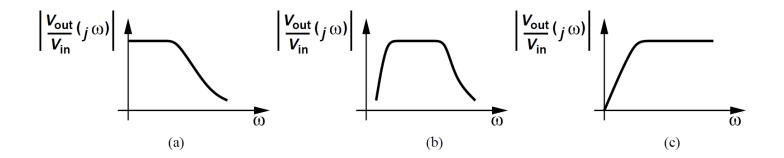
### Chapter 6: Frequency Response of Amplifiers

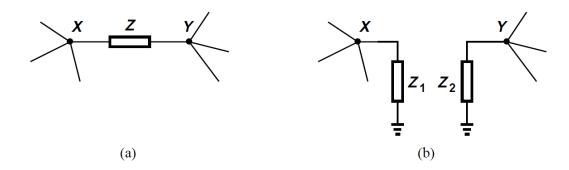
- **6.1 Basic Current Mirrors**
- **6.2 Common-Source Stage**
- **6.3 Source Followers**
- 6.4 Common-Gate Stage
- **6.5 Cascode Stage**
- **6.6 Differential Pair**
- 6.7 Gain-Bandwidth Trade-Offs

#### **General Considerations**



- In this chapter, we are primarily interested in the magnitude of the transfer function.
- The magnitude of a complex number a + jb is given by  $\sqrt{a^2 + b^2}$
- Zeros and poles are respectively defined as the roots of the numerator and denominator of the transfer function.

#### Miller effect

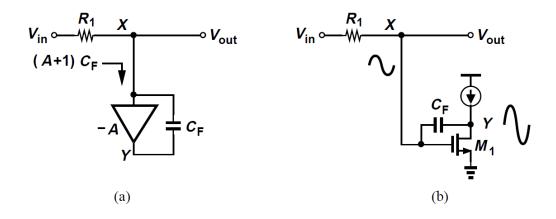


• If the circuit of Fig (a) can be converted to that of Fig (b), then

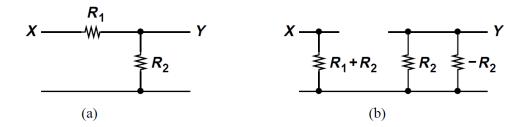
$$Z_1 = Z/(1 - A_v)$$

$$Z_2 = Z/(1 - A_v^{-1})$$

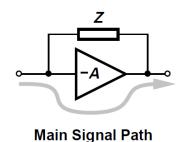
$$A_v = V_Y / V_X$$



- A student needs a large Capacitor and decides to utilize the Miller multiplication
- What is the issues in this approach?

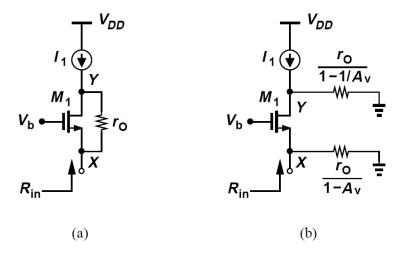


Improper application of Miller's theorem



Typical case for valid application of Miller's theorem.

- Miller's theorem does not stipulate the conditions under which this conversion is valid.
- If the impedance Z forms the only signal path between X and Y, then the conversion is often invalid.



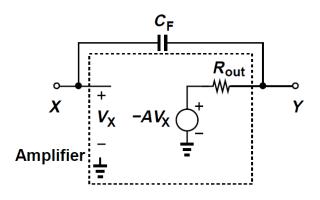
- Calculate the input resistance of the circuit shown.
- Since Av is usually greater than unity,  $r_O/(1-A_v)$  is a negative resistance.

• Av = 
$$1 + (g_m + g_{mb})r_O$$

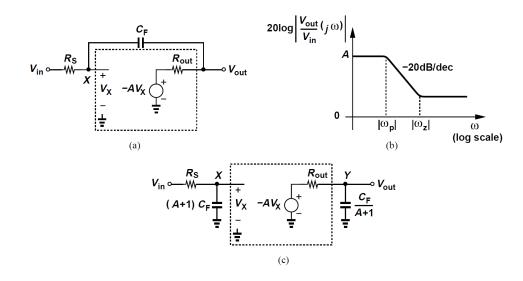
$$R_{in} = \frac{r_O}{1 - [1 + (g_m + g_{mb})r_O]} \left\| \frac{1}{g_m + g_{mb}} \right\|$$

$$= \frac{-1}{g_m + g_{mb}} \left\| \frac{1}{g_m + g_{mb}} \right\|$$

$$= \infty.$$



- The value of Av = VY / VX must be calculated at the frequency of interest.
- In the figure, the equivalent circui  $V_Y \neq -AV_X$  that at high frequencies.
- In many cases we use the low-frequency value of VY / VX to gain insight.
- We call this approach "Miller's approximation."



