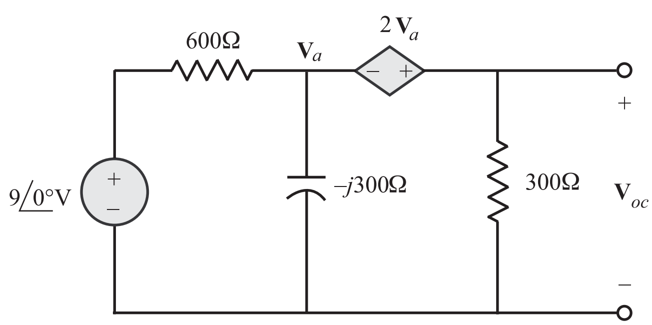


Figure 1.93

SOLUTION

Applying KVL clockwise for mesh 1 :

$$600\mathbf{I}_1 - j300(\mathbf{I}_1 - \mathbf{I}_2) - 9 = 0$$

$$\Rightarrow \quad (600 - j300)\mathbf{I}_1 + j300\mathbf{I}_2 = 9$$

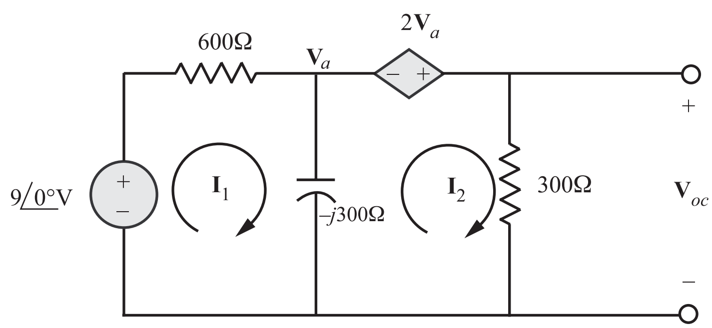


Figure 1.94

Applying KVL clockwise for mesh 2 :

$$-2\mathbf{V}_a + 300\mathbf{I}_2 - j300(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

Also, $$\mathbf{V}_a = -j300(\mathbf{I}_1 - \mathbf{I}_{2)}$$

Hence, $$-\,2(-j300(\mathbf{I}_1 - \mathbf{I}_2)) + 300\mathbf{I}_2 - j300\,(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$\Rightarrow \qquad j3\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 = 0$$

Putting the above two mesh equations in matrix form, we get

$$\begin{bmatrix} 600 - j300 & j300 \\ j3 & 1 - j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

Using Cramer's rule, we find that

$$\mathbf{I}_2 = 0.0124\,\underline{/-16°}\ \text{A}$$

Hence, $$\mathbf{V_{oc}} = \mathbf{300I_2} = \mathbf{3.72}\ \underline{\mathbf{/-16°}}\ \mathbf{V}$$

EXAMPLE 1.51

Find the steady current i_1 when the source voltage is $v_s = 10\sqrt{2}\cos(\omega t + 45°)$ V and the current source is $i_s = 3\cos\omega t$ A for the circuit of Fig. 1.95. The circuit provides the impedence in ohms for each element at the specified ω.

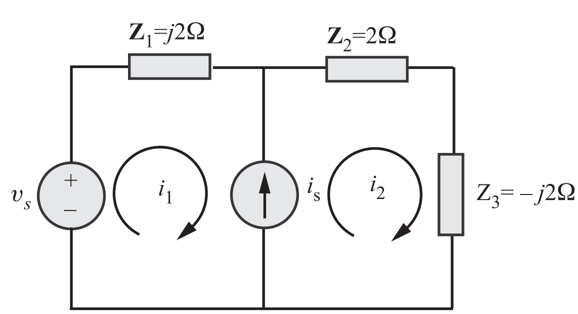


Figure 1.95

SOLUTION

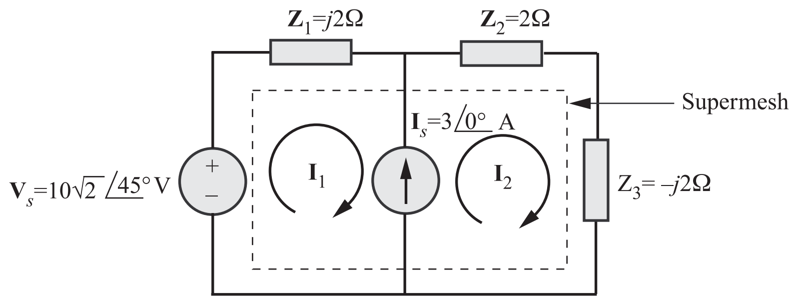


Figure 1.96

The first step is to convert the circuit of Fig. 1.95 into a phasor circuit. The phasor circuit is shown in Fig. 1.96.

$$v_s = 10\sqrt{2}\cos(\omega t + 45^\circ) \quad \Rightarrow \quad \mathbf{V}_s = 10\sqrt{2}\,\underline{/45^\circ} = 10(1+j)$$
$$i_s = 3\cos\omega t \quad \Rightarrow \quad \mathbf{I}_s = 3\,\underline{/0^\circ}$$

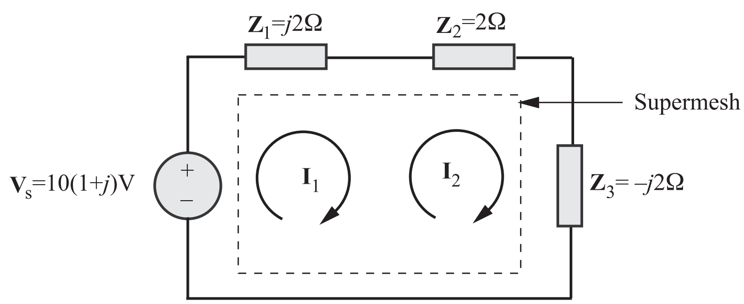


Figure 1.96(a)

Constraint equation:

$$\mathbf{I}_2 - \mathbf{I}_1 = \mathbf{I}_s = 3\,\underline{/0^\circ}$$

Applying KVL clockwise around the supermesh we get

$$\mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{V}_s = 0$$

Substituting $\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_s$ (from the constraint equation)

we get, $\mathbf{I}_1\mathbf{Z}_1 + (\mathbf{I}_1 + \mathbf{I}_s)(\mathbf{Z}_2 + \mathbf{Z}_3) = \mathbf{V}_s$

$$\Rightarrow \quad (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_1 = \mathbf{V}_s - (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_s$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{\mathbf{V}_s - (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_s}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(10 + j10) - (2 - j2)3}{2}$$
$$= 2 + j8 = 8.25\,\underline{/76^\circ}\text{ A}$$

Hence in time domain,

$$\boldsymbol{i_1 = 8.25\cos(\omega t + 76^\circ)\ \mathrm{A}}$$

EXAMPLE 1.52

Find the steady-state sinusoidal current i_1 for the circuit of Fig. 1.97, when $v_s = 10\sqrt{2}\cos$ $(100t + 45°)$ V.

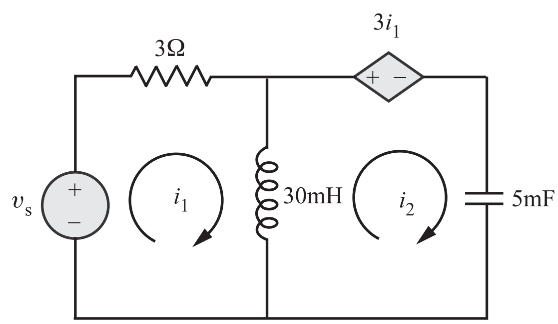


Figure 1.97

SOLUTION

The first step is to convert the circuit of Fig. 1.97 int to a phasor circuit. The phasor circuit is shown in Fig. 1.98.

$$v_s = 10\sqrt{2}\cos(100t + 45°)$$

$$\begin{aligned}
&\Rightarrow && \mathbf{V}_s = 10\sqrt{2}\,\underline{/45°}\,, \qquad \omega = 100 \text{ rad/ sec} \\
L = 30 \text{ mH} \quad &\Rightarrow && X_L = j\omega L \\
& && \quad\; = j100 \times 30 \times 10^{-3} = j3\Omega \\
C = 5 \text{ mF} \quad &\Rightarrow && X_C = \frac{1}{j\omega C} \\
& && \quad\; = \frac{1}{j100 \times 5 \times 10^{-3}} = -j2\Omega
\end{aligned}$$

KVL for mesh 1 :

$$(3 + j3)\mathbf{I}_1 - j3\mathbf{I}_2 = 10 + j10$$

KVL for mesh 2 :

$$(3 - j3)\mathbf{I}_1 + (j3 - j2)\mathbf{I}_2 = 0$$

Putting the above two mesh equations in matrix form, we get

$$\begin{bmatrix} 3 + j3 & -j3 \\ 3 - j3 & j1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10 + j10 \\ 0 \end{bmatrix}$$

Using Cramer's rule,we get

$$\mathbf{I}_1 = 1.05\,\underline{/71.6^\circ}\ \text{A}$$

Thus the steady state time response is,

$$\boldsymbol{i_1 = 1.05\cos(100t + 71.6^\circ)\,\mathrm{A}}$$

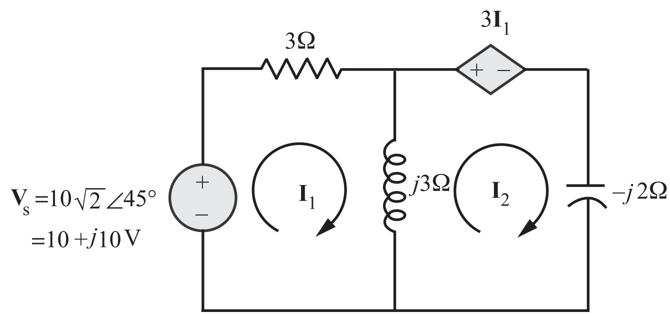


Figure 1.98

EXAMPLE 1.53

Determine $\mathbf{V}_o$ using mesh analysis.

