DATE Transient Characteristics

* Analysis purely resistive circuit have shown that by applying KVL, KCL produces algebraic eq^n.
* Analysis of circuits having R, L and C elements will lead to differential eq^{n}s of order 2/1

RL, RC – 1st order differential eq^n.

RLC – 2nd " "

Analysis of such differential eq^{n}s are quite difficult ~~to~~ compared to algebraic eq^{n}s.

* The solution of these differential eq^n will give equations that govern the behaviour of the current / voltage.
* Analysis of these differential eq^{n}s are done in two ways.

a) Source Free :- By using the initial energy stored is storage elements like C/L. Such response are due to initial condition / zero input response.

In this,

i) No external excitation used (No external excitation like voltage / current source used) ~~stat~~

ii) The stored energy causes the current flow in a closed path & this energy gets dissipated in the form of heat as current passes through resistor gradually. Over a time period stored energy becomes zero.

The responses obtained by such initial conditions are called as zero input response / natural response.

"The Response which is obtained due to initial energy stored due to physical characteristic of ckt elements but not due to external independent sources are called Natural response".

b) Using External independent source :- In this, an external excitation used along with initial condition. The initial condition may be in

i) Zero state :- No energy stored initially ~~due~~ in L/C.

ii) Non-zero state where energy storing elements in the n/w are holding ~~so~~ energy because of initial conditions in history.

Due to these different ~~sme~~ conditions of the ckt we get different types of responses.

Case (i) : If the initial state is zero state when external excitation is applied. The response obtained is called response due to forcing function. ie forced response

Case (ii) : If the initial state is non-zero (L/c holding energy) Then, the response obtained will have two - components

a) Natural response b) Forced response.

∴ The response is called complete response

Complete Response = Natural Response + Forced response
(zero input respo)

- When forced response is ~~done~~ being done with a external excitation over a time interval, the response reaches steady state (branch ~~voltage~~ current and ~~current~~ node voltages are not - changing with respect to time). The response obtained before the response reaches steady state is called transient response.

- The transient response is temporary, it decays to zero as time approches infinity i.e It dies out with time.

- steady state is one which remains after the transient response has died out.

∴ The total response can be rearranged into two parts as transient response & steady state response

Complete Response = transient Response + steady state response.

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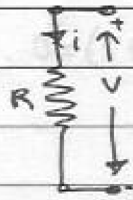
DATE Transient Response

- We have seen that applying Kirchoffs laws to purely resistive circuits results in algebraic eq^n which can be solved easily.
- Applying Kirchhoffs laws to RL, RC, RLC (serial & parallel) ckt leads to differential eq^n which are more difficult to solve.

* Resistor (R)

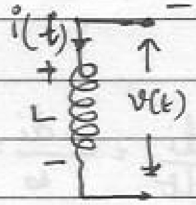


- It is linear, time invarient bilateral element which obeys ohm's law.

$$\{ v = iR \quad i = V/R = GV \quad \text{where } G = 1/R \}$$

- It is a memory less elements as it cannot hold energy. In fact it dissipates energy in the form heat.
- The current (i) through Resistor (R) changes instanteneauly if voltage 'V' changes.

* Inductor (L) :



- It is linear, time invarient bilateral element when it is in relaxed state
- v(t) is voltage across inductor, i(t) is current through inductor
- W.K.T

$$v(t) = L \cdot \frac{di}{dt} \quad \text{—(1)}$$

$$di = \frac{1}{L} v(t) \cdot dt \quad \text{—(2)}$$

Integrating eq^n (2) on both sides by limit (−∞ to t) we get i(t) at t.

$$\therefore i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\cdot d\tau = \cancel{\frac{1}{L}} \Bigg[\underbrace{\frac{1}{L}\int_{-\infty}^{0^-} v(\tau)\, d\tau}_{t<0 \text{ (history)}} + \underbrace{\frac{1}{L}\int_{0^-}^{t} v(\tau)\cdot d\tau}_{t \geq 0^+ \text{ (present)}} \Bigg]$$

term1 indicates current i(t) through inductor in history

$$\boxed{i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v(\tau)\cdot d\tau} \quad \text{—(3)}$$

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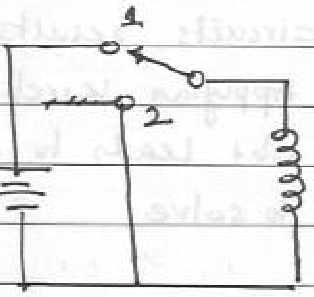
Now at $t = 0^+$ [instant after switch k is closed]



$$i(0^+) = i(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v(\tau)\cdot d\tau$$

0

$$\boxed{i(0^+) = i(0^-)}$$

~~As inductor~~ As current through inductor does not change instanteneouly

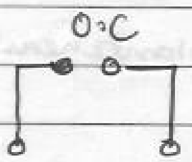
Case(i) : Assume L is in relaxed state initially ie $i(0^-) = 0$.

Case (a) : At the time of switching ie $t = 0^+$

$$i(0^+) = 0 + \int_{0^-}^{0^+} v(\tau)\, d\tau$$ As inductance can not energise instanteneously

0

O.C



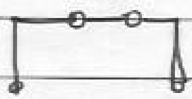
$$\boxed{i(0^+) = 0}$$

It means inductor acts as open circuit (O.C)

Case (b) : At steady state ($t = \infty$)

$$i(0^+) = 0 + \int_{0^-}^{\infty} v(\tau)\cdot d\tau \quad \{ v(t) = L\cdot\frac{di}{dt} = L\frac{di}{\infty} = 0V$$

S.C



$$i_{stedy} = \infty$$

It means inductor acts as short ckt (SC) in steady state.

Case (ii) : Assuming inductor L carries already some energised state carrying some current Io.

(a) At $t = 0^+$

$$i(0^+) = i(0^-) + \int_{0^-}^{0^+} v(\tau)\, d\tau.$$

0

$$\boxed{i(0^+) = I_0 = I(0^-)}$$

It means inductor acts as a current source

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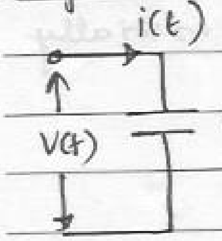
(b) At steady state $t=\infty$

$$i(\infty) = I_0 + \int_0^{\infty} v(\tau).d\tau. \qquad v(t) = L.\frac{di}{dt} = L.\frac{di}{\infty} = \underline{\underline{0}}$$

$$= I_0 + \text{short circuit}$$

∴, A current source I_0 is in parallel with short ckt

*Capacitor : We know that



$$i(t) = C.\frac{dv(t)}{dt}$$

$$\left[dv(t) = \frac{1}{C}\, i(t).dt\right] \quad -(1)$$

Integrating on both side from $[-\infty$ to $t]$, $v(t)$ is obtained

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau).d\tau.$$

$$\left[v(t) = \frac{1}{C}\int_{-\infty}^{0^-} i(\tau).d\tau + \frac{1}{C}\int_{0^-}^{t} i(\tau).d\tau.\right] \quad -(2)$$

Potential due to charge stored in history

$$\left[v(t) = v(0-) + \frac{1}{C}\int_{0^-}^{t} i(\tau).d\tau\right] \quad -(3)$$

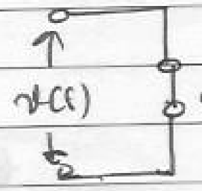
Case (i) : Capacitor initially is relaxed state

$$\therefore v(t) = \frac{1}{C}\int_{0^-}^{t} i(\tau).d\tau. \quad -(4)$$

(a) : At $t=0^+$

$$v(0^+) = \frac{1}{C}\int_{0}^{0^+} i(\tau).dt$$

Capacitor can't change instanteneously



$$\therefore \boxed{v(0^+) = 0} \quad -(5)$$

It means, capacitor 'C' at $t=0^+$ with initial relaxed state acts as short circuit.

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b) At steady state $t=\infty$

$$v(\infty)=\frac{1}{C}\int_{0^-}^{\infty} i(\tau)\cdot d\tau \quad — (6) \qquad i(t)=C\cdot\frac{dv(t)}{dt}=C\cdot\frac{dv(t)}{\infty}=\underline{0}$$

O.C

As $i(t)$ is zero at $t=\infty$ it means Capacitor can't be charged further ie Capacitor is charged to max & holding max energy. Acts as Open Circuit

Case (ii) Assume capacitor is not in relaxed state initially

$\therefore v(0^-)=V_0$ then

$$\boxed{v(t)=V_0+\int_{0^-}^{t} i(\tau)\cdot d\tau} \quad —(7)$$

a) At $t=0^+$

$$v(0^+)=V_0+\cancel{\int_{0^-}^{0^+} i(\tau)\cdot d\tau}^{\,0} \quad \text{Can't change instantaneously}$$

V_0

$\therefore \boxed{v(0^+)=V_0}$

It means acts as voltage source.

b) At steady state $(t=\infty)$

$$v(\infty)=V_0+\int_{0^-}^{\infty} i(\tau)\cdot d\tau \qquad i(t)=C\cdot\frac{dv(t)}{\infty}=\underline{0}$$

$i(t)$ through capacitor becomes 0. It means capacitor can't be charged further. Hence Capacitor acts as Voltage source in series with O.C

V_0 O.C

$\leftarrow v(t) \rightarrow$

Summary

Element	Initial $t=0^-$	$t=0^+$ (After switching)	$t=\infty$ (Steady state)
(1) R $v(t)=R\cdot i(t)$ $i(t)=\frac{v(t)}{R}$		$v(t)=R\cdot i(t)$ $i(t)=\frac{v(t)}{R}$ R	R
(2) L	$i(0^-)=0,$	$i(t)=i(0^-)+\frac{1}{L}\int_0^t v(\tau)\,d\tau$ & $v(t)=L\cdot\frac{di}{dt}$	
$v(t)=L\cdot\frac{di}{dt}$ $i(t)=\frac{1}{L}\int_{-\infty}^{0^-} v(\tau)\,d\tau+\frac{1}{L}\int_{0^-}^{\infty} v(\tau)\,d\tau$ $=i(0^-)+\frac{1}{L}\int_{0^-}^{\infty} v(\tau)\,d\tau$	$i(0^-)=0$	$i(t)=0+\frac{1}{L}\int_0^{0^+} v(\tau)\,d\tau$ $v(t)=L\cdot\frac{di}{0}=\infty$ Hence $i(t)=0$ O.C Inductor acts as open ckt	$i(t)=0+\frac{1}{L}\int_0^{\infty} v(\tau)\,d\tau$ $v(t)=L\cdot\frac{di}{\infty}=0$ Hence, $i(t)=\infty$ S.C Inductor acts as short ckt
	$i(0^-)=I_0$	$i(t)=I_0+0$ I_0 current source	$i(t)=I_0+\infty$ I_0 SC
(3) Capacitor (C) $i(t)=C\cdot\frac{dv(t)}{dt}$ $v(t)=\frac{1}{C}\int_{-\infty}^{0^-} i(\tau)\,d\tau+\frac{1}{C}\int_{0^-}^{\infty} i(\tau)\,d\tau$ $=v(0^-)+\frac{1}{C}\int_{0^-}^{\infty} i(\tau)\,d\tau$	$v(0^-)=0$	$v(t)=0+\frac{1}{C}\int_{0^-}^{0^+} i(\tau)\,d\tau$ $=0+\int_{0^-}^{0^+} i(\tau)\,d\tau$ $v(t)=0$ S.C $v(t)=0$ Capacitor acts as S.C	$v(t)=0+\frac{1}{C}\int_{0^-}^{\infty} i(\tau)\,d\tau$ $i(t)=C\cdot\frac{dv(t)}{dt}=\infty$ $v(t)=0+\infty$ (max) O.C $v(t)=\infty$ Cap acts O.C
	$v(0^-)=V_0$	$v(t)=V_0+\frac{1}{C}\int_{0^-}^{0^+} i(\tau)\,d\tau$ $\boxed{v(t)=V_0}$ V_0 acts as Voltage source	$v(t)=V_0+\frac{1}{C}\int_{0^-}^{\infty} i(\tau)\,d\tau$ $=V_0+\infty$ O.C

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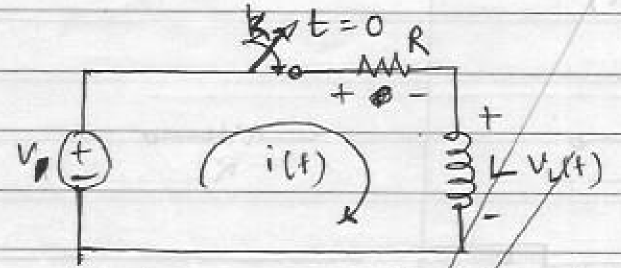
* Transient Response of Driven Series RL circuit

Transient response of driven Series RL ckt is used to find Expression of current through inductor (L) for $t > 0$



- It is important to know initial condition of current through inductor L.

Let us consider that, Initial current through inductor is I_0 at $t = 0^-$ (switch K is open).

$$\boxed{i(0^-) = I_0}$$

At $t = 0^+$ ie At the time of switching

$$i(0^+) = i(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v(t)\, dt$$

$$i(0^+) = i(0^-) \qquad 0$$

↳ current through inductor cannot change instantaneously

Now at ~~t = 0+~~ $t > 0^+$ switch K is closed.

Using KVL

$$-Ri(t) - L\frac{di(t)}{dt} + v = 0$$

$$V = Ri(t) + L\frac{di(t)}{dt} \qquad t > 0 \qquad \text{— (1)}$$

By dividing both side by R & rearranging

$$\frac{V}{R} = i(t) + \frac{L}{R}\frac{di(t)}{dt}$$

$$\frac{L}{R}\frac{di(t)}{dt} = \left(\frac{V}{R} - i(t)\right)$$

$$\frac{di(t)}{\left(\frac{V}{R} - i(t)\right)} = \frac{R}{L}\, dt \qquad \text{Separating variables.}$$

$$v - Ri(t) = L\frac{di(t)}{dt}$$

$$\cancel{v - Ri(t)} \quad \therefore \boxed{dt = \frac{L\, di(t)}{v - Ri(t)}} \qquad \text{— (2)}$$

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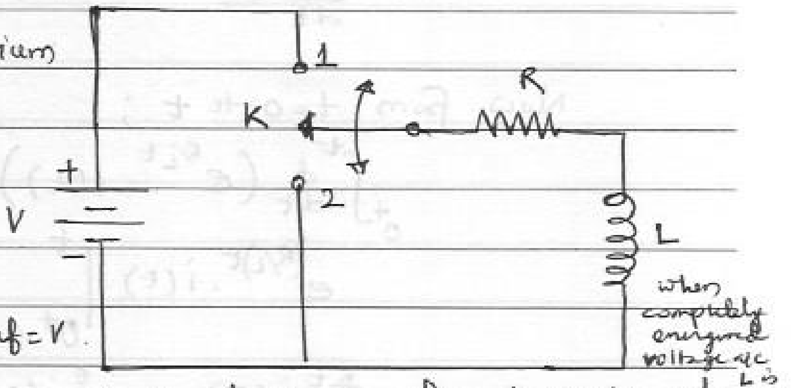
a) <u>A source free RL Circuit</u>:

- Assume that, the switch K is position 1 for a long duration of time. It means there is a equilibrium before switching. Under such equilibrium state the inductor acts as a short ckt (SC) ie Voltage a/c Inductor is zero ∵ back emf = V.



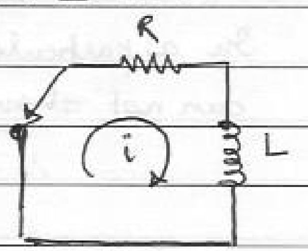
- Let $t=0$, be the reference instant of time when transition of switch takes place.
- <u>Objective of Analysis</u> – Obtain the equation that governs the behaviour of the current from the moment that equilibrium is altered.
- At $t=0^-$ → time immediately before transition of switch

Since there is a equilibrium at $t=0^-$, the current in the circuit is constant ie before transition, for all time $t \in (-\epsilon, 0^-)$ when $\epsilon > 0$.

$$\left.\begin{aligned} i(t) &= \frac{V}{R} \triangleq I_0 \\ \therefore\ i(0^-) &= I_0 \end{aligned}\right\} \quad -(1)$$

- After the transition of switch from position 1 to 2 Using KVL

$$L\frac{di}{dt} + Ri = 0 \quad -(2)$$



It is homogeneous first order ordinary differential eq² with constant coefficients.

The sol² for eq² ② can be obtained by solving eq² ②

$$\frac{di}{dt} + \frac{R}{L}.i = 0$$

Multiply throughout by integrating factor $e^{\left(\frac{R}{L}\right)t}$

$$e^{\left(\frac{R}{L}\right)t}.\frac{di}{dt} + e^{\left(\frac{R}{L}\right)t}\left(\frac{R}{L}\right)i = 0 \quad -(3)$$

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Clearly ~~the~~ eq² 3 is of the form $\frac{d(uv)}{dt} = u\cdot\frac{dv}{dt} + v\cdot\frac{du}{dt}$

$\therefore \frac{d}{dt}\left(e^{\frac{R}{L}t}\cdot i(t)\right) = 0$

Now from $t=0$ to t;

$$\int_{0^+}^{t}\frac{d}{dt}\left(e^{R/L\,t}\cdot i(t)\right) = 0.$$

$$e^{(R/L)t}\cdot i(t)\Big|_{0^+}^{t} = 0$$

$$e^{\frac{R}{L}t}\, i(t) - e^{0}\cdot i(0^+) = 0$$

$$e^{R/L\,t}\, i(t) = i(0^+)$$

$$\boxed{i(t) = i(0^+)\cdot e^{-\frac{R}{L}\cdot t}} \quad -(4)$$

(or) By seperating variables in eq² 2

$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{0^+}^{t}\frac{di}{i} = -\frac{R}{L}\int_{0}^{t}dt$$

$$\ln i\Big|_{0^+}^{t} = -\frac{R}{L}t$$

$$\ln(t) - \ln(0^+) = -\frac{R}{L}t$$

$$\ln\left(\frac{i(t)}{i(0^+)}\right) = -\frac{R}{L}\cdot t$$

xplyn by exponential on both s

$$\frac{i(t)}{i(0^+)} = e^{-R/L\,t}$$

$$i(t) = i(0^+)\cdot e^{-\frac{R}{L}t}$$

$$V(t) = L\cdot\frac{di}{dt}$$

$$= L\cdot\frac{d}{dt}\left(i(0^+)\, e^{-\frac{R}{L}t}\right)$$

$$= \cancel{L}\times\frac{V}{R}\times -\frac{R}{\cancel{L}}\, e^{-\frac{R}{L}t}$$

$$\boxed{V(t) = -V_0\cdot e^{-R/L\,t}}$$

In general, If the eq² obtained is a 1st order homogenous ~~eq~~ differential eq² of the form

$$\frac{dy}{dt} + \frac{a_1}{a_0}y = 0. \quad \cancel{\text{then}}$$

Sol² to any homogeneous ODE is of the form

then $\boxed{y(t) = y(0^+)\, e^{-\left(\frac{a_1}{a_0}\right)\cdot t}}$ $-(5)$ where $\tau = \frac{a_0}{a_1}$

where $y(0^+)$ & $\frac{a_1}{a_0}$ varies.

In a particular ckt, we know that current in the inductor can not change instantaneously

$$\therefore\ i(0^+) = i(0^-) = \frac{V}{R} = I_0.$$

Hence eq² (4) can be written as

$$\boxed{i(t) = \frac{V}{R}\, e^{-\frac{R}{L}t}} \quad \forall\, t \geq 0^+. \quad -(6)$$

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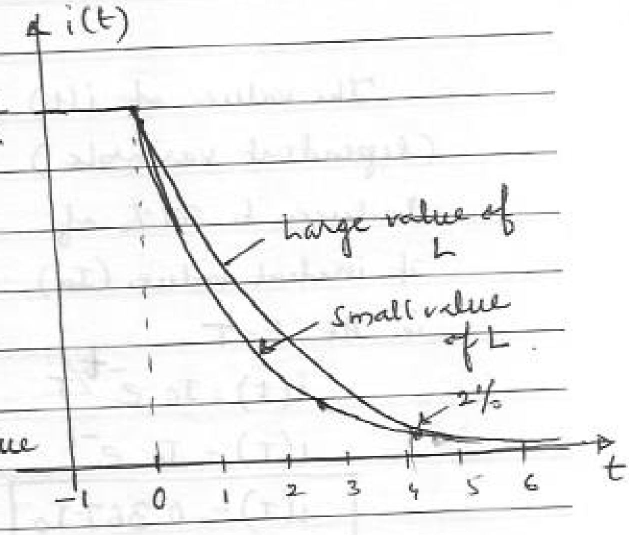
Now, eq² (6) can be written as

$$\boxed{i(t) = \frac{V}{R} \cdot e^{-t/T}} \quad \forall\, t \geq 0^+ \quad -(7)$$

where

$$\boxed{T = \frac{L}{R}} \quad -(8)$$

a) Response curve of i(t) where V and R are held constant & L is allowed to vary

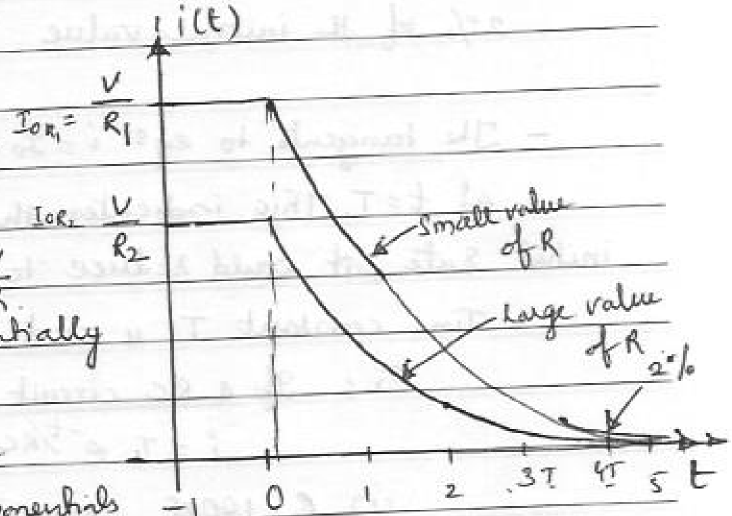


At $t < 0^+$; i(t) is constant & is equal to V/R

$i(0^+) = \frac{V}{R} \cdot e^0 = V/R$.

At $t \geq 0^+$; i(t) exponentially decreases. i(t) decreases less rapidly for larger value of L.

b) Response curve of i(t) where V & L are held constant and R is allowed to vary.



