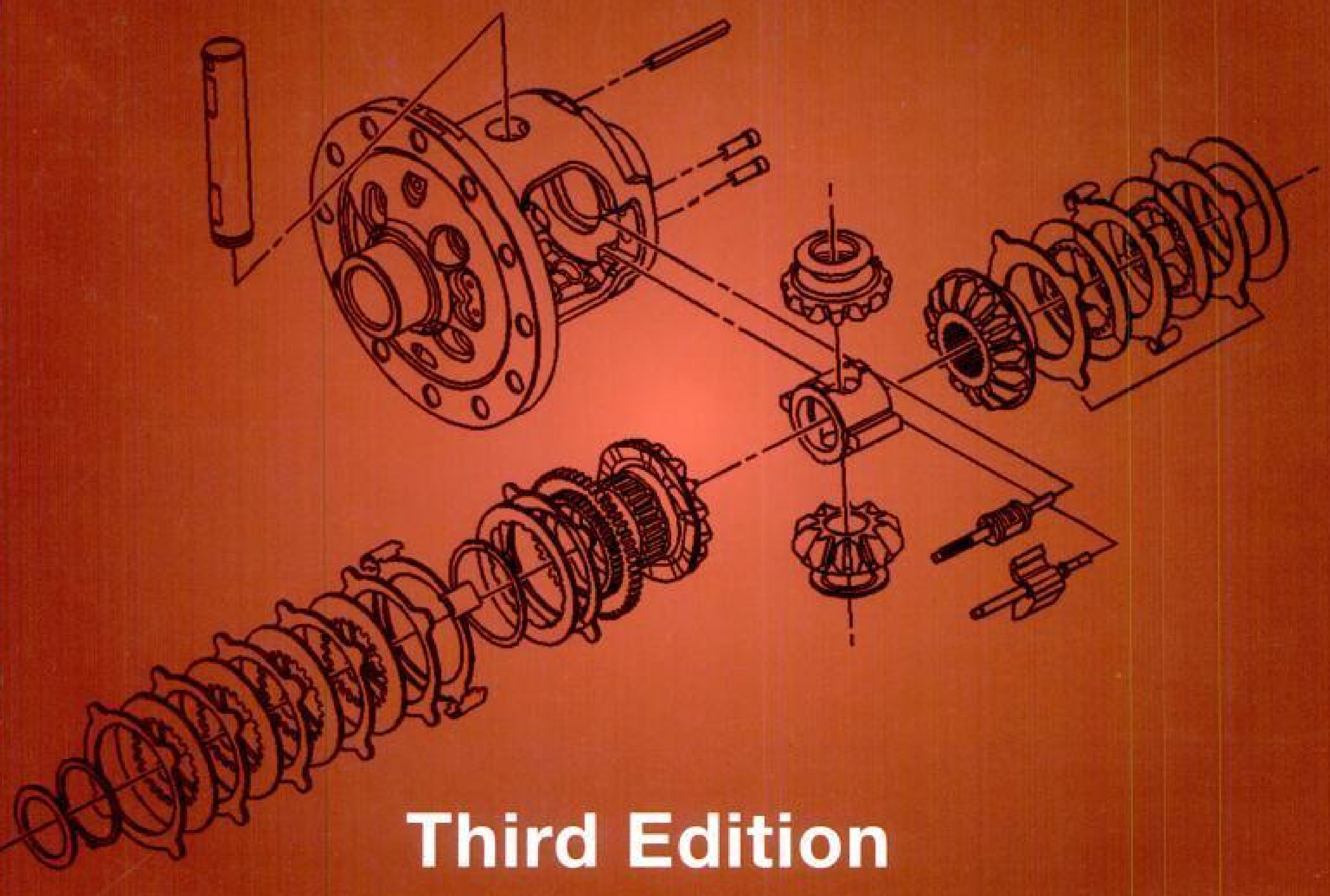

Third Edition

THEORY OF MACHINES

S S RATTAN

Tata McGraw-Hill

Published by Tata McGraw Hill Education Private Limited,
7 West Patel Nagar, New Delhi 110 008

Second reprint 2009
RYXZRRYFRDCCC

ISBN-13: 978-0-07-014477-4
ISBN-10: 0-07-014477-X

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Typeset at Bukprint India, B-180A, Guru Nanak Pura, Laxmi Nagar, Delhi-110 092 and printed at Gopal Jee Enterprises, 190/5 Main Road, Maujpur, Delhi 110053

Cover: SDR Printers

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PREFACE

Mechanisms and machines have considerable fascination for most students of mechanical engineering since the theoretical principles involved have immediate applications to practical problems. The main objective of writing this book has been to give a clear understanding of the concepts underlying engineering design. A sincere effort has been made to maintain the physical perceptions in the various derivations and to give the shortest comprehending solution to a variety of problems. The parameters kept in mind while writing the book are the coverage of contents, prerequisite knowledge of students, lucidity of writing, clarity of diagrams and the variety of solved and unsolved numerical problems.

The book is meant to be useful to the degree-level students of mechanical engineering as well as those preparing for AMIE and various other competitive examinations. However, diploma-level students will also find the book to be highly useful. The book will also benefit postgraduate students to some extent as it also contains advanced topics like curvature theory, analysis of rigid and elastic cam systems, complex number and vector methods, force balancing of linkages and field balancing. The salient features of the book are

- Concise and compact covering all major topics
- Presentation of concepts in a logical, innovative and lucid manner
- Evolving the basic theory from simple and readily understood principles
- A balanced presentation of the graphical and analytical approaches
- Computer programs in user-friendly C-language
- Large number of solved examples
- Summary, review questions as well as a number of unsolved problems at the end of each chapter
- An appendix containing objective-type questions
- Another appendix containing important relations and results

It is expected that the students using this book might have completed a course in applied mechanics. The book is divided broadly into two sections, kinematics and dynamics of machines. Kinematics involves study from the geometric point of view to know the displacement, velocity and acceleration of various components of mechanisms, whereas dynamics is the study of the effects of the applied and inertia forces. Chapters 1 to 11 are devoted to the study of the kinematics and the rest to that of dynamics. **Chapter 1** introduces the concepts of mechanisms and machines. **Chapters 2 and 3** describe graphical methods of velocity and acceleration analysis whereas the analytical approach is discussed in **Chapter 4**. Synthesis or designing of mechanisms is important to have the desirable motion of various components of machinery—the detail procedures for the same, both graphical and analytical, are given in **Chapter 5**. Various types of mechanisms with higher number of links are discussed in **Chapter 6**. Friction in various components of machines is very important as it affects their efficiency and is described in **Chapter 8**. Cams, belts, gears, gear trains are meant to transmit power from one shaft to another and are discussed in **chapters 7, 9, 10 and 11** respectively.

Forces are mainly of static and dynamic nature. **Chapters 12 and 13** are devoted to their effects on the components of the mechanisms. **Chapter 13** also includes the topic of flywheels which are essential components for rotary machines to regulate speeds. Speed regulation is also affected by governors which are described in **Chapter 16**. Unbalanced forces and vibrations in various components of rotating machines are mostly undesirable since the efficiency is reduced. A detailed study of these is undertaken in **chapters 14 and 18**. Brakes are essential for any moving components of machinery and are discussed in **Chapter 15**.

Moving bodies like aeroplanes, ships, two- and four-wheelers, etc., experience gyroscopic effect while taking turns. It is described in **Chapter 17**. Automatic control of machinery is very much desirable these days and an introduction of the same is given in **Chapter 19**.

The first edition of the book aimed at providing the fundamentals of the subject in a simple manner for easy comprehension by students. Simple mathematical methods were preferred instead of more elegant but less obvious methods so that those with limited mathematical skills could easily understand the expositions. However, to make the book more purposeful and acceptable to a wider section of users, the second edition also consisted of methods involving vector and complex numbers usually preferred by those who excel in mathematical skills. Such methods frequently lead to computer-aided solutions of the problems. The computer programs were rewritten in the more user-friendly C language. A Summary of each chapter was added at the end and theoretical questions were added to the exercises. One appendix containing objective-type questions was also included. All the previous figures were redrawn.

The present edition is aimed at making the book more exhaustive. Many more worked examples as well as unsolved problems have been added. Many new sections have been added in most of the chapters apart from rewriting some previous sections. Another appendix containing important relations and results has also been added. Effort has been made to remove all sorts of errors and misprints as far as possible. In spite of addition of a large amount of material, care has been taken to let the book remain concise and compact. Hints to most of the numerical problems at the end of each chapter have been provided at the publisher's website of the book for the benefit of average and weak students. Full solutions of the same are available to the faculty members at the same site. The facility can be availed by logging on to http://www.mhhe.com/rattan/tom3e.

I am grateful to all those teachers and students who pointed out errors and mistakes of the previous editions and also gave many valuable suggestions. I acknowledge the efforts of the editorial staff of Tata McGraw Hill Education Private Limited for bringing out the new edition in an excellent format.

Finally, I make an affectionate acknowledgement to my wife, Neena, and my children, Ravneet and Jasmeet, for their patience, support and putting up with it all so cheerfully. But for their sacrifice, I would not have been able to complete this work in the most satisfying way.

For further improvement of the book, readers are requested to post their comments and suggestions at ss_rattan@hotmail.com.

S S Rattan

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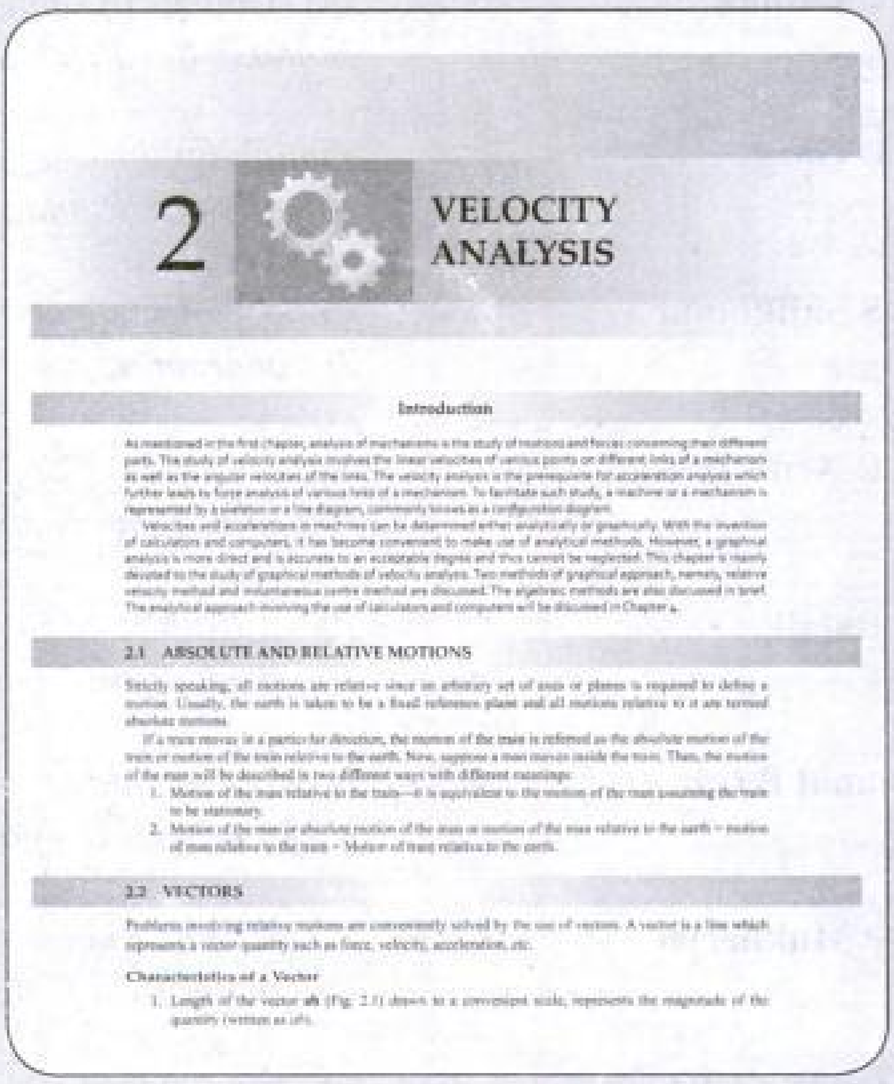
VISUAL WALKTHROUGH

Introduction at the beginning of each chapter sums up the aim and contents of the chapter.


2 VELOCITY ANALYSIS

Introduction

2.1 ABSOLUTE AND RELATIVE MOTIONS

2.2 VECTORS

Characteristics of a Vector

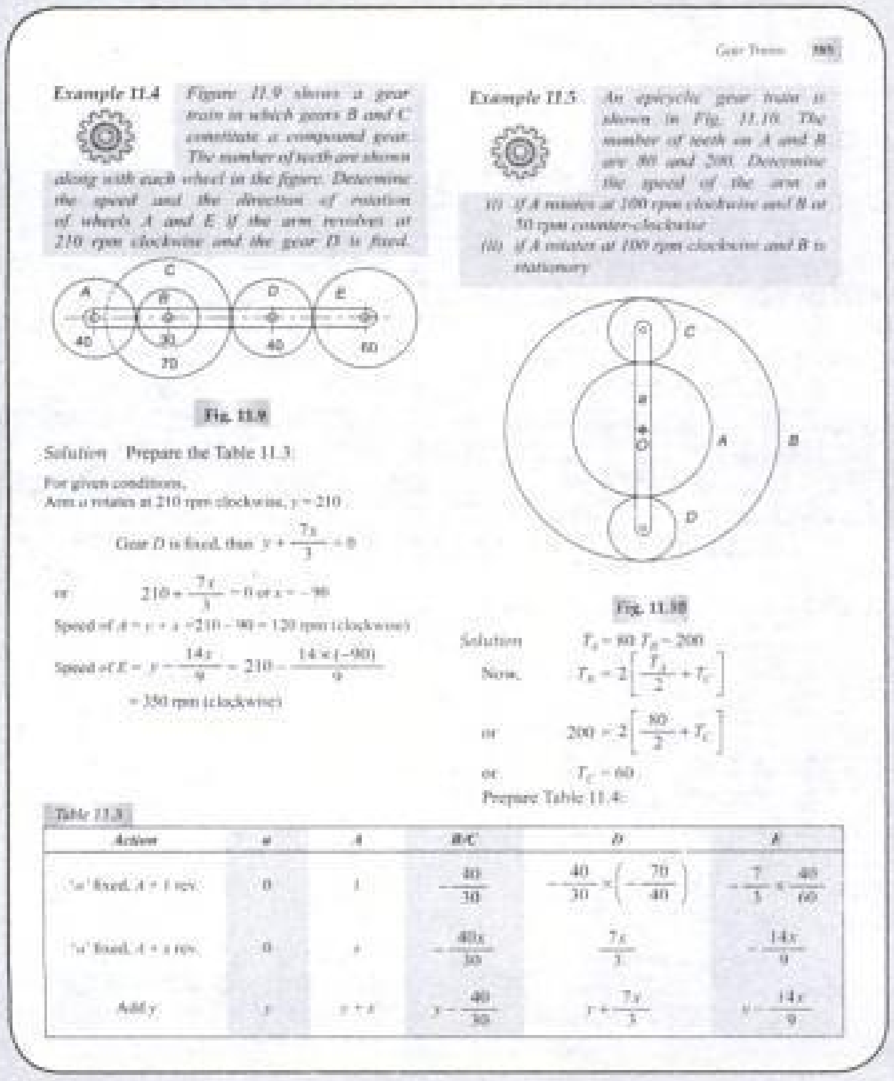

Example 11.4 Figure 11.9 shows a gear train in which gears B and C constitute a compound gear. The number of teeth are shown along with each wheel in the figure. Determine the speed and the direction of rotation of wheels A and E if the arm revolves at 210 rpm clockwise and the gear D is fixed.

Fig. 11.9

Solution Prepare the Table 11.3.

Example 11.5 An epicyclic gear train is shown in Fig. 11.10. The number of teeth on A and B are 80 and 200. Determine the speed of the arm a

(i) if A rotates at 100 rpm clockwise and B at 50 rpm counter-clockwise

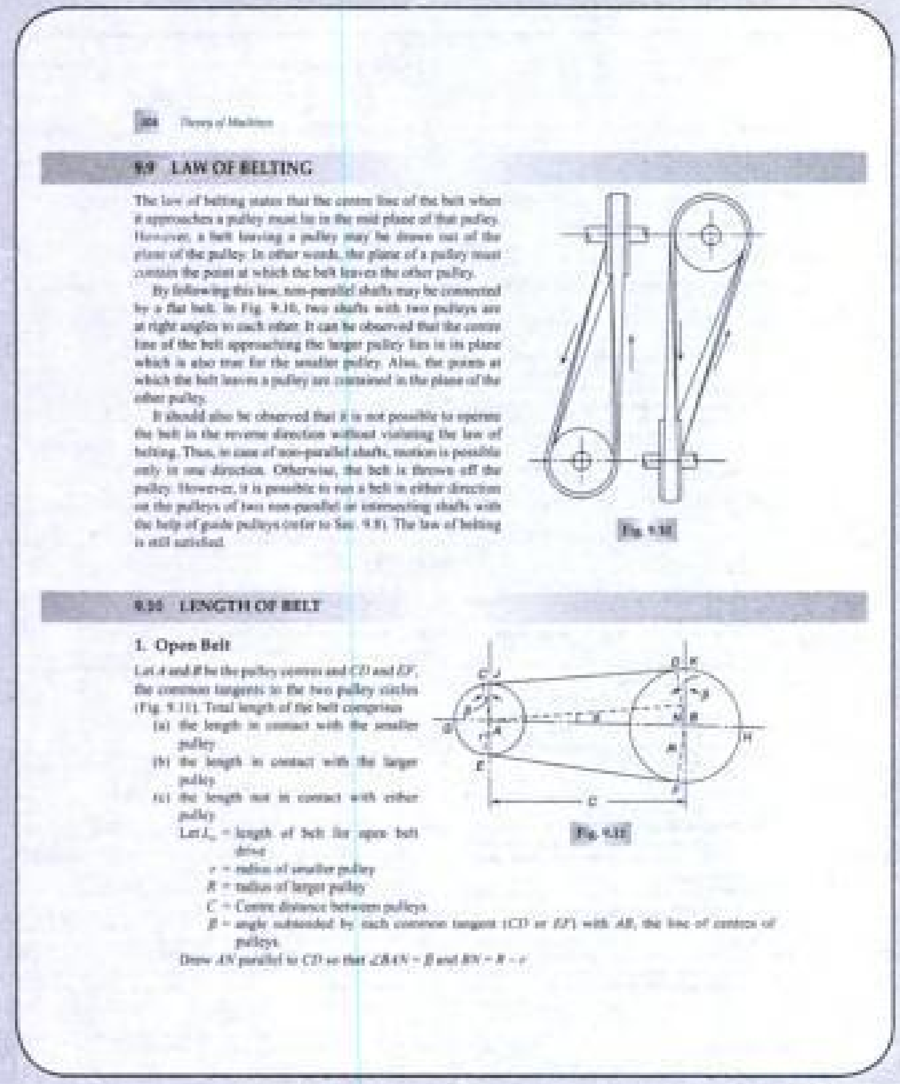
(ii) if A rotates at 100 rpm clockwise and B is stationary

Fig. 11.10

Prepare Table 11.4.

Table 11.3

A variety of solved examples are given to reinforce the concepts.



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9.9 LAW OF BELTING

The law of belting states that the centre line of the belt when it approaches a pulley must lie in the mid plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley. In other words, the plane of a pulley must contain the point at which the belt leaves the other pulley.

By following this law, non-parallel shafts may be connected by a flat belt. In Fig. 9.10, two shafts with two pulleys are at right angles to each other. It can be observed that the centre line of the belt approaching the larger pulley lies in its plane which is also true for the smaller pulley. Also, the points at which the belt leaves a pulley are contained in the plane of the other pulley.

It should also be observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Thus, in case of non-parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley. However, it is possible to run a belt in either direction on the pulleys of two non-parallel or intersecting shafts with the help of guide pulleys (refer to Sec. 9.8). The law of belting is still satisfied.

Fig. 9.10

9.10 LENGTH OF BELT

1. Open Belt

Let A and B be the pulley centres and CD and EF, the common tangents to the two pulley circles (Fig. 9.11). Total length of the belt comprises

(a) the length in contact with the smaller pulley

(b) the length in contact with the larger pulley

(c) the length not in contact with either pulley

Let L_o = length of belt for open belt drive

r = radius of smaller pulley

R = radius of larger pulley

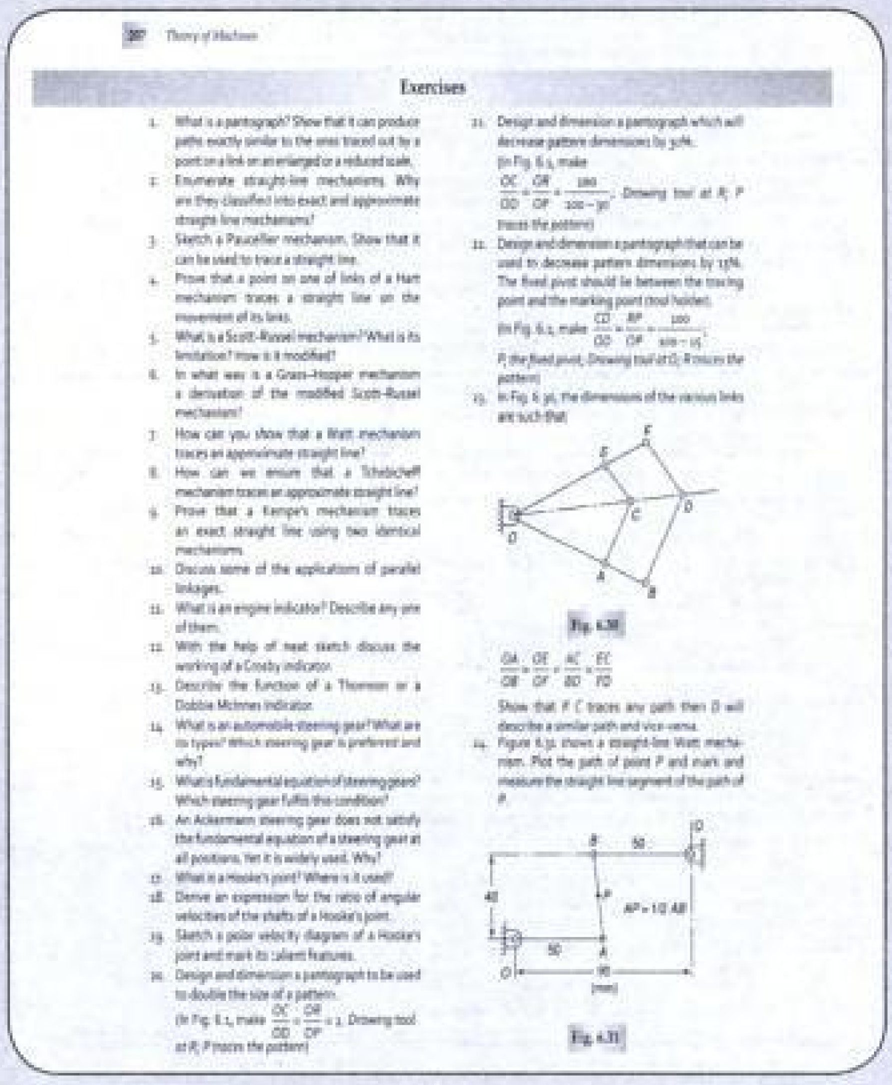
C = Centre distance between pulleys

β = angle subtended by each common tangent (CD or EF) with AB, the line of centres of pulleys.

Draw AN parallel to CD so that $\angle BAN = \beta$ and $BN = R - r$

Fig. 9.11

Each chapter has a concise and comprehensive treatment of topics with emphasis on fundamental concepts.



Theory of Machines

Exercises

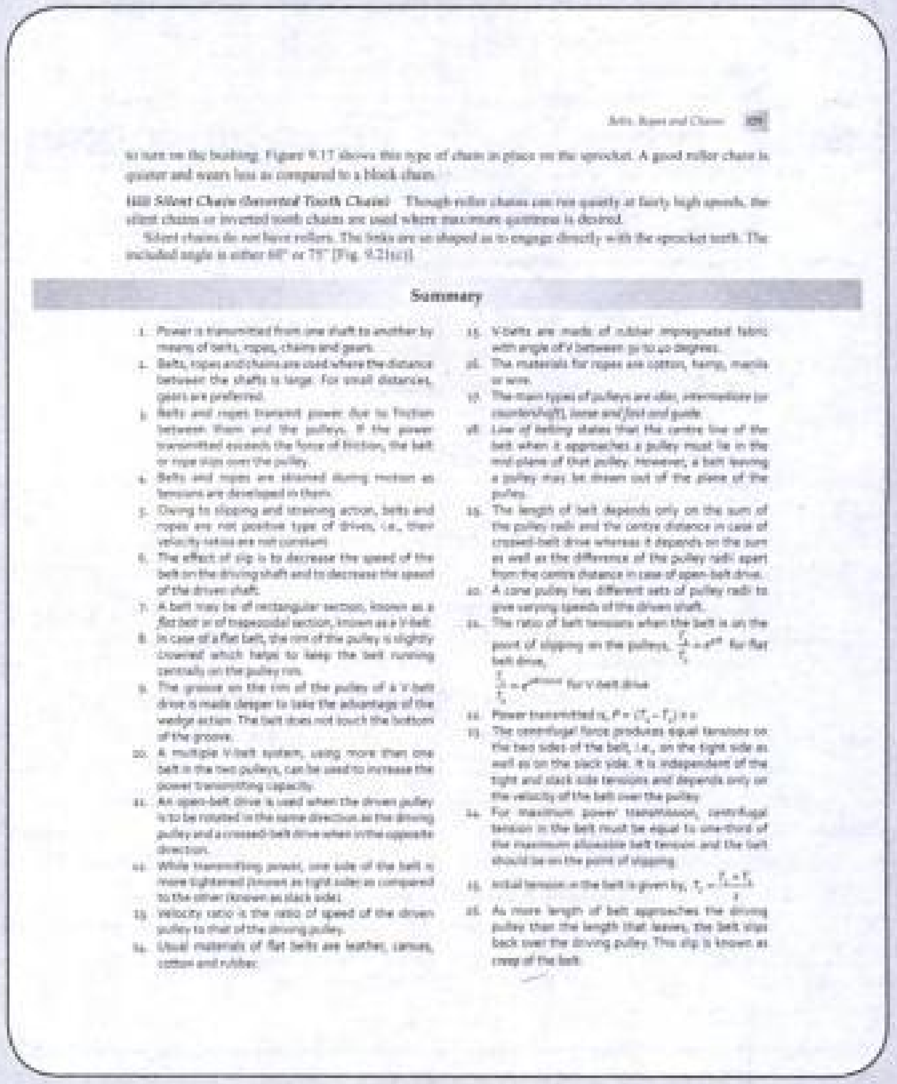
1. What is a pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or a reduced scale.
2. Enumerate straight-line mechanisms. Why are they classified into exact and approximate straight line mechanisms?
3. Sketch a Peaucellier mechanism. Show that it can be used to trace a straight line.
4. Prove that a point on one of links of a Hart mechanism traces a straight line on the movement of its links.
5. What is a Scott-Russel mechanism? What is its limitation? How is it modified?
6. In what way is a Grass-Hopper mechanism a derivation of the modified Scott-Russel mechanism?
7. How can you show that a Watt mechanism traces an approximate straight line?
8. How can we ensure that a Tchebicheff mechanism traces an approximate straight line?
9. Prove that a Kempe's mechanism traces an exact straight line using two identical mechanisms.
10. Discuss some of the applications of parallel linkages.
11. What is an engine indicator? Describe any one of them.
12. With the help of neat sketch discuss the working of a Crosby indicator.
13. Describe the function of a Thomson or a Dobbie McInnes indicator.
14. What is an automobile steering gear? What are its types?

Fig. 6.30

Fig. 6.31

A number of theoretical questions and unsolved exercises are given for practice to widen the horizon of comprehension of the topic.

A Summary at the end of each chapter recapitulates the inferences for quick revision.



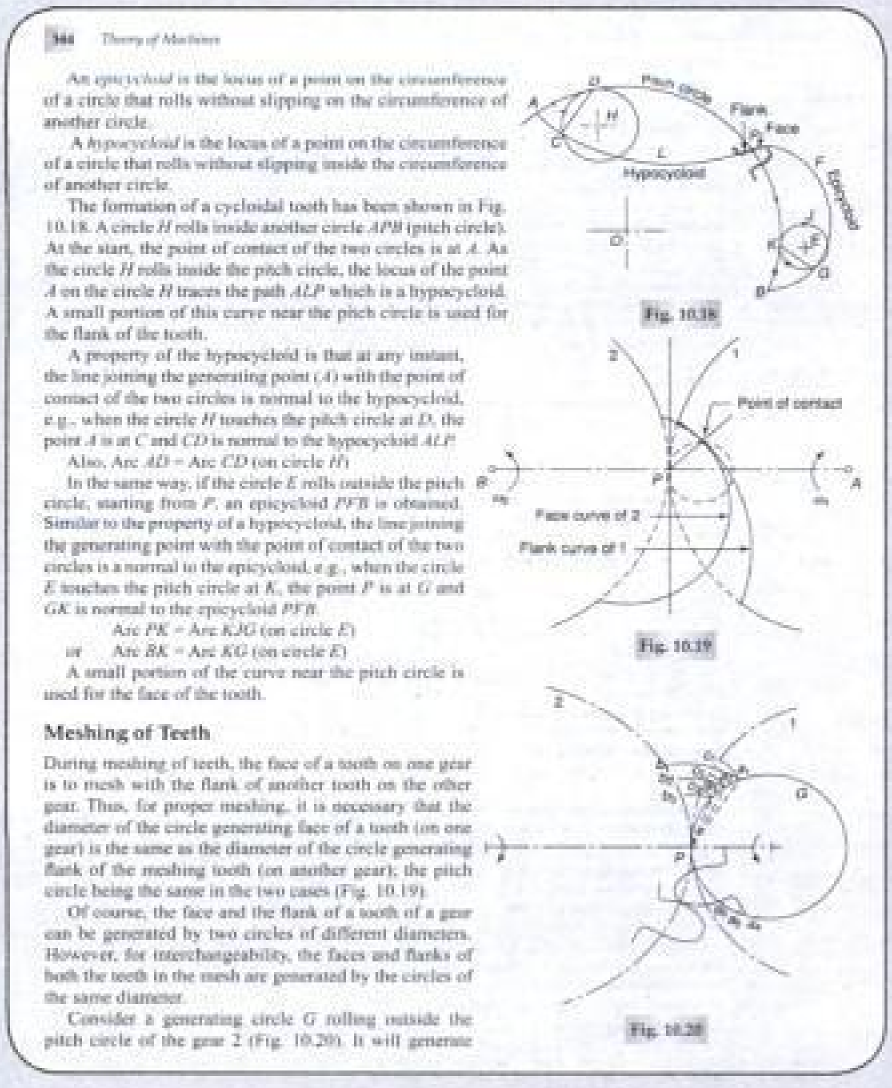
to turn on the bushing. Figure 9.17 shows this type of chain in place on the sprocket. A good roller chain is quieter and wears less as compared to a block chain.

(iii) Silent Chain (Inverted Tooth Chain) Though roller chains can run quietly at fairly high speeds, the silent chains or inverted tooth chains are used where maximum quietness is desired.

Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75° [Fig. 9.21(c)].

Summary

1. Power is transmitted from one shaft to another by means of belts, ropes, chains and gears.
2. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.
3. Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
4. Belts and ropes are strained during motion as tensions are developed in them.
5. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e., their velocity ratios are not constant.
6. The effect of slip is to decrease the speed of the belt on the driving shaft and to decrease the speed of the driven shaft.
7. A belt may be of rectangular section, known as a *flat belt* or of trapezoidal section, known as a *V-belt*.
8. In case of a flat belt, the rim of the pulley is slightly crowned which helps to keep the belt running centrally on the pulley rim.
9. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove.
10. A multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity.
11. An open-belt drive is used when the driven pulley is to be rotated in the same direction as the driving pulley and a crossed-belt drive when in the opposite direction.
12. While transmitting power, one side of the belt is more tightened (known as tight side) as compared to the other (known as slack side).
13. Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.
14. Usual materials of flat belts are leather, canvas, cotton and rubber.
15. V-belts are made of rubber impregnated fabric with angle of V between 30 to 40 degrees.
16. The materials for ropes are cotton, hemp, manila or wire.
17. The main types of pulleys are *idler*, *intermediate* (or *countershaft*), *loose and fast* and *guide*.
18. *Law of belting* states that the centre line of the belt when it approaches a pulley must lie in the mid-plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.
19. The length of belt depends only on the sum of the pulley radii and the centre distance in case of crossed-belt drive whereas it depends on the sum as well as the difference of the pulley radii apart from the centre distance in case of open-belt drive.
20. A cone pulley has different sets of pulley radii to give varying speeds of the driven shaft.
21. The ratio of belt tensions when the belt is on the point of slipping on the pulleys, $\frac{T_1}{T_2} = e^{\mu\theta}$ for flat belt drive,
$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec} \alpha}$ for V-belt drive
22. Power transmitted is, $P = (T_1 - T_2) \times v$
23. The centrifugal force produces equal tensions on the two sides of the belt, i.e., on the tight side as well as on the slack side. It is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.
24. For maximum power transmission, centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.
25. Initial tension in the belt is given by, $T_i = \frac{T_1 + T_2}{2}$
26. As more length of belt approaches the driving pulley than the length that leaves, the belt slips back over the driving pulley. This slip is known as creep of the belt.



An *epicycloid* is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.

A *hypocycloid* is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.

The formation of a cycloidal tooth has been shown in Fig. 10.18. A circle H rolls inside another circle APB (pitch circle). At the start, the point of contact of the two circles is at A. As the circle H rolls inside the pitch circle, the locus of the point A on the circle H traces the path ALP which is a hypocycloid. A small portion of this curve near the pitch circle is used for the flank of the tooth.

Fig. 10.18

A property of the hypocycloid is that at any instant, the line joining the generating point (A) with the point of contact of the two circles is normal to the hypocycloid, e.g., when the circle H touches the pitch circle at D, the point A is at C and CD is normal to the hypocycloid ALP.

Also, Arc AD = Arc CD (on circle H)

In the same way, if the circle E rolls outside the pitch circle, starting from P, an epicycloid PFB is obtained. Similar to the property of a hypocycloid, the line joining the generating point with the point of contact of the two circles is a normal to the epicycloid, e.g., when the circle E touches the pitch circle at K, the point P is at G and GK is normal to the epicycloid PFB.

Arc PK = Arc KJG (on circle E)

or Arc BK = Arc KG (on circle E)

A small portion of the curve near the pitch circle is used for the face of the tooth.

Fig. 10.19

Meshing of Teeth

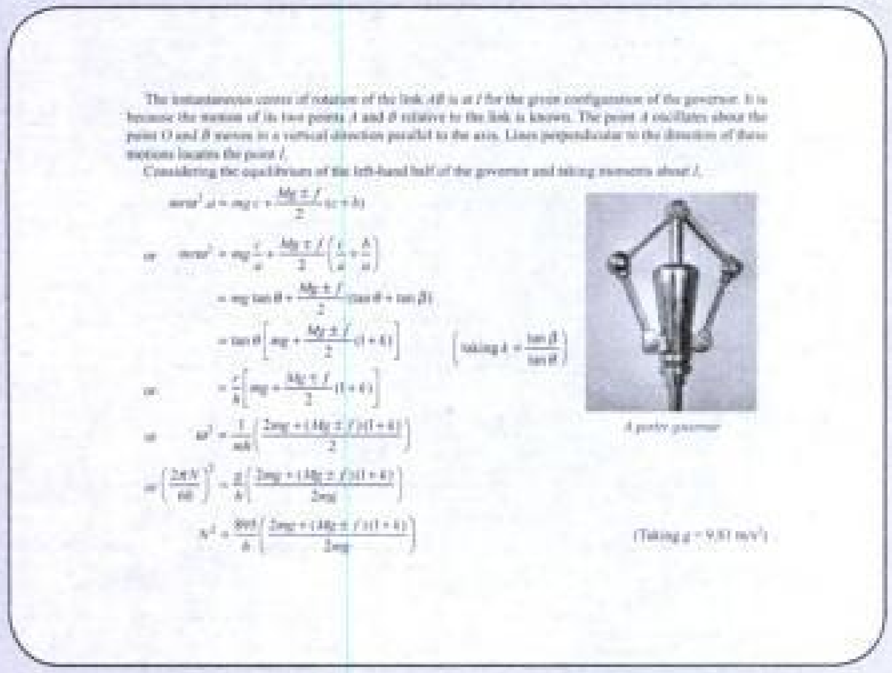
During meshing of teeth, the face of a tooth on one gear is to mesh with the flank of another tooth on the other gear. Thus, for proper meshing, it is necessary that the diameter of the circle generating face of a tooth (on one gear) is the same as the diameter of the circle generating flank of the meshing tooth (on another gear); the pitch circle being the same in the two cases (Fig. 10.19).

Of course, the face and the flank of a tooth of a gear can be generated by two circles of different diameters. However, for interchangeability, the faces and flanks of both the teeth in the mesh are generated by the circles of the same diameter.

Consider a generating circle G rolling outside the pitch circle of the gear 2 (Fig. 10.20). It will generate

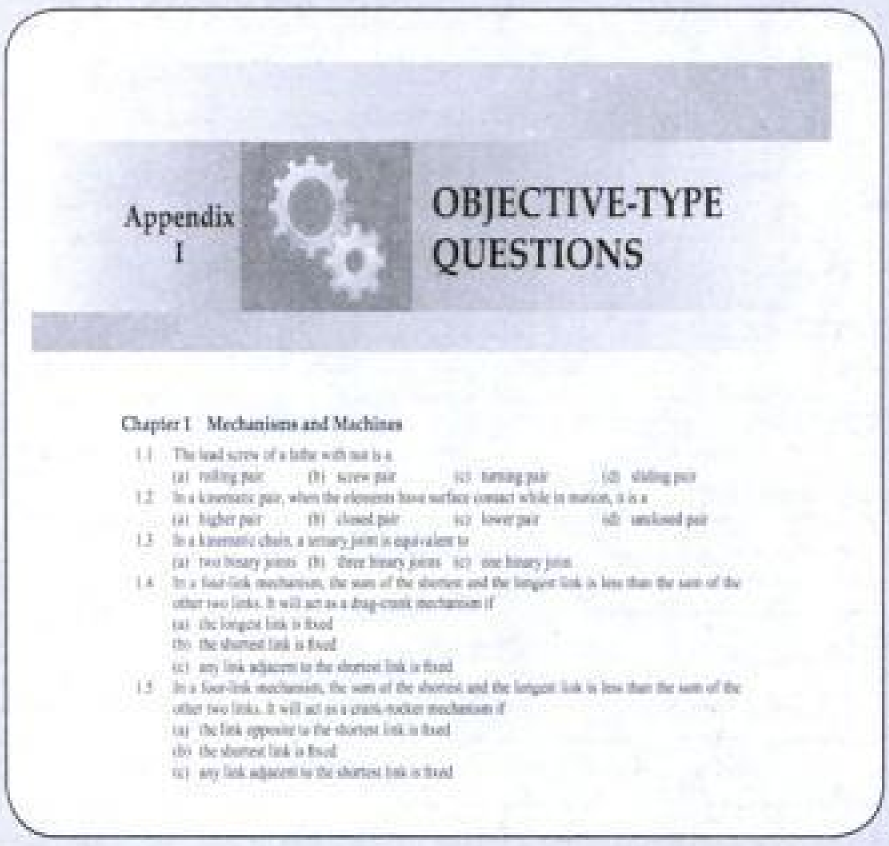
Fig. 10.20

Simple diagrams are given for easy visualization of the explanations.



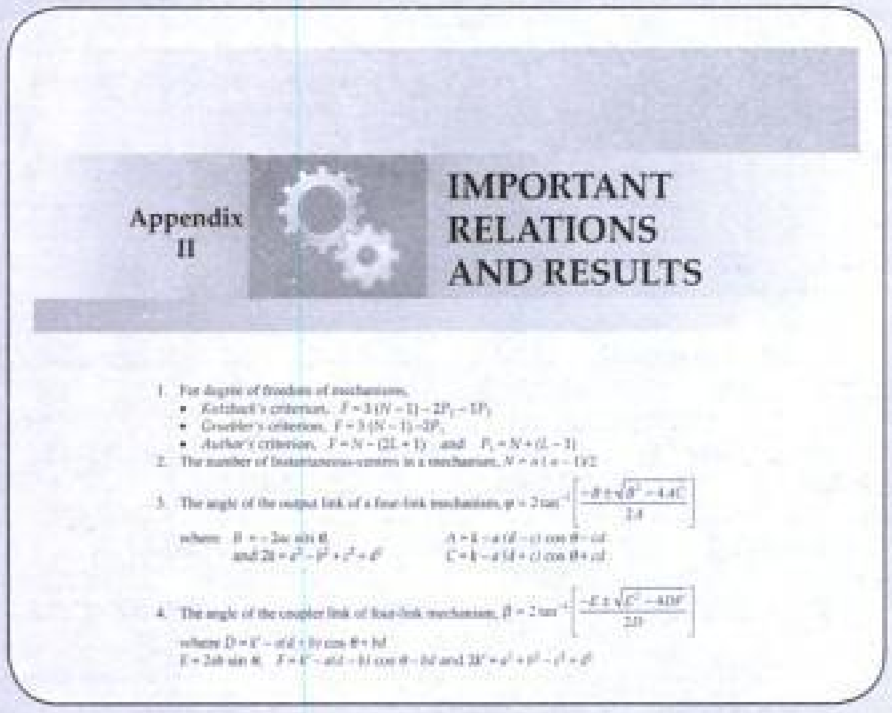
A number of photographs are given to emphasize the factual shape of various components.

An Appendix containing multiple choice questions is given at the end to help students prepare for competitive examinations.


Appendix I

OBJECTIVE-TYPE QUESTIONS

Chapter 1 Mechanisms and Machines


Appendix II

IMPORTANT RELATIONS AND RESULTS

An Appendix containing important relations is given for ready reference.

1 MECHANISMS AND MACHINES

Introduction

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*. A mechanism transmits and modifies a motion. A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. Thus, a mechanism is a fundamental unit and one has to start with its study.

The study of a mechanism involves its analysis as well as synthesis. *Analysis* is the study of motions and forces concerning different parts of an existing mechanism, whereas *synthesis* involves the design of its different parts. In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them. However, the study of the relative motions of the parts does not depend on the strength and the actual shapes of the parts.

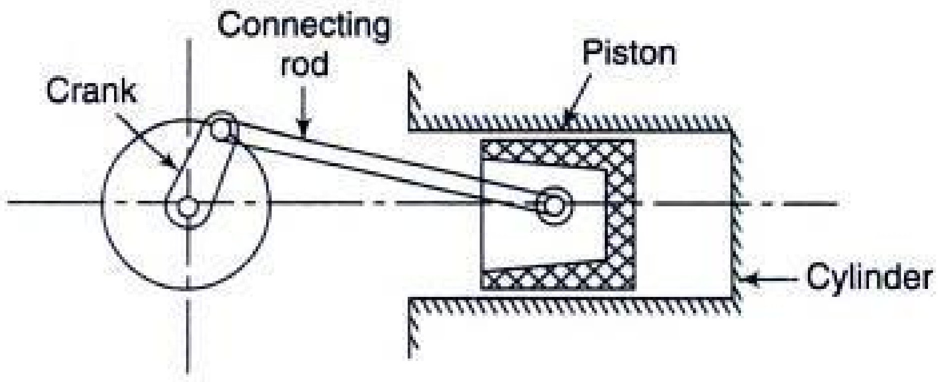


Fig. 1.1

In a reciprocating engine, the displacement of the piston depends upon the lengths of the connecting rod and the crank (Fig. 1.1). It is independent of the bearing strength of the parts or whether they are able to withstand the forces or not. Thus for the study of motions, it is immaterial if a machine part is made of mild steel, cast iron or wood. Also, it is not necessary to know the actual shape and area of the cross section of the part. Thus, for the study of motions of different parts of a mechanism, the study of forces is not necessary and can be neglected. The study of mechanisms, therefore, can be divided into the following disciplines:

Kinematics It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions. Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics It involves the calculations of forces impressed upon different parts of a mechanism. The forces can be either static or dynamic. Dynamics is further subdivided into *kinetics* and *statics*. Kinetics is the study of forces when the body is in motion whereas statics deals with forces when the body is stationary.

1.1 MECHANISM AND MACHINE

As mentioned earlier, a combination of a number of bodies (usually rigid) assembled in such a way that the motion of one causes constrained and predictable motion to the others is known as a *mechanism*. Thus, the function of a mechanism is to transmit and modify a motion.

A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. It is neither a source of energy nor a producer of work but helps in proper utilization of the same. The motive power has to be derived from external sources.

A slider-crank mechanism (Fig. 1.2) converts the reciprocating motion of a slider into rotary motion of the crank or vice-versa. However, when it is used as an automobile engine by adding valve mechanism, etc., it becomes a machine which converts the available energy (force on the piston) into the desired energy (torque of the crank-shaft). The torque is used to move a vehicle. Reciprocating pumps, reciprocating compressors and steam engines are other examples of machines derived from the slider-crank mechanism.

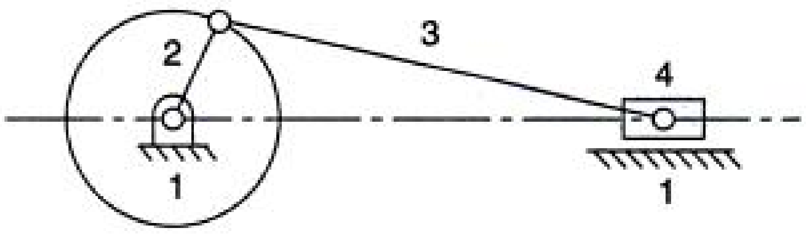


Fig. 1.2

Some other examples of mechanisms are typewriters, clocks, watches, spring toys, etc. In each of these, the force or energy provided is not more than what is required to overcome the friction of the parts and which is utilized just to get the desired motion of the mechanism and not to obtain any useful work.

1.2 TYPES OF CONSTRAINED MOTION

There are three types of constrained motion:

(i) ***Completely constrained motion*** When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion. The constrained motion may be linear or rotary. The sliding pair of Fig. 1.3(a) and the turning pair of Fig. 1.3(b) are the examples of the completely constrained motion. In sliding pair, the inner prism can only slide inside the hollow prism. In case of a turning pair, the inner shaft can have only rotary motion due to collars at the ends. In each case the force has to be applied in a particular direction for the required motion.

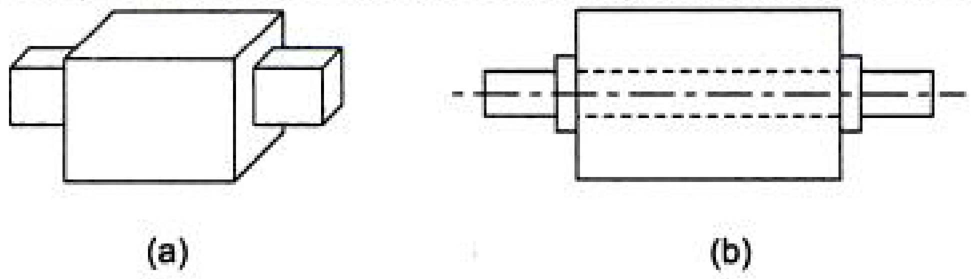


Fig. 1.3

(ii) ***Incompletely constrained motion*** When the motion between two elements of a pair is possible in more than one direction and depends upon the direction of the force applied, it is known as incompletely constrained motion. For example, if the turning pair of Fig. 1.4 does not have collars, the inner shaft may have sliding or rotary motion depending upon the direction of the force applied. Each motion is independent of the other.

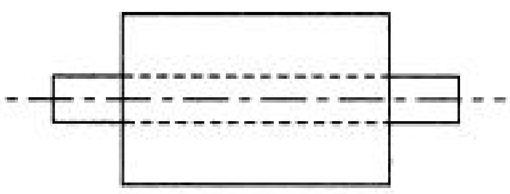
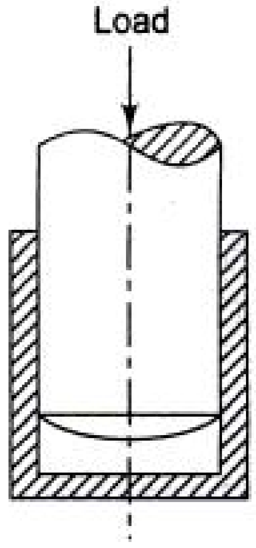


Fig. 1.4

(iii) ***Successfully constrained motion*** When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means, it is a successfully constrained motion. For example, a shaft in a footstep bearing may have vertical motion apart from rotary motion (Fig. 1.5). But due to load applied on the shaft it is constrained to move in



Footstep bearing

Fig. 1.5

that direction and thus is a successfully constrained motion. Similarly, a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion and no rotary motion due to constrain of the piston pin. Also, the valve of an IC engine is kept on the seat by the force of a spring and thus has successfully constrained motion.

1.3 RIGID AND RESISTANT BODIES

A body is said to be *rigid* if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.

Resistant bodies are those which are rigid for the purposes they have to serve. Apart from rigid bodies, there are some semi-rigid bodies which are normally flexible, but under certain loading conditions act as rigid bodies for the limited purpose and thus are resistant bodies. A belt is rigid when subjected to tensile forces. Therefore, the belt-drive acts as a resistant body. Similarly, fluids can also act as resistant bodies when compressed as in case of a hydraulic press. For some purposes, springs are also resistant bodies.

These days, resistant bodies are usually referred as rigid bodies.

1.4 LINK

A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a *link*. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them. Thus, a link may consist of one or more resistant bodies. A slider-crank mechanism consists of four links: frame and guides, crank, connecting-rod and slider. However, the frame may consist of bearings for the crankshaft. The crank link may have a crankshaft and flywheel also, forming one link having no relative motion of these.

A link is also known as *kinematic link* or *element*.

Links can be classified into *binary, ternary* and *quaternary* depending upon their ends on which revolute or turning pairs (Sec. 1.5) can be placed. The links shown in Fig. 1.6 are rigid links and there is no relative motion between the joints within the link.

Fig. 1.6

1.5 KINEMATIC PAIR

A kinematic pair or simply a pair is a joint of two links having relative motion between them. In a slider-crank mechanism (Fig. 1.2), the link 2 rotates relative to the link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (slider) reciprocates relative to the link 1 and is a sliding pair.

Types of Kinematic Pairs Kinematic pairs can be classified according to

- nature of contact
- nature of mechanical constraint
- nature of relative motion

Kinematic Pairs according to Nature of Contact

(a) Lower Pair A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint, etc.

(b) Higher Pair When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc.

Kinematic Pairs according to Nature of Mechanical Constraint

(a) Closed Pair When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs. A cam and follower pair (higher pair) shown in Fig. 1.7(a) and a screw pair (lower pair) belong to the closed pair category.

(b) Unclosed Pair When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g., cam and follower pair of Fig. 1.7(b).

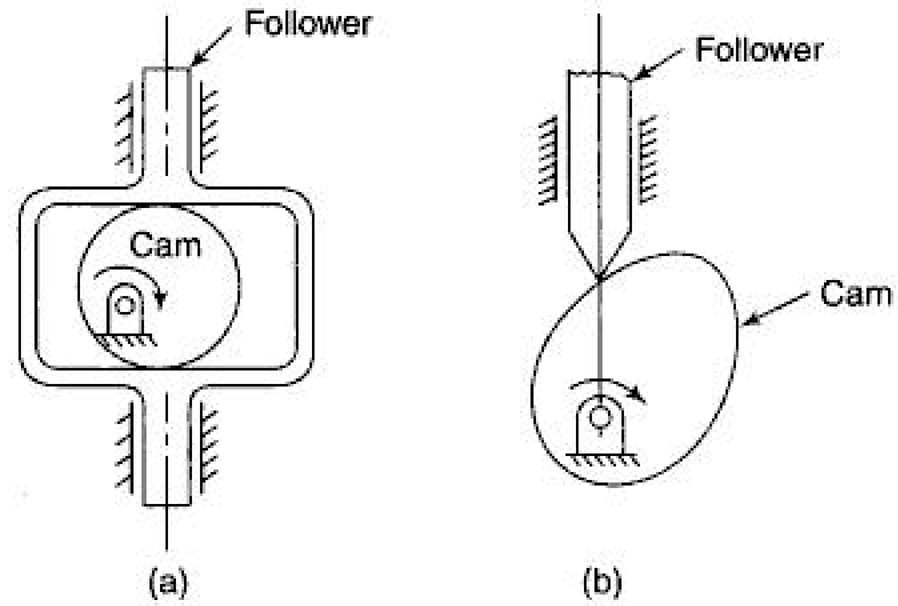


Fig. 1.7

Kinematic Pairs according to Nature of Relative Motion

(a) Sliding Pair If two links have a sliding motion relative to each other, they form a sliding pair.

A rectangular rod in a rectangular hole in a prism is a sliding pair [Fig. 1.8(a)].

(b) Turning Pair When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair [Fig. 1.8(b)].

In a slider-crank mechanism, all pairs except the slider and guide pair are turning pairs. A circular shaft revolving inside a bearing is a turning pair.

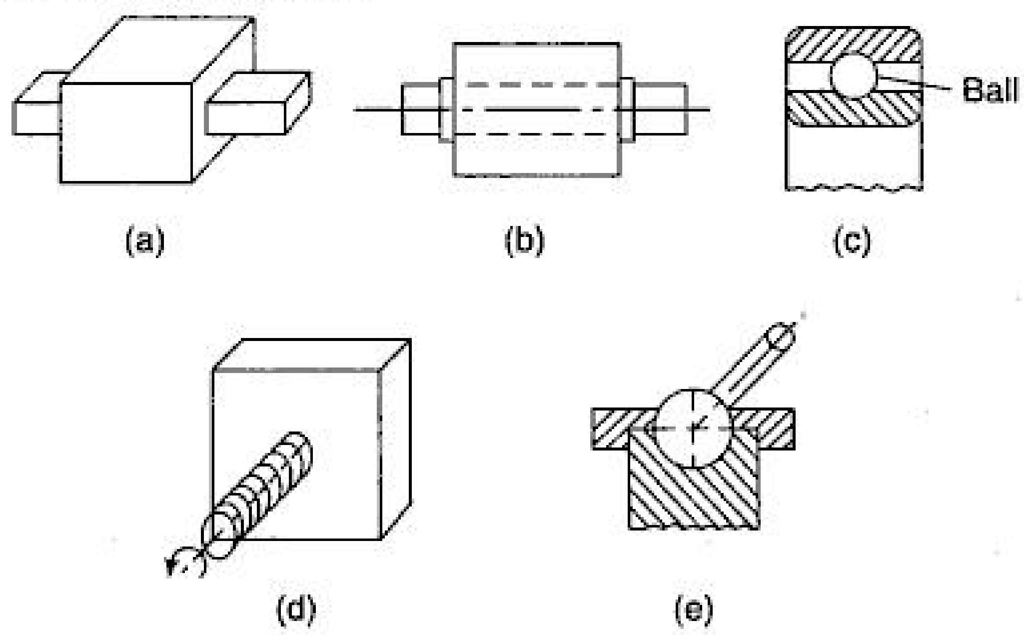


Fig. 1.8

(c) Rolling Pair When the links of a pair have a rolling motion relative to each other, they form a rolling pair, e.g., a rolling wheel on a flat surface, ball and roller bearings, etc. In a ball bearing [Fig. 1.8(c)], the ball and the shaft constitute one rolling pair whereas the ball and the bearing is the second rolling pair.

(d) Screw Pair (Helical Pair) If two mating links have a turning as well as sliding motion between them, they from a screw pair. This is achieved by cutting matching threads on the two links.

The lead screw and the nut of a lathe is a screw pair [Fig. 1.8(d)].

(e) Spherical Pair When one link in the form of a sphere turns inside a fixed link, it is a spherical pair.

The ball and socket joint is a spherical pair [Fig. 1.8(e)].

1.6 TYPES OF JOINTS

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

Binary Joint If two links are joined at the same connection, it is called a binary joint. For example, Fig. 1.9 shows a chain with two binary joints named *B*.

Ternary Joint If three links are joined at a connection, it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links. In Fig. 1.9 ternary links are mentioned as *T*.

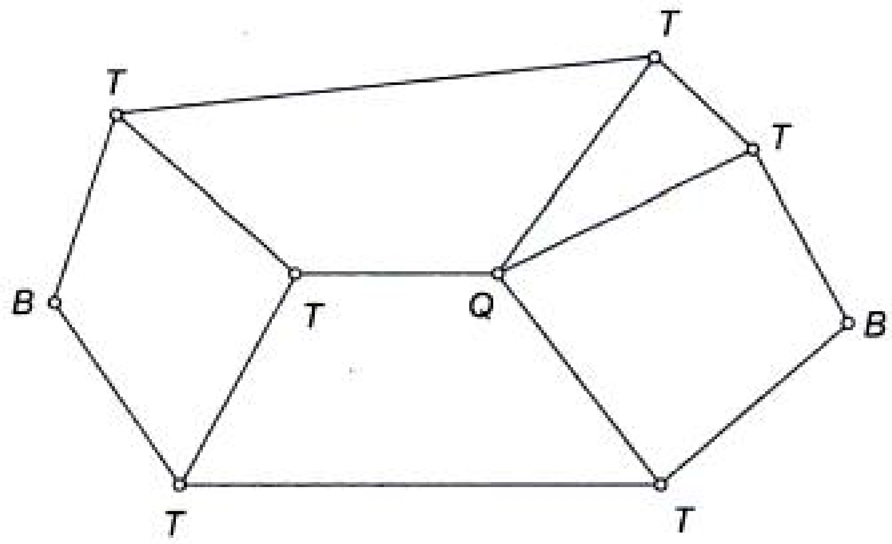


Fig. 1.9

Quaternary Joint If four links are joined at a connection, it is known as a quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints. Figure 1.9 shows one quaternary joint.

In general, if n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joints.

1.7 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – Number of restraints

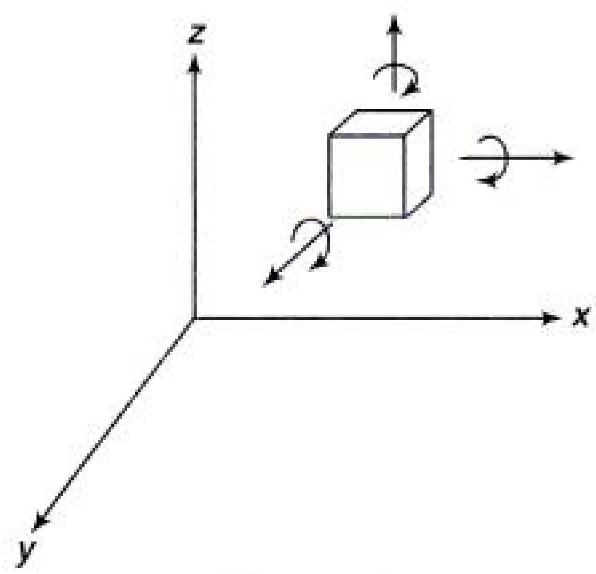


Fig. 1.10

1.8 CLASSIFICATION OF KINEMATIC PAIRS

Depending upon the number of restraints imposed on the relative motion of the two links connected together, a pair can be classified as given in Table 1.1 which gives the possible form of each class.

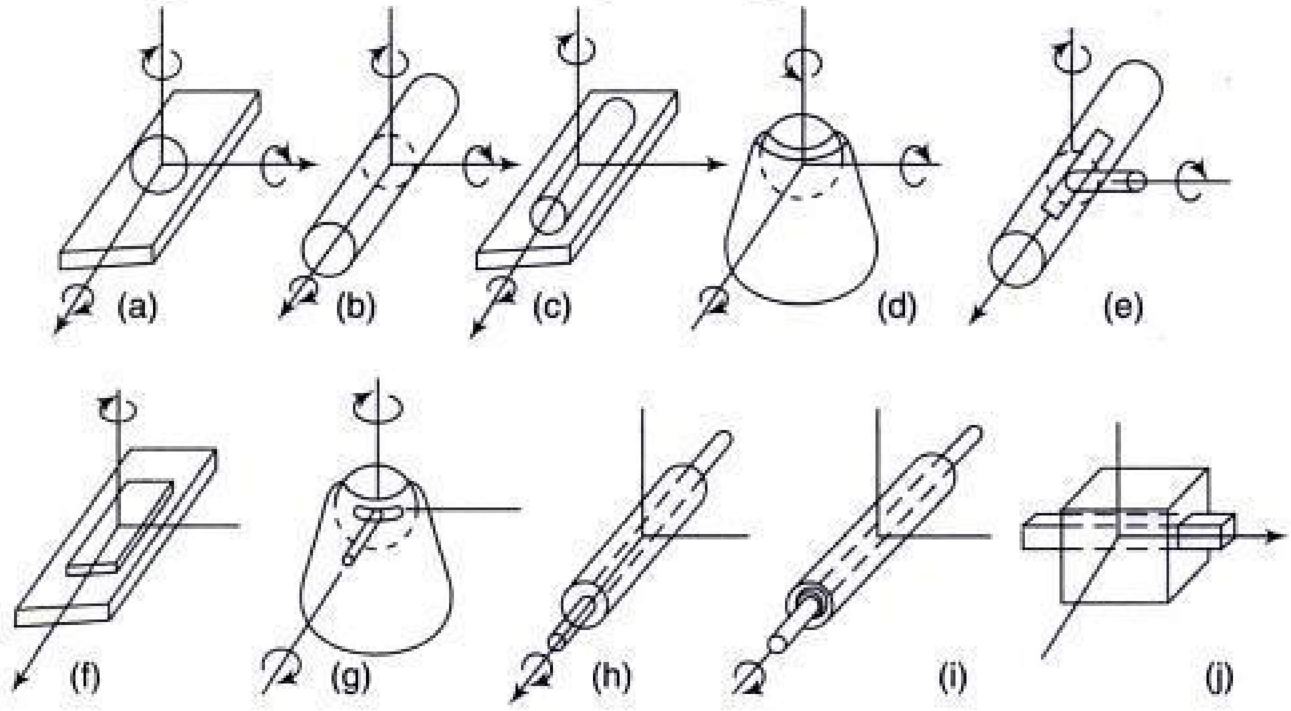


Fig. 1.11

Different forms of each class have also been shown in Fig. 1.11. Remember that a particular relative motion between two links of a pair must be independent of the other relative motions that the pair can have. A screw and nut pair permits translational and rotational motions. However, as the two motions cannot be accomplished independently, a screw and nut pair is a kinematic pair of the fifth class and not of the fourth class.

1.9 KINEMATIC CHAIN

A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite [Fig.1.12 (a), (b), and (c)].

Table 1.1

Class	Number of Restraints	Form	Restraints on		Kinematic pair	Fig. 1.11
			Translatory motion	Rotary motion		
I	1	1st	1	0	Sphere-plane	a
II	2	1st	2	0	Sphere-cylinder	b
		2nd	1	1	Cylinder-plane	c
III	3	1st	3	0	Spheric	d
		2nd	2	1	Sphere-slotted cylinder	e
		3rd	1	2	Prism-plane	f
IV	4	1st	3	1	Slotted-spheric	g
		2nd	2	2	Cylinder	h
V	5	1st	3	2	Cylinder (collared)	i
		2nd	2	3	Prismatic	j

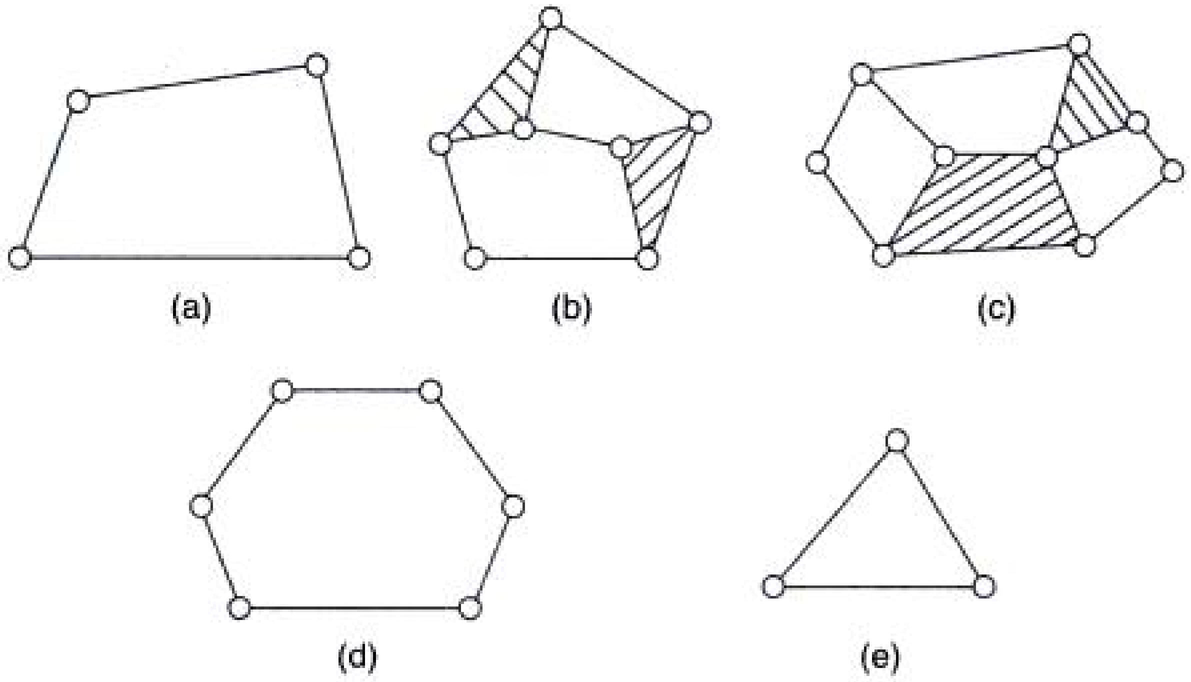


Fig. 1.12

In case the motion of a link results in indefinite motions of other links, it is a *non-kinematic chain* [Fig. 1.12(d)]. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

A *redundant chain* does not allow any motion of a link relative to the other [Fig. 1.12(e)].

1.10 LINKAGE, MECHANISM AND STRUCTURE

A *linkage* is obtained if one of the links of a kinematic chain is fixed to the ground. If motion of any of the moveable links results in definite motions of the others, the linkage is known as a *mechanism*. However, this distinction between a mechanism and a linkage is hardly followed and each can be referred in place of the other.

If one of the links of a redundant chain is fixed, it is known as a *structure* or *a locked system*. To obtain constrained or definite motions of some of the links of a linkage (or mechanism), it is necessary to know how many inputs are needed. In some mechanisms, only one input is necessary that determines the motions of other links and it is said to have one degree of freedom. In other mechanisms, two inputs may be necessary to get constrained motions of the other links and they are said to have two degrees of freedom, and so on.

The degree of freedom of a structure or a locked system is zero. A structure with negative degree of freedom is known as a *superstructure*.

1.11 MOBILITY OF MECHANISMS

A mechanism may consist of a number of pairs belonging to different classes having different number of restraints. It is also possible that some of the restraints imposed on the individual links are common or general to all the links of the mechanism. According to the number of these general or common restraints, a mechanism may be classified into a different order. A zero-order mechanism will have no such general restrain. Of course, some of the pairs may have individual restraints. A first-order mechanism has one general restraint; a second-order mechanism has two general restraints, and so on, up to the fifth order. A sixth-order mechanism cannot exist since all the links become stationary and no movement is possible.

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

P_1 = number of pairs having one degree of freedom

P_2 = number of pairs having two degrees of freedom, and so on

In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \tag{1.1}$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

Therefore, for plane mechanisms, the following relation may be used to find the degrees of freedom

$$F = 3(N - 1) - 2P_1 - 1P_2 \tag{1.2}$$

This is known as *Gruebler's criterion* for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom. Each pair with one degree of freedom imposes two further restraints on the mechanisms, thus reducing its degrees of freedom. Similarly, each pair with two degrees of freedom reduces the degrees of freedom of the mechanism at the rate of one restraint each.

Some authors mention the above relation as *Kutzback's criterion* and a simplified relation [$F = 3(N-1) - 2P_1$] which is applicable to linkages with a single degree of freedom only as Gruebler's criterion. However, many authors make no distinction between Kutzback's criterion and Gruebler's criterion.

Thus, for linkages with a single degree of freedom only, $P_2 = 0$

$$F = 3(N - 1) - 2P_1 \tag{1.3}$$

Most of the linkages are expected to have one degree of freedom so that with one input to any of the links, a constrained motion of the others is obtained.

Then,

$$1 = 3(N - 1) - 2P_1$$

or

$$2P_1 = 3N - 4 \tag{1.4}$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if N is even. For possible linkages made of binary links only,

$N = 4,$	$P_1 = 4$	No excess turning pair
$N = 6,$	$P_1 = 7$	One excess turning pair
$N = 8,$	$P_1 = 10$	Two excess turning pairs

and so on.

Thus, with the increase in the number of links, the number of excess turning pairs goes on increasing. Getting the required number of turning pairs from the required number of binary links is not possible. Therefore, the excess or the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links, etc.

For a six-link chain, some of the possible types are Watts six-bar chain, in which the ternary links are directly connected [Fig. 1.13(a)] and Stephenson's six-bar chain, in which ternary links are not directly connected [Fig. 1.13(b)]. Another possibility is also shown in Fig. 1.13(c). However, this chain is not a six-link chain but a four-link chain as links 1, 2 and 3 are, in fact, one link only with no relative motion of these links.

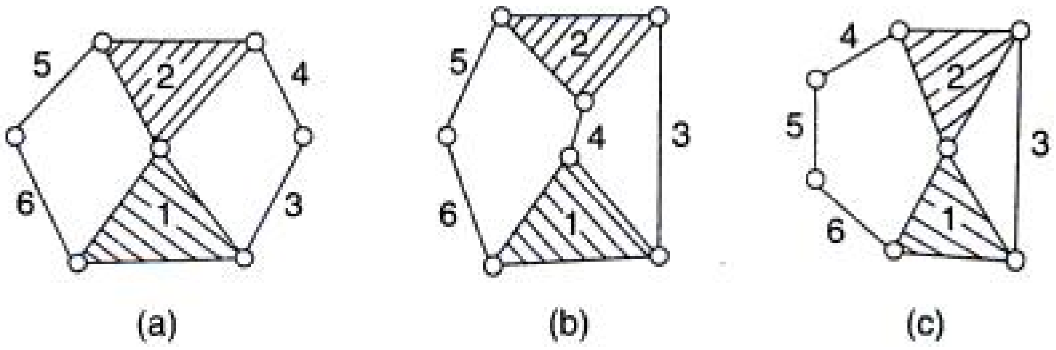


Fig. 1.13

Two excess turning pairs required for an eight-link chain can be obtained by using (apart from binary links):

four ternary links [Figs 1.14(a) and (b)]
two quaternary links [Fig.1.14(c)]
one quaternary and two ternary links [Fig. 1.14(d)].

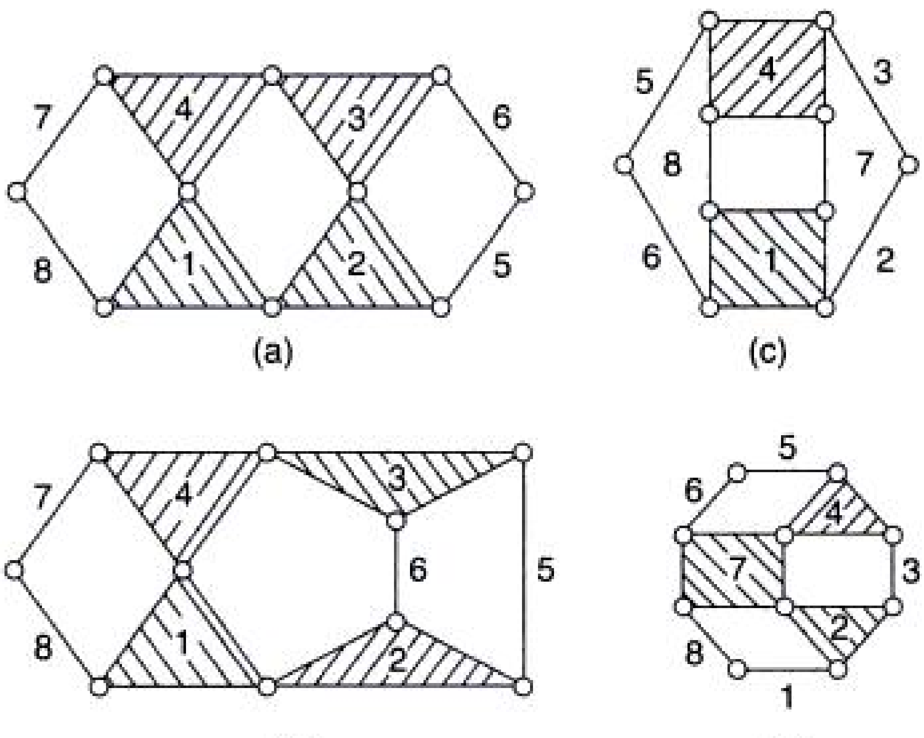


Fig. 1.14

Now, consider the kinematic chain shown in Fig. 1.15. It has 8 links, but only three ternary links. However, the links 6, 7 and 8 constitute a double pair so that the total number of pairs is again 10. The degree of freedom of such a linkage will be

$$F = 3\,(8 - 1) - 2 \times 10$$
$$= 1$$

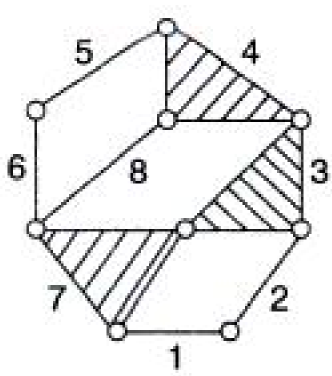


Fig. 1.15

This shows that the number of ternary or quaternary links in a chain can be reduced by providing double joints also.

The following empirical relations formulated by the author provide the degree of freedom and the number of joints in a linkage when the number of links and the number of loops in a kinematic chain are known. These relations are valid for linkages with turning pairs,

$$F = N - (2L + 1) \tag{1.5}$$

$$P_1 = N + (L - 1) \tag{1.6}$$

where

L = number of loops in a linkage.

Thus, for different number of loops in a linkage, the degrees of freedom and the number of pairs are as shown in Table 1.2.

Table 1.2

L	F	P_1
1	$N-3$	N
2	$N-5$	$N+1$
3	$N-7$	$N+2$
4	$N-9$	$N+3$
5	$N-11$	$N+4$
and so on		

For example, if in a linkage, there are 4 loops and 11 links, its degree of freedom will be 2 and the number of joints, 14. Similarly, if a linkage has 3 loops, it will require 8 links to have one degree of freedom, 9 links to have 2 degrees of freedom, 7 links to have –1 degree of freedom, etc.

Sometimes, all the above empirical relations can give incorrect results, e.g., Fig.1.16(a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom. However, if the links are arranged in such a way as shown in Fig. 1.16(b), a *double parallelogram linkage* with one degree of freedom is obtained. This is due to the reason that the lengths of the links or other dimensional properties are not considered in these empirical relations. So, exceptions are bound to come with equal lengths or parallel links.

Sometimes, a system may have one or more links which do not introduce any extra constraint. Such links are known as *redundant links* and should not be counted to find the degree of freedom. For example, the mechanism of Fig. 1.16(b) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 or 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus has one degree of freedom.

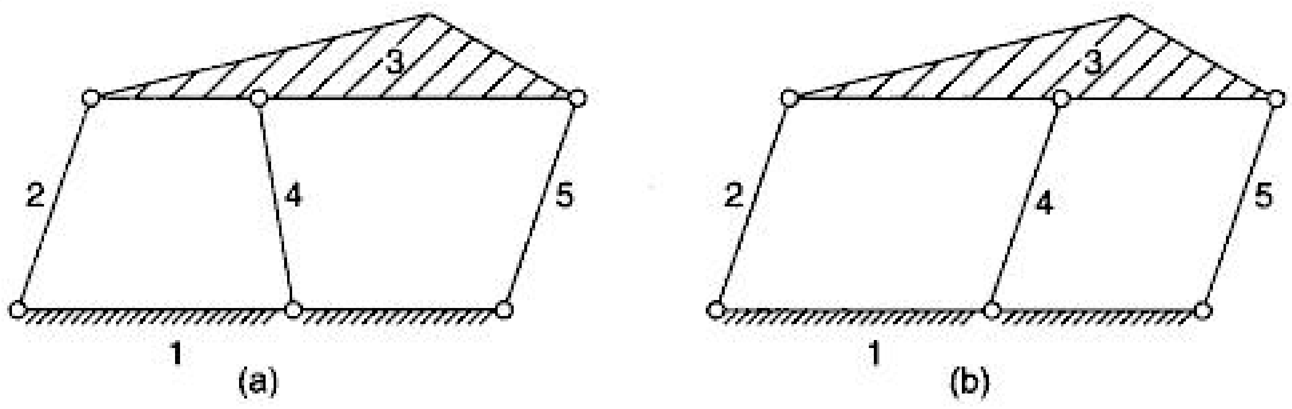


Fig. 1.16

Sometimes, one or more links of a mechanism can be moved without causing any motion to the rest of the links of the mechanism. Such a link is said to have a *redundant degree of freedom*. Thus in a mechanism, it is necessary to recognize such links prior to investigate the degree of freedom of the whole mechanism. For example, in the mechanism shown in Fig. 1.17, roller 3 can rotate about its axis without causing any movement to the rest of the mechanism. Thus, the mechanism represents a redundant degree of freedom.

In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by

$$F = 3(N-1) - 2P_1 - 1P_2 - F_r$$

where F_r is the number of redundant degrees of freedom. Now, as the above mechanism has a cam pair, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 4

Number of pairs with 1 degree of freedom = 3

Number of pairs with 2 degrees of freedom = 1

$$\begin{aligned} F &= 3(N-1) - 2P_1 - 1P_2 - F_r \\ &= 3(4-1) - 2\times 3 - 1\times 1 - 1 \\ &= 1 \end{aligned}$$

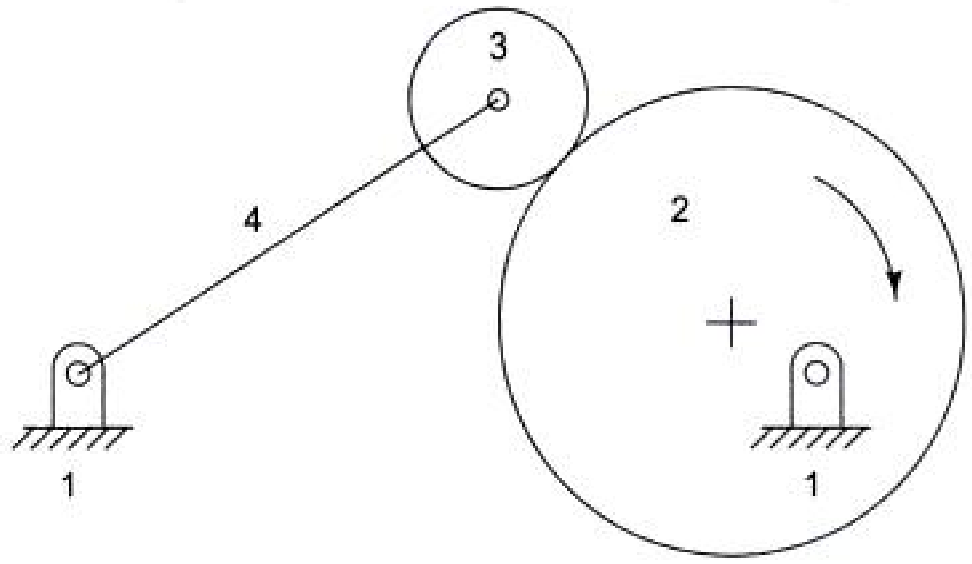


Fig. 1.17

Example 1.1 *For the kinematic linkages shown in Fig. 1.18, calculate the following:*

- *the number of binary links (N_b)*
- *the number of ternary links (N_t)*
- *the number of other (quaternary, etc.) links (N_o)*
- *the number of total links (N)*
- *the number of loops (L)*
- *the number of joints or pairs (P_1)*
- *the number of degrees of freedom (F)*

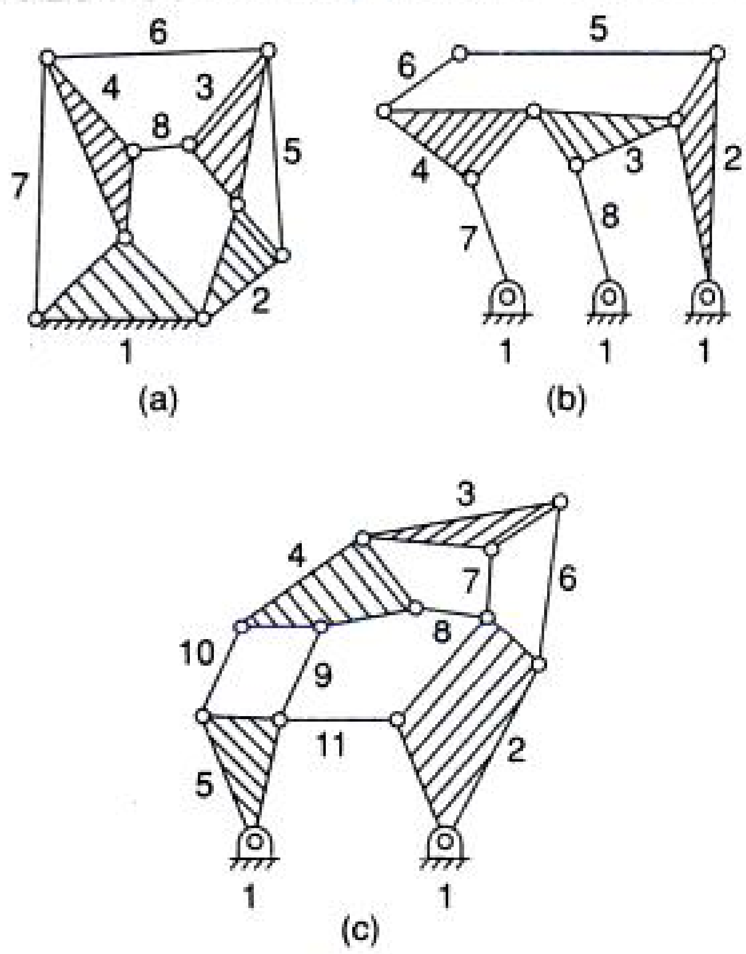


Fig. 1.18

Solution

(a) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 4$

$P_1 = 11$ by counting

or $P_1 = (N + L - 1) = 11$

$F = 3(N-1) - 2P_1$

$= 3(8-1) - 2\times 11 = -1$

or $F = N - (2L + 1)$

$= 8 - (2\times 4 + 1) = -1$

The linkage has negative degree of freedom and thus is a superstructure.

(b) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 3$

$P_1 = 10$ (by counting)

or $P_1 = (N + L - 1) = 10$

$F = N - (2L + 1) = 8 - (2\times 3 + 1) = 1$

or $F = 3(N-1) - 2P_1$

$= 3(8-1) - 2\times 10 = 1$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

(c) $N_b = 7$; $N_t = 2$; $N_o = 2$; $N = 11$

$L = 5$; $P_1 = 15$

$F = N - (2L + 1) = 11 - (2\times 5 + 1) = 0$

Therefore, the linkage is a structure.

Example 1.2 *State whether the linkages shown in Fig. 1.19 are mechanisms with one degree of freedom. If not, make suitable changes. The number of links should not be varied by more than 1.*

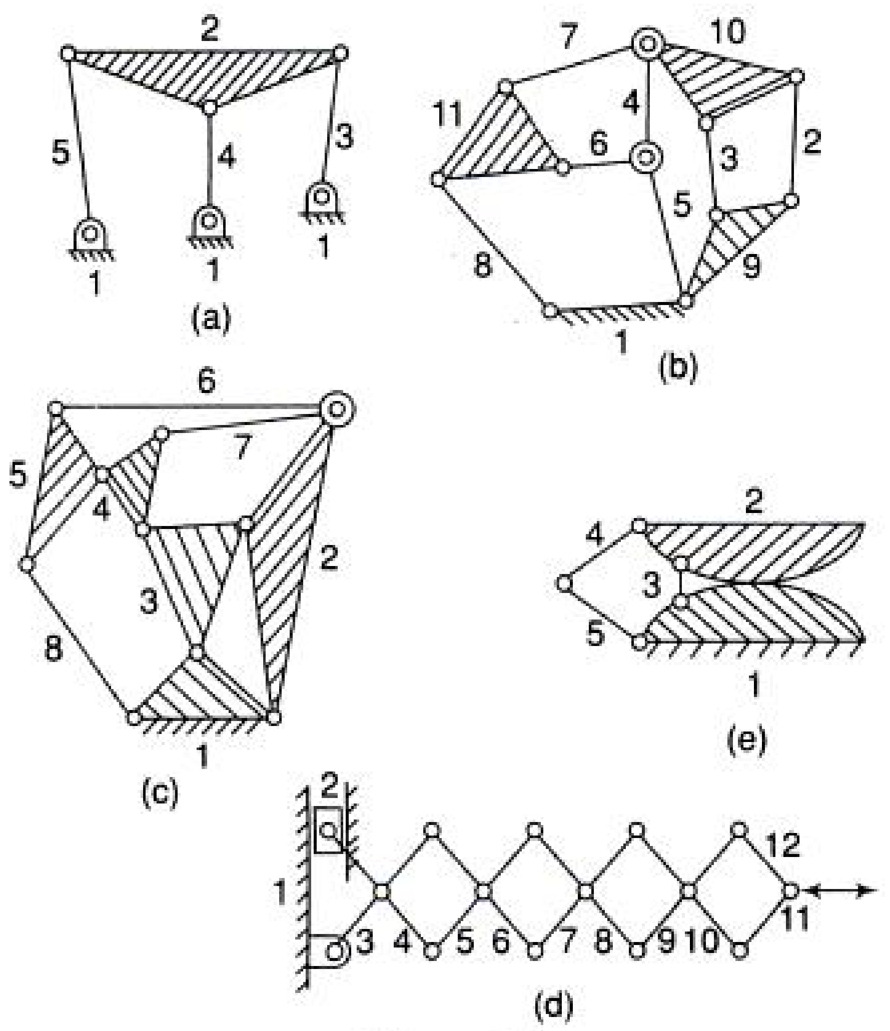


Fig. 1.19

Solution (a) The linkage has 2 loops and 5 links.

$F = N - (2L + 1) = 5 - (2 \times 2 + 1) = 0$

Thus, it is a structure. Referring Table 1.2, for a 2-loop mechanism, n should be six to have one degree of freedom. Thus, one more link should be added to the linkage to make it a mechanism of $F = 1$. One of the possible solutions has been shown in Fig. 1.20(a).

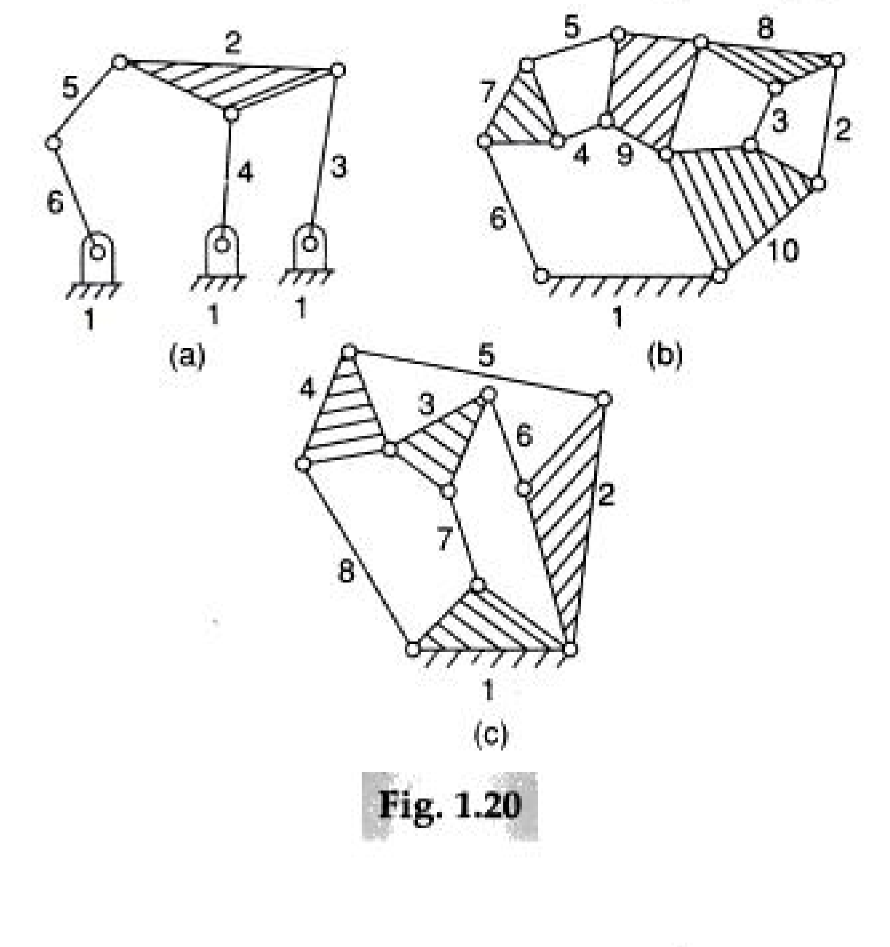


Fig. 1.20

(b) The linkage has 4 loops and 11 links. Referring Table 1.2, it has 2 degrees of freedom. With 4 loops and 1 degree of freedom, the number of links should be 10 and the number of joints 13. Three excess joints can be formed by

6 ternary links or
4 ternary links and 1 quaternary link or
2 ternary links, and 2 quaternary links, or
3 quaternary links, or
a combination of ternary and quaternary links with double joints.

Figure 1.20(b) shows one of the possible solutions.

(c) There are 4 loops and 8 links.