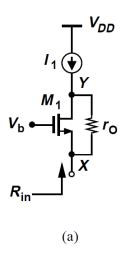
Direct Calculation:

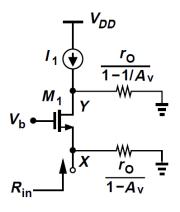
$$\frac{V_{out}}{V_{in}}(s) = \frac{R_{out}C_F s - A}{[(A+1)R_S + R_{out}]C_F s + 1}$$

Miller Aproximation:

$$\frac{V_{out}}{V_{in}}(s) = \frac{-A}{[(1+A)R_SC_F s + 1](\frac{1}{1+A^{-1}}C_F R_{out} s + 1)}$$

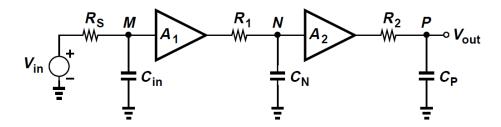
 Miller's approximation has eliminated the zero and predicted two poles for the circuit!





- Actual Rout =r0
- Miller's approximation:
- (1) it may eliminate zeros
- (2) it may predict additional poles
- (3) it does not correctly compute the "output" impedance.

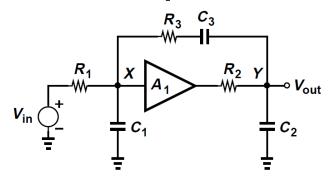
#### **Association of Poles with Nodes**

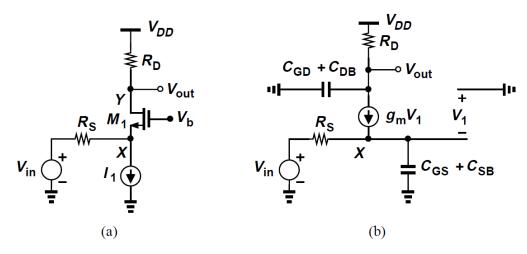


The overall transfer function can be written as

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$

- Each node in the circuit contributes one pole to the transfer function.
- Not valid in general. Example:





At node X:

$$\omega_{in} = \left[ \left( C_{GS} + C_{SB} \right) \left( R_S \left\| \frac{1}{g_m + g_{mb}} \right) \right]^{-1}$$

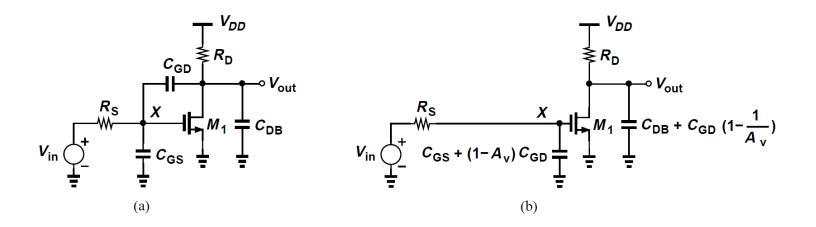
At node Y:

$$\omega_{out} = [(C_{DG} + C_{DB})R_D]^{-1}$$

• The overall transfer funtion:

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

## **Common-Source Stage**



• The magnitude of the "input" pole

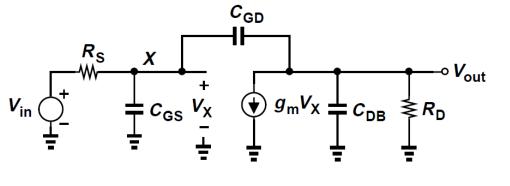
$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

At the output node

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right) \left(1 + \frac{s}{\omega_{out}}\right)}$$

## **Direct Analysis**

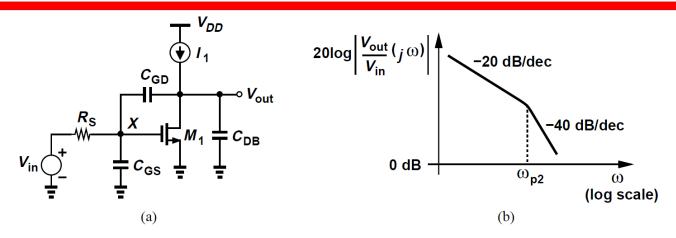


$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

- While the denominator appears rather complicated, it can yield intuitive expressions for the two poles.  $|\omega_{p1}| \ll |\omega_{p2}|$
- "Dominant pole" approximation.

$$\omega_{p1} = \frac{1}{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}$$

 The intuitive approach provides a rough estimate with much less effort.



$$\frac{V_{out}}{V_{in}}(s) = \frac{C_{GD}s - g_m}{R_S\xi s^2 + [g_m R_S C_{GD} + (C_{GD} + C_{DB})]s}$$

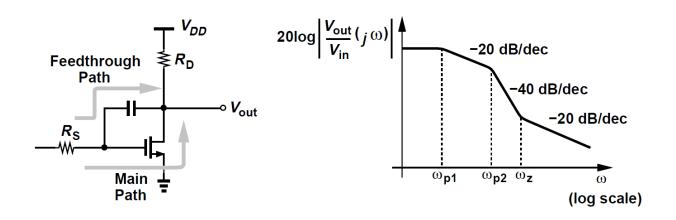
$$= \frac{C_{GD}s - g_m}{s[R_S(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})s + (g_m R_S + 1)C_{GD} + C_{DB}]}$$

- One pole is at the origin because the dc gain is infinity.
- For a large CDB or load capacitance

$$\omega_{p2} \approx \frac{1}{R_S(C_{GS} + C_{GD})}$$

No miller multiplication. Why?