At $t < 0^+$; i(t) is constant but if R varies $i(0^+)$ also varies ∵ $i(0^+) = i(0^-) = \frac{V}{R}$.

At $t \geq 0^+$; i(t) exponentially decreases

∴ These curves (a) & (b) are known as decaying exponentials where the characteristic of decaying is determined by R & L

∴ the rate of decaying is controlled by time constant T which is (L/R).

At $t \geq 0^+$; Eventually all energy gets disscipated in the form of heat through resistor R and current i(t) becomes zero.

t_0	t/T	$i(t) = V/R \cdot e^{-t/T}$	$i(t)/V/R$
0	0	V/R	1
T	1	$V/R \cdot e^{-1} = 0.367\frac{V}{R}$	0.367
2T	2	$0.14(V/R)$	0.14
3T	3	$0.05\, V/R$	0.05
4T	4	$0.018\, V/R$	0.018
5T	5	$0.0067\, V/R$	0.0067

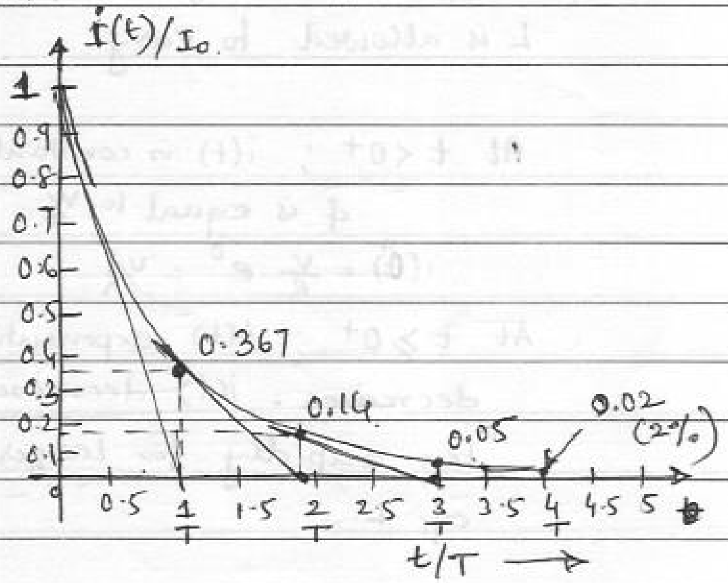
The value of $i(t)$ (dependent variable) reduces to 37% of it initial value (I_0)

ie At $t=T$

$$i(t) = I_0\, e^{-t/T}$$

$$i(T) = I_0 \cdot e^{-1}$$

$$\boxed{i(T) = 0.367 I_0}$$



By similar computation at $t=4T$, current decrease approximately 2% of the initial value.

- The tangents to eq² $i = I_0 \cdot e^{-t/T}$ at $t=0$ intersects the $i=0$ line at $t=T$. This indicates that if the current decreases at the initial rate, it would reduce to zero value is one time constant.
- Time constant T is used ~~for~~ as standard for comparision.

Ex: If a RC circuit consider which has general sol² as

$$i = I_0\, e^{-t/RC} \quad \text{where } T = RC.$$

(i) $C = 100\text{pF}$, $R = 100\,\Omega$ $\quad T = 100 \times 100 \times 10^{-12} = \underline{0.1\ \text{nsec}}$

(ii) $C = 1\,\mu\text{F}$, $R = 1000\,\text{M}\Omega$ $\quad T = 1000 \times 10^{6} \times 1 \times 10^{-6} = 1000\ \text{Sec}$

(or) 17 Sec

For combination (i) current would decrease to 37% of its initial value in small time interval 0.1 nsec where in case (ii) it takes 17 Sec to decrease to 37% of initial value.

~~Not~~ Approximately @ 4T constants transient approximately disappears

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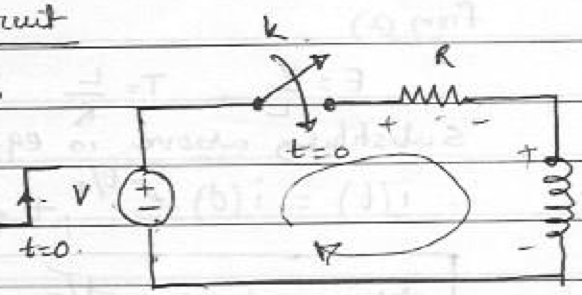
b) ~~RL~~ A Source Driven RL Circuit

Consider an RL ckt with constant applied voltage at $t=0$.



By KVL,

$$L\cdot\frac{di}{dt}+Ri=V \quad -(1)$$

$$\frac{di}{dt}+\frac{R}{L}\cdot i=\frac{V}{L} \quad -(2)$$

It is Non homogeneous first order differential eqn of the form

$$\frac{dy}{dt}+\frac{a_1}{a_0}y=E \quad -(3)$$

Now multiplying both sides by integrating factor $e^{t\left(\frac{a_1}{a_0}\right)}$

$$e^{\left(\frac{a_1}{a_0}\right)t}\cdot\frac{dy}{dt}+e^{\left(\frac{a_1}{a_0}\right)t}\left(\frac{a_1}{a_0}\right)y=E\cdot e^{\left(\frac{a_1}{a_0}\right)t}$$

$$\frac{d}{dt}\left(e^{\frac{a_1}{a_0}t}\cdot y\right)=E\cdot e^{\left(\frac{a_1}{a_0}\right)t} \qquad \int e^{zx}\,dt=\frac{e^{zx}}{z}$$

Integrating from 0 to t

$$e^{\frac{a_1}{a_0}t}y(t)\Big|_0^t=E\frac{a_0}{a_1}e^{\left(\frac{a_1}{a_0}\right)t}\Big|_0^t$$

$$e^{\frac{a_1}{a_0}t}y(t)-e^{0}y(0)=E\frac{a_0}{a_1}e^{(a_1/a_0)t}-E\left(\frac{a_0}{a_1}\right)\cdot e^{0}$$

$$e^{\frac{a_1}{a_0}t}y(t)-y(0)=E\left(\frac{a_0}{a_1}\right)\left[e^{\frac{a_1}{a_0}t}-1\right]$$

$$y(t)=\frac{y(0)+E\left(\frac{a_0}{a_1}\right)\left[e^{\frac{a_1}{a_0}t}-1\right]}{e^{(a_1/a_0)t}}$$

$$y(t)=y(0)\,e^{-\frac{a_1}{a_0}t}+E\left(\frac{a_0}{a_1}\right)\left[1-e^{-\frac{a_1}{a_0}t}\right]$$

if $T=\frac{a_0}{a_1}$ $\quad \boxed{y(t)=y(0)\,e^{-t/T}+ET\left(1-e^{-t/T}\right)} \quad -(4)$

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from (2)

$$E = \frac{V}{L}, \quad T = \frac{L}{R} \quad y(0) = I(0^+) \quad y(t) = i(t)$$

Substituting above in eqn (4)

$$\therefore\ i(t) = i(0^+) \cdot e^{-t/T} + \frac{V}{L} \times \frac{L}{R}\left(1 - e^{-t/T}\right), \forall t \geq 0^+$$

$$\boxed{i(t) = \underbrace{i(0^+) \cdot e^{-t/T}}_{A} + \underbrace{\frac{V}{R}\left(1 - e^{-t/T}\right)}_{B}} \quad -(5) \quad \forall t \geq 0^+$$

By rearranging

$$i(t) = i(0^+)\, e^{-t/T} + \frac{V}{R} - \frac{V}{R} \cdot e^{-t/T}$$

$$\boxed{i(t) = \underbrace{\left(i(0^+) - \frac{V}{R}\right) e^{-t/T}}_{C} + \underbrace{\frac{V}{R}}_{D}} \quad -(6)$$

$$(\text{or}) = \boxed{\frac{V}{R} - \left(\frac{V}{R} - i(0^+)\right) e^{-t/T}} = \boxed{i(\infty) - \left(i(\infty) - i(0)\right) e^{-t/T}}$$

A → zero input response (source free) – Natural

B → forced response (due to forcing function)

C → transient response

D → steady state response : $\lim_{t \to \infty} i(t)$

In general,

$$y(t) = \underbrace{y(0) \cdot e^{-t/T}}_{A} + \underbrace{ET\left(1 - e^{-t/T}\right)}_{B} \quad \text{where } T = \frac{a_0}{a_1}$$

By rearranging

$$y(t) = \underbrace{\left(y(0) - ET\right) e^{-t/T}}_{C} + \underbrace{ET}_{D} \quad -(7)$$

transient part must adjust itself ~~so~~ such that ~~the~~ it dies out then the response will be in ~~be~~ steady state.

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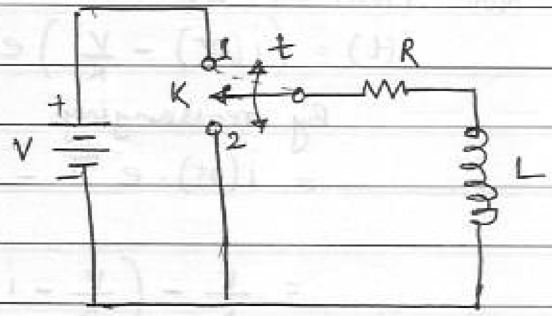
Forced Response (B)

- After a long time, the switch K transition from 2 to 1



- then,

$$i(0^-) = 0.$$

By KVL

$$L\frac{di}{dt} + Ri = V$$

$$i(t) = i(0^+).e^{-t/T} + \frac{V}{R}\left(1 - e^{-t/T}\right)$$

As $i(0^-) = 0 \quad \therefore i(0^+) = 0.$

Therefore.

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-t/T}\right)} \quad — (7)$$

$t \geq 0$

The steady state part is $i(\infty) = V/R$.

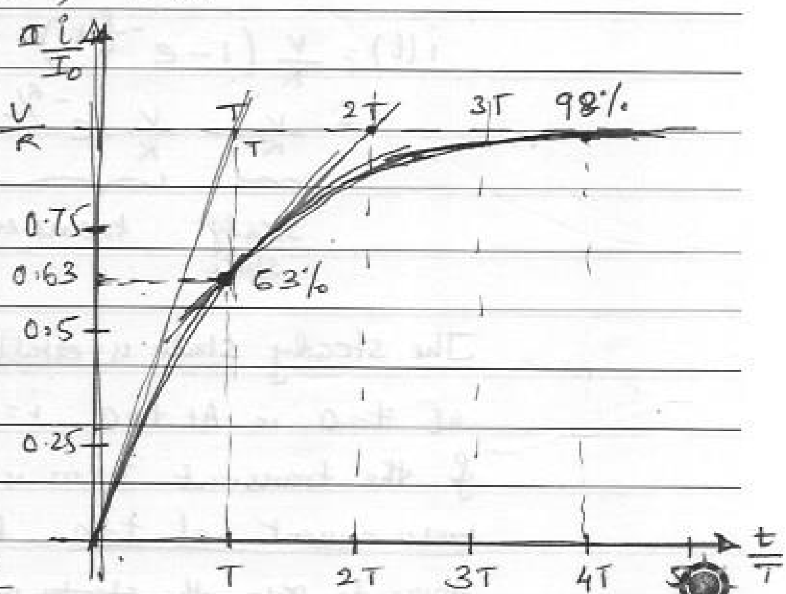
$t = T$

$$i(T) = \frac{V}{R}\left(1 - e^{-1}\right) = (1 - 0.37)I_0 = 0.63 I_0.$$

$t = 0$

$$i(0) = \frac{V}{R}\left(1 - e^{0}\right) = 0.$$

- It clear that at $t = T$ the current reaches 63% of it's final value. ie In one time constant & the current increases to approximately 98% of the final value is 4T constants.



- A tangent draw at $t = 0$, which intersects $i = I_0 = \frac{V}{R}$ at $t = T$ which means If the current i increases at initial rate, it reaches steady state at $t = T$

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Now from eqn (6)

$$i(t) = \left(i(0^+) - \frac{V}{R}\right)e^{-\frac{t}{T}} + \frac{V}{R}$$

By rearranging

$$= i(0^+)\cdot e^{-t/T} - \frac{V}{R}\cdot e^{-t/T} + \frac{V}{R}$$

$$= \frac{V}{R} - \left(\frac{V}{R} - i(0^+)\right)e^{-\frac{t}{T}}$$

$$\boxed{i(t) = \underbrace{i(\infty)}_{\text{steady state}} - \underbrace{\left(i(\infty) - i(0^+)\right)e^{-\frac{t}{T}}}_{\text{transient portion}}} \quad -(8)$$

It means, the response is made up of two parts of soln

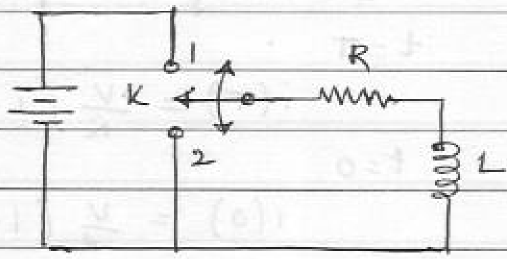
i) ~~particular~~ transient portion (complementary function)

ii) steady state value

The steady state value is ~~regarded~~ regarded as having been established at $t=0$, and the transient must adjust itself, mathematically, to account for the response at $t=0$ and all other time.

Example (1)

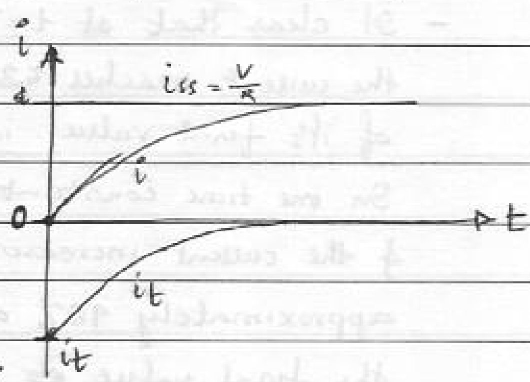
$$\boxed{i = i_{ss} + i_t}$$



Switch moved from position 2 to 1 at $t=0$. By KVL

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= \underbrace{\frac{V}{R}}_{\text{steady state}} - \underbrace{\frac{V}{R}\cdot e^{-R/L\,t}}_{\text{transient}}$$



The steady state is established at $t=0$ ie At $t=0$; $i = i_{ss} = \frac{V}{R}$

& the transient term is adjusted such that there is zero current at $t=0$. As transient part starts adjusting towards zero, the ~~steady~~ current i starts moving towards steady state i_{ss}.

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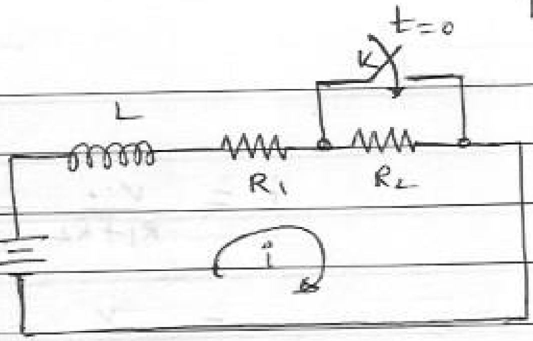
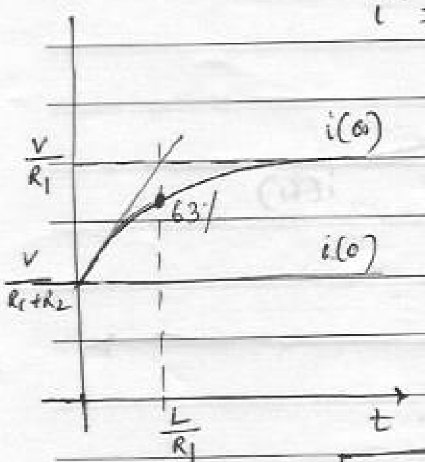
Example(2)



Case(i): We close the switch at $t=0$. This means switch was open for a long time.

$$\therefore \; i(0^-) = \frac{V}{R_1+R_2} \quad \text{and} \quad i(\infty) = \frac{V}{R_1} \qquad T = \frac{L}{R_1}$$

Now $i(t) = i(\infty) - \left(i(\infty) - i(0^+)\right) e^{-\frac{t}{T}}$ —



$$i = \frac{V}{R_1} - \left(\frac{V}{R_1} - \frac{V}{R_1+R_2}\right) \cdot e^{-\frac{R_1}{L}t}$$

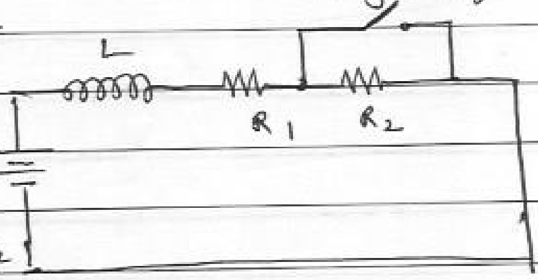
$i(\infty)$
$$= \frac{V}{R_1} - \left(\frac{V(R_1+R_2) - VR_1}{R_1(R_1+R_2)}\right) e^{-\frac{R_1}{L}t}$$

$i(0)$
$$= \frac{V}{R_1} - \left(\frac{VR_2}{R_1(R_1+R_2)}\right) e^{-\frac{R_1}{L}t}$$

$$= \frac{V}{R_1} - \frac{V}{R_1}\left(\frac{R_2}{R_1+R_2}\right) e^{-\frac{R_1}{L}t}$$

$$\boxed{i = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1+R_2} \cdot e^{-\frac{R_1}{L}t}\right)}$$

Case(ii): After the steady state is reached. (switch was closed for Long time)
Now, If switch is opened.



At $t=0^-$; $i(0^-) = \frac{V}{R_1}$

$$i(\infty) = \left(\frac{V}{R_1+R_2}\right) \quad \cancel{T = \frac{R_1+R_2}{L}}$$

& $T = \frac{L}{R_1+R_2}$ By substituting these in eq^n(8)

$$i = \left(\frac{V}{R_1+R_2}\right) - \left(\frac{V}{R_1+R_2} - \frac{V}{R_1}\right) e^{-t\left(\frac{R_1+R_2}{L}\right)}$$

$$= \left(\frac{V}{R_1+R_2}\right) - \left(\frac{VR_1 - V(R_1+R_2)}{R_1(R_1+R_2)}\right) e^{-t\left(\frac{R_1+R_2}{L}\right)}$$

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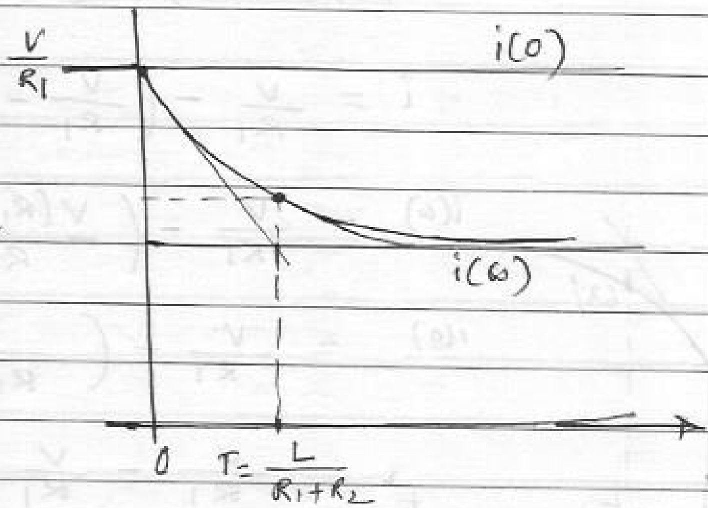
$$i = \frac{V}{R_1+R_2} - \left(\frac{-VR_2}{R_1(R_1+R_2)}\right)e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

$$= \frac{V}{R_1+R_2}\left(1 + \frac{R_2}{R_1}e^{-\left(\frac{R_1+R_2}{L}\right)t}\right)$$

$$\boxed{i = \frac{V}{R_1+R_2}\left(1 + \frac{R_2}{R_1}e^{-\left(\frac{R_1+R_2}{L}\right)t}\right)}$$



Here in case (i) & (ii) we observe that the time constant are different.

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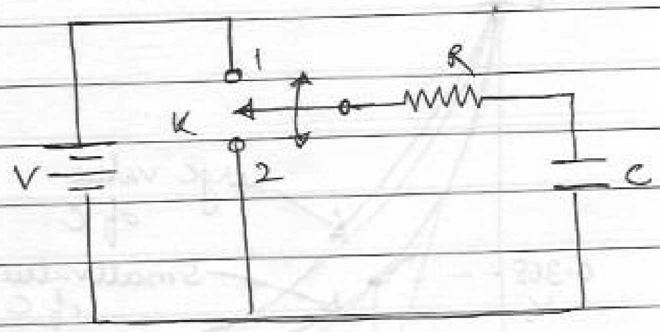
* Zero input | Source free RC n/w.



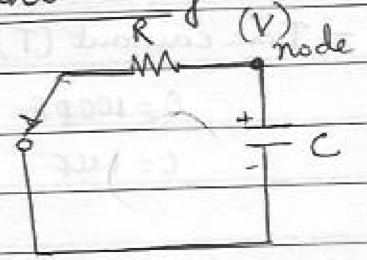
Switch K is at position 1 for long time ∴ Voltage is completely charged $V_c = V$ ie acting as open ckt (O.C)

Now, at $t = 0^-$

$$V_c(0^-) = V = V_0 \quad —(1)$$

At $t = 0^+$

$$V_c(0^+) = V_c(0^-) = V = V_0$$ as charge a/c capacitor cannot changed instanteneously



By KVL,

$$C\frac{dV}{dt} + \frac{V}{R} = 0.$$

$$\left[\frac{dv}{dt} + \frac{V}{RC} = 0\right] \quad —(2)$$

Eq² (2) is homogeneous 1st order differential eq²

W.K.T in general for homogeneous 1st order diff eq² of this form as response

$$y(t) = y(0) \cdot e^{-\frac{t}{(a_1/a_0)}}$$

∴ Now sol² for eq² (2) is

~~∴ V(t)~~

$$\boxed{v(t) = v(0^+) \cdot e^{-\frac{t}{RC}}} \quad —(3).$$

As at $t = 0$; $v(0^+) = v(0^-) = V_0$.

$$\boxed{v(t) = V_0 \cdot e^{-t/RC}} \quad —(4)$$

Time constant $T = RC$

$$\boxed{v(t) = V_0 \cdot e^{-t/T}} \quad —(5)$$

$$v(t) = V \cdot e^{-t/RC}.$$

$$i(t) = C \cdot \frac{dv(t)}{dt}$$

$$= C \cdot \frac{d}{dt}(V_0 \cdot e^{-t/RC})$$

$$= \cancel{C} \cdot V_0 \left(-\frac{1}{R\cancel{C}}\right) e^{-\frac{t}{RC}}$$

$$= -\frac{V_0}{R} \cdot e^{-\frac{t}{RC}}$$

$$\boxed{i(t) = -\frac{V}{R} \cdot e^{-t/RC}}$$

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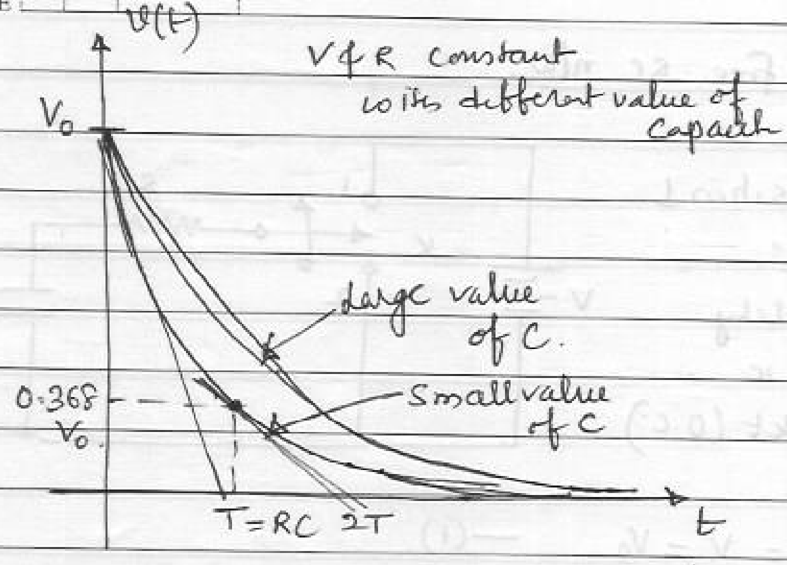

It means voltage a/c the capacitor decays by 37% of initial value in one time constant T.