$F = N - (2L +) = 8 - (4 \times 2 + 1) = -1$

It is a superstructure. With 4 loops, the number of links must be 10 to obtain one degree of freedom. As the number of links is not to be increased by more than one, the number of loops has to be decreased. With 3 loops, 8 links and 10 joints, the required linkage can be designed. One of the many solutions is shown in Fig. 1.20(c).

(d) It has 5 loops and 12 links. Referring Table 1.2, it has 1 degree of freedom and thus is a mechanism.

(e) The mechanism has a cam pair, therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 5

Number of pairs with 1 degree of freedom = 5

Number of pairs with 2 degrees of freedom = 1

$F = 3(N - 1) - 2P_1 - P_2$

$= 3(5 - 1) - 2 \times 5 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.3 *Determine the degree of freedom of the mechanisms shown in Fig. 1.21.*

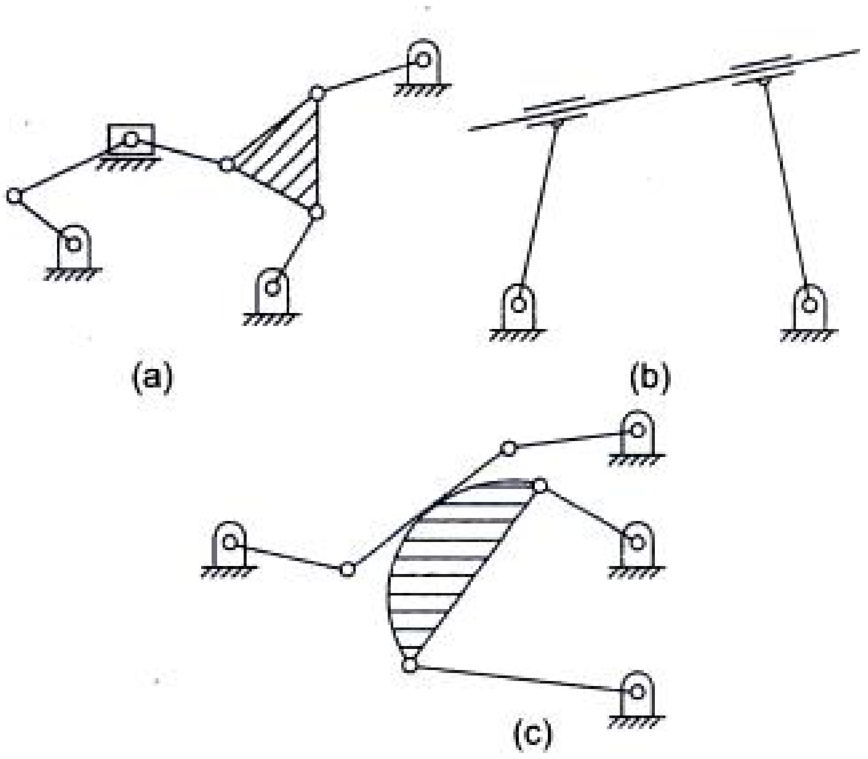


Fig. 1.21

Solution

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig. 1.22)

Number of pairs with 1 degree of freedom = 10

(At the slider, one sliding pair and two turning pairs)

$F = 3\,(N - 1) - 2P_1 - P_2$

$= 3(8 - 1) - 2 \times 10 - 0 = 1$

Thus, it is a mechanism with a single degree of freedom.

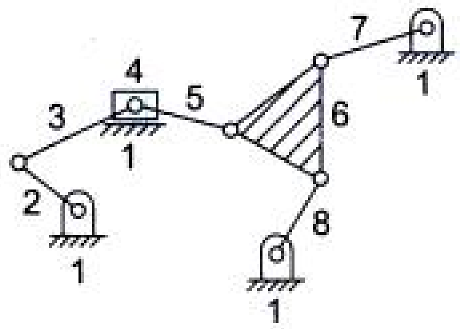


Fig. 1.22

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.

$\therefore$ effective degree of freedom

$= 3\,(N - 1) - 2P_1 - P_2 - F_r$

$= 3(4 - 1) - 2 \times 4 - 0 - 1 = 0$

As the effective degree of freedom is zero, it is a locked system.

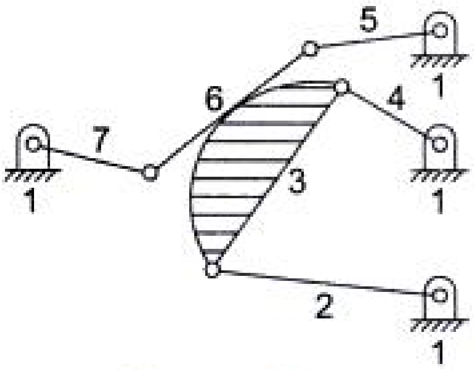


Fig. 1.23

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig. 1.23)

Number of pairs with 1 degree of freedom = 8

Number of pairs with 2 degrees of freedom = 1

$F = 3\,(N - 1) - 2P_1 - P_2$

$= 3(7 - 1) - 2 \times 8 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.4 *How many unique mechanisms can be obtained from the 8-link kinematic chain shown in Fig. 1.24?*

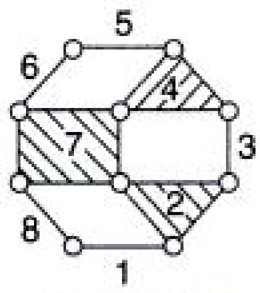


Fig. 1.24

Solution The kinematic chain has 8 links in all. A unique mechanism is obtained by fixing one of the links to the ground each time and retaining only one out of the symmetric mechanisms thus obtained.

The given kinematic chain is symmetric about links 3 or 7. Thus, identical inversions (mechanisms) are obtained if the links 2, 1, 8 or 4, 5, 6 are fixed. In addition, two more unique mechanisms can be obtained from the 8-link kinematic chain as shown in Fig. 1.25.

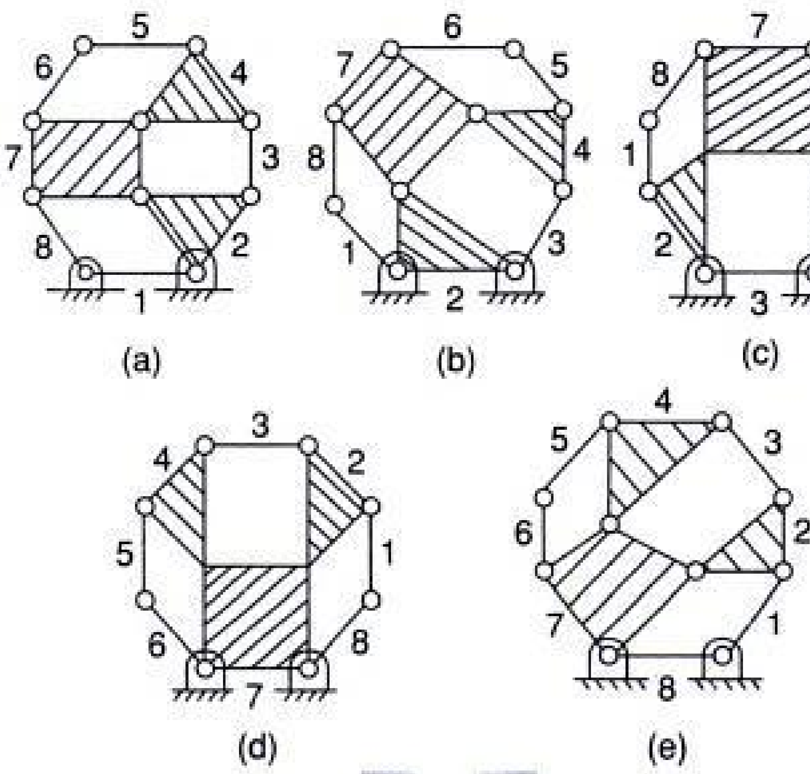


Fig. 1.25

Example 1.5 *A linkage has 11 links and 4 loops. Calculate its degree of freedom and the number of ternary and quaternary links it will have if it has only single turning pairs.*

Solution $F = N - (2L + 1) = 11 - (2 \times 4 + 1) = 2$

$P_1 = N + (L - 1) = 11 + (4 - 1) = 14$

The linkage has 3 excess joints and if all the joints are single turning pairs, the excess joints can be provided either by

- 6 ternary links or
- 4 ternary links and one quaternary link or
- 2 ternary links and two quaternary links or
- 3 quaternary links

1.12 EQUIVALENT MECHANISMS

It is possible to replace turning pairs of plane mechanisms by other types of pairs having one or two degrees of freedom, such as sliding pairs or cam pairs. This can be done according to some set rules so that the new mechanisms also have the same degrees of freedom and are kinematically similar.

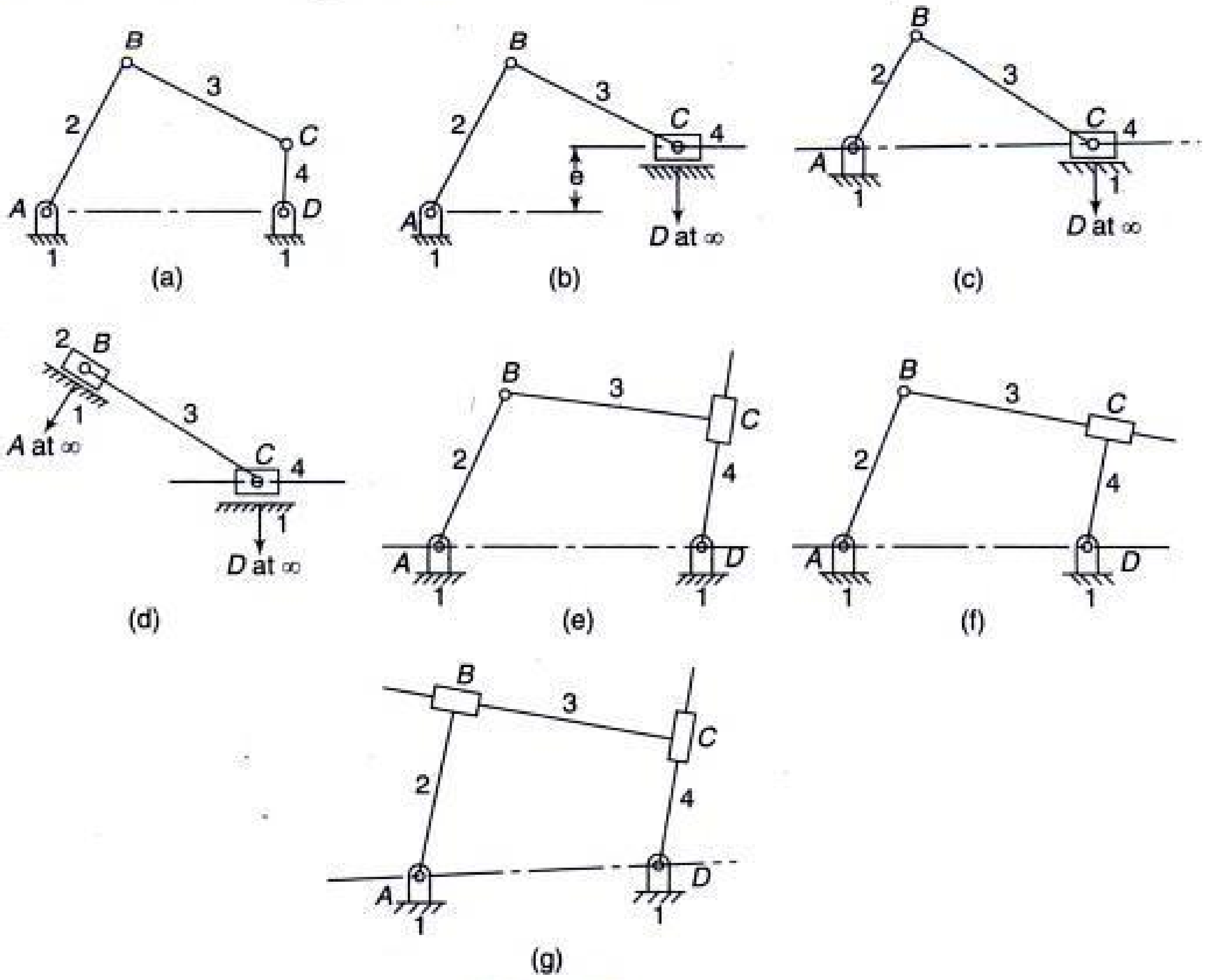


Fig. 1.26

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This shows that to have one cam pair in a mechanism with one degree of freedom, the number of links and turning pairs should be as below:

$N = 3, \quad P_1 = 2$
$N = 5, \quad P_1 = 5$
$N = 7, \quad P_1 = 8$
$N = 9, \quad P_1 = 11$ and so on.

A comparison of this with linkages having turning pairs only (Table 1.2) indicates that a cam pair can be replaced by one binary link with two turning pairs at each end.

Figure 1.28(a) shows link CD (of a four-link mechanism) with two turning pairs at its ends replaced by a cam pair. The centres of curvatures at the point of contact X of the two cams lie at D and C. Figures 1.28(b) and (c) show the link BC with turning pairs at B and C replaced by a cam pair. The centres of curvature at the point of contact X lie at B and C respectively. Figure 1.28(d) shows equivalent mechanism for a disc cam with reciprocating curved-face follower. The centres of curvature of the cam and the follower at the instant lie at A and B respectively.

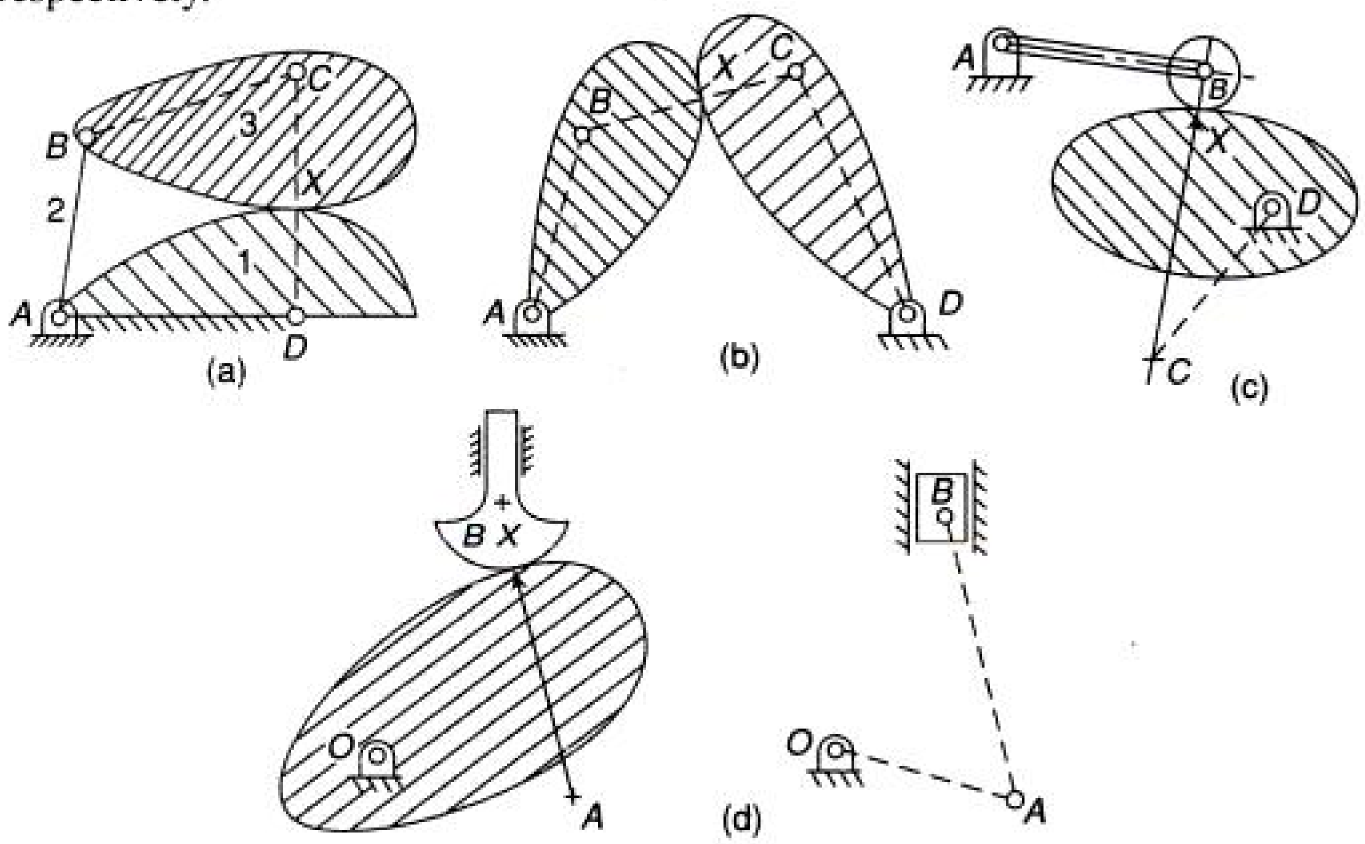


Fig. 1.28

Example 1.6 *Sketch a few slider-crank mechanisms derived from Stephenson's and Watt's six-bar chains.*

Solution Figure 1.29(a) shows a Stephenson's chain in which the ternary links are not directly connected. Thus, any of the binary links 3 or 6 can be replaced by a slider to obtain a slider-crank mechanism as shown in Fig. 1.29(b) and (c).

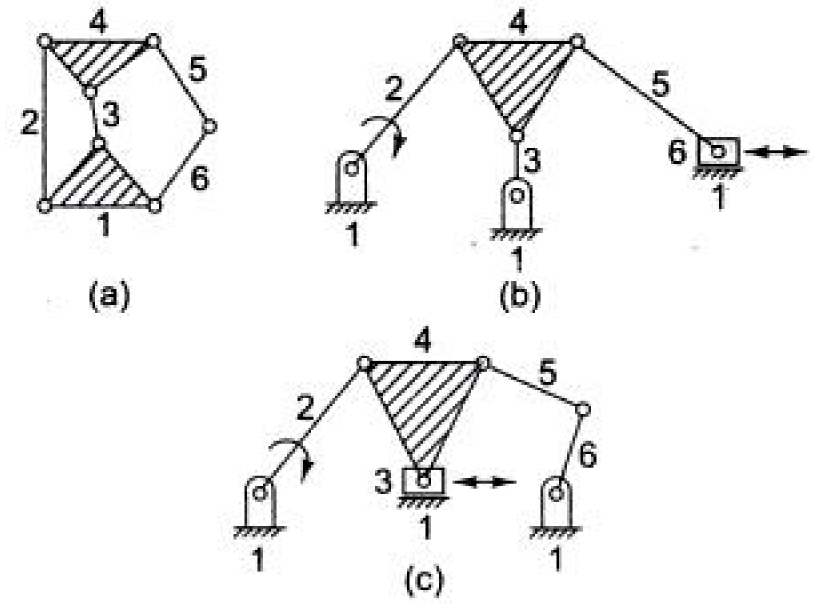


Fig. 1.29

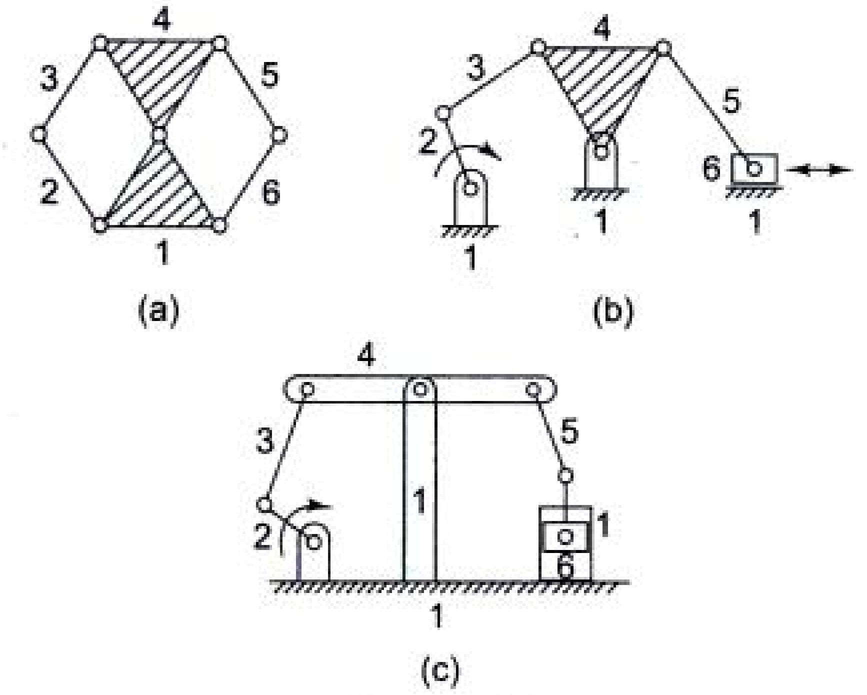


Fig. 1.30

Figure 1.30(a) shows a Watt's chain in which the ternary links are directly connected. Thus, any of the binary links 2 or 6 can be replaced by a slider to obtain a slider-crank mechanism. Figure 1.30 (b) and (c) show two variations of the slider obtained by replacing the binary link 6. The slider-crank mechanism of Fig. 1.30(c) is known as *beam engine*.

Example 1.7 *Sketch the equivalent kinematic chains with turning pairs for the chains shown in Fig. 1.31.*

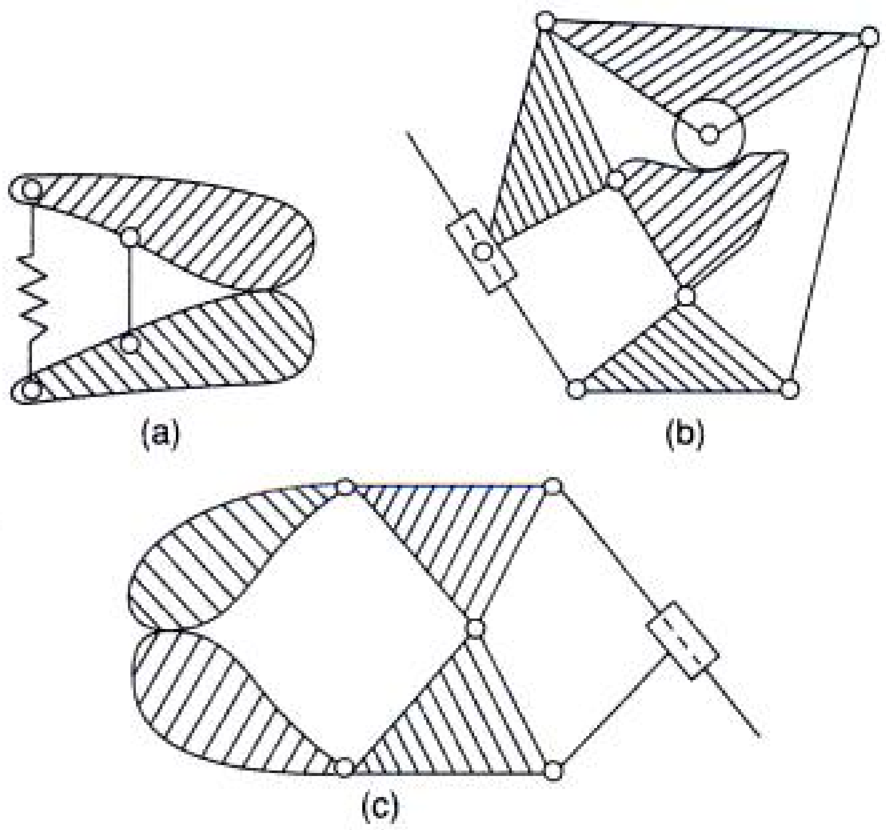


Fig. 1.31

Solution

(a) A spring is equivalent to two binary links connected by a turning pair. A cam pair is equivalent of one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig. 1.32(a).

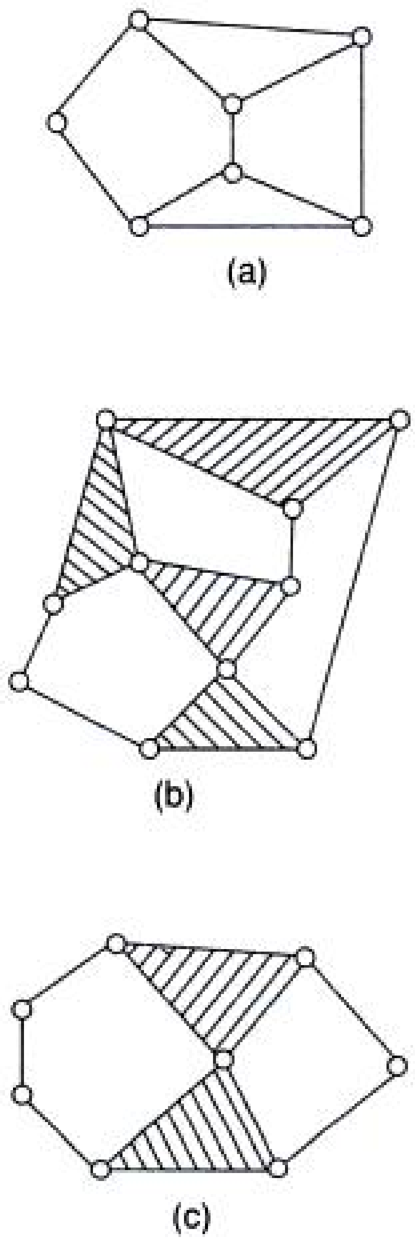


Fig. 1.32

(b) A slider pair can be replaced by one link with a turning pair at the other end. A cam pair with a roller follower can be replaced by a binary link with turning pairs at each end similar to the case of a curved-face follower of Fig. 1.28(d). the equivalent chain is shown in Fig. 1.32(b).

(c) The equivalent chain has been shown in Fig. 1.32(c).

1.13 THE FOUR-BAR CHAIN

A four-bar chain is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints. When one of the links is fixed, it is known as a *linkage* or *mechanism*. A link that makes complete revolution is called the *crank*, the link opposite to the fixed link is called the *coupler*, and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.

Note that it is impossible to have a four-bar linkage if the length of one of the links is greater than the sum of the other three. This has been shown in Fig. 1.33 in which the length of link d is more than the sum of lengths of a, b and c, and therefore, this linkage cannot exist.

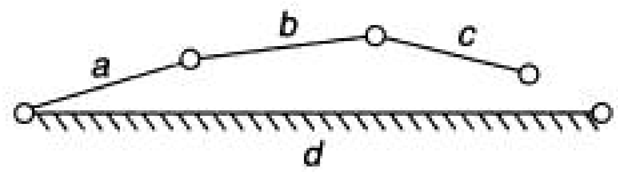


Fig. 1.33

Consider a four-link mechanism shown in Fig. 1.34(a) in which the length a of the link AB is more than d, the length of the fixed link AD. The linkage has been shown in various positions. It can be observed from these configurations that if the link a is to rotate through a full revolution, i.e., if it is to be a crank, then the following conditions must be met:

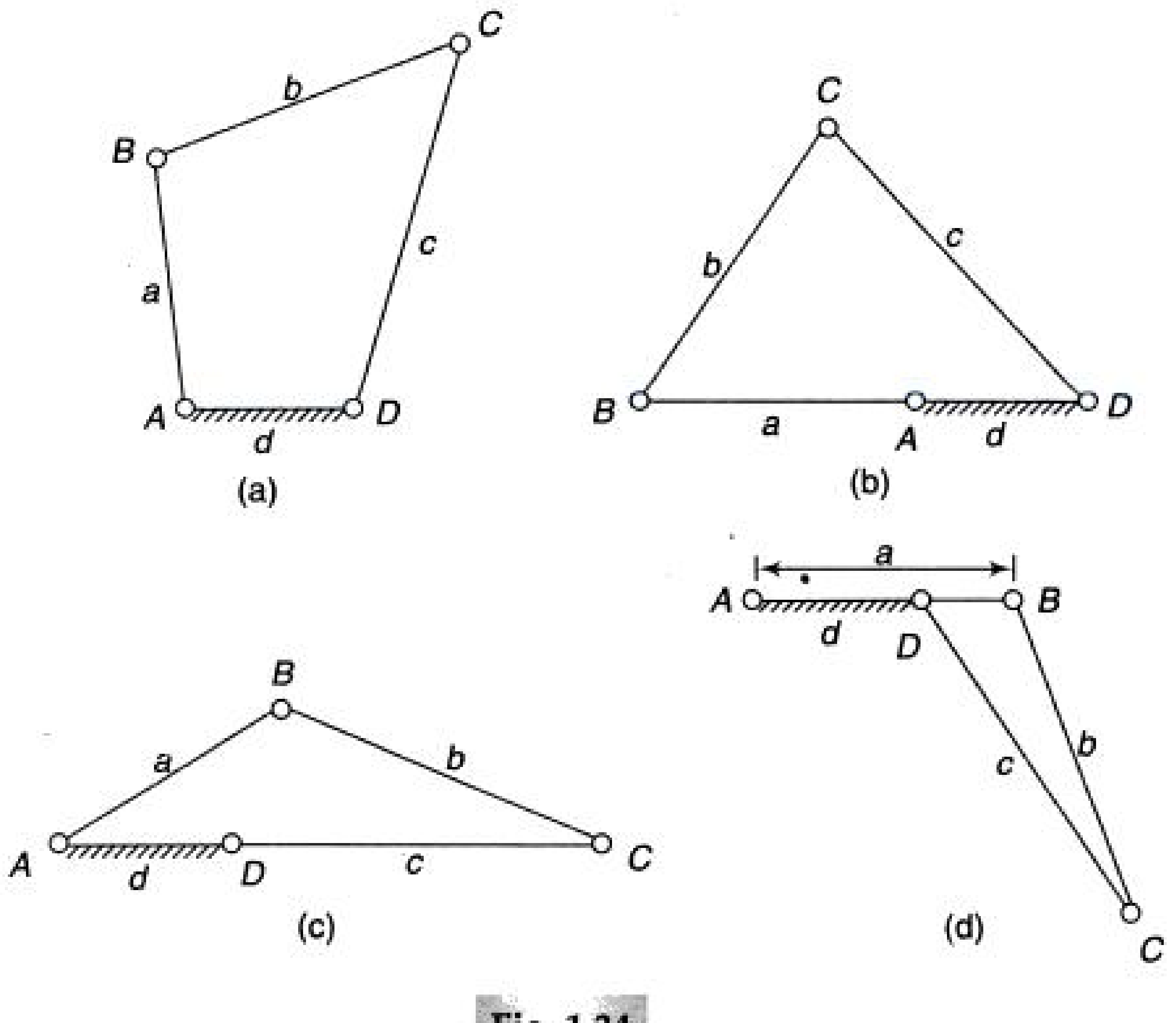


Fig. 1.34

From Fig. 1.34(b), $d + a < b + c$ (i)

From Fig. 1.34(c), $d + c < a + b$ (ii)

From Fig. 1.34(d), $b < c + (a - d)$ or $d + b < c + a$ (iii)

Adding (i) and (ii), $2d + a + c < 2b + a + c$

or $d < b$

Similarly, adding (ii) and (iii), and (iii) and (i) we get

$d < a$

and $d < c$

Thus, d is less than a, b and c, i.e., it is the shortest link if a is to rotate a full circle or act as a crank. The above inequalities also suggest that out of a, b and c, whichever is the longest, the sum of that with d, the shortest link will be less than the sum of the remaining two links. Thus, the necessary conditions for the link a to be a crank is

- the shortest link is fixed, and
- the sum of the shortest and the longest links is less than the sum of the other two links.

In a similar way, it can be shown that if the link c is to rotate through a full circle, i.e., if it is to be a crank then the conditions to be realised are the same as above. Also, it can be shown that if both the links a and c rotate through full circles, the link b also makes one complete revolution relative to the fixed link d.

The mechanism thus obtained is known as *crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter.* Figure 1.35 shows all the three links a, b and c rotating through one complete revolution.

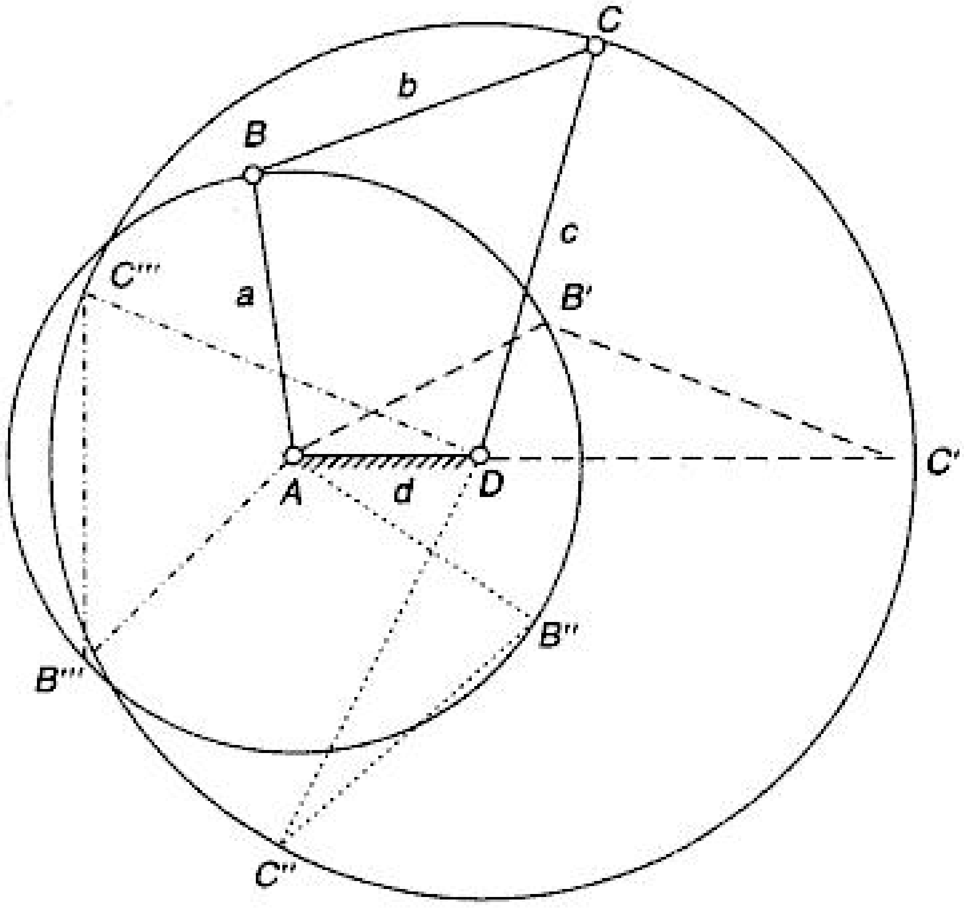


Fig. 1.35

In the above consideration, the rotation of the links is observed relative to the fixed link d. Now, consider the movement of b relative to either a or c. The complete rotation of b relative to a is possible if the angle $\angle ABC$ can be more than 180° and relative to c if the angle $\angle DCB$ more than 180°. From the positions of the links in Fig. 1.35(b) and (c), it is clear that these angles cannot become more than 180° for the above stated conditions.

Now, as the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as *inversions*) obtained by fixing different links of this kind of chain will be as follows:

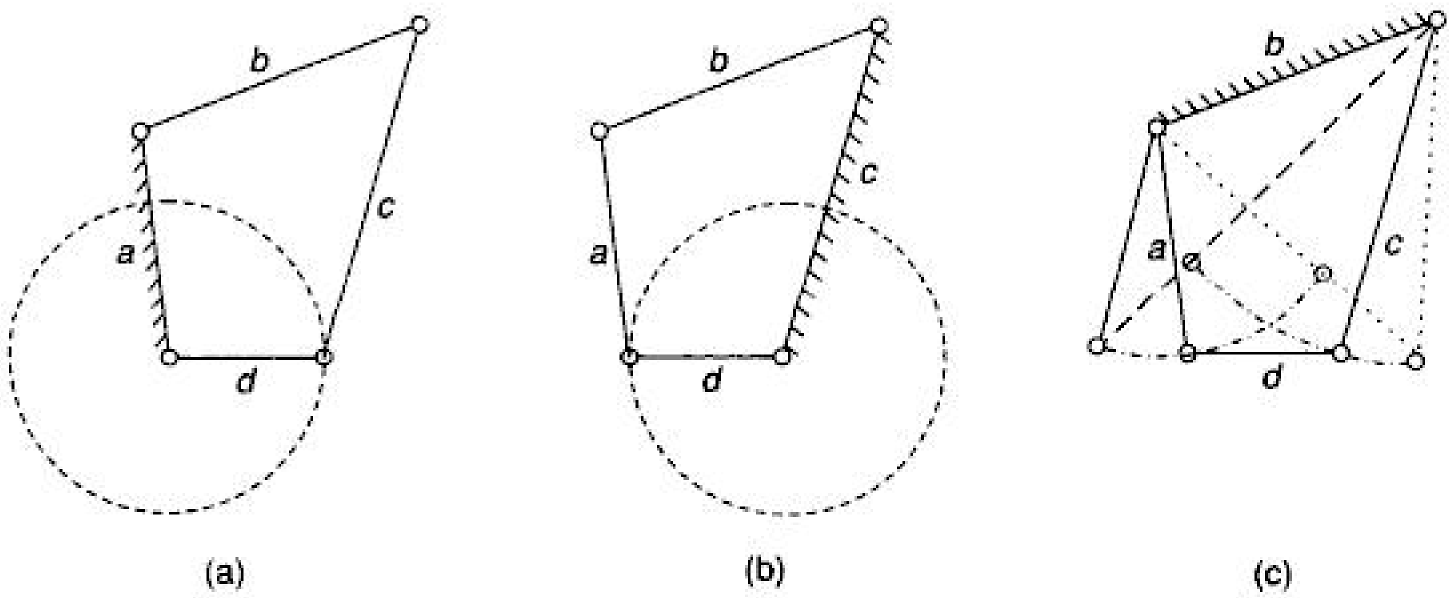


Fig. 1.36

1. If any of the adjacent links of link *d*, i.e., *a* or *c* is fixed, *d* can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig. 1.36(a), *a* is fixed, *d* is the crank and *b* oscillates whereas in Fig. 1.36(b), *c* is fixed, *d* is the crank and *b* oscillates. The mechanism is known as *crank-rocker* or *crank-lever mechanism* or *rotary-oscillating converter.*
2. If the link opposite to the shortest link, i.e., link *b* is fixed and the shortest link *d* is made a coupler, the other two links *a* and *c* would oscillate [Fig. 1.36(c)]. The mechanism is known as a *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscillating-oscillating converter*.

A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a *class-I*, four-bar linkage.

When the sum of the lengths of the largest and the shorted links is more than the sum of the lengths of the other two links, the linkage is known as a *class-II*, four-bar linkage. In such a linkage, fixing of any of the links always results in a *rocker-rocker* mechanism. In other words, the mechanism and its inversions give the same type of motion (of a *double-rocker* mechanism).

The above observations are summarised in *Grashof's law* which states *that a four-bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of the other two links.*

Further, if the *shortest link is fixed,* the chain will act as a double-crank mechanism in which links adjacent to the fixed link will have complete revolutions. If the *link opposite to the shortest link is fixed*, the chain will act as double-rocker mechanism in which links adjacent to the fixed link will oscillate. If the *link adjacent to the shortest link is fixed*, the chain will act as crank-rocker mechanism in which the shortest link will revolve and the link adjacent to the fixed link will oscillate.

If the sum of the lengths of the largest and the shorted links is equal to the sum of the lengths of the other two links, i.e., when equalities exist, the four inversions, in general, result in mechanisms similar to those as given by Grashof's law, except that sometimes the links may become collinear and may have to be guided in the proper direction. Usually, the purpose is served by the inertia of the links. A few special cases may arise when equalities exist. For example, *parallel-crank four-bar linkage* and *deltoid* linkage.

Parallel-Crank Four-Bar Linkage If in a four-bar linkage, two opposite links are parallel and equal in length, then any of the links can be made fixed. The two links adjacent to the fixed link will always act as two cranks. The four links form a parallelogram in all the positions of the cranks, provided the cranks rotate in the same sense as shown in Fig. 1.37.

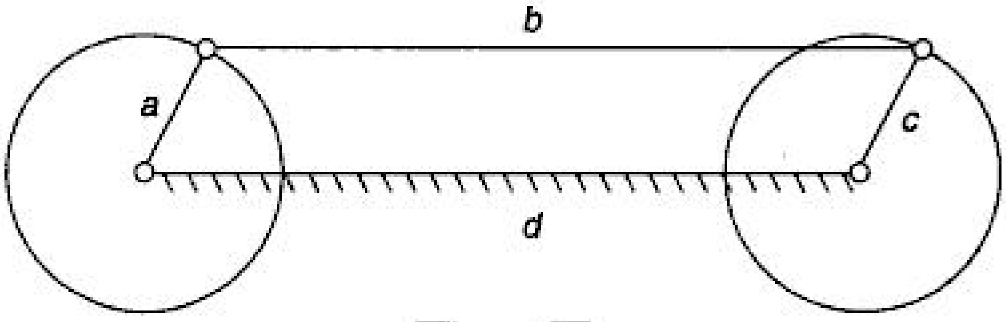


Fig. 1.37

The use of such a mechanism is made in the coupled wheels of a locomotive in which the rotary motion of one wheel is transmitted to the other wheel. For kinematic analysis, link *d* is treated as fixed and the relative motions of the other links are found. However, in fact, *d* has a translatory motion parallel to the rails.

Deltoid Linkage In a deltoid linkage (Fig. 1.38), the equal links are adjacent to each other. When any of the shorter links is fixed, a double-crank mechanism is obtained in which one revolution of the longer link causes two revolutions of the other shorter link. As shown in Fig. 1.38 (a), when the link *c* rotates through half a revolution and assumes the position *DC′*, the link *a* has completed a full revolution.

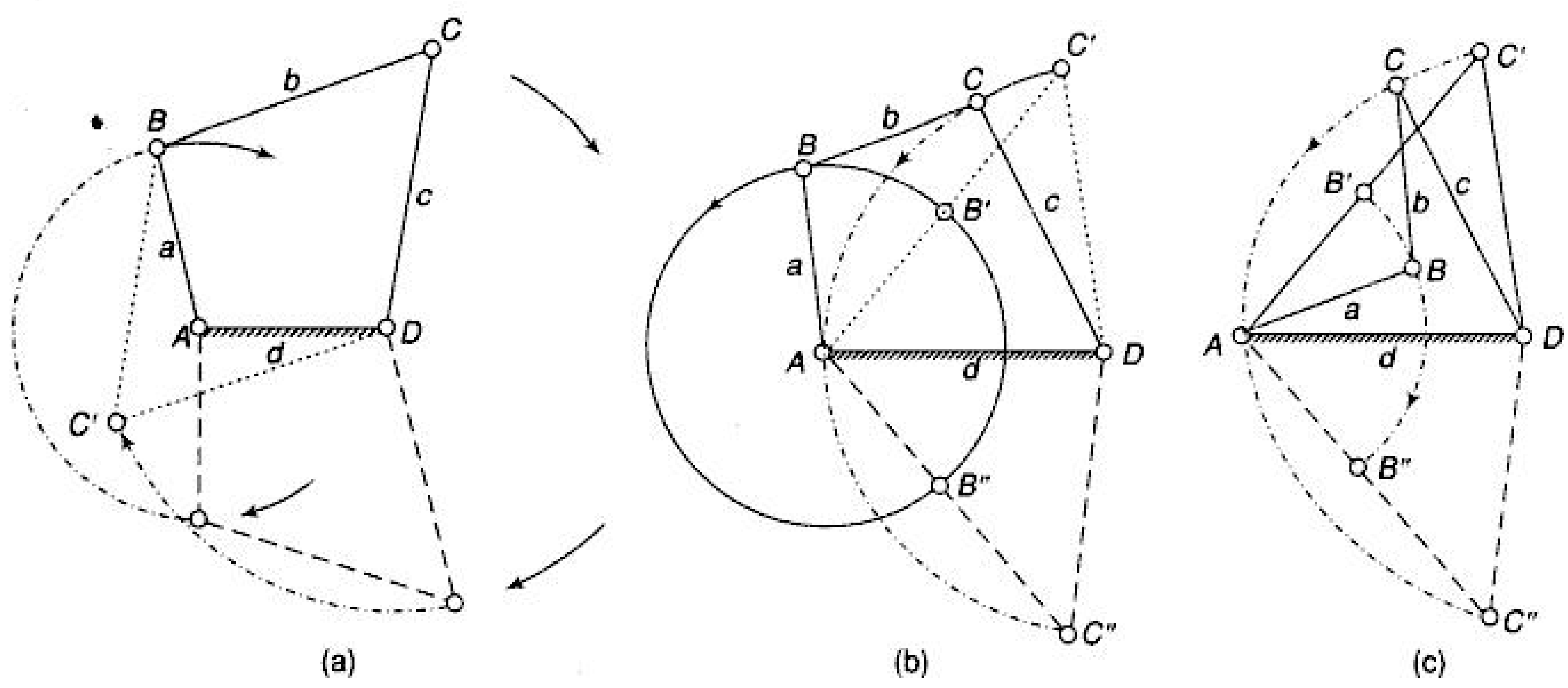


Fig. 1.38

When any of the longer links is fixed, two crank-rocker mechanisms are obtained [Fig. 1.38(b) and (c)]

Example 1.8 *Find all the inversion of the chain given in Fig. 1.39.*

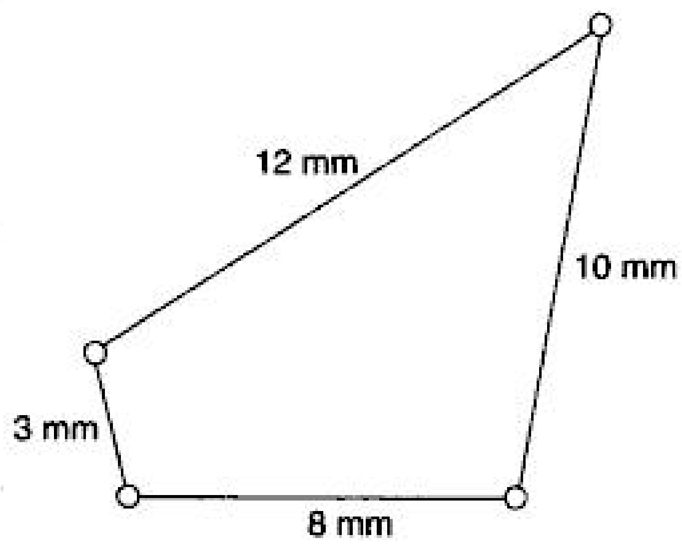


Fig. 1.39

Solution

(a) Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since $12 + 3 < 10 + 8$, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

Example 1.9 *Figure 1.40 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism whether crank-rocker or double-crank or double-rocker.*

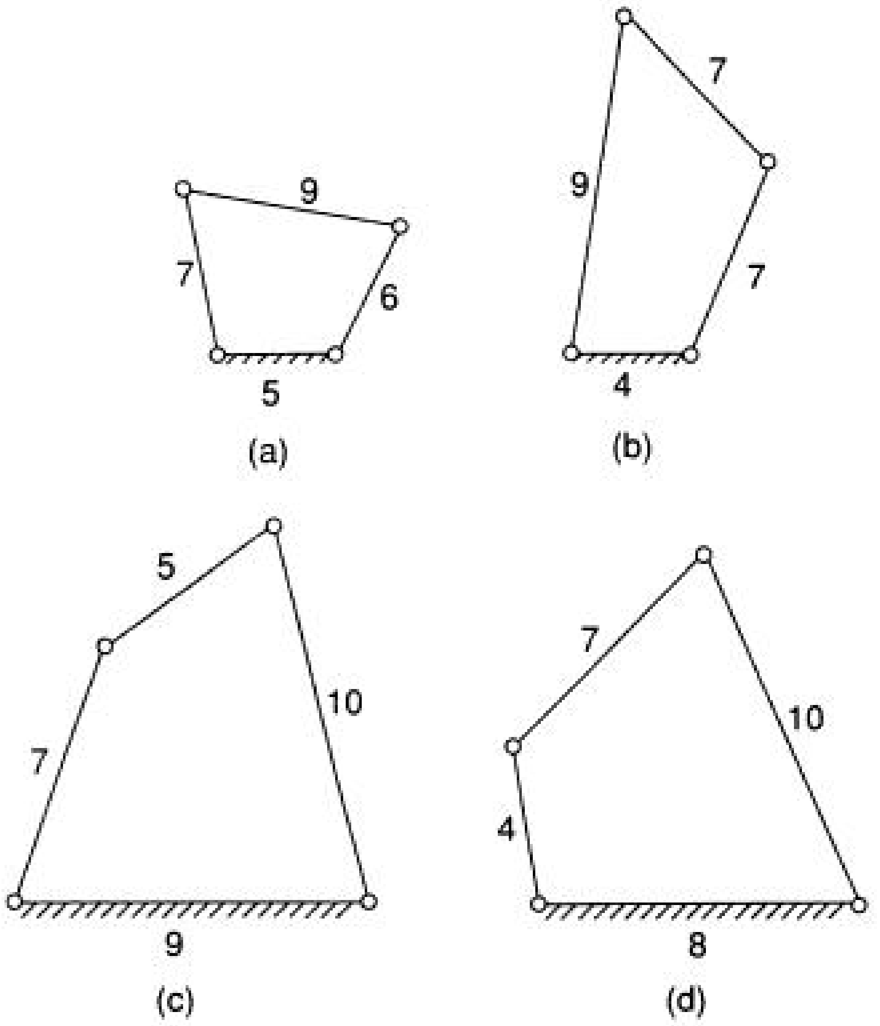


Fig. 1.40

Solution

(a) Length of the longest link = 9
Length of the shortest link = 5
Length of other links = 7 and 6
Since 9 + 5 > 7 + 6, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.

(b) Length of the longest link = 9
Length of the shortest link = 4
Length of other links = 7 and 7
Since 9 + 4 < 7 + 7, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

(c) Length of the longest link = 10
Length of the shortest link = 5
Length of other links = 9 and 7
Since 10 + 5 < 9 + 7, it belongs to the class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

(d) Length of the longest link = 10
Length of the shortest link = 4
Length of other links = 8 and 7
Since 10 + 4 < 8 + 7, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Example 1.10 *Figure 1.41 shows a plane mechanism in which the figures indicate the dimensions in standard units of length. The slider C is the driver. Will the link AG revolve or oscillate?*

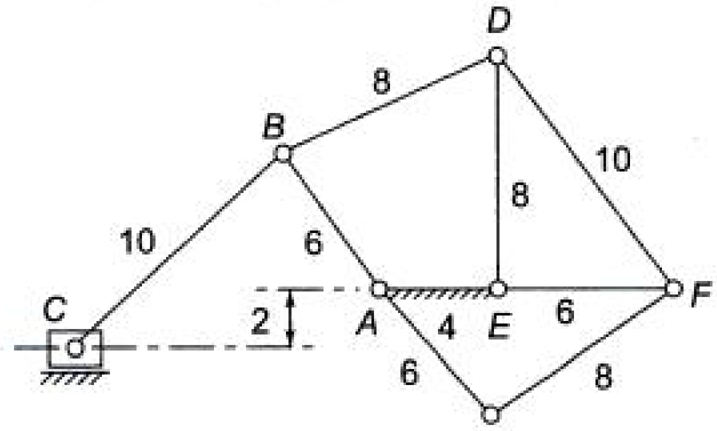


Fig. 1.41

Solution The mechanism has three sub-chains:

(i) *ABC*, a slider-crank chain
(ii) *ABDE*, a four-bar chain
(iii) *AEFG*, a four-bar chain
DEF is a locked chain as it has only three links.

- As the length *BC* is more than the length *AB* plus the offset of 2 units, *AB* acts as a crank and can revolve about *A*.
- In the chain *ABDE*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 8 and 6

Since 8 + 4 < 8 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus *AB* and *ED* can revolve fully.

- In the chain *AEFG*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 6 and 6

Since 8 + 4 = 6 + 6, it belongs to the class-I mechanism. As the shortest link is fixed, it is a double-crank mechanism and thus *EF* and *AG* can revolve fully.

As *DEF* is a locked chain with three links, the link *EF* revolves with the revolving of *ED*. With the revolving of *ED*, *AG* also revolves.

1.14 MECHANICAL ADVANTAGE

The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig. 1.42, if friction and inertia forces are ignored and the input torque T_2 is applied to the link 2 to drive the output link 4 with a resisting torque T_4 then

Power input = Power output

$$T_2\,\omega_2 = T_4\,\omega_4$$

or $$\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

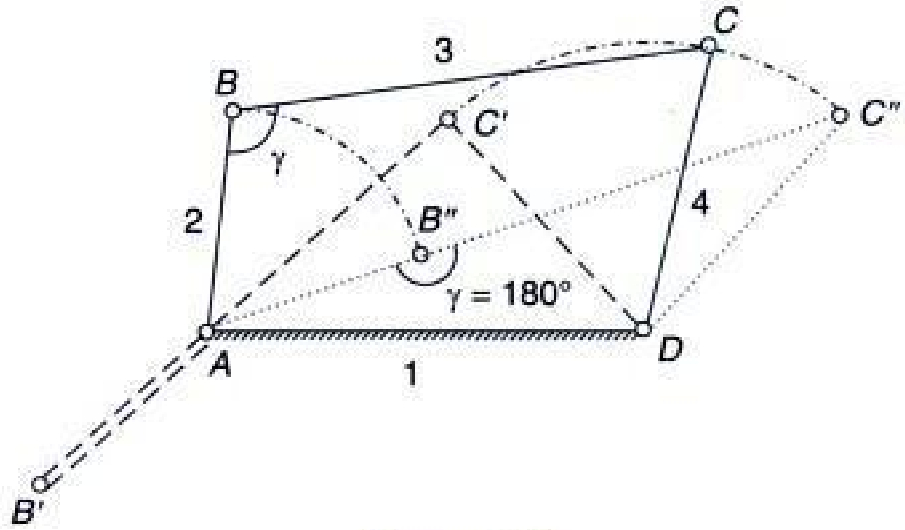


Fig. 1.42

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity ω_4 of the output link *DC* (rocker) becomes zero at the extreme positions (*AB′C′D* and *AB″C″D*), i.e., when the input link *AB* is in line with the coupler *BC* and the angle γ between them is either zero or 180°, it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as *toggle* positions.

1.15 TRANSMISSION ANGLE

The angle μ between the output link and the coupler is known as *transmission angle*. In Fig. 1.43, if the link *AB* is the input link, the force applied to the output link *DC* is transmitted through the coupler *BC*. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point *D*) is maximum when the transmission angle μ is 90°. If links *BC* and *DC* become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

Applying cosine law to triangles *ABD* and *BCD* (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \qquad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \qquad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

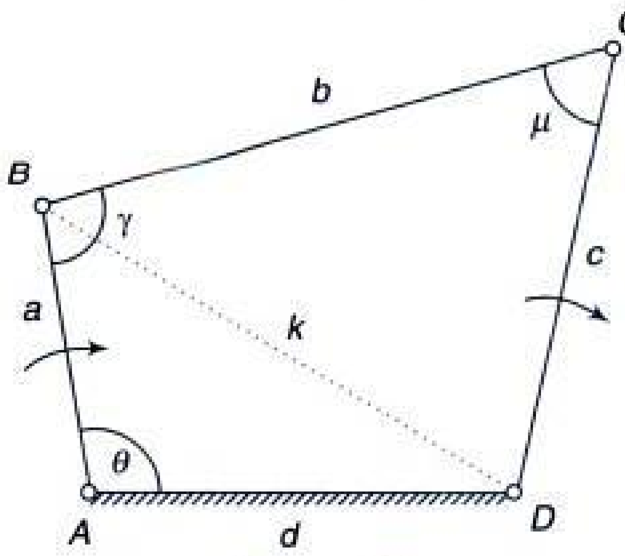


Fig. 1.43

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad\sin\theta - 2bc\sin\mu.\frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad \sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

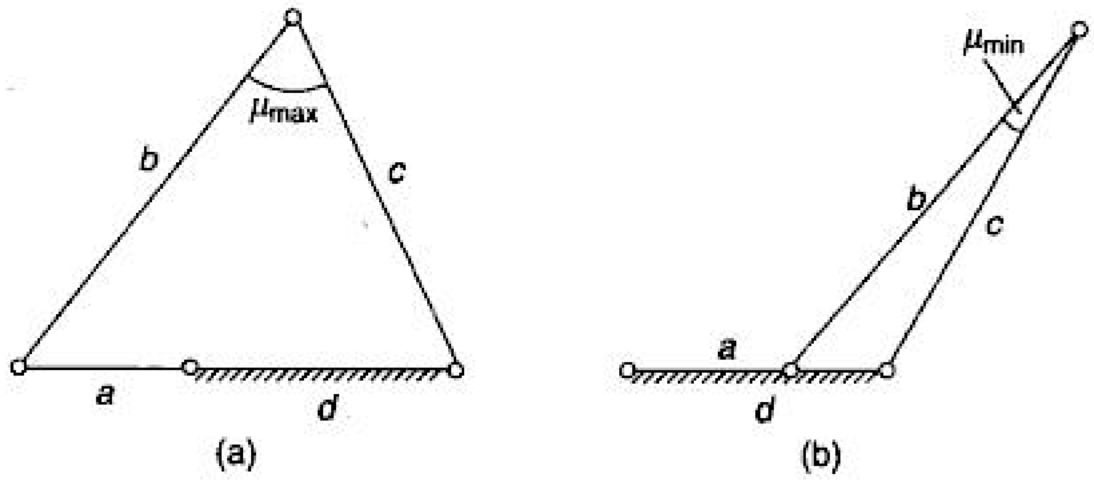


Fig. 1.44

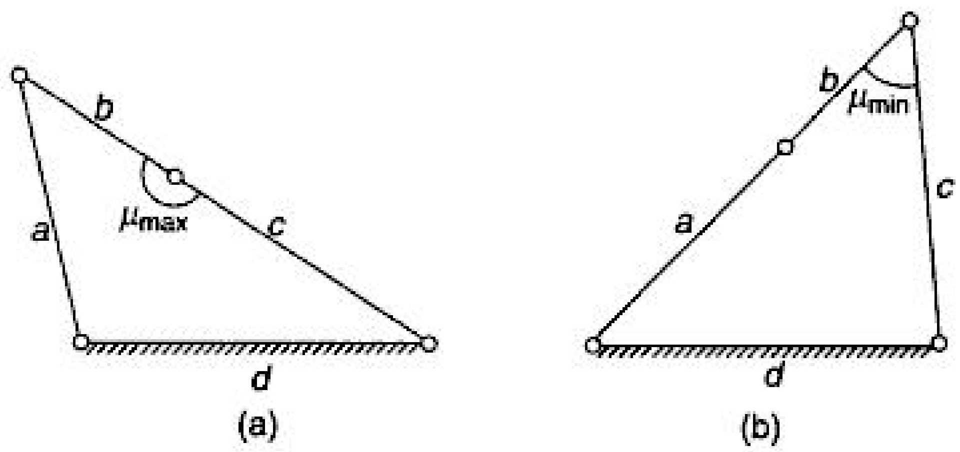


Fig. 1.45

Example 1.11 *Find the maximum and minimum transmission angles for the mechanisms shown in Fig. 1.46. The figures indicate the dimensions in standard units of length.*

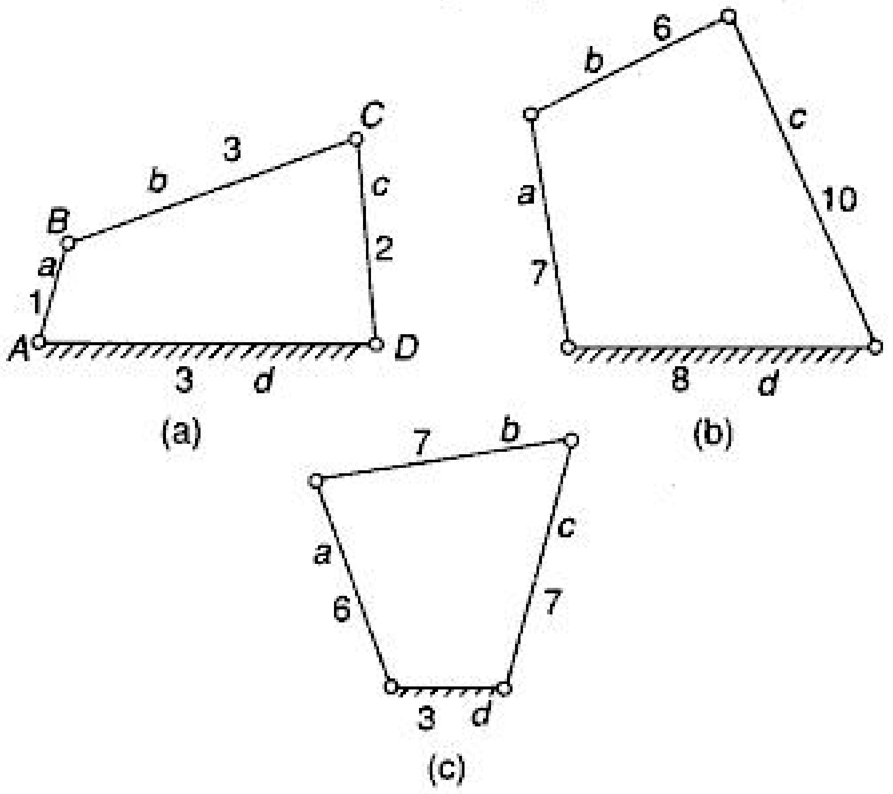


Fig. 1.46

Solution

(a) In this mechanism,

Length of the longest link = 3

Length of the shortest link = 1

Length of other links = 3 and 2

Since 3 + 1 < 3 + 2, it belongs to the class I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

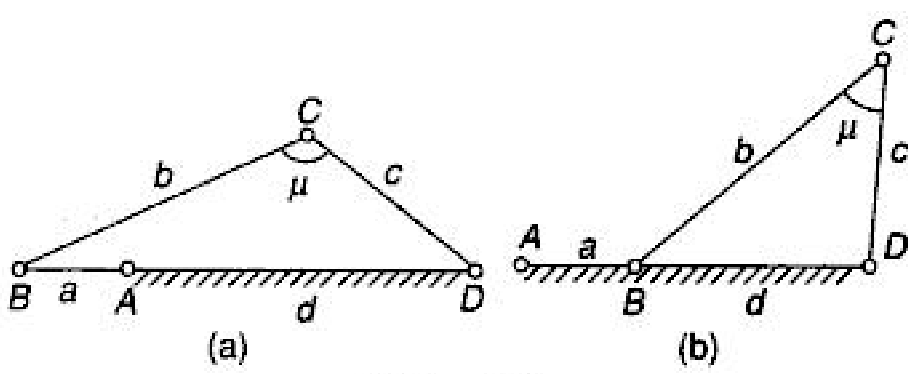


Fig. 1.47

Maximum transmission angle is when θ is 180° [Fig. 1.47(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$(1 + 3)^2 = 3^2 + 2^2 - 2 \times 3 \times 2\cos\mu$

$16 = 9 + 4 - 12\cos\mu$

$$\cos \mu = -\frac{3}{12} = -0.25$$

$$\mu = 104.5^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.47(b)],

Thus $(d - a)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(3 - 1)^2 = 3^2 + 2^2 - 2 \times 3 \times 2 \cos \mu$$

$$4 = 9 + 4 - 12 \cos \mu$$

$$\cos \mu = \frac{3}{4} = 0.75$$

$$\mu = 41.4^\circ$$

(b) In this mechanism,

Length of the longest link = 10

Length of the shortest link = 6

Length of other links = 8 and 7

Since 10 + 6 > 8 + 7, it belongs to the class-II mechanism and thus is a double-rocker mechanism.

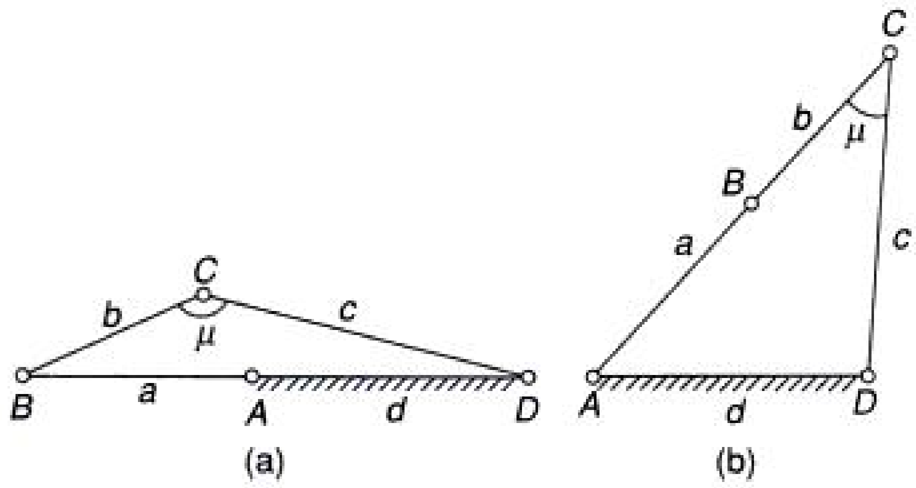


Fig. 1.48

Maximum transmission angle is when θ is 180° [Fig. 1.48(a)],

Thus, $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(7 + 8)^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos \mu$$

$$225 = 36 + 100 - 120 \cos \mu$$

$$\cos \mu = -\frac{89}{120} = -0.742$$

$$\mu = 137.9^\circ$$

Minimum transmission angle is when the angle at B is 180° [Fig. 1.48(b)],

Thus, $d^2 = (a + b)^2 + c^2 - 2(a + b)c \cos \mu$

$$8^2 = (7 + 6)^2 + 10^2 - 2(7 + 6) \times 10 \times \cos \mu$$

$$64 = 169 + 100 - 260 \cos \mu$$

$$\cos \mu = \frac{205}{260} = 0.788$$

$$\mu = 38^\circ$$

(c) In this mechanism,

Length of the longest link = 7

Length of the shortest link = 3

Length of other links = 6 and 6

Since 7 + 3 < 6 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank or drag-link mechanism.

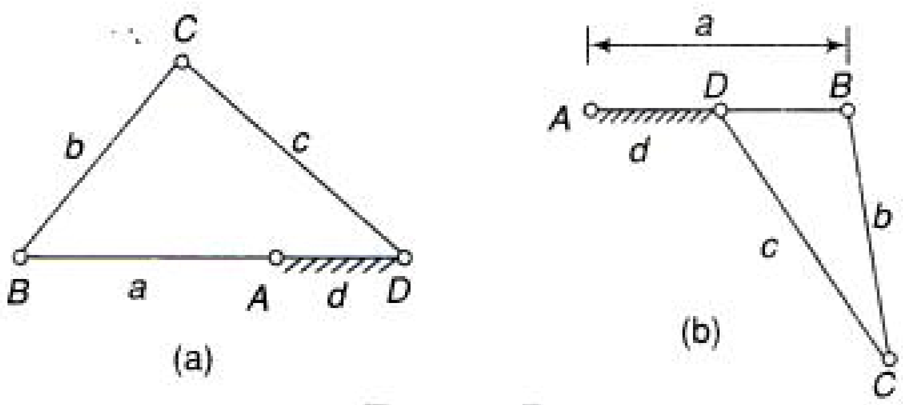


Fig. 1.49

Maximum transmission angle is when θ is 180° [Fig. 1.49(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(6 + 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu$$

$$81 = 36 + 49 - 84 \cos \mu$$

$$\cos \mu = \frac{4}{84} = 0.476$$

$$\mu = 87.27^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.49(b)],

Thus $(a - d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(6 - 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu$$

$$9 = 36 + 49 - 84 \cos \mu$$

$$\cos \mu = \frac{76}{84} = 0.9048$$

$$\mu = 25.2^\circ$$

Example 1.12 *A crank-rocker mechanism has a 70-mm fixed link, a 20-mm crank, a 50-mm coupler, and a 70-mm rocker. Draw the mechanism and determine the maximum and minimum values of the transmission angle. Locate the two toggle positions and find the corresponding crank angles and the transmission angles.*

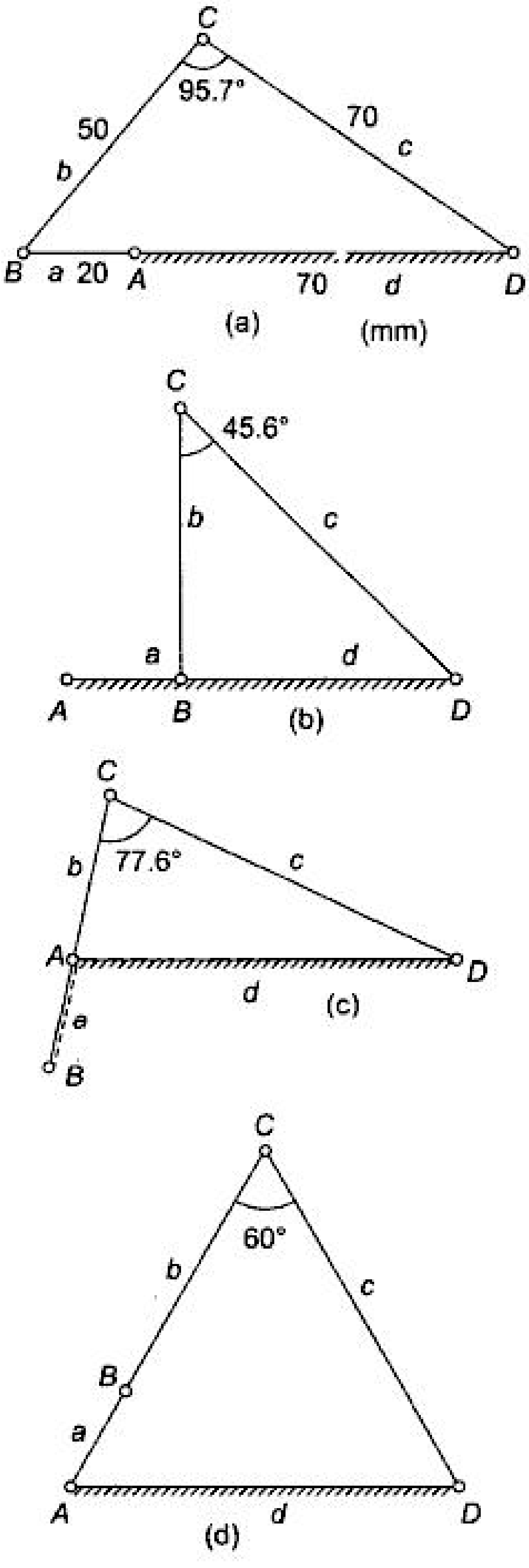


Fig. 1.50

Solution In this mechanism,

Length of the longest link = 70 mm
Length of the shortest link = 20 mm
Length of other links = 70 and 50 mm

Since 70 + 20 < 70 + 50, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Fig. 1.50(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$$
$$8100 = 2500 + 4900 - 7000 \cos \mu$$
$$\cos \mu = -0.1$$
$$\mu = 95.7^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.50(b)],

Thus $(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$

$$2500 = 2500 + 4900 - 7000 \cos \mu$$
$$\cos \mu = 0.7$$
$$\mu = 45.6^\circ$$

The two toggle positions are shown in Figs 1.50(c) and (d).

Transmission angle for first position,

$$d^2 = (b - a)^2 + c^2 - 2(b - a)c \cos \mu$$
$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70 \cos \mu$$
$$4900 = 900 + 4900 - 4200 \cos \mu$$
$$\cos \mu = 0.214$$
$$\mu = 77.6^\circ$$

As c and d are of equal length [Fig. 1.50(c)], it is an isosceles triangle and thus input angle $\theta = (77.6^\circ + 180^\circ) = 257.6^\circ$

Transmission angle for second position Fig. 1.50(d),

$$d^2 = (b + a)^2 + c^2 - 2(b + a)c \cos \mu$$
$$70^2 = 70^2 + 70^2 - 2 \times 70 \times 70 \cos \mu$$
$$4900 = 4900 + 4900 - 9800 \cos \mu$$
$$\cos \mu = 0.5$$
$$\mu = 60^\circ$$

(or as all the sides of the triangle of Fig. 1.50(d) are of equal length, it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60^\circ$

- The above results can also be obtained graphically by drawing the figures to scale and measuring the angles.

1.16 THE SLIDER-CRANK CHAIN

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a *single slider-crank chain* or simply a *slider-crank chain*. It is also possible to replace two sliding pairs of a four-bar chain to get a *double slider-crank chain* (Sec. 1.17). Further, in a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced. The distance e between the fixed pivot O and the straight line path of the slider is called the *offset* and the chain so formed an *offset slider-crank chain*.

Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*. A slider-crank chain has the following inversions:

First Inversion

This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively [Fig. 1.51(a)].

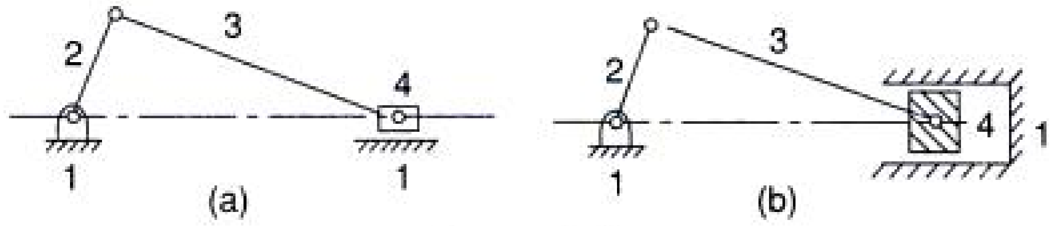


Fig. 1.51

Applications

1. Reciprocating engine
2. Reciprocating compressor

As shown in Fig. 1.51(b), if it is a reciprocating engine, 4 (piston) is the driver and if it is a compressor, 2 (crank) is the driver.

Second Inversion

Fixing of the link 2 of a slider-crank chain results in the second inversion.

The slider-crank mechanism of Fig. 1.51(a) can also be drawn as shown in Fig. 11.52(a). Further, when its link 2 is fixed instead of the link 1, the link 3 along with the slider at its end B becomes a crank. This makes the link 1 to rotate about O along with the slider which also reciprocates on it [Fig. 1.52(b)].

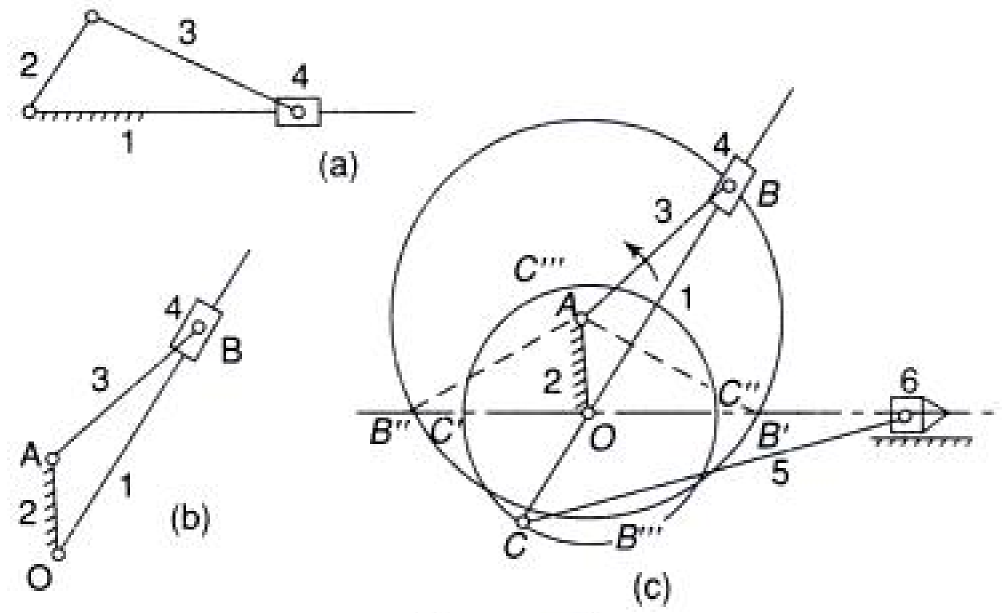


Fig. 1.52

Applications

1. Whitworth quick-return mechanism
2. Rotary engine

Whitworth Quick-Return Mechanism It is a mechanism used in workshops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

Slider 4 rotates in a circle about A and slides on the link 1 [Fig. 1.52(c)]. C is a point on the link 1 extended backwards where the link 5 is pivoted. The other end of the link 5 is pivoted to the tool, the forward stroke of which cuts the metal. The axis of motion of the slider 6 (tool) passes through O and is perpendicular to OA, the fixed link. The crank 3 rotates in the counter-clockwise direction.

Initially, let the slider 4 be at B' so that C be at C'. Cutting tool 6 will be in the extreme left position. With the movement of the crank, the slider traverses the path $B'BB''$ whereas the point C moves through $C'CC''$. Cutting tool 6 will have the forward stroke. Finally, the slider B assumes the position B'' and the cutting tool 6 is in the extreme right position. The time taken for the forward stroke of the slider 6 is proportional to the obtuse angle $B''AB'$ at A.

Similarly, the slider 4 completes the rest of the circle through the path $B''B'''B'$ and C passes through $C''C'''C'$. There is backward stroke of the tool 6. The time taken in this is proportional to the acute angle $B''AB'$ at A.

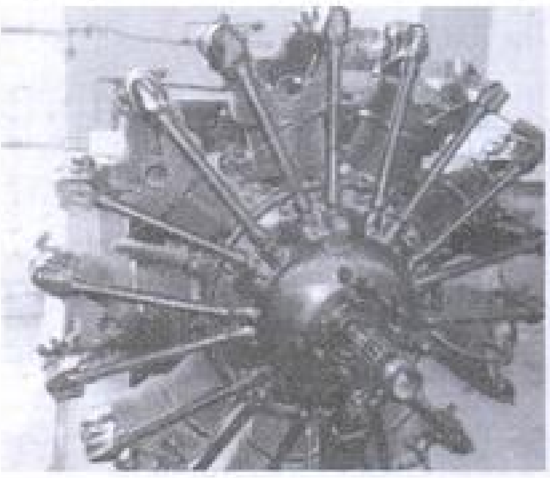
Let

θ = obtuse angle $B'AB''$ at A

β = acute angle $B'AB''$ at A

Then,

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$



A nine-cylinder rotary engine

Rotary Engine Referring Fig. 1.52(b), it can be observed that with the rotation of the link 3, the link 1 rotates about O and the slider 4 reciprocates on it. This also implies that if the slider is made to reciprocate on the link 1, the crank 3 will rotate about A and the link 1 about O.

In a rotary engine, the slider is replaced by a piston and the link 1 by a cylinder pivoted at O. Moreover, instead of one cylinder, seven or nine cylinders symmetrically placed at regular intervals in the same plane or in parallel planes, are used. All the cylinders rotate about the same fixed centre and form a balanced system. The fixed link 2 is also common to all cylinders (Fig. 1.53).

Thus, in a rotary engine, the crank 2 is fixed and the body 1 rotates whereas in a reciprocating engine (1st inversion), the body 1 is fixed and the crank 2 rotates.

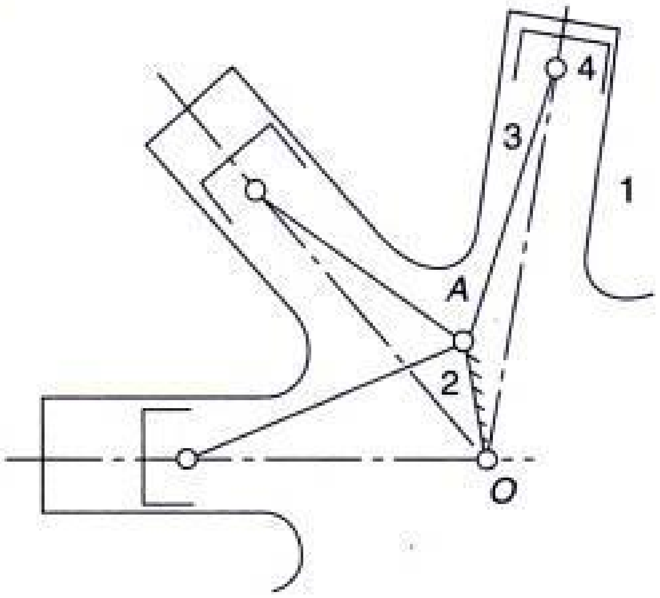


Fig. 1.53

Third Inversion

By fixing the link 3 of the slider-crank mechanism, the third inversion is obtained [Fig. 1.54(a)]. Now the link 2 again acts as a crank and the link 4 oscillates.

Applications

1. Oscillating cylinder engine
2. Crank and slotted-lever mechanism

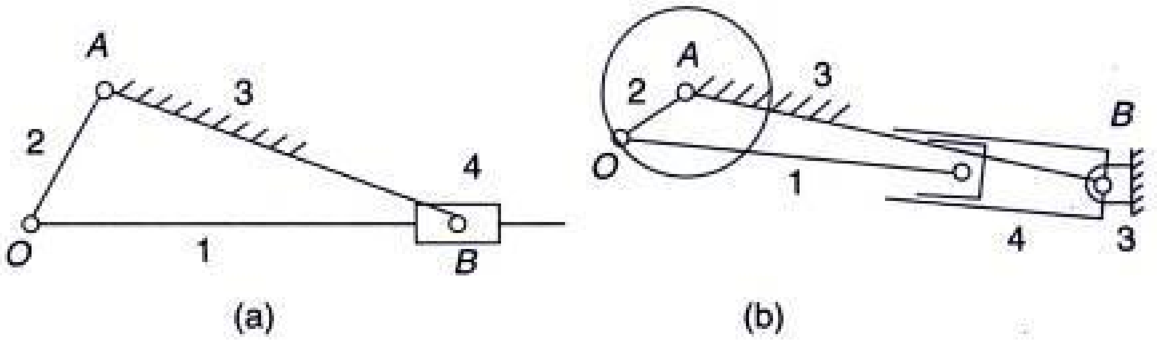


Fig. 1.54

Oscillating Cylinder Engine As shown in Fig. 1.54(b), the link 4 is made in the form of a cylinder and a piston is fixed to the end of the link 1. The piston reciprocates inside the cylinder pivoted to the fixed link 3. The arrangement is known as oscillating cylinder engine, in which as the piston reciprocates in the oscillating cylinder, the crank rotates.

Crank and Slotted-Lever Mechanism If the cylinder of an oscillating cylinder engine is made in the form of a guide and the piston in the form of a slider, the arrangement as shown in Fig. 1.55(a) is obtained. As the crank rotates about A, the guide 4 oscillates about B. At a point C on the guide, the link 5 is pivoted, the other end of which is connected to the cutting tool through a pivoted joint.

Figure 1.55(b) shows the extreme positions of the oscillating guide 4. The time of the forward stroke is proportional to the angle θ whereas for the return stroke, it is proportional to angle β, provided the crank rotates clockwise.

Comparing a crank and slotted-lever quick-return mechanism with a Whitworth quick-return mechanism, the following observations are made:

1. Crank 3 of the Whitworth mechanism is longer than its fixed link 2 whereas the crank 2 of the slotted-lever mechanism is shorter than its fixed link 3.
2. Coupler link 1 of the Whitworth mechanism makes complete rotations about its pivoted joint O with the fixed link. However, the coupler link 4 of the slotted-lever mechanism oscillates about its pivot B.
3. The coupler link holding the tool can be pivoted to the main coupler link at any convenient point C in both cases. However, for the same displacement of the tool, it is more convenient if the point C is taken on the extension of the main coupler link (towards the pivot with the fixed link) in case of the Whitworth mechanism and beyond the extreme position of the slider in the slotted-lever mechanism.

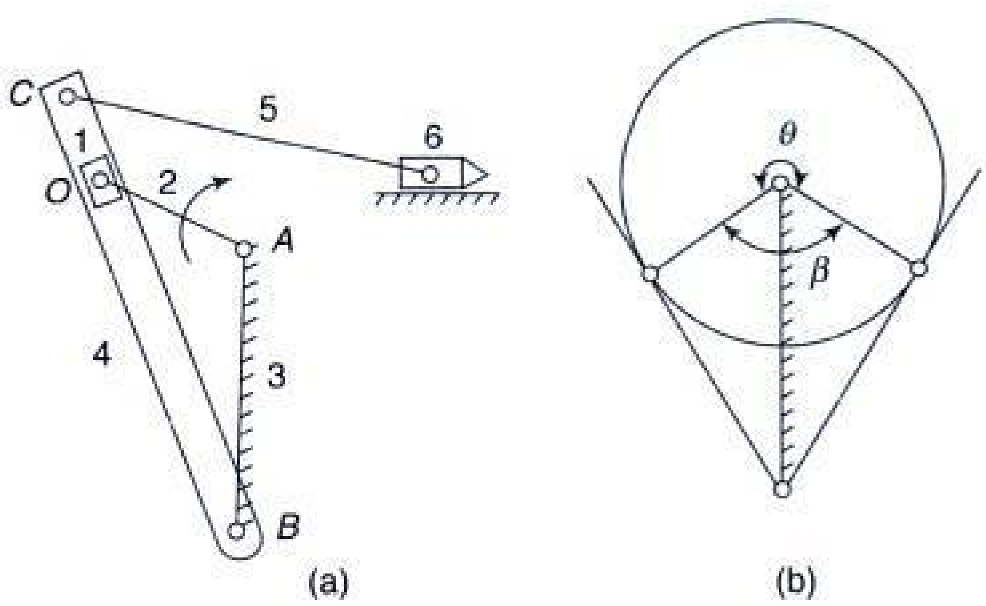


Fig. 1.55

A shaping machine. Shaping machines are fitted with quick-return mechanisms.

Fourth Inversion

If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained [Fig. 1.56(a)]. Link 3 can oscillate about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

Application Hand-pump

Figure 1.56(b) shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

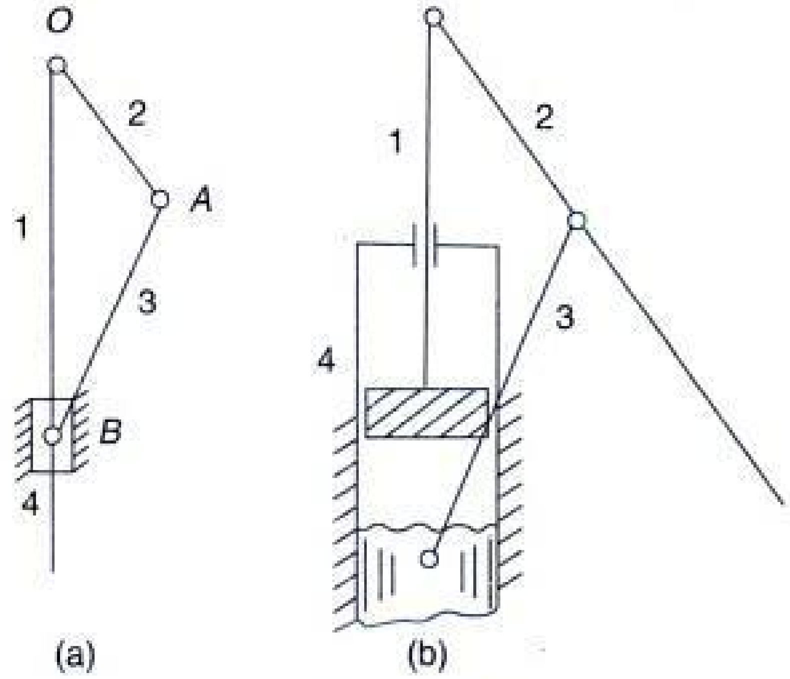


Fig. 1.56

Example 1.13 *The length of the fixed link of a crank and slotted-lever mechanism is 250 mm and that of the crank is 100 mm. Determine the*

(i) inclination of the slotted lever with the vertical in the extreme position,

(ii) ratio of the time of cutting stroke to the time of return stroke, and

(iii) length of the stroke, if the length of the slotted lever is 450 mm and the line of stroke passes through the extreme positions of the free end of the lever.

Solution Refer Fig. 1.57.

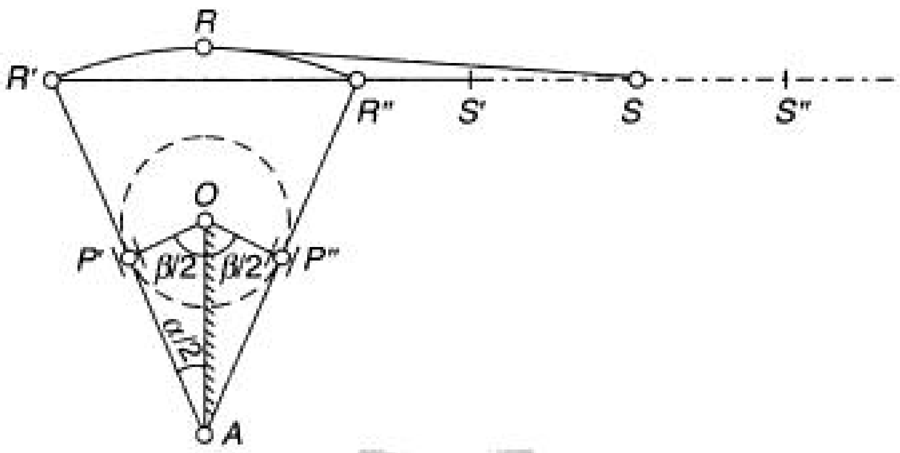


Fig. 1.57

$$OA = 250 \text{ mm} \qquad OP' = OP'' = 100 \text{ mm}$$
$$AR' = AR'' = AR = 450 \text{ mm}$$

$$\cos\frac{\beta}{2} = \frac{OP'}{OA} = \frac{100}{250} = 0.4$$

or $\frac{\beta}{2} = 66.4°$ or $\beta = 132.8°$

(i) Angle of the slotted lever with the vertical $\alpha/2 = 90° - 66.4° = 23.6°$

(ii) $\dfrac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$

$$= \frac{360° - \beta}{\beta} = \frac{360° - 132.8°}{132.8°} = 1.71$$

(iii) Length of stroke $= S'S'' = R'R''$

$$= 2\,AR'.\sin(\alpha/2)$$
$$= 2 \times 450 \sin 23.6°$$
$$= 360.3 \text{ mm}$$

1.17 DOUBLE SLIDER-CRANK CHAIN

A four-bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as a double-slider-crank chain [Fig. 1.58(a)]. The following are its inversions.

First Inversion

This inversion is obtained when the link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application Elliptical trammel

Elliptical Trammel Figure 1.58(b) shows an elliptical trammel in which the fixed link 1 is in the form of guides for sliders 2 and 4. With the movement of the sliders, any point C on the link 3, except the midpoint of AB will trace an ellipse on a fixed plate. The midpoint of AB will trace a circle.

