Network Analysis & Synthesis

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UE18EC201: Network Analysis & Synthesis

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Network Analysis and Synthesis

Part IV: Two-Ports

Overview of Syllabus

Unit IV (7+2 hours) Two Ports:

- Review of one-ports.
- *z*-parameters: open circuit analysis;
  y-parameters: short circuit analysis.
- *h*-parameters and *t*-parameters.
- Deriving two port network parameters from one another.
- Interconnection of two port networks.

Ref. A: Chapter 11.
Ref. B: Chapter 17.
Ref. C:[1] Chapter 9.

[1] F. F. Kuo, *Network Analysis and Synthesis*, 2nd edn., John Wiley, 2006.

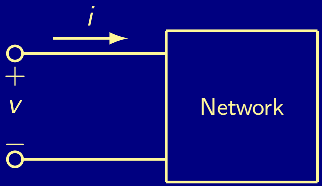
One-Port — Review (1)

- Suppose that we have bought an amplifier and speakers separately.

One-Port — Review (1)

- Suppose that we have bought an amplifier and speakers separately.
- The speaker system consists of woofers, mid-range speakers and tweeters.
- Filters are to be designed to direct the specific range of audio frequencies to the speakers, low frequency to woofer, and high-frequency to tweeter.
- The design of these filters requires only the following data — Thévenin equivalent circuit of the amplifier and the input impedance of the speakers.
- That is, for this application, we need not know what happens inside the amplifier, but only what happens at the terminals.

One-Port — Review (2)



- Equivalently, we treat the amplifier and the speaker as a two-terminal network or a one-port as we are interested only in the port properties.
- Or, the inside is treated as a black box.
- We have so far considered only one-ports.
- The pair of terminals is customarily connected to an energy source which is the driving force of the network.
- Hence, the pair of terminals is called the driving point of the network.

One-Port— Review (3)

- The transform impedance at a port is the ratio of the voltage transform to current transform for a network in a zero-state and no independent sources:

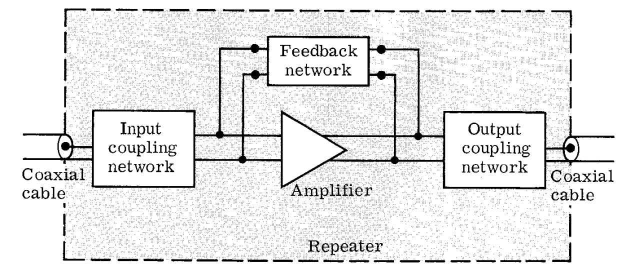
$$Z(s) = \frac{V(s)}{I(s)}$$

- Similarly, the transform admittance is the ratio

$$Y(s) = \frac{I(s)}{V(s)}$$

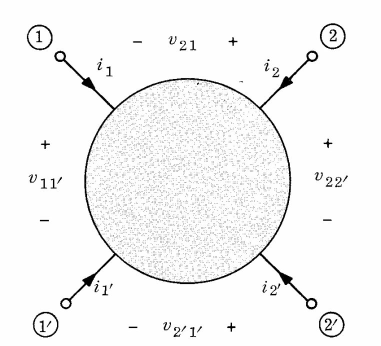
- The impedance or admittance at a port is called a driving-poing impedance or admittance.
- Often referred to as immittance.

Two-Ports (1)



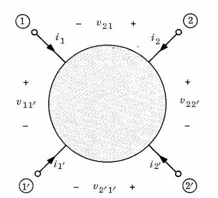
- Many practical examples fall into the category wherein one is interested in accessing two pairs of terminals; that is, two ports.
- The specifics or details of the network between these two ports are not relevant.

Two-Ports (2)



- The figure shows a network with four terminals.

Two-Ports (3)

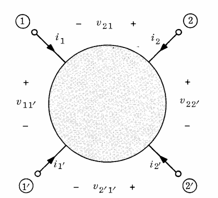


- In general, the four terminals must satisfy KVL and KCL:

$$i_1(t) + i_{1'}(t) + i_2(t) + i_{2'}(t) = 0$$
$$v_{11'}(t) + v_{21}(t) - v_{22'}(t) - v_{2'1'} = 0$$

for all instants t and all possible connections of the four-terminal network.