If a tangent is taken at t = 0, It indicates that the Voltage a/c capacitor can completely discharge ~~It~~ in one T state if it decays with initial rate.

– Time constant (T) is used as standard for comparision.

C = 100 pF R = 100 Ω RC = 0.1 μsec

C = 1 μF R = 1000 MΩ RC = 1000 sec ≅ 17 sec

Q – The energy stored in capacitor is $\overline{Cv^2/2}$, charge flows through the resistor from one plate ~~of~~ cap to the other & energy dissipated in the resistor at the rate $\underline{Ri^2}$. The reduction in ~~energy~~ stored energy due to dissipation reduces the voltage of capacitor to zero. Eventually current.

* Current $i_c(t)$.

$$i_c(t) = \frac{dq}{dt} = \frac{d}{dt}CV = \frac{dC}{dt}\cdot V \times 1$$

$$= C\cdot\frac{d}{dt}\left(V_0\cdot e^{-\frac{t}{RC}}\right)$$

$$= \cancel{C}V_0\left(\frac{-1}{R\cancel{C}}\right)e^{-\frac{t}{RC}}$$

$$= -\left(\frac{V_0}{R}\right)\cdot e^{-t/RC}$$

$$\boxed{i_c(t) = -\frac{V}{R}\cdot e^{-t/RC}} \quad -(6)$$

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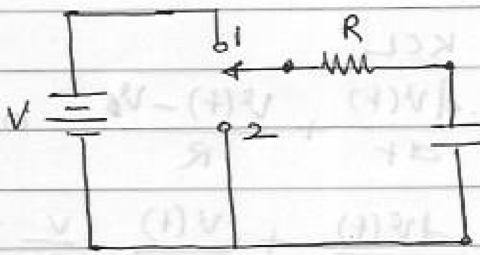
DATE * **Source Free RC circuit**

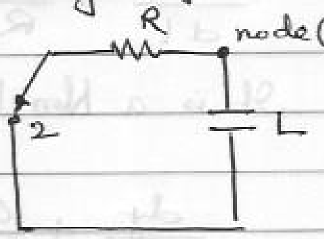
Switch is there in position 1 for long time. ∴ Capacitor is charged to applied voltage 'V' and acting as O.C.



$\therefore$ At $t<0; V(0^-) = V$ —(1)

Now At $t \geq 0$, switch position is changed from 1 to 2.

By KCL



$$C \cdot \frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0 \quad \text{—(2)}$$

W.K.T Above eq^n is a homogeneous 1st order differential eq^n of form $\frac{dy}{dt} + \frac{a_1}{a_0} \cdot y = 0$.

∴ Response of such homogeneous 1st differ eq^n is

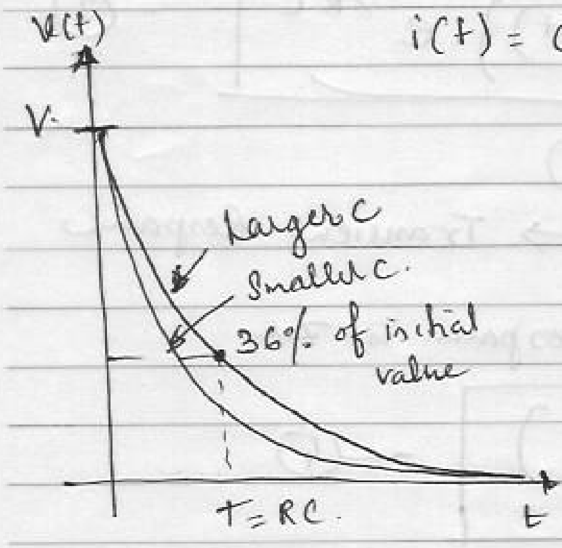
$$y(t) = y(0) \cdot e^{-\frac{a_1}{a_0} \cdot t}$$

$\frac{a_1}{a_0} = \frac{1}{RC}$

Hence $\boxed{v(t) = v(0^+) \cdot e^{-\frac{t}{RC}}}$ —(3)

$\therefore v(t) = V \cdot e^{-\frac{t}{RC}}$

From (3) $i(t)$ can be derived as



$$i(t) = C \cdot \frac{d\,v(t)}{dt} = C \frac{d}{dt}\left(v(0^+) \cdot e^{-t/RC}\right)$$

$$= -\frac{1}{R\cancel{C}} \times \cancel{C}\, v(0^+) \cdot e^{-\frac{t}{RC}}$$

$$= -\frac{v(0^+)}{R} \cdot e^{-\frac{t}{RC}}$$

$$\boxed{i(t) = -\frac{V}{R} \cdot e^{-\frac{t}{RC}}}$$

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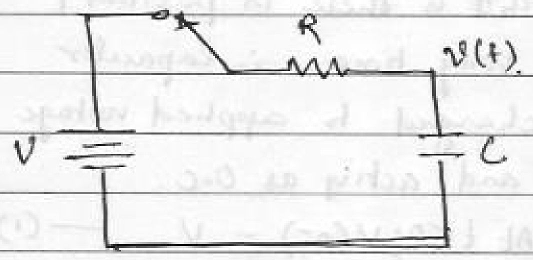
By KCL



$$C \cdot \frac{dv(t)}{dt} + \frac{v(t) - V_0}{R} = 0$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} - \frac{V}{RC} = 0$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{V}{RC} \quad \text{— (1)}$$

It is a Non homogeneous differential eq of the for

$$\frac{dy}{dt} + \frac{a_1}{a_0} y = E$$

The response of above eq^n is

$$y(t) = y(0) \cdot e^{-\frac{a_1}{a_0}t} + E\left(\frac{a_0}{a_1}\right)\left(1 - e^{-\frac{a_1}{a_0}t}\right)$$

$$v(t) = v(0^+) \cdot e^{-\frac{t}{RC}} + \frac{V}{RC} \times RC \left(1 - e^{-\frac{t}{T}}\right)$$

$$\boxed{v(t) = \underbrace{v(0^+) \cdot e^{-t/RC}}_{(A)} + V\underbrace{\left(1 - e^{-t/T}\right)}_{(B)}} \quad \text{— (2)}$$

Zero i/p Resp (A) (B) Forced Respons

By rearranging

$$\boxed{v(t) = \underbrace{V}_{(D)} - \underbrace{\left(V - v(0^+)\right) \cdot e^{-t/RC}}_{(C)}} \quad \text{— (3)}$$

(D) Steady state (C) ↳ Transient Respons

In eq^n (2) zero input respons is zero.

$$\boxed{v(t) = V\left(1 - e^{-\frac{t}{RC}}\right)} \quad \text{— (4)}$$

Forced Respons

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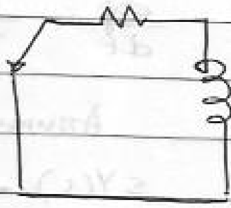
* Laplace transform

A source free RL ckt



$$L\frac{di}{dt} + Ri = 0 \quad \text{—} \ (1)$$

By appling Laplace transform

$$L\,\mathcal{L}\left(\frac{di}{dt}\right) + R\,\mathcal{L}(i) = 0$$

$$L\cdot\left[sI(s) - i(0^-)\right] + RI(s) = 0 \qquad \mathcal{L}\left(\frac{d}{dt}(f(t))\right) = L$$

$$LS\,I(s) - L\,i(0^-) + RI(s) = 0.$$

$$I_s(LS + R) = L\,i(0^+)$$

$$I(s)(LS+R) = L.\frac{V}{R}$$

$$I(s) = \frac{LV}{R(LS+R)}$$

$$= \frac{\cancel{L}V}{R} \times \frac{1}{\cancel{L}(S+\frac{R}{L})}$$

$$I(s) = \frac{V}{R}\left(\frac{1}{S + R/L}\right)$$

Taking Inverse Laplace transform

$$\boxed{i(t) = \frac{V}{R}e^{-\frac{R}{L}t}}$$

- for a Source driven RL ckt

$$L\frac{di}{dt} + Ri = V \quad -(1)$$

taking Laplace tranform

$$L\left[SI(s) - I(0^-)\right] + RI(s) = \frac{V}{S}$$

$$LS\,I(s) - L\,I(0^-) + R\,I(s) = V/S$$

$$I(s)\left[LS + R\right] - L\,\overset{I(0^-)}{0} = V/S$$

$$I(s)\left[LS + R\right] = V/S$$

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$$\frac{dy}{dt} + \frac{1}{T} y = E \quad -(1)$$

Assuming all signals are Laplace transformable

$$sY(s) - y(0^-) + \frac{1}{T} Y(s) = E(s)$$

$$Y(s)\left(s + 1/T\right) = E(s) + y(0^-)$$

$$Y(s) = \frac{y(0^-)}{(s + 1/T)} + \frac{E(s)}{(s + 1/T)}$$

E is constant

$$Y(s) = \frac{y(0^-)}{s + 1/T} + \frac{E}{s(s + 1/T)}$$

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General Non-homogeneous 1st order differential eq 2

$$\frac{dy}{dt} + \frac{1}{T}y = E$$

Assuming all signals are Laplace transform

$$sY(s) - y(0^-) + \frac{1}{T}Y(s) = E(s)$$

ie

$$Y(s)\left(s + \frac{1}{T}\right) - y(0^-) = E(s)$$

$$Y(s) = \frac{y(0^-)}{(s + 1/T)} + \frac{E(s)}{s(s + 1/T)}$$

E is a constant

$$Y(s) = \frac{y(0^-)}{(s + 1/T)} + \frac{E}{s(s + 1/T)}$$

finding roots

$$\frac{E}{s(s + 1/T)} = \frac{A}{s} + \frac{B}{(s + 1/T)}$$

$$E = A(s + 1/T) + Bs$$

$$s = 0 \; ; \quad E = \frac{A}{T} \qquad A = ET$$

$$s = -1/T \; ; \quad E = B(-1/T) \quad \therefore B = -ET$$

$$Y(s) = \frac{y(0^-)}{(s + 1/T)} + \left(\frac{ET}{s} - \frac{ET}{(s + 1/T)}\right)$$

$$= \frac{y(0^-)}{(s + 1/T)} + ET\left(\frac{1}{s} - \frac{1}{(s + 1/T)}\right)$$

ILT

$$\boxed{y(t) = y(0^-) \cdot e^{-\frac{t}{T}} + ET\left(1 - e^{-t/T}\right)} \quad t \geq 0$$

$$i(t) = i(0^-) \cdot e^{-\frac{t}{T}} + \frac{V}{\cancel{L}} \times \frac{\cancel{L}}{R}\left(1 - e^{-t/T}\right)$$

$$= i(0^-) \cdot e^{-\frac{t}{T}} + V/R\left(1 - e^{-t/T}\right)$$

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* First order differential eq2s have only one time constant
* These eq2s result from network with single inductor (or) capacitor in combination with any number of resistor (or).
* If more complex n/w can be reduced to a single equivalent resistor and single equivalent inductor (or) capacitor

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* 4~~23)~~ ~~Con~~ Consider RL driven by voltage Source

$$R\,i(t) + L\cdot\frac{di}{dt} = V \quad —(1)$$

Laplace on both side

$$R\,I(s) + L\left[s\,I(s) - I(0^-)\right] = V/s$$

$$I(s)\left[R + sL\right] - L\cdot I(0^-) = V/s$$

$$L\frac{d}{dt}f(t) = sF(s) - f(0^-)$$

Case(i) Assuming inductor is in relaxed state initially

$$I(s)\left[R+sL\right] = V/s$$

$$I(s) = \frac{V}{s(R+sL)}$$

$$I(s) = \frac{V}{Ls\left(\frac{R}{L}+s\right)} \quad —(2)$$

$$\frac{V}{Ls\left(s+\frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{(s+R/L)}$$

$$\frac{V}{L} = A(s+R/L) + Bs.$$

$s = 0$

$$\frac{V}{L} = A(R/L)$$

$$\boxed{A = \frac{V}{L}\times\frac{L}{R} = \frac{V}{R}}$$

$s = -R/L$

$$\frac{V}{L} = B(-R/L)$$

$$\frac{V}{\cancel{L}}\times\frac{\cancel{L}}{R} = -B$$

$$\boxed{B = -V/R}$$

$$\therefore \frac{V}{Ls(s+R/L)} = \frac{V}{Rs} - \frac{V}{R(s+R/L)}$$

from(1) $$I(s) = \frac{V}{Rs} - \frac{V}{R(s+R/L)}$$

Inverse Laplace transform

$$i(t) = \frac{V}{R} - \frac{V}{R}\cdot e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-R/L\,t}\right)}$$

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Case (ii) : Inductor L ~~not~~ not is relaxed state

$$I(s)\left[R+SL\right] - L\cdot I(0^-) = V/S$$

$$I(s) = \frac{V/S + L\cdot I(0^-)}{(R+SL)}$$

$$= \frac{V}{S(R+SL)} + \frac{L\cdot I(0^-)}{(R+SL)}$$

$$= \frac{V}{LS\left(\frac{R}{L}+S\right)} + \frac{L\,I(0^-)}{\cancel{L}\left(\frac{R}{L}+S\right)}$$

$$= \frac{\cancel{L}\,I(0^-)}{(S+R/L)} + \frac{V}{LS(S+R/L)}$$

$$I(s) = \frac{I(0^-)}{(S+R/L)} + \left(\frac{V}{RS} - \frac{V}{R(S+R/L)}\right)$$

By ILT ⟹

$$\boxed{i(t) = I(0^-)\cdot e^{-\frac{R}{L}t} + \frac{V}{R}\left(1 - e^{-R/L\,t}\right)}$$

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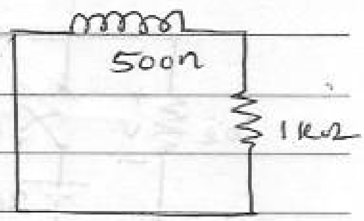
1) Determine the current i_R through resistor at $t = 1nsec$ if $i_R(0) = 6A$.



Sol: $i(t) = i(0)\cdot e^{-\frac{R}{L}t}$

$= 6\times e^{-\frac{1\times10^3}{500\times10^{-9}}t}$

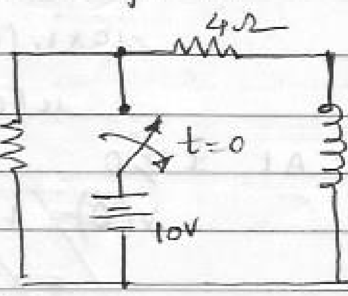
$i(t) = 6\times e^{-2\times10^9 t}$

At $t = 1nsec$

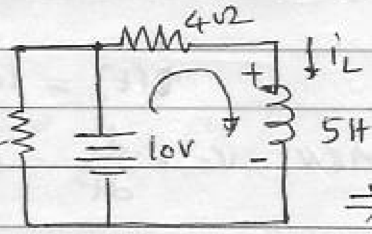
$i(1nsec) = 6\times e^{-2\times\cancel{10^9}\times\cancel{10^{-9}}\cancel{\times10^3}\cancel{\times10^{-3}}}$

$= .812A = 812mA$

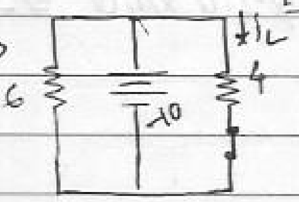
2) Determine the inductor voltage v in the ckt for $t>0$



It means $t<0$, K was closed



⇒

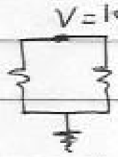


$\cancel{i_L(0^-) = \left(\frac{6}{6+4}\right)\times 4}$

$\frac{V}{6} + \frac{V}{4} = 0$

$\frac{10}{6} + \frac{10}{4}$

$.6 + .4 = 1$

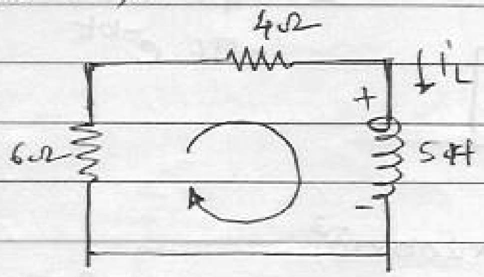


$\cancel{i_L(0^-) = \frac{10}{4} = 2.5A}$

KVL

$-10 + 4i_L = 0 \quad 4i_L = 10 \quad i_L = \frac{10}{4} = 2.5 \quad \cancel{10V = 4i_L} \quad \therefore\ i_L(0^-) = 2.5A$

At $t \geq 0$



$i(t) = I_L(0^-)\cdot e^{-\frac{10}{5}\times t}$

$\boxed{i(t) = 2.5\,e^{-2t}}$

$\cancel{V_L = 10\times2.5 = 25}$

→ $V_L = L\cdot\frac{di}{dt} = \cancel{5\times\frac{d(2.5)}{dt}}$

$= L\cdot\frac{d}{dt}\left(i(0)\cdot e^{-\frac{R}{L}t}\right)$

$= L\times\left(-\frac{R}{L}\right)i(0^+)\cdot e^{-R/L\,t}$

$= -R\cdot i(0^+)\cdot e^{-R/L\,t}$

$= -10\times2.5\times e^{-\frac{10}{5}t}$

$\boxed{V_L = -25\,e^{-2t}}$

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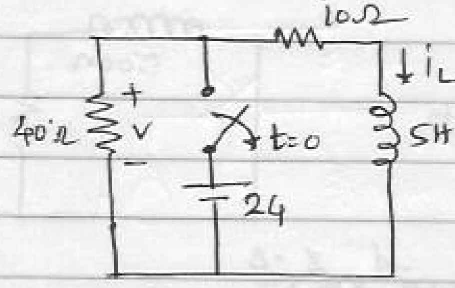
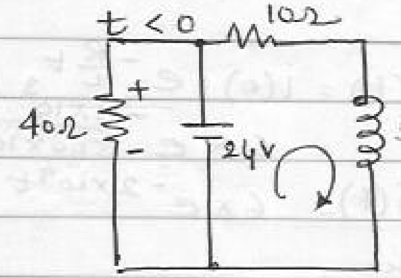
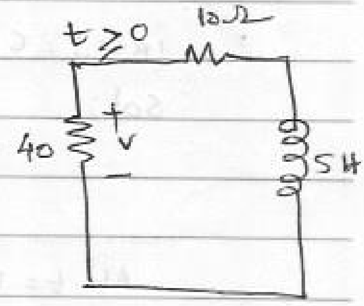
(3) find the voltage labeled v at $t = 200\,ms$







At $t < 0$ for long time, L is SC.