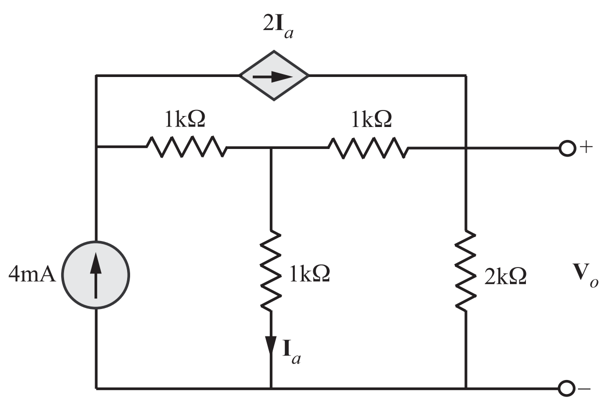


Figure 1.99

SOLUTION

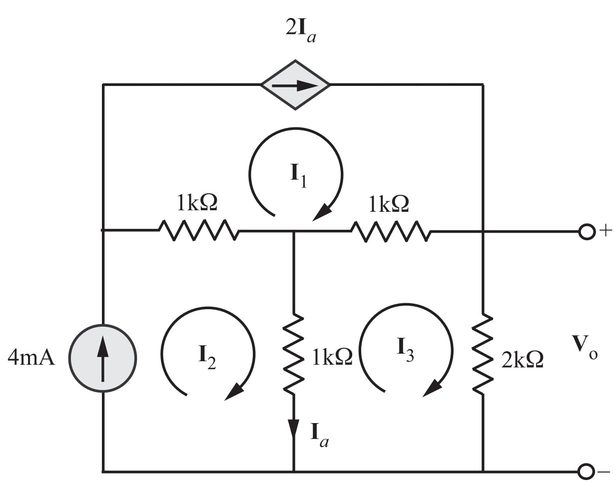


Figure 1.100

From Fig. 1.100, we find by inspection that,

$$\mathbf{I}_1 = 2\mathbf{I}_a = 2(\mathbf{I}_2 - \mathbf{I}_3)$$
$$\mathbf{I}_2 = 4 \text{ mA}$$

Applying KVL clockwise to mesh 3, we get

$$1 \times 10^3(\mathbf{I}_3 - \mathbf{I}_2) + 1 \times 10^3(\mathbf{I}_3 - \mathbf{I}_1) + 2 \times 10^3\mathbf{I}_3 = 0$$

Substituting $\mathbf{I}_1 = 2(\mathbf{I}_2 - \mathbf{I}_3)$ and $\mathbf{I}_2 = 4$ mA in the above equation and solving for $\mathbf{I}_3$,

we get, $$\mathbf{I}_3 = 2 \text{ mA}$$

Then, $$\mathbf{V}_o = 2 \times 10^3\mathbf{I}_3 = 4\text{V}$$

EXAMPLE 1.54

Find $\mathbf{V}_o$ in the network shown in Fig. 1.101 using mesh analysis.

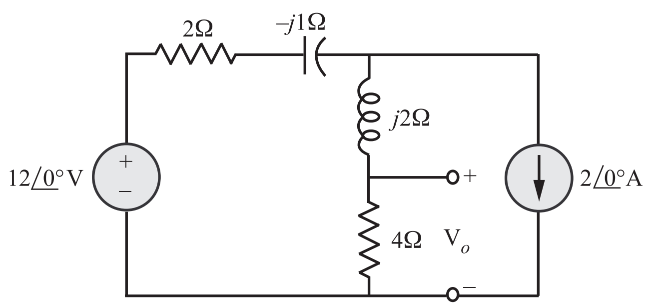


Figure 1.101

SOLUTION

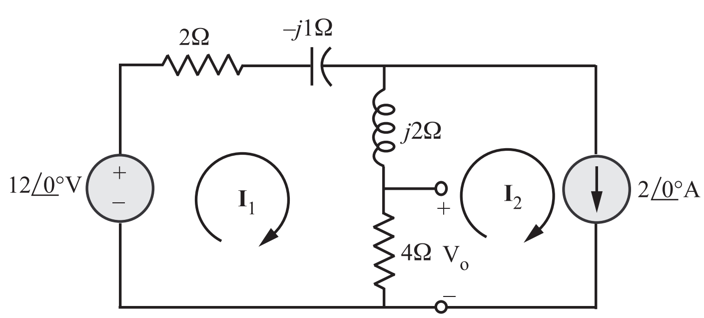


Figure 1.102

By inspection, we find that $\mathbf{I}_2 = 2\underline{/0^\circ}$ A.

Applying KVL clockwise to mesh 1, we get

$$-12 + \mathbf{I}_1(2 - j1) + (\mathbf{I}_1 - \mathbf{I}_2)(4 + j2) = 0$$

Substituting $\mathbf{I}_2 = 2\underline{/0^\circ}$ in the above equation yields,

$$-12 + \mathbf{I}_1(2 - j1 + 4 + j2) - 2(4 + j2) = 0$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{20 + j4}{6 + j1} = 3.35\underline{/1.85^\circ} \text{ A}$$

Hence
$$\mathbf{V}_o = 4(\mathbf{I}_1 - \mathbf{I}_2)$$
$$= 5.42\underline{/4.57^\circ} \text{ V}$$

Wye $\rightleftharpoons$ Delta transformation

For reducing a complex network to a single impedance between any two terminals, the reduction formulas for impedances in series and parallel are used. However, for certain configurations of network, we cannot reduce the interconnected impedances to a single equivalent impedance between any two terminals by using series and parallel impedance reduction techniques. That is the reason for this topic.

Consider the networks shown in Fig. 1.103 and 1.104.

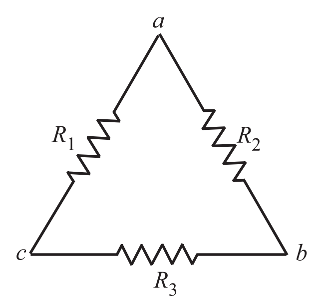


Figure 1.103 Delta resistance network

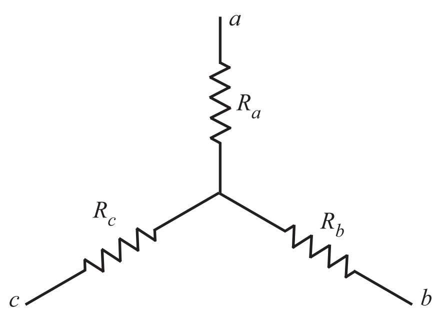


Figure 1.104 Wye resistance network

It may be noted that resistors in Fig. 1.103 form a Δ (delta), and resistors in Fig. 1.104. form a Υ (Wye). If both these configurations are connected at only the three terminals a, b and c, it would be very advantageous if an equivalence is established between them. It is possible to relate the resistances of one network to those of the other such that their terminal characteristics are the same. The relationship between the two configurations is called $\Upsilon - \Delta$ transformation.

We are interested in the relationship between the resistances R_1, R_2 and R_3 and the resitances R_a, R_b and R_c. For deriving the relationship, we assume that for the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal. That is, for example, resistance at terminals b and c with a open-circuited must be same for both networks. Hence, by equating the resistances for each corresponding set of terminals, we get the following set of equations :

(i) $$R_{ab}(\Upsilon) = R_{ab}(\Delta)$$

$$\Rightarrow \quad R_a + R_b = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3} \tag{1.57}$$

(ii) $$R_{bc}(\Upsilon) = R_{bc}(\Delta)$$

$$\Rightarrow \quad R_b + R_c = \frac{R_3(R_1 + R_2)}{R_3 + R_1 + R_2} \tag{1.58}$$

(iii) $$R_{ca}(\Upsilon) = R_{ca}(\Delta)$$

$$\Rightarrow \quad R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \tag{1.59}$$

Solving equations (1.57), (1.58) and (1.59) gives

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} \tag{1.60}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{1.61}$$

$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3} \tag{1.62}$$

Hence, each resistor in the Υ network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

To obtain the conversion formulas for transforming a *wye* network to an equivalent *delta* network, we note from equations (1.60) to (1.62) that

$$R_a R_b + R_b R_c + R_c R_a = \frac{R_1 R_2 R_3 (R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2} = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3} \tag{1.63}$$

Dividing equation (1.63) by each of the equations (1.60) to (1.62) leads to the following relationships :

$$R_1 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b} \tag{1.64}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} \tag{1.65}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a} \tag{1.66}$$

Hence each resistor in the Δ network is the sum of all possible products of Υ resistors taken two at a time, divided by the opposite Υ resistor.

Then Υ and Δ are said to be balanced when

$$R_{1=} R_2 = R_3 = R_\Delta \text{ and } R_a = R_b = R_c = R_\Upsilon$$

Under these conditions the conversions formula become

$$R_{\Upsilon} = \frac{1}{3} R_{\Delta}$$

and
$$R_{\Delta} = 3 R_{\Upsilon}$$

EXAMPLE 1.55

Find the value of resistance between the terminals $a - b$ of the network shown in Fig. 1.105.

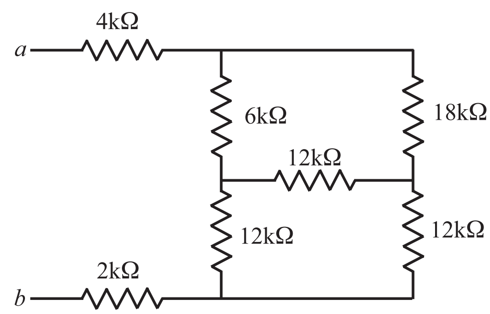


Figure 1.105

SOLUTION

Let us convert the upper Δ to Υ

$$R_{a_1} = \frac{(6\text{k})(18\text{k})}{6\text{k} + 12\text{k} + 18\text{k}} = 3 \text{ k}\Omega$$

$$R_{b_1} = \frac{(6\text{k})(12\text{k})}{6\text{k} + 12\text{k} + 18\text{k}} = 2 \text{ k}\Omega$$

$$R_{c_1} = \frac{(12\text{k})(18\text{k})}{6\text{k} + 12\text{k} + 18\text{k}} = 6 \text{ k}\Omega$$

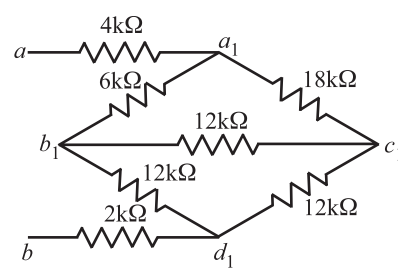
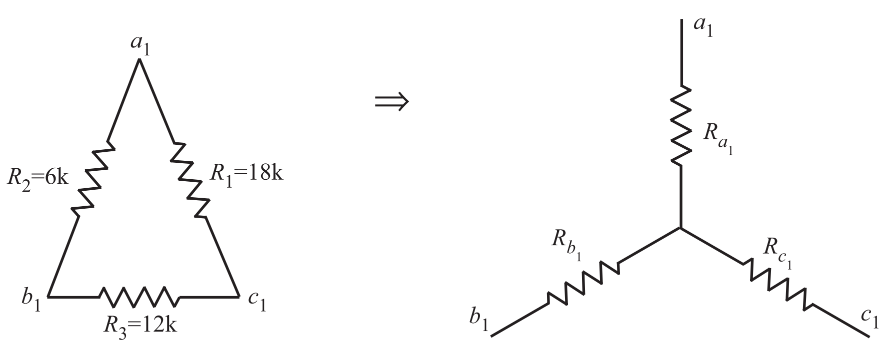