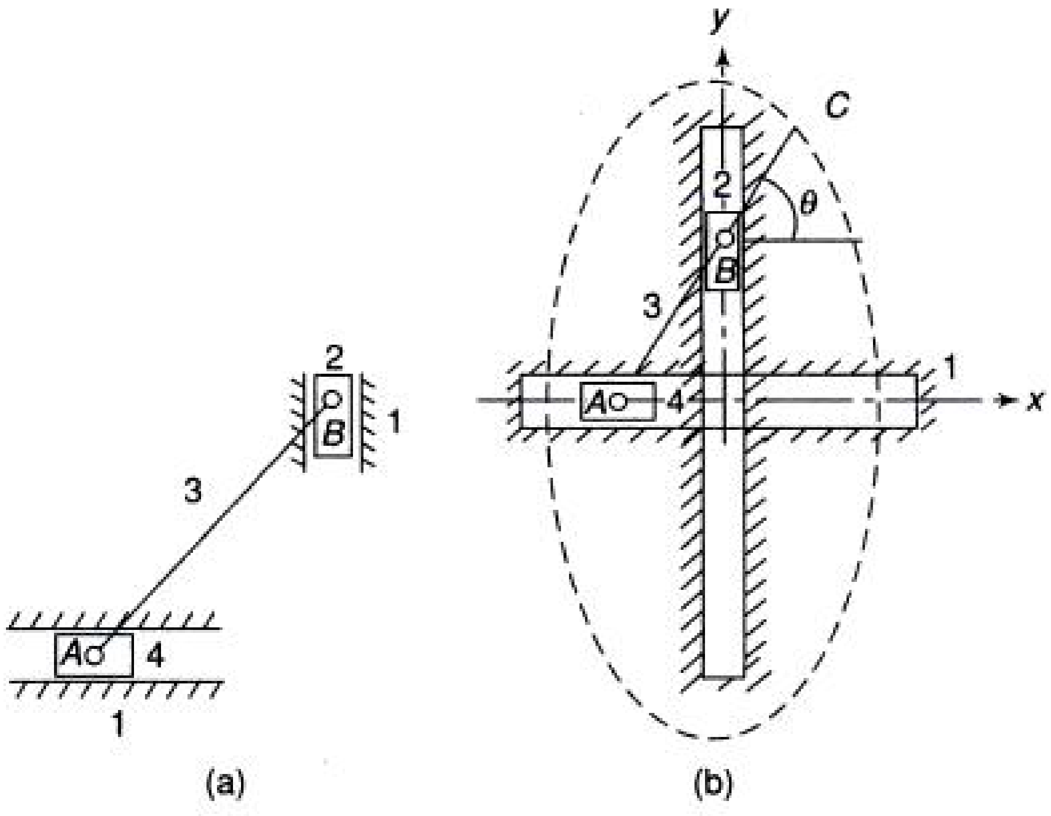


Fig. 1.58

Let at any instant, the link 3 make angle θ with the X-axis. Considering the displacements of the sliders from the centre of the trammel,

$$x = BC\cos\theta \text{ and } y = AC\sin\theta$$

$$\therefore \quad \frac{x}{BC} = \cos\theta \text{ and } \frac{y}{AC} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=\cos^2\theta+\sin^2\theta=1$$

This is the equation of an ellipse. Therefore, the path traced by C is an ellipse with the semi-major and semi-minor axes being equal to AC and BC respectively.

When C is the midpoint of AB; $AC = BC$,

and

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=1 \qquad \text{or} \qquad x^2+y^2=(AC)^2$$

which is the equation of a circle with AC (=BC) as the radius of the circle.

Second Inversion

If any of the slide-blocks of the first inversion is fixed, the second inversion of the double-slider-crank chain is obtained. When the link 4 is fixed, the end B of the crank 3 rotates about A and the link 1 reciprocates in the horizontal direction.

Application Scotch yoke

Scotch Yoke A scotch-yoke mechanism (Fig. 1.59) is used to convert the rotary motion into a sliding motion. As the crank 3 rotates, the horizontal portion of the link 1 slides or reciprocates in the fixed link 4.

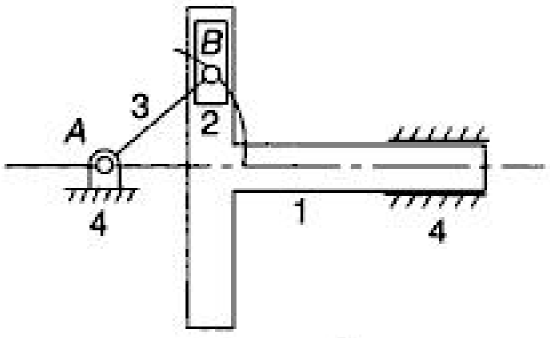


Fig. 1.59

Third Inversion

This inversion is obtained when the link 3 of the first inversion is fixed and the link 1 is free to move.

The rotation of the link 1 has been shown in Fig. 1.60 in which the full lines show the initial position. With rotation of the link 4 through 45° in the clockwise direction, the links 1 and 2 rotate through the same angle whereas the midpoint of the link 1 rotates through 90° in a circle with the length of link 3 as diameter. Thus, the angular velocity of the midpoint of the link 1 is twice that of links 2 and 4.

The sliding velocity of the link 1 relative to the link 4 will be maximum when the midpoint of the link 1 is at the axis of the link 4. In this position, the sliding velocity is equal to the tangential velocity of the midpoint of the link 1.

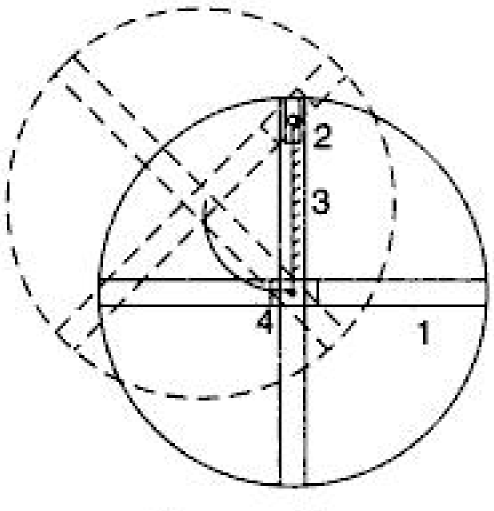


Fig. 1.60

Maximum sliding velocity = tangential velocity of midpoint of the link 1
= angular velocity of midpoint of the link 1 × radius
= (2 × angular velocity of the link 4) × (distance between axes of links 2 and 4)/2
= angular velocity of link 4 × distance between axes of links 2 and 4

The sliding velocity of the link 1 relative to the link 4 is zero when the midpoint of 1 is on the axis of the link 2.

Application Oldham's coupling

Oldham's Coupling If the rotating links 2 and 4 of the mechanism are replaced by two shafts, one can act as the driver and the other as the driven shaft with their axes at the pivots of links 2 and 4.

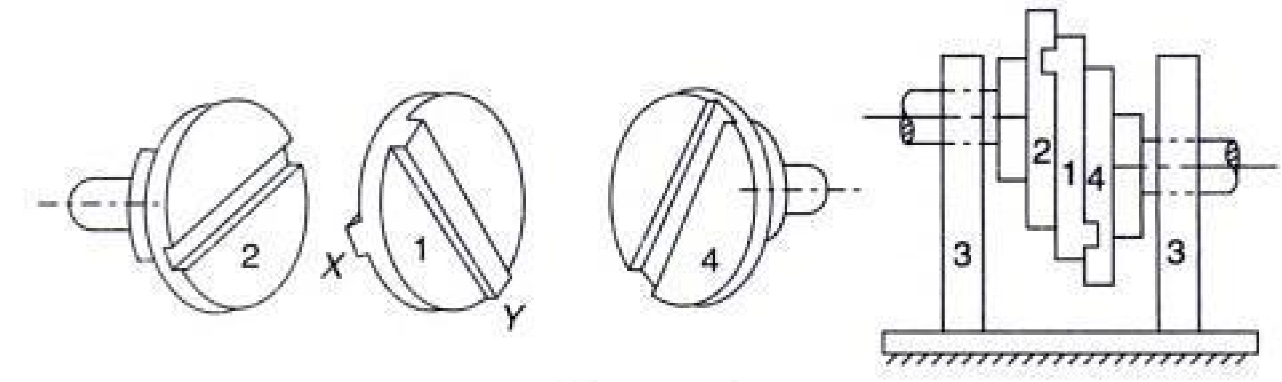


Fig. 1.61

Figure 1.61 shows an actual Oldham's coupling which is used to connect two parallel shafts when the distance between their axes is small. The two shafts have flanges at the ends and are supported in the fixed bearings representing the link 3. In the flange 2, a slot is cut in which the tongue X of the link 1 is fitted and has a sliding motion. Link 1 is made circular and has another tongue Y at right angles to the first and which fits in the recess of the flange of the shaft 4. Thus, the intermediate link 1 slides in the two slots in the two flanges while having the rotary motion.

As mentioned earlier, the midpoint of the intermediate piece describes a circle with distance between the axes of the shafts as diameter. The maximum sliding velocity of each tongue in the slot will be the peripheral velocity of the midpoint of the intermediate disc along the circular path.

Maximum sliding velocity = peripheral velocity along the circular path

= angular velocity of shaft × distance between shafts

Example 1.14 *The distance between two parallel shafts is 18 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?*

Solution $\omega = \dfrac{2\pi \times 160}{60} = 16.75$ rad/s

Maximum velocity of sliding $= \omega \times d$

$= 16.75 \times 0.018$

$= 0.302$ m/s

1.18 MISCELLANEOUS MECHANISMS

Snap-Action Mechanisms

The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms. They find their use in a variety of machines such as stone crushers, embossing presses, switches, etc. Figure 1.62(a) shows such a type of mechanism in which links of equal lengths 4 and 5 are connected by a pivoted joint at B. Link 4 is free to oscillate about the pivot C and the link 5 is connected to a sliding link 6. Link 3 joins links 4 and 5. When force is applied at the point B through the link 3, the angle α decreases and links 4 and 5 tend to become collinear. At this instant, the force is greatly multiplied at B, i.e., a very small force is required to overcome a great resistance R at the slider. This is because a large movement at B produces a relatively slight displacement of the slider at D. As the angle α approaches zero, reaction at the pivot becomes equal to R and for force balance in the link BC or BD,

$$\frac{F}{2\sin\alpha} = \frac{R}{\cos\alpha}$$

or $$2\tan\alpha = \frac{F}{R}$$

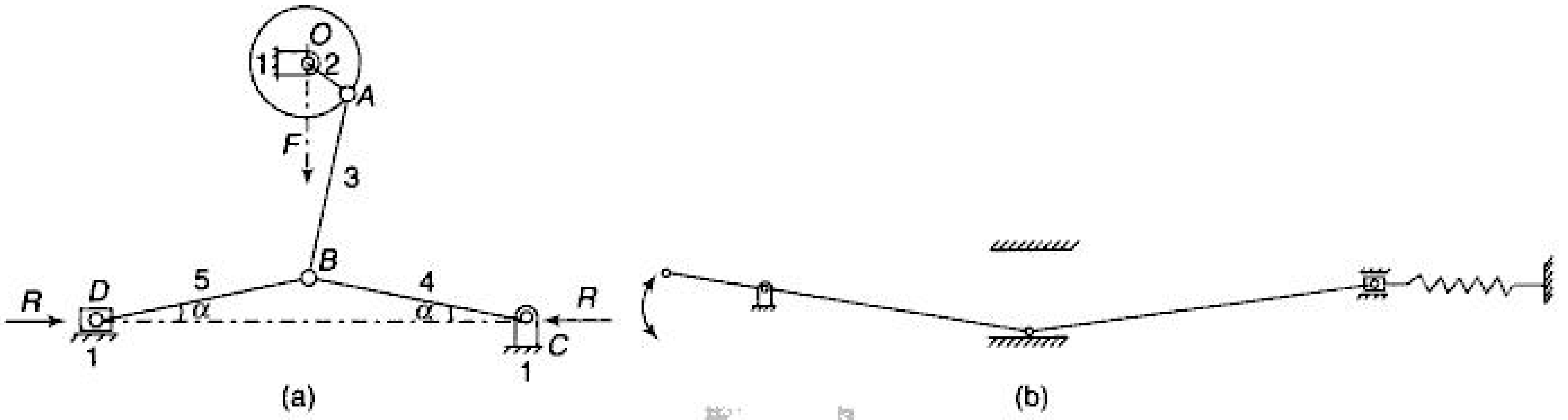


Fig. 1.62

As $\alpha \to 0$, $\tan\alpha \to 0$. Thus for a small value of the force F, R approaches infinity. In a stone crusher, a large resistance at D is overcome with a small force F in this way. Figure 1.62(b) shows another such mechanism.

Indexing Mechanisms

An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts. Indexing is generally done on gear cutting or milling machines.

An indexing mechanism consists of an index head in which a spindle is carried in a headstock [Fig. 1.63(a)]. The work to be indexed is held either between centres or in a chuck attached to the spindle. A 40-tooth worm wheel driven by a single-threaded right-hand worm is also fitted to the spindle. At the end of the worm shaft an adjustable index crank with a handle and a plunger pin is also fitted. The plunger pin can be made to fit into any hole in the index plate which has a number of circles of equally spaced holes as shown in Fig. 1.63(b). An index head is usually provided with a number of interchangeable index plates to cover a wide range of work. However, the figure shows only the circle of 17 holes for sake of clarity.

As the worm wheel has 40 teeth, the number of revolutions of the index crank required to make one revolution of the work is also 40. The number of revolutions of the crank, needed for a proper division of the work into the desired number of divisions, can be calculated as follows:

- If a work is to be divided into 40 divisions, the crank should be given one complete revolution; if 20 divisions, two revolutions for each division, and so on.
- If the work is to be divided into 160 divisions, obviously the crank should be rotated through one-fourth of a rotation. For such cases, an index plate with a number of holes divisible by 4 such as with 16 or 20 holes can be chosen.
- If the work is to be divided into 136 parts, the use of the index plate will be essential since the rotation of the crank for each division will be 40/136 or five-seventeenth of a turn. Thus, a plate with 17 holes is selected in this case. To obviate the necessity of counting the holes at each partial turn of the crank, an index sector with two arms which can be set and clamped together at any angle is also available. In this case, this can be set to measure off 5 spaces. Starting with the crankpin in the hole a, a cut would be made in the work. The crank is rotated and the pin is made to enter into the hole b, 5 divisions apart and a second cut is made in the work. In a similar way, a third cut is made by rotating the crank again through five divisions with the help of an index sector, and so on. Usually, index tables are provided to ascertain the number of turns of the crank and the number of holes for the given case.

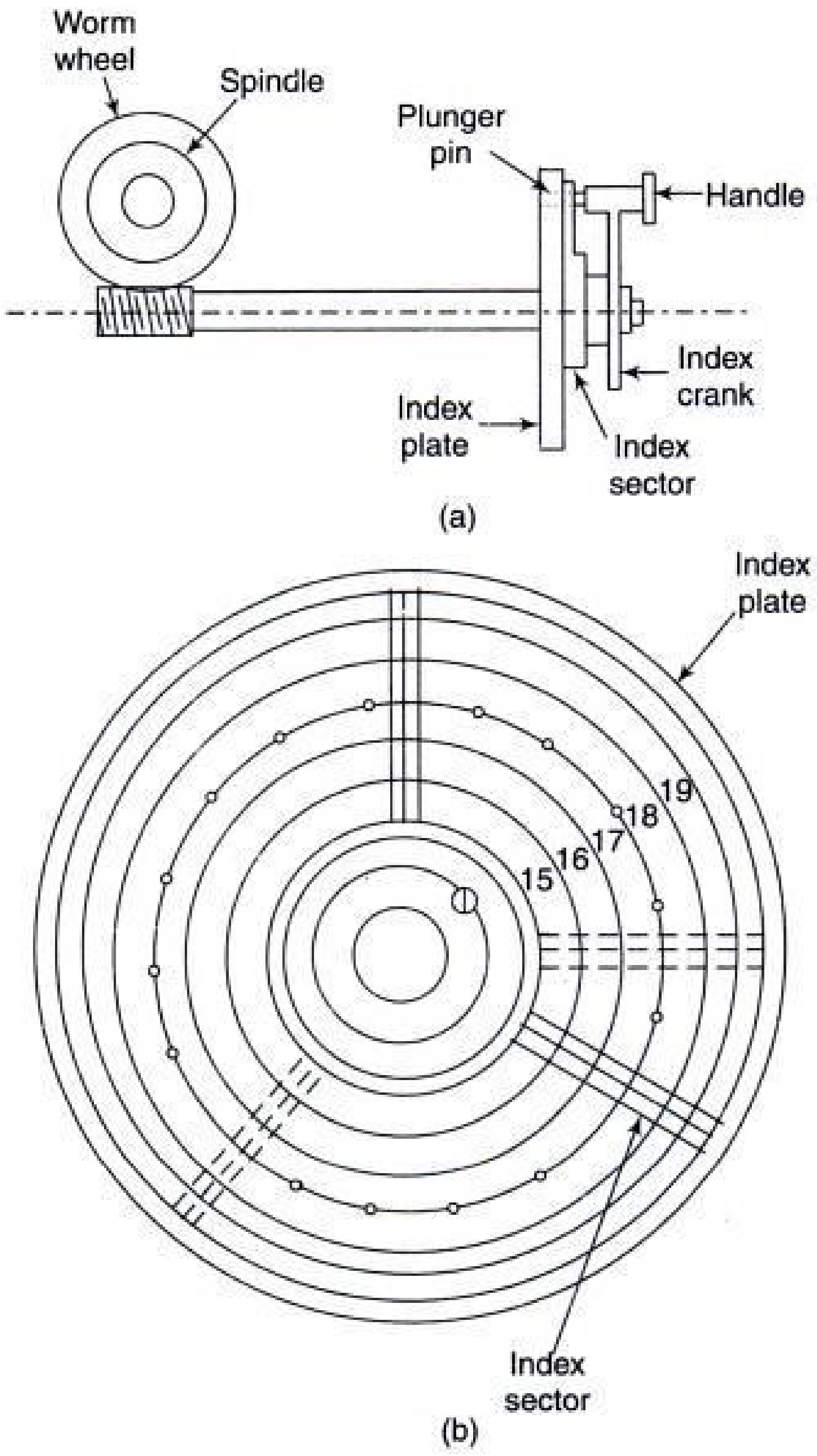
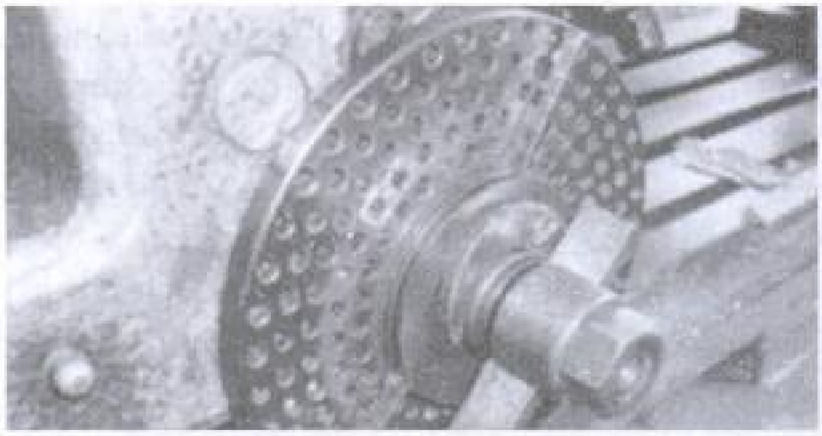


Fig. 1.63



Index plate of an indexing mechanism

Summary

1. *Kinematics* deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions whereas *dynamics* involves the calculation of forces impressed upon different parts of a mechanism.
2. *Mechanism* is a combination of a number of rigid bodies assembled in such a way that the motion of one causes constrained and predictable motion of the others whereas a *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of useful work.
3. There are three types of constrained motion: *completely constrained, incompletely constrained* and *successfully constrained.*
4. A *link* is a resistant body or a group of resistant bodies with rigid connections preventing their

relative movement. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.

5. A *kinematic pair* or simply a pair is a joint of two links having relative motion between them.
6. A pair of links having surface or area contact between the members is known as a *lower pair* and a pair having a point or line contact between the links, a *higher pair*.
7. When the elements of a pair are held together mechanically, it is known as a *closed pair*. The two elements are geometrically identical. When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an *unclosed pair*.
8. Usual types of joints in a chain are binary joint, ternary joint and quaternary joint
9. *Degree of freedom of a pair* is defined as the number of independent relative motions, both translational and rotational, a pair can have.
10. A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.
11. A *redundant chain* does not allow any motion of a link relative to the other.
12. A *linkage* or *mechanism* is obtained if one of the links of a kinematic chain is fixed to the ground.
13. *Degree of freedom of a mechanism* indicates how many inputs are needed to have a constrained motion of the other links.
14. *Kutzback's criterion* for the degree of freedom of plane mechanisms is

    $F = 3(N-1) - 2P_1 - 1P_2$
15. *Gruebler's criterion* for degree of freedom of plane mechanisms with single-degree of freedom joints only is

    $F = 3(N-1) - 2P_1$
16. *Author's criterion* for degree of freedom and the number of joints of plane mechanisms with turning pairs is

    $F = N - (2L + 1)$

    $P_1 = N + (L - 1)$
17. In a four-link mechanism, a link that makes a complete revolution is known as a *crank*, the link opposite to the fixed link is called the *coupler* and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.
18. In a Watts six-bar chain, the ternary links are direct connected whereas in a Stephenson's six-bar chain, they are not direct connected.
19. If a system has one or more links which do not introduce any extra constraint, it is known as *redundant link* and is not counted to find the degree of freedom.
20. If a link of a mechanism can be moved without causing any motion to the rest of the links of the mechanism, it is said to have a *redundant degree of freedom*.
21. The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant.
22. The angle μ between the output link and the coupler is known as *transmission angle*.
23. Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*.
24. The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms.
25. An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts.

Exercises

1. Distinguish between
   (i) mechanism and machine
   (ii) analysis and synthesis of mechanisms
   (iii) kinematics and dynamics
2. Define: kinematic link, kinematic pair, kinematic chain.
3. What are rigid and resistant bodies? Elaborate.
4. How are the kinematic pairs classified? Explain with examples.
5. Differentiate giving examples:
   (i) lower and higher pairs
   (ii) closed and unclosed pairs
   (iii) turning and rolling pairs
6. What do you mean by degree of freedom of a

kinematic pair? How are pairs classified? Give examples.

7. Discuss various types of constrained motion.
8. What is a redundant link in a mechanism?
9. How do a Watt's six-bar chain and Stephenson's six-bar chain differ?
10. What is redundant degree of freedom of a mechanism?
11. What are usual types of joints in a mechanism?
12. What is the degree of freedom of a mechanism? How is it determined?
13. What is Kutzback's criterion for degree of freedom of plane mechanisms? In what way is Gruebler's criterion different from it?
14. How are the degree of freedom and the number of joints in a linkage can be found when the number of links and the number of loops in a kinematic chain are known?
15. What is meant by equivalent mechanisms?
16. Define Grashof's law. State how is it helpful in classifying the four-link mechanisms into different types.
17. Why are parallel-crank four-bar linkage and deltoid linkage considered special cases of four-link mechanisms?
18. Define mechanical advantage and transmission angle of a mechanism.
19. Describe various inversions of a slider-crank mechanism giving examples.
20. What are quick-return mechanisms? Where are they used? Discuss the functioning of any one of them.
21. How are the Whitworth quick-return mechanism and crank and slotted-lever mechanism different from each other?
22. Enumerate the inversions of a double-slider-crank chain. Give examples.
23. Describe briefly the functions of elliptical trammel and scotch yoke.
24. In what way is Oldham's coupling useful in connecting two parallel shafts when the distance between their axes is small?
25. What are snap-action mechanisms? Give examples.
26. What is an indexing mechanism? Describe how it is used to divide the periphery of a circular piece into a number of equal parts.
27. For the kinematic linkages shown in Fig. 1.64, find the degree of freedom (F).

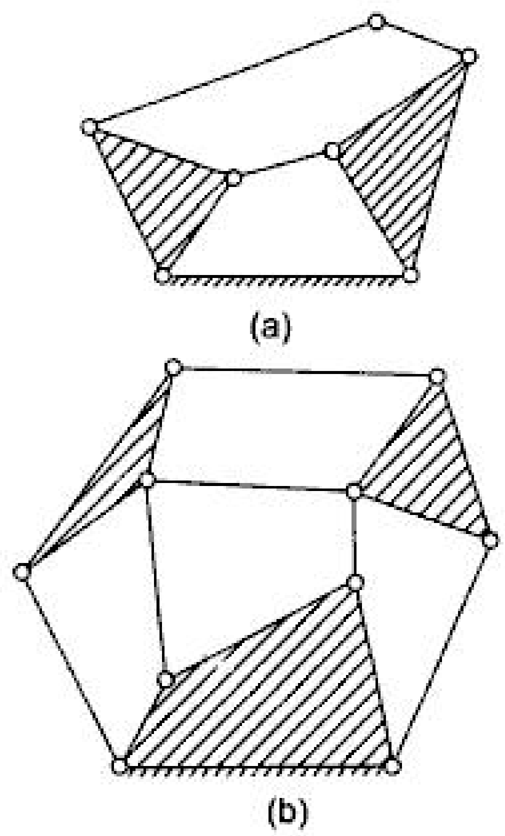


Fig. 1.64

[(a) 1 (b) 0]

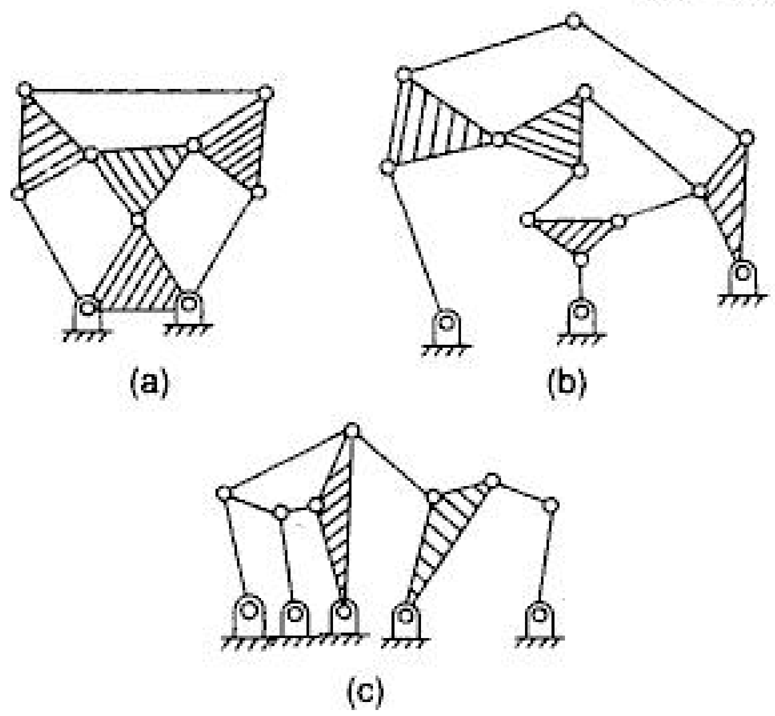


Fig. 1.65

28. For the kinematic linkages shown in Fig 1.65, find the number of binary links (N_b), ternary links (N_t), other links (N_o), total links N, loops L, joints or pairs (P_1), and degree of freedom (F).

[(a) $N_b = 3$; $N_t = 4$; $N_o = 0$; $N = 7$; $L = 3$; $P_1 = 9$; $F = 0$

(b) $N_b = 7$; $N_t = 5$; $N_o = 0$; $N = 12$; $L = 4$; $P_1 = 15$; $F = 3$

(c) $N_b = 8$; $N_t = 2$; $N_o = 1$; $N = 11$; $L = 5$; $P_1 = 15$; $F = 0$]

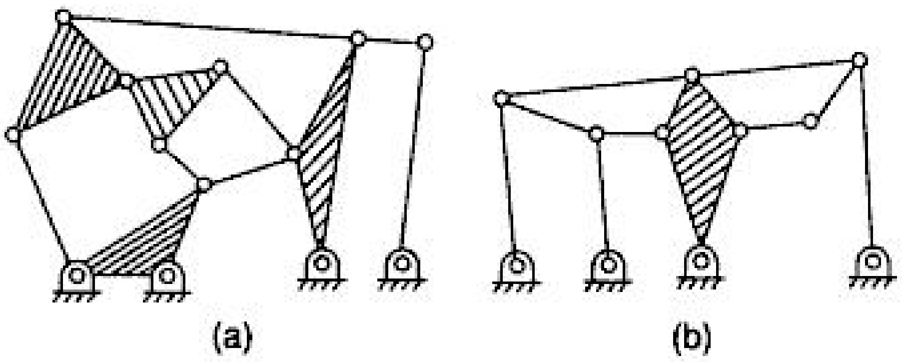


Fig. 1.66

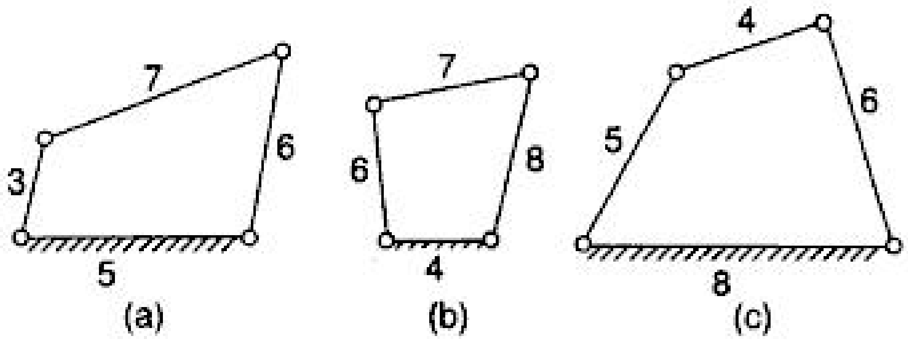


Fig. 1.67

29. Show that the linkages shown in Fig. 1.66 are structures. Suggest some changes to make them mechanisms having one degree of freedom. The number of links should not be changed by more than ±1.
30. A linkage has 14 links and the number of loops is 5. Calculate its
    (i) degrees of freedom
    (ii) number of joints
    (iii) maximum number of ternary links that can be had.

    Assume that all the pairs are turning pairs.

    (3; 18; 8)
31. Figure 1.67 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism, whether it is crank-rocker or double-crank or double-rocker.

    [(a) crank-rocker (b) double-crank (c) double-rocker]
32. A crank-rocker mechanism *ABCD* has the dimensions *AB* = 30 mm, *BC* = 90 mm, *CD* = 75 mm and *AD* (fixed link) = 100 mm. Determine the maximum and the minimum values of the transmission angle. Locate the toggle positions and indicate the corresponding crank angles and the transmission angles.

    (103°, 49°, θ = 228°, μ = 92°, θ = 38.5°, μ = 56°)

2 VELOCITY ANALYSIS

Introduction

As mentioned in the first chapter, analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a machine or a mechanism is represented by a skeleton or a line diagram, commonly knows as a *configuration diagram*.

Velocities and accelerations in machines can be determined either analytically or graphically. With the invention of calculators and computers, it has become convenient to make use of analytical methods. However, a graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected. This chapter is mainly devoted to the study of graphical methods of velocity analysis. Two methods of graphical approach, namely, relative velocity method and instantaneous centre method are discussed. The algebraic methods are also discussed in brief. The analytical approach involving the use of calculators and computers will be discussed in Chapter 4.

2.1 ABSOLUTE AND RELATIVE MOTIONS

Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

2.2 VECTORS

Problems involving relative motions are conveniently solved by the use of vectors. A vector is a line which represents a vector quantity such as force, velocity, acceleration, etc.

Characteristics of a Vector

1. Length of the vector **ab** (Fig. 2.1) drawn to a convenient scale, represents the magnitude of the quantity (written as *ab*).

Direction of the line is parallel to the direction in which the quantity acts.

The initial end **a** of the line is the tail and the final end **b**, the head. An arrowhead on the line indicates the direction-sense of the quantity which is always from the tail to the head, i.e., **a** to **b**.

If the sense is as shown in Fig. 2.1(a), the vector is read as **ab** and if the sense is opposite [Fig. 2.1 (b)], the vector is read as **ba**. This implies that **ab** = – **ba**

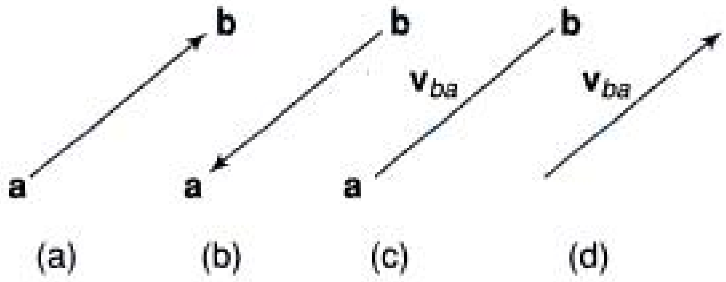


Fig. 2.1

2. Vector **ab** may also represent a vector quantity of a body B relative to a body A such as velocity of B relative to A.

   If the body A is fixed, **ab** represents the absolute velocity of B. If both the bodies A and B are in motion, the velocity of B relative to A means the velocity of B assuming the body A to be fixed for the moment.

   The vector **ab** can also be shown as $\mathbf{v}_{ba}$ [Fig. 2.1(c)], meaning the velocity of B relative to A provided a and b are indicated at the ends or an arrowhead is put on the vector [Fig. 2.1(d)].

3. Vector **ab** may also represent a vector quantity of a point B relative to a point A in the same body.

   If a vector $\mathbf{v}_{ba}$ or **ab** represents the velocity of B relative to A, the same vector in the opposite sense represents the velocity of A relative to B and will be read as $\mathbf{v}_{ab}$ or **ba**.

2.3 ADDITION AND SUBTRACTION OF VECTORS

Let

$\mathbf{v}_{ao}$ = velocity of A relative to O

$\mathbf{v}_{ba}$ = velocity of B relative to A

$\mathbf{v}_{bo}$ = velocity of B relative to O

The law of vector addition states that the velocity of B relative to O is equal to the vectorial sum of the velocity of B relative to A and the velocity of A relative to O.

Velocity of B relative to O = velocity of B relative to A + velocity of A relative to O (2.1)

i.e.
$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or
$$\mathbf{ob} = \mathbf{oa} + \mathbf{ab}$$

Take the vector **oa** and place the vector **ab** at the end of the vector **oa.** Then **ob** is given by the closing side of the two vectors (Fig. 2.2).

Note that the arrows of the two vectors to be added are in the same order and that of the resultant is in the opposite order.

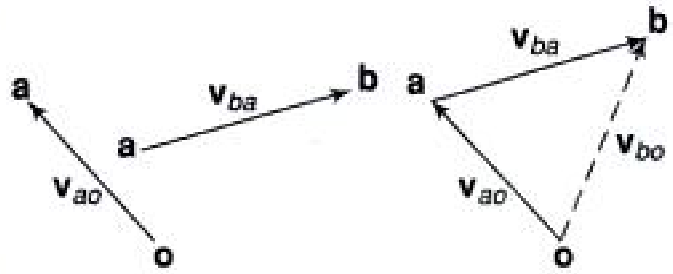


Fig. 2.2

Any number of vectors can be added as follows:

1. Take the first vector.
2. At the end of the first vector, place the beginning of the second vector.

3. At the end of the second vector, place the beginning of the third vector, and so on.
4. Joining of the beginning of the first vector and the end of the last vector represents the sum of the vectors. Figure 2.3 shows the addition of four vectors.

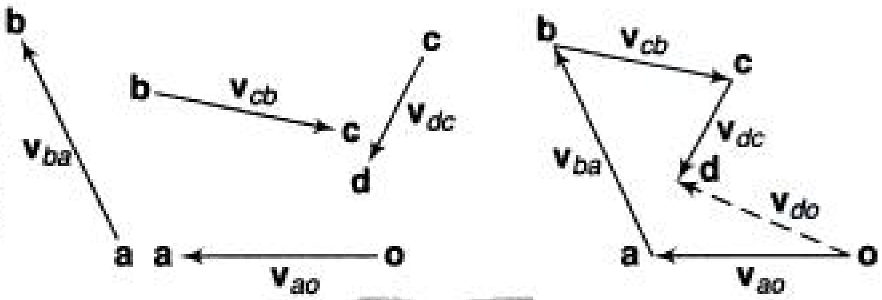


Fig. 2.3

$$\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{cb} + \mathbf{v}_{ba} + \mathbf{v}_{ao} \tag{2.2}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba} + \mathbf{v}_{cb} + \mathbf{v}_{dc}$$
$$\mathbf{od} = \mathbf{oa} + \mathbf{ab} + \mathbf{bc} + \mathbf{cd}$$

Equation 2.1 may be written as,

Vel. of B rel. to A = Vel. of B rel. to O – Vel. of A rel. to O (2.3)

$$\mathbf{v}_{ba} = \mathbf{v}_{bo} - \mathbf{v}_{ao}$$

or $$\mathbf{ab} = \mathbf{ob} - \mathbf{oa}$$

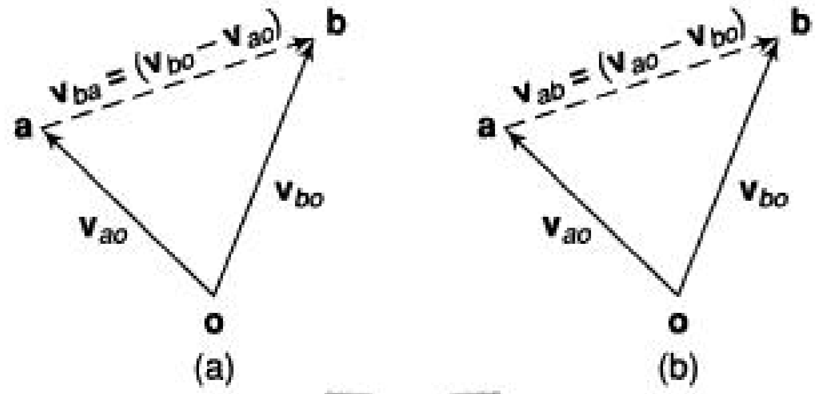


Fig. 2.4

This shows that in Fig. 2.2, **ab** also represents the subtraction of **oa** from **ob** [Fig. 2.4(a)]

Also $\mathbf{v}_{ab} = -\mathbf{v}_{ba} = \mathbf{v}_{ao} - \mathbf{v}_{bo}$

or $\mathbf{ba} = \mathbf{oa} - \mathbf{ob}$

This has been shown in Fig. 2.4 (b).

Thus, the difference of two vectors is given by the closing side of a triangle, the other two sides of which are formed by placing the two vectors tail to tail, the sense being towards the vector quantity from which the other is subtracted.

2.4 MOTION OF A LINK

Let a rigid link OA, of length r, rotate about a fixed point O with a uniform angular velocity ω rad/s in the counter-clockwise direction [Fig.2.5 (a)]. OA turns through a small angle $\delta\theta$ in a small interval of time δt. Then A will travel along the arc AA' as shown in [Fig.2.5(b)].

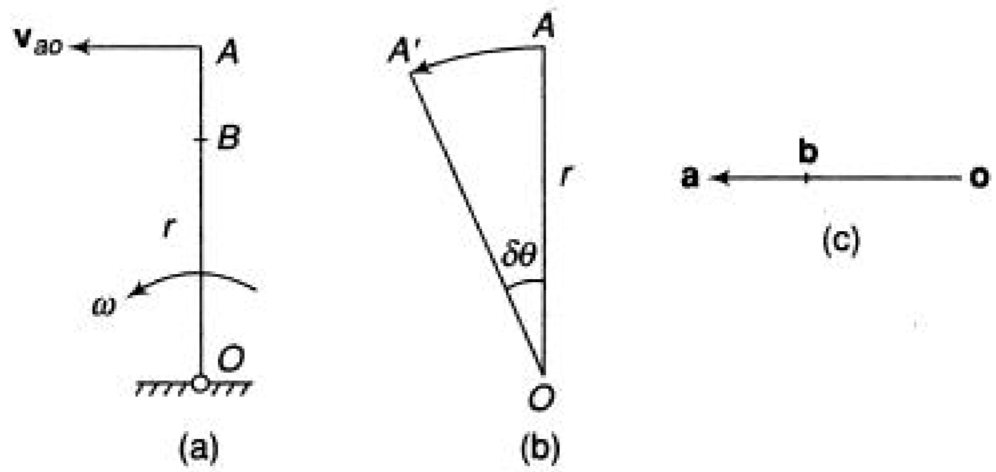


Fig. 2.5

Velocity of A relative to $O = \dfrac{\text{Arc}AA'}{\delta t}$

or
$$\mathbf{v}_{ao} = \frac{r\delta\theta}{\delta t}$$

In the limits, when $\delta t \to 0$

$$\mathbf{v}_{ao} = r\frac{d\theta}{dt} \tag{2.4}$$
$$= r\omega$$

The direction of v_{ao} is along the displacement of A. Also, as δt approaches zero ($\delta t \to 0$), AA' will be perpendicular to OA. Thus, velocity of A is ωr and is perpendicular to OA. This can be represented by a vector **oa** (Fig. 2.5 c). The fact that the direction of the velocity vector is perpendicular to the link also emerges from the fact that A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

Consider a point B on the link OA.

Velocity of $B = \omega \,.\, OB$ perpendicular to OB.

If **ob** represents the velocity of B, it can be observed that

$$\frac{\mathbf{ob}}{\mathbf{oa}} = \frac{\omega OB}{\omega OA} = \frac{OB}{OA} \tag{2.5}$$

i.e., **b** divides the velocity vector in the same ratio as B divides the link.

Remember, the velocity vector v_{ao} [Fig. 2.5(c)] represents the velocity of A at a particular instant. At other instants, when the link OA assumes another position, the velocity vectors will have their directions changed accordingly.

Also, the magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.

2.5 FOUR-LINK MECHANISM

Figure 2.6(a) shows a four-link mechanism (quadric-cycle mechanism) $ABCD$ in which AD is the fixed link and BC is the coupler. AB is the driver rotating at an angular speed of ω rad/s in the clockwise direction if it is a crank or moving at this angular velocity at this instant if it is a rocker. It is required to find the absolute velocity of C (or velocity of C relative to A).

Writing the velocity vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba} \tag{2.6}$$

The velocity of any point relative to any other point on a fixed link is always zero. Thus, all the points on a fixed link are represented by one point in the velocity diagram. In Fig. 2.6(a), the points A and D, both lie on the fixed link AD. Therefore, the velocity of C relative to A is the same as velocity of C relative to D.

Equation (2.6) may be written as,

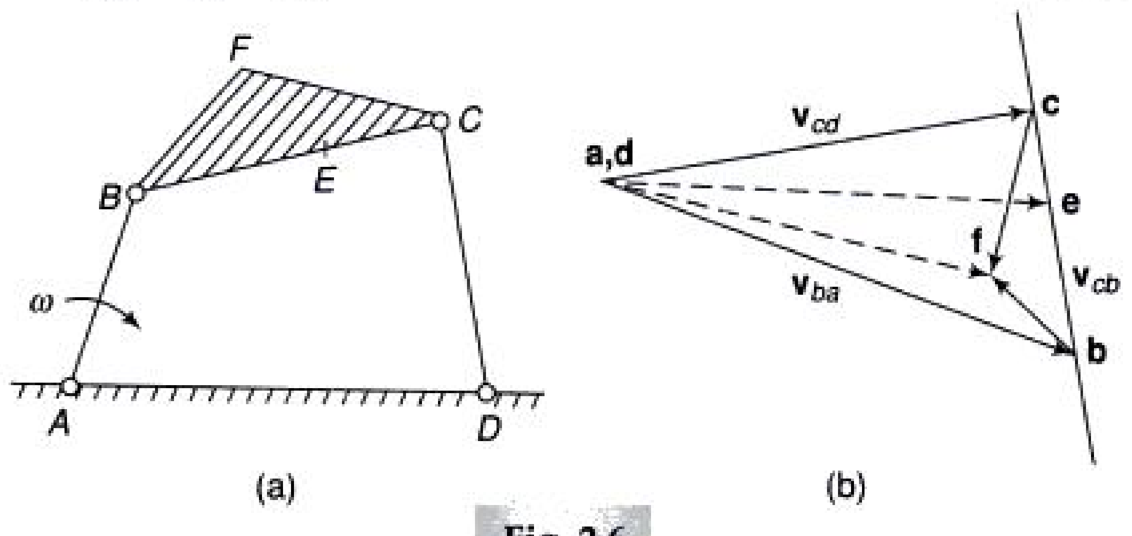


Fig. 2.6

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or

$$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

where $\mathbf{v}_{ba}$ or $\mathbf{ab} = \omega AB$; $\perp$ to AB

$\mathbf{v}_{cb}$ or **bc** is unknown in magnitude ; $\perp$ to BC

$\mathbf{v}_{cd}$ or **dc** is unknown in magnitude ; ⊥ to DC

The velocity diagram is constructed as follows:

1. Take the first vector **ab** as it is completely known.
2. To add vector **bc** to **ab**, draw a line ⊥ BC through **b,** of any length. Since the direction-sense of **bc** is unknown, it can lie on either side of **b.** A convenient length of the line can be taken on both sides of **b.**
3. Through **d,** draw a line ⊥ DC to locate the vector **dc.** The intersection of this line with the line of vector **bc** locates the point **c**.
4. Mark arrowheads on the vectors **bc** and **dc** to give the proper sense. Then **dc** is the magnitude and also represents the direction of the velocity of C relative to A (or D). It is also the absolute velocity of the point C (A and D being fixed points).
5. Remember that the arrowheads on vector **bc** can be put in any direction because both ends of the link BC are movable. If the arrowhead is put from **c** to **b**, then the vector is read as **cb**. The above equation is modified as

$$\mathbf{dc} = \mathbf{ab} - \mathbf{cb} \qquad (\mathbf{bc} = -\,\mathbf{cb})$$

or

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

Intermediate Point

The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link. For point E on the link BC,

$$\frac{\mathbf{be}}{\mathbf{bc}} = \frac{BE}{BC}$$

ae represents the absolute velocity of E.

Offset Point

Write the vector equation for point F,

$$\mathbf{v}_{fb} + \mathbf{v}_{ba} = \mathbf{v}_{fc} + \mathbf{v}_{cd}$$

or

$$\mathbf{v}_{ba} + \mathbf{v}_{fb} = \mathbf{v}_{cd} + \mathbf{v}_{fc}$$

or

$$\mathbf{ab} + \mathbf{bf} = \mathbf{dc} + \mathbf{cf}$$

The vectors $\mathbf{v}_{ba}$ and $\mathbf{v}_{cd}$ are already there on the velocity diagram.

$\mathbf{v}_{fb}$ is ⊥ BF, draw a line ⊥ BF through **b;**

$\mathbf{v}_{fc}$ is ⊥ CF, draw a line ⊥ CF through **c;**

The intersection of the two lines locates the point **f.**

af or **df** indicates the velocity of F relative to A (or D) or the absolute velocity of F.

2.6 VELOCITY IMAGES

Note that in Fig. 2.6, the triangle **bfc** is similar to the triangle BFC in which all the three sides **bc**, **cf** and **fb** are perpendicular to BC, CF and FB respectively. The triangles such as **bcf** are known as velocity images and are found to be very helpful devices in the velocity analysis of complicated shapes of the linkages. Thus, any offset point on a link in the configuration diagram can easily be located in the velocity diagram by drawing the velocity image. While drawing the velocity images, the following points should be kept in mind:

1. The velocity image of a link is a scaled reproduction of the shape of the link in the velocity diagram from the configuration diagram, rotated bodily through 90° in the direction of the angular velocity.
2. The order of the letters in the velocity image is the same as in the configuration diagram.
3. In general, the ratio of the sizes of different images to the sizes of their respective links is different in the same mechanism.

2.7 ANGULAR VELOCITY OF LINKS

1. Angular Velocity of BC

(a) Velocity of C relative to *B*, v_{cb} = **bc** (Fig. 2.6)

Point *C* relative to *B* moves in the direction-sense given by $\mathbf{v}_{cb}$ (upwards). Thus, *C* moves in the counter-clockwise direction about *B*.

$$\mathbf{v}_{cb} = \omega_{cb} \times BC = \omega_{cb} \times CB$$

$$\omega_{cb} = \frac{v_{cb}}{CB}$$

(b) Velocity of *B* relative to *C*, $\mathbf{v}_{bc} = \mathbf{cb}$

B relative to *C* moves in a direction-sense given by v_{bc} (downwards, opposite to **bc**), i.e., *B* moves in the counter-clockwise direction about *C* with magnitude ω_{bc} given by

$$\frac{v_{bc}}{BC}$$

It can be seen that the magnitude of $\omega_{cb} = \omega_{bc}$ as $v_{cb} = v_{bc}$ and the direction of rotation is the same. Therefore, angular velocity of a link about one extremity is the same as the angular velocity about the other. In general, the angular velocity of link *BC* is ω_{bc} (= ω_{cb}) in the counter-clockwise direction.

2. Angular Velocity of CD

Velocity of *C* relative to *D*,

$$v_{cd} = \mathbf{dc}$$

It is seen that *C* relative to *D* moves in a direction-sense given by v_{cd} or *C* moves in the clockwise direction about *D*.

$$\omega_{cd} = \frac{v_{cd}}{CD}$$

2.8 VELOCITY OF RUBBING

Figure 2.7 shows two ends of the two links of a turning pair. A pin is fixed to one of the links whereas a hole is provided in the other to fit the pin. When joined, the surface of the hole of one link will rub on the surface of the pin of the other link. The velocity of rubbing of the two surfaces will depend upon the angular velocity of a link relative to the other.

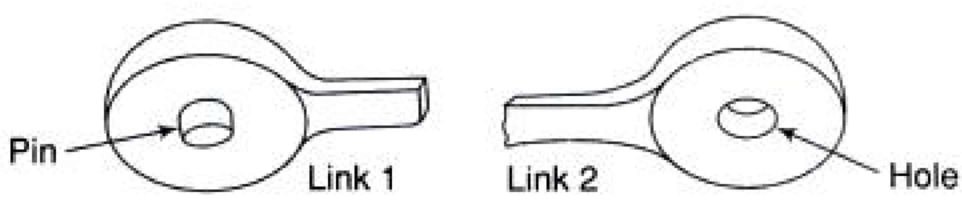


Fig. 2.7

Pin at *A* (Fig. 2.6a)

The pin at A joins links AD and AB. AD being fixed, the velocity of rubbing will depend upon the angular velocity of AB only.

Let r_a = radius of the pin at A
Then velocity of rubbing = $r_a . \omega$

Pin at *D*

Let r_d = radius of the pin at D
Velocity of rubbing = $r_d . \omega_{cd}$

Pin at *B*

$\omega_{ba} = \omega_{ab} = \omega$ clockwise

$\omega_{bc} = \omega_{cb} = \dfrac{v_{bc}}{BC}$ counter-clockwise

Since the directions of the two angular velocities of links AB and BC are in the opposite directions, the angular velocity of one link relative to the other is the sum of the two velocities.

Let r_b = radius of the pin at B
Velocity of rubbing = $r_b\,(\omega_{ab} + \omega_{bc})$

Pin at *C*

$\omega_{bc} = \omega_{cb}$ counter-clockwise
$\omega_{dc} = \omega_{cd}$ clockwise

Let r_c = radius of the pin at C
Velocity of rubbing = $r_c\,(\omega_{bc} + \omega_{dc})$

In case it is found that the angular velocities of the two links joined together are in the same direction, the velocity of rubbing will be the difference of the angular velocities multiplied by the radius of the pin.

2.9 SLIDER-CRANK MECHANISM

Figure 2.8(a) shows a slider-crank mechanism in which OA is the crank moving with uniform angular velocity ω rad/s in the clockwise direction. At point B, a slider moves on the fixed guide G. AB is the coupler joining A and B. It is required to find the velocity of the slider at B.

Writing the velocity vector equation,

Vel. of B rel. to O = Vel. of B rel. to A + Vel. of A rel. to O

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

or $\qquad$ **gb = oa + ab**

$\mathbf{v}_{bo}$ is replaced by v_{bg} as O and G are two points on a fixed link

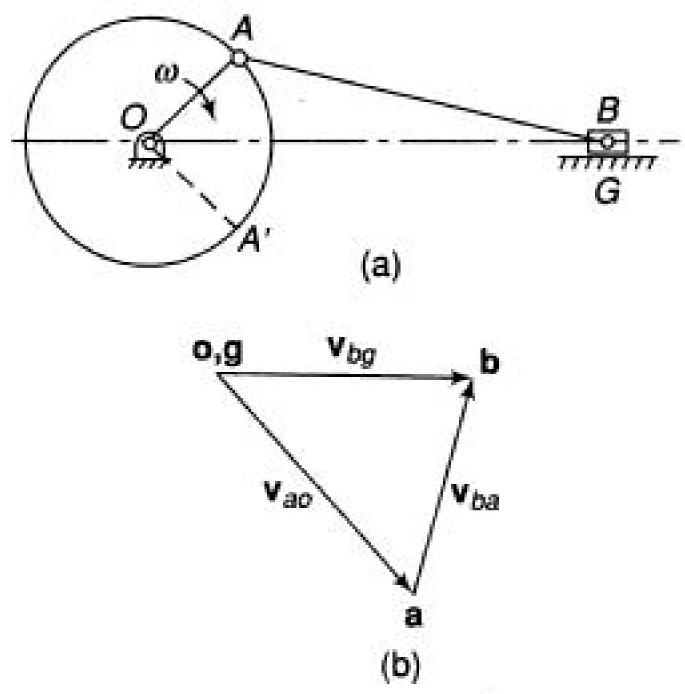


Fig. 2.8

with zero relative velocity between them.

Take the vector $\mathbf{v}_{ao}$ which is completely known.

$\mathbf{v}_{ao} = \omega . OA$; $\perp$ to OA

$\mathbf{v}_{ba}$ is $\perp$ AB, draw a line $\perp$ AB through **a**;

Through **g** (or **a**), draw a line parallel to the motion of B (to locate the vector $\mathbf{v}_{bg}$).

The intersection of the two lines locates the point **b**.

gb (or **ob**) indicates the velocity of the slider B relative to the guide G. This is also the absolute velocity of the slider (G is fixed). The slider moves towards the right as indicated by **gb.** When the crank assumes the position OA' while rotating, it will be found that the vector **gb** lies on the left of **g** indicating that B moves towards left.

For the given configuration, the coupler AB has angular velocity in the counter-clockwise direction, the magnitude being $\dfrac{v_{ba}}{BA(\text{or } AB)}$

Example 2.1 *In a four-link mechanism, the dimensions of the links are as under:*

$AB = 50$ mm, $BC = 66$ mm, $CD = 56$ mm and $AD = 100$ mm

At the instant when $\angle DAB = 60°$, the link AB has an angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine the

(i) velocity of the point C

(ii) velocity of the point E on the link BC when BE = 40 mm

(iii) angular velocities of the links BC and CD

(iv) velocity of an offset point F on the link BC if BF = 45 mm, CF = 30 mm and BCF is read clockwise

(v) velocity of an offset point G on the link CD if CG = 24 mm, DG = 44 mm and DCG is read clockwise

(vi) velocity of rubbing at pins A, B, C and D when the radii of the pins are 30, 40, 25 and 35 mm respectively.

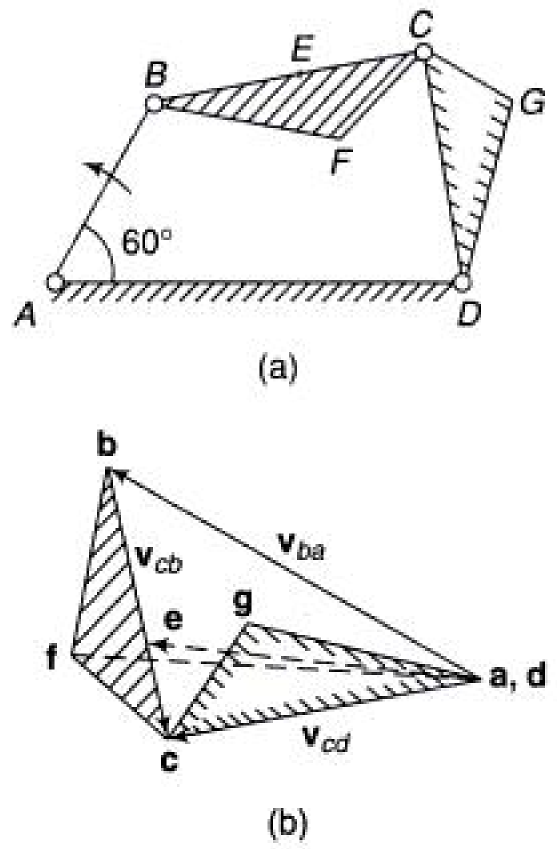


Fig. 2.9

Solution The configuration diagram has been shown in Fig. 2.9(a) to a convenient scale.

Writing the vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + Vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba}$$

or $$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or $$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

We have,

$$\mathbf{v}_{ba} = \omega_{ba} \times BA = 10.5 \times 0.05 = 0.525 \text{ m/s}$$

Take the vector $\mathbf{v}_{ba}$ to a convenient scale in the proper direction and sense [Fig. 2.9(b)].

$\mathbf{v}_{cb}$ is $\perp$ BC, draw a line $\perp$ BC through **b;**

$\mathbf{v}_{cd}$ is $\perp$ DC, draw a line $\perp$ DC through **d;**

The intersection of the two lines locates the point **c.**

Note In the velocity diagram shown in Fig. 2.9(b), arrowhead has been put on the line joining points b and c in such a way that it represents the vector for velocity of C relative to B. This satisfies the above equation. As the same equation

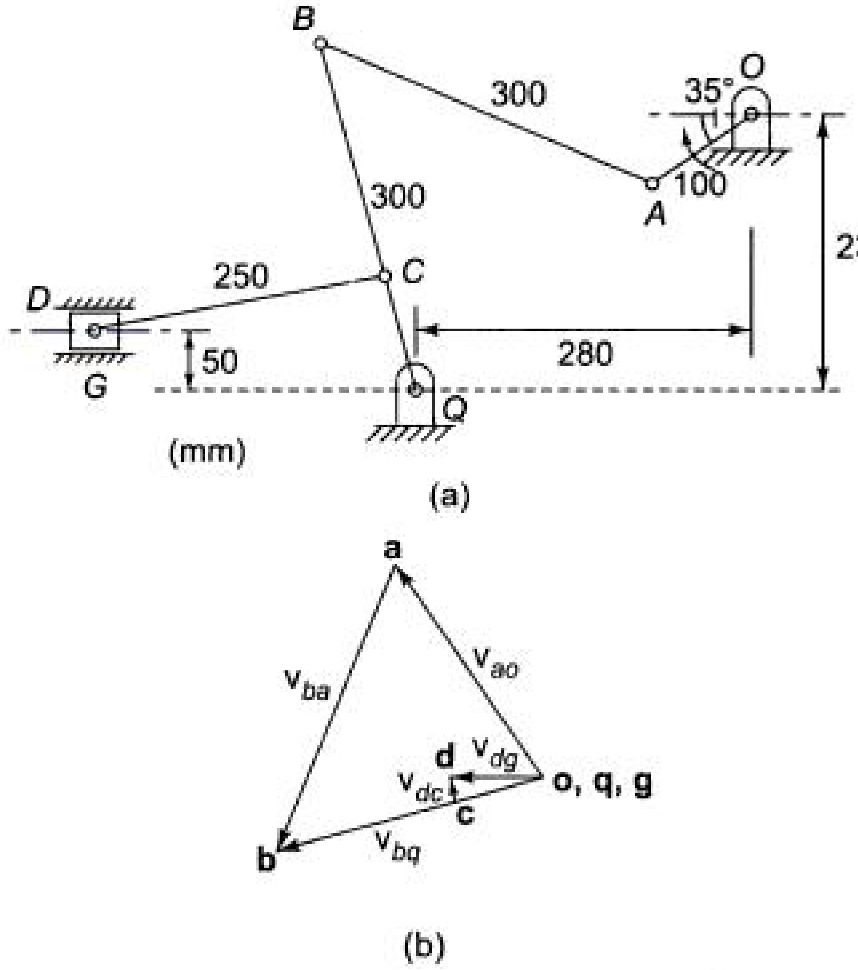


Fig. 2.11

$\mathbf{v}_{bq}$ is $\perp QB$, draw a line $\perp QB$ through **q;**

The intersection of the two lines locates the point **b.**

Locate the point **c** on **qb** such that

$$\frac{\mathbf{qc}}{\mathbf{qb}} = \frac{100}{300} = 0.3$$

The vector equation for the mechanism QCD,

$$\mathbf{v}_{dq} = \mathbf{v}_{dc} + \mathbf{v}_{cq} \quad \text{or} \quad \mathbf{v}_{dg} = \mathbf{v}_{cq} + \mathbf{v}_{dc}$$

or $\quad$ **gd = qc + cd**

$\mathbf{v}_{dc}$ is $\perp DC$, draw a line $\perp DC$ through **c;**

For $\mathbf{v}_{dg}$, draw a line through **g,** parallel to the line of stroke of the slider in the guide G.

The intersection of the two lines locates the point **d.**

(i) The velocity of slider at D, v_d = **gd** = 0.56 m/s

(vi) $\omega_{bq} = \dfrac{v_{bq}}{QB} = \dfrac{1.69}{0.3} = 5.63$ rad/s counter-clockwise

(vii) $\omega_{ba} = \dfrac{v_{ba}}{AB} = \dfrac{1.89}{0.3} = 6.3$ rad/s counter-clockwise

As both the links connected at B have counter-clockwise angular velocities,

velocity of rubbing at the crank pin

$$B = (\omega_{ba} - \omega_{bq})\, r_b$$
$$= (6.3 - 5.63) \times 0.04 = 0.0268 \text{ m/s}$$

Example 2.4 *An engine crankshaft drives a reciprocating pump through a mechanism as shown in Fig. 2.12(a). The crank rotates in the clockwise direction at 160 rpm. The diameter of the pump piston at F is 200 mm. Dimensions of the various links are*

OA = 170 mm (crank) CD = 170 mm
AB = 660 mm DE = 830 mm
BC = 510 mm

For the position of the crank shown in the diagram, determine the

(i) velocity of the crosshead E

(ii) velocity of rubbing at the pins A, B, C and D, the diameters being 40, 30, 30 and 50 mm respectively

(iii) torque required at the shaft O to overcome a pressure of 300 kN/m² at the pump piston at F

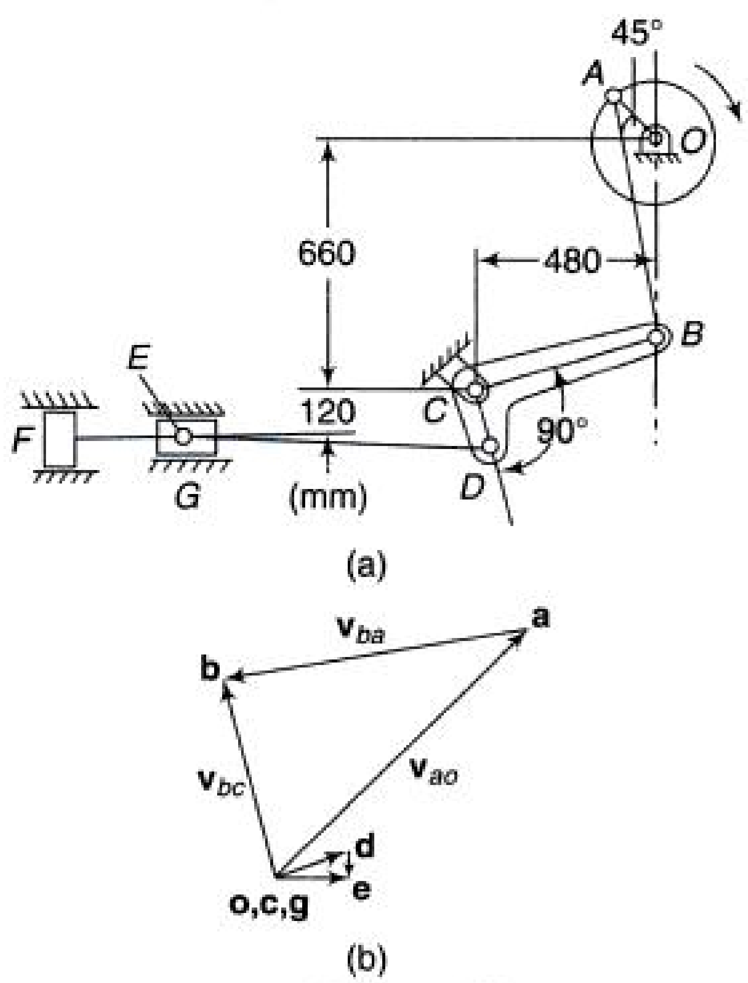


Fig. 2.12

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The crank OA rotates at 60 rpm in the counter-clockwise direction. Determine the velocity of the slider B and the angular velocity of the link BD when the crank has turned an angle of 45° with the vertical.

Solution

$$v_a = \frac{2\pi N}{60} \times OA = \frac{2\pi \times 60}{60} \times 0.15 = 0.94 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.16(b)] and complete the velocity diagram for the mechanism *OACQ*.

Now *CQD* is one link. Make Δ **cqd** similar to Δ *CQD* such that **cqd** reads clockwise as *CQD* is clockwise. This locates the point **d**. Complete the velocity diagram for the mechanism *QDB*.

v_b = **ob** = <u>0.9 m/s</u>

$$\omega_{bd} = \frac{v_{bd}}{BD} = \frac{0.49}{0.50} = \underline{0.98 \text{ rad/s}} \text{ clockwise}$$

Example 2.9 *The configuration diagram of a wrapping machine is given in Fig. 2.17(a). The crank OA rotates at 6 rad/s clockwise. Determine the*

(i) velocity of the point P on the bell-crank lever DCP

(ii) angular velocity of the bell-crank lever DCP

(iii) velocity of rubbing at B if the pin diameter is 20 mm

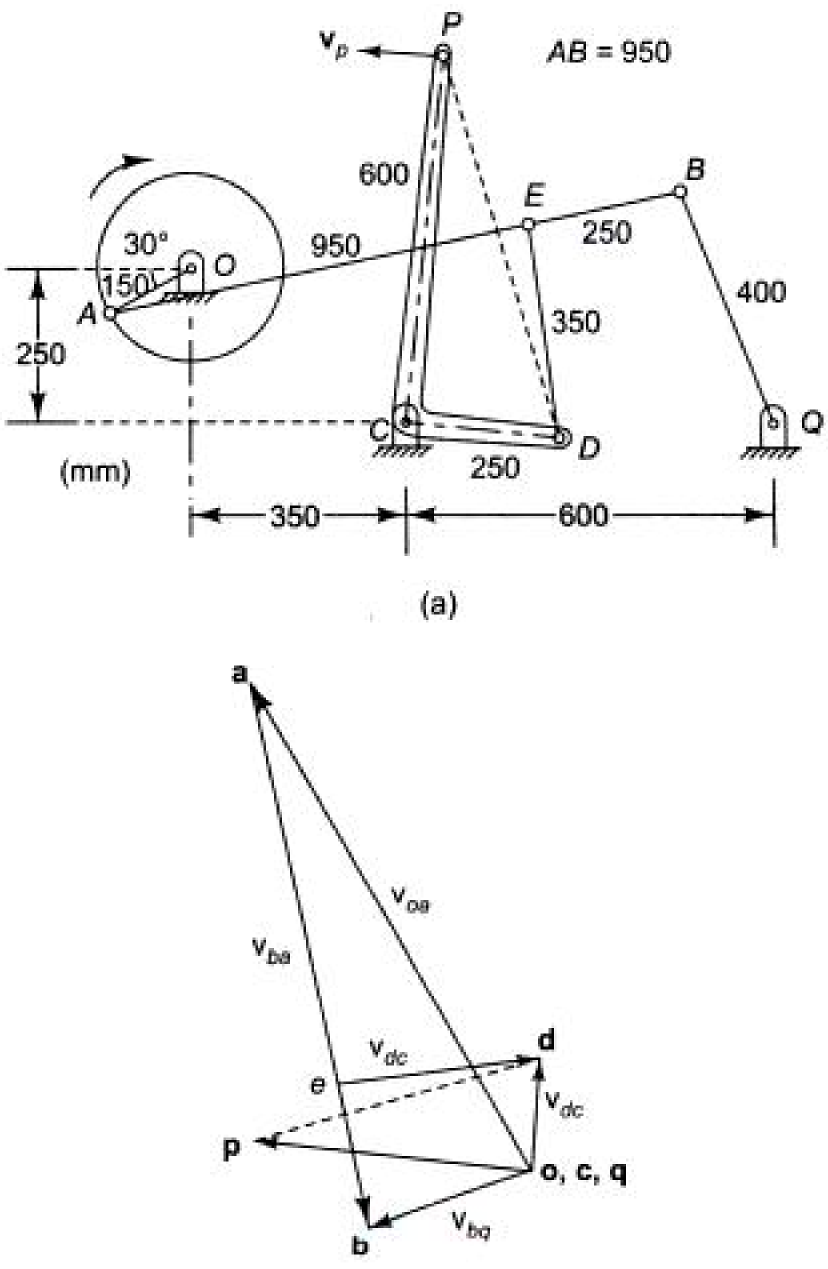

Fig. 2.17

Solution

$$v_a = 6 \times 0.15 = 0.9 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.17(b)] and complete the velocity diagram for the mechanism *OABQ*.

Now locate point **e** on the vector **ab**.

$\mathbf{v}_{de}$ is ⊥ *DE*, draw **de** ⊥ *DE* through **e;**

$\mathbf{v}_{dc}$ is ⊥ *CD*, draw **cd** ⊥ *CD* through **c.**

The intersection locates the point **d.**

Now, *DCP* is one link. Make Δ**dcp** similar to Δ*DCP* such that **dcp** reads clockwise as *DCP* is clockwise. This locates the point **p**. Then

(i) v_p = **cp** = <u>0.44 m/s</u>

(ii) $$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.182}{0.25} = \underline{0.73 \text{ rad/s}}$$

counter clockwise

(iii) $$\omega_{ab} = \frac{v_{ab}}{AB} = \frac{0.91}{0.95} = \underline{0.96 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{qb} = \frac{v_{qb}}{QB} = \frac{0.28}{0.4} = \underline{0.7 \text{ rad/s}}$$

counter-clockwise

Thus, velocity of rubbing at $B = (\omega_{ab} + \omega_{bq})r_b$

$$= (0.96 + 0.7) \times 0.02 = \underline{0.0332 \text{ m/s}}$$

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The component of velocity along AR, i.e., **qp** indicates the relative velocity between Q and P or the velocity of sliding of the block on link AR.

Now, the velocity of R is perpendicular to AR. As the velocity of Q perpendicular to AR is known, the point **r** will lie on vector **aq** produced such that **ar**/**aq** = AR/AQ

To find the velocity of ram S, write the velocity vector equation,

$$\mathbf{v}_{so} = \mathbf{v}_{sr} + \mathbf{v}_{ro}$$

or

$$\mathbf{v}_{sg} = \mathbf{v}_{ro} + \mathbf{v}_{sr}$$

or

$$\mathbf{gs} = \mathbf{or} + \mathbf{rs}$$

$\mathbf{v}_{ro}$ is already there in the diagram. Draw a line through **r** perpendicular to RS for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider S on the guide G, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram S = **os** (or **gs**) towards right for the given position of the crank.

Also, $\omega_{rs} = \dfrac{v_{rs}}{RS}$ clockwise

Usually, the coupler RS is long and its obliquity is neglected.

Then **or** ≈ **os**

Referring Fig. 2.20 (c),

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

When the crank assumes the position OP' during the cutting stroke, the component of velocity along AR (i.e., **pq**) is zero and **oq** is maximum (= **op**)

Let r = length of crank (= OP)
l = length of slotted lever (= AR)

c = distance between fixed centres (= AO)

ω = angular velocity of the crank

Then, during the cutting stroke,

$$v_{s\,\max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

This is by neglecting the obliquity of the link RS, i.e. assuming the velocity of S equal to that of R.

Similarly, during the return stroke,

$$v_{s\,\max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,\max}(\text{cutting})}{v_{s\,\max}(\text{return})} = \frac{\omega r \dfrac{1}{c+r}}{\omega r \dfrac{1}{c-r}} = \frac{c-r}{c+r}$$

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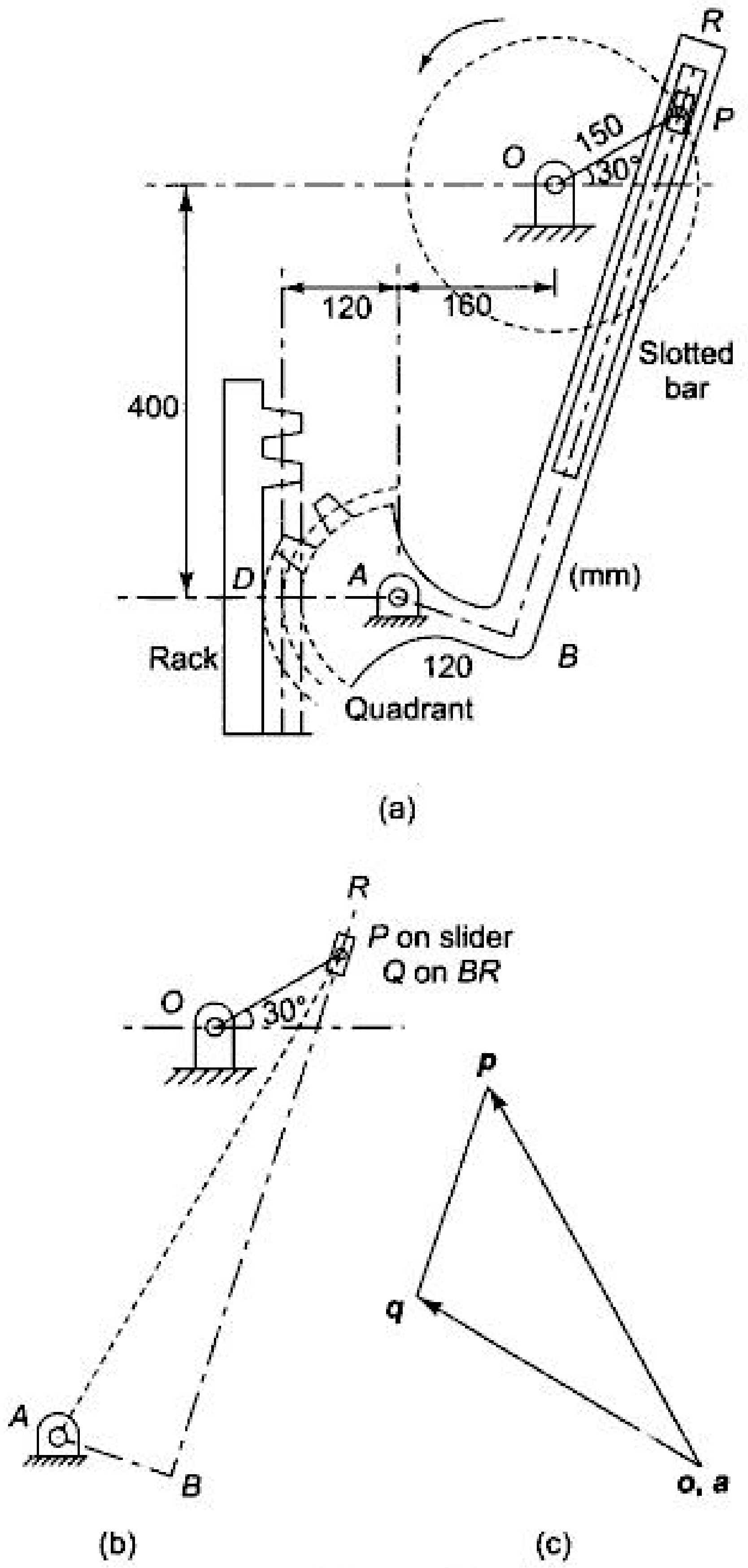


Fig. 2.26

Solution $\omega_{po} = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 210}{60} = 22 \text{ rad/s}$

$$v_{po} = 22 \times 0.15 = 3.3 \text{ m/s}$$

Draw the configuration diagram to a suitable scale [Fig. 2.26(b)].

Locate a point Q on BR beneath point P on the slider.

Then the vector equation is

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po} \quad \text{or} \quad \mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

Take the vector $\mathbf{v}_{po}$ to a convenient scale in the proper direction and sense [Fig. 2.26(c)].

$\mathbf{v}_{qp}$ is along BR, draw a line parallel to BR through **p;**

Now, Q is a point on the link ABR which is pivoted at point A. The direction of velocity of any point on the link is perpendicular to the line joining that point with the pivoted point A.

$\mathbf{v}_{qa}$ is $\perp QA$, draw a line $\perp QA$ through **a;**

The intersection of the two lines locates the point **q.**

Now angular velocity of the quadrant and the lever ABQ,

$$\omega_{aq} = \frac{v_{aq}}{AQ} = \frac{2.5}{0.577} = 4.33 \text{ rad/s}$$

counter-clockwise

(i) The linear velocity of the rack will be equal to the tangential velocity of the quadrant at the teeth, i.e.,

$$v_r = \omega \times AD = \omega \times 120 = 4.33 \times 120 = \underline{519.6 \text{ mm/s}}$$

(ii) The reciprocating rack changes the direction when the crank OP assumes a position such that the tangent at P to the circle at O is also a tangent to the circle at A with radius AB as shown in Fig. 2.27. The rack is lowered during the rotation of the crank from P to P' and is raised when P' moves to P counter-clockwise.

Thus,

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\theta}{\beta} = \frac{215^\circ}{135^\circ} = 1.59$$

In the polar form **r** can be expressed as

$$\mathbf{r} = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

In this equation r is said to be magnitude of **r**, denoted by $|\mathbf{r}|$ and θ is called the argument of **r**, denoted by arg(**r**).

Since r is the magnitude of vector **r**, the term in the parenthesis in the above equation plays the role of a unit vector which points in the direction of OP.

From trigonometry, it can be written that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Therefore,

$$\mathbf{r} = re^{i\theta} \text{ which is the complex polar form.} \qquad \text{(i)}$$

Complex numbers are assumed to follow all the formal rules of real algebra.

Velocity

Differentiating Eq. (i) with respect to time,

$$\begin{aligned}\mathbf{v} &= \dot{r}e^{i\theta} + ir\dot{\theta}e^{i\theta}\\ &= (\dot{r} + ir\dot{\theta})e^{i\theta}\end{aligned} \qquad (2.8)$$

2.12 INSTANTANEOUS CENTRE (I-CENTRE)

Let there be a plane body p having a non-linear motion relative to another plane body q. At any instant, the linear velocities of two points A and B on the body p are $\mathbf{v}_a$ and $\mathbf{v}_b$ respectively in the directions as shown in Fig. 2.31.

If a line is drawn perpendicular to the direction of $\mathbf{v}_a$ at A, the body can be imagined to rotate about some point on this line. Similarly, the centre of rotation of the body also lies on a line perpendicular to the direction of $\mathbf{v}_b$ at B. If the intersection of the two lines is at I, the body p will be rotating about I at the instant. This point I is known as the *instantaneous centre of velocity* or more commonly *instantaneous centre of rotation* for the body p. This property is true only for an instant and a new point will become the instantaneous centre at the next instant. Thus, it is a misnomer to call this point the centre of rotation, as generally this point is not located at the centre of curvature of the apparent path taken by a point

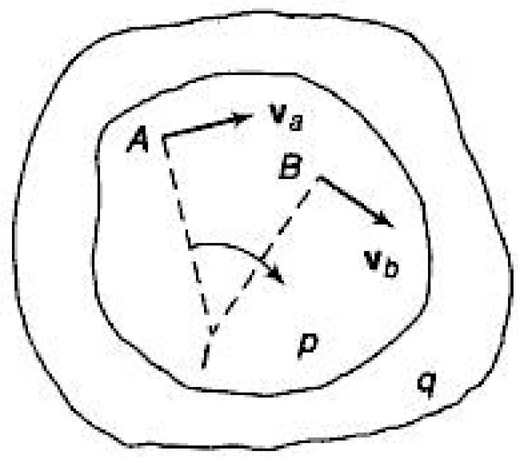
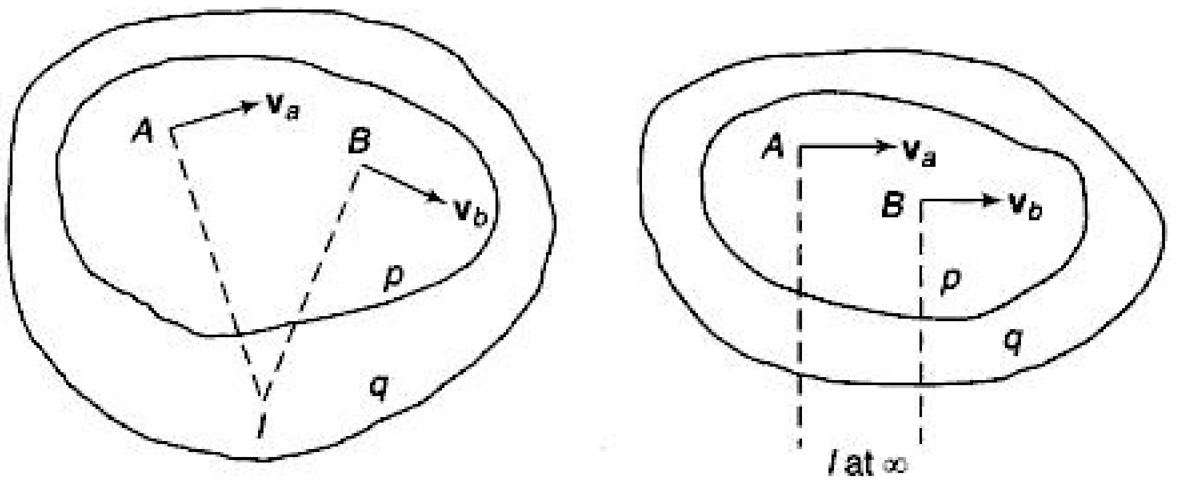


Fig. 2.31



(a) (b)

Fig. 2.32

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3. In a pure rolling contact of the two links, the I-centre lies at the point of contact at the given instant [Fig. 2.37(c)]. It is because the two points of contact on the two bodies have the same linear velocity and thus there is no relative motion of the two at the point of contact which is the I-centre (Refer Sec. 2.12).

2.15 ANGULAR-VELOCITY-RATIO THEOREM

When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre. The velocity of this I-centre relative to a fixed third link is the same whether the I-centre is considered on the first or the second link (Sec. 2.13). First consider the I-centre to be on the first link and obtain the velocity of the I-centre. Then consider the I-centre to be on the second link and find its angular velocity.

For example, if it is required to find the angular velocity of the link 4 when the angular velocity of the link 2 of a four-link mechanism is known, locate the I-centre 24 (Fig. 2.35). Imagine link 2 to be in the form of a flat disc containing point 24 and revolving about 12 or A. Then

$$v_{24} = \omega_2\,(24 - 12)$$

Now, imagine the link 4 to be large enough to contain point 24 and revolving about 14 or D. Then

$$v_{24} = \omega_4\,(24 - 14)$$

or

$$\omega_4 = \frac{v_{24}}{24-14} = \omega_2\left(\frac{24-12}{24-14}\right)$$

or

$$\frac{\omega_4}{\omega_2} = \frac{24-12}{24-14}$$

The above equation is known as the *angular-velocity-ratio theorem*. In words, the angular velocity ratio of two links relative to a third link is inversely proportional to the distances of their common I-centre from their respective centres of rotation.

In the above case, the points 12 and 14 lie on the same side of 24 on the line 24–14 and the direction of rotation of the two links (2 and 4) is the same, i.e., clockwise or counter-clockwise. Had they been on the opposite sides of the common I-centre, the direction would have been opposite.

Example 2.19 *In a slider-crank mechanism, the lengths of the crank and the connecting rod are 200 mm and 800 mm respectively. Locate all the I-centres of the mechanism for the position of the crank when it has turned 30° from the inner dead centre. Also, find the velocity of the slider and the angular velocity of the connecting rod if the crank rotates at 40 rad/s.*

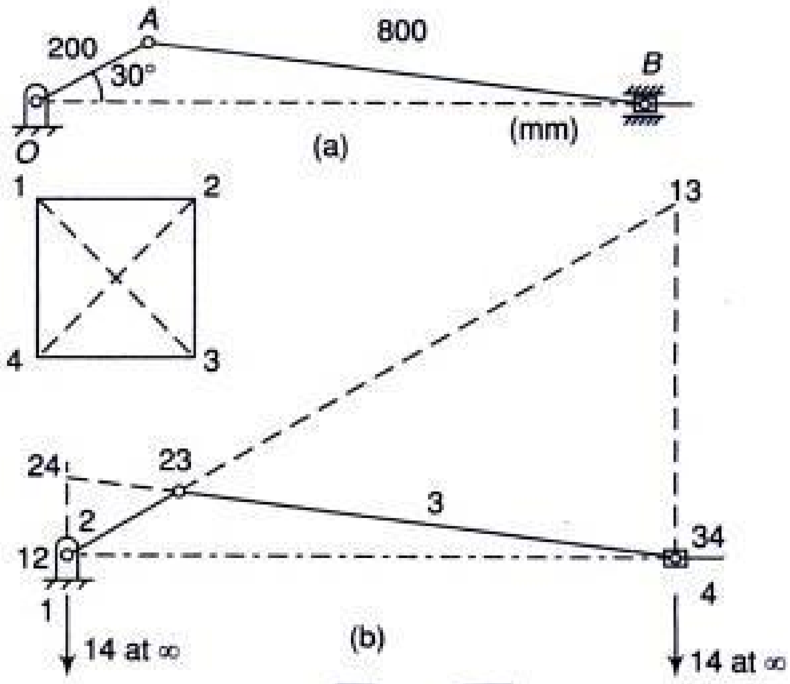

Fig. 2.38

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intersection of lines joining I-centres 12, 16 and 25, 56.

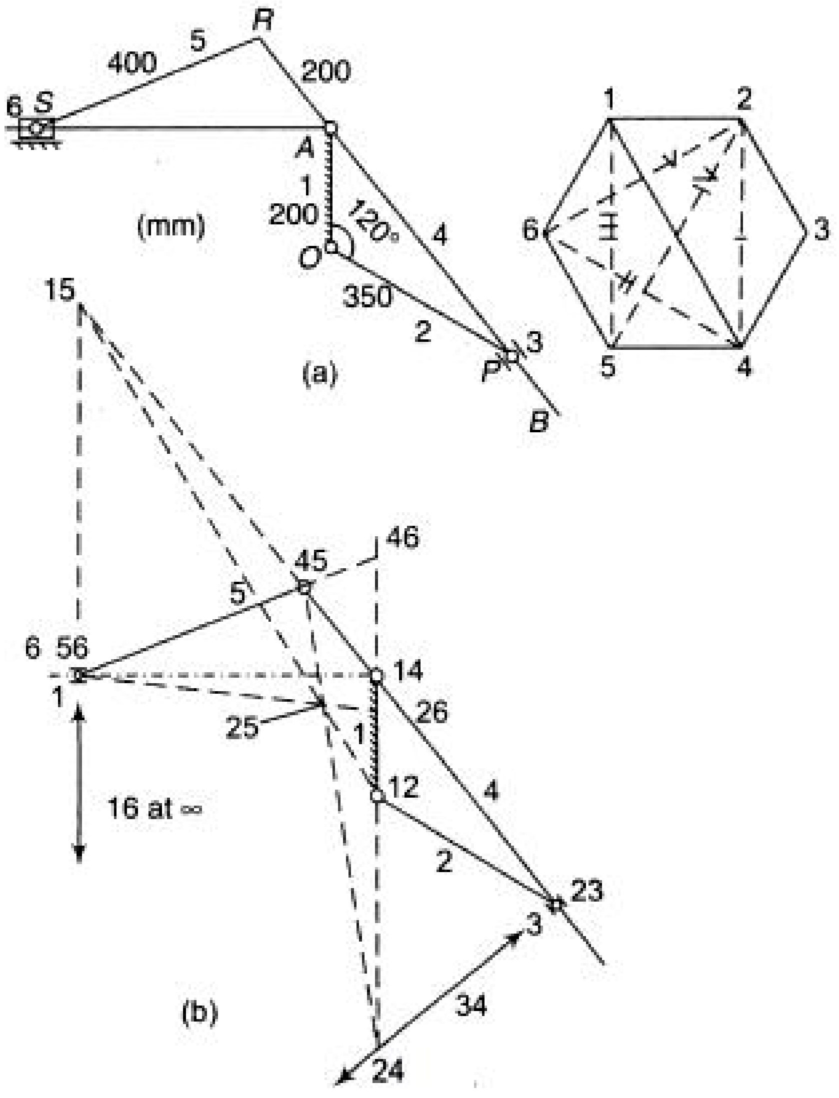

Fig. 2.42

Now, as the velocity of the I-centre 26 is the same whether it is considered to lie on the link 2 or 6,

$$v_{26} = \omega_2 .(12\text{–}26) = v_s$$

or $\quad v_s = \omega_2 .(12\text{–}26) = 10 \times 0.137 = 1.37 \text{ m/s}$

Example 2.24 *Solve Example 2.4 by the instantaneous centre method.*

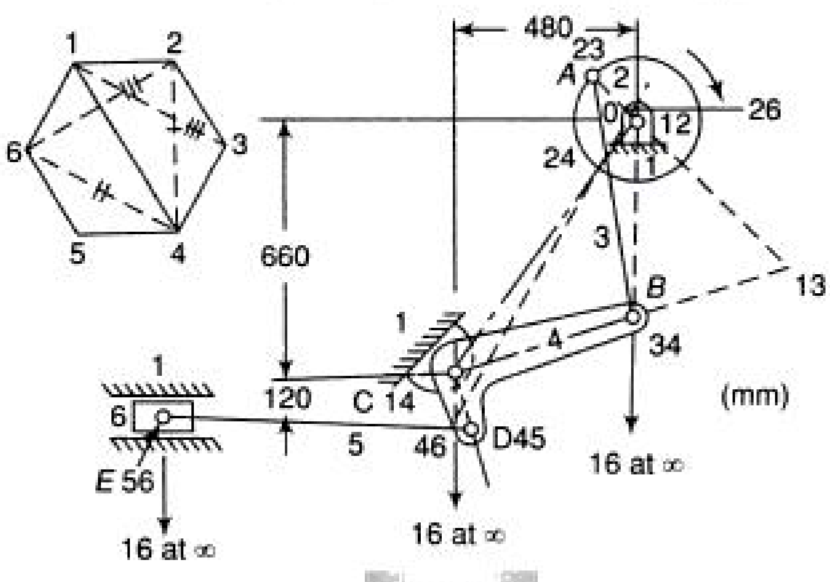

Fig. 2.43

Solution Draw the configuration to a suitable scale as shown in Fig. 2.43.