KVL to loop 2

$10 \times i_L(0^-) = 24$

$i_L(0^-) = \frac{24}{10} = 2.4\,A.$

At $t \geq 0$

$\cancel{V(t) = i_L(0^-) \times 40}$

$\cancel{= -2.4 \times 40 = -96V}$

$i(t) = I_0(0^-) \cdot e^{-\frac{R}{L}t}$

$V = L \cdot \frac{di}{dt} = L \cdot \frac{d}{dt}\left(I_{(0^-)} \cdot e^{-\frac{R}{L}t}\right)$

$= I_0(0^-) \cdot \cancel{L}\left(-\frac{R}{\cancel{L}}\right) \cdot e^{-\frac{R}{L}t}$

$= -R\,I(0^-) \cdot e^{-R/L\,t}$

$= -(R_1 + R_2)\,I(0^-) \cdot e^{-\frac{R}{L}t}$

$V(t) = -R_1 \cdot I(0^-) \cdot e^{-\frac{R}{L}t} - R_2\,I(0^-) \cdot e^{-R/L\,t}$

$V_{40}(t) = -R_2 I(0^-) \cdot e^{-R/L\,t} = -40 \times 2.4 \times e^{-\frac{50}{5}t}$

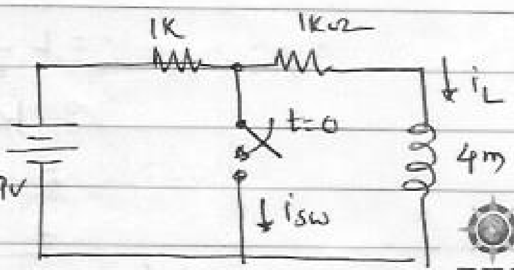
$= -96 \cdot e^{-10t}$

$$\boxed{V_{40}(t) = -96 \cdot e^{-10t}}$$

At $t = 200\,msec$

$V_{40}(t) = -96 \times e^{-10 \times 200 \times 10^{-3}} = -12.992\ V$

* (4) After being in the configuration shown for hours, the switch in the ckt is closed at $t=0$. At $t = 5\,\mu sec$ Calculate a) i_L b) i_{sw}



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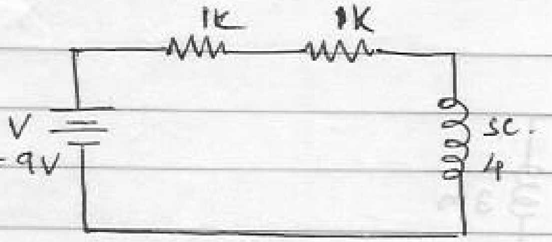
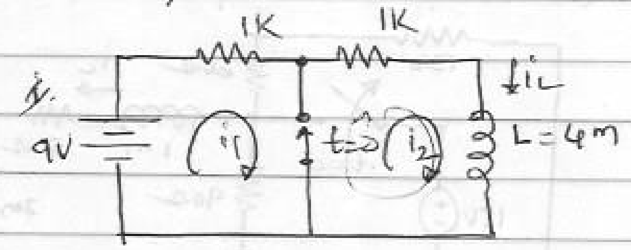
At $t<0$



$t \geq 0$



$i_L(0^-) = \frac{V}{R} = \frac{9V}{2K} = 4.5mA$

$i(t) = i(0^+) . e^{-\frac{R}{L}.t}$

$= 4.5\times10^{-3} e^{-\frac{1K}{4m}t}$

$i(t) = 4.5\times10^{-3} e^{-0.25\times10^{6}t}$

At $t = 5\mu sec$ $\quad i(500\mu) = 4.5\times10^{-3} e^{-0.25\times10^{6}\times 5\times10^{-6}}$

$= \underline{\underline{1.289mA}}$

$i_{sw} = i_1 - i_L = \frac{9V}{1\times10^{3}} - 1.289mA$

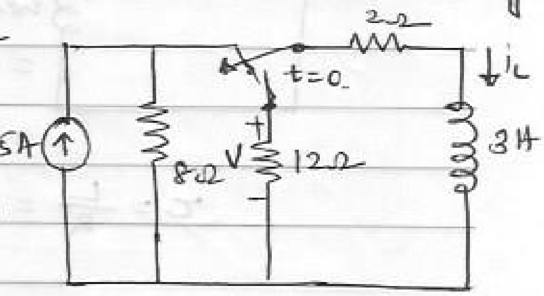
$= 9m - 1.289mA$

$\boxed{i_{sw} = 7.71mA}$

*(5) Assuming the switch has been in position drawn in fig for long time, Determine the value of v & i_L

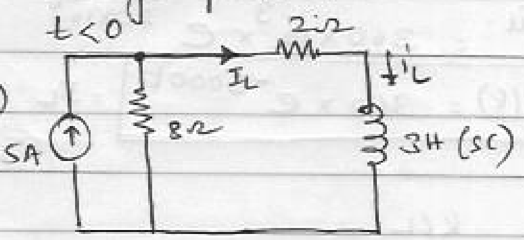
a) the instant just ~~before~~ prior of changing position of switch

b) the Instant just after the switch changes position



Ans: $t<0$

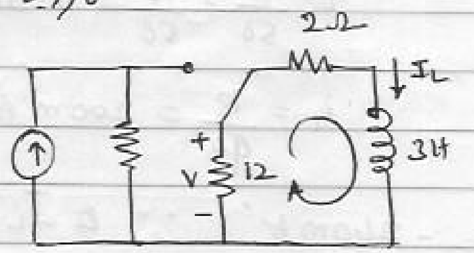
a)



$I_L = \left(\frac{8}{8+2}\right)\times5 = 4A$

$i_L(0^-) = 4A$ ✓ $\quad V = 0V$.

b) $t \geq 0$



$i_L(0^+) = i_L(0^-) = 4A$

$V = -I_L(0^+)\times12$

$= -4\times12 = -\underline{48V}$

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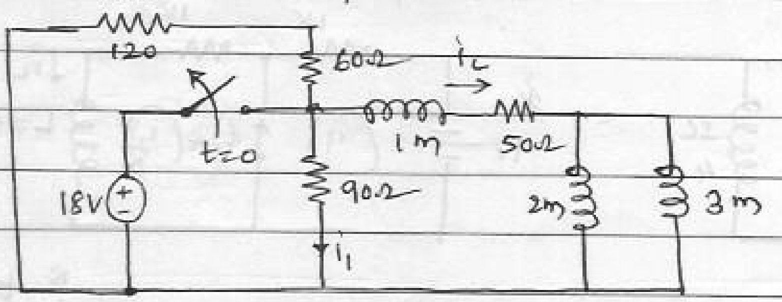
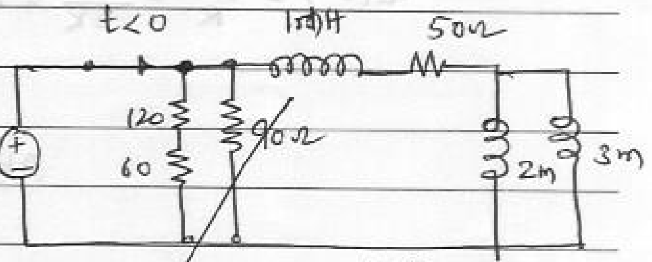
(6) Determine both i_1 and i_L in the ckt



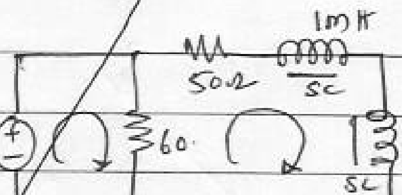
At $t<0$



$$L_{eq} = \left(\frac{2m \times 3m}{2m+3m} + 1\right)$$

$$= 2.2\,mH$$

$$R_{eq} = \left(\frac{90 \times 180}{90+180}\right) + 50 = 110.$$



$60 i_1 = 18$

$i_1 = \frac{18}{60}$

$i_1 = 0.3A$

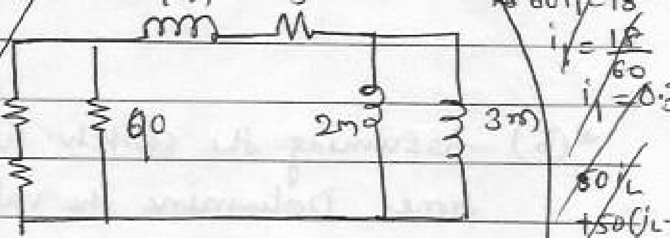
$50 i_L + 50(i_L - i_1) = 0$

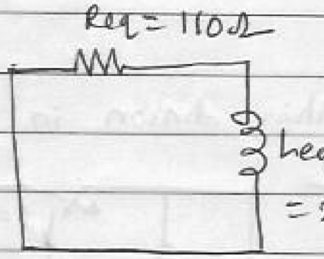
$100 i_L = 50 i_1$

$i_L = \frac{50 i_1}{100}$

$i_L = 0.15$

At $t=0$; switch is disconnected



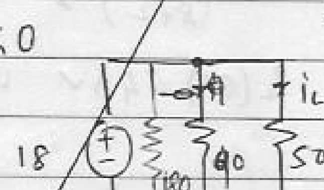


$$\tau = \frac{L}{R} = \frac{2.2m}{110} = \underline{20\mu\,sec}$$

$$i(t) = I(0^+) \cdot e^{-\frac{t}{20\mu}} = 360 \times 10^{-3} \times e^{-\frac{t}{20 \times 10^{-6}}}$$

$$\boxed{i_L(t) = 360 \times e^{-5000t}} = I_L$$

At $t<0$

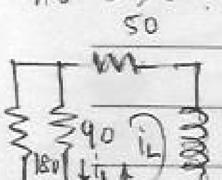


KCL

$$i_L = \frac{V}{50} = \frac{18}{50} = 0.36\,A = 360\,mA$$

$$i_1 = \frac{18}{90} = 200\,mA$$

At $t>0$



Current Divider

$$i_1(0^+) = -\left(\frac{I_L(0^-) \times 180}{180+90}\right) = -240\,mA \quad \therefore \; I_L \text{ opposite to } i_1$$

$$i_1(t) = -240\, e^{-5000t}$$

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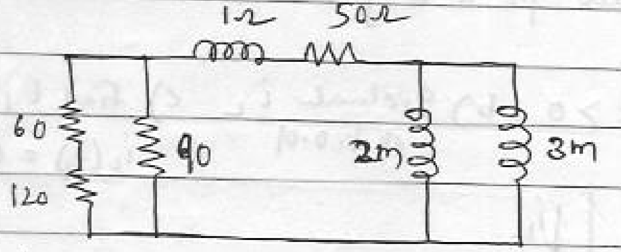
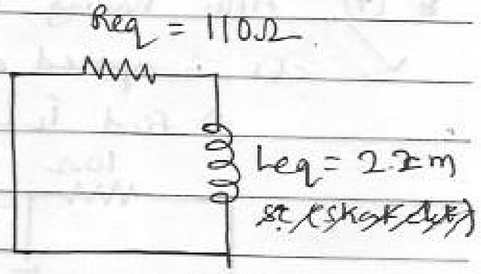
(6) continued At $t=0$

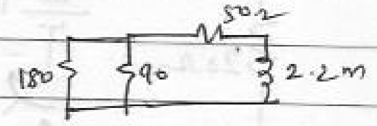


$\Rightarrow$



At $t \geq 0$

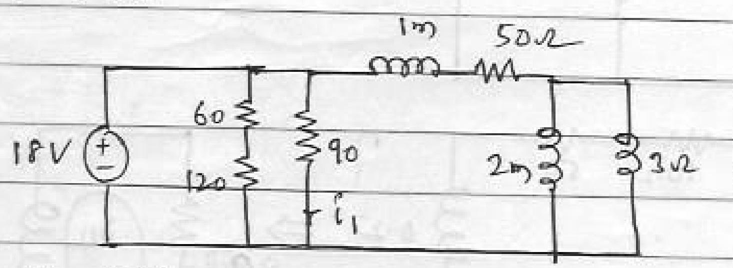
$$Req = \left(\frac{90 \times 180}{90+180}\right) + 50 = 110\,\Omega$$



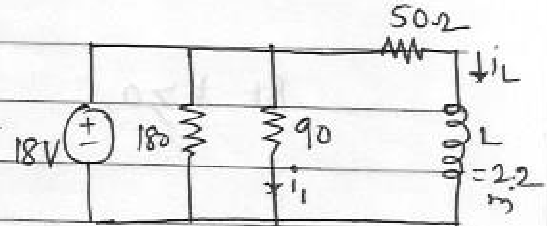
$$Leq = \frac{3m \times 2m}{(3+2)\,m} \cancel{*1800}\,H = 2.2\,mH \qquad T = \frac{L}{R} = \frac{2.2m}{110} = \underline{\underline{20\mu sec}}$$

$$i_L(t) = i_L(0^+) \cdot e^{\frac{-t}{T}} = \overset{I(0^+)}{\cancel{360}} \times e^{\frac{-t}{20\mu}} \cancel{mA} = I(0^+) \cdot e^{-5000t}$$

At $t<0$



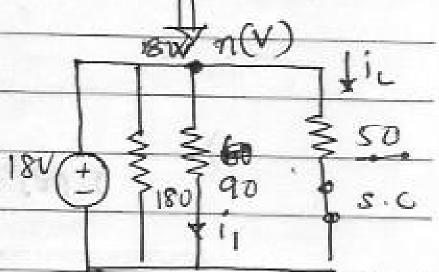
$\Rightarrow$



By KCL

$$I_L = i_L(0^-) = \cancel{\frac{V}{R}}\; \cancel{I_L = \left(\frac{60}{60+50}\right)} \frac{V}{50}$$

$$= 18/50 = \underline{360\,mA}$$



& $\boxed{i_1(0^-) = \frac{V}{90} = \frac{18}{90} = 200mA}$ ✓

$i_1(0^-)$

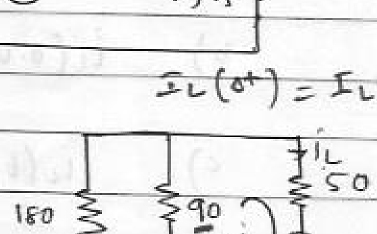
$\therefore\ \boxed{i_L(0^+) = i_L(0^-) = 360\,mA}$

Hence @ $t \geq 0$ $\boxed{i_L(t) = 360\,e^{-5000t}\ mA}$

At $t>0$: Using current divider

~~$i_1(0^+)$~~

$I_L(0^+) = I_L(0^-)$



There is No restriction on i_1 changing instantaneously at $t=0$ as it current through Resistor. So it value @ $t=0^-$ is $i_1(0^-) = 200mA$

$$i_1(0^+) = -\left(\frac{180}{90+180}\right) i_L(0^+)$$

$$= -\left(\frac{180}{270} \times 360\,mA\right)$$

$$i_1 = -\left(\frac{180}{180+90}\right) \cdot 36$$

$$i_1(0^+) = -240\,mA$$

$$\boxed{i_1(t) = -240\,e^{-5000t}}$$

& $i_1(0^+) = \underline{-240\,mA}$

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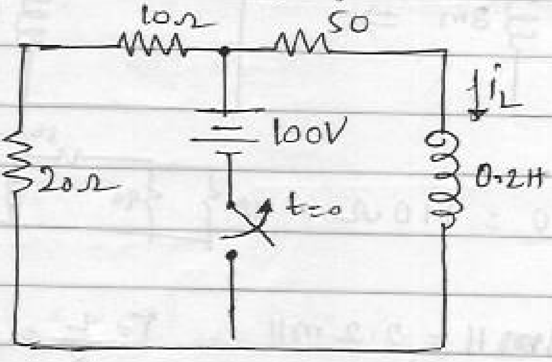
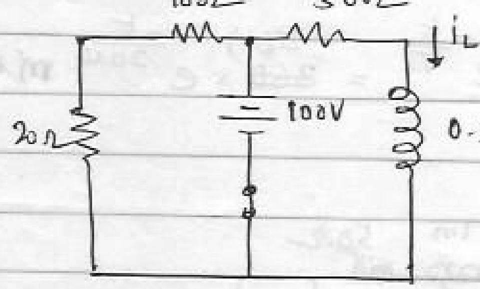
* (7) After having been closed for a long time, the switch in the ckt is opened at $t=0$.

a) find $i_L(t)$ for $t>0$ b) Evaluate i_L at $t=0.01$ c) find t_1 if $i_L(t_1) = 0.5\, i_L(0)$

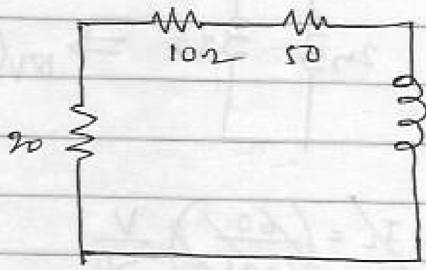


At $t<0$

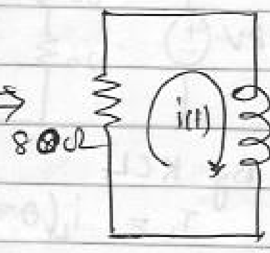


$$i_L(0^-) = \frac{V}{R} = \frac{100}{50} = 2A$$

At $t \geq 0$



$\Rightarrow$



a)
$$I_L(t) = I(0^+)\cdot e^{-\frac{t}{T}}$$
$$= 2\cdot e^{-t/2.5\times10^{-3}}$$
$$\boxed{i_L(t) = 2\cdot e^{-400t}\ A}$$

$$T = L/R = \frac{0.2H}{80} = 2.5\times10^{-3}\ \text{sec} = \underline{2.5\ \text{msec}}$$

b) $i_L(0.01) = 2e^{-400\times0.01} = 0.0366\ A = 36.6\ mA$

c)
$$i_L(t_1) = 0.5\, i_L(0^+)$$
$$= 0.5\times2 = 1$$
$$\therefore\ 1 = 2\cdot e^{-400t}$$
$$\tfrac{1}{2} = e^{-400t}$$
$$e^{400t} = 2$$

$$i_L(t_1) = 2\cdot e^{-400t_1}$$
$$0.5\, i_L(0^+) = 2\cdot e^{-400t_1}$$
$$0.5(2) = 2\, e^{-400t_1}$$
$$\tfrac{1}{2} = e^{-400t_1}$$
$$e^{400t_1} = 2$$

$$400t = \ln(2)$$
$$400t = 0.6931 \quad \therefore\ t = \underline{1.7328\ \text{sec}}$$

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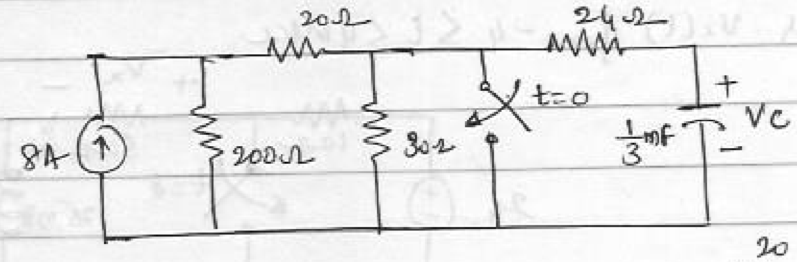
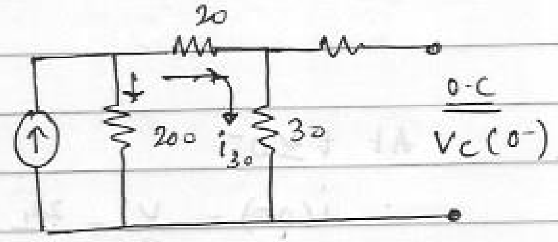
*(8) a) Find $V_c(t)$ for all time in the ckt
b) At what time is $V_c = 0.1 V_c(0)$



?

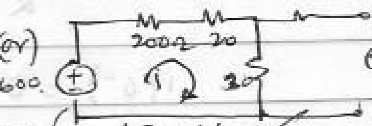
At $t \leq 0^-$ $\quad V_c(0^-)$



a)

$$i_{30} = \left(\frac{200}{200+50}\right) 8 = 6.4A$$

(or)

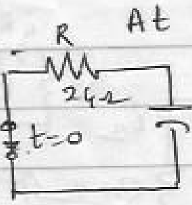


$$V_c(0^-) = i_{30} \times 30 = 6.4 \times 30 = 192V.$$

$$i = \frac{1600}{200+20+30} = 6.4A$$

$V_c(0^-) = 6.4 \times 30 = 192V.$

At $t = 0^+$



$$V_c(t) = V_c(0^+) \cdot e^{-\frac{t}{RC}}$$

$$= 192 \cdot e^{\frac{-t}{8\times10^{-3}}}$$

?

$$V_c(t) = 192 \, e^{-125t}$$

$$T = RC = 24 \times \frac{1}{3} \times 10^{-3}$$
$$= 8 \times 10^{-3}.$$

b) $V_c(t) = 192 \cdot e^{-125t}$

$$0.1 V_c(0) = 192 \cdot e^{-125t}$$

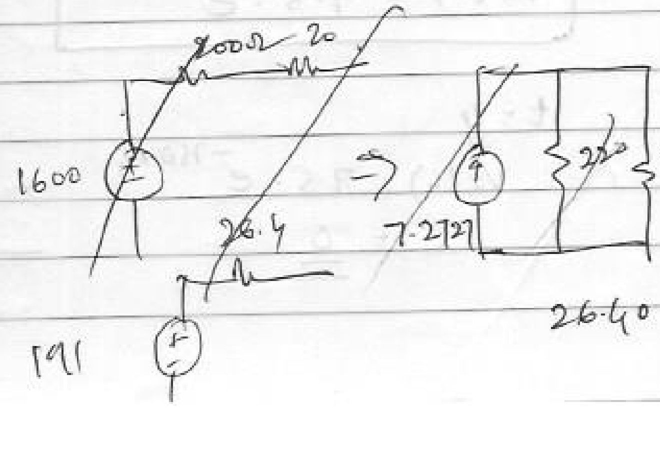
$$0.1 \times \cancel{192} = \cancel{192} \cdot e^{-125t}$$

$$e^{-125t} = 0.1 \qquad e^{125t} = 10.$$

$$125t = \ln(10)$$

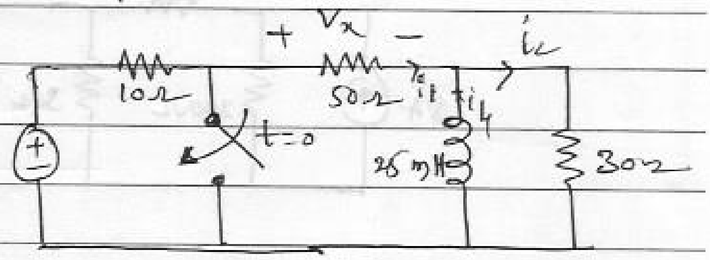
$$t = 2.30/125 = 0.0184 sec$$
$$= 18.4 \text{ msec}$$

$t = 18.4$ msec



$i_c(t) = 0.5 \, i_y(0)$
$= 0.5x$

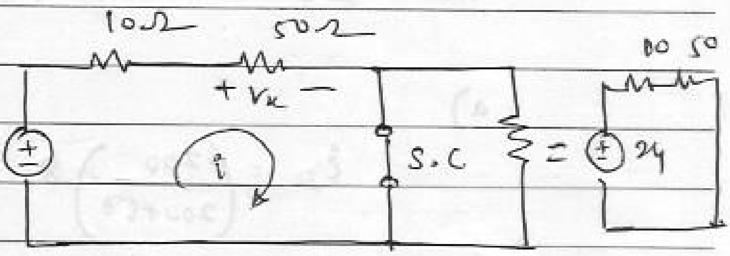
* (9) Switch has been open for long time before it closes at $t=0$.
a) Find $i_L(t)$ for $t>0$.
b) Sketch $v_x(t)$ for $-4 < t < 4$ msec



At $t \leq 0^-$

$\therefore i(0^-) = \frac{V}{R} = \frac{24}{60}$

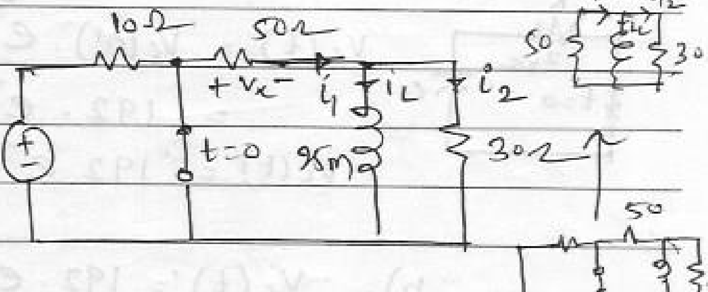
$i(0^-) = 0.4\,A$



$V_x(0^-) = 0.4 \times 50 = 20V$

At $t \geq 0^+$

$i_L(0^+) = i_L(0^-)$



$i_L(t) = i_L(0^+) \cdot e^{-t/T}$

$= 0.4 \cdot e^{-\frac{t}{1.333\,msec}}$

$\boxed{i_L(t) = 0.4\, e^{-750t}}$

$R_{th} = \frac{50 \times 30}{50+30} = 18.75\Omega$

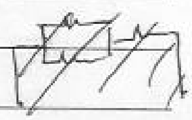
$T = \frac{L}{R} = \frac{25 \times 10^{-3}}{18.75} = 1.333\,msec$