Two-Ports (3)



- In general, the four terminals must satisfy KVL and KCL:

$$i_1(t) + i_{1'}(t) + i_2(t) + i_{2'}(t) = 0$$
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  for all instants t and all possible connections of the four-terminal network.
- The four-terminal network becoms a two-port only if the following conditions are imposed for all t:

$$i_{1'}(t) = -i_1(t), \quad i_{2'}(t) = -i_2(t)$$

Two-Ports (4)

- The four-terminals are paired into ports 1–$1'$ and 2–$2'$.

Two-Ports (4)

- The four-terminals are paired into ports 1–1′ and 2–2′.
- Everything between these four terminals is a black box.

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- A two-port is any four-terminal network such that
  - At every instant of time t, the current entering terminal 1 (i.e., i_1) is equal to the current leaving the terminal $1'$.

Two-Ports (4)

- The four-terminals are paired into ports 1–1′ and 2–2′.
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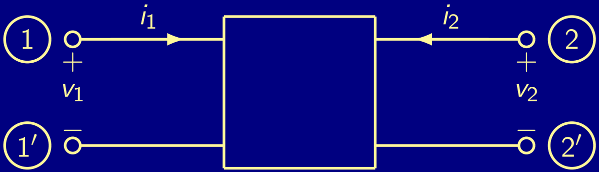
- A two-port is any four-terminal network such that
  - At every instant of time t, the current entering terminal 1 (i.e., i_1) is equal to the current leaving the terminal 1′.
  - Similarly, at every instant of time t, the current entering terminal 2 (i.e., i_2) is equal to the current leaving the terminal 2′.

Two-Ports (4)

- The four-terminals are paired into ports 1–$1'$ and 2–$2'$.
- Everything between these four terminals is a black box.
- A two-port is any four-terminal network such that
  - At every instant of time t, the current entering terminal 1 (i.e., i_1) is equal to the current leaving the terminal $1'$.
  - Similarly, at every instant of time t, the current entering terminal 2 (i.e., i_2) is equal to the current leaving the terminal $2'$.

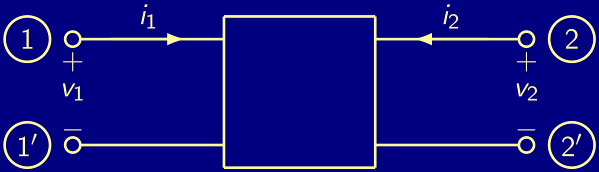
Two-Ports (5)



Network vs. Two-ports:

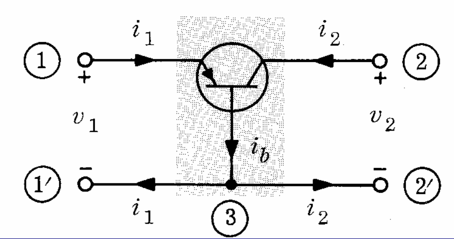
- To solve a network implies that all branch voltages and branch currents are calculated.
- In a two-port, the only variables of interest are the port variables — v_1, v_2, i_1 and i_2.
- Also, in a two-port, the only places independent sources are connected are the ports.

Two-Ports (6)



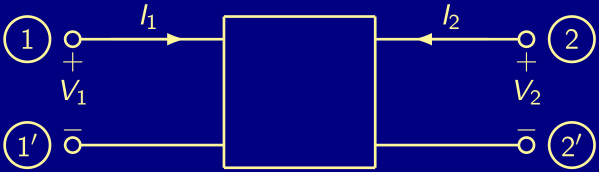
- The port at the left is usually referred to as the input port and has voltage v_1 and current i_1.
- The port at the right is usually referred to as the output port and has voltage v_2 and current i_2.
- Two-ports are completely characterised by all possible waveforms $v_1(\cdot)$, $v_2(\cdot)$, $i_1(\cdot)$ and $i_2(\cdot)$.

Two-Ports (7)



- Common-base transistor: An example of a two-port.

Two-Ports (8)



- Port 1 is the input port with transform quantities V_1 and I_1.
- Port 2 is the output port with transform quantities V_2 and I_2.
- Only two of these four variables are independent.
- The dependence of the other two variables on the independent variables can be expressed in a number of ways.
- Here, we consider six such combinations.