(a) To find the velocity of E or the link 6, it is required to locate the I-centre 26 as the velocity of a point A on the link 2 is known. After locating I-centres by inspection, locate I-centres 24, 46 and 26 by Kennedy's theorem.

First consider 26 to be on the crank 2.

$v_{26} = \omega\,(12\text{–}26) = 16.76 \times 0.032 = 0.536$ m/s (horizontal)

When the point 26 is considered on the link 6, all points on it will have the same velocity as the point 26.

Velocity of the crosshead = <u>0.536 m/s</u>

(b) (i) To find the velocity of rubbing at A (or 23), ω_2 and ω_3 are required.

Locate I-centre 13. Then

$$\omega_3(23\text{–}13) = \omega_2\,(23\text{–}12)$$

$$\therefore \quad \omega_3 = 16.76 \times \frac{0.17}{0.756}$$

$$= \underline{3.77 \text{ rad/s}}$$

ω_3 is clockwise as 13 and 12 lie on the same side of 23.

Velocity of rubbing at $A = (\omega_3 - \omega_2)\, r_a$

$$= (16.76 - 3.77) \times \frac{0.04}{2} = \underline{0.26 \text{ m/s}}$$

(ii) For velocity of rubbing at B, ω_2 and ω_4 are required. ω_3 was calculated above.

$$\omega_4(34 - 14) = \omega_3(34 - 13)$$

$$\omega_4 = 3.77 \times \frac{0.45}{0.51} = 3.33 \text{ rad/s}$$

ω_4 is counter-clockwise as 14 and 13 lie on the opposite sides of 34 and ω_3 is clockwise.

Thus, velocity of rubbing at B can be calculated.

(iii) As ω_4 is known, the velocity of rubbing at C can be known.

Similarly, locate the I-centre 15 and obtain ω_5 from the relation,

Summary

1. A machine or a mechanism, represented by a skeleton or a line diagram, is commonly knows as a *configuration diagram*.
2. Velocity is the derivative of displacement with respect to time and is proportional to the slope of the tangent to the displacement–time curve at any instant.
3. A vector is a line which represents a vector quantity such as force, velocity and acceleration.
4. The magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.
5. The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link.
6. *Velocity images* are found to be very helpful devices in the velocity analysis of complicated linkages. The order of the letters in the velocity image is the same as in the configuration diagram.
7. The angular velocity of a link about one extremity is the same as the angular velocity about the other.
8. The *instantaneous centre of rotation* of a body relative to another body is the centre about which the body rotates at the instant.
9. In a mechanism, the number of I-centres is given by $N = n(n-1)/2$
10. If three plane bodies have relative motion among themselves, their I-centres must lie on a straight line. This is known as *Kennedy's theorem.*
11. When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre. The velocity of this I-centre relative to a fixed third link is the same whether the I-centre is considered on the first or the second link.
12. A *centrode* is the locus of the I-centre of a plane body relative to another plane body for the range of motion specified or during a finite period of time.
13. The plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.

Exercises

1. What is a configuration diagram? What is its use?
2. Describe the procedure to construct the diagram of a four-link mechanism.
3. What is a velocity image? State why it is known as a helpful device in the velocity analysis of complicated linkages.
4. What is velocity of rubbing? How is it found?
5. What do you mean by the term 'coincident points'?
6. What is *instantaneous centre of rotation?* How do you know the number of *instantaneous centres* in a mechanism?
7. State and prove Kennedy's theorem as applicable to instantaneous centres of rotation of three bodies. How is it helpful in locating various instantaneous centres of a mechanism?
8. State and explain *angular-velocity-ratio theorem* as applicable to mechanisms.
9. What do you mean by *centrode* of a body? What are its types?
10. What are fixed centrode and moving centrode? Explain.
11. Show that the plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.
12. In a slider-crank mechanism, the stroke of the slider is one-half the length of the connecting rod. Draw a diagram to give the velocity of the slider at any instant assuming the crankshaft to turn uniformly.
13. In a four-link mechanism, the crank AB rotates at 36 rad/s. The lengths of the links are AB = 200 mm, BC = 400 mm, CD = 450 mm and AD = 600 mm. AD is the fixed link. At the instant when AB is at right angles to AD, determine the velocity of
    (i) the midpoint of link BC
    (ii) a point on the link CD, 100 mm from the pin connecting the links CD and AD.

    (6.55 m/s; 1.45 m/s)
14. For the mechanism shown in Fig. 2.51, determine the velocities of the points C, E and F and the angular velocities of the links BC, CDE and EF.

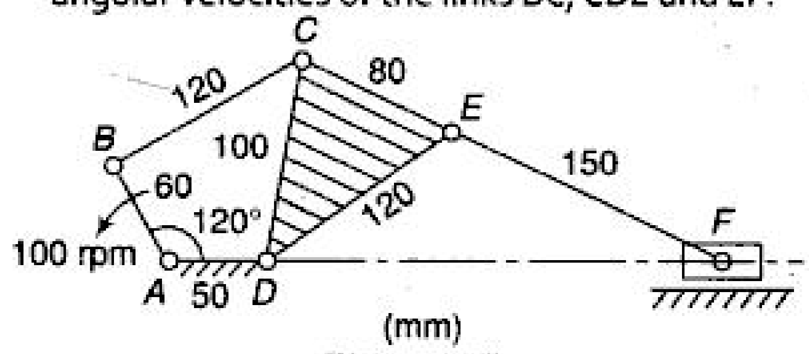

Fig. 2.51

(0.83 m/s; 0.99 m/s; 0.81 m/s; 5.4 rad/s ccw; 8.3 rad/s ccw; 6.33 rad/s ccw)

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Tangential velocity of A', $v'_a = (\omega + \alpha\,\delta t)\,r$

The tangential velocity of A' may be considered to have two components; one perpendicular to OA and the other parallel to OA.

Change of Velocity Perpendicular to OA

Velocity of $A \perp$ to $OA = v_a$

Velocity of $A' \perp$ to $OA = v'_a \cos\delta\theta$

$\therefore$ change of velocity $= v'_a \cos\delta\theta - v_a$

$$\text{Acceleration of } A \perp \text{ to } OA = \frac{(\omega + \alpha.\delta t)r\cos\delta\theta - \omega r}{\delta t}$$

In the limit, as $\delta t \to 0$, $\cos\delta\theta \to 1$

$\therefore$ acceleration of $A \perp$ to $OA = \alpha\,r$

$$= \left(\frac{d\omega}{dt}\right)r \qquad \ldots\left(\alpha = \frac{d\omega}{dt}\right)$$

$$= \frac{dv}{dt} \tag{3.1}$$

This represents the rate of change of velocity in the tangential direction of the motion of A relative to O, and thus is known as the *tangential acceleration* of A relative to O. It is denoted by f^t_{ao}.

Change of Velocity Parallel to OA

Velocity of A parallel to $OA = 0$

Velocity of A' parallel to $OA = v'_a \sin\delta\theta$

$\therefore$ change of velocity $= v'_a \sin\delta\theta - 0$

$$\text{Acceleration of } A \text{ parallel to } OA = \frac{(\omega + \alpha\,\delta t)r\sin\delta\theta}{\delta t}$$

In the limit, as $\delta t \to 0$, $\sin\delta\theta \to \delta\theta$

$$\text{Acceleration of } A \text{ parallel to } OA = \omega r\frac{d\theta}{dt}$$

$$= \omega\,r.\omega \qquad \ldots\left(\omega = \frac{d\theta}{dt}\right)$$

$$= \omega^2 r \tag{3.2}$$

$$= \frac{v^2}{r} \ldots\ldots \quad (v = \omega\,r) \tag{3.3}$$

This represents the rate of change of velocity along OA, the direction being from A towards O or towards the centre of rotation. This acceleration is known as the *centripetal* or the *radial acceleration* of A relative to O and is denoted by f^c_{ao}.

Figure 3.1(b) shows the centripetal and the tangential components of the acceleration acting on A. Note the following:

$$\mathbf{f}_{fb} + \mathbf{f}_{ba} = \mathbf{f}_{fc} + \mathbf{f}_{cd}$$

or
$$\mathbf{f}_{ba} + \mathbf{f}_{fb} = \mathbf{f}_{cd} + \mathbf{f}_{fc}$$

or
$$\mathbf{f_{ba}} + \mathbf{f^c_{fb}} + \mathbf{f^t_{fb}} = \mathbf{f_{cd}} + \mathbf{f^c_{fc}} + \mathbf{f^c_{fc}}$$

or
$$\mathbf{a_1b_1} + \mathbf{b_1f_b} + \mathbf{f_bf_1} = \mathbf{d_1c_1} + \mathbf{f_1f_c} + \mathbf{f_cf_1}$$

The equation can be easily solved graphically as shown in Fig. 3.2(d). $\mathbf{a_1f_1}$ represents the acceleration of F relative to A or D.

2. Writing the vector equation,

$$\mathbf{f}_{fa} = \mathbf{f}_{fb} + \mathbf{f}_{ba}$$
$$= \mathbf{f}_{ba} + \mathbf{f}_{fb}$$
$$= \mathbf{f}_{ba} + \mathbf{f}^c_{fb} + \mathbf{f}^t_{fb}$$

or
$$\mathbf{a_1f_1} = \mathbf{a_1b_1} + \mathbf{b_1f_b} + \mathbf{f_bf_1}$$

$\mathbf{f}_{ba}$ already exists on the acceleration diagram.

$$\mathbf{f^c_{fb}} = \frac{(bf)^2}{BF}, \parallel FB, \text{ direction towards } B.$$

$$\mathbf{f^t_{fb}} = \alpha_{fb} \times FB = \alpha_{cb} \times FB$$

$$= \frac{\mathbf{f}^t_{cb}}{CB} \times FB \perp \text{ to } FB; \text{ direction } \mathbf{b} \text{ to } \mathbf{f}$$

$\alpha_{fb} = \alpha_{cb}$, because angular acceleration of all the points on the link BCF about the point B is the same (counter-clockwise).

$\mathbf{f}_{fa}$ can be found in this way.

3. ***By acceleration image method*** In the previous chapter, it was mentioned that velocity images are useful in finding the velocities of offset points of links. In the same way, *acceleration images* are also helpful to find the accelerations of offset points of the links. The acceleration image of a link is obtained in the same manner as a velocity image. It can be proved that the triangle $\mathbf{b_1c_1f_1}$ is similar to the triangle BCF in Figs 3.2(d) and (a).

Let ω' = angular velocity of the link BCF

α = angular acceleration of the link BCF

Referring to Figs 3.2(a) and 3.3,

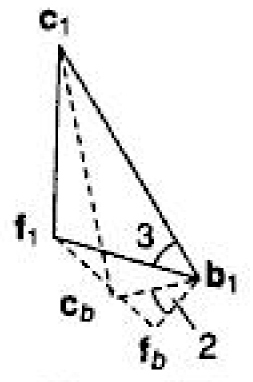

Fig. 3.3

$$\frac{\mathbf{b_1 f_b}}{\mathbf{b_1 c_b}} = \frac{\omega'^2 BF}{\omega'^2 BC} = \frac{BF}{BC} = \frac{\alpha BF}{\alpha BC} = \frac{\mathbf{f_b f_1}}{\mathbf{c_b c_1}}$$

$\mathbf{b_1f_bf_1}$ and $\mathbf{b_1c_bc_1}$ are two right-angled triangles in which the ratio of the two corresponding sides is the same as proved above. Therefore, the two triangles are similar.

$$\frac{\mathbf{b_1 f_1}}{\mathbf{b_1 c_1}} = \frac{BF}{BC} = k$$

Also, $\angle \mathbf{f_b b_1 f_1} = \angle \mathbf{c_b b_1 c_1}$

or
$$\angle \mathbf{f_bb_1f_1} = \angle \mathbf{c_bb_1f_1} = \angle \mathbf{c_bb_1c_1} - \angle \mathbf{c_bb_1f_1}$$

or
$$\angle 3 = \angle 2 = \angle 1 \quad (\because \mathbf{b_1f_b} \parallel BF, \mathbf{b_1c_b} \parallel BC)$$

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Example 3.2 *For the configuration of a slider-crank mechanism shown in Fig. 3.6(a), calculate the*

(i) acceleration of the slider at B
(ii) acceleration of the point E
(iii) angular acceleration of the link AB
OA rotates at 20 rad/s counter-clockwise.

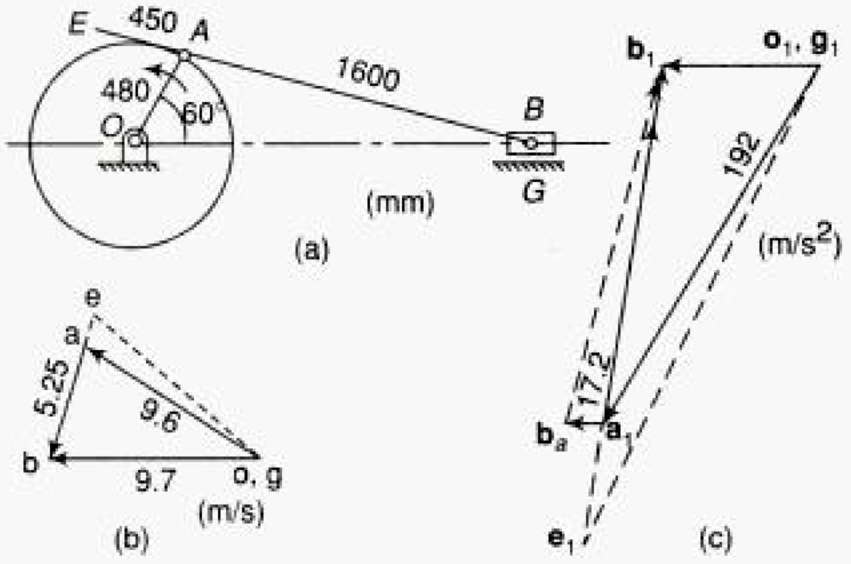

Fig. 3.6

Solution $v_a = 20 \times 0.48 = 9.6$ m/s

Complete the velocity diagram as shown in Fig. 3.6(b).

Writing the vector equation,

$$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao}$$

or $$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba}$$
$$= \mathbf{f}^c_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$$

or $$\mathbf{g_1 b_1} = \mathbf{o_1 a_1} + \mathbf{a_1 b_a} + \mathbf{b_a b_1}$$

Set the vector table (Table 2) as given below.

The acceleration diagram is drawn as follows:

(a) Take the pole point $\mathbf{o_1}$ or $\mathbf{g_1}$ [Fig. 3.6 (c)].

(b) Take the first vector $\mathbf{o_1 a_1}$ and add the second vector.

(c) For the third vector, draw a line ⊥ to *AB* through the head $\mathbf{b_a}$ of the second vector.

(d) For the fourth vector, draw a line ∥ to the line of motion of the slider through $\mathbf{g_1}$. The intersection of this line with the line drawn in the step (d) locates point $\mathbf{b_1}$.

(e) Join $\mathbf{a_1 b_1}$.

(i) $f_b = \mathbf{g_1 b_1} = 72$ m/s²

As the direction of acceleration $\mathbf{f}_b$ is the same as of $\mathbf{v}_b$, this means the slider is accelerating at the instant.

(ii) Locate point $\mathbf{e_1}$ on $\mathbf{b_1 a_1}$ produced such that

$$\frac{\mathbf{b_1 e_1}}{\mathbf{b_1 a_1}} = \frac{\mathbf{BE}}{\mathbf{BA}}$$

$f_e = \mathbf{o_1 e_1} = \underline{236 \text{ m/s}^2}$

(iii) $$\alpha_{ab} = \frac{\mathbf{f}^t_{ba}}{AB} = \frac{\mathbf{b_a b_1}}{AB} = \frac{167}{1.6}$$

$= \underline{104 \text{ rad/s}^2}$ counter-clockwise

Example 3.3 *Figure 3.7(a) shows configuration of an engine mechanism. The dimensions are the following:*

Crank OA = 200 mm; Connecting rod AB = 600 mm; distance of centre of mass from crank end, AD = 200 mm. At the instant, the crank has an angular velocity of 50 rad/s clockwise and an angular acceleration of 800 rad/s². Calculate the

(i) velocity of D and angular velocity of AB
(ii) acceleration of D and angular acceleration of AB

Table 2

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}^c_{ao}$ or $\mathbf{o_1 a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(9.6)^2}{0.48} = 192$	∥ *OA*	→ *O*
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1 b_a}$	$\frac{(\mathbf{ab})^2}{AB} = \frac{(5.25)^2}{1.60} = 17.2$	∥ *AB*	→ *A*
3.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_a b_1}$	-	⊥*AB*	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1 b_1}$	-	∥ to slider motion	-

Links AC and CQ each can have centripetal and tangential components.

$$\mathbf{f}^t_{cq} + \mathbf{f}^t_{cq} = \mathbf{f}^t_{ao} + \mathbf{f}^t_{ca} + \mathbf{f}^t_{ca}$$

or $$\mathbf{q_1\,c_q + c_q\,c_1 = o_1\,a_1 + a_1\,c_a + c_a\,a_1}$$

Set the following vector table (Table 5).

Complete the acceleration vector diagram $\mathbf{o_1a_1c_1q_1}$ as usual [Fig. 3.10(c)].

Draw $\Delta\mathbf{c_1q_1d_1}$ similar to ΔCQD such that both are read in the same sense, i.e., clockwise.

Write the vector equation for the slider-crank mechanism QDB,

$$\mathbf{f}_{bq} = \mathbf{f}_{bd} + \mathbf{f}_{dq}$$

or $$\mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}_{bd}$$

or $$\mathbf{g_1b_1 = q_1d_1 + d_1b_1}$$

From this equation $\mathbf{q_1d_1}$ is already drawn in the diagram and $\mathbf{g_1b_1}$ is a linear acceleration component.

$$\mathbf{f}_{bg} = \mathbf{f_{dq}} + \mathbf{f}^c_{bd} + \mathbf{f}^t_{bd}$$

or $$\mathbf{g_1b_1 = q_1d_1 + d_1b_d + b_db_1}$$

Set the following vector table (Table 6).

Complete the acceleration vector diagram $\mathbf{q_1\,d_1\,b_1\,g_1}$.

(i) $f_g = \mathbf{g_1\,b_1} = 7$ m/s^2 towards left

As the acceleration $\mathbf{f}_b$ is opposite to $\mathbf{v}_b$, the slider is decelerating.

(ii) $$\alpha_{ac} = \frac{\mathbf{f}^t_{ca} \text{ or } \mathbf{c}_a\mathbf{c}_1}{AC} = \frac{13.8}{0.6} = \underline{23 \text{ rad/s}^2}$$

counter-clockwise

Table 5

SN	Vector	Magnitude (m/s^2)	Direction	Sense
1.	$\mathbf{f}_{ao}$ or $\mathbf{o_1\,a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(0.94)^2}{0.15} = 5.92$	$\parallel OA$	$\to O$
2.	$\mathbf{f}^c_{ca}$ or $\mathbf{a_1c_a}$	$\frac{(\mathbf{ac})^2}{AC} = \frac{(1.035)^2}{0.60} = 1.79$	$\parallel AC$	$\to A$
3.	$\mathbf{f}^t_{ca}$ or $\mathbf{c_ac_1}$	-	$\perp AC$	-
4.	$\mathbf{f}^c_{cq}$ or $\mathbf{q_1c_q}$	$\frac{(\mathbf{qc})^2}{QC} = \frac{(1.14)^2}{0.145} = 8.96$	$\parallel QC$	$\to Q$
5.	$\mathbf{f}^t_{cq}$ or $\mathbf{c_qc_1}$	-	$\perp QC$	-

Table 6

SN	Vector	Magnitude (m/s^2)	Direction	Sense
1.	$\mathbf{f}_{dq}$ or $\mathbf{q_1\,d_1}$	Already drawn	-	-
2.	$\mathbf{f}^c_{bd}$ or $\mathbf{d_1b}_d$	$\frac{(\mathbf{db})^2}{DB} = \frac{(0.495)^2}{0.50} = 0.49$	$\parallel DB$	$\to D$
3.	$\mathbf{f}^t_{bd}$ or $\mathbf{b}_d\,\mathbf{b}_1$	-	$\perp DB$	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1\,b_1}$	-	$\parallel$ to slider motion	-

Example 3.8 *An Andrew variable-stroke engine mechanism is shown in Fig. 3.13(a). The crank OA rotates at 100 rpm. Find the*

(i) linear acceleration of the slider at D

(ii) angular acceleration of the links AC, BC and CD.

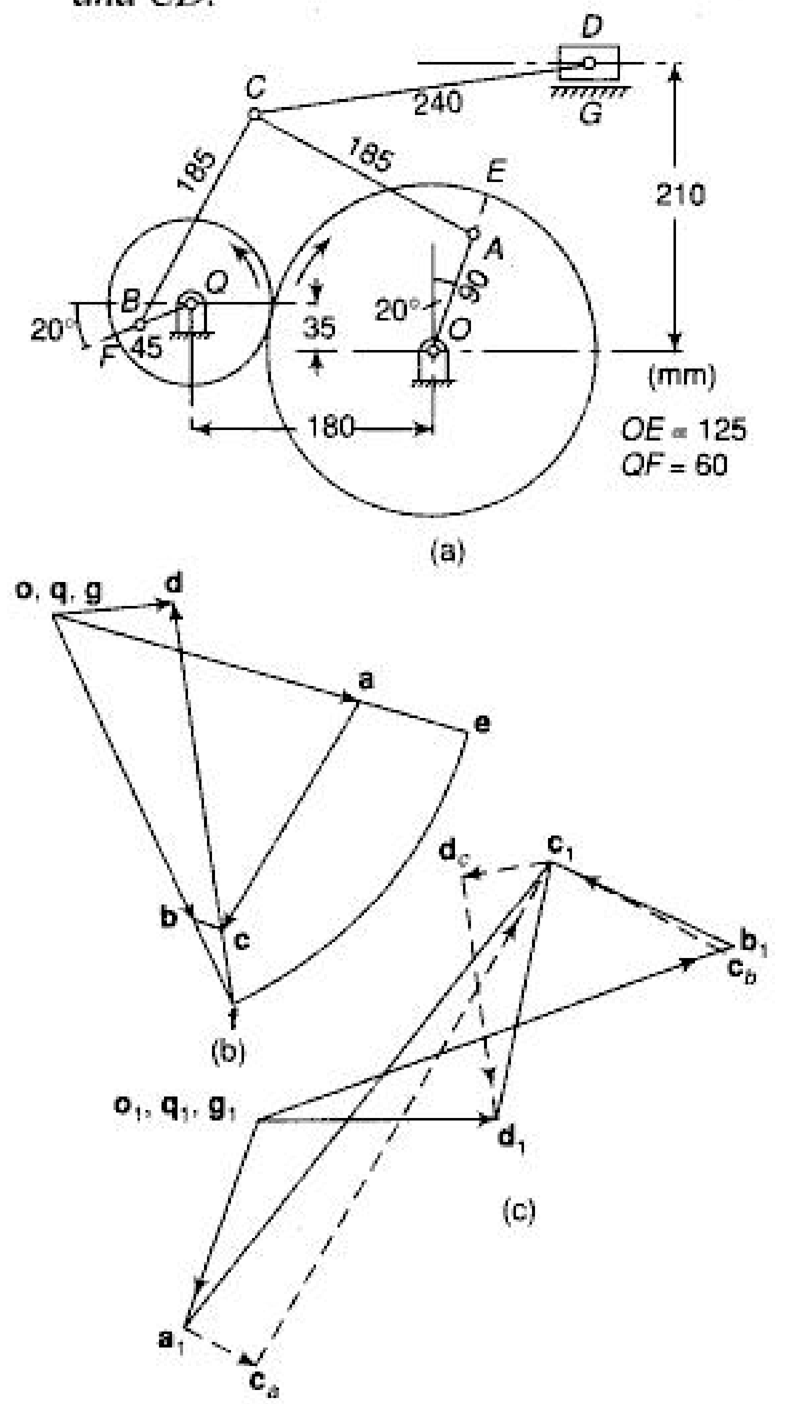

Fig. 3.13

Solution $v_a = \dfrac{2\pi \times 100}{60} \times 0.09 = 0.94$ m/s

Complete the velocity diagram as shown in Fig. 3.13(b). The procedure is explained in Example 2.10.

Write the acceleration vector equation noting that the cranks *OA* and *QB* rotate at different uniform speeds.

For the linkage *OACBQ*,

$\mathbf{f}_{ca} + \mathbf{f}_{ao} = \mathbf{f}_{cb} + \mathbf{f}_{bq}$ or $\mathbf{f}_{ao} + \mathbf{f}_{ca} = \mathbf{f}_{bq} + \mathbf{f}_{cb}$

$\mathbf{o_1\,a_1 + a_1\,c_1 = q_1\,b_1 + b_1\,c_1}$

Links *AC* and *BC* each have two components,

$\mathbf{f}_{ao} + \mathbf{f}^{c}_{ca} + \mathbf{f}^{t}_{ca} = \mathbf{f}_{bq} + \mathbf{f}^{c}_{cb} + \mathbf{f}^{t}_{cb}$

or $\mathbf{o_1\,a_1 + a_1\,c_a + c_a\,c_1 = q_1\,b_1 + b_1\,c_b + c_b\,c_1}$

Set the following vector table (Table 9).

Draw the acceleration diagram as follows:

(a) From the pole point $\mathbf{o_1}$, take the first vector and add the second vector to it as shown in Fig. 3.13 (c).

(b) Through the head $\mathbf{c_a}$ of the second vector, draw a line ⊥ to *AC* for the third vector.

(c) From $\mathbf{q_1}$ (or $\mathbf{o_1}$), take the fourth vector and add the fifth vector to it.

(d) Through the head $\mathbf{c_b}$ of the fifth vector, draw a line ⊥ to *BC* for the sixth vector.

The intersection of the lines drawn in steps (b) and (d) locates the point $\mathbf{c_1}$.

Now,

$\mathbf{f}_{do} = \mathbf{f}_{dc} + \mathbf{f}_{co}$ or $\mathbf{f}_{dg} = \mathbf{f}_{co} + \mathbf{f}_{dc}$

Since $\mathbf{f}_{dc}$ has two components,

$\mathbf{f}_{dg} = \mathbf{f}_{co} + \mathbf{f}^{c}_{dc} + \mathbf{f}^{t}_{dc}$

or $\mathbf{g_1\,d_1 = o_1\,c_1 + c_1\,d_c + d_c\,d_1}$

Set the following vector table (Table 10).

From $\mathbf{c_1}$ draw the second vector and draw a line ⊥ to *CD* through the head of the second vector. Draw a line parallel to the line of motion of the slider through $\mathbf{g_1}$. Thus, the point $\mathbf{d_1}$ is located.

(i) $f_d = \mathbf{o_1 d_1} = \underline{10.65 \text{ m/s}^2}$

(ii) $\alpha_{ac} = \dfrac{\mathbf{f}^{t}_{ca} \text{ or } \mathbf{c_a c_1}}{AC} = \dfrac{26.4}{0.185} = \underline{142.7 \text{ rad/s}^2}$ clockwise

$\alpha_{bc} = \dfrac{\mathbf{f}^{t}_{cb} \text{ or } \mathbf{c_b\,c_1}}{BC} = \dfrac{8.85}{0.185} = \underline{47.8 \text{ rad/s}^2}$ counter-clockwise

$\alpha_{cd} = \dfrac{\mathbf{f}^{t}_{dc} \text{ or } \mathbf{d_c\,d_1}}{CD} = \dfrac{11.2}{0.24} = \underline{46.7 \text{ rad/s}^2}$ clockwise

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Writing the vector equation,

$$\mathbf{f}_{pa} = \mathbf{f}_{pq} + \mathbf{f}_{qa} \qquad \text{or} \qquad \mathbf{f}_{qo} = \mathbf{f}_{qp} + \mathbf{f}_{po}$$

Any of these equations can be solved graphically. Both will lead to the same acceleration diagram except for the direction of the vectors $\mathbf{f}_{pq}$ and $\mathbf{f}_{qp}$.

Considering the first equation,

$$\mathbf{f}_{pa} = \mathbf{f}_{pq} + \mathbf{f}_{qa} \qquad \text{or} \qquad \mathbf{f}_{po} = \mathbf{f}_{qa} + \mathbf{f}_{pq}$$

$$= f_{qa}^{c} + f_{qa}^{t} + f_{pq}^{s} + f_{pq}^{cr}$$

or

$$\mathbf{o}_1\,\mathbf{p}_1 = \mathbf{a}_1\,\mathbf{q}_a + \mathbf{q}_a\,\mathbf{q}_1 + \mathbf{q}_1\,\mathbf{p}_q + \mathbf{p}_q\,\mathbf{p}_1$$

Set the following vector table (Table 11):

The direction is obtained by rotating the vector $\mathbf{v}_{pq}$ (or qp) through 90° in the direction of ω_1 (clockwise in the present case).

Construct the acceleration diagram as follows [Fig. 3.15(c)]:

1. Take the first vector f_{po} which is completely known.
2. Take the second vector from the point a_1 (or o_1). This vector is also completely known.
3. Only the direction of the third vector f_{qa}^{t} is known. Draw a line ⊥ to AQ through the head $\mathbf{q}_a$ of the second vector.
4. As the head of the third vector is not available, the fourth vector cannot be added to it.
   Take the last vector f_{pq}^{cr} which is completely known. Place this vector in the proper direction and sense so that $\mathbf{p}_1$ becomes the head of the vector. In Fig. 3.15(d), $\mathbf{p}_q$ cannot lie on the right side of $\mathbf{p}_3$ because then the vector would become $\mathbf{p}_1\,\mathbf{p}_q$ and not $\mathbf{p}_q\,\mathbf{p}_1$.
5. For the fourth vector, draw a line parallel to AR through the point $\mathbf{p}_q$ of the fifth vector.
   The intersection of this line with the line drawn in the step 3 locates the point $\mathbf{q}_1$.
   Total acc. of P rel. to Q, $\mathbf{f}_{pq} = \mathbf{q}_1\,\mathbf{p}_1$
   Total acc. of Q rel. to A, $\mathbf{f}_{qa} = \mathbf{a}_1\,\mathbf{q}_1$
   The acceleration of R relative to A is given on $\mathbf{a}_1\,\mathbf{q}_1$ produced such that

$$\frac{\mathbf{a}_1\mathbf{r}_1}{\mathbf{a}_1\mathbf{q}_1} = \frac{AR}{AQ}$$

Table 11

SN	Vector	Magnitude	Direction	Sense
1.	$\mathbf{f}_{po}$ or $\mathbf{o}_1\,\mathbf{p}_1$	$\omega \times OP$	$\parallel OP$	$\to O$
2.	$\mathbf{f}_{qa}^{c}$ or $\mathbf{a}_1\,\mathbf{q}_a$	$\dfrac{(\mathbf{aq})^2}{AQ}$	$\parallel AQ$	$\to A$
3.	$\mathbf{f}_{qa}^{t}$ or $\mathbf{q}_a\,\mathbf{q}_1$	-	$\perp AQ$ or $\mathbf{a}_1\,\mathbf{q}_a$	-
4.	$\mathbf{f}_{pq}^{s}$ or $\mathbf{q}_1\,\mathbf{p}_q$	-	$\parallel AR$	-
5.	$\mathbf{f}_{pq}^{cr}$ or $\mathbf{p}_q\,\mathbf{p}_1$	Coriolis component*	$\perp AR$	Refer*

* $f_{pq}^{cr} = 2\omega_1 v_{pq}$ (ω_1 = angular vel. of AR) $= 2\left(\dfrac{\mathbf{aq}}{AQ}\right)qp$

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1. Take the vector **op** equal to 2 m/s to some suitable scale.
2. The velocity of Q relative to P is along the guide path. Therefore, draw a line parallel to this path (vertical) through **p** to locate the point **q**.
3. The velocity of C relative to G is along the guide path at G or is horizontal. Thus, draw a horizontal line through **g** to locate point **c**.
4. Now, Q and C are two fixed points on the same link and the distance between them does not vary. Therefore, the points **q** and **c** in the velocity diagram coincide. Thus, the intersection of lines drawn in steps 2 and 3 locates points **q** or **c**.

Now, $\mathbf{f}^c_{po}$ or $\mathbf{o}_1\,\mathbf{p}_o = \dfrac{(\mathbf{op})^2}{OP} = \dfrac{2^2}{0.2} = 20\ \text{m/s}^2$

$\mathbf{f}^t_{po}$ or $\mathbf{p}_o\,\mathbf{p}_1 = 30 \times 0.2 = 6\ \text{m/s}^2$

Draw acceleration diagram as follows [Fig. 3.18(c)]:

1. First take the centripetal acceleration component $\mathbf{f}^c_{po}$ or $\mathbf{o}_1\mathbf{p}_o$ and add the tangential component $\mathbf{f}^t_{po}$ or $\mathbf{p}_o\mathbf{p}_1$ to it.
2. Now, the linear acceleration of sliding of Q relative to P is vertical. Thus, draw a line to locate point $\mathbf{q}_1$ on that.
3. Draw a horizontal line through $\mathbf{g}_1$ to locate the point $\mathbf{c}_1$ on that.
4. As there is zero velocity between Q and P, they are to be the coinciding points in the acceleration diagram also. Thus, the intersection of lines drawn in steps 2 and 3 locates the point $\mathbf{q}_1$ or $\mathbf{c}_1$.

Acceleration of slider P = $\mathbf{f}_{pq}$ or $\mathbf{q}_1\mathbf{p}_1$ = 4.75 m/s^2

and horizontal acceleration of guide = $\mathbf{f}_{cg}$ or $\mathbf{g}_1\,\mathbf{c}_1 = 20.5\ \text{m/s}^2$

It is to be noted that in this example, Q and P are two coincident points, but still there is no Coriolis component. This is because the link (guide) on which the slider is moving does not have any angular motion and thus ω for that is zero.

Example 3.12 *A Whitworth quick-return mechanism has been shown in Fig. 3.19(a). The dimensions of the links are OP (crank) = 240 mm, OA = 150 mm, AR = 165 mm and RS = 430 mm. The crank OP has an angular velocity of 2.5 rad/s and an angular deceleration of 20 rad/s^2 at the instant. Determine the*

(i) acceleration of the slider S

(ii) angular acceleration of links AR and RS

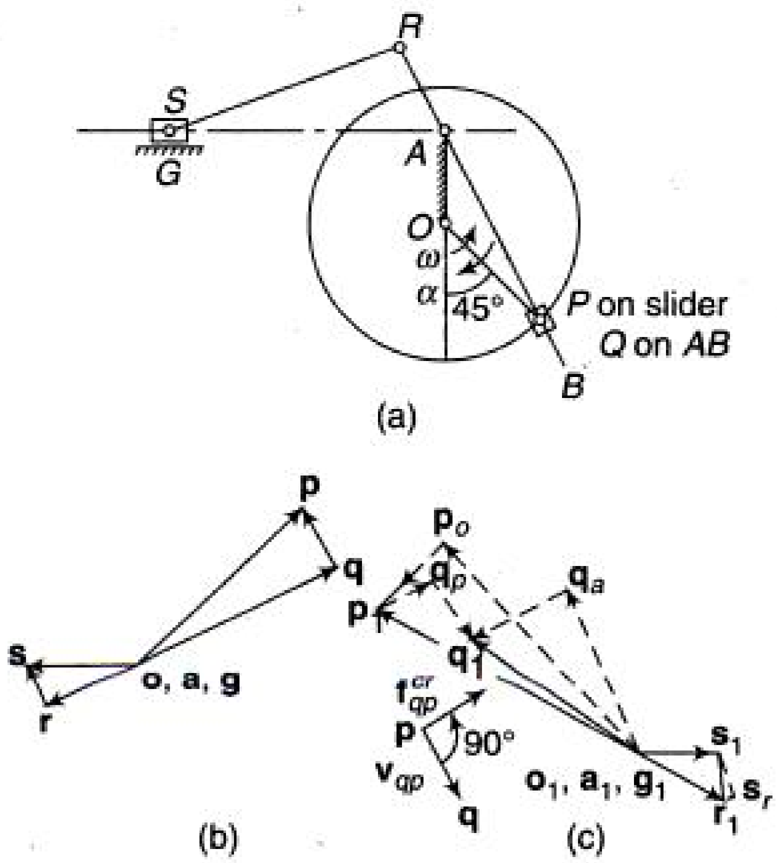

Fig. 3.19

Solution The velocity diagram has been reproduced in Figs 3.19(b) from Fig. 2.25(b).

Writing the acceleration vector equation,

$$\mathbf{f}_{qo} = \mathbf{f}_{qp} + \mathbf{f}_{po} \quad \text{or} \quad \mathbf{f}_{pa} = \mathbf{f}_{pq} + \mathbf{f}_{qa}$$

Both the equations lead to the same acceleration diagram except that the direction sense of the acceleration vectors between P and Q are reversed.

Taking the first one,

$$\mathbf{f}_{qo} = \mathbf{f}_{qp} + \mathbf{f}_{po}$$

$$\mathbf{f}_{qa} = \mathbf{f}_{po} + \mathbf{f}_{qp}$$

or $\quad \mathbf{a}_1\,\mathbf{q}_1 = \mathbf{o}_1\,\mathbf{p}_1 + \mathbf{p}_1\,\mathbf{q}_1$

Each has two components,

$$\mathbf{f}^c_{qa} + \mathbf{f}^t_{qa} = \mathbf{f}^c_{po} + \mathbf{f}^t_{po} + \mathbf{f}^{cr}_{qp} + \mathbf{f}^s_{qp}$$

$$= 2 \times \frac{1.95}{0.16} \times 1.85 = 45.1 \text{ m/s}^2$$

ω_{qe} is found to be counter-clockwise.

The direction for Coriolis component is taken by rotating v_{qe} through 90° in the direction of angular movement of the link *QE* (counter-clockwise in this case). The acceleration diagram is completed as usual.

Acceleration of sliding of link *EF* in the trunnion $= \mathbf{q_o q_1} = \underline{4.86 \text{ m/s}^2}$

This shows that it is downwards or opposite to the velocity. Thus, it is deceleration.

Example 3.15 *In the pump mechanism shown in Fig. 3.22(a), OA = 320 mm, AC = 680 mm and OQ = 650 mm. For the given configuration, determine*

(i) linear (sliding) acceleration of slider C relative to cylinder walls

(ii) angular acceleration of the piston rod

Solution The velocity diagram has been reproduced in Fig. 3.22(b) from Example 2.15.

The problem can be solved by either of the two methods discussed for velocity diagram in Example 2.15.

Writing the acceleration vector equation for the latter configuration,

$$\mathbf{f}_{aq} = \mathbf{f}_{ab} + \mathbf{f}_{bq}$$

or $$\mathbf{f}_{ao} = \mathbf{f}_{bq} + \mathbf{f}_{ab}$$

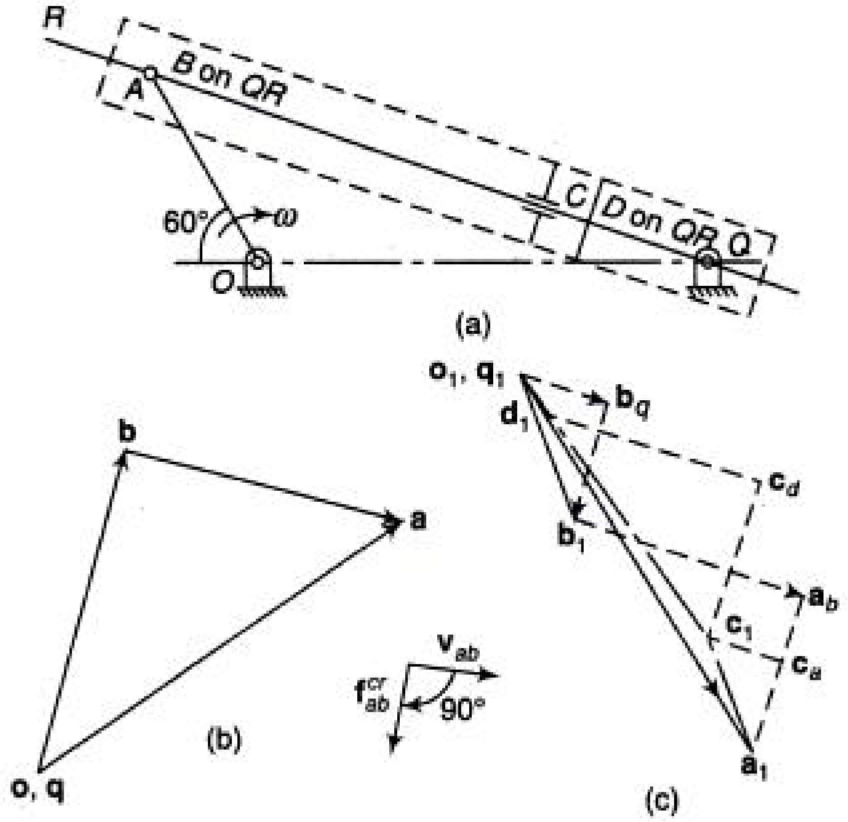

Fig. 3.22

$$= \mathbf{f}^c_{bq} + \mathbf{f}^t_{bq} + \mathbf{f}^s_{ab} + \mathbf{f}^{cr}_{ab}$$

or $$\mathbf{o_1 a_1} = \mathbf{q_1 b_q} + \mathbf{b_q b_1} + \mathbf{b_1 a_b} + \mathbf{a_b a_1}$$

Set the vector table (Table 17)

The direction of $\mathbf{f}^{cr}_{ab}$ is obtained by rotating $\mathbf{v}_{ba}$ through 90° in the direction of ω_{bq} (clockwise).

Draw the acceleration diagram as given below:

1. Take the first vector [Fig. 3.22(c)].
2. From point $\mathbf{q_1}$ (pole point), take the second vector and through the head of it, draw a line perpendicular to *QB* for the third vector.

Table 17

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}_{ao}$ or $\mathbf{o_1 a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(6.4)^2}{0.32} = 128$	$\parallel OA$	$\rightarrow O$
2.	$\mathbf{f}^c_{bq}$ or $\mathbf{q_1 b_q}$	$\frac{(\mathbf{bq})^2}{BQ} = \frac{(4.77)^2}{0.85} = 26.8$	$\parallel QB$	$\rightarrow Q$
3.	$\mathbf{f}^t_{bq}$ or $\mathbf{b_q b_1}$	-	$\perp QB$	-
4.	$\mathbf{f}^s_{ab}$ or $\mathbf{b_1 f_b}$	-	$\parallel QB$	-
5.	$\mathbf{f}^{cr}_{ab}$ or $\mathbf{a_b a_1}$	47.1*	$\perp QB$	-

$$*\mathbf{f}^{cr}_{ab} = 2\omega_{rq} v_{ab} = 2\frac{v_{bq}}{BQ}\mathbf{ba} \quad (\omega_{rq} = \omega_{bq}) = 2 \times \frac{4.77}{0.85} \times 4.2 = 47.1 \text{ m/s}^2$$

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3.8 KLEIN'S CONSTRUCTION

In Klein's construction, the velocity and the acceleration diagrams are made on the configuration diagram itself. The line that represents the crank in the configuration diagram also represents the velocity and the acceleration of its moving end in the velocity and the acceleration diagrams respectively. For a slider-crank mechanism, the procedure to make the Klein's construction is described below.

Slider-Crank Mechanism

In Fig. 3.25, *OAB* represents the configuration of a slider-crank mechanism. Its velocity and acceleration diagrams are as shown in Figs. 3.4(b) and (c). Let r be the length of the crank *OA*.

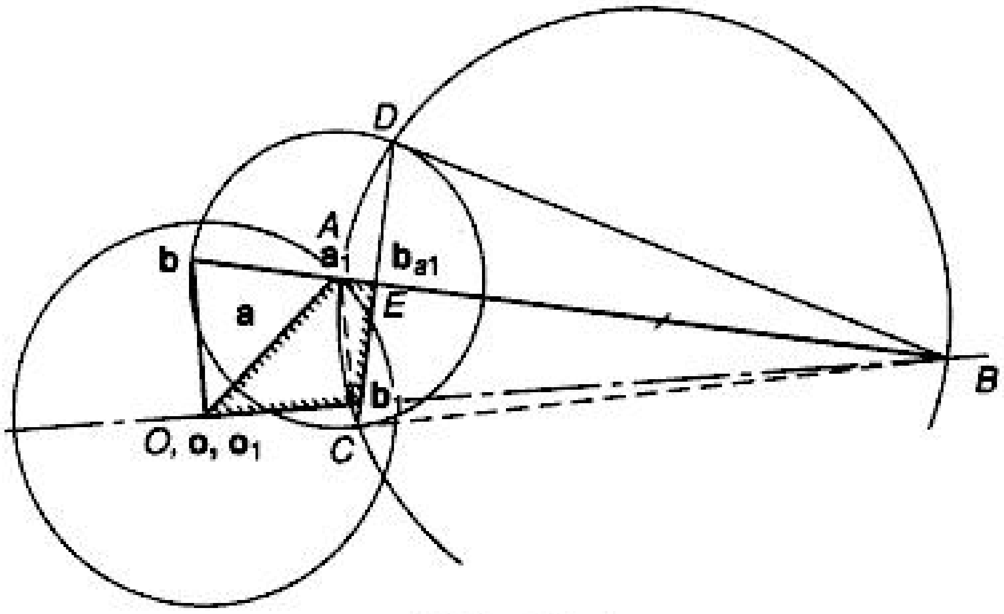

Fig. 3.25

Velocity Diagram For velocity diagram, let r represent v_{ao}, to some scale. Then for the velocity diagram, length $oa = \omega r = OA$.

From this, the scale for the velocity diagram is known.

Produce *BA* and draw a line perpendicular to *OB* through *O*. The intersection of the two lines locates the point **b**. the figure, **oab** is the velocity diagram which is similar to the velocity diagram of Fig. 3.4(b) rotated through 90° in a direction opposite to that of the crank.

Acceleration Diagram For acceleration diagram, let r represent f_{ao}.

$$\therefore \quad \mathbf{o_1\, a_1} = \omega^2 r = OA$$

This provides the scale for the acceleration diagram.

Make the following construction:

1. Draw a circle with **ab** as the radius and **a** as the centre.
2. Draw another circle with *AB* as diameter.
3. Join the points of intersections *C* and *D* of the two circles. Let it meet *OB* at $\mathbf{b_1}$ and *AB* at *E*.

Then $\mathbf{o_1 a_1 b_a b_1}$ is the required acceleration diagram which is similar to the acceleration diagram of Fig. 3.4(c) rotated through 180°.

The proof is as follows:

Join *AC* and *BC*.

AEC and *ABC* are two right-angled triangles in which the angle *CAB* is common. Therefore, the triangles are similar.

$$\frac{AE}{AC} = \frac{AC}{AB} \quad \text{or} \quad AE = \frac{(AC)^2}{AB} \quad \text{or} \quad \mathbf{a_1\, b_a} = \frac{(\mathbf{ab})^2}{AB} = f_{ba}^c$$

Thus, this acceleration diagram has all the sides parallel to that of acceleration diagram of Fig. 3.4(c) and also has two sides $\mathbf{o_1a_1}$ and $\mathbf{a_1b_a}$ representing the corresponding magnitudes of the acceleration. Thus, the two diagrams are similar.

3.12 EULER–SAVARY EQUATION

An analytical expression known as the *Euler–Savary equation* for the location of the conjugate point of A is derived as follows:

If α is the angle between the centrode tangent and the line AP (Fig. 3.27), then as $v = \dfrac{IO'}{OO'} v_o$,

$$u = v\sin\alpha = \frac{IO'}{OO'} v_o . \sin\alpha = \frac{IO'}{OO'} .(\omega . OP)\sin\alpha = \frac{IO'.OI}{OO'} .\omega . \sin\alpha \qquad \text{(i)}$$

Also,

$$u = \frac{IA'}{AA'} v_a = \frac{IA'}{AA'} .(\omega . AI) = \frac{IA'.AI}{AA'} .\omega \qquad \text{(ii)}$$

From (i) and (ii), $\dfrac{IO'.OI}{OO'} .\omega . \sin\alpha = \dfrac{IA'.AI}{AA'} .\omega$

or

$$\frac{AA'}{AI.IA'} \sin\alpha = \frac{OO'}{OI.IO'}$$

or

$$\left(\frac{AI}{AI.IA'} + \frac{IA'}{AI.IA'}\right) \sin\alpha = \frac{OI}{OI.IO'} + \frac{IO'}{OI.IO'}$$

or

$$\left(\frac{1}{IA'} + \frac{1}{AI}\right) \sin\alpha = \frac{1}{IO'} + \frac{1}{OI}$$

or

$$\left(\frac{1}{AI} - \frac{1}{A'I}\right) \sin\alpha = \frac{1}{OI} - \frac{1}{O'I} \qquad \text{(iii)}$$

This is known as one form of the Euler–Savary equation. This is useful to locate the conjugate point A' of the point A when the radii of curvature of the two centrodes are known.

For any other point B at an angle β with the centrode tangent whose conjugate point is B' (Fig. 3.28), the above equation may be written as

$$\left(\frac{1}{BI} - \frac{1}{B'I}\right) \sin\beta = \frac{1}{OI} - \frac{1}{O'I}$$

Let this point be a particular point in the moving centrode such that it satisfies the equation

$$\frac{\sin\beta}{BI} = \frac{1}{OI} - \frac{1}{O'I}$$

This means that the term $1/B'I$ is zero which indicates that the point B is such that its conjugate point lies at infinity on the line joining BI.

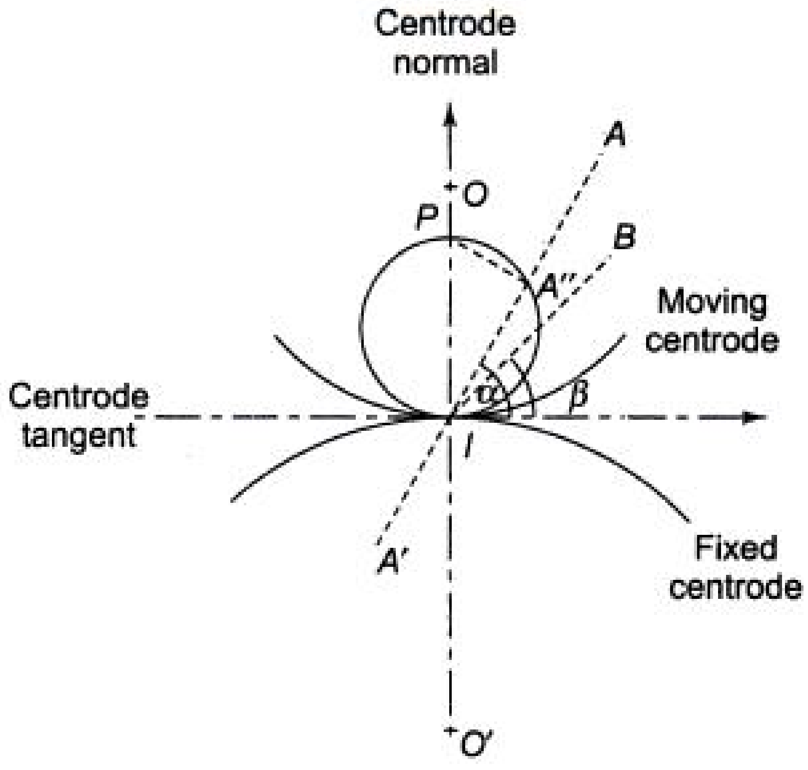

Fig. 3.28

Similarly, for a point P on the centrode normal whose conjugate point is at infinity on the line IO, $\dfrac{1}{PI} = \dfrac{1}{OI} - \dfrac{1}{O'I}$ as angle β is 90° and $\sin\beta$ is 1.

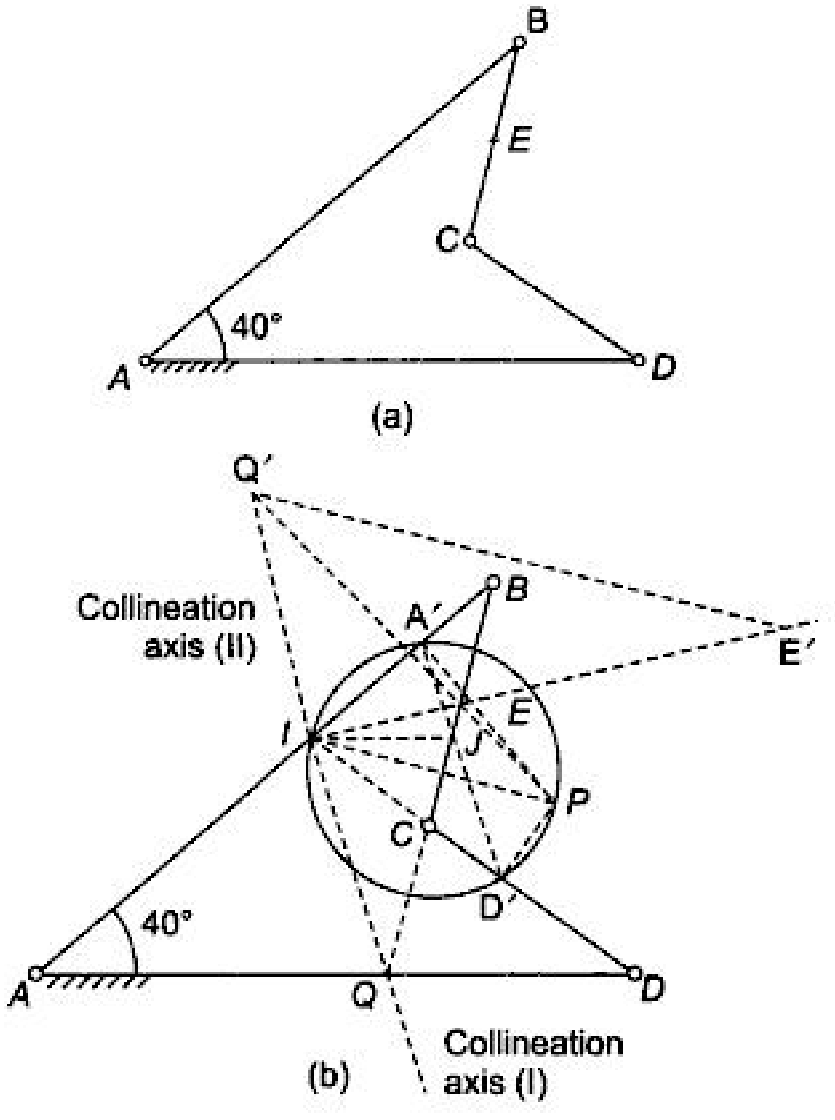

Fig. 3.33

Solution Proceed as follows:

1. Locate points I and Q. Join IQ which is the collineation axis [Fig. 3.33(b)].
2. Draw IJ parallel to AD intersecting BC at J. Draw $A'D'$ through J parallel to IQ and obtain points A' and D' on AB and DC respectively.
3. Through A' draw a perpendicular to AB and through D' draw a perpendicular to DC. Let these perpendiculars intersect at the inflection point P. Draw the inflection circle with IP as the diameter.
4. To find the conjugate point of E, draw the ray IE. Then obtain the new collineation axis by drawing a line perpendicular to IE through I. Locate Q' at the intersection of PE with the collineation axis.
5. Draw a line parallel to IP through Q' intersecting the line IE at E', the requisite conjugate point of E.

On measurement, $EE' = 33$ mm

3.14 CUBIC OF STATIONARY CURVATURE

Usually, the coupler curve (the locus or path of a point on the coupler) is a sixth-order curve whose radius of curvature changes continuously. However, it is observed that in certain situations, the path has a stationary curvature. Thus, if R is the radius of curvature and s is the distance traveled along the path, then $dR/ds = 0$ indicates a stationary curvature of the curve. The locus of all such points on the coupler which have stationary curvature at the instant is known as the *cubic of the stationary curvature* or the *circling-point curve*. Note that the stationary curvature does not mean only a constant radius, but also that the continuously varying radius passes through a maximum or minimum value.

Graphical Method

Let the four-link mechanism be $ABCD$ as shown in Fig. 3.34(a) in which AD is the fixed link. Now, as the link AB can rotate about A only, therefore, A is also the conjugate of B with a constant radius of curvature AB. Thus, A lies on the cubic curve. Similarly, C also lies on the cubic as it has a constant radius of curvature CC'.

Now, adopt the following procedure:

1. Locate points I and Q as usual. Join IQ which is the collineation axis.
2. Let the angle subtended by the ray AB with the collineation axis be α. The same angle is subtended by the other ray CD with the centrode tangent at the point I in the opposite direction. Thus, make angle DIT equal to α with IA in the counter-clockwise direction as the angle made by AB with the collineation axis is clockwise. Then, IT is the centrode tangent.

4 COMPUTER-AIDED ANALYSIS OF MECHANISMS

Introduction

The analyses of the velocity and the acceleration, given in chapters 2 and 3, depend upon the graphical approach and are suitable for finding out the velocity and the acceleration of the links of a mechanism in one or two positions of the crank. However, if it is required to find these values at various configurations of the mechanism or to find the maximum values of maximum velocity or acceleration, it is not convenient to draw velocity and acceleration diagrams again and again. In that case, analytical expressions for the displacement, velocity and acceleration in terms of the general parameters are derived. A desk-calculator or digital computer facilitates the calculation work.

4.1 FOUR-LINK MECHANISM

Displacement Analysis

A four-link mechanism shown in Fig. 4.1 is in equilibrium. *a*, *b*, *c* and *d* represent the magnitudes of the links *AB*, *BC*, *CD* and *DA* respectively. θ, β and φ are the angles of *AB*, *BC* and *DC* respectively with the *x*-axis (taken along *AD*). *AD* is the fixed link. *AB* is taken as the input link whereas *DC* as the output link.

As in any configuration of the mechanism, the figure must enclose, the links of the mechanism can be considered as vectors. Thus, vector displacement relationships can be derived as follows.

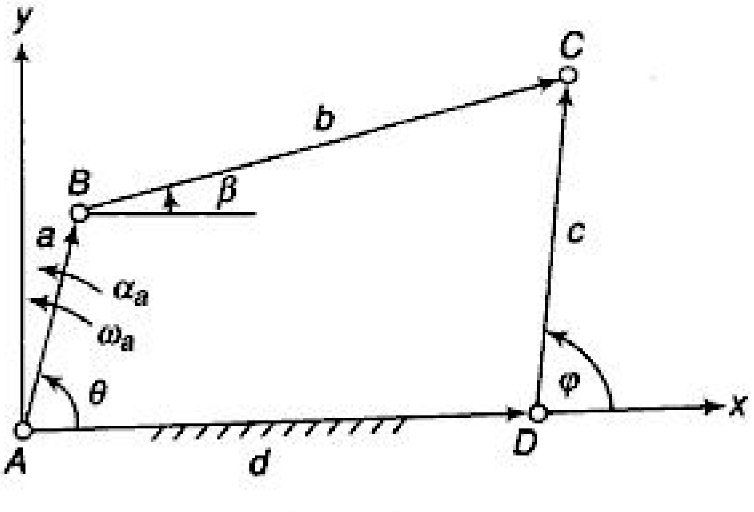

Fig. 4.1

Displacement along x-axis

$$a\cos\theta + b\cos\beta = d + c\cos\phi \tag{4.1}$$

(The equation is valid for $\angle\phi$ more than 90° also.)

or

$$b\cos\beta = c\cos\varphi - a\cos\theta + d$$

or

$$(b\cos\beta)^2 = (c\cos\varphi - a\cos\theta + d)^2$$

$$= c^2\cos^2\varphi + a^2\cos^2\theta + d^2 - 2ac\cos\theta\cos\varphi - 2ad\cos\theta + 2cd\cos\varphi \tag{4.2}$$

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Acceleration Analysis

Let α_a, α_b, and α_c be the angular accelerations of the links a, b and c respectively.

Differentiating equations (4.13) and (4.15) with respect to time in the above manner or rewriting in the following form,

$$-a\,\omega_a \sin\omega_a t - b\omega_b \sin\omega_b t + c\,\omega_c \sin\omega_c t = 0 \tag{4.18}$$

$$a\,\omega_a \cos\omega_a t + b\omega_b \cos\omega_b t - c\,\omega_c \cos\omega_c t = 0 \tag{4.19}$$

Differentiating these equations with respect to time,

$$(-a\alpha_a \sin\theta - a\omega_a^2 \cos\theta) - (b\alpha_b \sin\beta - b\omega_b^2 \cos\beta) + (c\alpha_c \sin\varphi + c\omega_c^2 \cos\varphi) = 0 \tag{4.20}$$

$$(a\alpha_a \cos\theta - a\omega_a^2 \sin\theta) + (b\alpha_b \cos\beta - b\omega_b^2 \sin\beta) - (c\alpha_c \cos\varphi + c\omega_c^2 \sin\varphi) = 0 \tag{4.21}$$

where $$\alpha_a = \frac{d\omega_a}{dt}, \alpha_b = \frac{d\omega_b}{dt} \text{ and } \alpha_c = \frac{d\omega_c}{dt}$$

Multiply Eq. (4.20) by $\cos\varphi$ and Eq. (4.21) by $\sin\varphi$ and add,

$$a\alpha_a (\sin\varphi\cos\theta - \cos\varphi\sin\theta) - a\omega_a^2 (\cos\theta\cos\varphi + \sin\theta\sin\varphi)$$
$$-b\alpha_b (\sin\beta\cos\varphi - \cos\beta\sin\varphi) - b\omega_b^2 (\cos\beta\cos\varphi + \sin\beta\sin\varphi) + c\omega_c^2 = 0$$

or

$$a\alpha_a \sin(\varphi-\theta) - a\omega_a^2 \cos(\varphi-\theta) - b\alpha_b \sin(\beta-\varphi) - b\omega_b^2 \cos(\varphi-\beta) + c\omega_c^2 = 0$$

or

$$\alpha_b = \frac{a\alpha_a \sin(\varphi-\theta) - a\omega_a^2 \cos(\varphi-\theta) - b\omega_b^2 \cos(\varphi-\beta) + c\omega_c^2}{b\sin(\beta-\varphi)} \tag{4.22}$$

Multiply Eq. (4.20) by $\cos\beta$ and Eq. (4.21) by $\sin\beta$ and add,

$$a\alpha_a (\sin\beta\cos\theta - \cos\beta\sin\theta) - a\omega_a^2 (\cos\beta\cos\theta + \sin\beta\sin\theta) - b\omega_b^2$$
$$+c\alpha_c (\sin\varphi\cos\beta - \cos\varphi\sin\beta) + c\omega_c^2 (\cos\beta\cos\varphi + \sin\beta\sin\varphi) = 0$$

or

$$a\alpha_a \sin(\beta-\theta) - a\omega_a^2 \cos(\beta-\theta) - b\omega_b^2 - c\alpha_c \sin(\beta-\varphi) + c\omega_c^2 \cos(\beta-\varphi) = 0$$

or

$$\alpha_c = \frac{a\alpha_a \sin(\beta-\theta) - a\omega_a^2 \cos(\beta-\theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta-\varphi)}{c\sin(\beta-\varphi)} \tag{4.23}$$

which are the same equations as 4.1 and 4.3 and thus can be solved to find β and θ.

Differentiating Eq. (4.25) with respect to t,

$$ia\dot{\theta}\, e^{i\theta} + ib\dot{\beta}\, e^{i\beta} - ic\dot{\varphi}\, e^{i\varphi} = 0 \tag{4.28}$$

or

$$ia\ \omega_a\, e^{i\,\theta} + ib\ \omega_b\, e^{i\,\beta} - ic\ \omega_c\ e^{i\phi} = 0 \tag{4.29}$$

Again, transforming this equation into complex rectangular form and separating the real and imaginary terms,

$$a\ \omega_a \cos\theta + b\omega_b \cos\beta - c\ \omega_c \cos\varphi = 0 \tag{4.30}$$

$$-a\ \omega_a \sin\theta - b\omega_b \sin\beta + c\ \omega_c \sin\varphi = 0 \tag{4.31}$$

which are the same equations as 4.13 and 4.15 and thus can be solved to find ω_b and ω_c.

Differentiating Eq. (4.28) with respect to t,

$$ia(\ddot{\theta}e^{i\theta} + i\dot{\theta}^2 e^{i\theta}) + ib(\ddot{\beta}\ e^{i\beta} + i\dot{\beta}^2 e^{i\beta}) - ic(\ddot{\varphi}e^{i\varphi} + i\dot{\varphi}^2 e^{i\varphi}) = 0 \tag{4.32}$$

or

$$ia(\alpha_a e^{i\theta} + i\omega_a{}^2 e^{i\theta}) + ib(\alpha_b\ddot{\beta}\ e^{i\beta} + i\omega_b{}^2 e^{i\beta}) - ic(\alpha_c e^{i\varphi} + i\omega_c{}^2 e^{i\varphi}) = 0 \tag{4.33}$$

Transforming this equation into complex rectangular form and separating the real and imaginary terms,

$$-a\alpha_a \sin\theta - a\omega_a^2 \cos\theta - b\alpha_b \sin\beta - b\omega_b^2 \cos\beta + c\alpha_c \sin\varphi + c\omega_c^2 \cos\varphi = 0 \tag{4.34}$$

$$a\alpha_a \cos\theta - a\omega_a^2 \sin\theta + b\alpha_b \cos\beta - b\omega_b^2 \sin\beta - c\alpha_c \cos\varphi + c\omega_c^2 \sin\varphi = 0 \tag{4.35}$$

which are the same equations as 4.20 and 4.21 and can be solved as before.

4.3 THE VECTOR METHOD

We have

$$\mathbf{a} + \mathbf{b} - \mathbf{c} - \mathbf{d} = \mathbf{0}$$

Assuming that the angles β and ϕ have been determined by any of the above methods, differentiate the above equation with respect to time,

$$\omega_{\mathbf{a}} \times \mathbf{a} + \omega_{\mathbf{b}} \times \mathbf{b} - \omega_{\mathbf{c}} \times \mathbf{c} = \mathbf{0} \qquad (a,\ b,\ c \text{ and } d \text{ are constants}) \tag{4.36}$$

Let $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ be the unit vectors along **a**, **b** and **c** vectors. In plane–motion mechanisms, all the angular velocities are in the **k** direction. Therefore,

$$a\omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}) + b\ \omega_b\,(\mathbf{k} \times \hat{\mathbf{b}}) - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}}) = \mathbf{0} \tag{4.37}$$

Take the dot product with $\hat{\mathbf{b}}$,

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}} + b\omega_b\,(\mathbf{k} \times \hat{\mathbf{b}})\ \hat{\mathbf{b}} - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{b}} = 0$$

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}} + 0 - c\omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{b}} = 0$$

or

$$\omega_c = -\frac{a\omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}}}{c(\mathbf{k} \times \hat{\mathbf{b}}).\ \hat{\mathbf{b}}} \tag{4.38}$$

Taking the dot product with $\hat{\mathbf{c}}$,

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{c}} + b\ \omega_b\,(\mathbf{k} \times \hat{\mathbf{b}}).\ \hat{\mathbf{c}} - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{c}} = 0$$

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120	10.5	0.0	-145.28	-48.22	-0.77	3.20	26.64	4.54
120	10.5	0.0	123.49	26.44	3.77	-0.20	-0.66	21.44
160	10.5	0.0	-144.69	-36.44	1.12	2.75	29.94	-16.40
160	10.5	0.0	136.77	28.52	2.96	1.32	-22.41	23.99
200	10.5	0.0	-136.77	-28.52	2.96	1.32	22.41	-23.99
200	10.5	0.0	144.69	36.44	1.12	2.75	-29.94	16.40
240	10.5	0.0	-123.49	-26.44	3.77	-0.20	0.66	-21.44
240	10.5	0.0	145.28	48.22	-0.77	3.20	-26.64	-4.54
280	10.5	0.0	-110.16	-29.87	2.92	-1.62	-27.30	22.42
280	10.5	0.0	139.02	58.74	-2.51	2.03	-26.17	-31.04
320	10.5	0.0	-103.82	-38.99	0.07	-3.15	-56.46	-20.96
320	10.5	0.0	126.30	61.47	-4.06	-0.83	-15.50	-50.99

Fig. 4.4

4.4 SLIDER-CRANK MECHANISM

Figure 4.5 shows a slider-crank mechanism in which the strokeline of the slider does not pass through the axis of rotation of the crank. Angle β in clockwise direction from the x-axis is taken as negative.

Let e = eccentricity (distance CD).

Displacement along x-axis,

$$a\cos\theta + b\cos(-\beta) = d \qquad (4.46)$$

or

$$b\cos\beta = d - a\cos\theta \qquad (4.46a)$$

Displacement along y-axis,

$$a\sin\theta + b\sin(-\beta) + e \qquad (4.47)$$

or

$$b\sin\beta = e - a\sin\theta \qquad (4.47a)$$

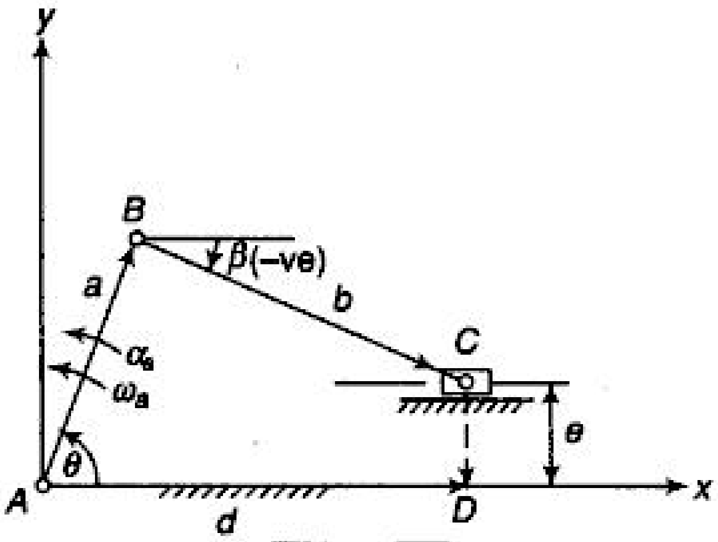

Fig. 4.5

Squaring Eqs (4.46a) and (4.47a) and adding,

$$b^2 = a^2\cos^2\theta + d^2 - 2ad\cos\theta + a^2\sin^2\theta + e^2 - 2ae\sin\theta$$
$$= a^2 + e^2 + d^2 - 2ae\sin\theta - 2ad\cos\theta$$

or

$$d^2 - (2a\cos\theta)d + a^2 - b^2 + e^2 - 2ae\sin\theta = 0$$

or

$$d^2 + C_1 d + C_2 = 0 \qquad (4.48)$$

where $C_1 = -2a\cos\theta$

$C_2 = a^2 - b^2 + e^2 - 2ae\sin\theta$

Equation (4.48) is a quadric in d. Its two roots are,

4.5 COUPLER CURVES

A coupler curve is the locus of a point on the coupler link. A four-link mechanism $ABCD$ with a coupler point E (offset) is shown in Fig. 4.8. Let the x-axis be along the fixed link AD.

Let $BE = e$ and $\angle CBE = \alpha$

Angles β and γ are defined as shown in the diagram.

Let X_e and Y_e be the coordinates of the point E.

Then,

$$X_e = a\cos\theta + e\cos(\alpha+\beta) \qquad (4.59)$$

$$Y_e = a\sin\theta + e\sin(\alpha+\beta) \qquad (4.60)$$

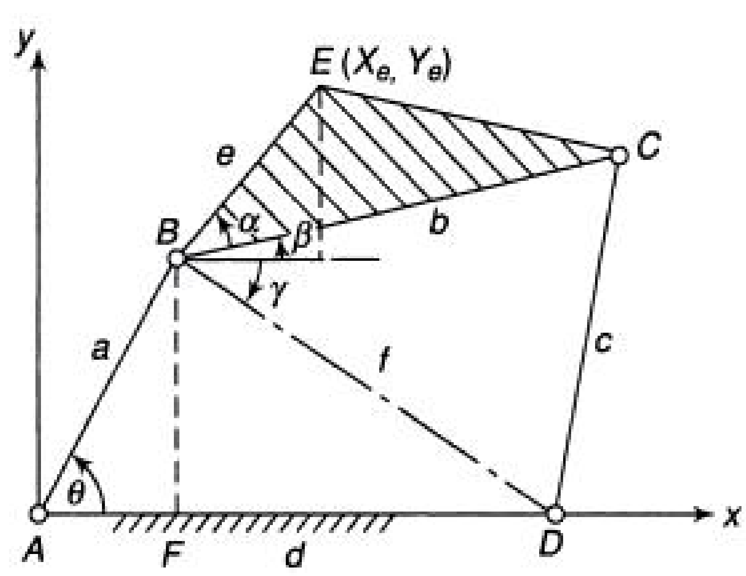

Fig. 4.8

In these equations a, e, θ and α are known. To know the coordinates X_e and Y_e, it is necessary to express β in terms of known parameters, i.e., a, b, c, d, e, θ and α.

In ΔBDC, applying cosine law,

$$\cos(\beta+\gamma) = \frac{b^2+f^2-c^2}{2bf}$$

or

$$\beta+\gamma = \cos^{-1}\left[\frac{b^2+f^2-c^2}{2bf}\right]$$

$$\beta = \cos^{-1}\left[\frac{b^2+f^2-c^2}{2bf}\right] - \gamma \qquad (4.61)$$

where

$$\tan\gamma = \frac{BF}{FD} = \frac{BF}{AD-AF} = \frac{a\sin\theta}{d-a\cos\theta}$$

or

$$\gamma = \tan^{-1}\left[\frac{a\sin\theta}{d-a\cos\theta}\right] \qquad (4.62)$$

f^2 can be found by applying the cosine law to ΔABD,

i.e.,

$$f^2 = a^2 + d^2 - 2ad\cos\theta$$

Having found the value of the angle β, the coordinates of the point E can be known for different values of θ from Eqs (4.59) and (4.60).

A coupler curve can also be obtained in case of a slider-crank mechanism (Fig. 4.9). The angle CBE is α and the eccentricity is e.

Draw $BL \perp AD$ and $CF \perp BL$

$$X_e = a\cos\theta + e\cos(\alpha-\beta) \qquad (4.63)$$

$$Y_e = a\sin\theta + e\sin(\alpha-\beta) \qquad (4.64)$$

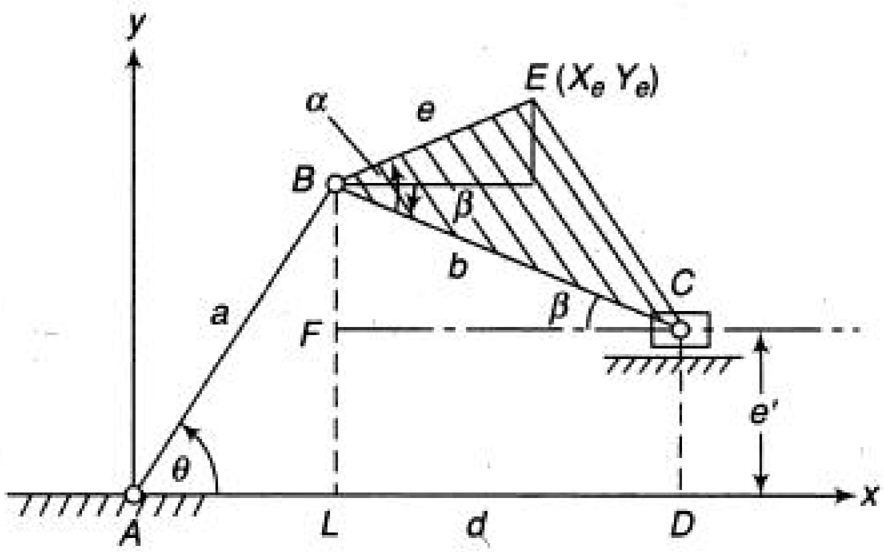

Fig. 4.9

5 GRAPHICAL AND COMPUTER-AIDED SYNTHESIS OF MECHANISMS

Introduction

Dimensional synthesis of a pre-conceived type mechanism necessitates determining the principal dimensions of various links that satisfy the requirements of motion of the mechanism. A mechanism of preconceived type may be a four-link or a slider-crank mechanism. Principal dimensions involve link lengths, angular positions, position of pivots, eccentricities, angle between bell-crank levers and linear distance of sliders, etc. Synthesis of mechanisms may be done by graphical methods or by analytical means that involves the use of calculators and computers. In general, the types of synthesis may be classified as under:

1. *Function generation* It requires correlating the rotary or the sliding motion of the input and the output links. The motion of the output and the input links may be prescribed by an arbitrary function $y = f(x)$. This means if the input link moves by x, the output link moves by $y = f(x)$ for the range $x_o \leq x \leq x_{n+1}$ There lies n values of independent parameters $(x_1, x_2, \ldots..x_n)$ in the range between x_o and $x_{n=1}$. In case of rotary motions of the input and the output links, when the input link rotates through an angle θ, the output link moves through an angle ϕ corresponding to the value of the dependent variable $y = f(x)$. In case of slider-crank mechanism, the output is in the form of displacement s of the slider. It is to be noted that a four-link mechanism can match the function at only a limited number of prescribed points. However, it is a widely used mechanism in the industry since a four-bar is easy to construct and maintain and in most of the cases exact precision at many points is not required.
2. *Path generation* When a point on the coupler or the floating link of a mechanism is to be guided along a prescribed path, it is said to be a path generation problem. This guidance of the path of the point may or may not be coordinated with the movement of the input link and is generally called *with prescribed timing* or *without prescribed timing*.
3. *Motion generation* In this type, a mechanism is designed to guide a rigid body in a prescribed path. This rigid body is considered to be the coupler or the floating link of a mechanism.

If the above tasks are to be accomplished at fewer positions, it is simple to design a mechanism. However, when it is required to synthesize a mechanism to satisfy the input and the output links at larger number of positions, only an approximated solution can be obtained giving least deviation from the specified positions. In this chapter, both graphical as well as analytical methods to design a four-link mechanism and a slider-crank mechanism are being discussed.

PART A: GRAPHICAL METHODS

5.1 POLE

If it is desired to guide a body or link in a mechanism from one position to another, the task can easily be accomplished by simple rotation of the body about a point known as the *pole*. In Fig. 5.1, a link B_1C_1 is

5.3 FUNCTION GENERATION BY RELATIVE POLE METHOD

The problems of function generation for two and three accuracy positions are easily solved by the relative pole method as discussed below:

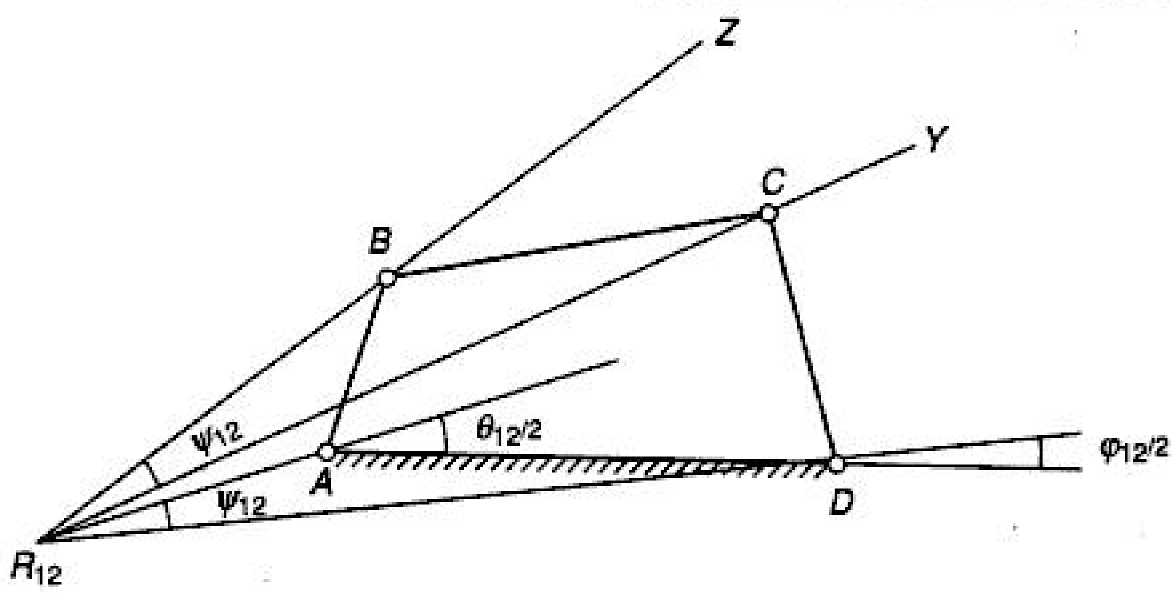

Fig. 5.4

(a) Four-link Mechanisms

Two-position synthesis Let for a four-link mechanism, the positions of the pivots A and D along with the angular displacements θ_{12} (angle between θ_1 and θ_2) and φ_{12} (angle between φ_1 and φ_2) of the driver and the driven links respectively be known.

To design the mechanism (Fig. 5.4), first locate the relative pole R_{12} by the procedure given in Sec. 5.2.

Now, angle subtended by the coupler BC at R_{12}

= angle subtended by the fixed pivots A and D at R_{12}

$$= \frac{1}{2}\angle\theta_{12} - \frac{1}{2}\angle\varphi_{12} \qquad \text{(assuming } DC > AB\text{)}$$

$$= \angle\psi_{12}$$

Adopt any of the following alternatives to design the required mechanism:

1. At point R_{12}, construct an angle ψ_{12} at an arbitrary position. Join any two points on the two arms of the angle to obtain the coupler link BC of the mechanism. Join AB and DC to have the driver and the driven links respectively.
2. Locate the point C arbitrary so that DC is the output link. Construct an angle $CR_{12}Z = \psi_{12}$. Take any point B on $R_{12}Z$. Join AB and BC.
3. Instead of locating the point C as above, locate the point B arbitrary so that AB is the input link. Construct an angle $BR_{12}Y = \psi_{12}$. Take any point C on $R_{12}Y$. Join BC and DC.

Then $ABCD$ is the required four-link mechanism.

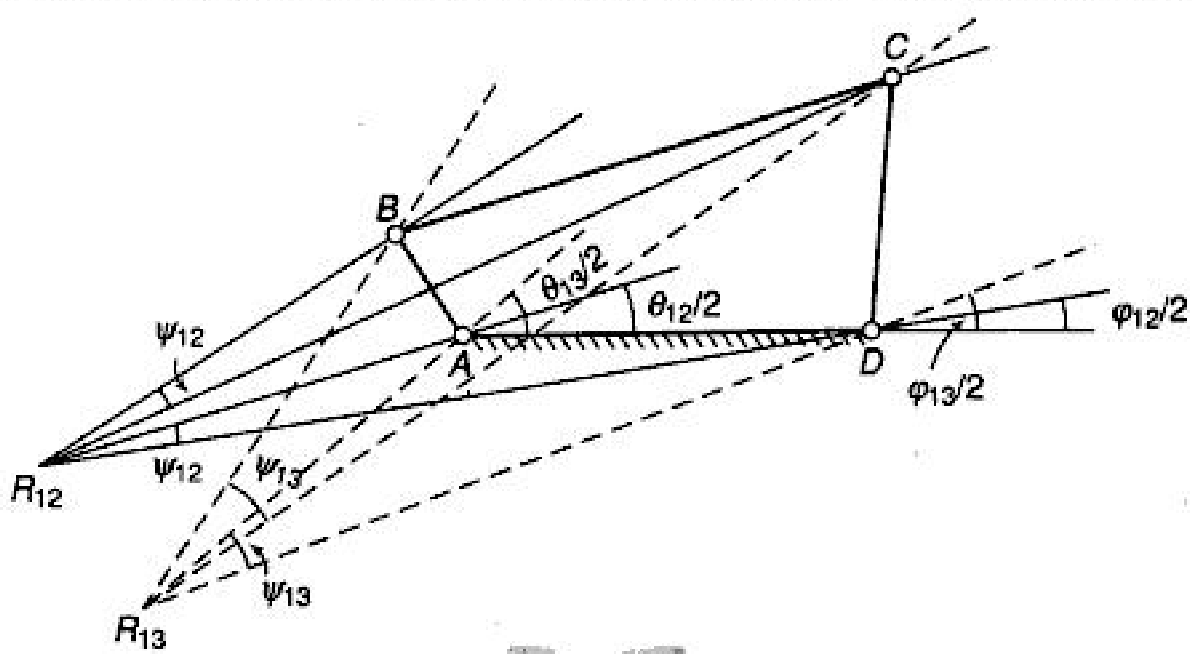

Fig. 5.5

Three-position synthesis If instead of one angular displacement of the input and of the output link, two displacements of the input (θ_{12} and θ_{13}) and two of the output (φ_{12} and φ_{13}) are known, find R_{12} and R_{13} as shown in Fig. 5.5.

Let ψ_{12} and ψ_{13} = angles made by the fixed link at R_{12} and R_{13} respectively.

Construct the angles ψ_{12} and ψ_{13} at the points R_{12} and R_{13} respectively in arbitrary positions such that the arms of the angles intersect at B and C in convenient positions.

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Let the input and the output are related by some function such as $y = f(x)$ and for the specified positions

$\theta_1, \theta_2, \theta_3$ = three positions of input link (given)

and $\varphi_1, \varphi_2, \varphi_3$ = three positions of output link (given)

It is required to find the values of a, b, c and d to form a four-link mechanism giving the prescribed motions of the input and the output links.

Equation (5.1) can be written as,

$$k_1\cos\varphi_1 + k_2 \cos \theta_1 + k_3 = \cos (\theta_1 - \varphi_1)$$
$$k_1\cos\varphi_2 + k_2 \cos \theta_2 + k_3 = \cos (\theta_2 - \varphi_2)$$
$$k_1\cos\varphi_3 + k_2 \cos \theta_3 + k_3 = \cos (\theta_3 - \varphi_3)$$

k_1, k_2, and k_3 can be evaluated by Gaussian elimination method or by the Cramer's rule.

$$\Delta = \begin{vmatrix} \cos\varphi_1 & \cos\theta_1 & 1 \\ \cos\varphi_2 & \cos\theta_2 & 1 \\ \cos\varphi_3 & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \cos(\theta_1-\varphi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2-\varphi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3-\varphi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \cos\varphi_1 & \cos(\theta_1-\varphi_1) & 1 \\ \cos\varphi_2 & \cos(\theta_2-\varphi_2) & 1 \\ \cos\varphi_3 & \cos(\theta_3-\varphi_3) & 1 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \cos\varphi_1 & \cos\theta_1 & \cos(\theta_1-\varphi_1) \\ \cos\varphi_2 & \cos\theta_2 & \cos(\theta_2-\varphi_2) \\ \cos\varphi_3 & \cos\theta_3 & \cos(\theta_3-\varphi_3) \end{vmatrix}$$

k_1, k_2 and k_3 are given by,

$$k_1 = \frac{\Delta_1}{\Delta}; \quad k_2 = \frac{\Delta_2}{\Delta}; \quad k_3 = \frac{\Delta_3}{\Delta}$$

Knowing k_1, k_2 and k_3, the values of a, b, c and d can be computed from the relations

$$k_1 = \frac{d}{a}; \quad k_2 = -\frac{d}{c}; \quad k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

Value of either a or d can be assumed to be unity to get the proportionate values of other parameters.