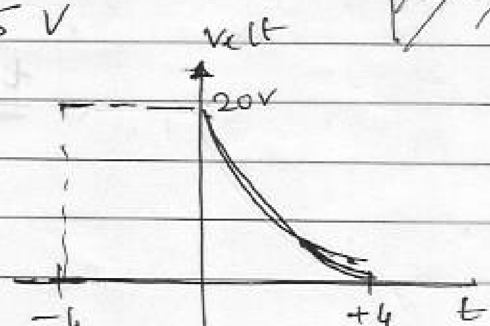
Current through Resistor changes instantaneously

$i_1 = \left(\frac{30}{30+50}\right) \times i_L(0^+) = \frac{30}{80} \times 0.4 = 0.15\,A$

$Vx(0^+) = 0.15 \times 50 = 7.5\,V$

$\boxed{Vx(t) = 7.5 \cdot e^{-750t}}$





$t = 4$

$Vx(4) = 7.5 \cdot e^{-750 \times 4}$

$= \underline{0}$

$\frac{V-24}{60} + \frac{1}{L}\frac{di}{dt} + \frac{V}{30} = 0$

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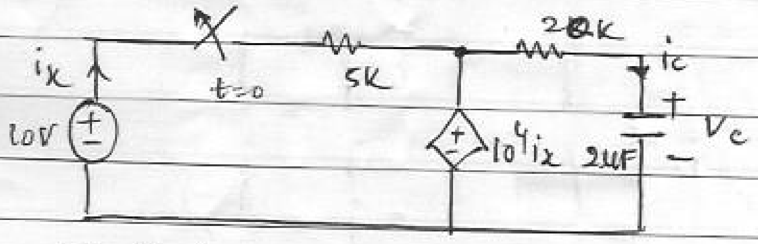
(5) Determine $V_c(t)$ and $i_c(t)$ for the ckt and sketch the curves



KVL @ Loop 1

$$-10 + 5000\, i_x + 10^4 i_x = 0.$$

$$(5000 + 10^4)\, i_x = 10$$

$$i_x = 10/(5000+10^4) = \underline{0.666\, mA}$$

$$\therefore\ V_c(0^-) = 10^4 i_x = 10^4 \times 0.666\, mA = 6.667V = V_c(0^+)$$

At $t = 0^+$

$$V_c(t) = V_c(0^+) \cdot e^{-\frac{1}{RC}t}$$

$$= 6.667\, e^{-\frac{1}{20\times10^3 \times 2\times10^{-6}}t}$$

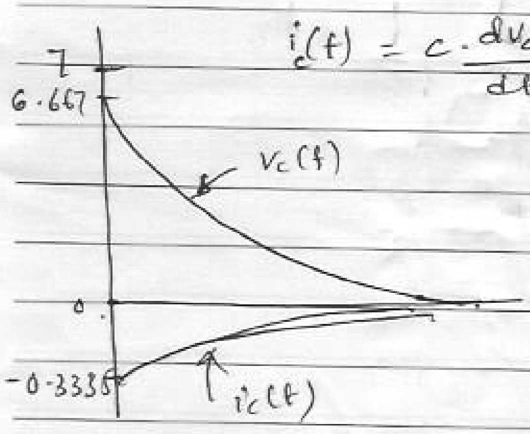
$$= 6.667\, e^{-\frac{1}{0.04}t}$$

$$\boxed{V_c(t) = 6.667\, e^{-25t}}$$

$$i_c(t) = C \cdot \frac{dV_c(t)}{dt} = 2\times10^{-6} \frac{d}{dt}\left(6.667\, e^{-25t}\right)$$

$$= 2\times10^{-6} \times 6.667 \times (-25) \cdot e^{-25t}$$

$$= -0.3335\times10^{-3}\, e^{-25t}$$



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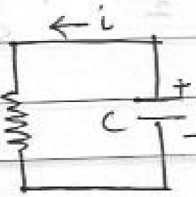
(10) $v(t) = 56e^{-200t}$ & $i(t) = 8 \cdot e^{-200t}$ for $t > 0$.

a) Find the value of R & C. b) Calculate τ c) Determine time required for the voltage to decay half its initial value at $t = 0$.



a) $R = \frac{v(t)}{i(t)} = \frac{56e^{-200t}}{8e^{-200t}} = 7K\Omega$

$1/RC = 200 \quad \therefore C = 1/R\,200 = 1/7K \times 200 = 0.714\ \mu F$.

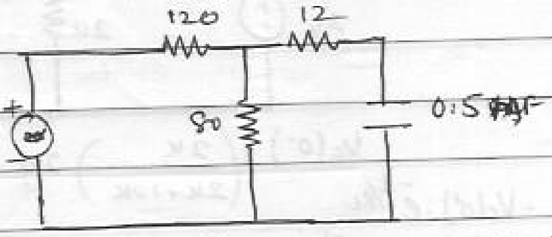
b) $\frac{1}{RC} = 200 \quad \therefore \tau = RC = \frac{1}{200} = 5\,m\,sec$

c) $v(t) = 56e^{-200t}$ $\quad 28 = 56e^{-200(t)}$ $\quad \frac{1}{2} = e^{-200t}$

$\ln(0.5) = -200t$

$t = \underline{3.4657\ m\,sec}$

(11)(2) Find time constant for RC ckt

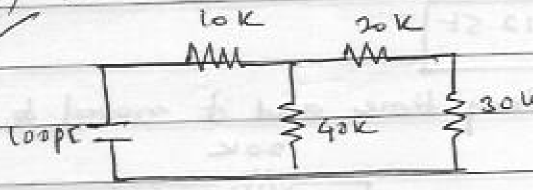


$\tau = RC = R_{eq} \cdot C$

$= \left[\left(\frac{80 \times 120}{80 + 120}\right) + 12\right] \times 0.5 \times 10^{-3}$

$\boxed{RC = 30\ msec}$

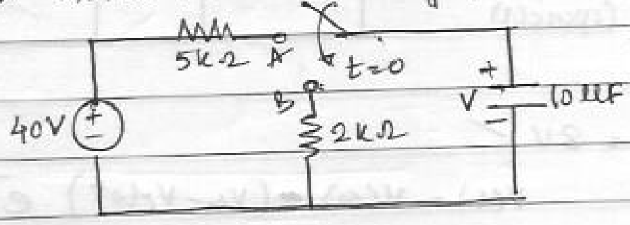
(12)(3) Determine the time constant for the ckt



$\tau = R_{eq}C = \left[\left(\frac{50 \times 40}{50 + 40}\right) + 10K\right] \times 100 \times 10^{-12}$

$\boxed{\tau = 3.223 \times 10^{-6}\ sec}$

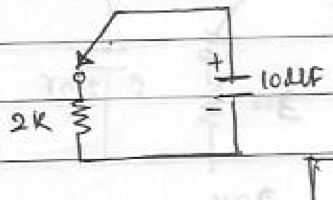
(13)(4) The switch has been in position A for long time. Assume the switch moves instantaneously from A to B at $t = 0$. Find V for $t > 0$



$t < 0$ $\quad V_c(0^-) = 40V \neq \cancel{V_c(0^+)}$

& $t = 0^+$ $\quad V_c(0^+) = V_c(0^-) = 40V$

$t \geq 0$



$V_c(t) = V_0(0^+) \cdot e^{-\frac{t}{RC}}$

$= 40e^{-t/2 \times 10^3 \times 10 \times 10^{-6}}$

$= 40e^{-t/0.2}$

$\boxed{V_c(t) = 40e^{-50t}}$

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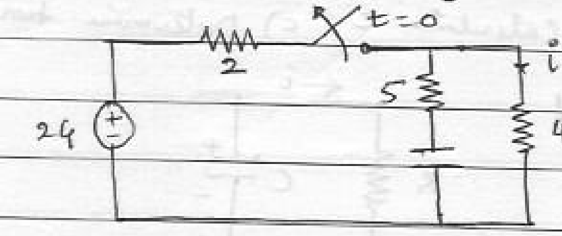
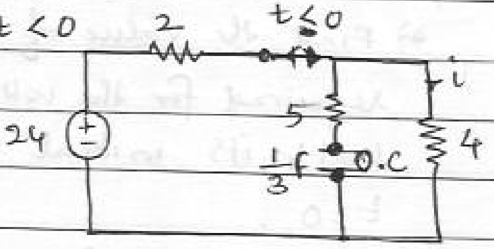
(14) (5) Circuit shown in fig, find $i(t), t>0$.

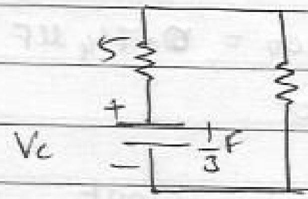


At $t<0$



C is O.C

At $t>0$



$V_c(t) = V_c(0^+) \cdot e^{-\frac{t}{RC}}$

$= 16 \cdot e^{-t/9\times\frac{1}{3}}$

$\boxed{V_c(t) = 16 \cdot e^{-t/3}}$

$i = 24/2+4 = 4A$

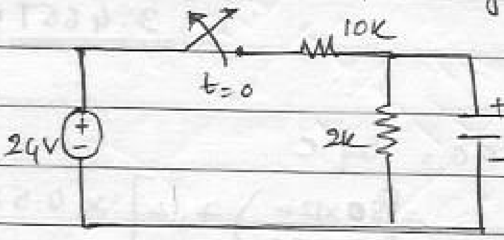
$i(0^-) = 4A = i(0^+)$

$\therefore V_c(0^-) =$

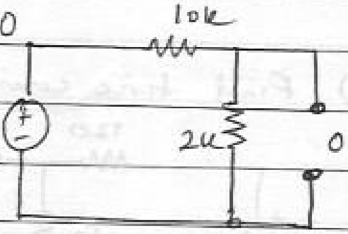
$-24 + 2\times4 + V_c(0^-) = 0$

$V_c(0^-) = 24 - 8 = \underline{16V}$?

(15) (6) Switch is closed for long time and it opens at $t=0$. find $v(t)$ for $t\geq0$

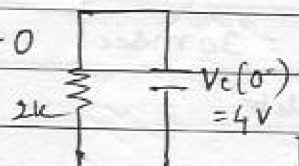


At $t<0$



$V_c(0^-) = \left(\frac{2K}{2K+10K}\right)24 = \underline{4V}$

At >0



$V_c(t) = V_c(0^+) \cdot e^{-t/RC}$

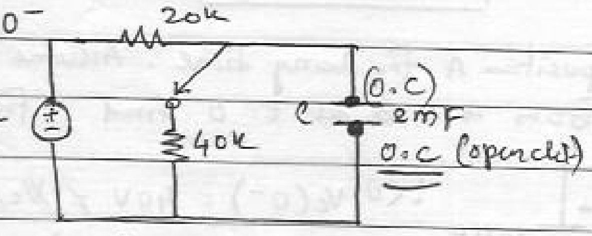
$= 4 \cdot e^{-t/2K\times40\times10^{-6}}$

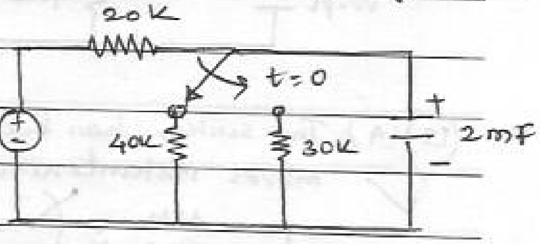
$\therefore V_c(0^+) = V_c(0^-) = \underline{4V}$

$\boxed{V_c(t) = 4 \cdot e^{-12.5t}}$

(16) (7) Switch has been in position A for long time and it moved to position B at $t=0$. Find $V_c(t)$ for $t\geq0$.

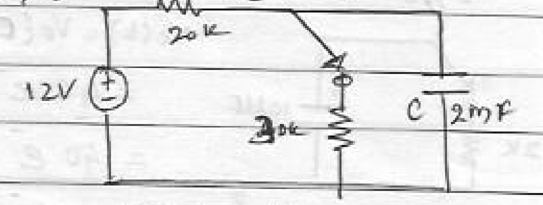
$t \leq 0^-$





$V_c(0^-) = \frac{40K}{(40+20)K} \times 12 = 8V$

At >0



$V(\infty) = \frac{30K}{(30+20)K} \times 12 = 7.2$

$V(t) = V(\infty) + (V_\infty - V_c(0^+)) \cdot e^{-t/RC}$

$= 7.2 + (7.2 - 8)\, e^{-t/RC}$

$= 7.2 + 0.8\, e^{-t/RC}$

(or) $V(t) = V_{(0^+)}\, e^{-t/RC} + V_\infty(1 - e^{-t/RC})$

$= 8e^{-t/RC} + 7.2\left(1 - e^{-t/RC}\right)$

$= 8e^{-t/RC} + 7.2 - 7.2\, e^{-t/RC}$

$= 7.2 + (8 - 7.2)\, e^{-t/RC}$

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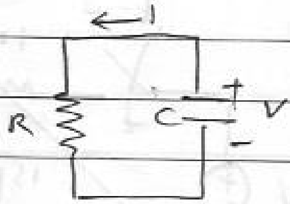
(17)(8) $v = 10 \cdot e^{-4t}$ $i = 0.2\, e^{-4t}$ A $t > 0$.



a) find R and C b) time constant τ

c) Calculate initial energy

d) Time it takes to dissipate 50% of initial energy

a) $R = V/i = \frac{10}{0.2} = 50\,\Omega$ & $RC = 4$ $C = 4/50 = 5\,mF$.

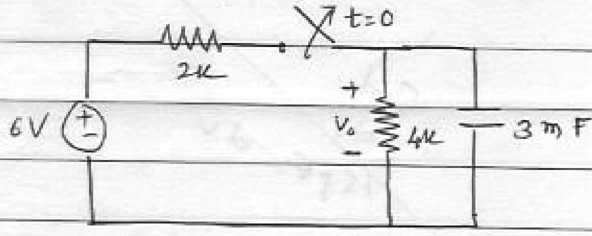
b) $\tau = RC$ $\frac{1}{RC} = 4$ $\therefore RC = 1/4 = 0.25$ msec

c) Initial energy

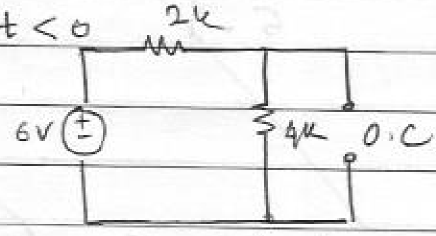
$$w_c(0) = \frac{1}{2} C V^2 = \frac{1}{2} \times 5 \times 10^{-3} \times (10)^2 = 0.25$$

d) ?

(18)(9) Switch is open at $t=0$. find V_0 for $\forall t > 0$

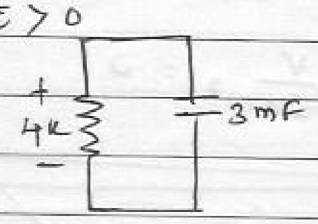


t < 0



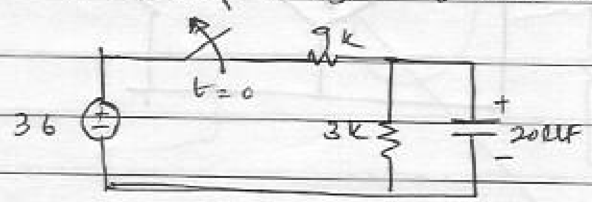
$$V_c(0^-) = \left(\frac{4k}{4k+2k}\right) \times 6 = 4V$$

At $t > 0$

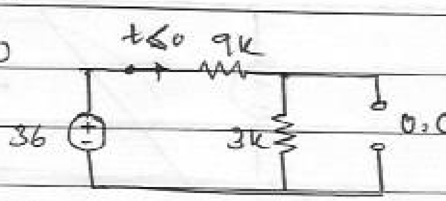


$$V_c(t) = V_0(0^+) \cdot e^{-t/RC}$$
$$= 4 \cdot e^{-t/4k \times 3 \times m}$$
$$= 4 \cdot e^{-t/12} = 4 \cdot e^{-0.0833t}$$

(19)(10) Find $V_c(t)$ for $t > 0$ & find the time necessary for cap voltage to decay to 1/3rd of the value at $t = 0$

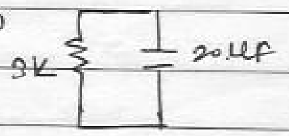


At $t < 0$



$$V_c(0^-) = \left(\frac{3k}{3k+9k}\right) \times 36 = 9V$$

At $t > 0$



$$V_c(t) = V_c(0^+) \cdot e^{-t/RC}$$
$$= 9 \cdot e^{-t/3 \times 20 \times 10^{-3}} = 9 \cdot e^{-t/60m}$$

$$\boxed{V_c(t) = 9 \cdot e^{-16.67t}}$$

$\frac{1}{3} \times 9 = 9 \cdot e^{-t/60m}$

$3 = 9\, e^{-16.67t}$ $\Rightarrow \frac{3}{9} = e^{-t\, 16.67t}$

$\therefore \boxed{t = 65.89 \text{ msec}}$

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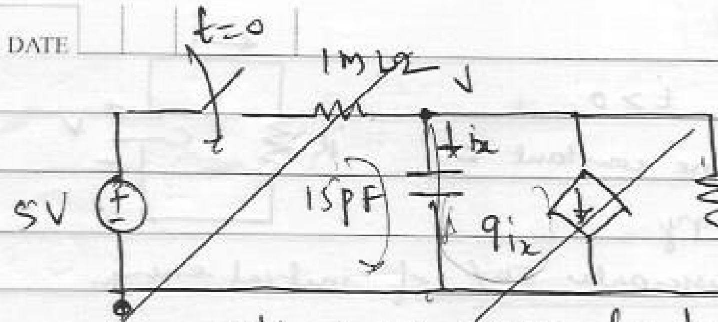
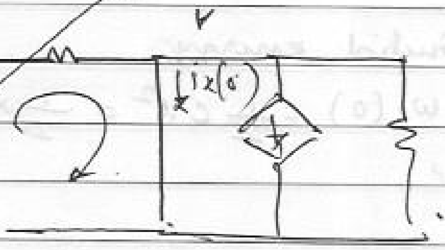


After having open for long time, the switch closes at $t=0$. Find $v_1(t)$ for $t>0$

$i(0^-) = 0$; $i(0^+)$

$i(0)$ $t=0^-$;

$i_x(0^-) = 0$



$t=0^-$

$5 - i$

$C\frac{dv}{dt}$

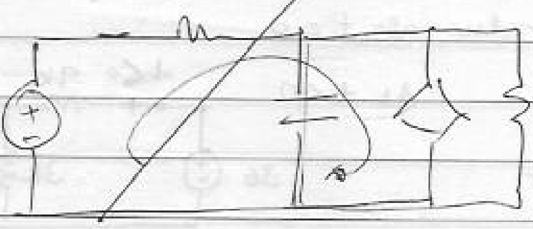
$15pF.\ dv$

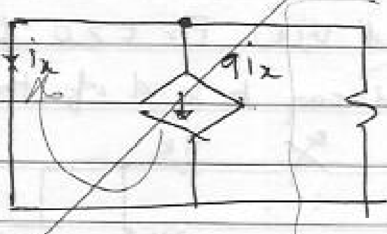
$$\frac{v-5}{1} + C.\frac{dv}{dt} + 9i_x + \frac{v}{2m} = 0$$

$$v-5 + i_x + 9i_x + \frac{v}{2\times10^3} = 0$$

$C.\frac{dv}{dt}$ =

$V_1(t) =$





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4.21) The ckt shown in fig consists of a resistor and a relay with inductance L. The relay is adjusted so that it is actuated when current through the coil is 0.008 Amps. The switch K is closed at t=0. and observed that the relay is actuated when t = 0.1 sec.
Find a) Inductance L of coil b) the equation i(t) with ~~other~~ all terms evaluated

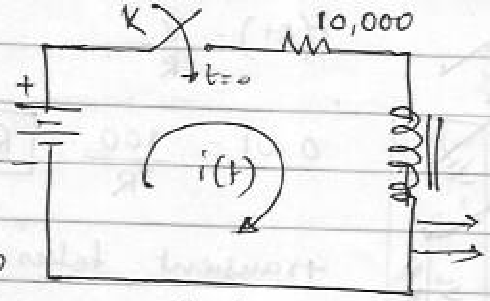
Sol: $i(t) = 0.008$ A

$t = 0.1$ Sec.

$\cancel{4T = 0.1}$

$\cancel{4\frac{L}{R} = 0.1}$

$\cancel{L = \frac{0.1 \times R}{4} = \frac{0.1 \times 10000}{4}}$



$i(0^-) = 0$ as k is open.