#### **Zero in Transfer Function**



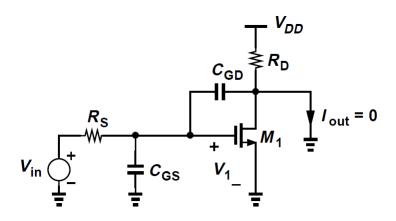
$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

The transfer function of exhibits a zero given by

$$\omega_z = +g_m/C_{GD}$$

 CGD provides a feedthrough path that conducts the input signal to the output at very high frequencies.

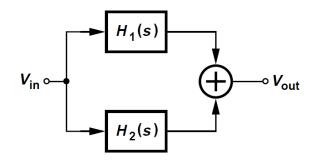
# Calculating zero in a CS stage



- The transfer function Vout(s)=Vin(s) must drop to zero for  $s = s_z$ .
- Therefore, the currents through C<sub>GD</sub> and M1 are equal and opposite:

$$V_1 C_{GD} s_z = g_m V_1$$

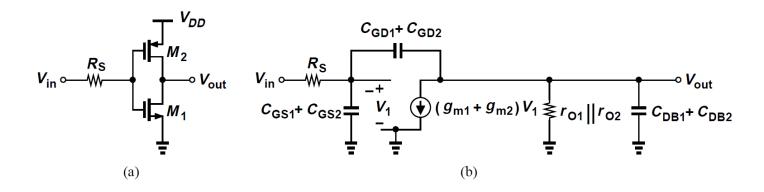
• That is  $s_z = +g_m/C_{GD}$ 



- Can this (the zero) occur if H1(s) and H2(s) are first-order low-pass circuits?
- H1=  $A_1/(1+s/\omega_{p1})$  and H2=  $A_2/(1+s/\omega_{p2})$

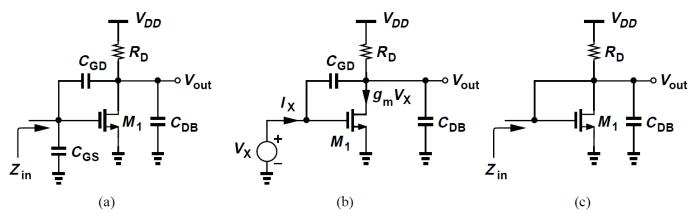
$$\frac{V_{out}}{V_{in}}(s) = \frac{(\frac{A_1}{\omega_{p2}} + \frac{A_2}{\omega_{p1}})s + A_1 + A_2}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}$$

The overall transfer function contains a zero.



- Since the corresponding terminals ofM1 and M2 are shorted to one another in the small-signal model, we merge the two transistors.
- The circuit thus has the same transfer function as the simple CS stage.

## Miller's Approximation



With the aid of Miller's approximation,

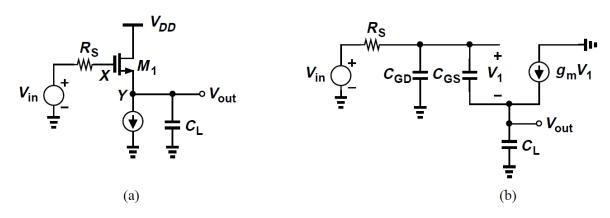
$$Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

- But at high frequencies, the effect of the output node capacitance must be taken into account.
- Ignore Cgs

$$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)}$$

• if Cgd is large, it provides a low impedance path between the gate and drain of M1.

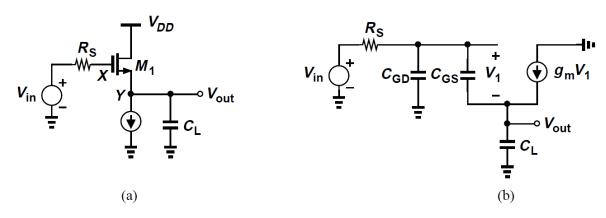
#### **Source Followers**



 The strong interaction between nodesX and Y through CGs in makes it difficult to associate a pole with each node.

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Contains a zero in the left half plane. Why?



Transfer function if CL = 0?

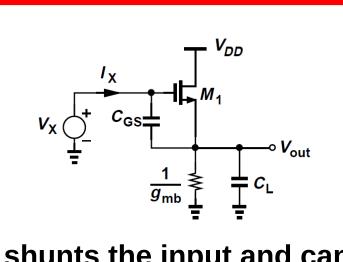
$$\frac{V_{out}}{V_{in}} = \frac{g_m + C_{GS}s}{R_S C_{GS} C_{GD} s^2 + (g_m R_S C_{GD} + C_{GS})s + g_m}$$

$$= \frac{g_m + C_{GS}s}{(1 + R_S C_{GD}s)(g_m + C_{GS}s)}$$

$$= \frac{1}{1 + R_S C_{GD}s}.$$

- C<sub>GS</sub> disappear
- In the absence of channel-length modulation and body effect, the voltage gain from the gate to the source is equal to unity.