Figure 1.106



The network shown in Fig. 1.106 is now reduced to that shown in Fig. 1.106(a)

Hence, $$R_{ab} = 4 + 3 + 7.875 + 2$$
$$= \mathbf{16.875k\Omega}$$

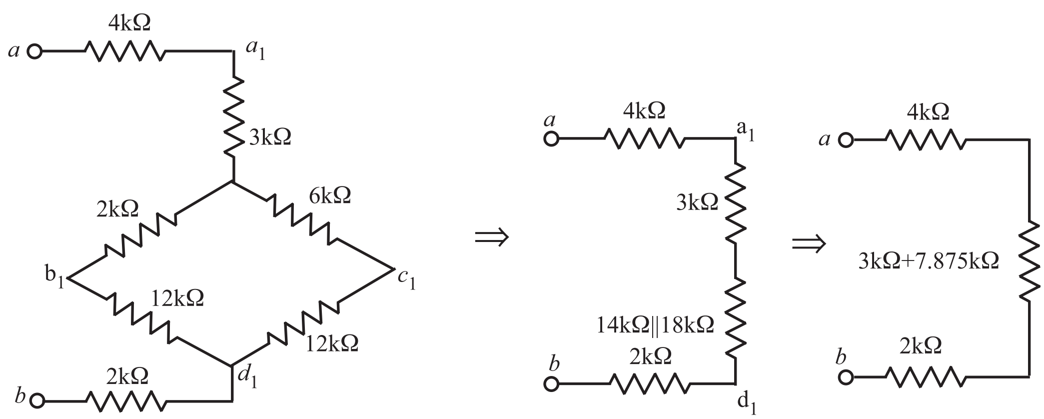


Figure 1.106(a)

EXAMPLE 1.56

Find the resistance R_{ab} using $\Upsilon - \Delta$ transformation.

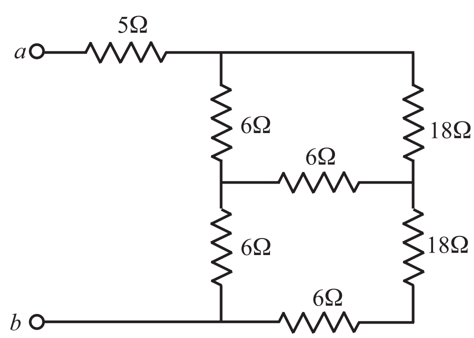


Figure 1.107

SOLUTION

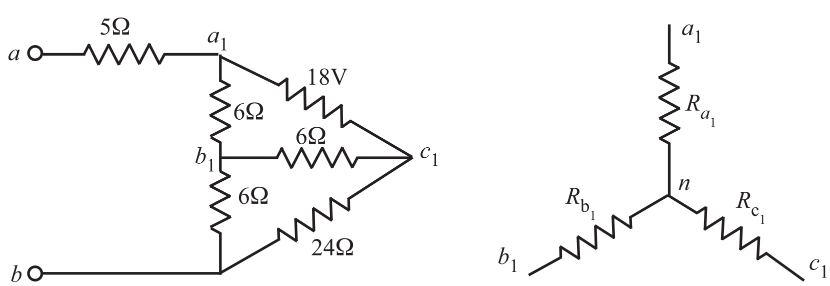
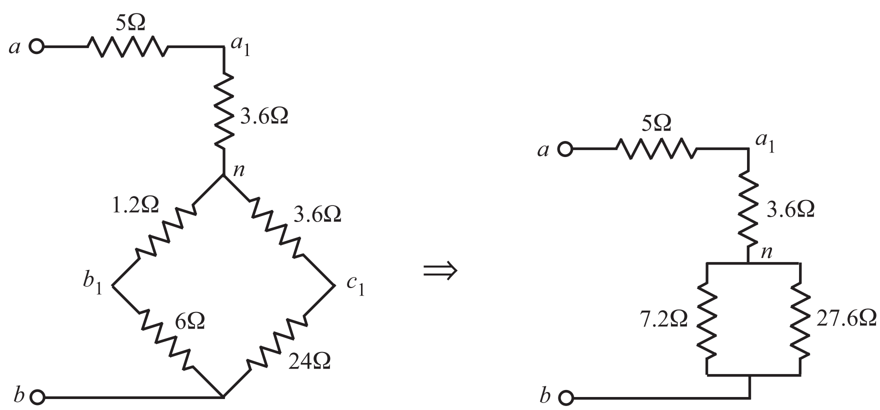


Figure 1.108

Let us convert the upper Δ between the points a_1, b_1 and c_1 into an equivalent Υ.

$$R_{a_1} = \frac{6 \times 18}{6 + 18 + 6} = 3.6\Omega$$
$$R_{b_1} = \frac{6 \times 6}{6 + 18 + 6} = 1.2\Omega$$
$$R_{c_1} = \frac{6 \times 18}{6 + 18 + 6} = 3.6\Omega$$

Figure 1.108 now becomes



$$R_{ab} = 5 + 3.6 + 7.2||27.6$$
$$= 8.6 + \frac{7.2 \times 27.6}{7.2 + 27.6}$$
$$= \mathbf{14.31\Omega}$$

EXAMPLE 1.57

Obtain the equivalent resistance R_{ab} for the circuit of Fig. 1.109 and hence find i.

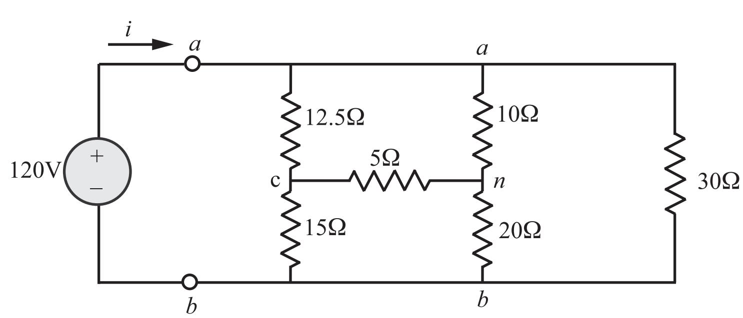
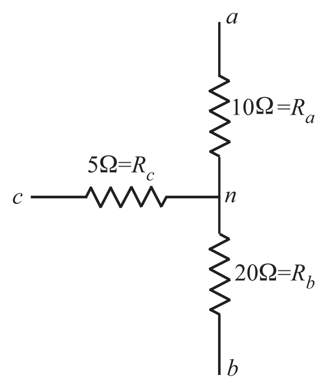


Figure 1.109

SOLUTION

Let us convert Υ between the terminals a, b and c into an equivalent Δ.

$$
\begin{aligned}
R_{ab} &= \frac{R_aR_b + R_bR_c + R_cR_a}{R_c} \\
&= \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{5} = 70\Omega \\
R_{bc} &= \frac{R_aR_b + R_bR_c + R_cR_a}{R_a} \\
&= \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = 35\Omega \\
R_{ca} &= \frac{R_aR_b + R_bR_c + R_cR_a}{R_b} \\
&= \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{20} = 17.5\Omega
\end{aligned}
$$



The circuit diagram of Fig. 1.109 now becomes the circuit diagram shown in Fig. 1.109(a). Combining three pairs of resistors in parallel, we obtain the circuit diagram of Fig. 1.109(b).

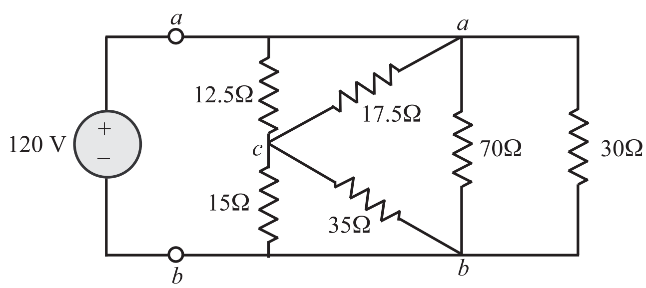


Figure 1.109(a)

$$
\begin{aligned}
70||30 &= \frac{70 \times 30}{70 + 30} = 21\Omega \\
12.5||17.5 &= \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\Omega \\
15||35 &= \frac{15 \times 35}{15 + 35} = 10.5\Omega \\
R_{ab} &= (7.292 + 10.5)||21 = 9.632\Omega
\end{aligned}
$$

Thus, $i = \dfrac{v_s}{R_{ab}} = \mathbf{12.458\ A}$

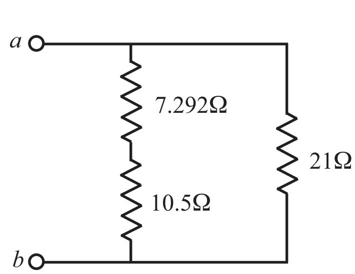


Figure 1.109(b)

Nodal versus mesh analysis

The analysis of a complex circuit can usually be accomplished by either the node voltage or mesh current method. One may ask : Given a network to be analyzed, how do we know which method is better or more efficient? The choice is dictated by two factors.

When a circuit contains only voltage sources, it is probably easier to use the mesh current method. Conversely, when the circuit contains only current sources, it will be easier to use the node voltage method. Also, a circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis. In other words, the best technique is one which gives smaller number of equations.

Another point to consider while choosing between the two methods is, what information is required. If node voltages are required, it may be advantageous to apply nodal analysis. On the other hand, if you need to know several currents, it may be wise to proceed directly with mesh current analysis.

It is often advantageous if we know both the techniques. The first advantage lies in the fact that the second method can verify the results of the first method. Also, both the methods have limitations. For example, while analysing a transistor circuit, only mesh method is suited and while analysing an Op-amp circuit, nodal method is only applicable. Mesh technique is applicable for planar[1] networks. However, nodal method suits to both planar and nonplanar [2] networks.

Reinforcement Problems

R.P 1.1

Find the power dissipated in the 80Ω resistor using mesh analysis.