Two-Ports (9)

Name	Dependent Variables	Independent Variables
Open-circuit impedance	$V_1,\ V_2$	$I_1,\ I_2$
Short-circuit admittance	$I_1,\ I_2$	$V_1,\ V_2$
Transmission	$V_1,\ I_1$	$V_2,\ I_2$
Inverse transmission	$V_2,\ I_2$	$V_1,\ I_1$
Hybrid	$V_1,\ I_2$	$I_1,\ V_2$
Inverse hybrid	$I_1,\ V_2$	$V_1,\ I_2$

Two-Ports (10)

Short-Circuit Admittance Parameters: Assuming no dependent (controlled) sources,

$$
\begin{aligned}
I_1 &= y_{11}V_1 + y_{12}V_2 \\
I_2 &= y_{21}V_1 + y_{22}V_2
\end{aligned}
$$

Two-Ports (10)

Short-Circuit Admittance Parameters: Assuming no dependent (controlled) sources,

$$\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}$$

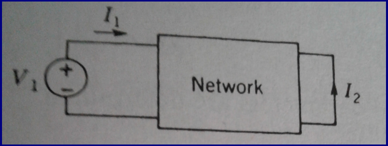
Observe that the four parameters may be defined as follows:

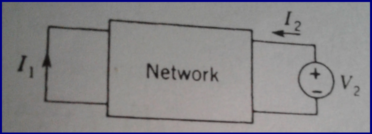
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

- The conditions $V_1 = 0$ or $V_2 = 0$ is equivalent to shorting port 1 or port 2, respectively.

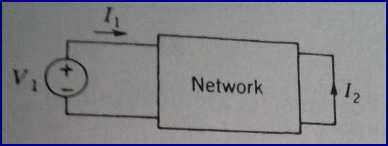
Two-Ports (11)

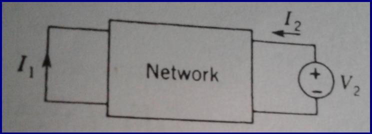




- (a): y_{11} and y_{21}.
- (b): y_{12} and y_{22}.
  - The terminology "short-circuit admittance" is obvious.

Two-Ports (11)





- (a): y_{11} and y_{21}.
- (b): y_{12} and y_{22}.
  - The terminology "short-circuit admittance" is obvious.
  - For a reciprocal network,

$$y_{12} = y_{21}$$

Two-Ports (12)

Open-Circuit Impedance Parameters: Assuming no dependent (controlled) sources,

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Two-Ports (12)

Open-Circuit Impedance Parameters: Assuming no dependent (controlled) sources,

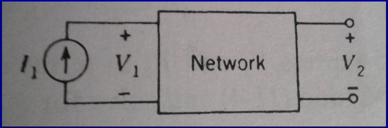
$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

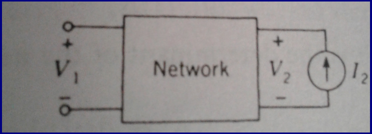
Observe that the four parameters may be defined as follows:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- The conditions $I_1 = 0$ or $I_2 = 0$ is equivalent to open circuits at port 1 or port 2, respectively.

Two-Ports (13)





- (a): z_{11} and z_{21}.
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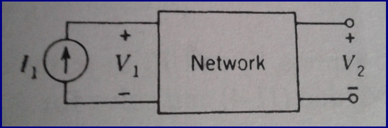
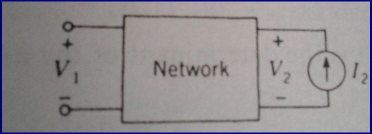
  - The terminology "open-circuit impedance" is obvious.

Two-Ports (13)





- (a): z_{11} and z_{21}.
- (b): z_{12} and z_{22}.
  - The terminology "open-circuit impedance" is obvious.
  - For a reciprocal network,

$$z_{12} = z_{21}$$

Two-Ports (14)

Note that for short-circuit admittance parameters,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

or

$$I = YV$$

Two-Ports (14)

Note that for short-circuit admittance parameters,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

or

$$I = YV$$

Similarly, for open-circuit impedance parameters,

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

or

$$V = ZI$$

Two-Ports (14)