## Input impedance



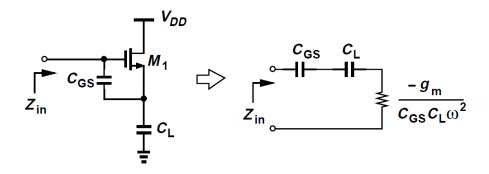
• C<sub>GD</sub> simply shunts the input and can be ignored initially.  $\frac{1}{1-(1-q_m)}$ 

 $Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right) \frac{1}{g_{mb} + C_L s}$ 

- If gmb = 0 and CL = 0, then  $Zin = \infty$
- CGS is entirely bootstrapped by the source follower and draws no current from the input.
- At Low frequency the overall input capacitance is equal to CgD plus a fraction of Cgs.

$$Z_{in} \approx \frac{1}{C_{GS}s} \left( 1 + \frac{g_m}{g_{mb}} \right) + \frac{1}{g_{mb}}$$

## **Input Impedance**

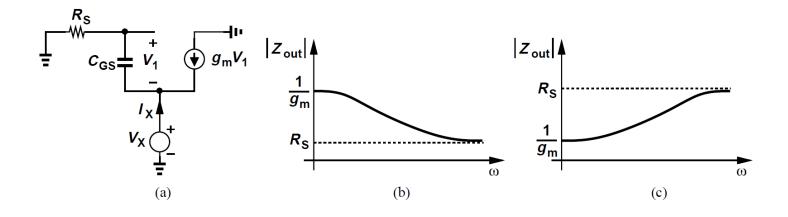


• At high frequencies,  $g_{mb} \ll |C_L s|$ 

$$Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_{L}s} + \frac{g_m}{C_{GS}C_{L}s^2}$$

 A source follower driving a load capacitance exhibits a negative input resistance, possibly causing instability.

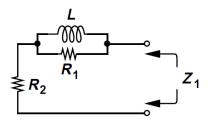
# **Output Impedance**



$$Z_{out} = \frac{V_X}{I_X}$$
$$= \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

- At low frequency:  $Z_{out} \approx 1/g_m$
- At very high frequencies,  $Z_{out} \approx R_S$
- Because Cgs shorts the gate and the source.
- Which one of these variations is more realistic?

## **Output Impedance**

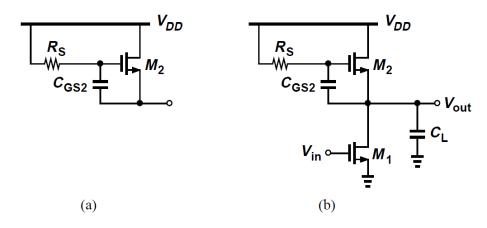


- Equivalent output impedance of a source follower
- Since the output impedance increases with frequency, we postulate that it contains an inductive component.

$$Z_{out} - \frac{1}{g_m} = \frac{C_{GS}s\left(R_S - \frac{1}{g_m}\right)}{g_m + C_{GS}s}$$

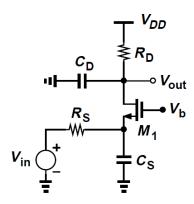
$$\frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{1}{R_S - \frac{1}{g_m}} + \frac{1}{\frac{C_{GS}s}{g_m}\left(R_S - \frac{1}{g_m}\right)}$$

$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$



- Can we construct a (two-terminal) inductor from a source follower?
- Yes, but non-ideal.
- It also incurs a parallel resistance and a series resistance.
- The inductance can partially cancel the load capacitance, CL, at high frequencies, thus extending the bandwidth.

## **Common-Gate Stage**

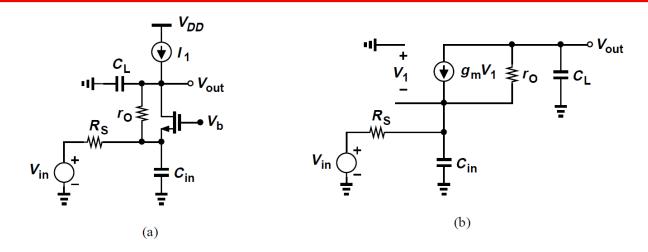


Common-gate stage at high frequencies

A transfer function

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}}s\right)(1 + R_D C_D s)}$$

- No Miller multiplication of capacitances.
- RD is typically maximized, so the dc level of the input signal must be quite low.
- As an amplifier in cases where a low input impedance is required
- In cascode stages.

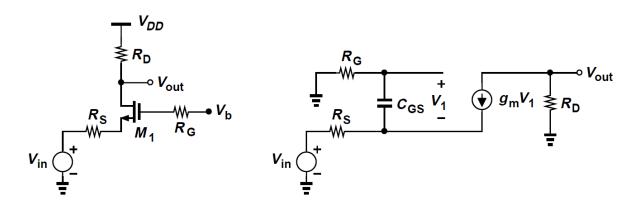


• Why Zin becomes independent of C<sub>L</sub> as this capacitance increases?

$$Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{C_L s} \cdot \frac{1}{(g_m + g_{mb})r_O}$$

- As C<sub>L</sub> or s increases, Zin approaches 1/(gm + gmb)
- C<sub>L</sub> lowers the voltage gain of the circuit, thereby suppressing the effect of the negative resistance introduced by Miller effect through rO.