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()
{
    float a,b,c,p1,p2,p3,t1,t2,t3,th2,th3,a1,a2,a3,del,
    rad,ph1,ph2,ph3,del1,del2,del3;
    clrscr();
    printf("enter values of th1,th2,th3,ph1,ph2,ph3;\n");
    scanf("%f%f%f%f%f%f",&th1,&th2,&th3,&ph1,&ph2,&ph3);
```

a	b	c	d
1.56	0.66	1.33	1.00

Fig. 5.28

The mechanism is shown in Fig. 5.29.

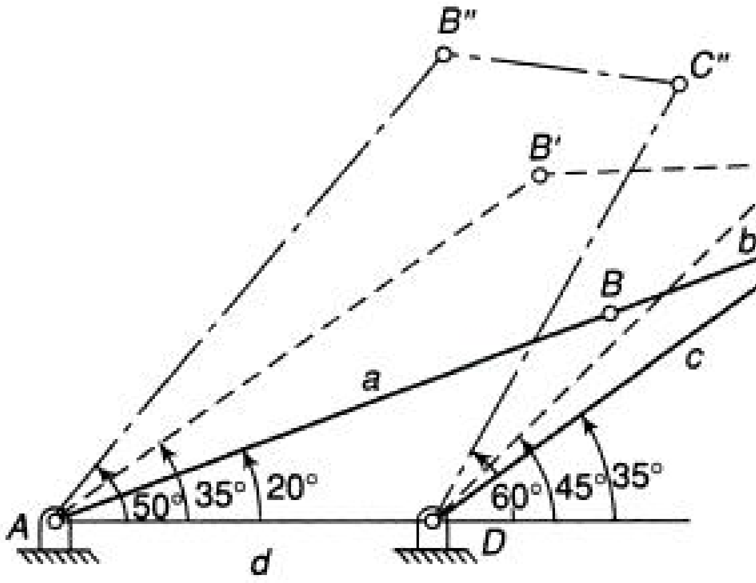

Fig. 5.29

Example 5.8 *Design a four-link mechanism when the motions of the input and the output links are governed by a function $y = x^2$ and x varies from 0 to 2 with an interval of 1. Assume θ to vary from 50° to 150° and ϕ from 80° to 160°.*

Solution The angular displacement of the input link is governed by x whereas that of the output link, by y.

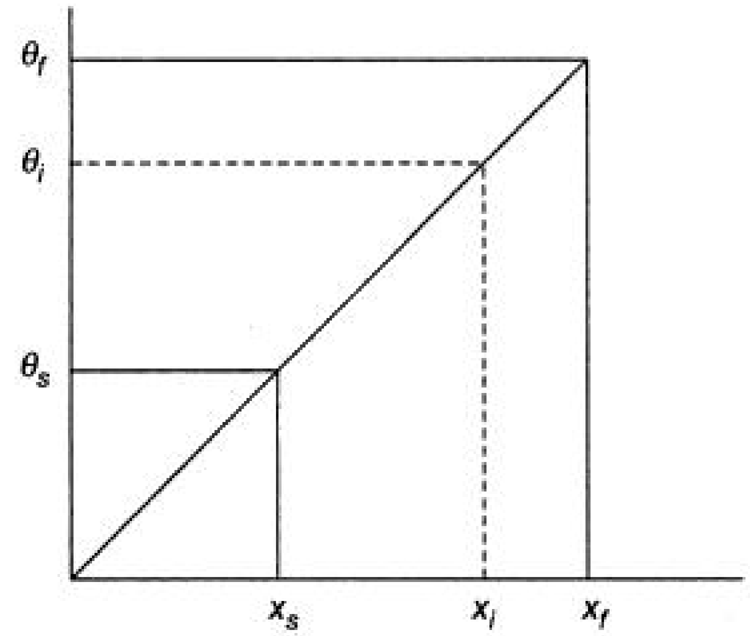

Fig. 5.30

Let subscripts s, f and i indicate the start, final and any value in the range.

Range of $x = x_f - x_s = 2 - 0 = 2$ and thus

$x_1 = 0;\quad x_2 = 1;\quad x_3 = 2$

The corresponding values of y are according to function, $y = x^2$

Range of $y = y_f - y_s = 4 - 0 = 4$ and ;

$y_1 = 0;\quad y_2 = 1;\quad y_3 = 4$

Range of $\theta = \theta_f - \theta_s = 150° - 50° = 100°$

Range of $\phi = \phi_f - \phi_s = 160° - 80° = 80°$

Refer Fig. 5.30 which indicates a linear relationship between x and θ. Thus

$$\frac{\theta_i - \theta_s}{\theta_f - \theta_s} = \frac{x_i - x_s}{x_f - x_s}$$

or

$$\theta_i = \theta_s + \frac{\theta_f - \theta_s}{x_f - x_s}(x_i - x_s) = \theta_s + \frac{\Delta\theta}{\Delta x}(x_i - x_s);$$

Thus, $\theta_1 = 50° + \frac{100°}{2} \times 0 = 50°$;

$\theta_2 = 50° + \frac{100°}{2} \times 1 = 100°$;

$\theta_3 = 50° + \frac{100°}{2} \times 2 = 150°$

Similarly,

$$\varphi_i = \varphi_s + \frac{\varphi_f - \varphi_s}{y_f - y_s}(y_i - y_s) = \varphi_s + \frac{\Delta\varphi}{\Delta y}(y_i - y_s);$$

or $\varphi_1 = 80° + \frac{80°}{4} \times 0 = 80°$;

$\varphi_2 = 80° + \frac{80°}{4} \times 1 = 100°$;

$\varphi_3 = 80° + \frac{80°}{4} \times 4 = 160°$

This can be written in a tabular form:

Position	x	y	θ	φ
1	0	0	50°	80°
2	1	1	100°	100°
3	2	4	150°	160°

Thus, we have the following equations,

$k_1 \cos 80° + k_2 \cos 50° + k_3 \quad = \cos 30°$

$k_1 \cos 100° + k_2 \cos 100° + k_3 = \cos 0°$

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Solution

$$x_i = \frac{x_{n+1}+x_o}{2} - \frac{x_{n+1}-x_o}{2}\cos\frac{(2i-1)\pi}{2n}$$

$$x_1 = \frac{4+1}{2} - \frac{4-1}{2}\cos\frac{(2-1)\pi}{2\times 3} = 2.5 - 1.5\cos\frac{\pi}{6} = 1.2$$

$$x_2 = 2.5 - 1.5\cos\frac{3\pi}{6} = 2.5$$

$$x_3 = 2.5 - 1.5\cos\frac{5\pi}{6} = 3.8$$

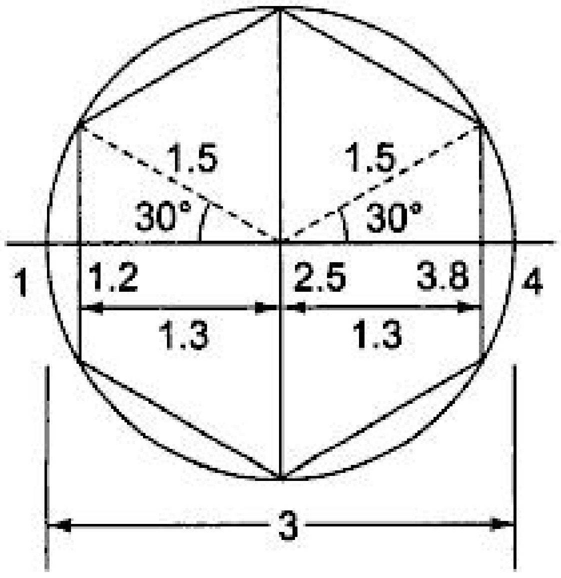

Fig. 5.36

Figure 5.36 shows Chebychev spacing of accuracy points by graphical method.

Let subscripts s and f indicate the start and final values in the range.

The corresponding values of y,

$y_1 = 1.315;\quad y_1 = 3.953;\quad y_1 = 7.408;$

Also, $y_s = 1^{1.5} = 1$; and $y_f = 4^{1.5} = 8$

Range of $x = x_f - x_s = 4 - 1 = 3$

Range of $y = y_f - y_s = 8 - 1 = 7$

$$\theta_1 = 30^\circ + \frac{120^\circ - 30^\circ}{4-1}(1.2-1) = 36^\circ;$$

$$\varphi_1 = 60^\circ + \frac{130^\circ - 60^\circ}{8-1}(1.315-1) = 63.2^\circ;$$

$$\theta_2 = 30^\circ + \frac{90^\circ}{3}(2.5-1) = 75^\circ;$$

$$\varphi_2 = 60^\circ + \frac{70^\circ}{7}(3.953-1) = 89.5^\circ;$$

$$\theta_3 = 30^\circ + \frac{90^\circ}{3}(3.8-1) = 114^\circ;$$

$$\varphi_3 = 60^\circ + \frac{70^\circ}{7}(7.408-1) = 124.1^\circ;$$

Now, $k_1\cos 63.2^\circ + k_2\cos 36^\circ + k_3 = \cos(36^\circ - 63.2^\circ) = \cos 27.2^\circ$

$k_1\cos 89.5^\circ + k_2\cos 75^\circ + k_3 = \cos(75^\circ - 89.5^\circ) = \cos 14.5^\circ$

$k_1\cos 124.1^\circ + k_2\cos 114^\circ + k_3 = \cos(114^\circ - 124.1^\circ) = \cos 10.1^\circ$

Solving by Cramer's rule,

$k_1 = 2.286$; $k_2 = -1.98$; $k_3 = 1.461$

Now, $d = 30$ mm

$k_1 = \dfrac{30}{a} = 2.286$ or $a = 13.1$ mm

$k_2 = -\dfrac{30}{c} = -1.98$ or $c = 15.2$ mm

$k_3 = \dfrac{13.1^2 - b^2 + 15.2^2 + 30^2}{2\times 13.1\times 15.2}$ or $b = 26.8$ mm

Thus, a, b, c and d are 13.1 mm, 26.5 mm, 15.2 mm and 30 mm respectively.

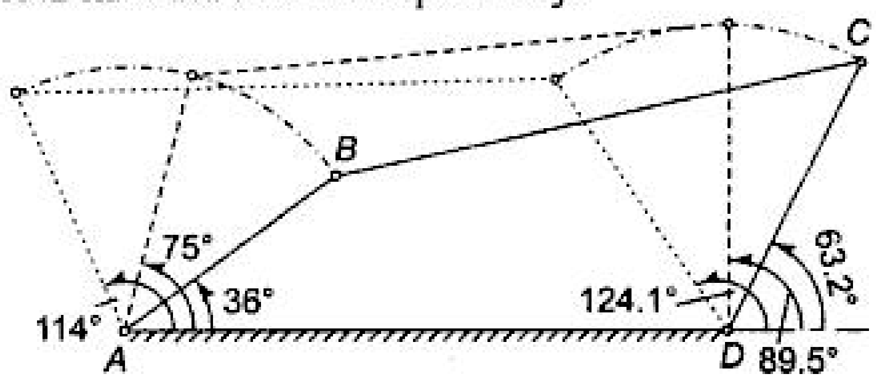

Fig. 5.37

The mechanism is shown in Fig. 5.37 in three positions.

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```
    ak2=del2/del;
    ak3=del3/del;
    if(k==0)
    {
          ala=ak1;
          alg=ak2;
          alk=ak3;
          c1=2*cos(t11-gg);
          c2=2*cos(t22-gg);
          c3=2*cos(t33-gg);
          }
    }
   ama=ak1;
   amg=ak2;
   amk=ak3;
   aa=ama*amg;
   bb=ala*amg+alg*ama-1;
   cc=ala*alg;
   squ=bb*bb-4*aa*cc;
   if(squ>0)
    {
    all=sqrt(squ);
    all=(-bb-all)/(2*aa);
    al2=(-bb+all)/(2*aa);
    a1=ala+all*ama;
    g1=alg+all*amg;
    a2=ala+al2*ama;
    g2=alg+al2*amg;
    e1=sqrt(alk+all*amk+a1*a1+g1*g1);
    e2=sqrt(alk+al2*amk+a2*a2+g2*g2);
    if(j==0){printf("   g      a      e");
    printf("     h      c      f\n"); }
    if(j==1){printf("%8.2f %8.2f %8.2f %8.2f %8.2f %8.2f
    \n",g12,a12,e12,g1,e1,a1);
    printf("%8.2f %8.2f %8.2f %8.2f %8.2f %8.2f\n",
    g12,a12,e12,g2,e2,a2); }
    if (j==2) {printf("%8.2f %8.2f %8.2f %8.2f %8.2f
    %8.2f \n",g21,a21,e21,g1,e1,a1);
    printf("%8.2f %8.2f %8.2f %8.2f %8.2f %8.2f \n",
    g21,a12,e21,g2,e2,a2); }
     if(j==0)
     {
          g12=g1;
          a12=a1;
          e12=e1;
          g21=g2;
          a21=a2;
          e21=e2;
          gs=gg;
          p11=t11;
```

from Eq. (5.5) and (5.6) instead of β. The equations formed are exactly the same if θ is replaced by β in Eqs (5.11) and (5.12). Also, as β is directly known, there is no need of using Eq. 5.16.

The program given in Fig. 5.43 solves this type of problem. The input variables are