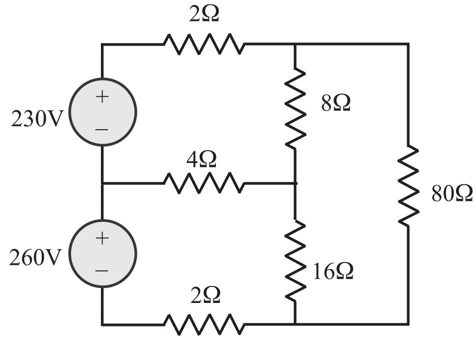
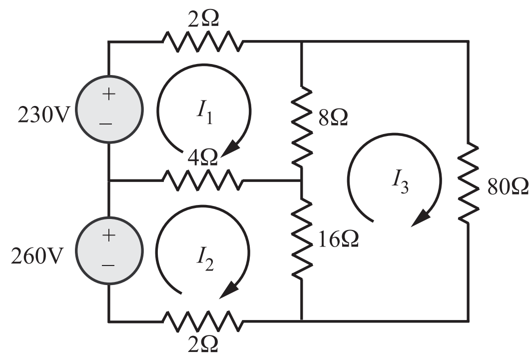


Figure R.P.1.1

[1]A planar network can be drawn on a plane without branches crossing each other.

[2]A nonplanar network is one in which crossover is identified and cannot be eliminated by redrawing the branches.

SOLUTION



$KVL\ clockwise\ to\ mesh\ 1:$

$$14I_1 - 4I_2 - 8I_3 = 230$$

$KVL\ clockwise\ to\ mesh\ 2:$

$$-4I_1 + 22I_2 - 16I_3 = 260$$

$KVL\ clockwise\ to\ mesh\ 3:$

$$-8I_1 - 16I_2 + 104I_3 = 0$$

Putting the above mesh equations in matrix form, we get

$$\begin{bmatrix} 14 & -4 & -8 \\ -4 & 22 & -16 \\ -8 & -16 & 104 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 230 \\ 260 \\ 0 \end{bmatrix}$$

The current I_3 is found from the above matrix equation by using Cramer's rule.

$$I_3 = 5\text{A}$$

Thus, $$P_{80} = I_3^2 R_{80} = 5^2 \times 80 = \mathbf{2000W(dissipated)}$$

R.P 1.2

Refer the circuit shown in Fig. R.P. 1.2. The current $i_o = 4$A. Find the power dissipated in the 70 Ω resistor.

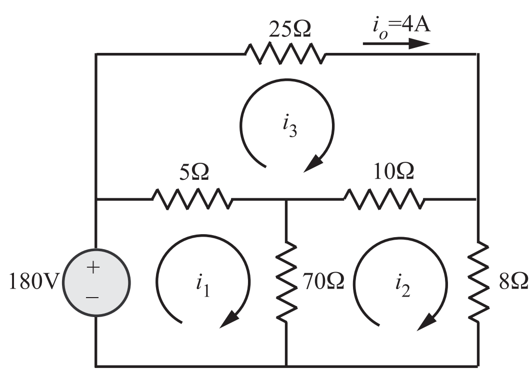


Figure R.P.1.2

SOLUTION

By inspection, we find that the mesh current $i_3 = i_o = 4$A

$KVL\ clockwise\ to\ mesh\ 1:$ $\quad 75i_1 - 70i_2 - 5i_3 = 180$

Substituting $i_3 = 4$A, we get $\quad 75i_1 - 70i_2 = 200$
KVL clockwise to mesh 2 : $\quad -70i_1 + 88i_2 - 10i_3 = 0$
Substituting the value $i_3 = 4$A, we get $\quad -70i_1 + 88i_2 = 40$
Puting the two mesh equations in matrix from, we get

$$\begin{bmatrix} 75 & -70 \\ -70 & 88 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 40 \end{bmatrix}$$

Using Cramer's rule, we get

$$\begin{aligned} i_1 &= 12\text{A}, i_2 = 10\text{A} \\ P_{70} &= (i_1 - i_2)^2 70 = 4 \times 70 \\ &= \mathbf{280\ W\ (dissipated)} \end{aligned}$$

R.P 1.3

Solve for current **I** in the circuit of Fig. R.P. 1.3 using nodal analysis.

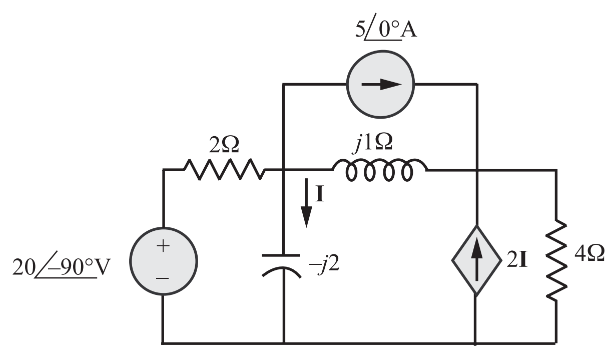


Figure R.P.1.3

SOLUTION

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1 - 20\,\underline{/-90^\circ}}{2} + \frac{\mathbf{V}_1}{-j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j1} + 5\,\underline{/0^\circ} = 0$$

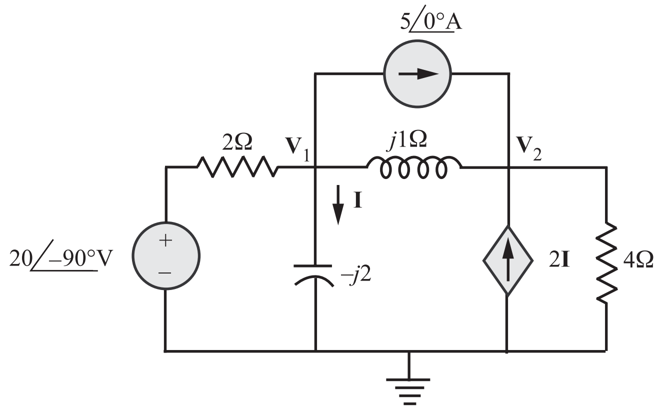
$$\Rightarrow \qquad (0.5 - j0.5)\mathbf{V}_1 + j\mathbf{V}_2 = -5 - j10$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j1} + \frac{\mathbf{V}_2}{4} - 2\mathbf{I} - 5\,\underline{/0^\circ} = 0$$

Also, $\qquad \mathbf{I} = \dfrac{\mathbf{V}_1}{-j2}$

Hence, $\qquad \dfrac{\mathbf{V}_2 - \mathbf{V}_1}{j1} + \dfrac{\mathbf{V}_2}{4} + \dfrac{2}{j2}\mathbf{V}_1 - 5\,\underline{/0^\circ} = 0$



$$\Rightarrow \qquad (0.25 - j)\mathbf{V}_2 = 5$$

$$\Rightarrow \qquad \mathbf{V}_2 = \frac{5}{0.25 - j}$$

Making use of $\mathbf{V}_2$ in the nodal equation at node $\mathbf{V}_1$, we get

$$-5 - j10 - \frac{j5}{0.25 - j} = 0.5(1 - j)\mathbf{V}_1$$

$$\Rightarrow \qquad (1 - j)\mathbf{V}_1 = -10 - j20 - \left(\frac{j40}{1 - j4}\right)$$

$$\Rightarrow \qquad \mathbf{V}_1 = 15.81\,\angle{-46.5^\circ}\ \text{V}$$

Hence,

$$\mathbf{I} = \frac{\mathbf{V}_1}{-j2} = \frac{15.81\,\angle{-46.5^\circ}}{2\,\angle{-90^\circ}}$$

$$= \mathbf{7.906\,\angle{43.5^\circ}\ A}$$

R.P 1.4

Find $\mathbf{V}_o$ shown in the Fig. R.P. 1.4 using Nodal technique.

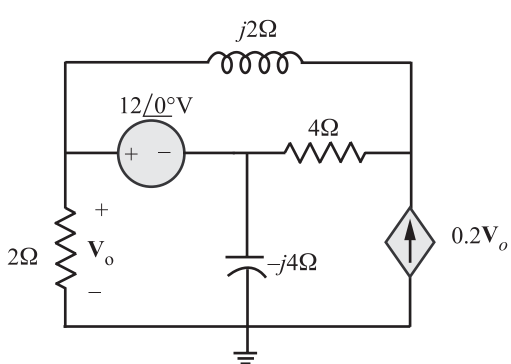


Figure R.P.1.4

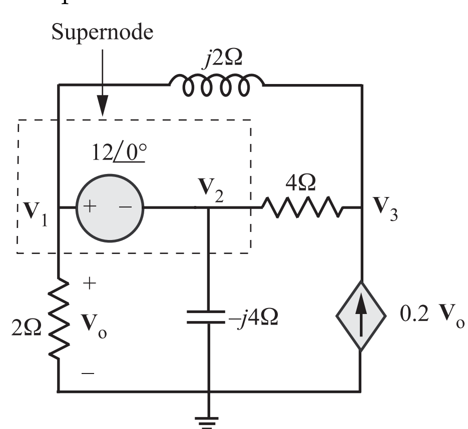


Figure R.P.1.4(a) .

SOLUTION

We find from Fig RP 1.4(a) that,

$$\mathbf{V}_1 = \mathbf{V}_o$$

Constraint equation:

Applying KVL clockwise along the path consisting of voltage source, capacitor, and 2Ω resistor, we find that

$$12\,\underline{/0^\circ} + \mathbf{V}_2 - \mathbf{V}_1 = 0$$

$$\Rightarrow \qquad \mathbf{V}_1 = \mathbf{V}_2 + 12\,\underline{/0^\circ}$$

or

$$\mathbf{V}_2 = \mathbf{V}_1 - 12$$

KCL at Supernode :

$$\frac{\mathbf{V}_1 - \mathbf{V}_3}{j2} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_2}{-j4} + \frac{\mathbf{V}_2 - \mathbf{V}_3}{4} = 0$$

$$\Rightarrow \quad (2 - j2)\mathbf{V}_1 + (1 + j)\mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 = 0$$

KCL at node 3 :

$$\frac{\mathbf{V}_3 - \mathbf{V}_1}{j2} + \frac{\mathbf{V}_3 - \mathbf{V}_2}{4} - 0.2\mathbf{V}_o = 0 \tag{1.67}$$

Substituting $\mathbf{V}_o = \mathbf{V}_1$, we get

$$(0.8 - j2)\mathbf{V}_1 + \mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 = 0 \tag{1.68}$$

Subtracting equation (1.68) from (1.67), we get

$$1.2\mathbf{V}_1 + j\mathbf{V}_2 = 0 \tag{1.69}$$

Substituting $\mathbf{V}_2 = \mathbf{V}_1 - 12$ (from the constraint equation), we get

$$1.2\mathbf{V}_1 + j(\mathbf{V}_1 - 12) = 0$$

$$\Rightarrow \qquad \mathbf{V}_1 = \frac{j12}{1.2 + j} = \mathbf{V}_o$$

Hence

$$\mathbf{V}_o = \mathbf{7.68}\,\underline{/\mathbf{50.2^\circ}}\ \mathbf{V}$$

R.P 1.5

Solve for i_o using mesh analysis.