## **CG Stage**

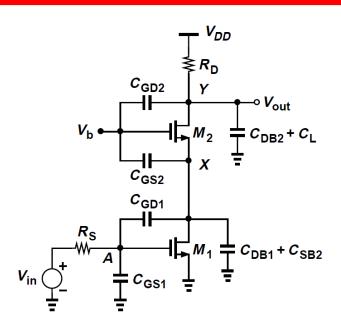


- The bias network providing the gate voltage exhibits a finite impedance.
- Consider only Cgs here.

$$rac{V_{out}}{V_{in}} = rac{g_m R_D}{(R_G + R_S)C_{GS}s + 1 + g_m R_S},$$
  $\omega_p = rac{1 + g_m R_S}{(R_G + R_S)C_{GS}}.$ 

- Lowering the pole magnitude.
- Output impedance of the circuit drops at high frequencies.

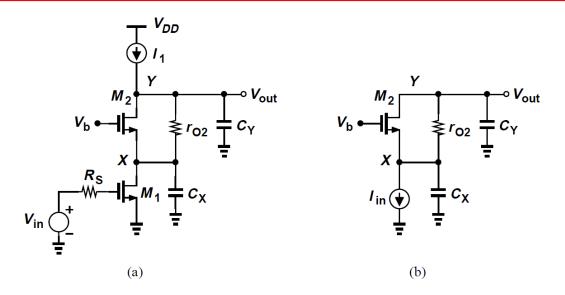
# **Cascode Stage**



$$\omega_{p,A} = rac{1}{R_S \left[ C_{GS1} + \left( 1 + rac{g_{m1}}{g_{m2} + g_{mb2}} \right) C_{GD1} \right]}$$
 $\omega_{p,X} = rac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$ 
 $\omega_{p,Y} = rac{1}{R_D (C_{DB2} + C_L + C_{GD2})}$ 

High-frequency model of a cascode stage.

- Miller effect is less significant in cascode amplifiers than in common-source stages.
- But  $\omega_{p,X}$  is typically quite higher than the other two.
- What if RD is replaced by a current source?
- Pole at node X may be quite lower, but transfer function will not affect much by this. See example.



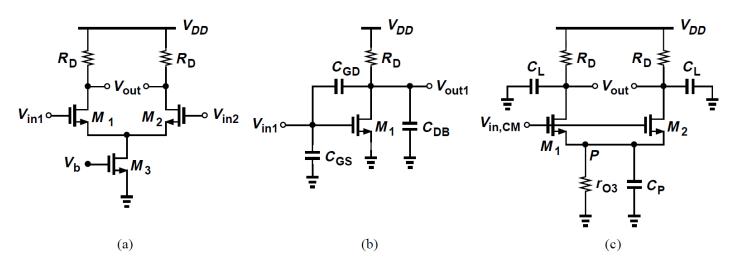
Compute the transfer function.

$$\frac{V_{out}}{I_{in}} = -\frac{g_{m2}r_{O2} + 1}{C_X s} \cdot \frac{1}{1 + (1 + g_{m2}r_{O2})\frac{C_Y}{C_X} + C_Y r_{O2} s}$$

• For 
$$g_{m2}r_{O2}\gg 1$$
 
$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}g_{m2}}{C_{V}C_{X}s}\frac{1}{g_{m2}/C_{X}+s}$$

• The magnitude of the pole at node X is still given by gm2/Cx. Why?

#### **Differential Pair**

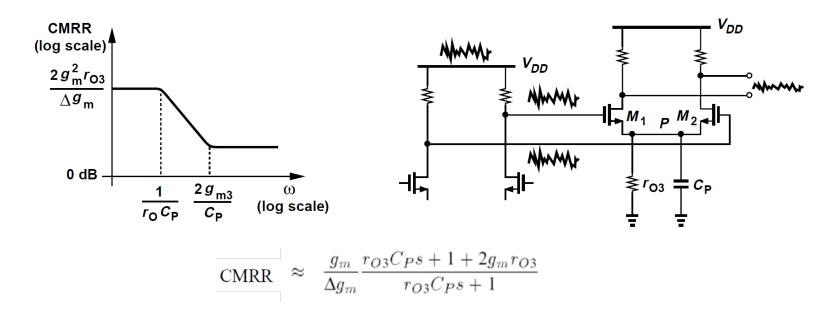


- (a) Differential pair, (b) half-circuit equivalent, (c) equivalent circuit for common-mode inputs.
- For differential signals, the response is identical to that of a common-source stage.
- the common-mode rejection of the circuit degrades considerably at high frequencies.

$$A_{v,CM} = -\frac{\Delta g_m \left[ R_D \left\| \left( \frac{1}{C_L s} \right) \right]}{\left( g_{m1} + g_{m2} \right) \left[ r_{O3} \left\| \left( \frac{1}{C_P s} \right) \right] + 1}$$

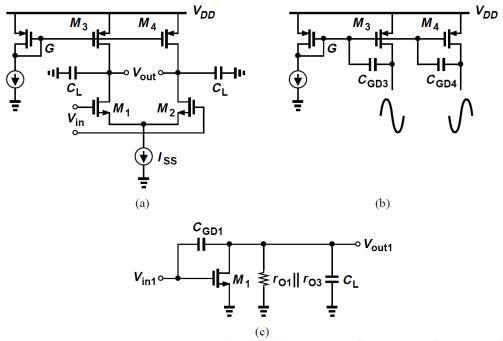
 Channel-length modulation, body effect, and other capacitances are neglected.