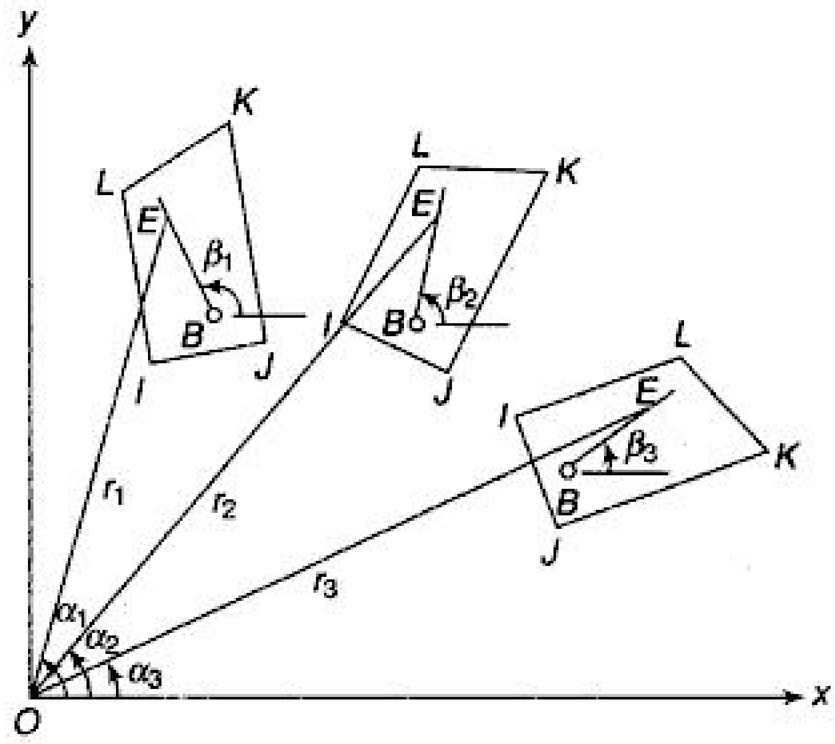

Fig. 5.42

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main ( )
 {
  FILE *fp;
  int k,j;
  float a1,a2,a3,a11,a22,a33,a12,a21,g12
  ,g21,e12,e21,ak1,
  ak2,ak3,al1,al2,al3,aa,g1,g2,g3,t11,t
  22,t33,tb1,tb2,
  tb3,gg,gamm,ss,si,d11,d22,d33,r1,r2,r
  3, p1,p2,p3,t1,t2,
  t3,c1,c2,c3,alg,ala,alk,ama,amg,amk,bb,cc,all,e1,e2,
  squ,bet1,bet2,bet3,p11,p22,p33,e3,gs,del,del1,del2,
  del3,rad;
  clrscr( ) ;
  printf("Enter values of tb1,tb2,tb3,r1,r2,r3,al1,al1,");
  printf("al2,al3,gamm,si,del1\n");
  scanf("%f%f%f%f%f%f%f%f%f%f%f%f",&tb1,&tb2,&tb3,&r1,&r2,
  &r3,&al1,&al2,&al3,&gamm,&si,&del1);
  rad=4*atan(1)/180;
  t11=tb1*rad;
  t22=tb2*rad;
  t33=tb3*rad;
  a11=al1*rad;
  a22=al2*rad;
  a33=al3*rad;
  gg=gamm*rad;
  ss=si*rad;
  d11=del1*rad;
  for (j=0; j<3; j++)
  {
   p1=2*r1*cos(t11-a11);
   p2=2*r2*cos(t22-a22);
   p3=2*r3*cos(t33-a33);
   t1=2*r1*cos(a11-gg);
   t2=2*r2*cos(a22-gg);
   t3=2*r3*cos(a33-gg);
   c1=r1*r1;
   c2=r2*r2 ;
   c3=r3*r3;
    for(k=0;k<2;k++)
```

3. What is the pole of a coupler link of four-link mechanism? Enumerate its properties. What is a relative pole?
4. Describe the procedure to design a four-link mechanism by relative pole method when three positions of the input $(\theta_1, \theta_2, \theta_3)$ and the output link $(\varphi_1, \varphi_2, \varphi_3)$ are known.
5. Describe the procedure to design a slider-crank mechanism by relative pole method when three positions of the input link $(\theta_1, \theta_2, \theta_3)$ and the slider (s_1, s_2, s_3) are known.
6. Discuss the procedure to design the mechanisms by inversion method.
7. What is *Freudenstein's* equation? How is it helpful in designing a four-link mechanism when three positions of the input $(\theta_1, \theta_2, \theta_3)$ and the output link $(\varphi_1, \varphi_2, \varphi_3)$ are known?
8. What is least-square technique? When is it useful in designing a four-link mechanism?
9. What do you mean by *precision* or *accuracy* points in the design of mechanisms? What is *structural error*?
10. What is *Chebychev spacing*? What is its significance?
11. Design a four-link mechanism to coordinate three positions of the input and the output links for the following angular displacements using relative pole method:

    $\theta_{12} = 50°$ $\quad\varphi_{12} = 40°$
    $\theta_{13} = 70°$ $\quad\varphi_{13} = 75°$
12. Design a slider-crank mechanism to coordinate three positions of the input link and the slider for the following angular and linear displacements of the input link and the slider respectively:

    $\theta_{12} = 30°$ $\quad s_{12} = 100$ mm
    $\theta_{13} = 90°$ $\quad s_{13} = 200$ mm

    Take eccentricity of the slider as 10 mm. Use the relative pole method.
13. In a four-link mechanism, the angular displacements of the input link are 30° and 75° and of the output link, 40° and 65° respectively. Design the mechanism using the inversion method.
14. Design a slider-crank mechanism to coordinate three positions of the input and of the slider when the angular displacements of the input link are 40° and 75° and linear displacements of the slider are 55 mm and 90 mm respectively with an eccentricity of 20 mm. Use the inversion method.
15. For the following angular displacements of the input and the output links, design a four-link mechanism:

    $\theta_{12} = 40°$ $\quad\varphi_{12} = 45°$
    $\theta_{13} = 85°$ $\quad\varphi_{13} = 75°$
    $\theta_{13} = 120°$ $\quad\varphi_{13} = 110°$
16. Design a four-link mechanism that coordinates the following three positions of the coupler point if the positions are indicated with respect to coordinate axes:

    $r_1 = 60$ mm $\quad\alpha_1 = 75°$
    $r_2 = 75$ mm $\quad\alpha_2 = 60°$
    $r_3 = 85$ mm $\quad\alpha_3 = 50°$

    The angular displacements of the input link are $\theta_{12} = 40°$ and $\theta_{13} = 75°$.
17. Design a four-link mechanism to coordinate three positions of the input and the output links given by

    $\theta_1 = 25°$ $\quad\varphi_1 = 30°$
    $\theta_2 = 35°$ $\quad\varphi_2 = 40°$
    $\theta_3 = 50°$ $\quad\varphi_3 = 60°$ (5.6, 0.17, 4.88, 1)
18. Design a four-link mechanism when the motions of the input and the output links are governed by the function $y = 2x^2$ and x varies from 2 to 4 with an interval of 1. Assume θ to vary from 40° to 120° and ϕ from 60° to 132°. (1.73, 0.70, 1.78, 1.00)
19. Design a four-link mechanism to coordinate the motions of the input and the output links governed by a function $y = 2 \log x$ for $2 < x < 12$. Take $\Delta x = 1$. Assume suitable ranges for θ and φ.
20. Design a four-link mechanism if the motions of the input and the output links are governed by a function $y = x^{1.5}$ and x varies from 1 to 4. Assume θ to vary from 30° to 120° and φ from 60° to 130°. The length of the fixed link is 30 mm. Use Chebychev spacing of accuracy points.
21. Design a four-link mechanism to guide a rigid body through three positions of the input link with three positions of the coupler point, the data for which is given below:

$\theta_1 = 40°$	$r_1 = 90$ mm	$\alpha_1 = 78°$
$\theta_2 = 55°$	$r_2 = 40$ mm	$\alpha_2 = 90°$
$\theta_3 = 70°$	$r_3 = 75$ mm	$\alpha_3 = 95°$

22. Design a four-link mechanism, the coupler point of which traces a coupler curve that is approximated by ten positions given by the following data

$\theta_1 = 160°$	$r_1 = 57$ mm	$\alpha_1 = 70°$
$\theta_2 = 130°$	$r_2 = 76$ mm	$\alpha_2 = 65°$
$\theta_3 = 98°$	$r_3 = 88$ mm	$\alpha_3 = 55°$
$\theta_4 = 73°$	$r_4 = 98$ mm	$\alpha_4 = 45°$
$\theta_5 = 32°$	$r_5 = 92$ mm	$\alpha_5 = 30°$
$\theta_6 = -15°$	$r_6 = 89$ mm	$\alpha_6 = 20°$
$\theta_7 = -25°$	$r_7 = 82$ mm	$\alpha_7 = 19°$
$\theta_8 = -70°$	$r_8 = 53$ mm	$\alpha_8 = 25°$
$\theta_9 = -125°$	$r_9 = 38$ mm	$\alpha_9 = 50°$
$\theta_{10} = -165°$	$r_{10} = 42$ mm	$\alpha_{10} = 70°$

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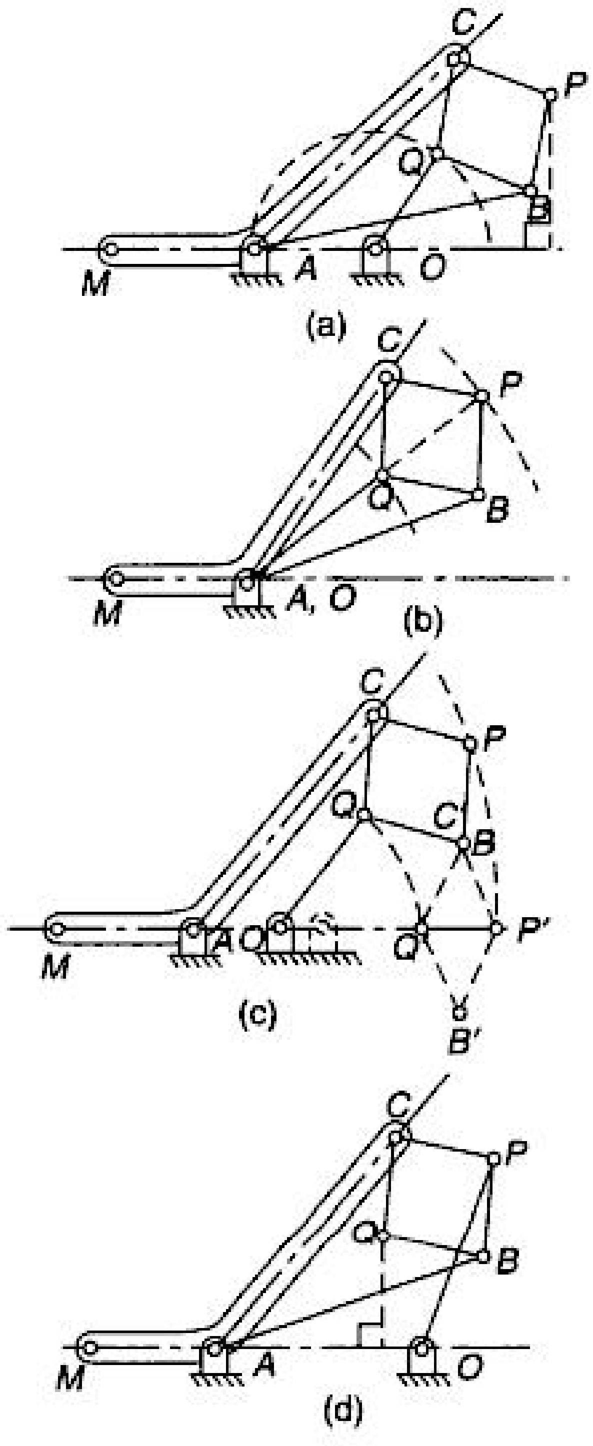

Fig. 6.3

Solution

(i) As in the Paucellier mechanism, O is located by drawing a straight line through A and perpendicular to the motion of P such that $AO = OQ$ [Fig. 6.3(a)].

(ii) If O is made to coincide with A, AQ would be equal to OQ. Thus, Q and P will be fixed on AP. Q will rotate about A and thus P will also rotate in a circle about A with AP as the radius [Fig. 6.3(b)].

(iii) From the above two cases, it can be observed that in (i) P moves in a circle with the centre at infinity on OA produced and in (ii) P moves in circle with the centre at A. Thus, if P is to move in a circle with the centre in-between A and infinity on OA produced, O must lie in-between O and A or in other words OQ should be greater than OA [Fig. 6.3(c)].

(iv) The mechanism will be similar to the Paucellier mechanism. P is to be joined with O by a link so that P moves in a circle about O and $OA = OP$. The lengths can be modified in two ways [Fig. 6.3(d)].

(a) OA is increased and OA and OP are made equal.

(b) Lengths AB and AC are reduced in such a way that $OA = OP$.

2. Hart Mechanism

A Hart mechanism consists of six links as shown in Fig. 6.4 such that

$$AB = CD; \quad AD = BC \quad \text{and } OE = OQ$$

OE is the fixed link and OQ, the rotating link. The links are arranged in such a way that $ABDC$ is a trapezium (AC parallel to BD). Pins E and Q on the links AB and AD respectively, and the point P on the link CB are located in such a way that

$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB} \qquad \text{(i)}$$

It can be shown that as OQ rotates about O, P moves in a line perpendicular to EO produced.

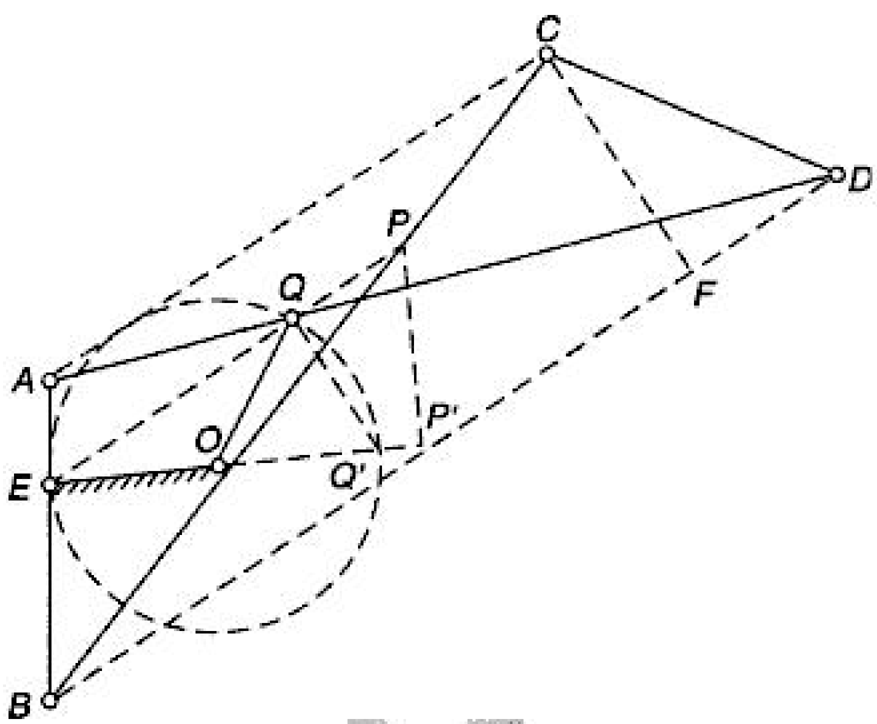

Fig. 6.4

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6. Tchebicheff Mechanism

It consists of four links OA, QB, AB and OQ (fixed) as shown in Fig. 6.11. The links OA and QB are equal and crossed. P, the mid-point of AB, is the tracing point. The proportions of the links are taken in such a way that P, A and B lie on vertical lines when on extreme positions, i.e., when directly above O or Q.

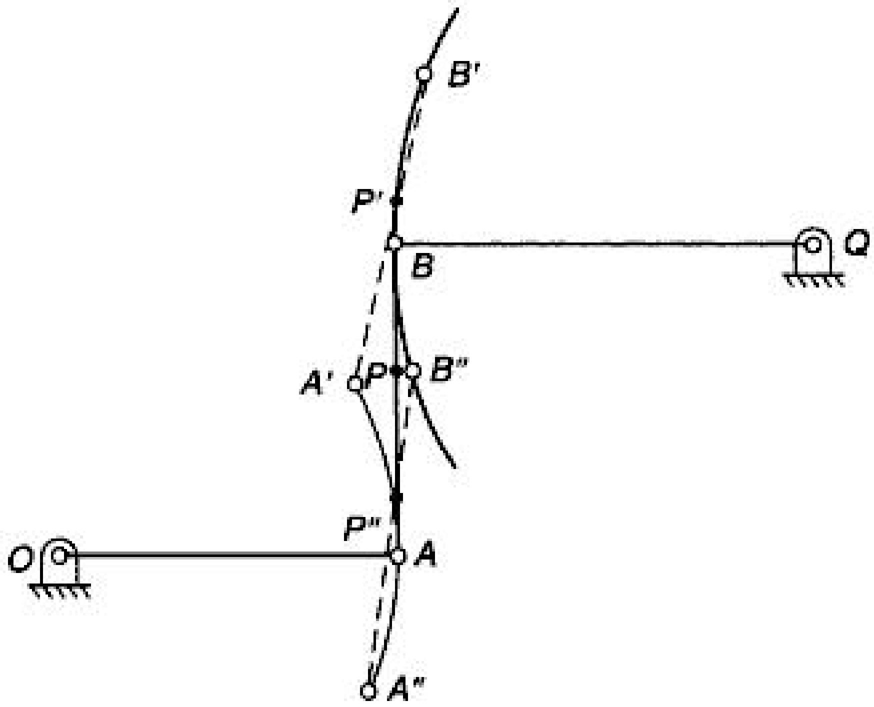

Fig. 6.10

Let $AB = 1$ unit

$OA = QB = x$ units

and $OQ = y$ units

When AB is on the extreme left position, A and B assume the positions A' and B', respectively.

In $\Delta OQB'$,

$$(QB')^2 - (OQ)^2 = (OB')^2$$

$$(QB)^2 - (OQ)^2 = (OB')^2 \qquad (OB' = OB)$$

$$x^2 - y^2 = (OA' - A'B')^2$$

$$= (x-1)^2$$

$$= x^2 - 2x + 1$$

or $\quad 2x - 1 = y^2$

$$x = \frac{y^2+1}{2} \qquad \text{(i)}$$

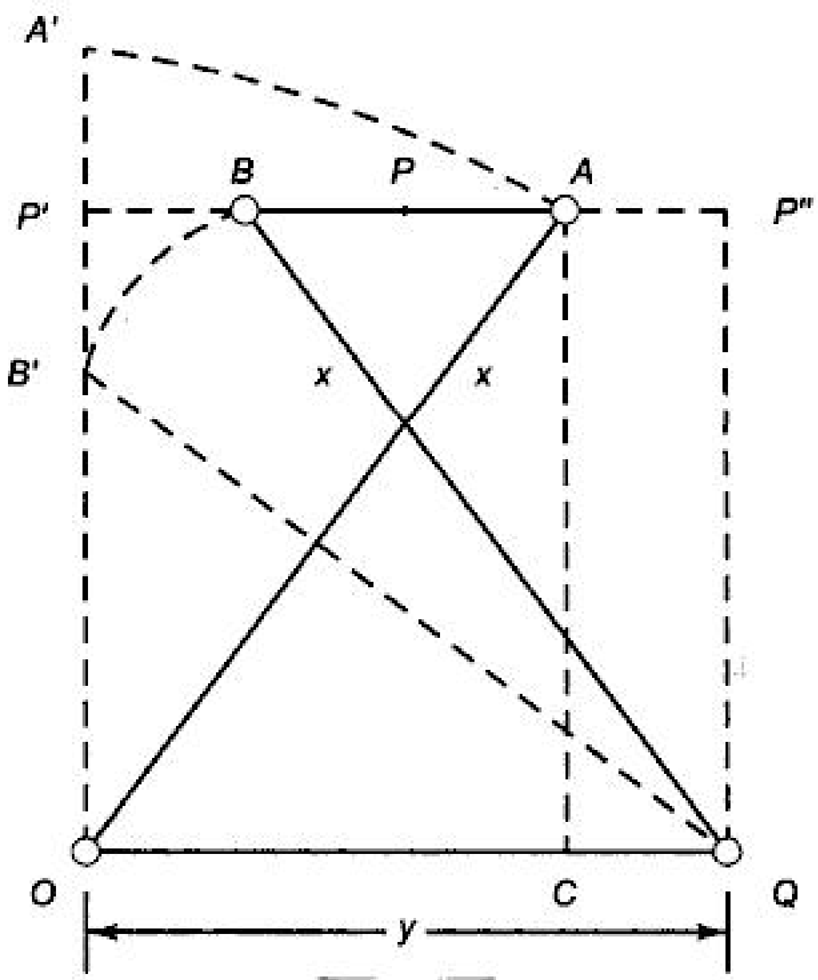

Fig. 6.11

In ΔOAC,

$$(OA)^2 - (AC)^2 = (OC)^2$$

$$(OA)^2 - (OP')^2 = (AP')^2$$

$$(OA)^2 - (OA' - A'P')^2 = (PP' + AP)^2$$

$$x^2 - \left(x - \frac{1}{2}\right)^2 = \left(\frac{y}{2} + \frac{1}{2}\right)^2$$

or $\quad x^2 - \left(x^2 + \frac{1}{4} - x\right) = \frac{y^2}{4} + \frac{1}{4} + \frac{y}{2}$

$$x = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2} \qquad \text{(ii)}$$

From Eqs (i) and (ii),

$$\frac{y^2}{2} + \frac{1}{2} = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2}$$

$$\frac{y^2}{4} = \frac{y}{2}$$

or $\quad y = 2$

and $\quad x = \frac{y^2+1}{2} = 2.5$

Thus, AB: OQ: OA = 1:2:2.5

This ratio of the links ensures that P moves approximately in a horizontal straight line parallel to OQ.

Thus, the link 3 has its centre of rotation at 31 (link 1 is fixed) and the velocity of any point on the link is proportional to its distance from 31, the direction being perpendicular to a line joining the point with the I-centre.

To locate 51, the directions of velocities of B and C are known.

- The direction of velocity of B is $\perp$ to $31-B$. Therefore, 51 lies on $31-B$.
- The direction of velocity of C is $\perp$ to OC. Therefore, 51 lies on OC.

Thus, 51 can be located.

Now, the link 5 has its centre of rotation at 51. The direction of velocity of the point R on this link will be perpendicular to 51-R. To have a vertical motion of R, it must lie on a horizontal line through 51.

The ratio of the velocities of R and P is given by,

$$\frac{v_r}{v_p}=\frac{v_r}{v_b}\cdot\frac{v_b}{v_p} \qquad (B \text{ is common to 3 and 5})$$

$$=\frac{51-R}{51-B}\cdot\frac{31-B}{31-P}$$

$$=\frac{51-R}{51-B}\cdot\frac{51-B}{51-F} \qquad (\because \Delta\text{s } BPG \text{ and } BFH \text{ are similar})$$

$$=\frac{51-R}{51-F}$$

$$=\frac{CR}{CB} \qquad (\because \Delta\text{s } CRH \text{ and } BRF \text{ are similar})$$

$$=\text{constant}$$

This shows that the velocity or the displacement of R will be proportional to that of P.

Alternatively, locate the I-centre 56 by using Kennedy's theorem. It will be at the point F (the intersection of lines joining I-centres 16, 15 and 35, 36, not shown in the figure).

First, consider this point 56 to lie on the link 6. Its absolute velocity is the velocity of 6 in the vertical direction (1 being fixed).

Now, consider the point 56 to lie on the link 5. The motion of 5 is that of rotation about 51 (1 being fixed). Thus, velocity of R on the link 5 can be found as the velocity of 56, another point on the same link is known.

$$\frac{v_r}{v_f}=\frac{51-R}{51-F}$$

or

$$\frac{v_r}{v_p}=\frac{51-R}{51-F} \qquad (v_f=v_p)$$

$$=\frac{CR}{CB}$$

3. Thomson Indicator

A Thomson indicator employs a Grass–Hopper mechanism $OCEQ$. R is the tracing point which lies on CE produced as shown in Fig. 6.18.

The best position of the tracing point R is obtained as discussed below:

Locate the I-centres 31 and 51 as in case of a Crosby indicator. The directions of velocities of two points C and E on the link 5 are known; therefore, first locate the I-centre 51.

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$$yh - h^2 \tan\theta = hy - hx + y^2 \tan\theta - xy \tan\theta$$

$$hx = (y^2 - xy + h^2)\tan\theta$$

$$\tan\theta = \frac{hx}{y^2 - xy + h^2}$$

Also, $\tan(\alpha + \varphi) = \dfrac{y + x}{h}$

and it can be proved that $\tan\varphi = \dfrac{hx}{y^2 + xy + h^2}$

For correct steering action, $\cot\varphi - \cot\theta = \dfrac{w}{l}$

or

$$\frac{y^2 + xy + h^2}{hx} - \frac{y^2 - xy + h^2}{hx} = \frac{w}{l}$$

or $\quad \dfrac{2xy}{hx} = \dfrac{w}{l}$

or $\quad \dfrac{y}{h} = \dfrac{w}{2l}$

or $\quad \tan\alpha = \dfrac{w}{2l} \qquad (6.2)$

The usual value of w/l is between 0.4 to 0.5 and that of α from 11 or 14 degrees.

Example 6.5 *The ratio between the width of the front axle and that of the wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18°, what should be the angle turned by the outer front wheel for perfect steering?*

Solution

$w/l = 0.44 \qquad \theta = 18°$

As $\quad \cot\varphi - \cot\theta = \dfrac{w}{l}$

$\therefore \quad \cot\varphi - \cot 18° = 0.44$

$\cot\phi = 0.44 + 3.078 = 3.518$

or $\quad \phi = \underline{15.9°}$

Example 6.6 *The distance between the steering pivots of a Davis steering gear is 1.3 m. The wheel base is 2.75 m. What will be the inclination of the track arms to the longitudinal axis of the vehicle if it is moving in a straight path?*

Solution

$w = 1.3$ m $\qquad l = 2.75$ m

$$\tan\alpha = \frac{w}{2l} = \frac{1.3}{2 \times 2.75} = 0.236$$

$\therefore \quad \alpha = 13.3°$ or $13° 18'$

Example 6.7 *The track arm of a Davis steering gear is at a distance of 192 mm from the front main axle whereas the difference between their lengths is 96 mm. If the distance between steering pivots of the main axle is 1.4 m, determine the length of the chassis between the front and the rear wheels. Also, find the inclination of the track arms to the longitudinal axis of the vehicle.*

Solution

$w = 1.4$ m $\qquad h = 192$ mm $\qquad y = 96/2 = 48$ mm

$$\tan\alpha = \frac{y}{h} = \frac{48}{192} = 0.25$$

$\therefore \quad \alpha = \underline{14°}$

Also $\quad \tan\alpha = \dfrac{w}{2l}$

$\therefore \quad \tan 14° = \dfrac{1.4}{2l}$

or $\quad l = \underline{2.8 \text{ m}}$

Angular Velocity Ratio

Let ω_1 = angular velocity of driving shaft $\left(=\dfrac{d\theta}{dt}\right)$

ω_2 = angular velocity of driving shaft $\left(=\dfrac{d\varphi}{dt}\right)$

Differentiating Eq. 6.4 with respect to time t,

$$\sec^2\theta\,\frac{d\theta}{dt} = \cos\alpha\sec^2\varphi\,\frac{d\varphi}{dt}$$

or

$$\frac{d\varphi/dt}{d\theta/dt} = \frac{\sec^2\theta}{\cos\alpha\sec^2\varphi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta\cos\alpha\,(1+\tan^2\varphi)}$$

$$= \frac{1}{\cos^2\theta\cos\alpha\left(1+\dfrac{\tan^2\theta}{\cos^2\alpha}\right)}\ldots \qquad \left(\tan\varphi = \frac{\tan\theta}{\cos\alpha}\right)$$

$$= \frac{1}{\cos^2\theta\cos\alpha\left(1+\dfrac{\sin^2\theta}{\cos^2\theta\cos^2\alpha}\right)}$$

$$= \frac{\cos^2\theta\cos^2\alpha}{\cos^2\theta\cos\alpha\,(\cos^2\theta\cos^2\alpha+\sin^2\theta)}$$

$$= \frac{\cos\alpha}{\cos^2\theta\,(1-\sin^2\alpha)+\sin^2\theta}$$

$$= \frac{\cos\alpha}{\cos^2\theta-\cos^2\theta\sin^2\alpha+\sin^2\theta}$$

$$= \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} \tag{6.5}$$

(i) $\dfrac{\omega_2}{\omega_1}$ is unity when $\dfrac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} = 1$

or $\cos\alpha = 1 - \sin^2\cos^2\theta$

or $\cos^2\theta = \dfrac{1-\cos\alpha}{\sin^2\alpha}$

or $2\theta \approx 78°42'$, $(360° - 78°42')$, $(360° + 78°42')$, $(720° - 78°42')$

or $\theta \approx 39°21'$, $140°39'$, $219°21'$ and $320°39'$

Now,

$$\text{acceleration} = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

Thus, acceleration is positive when $\sin 2\theta$ is negative and is negative when $\sin 2\theta$ is positive.

Corresponding to four values of 2θ found above, $\sin 2\theta$ will be +ve, –ve, +ve and –ve respectively.

Maximum acceleration will be at $140°39'$ and $320°39'$ and minimum acceleration (–ve) will be at $39°21'$ and $219°21'$.

Acceleration is zero when ω_2/ω_1 is maximum or minimum, i.e., at 0°, 90°, 180° and 270°.

Or acceleration is zero when 2θ is zero or when 2θ is 0°, 180°, 360°, 540° or when θ is 0°, 90°, 180° and 270°.

Example 6.11 *The angle between the axes of two shafts joined by Hooke's joint is 25°. The driving shaft rotates at a uniform speed of 180 rpm. The driven shaft carries a steady load of 7.5 kW. Calculate the mass of the flywheel of the driven shaft if its radius of gyration is 150 mm and the output torque of the driven shaft does not vary by more than 15% of the input shaft.*

Solution:

$\alpha = 25°$ $\quad N_1 = 180$ rpm

$P = 7.5$ kW $\quad \omega_1 = \dfrac{2\pi \times 180}{60} = 6\pi$

$k = 0.15$ m $\quad \Delta T = 15\%$

Maximum torque on the driven shaft will be when the acceleration is maximum, i.e., when

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \approx \frac{2\sin^2 25°}{2-\sin^2 25°} \approx 0.196$$

or

$2\theta = 78°42'$ or $281°18'$

$\therefore$ Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

$$= \frac{-(6\pi)^2 \cos 25° \sin^2 25° \sin 281°18'}{(1-\sin^2 25° \cos^2 140°39')^2}$$

$= 70.677$ rad/s^2

$P = T\omega_1$

$7500 = T \times 6\pi$

Input torque, $T = 397.9$ N.m

Permissible variation = torque due to acceleration of driven shaft

$397.9 \times 0.15 = I\alpha = mk^2\alpha$

or $\quad 397.9 \times 0.15 = m \times (0.15)^2 \times 70.677$

$\underline{m = 37.53 \text{ kg}}$

Example 6.12 *A Hooke's joint connects two shafts whose axes intersect at 18°. The driving shaft rotates at a uniform speed of 210 rpm. The driven shaft with attached masses has a mass of 60 kg and radius of gyration of 120 mm. Determine the*

(i) torque required at the driving shaft if a steady torque of 180 N.m resists rotation of the driven shaft and the angle of rotation is 45°

(ii) angle between the shafts at which the total fluctuation of speed of the driven shaft is limited to 18 rpm

Solution:

$m = 60$ kg $\quad k = 120$ mm $\quad N = 210$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

$$= \frac{-(22)^2 \cos 18° \sin^2 18° \sin 90°}{(1-\sin^2 18° \cos^2 45°)^2}$$

$$= -\frac{43.956}{0.907}$$

$= -48.47$ rad/s^2

The negative sign indicates that it is retardation at the instant.

Torque required for retardation of the driven shaft $= I\,\alpha = mk^2.\alpha$

$= 60 \times 0.12^2 \times (-48.47)$

$= -41.88$ N.m

25. Figure 6.32 shows a Robert straight-line mechanism in which *ABCD* is a four-bar linkage. The cranks *AB* and *DC* are equal and the connecting rod *BC* is one-half as long as the line of centres *AD*. *P* is a point rigidly attached to the connecting rod and lying on the midpoint of *AD* when *BC* is parallel to *AD*. Show that the point *P* moves in an approximately straight line for small displacement of the cranks.
(Note: *For better results take AB or DC > 0.6 AD*)

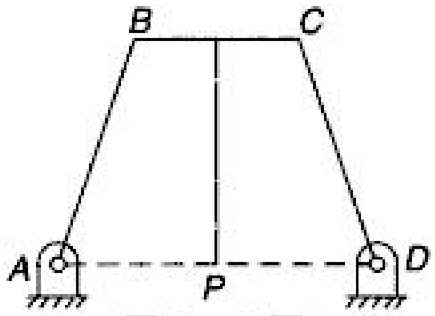

Fig. 6.32

26. In the Robert mechanism (Fig. 6.32) if *AB* = *BC* = *CD* = *AD*/2, locate the point *P* on the central vertical arm that approximately describes a straight line. (At a length 1.3 *BC* below *BC*)
27. In a Watt parallel motion (Fig. 6.10), the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. The lengths of the moving links are *OA* = 120 mm, *QB* = 200 mm and *AB* = 175 mm.
Locate the position of a point *P* on *AB* to trace approximately a straight line motion. Also, trace the locus of *P* for all possible movements. (*AP* = 109.3 mm)
28. In a Watt mechanism of the type shown in Fig. 6.33, the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. If *OA* = 45 mm, *QB* = 90 mm and *AB* = 60 mm, find the point *P* on the link *AB* produced for approximate straight-line motion of point *P*.

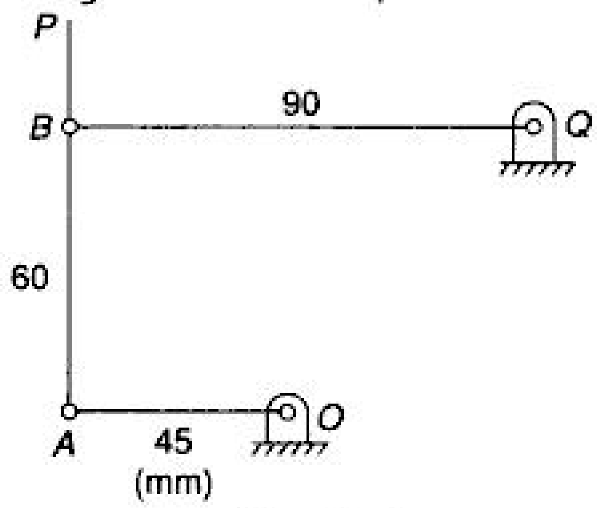

Fig. 6.33

(*AP* = 120 mm)

29. In a Davis steering gear, the length of the car between axles is 2.4 m, and the steering pivots are 1.35 m apart. Determine the inclination of the track arms to the longitudinal axis of the car when the car moves in a straight path. (15°42′)
30. In a Hooke's joint, the angle between the two shafts is 15°. Find the angles turned by the driving shaft when the velocity of the driven shaft is maximum, minimum and equal to that of the driving shaft. Also, determine when the driven shaft will have the maximum acceleration and retardation.
(Max. vel. at 0° and 180°; min. at 90° and 270°; equal to 44°30′. 135°30′, 224°30′ and 315°30′; Max. acc. at 137° and 317°; and Max. ret. at 43° and 223°)
31. The driving shaft of a Hooke's joint has a uniform angular speed of 280 rpm. Determine the maximum permissible angle between the axes of the shafts to permit a maximum variation in speed of the driven shaft by 8% of the mean speed. (22.6°)
32. The two shafts of a Hooke's coupling have their axes inclined at 20°. The shaft *A* revolves at a uniform speed of 1000 rpm. The shaft *B* carries a flywheel of mass 30 kg. If the radius of gyration of the flywheel is 100 mm, find the maximum torque in shaft *B*. (411 N.m)
33. In a double universal coupling joining two shafts, the intermediate shaft is inclined at 10° to each. The input and the output forks on the intermediate shaft have been assembled inadvertently at 90° to one another. Determine the maximum and the least velocities of the output shaft if the speed of the input shaft is 500 rpm. Also, find the coefficient of fluctuation in speed. (515.5 rpm; 484.9 rpm; 0.06)

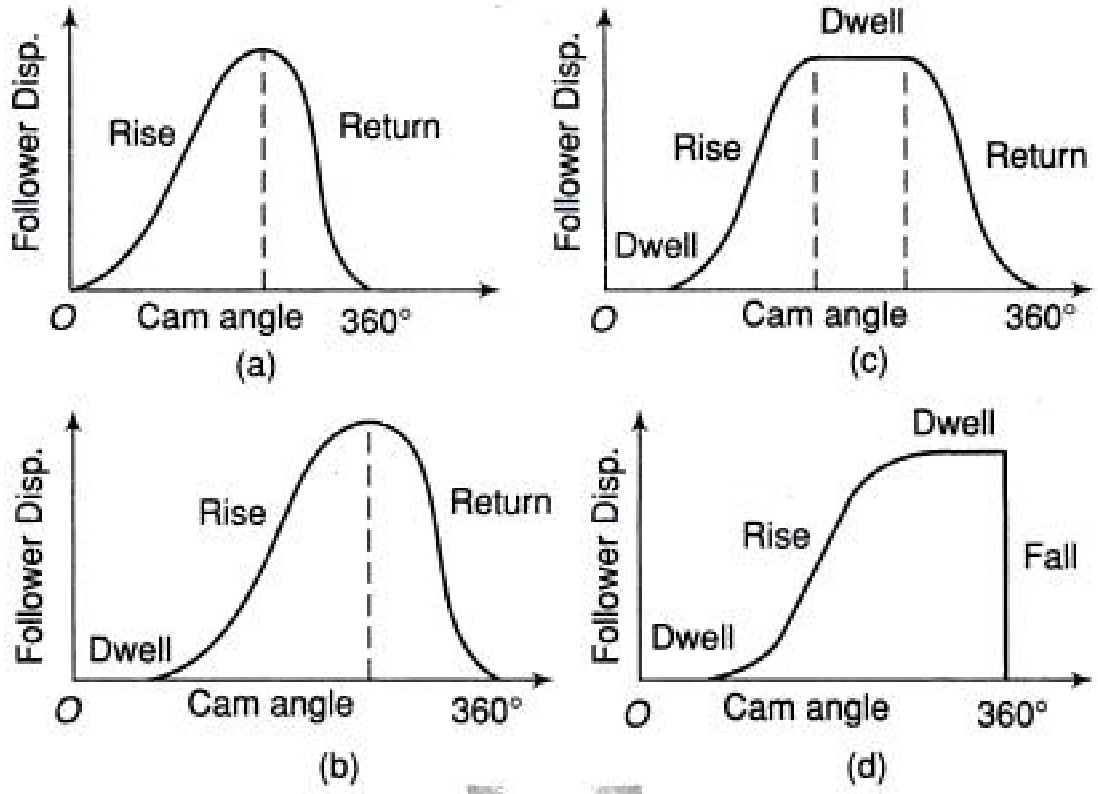

Fig. 7.9

1. ***Rise-Return-Rise* (R-R-R)** In this, there is alternate rise and return of the follower with no periods of dwells (Fig. 7.9a). Its use is very limited in the industry. The follower has a linear or an angular displacement.
2. ***Dwell-Rise-Return-Dwell (D-R-R-D)*** In such a type of cam, there is rise and return of the follower after a dwell [Fig. 7.9(b)]. This type is used more frequently than the R-R-R type of cam.
3. ***Dwell-Rise-Dwell-Return-Dwell* (D-R-D-R-D)** It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in Fig. 7.9(c). In case the return of the follower is by a fall [Fig. 7.9(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).

According to Manner of Constraint of the Follower

To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times. The cams can be classified according to the manner in which this is achieved.

1. ***Pre-loaded Spring Cam*** A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower [Figs 7.1(a) and (b), 7.3(a), 7.5(b) and 7.8].
2. ***Positive-drive Cam*** In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam [Figs 7.2, 7.3(b). 7.4, 7.5(a) and 7.7]. The follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam (Fig. 7.6).
3. ***Gravity Cam*** If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. Figure 7.2(c) shows such a cam. However, these cams are not preferred due to their uncertain behaviour.

7.2 TYPES OF FOLLOWERS

Cam followers are classified according to the

1. shape,
2. movement, and
3. location of line of movement.

According to Shape

1. ***Knife-edge Follower*** It is quite simple in construction. Figure 7.1(a) shows such a follower. However, its use is limited as it produces a great wear of the surface at the point of contact.
2. ***Roller Follower*** It is a widely used cam follower and has a cylindrical roller free to rotate about a pin joint [Figs 7.1(b), 7.2, 7.5, 7.8]. At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs.

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The second derivative is

$$\ddot{s} = \frac{d^2 s}{dt^2} = \omega^2 \frac{d^2 s}{d\theta^2}$$

It represents the *acceleration* of the follower. A higher value of acceleration means a higher inertia force. A third derivative is known as the *jerk*.

$$\dddot{s} = \frac{d^3 s}{dt^3} = \omega^3 \frac{d^3 s}{d\theta^3}$$

For smooth movement of the follower, even the high values of the jerk are undesirable in case of high-speed cams.

7.6 HIGH-SPEED CAMS

A real follower always has some mass and when multiplied by acceleration, inertia force of the follower is obtained. This force is always felt at the contact point of the follower with the cam surface and at the bearings. An acceleration curve with abrupt changes exerts abrupt stresses on the cam surfaces and at the bearings accompanied by detrimental effects such as surface wear and noise. All this may lead to an early failure of the cam system. Thus, it is very important to give due consideration to velocity and acceleration curves while choosing a displacement diagram. They should not have any step changes.

In low-speed applications, cams with discontinuous acceleration characteristics may not show any undesirable characteristic, but at higher speeds such cams are certainly bound to show the same. The higher the speed, the higher is the need for smooth curves. At very high speeds, even the jerk (related to rate of change of acceleration or force) is made continuous as well. For most of the applications, however, this may not be needed. In Section 7.8, standard cam motions have been discussed from which some comparison can easily be made for suitable selection.

7.7 UNDERCUTTING

Sometimes, it may happen that the prime circle of a cam is proportioned to provide a satisfactory pressure angle; still the follower may not be completing the desired motion. This can happen if the curvature of the pitch curve is too sharp.

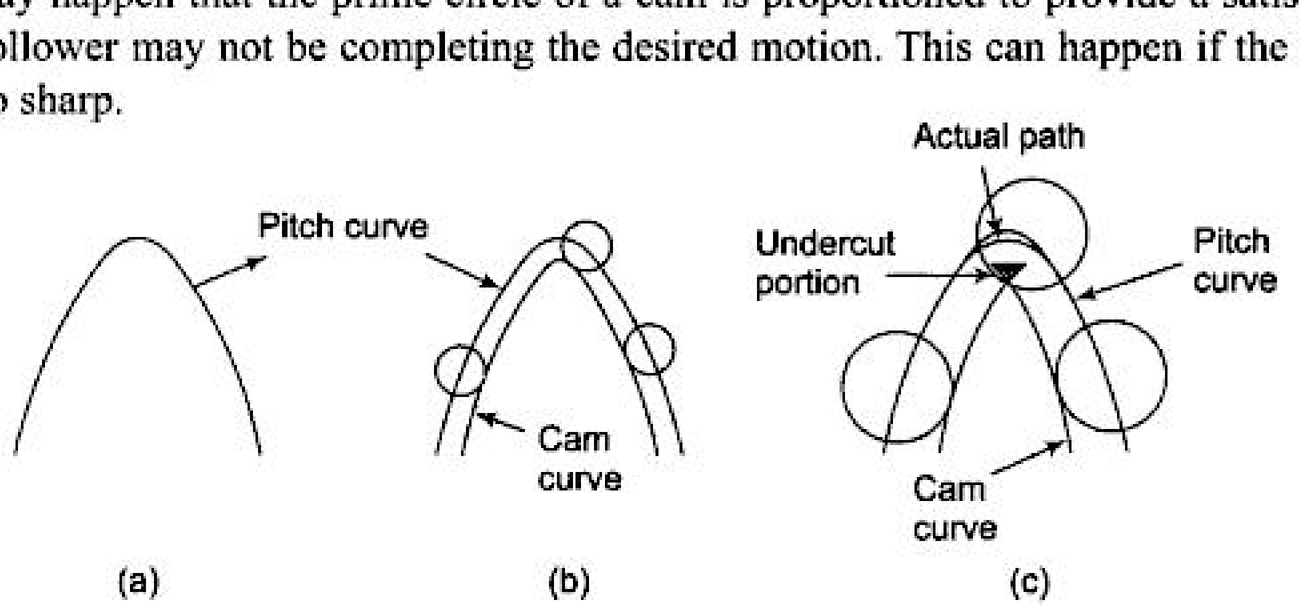

Fig. 7.14

Figure 7.14(a) shows the pitch curve of a cam. In Fig. 7.14(b), a roller follower is shown generating this curve. In Fig. 7.14(c), a larger roller is shown trying to generate this curve. It can easily be observed that the

or $$f = \frac{2s}{t^2} = \text{constant} \tag{7.6}$$

As f is constant during the accelerating period, considering the follower at the midway,

$$s = \frac{h}{2} \quad \text{and} \quad t = \frac{\varphi / 2}{\omega}$$

$$\therefore \quad f = \frac{2h/2}{\varphi^2 / 4\omega^2} = \frac{4h\omega^2}{\varphi^2} \tag{7.6a}$$

The velocity is linear during the period and is given by

$$v = \frac{ds}{dt} = \frac{1}{2} \times 2ft = ft \tag{7.7}$$

$$= \frac{4h\omega^2}{\varphi^2}\frac{\theta}{\omega} \qquad (\theta = \omega t)$$

$$= \frac{4h\omega}{\varphi^2}\theta \tag{7.7a}$$

The velocity is maximum when θ is maximum or the follower is at the midway, i.e., when $\theta = \varphi/2$.

$$v_{\max} = \frac{4h\omega}{\varphi^2}\frac{\varphi}{2} = \frac{2h\omega}{\varphi} \tag{7.8}$$

During the second half of the follower motion, the follower is decelerated at constant rate so that the velocity reduces to zero at the end.

It can be observed from the plots shown in Fig. 7.18 that there are abrupt changes in the acceleration at the beginning, midway and the end of the follower motion. At midway, an infinite jerk is produced. Thus, this programme of the follower is adopted only up to moderate speeds.

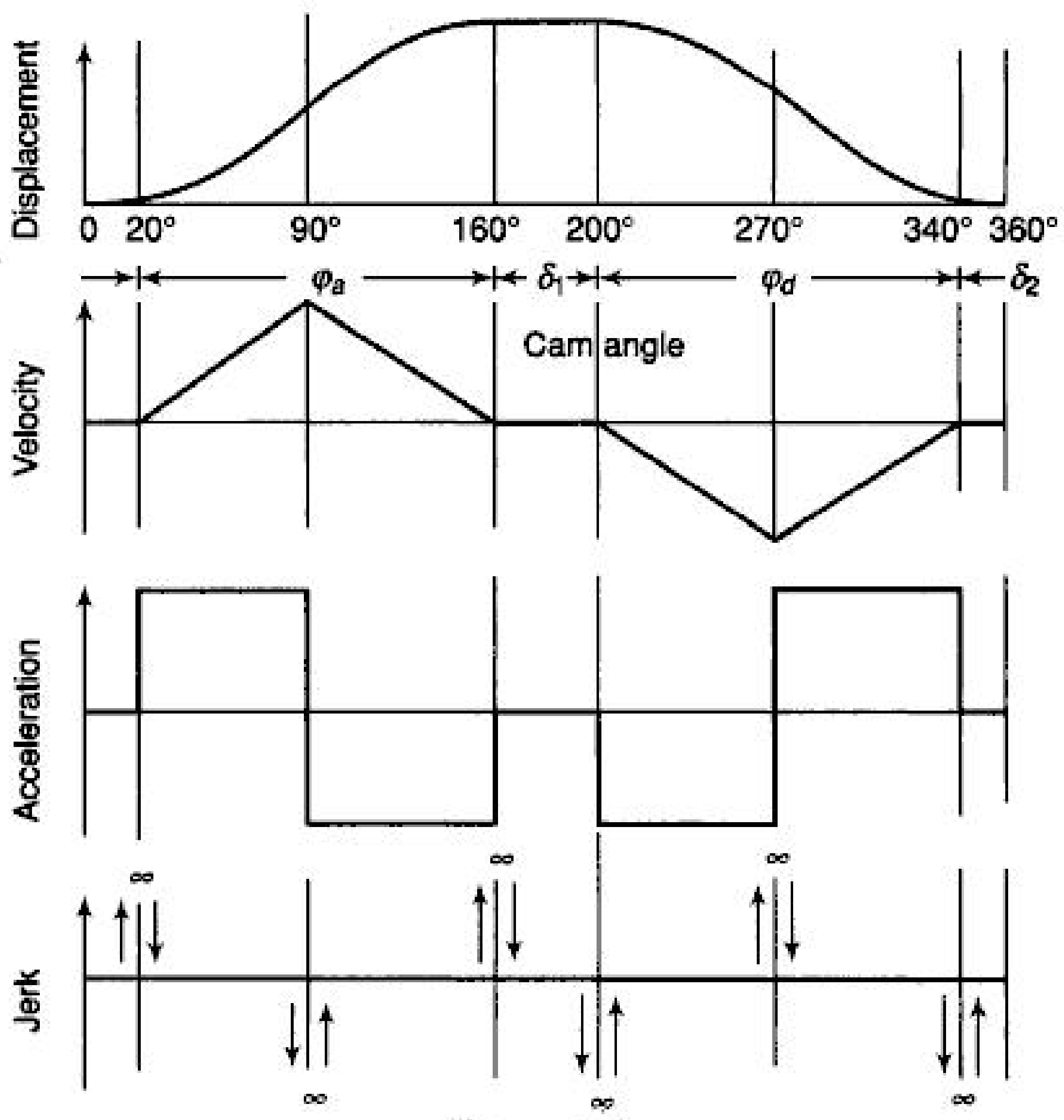

Fig. 7.18

3. Constant Velocity

Constant velocity of the follower implies that the displacement of the follower is proportional to the cam displacement and the slope of the displacement curve is constant (Fig. 7.19).

Displacement of the follower for the

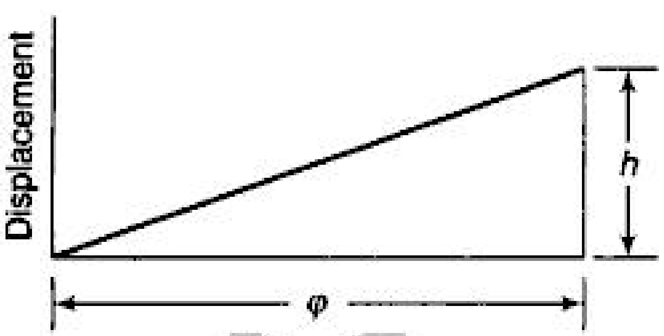

Fig. 7.19

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$r_c = 20$ mm $\quad \varphi_d = 100°$

$\delta_2 = (360° - 150° - 100° - 60°) = 50°$

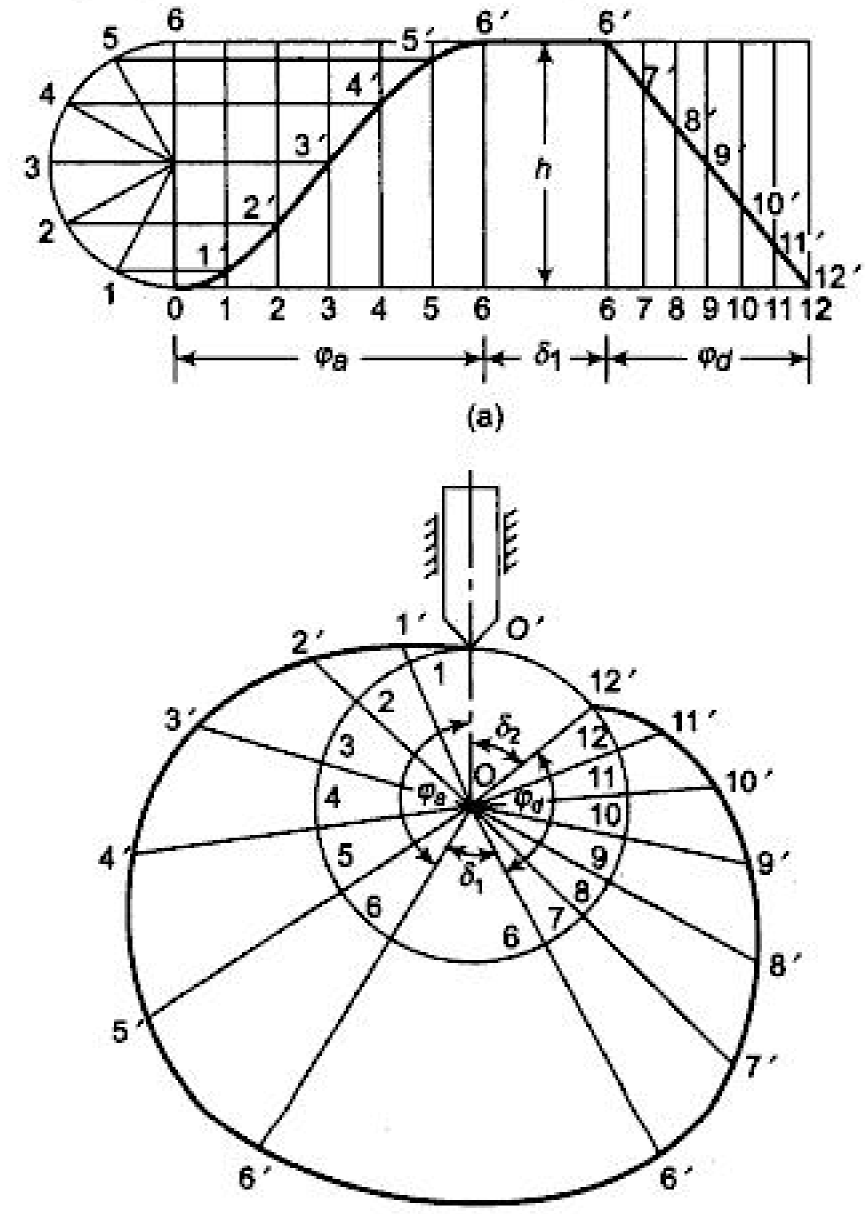


Fig. 7.23

Draw the displacement diagram of the follower as discussed earlier [Fig. 7.23(a)] taking a convenient scale. Construct the cam profile as follows [refer Fig. 7.23(b)]:

(i) Draw a circle with radius r_c.

(ii) If the cam rotates clockwise and the follower remains in vertical direction, the cam profile can be drawn by assuming that the cam is stationary and the follower rotates about the cam in the counter-clockwise direction.
From the vertical position, mark angles φ_a, δ_1, φ_d, and δ_2 in the counter-clockwise direction, representing angles of ascent, rest or dwell, descent and rest respectively.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, each has been divided into 6 equal parts.

(iv) Draw radial lines O-1, O-2, O-3, etc., O-1 represents that after an interval of $\varphi_d/6$ of the cam rotation in the clockwise direction it will take the vertical position of O-O'.

(v) On the radial lines produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e., 1-1′, 2-2′, 3-3′, etc.

(vi) Draw a smooth curve passing through O', 1′, 2′,…., 10′, 11′ and 12′. Draw an arc of radius O-6′ for the dwell period δ_1.

During ascent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a} \qquad \text{[refer Eq. (7.3)]}$$

or

$$v_{max} = \frac{30}{2} \times \frac{\pi \times 12.57}{150 \times \frac{\pi}{180}} = 226.3 \text{ mm/s}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{[refer Eq. (7.5)]}$$

or

$$f_{max} = \frac{30}{2} \times \left(\frac{\pi \times 12.57}{150 \times \frac{\pi}{180}}\right)^2$$

$= 3413$ mm/s^2 or $\underline{3.413 \text{ m/s}^2}$

During desent

$$v_{max} = h\frac{\omega}{\varphi_d} \qquad \text{[refer Eq. (7.10)]}$$

$$v_{max} = 30 \times \frac{12.57}{100 \times \frac{\pi}{180}} = \underline{216 \text{ mm/s}}$$

$f_{max} = f = 0$

Note that to draw the cam profile, it is not necessary that the interval δ_1 is taken in the displacement diagram. Also, the scales of φ_a and φ_d can be taken different and of any magnitudes.

Draw the displacement diagram of the follower as shown in Fig. 7.27(a). Construct the cam profile as described below [Fig. 7.27(b)].

(i) Draw a circle with radius $(r_c + r_r)$.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the counter-clockwise direction (assuming that the cam is to rotate in the clockwise direction).

(iii) Divide the angles φ_a and φ_d into the same number of parts as is done in the displacement diagram. In this case, φ_a has been divided into 6 equal parts whereas φ_d is divided into 8 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a series of arcs of radii equal to r_r, as shown in the diagram from the points 1′, 2′, 3′, etc.

(vi) Draw a smooth curve tangential to all the arcs which is the required cam profile.

During the descent period, the acceleration and the deceleration are uniform. Therefore, the maximum velocity is at the end of the acceleration period.

$$v_{max} = 2h\frac{\omega}{\varphi_d} \qquad \text{Eq. (7.8)}$$

or

$$v_{max} = 2 \times 30 \times \frac{\dfrac{2\pi \times 150}{60}}{150 \times \dfrac{\pi}{180}} = \underline{360 \text{ m/s}}$$

$$f_{max} = f_{uniform} = \frac{4h\omega^2}{\varphi_d^2} \qquad \text{Eq. (7.6)}$$

$$f_{max} = \frac{4 \times 30 \times \left(\dfrac{2\pi \times 150}{60}\right)^2}{\left(150 \times \dfrac{\pi}{180}\right)^2} = 4320 \text{ mm/s}^2$$

or $\underline{4.32 \text{ m/s}^2}$

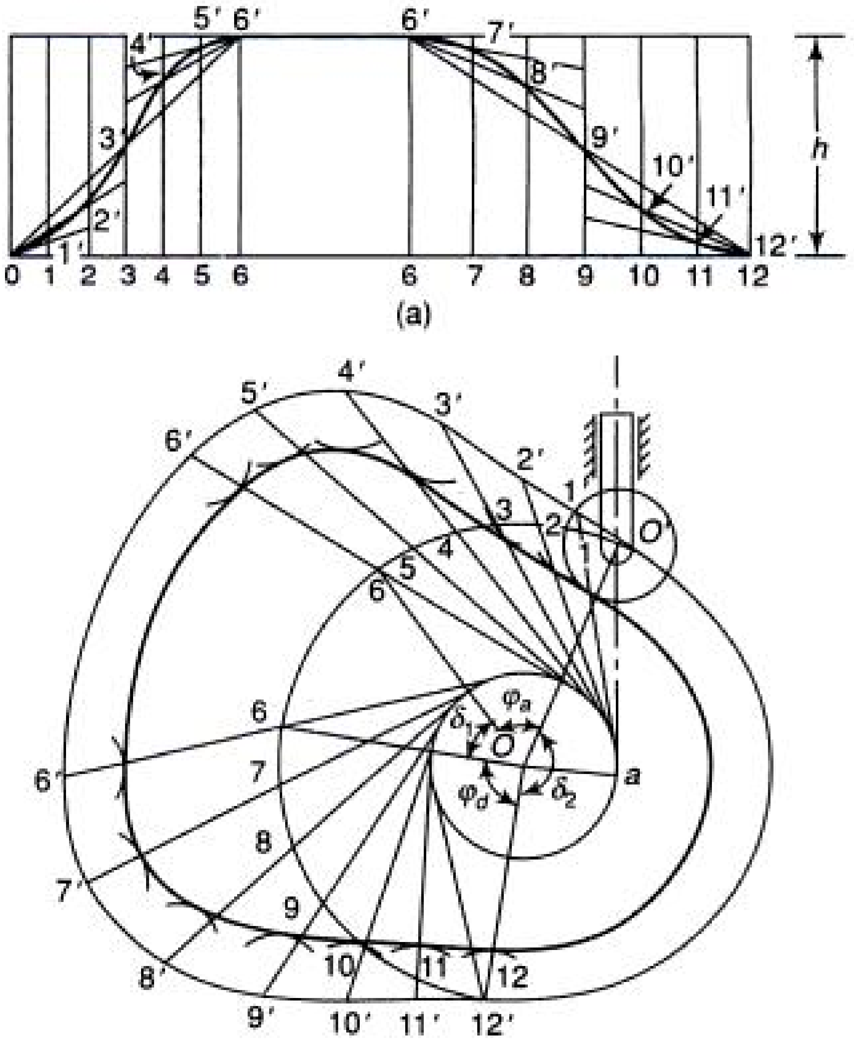
Example 7.5 *The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent.*

Minimum radius of cam = 25 mm
Roller diameter = 7.5 mm
Lift = 28 mm
Offset of follower axis = 12 mm towards right
Angle of ascent = 60°
Angle of descent = 90°
Angle of dwell between ascent and descent = 45°
Speed of the cam = 200 rpm

Draw the profile of the cam and determine the maximum velocity and the uniform acceleration of the follower during the outstroke and the return stroke.

Solution:

$h = 28$ mm $\quad \varphi_a = 60°$
$r_c = 25$ mm $\quad \delta_1 = 45°$
$r_r = 7.5$ mm $\quad \varphi_d = 90°$
offset, $x = 12$ mm $\quad \delta_2 = (360° - 60° - 45° - 90°)$
$N = 200$ mm $\quad = 165°$



(a)

(b)

Fig. 7.28

From the given data, construct the displacement diagram as usual [Fig. 7.28(a)]. For the cam profile [Fig. 7.28(b)], the procedure is as follows:

(i) Draw a circle with radius $(r_c + r_r)$.

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these arcs as shown in the diagram. It is on the assumption that for small angular displacements, the linear displacements on the arcs and on the straight lines are the same.

(viii) With 1′, 2′, 3′, etc., draw a series of arcs of radii equal of r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

7.10 CAMS WITH SPECIFIED CONTOURS

It is always desired that a cam is made to provide a smooth motion of the follower and for that a follower motion programme is always selected first. However, sometimes it becomes difficult to manufacture the cams in large quantities of the specified contours. Under such circumstances, it becomes necessary that the cam is designed first and then some improvements are made in that if possible. Such cams are generally made up of some combination of curves such as straight lines, circular arcs, etc. In the present section, some cams with specified contours are analysed.

1. Tangent Cam (with Roller Follower)

A tangent cam is symmetrical about the centre line. It has straight flanks (such as AK in Fig. 7.31) with a circular nose. The centre of the cam is at O and that of the nose at Q. A tangent cam is used with a roller cam since there is no meaning of using flat-faced followers with straight flanks.

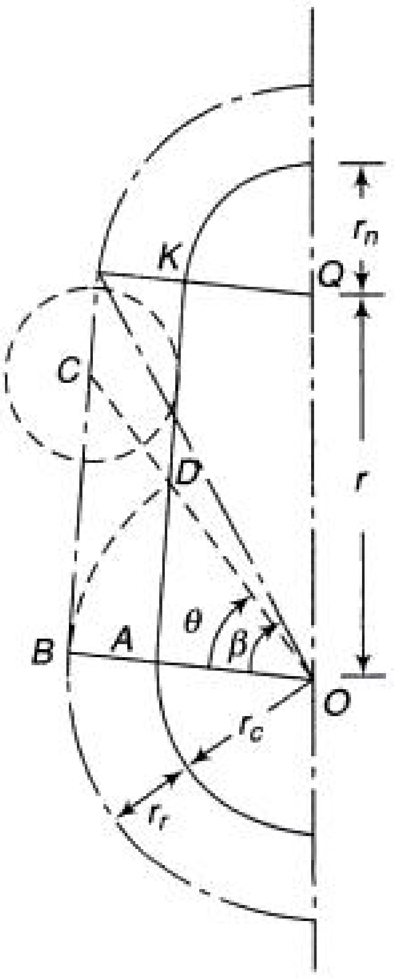

Fig. 7.31

Let r_c = least radius of cam

r_n = radius of nose

r_r = radius of roller

r = distance between the cam and the nose centres.

Roller on the Flank When the roller is on the straight flank, the centre of the roller is at C on the pitch profile as shown in Fig. 7.31.

Let θ = angle turned by the cam from the beginning of the follower motion

Let,
$$x = OC - OD = OC - OB = \frac{OB}{\cos\theta} - OB = OB\left(\frac{1}{\cos\theta} - 1\right)$$

or
$$x = (r_c + r_r)\left(\frac{1}{\cos\theta} - 1\right) \tag{7.17}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\,\frac{d\theta}{dt} = (r_c + r_r)\left(\frac{\sin\theta}{\cos^2\theta} - 0\right)\omega$$

$$= \omega\,(r_c + r_r)\frac{\sin\theta}{\cos^2\theta} \tag{7.18}$$

$\sin\theta$ increases with the increase in θ whereas $\cos\theta$ decreases. Hence the velocity increases with θ and it is maximum when θ is maximum. This will happen when the point of contact leaves the straight flank.

Let β = angle turned by the cam when the roller leaves the flank.

$$\therefore \quad v_{max} = \omega\,(r_c + r_r)\frac{\sin\beta}{\cos^2\beta} \text{ and } v_{min} = 0 \text{ at } \theta = 0$$

$$= \sqrt{1-\left[\frac{(r_f - r_c)\sin\theta}{(r_f + r_r)}\right]^2}$$

or $$\cos\varphi = \sqrt{1-\left(\frac{A\sin\theta}{B}\right)^2} = \frac{1}{B}\sqrt{B^2 - A^2\sin^2\theta}$$

where $A = r_f - r_c$ and $B = r_f + r_r$

$$x = \sqrt{B^2 - A^2\sin^2\theta} - A\cos\theta - (r_c + r_r) \tag{7.30}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \left[\frac{1}{2}(B^2 - A^2\sin^2\theta)^{-1/2}(-A^2 2\sin\theta\cos\theta) + A\sin\theta)\right]\omega$$

$$= \omega A\left[\sin\theta - \frac{A\sin 2\theta}{2\sqrt{B^2 - A^2\sin^2\theta}}\right] \tag{7.31}$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \omega^2 A\left[\cos\theta - \frac{A}{2}\left\{\frac{2\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} + \sin 2\theta\left(-\frac{1}{2}\right)(B^2 - A^2\sin^2\theta)^{-3/2}(-A^2)(2\sin\theta\cos\theta)\right\}\right]$$

$$= \omega^2 A\left[\cos\theta - \frac{A\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} - \frac{A^3\sin 2\theta}{4(B^2 - A^2\sin^2\theta)^{3/2}}\right] \tag{7.32}$$

Follower on the Nose This case has already been discussed for the tangent cam when the roller follower is on the nose. Same expressions for the displacement, velocity and the acceleration hold good.

Example 7.8 *A tangent cam with straight working faces tangential to a base circle of 120 mm diameter has a roller follower of 48-mm diameter. The line of stroke of the roller follower passes through the axis of the cam. The nose circle radius of the cam is 12 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 180 rpm, determine the acceleration of the follower when*

(i) during the lift, the roller just leaves the straight flank

(ii) the roller is at the outer end of its lift, i.e., at the top of the nose

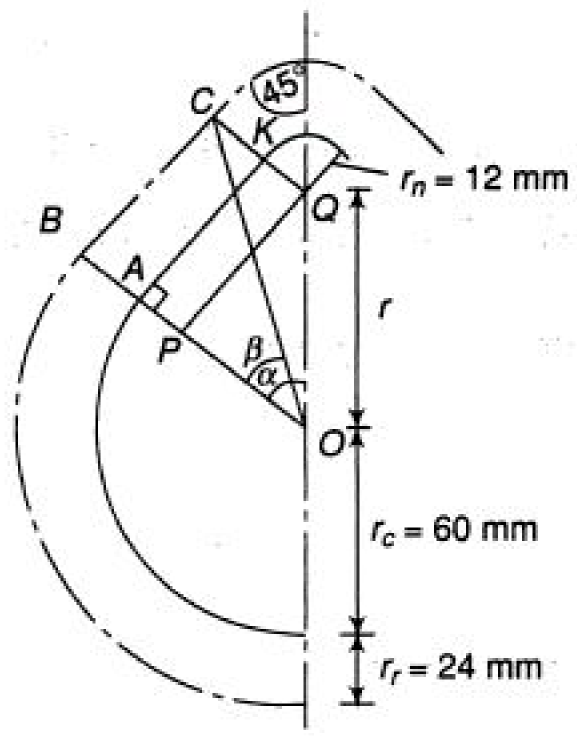

Fig. 7.36

Solution:

$r_c = 60$ mm $\quad r_n = 12$ mm

$r_r = 24$ mm $\quad N = 180$ rpm

$$\omega = \frac{2\pi \times 180}{60} = 6\pi \text{ rad/s}$$

Refer Fig. 7.36,

$$(r_f - 8)^2 = (r_f - 40)^2 + (56)^2 - 2(r_f - 40)(56)\cos(180° - 75°)$$

$$r_f^2 + 64 - 16r_f = r_f^2 + 1600 - 80r_f + 3136 + 29r_f - 1160$$

$35\, r_f = 3512$

$r_f = 100.3$ mm

$OP = 100.3 - 40 = 60.3$ mm

$PQ = 100.3 - 8 = 92.3$ mm

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin\beta} = \frac{PQ}{\sin(180° - \alpha)}$$

or $$\frac{r}{\sin\beta} = \frac{r_f - r_n}{\sin 105°}$$

or $$\frac{56}{\sin\beta} = \frac{100.3 - 8}{\sin 105°}$$

$\sin\beta = 0.586$

$\beta = 35.9°$

Acceleration when the follower is on the circular flank, $f = \omega^2 (r_f - r_c)\cos\theta$

(i) At the beginning of lift, $\theta = 0°$,

$f = \omega^2 (r_f - r_c) = 44^2\,(100.3 - 40)$
$= 116\,740\ \text{mm/s}^2 = \underline{116.74\ \text{m/s}^2}$

(ii) At the end of contact with the circular flank,

$f = \omega^2 (r_f - r_c)\cos\theta = 44^2\,(100.3 - 40)$
$\cos 35.9° = 94\,565\ \text{mm/s}^2 = \underline{94.565\ \text{m/s}^2}$

Acceleration when the follower is on the nose, $f = -\omega^2 r\cos(\alpha - \beta)$

(iii) At the beginning of contact with the nose

$f = -\,\omega^2 r\cos(\alpha - \beta) = -\,44^2 \times 56$
$\times \cos(75° - 35.9°) = -84\,136\ \text{mm/s}^2$
or $\underline{-84.136\ \text{m/s}^2}$

(iv) At the apex of the nose, $\alpha = \beta$

$f = -\omega^2 r = -\,44^2 \times 56 = -\,108\,416\ \text{mm/s}^2$

or $-\,\underline{108.416\ \text{m/s}^2}$

Example 7.12 *In a four-stroke petrol engine, the exhaust valve opens 45° before the t.d.c. and closes 15° after the b.d.c. The valve has a lift of 12 mm. The least radius of the circular-arc-type cam operating a flat-faced follower is 25 mm. The nose radius is 3 mm. The camshaft rotates at 1500 rpm. Calculate the maximum velocity of the valve and the minimum force exerted by the spring to overcome the inertia of the moving parts that weigh 300 g.*

Solution:

$r_c = 25$ mm $N = 1500$ rpm

$h = 12$ mm $r_n = 3$ mm

$m = 0.3$ kg

Crank rotation during of the exhaust valve = 45° + 180° + 15° = 240°

In four-stroke engines, the camshaft speed is half that of the crankshaft.

Angle of action of the camshaft,

$$2\alpha = \frac{240}{2} = 120°$$

$$\therefore \quad \alpha = \frac{120}{2} = 60°$$

Refer to Fig. 7.37,

$r + r_n = r_c + h$

or $r = 25 + 12 - 3 = 34$ mm

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2\,(OP)\,(OQ)\cos\angle POQ$$

$$(r_f - 3)^2 = (r_f - 25)^2 + (34)^2 - 2\,(r_f - 25)\,(34)\cos(180° - 60°)$$

$$r_f^2 + 9 - 6r_f = r_f^2 + 625 - 50r_f + 1156 + 34r_f - 850$$

$r_f = 92.2$ mm

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin\beta} = \frac{PQ}{\sin(180° - \alpha)}$$

$$\frac{r}{\sin\beta} = \frac{r_f - r_n}{\sin 120°}$$

$$\frac{34}{\sin\beta} = \frac{92.2 - 3}{\sin 120°}$$

$\sin\beta = 0.33$

$\alpha = 19.27°$

Velocity is maximum when the contact is on the point where the circular flank meets the circular nose.

$$v_{max} = \omega\,(r_f - r_c)\sin\beta$$
$$= \frac{2\pi \times 1500}{60}(92.2 - 25)\sin 19.27°$$
$$= 157.08 \times 67.2 \times 0.33$$
$$= 3480\ \text{mm/s or } 3.48\ \text{m/s}$$

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$$= \sqrt{\frac{2\times5000\times0.04+60+3\times9.81}{3\times0.04}} \qquad = 63.86 \text{ rad/s}$$

$$\text{or} \quad \frac{2\pi N}{60} = 63.86$$

$$= \sqrt{4078.6} \qquad \text{or} \quad N = \underline{609.9 \text{ rpm}}$$

7.12 ANALYSIS OF AN ELASTIC CAM SYSTEM

To illustrate the effect of follower elasticity upon its displacement and velocity, consider a simplified model of a cam system with a linear motion used for low-speed cams (Fig. 7.44).

Let

m = lumped mass of the follower
s_1 = stiffness of the retaining spring
s_2 = stiffness of the follower
x = displacement of the lumped mass of the follower
y = motion machined into the cam surface
h = lift of the follower

Assume a cam profile that gives a uniform rise for an angle of rotation ϕ followed by a dwell. Thus,

$$y = h\frac{\theta}{\varphi} = h\frac{\omega t}{\varphi}$$

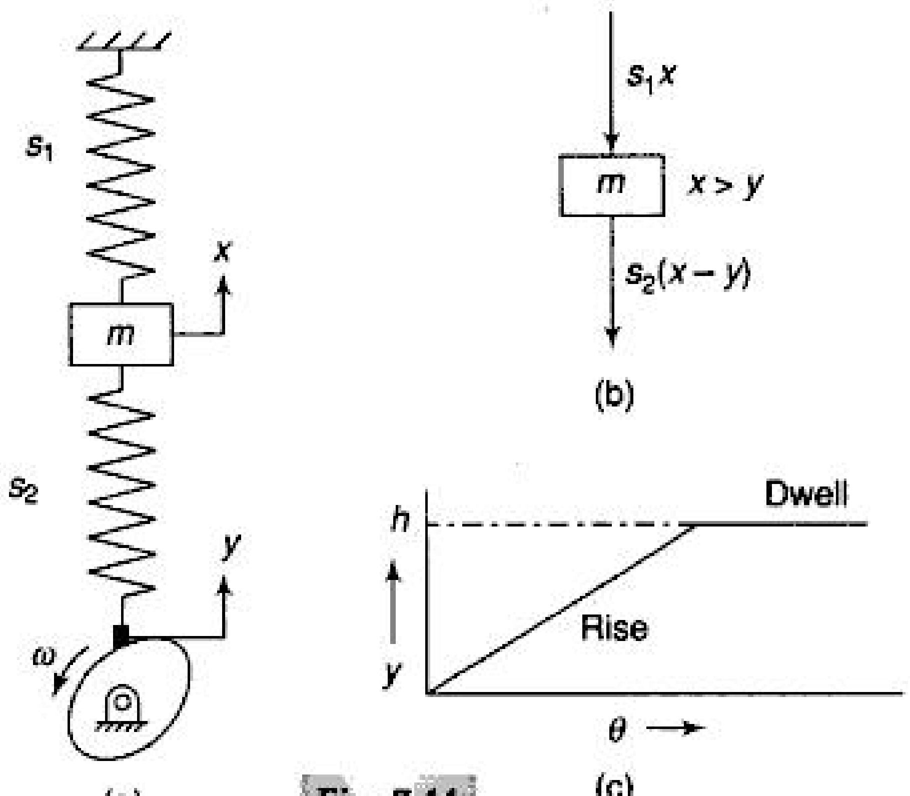

Fig. 7.44

As the follower is usually a rod, its stiffness s_2 is far greater than the stiffness s_1 of the spring. The spring is assembled in such a way that it exerts a preload force. The displacement x of the lumped mass is taken from the equilibrium position after the spring is assembled. In the equilibrium position, the spring and the follower exert equal and opposite preload forces on the mass.

Assuming the displacement x to be more than y, the various forces acting on the follower mass are

Inertia force $= m\ddot{x}$ (downwards)
Spring force $= s_1x$ (downwards)
Force of elastic follower $= s_2(x - y)$ (downwards)

Thus

$$m\ddot{x} + s_1x_1 + s_2(x - y) = 0$$

$$\ddot{x} + \frac{s_1 + s_2}{m}x = \frac{s_2}{m}y$$

$$\ddot{x} + \omega_n^2 x = \frac{s_2}{m}y$$

During ascent, the displacement of the follower mass is given by the solution of the above equation, i.e.,

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^2}y \qquad \text{(i)}$$

Differentiating with respect to t,

$$\dot{x} = -A\omega_n\sin\omega_n t + B\omega_n\cos\omega_n t + \frac{s_2}{m\omega_n^2}\dot{y} \qquad \text{(ii)}$$

revolution. Determine the maximum velocity and maximum acceleration during the lifting. The cam rotates at 60 rpm.

(0.25 m/s; 3.155 m/s^2)

15. Lay out the profile of a cam so that the follower
    - is moved outwards through 30 mm during 180° of cam rotation with cycloidal motion
    - dwells for 20° of the cam rotation
    - returns with uniform velocity during the remaining 160° of the cam rotation

    The base circle diameter of the cam is 28 mm and the roller diameter 8 mm. The axis of the follower is offset by 6 mm to the left. What will be the maximum velocity and acceleration of the follower during the outstroke if the cam rotates at 1500 rpm counter-clockwise?

    (3 m/s; 471.2 m/s^2)

16. Use the following data in drawing the profile of a cam in which a knife-edged follower is raised with uniform acceleration and deceleration and is lowered with simple harmonic motion:

    Least radius of cam = 60 mm
    Lift of follower = 45 mm
    Angle of ascent = 60°
    Angle of dwell between ascent and descent = 40°
    Angle of descent = 75°

    If the cam rotates at 180 rpm, determine the maximum velocity and acceleration during ascent and descent.

    (*Ascent:* 1.62 m/s; 58.3 m/s^2)
    Descent: 1.02 m/s; 46.05 m/s^2)

17. Draw the profile of a cam which is to give oscillatory motion to the follower with uniform angular velocity about its pivot. The base circle diameter is 50 mm, angle of oscillation of the follower is 30° and the distance between the cam centre and the pivot of the follower is 60 mm. The oscillating lever is 60 mm long with a roller of 8-mm diameter at the end. One oscillation of the follower is completed in one revolution of the cam.

18. Set out the profile of a cam to give the following motion to a flat mushroom contact face follower:
    - Follower to rise through 24 mm during 150° of cam rotation with SHM
    - Follower to dwell for 30° of the cam rotation
    - Follower to return to the initial position during 90° of the cam rotation with SHM
    - Follower to dwell for the remaining 90° of cam rotation

    Take minimum radius of the cam as 30 mm.

19. A cam is required to give motion to a follower fitted with a roller that is 50 mm in diameter. The lift of the follower is 30 mm and is performed
    - with uniform acceleration for 12 mm, the cam turns through 45°
    - with uniform velocity for 12 mm, the cam turns through the next 30°
    - with uniform deceleration for the remainder of the lift, the cam turns through the next 45°

    The follower falls through immediately with simple harmonic motion while the cam turns through 120°. Then a period of dwell is followed for 120° of the cam angle. Construct a lift and fall diagram on a cam angle base. Also, draw the outline of the cam. The least radius of the cam is 35 mm. The line of motion of the follower passes through the centre of the cam axis.

20. Draw the profile of a cam operating a roller reciprocating follower having a lift of 35 mm. The line of stroke of the follower passes through the axis of the cam shaft. The radius of the roller is 10 mm and the minimum radius of the cam is 40 mm. The cam rotates at 630 rpm counter-clockwise. The follower is raised with simple harmonic motion for 90° of the cam rotation, dwells for next 60° and then lowers with uniform acceleration and deceleration for the next 150°. The follower dwells for the rest of the cam rotation.

    Also, draw the displacement, velocity and the acceleration diagrams for the motion of the follower for one complete revolution of the cam indicating main values.

    (2.31 m/s, 304.9 m/s^2, 5.54 m/s, 878.2 m/s^2)

21. A tangent cam with straight working faces is tangential to a base circle of 80 mm diameter. It operates a roller follower of 32 mm diameter. The line of stroke of the follower passes through the axis of the cam. The nose circle radius of the cam is 10 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 315 rpm, determine the acceleration of the follower when (i) during the lift, the roller just leaves the straight flank, and (ii) the roller is at the outer end of its lift, i.e., at the top of the nose.

    (105.1 m/s^2, 799.1 m/s^2)

22. The following particulars relate to a symmetrical tangent cam having a roller follower:

    Minimum radius of the cam = 40 mm
    Lift = 20 mm
    Speed = 360 rpm

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$$\text{Efficiency} = \frac{\tan\alpha}{\tan(\alpha+\varphi)} = \frac{\tan 3.25°}{\tan(3.25°+6.28°)}$$

$= 0.338$ or $\underline{33.8\%}$

- Efficiency can also be found from

$$\eta = \frac{\text{Work done in lifting the load/rev.}}{\text{Work done by the applied force/rev.}}$$

$$= \frac{W \times p}{F \times \pi d} = \frac{W}{F}\tan\alpha$$

where $F = \dfrac{T}{r} = \dfrac{75.211}{0.028} = 2686$ N

$$\eta = \frac{16\,000}{2686} \times 0.0568 = 0.338 \text{ or } \underline{33.8\%}$$

(a) *When load does not rotates with the screw*
Mean radius of the bearing surface,

$$r = \frac{1}{2}\left(\frac{50+10}{2}\right) = 15 \text{ mm}$$

Torque due to collar friction $= (\mu W)r$
$= 0.11 \times 16\,000 \times 0.015$
$= 26.4$ N.m

Total friction torque required to raise the load $= 75.211 + 26.4 = 101.61$ N.m

Work done in raising the load =
$T.2\pi N = 101.611 \times 2\pi \times 15 = \underline{9577 \text{ N.m}}$

$$\eta = \frac{\text{Work done in lifting the load/rev.}}{\text{Work done by the applied force/rev.}}$$

$$= \frac{W \times p}{F \times \pi d} = \frac{W}{F}\tan\alpha$$

where $F = \dfrac{T}{r} = \dfrac{101.61}{0.028} = 3629$ N

$$\eta = \frac{16\,000}{3629} \times 0.0568 = 0.25 \text{ or } \underline{25\%}$$

8.7 WEDGE

A wedge is used to raise loads like a screw jack. It consists of three sliding pairs as shown in Fig. 8.10(a) formed by the frame *A*, wedge *B* and the slider *S*. When a force **F** is applied to the wedge, the slider is raised in the guides raising the load.

Mechanical efficiency of the wedge is defined as the ratio of the load raised when friction is considered to the load raised when friction is neglected while the force applied is the same.

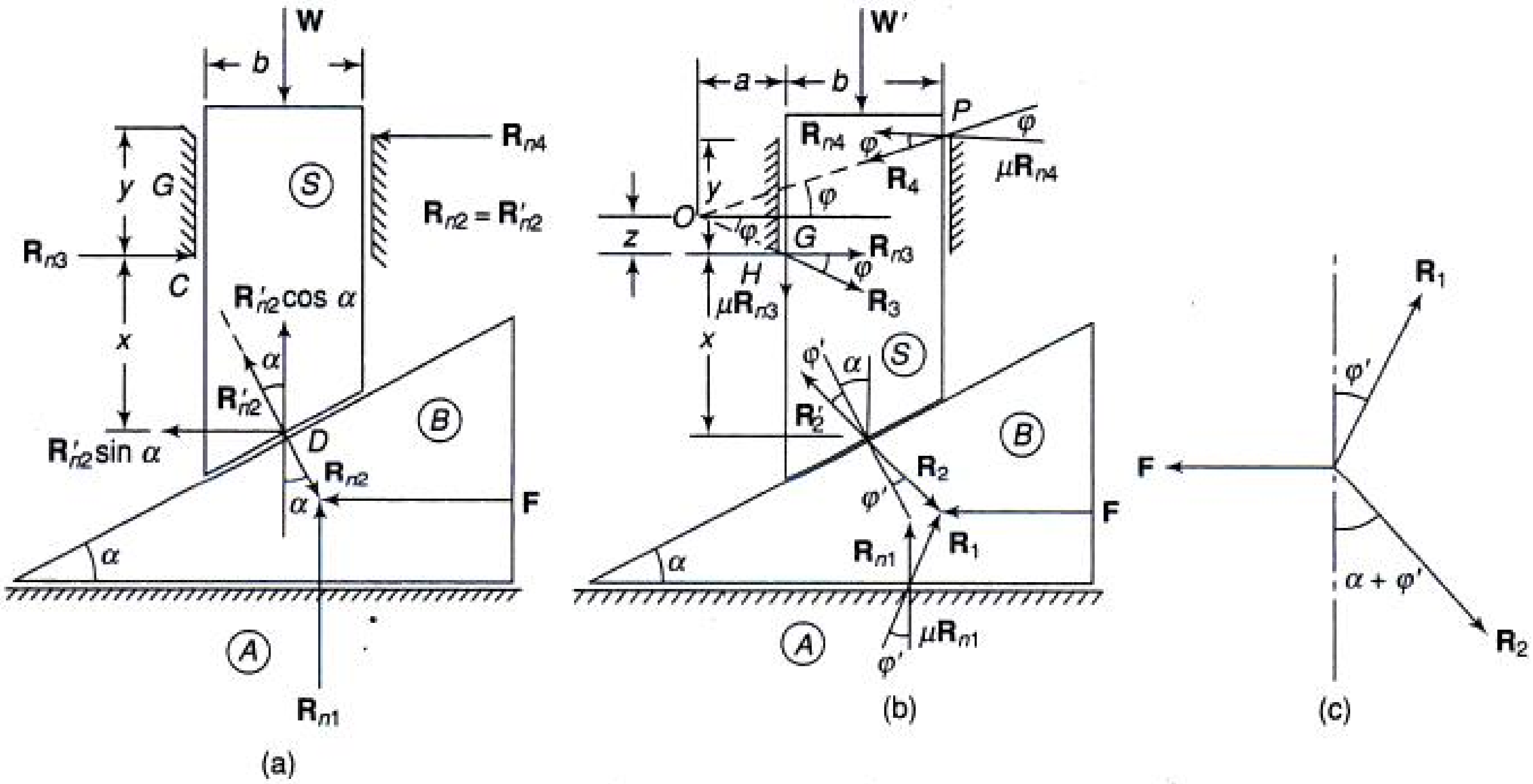

Fig. 8.10

$$= \int_{R_i}^{R_o} p \times 2\pi r dr$$

$$= \int_{R_i}^{R_o} \frac{C}{r} \times 2\pi r dr \qquad (p.r = C)$$

$$= \int_{R_i}^{R_o} 2\pi C.dr$$

$$= (2\pi Cr)_{R_i}^{R_o}$$

$$= 2\,\pi\,C\,(R_o - R_i)$$

$$= 2\,\pi\,pr\,(R_o - R_i)$$

or pressure intensity p at a radius r of the collar,

$$p = \frac{F}{2\pi r\,(R_o - R_i)} \qquad (8.28)$$

In a flat pivot, in which $R_i = 0$, the pressure would be infinity at the centre of the bearing ($r = 0$), which cannot be true. Thus, the uniform wear theory has a flaw in it.

Collars and pivots, using the above two theories, have been analysed below:

Collars

(i) Flat Collar

Let p = uniform normal pressure over an area

F = axial thrust

N = speed of the shaft

μ = coefficient of friction between the two surfaces

Consider an element of width δr of the collar at radius r. Friction force on the element (Fig. 8.10),

$$\delta F = \mu \times \text{axial force}$$

$$= \mu \times p \times \text{area of the element}$$

$$= \mu \times p \times 2\,\pi\,r\,\delta r$$

Friction torque about the shaft axis,

$$\delta T = \delta F \times r$$

$$= \mu \times p \times 2\,\pi\,r\,\delta r \times r$$

$$= 2\,\mu\,p\,\pi\,r^2\,\delta r$$

$$\text{Total friction torque, } T = \int_{R_i}^{R_o} 2\mu p \pi r^2 dr \qquad (8.29)$$

(a) With Uniform Pressure Theory Pressure is uniform over the whole area and is given by

$$p = \frac{F}{\pi(R_o^2 - R_i^2)}$$

$$T = \int_{R_i}^{R_o} 2\mu\pi r^2 \frac{F}{\pi\,(R_o^2 - R_i^2)} dr$$

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or $\quad R_o = 1.296\, R_i$

Now,

$$P = T.\ \omega$$

$$10\ 000 = T.\ \frac{2\pi \times 600}{60}$$

$$T = 159 \text{ N.m}$$

Intensity of pressure is maximum at the inner radius (uniform wear theory).

$$T = \frac{\mu F_n}{2}(R_o + R_i)$$

$$= \frac{\mu F}{2\sin\alpha}(R_o + R_i)$$

$$= \frac{\mu}{2\sin\alpha}[2\pi p_i R_i (R_o - R_i)](R_o + R_i)$$

$$= \frac{\mu\pi p_i}{\sin\alpha} R_i (R_o^2 - R_i^2)$$

$$159 = \frac{0.25\pi \times 100 \times 10^3}{\sin 15°} R_i[(1.296 R_i)^2 - R_i^2]$$

$$= \frac{0.25\pi \times 100 \times 10^3}{\sin 15°} \times 0.6796 R_i^3$$

$R_i = 0.0917$ m $\quad$ or $\quad$ <u>91.7mm</u>

$R_o = 91.7 \times 1.296 =$ <u>118.8 mm</u>

(ii) $b = \dfrac{R_o - R_i}{\sin\alpha} = \dfrac{118.8 - 91.7}{\sin 15°} = \underline{105 \text{ mm}}$

(iii) Axial force, $F = 2\ \pi p_i R_i\ (R_o - R_i)$

$= 2\pi \times 100 \times 10^3 \times 0.0917\ (0.1188 - 0.0917)$

$=$ <u>1561 N</u>

Example 8.27 *A centrifugal clutch transmits 20 kW of power at 750 rpm. The engagement of the clutch commences at 70 per cent of the running speed. The inside diameter of the drum is 200 mm and the distance of the centre of mass of each shoe is 40 mm from the contact surface. Determine the*

(i) mass of each shoe

(ii) net force exerted by each shoe on the drum surface

(iii) power transmitted when the shoe is worn 2 mm and is not readjusted

Assume μ to be 0.25 and stiffness of the spring 150 kN/m.

Solution:

$P = 20$ kW $\qquad R = 0.2$ m

$N = 750$ rpm $\qquad r = 0.2 - 0.04 = 0.16$ mm

$\mu = 0.25$

(i) $\omega = \dfrac{2\pi \times 750}{60} = 78.5$ rad/s

$\omega' = 0.7 \times 78.5 = 55$ rad/s

$P = T\,\omega$

$20\ 000 = T \times 78.5$

T = 254.8 N.m

Total frictional torque acting

$= \mu\ mr\ (\omega^2 - \omega'^2).R.n$

$254.8 = 0.25 \times m \times 0.16(78.5^2 - 55^2) \times 0.2 \times 4$

$= 100.4 \times m$

$m =$ <u>2.538 kg</u>

(ii) Net force exerted by each shoe on the drum surface

$= mr(\omega^2 - \omega'^2)$

$= 2.538 \times 0.16\ (78.5^2 - 55^2)$

$=$ <u>1274 N</u>

(iii) Spring force exerted by each spring

$= 2.538 \times 0.16 \times 55^2 = 1228.4$ N

When the shoe wears 2 mm, each shoe has to move forward by 2 mm. This increases the distance of the centre of mass of the shoe from 160 mm to 162 mm. Also, the spring force is increased due to its additional displacement of 2 mm.

Additional spring force = Stiffness × Displacement

$= 150 \times 10^3 \times 0.002$

$= 300$ N

Total spring force $= 1228.4 + 300 = 1528.4$ N

Net force exerted by each shoe on the drum surface

$= mr\omega^2 - 1528.4$

$= 2.538 \times 0.162 \times 78.5^2 - 1528.4$

$=$ <u>1005.2 N</u>

Total frictional torque acting $= \mu F.R.n$

$T = 0.25 \times 1005.2 \times 0.2 \times 4$

$= 201.04$ N.m

$P = T.\ \omega$

$= 201.04 \times 78.5$

$= 15\ 782$ W or <u>15.782 kW</u>

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The turning moment of the crankshaft = $F_c \times D'E$
where $D'E$ is drawn perpendicular to the friction axis.

Dead-angle and Dead-centre Positions During rotation, when the crank is near the inner dead centre, a position is obtained where the reaction at the crankshaft bearing and the friction axis of the connecting rod become in the same straight line [Fig. 8.22(d)]. These positions of the crank have been shown as OA_1 and OA_2 and are known as *dead-centre positions*. In these positions, the length $D'E$ is zero and, therefore, the turning moment or the torque transmitted is zero.

The angle A_1OA_2 is known as the *dead angle*. A corresponding dead angle at the outer dead-centre position of the crank can also be found in the same way.

2. Four-bar Mechanism

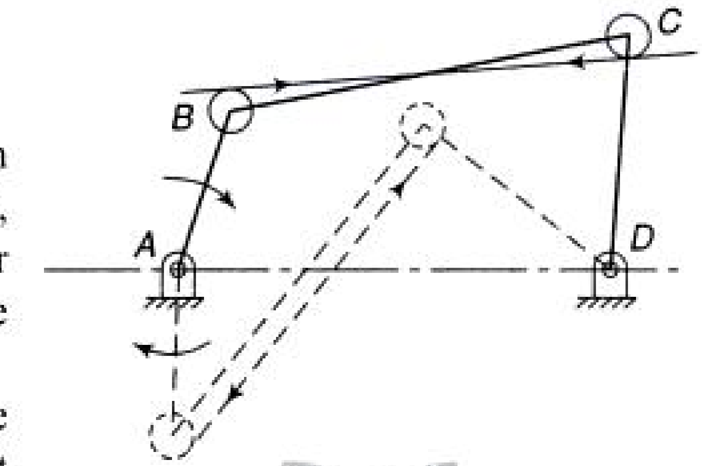

Fig. 8.23

Consider a four-bar mechanism shown in Fig. 8.23. Let the rotation of the driving link AB be clockwise. In the absence of friction, the driving torque induces compressive axial force in the coupler along its axis BC. When friction is considered, it is tangential to the friction circles at B and C.

The angle ABC at B is increasing. Thus, the coupler rotates in the counter-clockwise direction relative to AB. Therefore, the tangent at B is on the upper side of the circle to give a clockwise friction couple. Similarly at C, the angle BCD is decreasing. Thus, BC rotates in the counter-clockwise direction relative to CD. The tangent at C will be on the lower side of the circle so that the resulting friction couple is clockwise. Similarly, the friction axis for the links AB and DC can be drawn.

Example 8.28 *The length of crank of a slider-crank mechanism is 300 mm and of the connecting rod is 1.25 m. The diameters of the journals at the crosshead, crankpin and the crankshaft are 80 mm, 120 mm and 140 mm respectively. The steam pressure on the piston is 450 kN/m² which has a diameter of 250 mm. Coefficient of friction between the crosshead and the guides is 0.07 and for journals, it is 0.05.*

Find the reduction in the turning moment available at the crankshaft due to friction of the crosshead guides and the journals when the crank has rotated 50° from the inner-dead centre.

Solution: Refer to Fig. 8.24(a).

$AB = 1.25$ m $\qquad \theta = 50°$

$OA = 0.3$ m $\qquad p = 450$ kN/m²

$d = 250$ mm

Neglecting Friction

$$T = F_c \times OC = F_c \times OA \sin(\theta + \beta)$$

Piston at B is in equilibrium under the forces, **F**, $\mathbf{F_c}$ and **R** (reaction of guides).

$$\therefore\ F_c \cos\beta = F \quad \text{or} \quad F_c = \frac{F}{\cos\beta} = \frac{\frac{\pi}{4}d^2 \times p}{\cos\beta}$$

$$\text{Thus } T = \left(\frac{\pi}{4}d^2 p\right) \times OA\,\frac{\sin(\theta+\beta)}{\cos\beta}$$

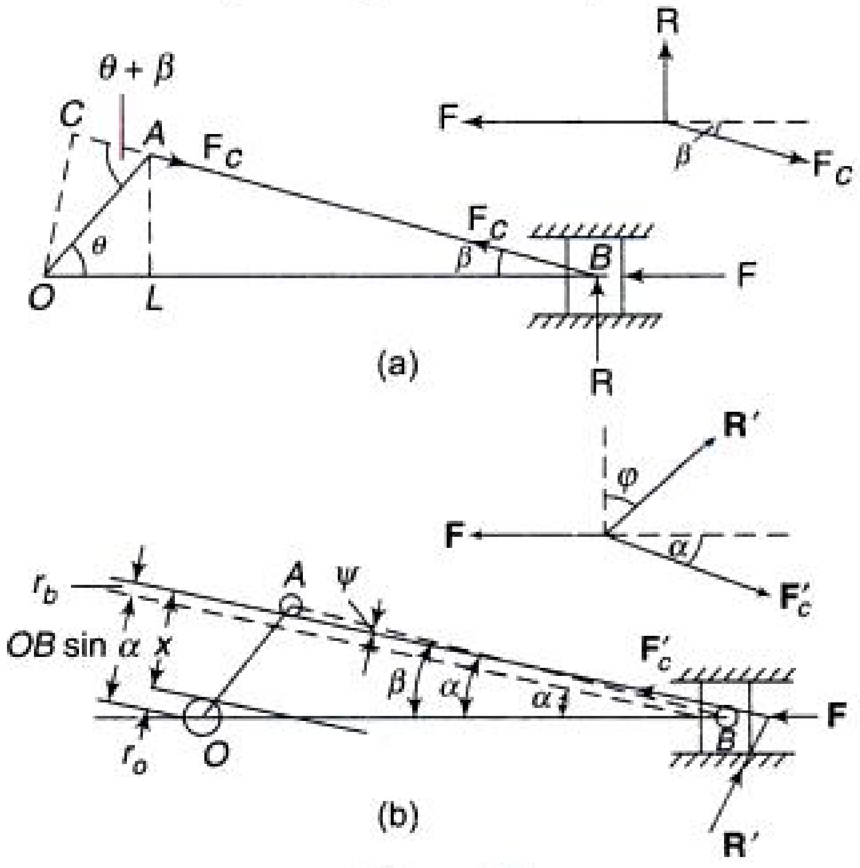

Fig. 8.24

11. The reversal of the nut is avoided if the efficiency of the thread is less than 50%
12. A wedge is used to raise loads like a screw jack.
13. Efficiency of a wedge

$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \times \frac{\cos(\alpha + \varphi + \varphi')}{\cos\varphi}$$

If $\varphi = \varphi'$, $\eta = \dfrac{\tan\alpha}{\tan(\alpha + 2\varphi)}$

14. A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.
15. When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.
16. In uniform pressure theory, pressure is assumed to be uniform over the surface area.
17. For uniform wear over an area, the intensity of pressure varies inversely proportional to the elementary areas and the product of the normal pressure and the corresponding radius is constant. Pressure intensity p at a radius r of the collar,

$$p = \frac{F}{2\pi r(R_o - R_i)}$$

18. For flat collars, friction torque is

$$T = \frac{2\mu F(R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)} \text{ with uniform pressure theory}$$

$$= \frac{\mu F}{2}(R_o^2 + R_i^2) \text{ with uniform wear theory}$$

19. For conical collars, friction torque is increased by $1/\sin\alpha$ times from that for flat collars.
20. Expressions for torque in case of pivots can directly be obtained from the expressions for collars by inserting the values $R_i = 0$ and $R_o = R$.
21. To be on safer side, friction torque in clutches is calculated on the basis of uniform wear theory and in bearings on the basis of uniform pressure theory.
22. A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.
23. In a multi-plate clutch, the number of frictional linings and the metal plates is increased which increases the capacity of the clutch to transmit torque.
24. Ball and roller bearings are known as *anti-friction bearings*.
25. The property of a lubricant to form a layer of molecular thickness (adsorbed film) on a metallic surface is known as its *oiliness*.
26. Greasy or boundary friction occurs in heavily loaded, slow-running bearings.
27. A circle drawn with μr as radius is known as the *friction circle* of the journal.
28. Friction couple (torque) = $Wr\mu$
29. During rotation, the positions of the crank where the reaction at the crankshaft bearing and the friction axis of the connecting rod become aligned in the same straight line are known as *dead centre positions*.
30. For a journal rotating in a bearing under the film lubrication conditions, the frictional resistance is proportional to the area, the viscosity of the lubricant, the speed and is independent of the pressure and the materials of the journal and the bearing.

Exercises

1. What is friction? Is it a blessing or curse? Justify your answer giving examples.
2. What are various kinds of friction? Discuss each in brief.
3. What are the laws of solid dry friction?
4. Define the terms *coefficient of friction* and *limiting angle of friction*.
5. Deduce an expression for the efficiency of an inclined plane when a body moves
   (i) up a plane
   (ii) down a plane
6. Find expression for the screw efficiency of a square thread. Also, determine the condition for maximum efficiency.
7. Show that the reversal of the nut is avoided if the efficiency of square thread is less than 50% (approximately).
8. What is a wedge? Deduce an expression for its efficiency.
9. What are uniform pressure and uniform wear theories? Deduce expressions for the friction torque considering both the theories for a flat collar.
10. In what way are the expressions for the friction

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effective radius of rotation of a pulley is obtained by adding half the belt thickness to the radius of the pulley.

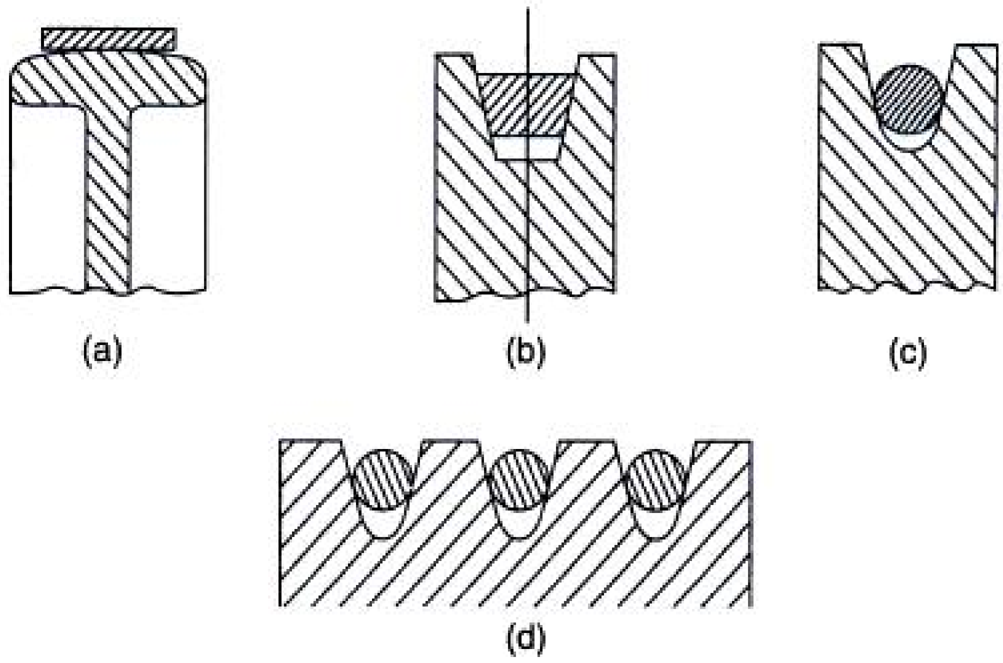

Fig. 9.2

Belt Drive A belt may be of rectangular section, known as a *flat belt* [Fig. 9.2(a)] or of trapezoidal section, known as a *V-belt* [Fig. 9.2(b)]. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove. Owing to wedging action, V-belts need little adjustment and transmit more power, without slip, as compared to flat belts. Also, a multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity. Generally, these are more suitable for shorter centre distances.

Some advantages of V-belts are

- Positive drive as slip between belt and pulley is negligible
- No joint troubles as V-belts are made endless
- Operation is smooth and quite
- High velocity ratio up to 10 can be obtained
- Due to wedging action in the grooves, limiting ratio of tensions is higher and thus, more power transmission
- Multiple V-belt drive increases the power transmission manifold
- May be operated in either direction with tight side at the top or bottom
- Can be easily installed and removed.

Disadvantages of V-belts are

- Cannot be used for large centre distances
- Construction of pulleys is not simple
- Not as durable as flat belts
- Costlier as compared to flat belts.

Rope Drive For power transmission by ropes, grooved pulleys are used [Fig. 9.2(c)]. The rope is gripped on its sides as it bends down in the groove reducing the chances of slipping. Pulleys with several grooves can also be employed to increase the capacity of power transmission [Fig. 9.2(d)]. These may be connected in either of the two ways:

1. Using a continuous rope passing from one pulley to the other and back again to the same pulley in the next groove, and so on.
2. Using one rope for each pair of grooves.

The advantage of using continuous rope is that the tension in it is uniformly distributed. However, in case of belt failure, the whole drive is put out of action. Using one rope for each groove poses difficulty in tightening the ropes to the same extent but with the advantage that the system can continue its operation even if a rope fails. The repair can be undertaken when it is convenient.

Rope drives are, usually, preferred for long centre distances between the shafts, ropes being cheaper as compared to belts. These days, however, long distances are avoided and thus, the use of ropes has been limited.

9.6 MATERIAL FOR BELTS AND ROPES

Choice of materials for the belts and ropes is influenced by climate or environmental conditions along with the service requirements. The common materials are as given below:

1. Flat Belts

Usual materials for flat belts are leather, canvas, cotton and rubber. These belts are used to connect shafts up to 8–10 m apart with speeds as high as 22 m/s.

Leather belts are made from 1.2 to 1.5 m long strips. The thickness of a belt may be increased by cementing the strips together. The belts are specified by the number of layers, i.e., single, double or triple ply. The leather belts are cleaned and dressed periodically with suitable oils to keep them soft and flexible.

Fabric belts are made by folding cotton or canvas layers to three or more layers and stitching together. The belts are made waterproof by impregnating with linseed oil. These are mostly used in belt conveyors and farm machinery.

Rubber belts are very flexible and are destroyed quickly on coming in contact with heat, grease or oil. Usually, these are made endless. Rubber belts are used in paper and saw mills as these can withstand moisture.

2. V-Belts

These are made of rubber impregnated fabric with the angle of V between 30 to 40 degrees. These are used to connect shafts up to 4 m apart. Speed ratios can be up to 7 to 1 and belt speeds up to 24 m/s.

3. Ropes

The materials for ropes are cotton, hemp, manila or wire. Ropes may be used to connect shafts up to 30 m apart with operating speed less than 3 m/s.

Hemp and *manila* fibres are rough and thus, the ropes made from such materials are not very flexible. Manila ropes are stronger as compared to hemp ropes. Generally, the rope fibres are lubricated with tar, tallow or graphite to prevent sliding of fibres when the ropes are bent over the pulleys. The cotton ropes are soft and smooth and do not require lubrication. These are not as strong and durable as manila ropes.

Wire ropes are used when the power transmitted is large over long distances, may be up to 150 m such as cranes, conveyors, elevators, etc. Wire ropes are lighter in weight, have silent operation, do not fail suddenly, more reliable and durable, less costly and can withstand shock loads.

9.7 CROWING OF PULLEYS

As mentioned is Section 9.2, the rim of the pulley of a flat-belt drive is slightly crowned to prevent the slipping off the belt from the pulley. The crowing can be in the form of conical surface or a convex surface.

Assume that somehow a belt comes over the conical portion of the pulley and takes the position as shown in Fig. 9.5(a), i.e., its centre line remains in a plane, the belt will touch the rim surface at its one edge only. This is impractical. Owing to the pull, the belt always tends to stick to the rim surface. The belt also has a lateral stiffness. Thus, a belt has to bend in the way shown in Fig. 9.5(b) to be on the conical surface of the pulley.

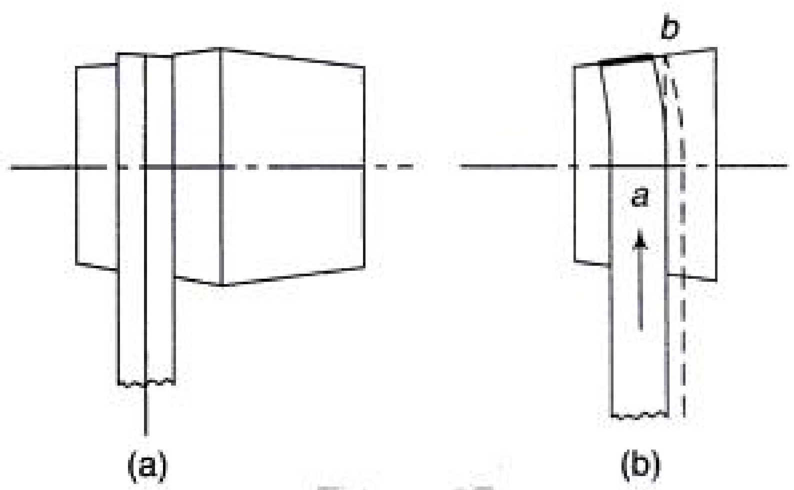

Fig. 9.5

As CD is tangent to two circles, AC and BD both are perpendicular to CD or AN.

Now, $AB \perp BK$ and $AN \perp BD$.

$\therefore \quad \angle DBK = \angle NAB = \beta$

Similarly, as $BA \perp AJ$, $NA \perp AC$

$\angle CAJ = \angle NAB = \beta$

$$L_o = 2\,[\text{Arc } GC + CD + \text{arc } DH]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right) r + AN + \left(\frac{\pi}{2} + \beta\right) R\right]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right) r + C\cos\beta + \left(\frac{\pi}{2} + \beta\right) R\right]$$

$$= \pi (R + r) + 2\beta (R - r) + 2C\cos\beta \tag{9.4}$$

This relation gives the exact length of belt required for an open belt drive. In this relation,

$$\beta = \sin^{-}\left(\frac{R - r}{C}\right) \tag{9.5}$$

An approximate relation for the length of belt can also be found in terms of R, r and C eliminating β, if β is small, i.e., if the difference in radii of the two pulleys is small and the centre distance is large.

For small angle of β, $\sin\beta \approx \beta$

$$\therefore \qquad \beta = \frac{R - r}{C}$$

and

$$\cos\beta = \sqrt{1 - \sin^2\beta}$$

$$= (1 - \sin^2\beta)^{1/2}$$

$$= \left(1 - \frac{1}{2}\sin^2\beta + \ldots\ldots\right) \qquad \text{[By binomial theorem]}$$

or

$$\cos\beta = \left(1 - \frac{1}{2}\beta^2\right) = 1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2$$

$$L_o = \pi (R + r) + 2\left(\frac{R - r}{C}\right)(R - r) + 2C\left[1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2\right]$$

$$= \pi (R + r) + 2\frac{(R - r)^2}{C} + 2C - \frac{2C}{2}\frac{(R - r)^2}{C^2}$$

$$= \pi (R + r) + 2\frac{(R - r)^2}{C} - \frac{(R - r)^2}{C} + 2C$$

$$= \pi (R + r) + \frac{(R - r)^2}{C} + 2C \tag{9.6}$$

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$$\frac{r_n}{R_n} = \frac{R_1}{r_1} \text{ or } K^{n-1}\frac{r_1}{R_1} = \frac{R_1}{r_1} \text{ or } \left(\frac{R_1}{r_1}\right)^2 = K^{n-1} \text{ or } \frac{R_1}{r_1} = \sqrt{K^{n-1}} \tag{9.11}$$

This gives the ratio R_1/r_1. After deciding its value, ratio r_2/R_2, r_3/R_3, etc., can be obtained and then from the relation for the length of belt, the values of r_2, R_2, and r_3, R_3, etc., can be obtained.

If cone pulleys are being used for a crossed-belt drive, it is very easy to obtain the dimensions of the radii of different pairs of steps. In this case, the length of the belt is same if the sums of the radii of different pairs of steps are constant for a given centre distance between the pulleys, i.e.,

$$R_1 + r_1 = R_2 + r_2 = \ldots\ldots = R_n + r_n \tag{9.12}$$

Example 9.3 *Design a set of stepped pulleys to drive a machine from a countershaft that runs at 220 rpm. The distance between centres of the two sets of pulleys is 2 m. The diameter of the smallest step on the countershaft is 160 mm. The machine is to run at 80, 100 and 130 rpm and should be able to rotate in either direction.*

Solution: As the driven shaft is to rotate in either direction, both the cases of a crossed-belt and an open-belt are to be considered.

(*i*) ***For Crossed-belt System***

The smallest step on the countershaft will correspond to the biggest step on the machine shaft (or the minimum speed of the machine shaft).

$n_1 = n_2 = n_3 = 220$ rpm $\qquad r_1 = 80$ mm

$N_1, N_2, N_3 = 80, 100, 130$ rpm respectively

(*a*) *For First Step*

$$\frac{R_1}{r_1} = \frac{n_1}{N_1} \text{ or } \frac{R_1}{80} = \frac{220}{80} \text{ or } R_1 = \underline{220 \text{ mm}}$$

(*b*) *For Second Step*

$$\frac{R_2}{r_2} = \frac{n_2}{N_2} = \frac{220}{100} \text{ or } R_2 = 2.2r_2$$

Also $R_2 + r_2 = R_1 + r_1$

$2.2r_2 + r_2 = 220 + 80$

$3.2r_2 = 300$

$r_2 = \underline{93.75 \text{ mm}}$

$R_2 = 93.7 \times 2.2 = \underline{206.3 \text{ mm}}$

(*c*) *For third Step*

$$\frac{R_3}{r_3} = \frac{220}{130} \text{ or } R_3 = 1.69r_3$$

Also

$R_3 + r_3 = R_1 + r_1$

$1.69\,r_3 + r_3 = 220 + 80 = 300$

$r_3 = \underline{111.5 \text{ mm}}$

$R_3 = 111.5 \times 1.69 = \underline{188.5 \text{ mm}}$

(*ii*) ***For Open-belt System***

(*a*) *For First Step* $r_1 = 80$ mm $R_1 = 220$ mm as before

(*b*) *For Second Step*

$$\pi(R_2 + r_2) + \frac{(R_2 - r_2)^2}{C} = \pi(R_1 + r_1) + \frac{(R_1 - r_1)^2}{C}$$

$$\text{or} \quad \pi(2.2r_2 + r_2) + \frac{(2.2r_2 - r_2)^2}{2} = \pi(0.22 + 0.08) + \frac{(0.22 - 0.08)^2}{2}$$

$$10.05r_2 + \frac{1.44}{2}r_2^2 = 0.9523$$

$$r_2^2 + 13.958\,r_2 = 1.323$$

$(r_2 + 6.979)^2 = 1.323 + (6.979)^2$

$= 50.029 = (7.073)^2$

$r_2 = 7.073 - 6.979 = 0.094$ m or 94 mm

$R_2 = 2.2 \times 94 = \underline{206.8 \text{ mm}}$

(*c*) *For Third Step*

$$\pi(1.69r_3 + r_3) + \frac{(1.69r_3 - r_3)^2}{2} = 0.9523$$

$$\text{or} \quad 8.451r_3 + \frac{0.476}{2}r_3^2 = 0.9523$$

$$\text{or} \quad r_3^2 + 35.508r_3 = 4.001$$