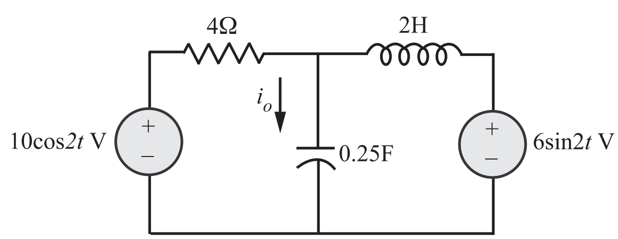


Figure R.P. 1.5

SOLUTION

The first step in the analysis is to draw the phasor circuit equivalent of Fig. R.P.1.5.

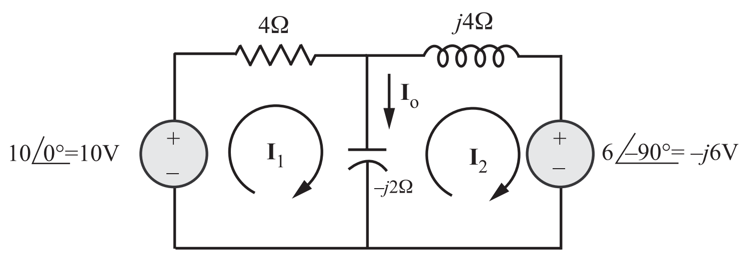


Figure R.P. 1.5(a)

$$\omega = 2$$
$$10\cos 2t \Rightarrow 10\,\underline{/0^\circ}\ \mathbf{V}$$
$$6\sin 2t = 6\cos(2t-90) \Rightarrow 6\,\underline{/-90^\circ} = -j6\mathbf{V}$$
$$L = 2H \Rightarrow X_L = j\omega L = j4\Omega$$
$$C = 0.25F \Rightarrow X_C = \frac{1}{j\omega C} = \frac{1}{j2\left(\frac{1}{4}\right)} = -j2\Omega$$

Applying KVL clockwise to mesh 1 :

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$\Rightarrow \quad (2 - j1)\mathbf{I}_1 + j\mathbf{I}_2 = 5$$

Applying KVL clockwise to mesh 2 :

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3$$

Putting the above mesh equations in a matrix form, we get

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Using Cramer's rule, we get

$$\mathbf{I}_1 = 2 + j0.5,$$
$$\mathbf{I}_2 = 1 - j0.5,$$
$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = 1 + j = 1.414\,\underline{/45^\circ}$$

Hence $\qquad i_o(t) = \mathbf{1.414}\cos\left(\mathbf{2}\boldsymbol{t} + \mathbf{45^\circ}\right)\ \mathbf{A}$

R.P 1.6

Refer the circuit shown in Fig. R.P. 1.6. Find **I** using mesh analysis.

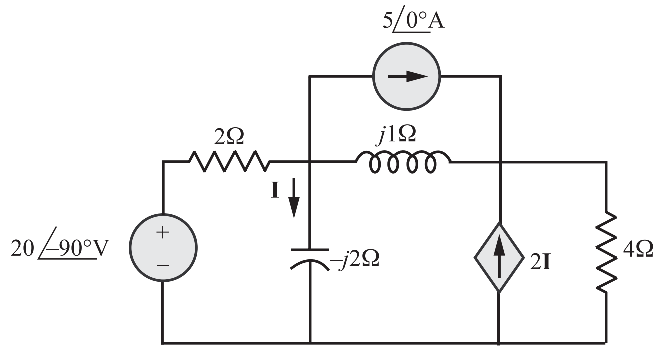


Figure R.P.1.6

SOLUTION

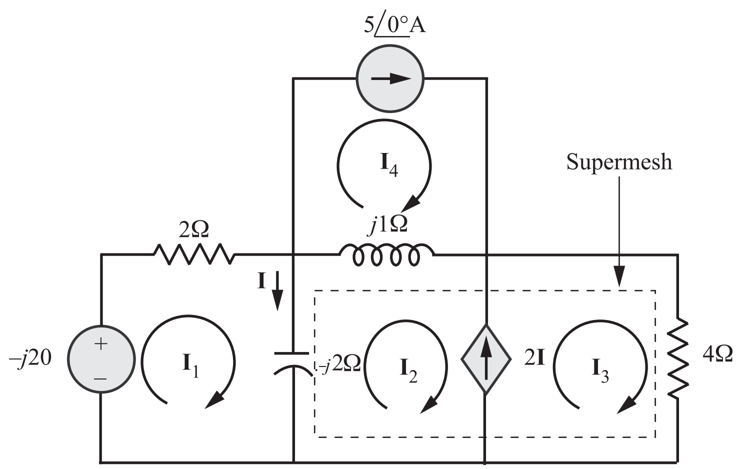


Figure R.P.1.6(a)

Constraint equation:

$$\mathbf{I}_3 - \mathbf{I}_2 = 2\mathbf{I}$$

$$\Rightarrow \qquad \mathbf{I}_3 - \mathbf{I}_2 = 2(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\Rightarrow \qquad \mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2$$

Also, for mesh 4, $\mathbf{I}_4 = 5 \text{ A}$

Applying KVL clockwise for mesh 1 :

$$-(-j20) + (2 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$\Rightarrow \qquad (1 - j)\mathbf{I}_1 + j\mathbf{I}_2 = -j10 \tag{1.70}$$

Applying KVL clockwise for the supermesh :

$$(j - j2)\mathbf{I}_2 + j2\mathbf{I}_1 + 4\mathbf{I}_3 - j\mathbf{I}_4 = 0$$

Substituting $\mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2$ and $\mathbf{I}_4 = 5\text{A}$

we get

$$(8 + j2)\mathbf{I}_1 - (4 + j)\mathbf{I}_2 = j5 \tag{1.71}$$

Putting equations (1.70) and (1.71) in matrix form, we get

$$\begin{bmatrix} 1 - j & j \\ 8 + j2 & -(4 + j) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -j10 \\ j5 \end{bmatrix}$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$, we get

$$\begin{aligned} \mathbf{I}_1 &= -(5.44 + j4.26) \text{ A} \\ \mathbf{I}_2 &= -(11.18 + j9.7) \text{ A} \\ \mathbf{I} &= \mathbf{I}_1 - \mathbf{I}_2 \\ &= 5.735 + j5.44 \\ &= \mathbf{7.9 \angle 43.49^\circ \ A} \end{aligned}$$

R.P 1.7

Calculate $\mathbf{V}_o$ in the circuit of Fig. R.P. 1.7 using the method of source transformation.

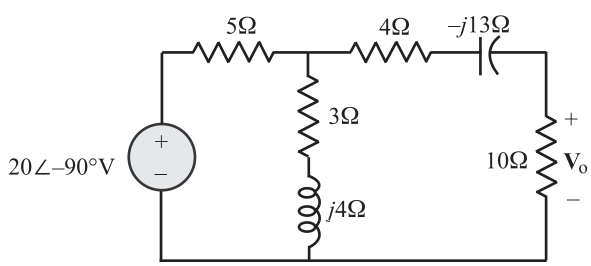


Figure R.P. 1.7

SOLUTION

Transform the voltage source to a current source and obtain the circuit shown in Fig. R.P.1.7(a).

$$\mathbf{I}_s = \frac{20 \angle -90^\circ}{5} = 4 \angle -90^\circ \text{ A}$$

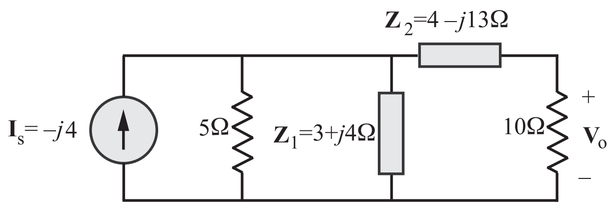


Figure R.P.1.7(a)

$$\mathbf{Z}_p = 5\Omega || 3 + j4 = \frac{5 \times (3 + j4)}{5 + (3 + j4)} = 2.5 + j1.25\Omega$$

Converting the current source in Fig. R.P. 1.7(b) to a voltage source gives the circuit as shown in Fig. R.P. 1.7(c).

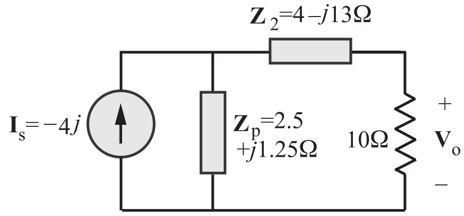


Figure R.P.1.7(b)

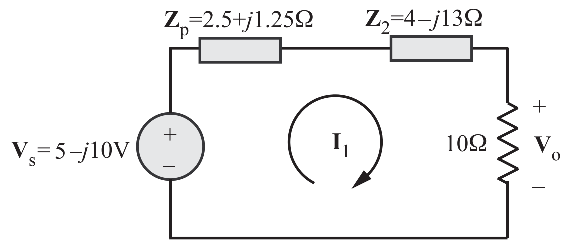


Figure R.P.1.7(c)

$$
\begin{aligned}
\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_p &= -4j(2.5 + j1.25) \\
&= 5 - j10\mathbf{V} \\
\mathbf{V}_o &= 10\mathbf{I} \\
&= \left[\frac{\mathbf{V}_s}{\mathbf{Z}_p + \mathbf{Z}_2 + 10}\right] 10 \\
&= \frac{5 - j10}{[2.5 + j1.25 + 4 - j13 + 10]} \times 10 \\
&= \mathbf{5.519\,\underline{/-28^\circ}\ V}
\end{aligned}
$$

R.P 1.8

Find v_x and i_x in the circuit shown in Fig. R.P. 1.8.