### **Differential Pair**



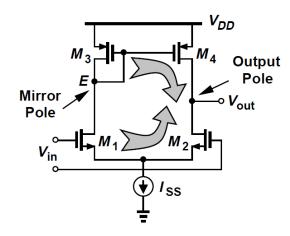
- This transfer function contains a zero and a pole.
- The magnitude of the zero is much greater than the pole.
- Common-mode disturbance at node P translates to a differential noise component at the output, if the supply voltage contains high-frequency noise and the circuit exhibits mismatches.
- Trade-off between voltage headroom and CMRR.

#### Diff Pair



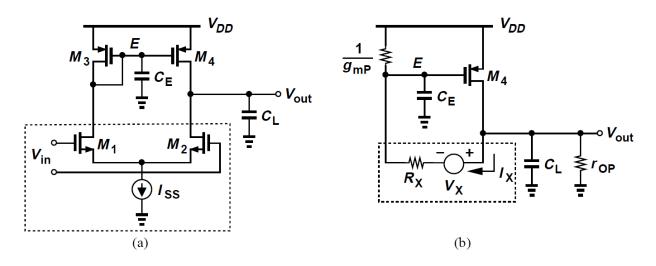
- Frequency response of differential pairs with high-impedance loads.
- Fig (b) CGD3 and CGD4 conduct equal and opposite currents to node G, making this node an ac ground.
- The differential half circuit is depicted in Fig. (c).
- More on chapter 10

#### **Differential Pair with Active Load**



- How many poles does this circuit have?
- The severe trade-off between gm and CGS of PMOS devices results in a pole that impacts the performance of the circuit.
- The pole associated with node E is called a "mirror pole."

#### **Active Load**



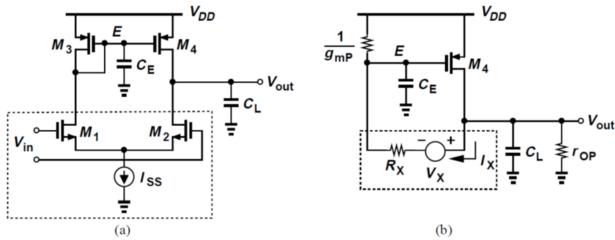
 Replacing Vin, M1, and M2 by a Thevenin equivalent.

$$V_X = g_{mN} r_{ON} V_{in}$$
  $R_X = 2 r_{ON}$ 

$$egin{split} rac{V_{out}}{V_{in}} \ &= rac{g_{mN}r_{ON}(2g_{mP}+C_{E}s)r_{OP}}{2r_{OP}r_{ON}C_{E}C_{L}s^{2}+[(2r_{ON}+r_{OP})C_{E}+r_{OP}(1+2g_{mP}r_{ON})C_{L}]s+2g_{mP}(r_{ON}+r_{OP})}. \end{split}$$

$$\omega_{p1} pprox rac{1}{(r_{ON} || r_{OP}) C_L}$$
  $\omega_{p2} pprox rac{g_{mP}}{C_E}$ 

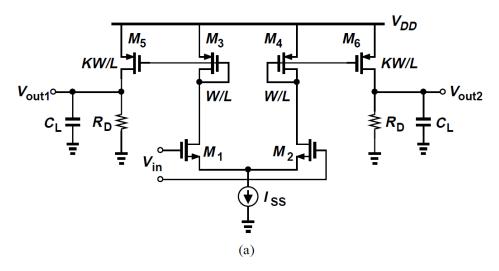
#### **Active Load**



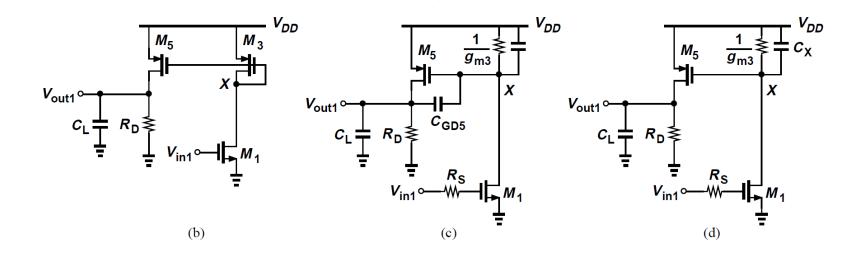
- A zero with a magnitude of  $2g_{mP}/C_E$ in the left half plane.
- The appearance of such a zero can be understood by noting that the circuit consists of a "slow path" (M1,M3 and M4) in parallel with a "fast path" (M1 and M2) by  $A_0/[(1+s/\omega_{p1})(1+s/\omega_{p2})]$  and  $A_0/(1+s/\omega_{p1})$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + s/\omega_{p1}} \left( \frac{1}{1 + s/\omega_{p2}} + 1 \right)$$

$$= \frac{A_0(2 + s/\omega_{p2})}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}.$$



- Not all fully differential circuits are free from mirror poles.
- Estimate the low-frequency gain and the transfer function of this circuit.