$$\text{or} \quad (r_3^2 + 17.754)^2 = 4.001 + (17.754)^2 = 319.206 = (17.866)^2$$

or $r_3 = 17.866 - 17.754 = 0.112$ m or 112 mm

$R_3 = 112 \times 1.69 = \underline{189.3 \text{ mm}}$

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or $\theta = \pi + 2\sin^{-}\left(\dfrac{450+300}{5000}\right)$

$= \pi + 17.254° = \pi + 0.301 = 3.443$ rad

Also $\dfrac{T_1}{T_2} = e^{\mu\theta}$

or $\dfrac{T_1}{T_2} = e^{0.25\times3.443} = 2.365$ or $T_1 = 2.365T_2$ (iii)

From (i) and (iii),

$2.365T_2 - T_2 = 1061$

$T_2 = 777$ N

$T_1 = 1838$ N

The maximum tension, $T_1 = \sigma.b.t$

or $1838 = 3b \times 8$ or $b = 76.6$ mm

Example 9.7 *A 100-mm wide and 10-mm thick belt transmits 5 kW of power between two parallel shafts. The distance between the shaft centres is 1.5 m and the diameter of the smaller pulley is 440 mm. The driving and the driven shafts rotate at 60 rpm and 150 rpm respectively. The coefficient of friction is 0.22. Find the stress in the belt if the two pulleys are connected by (i) an open belt, and (ii) a cross belt. Take μ = 0.22.*

Solution Speed of the driving pulley, $N_1 = 60$ rpm

Speed of the driven pulley, $N_2 = 150$ rpm

Thus, smaller pulley is the driven pulley and $d = 440$ mm

$P = 5$ kW; $b = 100$ mm; $C = 1.5$ m; $t = 10$ mm; $\mu = 0.22$; $r = 220$ mm;

$$v = \omega_2\left(r + \frac{t}{2}\right) = \frac{2\pi N_2}{60}\left(r + \frac{t}{2}\right)$$

$$= \frac{2\times\pi\times150}{60}\left(220 + \frac{10}{2}\right)$$

$= 3535$ mm/s or 3.535 m/s

$P = (T_1 - T_2)v$ or $5000 = (T_1 - T_2) \times 3.535$

or $T_1 - T_2 = 1414.5$ N (i)

(i) Open-belt Drive

Angle of contact on the smaller pulley,

$$\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-1}\left(\frac{220\times150/60 - 220}{1500}\right)$$

$= \pi - 25.4°$

$= \pi - 0.443$ (β is to be in radians)

$= 2.698$ rad.

$\dfrac{T_1}{T_2} = e^{\mu\theta}$

or $\dfrac{T_1}{T_2} = e^{0.22\times2.698} = 1.81$ or $T_1 = 1.81T_2$ (ii)

From (i) and (ii), $T_1 - T_2 = 1414.5$

$\therefore$ $1.81T_2 - T_1 = 1414.5$

or $0.81T_2 = 1414.5$

$T_2 = 1746.3$ N and $T_1 = 1746.3 \times 1.81 = 3160.8$ N

$\therefore$ stress in the belt,

$$\sigma_t = \frac{T_1}{b\times t} = \frac{3160.8}{100\times10} = \underline{3.16 \text{ N/mm}^2}$$

(ii) Cross-belt Drive

$$\theta = \pi + 2\beta = \pi + 2\sin^{-1}\left(\frac{R+r}{C}\right)$$

or $\theta = \pi + 2\sin^{-1}\left(\dfrac{220\times150/60 + 220}{1500}\right)$

$= \pi + 61.8°$

or $\theta = \pi + 1.08 = 4.22$ rad

$\dfrac{T_1}{T_2} = e^{0.22\times4.22} = 2.53$

$T_1 - T_2 = 3160.8 - 1746.3 = 1414.5$ (iii)

From (i) and (iii), $2.53T_2 - T_1 = 1414.5$

$T_2 = 924.5$ N

$T_1 = 924.5 \times 2.53 = 2339$ N

$$\sigma_t = \frac{2339}{100\times10} = \underline{2.339 \text{ N/mm}^2}$$

Example 9.9 *Two parallel shafts that are 3.5m apart are connected by two pulleys of 1-m and 400-mm diameters, the larger pulley being the driver runs at 220 rpm. The belt weighs 1.2 kg per metre length. The maximum tension in the belt is not to exceed 1.8 kN. The coefficient of friction is 0.28. Owing to slip on one of the pulleys, the velocity of the driven shaft is 520 rpm only. Determine the*

(i) torque on each shaft
(ii) power transmitted
(iii) power lost in friction
(iv) efficiency of the drive

Solution The larger pulley is the driver, $D = 1$ m; $N_1 = 220$ rpm;

$R = 500$ mm; $d = 400$ mm; $r = 200$ mm; $N_2 = 520$ rpm; $C = 3.5$ m; $\mu = 0.28$; $m = 1.2$ kg/m; $T = 1800$ N

$$v = \frac{\pi D N_1}{60} = \frac{\pi \times 1 \times 220}{60} = 11.52 \text{ m/s}$$

Also $T_c = mv^2 = 1.2 \times 11.52^2 = 159$ N

$\therefore$ tension on the tight side, $T_1 = T - T_c$

$= 1800 - 159 = 1641$ N

$$\text{Now, } \theta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-}\left(\frac{500-200}{3500}\right) = \pi - 9.834°$$

$$= \pi - 0.172 = 2.97 \text{ rad}$$

$$\text{Also } \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.28\times 2.97} = 2.297 \text{ or } T_1 = 2.297\, T_2$$

or $T_2 = 1641/2.297 = 714$ N

(i) Torque on larger pulley $= (T_1 - T_2)R$
$= (1641 - 714) \times 0.5 = 463.5$ N.m
Torque on smaller pulley $= (T_1 - T_2)r$
$= (1641 - 714) \times 0.2 = 185.4$ N.m

(ii) $P = (T_1 - T_2)v = (1641 - 714) \times 11.52$
$= 10679$ W $= 10.679$ kW

(iii) $$\text{Power input} = \frac{2\pi N_1 T_1}{60} = \frac{2\pi \times 220 \times 463.5}{60}$$
10 678 W…(T_1 is the torque)

$$\text{Power output} = \frac{2\pi N_2 T_2}{60} = \frac{2\pi \times 520 \times 185.4}{60}$$
$= 10\,096$ W…(T_2 is the torque)

Power loss = 10 678 − 10 096 = 582 W

(iv) $$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{10096}{10678}$$
$= 0.945$ or 9.45%

Example 9.10 *A V-belt drive with the following data transmits power from an electric motor to a compressor:*

Power transmitted	=	*100 kW*
Speed of the electric motor	=	*750 rpm*
Speed of the compressor	=	*300 rpm*
Diameter of compressor pulley	=	*800 mm*
Centre distance between pulleys	=	*1.5 m*
Maximum speed of the belt	=	*30 m/s*
Mass density of the belt	=	*900 kg/m³*
Cross-sectional area of belt	=	*350 mm²*
Allowable stress in the belt	=	*2.2 N/mm²*
Groove angle of the pulley	=	*38°*
Coefficient of friction	=	*0.28*

Determine the number of belts required and the length of each belt.

Solution Speed of driving pulley (electric motor), $N_1 = 750$ rpm

Speed of the driven pulley, $N_2 = 300$ rpm

Thus, larger pulley is the driven pulley and $D = 800$ mm

$$\therefore \quad \frac{d}{D} = \frac{N_2}{N_1} \text{ or } \frac{d}{800} = \frac{300}{750} \text{ or } d = 320 \text{ mm or}$$
$r = 160$ mm

Mass of belt/m length = area × length × density
$= 350 \times 10^{-6} \times 1 \times 900 = 0.315$ kg

Centrifugal tension, $T_c = mv^2 = 0.315 \times 30^2$
$= 283.5$ N

Maximum tension in the belt,

$T = \sigma \times \text{area} = 2.2 \times 350 = 770$ N

$T_1 = T - T_c = 770 - 283.5 = 486.5$ N

$$\text{Now, } \theta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

Solution $P = 2.5$ kW $\mu = 0.3$

$\theta = 165°$ $v = 2.5$ m/s

$P = (T_1 - T_2)v$

$$2500 = (T_1 - T_2) \times 2.5$$

$$T_1 - T_2 = 1000 \text{ N}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 165\pi/180} = 2.37$$

or $T_1 = 2.37T_2$

$2.37T_2 - T_2 = 1000$

or $T_2 = 729.9$ N

$T_1 = 729.9 \times 2.37 = 1729.9$ N

Initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1729.3 + 729.9}{2} = 1229.9 \text{ N}$$

(i) When initial tension is increased by 8%

$T'_0 = 1229.9 \times 1.08 = 1328.3$ N

or $\frac{T_1 + T_2}{2} = 1328.3$ or $T_1 + T_2 = 2656.6$

As μ and θ remain unchanged, $e^{\mu\theta}$ or $\frac{T_1}{T_2}$ is same.

$$2.37\,T_2 + T_2 = 2556.6$$

$$T_2 = 788.3 \text{ N}$$

$$T_1 = 1868.3 \text{ N}$$

$$P = (T_1 - T_2)v = (1868.3 - 788.3) \times 2.5$$

$$= 2700 \text{ W or } 2.7 \text{ kW}$$

$\therefore$ increase in power $= \frac{2.7 - 2.5}{2.5} = 0.08$ or 8%

(ii) When initial tension is decreased by 8%

$T'_0 = 1229.9 \times (1 - 0.08) = 1131.5$

or $\frac{T_1 + T_2}{2} = 1131.5$ or $T_1 + T_2 = 2263$

$3.37T_2 = 2263$

$T_2 = 671.5$ N

$T_1 = 1591.5$ N

$P = (1591.5 - 671.5) \times 2.5 = 2300$ W or 2.3 kW

$\therefore$ Decrease in power $= \frac{2.5 - 2.3}{2.5} = 0.08$ or 8%

(iii) $\frac{T_1}{T_2} = e^{\mu\theta}$

T_1 is the same as before whereas θ increases by 8%

$$\frac{1729.9}{T_2} = e^{0.3 \times \frac{165 \times 1.08 \times \pi}{180}} = 2.54$$

$T_2 = 680.5$ N

$P = (1729.9 - 680.5) \times 2.5 = 2624$ W

or 2.624 kW

$\therefore$ Increase in power

$$= \frac{2.624 - 2.5}{2.5} = 0.0496 \text{ or } 4.96\%$$

(iv) $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 1.08 \times \frac{165 \times \pi}{180}} = 2.54$

or $T_1 = 2.54\,T_2$

$T_1 + T_2 = 1229.9 \times 2 = 2459.8$

$T_2 = 694.9$ N

$T_1 = 694.9 \times 2.54 = 1764.9$ N

$P = (1764.9 - 694.9) \times 2.5$

$= 2675$ W or 2.675 kW

$\therefore$ Increase in power

$$= \frac{2.675 - 2.5}{2.5} = 0.07 \text{ or } 7\%$$

Example 9.15 *In a belt drive, the mass of the belt is 1 kg/m length and its speed is 6 m/s. The drive transmits 9.6 kW of power. Determine the initial tension in the belt and the strength of the belt. The coefficient of friction is 0.25 and the angle of lap is 220°.*

Solution $P = (T_1 - T_2)v$

or $9600 = (T_1 - T_2) \times 6$

or $T_1 - T_2 = 1600$ (i)

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times \frac{220\pi}{180}} = e^{0.96} = 2.61$$

or $T_1 = 2.61\,T_2$ (ii)

From (i) and (ii),

$2.61\,T_2 - T_2 = 1600$

$T_2 = 994$ N

$T_1 = 2594$ N

Centrifugal tension $= mv^2 = 1 \times 12^2 = 144$ N

Initial tension $T_o = \frac{T_1 + T_2}{2} + T_c$

$$= \frac{2594 + 994}{2} + 144 = 1938 \text{ N}$$

are known as *sprockets*. The surfaces of sprockets conform to the type of chain used. Usually, a sprocket has projected teeth that fit into the recesses in the chain. Thus, the chain passes round the sprockets as a series of chordal links (Fig. 9.17).

The distance between roller centres of two adjacent links is known as the pitch (p) of the chain. A circle through the roller centres of a wrapped chain round a sprocket is called the *pitch circle* and its diameter as *pitch circle diameter*.

Observe that a chain is wrapped round the sprocket in the form of a pitch polygon and not in the form of a pitch circle.

Let T = number of teeth on a sprocket.

φ = angle subtended by chord of a link at the centre of sprocket

r = radius of the pitch circle

Then
$$p = 2r\sin\frac{\varphi}{2} = 2r\sin\frac{1}{2}\left(\frac{360^\circ}{T}\right) = 2r\sin\frac{180^\circ}{T}$$

or
$$r = \frac{p}{2\sin\frac{180^\circ}{T}} = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{T} \tag{9.21}$$

9.19 CHAIN LENGTH

For a given pair of sprockets at a fixed distance apart, the length of the chain may be calculated in the same way as for an open belt. Since the pitch line of a sprocket is a polygon, Eq. 9.6 will give a length slightly more than the actual length.

Let R and r be the radii of the pitch circles of the two sprockets having T and t teeth respectively. Also,

let L = length of the chain

C = centre distance between sprockets = kp

p = pitch of chain

From Eq. (9.6),

$$L = \pi(R + r) + \frac{(R - r)^2}{C} + 2C$$

The first term in the equation is half the sum of the circumference of the pitch circles. In case of a chain it will be $(pT + pt)/2$.

Replacing R and r in the second term by

$$R = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{T} \text{ and } r = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{t}$$

$$L = \frac{pT + pt}{2} + \frac{\left(\frac{p}{2}\operatorname{cosec}\frac{180^\circ}{T} - \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{t}\right)^2}{kp} + 2kp$$

$$= p\left[\frac{T + t}{2} + \frac{\left(\operatorname{cosec}\frac{180^\circ}{T} - \operatorname{cosec}\frac{180^\circ}{t}\right)^2}{4k} + 2k\right] \tag{9.22}$$

Exercises

1. What are different modes of transmitting power from one shaft to another? Compare them.
2. Discuss the effect of slip of belt on the pulleys on the velocity ratio of a belt drive.
3. Name the materials of the flat belts, V-belts and ropes.
4. What do you mean by crowning of pulleys in flat-belt drives? What is its use?
5. What are different types of pulleys? Explain briefly.
6. Explain the following:
   (i) Idler pulleys
   (ii) Intermediate pulleys
   (iii) Loose and fast pulleys
   (iv) Guide pulleys
7. Define and elaborate the law of belting.
8. Deduce expressions for the exact and approximate lengths of belt in an open-belt drive.
9. What is meant by cross-belt drive? Find the length of belt in a cross-belt drive.
10. Where do we use cone (stepped) pulleys? Explain the procedure to design them.
11. Derive the relation for ratio of belt tensions in a flat-belt drive.
12. Derive the relation, $\frac{T_1}{T_2} = e^{\mu\theta}$ for a flat-belt drive with usual notations.
13. Deduce an expression for the ratio of tight and slack side tensions in case of a V-belt drive.
14. What is the effect of centrifugal tension on the tight and slack sides of a belt drive? Show that it is independent of the tight and slack-side tensions and depends only on the velocity of the belt over the pulley.
15. What is the effect of centrifugal tension on the power transmitted?
16. Derive the condition for maximum power transmission by a belt drive considering the effect of centrifugal tension.
17. What is meant by initial tension in a belt drive?
18. What is creep in a belt drive?
19. A motor shaft drives a main shaft of a workshop by means of a flat belt, the diameters of the pulleys being 500-mm and 800-mm respectively. Another pulley of 600 mm diameter on the main shaft drives a counter-shaft having a 750-mm diameter pulley. If the speed of the motor is 1600 rpm, find the speed of the countershaft neglecting the thickness of the belt and considering a slip of 4% on each drive.
    (737.3 rpm)
20. Two pulleys on two shafts are connected by a flat belt. The driving pulley is 250 mm in diameter and runs at 150 rpm. The speed of the driven pulley is to be 90 rpm. The belt is 120 mm wide, 5-mm thick and weighs 1000 kg/m^3. Assuming a slip of 2% between the belt and each pulley, determine the diameter of the driven pulley. Also, find the total effective slip.
    (403 mm; 3.96%)
21. The pulleys of two parallel shafts that 8 m apart are 600 mm and 800 mm in diameters and are connected by a crossed belt. It is needed to change the direction of rotation of the driven shaft by adopting the open-belt drive. Calculate the change in length of the belt.
    (Shorten the belt by 60 mm)
22. Determine the diameters of the cone pulley joined by a crossed belt. The driven shaft is desired to be run at speeds of 60, 90 and 120 rpm while the driving shaft rotates at 160 rpm. The centre distance between the axes of the two shafts is 2.5 m. The smallest pulley diameter can be taken as 150 mm.
    (150 mm and 400 mm; 198 mm and 352 mm; 236 mm and 314 mm)
23. Design a set of stepped pulleys to drive a machine from a countershaft running at 300 rpm. It is needed to have the following speeds of the driven shaft: 140 rpm, 180 rpm and 220 rpm. The centre distance between the axes of the two shafts is 5 m. The diameter of the smallest pulley is 300 mm. The two shafts rotate in the same direction.
    (300 mm and 642 mm; 354 mm and 590 mm; 400 mm and 545 mm)
24. A countershaft is to be driven at 240 rpm from a driving shaft rotating at 100 rpm by an open-belt drive. The diameter of the driving pulley is 480 mm. The distance between the centre line of shafts is 2 m. Find the width of the belt to transmit 3 kW of power if the safe permissible stress in tension is 15 N/mm width of the belt. Take $\mu = 0.3$.
    (134 mm)
25. A casting having a mass of 100 kg is suspended freely from a rope. The rope makes 2 turns round a drum of 300 mm diameter rotating at 24 rpm. The other end of the rope is pulled by a man. Calculate

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eliminated in double-helical gears. This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at high speeds with less noise and vibrations.

If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as *herringbone* gear (Fig. 10.5).

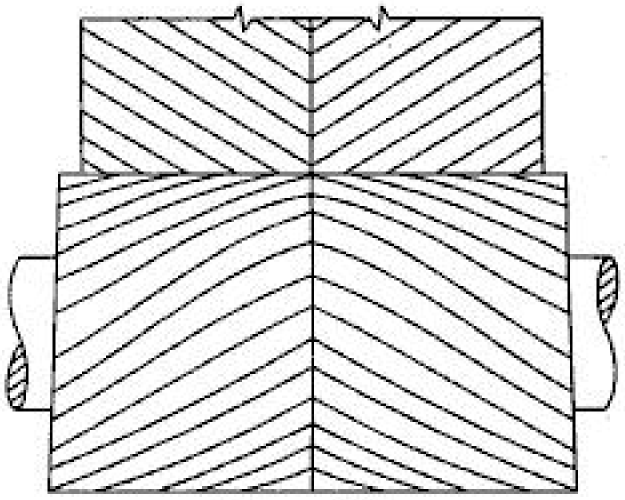

Fig. 10.5

2. Intersecting Shafts

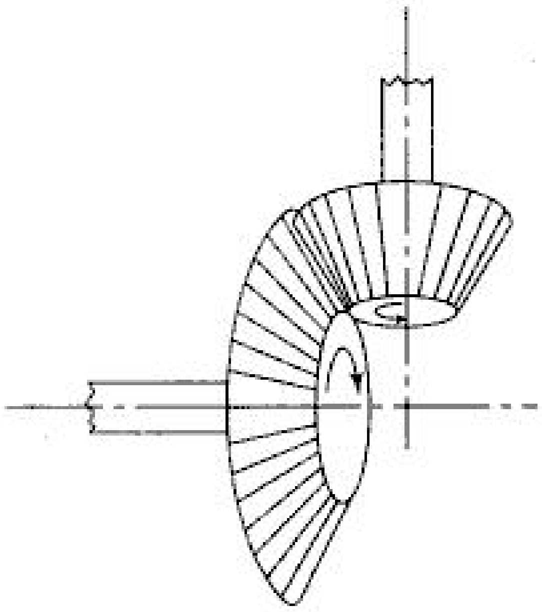

Fig. 10.6

Kinematically, the motion between two intersecting shafts is equivalent to the rolling of two cones, assuming no slipping. The gears, in general, are known as *bevel* gears.

When teeth formed on the cones are straight, the gears are known as *straight bevel* and when inclined, they are known as *spiral* or *helical bevel*.

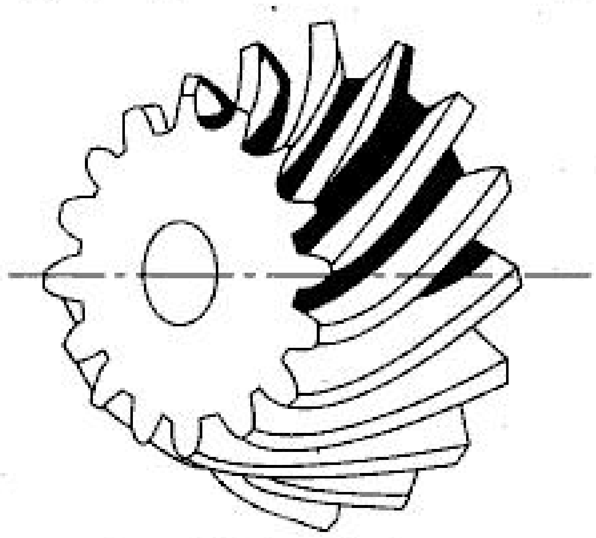

Fig. 10.7

Straight Bevel Gears The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout their length. Usually, they are used to connect shafts at right angles which run at low speeds (Fig. 10.6). Gears of the same size and connecting two shafts at right angles to each other are known as *mitre* gears.

At the beginning of engagement, straight bevel gears make the line contact similar to spur gears. There can also be *internal bevel* gears analogous to internal spur gears.

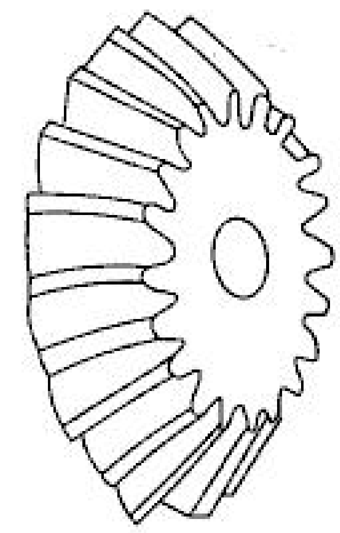

Fig. 10.8

Spiral Bevel Gears When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as *spiral bevels* or *helical bevels* (Fig. 10.7). They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supporting assemblies.

These are used for the drive to the differential of an automobile.

Zerol Bevel Gears Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zerol bevel gears (Fig. 10.8). Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the same mountings. However, they are quieter in action than the straight bevel type as the teeth are curved.

3. Skew Shafts

In case of parallel and intersecting shafts, a uniform rotary motion is possible by pure rolling contact. But in case of skew (non-parallel, non-intersecting) shafts, this is not possible.

Observe a hyperboloid shown in Fig. 10.9(a). It is a surface of revolution generated by a skew line *AB* revolving around an axis *O-O* in another plane, keeping the angle ψ_1 between them as constant. The minimum

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(c) *Pitch Diameter* It is the diameter of the pitch cylinder.

(d) *Pitch Surface* It is the surface of the pitch cylinder.

(e) *Pitch Point* The point of contact of two pitch circles is known as the pitch point.

(f) *Line of Centres* A line through the centres of rotation of a pair of mating gears is the line of centres.

(g) *Pinion* It is the smaller and usually the driving gear of a pair of mated gears.

2. (a) *Rack* It is a part of a gear wheel of infinite diameter (Fig. 10.15).

(b) *Pitch Line* It is a part of the pitch circle of a rack and is a straight line (Fig. 10.15).

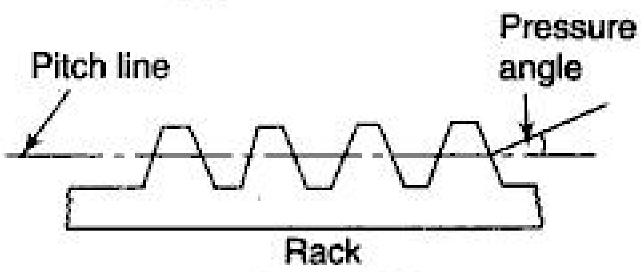

Fig. 10.15

3. *Pitch* It is defined as follows:

(a) *Circular Pitch (p)* It is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth (Fig. 10.13).

$$p = \frac{\pi d}{T}$$

where p = circular pitch

d = pitch diameter

T = number of teeth

As the expression for p involves π, an indeterminate number, p, cannot be expressed precisely. The angle subtended by the circular pitch at the centre of the pitch circle is known as the *pitch angle* (γ).

(b) *Diametral Pitch (P)* It is the number of teeth per unit length of the pitch circle diameter in inches.

$$P = \frac{T}{d}$$

The limitations of the diametral pitch is that it is not in terms of units of length, but in terms of teeth per unit length.

Also, it can be seen that

$$pP = \frac{\pi d}{T} \cdot \frac{T}{d} = \pi$$

The term *diametral pitch* is not used in SI units.

(c) *Module (m)* It is the ratio of the pitch diameter in mm to the number of teeth. The term is used in SI units in place of diametral pitch.

$$m = \frac{d}{T}$$

Also,

$$p = \frac{\pi d}{T} = \pi m$$

Pitch of two mating gears must be same.

4. (a) *Gear Ratio (G)* It is the ratio of the number of teeth on the gear to that on the pinion.

10.3 LAW OF GEARING

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears. Figure 10.17 shows two bodies 1 and 2 representing a portion of the two gears in mesh.

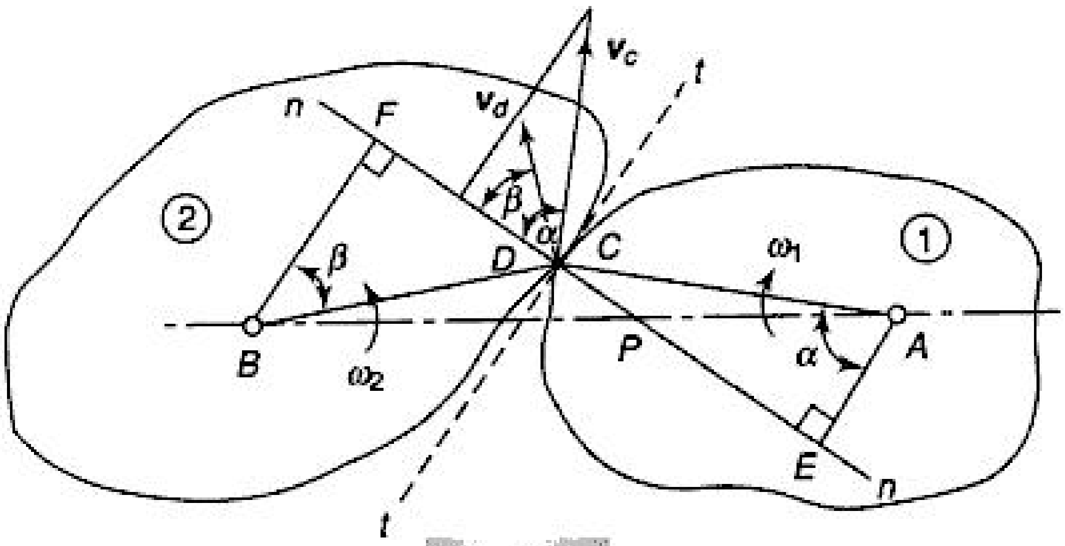

Fig. 10.17

A point C on the tooth profile of the gear 1 is in contact with a point D on the tooth profile of the gear 2. The two curves in contact at points C or D must have a common normal at the point. Let it be $n - n$.

Let ω_1 = instantaneous angular velocity of the gear 1 (clockwise)

ω_2 = instantaneous angular velocity of the gear 2 (counter-clockwise)

v_c = linear velocity of C

v_d = linear velocity of D

Then $v_c = \omega_1.AC$ in a direction perpendicular to AC or at an angle α to $n - n$.

$v_c = \omega_2.BD$ in a direction perpendicular to BD or at an angle β to $n - n$.

Now, if the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent $t - t$. The relative motion between the surfaces along the common normal $n - n$ must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along $n - n = v_c \cos\alpha$

Component of v_d along $n - n = v_d \cos\beta$

Relative motion along $n - n = v_c \cos\alpha - v_d \cos\beta$

Draw perpendiculars AE and BF on $n - n$ from points A and B respectively. Then $\angle CAE = \alpha$ and $\angle DBF = \beta$. For proper contact,

$$v_c \cos \alpha - v_d \cos \beta = 0$$

or
$$\omega_1 AC \cos \alpha - \omega_2 BD \cos \beta = 0$$

or
$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

or
$$\omega_1 AE - \omega_2 BF = 0$$

or
$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$$= \frac{BP}{AP}$$

[$\because$ ΔAEP and BEP are similar]

Thus, it is seen that the centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears. If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears.

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the circumference of a circle. The circle on which the straight line rolls or from which the cord is unwound is known as the *base circle.*

Figure 10.21 shows an involute generated by a line rolling over the circumference of a base circle with centre at O. At the start, the tracing point is at A. As the line rolls on the circumference of the circle, the path ABC traced out by the point A is the involute.

Note that as D can be regarded as the instantaneous centre of rotation of B, the motion of B is perpendicular to BD. Since BD is tangent to the base circle, the normal to the involute is a tangent to the base circle.

A short length EF of the involute drawn from A can be utilized to make the profile of an involute tooth. The other side HJ of the tooth has been taken from the involute drawn from G in the reverse direction. The profile of an involute tooth is made up of a single curve, and teeth, usually, are termed as single curve teeth.

Owing to the ease of standardization and manufacture, and low cost of production, the use of involute teeth has become universal by entirely superseding the cycloidal shape. Only one cutter or tool is necessary to manufacture a complete set of interchangeable gears. The cutter is in the form of a rack as all gears will gear with their corresponding rack. Moreover, the cutters of this form can be made to a higher degree of accuracy as the teeth of an involute rack are straight.

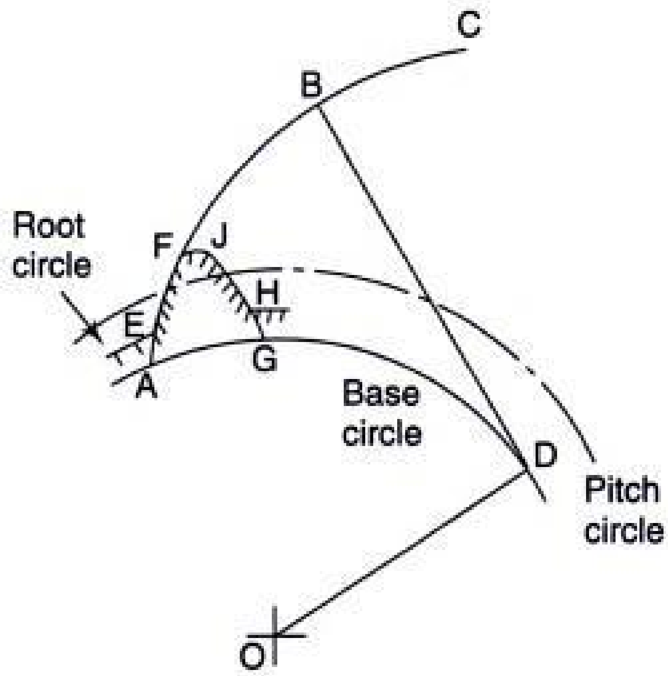
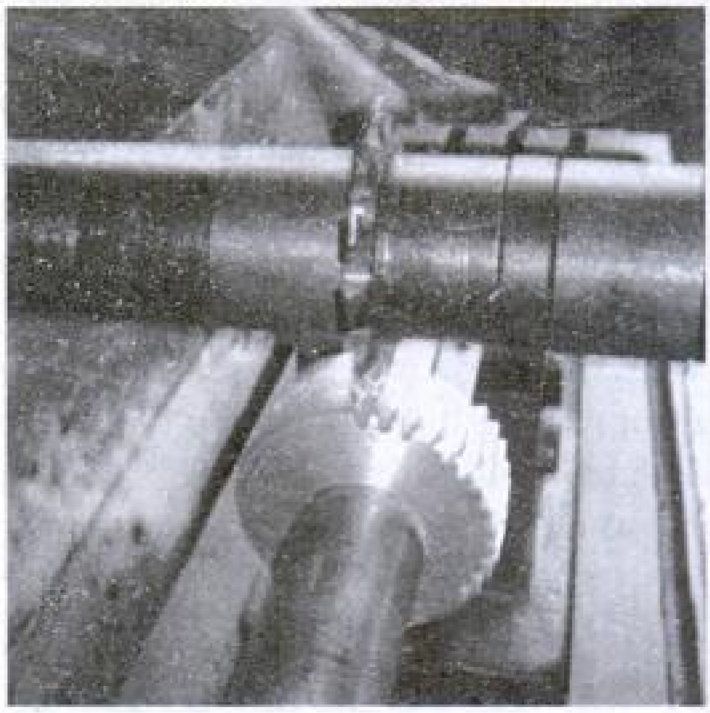


Fig. 10.21

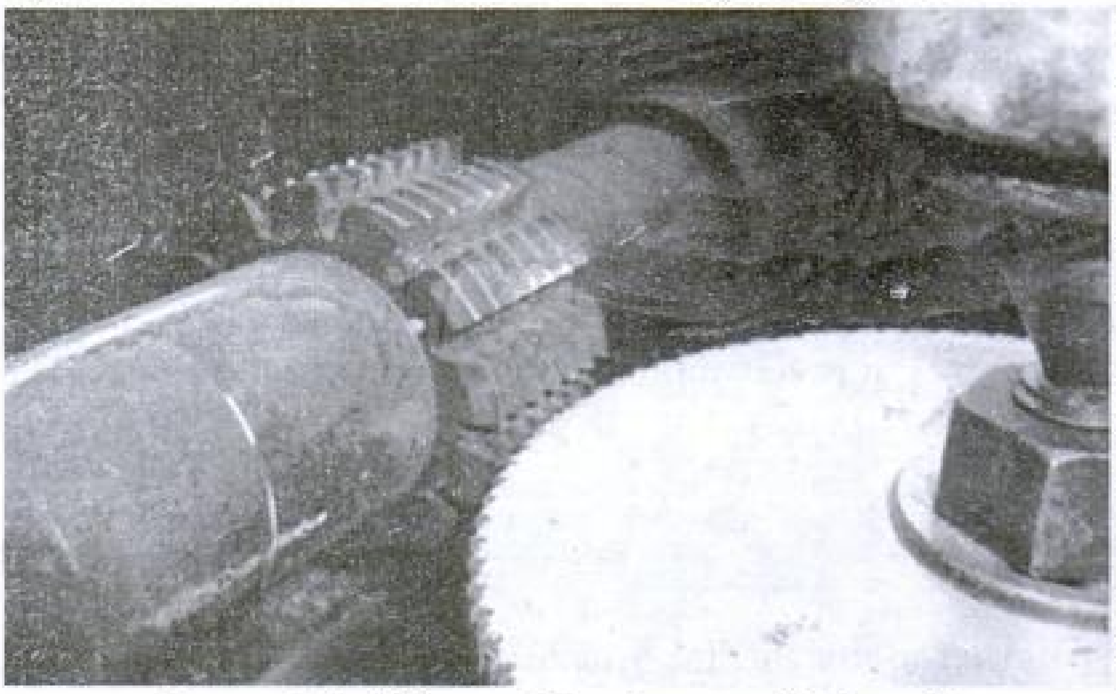


Gear cutter of a milling machine. It cuts involute teeth.

Meshing of Teeth

In Fig. 10.22, two gear wheels 1 and 2 with centres of rotation at A and B respectively are in contact at C. CE and CF are the tangents to the two base circles 1 and 2 respectively. t-t is the common tangent to the two involutes DC and GCH of the two meshing teeth. The involute DC is traced by rolling line EF on the base circle of the gear 2 while the involute GCH is obtained by rolling line EF on the base circle of the gear 1.

From the property of the involute, the tangent CF to the base circle of the gear 2 is normal to the involute DC or the tangent $t - t$. Similarly, the tangent CE to the base circle of the gear 1 is normal to the involute GC or the tangent $t - t$. As CE and CF both are normal to the common tangent t–t at the point C, CE and CF lie



Cutter of a hobbing machine. It cuts multiple teeth.

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Similarly, if the addendum radius of the wheel is made greater than BE, the tip of the wheel tooth will be in contact with a portion of the non-involute profile of the pinion tooth for some of the engagement. The conclusion is that to have no interference of the teeth, the addendum circles of the wheel and the pinion must intersect the line of action between E and F. The points E and F are called *interference points*.

Note that to avoid interference, the limiting value of the addendum of the wheel is GE whereas that of the pinion is HF and the latter is clearly greater than the former. Thus, if the addenda of the wheel and the pinion are to be equal, the addendum circle of the wheel passes through the limiting point E on the line of action before the addendum circle of the pinion passes through the limiting point F on the same line. Thus, for equal addenda of the wheel and the pinion, the addendum radius of the wheel decides whether the interference will occur or not.

10.14 MINIMUM NUMBER OF TEETH

As explained in the previous section, the maximum value of the addendum radius of the wheel to avoid interference can be up to BE. Referring Fig. 10.24,

$$\begin{aligned}
(BE)^2 &= (BF)^2 + (FE)^2 \\
&= (BF)^2 + (FP + PE)^2 \\
&= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2 \\
&= R^2\cos^2\varphi + R^2\sin^2\varphi + r^2\sin^2\varphi + 2\,r\,R\sin^2\varphi \\
&= R^2(\cos^2\varphi + \sin^2\varphi) + \sin^2\varphi\,(r^2 + 2rR) \\
&= R^2 + (r^2 + 2rR)\sin^2\varphi \\
&= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\varphi\right] \\
&= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\varphi\right] \\
BE &= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}
\end{aligned}$$

Therefore, the maximum value of the addendum of the wheel can be equal to (BE – Pitch circle radius) or

$$\begin{aligned}
a_{w\max} &= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - R \\
&= R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - 1\right]
\end{aligned}$$

Let t = number of teeth on the pinion

T = number of teeth on the wheel

Now, $R = \dfrac{mT}{2}, r = \dfrac{mt}{2}$ and $G = \dfrac{T}{t}$ = Gear ratio

Hence, $$a_{w\max} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi} - 1\right] = \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$

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Solution $\varphi=20°$; $VR=10$; $C=275$ mm; $P=360$ kW; $p = 1$ kN/mm of width; $N_p = 1800$ rpm

$$VR = \frac{N_p}{N_g} = \frac{T_g}{T_p} = \frac{d_g}{d_p} \text{ or } d_g = 10\, d_p$$

$$C = \frac{d_p + d_g}{2} \text{ or } d_p + d_g = 2 \times 275 = 550 \text{ mm}$$

or $\quad 11 d_p = 550$ or $d_p = 50$ mm and $d_g = 50 \times 10 = 500$ mm

Minimum number of teeth on gear wheel,

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

$$= \frac{2\times 1}{\sqrt{1+\frac{1}{10}\left(\frac{1}{10}+2\right)\sin^2 20^\circ} - 1} = 164$$

(i) Number of teeth on pinion = 164/10 = 16.4 say 17

$\therefore$ number of teeth on gear wheel $= 17 \times 10 = 170$

(ii) Now $m = \dfrac{d_p}{t} = \dfrac{50}{17} \approx 3\text{ mm}$

Exact $d_p = m\,T_p = 3 \times 17 = 51$ mm and $d_g = m\,T_g = 3 \times 170 = 510$ mm

Exact centre distance,

$$C = \frac{d_p + d_g}{2} = \frac{51+510}{2} = 280.5 \text{ mm}$$

(iii) $P = \dfrac{2\pi NT}{60}$ or $360 \times 1000 = \dfrac{2\pi \times 1800 \times T}{60}$

or $T = 1909.9$ N.m

$$\text{Tangential force} = \frac{1909.9\times 10^3}{51/2} = 74\,896 \text{ N}$$

Normal pressure on the tooth

$$= \frac{F}{\cos\varphi} = \frac{74\,896}{\cos 20^\circ} = 79\,700 \text{ N}$$

$$\text{Width of pinion} = \frac{F_n}{\text{Limiting normal pressure}}$$

$$= \frac{79\,700}{1000} = 79.7 \text{ mm}$$

10.15 INTERFERENCE BETWEEN RACK AND PINION

Figure 10.27 shows a rack and pinion in which pinion is rotating in the clockwise direction and driving the rack. P is the pitch point and PE is the line of action. Engagement of the rack tooth with the pinion tooth occurs at C. To avoid interference, the maximum addendum of the rack can be increased in such a way that C coincides with E. Thus, the addendum of the rack must be less than GE.

Let the adopted value of the addendum of the rack be $a_r m$ where a_r is the *addendum coefficient* by which the standard value of the addendum has been multiplied.

$$GE = PE \sin\phi = (r\sin\phi)\sin\phi = r\sin^2\phi$$

$$= \frac{mt}{2}\sin^2\varphi$$

To avoid interference,

$$GE \ge a_r m \text{ or } \frac{mt}{2}\sin^2\varphi \ge a_r m \text{ or } t \ge \frac{2a_r}{\sin^2\varphi}$$

When $a_r = 1$, i.e., for standard addendum,

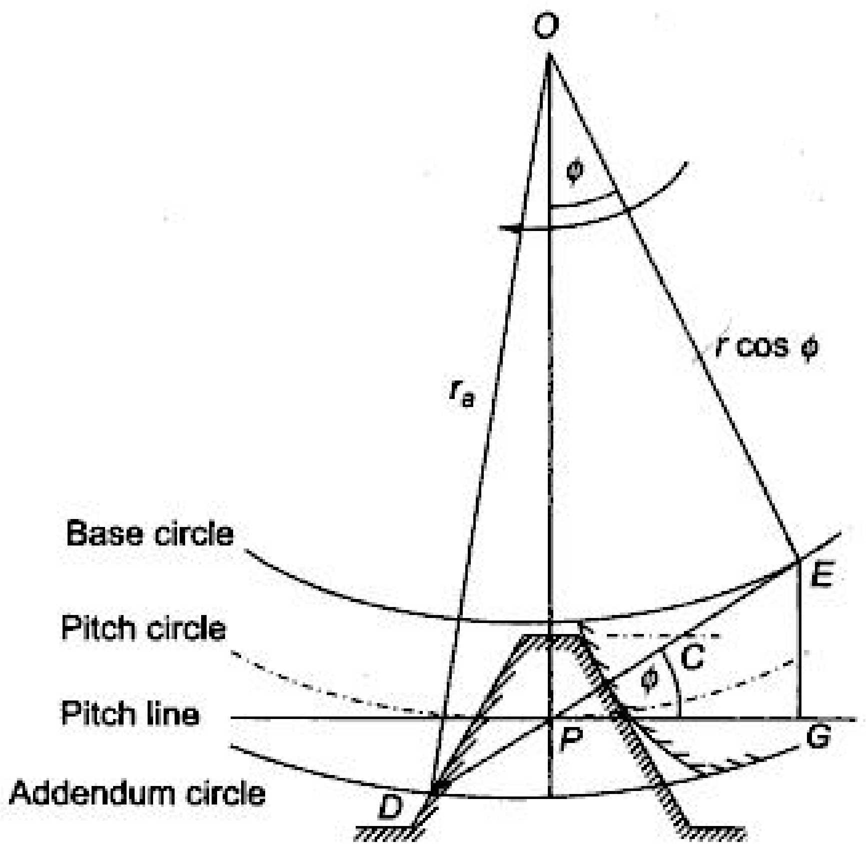

Fig. 10.27

10.19 TERMINOLOGY OF HELICAL GEARS

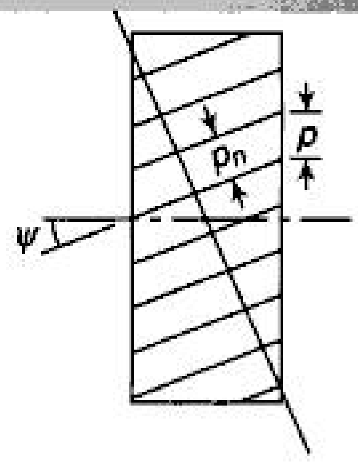

Fig. 10.32

Refer Fig. 10.32.

Helix Angle (ψ) It is the angle at which the teeth are inclined to the axis of a gear. It is also known as *spiral angle*.

Circular Pitch (p) It is the distance between the corresponding points on adjacent teeth measured on the pitch circle. It is also known as *transverse circular pitch*.

Normal Circular Pitch (p_n) Normal circular pitch or simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth. The normal circular pitch of two mating gears must be same.

$$P_n = p \cos \psi$$

Also, we have, $p = \pi m$ as for spur gears

$$P_n = \pi m_n$$

and $$m_n = m \cos \psi$$

10.20 VELOCITY RATIO AND CENTRE DISTANCE OF HELICAL GEARS

Velocity Ratio Refer Fig. 10.30(e),

$$v_n = v_1 \cos \psi_1 = v_2 \cos \psi_2$$

or $$\frac{v_2}{v_1} = \frac{\cos\psi_1}{\cos\psi_2}$$

$$VR = \frac{\omega_2}{\omega_1} = \frac{v_2/r_2}{v_1/r_1} = \frac{v_2/d_2}{v_1/d_1} = \frac{d_1/v_2}{d_2/v_1}$$

or $$VR = \frac{d_1}{d_2}\frac{\cos\psi_1}{\cos\psi_2}$$

$$= \frac{m_1T_1}{m_2T_2}\frac{\cos\psi_1}{\cos\psi_2}$$

$$= \frac{m_n/\cos\psi_1}{m_n/\cos\psi_2}\frac{T_1}{T_2}\frac{\cos\psi_1}{\cos\psi_2}$$

$$= \frac{T_1}{T_2} \tag{10.11}$$

Centre Distance Let C be the centre distance between two skew shaft axes which is the shortest distance between them.

$$C = r_1 + r_2 = \frac{1}{2}(d_1 + d_1) = \frac{1}{2}(m_1T_1 + m_2T_2)$$

or $$C = \frac{1}{2}\left(\frac{m_n}{\cos\psi_1}T_1 + \frac{m_n}{\cos\psi_2}T_2\right)$$

so that the pressure angle is increased to 22°?

(222 mm)

32. A pinion has 24 teeth and drives a gear with 64 teeth. The teeth are of involute type with 20° pressure angle. The addendum and the module are 8 mm and 10 mm respectively. Determine path of contact, arc of contact and the contact ratio.

(41.08 mm, 43.72, 1.39)

33. Two gears in mesh have a module of 10 mm and a pressure angle of 25°. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to one module. Determine the
    (i) number of pairs of teeth in contact
    (ii) angles of action of the pinion and the wheel
    (iii) ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement

(1.475; $\delta_p = 26°36'$, $\delta_g = 10°13'$; zero, 0.304, 0.278)

34. The number of teeth on the gear and the pinion of two spur gears in mesh are 30 and 18 respectively. Both the gears have a module of 6 mm and a pressure angle of 20°. If the pinion rotates at 400 rpm, what will be the sliding velocity at the moment the tip of the tooth of pinion has contact with the gear flank? Take addendum equal to one module. Also, find the maximum velocity of sliding.

(908 mm/s; 981.5 mm/s)

35. Two 20° involute spur gears have a module of 6 mm. The larger wheel has 36 teeth and the pinion has 16 teeth. If the addendum be equal to one module, will the interference occur? What will be the effect if the number of teeth on the pinion is reduced to 14?

(No; interference occurs)

36. The addendum on each wheel of two mating gears is to be such that the line of contact on each side of the pitch point is half the maximum possible length. The number of teeth on the two gears is 24 and 48. The teeth are of 20° pressure angle involute with a module of 12 mm. Determine the addendum for the pinion and the gear. Also, find the arc of contact and the contact ratio.

(23.4 mm, 9.3 mm, 78.6 mm, 2.08)

37. The following data refer to two meshing gears having 20° involute teeth:
    Number of teeth of gear wheel = 52
    Number o teeth of pinion =20
    Speed of pinion = 360 rpm
    Module = 8 mm
    If the addendum of each gear is such that the path of approach and path of recess are half of their maximum possible values, determine the addendum for the gear and the pinion and the length of arc of contact.

(5.07 mm; 18.04 mm; 52.4 mm)

38. Determine the minimum number of teeth and the arc of contact (in terms of module) to avoid interference in the following cases:
    (a) Gear ratio is unity.
    (b) Gear ratio is 3.
    (c) Pinion gears with a rack.
    The addendum of the teeth is 0.88 module and the power component is 0.94 times the normal thrust.

(11, 3.96 m; 14, 4.48 m; 16, 4.24 m)

39. Two 20° involute spur gears having a velocity ratio of 2.5 mesh externally. The module is 4 mm and the addendum is equal to 1.23 module. The pinion rotates at 150 rpm. Find the
    (i) minimum number of teeth on each wheel to avoid interference
    (ii) number of pairs of teeth in contact.

(45, 18; 1.95)

40. If the angle of obliquity of a pair of gear wheels is 20°, and the arc of approach or recess not less than the pitch, what will be the least number of teeth on the pinion?

(18)

41. Two 20° full-depth involute spur gears having 30 and 48 teeth are in mesh. The pinion rotates at 800 rpm. The module is 4 mm. Find the sliding velocities at the engagement and at the disengagement of a pair of teeth and the contact ratio. If the interference is just avoided, find (i) the addenda on the wheel and the pinion, (ii) the path of contact, (iii) the maximum velocity of sliding at engagement and disengagement of a pair of teeth, and (iv) contact ratio.

(8.8 mm, 17.6 mm; 53.35 mm;
2.934 m/s, 4.695 m/s; 4.52)

42. A rack is driven by a pinion having 24 involute teeth and a 140 mm pitch circle diameter. The addendum of both pinion and the rack is 6 mm. Determine the least pressure angle which can be used to avoid interference. For this pressure angle, find the minimum number of teeth in contact at a time.

(17.02°, 2.11)

43. The centre distance between two meshing spiral gears is 150 mm and the angle between the shafts is 60°. The gear ratio is 2 and the normal circular pitch is 10 mm. The driven gear has a helix angle of 25°, determine the

Table 11.5

Action	a	A/B	E	F	C	D
'a' fixed, A + 1 rev.	0	1	$-\frac{56}{36}$	$-\frac{52}{36}$	$\frac{52}{36}\times\frac{36}{124}$	$\frac{56}{36} X \frac{36}{128}$
'a' fixed, A + x rev.	0	x	$-\frac{14x}{9}$	$-\frac{13x}{9}$	$\frac{13x}{31}$	$\frac{7x}{16}$
Add y	y	$y + x$	$y-\frac{14x}{9}$	$y-\frac{13x}{9}$	$y-\frac{13x}{31}$	$y-\frac{7x}{16}$

(i) From given conditions,

Arm a rotates at 200 rpm clockwise, $\therefore y = 200$

D is fixed, $\therefore\ y-\frac{7x}{16}=0$

or $\quad 200-\frac{7x}{16}=0$

or $\quad x = 457.1$

Speed of $C = y-\frac{13x}{31}$

$= 200-\frac{13\times 457.1}{31}$

$=$ 8.31 rpm (clockwise)

(ii) $y = 200$,

$\therefore \quad y-\frac{7x}{16}=-20$

or $\quad 200-\frac{7x}{16}=-20$

or $\quad x = 502.86$

Speed of $C = y-\frac{13x}{31}$

$= 200-\frac{13\times 502.86}{31}$

$= -10.9$ rpm

or $\quad$ 10.9 rpm counter-clockwise

Example 11.7 *The annulus A in the gear shown in Fig. 11.12 rotates at 300 rpm about the axis of the fixed wheel S which has 80 teeth. The three-armed spider (only one arm a is shown in Fig.11.12a) is driven at 180 rpm. Determine the number of teeth required on the wheel P.*

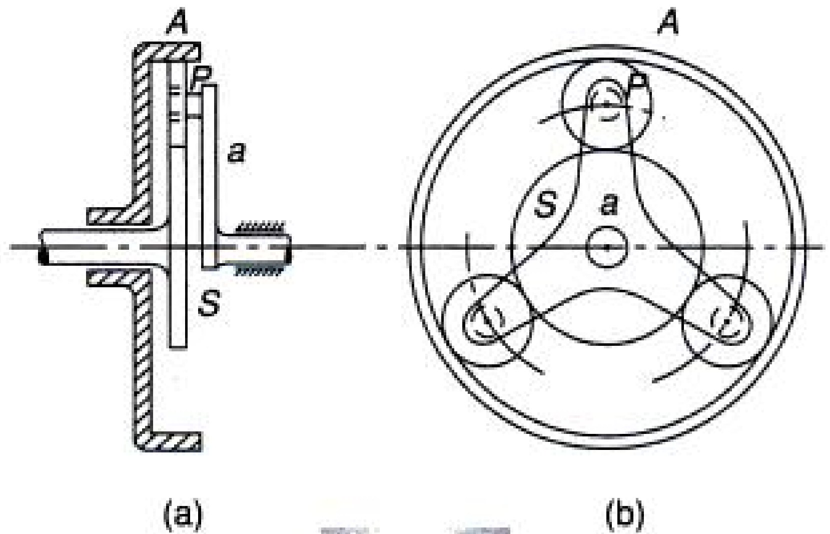

Fig. 11.12

Solution: $N_s = 0$ $\quad N_A = 300$ rpm

$N_a = 180$ rpm $\quad T_S = 80$

Prepare Table 11.6.

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$$N_a = y = \frac{N_S T_S + N_A T_A}{T_S + T_A} \tag{11.11}$$

If the sun wheel S is fixed, $N_S = 0$

Speed of the arm,

$$N_a = \frac{N_A T_A}{T_S + T_A} \text{ or } \frac{N_a}{N_A} = \frac{1}{T_S / T_A + 1}$$

If the annular wheel A is fixed, $N_A = 0$.

Speed of the arm,

$$N_a = \frac{N_S T_S}{T_S + T_A} \text{ or } \frac{N_a}{N_S} = \frac{T_S / T_A}{1 + T_S / T_A}$$

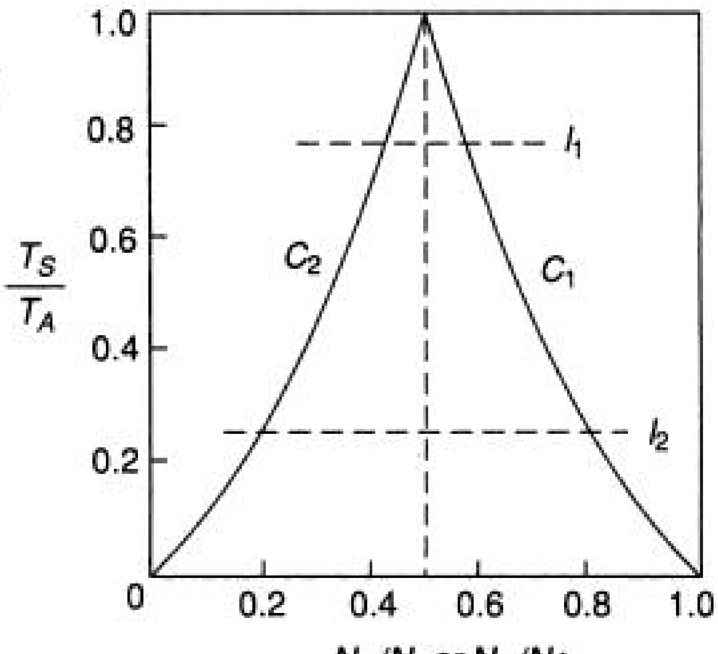

Fig. 11.21

The number of teeth on the sun wheel can vary from 0 to T_A, i.e., from zero to the number of teeth on the annular wheel. Therefore, the ratio of the number of teeth on the sun wheel to that on the annular wheel, i.e., T_S/T_A can vary from 0 to 1. If a graph T_S/T_A vs. N_a/N_A (S is fixed) is plotted, the curve C_1 is obtained (Fig. 11.21) which shows that the sun and the planet gear always acts as a reduction gear. The speed of the arm decreases from N_A to 1/2 N_A as the number of teeth of the sun wheel increases from zero to T_A. Similarly, if A is fixed T_S/T_A vs. N_a/N_S is plotted, the curve C_2 is obtained. This shows that it again acts as a reduction gear in which the speed of the arm varies from zero to $1/2N_S$.

In both cases, the direction of the arm is the same as that of the driving member.

Horizontal dotted lines l_1 and l_2 show the practical limits of the ratio of T_S/T_A and of the corresponding speeds of the arm, where middle portion shows the range.

Example 11.14 *Determine the velocity ratio of the two shafts B and C of the compound gear shown in Fig. 11.22a in which the sun wheel S_2 is fixed. The numbers of teeth on different gears are mentioned alongside the respective gear. Also, find the torque required to fix the gear S_2 when a clockwise torque of 160 N.m is applied to the gear S_1.*

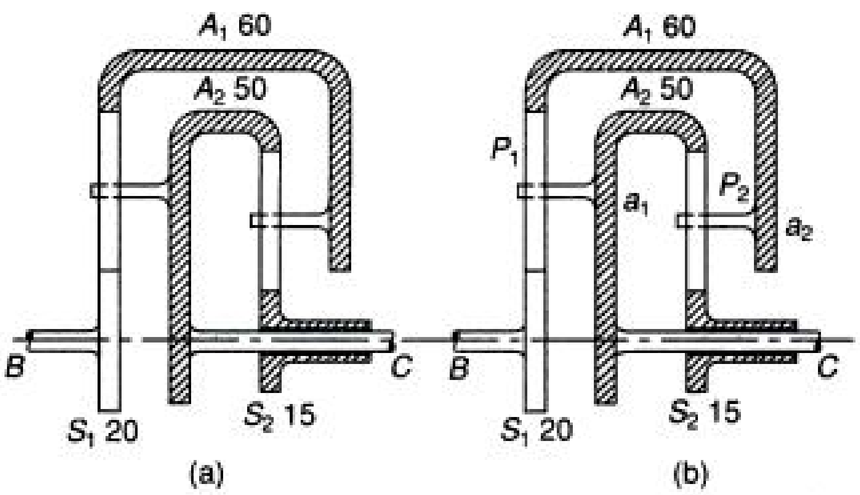

Fig. 11.22

Solution For the first sun and planet gear [Fig. 11.22(b)]

$$N_{a1} = \frac{N_{A1} T_{A1} + N_{S1} T_{S1}}{T_{A1} + T_{S1}} = \frac{60N_{A1} + 20N_{S1}}{60 + 20} = \frac{3N_{A1} + N_{S1}}{4} \tag{i}$$

For the second sun and planet gear (S_2 is fixed),

$$N_{a2} = \frac{50N_{A2} + 0}{50 + 15} = \frac{10N_{A2}}{13}$$

or $N_{a2} = 0.769\ N_{A2}$ (ii)

From the figure of the gear, it can be seen that

- Annular wheel A_1 is the arm a_2 of the second sun and planet gear
  Thus, $N_{A1} = N_{a2}$
- Annular wheel A_2 is the arm a_1 of the first sun and planet gear
  Thus, $N_{A2} = N_{a1}$
- Also $N_B = N_{S1}$ and $N_c = N_{A2}$

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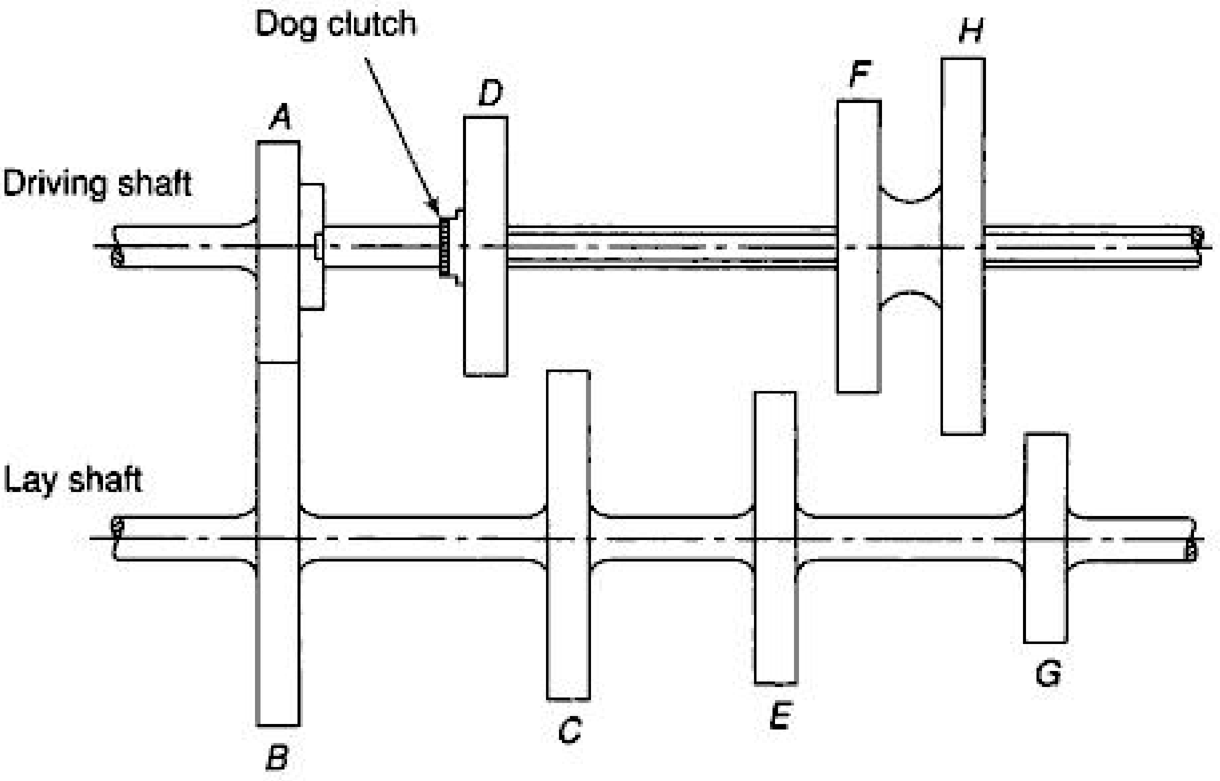

Fig. 11.27

First Gear The first gear is engaged by the sliding gear H towards the right and meshing it with the gear G of the lay shaft. The transmission will be from A to B and from G to H and the train value $\frac{T_A}{T_B}.\frac{T_G}{T_H}$.

Second Gear The vehicle is engaged in the second gear by the sliding gear F towards left and engaging it with the gear E of the lay shaft. The transmission is from A to B and from E to F and the train value $\frac{T_A}{T_B}.\frac{T_E}{T_F}$.

Third Gear By sliding the gear D towards the right and engaging with the gear C of the lay shaft, the transmission in the third gear is obtained which is from A to B and from C to D and the train value $\frac{T_A}{T_B}.\frac{T_C}{T_D}$.

Top Gear The gear D is engaged directly with the gear A through a dog clutch. This way the power is transmitted directly to the driven shaft. The lay shaft along with its gear wheels revolves idly and the driven shaft runs at the same speed as the driving shaft.

To put the vehicle in the reverse gear, an idler is made to mesh with G and H (not shown in the figure) so that both of them rotate in the same direction, thus rotating the driven shaft in the opposite direction.

2. Pre-Selective Gear Box

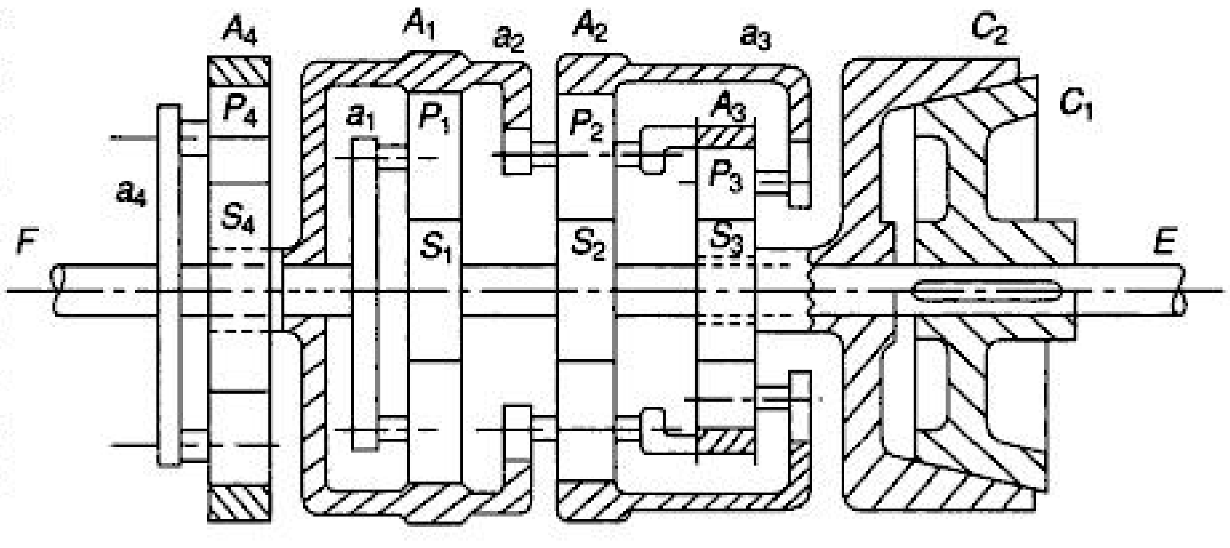

Fig. 11.28

A pre-selective gear-box is a device by which three or four different speeds of the automobile can be obtained. It makes use of sun and planet gears. The number of sun and planet gears to be used will be equal to the number of gear ratios required, except for the top gear which is obtained by direct drive from the crankshaft to the propeller shaft through the clutch.

A typical pre-selective gear-box known as *Wilson gear box* is shown in Fig. 11.28. It consists of four sets of the sun and planet gears out of which the first three are for speed reduction and the fourth, for the reverse gear. The first set of the sun and planet gear gives the minimum speed of the propeller shaft whereas the third, the maximum, but lesser than that obtained with the top gear.

The engine shaft E is made integral with the sun wheels S_1 and S_2 and is keyed to the inner element C_1 of

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From the above table, it can be observed that the speed of B is the arithmetical mean of the speeds of C and D, because $y = \{(y + x) + (y - x)\}/2$. This shows that while taking a turn, if the speed of C decreases than that of B, there will be a corresponding increase in the speed of D.

Example 11.20 *In the differential gear of a car shown in Fig. 11.30, the number of teeth on the pinion A on the propeller shaft is 18 whereas the crown gear B has 90 teeth. If the propeller shaft rotates at 1200 rpm and the wheel attached to the shaft S_2 has a speed of 255 rpm, while the vehicle takes a turn, determine the speed of the wheel attached to the shaft S_1.*

Solution

Speed of the gear $B = N_A \times \dfrac{T_B}{T_A} = 1200 \times \dfrac{18}{90}$

$= 240$ rpm

Thus, $y = 240$

Prepare the table as in the above section.

Speed of $S_2 = y - x = 255$

or $240 - x = 255$

or $x = -15$

Speed of $S_1 = y + x = 240 - 15 = \underline{225 \text{ rpm}}$

Summary

1. A gear train is a combination of gears used to transmit motion from one shaft to another.
2. The main types of gear trains are *simple*, *compound*, *reverted* and *planetary* or *epicyclic*.
3. A series of gears, capable of receiving and transmitting motion from one gear to another is called a *simple gear train*. All the gear axes remain fixed relative to the frame and each gear is on a separate shaft.
4. When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as a *compound gear train*.
5. If the axes of the first and the last wheels of a compound gear coincide, it is called a *reverted gear train*.
6. A gear train having a relative motion of axes is called a *planetary* or an *epicyclic gear train* (or simply epicyclic gear or train). Thus, in an epicyclic train, the axis of at least one of the gears also moves relative to the frame.
7. In general, gear trains have two degrees of freedom. However, the number of inputs can be reduced to one, if one wheel of the train is fixed. That amounts to reducing the speed of that gear wheel to zero.
8. Large speed reductions are possible with epicyclic gears and if the fixed wheel is annular, a more compact unit could be obtained.
9. When an annular wheel is added to the epicyclic gear train, the combination is usually referred as *sun and planet gear*.
10. When an epicyclic gear consists of a number of sun and planet gears in series such that the pin of the arm of the first drives an element of another, it is known as a *compound epicyclic gear*.
11. A simple sliding gear box makes use of a compound gear train and is engaged by sliding the gears on the driven shaft to mesh with the gears on a lay shaft.
12. A pre-selective gear-box is a device by which three or four different speeds of the automobile can be obtained. It makes use of sun and planet gears.
13. A differential of an automobile permits the two wheels of a vehicle to rotate at the same speed when driving in straight while allowing the wheels to rotate at different speeds when taking a turn. Thus, a differential gear is a device which adds or subtracts angular displacements.

Exercises

1. What is a gear train? What are its main types?
2. What is the difference between a simple gear train and a compound gear train? Explain with the help of sketches.
3. What is a reverted gear train? Where is it used?
4. Explain the procedure to analyse an epicyclic gear train.

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12.4 MEMBER WITH TWO FORCES AND A TORQUE

A member under the action of two forces and an applied torque will be in equilibrium if

- the forces are equal in magnitude, parallel in direction and opposite in sense, and
- the forces form a couple which is equal and opposite to the applied torque.

Figure 12.3 shows a member acted upon by two equal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and an applied torque $\mathbf{T}$. For equilibrium,

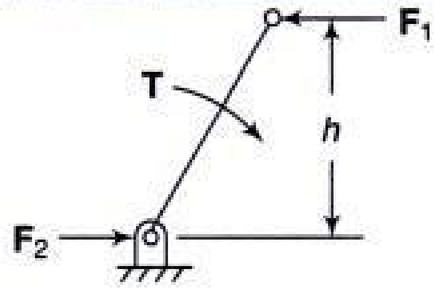

Fig. 12.3

$$T = F_1 \times h = F_2 \times h \tag{12.6}$$

where T, F_1 and F_2 are the magnitudes of $\mathbf{T}$, $\mathbf{F}_1$ and $\mathbf{F}_2$ respectively. $\mathbf{T}$ is clockwise whereas the couple formed by $\mathbf{F}_1$ and $\mathbf{F}_2$ is counter-clockwise.

12.5 EQUILIBRIUM OF FOUR-FORCE MEMBERS

Normally, in most of the cases the above conditions for equilibrium of a member are found to be sufficient. However, in some problems, it may be found that the number of forces on a member is four or even more than that. In such cases, first look for the forces completely known and combine them into a single force representing the sum of the known forces. This may reduce the number of forces acting on a body to two or three. However, in planer mechanisms, a four-force system is also solvable if one force is known completely along with lines of action of the others. The following examples illustrate the procedure.

Example 12.1 *Figure 12.4(a) shows a quaternary link ABCD under the action of forces F_1, F_2, F_3, and F_4 acting at A, B, C and D respectively. The link is in static equilibrium. Determine the magnitude of the forces F_2 and F_3 and the direction of F_3.*

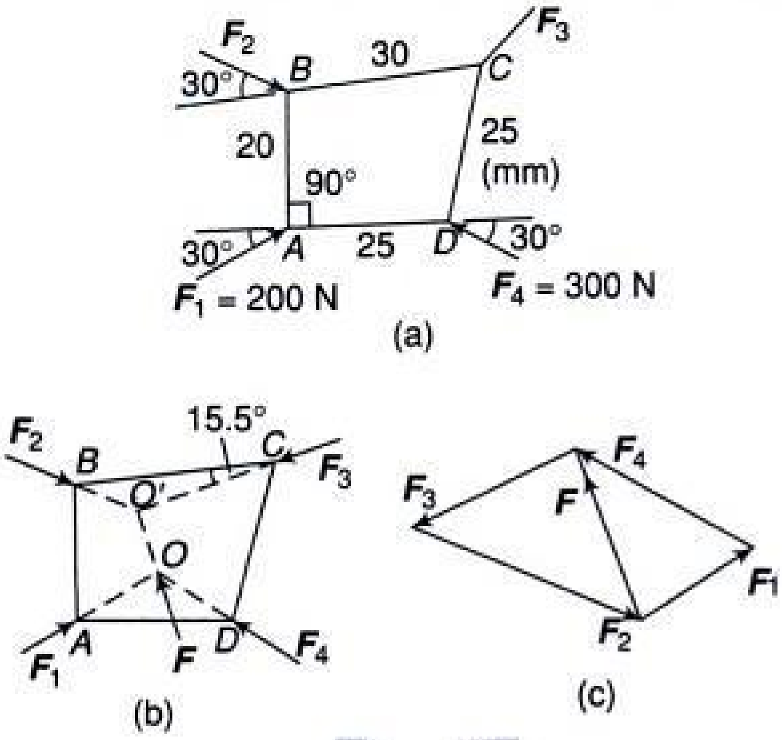

Fig. 12.4

Solution The forces $\mathbf{F}_1$ and $\mathbf{F}_4$ can be combined into a single force $\mathbf{F}$ by obtaining their resultant [Figs 12.4(b) and (c)]. The force $\mathbf{F}$ acts through O, the point where lines of action of $\mathbf{F}_1$ and $\mathbf{F}_4$ meet.

Now, the four-force member *ABCD* is reduced to a three-force member under the action of forces $\mathbf{F}$ (completely known), $\mathbf{F}_2$ (only the direction known) and $\mathbf{F}_3$ (completely unknown).

Let $\mathbf{F}$ and $\mathbf{F}_2$ meet at O'. Then CO' is the line of action of force $\mathbf{F}_3$. By completing the force triangle, obtain the magnitude of $\mathbf{F}_2$ and $\mathbf{F}_3$.

Magnitude of $\mathbf{F}_2$ = 380 N

Magnitude of $\mathbf{F}_3$ = 284 N

Line of action of force $\mathbf{F}_3$ makes an angle of 15.5° with *CB*.

Example 12.2 *Figure 12.5(a) shows a cam with a reciprocating-roller follower system. Various forces acting on the follower are indicated in the figure. At the instant, an external force F_1 of 40 N, a spring force F_2 of*

$$\text{or}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -69.28 & 40 & 0 \\ 0.108 & 0.333 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.93F_{34} & 0.367F_{34} & 0 \\ 0.232 & 0.713 & 0 \end{vmatrix} = 0$$

or $(-69.28 \times 0.333 - 40 \times 0.108) + (0.93\ F_{34} \times 0.713 - 0.367F_{34} \times 0.232)$

or $-27.4 + 0.58\ F_{34} = 0$ or $F_{34} = 47.3$ N

Thus, $\mathbf{F_{34}} = 47.3\angle 21.5^\circ$

Now, $\mathbf{F_{32}} = -\mathbf{F_{23}} = \mathbf{F_{43}} = -\mathbf{F_{34}} = 47.3\angle 21.5^\circ$

$\mathbf{F_{12}} = -\mathbf{F_{32}} = 47.3\ \angle 21.5^\circ$

$T_{2c} = \mathbf{F_{12}} \times \mathbf{AB} = 47.3\angle 21.5^\circ \times 0.4\ \angle 120^\circ = 18.9$ N.m

Example 12.5 *A four-link mechanism with the following dimensions is acted upon by a force of 50 N on the link DC at the point E (Fig. 12.10a):*

AD = 300 mm, AB = 400 mm, BC = 600 mm, DC = 640 mm, DE = 840 mm

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

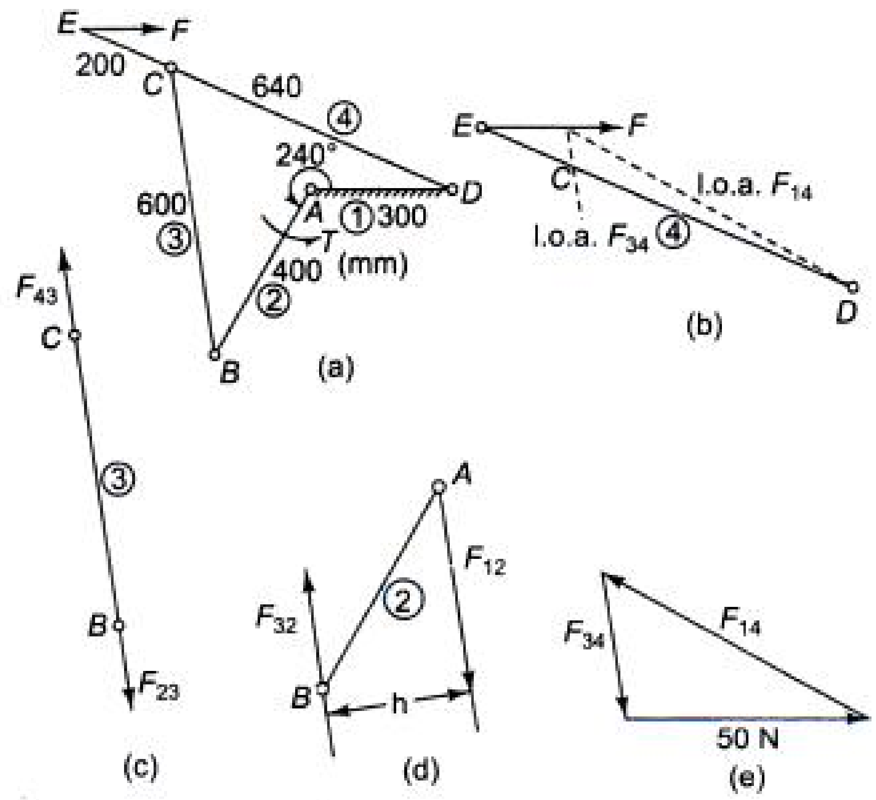

Fig. 12.10

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.10(b)]

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ [Fig. 12.10(c)]

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T** [Fig. 12.10(d)]

Initially, the direction and the sense of some of the forces are not known.

The procedure to solve the problem graphically is exactly similar to the previous example. In brief, the link 3 is a two-force member, so it provides the line of action of force $\mathbf{F}_{34}$ on the link 4. Since the link 4 is a three-force member and forces are to be concurrent, the lines of action of all the forces on the link 4 can be drawn. Then the force diagram provides the magnitude of various forces [Fig. 12.10(e)]. The rest of the procedure is self-explanatory.

From force triangle,

$F_{34} = 30.5$ N

Now, $\mathbf{F_{32}} = -\mathbf{F_{23}} = \mathbf{F_{43}} = -\mathbf{F_{34}}$ or $F_{32} = 30.5$ N

$T = F_{32} \times h = 30.5 \times 249 = 7595$ N.mm

($h = 249$ mm, on measurement)

The input torque has to be equal and opposite to the couple obtained by parallel forces i.e.,

$T = \underline{7.595 \text{ N.m}}$ (counter clockwise)

Example 12.6 *For the mechanism shown in Fig. 12.11a, determine the torque on the link AB for the static equilibrium of the mechanism.*

Solution

(i) Composite Graphical Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

- Member 4 is acted upon by three forces $\mathbf{F}_1$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.11(b)].
- Member 3 is acted upon by three forces $\mathbf{F}_2$, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.
- Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

To solve the problem graphically, proceed as follows:

- Force $\mathbf{F}_1$ on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. However, as the link 3 now is a three-force member, it is not possible to know the direction of the force $\mathbf{F}_{34}$ from that also.

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Now, consider the link 4. The line of action of the force $\mathbf{F}_{34}$ will be opposite to that of $\mathbf{F}_{43}$. Intersection of this line with the line of action of $\mathbf{F}_4$ gives the point of concurrency O for the forces acting on the link 4. As the link 4 rotates counter-clockwise, the tangent to the friction circle at D drawn from point O is such that a clockwise friction couple is obtained.

By drawing a force triangle for the forces acting on link 4 ($\mathbf{F}_4$ is completely known), $\mathbf{F}_{34}$ is obtained.

$$F_{34} = F_{43} = F_{23} = F_{32}$$

The point of concurrency for the forces acting on the link 2 is at O' which is the intersection of $\mathbf{F}_{32}$ and $\mathbf{F}_2$. As the link 1 rotates counter-clockwise, draw a tangent to the friction circle at A from O' such that a clockwise friction couple is obtained.

Draw a force diagram for the forces acting on the link 2 ($\mathbf{F}_{32}$ is completely known) and obtain the value of $\mathbf{F}_2$.

$$F_2 = 20.3 \text{ N}$$

When the motion of AB is clockwise, DC also moves clockwise. For the equilibrium of the link 4, the friction couples at D and C are to be counter-clockwise. For the equilibrium of the link 2, friction couples at A and B are also to be counter-clockwise. Obtain $\mathbf{F}_{32}$ in the manner discussed above and shown in Fig. 12.27(b) $\mathbf{F}_{12}$ will be equal, parallel and opposite to $\mathbf{F}_{32}$.

$$T_2 = F_{32} \times h = 8.6 \times 208 = 1789 \text{ N.mm}$$

or $\underline{1.789 \text{ N. m}}$

Example 12.14 *Find the minimum value of force F_5 to be applied for the static equilibrium of the follower of Example 12.2 if the friction is also considered of the sliding bearings at B and C. Assume the coefficient of friction as 0.15. Ignore the thickness of the follower.*

Solution When a force analysis with friction is to be made, it is always convenient to seek a rough solution of the problem first without friction. This may be obtained by drawing freehand sketches. The purpose is to know the direction-sense of the normal reactions at B and C as these have to be combined with the friction forces at the sliders. Adopting the procedure of Example 12.2, the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ at the bearings are found to be towards right.

As the force $\mathbf{F}_5$ required for the static equilibrium is to be the least, i.e., any force smaller than that will make the follower move down due to the applied force. Thus, the impending motion of the follower is downwards. (If it is desired to have the maximum force for the static equilibrium, any force greater than that will make the follower move up and the impending motion of the follower will be upwards).

Now, as the impending motion of the follower is downwards, the friction forces at the bearings are upwards. Combining these forces with the reaction forces which are towards right, the lines of action of both the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ are tilted through an angle ϕ given by

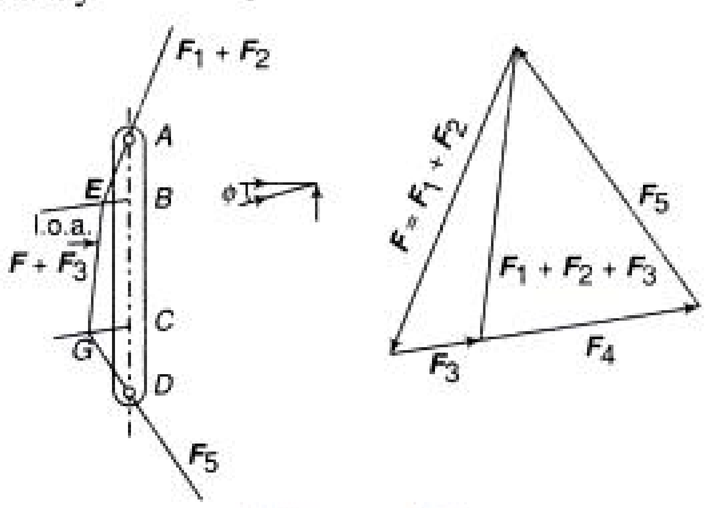

Fig. 12.28

$$\mu = 0.15$$

or $\tan\phi = 0.15$

or $\phi = 8.5^\circ$

On knowing the new lines of action of $\mathbf{F}_3$ and $\mathbf{F}_4$ [Fig. 12.28(a)], the exact solution can be easily obtained as before [Fig. 12.28(b)]. The values obtained are

Magnitude of $\mathbf{F}_3 = 14.5$ N
Magnitude of $\mathbf{F}_4 = 35.5$ N
Magnitude of $\mathbf{F}_5 = 51$ N

Example 12.15 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.29a, find the maximum input torque T_2 required for a force of 300 N on the slider D. Angle θ is 105°. Coefficient of friction $\mu = 0.15$ for each sliding pair.*

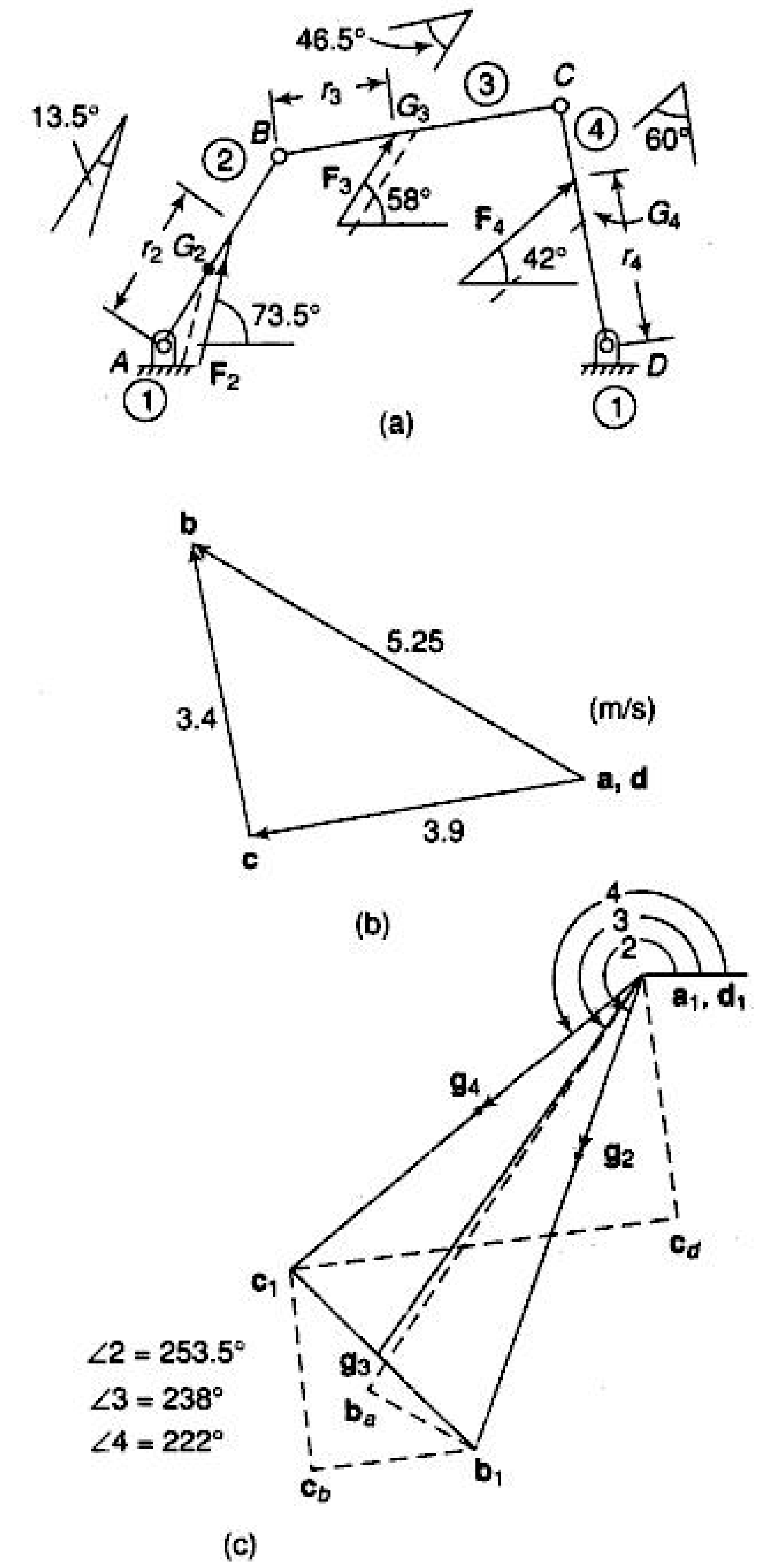

Fig. 13.1

Solution Draw the configuration diagram *ABCD* of the mechanism to a suitable scale [Fig.13.1(a)]. The velocity and acceleration diagrams of the same have been shown in Figs 13.1 (b) and (c).

From the velocity diagram,

v_b or $ab = \omega_{ba} \times AB = 10.5 \times 0.5 = 5.25$ m/s

v_{cd} or $bc = 3.4$ m/s and v_c or $dc = 3.9$ m/s

From the acceleration diagram,

$$f_{ba}^c = \frac{(ab)^2}{AB} = \frac{(5.25)^2}{0.5} = 55.1 \text{ m/s}^2$$

$$f_{ba}^t = \alpha \times AB = 26 \times 0.5 = 13 \text{ m/s}^2$$

$$f_{cb}^c = \frac{(bc)^2}{BC} = \frac{(3.4)^2}{0.66} = 17.5 \text{ m/s}^2$$

$$f_{cd}^c = \frac{(dc)^2}{DC} = \frac{(3.9)^2}{0.56} = 27.2 \text{ m/s}^2$$

Mass of the links

$m_2 = 3.54$ kg

$m_3 = 0.66 \times 4.2 = 2.77$ kg

$m_4 = 0.56 \times 4.2 = 2.35$ kg

Let G_2, G_3 and G_4 denote the centres of masses of links *AB*, *BC* and *CD* respectively. G_2 lies at 200 mm from *A*, and G_3 and G_4 at the midpoints of *BC* and *CD* respectively. Locate these points in the acceleration diagram. Measure the accelerations of G_2, G_3 and G_4.

$F_{g2} = 22.6 \text{ m/s}^2 \angle 253.5°$

$F_{g3} = 52.0 \text{ m/s}^2 \angle 238°$

$F_{g4} = 25.7 \text{ m/s}^2 \angle 222°$

Now find the inertia on the links. These act through their respective centres of mass in the directions opposite to that of accelerations.

$\mathbf{F}_2 = m_2 f_{g2} = 80 \text{ N} \angle 73.5°$ (253.5° – 180°)

$\mathbf{F}_3 = m_3 f_{g3} = 144 \text{ N} \angle 58°$ (238° – 180°)

$\mathbf{F}_4 = m_4 f_{g4} = 60 \text{ N} \angle 42°$ (222° – 180°)

To determine the inertia couples, angular accelerations of the links are to be found.

$\alpha_2 = 26$ rad/s² clockwise

$$\alpha_3 = \frac{f_{cb}^t}{CB} = \frac{22.5}{0.66} = 34.1 \text{ rad/s}^2 \text{counter-clockwise}$$

$$\alpha_4 = \frac{f_{cd}^t}{CD} = \frac{44.3}{0.56} = 79.1 \text{ rad/s}^2 \text{counter-clockwise}$$

Then $C_i = I_g \alpha$

However, the inertia couples can be taken into account by replacing the inertia forces with equivalent offset inertia forces.

Now,

$$k_2^2 = \frac{I_g}{m_2} = \frac{88\,500}{3.54} = 25\,000 \text{ mm}^2$$

Links 3 and 4 have uniform cross sections,

$$k_3^2 = \frac{l^2}{12} = \frac{(660)^2}{12} = 36\,300 \text{ mm}^2$$

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$$= Fr\left(\sin\theta + \cos\theta\,\frac{\sin\theta}{n}\,\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{2\sin\theta\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right) \tag{13.21}$$

Also, as $r\sin(\theta+\beta) = OD\cos\beta$

$$T = F_t \times r$$

$$= \frac{F}{\cos\beta}\, r\sin(\theta+\beta) \qquad \text{[From (13.20)]}$$

$$= \frac{F}{\cos\beta}(OD\cos\beta)$$

$$= F \times OD \tag{13.22}$$

Example 13.2 *A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m² and 60 kN/m² respectively. Diameter of the piston rod is 40 mm. Determine*

(i) turning moment on the crank shaft
(ii) thrust on the bearings
(iii) acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW

Solution

$r = 0.44/2 = 0.22$ m $\qquad l = 0.924$ m

$N = 210$ rpm $\qquad m = 20$ kg

$\theta = 30°$

$$n = l/r = 0.924/0.22 = 4.2$$

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 30°}{4.2} = 0.119$$

or $\beta = 6.837°$

$$F_p = (p_1A_1 - p_2A_2)$$

$$= (500\times10^3 \times \frac{\pi}{4}\times 0.22^2$$

$$-60\times10^3\times\frac{\pi}{4}\times(0.22^2-0.04^2)$$

$$= 19\,007 - 2206$$

$$= 16\,801 \text{ N}$$

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 20\times0.22\times(22)^2\left(\cos 30° + \frac{\cos 60°}{4.2}\right)$$

$= 2098$ N

Piston effort, $F = F_p - F_b$

$= 16\,801 - 2098 = 14\,703$ N

(i) Turning moment, $T = \dfrac{F}{\cos\beta}\sin(\theta+\beta)\times r$

$$= \frac{14\,703}{\cos 6.837}\sin(30° + 6.837°)\times 0.22$$

$= 1953$ N.m

(ii) Thrust on the bearings, F_r

$$= \frac{F}{\cos\beta}\cos(\theta+\beta)$$

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Angular acceleration of the body, $\alpha = \dfrac{F.e}{I}$

where e = perpendicular distance of F from G

and I = moment of inertia of the body about perpendicular axis through G

Now to have the dynamically equivalent system, let the replaced massless link [Fig. 13.6(b)] has two point masses m_1 (at B and m_2 at D) at distances b and d respectively from the centre of mass G as shown in Fig. 13.6 (b).

1. To satisfy the first condition, as the force F is to be same, the sum of the equivalent masses m_1 and m_2 has to be equal to m to have the same acceleration. Thus, $m = m_1 + m_2$.
2. To satisfy the second condition, the numerator $F.e$ and the denominator I must remain the same. F is already taken same, Thus, e has to be same which means that the perpendicular distance of F from G should remain same or the combined centre of mass of the equivalent system remains at G. This is possible if

$$m_1 b = m_2 d$$

To have the same moment of inertia of the equivalent system about perpendicular axis through their combined centre of mass G, we must have

$$I = m_1 b^2 + m_2 d^2$$

Thus, any distributed mass can be replaced by two point masses to have the same dynamical properties if the following conditions are fulfilled:

(i) The sum of the two masses is equal to the total mass.
(ii) The combined centre of mass coincides with that of the rod.
(iii) The moment of inertia of two point masses about the perpendicular axis through their combined centre of mass is equal to that of the rod.

13.9 INERTIA OF THE CONNECTING ROD

Let the connecting rod be replaced by an equivalent massless link with two point masses as shown in Fig. 13.7. Let m be the total mass of the connecting rod and one of the masses be located at the small end B. Let the second mass be placed at D and

m_b = mass at B

m_d = mass at D

Take, $BG = b$ and $DG = d$

Then

$$m_b + m_d = m \quad \text{(i)}$$

and

$$m_b.b = m_d.d \quad \text{(ii)}$$

From (i) and (ii)

$$m_b + \left(m_b \frac{b}{d}\right) = m$$

or

$$m_b\left(1 + \frac{b}{d}\right) = m$$

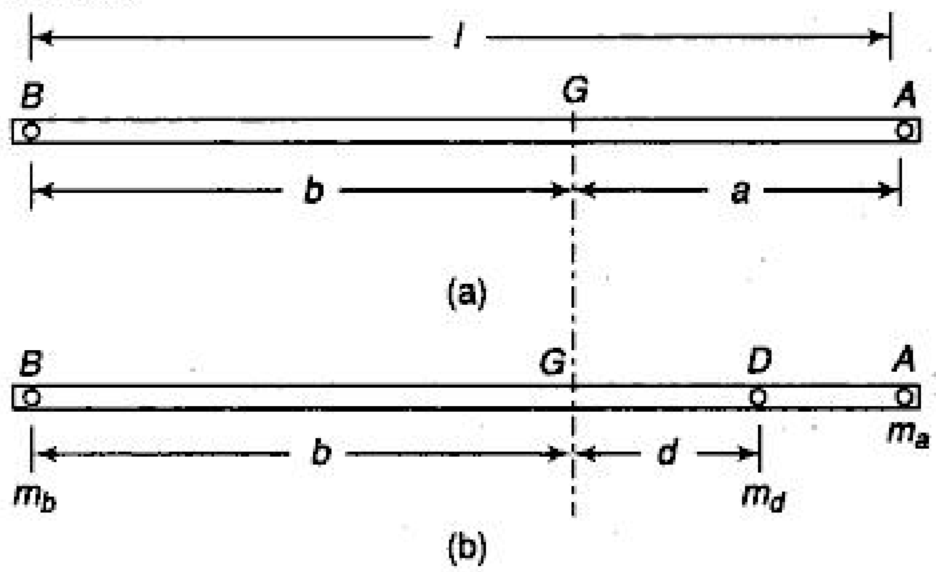

Fig. 13.7

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As θ is less than 90°, it is towards the right and Thus, the inertia force is towards left.

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

$= 156 \times 0.09 \times (62.8)^2\left(\cos 30° + \dfrac{\cos 60°}{5}\right)$

$= 53\ 490$ N

Inertia torque due to reciprocating parts

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \text{[Eq. (13.21)]}$$

$$= 53\,490 \times 0.09\left(\sin 30° + \frac{\sin 60°}{2\sqrt{(5)^2 - \sin^2 30°}}\right)$$

$= 2826$ N.m

(counter-clockwise as inertia force is towards left)

Correction couple due to assumed second mass of connecting rod at A,

$$\Delta T = m\alpha_c b(l - L) \quad \text{[Eq. (13.25)]}$$

where $b = 450 - 180 = 270$ mm

$l = 450$ mm

and $L = b + \dfrac{k^2}{b} = 270 + \dfrac{(150)^2}{270} = 353.3$ mm

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right] \quad \text{[Eq. (13.16)]}$$

$$= -(62.8)^2 \sin 30°\left[\frac{5^2 - 1}{(25 - \sin^2 30°)^{3/2}}\right]$$

$= -384.7$ rad/s^2

$\therefore\ \Delta T = 90 \times (-384.7) \times 0.27 \times (0.45 - 0.3533)$

$= -903.97$ N.m

The direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β as discussed in Section 13.9. Thus, it is clockwise.

$\therefore$ correction torque on the crankshaft,

$$T_c = \Delta T \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -903.97 \times \frac{\cos 30°}{\sqrt{25 - \sin^2 30°}}$$

$= -157.4$ N.m

Correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards left of the crankshaft, the correction torque is counter-clockwise.

Torque due to weight of mass at A,

$T_a = (m_a g)\, r \cos\theta$

$= 54 \times 9.81 \times 0.09 \times \cos 30°$

$= 41.3$ N.m counter-clockwise

$\therefore$ total inertia torque on the crankshaft

$= T_b - T_c + T_a$

$2826 - (-157.4) + 41.3$

$= \underline{3024.7 \text{ N. m}}$ counter-clockwise

Graphical Method

Draw the configuration diagram OAB of the engine mechanism to a convenient scale (Fig. 13.12) and its velocity and acceleration diagrams by Klein's construction (refer Section 13.10).

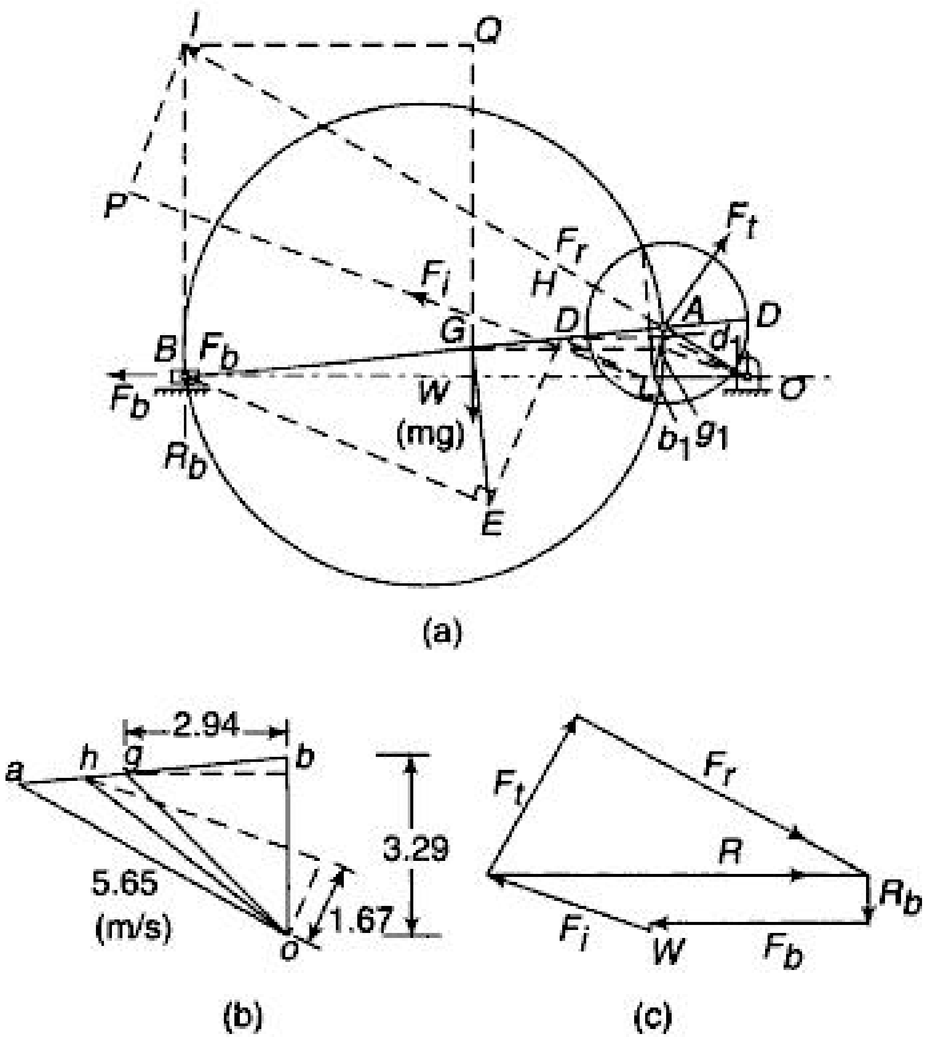

Fig. 13.12

$v_a = \omega r = 62.8 \times 0.09 = 5.65$ m/s

$f_a = \omega^2 r = (62.8)^2 \times 0.09 = 355$ m/s^2

Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram

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Inertia torque due to reciprocating parts

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \text{[Eq. (13.21)]}$$

$$= -4236 \times 0.11\left(\sin 140° + \frac{\sin 280°}{2\sqrt{(4.5)^2 - \sin^2 140°}}\right)$$

$= -248$ N.m

(clockwise as inertia force is towards left)

Correction couple due to assumed second mass of connecting rod at A,

$$\Delta T = m\alpha_c b(l - L) \quad \text{[Eq. (13.25)]}$$

where $b = 495 - 170 = 325$ mm

$l = 495$ mm

and $L = b + \dfrac{k^2}{b} = 325 + \dfrac{(148)^2}{325} = 392.4$ mm

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

[Eq. (13.16)]

$$= -(33.5)^2 \sin 140°\left[\frac{4.5^2 - 1}{(4.5 - \sin^2 140°)^{3/2}}\right]$$

$= -157.17$ rad/s^2

$\therefore \Delta T = 50 \times (-157.17) \times 0.325 \times (0.495 - 0.3924)$

$= -262.04$ N.m

The direction of the correction couple will be the same as that of the angular acceleration, i.e., in the direction of decreasing angle β as discussed in Section 13.6. Thus, it is clockwise.

$\therefore$ correction torque on the crankshaft,

$$T_c = \Delta T \frac{\cos\vartheta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -262.04 \times \frac{\cos 140°}{\sqrt{4.5^2 - \sin^2 140°}}$$

$= 45.07$ N.m

The correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards right of the crankshaft, the correction torque is clockwise.

Torque due to weight of mass at A,

$T_a = (m_a g)\, r \cos\theta$

$= 32.83 \times 9.81 \times 0.11 \times \cos 140°$

$= -27.14$ N.m counter-clockwise

$\therefore$ total inertia torque on the crankshaft $= T_b - T_c + T_a$

$= -248 - 45.07 - 27.14$

$= 320.2$ clockwise

Graphical Method

Draw the configuration diagram OAB of the engine mechanism to a convenient scale (Fig. 13.16) and its velocity and acceleration diagrams by Klein's construction.

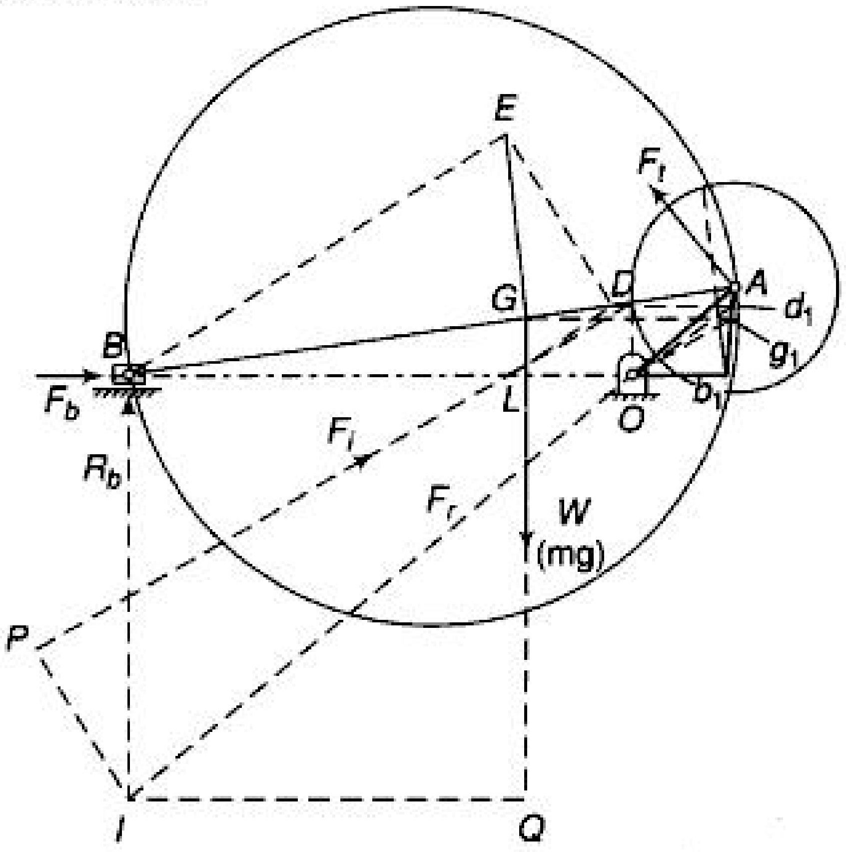

Fig. 13.16

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} = 33.5 \text{ rad/s}$$

$v_a = \omega r = 33.5 \times 0.11 = 3.685$ m/s

$f_a = \omega^2 r = (33.5)^2 \times 0.11 = 123.4$ m/s^2

Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram represents the acceleration of A relative to O, i.e., 123.4 m/s^2, therefore, f_b can be obtained from

$$f_b = 123.4 \times \frac{b_1O}{OA}$$

It is found to be $f_b = 89.6$ m/s^2

Similarly, $f_g = 106.7$ m/s^2

$\therefore$ $F_b = m_b \times f_b = 30 \times 89.6 = 2688$ N

$F_i = m \times f_g = 50 \times 106.7 = 5335$ N

Complete the diagram of Fig. 13.16 as discussed in Section 13.10. Taking moments about I,

$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$

$F_t \times 0.64 = 2688 \times 0.340 + 5335 \times 0.138 + 50 \times 9.81 \times 0.322$

$F_t = 2825.1$ N

$T = F_t \times r = 2825.1 \times 0.11 = 310.7$ N.m

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Work done /stroke

$$= \frac{\text{Work done per second}}{\text{Number of working strokes/second}}$$

$$= \frac{56\,000}{420/60} = 8000 \text{ N.m}$$

Fluctuation of energy = 8000 × 0.3 = 2400 N.m

$$K = \frac{\omega_1 - \omega_2}{\omega} = \frac{1.01\omega - 0.99\omega}{\omega} = 0.02$$

Also, $K = \dfrac{e}{I\omega^2} = \dfrac{e}{mk^2\omega^2}$

or $0.02 = \dfrac{2400}{m \times 0.5^2 \times 22^2}$

or $m = 992$ kg

Example 13.15 *A flywheel fitted to a steam engine has a mass of 800 kg. Its radius of gyration is 360 mm. The starting torque of the engine is 580 N.m and may be assumed constant. Find the kinetic energy of the flywheel after 12 seconds.*

Solution Angular acceleration,

$$\alpha = \frac{T}{I} = \frac{T}{mk^2} = \frac{580}{800 \times 0.36^2} = 5.59 \text{ rad/s}^2$$

$$\omega_2 = \omega_1 + \alpha t = 0 + 5.59 \times 12 = 67.08 \text{ rad/s}$$

Kinetic energy

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2\omega^2 = \frac{1}{2} \times 800 \times 0.36^2 \times 67.08^2$$

= 233 270 N.m or 233.27 kJ

Example 13.16 *The turning-moment diagram for a petrol engine is drawn to a vertical scale of 1 mm = 500 N.m and a horizontal scale of 1mm = 3°. The turning-moment diagram repeats itself after every half revolution of the crankshaft. The areas above and below the mean torque line are 260, –580, 80, –380, 870, and –250 mm². The rotating parts have a mass of 55 kg and radius of gyration of 2.1 m. If the engine speed is 1600 rpm, determine the coefficient of fluctuation of speed.*

Solution Let flywheel KE at $a = E$

(refer Fig. 13.21)

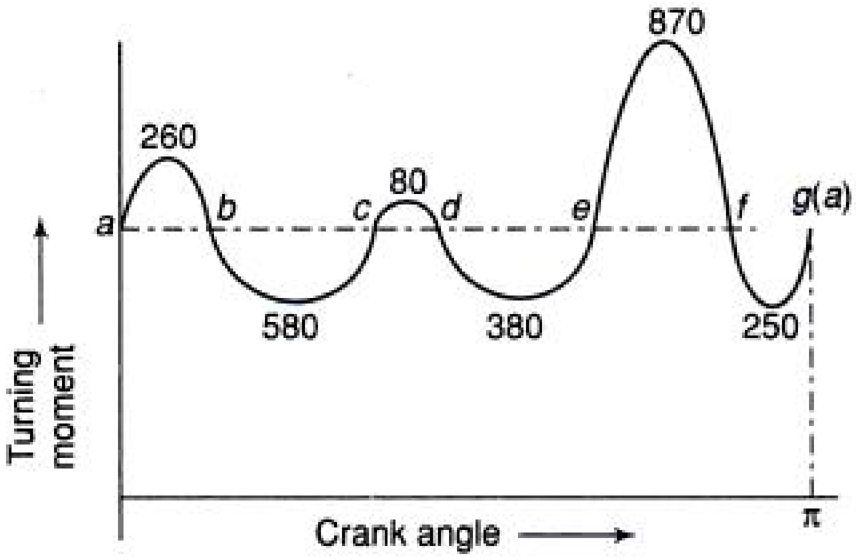

Fig. 13.21

at $b = E + 260$
at $c = E + 260 - 580 = E - 320$
at $d = E - 320 + 80 = E - 240$
at $e = E - 240 - 380 = E - 620$
at $f = E - 620 + 870 = E + 250$
at $g = E + 250 - 250 = E$

Maximum energy = $E + 260$ (at b)
Minimum energy = $E - 620$ (at e)

Maximum fluctuation of energy,

$e_{max} = (E + 260) - (E - 620) \times$ Hor. scale × Vert. scale

$$= 880 \times \left(3 \times \frac{\pi}{180}\right) \times 500$$

= 23 038 N.m

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2} = \frac{23\,038}{55 \times 2.1^2 \times \left(\dfrac{2\pi \times 1600}{60}\right)^2}$$

$K = 0.0034$ or 0.34%

Example 13.17 *A three–cylinder single-acting engine has its cranks at 120°. The turning-moment diagram for each cycle is a triangle for the power stroke with a maximum torque of 60 N.m at 60° after the dead centre of the corresponding crank. There is no torque on the return stroke. The engine runs at 400 rpm. Determine the*

(i) power developed

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$$= [-150\cos 2\theta - 250\sin 2\theta]_{29.5°}^{119.5°}$$

$$= 583.1 \text{ N.m}$$

$$K = \frac{e}{mk^2\omega^2}$$

$$= \frac{583.1}{400 \times (0.4)^2 \times \left(\frac{2\pi \times 250}{60}\right)^2}$$

$$= 0.013\,29 \quad \text{or} \quad 1.329\%$$

(iii) Acceleration or deceleration is produced by excess or deficit torque than the mean value at any instant.

$$\Delta T = 300\sin 2\theta - 500\cos 2\theta$$

when $\theta = 60°$,

$$\Delta T = 259.8 - (-250) = 509.8 \text{ N.m}$$

or $I\alpha = mk^2\,\alpha = 509.8$

or $400 \times (0.4)^2 \times \alpha = 509.8$

or $\alpha = 7.966 \text{ rad/s}^2$

(iv) For ΔT_{max} and ΔT_{min},

$$\frac{d}{d\theta}(\Delta T) = \frac{d}{d\theta}(300\sin 2\theta - 500\cos 2\theta) = 0$$

or $2 \times 300\cos 2\theta + 2 \times 500\sin 2\theta = 0$

or $600\cos 2\theta = -1000\sin 2\theta$

or $\tan 2\theta = -0.6$

or $2\theta = 149.04°$ and $329.04°$

or $\theta = 74.52°$ and $164.52°$

when $2\theta = 149.04°$, $T = 1583.1$ N.m,

$\Delta T = 583.1$ N.m

when $2\theta = 329.04°$, $T = 416.9$ N.m,

$\Delta T = -583.1$ N.m

As values of ΔT at maximum and minimum torque T are same, maximum acceleration is equal to maximum retardation.

or $\Delta T = mk^2\alpha = 583.1$

or $400 \times (0.4)^2 \times \alpha = 583.1$

Maximum acceleration or retardation, $\alpha = 9.11$ rad/s^2

Example 13.21 *A machine is coupled to a two-stroke engine which produces a torque of $(800 + 180\sin 3\theta)$ N.m, where θ is the crank angle. The mean engine speed is 400 rpm. The flywheel and the other rotating parts attached to the engine have a mass of 350 kg at a radius of gyration of 220 mm. Calculate the*