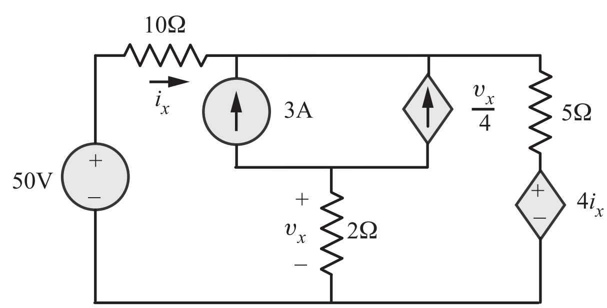


Figure R.P. 1.8

SOLUTION

Constraint equation: $$i_2 - i_1 = 3 + \frac{v_x}{4}$$

$$\Rightarrow \qquad i_2 = i_1 + 3 + \frac{v_3}{4}$$

The above equation becomes very clear if one writes *KCL equation* at node B of Fig. R.P. 1.8(a).

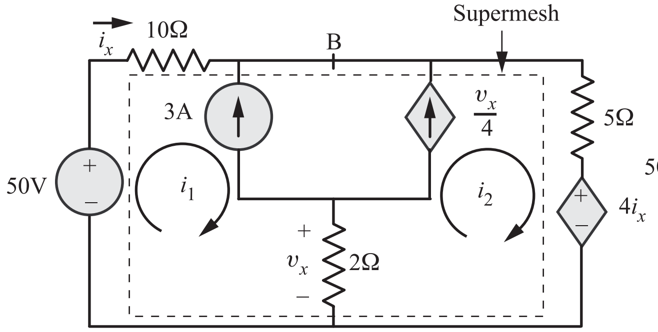


Figure R.P. 1.8(a)

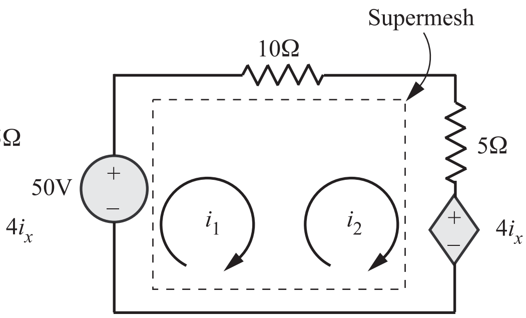


Figure R.P. 1.8(b)

Applying KVL clockwise to the supermesh in Fig. R.P. 1.8(b), we get

$$-50 + 10i_1 + 5i_2 + 4i_x = 0$$

But $i_x = i_1$. Hence, $-50 + 10i_1 + 5i_2 + 4i_1 = 0$

$$\Rightarrow \qquad 14i_1 + 5i_2 = 50 \tag{1.72}$$

Making use of $v_x = (i_1 - i_2) \times 2$ in the constraint equation, we get

$$i_2 = i_1 + 3 + \frac{(i_1 - i_2) \times 2}{4}$$

$$\Rightarrow \qquad i_2 = i_1 + 3 + \frac{i_1 - i_2}{2}$$

$$\Rightarrow \qquad 2i_2 = 2i_1 + 6 + i_1 - i_2$$

$$\Rightarrow \qquad 3i_1 - 3i_2 + 6 = 0$$

$$\Rightarrow \qquad i_1 - i_2 = -2 \tag{1.73}$$

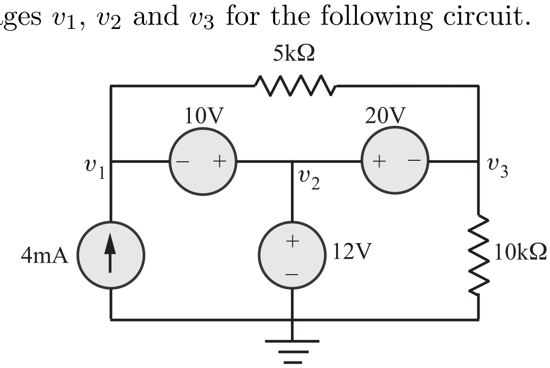
Solving equations (1.72) and (1.73) gives $i_1 = 2.105$ A, $i_2 = 4.105$ A

Thus, $$v_x = 2(i_1 - i_2) = \mathbf{-4\ V}$$

and $$i_x = i_1 = \mathbf{2.105\ A}$$

R.P 1.9

Obtain the node voltages v_1, v_2 and v_3 for the following circuit.



SOLUTION

We have a supernode as shown in Fig. R.P. 1.9(a). By inspection, we find that $\mathbf{V}_2 = 12$V.
Refer Fig. R.P. 1.9(b) for further analysis.

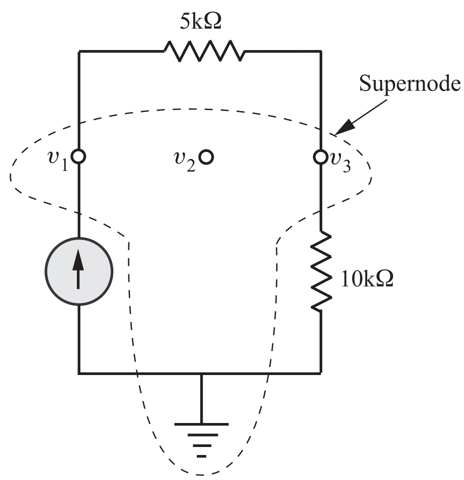


Figure R.P.1.9(a)

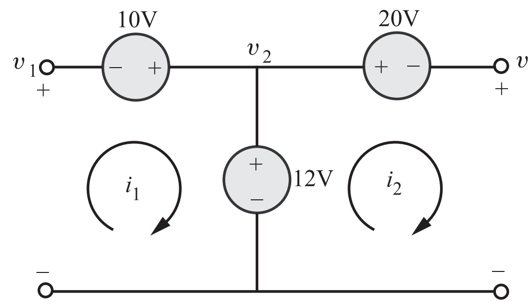


Figure R.P.1.9(b) .

KVL *clockwise to mesh* 1 :

$$-v_1 - 10 + 12 = 0 \quad \Rightarrow \quad v_1 = 2$$

KVL *clockwise to mesh* 2 :

$$-12 + 20 + v_3 = 0$$

$$\Rightarrow \qquad v_3 = -8 \text{ V}$$

Hence, $\qquad \boldsymbol{v_1 = 2 \text{ V}, v_2 = 12 \text{ V}, v_3 = -8 \text{ V}}$

R.P 1.10

Find the equivalent resistance R_{ab} for the circuit shown in Fig. R.P.1.10.

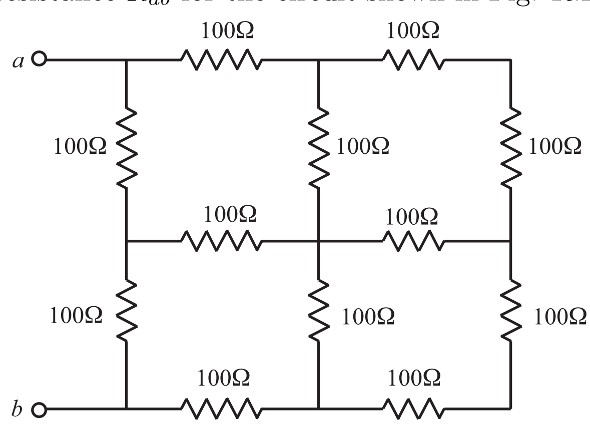


Figure R.P. 1.10

SOLUTION

The circuit is redrawn marking the nodes c to j in Fig. R.P. 1.10(a). It can be seen that the network consists of four identical stars :

(i) ae, ef, cb
(ii) ac, cf, cd
(iii) dg, gf, gj
(iv) bh, fh, hj

Converting each stars in to its equivalent delta, the network is redrawn as shown in Fig. R.P. 1.10(b), noting that each resistance in delta is $100 \times 3 = 300\Omega$, eliminating nodes c, e, g, h.

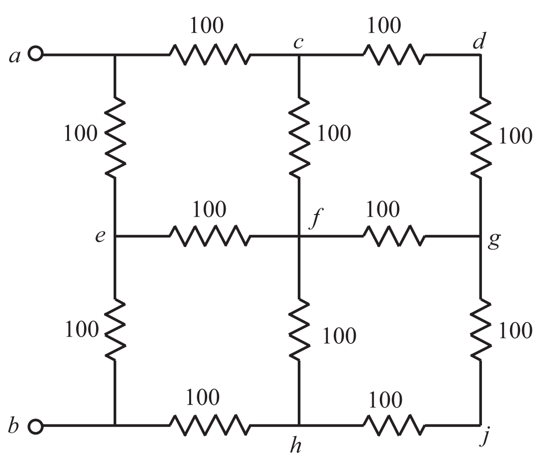


Figure R.P.1.10(a)

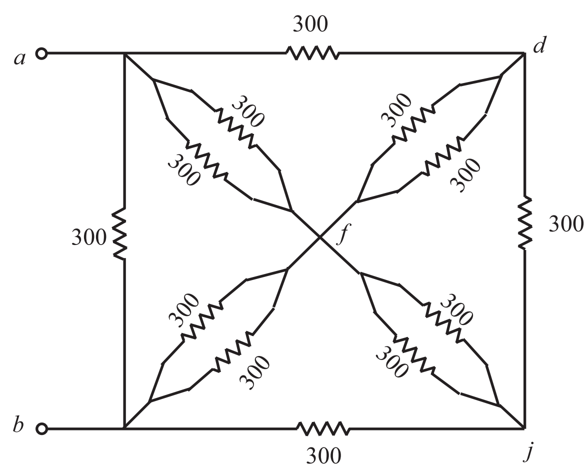


Figure R.P.1.10(b)

Reducing the parallel resistors, we get the circuit as in Fig. R.P. 1.10(c).