At $t > 0$

$$i(\infty) = V/R = 10,\cancel{000}/10000 = 0.01 \text{ Amps.}$$

$$i(t) = i(\infty) - (i(\infty) - i(0^+)) \cdot e^{-\frac{R}{L}t}$$

$$= 0.01 - (0.01 - 0) \cdot e^{-\frac{10000}{L}t}$$

$$i(t) = 0.01 - 0.01 \cdot e^{-\frac{10000}{L} \cdot t}$$

At $t = 0.1$ sec , $i(0.1) = 0.0008$ A

$$0.008 = 0.01 - 0.01 \cdot e^{-\frac{10000 \times 0.1}{L}}$$

$$\cancel{-}\, 2 \times 10^{-3} = \cancel{-}\, 0.01\, e^{-\frac{1000}{L}}$$

$$\cancel{-}\, 0.2 \quad \cancel{\frac{20 \times 10^{-3}}{}} = e^{-1000/L}$$

$$\ln(0.2) = -1000/L$$

$$\cancel{-}\, 1.6094 = \cancel{-}\, 1000 L$$

$$\boxed{L = \frac{1000}{1.6094} = 621.349 \text{ H}}$$

$$\boxed{i(t) = 0.01 (1 - e^{-16.09t})}$$

b) $i(t) = \frac{V}{R} - \left(\frac{V}{R} - i(0^+)\right) e^{-R/L\, t}$

$$= \frac{100}{10000} - \left(\frac{100}{10000} - 0\right) e^{-\frac{10000}{621.349}t}$$

$$\boxed{i(t) = 0.01 - 0.01 \times e^{-16.09t}}$$

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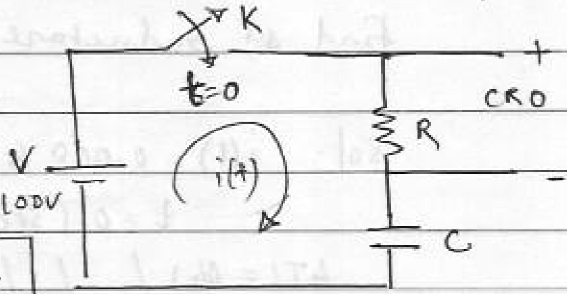
4.20) In the ~~ckt~~ n/w shown, switch k is closed at $t=0$. The current waveform is observed with CRO. The initial value of current is measured to be 0.01 Amps. The transient appears to dissappear in 0.1 sec. find

a) the value of R b) the eqn of $i(t)$

Sol:



At $t=0$; $i(0^+) = 0.01$ Amps.

$$i(0^+) = \frac{V}{R}$$

$$0.01 = \frac{100}{R} \qquad \boxed{R = 10k\Omega}$$

transient takes $4T$ approximately to dissappear

$$4T = 0.1 \text{ sec}$$

$$4RC = 0.1$$

$$4 \times 10 \times 10^3 \, C = 0.1$$

$$\boxed{C = \frac{0.1}{4 \times 10 \times 10^3} = 2.5 \mu F}$$

$$i(t) = \frac{V_0}{R} e^{-t/RC} + \frac{V}{R} e^{-t/RC}$$

$$= \frac{100}{10000} \cdot e^{-t/RC}$$

$$\cancel{i(t) = \underbrace{0.01 \cdot e^{-t/RC}}_{A} - \underbrace{\frac{100}{10 \times 10^3}\left(1 - e^{-t/0.25}\right)}_{B}}$$

$$\cancel{v(t) = V - (V - v(0^+))\, e^{-t/RC}}$$

$$\cancel{= 100 - (100 - v(0^+)) \cdot e^{-\frac{t}{0.25}}}$$

$$\cancel{v(t) = 100 - (100 - v(0^+))\, e^{-4t}}$$

$$\cancel{C \cdot \frac{dv}{dt} = (C \cdot 100 \times -4) - v(0^+)(-4) \cdot e^{-4t}}$$

$$\cancel{= -400C + 4\, v(0^+) \cdot e^{-4t}}$$

$$\cancel{v(t) = v(0^+) \cdot e^{-t/RC} + V(1 - e^{-t/RC})}$$

$$\cancel{i(t) = C \cdot \frac{dv(t)}{dt}}$$

$$\cancel{= C\, v(0^+)\left(-\frac{1}{RC}\right) e^{-t/RC} + C \cdot V\left(-\frac{1}{RC}\right) \cdot e^{-t/RC}}$$

$$\cancel{= -\frac{V(0^+)}{R}}$$

$$\cancel{= -\frac{1}{R}\left(V(0^+) + V\right) \cdot e^{-\frac{t}{RC}}}$$

$$\cancel{= -\frac{V(0^+)}{R} \cdot e^{-t/RC} - \frac{V}{R} \cdot e^{-t/RC}}$$

$$\cancel{v(t) = v(0^+) \cdot e^{-t/RC} + V_s\left(1 - e^{-t/RC}\right)}$$

$$\cancel{i(t) = C \frac{dv(t)}{dt} = C\, v(0^+)\left(-\frac{1}{RC}\right) e^{-t/RC} - C V_s\left(-\frac{1}{RC}\right) e^{-t/RC}}$$

$$\cancel{i(t) = -\frac{v(0^+)}{RC} \cdot e^{-t/RC} + \frac{V_s}{RC}\, e^{-t/RC}}$$

$$i(t) = +\frac{V}{R} \cdot e^{-t/RC}$$

$$= \frac{-100}{10 \times 10^3} \cdot e^{-t/10 \times 10^3 \times 2.5 \times 10^{-6}}$$

$$= +0.01 \cdot e^{-\frac{t}{0.025}}$$

$$\boxed{i(t) = +0.01 \cdot e^{-40t}} \checkmark$$

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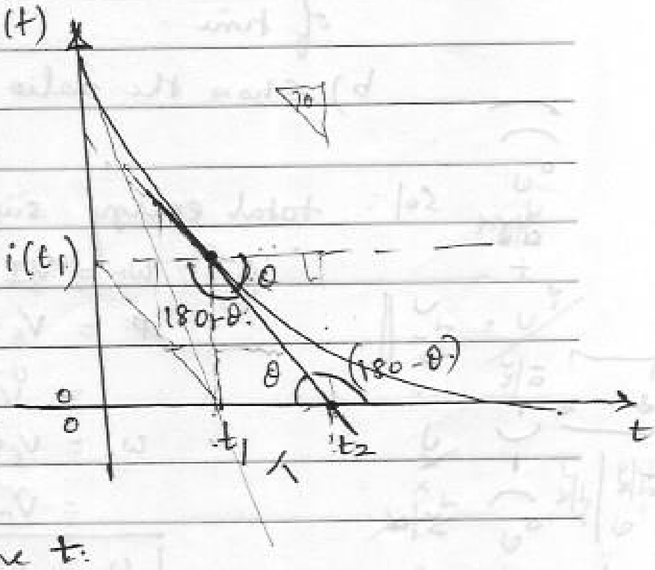
4.23) Consider the exponentially decreasing function $i = K \cdot e^{-t/T}$ where T is time constant. Let the tangent drawn from the curve at $t = t_1$ intersect the line $i = 0$ at t_2. Show that for any such point, $i(t_1)$, $t_2 - t_1 = T$.



$i = K \cdot e^{-t/T}$

slope of i can be found by taking derivative of i

$$\therefore \frac{di}{dt} = \frac{d}{dt}\left(K \cdot e^{-t/T}\right)$$

$$\left[\frac{di}{dt} = \frac{-K}{T} \cdot e^{-t/T}\right] \quad -(1)$$

eqn (1) is slope at any time t.

$\therefore$ Slope at time t_1.

$$\boxed{\frac{di}{dt_1} = -\frac{K}{T} e^{-t_1/T}} \quad -(2) \qquad \& \quad i(t_1) = K \cdot e^{-\frac{t_1}{T}}$$

$$\boxed{\frac{\cancel{\tan\theta} =}{\tan(180-\theta)} \frac{i(t_1)}{(t_2 - t_1)} = \frac{K \cdot e^{-t_1/T}}{(t_2 - t_1)}} \quad -(3) \qquad \therefore -\tan\theta = \frac{K \cdot e^{-t_1/T}}{(t_2 - t_1)}$$

equating (2) & (3)

$$\cancel{-}\left(\frac{K}{T} e^{-\frac{t_1}{\cancel{T}}}\right) = \cancel{-}\frac{\left(\cancel{K \cdot e^{-t_1/T}}\right)}{t_2 - t_1}$$

$$\therefore \boxed{T = t_2 - t_1}$$

(-)ve sign indicates that the curve is decreasing.

t_2 — time at which the tangent taken on curve at t_1 intersects the $i=0$ line.

t_1 — time at which tangent of curve $i(t)$ is taken.

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4.22) A switch is closed at $t=0$, connecting battery of voltage V with series RC circuit.

a) Determine the ratio of energy delivered to the capacitor to the total energy supplied by source as a function of time.

b) Show the ratio approaches 0.50 at $t \to \infty$

Sol: total energy supplied by source

$$W_s = V_s I \cdot t$$

$$\cancel{P} = V_s \int_0^t I \, dt \qquad \text{limit } (0-t) \text{ i.e. After k is closed at } t=0$$

$$= V_s \int_0^q dq$$

$$W_s = V_s q \qquad \because q = \int_0^t I \, dt \qquad i = \frac{dq}{dt}$$

$$= V_s (CV) \qquad q = \underline{CV} \qquad i\,dt = dq$$

$$\boxed{W_s = C V_s^2} \qquad q = \int_0^t i \, dt$$

Voltage a/c the Capacitor

$$V_C(t) = V_s\left(1 - e^{-t/RC}\right)$$

Power absorbed Capacitor

$$P = V_C(t) \times i_C(t)$$

$$= V_s\left(1 - e^{-t/RC}\right) \times C \frac{d}{dt}\left(V_s\left(1 - e^{-t/RC}\right)\right)$$

$$= V_s\left(1 - e^{-t/RC}\right) \times \cancel{C} \times \left(+\frac{1}{R\cancel{C}}\right) V_s \cdot e^{-t/RC}$$

$$= V_s\left(1 - e^{-t/RC}\right) \times \frac{+V_s}{R} \cdot e^{-t/RC}$$

$$= \cancel{\left(\frac{V_s^2}{R} - \frac{V_s^2}{R} e^{-t/RC}\right)}$$

$$= \frac{V_s^2}{R} \cdot e^{-t/RC} - \frac{V_s}{R} \cdot e^{-2t/RC}$$

$$P = \frac{V_s^2}{R}\left[e^{-t/RC} - e^{-2t/RC}\right]$$

b) Ratio = $\dfrac{\text{Energy stored in Cap}}{\text{Energy supplied by the}}$

$$= \frac{\frac{1}{2} C V_s^2}{C V_s^2}$$

$$= \frac{1}{2} = \underline{\underline{0.5}}$$

Energy stored in capacitor

$$\text{Energy} = P \times t$$

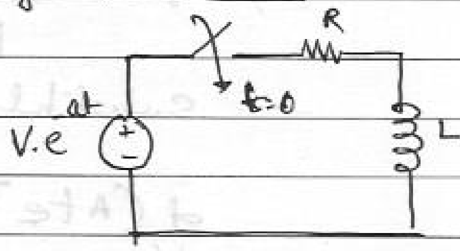
$$= \frac{V_s^2}{R}\int_0^\infty e^{-t/RC} \, dt - \frac{V_s^2}{2}\int_0^\infty e^{-2t/RC} \, dt$$

$$\frac{V_s^2}{2}\left[\frac{e^{-t/RC}}{-1/RC}\right]_0^\infty - \frac{V_s^2}{2}\left[\frac{e^{-2t/RC}}{\frac{-2}{RC}}\right]_0^\infty$$

$$E = \frac{V_s^2}{R}\left[\frac{RC}{1} - \frac{RC}{2}\right] = \frac{V_s^2}{\cancel{R}} \times \frac{+RC}{2} = \left(+\frac{1}{2} C V_s^2\right) \quad (2)$$

$$= \frac{V_s^2}{\cancel{R}}\left[\frac{e^{-t/RC}}{\frac{-1}{RC}}\Big|_0^\infty - \frac{e^{-2t/RC}}{\frac{-2}{RC}}\Big|_0^\infty\right] = \frac{V_s^2}{\cancel{R}}\left[-RC\left(e^{-\infty} - e^{0}\right) + \frac{RC}{2}\left(e^{-\infty} - e^{0}\right)\right] = \frac{V_s^2}{\cancel{R}}\left[RC - \frac{RC}{2}\right] = \frac{V_s^2}{\cancel{R}} \times \frac{RC}{2} = \frac{V_s^2 C}{2}$$

(i) A RL circuit with driving force voltage $v(t) = V \cdot e^{-at}$



By KVL

$$\left[L \frac{di}{dt} + Ri = V \cdot e^{-at} \right]$$

$$\left[\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \cdot e^{-at} \right] \quad -(1)$$

For this, complete Response $i(t) = i_c(t) + i_p(t)$.

Case (i) : complementary sol is obtained for zero i/p.

$\therefore \frac{di}{dt} + \left(\frac{R}{L}\right) i = 0$. which has ~~of~~ characteristic eq² of the for $\left(s + \frac{R}{L}\right)$

The sol² of such characteristic eq² is given by

$$i_c(t) = K \cdot e^{+st} \quad -(2)$$

for given differential eq² $s = -R/L$.

$$\therefore \left[i_c(t) = K \cdot e^{-\frac{R}{L}t} \right] \quad -(3)$$

Case (ii) : Particular sol²

a) If $a \neq R/L$; then particular sol² can be

$\left[i_p(t) = A \cdot e^{-at} \right] -(4)$ $\because$ for a LTI system the particular sol² will be in the same form as that of input.

substitute (4) in (1) and differentiate

$$\frac{d}{dt}\left(A \cdot e^{-at}\right) + \frac{R}{L}\left(A \cdot e^{-at}\right) = \frac{V}{L} e^{-at}$$

$$-Aa \cdot \cancel{e^{-at}} + \frac{R}{L} \cdot A \cancel{e^{-at}} = V/L \cdot \cancel{e^{-at}}$$

$$-Aa + R/L A = V/L$$

$$\therefore A\left(R/L - a\right) = V/L$$

$$A = \frac{V}{L\left(\frac{R}{L} - a\right)} = \frac{V}{(R - La)}$$

$$\therefore \boxed{i_p(t) = A \cdot e^{-at} = \frac{V}{(R - La)} \cdot e^{-at}} \quad -(5)$$

$$\boxed{\begin{aligned} i(t) &= i_c(t) + i_p(t) \\ &= K e^{-\frac{R}{L}t} + \left(\frac{V}{R - La}\right) \cdot e^{-at} \end{aligned}} \quad -(6)$$

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(case b) if $a = R/L$, then particular sol^n be

$$i_p(t) = A \cdot t e^{-at}$$

Substituting in differential eq^n and solving

$$\frac{d}{dt}\left(Ate^{-at}\right) + \frac{R}{L}\left(A \cdot t \cdot e^{-at}\right) = \frac{V}{L} e^{-at}$$

$$A \cdot \left[t(-a) \cdot e^{-at} + e^{-at} \times 1\right] + \frac{R}{L} A t e^{-at} = \frac{V}{L} e^{-at}$$

$$-aAt\cancel{e^{-at}} + A \cdot \cancel{e^{-at}} + \frac{R}{L} A t \cancel{e^{-at}} = \frac{V}{L}\cancel{e^{-at}}$$

$$A\left(-at + 1 + \frac{R}{L}t\right) = \frac{V}{L}$$

As $a = \frac{R}{L}$

$$A\left(-\cancel{\frac{R}{L}t} + 1 + \cancel{\frac{R}{L}t}\right) = V/L$$

$$A = \frac{V}{L}$$

Hence $i_p(t) = Ate^{-at}$

$$\boxed{i_p(t) = \left(\frac{V}{L}\right) t \cdot e^{-at}} \quad (7)$$

$\therefore \quad i(t) = i_c(t) + i_p(t)$

$$\cancel{= K \cdot e^{-\frac{R}{L}t}}$$

$$\boxed{i(t) = K \cdot e^{-R/L\,t} + \left(\frac{V}{L}\right) t \cdot e^{-at}} \quad -(8)$$

Arbitrary constant K is then determined by the initial condition in each case.

by substituting $i(0+)$ at $t=0$ in eq^n (6) & (8).

Ex: In (8)

$$i(t) = k \cdot e^{-\frac{R}{L}t} + \frac{V}{L} t \cdot e^{-at}$$

$$i(0^+) = k \cdot e^0 + \frac{V}{L}\cancel{(0)}\, e^0$$

$$k = i(0^+) = \underline{0}$$

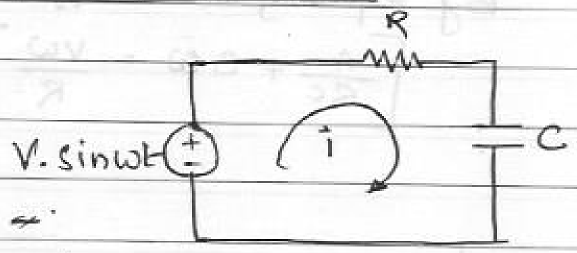
In (6)

$$i(0^+) = k \cdot e^0 + \frac{V}{(R - La)} e^0$$

$$0 = k + \left(\frac{V}{R - La}\right)$$

$$\left(k = -\frac{V}{R - La}\right)$$

* Consider an RC network with the excitation $V \sin(\omega t)$



By KVL

$$Ri + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = V.\sin\omega t$$

By differentiating and rearranging

$$R.\frac{di}{dt} + \frac{1}{C}i = V.\omega.\cos\omega t$$

$$\left[\frac{di}{dt} + \frac{1}{RC}i = \frac{V.\omega.\cos\omega t}{R}\right] \quad -(1)$$

Case (i) : Complementary sol is obtained for zero input condn

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

Auxillary eqn $\left[si + \frac{1}{RC}i = 0\right]$ $-(2)$ which is ~~hence~~

The characteristic eqn is of the form $s + \frac{1}{RC} = 0$

For the soln for Auxillary eqn of from given in (2)

$$\left[i_c(t) = k.e^{st}\right] -(3)$$

$$\therefore \quad i_c(t) = k.e^{-t\left(\frac{1}{RC}\right)} \quad \because s = -\frac{1}{RC}$$

Case (ii) : Particular soln for eqn (1) will be of the form

$$i_p(t) = A\cos\omega t + B\sin\omega t \quad -(4)$$

$\because$ for any LTI system, the particular soln will be in the same form as that of input

By substituting (4) in (1) and differentiating

$$\frac{d(A\cos\omega t + B\sin\omega t)}{dt} + \frac{1}{RC}(A\cos\omega t + B\sin\omega t) = \frac{V}{R}\omega\cos\omega t$$

$$-\omega A.\sin\omega t + B\omega\cos\omega t + \frac{A}{RC}\cos\omega t + \frac{B}{RC}\sin\omega t = \frac{V}{R}\omega.\sin\omega t$$

By rearranging

$$\left[\left(B\omega + \frac{A}{RC}\right)\cos\omega t + \left(\frac{B}{RC} - \omega A\right)\sin\omega t = \frac{V}{R}\omega.\cos\omega t + 0.\sin\omega t\right] \quad -(5)$$

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By equating the coefficient on LHS with RHS:

$$\left[\frac{A}{RC} + B\omega = \frac{V\omega}{R}\right] -(5) \qquad \left[\frac{B}{RC} - A\omega = 0\right] -(6)$$

$$\frac{B}{RC} = A\omega$$

$$\therefore \left[B = A\omega RC\right] -(7)$$

(7) in (5)

$$\frac{A}{RC} + (A\omega RC).\omega = \frac{V\omega}{R}$$

$$A\left(\frac{1}{RC} + \omega^2 RC\right) = \frac{V\omega}{R}$$

$$A\left(\frac{1+\omega^2R^2C^2}{RC}\right) = \frac{V\omega}{R}$$

$$\left[A = \frac{V\omega}{\cancel{R}} \times \frac{\cancel{R}C}{(1+\omega^2R^2C^2)} = \frac{V\omega C}{(1+\omega^2R^2C^2)}\right] -(8)$$

$$\therefore \left[B = A\omega RC = \left(\frac{V\omega C \times \omega RC}{1+\omega^2R^2C^2}\right) = \frac{\omega^2 RC^2 V}{1+\omega^2R^2C^2}\right] (9)$$

Now substituting (8) & (9) in eq² (4)

$$i_p(t) = A \cdot \cos\omega t + B \cdot \sin\omega t$$

$$= \frac{V\omega C}{(1+\omega^2R^2C^2)} \cdot \cos\omega t + \frac{\omega^2RC^2V}{(1+\omega^2R^2C^2)} \cdot \sin\omega t$$

$$= \frac{V}{(1+\omega^2R^2C^2)}\left[\omega C \cdot \cos\omega t + \omega^2 R C^2 \cdot \sin\omega t\right]$$

$$= \frac{V}{\left(\frac{1}{\omega^2C^2} + R^2\right)}\left[\frac{\cancel{\omega C} \cdot \cos\omega t}{\omega^{\cancel{2}} C^{\cancel{2}}} + \frac{\cancel{\omega^2} \cancel{C^2} R \cdot \sin\omega t}{\cancel{\omega^2 C^2}}\right]$$

$$\boxed{i_p(t) = \frac{V}{\left(R^2 + \frac{1}{\omega^2C^2}\right)}\left[\frac{\cos\omega t}{\omega C} + R \cdot \sin\omega t\right]} -(10)$$

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In eq² (10) Substitute $\frac{1}{\omega C} = k \cdot \cos\theta$ and $R = k \cdot \sin\theta$

$$i_p(t) = \left(\frac{V}{R^2 + \frac{1}{\omega^2 C^2}}\right)\left[k \cdot \cos\theta \cdot \cos\omega t + k\sin\theta \cdot \sin\omega t\right]$$

$$\left[i_p(t) = \left(\frac{KV}{R^2 + \frac{1}{\omega^2 C^2}}\right)\left[\cos(\omega t - \theta)\right]\right] \quad \text{— (11)}$$

Now,

$$\frac{\cancel{K} \cdot \sin\theta}{\cancel{K}\cos\theta} = \frac{R}{1/\omega C} = R\omega C$$

$$\therefore \tan\theta = R\omega C \quad \boxed{\theta = \tan^{-1}(R\omega C)} \quad \text{— (a)}$$

$$K^2 \sin^2\theta + k^2\cos^2\theta = R^2 + \frac{1}{\omega^2 C^2}$$

$$K^2 = R^2 + \frac{1}{\omega^2 C^2}$$

$$K = \sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)} \quad \text{— (b)}$$

substituting K and θ from (a), (b) to eq² (11)

$$i_p(t) = \left[\frac{V\left(\sqrt{R^2 + \frac{1}{\omega^2 C^2}}\right)}{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}\right]\left(\cos(\omega t - \tan^{-1}(\omega RC)\right)$$

$$\boxed{i_p(t) = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos\left(\omega t - \tan^{-1}(\omega RC)\right)} \quad \text{— (12)}$$

$$i(t) = i_c(t) + i_p(t) \quad \text{— (13)}$$

& k in $i_c(t)$ is found by using initial conditions applied to eq² ~~(12)~~ (13) at $t = 0$ ∴ $i(0^+) = 0$.

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$$i(t) = k \cdot e^{-\frac{t}{RC}} + \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos\left(\omega t - \tan^{-1}\omega RC\right)$$

At $t=0$, $i(0^+) = 0$.

$$0 = k \cdot e^{0} + \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos\left(0 - \tan^{-1}\omega RC\right)$$

$$\boxed{k = + \left(\frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}\right) \cos\left(\tan^{-1}\omega RC\right)}$$

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Step 1 – Draw the equivalent ckt for $t=0^+$, based on equivalent element representation for L, C based on initial cond^n

Element	$i(0^-)$	$i(0^+)$ [Initial]	$i(\infty)$ [final]
L	0, $\rightarrow i=0$	$i(0^+)=0$	SC
	$\rightarrow I_0$	I_0	
C	$i(0^-)$ relaxed	$i(0^+)$ S.C	$i(\infty)$ O.C — Capacitor is charged to CV
	V_0	V_0	V_0 O.C.

