REVISION

MAXWELL'S EQN:

∇ - (DEL) represent how something changes in space.
(GRADIENT) (Function)

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$\nabla \cdot$ - represent how something changes in space in an expanding or contracting way.
(DIVERGENCE)

$\nabla \times$ - represent how something changes in space in a turning kind of way (degree of rotation).
(CURL) (Function!)

$\partial/\partial t$ - time derivative operator. (how something changes in time).

Gauss's Law:

* $\nabla \cdot \vec{E} = 0$ (Source Free Region) & $\nabla \cdot E = \rho/\epsilon_0$

+ve charge acts like end of a hose pipe and $\bar{E}$ bursts out in all directions.

-ve charge - $\vec{E}$ get sucked in (Sink).

* $\nabla \cdot \vec{B} = 0$ ($\vec{B}$ enters at one point and has to leave again at another point. magnetic monopoles doesn't exist).

* $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ -> if $\bar{B}$ changes in time, then it will cause E to move in space. ($\nabla\times$).

* $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ J = current density (movement of carriers).

Source free region, $\vec{J} = 0$, then it looks like previous eqn.

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

TO PROVE MAXWELL'S EQN:S HAVE WAVE SOLUTIONS:

Wave eqn: $\frac{\partial^2 A}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$; in 3D => $\nabla^2 A = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$

∇^2 is 3D analog of $\frac{\partial^2}{\partial x^2}$ (Laplacian)

Any system that ~~behave~~ obeys the above behave with wavelike properties.

=> Using $\nabla \times \nabla \times E = -\nabla \times \frac{\partial B}{\partial t}$

(A theorem in calculus states that 'curl of a curl of a field' can be written as: $\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$.

$\therefore \nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ (For source free region).

Comparing with original wave eqn $\rightarrow$ '$\mu_0\epsilon_0$' corresponds to '$\frac{1}{v^2}$'.

So the speed at which the wave travels is, $v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$= 3 \times 10^8$ m/s

* This derivation can be repeated for magnetic field.

$\therefore \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$.

The two field (E & B) always come in pairs perpendicular to each other and to the direction that they are going.

* Conclusion: changing electric field creates changing magnetic field which creates a changing electric field, etc.

Integral and Differential form of Maxwell's eqn:

Integral forms are useful when dealing with macroscopic problems with high degree of symmetry. (Formulated in terms of integrals, so we need curve, surface or volume)

Differential forms are local, deal with changes of fields at a point in space and time.

Power or Power Density or Poynting Vector

$\text{Power} = \frac{\text{Energy}}{\text{time}}$

Power Density of EM wave = Strength of power of EM wave

$$\boxed{\frac{1}{\mu_0} E \times B = \frac{EB}{\mu_0} = c \cdot \epsilon_0 E^2}$$

$S = E \times H = \frac{1}{\mu_0}(E \times B)$

= Poynting vector of EM wave.

$\boxed{\text{Propagation of EM wave} = \text{Power Flow}}$

$\boxed{\text{AVG: ENERGY } \frac{1}{2} \frac{E_0 B_0}{\mu_0}}$

ENERGY OF EM WAVES

$E = E_{electric} + E_{magnetic}$

$= \frac{1}{2}\epsilon_0 E_x^2 + \frac{1}{2}\frac{B_y^2}{\mu_0}$

$= \epsilon_0 E_x^2$

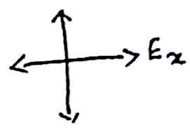
Polarization of EM waves:

It Represents orientation of Electric field of EM waves.

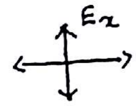
case: 1 Linear polarization

Consider an EM wave propagating in 'z' direction; If the Electric field component is along x-direction, its linearly polarized in x-direction. (Horizontal polarization)

i.e, $E_{(z,t)_x} = E_0 \cos(\omega t + kz)$



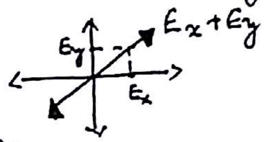
||'y for y-direction,



$E_{(z,t)_y} = E_0 \cos(\omega t + kz)$ (Vertical polarizn)

Also, Addition of horizontally polarized and vertically polarized

$$E_{(z,t)} = E_{x_0} \cos(\omega t + kz)\hat{i} + E_{y_0} \cos(\omega t + kz)\hat{j}$$

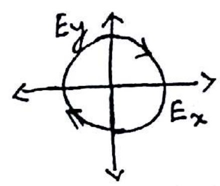


Case: 2

If EM wave propagates in 'z direction' with Electric-field along both x dirn and y dirn with phase difference of 90° and same amplitude, its called circularly polarized.

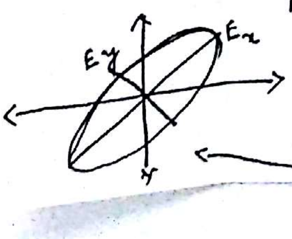
$$E_{(z,t)} = E_0 \cos(\omega t + kz)\hat{i} + E_0 \sin(\omega t + kz)\hat{j}$$

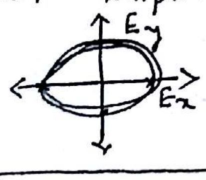
Same amplitude and 90° phase diff.



Case: 3

Different amplitude and 90° phase diff: represents Elliptically polarized.





$$E_{(z,t)} = E_1 \cos(\omega t + kz)\hat{i} + E_2 \sin(\omega t + kz)\hat{j}$$

$E_1 > E_2$ or $E_2 > E_1$

OR

$$E_{(z,t)} = E_1 \cos(\omega t + kz)\hat{i} + E_2 \sin(\omega t + kz \pm \phi)\hat{j}$$

ALL THE BEST!