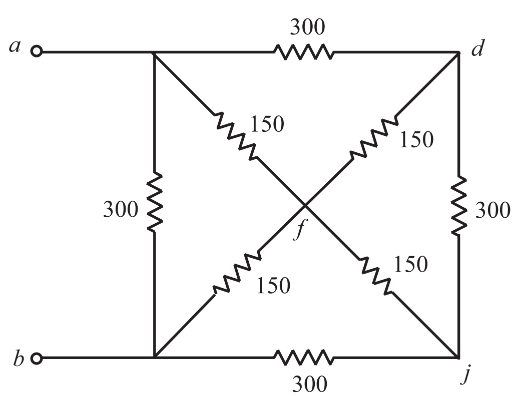


Figure R.P.1.10(c)

Hence, there are two identical deltas afd and bfj. Converting them to their equivalent stars, we get the circuit as shown in Fig. R.P.1.10(d).

$$R_{ak} = R_{bl} = R_{kd} = R_{lj} = \frac{300 \times 150}{600} = 75\ \Omega$$

$$R_{kf} = R_{fl} = \frac{150^2}{600} = 37.5\ \Omega$$

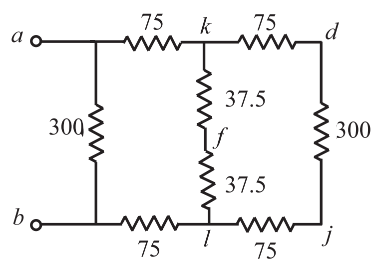


Figure R.P.1.10(d)

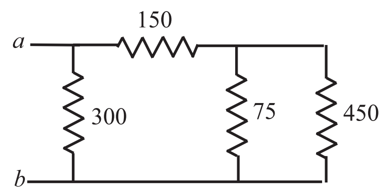


Figure R.P.1.10(e)

The circuit is further reduced to Fig. R.P. 1.10(e) and then to Fig. R.P. 1.10(f) and (g). Then the equivalent resistance is

$$R_{ab} = \frac{214.286 \times 300}{514.286} = 125\ \Omega$$

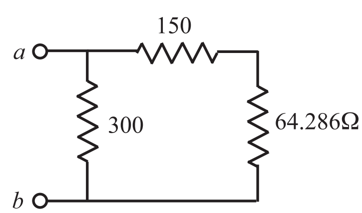


Figure R.P.1.10(f)

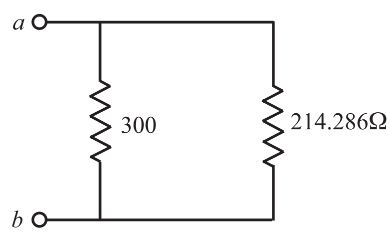


Figure R.P.1.10(g)

R.P 1.11

Obtain the equivalent resistance R_{ad} for the circuit shown in Fig. R.P.1.11.

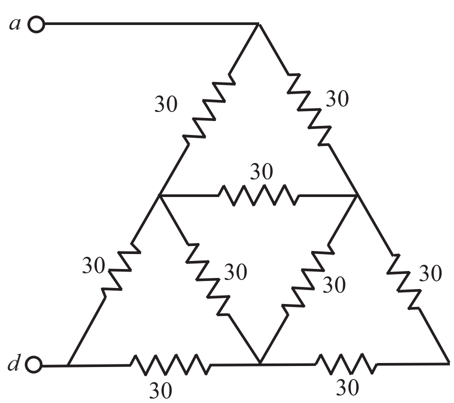


Figure R.P.1.11

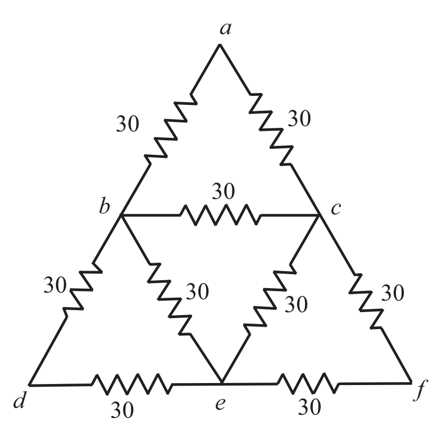


Figure R.P.1.11(a)

SOLUTION

The circuit is redrawn as shown Fig. 1.11(a), marking the nodes a to f to identify the deltas in it. It contains 3 deltas abc, bde and def with 3 equal resistors of 30 Ω each. For each delta, their equivalent star contains 3 resistors each of value $\dfrac{30}{3} = 10\Omega$. Then the circuit becomes as shown in Fig. R.P. 1.11(b) where f is isolated.

On simplification, we get the circuit as shown in Fig. R.P.1.11(c) and further reduced to Fig. R.P.1.11(d).

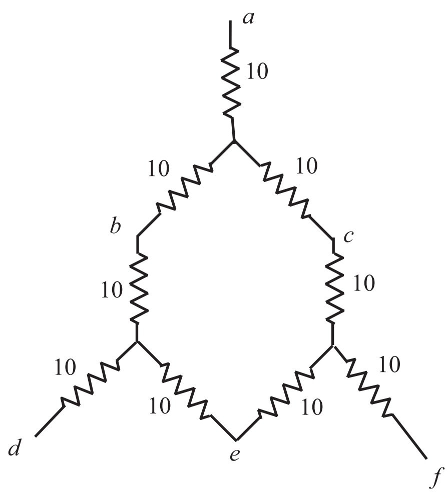


Figure R.P.1.11(b)

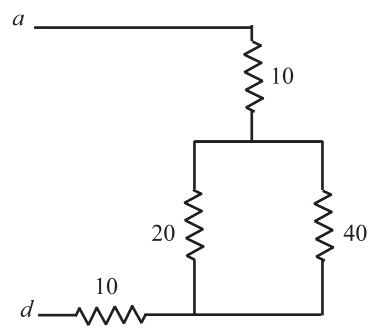


Figure R.P.1.11(c)

Figure R.P.1.11(d)

Then the equivalent ressitance,

$$R_{ad} = 10 + 13.33 + 10 = 33.33\ \Omega$$

R.P 1.12

Draw a network for the following mesh equations in matrix form :

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 30\,\underline{/-0^\circ} \\ 0 \\ -20\,\underline{/-0^\circ} \end{bmatrix}$$

SOLUTION

The general form of the mesh equations in matrix form for a network having three mashes is given by

$$\begin{bmatrix} \mathbf{Z}_{11} & -\mathbf{Z}_{12} & -\mathbf{Z}_{13} \\ -\mathbf{Z}_{21} & \mathbf{Z}_{22} & -\mathbf{Z}_{23} \\ -\mathbf{Z}_{31} & -\mathbf{Z}_{32} & \mathbf{Z}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \underline{/\theta_1} \\ \mathbf{V}_2 \underline{/\theta_2} \\ \mathbf{V}_3 \underline{/\theta_3} \end{bmatrix}$$

and, $$\mathbf{Z}_{11} = \mathbf{Z}_{10} + \mathbf{Z}_{12} + \mathbf{Z}_{13}$$

where $\mathbf{Z}_{10} =$ Sum of the impedances confined to mesh 1 alone

$\mathbf{Z}_{12} =$ Sum of the impedances common to meshes 1 and 2

$\mathbf{Z}_{13} =$ Sum of the impedances common to meshes 1 and 3

Similiar difenitions hold good for $\mathbf{Z}_{22}$ and $\mathbf{Z}_{33}$. Also, $\mathbf{Z}_{ij} = \mathbf{Z}_{ji}$

For the present problem,

$$\mathbf{Z}_{11} = 5 + j5\Omega$$
$$\mathbf{Z}_{12} = \mathbf{Z}_{21} = j5\Omega$$
$$\mathbf{Z}_{13} = \mathbf{Z}_{31} = 0\Omega$$
$$\mathbf{Z}_{23} = \mathbf{Z}_{32} = 6\Omega$$

We know that, $$\mathbf{Z}_{11} = \mathbf{Z}_{10} + \mathbf{Z}_{12} + \mathbf{Z}_{13}$$

$\Rightarrow$ $$5 + j5 = \mathbf{Z}_{10} + j5 + 0$$

$\Rightarrow$ $$\mathbf{Z}_{10} = 5\Omega$$

Similarly, $$\mathbf{Z}_{22} = \mathbf{Z}_{20} + \mathbf{Z}_{21} + \mathbf{Z}_{23}$$

$\Rightarrow$ $$8 + j8 = \mathbf{Z}_{20} + j5 + 6$$

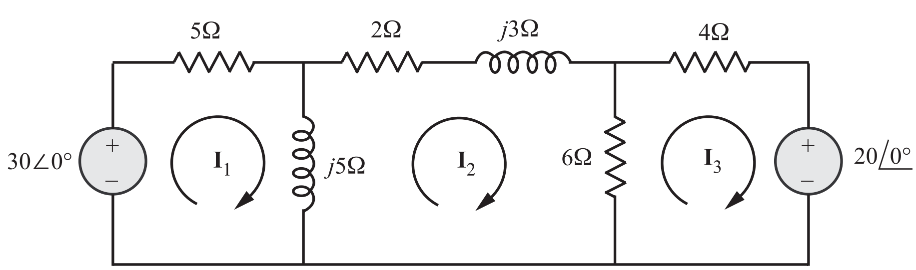
$\Rightarrow$ $$\mathbf{Z}_{20} = 2 + j3\Omega$$

Finally, $$\mathbf{Z}_{33} = \mathbf{Z}_{30} + \mathbf{Z}_{31} + \mathbf{Z}_{32}$$

$\Rightarrow$ $$10 = \mathbf{Z}_{30} + 0 + 6$$

$\Rightarrow$ $$\mathbf{Z}_{30} = 4\Omega$$

Making use of the above impedances, we can configure a network as shown below :



R.P 1.13

Draw a network for the following nodal equations in matrix form.

$$\begin{bmatrix} \left(\dfrac{1}{-j10}+\dfrac{1}{10}\right) & -\dfrac{1}{10} \\ -\dfrac{1}{10} & \left(\dfrac{1}{5}(1-j)+\dfrac{1}{10}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 10\,\underline{/0^\circ} \\ 0 \end{bmatrix}$$

SOLUTION

The general form of the nodal equations in matrix form for a network having two nodes is given by

$$\begin{bmatrix} \mathbf{Y}_{11} & -\mathbf{Y}_{12} \\ -\mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_1\,\underline{/\theta_1} \\ \mathbf{I}_2\,\underline{/\theta_2} \end{bmatrix}$$

where $\mathbf{Y}_{11} = \mathbf{Y}_{10} + \mathbf{Y}_{12}$ and $\mathbf{Y}_{22} = \mathbf{Y}_{20} + \mathbf{Y}_{21}$.

$\mathbf{Y}_{10} =$ sum of admittances connected at node 1 alone.

$\mathbf{Y}_{12} = \mathbf{Y}_{21} =$ sum of admittances common to nodes 1 and 2.

$\mathbf{Y}_{20} =$ sum of admittances connected at node 2 alone.