2) For each ~~nfw~~ element in the nfw, we must determine just what will happen when switching action takes place. From this analysis, a new schematic of an equivalent nfw for $t=0^+$ may be constructed according to these rules.

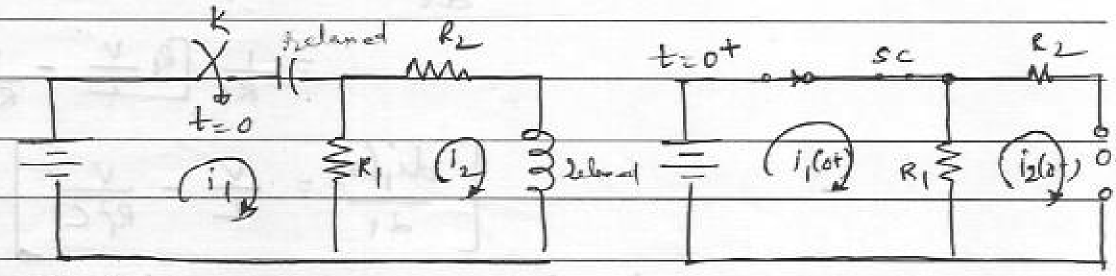
a) Replace all inductor with open circuit (or) with current generator having the value of current flowing at $t=0^+$.

b) Replace all capacitor with S.C (or) with voltage src of voltage $V_0 = q_0/C$ if there is an initial charge.

c) Resistor as it is – Left without change

3) Using KVL/KCL get the ~~derivatives~~. Integrodifferential eq^{n}s are written based on ckt obtained in step 2

Example: find $\frac{di_1(0^+)}{dt}$ & $\frac{di_2(0^+)}{dt}$



At $t=0^+$

from the equivalent ckt is clear that

$$i_1(0^+) = V/R_1 \quad \text{and} \quad i_2(0^+) = 0$$

Apply KVL to both the loops

$$\left.\begin{aligned} &\frac{1}{C}\int_{-\infty}^{t} i_1(\tau)\,d\tau + R_1(i_1 - i_2) = V \quad -(1) \\ &R_1(i_2 - i_1) + R_2 i_2 + L\frac{di_2}{dt} = 0 \quad -(2) \end{aligned}\right\} \text{these eq}^n \text{ hold good for all } t.$$

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Eq^n (2) can be written as

$$L\frac{di_2}{dt} = -R_1(i_2-i_1) - R_2 i_2$$

$$= -R_1 i_2 + R_1 i_1 - R_2 i_2$$

$$L\cdot\frac{di_2}{dt} = R_1 i_1 - i_2(R_1+R_2)$$

$$\boxed{\frac{di_2}{dt} = \frac{1}{L}\left[R_1 i_1 - \cancel{i_2}(R_1+R_2)\,i_2\right]} \quad -\text{(general)}$$

$-(3)$

At $t=0+$

$$\frac{di_2}{dt}(0^+) = \frac{1}{L}\left[R_1\, i_1(0^+) - (R_1+R_2)\, i_2(0^+)\right] \quad - t=0+$$

$$\frac{di_2}{dt}(0^+) = \frac{1}{L}\left[\cancel{R_1}\times\frac{V}{\cancel{R_1}} - (R_1+R_2)\cdot 0\right]$$

$$\left[\frac{di_2}{dt}(0^+) = \frac{V}{L}\right]$$

For (1), take differentiation

$$\boxed{\frac{i_1}{C} + R_1\frac{di_1}{dt} - R_1\frac{di_2}{dt} = 0} \quad -\text{(general)}$$

$-(4)$

At $t=0+$

$$\cancel{\frac{i_1(0^+)}{C}} \cdot R_1\frac{di_1}{dt} = R_1\cdot\frac{di_2}{dt} - \frac{i_1}{C}$$

$$R_1\frac{di_1(0^+)}{dt} = \frac{1}{R_1}\left[R_1\cdot\frac{di_2(0^+)}{dt} - \frac{i_1(0^+)}{C}\right]$$

$$\frac{di_1(0^+)}{dt} = \frac{1}{\cancel{R_1}}\left(\cancel{\frac{V}{R_1 C}}\right)$$

$$= \frac{1}{R_1}\left[R_1\frac{V}{L} - \frac{V}{R_1 C}\right]$$

$$\left[\frac{di_1(0^+)}{dt} = \frac{V}{L} - \frac{V}{R_1^2 C}\right]$$

By taking differentiation of (3) & (4) 2nd order differential eq^ns can be obtained.

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* Second order System

Consider

$$a_0\frac{d^2i}{dt^2}+a_1\frac{di}{dt}+a_2 i=0 \quad \text{— (1)}$$

if $i(t)=k\cdot e^{st}$

then,

$$a_0\frac{d^2(k\cdot e^{st})}{dt^2}+a_1\frac{d}{dt}(k\cdot e^{st})+a_2(k\cdot e^{st})=0$$

$$a_0 s^2 k\cdot e^{st}+a_1 s\cdot k\cdot e^{st}+a_2 k\cdot e^{st}=0$$

$$(a_0 s^2+\underline{\underline{a_1 s}}+a_2)\,k\cdot e^{st}=0 \quad \text{— (2)}$$

In this,

$a_0s^2+a_1s+a_2$, is the characteristic or auxillary polynomial

Evidently, the roots are

$$s_1,s_2=-\frac{a_1}{2a_0}\pm\frac{\sqrt{a_1^2-4a_0a_2}}{2a_0}$$

∴ with roots s_1 & s_2, the characteristic eqn (2) can be written

$i_1(t)=k\cdot e^{s_1t}$ & $i_2(t)=k\cdot e^{s_2t}$

result is

$$(a_0s_i^2+a_1s_i+a_2)\,k\cdot e^{s_it}=0 \quad \text{— (3)}$$

and hence are soln of the second order differential eqns.

i_1 & i_2 are solutions, then so is any linear combo.

αi_1 and βi_2 for all $\alpha,\beta\in\mathbb{R}$

∴ The general soln

$$\boxed{i(t)=k_1e^{s_1t}+k_2e^{s_2t}} \quad \text{— (4)}$$

the roots s_1,s_2 may be

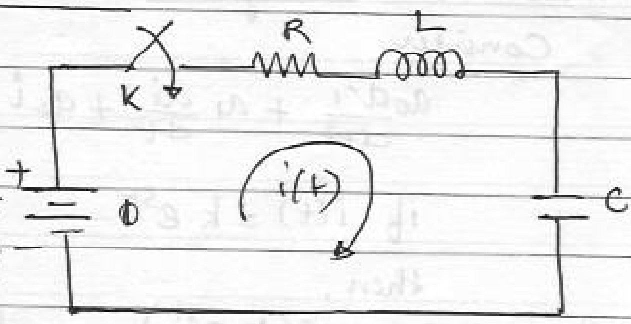
(i) Real and different

(ii) Real and equal or

(iii) Complex conjugate pair.

Let us consider a series RLC n/w

By applying KVL



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = V \quad \text{—(1)}$$

Differentiating

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \quad \text{—(2)}$$

Thus auxillary equations is

$$\left(LS^2 + RS + \frac{1}{C}\right)k\,e^{st} = 0$$

where $LS^2 + RS + \frac{1}{C}$ is auxillary eqn

÷ By L to characteristic (auxillary eqn)

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

Clearly

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}}{2\times 1}$$

$\therefore i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$

$$= \frac{-R}{2L} \pm \sqrt{\frac{1}{4}\left(\frac{R}{L}\right)^2 - \frac{4}{4}\left(\frac{1}{LC}\right)}$$

Xplyng by 2 $= \frac{-R}{2L}\times 2 \pm \sqrt{\frac{4}{4}\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}$

$$\boxed{S_1, S_2 = -\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}} \quad \text{—(3)}$$

Example: If $R = 3\Omega$ $L = 1H$ and $C = \frac{1}{2}F$ in the ckt

$$1S^2 + 3S + \frac{1}{(1/2)} = 0 \qquad S^2 + 3S + 2 = 0.$$

Roots are

$$S^2 + 3S + 2 = (S+1)(S+2) = 0$$

$$\therefore S_1 = -1, \; S_2 = -2$$

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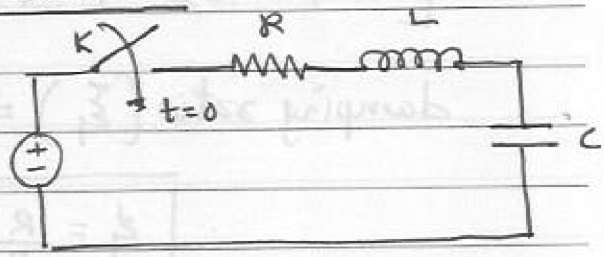
* Series RLC Circuit driven by D.C Excitation



$t=0^-$;

$i_0(0^-) = 0 = i(0^+)$

~~t>0~~; K is closed at $t=0$

$\therefore \quad i(0^+) = i(0^-)$ as current through inductor cannot change instantaneously

By KVL,

$$L\cdot\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i\cdot d\tau = V \quad -(1)$$

$$L\cdot\frac{di}{dt} + Ri + \frac{1}{C}\cancel{\int_{-\infty}^{0} i\cdot d\tau} + \frac{1}{C}\int_{0}^{t} i\cdot d\tau = 0$$

By differentiating

$$\left[L\frac{d^2i}{dt^2} + R\cdot\frac{di}{dt} + \frac{1}{C}\,i = 0\right] \quad -(2)$$ Homogeneous 2st order diff eq²

The characteristic eq² can be written as

$$\cancel{L}\cdot s^2 i + \left(\frac{R}{L}\right)si + \frac{1}{LC}\,i = 0$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)i = 0$$ where $i = k\cdot e^{st}$ as it is general sol² for homogeneous diff eq²

The roots of $\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$ ~~are~~ are

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-\left(\frac{R}{L}\right) \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\times\frac{1}{LC}}}{2}$$

$$s_1, s_2 = \left[-\left(\frac{R}{2L}\right) \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}\right] \quad -(3)$$

where α = damping factor $= \frac{R}{2L}$

ω_s = Natural freq $= \frac{1}{\sqrt{LC}}$

$$\therefore \quad s_1, s_2 = \left[-\alpha \pm \sqrt{\alpha^2 - \omega_s^2}\right] \quad -(4)$$

when ~~cotate~~ char eq² $\left(s^2 + 2\left(\frac{R}{2L}\right)s + \left(\frac{1}{\sqrt{LC}}\right)^2\right)i = 0$

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$$\left[s^2 + 2\alpha s + \omega_s^2 = 0 \right] \quad -(4)$$

$$\text{damping ratio } (\xi_1) = \frac{\alpha}{\omega_s} = \frac{R/2L}{1/\sqrt{LC}} = \frac{R}{2L} \times \sqrt{L}\sqrt{C} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\boxed{\xi_1 = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

Eq^n (4) can be written as.

$$\left[s^2 + 2\xi_1 \omega_s s + \omega_s^2 = 0 \right] \quad -(5)$$

and s_1, s_2 can be written as

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$$

$$= -(\xi_1 \omega_s) \pm \sqrt{(\xi_1 \omega_s)^2 - \omega_s^2}$$

$$\boxed{s_1, s_2 = -\xi_1 \omega_s \pm \omega_s\sqrt{\xi_1^2 - 1}} \quad -(6)$$

The roots s_1, s_2

i) Real, Negative and Unequal.	$\alpha > \omega_s$, $\xi_1 > 1$
ii) Real, Negative and Equal	$\alpha = \omega_s$, $\xi_1 = 1$
iii) Complex conjugate with α negative	$\alpha < \omega_s$ $0 < \xi_1 < 1$
iv) Imaginary conjugate.	$\alpha = 0$, $\xi_1 = 0$.

(i) s_1, s_2 are real, Negative and Unequal.

$\therefore$ General sol^n of $i(t)$ w.r.t s_1, s_2 is

$$\left[i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} \right] \quad -(7)$$

where k_1 & k_2 are found based on initial conditions.

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Case (i) : $S_1, S_2 \rightarrow$ Real, Negative and Unequal.

These roots are obtained when

$\alpha > \omega_s$ ie $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$ $\therefore$ $\xi_1 > 1$

$\rightarrow R > \frac{2L}{\sqrt{LC}}$

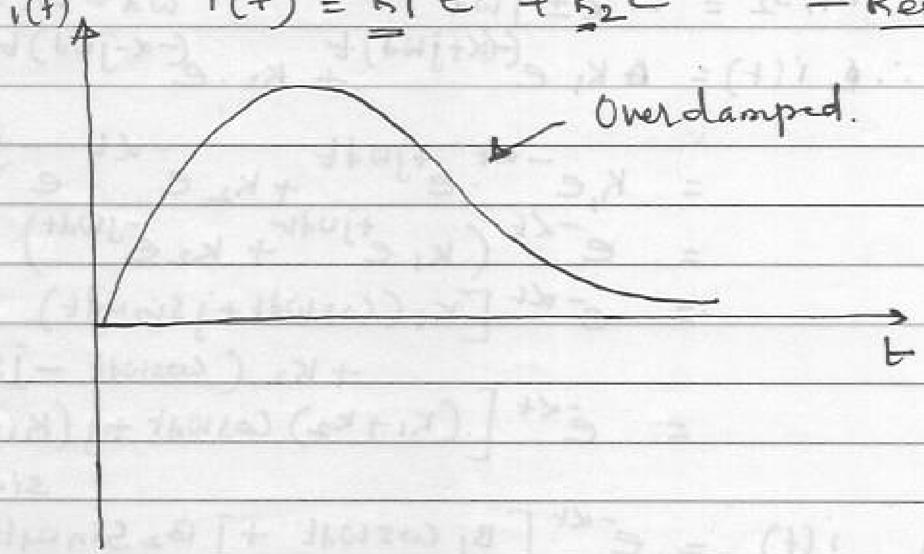
$R^2 > \frac{4L^2}{LC} > \frac{4L}{C}$

$$\therefore \; i(t) = k_1 e^{+s_1 t} + k_2 e^{s_2 t}$$

Ex: $S_1 = -1$ & $S_2 = -2$, then

$$i(t) = k_1 e^{-1t} + k_2 e^{-2t}$$

— Nature of Response overdamped



Case (ii) : S_1, S_2 are real, negative and equal

These roots are obtained where

$\alpha = \omega_s$; $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ $\xi_1 = \frac{\alpha}{\omega_s} = 1$.

$$\left[R = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} = R_{cr} \right]$$

Now,

$$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$$

$$= -\alpha \pm \sqrt{\omega_s^2 - \omega_s^2} \qquad \alpha = \omega_s.$$

$$S_1, S_2 = -\alpha$$

$$\therefore \; i(t) = k_1 e^{-\alpha t} + k_2 e^{-\alpha t}$$

$$\boxed{i(t) = e^{-\alpha t}(k_1 + k_2)}$$

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Case (iii) : Roots (s_1, s_2) are complex conjugate

- When $\alpha < \omega_s$.

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}} \qquad R < \frac{2L}{\sqrt{LC}} \qquad R < 2\sqrt{\frac{L}{C}} = R_{cr}$$

$$\rightarrow s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$$

$$= -\alpha \pm \sqrt{-(\omega_s^2 - \alpha^2)}$$

$$= -\alpha \pm j\sqrt{\omega_s^2 - \alpha^2}$$

$$s_1, s_2 = -\alpha \pm j\omega d \qquad \omega d = \omega_s^2 - \alpha^2$$

$$\therefore i(t) = K_1 e^{(-\alpha + j\omega d)t} + K_2 \cdot e^{(-\alpha - j\omega d)t}$$

$$= K_1 e^{-\alpha t} \cdot e^{+j\omega dt} + K_2 e^{-\alpha t} \cdot e^{-j\omega dt}$$

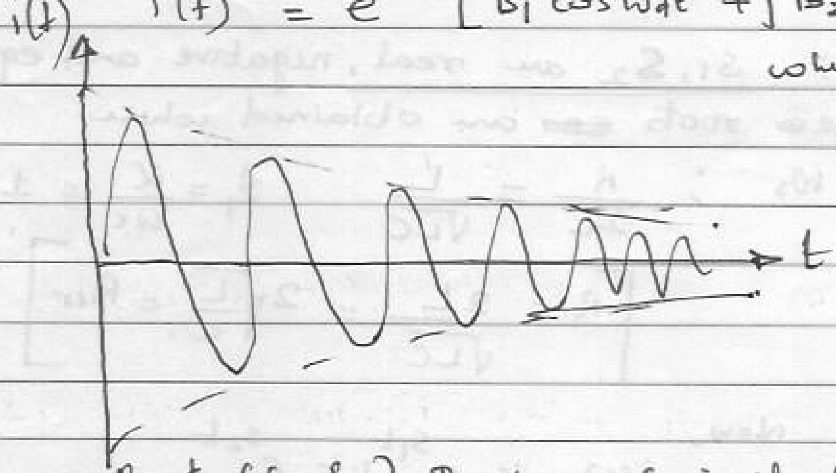
$$= e^{-\alpha t}\left(K_1 e^{+j\omega dt} + K_2 e^{-j\omega dt}\right)$$

$$= e^{-\alpha t}\left[K_1(\cos \omega dt + j\sin \omega dt) + K_2(\cos \omega dt - j\sin \omega dt)\right]$$

$$= e^{-\alpha t}\left[(K_1 + K_2)\cos \omega dt + j(K_1 - K_2)\sin \omega dt\right]$$

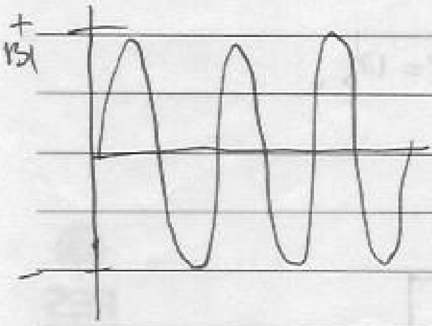
$$i(t) = e^{-\alpha t}\left[B_1 \cos \omega dt + j B_2 \sin \omega dt\right]$$

where $\omega d = \omega_s - \alpha$.



Case (iv): Roots (s_1, s_2) Imaginary Conjugate $(\pm j\omega_s)$

When $\alpha = 0$



$$s_1, s_2 = -0 \pm \sqrt{0^2 - \omega_s^2}$$

$$= \pm j\sqrt{\omega_s^2} = \pm j\omega_s.$$

$$\therefore i(t) = K_1 e^{+j\omega_s t} + K_2 e^{-j\omega_s t}$$

$$= K_1(\cos \omega_s t + j\sin \omega_s t) + K_2(\cos \omega_s t - j\sin \omega_s t)$$

$$i(t) = (K_1 + K_2)\cos \omega_s t + j(K_1 - K_2)\sin \omega_s t$$

$$= B_1 \cos \omega_s t + j B_2 \sin \omega_s t.$$

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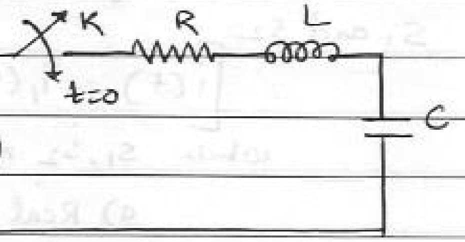
Transient Response of Series RLC with DC Excitation



At $t = 0^-$

$i(0^-) = 0$

At $t = 0^+$

$i(0^+) = i(0^-) = 0 \quad , \quad V_C(0^+) = 0$

By KVL,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i \cdot d\tau = V \quad -(1)$$

$$L\frac{di(0^+)}{dt} = V - R\cancel{i} - \frac{1}{C}\cancel{\int_{-\infty}^{t} i\, d\tau\, dt}$$

$$\boxed{\frac{di(0^+)}{dt} = \frac{V}{L}}$$

Differentiating

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad -(2)$$

It eq² is a Homogenous 2nd order differential eq² where $i(t) = k \cdot e^{st}$ form.