M5 multiplies the drain current of M3 by K.

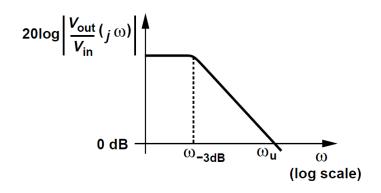
$$A_v = g_{m1} K R_D$$

• Assume RDCL is relatively small so that the Miller multiplication of CgD5 can be approximated as

$$C_{GD5}(1+g_{m5}R_D)$$
 
$$\frac{V_{out1}}{V_X}(s) = -g_{m5}R_D\frac{1}{1+R_DC_Ls}$$

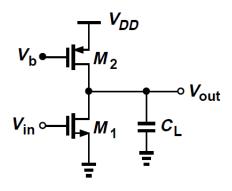
 The overall transfer function is then equal to Vx/Vin1 multiplied by Vout1/Vx.

#### **Gain-Bandwidth Trade-Offs**



- We wish to maximize both the gain and the bandwidth of amplifiers.
- we are interested in both the 3-dB bandwidth,  $\omega_{-3dB}$  and the "unity-gain" bandwidth,  $\omega_u$ .

## One pole circuit

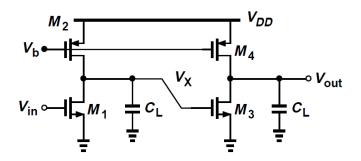


$$GBW = A_0 \omega_p$$
  
=  $g_{m1}(r_{O1}||r_{O2}) \frac{1}{2\pi (r_{O1}||r_{O2})C_L}$   
=  $\frac{g_{m1}}{2\pi C_L}$ .

$$\omega_u = \sqrt{A_0^2 - 1} \omega_p$$

$$\approx A_0 \omega_p$$

#### **Multi-Pole Circuits**



- It is possible to increase the GBW product by cascading two or more gain stages.
- Assume the two stages are identical and neglect other capacitances.

$$\frac{V_{out}}{V_{in}} = \frac{A_0^2}{(1 + \frac{s}{\omega_p})^2}$$

$$\omega_{-3-dB} = \sqrt{\sqrt{2} - 1}\omega_p$$

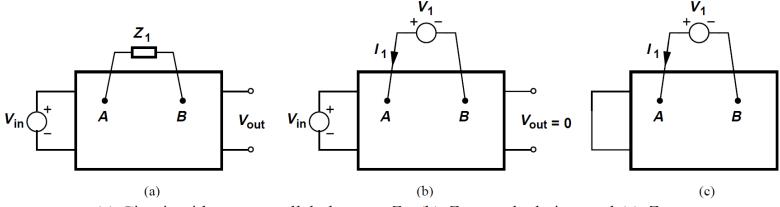
$$\approx 0.64\omega_p.$$

$$GBW = \sqrt{\sqrt{2} - 1}A_0^2\omega_p$$

 While raising the GBW product, cascading reduces the bandwidth.

## Appendix A: Extra Element

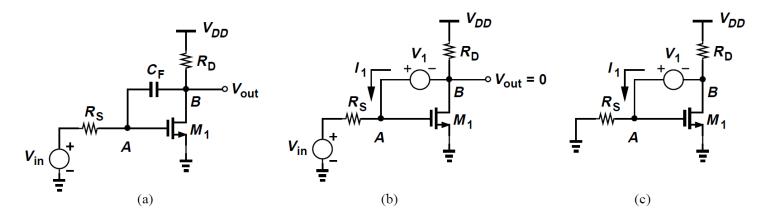
Theorem



- (a) Circuit with extra parallel element,  $Z_1$ , (b)  $Z_{out,0}$  calculation, and (c)  $Z_{in,0}$  calculation.
- Suppose the transfer function of a circuit is known and denoted by H(s). Add an extra impedance Z1 between two nodes of the circuit.
- New transfer function:

$$G(s) = H(s) \frac{1 + \frac{Z_{out,0}}{Z_1}}{1 + \frac{Z_{in,0}}{Z_1}}$$

 Particularly useful for frequency response analysis.



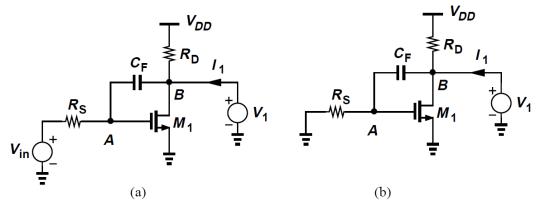
Find the transfer function.

$$H(s) = -g_m(R_D||r_O)$$
$$Z_{out.0} = -1/g_m$$

• The negative sign of Zout,0 does not imply a negative impedance between A and B, since  $V_{in} \neq 0$ 

$$Z_{in,0} = (1 + g_m R_S)R_D + R_S = (1 + g_m R_D)R_S + R_D$$

$$\frac{1 - \frac{1}{g_m}C_F s}{1 + [(1 + g_m R_D)R_S + R_D]C_F s}$$

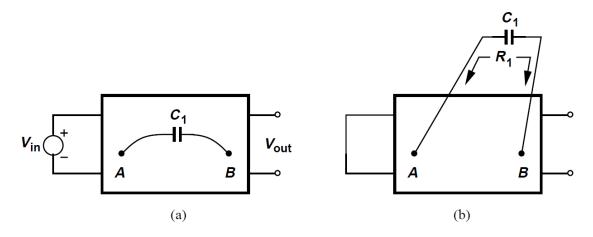


• Include CB, from node B to ground.