(i) power of the engine

(ii) total fluctuation of speed of the flywheel when the

(a) resisting torque is constant

(b) resisting torque is $(800 + 80\sin\theta)$ N.m

Solution

$m = 350$ kg $\qquad N = 400$ rpm

$k = 220$ mm $\qquad \omega = \frac{2\pi \times 400}{60} = 41.89$ rad/s

For the expression for torque being a function of 3θ, the cycle is repeated after every 120° of the crank rotation (Fig. 13.26).

$$\text{(i)}\quad T_{mean} = \frac{1}{2\pi/3}\int_0^{2\pi/3} T\,d\theta$$

$$= \frac{3}{2\pi}\int_0^{2\pi/3}(800 + 180\sin 3\theta)\,d\theta$$

$$= \frac{3}{2\pi}\left[800\theta - \frac{180}{3}\cos 3\theta\right]_0^{2\pi/3}$$

$$= 800 \text{ N.m}$$

$$P = T\omega = 800 \times 41.89 = 33\,512 \text{ W}$$

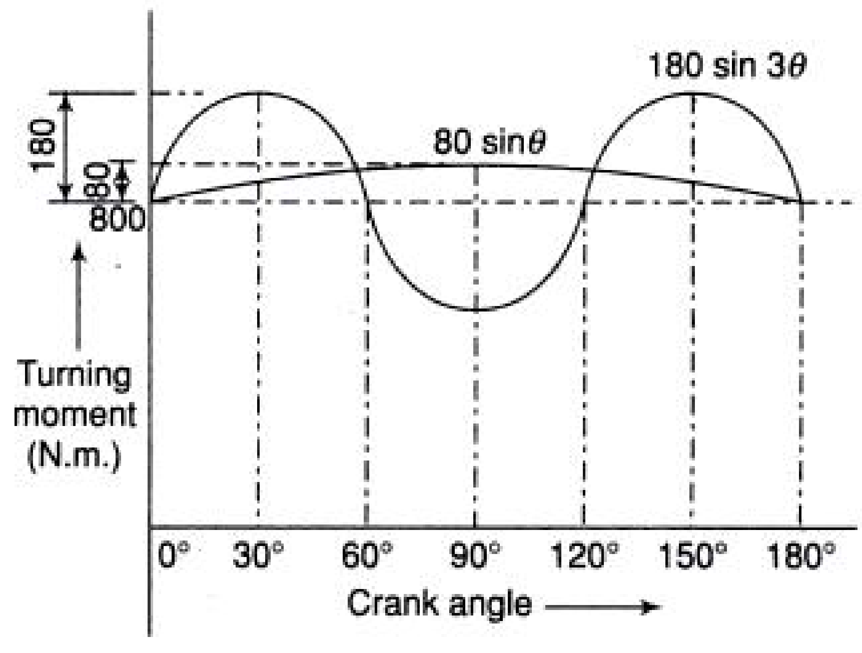

Fig. 13.26

(ii) (a) At any instant, $\Delta T = T - T_{mean}$

$= 800 + 180\sin 3\theta - 800$

$= 180\sin 3\theta$

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of 1 mm = 4.5°. The areas above and below the mean torque line are –28, + 380, –260, + 310, –300, +242, –380, +265 and –229 mm².

The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m³ and width of the rim is 4.5 times its thickness. The centrifugal stress (hoop stress) in the rim material is limited to 6 N/mm². Neglecting the effect of the boss and arms, determine the diameter and cross section of the flywheel rim.

Solution

$\rho = 7000$ kg/m³ $\qquad \sigma = 6 \times 10^6$ N/m²

$N = 400$ rpm $\qquad K = 0.018 + 0.018 = 0.036$

$b = 4.5t$

Now,

$$\sigma = \rho v^2 \qquad \text{(Eq. 13.31)}$$

$$6 \times 10^6 = 7000 \times v^2$$

$$v = 29.28 \text{ m/s}$$

or $$\frac{\pi dn}{60} = \frac{\pi \times d \times 400}{60} = 29.28$$

or $$d = 1.398 \text{ m}$$

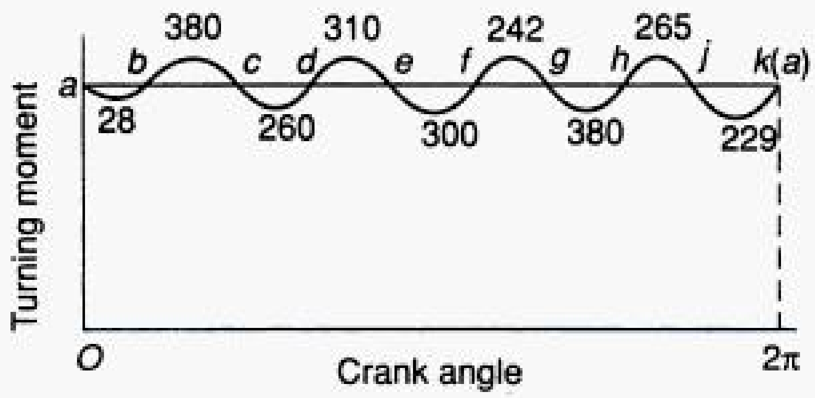

Fig. 13.30

Refer the turning-moment diagram of Fig. 13.30,

Let the flywheel KE at $a = E$

at $b = E - 28$

at $c = E - 28 + 380 = E + 352$

at $d = E + 352 - 260 = E + 92$

at $e = E + 92 + 310 = E + 402$

at $f = E + 402 - 300 = E + 102$

at $g = E + 102 + 242 = E + 344$

at $h = E + 344 - 380 = E - 36$

at $j = E - 36 + 265 = E + 229$

at $k = E + 229 - 229 = E$

Maximum energy = $E + 402$ (at e)

Minimum energy = $E - 36$ (at h)

Maximum fluctuation of energy,

$$e_{max} = (E + 402) - (E - 36) \times \text{hor. scale} \times \text{vert. scale}$$

$$= 438 \times \left(4.5 \times \frac{\pi}{180}\right) \times 650$$

$$= 22\,360 \text{ N.m}$$

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

$$0.036 = \frac{22\,360}{m\left(\frac{1.398}{2}\right)^2 \left(\frac{2\pi \times 400}{60}\right)^2}$$

$$m = 724.5 \text{ kg}$$

or density × volume = 724.5

or $\rho \times (\pi d) \times t \times 4.5t = 724.5$

or $7000 \times \pi \times 1.398 \times t \times 4.5t = 724.5$

or $t = 0.0512$ m or 51.2 mm

$b = 4.5 \times 51.2 = 230.3$ mm

Example 13.25 *The speed variation of an Otto cycle engine during the power stroke is limited to 0.8% of the mean speed on either side. The engine develops 40 kW of power at a speed of 130 rpm with 65 explosions per minute. The work done during the power stroke is 1.5 times the work done during the cycle. If the hoop stress in the rim of the flywheel is not to exceed 3.5 MPa and the width is three times the thickness, determine the mean diameter and the cross section of the rim. Assume that the energy stored by the flywheel is 1.1 times the energy stored by the rim and the density of the rim material is 7300 kg/m³. The turning-moment diagram during the expansion stroke may be assumed to be triangular in shape.*

14 BALANCING

Introduction

Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses. Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

A particle or mass moving in a circular path experiences a centripetal acceleration and a force is required to produce it. An equal and opposite force acting radially outwards acts on the axis of rotation and is known as centrifugal force [Fig. 14.1(a)]. This is a disturbing force on the axis of rotation, the magnitude of which is constant but the direction changes with the rotation of the mass.

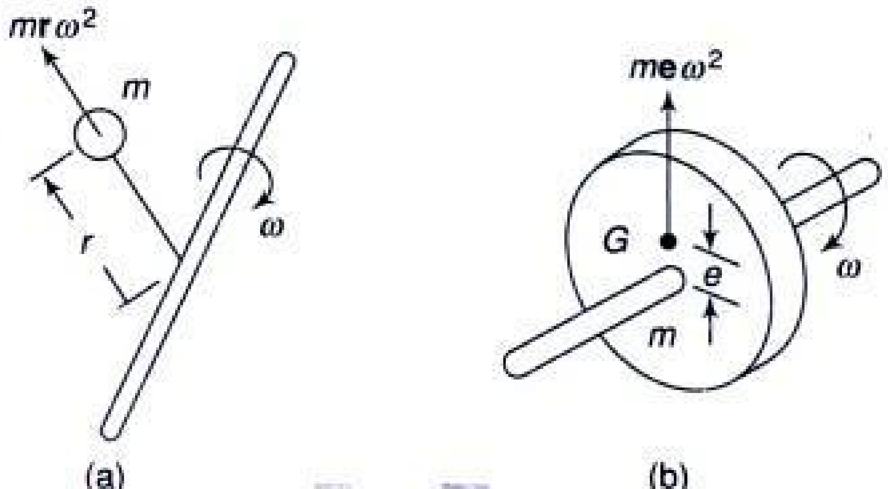

Fig. 14.1

In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of the rotor lies on the axis of the shaft. When the centre of mass does not lie on the axis or there is an eccentricity, an unbalanced force is produced (Fig. 14.1b). This type of unbalance is very common. For example, in steam turbine rotors, engine crankshafts, rotary compressors and centrifugal pumps.

Most of the serious problems encountered in high-speed machinery are the direct result of unbalanced forces. These forces exerted on the frame by the moving machine members are time varying, impart vibratory motion to the frame and produce noise. Also, there are human discomfort and detrimental effects on the machine performance and the structural integrity of the machine foundation.

The most common approach to balancing is by redistributing the mass which may be accomplished by addition or removal of mass from various machine members.

There are two basic types of unbalance—rotating unbalance and reciprocating unbalance—which may occur separately or in combination.

14.1 STATIC BALANCING

A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.

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Let the two countermasses be placed in transverse planes at axial locations O and Q, i.e., the countermass m_{c1} be placed in the reference plane and the distance of the plane of m_{c2} be l_{c2} from the reference plane.

Equation (14.6) modifies to (taking moments about O)

$$m_1\mathbf{r}_1 l_1\omega^2 + m_2\mathbf{r}_2 l_2\omega^2 + m_3\mathbf{r}_3 l_3\omega^2 + m_{c2}\mathbf{r}_{c2} l_{c2}\omega^2 = 0 \tag{14.9}$$

or

$$m_1\mathbf{r}_1 l_1 + m_2\mathbf{r}_2 l_2 + m_3\mathbf{r}_3 l_3 + m_{c2}\mathbf{r}_{c2} l_{c2} = 0 \tag{14.9a}$$

In general,

$$\Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2} l_{c2} = 0 \tag{14.10}$$

Thus, Eqs (14.8) and (14.10) are the necessary conditions for dynamic balancing of the rotor. Again the equations can be solved mathematically or graphically.

Dividing Eq. (14.10) into component form

$$\Sigma m\mathbf{r}l\cos\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = 0$$

and

$$\Sigma m\mathbf{r}l\sin\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = 0$$

or

$$m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = -\Sigma m\mathbf{r}l\cos\theta \tag{i}$$

and

$$m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = -\Sigma m\mathbf{r}l\sin\theta \tag{ii}$$

Squaring and adding (i) and (ii)

$$m_{c2}r_{c2}l_{c2} = \sqrt{(\Sigma\, mrl\cos\theta)^2 + (\Sigma mrl\sin\theta)^2} \tag{14.11}$$

Dividing (ii) by (i),

$$\tan\theta_{c2} = \frac{-\Sigma\, mrl\sin\theta}{-\Sigma\, mrl\cos\theta} \tag{14.12}$$

After obtaining the values of m_{c2} and θ_{c2} from the above equations, solve Eq. (14.8) by taking its components,

$$m_{c1}r_{c1} = \sqrt{(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})^2 + (\Sigma\, mrl\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})^2} \tag{14.13}$$

and

$$\tan\theta_{c1} = \frac{-(\Sigma\, mr\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})}{-(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})} \tag{14.14}$$

To solve Eqs (14.8) and (14.10) graphically, Eq. (14.10) is solved first and a couple polygon is made by adding the known vectors and considering each vector parallel to the radial line of the mass. Then the closing vector will be $m_{c2}\mathbf{r}_{c2}l_{c2,}$ the direction of which specifies the angular position of the countermass m_{c2} [Fig. 14.6(c)] in the plane at the point Q. Then solve Eq. (14.8) and make a force polygon by adding the known vectors (along with the vector $m_{c2}\mathbf{r}_{c2}$). The closing vector is $m_{c1}\mathbf{r}_{c1}$, identifying the magnitude and the direction of the countermass m_{c1} [Fig. 14.6(d)]. Figure 14.6(b) represents the position of the balancing masses on the rotating shaft.

Example 14.10 *The following data refer to a two-cylinder uncoupled locomotive:*

Rotating mass per cylinder	*= 280 kg*
Reciprocating mass per cylinder	*= 300 kg*
Distance between wheels	*= 1400 mm*
Distance between cylinder centres	*= 600 mm*
Diameter of treads of driving wheels	*= 1800 mm*
Crank radius	*= 300 mm]*
Radius of centre of balance mass	*= 620 mm*
Locomotive speed	*= 50 km/hr*
Angle between cylinder cranks	*= 90°*
Dead load on each wheel	*= 3.5 tonne*

Determine the

(i) balancing mass required in the planes of driving wheels if whole of the revolving and two-third of the reciprocating mass are to be balanced

(ii) swaying couple

(iii) variation in the tractive force

(iv) maximum and minimum pressure on the rails

(v) maximum speed of locomotive without lifting the wheels from the rails

Solution Total mass to be balanced $= m_p + cm$

$$= 280 + \frac{2}{3} \times 300$$

$$= 480 \text{ kg}$$

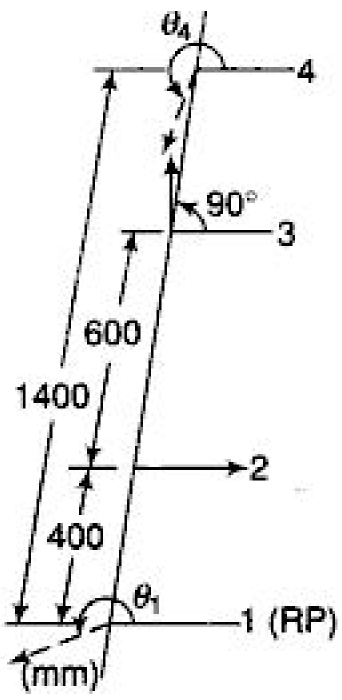

Fig. 14.16

(i) Take 1 as the reference plane and angle $\theta_2 = 0°$ (Fig.14.16). Writing the couple equations,

$$m_2 \mathbf{r}_2 l_2 \cos\theta_2 + m_3 \mathbf{r}_3 l_3 \cos\theta_3 + m_4 \mathbf{r}_4 l_4 \cos\theta_4 = 0$$

or $480 \times 300 \times 400 \cos 0° + 480 \times 300 \times 1000 \cos 90° + m_4 \times 620 \times 1400 \cos\theta_4 = 0$

or $m_4 \cos\theta_4 = -66.36$ (i)

and $m_2 \mathbf{r}_2 l_2 \sin\theta_2 + m_3 \mathbf{r}_3 l_3 \sin\theta_3 + m_4 \mathbf{r}_4 l_4 \sin\theta_4 = 0$

or $480 \times 300 \times 400 \sin 0° + 480 \times 300 \times 1000 \sin 90° + m_4 \times 620 \times 1400 \sin\theta_4 = 0$

or

$m_4 \sin\theta_4 = -165.9$ (ii)

Squaring and adding (i) and (ii), $m_4 = \underline{178.7 \text{ kg}}$

Dividing (ii) by (i), $\tan\theta_4 = \dfrac{-165.9}{-66.36} = 2.5$

$$\theta_4 = \underline{248.2°}$$

Taking 4 as the reference plane and writing the couple equations,

$$m_2 \mathbf{r}_2 l_2 \cos\theta_2 + m_3 \mathbf{r}_3 l_3 \cos\theta_3 + m_1 \mathbf{r}_1 l_1 \cos\theta_1 = 0$$

$480 \times 300 \times 1000 \cos 0° + 480 \times 300 \times 400 \cos 90° + m_1 \times 620 \times 1400 \sin\theta_1 = 0$

or

$m_1 \sin\theta_1 = -165.9$ (iii)

Similarly, $m_1 \sin\theta_1 = -66.36$ (iv)

From (iii) and (iv), $m_1 = \underline{178.7 \text{ kg}} = m_4$

$$\tan\theta_1 = \frac{-66.36}{-165.9} = 0.4 \text{ or } \theta_1 = \underline{201.8°}$$

The treatment shows that the magnitude of m_1 could have directly been written equal to m_4.

(ii) $$\omega = \frac{50 \times 1000 \times 1000}{60 \times 60} \times \frac{1}{\frac{1800}{2}} = 15.43 \text{ rad/s}$$

$$\text{Swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)\, mr\omega^2 l$$

$$= \pm\frac{1}{\sqrt{2}}\left(1-\frac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2 \times 0.6$$

$$= \underline{3030.3 \text{ N.m}}$$

(iii) Variation in tractive force $= \pm\sqrt{2}(1-c)mr\omega^2$

$$= \pm\sqrt{2}\left(1-\frac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2$$

$$= \underline{10\,100 \text{ N}}$$

(iv) Balance mass for reciprocating parts only

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For convenience, choose a plane passing through the middle bearing about which the arrangement is symmetrical as the reference plane.

Primary force

$= mr\omega^2 [\cos\theta + \cos(180° + \theta) + \cos(180° + \theta) + \cos\theta] = 0$

Primary couple

$$= mr\omega^2 \left[\frac{3l}{2}\cos\theta + \frac{l}{2}\cos(180° + \theta) + \left(-\frac{l}{2}\right)\cos(180° + \theta) + \left(-\frac{3l}{2}\right)\cos\theta\right]$$

$$= 0$$

Secondary force

$$= \frac{mr\omega^2}{n}[\cos 2\theta + \cos(360° + 2\theta) + \cos(360° + 2\theta) + \cos 2\theta]$$

$$= \frac{4mr\omega^2}{n}\cos 2\theta$$

Maximum value $= \dfrac{4mr\omega^2}{n}$ at $2\theta = 0°$, 180°, 360° and 540° or $\theta = 0°$, 90°, 180° and 270°

Secondary couple

$$= \frac{mr\omega^2}{n}\left[\frac{3l}{2}\cos 2\theta + \frac{l}{2}\cos(360° + 2\theta) + \left(-\frac{l}{2}\right)\cos(360° + 2\theta) + \left(-\frac{3l}{2}\right)\cos 2\theta\right]$$

$$= 0$$

Graphical solution has been shown in Fig. 14.21. Thus this engine is not balanced in secondary forces.

3. Six-cylinder Four-stroke Engine

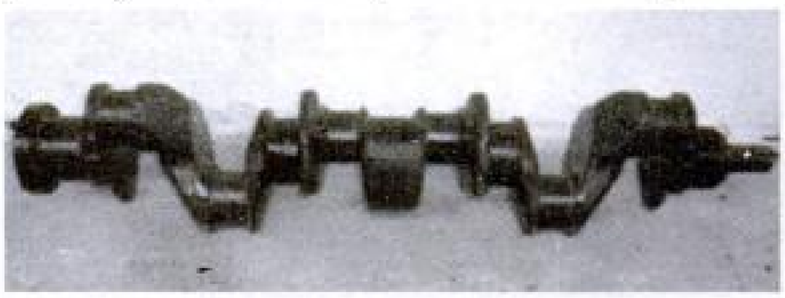
Only a graphical solution is being given for simplicity. In a four-stroke engine, the cycle is completed in two revolutions of the crank and the cranks are 120° apart.

Crank positions for different cylinders for the firing order 142635 for clockwise rotation of the crankshaft are

For first, $\theta = 0°$ For fourth, $\theta = 120°$
For second, $\theta = 240°$ For fifth, $\theta = 240°$
For third, $\theta = 120°$ For sixth, $\theta = 0°$

Assuming m and r equal for all cylinders and taking a vertical plane passing through the middle of the shaft as the reference plane, the force and the couple polygons are drawn as shown in Fig. 14.22.

Since all the force and couple polygons close, it is an inherently balanced engine for primary and secondary forces and couples.



Crankshaft of a six-cylinder engine

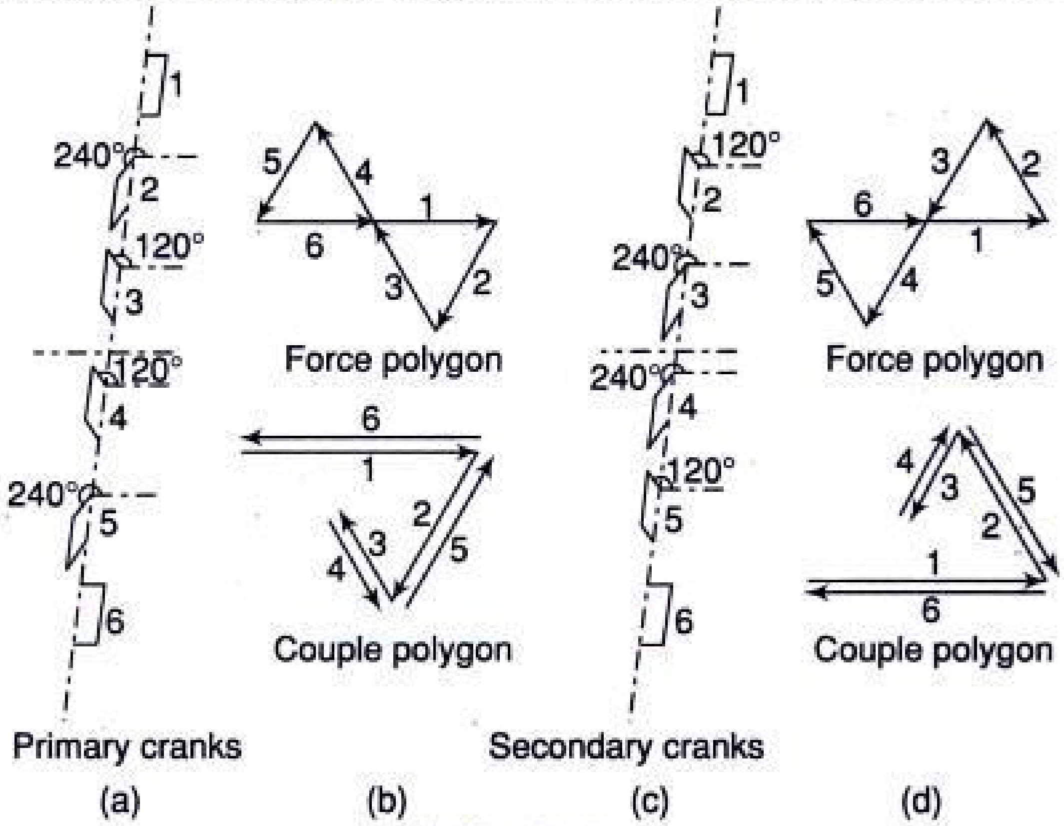


Fig. 14.22

or out of balance secondary couple

$$= \frac{2r\omega^2}{n}[m_1 l_1 \sin 2\alpha + m_2 l_2 \sin 2\beta]$$

Example 14.14 *Each crank and the connecting rod of a four-crank in-line engine are 200 mm and 800 mm respectively. The outer cranks are set at 120° to each other and each has a reciprocating mass of 200 kg. The spacing between adjacent planes of cranks are 400 mm, 600 mm and 500 mm. If the engine is in complete primary balance, determine the reciprocating masses of the inner cranks and their relative angular positions. Also find the secondary unbalanced force if the engine speed is 210 rpm.*

Solution

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$n = 800/200 = 4$

Figure 14.26 represents the relative position of the cylinders and the cranks.

Taking 2 as the reference plane,
primary couples about the *RF*,

$m_1 r_1 l_1 = 200 \times 0.2 \times 0.4 = 16$

$m_2 r_2 l_2 = 0$

$m_3 r_3 l_3 = m_3 \times 0.2 \times (-0.6) = -0.12\, m_3$

$m_4 r_4 l_4 = 200 \times 0.2 \times (-1.1) = -44$

The couple polygon is drawn in Fig. 14.26.

$m_3 r_3 l_3$ of the crank 3 from the diagram = 53.7 at 135°

$\therefore m_3 r_3 l_3 = m_3 \times 0.12 = 53.7$ or $m_3 = 448$ kg

As its direction is to be negative, its direction is (135° + 180°) or 315°.

Primary force (*mr*) along each of outer cranks = $200 \times 0.2 = 40$

Primary force (*mr*) along crank 3 = $448 \times 0.2 = 89.6$

The force polygon is drawn in Fig. 14.26.

$m_2 r_2$ of crank 2 from the diagram = 87.6 at 161.4°

$\therefore m_2 r_2 = m_2 \times 0.2 = 87.6$ or $m_2 = 438$ kg

Its angular position is 161.4°.

Figure 14.26(b) represents the relative position of the cylinders and the cranks.

From secondary unbalanced force polygon,

$mr = 198$

Maximum unbalanced force

$$= 198 \times \frac{\omega^2}{n} = 198 \times \frac{22^2}{4} = 23\,958 \text{ N}$$

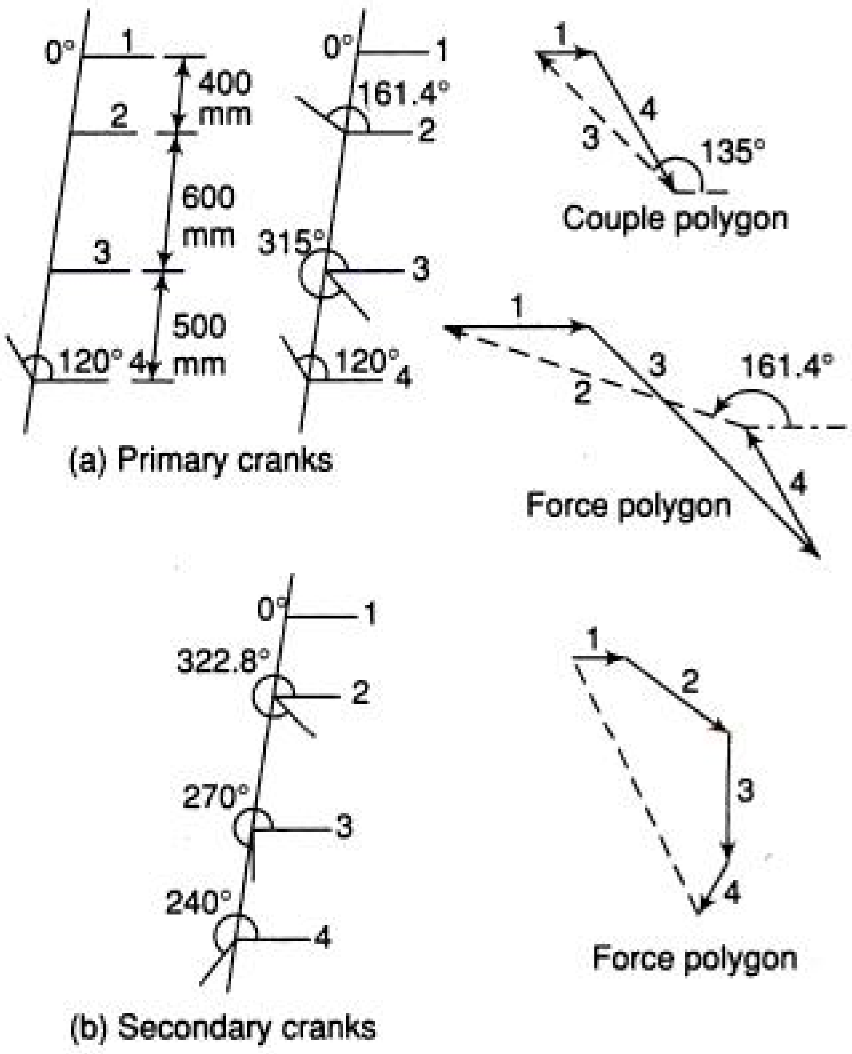


Fig. 14.26

Example 14.15 *The successive cranks of a five-cylinder in-line engine are at 144° apart. The spacing between cylinder centre lines is 400 mm. The lengths of the crank and the connecting rod are 100 mm and 450 mm respectively and the reciprocating mass for each cylinder is 20 kg. The engine speed is 630 rpm. Determine the maximum values of the primary and secondary forces and couples and the position of the central crank at which these occur.*

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It can easily be balanced by a revolving mass in a direction opposite to that of crank.

Countermass m_r at a radial distance of 100 mm,

$m_r \times 100 \times \omega^2 = 8.7 \times (160/2)\omega^2$

$m_r = 6.96$ kg

The rotating masses do not affect the secondary forces as they are only due to second harmonics of the piston acceleration.

Secondary force $= \sqrt{2}\dfrac{mr\omega^2}{n}\sin 2\theta$ (Eq. 14.40)

$= \sqrt{2} \times \dfrac{2.7 \times 0.08 \times 88^2}{5}\sin 2$

$= 473.1 \sin 2\theta$

Maximum value at $\theta = 45° = 473.1$ N

Example 14.21 *The cylinders of a twin V-engine are set at 60° angle with both pistons connected to a single crank through their respective connecting rods. Each connecting rod is 600 mm long and the crank radius is 120 mm. The total rotating mass is equivalent to 2 kg at the crank radius and the reciprocating mass is 1.2 kg per piston. A balance mass is also fitted opposite to the crank equivalent to 2.2 kg at a radius of 150 mm. Determine the maximum and minimum values of the primary and secondary forces due to inertia of the reciprocating and the rotating masses if the engine speed is 800 rpm.*

Solution Refer Fig. 14.33.

$m = 1.2$ kg	$M = 2$ kg
$l = 600$ mm	$r = 120$ mm
$m' = 2.2$ kg	$r' = 150$ mm
$N = 800$ rpm	

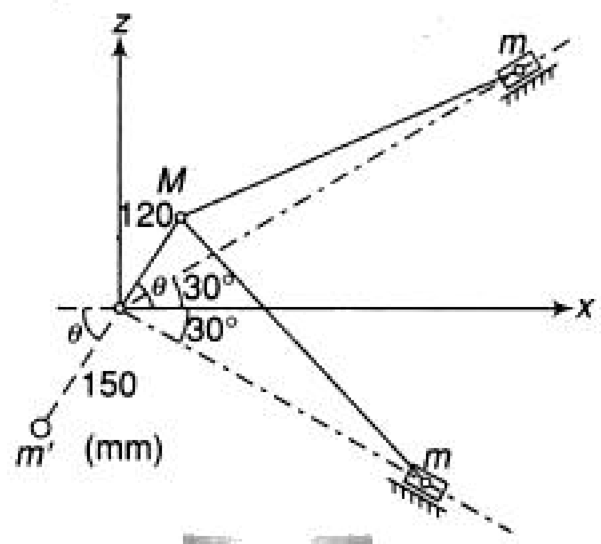


Fig. 14.33

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1050}{60} = 110 \text{ rad/s}$$

$$n = \frac{400}{80} = 5$$

Primary force

Total primary force along x-axis

$= 2mr\omega^2\cos^2\alpha\cos\theta$ (Eq. 14.29)

Centrifugal force due to rotating mass along x-axis $= Mr\omega^2\cos\theta$

Centrifugal force due to balancing mass along x-axis $= -m'r\omega^2\cos\theta$

Total unbalanced force along x-axis

$= 2mr\omega^2\cos^2\alpha\cos\theta + Mr\omega^2\cos\theta - m'r'\omega^2\cos\theta$

$= \omega^2\cos\theta(2mr\cos^2\alpha + Mr - m'r')$

$= 110^2 \times \cos\theta(2 \times 1.2 \times 0.12\cos^2 30° + 2 \times 0.12 - 2.2 \times 0.15)$

$= 110^2 \times \cos\theta(0.216 + 0.24 - 0.33)$

$= 1524.6\cos\theta$ N

Total primary force along z-axis

$= 2mr\omega^2\sin^2\alpha\sin\theta$ (Eq. 14.30)

Centrifugal force due to rotating mass along z-axis $= Mr\omega^2\sin\theta$

Centrifugal force due to balancing mass along z-axis $= -m'r\omega^2\sin\theta$

Total unbalanced force along z-axis

$= 2mr\omega^2\sin^2\alpha\sin\theta + Mr\omega^2\sin\theta - m'r'\omega^2\sin\theta$

$= \omega^2\sin\theta(2mr\sin^2\alpha + Mr - m'r')$

$= 110^2 \times \sin\theta(2 \times 1.2 \times 0.12\sin^2 30° + 2 \times 0.12 - 2.2 \times 0.15)$

$= 110^2 \times \sin\theta(0.072 + 0.24 - 0.33)$

$= -217.8\sin\theta$ N

Resultant primary force

$= \sqrt{1524^2\cos^2\theta + (-217.8)^2\sin^2\theta}$

$= \sqrt{2\ 322\ 576\cos^2\theta + 47\ 437\sin^2\theta}$

$= \sqrt{\begin{array}{l}2\ 275\ 139\cos^2\theta + 47\ 437 \\ \cos^2\theta + 47\ 437\sin^2\theta\end{array}}$

$= \sqrt{2\ 275\ 139\cos^2\theta + 47\ 437}$

This is maximum when θ is 0° and minimum when $\theta = 90°$.

Maximum primary force

$= \sqrt{2\ 275\ 139 + 47\ 437} = 1524$ N

Minimum primary force

Example 14.24 *A radial aero-engine has seven cylinders equally spaced with all the connecting rods coupled to a common crank. The crank and each of the connecting rods are 200 mm and 800 mm respectively. The reciprocating mass per cylinder is 3 kg. Determine the magnitude and the angular position of the balance masses required at the crank radius for complete primary and secondary balancing of the engine.*

Solution The position of the seven cylinders is shown in Fig. 14.42.

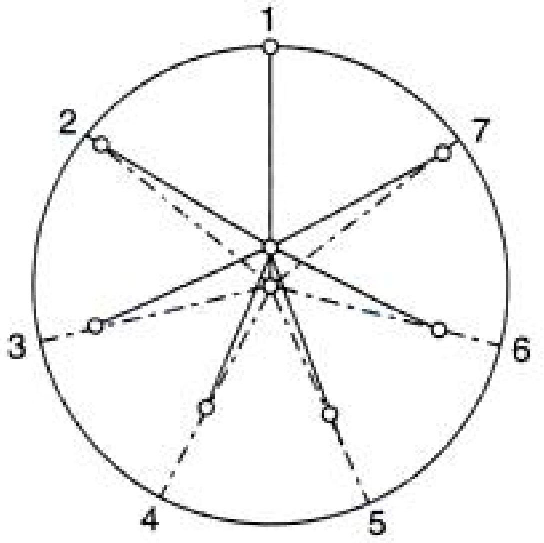


Fig. 14.42

Primary Cranks

The primary direct and reverse crank positions are shown in Fig. 14.43(a) and (b) respectively. Refer Table 14.5.

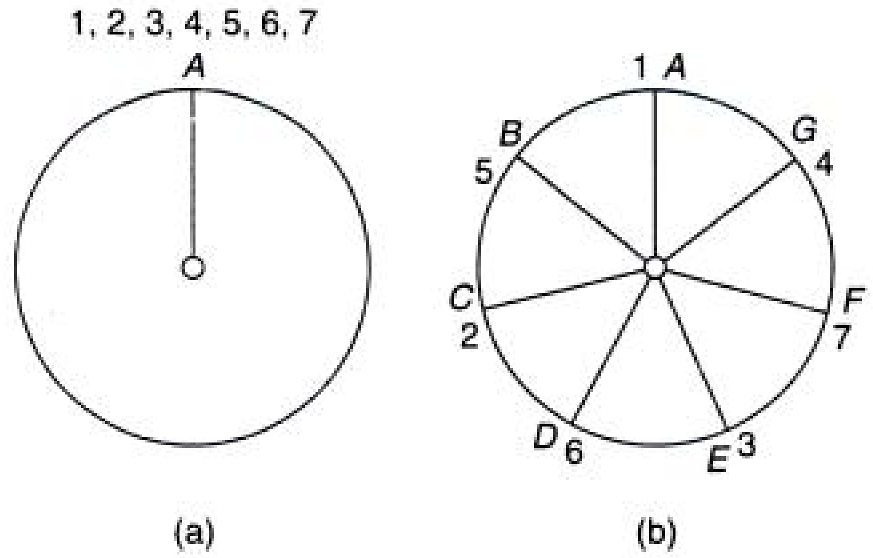


Fig. 14.43

Let $360°/7 = X$

This shows that there is primary unbalance due to direct cranks.

Secondary Cranks

The secondary direct and reverse crank positions are shown in Fig. 14.44 (a) and (b) respectively. Refer Table 14.6.

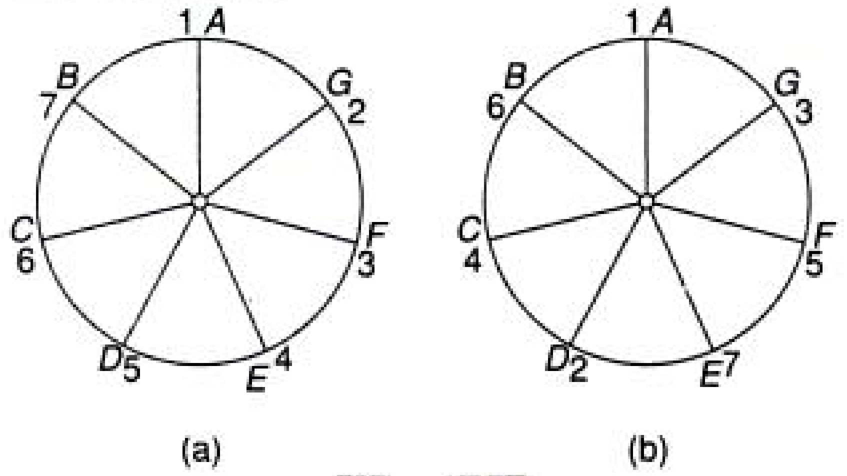


Fig. 14.44

Table 14.5

Cylinder	*Crank angle (counter-clockwise) deg.*	*Angle of rotation of the crank, deg.*	*Position of direct crank on clockwise rotation*	*Position of reverse crank on counter-clockwise rotation*
1	0	0	*OA*	*OA*
2	X	X	*OA*	*OC*
3	$2X$	$2X$	*OA*	*OE*
4	$3X$	$3X$	*OA*	*OG*
5	$4X$	$4X$	*OA*	*OB*
6	$5X$	$5X$	*OA*	*OD*
7	$6X$	$6X$	*OA*	*OF*

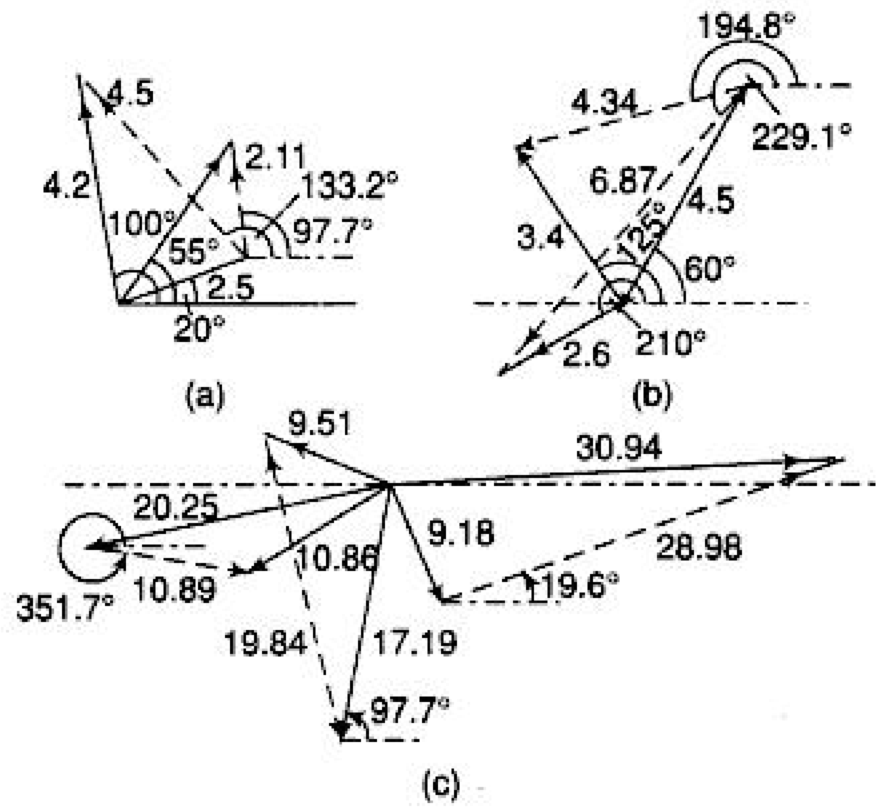


Fig. 14.51

Now, $$\alpha = \frac{B(A_2 - A) - A(B_2 - B)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)}$$ [Eq. 14.42]

or $$\alpha = \frac{\begin{bmatrix} 4.5e^{i(60^\circ)} \times 2.113e^{i(97.7^\circ)} \\ -2.5e^{i(20^\circ)} \times 6.875e^{i(229.1^\circ)} \end{bmatrix}}{\begin{bmatrix} 4.5e^{i(133.2^\circ)} \times 6.875e^{i(229.1^\circ)} \\ -2.113e^{i(97.7^\circ)} \times 4.345e^{i(194.8^\circ)} \end{bmatrix}}$$

$$= \frac{9.51e^{i(157.7^\circ)} - 17.188e^{i(249.1^\circ)}}{30.94e^{i(2.3^\circ)} - 9.18e^{i(292.5^\circ)}}$$

The numerator and the denominator can be solved analytically or graphically [Fig.14.51(c)].

i.e.,

$$\alpha = \frac{19.84e^{i(97.7^\circ)}}{28.98e^{i(19.6^\circ)}} = 0.685\, e^{i(78.1^\circ)}$$

Similarly,

$$\beta = \frac{A(B_1 - B) - B(A_1 - A)}{(A_1 - A)(B_2 - B) - (A_2 - A)(B_1 - B)}$$ [Eq. 14.43)]

or $$\beta = \frac{\begin{bmatrix} 2.5e^{i(20^\circ)} \times 4.345e^{i(194.8^\circ)} \\ -4.5e^{i(60^\circ)} \times 4.5e^{i(133.2^\circ)} \end{bmatrix}}{\begin{bmatrix} 4.5e^{i(133.2^\circ)} \times 6.875e^{i(229.1^\circ)} \\ -2.113e^{i(97.7^\circ)} \times 4.345e^{i(194.8^\circ)} \end{bmatrix}}$$

$$= \frac{10.86e^{i(214.8^\circ)} - 20.25e^{i(193.2^\circ)}}{30.94e^{i(2.3^\circ)} - 9.18e^{i(292.5^\circ)}}$$

$$= \frac{10.895e^{i(351.7^\circ)}}{28.98e^{i(19.6^\circ)}} = 0.376\, e^{i(332.1^\circ)}$$

Thus, the balance mass in the plane A
$= 0.685 \times 3 = 2.055$ kg

Angular position = 78.1° counter-clockwise with the direction of trial mass in the plane A.

Similarly, the balance mass in plane B
$= 0.376 \times 3 = 1.128$ kg

Angular position = 332.1° counter-clockwise with the direction of trial mass in the plane B.

Summary

1. A system of rotating masses is said to be in *static balance* if the combined mass centre of the system lies on the axis of rotation.
2. Several masses rotating in different planes are said to be in *dynamic balance* when there does not exist any resultant centrifugal force as well as the resultant couple.
3. Balancing of a *linkage* implies that the total centre of its mass remains stationary so that the vector sum of all the frame forces always remains zero.
4. *Primary accelerating force* in a reciprocating engine is $mr\omega^2 \cos\theta$ along the line of stroke.
5. *Secondary accelerating force* in a reciprocating engine is $mr\omega^2 \cos(2\theta)/n$ along the line of stroke.
6. In reciprocating engines, unbalanced forces along the line of stroke are more harmful than the forces perpendicular to the line of stroke.
7. In locomotives, *hammer-blow* is the maximum vertical unbalanced force caused by the mass to balance the reciprocating masses and *swaying couple* tends to make the leading wheels sway from side to side due to unbalanced primary forces along the lines of stroke.
8. The effect of the secondary force is equivalent

A material softer than that of the drum or the rim of the wheel is used to make the blocks so that these can be replaced easily on wearing. Wood and rubber are used for light and slow vehicles and cast steel for heavy and fast ones.

Let r = radius of the drum

μ = coefficient of friction

F_r = radial force applied on the drum (not shown in the figure)

R_n = normal reaction on the block (= F_r)

F = force applied at the lever end

F_f = frictional force = μR_n

Assuming that the normal reaction R_n and frictional force F_f act at the mid-point of the block,

Braking torque on the drum = frictional force × radius

or

$$T_B = \mu R_n \times r \tag{15.1}$$

To obtain R_n consider the equilibrium of the block as follows.

The direction of the frictional force on the drum is to be opposite to that of its rotation while on the block it is in the same direction. Taking moments about the pivot O [Fig. 15.1(a)],

$$F \times a - R_n \times b + \mu R_n \times c = 0$$

$$R_n = \frac{Fa}{b - \mu c} \tag{15.2}$$

Also

$$F = R_n \frac{b - \mu c}{a} \tag{15.3}$$

- When $b = \mu c$, $F = 0$, which implies that the force needed to apply the brake is virtually zero, or that once contact is made between the block and the drum, the brake is applied itself. Such a brake is known as a *self-locking brake*.
- As the moment of the force F_f about O is in the same direction as that of the applied force F, F_f aids in applying the brake. Such a brake is known as a *self-energised brake*.
- If the rotation of the drum is reversed, i.e., it is made clockwise,

  $$F = R_n [(b + \mu c)/a]$$

  which shows that the required force F will be far greater than what it would be when the drum rotates counter-clockwise.
- If the pivot lies on the line of action of F_f, i.e., at O', $c = 0$ and $F = R_n \dfrac{a}{b}$, which is valid for clockwise as well as for counter-clockwise rotation.
- If c is made negative, i.e., if the pivot is at O'',

  $$F = R_n \left(\frac{b + \mu c}{a}\right) \text{ for counter-clockwise rotation}$$

  and

  $$F = R_n \left(\frac{b - \mu c}{a}\right) \text{ for clockwise rotation}$$

If the force applied is not as above, the band is further loosened on the drum which means no braking effect is possible.

(i) $a > b$, F Downwards

(a) Rotation Counter-clockwise For counter-clockwise rotation of the drum, the tight and the slack sides of the band will be as shown in Fig. 15.7.

Considering the forces acting on the lever and taking moments about the pivot,

$$F\,l - T_1\,a + T_2\,b = 0$$

or
$$F = \frac{T_1 a - T_2 b}{l} \tag{15.4}$$

As $T_1 > T_2$ and $a > b$ under all conditions, the effectiveness of the brake will depend upon the force F.

(b) Rotation Clockwise In this case, the tight and the slack sides are reversed as shown in Fig. 15.8.

Now, $F\,l - T_2\,a + T_1\,b = 0$ or $F = \dfrac{T_2 a - T_1 b}{l}$

As $T_2 < T_1$ and $a > b$, the brake will be effective as long as $T_2.a$ is greater than $T_1.b$

or $T_2\,a > T_1\,b$ or $\dfrac{T_2}{T_1} > \dfrac{b}{a}$

i.e., as long as the ratio of T_2 to T_1 is greater than the ratio b/a.

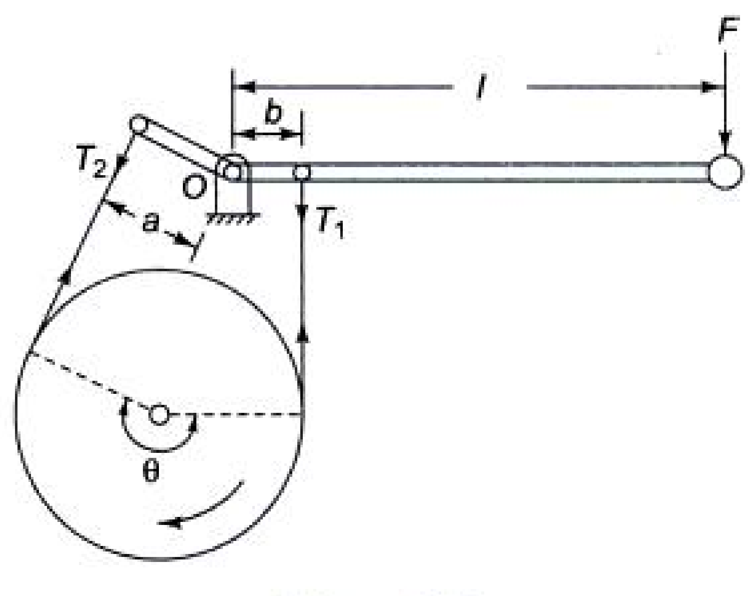
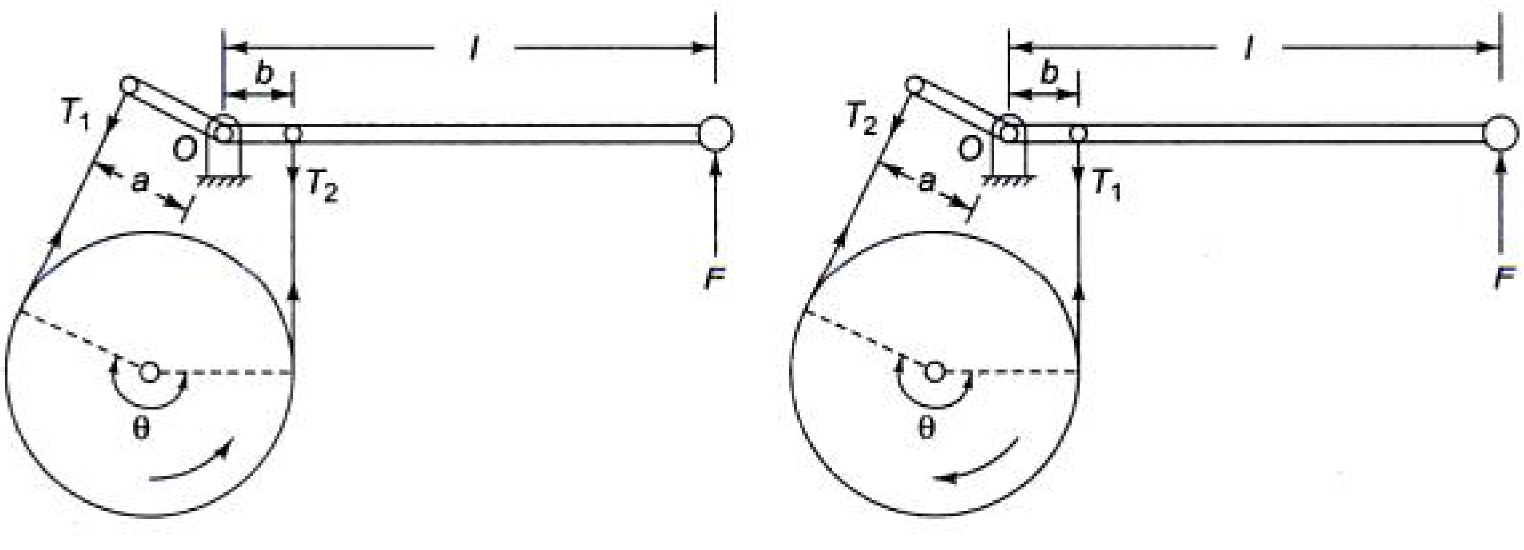


Fig. 15.8

When $\dfrac{T_2}{T_1} \le \dfrac{a}{b}$, F is zero or negative, i.e., the brake becomes self-locking as no force is needed to apply the brake. Once the brake has been engaged, no further force is required to stop the rotation of the drum.

(ii) $a < b$, F upwards



(a) (b)

Fig. 15.9

at a distance of 600 mm from the fulcrum if the power absorbed by the blocks is 180 kW at 175 rpm. Coefficient of friction between the blocks and the drum is 0.35.

Solution

$N = 175$ rpm, $d = 750$ mm, $\theta = 7°$, $\mu = 0.35$, $P = 180$ kW, $t = 65$ mm, $l = 600$ mm

Refer Fig. 15.15.

$$P = (T_{14} - T_0).v = (T_{14} - T_0).\frac{\pi DN}{60}$$

$$\therefore 180\,000 = (T_{14} - T_0) \times \frac{\pi \times (0.75 + 2 \times 0.065) \times 175}{60}$$

or $T_{14} - T_0 = 22\,323$ N

$$\frac{T_{14}}{T_0} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n = \left(\frac{1 + 0.35 \tan 7°}{1 - 0.35 \tan 7°}\right)^{14} = 3.334$$

or $2.334\, T_0 = 22\,323$ or $T_0 = 9564$ N

and $T_{14} = 22\,323 + 9564 = 31\,887$ N

Assume $a = 210$ mm and $b = 50$ mm (Fig. 15.15)

As $a > b$, F must be downwards and rotation clockwise for maximum braking torque. Taking moments about the fulcrum,

$$F \times l - T_0\, a + T_{12}\, b = 0$$

$$F \times 600 - 9564 \times 210 + 31887 \times 50 = 0$$

$$600\, F = 414\,090 \quad \text{or} \quad F = 690 \text{ N}$$

Example 15.12 *A band and block brake having 12 blocks, each of which subtends an angle of 16° at the centre, is applied to a rotating drum with a diameter of 600 mm. The blocks are 75 mm thick. The drum and the flywheel mounted on the same shaft have a mass of 1800 kg and have a combined radius of gyration of 600 mm. The two ends of the band are attached to pins on the opposite sides of the brake fulcrum at distances of 40 mm and 150 mm from it. If a force of 250 N is applied on the lever at a distance of 900 mm from the fulcrum, find the*

(i) maximum braking torque

(ii) angular retardation of the drum

(iii) time taken by the system to be stationary from the rated speed of 300 rpm.

Take coefficient of friction between the blocks and the drum as 0.3.

Solution

$F = 250$ N, $d = 600$ mm, $\theta = 8°$, $t = 75$ mm, $l = 900$ mm, $k = 600$ mm, $m = 1800$ kg, $n = 12$, $N = 300$ rpm, $\mu = 0.3$

Refer Fig. 15.15.

(i) $$\frac{T_{12}}{T_0} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n$$

$$= \left(\frac{1 + 0.3 \tan 8°}{1 - 0.3 \tan 8°}\right)^{12} = 2.752$$

Assume $a = 150$ mm and $b = 40$ mm

As $a > b$, F must be downwards and the rotation is clockwise for maximum braking torque. Taking moments about the fulcrum,

$$F \times l - T_0\, a + T_{12}\, b = 0$$

$$250 \times 900 - T_0 \times 150 + 2.752\, T_0 \times 40 = 0$$

$$T_0 (150 - 2.752 \times 40) = 250 \times 900$$

$$T_0 = 5636 \text{ N}$$

$$T_{12} = 5636 \times 2.752 = 15\,511 \text{ N}$$

Maximum braking torque, $T_B = (T_{12} - T_0) \times \dfrac{d}{2}$

$$= (15\,511 - 5636) \times \left(\frac{0.6 + 0.075 \times 2}{2}\right)$$

$$= \underline{3703 \text{ N.m}}$$

(ii) $T_B = I\alpha = mk^2\alpha$

$$3703 = 1800 \times (0.6)^2 \times \alpha$$

$$\alpha = \underline{5.71 \text{ rad/s}^2}$$

(iii) Initial angular speed,

$$\omega_0 = \frac{2\pi \times 300}{60} = 31.4 \text{ rad/s}$$

Final angular speed, $\omega = 0$

$\therefore$ $\omega = \omega_0 - \alpha t$ (α negative due to retardation)

or $0 = 31.4 - 5.71\, t$

$$t = \underline{5.5 \text{ s}}$$

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Solution Let s be the distance moved by the car before coming to rest.

$$u = 54 \text{ km/h} = \frac{54\ 000}{3600} = 15 \text{ m/s}$$

(i) Brakes applied to rear wheels

$$f = g\frac{\mu(l-x)}{l+\mu h} = 9.81 \times \frac{0.5(3.2-1.4)}{3.2+0.5\times 0.55}$$
$$= 2.54 \text{ m/s}^2$$

If retardation is uniform, $v^2 - u^2 = -2fs$

$$0 - u^2 = -2fs$$

$$s = \frac{u^2}{2f} = \frac{15^2}{2\times 2.54} = \underline{44.3 \text{ m}}$$

(ii) Brakes applied to front wheels

$$f = g\frac{\mu x}{l-\mu h} = 9.81 \times \frac{0.5\times 1.4}{3.2-0.5\times 0.55}$$
$$= 2.35 \text{ m/s}^2$$

$$s = \frac{u^2}{2f} = \frac{15^2}{2\times 2.35} = 47.9 \text{ m}$$

(iii) Brakes applied to all the four wheels

$$f = gu = 9.81 \times 0.5 = 4.905 \text{ m/s}^2$$

$$s = \frac{u^2}{2fs} = \frac{15^2}{2\times 4.90} = \underline{22.9 \text{ m}}$$

Example 15.15 *A vehicle moves on a road that has a slope of 15°. The wheel base is 1.6 m and the centre of mass is at 0.72 m from the rear wheels and 0.8 m above the inclined plane. The speed of the vehicle is 45 km/h. The brakes are applied to all the four wheels and the coefficient of friction is 0.4. Determine the distance moved by the vehicle before coming to rest and the time taken to do so if it moves*

(i) up the plane

(ii) down the plane

Solution Let s be the distance moved by the car before coming to rest.

$$u = 45 \text{ km/h} = \frac{45\ 000}{3600} = 12.5 \text{ m/s}$$

(i) The vehicle moves up

$$f = g \cos\alpha\,(\mu + \tan\alpha)$$
$$= 9.81 \times \cos 15^\circ\,(0.4 + \tan 15^\circ) = 6.33 \text{ m/s}^2$$

If retardation is uniform, $v^2 - u^2 = -2fs$

$$0 - u^2 = -2fs$$

$$s = \frac{u^2}{2f} = \frac{12.5^2}{2\times 6.33} = \underline{12.34 \text{ m}}$$

Also, $v = u - ft$

or $0 = 12.5 - 6.33 \times t$

or $t = 1.97$ s

(ii) The vehicle moves down

$$f = g \cos\alpha\,(\mu - \tan\alpha)$$
$$= 9.81 \times \cos 15^\circ\,(0.4 - \tan 15^\circ) = 1.25 \text{ m/s}^2$$

$$s = \frac{u^2}{2f} = \frac{12.5^2}{2\times 1.25} = \underline{62.5 \text{ m}}$$

Also, $0 = 12.5 - 1.25 \times t$

or $t = \underline{10 \text{ s}}$

15.7 TYPES OF DYNAMOMETERS

There are mainly two types of dynamometers:

(i) Absorption Dynamometers In this type, the work done is converted into heat by friction while being measured. They can be used for the measurement of moderate powers only. Examples are prony brake dynamometer and rope brake dynamometer.

(ii) Transmission Dynamometers In this type, the work is not absorbed in the process, but is utilised after the measurement. Examples are the belt-transmission dynamometer and the torsion dynamometer.

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Now,

$$\text{Brake power, } P = \frac{R_t.v}{\eta} = \frac{6061}{0.82}\left(\frac{32 \times 1000}{3600}\right)$$
$$= 67\ 700 \text{ W or } 67.7 \text{ kW}$$

$$G = \frac{\omega_e}{\omega_w \times \text{Back axle ratio}}$$
$$= \frac{\omega_e}{(v/r) \times \text{Back axle ratio}}$$
$$= \frac{80\pi}{(8.889/0.4) \times 4.02}$$
$$= 2.81$$

In the top gear on a level road

$$R_t = 0.015\ Mg + 0.038AV^2$$
$$= 0.015 \times 4400 \times 9.81 + 0.038 \times 6 \times V^2$$
$$= 5827 + 234$$
$$= 6061 \text{ N}$$

$$P = \frac{R_t.v}{\eta}$$
$$67\ 700 = \frac{(0.015 \times 4400 \times 9.81 + 0.038 \times 6 \times V^2)V}{0.92}$$
$$647.46V + 0.228V^2 = 224\ 222$$
$$V = 90 \text{ km/h}$$
$$= \frac{90 \times 1000}{3600}$$
$$= 25 \text{ m/s}$$
$$G = \frac{\omega_e}{\omega_w \times \text{Back axle ratio}}$$
$$= \frac{\omega_e}{(V/r) \times \text{Back axle ratio}}$$
$$= \frac{80\pi \times 0.4}{25 \times 4.02} = 1$$

Summary

1. A brake is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy.
2. The functional difference between a clutch and a brake is that a clutch connects two moving members of a machine whereas a brake connects a moving member to a stationary member.
3. The main types of mechanical brakes are *block* or *shoe brake, band brake, band and block brake* and *internal expanding shoe brake*.
4. A *block* or *shoe brake* consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever.
5. A *band brake* consists of a rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied.
6. A *band and block brake* consists of a number of wooden blocks secured inside a flexible steel band which are pressed against the drum when the brake is applied.
7. An *internal expanding shoe brake* consists of two semi-circular shoes which are lined with a friction material such as *ferodo*. The shoes press against the inner flange of the drum when the brakes are applied.
8. The power required for propulsion of a wheeled vehicle depends mainly on the *tractive resistance*, i.e., the resistance faced by the vehicle on the road.
9. The main components of the tractive resistance are the *road resistance, aerodynamic resistance* and *gradient resistance*.
10. Road resistance consists of two types of resistances: *rolling resistance* and *frictional resistance*.
11. Aerodynamic resistance is the resistance posed by air or wind and depends upon the speed of the vehicle, its shape and the wind velocity.
12. Gradient resistance depends upon the weight of the vehicle and the gradient of the surface and is independent of the vehicle speed.

Exercises

1. What is a brake? What is the difference between a brake and a clutch?
2. What are various types of brakes? Describe briefly.
3. With the help of a neat sketch explain the working of a block or shoe brake.
4. What is meant by a self-locking and a self-energised brake.
5. Discuss the effectiveness of a band brake under various conditions.
6. Describe the working of a band and block brake

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16.3 PORTER GOVERNOR

If the sleeve of a Watt governor is loaded with a heavy mass, it becomes a Porter governor [Fig.16.4(a)]

Ler M = mass of the sleeve

m = mass of each ball

f = force of friction at the sleeve

The force of friction always acts in a direction opposite to that of the motion. Thus when the sleeve moves up, the force of friction acts in the downward direction and the downward force acting on the sleeve is $(Mg + f)$. Similarly, when the sleeve moves down, the force on the sleeve will be $(Mg - f)$. In general, the net force acting on the sleeve is $(Mg \pm f)$ depending upon whether the sleeve moves upwards or downwards.

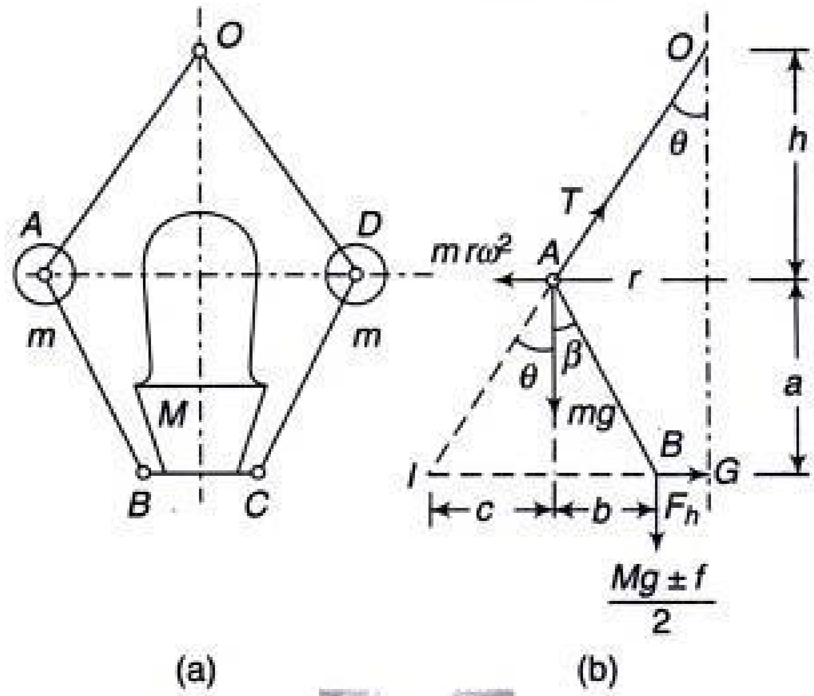


Fig. 16.4

Forces acting on the sleeve and on each ball have been shown in Fig.16.4(b).

Let h = height of the governor

r = distance of the centre of each ball from axis of rotation

The instantaneous centre of rotation of the link AB is at I for the given configuration of the governor. It is because the motion of its two points A and B relative to the link is known. The point A oscillates about the point O and B moves in a vertical direction parallel to the axis. Lines perpendicular to the direction of these motions locates the point I.

Considering the equilibrium of the left-hand half of the governor and taking moments about I,

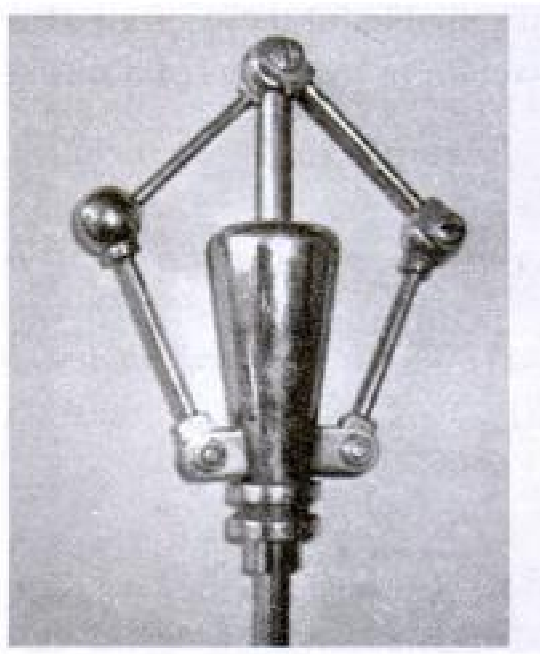
$$mr\omega^2 . a = mg\, c + \frac{Mg \pm f}{2}(c + b)$$

or $$mr\omega^2 = mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right)$$

$$= mg \tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \qquad \left(\text{taking } k = \frac{\tan\beta}{\tan\theta}\right)$$

or $$= \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1 + k)\right]$$



A porter governor

or $$\omega^2 = \frac{1}{mh}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2}\right)$$

or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right) \qquad \text{(Taking } g = 9.81 \text{ m/s}^2\text{)}$$

$$mr'\omega^2 e = mg\,(c + r - r') + \frac{Mg \pm f}{2}(c + b)$$

where b, c, d and r are the dimensions as indicated in the diagram.

$$mr'\omega^2 = \frac{1}{e}\left[mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)\right] \tag{16.4}$$

In the position when AE is vertical, i.e., neglecting its obliquity

$$mr'\omega^2 = \frac{1}{e}\left[mgc + \frac{Mg \pm f}{2}(c + b)\right]$$

$$= \frac{a}{e}\left[mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right)\right]$$

$$= \frac{a}{e}\left[mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)\right]$$

$$= \frac{a}{e}\tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \tag{16.5}$$

$$= \frac{a}{e}\frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1 + k)\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{a}{e}\frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right) \qquad \text{(Taking } g = 9.81 \text{ m/s}^2\text{)}$$

- If $k = 1$, $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{mg + (Mg \pm f)}{mg}\right)$$
- If $f = 0$, $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{2m + M(1 + k)}{2m}\right)$$
- If $k = 1, f = 0$ $$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{m + M}{m}\right)$$

Example 16.6 *Each arm of a Proell governor is 240 mm long and each rotating ball has a mass of 3 kg. The central load acting on the sleeve is 30 kg. The pivots of all the arms are 30 mm from the axis of rotation. The vertical height of the governor is 190 mm. The extension links of the lower arms are vertical and the governor speed is 180 rpm when the sleeve is in the mid-position. Determine the lengths of the extension links and the tension in the upper arms.*

Solution Refer Fig. 16.10.

$$(F_2 - F_1)\,a = \frac{1}{2}(F_{s2} - F_{s1})b$$

or
$$F_{s2} - F_{s1} = \frac{2a}{b}(F_2 - F_1)$$

Let s = stiffness of the spring
h_1 = movement of the sleeve

$$F_{s2} - F_{s1} = h_1 s = \frac{2a}{b}(F_2 - F_1) \quad \text{or} \quad s = \frac{2}{h_1}.\frac{a}{b}.(F_2 - F_1)$$

But
$$h_1 = \theta.b = \frac{r_2 - r_1}{a}.b$$

$\therefore$
$$s = \frac{2}{r_2 - r_1}.\left(\frac{a}{b}\right)^2.(F_2 - F_1) = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right) \tag{16.7}$$

Example 16.8 *In a Hartnell governor, the extreme radii of rotation of the balls are 40 mm and 60 mm, and the corresponding speeds are 210 rpm and 230 rpm. The mass of each ball is 3 kg. The lengths of the ball and the sleeve arms are equal. Determine the initial compression and the constant of the central spring.*

Solution

$$\omega_1 = \frac{2\pi \times 210}{60} = 22 \text{ rad/s};$$
$$\omega_2 = \frac{2\pi \times 230}{60} = 23.04 \text{ rad/s}$$
$$F_1 = m r_1 \omega_1^2 = 3 \times 0.04 \times 22^2 = 58.1 \text{ N}$$

and
$$F_2 = m r_2 \omega_2^2 = 3 \times 0.06 \times 23.04^2 = 95.6 \text{ N}$$

Spring constant,
$$s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$
$$= 2\,(1)^2\left(\frac{95.6 - 58.1}{60 - 40}\right) = \underline{3.75 \text{ N/mm}}$$

We have, $F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b$ or $F_1 = \frac{F_{s1}}{2}$

$(M = 0, f = 0, a = b)$

or $F_{s1} = 2 \times 58.1 = 116.2$ N

Initial compression $= \frac{116.2}{3.75} = \underline{31 \text{ mm}}$

Example 16.9 *In a spring-loaded governor of the Hartnell type, the lengths of the horizontal and the vertical arms of the bell-crank lever are 40 mm and 80 mm respectively. The mass of each ball is 1.2 kg. The extreme radii of rotation of the balls are 70 mm and 105 mm. The distance of the fulcrum of each bell-crank lever is 75 mm from the axis of rotation of the governor. The minimum equilibrium speed is 420 rpm and the maximum equilibrium speed is 4% higher than this. Neglecting the obliquity of the arms, determine the*

(i) spring stiffness,
(ii) initial compression, and
(iii) equilibrium speed corresponding to radius of rotation of 95 mm.

Solution

$$\omega_1 = \frac{2\pi \times 420}{60} = 44 \text{ rad/s};$$
$$\omega_2 = 44 \times 1.04 = 45.76 \text{ rad/s}$$
$$F_1 = m r_1 \omega_1^2 = 1.2 \times 0.07 \times 44^2 = 162.6 \text{ N}$$

and $F_2 = m r_2 \omega_2^2 = 1.2 \times 0.105 \times 45.76^2 = 263.8$ N

(i) Spring constant,
$$s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right) = 2\left(\frac{80}{40}\right)^2\left(\frac{263.8 - 162.6}{105 - 70}\right)$$
$$= \underline{23.14 \text{ N/mm}}$$

16.8 PICKERING GOVERNOR

A Pickering governor consists of three leaf springs which are arranged at equal angular intervals around the governor spindle (Fig. 16.17), only one leaf spring is shown in the figure. The upper end of each spring is fixed by a screw to a hexagonal nut attached to the spindle. The lower end is fastened to the sleeve which can move up and down the governor spindle. Each spring has a fly mass m attached at its centre. As the spindle rotates, a centrifugal force is exerted on the leaf spring at the centre which causes it to deflect. This deflection makes the sleeve move up. A stop is also provided to limit the movement of the sleeve.

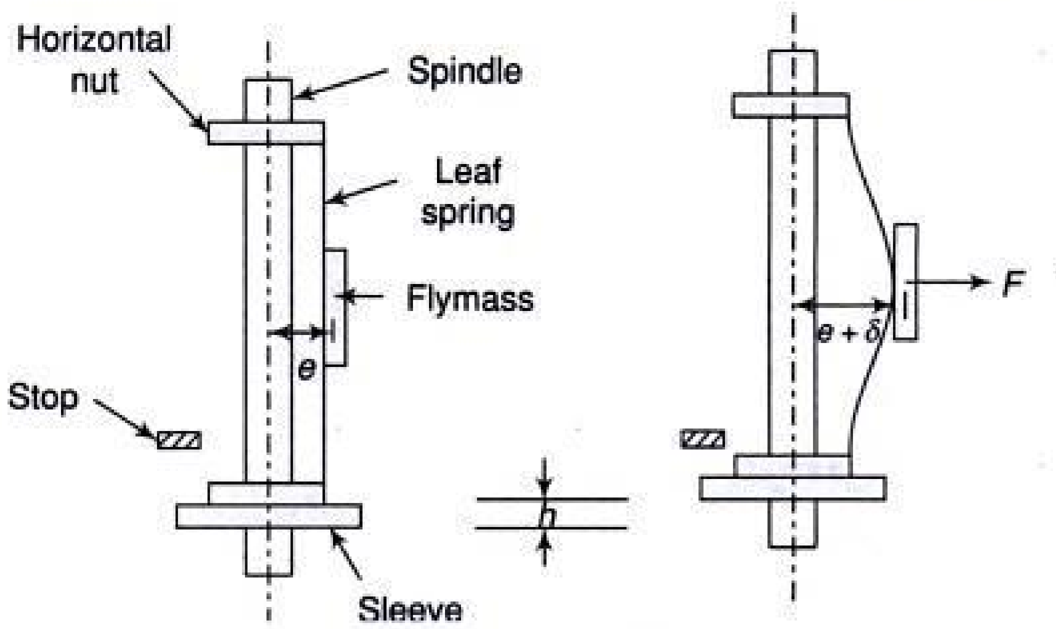


Fig. 16.17

Let m = mass fixed to each spring

e = distance between spindle axis and centre of mass when the governor is at rest

ω = Angular speed of the sleeve

δ = deflection of the centre of the leaf spring for spindle speed ω

Centrifugal force, $F = m(e + \delta)\,\omega^2$

To find δ, the leaf spring is treated as a beam of uniform cross section fixed at both ends and carrying a load at the centre.

$$\delta = \frac{Fl^3}{192EI} = \frac{m(e+\delta)\omega^2 l^3}{192EI}$$

where

E = modulus of elasticity of the spring material

I = moment of inertia of the cross-section of the spring about neutral axis = $\frac{bt^3}{12}$, b and t being the width and the thickness of the leaf spring.

An empirical relation to between the deflection δ and the lift h of the sleeve may also be used as follows:

$$h = 2.4\frac{\delta^2}{l}$$

A Pickering governor is used in gramophones to adjust the speed of the turn table.

Example 16.14 *Each spring of a Pickering governor of a gramophone is 6 mm wide and 0.12 mm thick with a length of 48 mm. A mass of 25 g is attached to each leaf spring at the centre. The distance between the spindle axis and the centre of mass when the governor is at rest is 8 mm. The ratio of the governor speed to the turn table speed is 10. Determine the speed of the turn table for a sleeve lift of 0.6 mm. Take E = 200 GN/m².*

Solution

$m = 0.025$ kg $\qquad b = 0.006$ m

displacement of sleeve $\approx (1 + k)(h - h_1) \approx (1 + k)\, h\left(\dfrac{2c}{1+2c}\right)$

and thus power $= \dfrac{cg}{1+k}[2m + M(1+k)] \times (1+k)h\left(\dfrac{2c}{1+2c}\right)$

$$= \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$

Example 16.17 *Each ball of a Porter governor has a mass of 3 kg and the mass of the sleeve is 15 kg. The governor has equal arms, each of 200-mm length and pivoted on the axis of rotation. When the radius of rotation of the balls is 120 mm, the sleeve begins to rise up 160 mm at the maximum speed. Determine the*

(i) range of speed
(ii) lift of the sleeve
(iii) effort of the governor
(iv) power of the governor

What will be the effect of friction at the sleeve if it is equivalent to 8 N?

Solution

Refer Fig. 16.22.

$$h_1 = \sqrt{0.2^2 - 0.12^2} = 0.16 \text{ m}$$

$$h_1 = \sqrt{0.2^2 - 0.16^2} = 0.12 \text{ m}$$

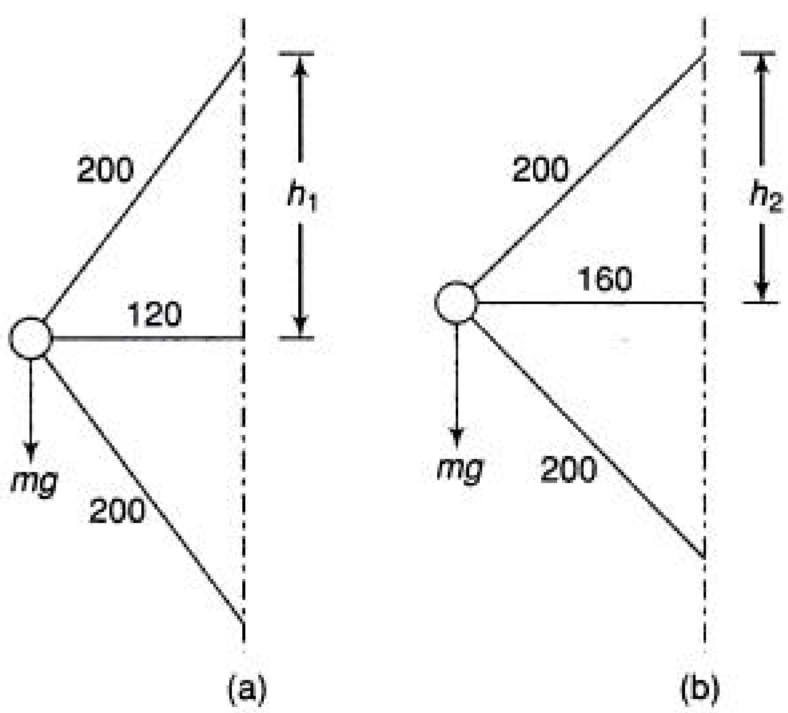


Fig. 16.22

$$N_1^2 = \frac{895}{h_1}\left(\frac{m+M}{m}\right) = \frac{895}{0.16}\left(\frac{3+15}{3}\right) = 33\,563$$

$N_1 = 183.2$ rpm

And

$$N_2^2 = \frac{895}{0.12}\left(\frac{3+15}{3}\right) = 44750 \text{ or } N_2 = 212.5 \text{ rpm}$$

or $N_2 = 212.5$ rpm

(i) Range of speed = 212.5 – 183.2 = 29.3 rpm

(ii) Lift of sleeve = 2 $(h_1 - h_2)$ = 2(0.16 – 0.12) = 0.08 m

(iii) Effort = $(m + M)cg$

where $c\,N$ = (212.5 – 183.2) = 29.2

or c = 29.2/183.2 =0.16

or Effort = (3 + 15) × 0.16 × 9.81 = 28.3 N

(iv) Power $= (m+M)gh\left(\dfrac{4c^2}{1+2c}\right)$

$$= (3+15)\times 9.81 \times 0.16\left(\frac{4\times 0.16^2}{1+2\times 0.16}\right)$$

= 2.26 N.m

or Power = Effort × Displacement

= 28.3 × 0.08 = 2.19 N.m

(The difference in the two values is due to the approximations taken in the derivation of relations.)

When friction is considered

$$N_1^2 = \frac{895}{h_1}\left(\frac{mg + (Mg - f)}{mg}\right)$$

$$= \frac{895}{0.16}\left(\frac{3\times 9.81 + (15\times 9.81 - 8)}{3\times 9.81}\right)$$

= 32 042

N_1 = 179 rpm

Summary

1. The function of a governor is to maintain the speed of an engine within specified limits whenever there is variation of load.
2. The variation in the output torque of the engine during a cycle can be regulated by mounting a suitable flywheel on the shaft whereas the speed variation over a number of cycles due to load variation is regulated by governors which regulate the fuel supply according to the load.
3. The action of *centrifugal governors* depends upon the change in the centrifugal force of balls due to change in speed, and that of *inertia governors* on the acceleration or deceleration of spindle apart from change of centrifugal force:
4. The height of a *Watt governor* is inversely proportional to the square of the speed. At high speeds, the movement of the sleeve becomes very small and thus this type of governor is unsuitable for high speeds.
5. In a *Porter governor*, the sleeve is loaded with a heavy mass which improves the action of the governor.
6. Friction makes the governor inactive for a small range of speed on changing the direction of the sleeve movement.
7. In a *Proell governor*, the two balls are fixed on the upward extensions of the lower links which are in the form of bent links.
8. Further improvement in the action of governors is brought about by using springs. Spring controlled governors are *Hartnell, Hartung, Wilson Hartnell, gravity* and *Pickering*.
9. A *Wilson–Hartnell governor* uses two parallel springs alongwith an auxiliary spring.
10. A *Pickering governor* consists of three leaf springs which are arranged at equal angular intervals around the governor spindle.
11. A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of *sensitivity.*
12. *Hunting* is the process of continuous fluctuating of sleeve for longer periods whenever there is change in speed. This happens if the governor is too sensitive.
13. A governor with a zero speed range is known as an *isochronous governor*. This means that for all positions of the sleeve or the balls, the governor has the same equilibrium speed.
14. A governor is said to be *stable* if it brings the speed of the engine to the required value without much hunting and for each speed there is only one radius of rotation of the balls.
15. The *effort* of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed.
16. The *power* of a governor is the work done at the sleeve for a given percentage change of speed, i.e., it is the product of the effort and the displacement of the sleeve.
17. The centrifugal force on each ball of a governor is balanced by an equal and opposite force acting radially inwards known as *controlling force*.

Exercises

1. What is the function of a governor? How does it differ from that of a flywheel?
2. What are centrifugal governors? How do they differ from inertia governors?
3. Describe the function of a simple Watt governor. What are its limitations?
4. How does a Porter governor differ from that of a Watt governor?
5. Discuss the effect of friction on the functioning of a Porter governor? Deduce its governing equation taking into account the friction at the sleeve.
6. Describe the function of a Proell governor with the help of a neat sketch. Establish a relation among various forces acting on the bent link.
7. What are spring-controlled governors? Describe the function of any one of them.
8. Sketch a Hartnell governor. Describe its function and deduce a relation to find the stiffness of the spring.
9. Explain the working of a Hartung governor with a neat sketch.
10. Why is an auxiliary spring used along with main

17.3 GYROSCOPIC TORQUE (COUPLE)

Let I be the moment of inertia of a rotor and ω its angular velocity about a horizontal axis of spin Ox in the direction as shown in Fig.17.3(a). Let this axis of spin turn through a small angle $\delta\theta$ in the horizontal plane (xy) to the position Ox' in time δt.

Figure 17.3(b) shows the vector diagram. **oa** represents the angular velocity vector when the axis is Ox and **ob** when the axis is changed to Ox'. Then **ab** represents the change in the angular velocity due to change in direction of the axis of spin of the rotor. This change in the angular velocity is clockwise when viewed from **a** towards **b** and is in the vertical plane xz. This change results in an angular acceleration, the sense and direction of which are the same as that of the change in the angular velocity.

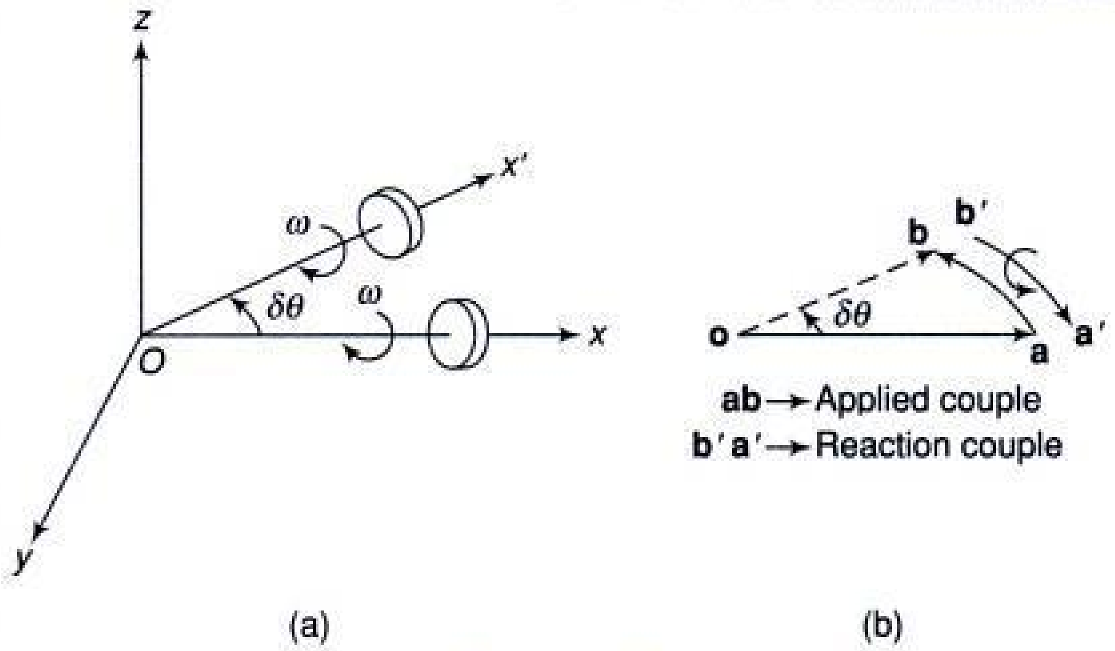


Fig. 17.3

Change in angular velocity, **ab** = $\omega \times \delta\theta$

Angular acceleration, $\alpha = \omega \dfrac{\delta\theta}{\delta t}$

In the limit, when $\delta t \to 0$, $\alpha = \omega \dfrac{\delta\theta}{\delta t}$

Usually, $d\theta/dt$, the angular velocity of the axis of spin is called the *angular velocity of precession* and is denoted by ω_p.

$\therefore$ Angular acceleration, $\alpha = \omega.\omega_p$.

The torque required to produce this acceleration is known as the gyroscopic torque and is a couple which must be applied to the axis of spin to cause it to rotate with angular velocity ω_p about the axis of precession Oz.

$$\text{Acceleration torque, } T = I$$
$$a = I\,\omega\,\omega_p \qquad (17.2)$$

For the configuration of Fig.17.3(a);

Ox is known as the axis of spin

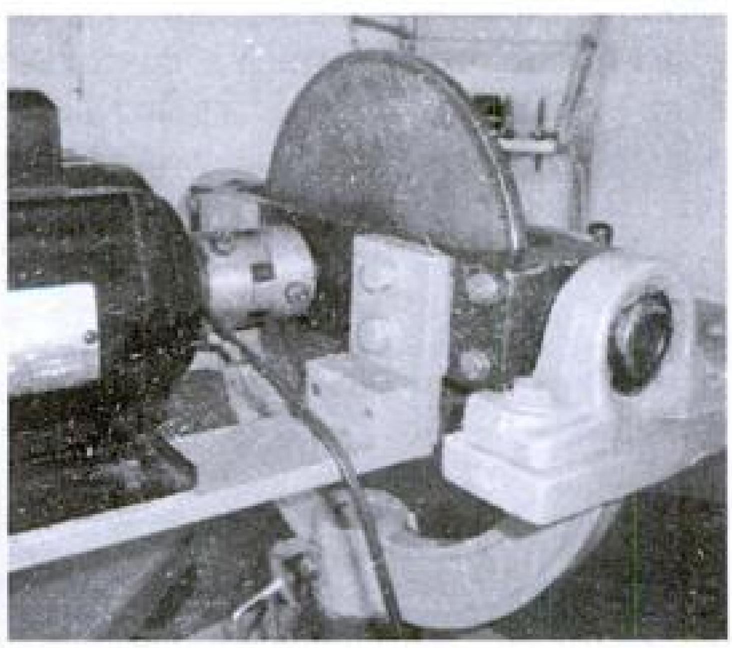
Oz is known as the axis of precession

Oy is known as the axis of gyroscopic couple

yz is the plane of spin (parallel to plane of rotor)

xy is the plane of precession

yz is the plane of gyroscopic couple00



A gyroscope

The torque obtained above is that which is required to cause the axis of spin to precess in the horizontal plane and is known as the *active gyroscopic torque* or the applied torque. A *reactive gyroscopic torque* or reaction torque is also applied to the axis which tends to rotate the axis of spin in the opposite direction, i.e., in the counter-clockwise direction in the above case. Just as the centrifugal force on a rotating body tends to move the body outwards, while a centripetal acceleration (and thus centripetal force) acts on it inwards, in the same way, the effects of active and reactive gyroscopic torques can be understood.

Reaction of ground on each inner wheel, $R_c = \dfrac{C_c}{2w}$ (downwards)

Vertical reaction on each outer wheel $= \dfrac{W}{4} + \dfrac{C_G}{2w} + \dfrac{C_c}{2w}$ (upwards)

Vertical reaction on each inner wheel $= \dfrac{W}{4} + \dfrac{C_G}{2w} + \dfrac{C_c}{2w}$ (upwards)

It can be observed that there are chances that the reaction of the ground on the inner wheels may not be upwards and thus the wheels are lifted from the ground. For positive reaction, the conditions will be

$$\frac{W}{4} - \frac{C_G}{2w} - \frac{C_c}{2w} \geq 0$$

or

$$\frac{W}{4} \geq \frac{C_G + C_c}{2w}$$

or

$$R_w \geq R_G + R_c \tag{17.7}$$

Example 17.6 *Each wheel of a four-wheeled rear engine automobile has a moment of inertia of 2.4 kg.m² and an effective diameter of 660 mm. The rotating parts of the engine have a moment of inertia of 1.2 kg.m². The gear ratio of engine to the back wheel is 3 to 1. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The mass of the vehicle is 2200 kg and the centre of the mass is 550 mm above the road level. The track width of the vehicle is 1.5 m. Determine the limiting speed of the vehicle around a curve with 80 m radius so that all the four wheels maintain contact with the road surface.*

Solution

$I_w = 2.4$ kg.m²	$m = 2200$ kg
$r = 0.33$ m	$h = 0.55$ m
$I_e = 1.2$ kg.m²	$w = 1.5$ m
$G = \dfrac{\omega_e}{\omega_w} = 3$	$R = 80$ m

(i) Reaction due to weight

$$R_w = \frac{mg}{4} = \frac{2200 \times 9.81}{4} = 5395.5 \text{ N (upwards)}$$

(ii) Reaction due to gyroscopic couple

$$C_w = 4I_w \frac{v^2}{rR} = 4 \times 2.4 \times \frac{v^2}{0.33 \times 80} = 0.364v^2$$

$$C_e = I_e G\omega_w \omega_p = 1.2 \times 3 \times \frac{v^2}{0.33 \times 80} = 0.136v^2$$

$\therefore$ $C_G = C_w + C_e = 0.364\,v^2 + 0.136\,v^2 = 0.5\,v^2$

Reaction on each outer wheel,

$$R_{Go} = \frac{C_G}{2w} = \frac{0.5v^2}{2 \times 1.5} = 0.167v^2 \text{ (upwards)}$$

Reaction on each inner wheel,

$R_{Gi} = 0.167\,v^2$ (downwards)

(iii) Reaction due to centrifugal couple

$$C_c = \frac{mv^2}{R} h = \frac{2200 \times v^2}{80} \times 0.55 = 15.125v^2$$

Reaction on each outer wheel,

$$R_{co} = \frac{C_e}{2w} = \frac{15.125v^2}{2 \times 1.5} = 5.042v^2 \text{ (upwards)}$$

Reaction on each inner wheel,

$R_{ci} = 5.042\,v^2$ (downwards)

For maximum safe speed, the condition is

$R_w = R_{Gi} + R_{ci}$

$5395.5 = (0.167 + 5.042)\,v^2$

$v^2 = 1035.8$

$v = 32.18$ m/s

or $v = \dfrac{32.18 \times 3600}{1000} = 115.9$ km/h

18.2 TYPES OF VIBRATIONS

Consider a vibrating body, e.g., a rod, shaft or spring. Figure 18.1 shows a massless shaft, one end of which is fixed and the other end carrying a heavy disc. The system can execute the following types of vibrations.

(i) ***Longitudinal Vibrations*** If the shaft is elongated and shortened so that the same moves up and down resulting in tensile and compressive stresses in the shaft, the vibrations are said to be longitudinal. The different particles of the body move parallel to the axis of the body [Fig.18.1(a)].

(ii) ***Transverse Vibrations*** When the shaft is bent alternately [Fig.18.1(b)] and tensile and compressive stresses due to bending result, the vibrations are said to be transverse. The particles of the body move approximately perpendicular to its axis.

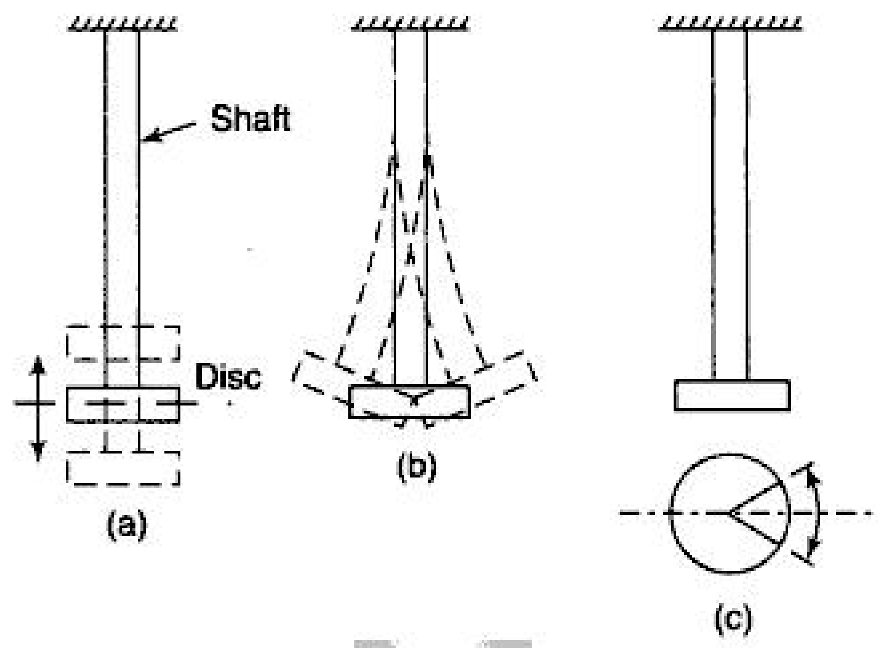


Fig. 18.1

(iii) ***Torsional Vibrations*** When the shaft is twisted and untwisted alternately and torsional shear stresses are induced, the vibrations are known as torsional vibrations. The particles of the body move in a circle about the axis of the shaft [Fig.18.1(c)].

18.3 BASIC FEATURES OF VIBRATING SYSTEMS

For mathematical analysis of a vibratory system, it is necessary to have an *idealized model* of the same which appropriately represents the system.

Basic Elements

For a system to vibrate, it must possess inertial and restoring elements whereas it may possess some damping element responsible for dissipating the energy.

Inertial elements These are represented by lumped masses for rectilinear motion and by lumped moment of inertia for angular motion.

Restoring Elements Massless linear or torsional springs represent the restoring elements for rectilinear and torsional motions respectively.

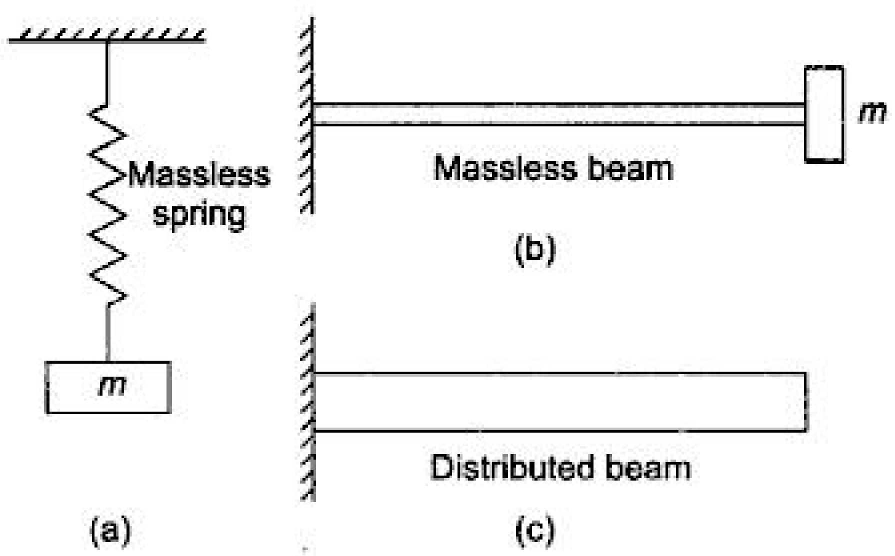


Fig. 18.2

Damping Elements Massless dampers of rigid elements may be considered for energy dissipation in a system.

It is to be noted that lumping of quantities depends upon the distribution of these quantities in the systems. In a spring-mass vibrating system, the spring can be considered massless only if its mass is very less as compared to the suspended mass [Fig.18.2 (a)]. Similarly, if the mass of the beam is negligible as compared to the end mass, lumping is possible [Fig.18.2 (b)], otherwise not [Fig.18.2 (c)].

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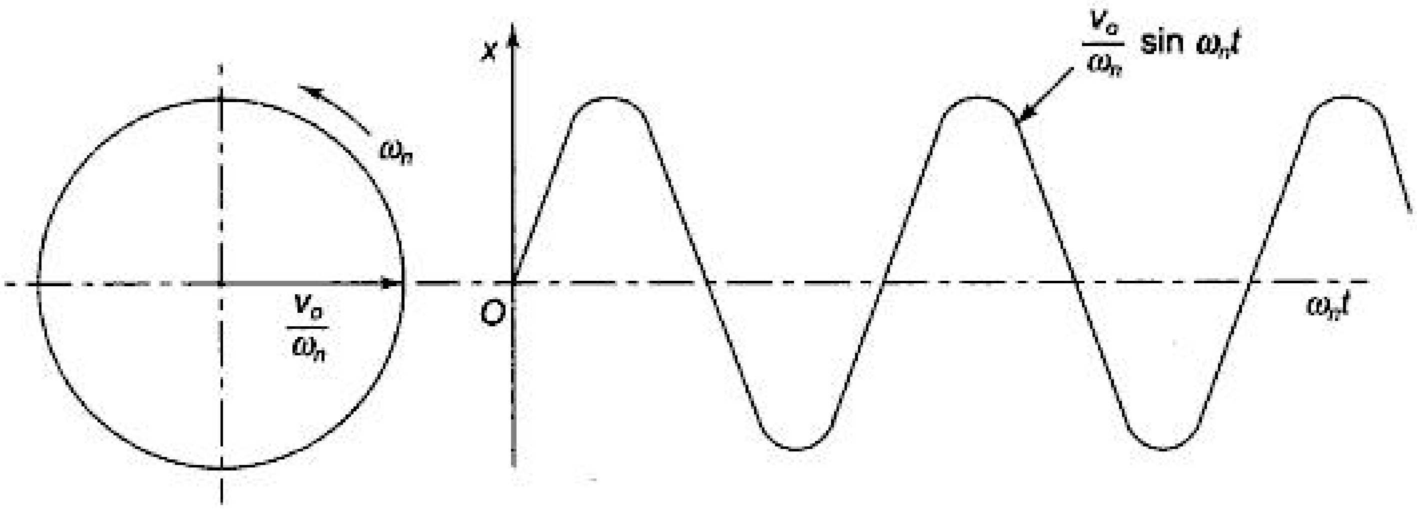


Fig. 18.8

The solution is represented graphically in Fig. 18.8.

- For the sol of Eq. 18.6 and for the same initial conditions

$$0 = X \sin \varphi \quad \text{or} \quad \phi = 0^o$$

and $\qquad \dot{x} = X\,\omega_n \cos(\omega_n t + \varphi)$

or $\qquad v_o = X\,\omega_n \qquad (\phi = 0)$

or $\qquad X = \dfrac{v_o}{\omega_n}$

Therefore, equation of motion, $x = \dfrac{v_o}{\omega_n} \sin \omega_n t$

which is the same equation as Eq. 18.17.

- For the solution of Eq. 18.7 and for the same initial conditions, the equation of motion can be obtained which will be same as Eq. 18.17.

Equation 18.6 is considered a more convenient form of the equation. In this equation, the coefficient is the *amplitude* (maximum displacement) of the vibration. φ is called the *phase angle* and is the angular advance of the vector with respect to the sine function.

Equation 18.7 is also a convenient form of the equation.

2. Energy Method

In a conservative system (a system with no damping), the total mechanical energy, i.e., the sum of the kinetic and the potential energies, remains constant and therefore,

$$\frac{d}{dt}(KE + PE) = 0$$

We have

$$KE = \frac{1}{2} m \dot{x}^2$$

and

$$PE = \text{mean force} \times \text{displacement}$$
$$= \frac{0 + sx}{2} \times x$$
$$= \frac{sx^2}{2}$$

$$= mg\left[\frac{a}{a+b}\frac{1}{s_2} - \left(\begin{matrix}\frac{a}{a+b}\frac{1}{s_2} \\ -\frac{a}{a+b}\frac{1}{s_1}\end{matrix}\right)\frac{b}{a+b}\right]$$

$$= \frac{mg}{a+b}\left[\frac{a}{s_2} - \frac{ab}{(a+b)s_2} + \frac{b^2}{(a+b)s_1}\right]$$

$$= \frac{mg}{a+b}\left[\frac{a^2 s_1 + abs_1 - abs_1 + b^2 s_2}{(a+b)s_1 s_2}\right]$$

$$= \frac{mg}{(a+b)^2}\left[\frac{a^2}{s_2} + \frac{b^2}{s_1}\right]$$

Total spring force = force in Spring 1 + force in Spring 2

$$s\,\Delta = s_1\,\Delta_1 + s_2\,\Delta_2$$

$$s\frac{mg}{(a+b)^2}\left[\frac{a^2}{s_2} + \frac{b^2}{s_2}\right] = s_1 mg\frac{b}{a+b} \times \frac{1}{s_1} + s_2 mg\frac{a}{a+b} \times \frac{1}{s_2}$$

$$s\frac{1}{a+b}\left(\frac{a^2}{s_2} + \frac{b^2}{s_1}\right) = b + a$$

or

$$s = \frac{(a+b)^2}{\left(\frac{a^2}{s_2} + \frac{b^2}{s_1}\right)}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(a+b)^2}{\left(\frac{a^2}{s_2} + \frac{b^2}{s_1}\right)m}}$$

$$= \sqrt{\frac{(0.02+0.012)^2}{\frac{(0.02)^2}{8\times 10^3} + \frac{(0.012)^2}{5\times 10^3}} \times \frac{1}{10}} = \underline{36\text{ Hz}}$$

Alternatively,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{g}{\frac{mg}{(a+b)^2}\left(\frac{a^2}{s_2} + \frac{b^2}{s_1}\right)}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(a+b)^2}{\left(\frac{a^2}{s_2} + \frac{b^2}{s_1}\right)m}}$$

i.e., the same expression.

Example 18.2 *Determine the frequency (circular) of vibration of the systems shown in Figs 18.12(a) and (b). Neglect the mass of the pulleys.*

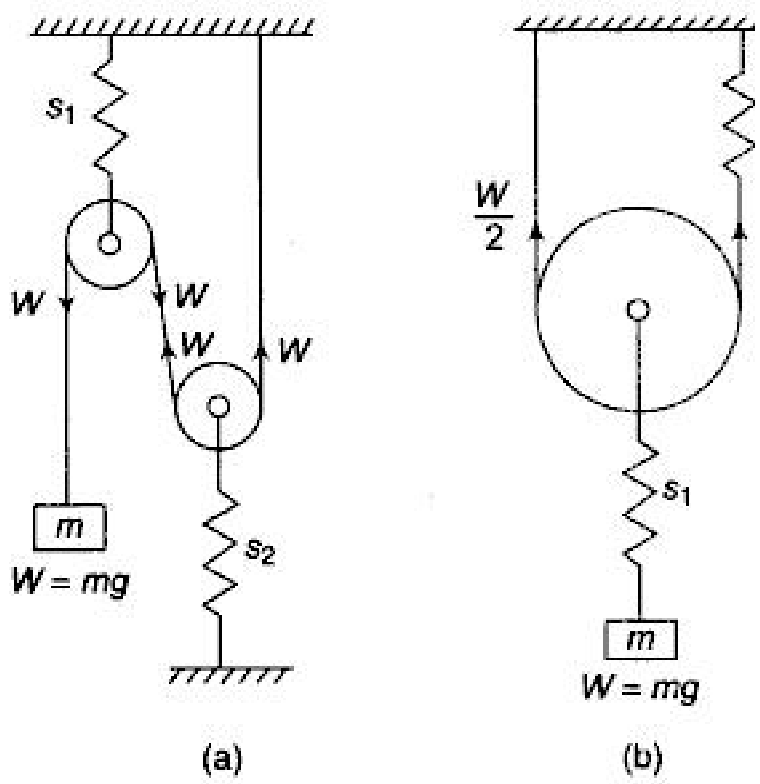


Fig. 18.12

Solution

(a) Force in each spring = $2W$

Deflection on mass m, Δ = 2 (deflection of Spring 1 + deflection of Spring 2)

$$= 2\left(\frac{2W}{s_1} + \frac{2W}{s_2}\right)$$

$$= 4mg\left(\frac{s_1 + s_2}{s_1 s_2}\right)$$

$$\omega_n = \frac{g}{\Delta} = \sqrt{\frac{g(s_1 s_2)}{4mg(s_1 + s_2)}} = \sqrt{\frac{s_1 s_2}{4(s_1 + s_2)m}}$$

(b) Force in Spring 1 = W

Force in Spring 2 = $W/2$

Deflection of mass = deflection of Spring 1 + deflection of Spring 2

The series given by this equation is known as *Fourier series*. The various amplitudes $a_1, a_2 \ldots b_1, b_2 \ldots$, etc., of sine and cos waves can be found analytically when $f(t)$ is known. The harmonic of frequency ω is known as the fundamental or the first harmonic of $f(t)$ and the harmonic of frequency $n\omega$, the nth harmonic.

Thus, a periodic force is represented by

$$F(t) = F_o + F_1 \sin \omega t + F_2 \sin 2\omega t + F_3 \sin 3\omega t + \ldots. F_n \sin n\omega t + \ldots.$$
$$+ F_1 \cos \omega t + F_2 \cos 2\omega t + F_3 \cos 3\omega t + \ldots.F_n \cos n\omega t + \ldots.$$

and the differential equation of the system becomes

$$m\ddot{x} + sx = F_o + F_1 \sin \omega t + F_2 \sin 2\omega t + F_3 \sin 3\omega t + \ldots. F_n \sin n\omega t + \ldots.$$
$$+ F_1 \cos \omega t + F_2 \cos 2\omega t + F_3 \cos 3\omega t + \ldots.F_n \cos n\omega t + \ldots.$$

The response of the complete periodic forcing is the vector sum of the responses to the complimentary functions and particular solutions of the individual forcing functions as on the right-hand side of the equation.

18.11 FORCED-DAMPED VIBRATIONS

A mass m is attached to a helical spring and is suspended from a fixed support as before. Damping is also provided in the system with a dashpot (Fig. 18.20).

Before the mass is set in motion, let B-B be the static equilibrium position under the weight of the mass. Now, if the mass is subjected to an oscillating force $F = F_0 \sin \omega t$, the forces acting on the mass at any instant will be

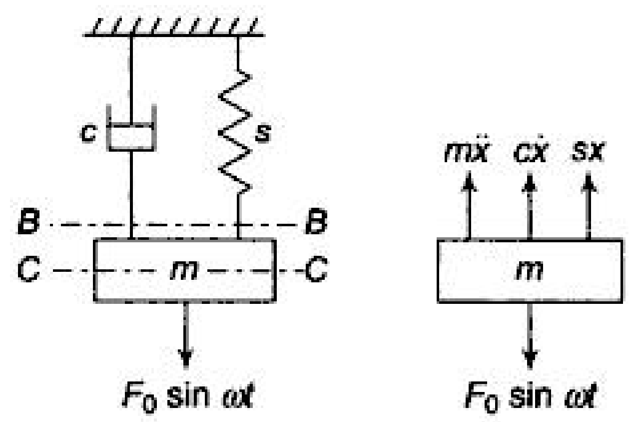


Fig. 18.20

- Impressed oscillating force (downwards) $F = F_0 \sin \omega t$
- Inertia force (upwards) $= m\ddot{x}$
- Damping force (upwards) $= c\dot{x}$
- Spring force (restoring force) $= sx$ (upwards)

Thus the equation of motion will be

$$m\ddot{x} + c\dot{x} + sx - F_0 \sin\omega t = 0$$

or

$$m\ddot{x} + c\dot{x} + sx = F_0 \sin\omega t \quad (18.36)$$

Complete solution of this equation consists of two parts, the complementary function (CF) and the particular integral (PI).

$$CF = Xe^{-\zeta\omega_n t} \sin(\omega_d t + \varphi_1) \quad \text{[refer to Eq. (18.30)]}$$

To obtain the PI, let

$$\frac{c}{m} = a, \frac{s}{m} = b, \text{ and } \frac{F_0}{m} = d$$

Then, using the operator D, the equation becomes

$$(D^2 + aD + b)\,x = d \sin \omega t$$
$$PI = \frac{d \sin \omega t}{D^2 + aD + b}$$

(i) when $\omega/\omega_n < \sqrt{2}$, ε is more than 1, i.e., the transmitted force is always more than the exciting force

(ii) when $\omega/\omega_n > \sqrt{2}$, ε is less than 1, i.e., the transmitted force is always less than the exciting force

(iii) when $\omega/\omega_n = \sqrt{2}$, ε is 1, i.e., the transmitted force is equal to the exciting force

(iv) when $\omega/\omega_n > 1$, the transmitted force is infinite; if damping is used, the magnitude of the transmitted force can be reduced

(v) when $\omega/\omega_n = \sqrt{2}$, ε increases as the damping is increased

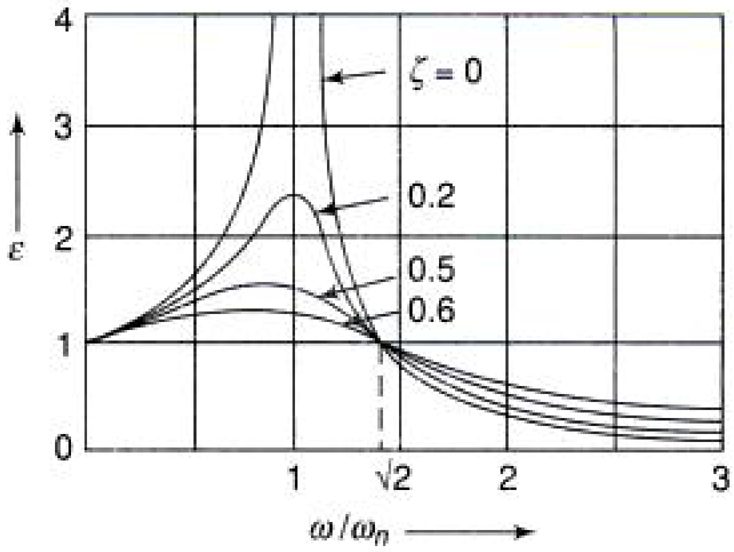


Fig. 18.23

Thus in a system where ω/ω_n can vary from zero to higher values, dampers should not be used. Instead, stops may be provided to limit the resonance amplitude (at resonance, the amplification factor is infinitely).

Example 18.14 *A refrigerator unit having a mass of 35 kg is to be supported on three springs, each having a spring stiffness s. The unit operates at 480 rpm. Find the value of stiffness s if only 10% of the shaking force is allowed to be transmitted to the supporting structure.*

Solution As no damper is used,

$$\varepsilon = \frac{1}{\pm\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$\omega = \frac{2\pi \times 480}{60} = 16\pi \quad \text{and} \quad \varepsilon = 0.1$$

$$\therefore \quad 0.1 = \frac{1}{\pm\left[1-\left(\frac{16\pi}{\omega_n}\right)^2\right]}$$

$$\pm\left[0.1-0.1\left(\frac{16\pi}{\omega_n}\right)^2\right] = 1$$

If the positive sign is taken, $\frac{16\pi}{\omega_n} = \sqrt{-9}$ which is not possible.

Therefore taking the negative sign, $\frac{16\pi}{\omega_n} = \sqrt{11}$

or $\quad \omega_n = 15.15$ rad/s

or $\quad \sqrt{\frac{s}{m}} = \sqrt{\frac{s}{35}} = 15.15$

Equivalent stiffness,

$$s = 8037 \text{ N/m} = 8.037 \text{ N/mm}$$

Stiffness of each spring $= \frac{8.037}{3} = \underline{2.679 \text{ N/mm}}$

Example 18.15 *A machine supported symmetrically on four springs has a mass of 80 kg. The mass of the reciprocating parts is 2.2 kg which move through a vertical stroke of 100 mm with simple harmonic motion. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is 1/20th of the impressed force. The machine crankshaft rotates at 800 rpm.*

If, under actual working conditions, the damping reduces the amplitudes of successive vibrations by 30%, find the

(i) force transmitted to the foundation at 800 rpm

(ii) force transmitted to the foundation at resonance

(iii) amplitude of the vibrations at resonance

Solution

$M = 80$ kg $\qquad \varepsilon = \frac{1}{20} = 0.05$

$m = 2.2$ kg $\qquad N = 800$ rpm

$r = \frac{100}{2} = 50$ mm

This is the general expression for the deflection in case of uniformly loaded shafts. Constants A, B, C and D have to be found from the end conditions.

Simply Supported Shaft

The boundary conditions are

(a) $y = 0$ at $x = 0$ and l

(b) $\dfrac{d^2y}{dx^2} = 0$ at $x = 0$ and l (bending moment is zero at ends)

When $x = 0$, $y = 0$; $B + D = 0$ (ii)

When $x = l$, $y = 0$

$$A \sin \lambda l + B \cos \lambda l + C \sinh \lambda l + D \cosh \lambda l = 0 \quad \text{(iii)}$$

Differentiating (i) with respect to x twice,

$$\frac{dy}{dx} = \lambda (A \cos \lambda x - B \sin \lambda x + C \cosh \lambda x + D \sinh \lambda x)$$

$$\frac{d^2y}{dx^2} = \lambda^2(-A \sin \lambda x - B \cos \lambda x + C \sinh \lambda x + D \cosh \lambda x)$$

When $x = 0$,

$$\frac{d^2y}{dx^2} = 0$$

$\therefore$
$$\lambda^2 (-B + D) = 0 \quad \text{(iv)}$$

When $x = l$,

$$\frac{d^2y}{dx^2} = 0$$

$$\lambda^2(-A \sin \lambda l - B \cos \lambda l + C \sinh \lambda l + D \cosh \lambda l) = 0 \quad \text{(v)}$$

From (ii) and (iv)

$B = 0$ and $D = 0$

Thus (iii) and (v) can be written as

$$A \sin \lambda l + C \sinh \lambda l = 0$$

and
$$-A \sin \lambda l + C \sinh \lambda l = 0$$

Adding these, we get

$$C \sinh \lambda l = 0$$

Subtracting,

$$A \sin \lambda l = 0$$

$\sinh \lambda l$ cannot be zero, because if $\lambda = 0$, $\lambda^4 = 0$

or
$$\frac{m\omega^2}{EI} = 0$$

$$\ddot{\theta} + \frac{gab}{lk^2}\theta = 0$$

$\therefore$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{gab}{lk^2}}$$

or

$$k = \frac{1}{2\pi f_n}\sqrt{\frac{gab}{l}} \quad (18.61)$$

Thus radius of gyration can be found out by finding the natural frequency of vibration of the body.

(ii) Trifilar Suspension

Consider a disc of mass m (weight W), suspended by three vertical cords, each of length l, from a fixed support as shown in Fig. 18.35. Each cord is symmetrically attached to the disc at the same distance from the centre of mass of the disc.

If the disc is now turned through a small angle about its vertical axis, the cords become inclined. On being released, the disc will perform oscillations about the vertical axis. At any instant

let θ = angular displacement of the disc

φ = inclination of the cords to the vertical

F = tension in each cord = $W/3$

Inertia torque = $-I\ddot{\theta} = -mk^2\ddot{\theta}$

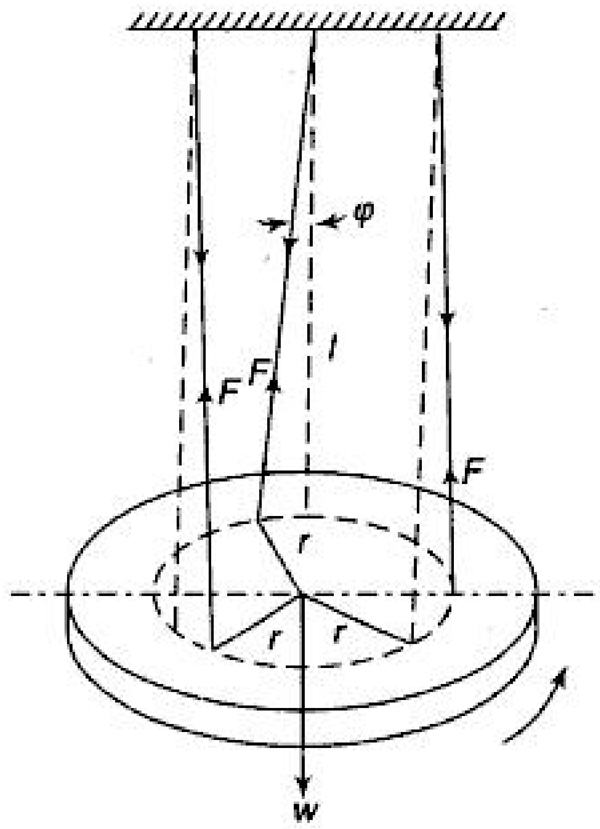


Fig. 18.35

Restoring torque = $-3 \times$ (Horizontal component of force in each string $\times r$)

$$= -3 \times Fr \sin\varphi$$

$$= -3\, Fr\,\varphi \quad \text{(as } \varphi \text{ is small)}$$

$$= -3Fr\frac{\theta r}{l} \quad (\because \varphi l = \theta r)$$

$$= -\frac{3W}{3} \times \frac{r^2}{l}\theta$$

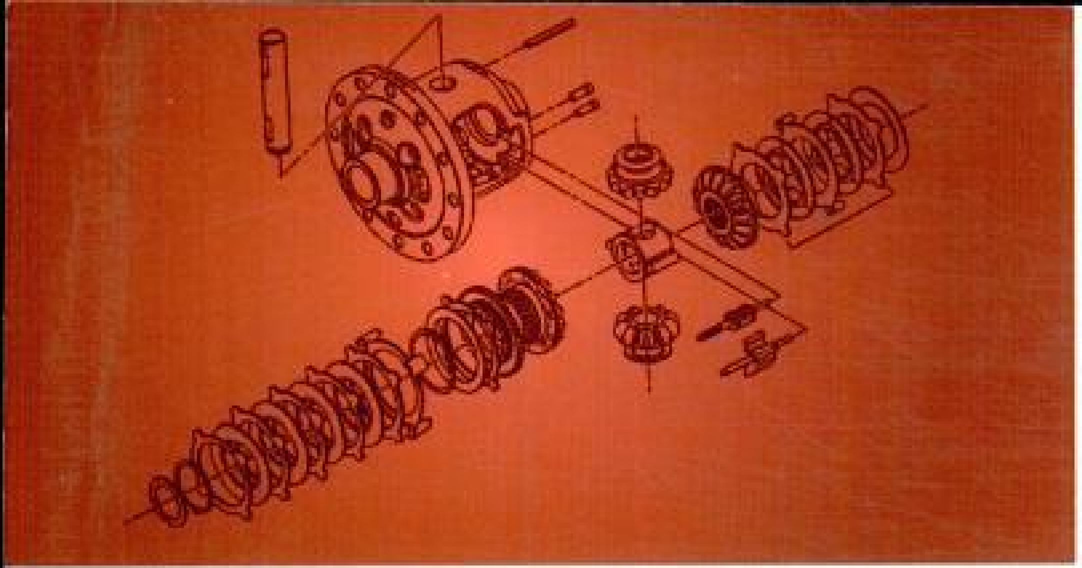
$$= -\frac{mgr^2}{l}\theta$$

$$mk^2\ddot{\theta} + \frac{mgr^2}{l}\theta = 0$$

$$\ddot{\theta} + \frac{gr^2}{lk^2}\theta = 0$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{gr^2}{lk^2}}$$

$$k = \frac{r}{2\pi f_n}\sqrt{\frac{gr^2}{l}} \quad (18.62)$$



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ISBN-13: 978-0-07-014477-4
ISBN-10: 0-07-014477-X