For the present problem,

$$\mathbf{Y}_{11} = \frac{1}{-j10} + \frac{1}{10}\ \text{S}$$

$$\mathbf{Y}_{12} = \mathbf{Y}_{21} = \frac{1}{10}\ \text{S}$$

$$\mathbf{Y}_{22} = \frac{1}{5}(1-j) + 10\ \text{S}$$

We know that, $\mathbf{Y}_{11} = \mathbf{Y}_{10} + \mathbf{Y}_{12}$

$$\Rightarrow \quad \frac{1}{-j10} + \frac{1}{10} = \mathbf{Y}_{10} + \frac{1}{10}$$

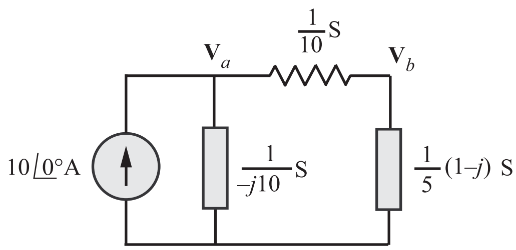
$$\Rightarrow \quad \mathbf{Y}_{10} = \frac{-1}{j10}\ \text{S}$$

Similarly, $\mathbf{Y}_{22} = \mathbf{Y}_{20} + \mathbf{Y}_{21}$

$$\Rightarrow \quad \frac{1}{5}(1-j) + \frac{1}{10} = \mathbf{Y}_{20} + \frac{1}{10}$$

$$\Rightarrow \quad \mathbf{Y}_{20} = \frac{1}{5}(1-j)\ \text{S}$$

Making use of the above admittances, we can configure a network as shown below :



Exercise problems

E.P 1.1

Refer the circuit shown in Fig. E.P.1.1. Using mesh analysis, find the current delivered by the source. Verify the result using nodal technique.

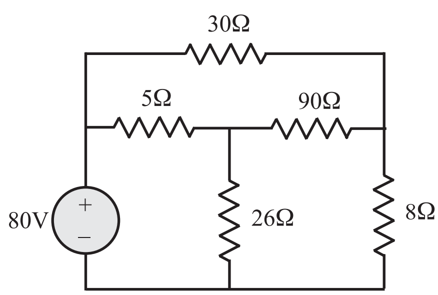


Figure E.P. 1.1

Ans : 5A

E.P 1.2

For the resistive circuit shown in Fig. E.P. 1.2. by using source transformation and mesh analysis, find the current supplied by the 20 V source.

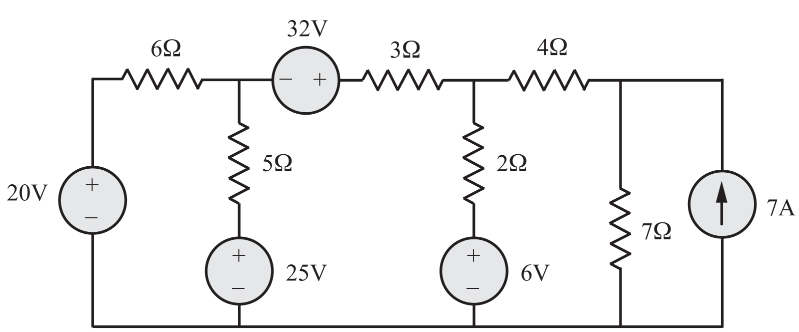


Figure E.P. 1.2

Ans : 2.125A

E.P 1.3

Find the voltage v using nodal technique for the circuit shown in Fig. E.P. 1.3.

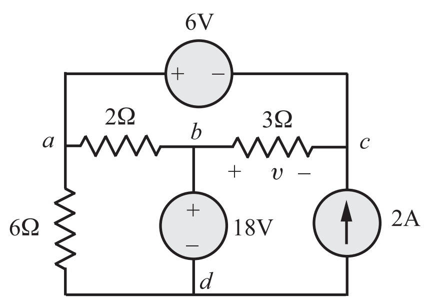


Figure E.P. 1.3

Ans : $\boldsymbol{v = 5\text{V}}$

E.P 1.4

Refer the network shown in Fig. E.P. 1.4. Find the currents i_1 and i_2 using nodal analysis.

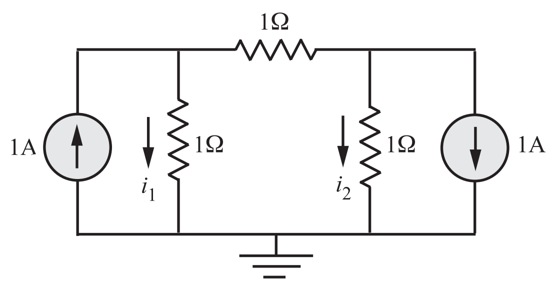


Figure E.P. 1.4

Ans : $\boldsymbol{i_1 = 1\ \text{A}, i_2 = -1\ \text{A}}$

E.P 1.5

For the network shown in Fig. E.P. 1.5, find the currents through the resistors R_1 and R_2 using nodal technique.

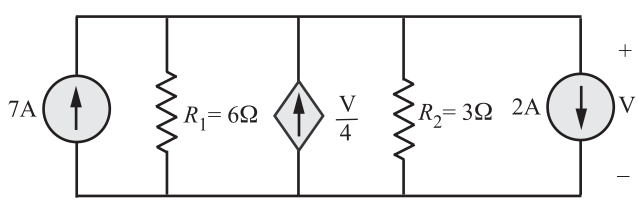


Figure E.P. 1.5

Ans : **3.33A, 6.67A**

E.P 1.6

Use the mesh-current method to find the branch currents i_1, i_2 and i_3 in the circuit of Fig. E.P. 1.6.

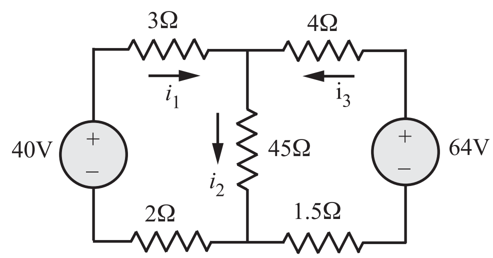


Figure E.P. 1.6

Ans : $\boldsymbol{i_1 = -1.72\text{A}, i_2 = 1.08\text{A}, i_3 = 2.8\text{A}}$

E.P 1.7

Refer the network shown in Fig. E.P. 1.7. Find the power delivered by the dependent voltage source in the network.

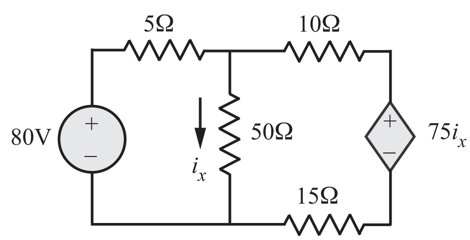


Figure E.P. 1.7

Ans : **−375 Watts**

E.P 1.8

Find the current I_x using (i) nodal analysis and (ii) mesh analysis.

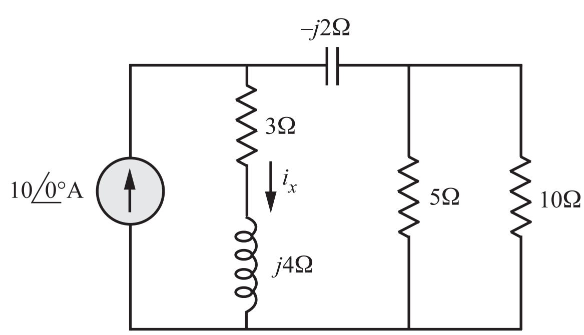


Figure E.P. 1.8

Ans : $\boldsymbol{I_x = \dfrac{150(3+j4)}{95+j30}\text{A}}$

E.P 1.9

Determine the current i_x in the circuit shown in Fig. E.P. 1.9

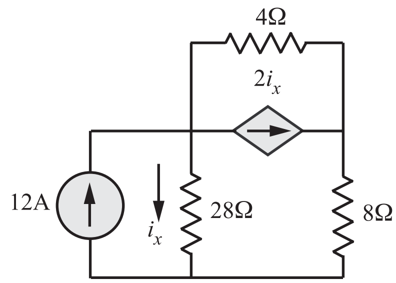


Figure E.P. 1.9

Ans : $\boldsymbol{i_x = 3A}$

E.P 1.10

Determine the resistance between the terminals $a - b$ of the network shown in Fig. E.P. 1.10.

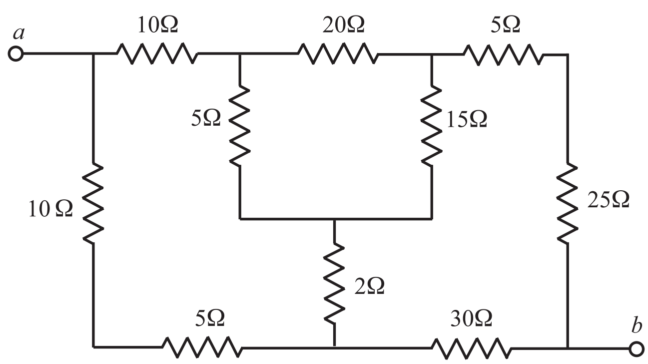


Figure E.P. 1.10

Ans : **23.6 Ω**

E.P 1.11

Determine the resistance between the points A and B in the network shown in Fig. E.P. 1.11.

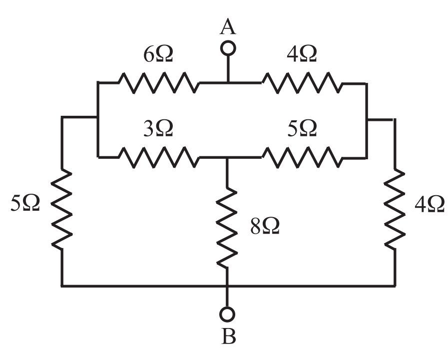


Figure E.P. 1.11

Ans : **4.23 Ω**

E.P 1.12

Determine the current in the galvanometer branch of the bridge network shown in Fig. E.P. 1.12.

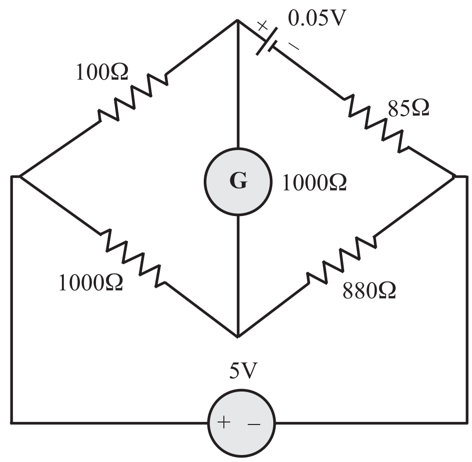


Figure E.P. 1.12

Ans : **10.62μA**