Note that for short-circuit admittance parameters,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

or

$$I = YV$$

Similarly, for open-circuit impedance parameters,

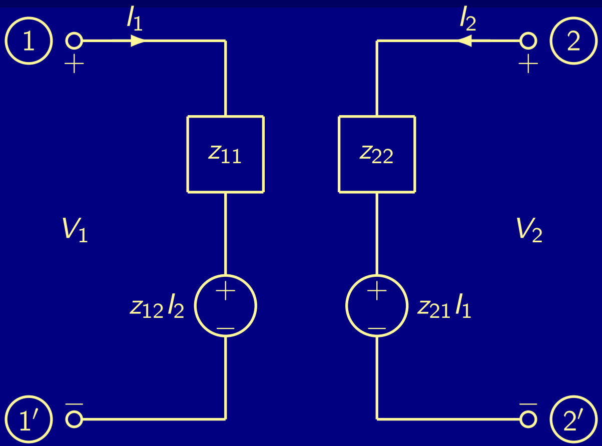
$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

or

$$V = ZI$$

- Clearly, $YZ = ZY = I$.
- Also, $y_{11}z_{11} = y_{22}z_{22}$.

Two-Ports (15)



- Two-generator equivalent in terms of open-circuit impedance parameters.

Two-Ports (16)

Recall:

$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

Two-Ports (16)

Recall:

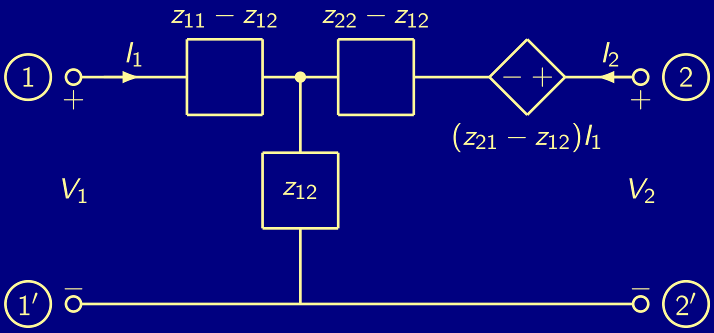
$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

Equivalently,

$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{12} I_1 + z_{22} I_2 + (z_{21} - z_{12}) I_1 \end{aligned}$$

- The latter leads to the one-generator equivalent in terms of the open-circuit impedance parameters.

Two-Ports (17)



- One-generator equivalent in terms of open-circuit impedance parameters.
- For reciprocal networks $z_{12} = z_{21}$, and the dependent source vanishes, leaving a T-network.

Two-Ports (18)

Recall:

$$\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}$$

Two-Ports (18)

Recall:

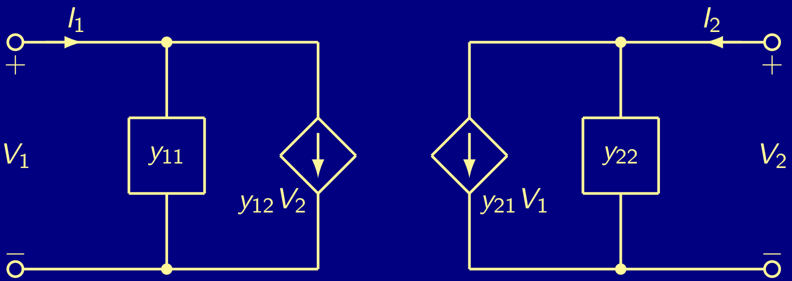
$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}$$

Equivalently,

$$\begin{aligned} I_1 &= (y_{11} + y_{12})V_1 - y_{12}(V_1 - V_2) \\ I_2 &= (y_{21} - y_{12})V_1 + (y_{12} + y_{22})V_2 - y_{12}(V_2 - V_1) \end{aligned}$$

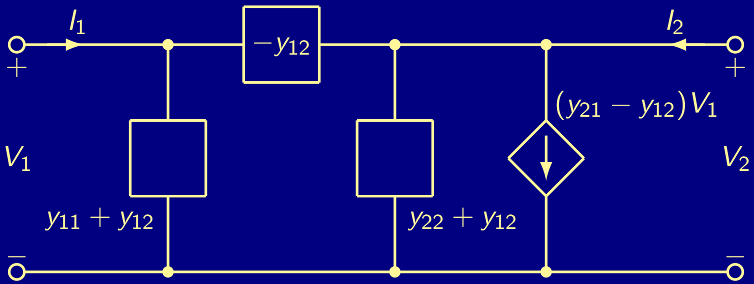
- The latter leads to the one-generator equivalent in terms of the short-circuit admittance parameters.

Two-Ports (19)



- Two-generator equivalent in terms of short-circuit admittance parameters.

Two-Ports (20)



- One-generator equivalent in terms of short-circuit admittance parameters.
- For reciprocal networks $y_{12} = y_{21}$.