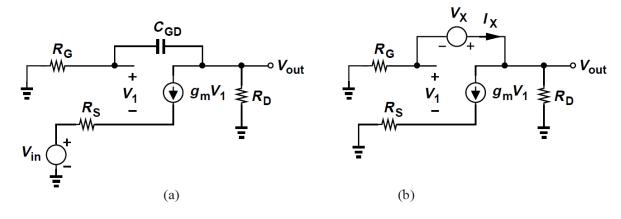
$$Z_{out,0} = 0$$
 
$$Z_{in,0} = \frac{R_D(R_SC_Fs+1)}{R_S(1+q_mR_D)+R_D|C_Fs+1}$$

$$G(s) = -g_m(R_D||r_O) \frac{1 - \frac{C_F}{g_m}s}{1 + [(1 + g_m R_D)R_S + R_D]C_F s} \frac{1}{1 + \frac{R_D(R_S C_F s + 1)C_B s}{[R_S(1 + g_m R_D) + R_D]C_F s + 1}}$$

# Appendix B: Zero-value Time Constant



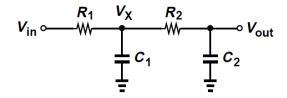
- Suppose a circuit contains one capacitor and no other storage elements.
- Wish to determine the pole of the system.
- Set the input to zero, compute the resistance, R1, seen by C1, and express the pole as 1/(R1C1).



 If only CGD is considered, determine the pole frequency.

$$g_m V_1 R_S + V_1 = -I_X R_G$$
  $rac{V_X}{I_X} = R_D + \left(rac{g_m R_D}{1 + g_m R_S} + 1
ight) R_G = R_{eq}$ 

• The pole is given by  $1/(R_{eq}C_{GD})$ 



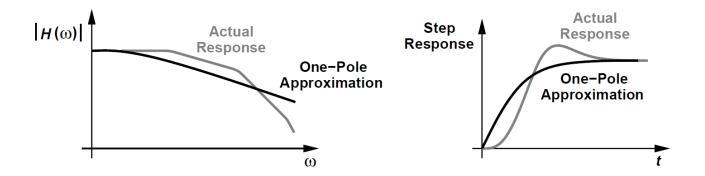
Writing a KVL around Vin, R1, R2, and Vout gives

$$\frac{V_{out}}{V_{in}}(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + [R_1 C_1 + (R_1 + R_2) C_2]s + 1}$$

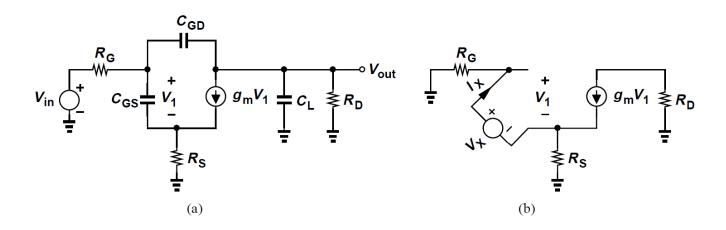
 The dominant pole is indeed equal to the inverse of the sum of the zero-value time constants. (need to prove)

$$B_s = \omega_{p1}^{-1} + \omega_{p2}^{-1} + \dots + \omega_{p1}^{-n} = R_1 C_1 + R_2 C_2 + \dots + R_n C_n$$

• The ZVTC method proves useful if we wish to estimate the 3-dB bandwidth of a circuit.



 Approximation of the frequency and time responses by one-pole counterparts.

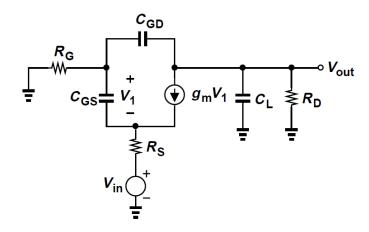


- Estimate the 3-dB bandwidth.
- Begin with the time constant associated with CGS and Set CGD and CL to zero.

$$R_{CGS} = rac{R_G + R_S}{1 + g_m R_S}$$
  $R_{CGD} = R_D + \left(rac{g_m R_D}{1 + g_m R_S} + 1
ight)R_G$ 

The resistance seen by C<sub>L</sub> is simply equal to R<sub>D</sub>.

$$\omega_{-3dB}^{-1} = \frac{R_G + R_S}{1 + g_m R_S} C_{GS} + \left[ R_D + \left( \frac{g_m R_D}{1 + g_m R_S} + 1 \right) R_G \right] C_{GD} + R_D C_L$$



- Find 3-dB band width of a common-gate stage containing a gate resistance of RG and a source resistance of RS.
- The resulting equivalent circuits are identical for CS and CG stages, yielding the same time constants and hence the same bandwidth.
- Does this result contradict our earlier assertion that the CG stage is free from the Miller effect?
- No. Why?