Auxillary eq² / characteristic eq² can be written as

$$s^2 i + \left(\frac{R}{L}\right)s i + \frac{1}{LC} i = 0$$

$$\left(s^2 + (R/L)s + (1/LC)\right)i = 0 \quad \left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)k e^{st} = 0 \quad -(3)$$

Now root of $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ given by

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\left(\frac{R}{2L}\right) \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2} \quad -(4)$$

Damping Ratio

$$\boxed{\xi = \frac{\alpha}{\omega_s} = \frac{R/2L}{1/\sqrt{LC}} = \frac{R}{2}\sqrt{\frac{L}{C}}}$$

where $\frac{1}{\sqrt{LC}} = \omega_s$ = Natural frequency
$\frac{R}{2L} = \alpha$ = Damping factor. $\Big\}$ $-(5)$

$$\boxed{s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_s^2}} \quad -(6)$$

$$s^2 + 2\left(\frac{R}{2L}\right) \pm \cancel{\sqrt{\left(\frac{R}{2L}\right)^2}} \left(\frac{1}{\sqrt{LC}}\right)^2 = 0$$

$$\left[s^2 + 2\alpha + \omega_s^2 = 0\right] - (7)$$

$\alpha = \xi\omega_s$ then $\left[s^2 + 2\xi\omega_s + \omega_s^2 = 0\right] - (8)$

then

$$s_1, s_2 = -\xi\omega_s \pm \sqrt{(\xi\omega_s)^2 - \omega_s^2}$$

$$\boxed{s_1, s_2 = -\xi\omega_s \pm \omega_s\sqrt{(\xi)^2 - 1}} \quad -(9)$$

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Roots s_1, s_2 ~~may be~~ will lead to two components of i w.r.t s_1 and s_2

$$\therefore \boxed{i(t) = i_1(t) + i_2(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}} \quad -(10)$$

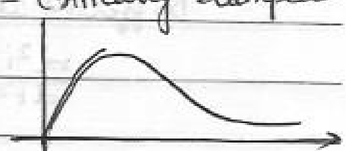
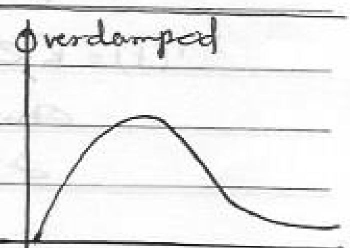
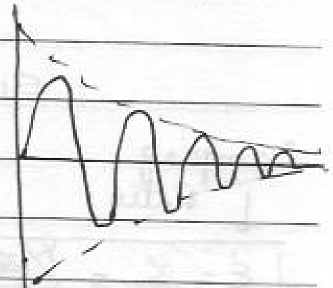
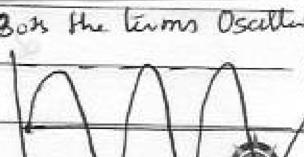
while s_1, s_2 may

a) Real, Negative and Unequal

b) Real, Negative and Equal

$-\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$ c) Complex conjugate with Negative α.

d) Complex Conjugate with $\alpha = 0$; ie Imag Conjugate

	Damping Ratio	Nature of Roots	i(t)	Nature of Response
$\alpha = \omega_s$ $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ $\oint R = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} = R_{cr}$	$\xi = 1$	Real, Negative & Equal $0 - \alpha \pm \sqrt{\omega_s^2 - \omega_s^2}$ $\therefore s_1, s_2 = -\alpha$	$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$ $i(t) = k_1 e^{-\alpha t} + k_2 e^{-\alpha t}$ $= e^{-\alpha t}(k_1 + k_2)$	- Critically damped 
$\alpha > \omega_s$ $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$	$\xi > 1$	Real, Negative & Not equal $-s_1, s_2$ where $s_1 \neq s_2$ Ex: $s_1 = -1$ & $s_2 = -2$	$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$ $i(t) = k_1 e^{-s_1 t} + k_2 e^{-s_2 t}$ Ex: $= k_1 e^{-1t} + k_2 e^{-2t}$	Overdamped 
$\alpha < \omega_s$ $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$	$\xi < 1$	$-\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$ $-\alpha \pm \sqrt{-(\omega_s^2 - \alpha^2)}$ $-\alpha \pm j\sqrt{\omega_s^2 - \alpha^2}$ $s_1 s_2 (-\alpha \pm j\omega_d)$ $s_1, s_2 = -\alpha \pm j\omega_d$ where $\omega_d = \sqrt{\omega_s^2 - \alpha^2}$ Complex Conjugate	$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$ $= k_1 e^{[-\alpha + j\omega_d]t} + k_2 e^{[-\alpha - j\omega_d]t}$ $= k_1 e^{-\alpha t} \cdot e^{j\omega_d t} + k_2 e^{-\alpha t} e^{-j\omega_d}$ $= e^{-\alpha t}(k_1 e^{j\omega_d t} + k_2 e^{-j\omega_d t})$ $= e^{-\alpha t}(k_1(\cos\omega_d t + j\sin\omega_d t) + k_2(\cos\omega_d t - j\sin\omega_d t))$ $= e^{-\alpha t}[(k_1 + k_2)\cos\omega_d t + j(k_1 - k_2)\sin\omega_d t]$ $i(t) = e^{-\alpha}[B_1 \cos\omega_d t + B_2 \sin\omega_d t]$ when $\omega_d = \omega_s - \alpha$	underdamped 
$\alpha = 0$	$\xi = 0$	$s_1 s_2 = \pm\sqrt{-j\omega_s^2}$ $= \pm j\sqrt{\omega_s^2}$ $= \pm j\omega_s$	$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$ $= k_1 e^{+j\omega_s t} + k_2 e^{-j\omega_s t}$ $= k_1(\cos\omega_s t + j\sin\omega_s t) + k_2(\cos\omega_s t - j\sin\omega_s t)$ $= (k_1 + k_2)\cos\omega_s t + j(k_1 - k_2)\sin\omega_s t$ $= B_1 \cos\omega_s t + jB_2 \sin\omega_s t$	Both the terms Oscillatory 

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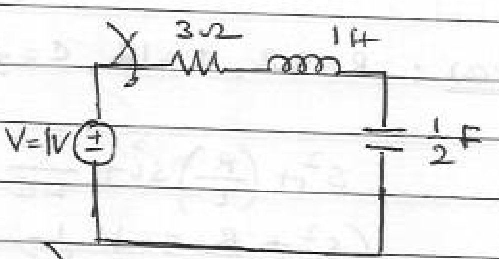
$\boxed{Ex (1)}$: $i(0^-) = 0$



At $t = 0^+$; $i(0^+) = i(0^-)$

$$\frac{di}{dt}(0^+) = \frac{V}{L} = \frac{1}{1} = 1 \text{ Amp/sec}$$

KVL
$$\left(\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0\right) \quad -(1)$$

$$s^2 i + \left(\frac{R}{L}\right)si + \frac{1}{LC}i = 0 \qquad \left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)i = 0.$$

∴ Chara eqn will be

$$\left[s^2 + \left(\frac{3}{1}\right)s + \left(\frac{1}{1\times\frac{1}{2}}\right)\right] i = 0.$$

$$\alpha = \frac{R}{2L} = \frac{3}{2\times1} = 1.5$$

$$(s^2 + 3s + 2)k \cdot e^{st} = 0.$$

$$\omega_s = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times0.5}} = \sqrt{2}$$

Roots of $s^2 + 3s + 2$

$\omega_s = 1.414$

$s_1 = -1$ & $s_2 = -2$

$\alpha > \omega_s$.

$$\therefore \boxed{i(t) = k_1 e^{-t} + k_2 e^{-2t}} \quad -(2)$$

∴ Overdamped

Using initial condn k_1 & k_2 can be determined.

$$i(0^+) = k_1 e^{1(0)} + k_2 e^{-2(0)}$$

$$i(0^+) = k_1 + k_2$$

$$\therefore \boxed{k_1 + k_2 = 0} \quad \because i(0^+) = 0. \quad -(a)$$

$$\frac{di(0^+)}{dt} = k_1(-1)e^{-t} + k_2(-2)e^{-2t}$$
$$= -k_1 e^{-t} - 2k_2 e^{-2t}$$
$$= -k_1 e^{0} - 2k_2 e^{0}.$$

$$\frac{di(0^+)}{dt} = -k_1 - 2k_2$$

$$\therefore \frac{V}{L} = -k_1 - 2k_2$$

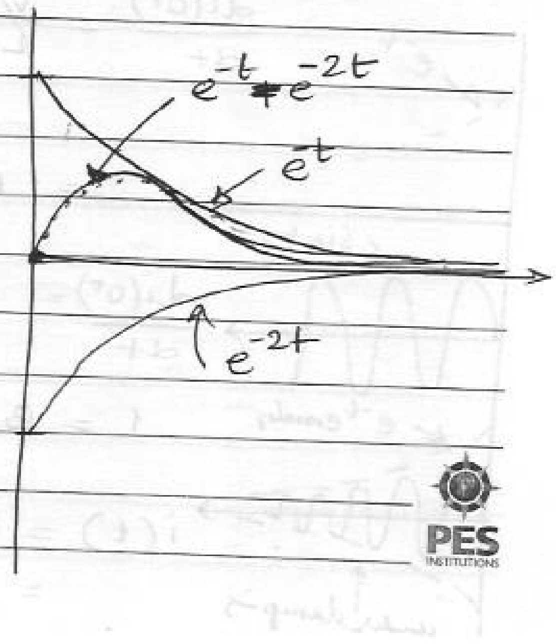
$$\boxed{-k_1 - 2k_2 = 1} \quad -(b)$$

Solving (a) and (b)

$k_1 = 1$ and $k_2 = -1$

$$\therefore i(t) = k_1 e^{-t} + k_2 e^{-2t}$$
$$= 1e^{-t} - e^{-2t}$$

$$\boxed{i(t) = e^{-t} - e^{-2t}}$$



∴ n/w response is Overdamping

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Ex(2) : $R = 2\Omega$, $L = 1H$, $C = \frac{1}{2}F$, $V = 1V$

$$s^2 i + \left(\frac{R}{L}\right) s i + \frac{1}{LC} i = 0.$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) k e^{st} = 0$$

$\alpha = \frac{R}{2L} = \frac{2}{2} = 1$

$\omega_s = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.5}}$

$= \underline{\underline{1.4142}}$

$$s^2 + 2s + 2 = 0$$

Roots will be $s_1 = (-1+j)$ & $s_2 = (-1-j)$ $\alpha < \omega_s$

It indicated that nature is Undamped resp. - underdamped

$$i(t) = e^{-\alpha t}\left[B_1 \cdot \cos\omega_d t + B_2 \sin\omega_d t\right] \qquad \omega_d = \sqrt{\omega_s^2 - \alpha^2}$$

$= \sqrt{(1.4142)^2 - (1)^2}$

$$= e^{-t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right]$$

$= 0.9999 = \underline{1}$

$$\cancel{i(t) = K_1 e^{(-1+j)t} + K_2 e^{(-1-j)t}}$$

$$\cancel{= K_1(e^{-t} \cdot e^{jt}) + K_2(e^{-t} \cdot e^{-jt})}$$

$$\cancel{= e^{-t}\left[K_1 e^{jt} + K_2 e^{-jt}\right]}$$

$$\cancel{= e^{-t}\left[K_1 (\cos\right.}$$

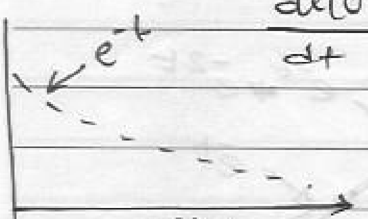
$$i(t) = e^{-t}\left(B_1 \cos(1) \cdot t + B_2 \sin(1) t\right)$$

$$\boxed{i(t) = e^{-t}\left(B_1 \cos t + j B_2 \sin t\right)}$$

$t = 0^+$

$$i(0^+) = e^0\left(B_1 \cos(0) + j B_2 \sin(0)\right)$$

$$i(0^+) = B_1 \times 1 \qquad \therefore \quad B_1 = 0$$

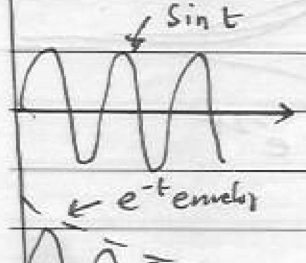


$$\frac{di(0^+)}{dt} = \cancel{\frac{V}{L}} = \cancel{\frac{1}{1}} = \cancel{1}\; B_1\left[e^{-t} \cdot (-\sin t) + j \cos t \cdot (-1) \cdot e^{-t}\right]$$

$$B_2\left[e^{-t} \cos t + j \sin t \cdot (-1) \cdot e^{-t}\right]$$

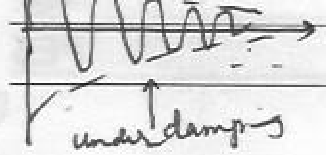
$$= B_1\left[e^0 (0) - j 0 \cdot e^0\right]$$

$$+ B_2\left[1 + 0\right]$$



$$\frac{di(0^+)}{dt} = B_2[1]$$

$$1 = B_2 \qquad \therefore \boxed{B_2 = 1}$$



$$i(t) = e^{-t}\left(B_1 \cos t + B_2 \sin t\right)$$

$$= e^{-t} \times 1 \times \sin t = \underline{e^{-t} \sin t}$$

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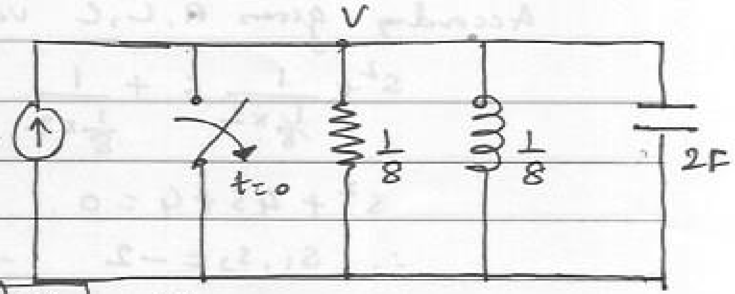
* Parallel R-L-C



At $t=0^-$, Switch K is closed

$\boxed{V(0^-) = 0}$ ⓐ, $I_L(0^-)=0$

At $t=0^+$, K is open

$\boxed{V_c(0^+) = V_c(0^-) = 0.}$ — ⓐ

By KCL

$$C \cdot \frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} V \cdot d\tau = I \quad -(1)$$

$$C\frac{dv(0^+)}{dt} + \cancel{\frac{V(0^+)}{R}} + \frac{1}{L}\cancel{\int_{-\infty}^{0^+} V(0^+) \cdot d\tau} = I$$

0 $\quad$ $I_L(0^-) = 0$

$$C \cdot \frac{dv(0^+)}{dt} = I.$$

$$\boxed{\frac{dv(0^+)}{dt} = \frac{I}{C} = \frac{1}{2} = \underline{\underline{0.5}}} \quad -ⓑ$$

ⓐ and ⓑ are initial conditions.

Differentialy eq² (1)

$$C\frac{d^2V}{dt^2} + \frac{1}{R} \cdot \frac{dv}{dt} + \frac{1}{L}V = I.$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC} \cdot \frac{dv}{dt} + \frac{1}{LC}V = I$$

$$\left(S^2 + \frac{1}{RC}S + \frac{1}{LC}\right)V = I \quad \text{where } V = K \cdot e^{st}.$$

for homogeneous differential eq²

Roots :

$$S_1, S_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4\frac{1}{LC}}}{2 \times 1}$$

$$S_1, S_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$\alpha = \frac{1}{2RC} \quad \& \quad \omega_s = \frac{1}{\sqrt{LC}}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_s^2}$$

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According given R, L, C values

$$s^2 + \frac{1}{\frac{1}{8}\times 2} s + \frac{1}{\frac{1}{8}\times 2} = 0$$

$$s^2 + 4s + 4 = 0.$$

$\therefore\ s_1, s_2 = -2$ — Real and equal roots.

W.K.T

General sol^n of 2nd order differential eqn will have two elements.

$$y(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$$\therefore\ v(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

As root $s_1, s_2 = s = -2$

$$v(t) = k_1 e^{-2t} + k_2 e^{-2t}$$
$$= (k_1 + k_2) e^{-2t}$$
$$\boxed{y(t) = \underline{\underline{k \cdot e^{-2t}}}}$$

which has only one element in the sol^n $\therefore$ which is not a correct sol^n. We need to get two independent sol^n somehow. Hence for 2nd element in the sol^n multiplied by linear factor t

** $v(0^+) = k \cdot e^0$

$v(0^+) = k$

$\therefore\ k = 0.$

$\therefore\ \frac{dv(0^+)}{dt} = -2k e^{-2t}$

$= \underline{0}$

if $v(0^+) = 2$

$k = 2$

$\frac{dv(0^+)}{dt} = k e^{-2t}$

$= -2k \cdot e^{-2t}$

$= -2\times 2 e^{0}$

$\boxed{1 \neq -4}$

So, for repeated roots, the general sol^n will be of the form

$$\left[y(t) = k_1 e^{s_0 t} + k_2 \cdot t \cdot e^{st} \right]$$

$$\text{Now, } \left[v(t) = k_1 e^{-2t} + k_2 t e^{-2t} \right] \quad —③$$

$$\boxed{\begin{array}{l} \frac{d^2 v}{dt} = 0 \quad ** \\ \text{1st integral} \\ \frac{dv}{dt} = k_2 \\ \text{2nd integral} \\ \boxed{v(t) = k_1 + k_2 t} \end{array}}$$

Using initial cond^n. $t = 0^+$

$v(0^+) = k_1 e^{-2t} + k_2 t e^{-2t}$

$= k_1 e^0 + k_2 \times 0 \times e^0$

$v(0^+) = k_1$

$\therefore\ \boxed{k_1 = 0}$ as $v(0^+) = 0$

$\frac{dv}{dt} = k_1 (-2) \cdot e^{-2t} + k_2 \left[t \cdot (-2) e^{-2t} + e^{-2t}(1) \right]$

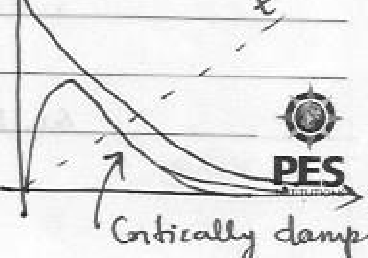
$\frac{dv(0^+)}{dt} = 0 \times (-2) e^0 + k_2 \left[0 \times (-2) e^{-2t} + e^0 \right]$

$\frac{dv(0^+)}{dt} = k_2$

$\therefore\ k_2 = \underline{0.5}$

Using k_1 & k_2

$$\boxed{v(t) = 0.5 t e^{-2t}}$$



Critically damped Cur

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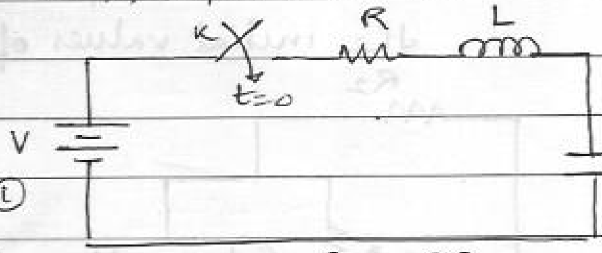
Example (1): In the ckt shown, $V = 10V$, $R = 10\Omega$, $L = 1H$, $C = 10\mu F$ and $v_c(0) = 0$. ~~Let~~ Find $i(0^+)$, $di/dt(0^+)$, and $d^2i(0^+)/dt^2$

At $t=0$; switch is closed

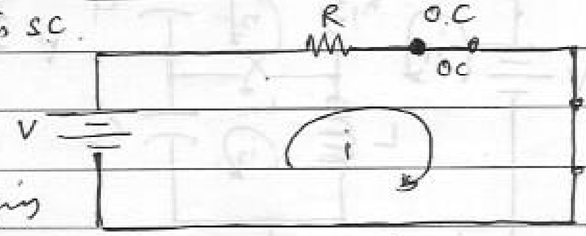


By KVL

$$\left[L\frac{di}{dt} + Ri + \frac{1}{C}\int_0 i\,dt = V\right] \quad -(1)$$

(i) At $t=0^+$, L is O.C & C is SC.

$$\therefore\ i(0^+) = \cancel{\frac{V}{R}}\ 0$$



At $t=0^+$

(ii) From eq^n (1), By rearranging we get

$$L\frac{di}{dt} = V - Ri - \frac{1}{C}\int i\,dt$$

L acts as O.C as ~~St~~ current/inchise

$$\frac{di}{dt} = \frac{V}{L} - \frac{R}{L}i - \frac{1}{RC}\int i\,dt$$

$V_C(0^+) = 0$.

At $t=0^+$

$$\frac{di(0^+)}{dt} = \frac{V}{L} - \frac{R}{L}\cancel{i(0^+)}_{0} - \frac{1}{RC}\cancel{\int i(0^+)\,dt}_{0}$$

$$\boxed{\frac{di(0^+)}{dt} = \frac{V}{L}} \quad -(2) \qquad \therefore\ \frac{di(0^+)}{dt} = \frac{10}{1} = 10\ \text{Amp/sec}$$

(iii) By taking derivative on both sides of eq^n (1)

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0.$$

At $t=0^+$

$$\frac{d^2i(0^+)}{dt^2} = -\frac{R}{L}\cdot\frac{di(0^+)}{dt} - \frac{i(0^+)}{C}$$

$$= -\frac{R}{L}\times\frac{V}{L} - \frac{0}{C}$$

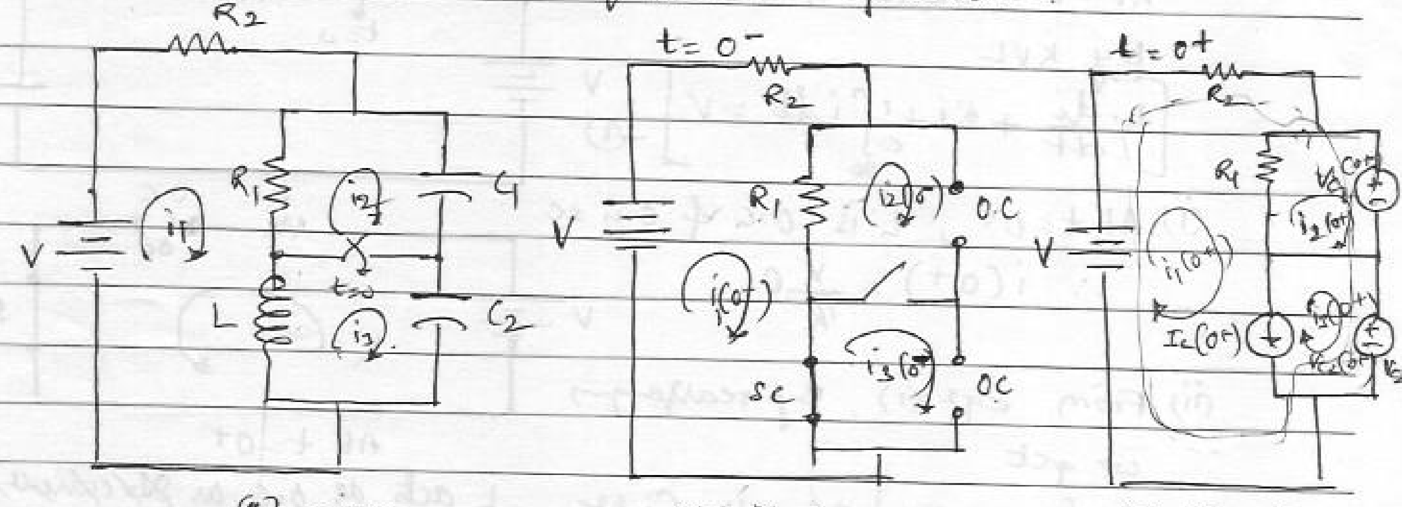
$$\frac{d^2i(0^+)}{dt^2} = -\frac{10}{1}\times 10 = -100\ \text{Amp/sec}$$

$i(0^+) = 0$ $\qquad \dfrac{di(0^+)}{dt} = \dfrac{V}{L} = 10\ \text{Amp/sec}$ $\qquad \dfrac{d^2i(0^+)}{dt^2} = -\dfrac{R}{L}\cdot\dfrac{di(0^+)}{dt} - \dfrac{i(0^+)}{C}$

$= -10\times 10$

$= -100\ \text{Amp/sec}$

Example (2): In the n/w shown, steady state is reached with the switch K open and at $t=0$, the switch is closed. Find the initial values of all the loop currents.

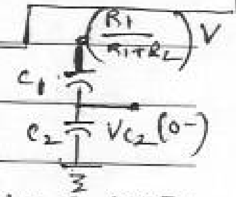


(a) given (b) At $t=0^-$ (c) At $= 0^+$

(i) At $t=0^-$

$$\boxed{i_1(0^-) = \left(\frac{V}{R_1+R_2}\right) = i_{R_1}(0^-) = i_L(0^-) \;\&\; i_2(0^-)=0,\; i_3(0^-)=0}$$

$$V_{C1}(0^-) + V_{C2}(0^-) = \left(\frac{R_1}{R_1+R_2}\right)V \quad \text{— (1)}$$



Charges on the capacitor will be same as they are in series.

$$q_1(0^-) = q_2(0^-)$$

$$C_1 V_{C_1}(0^-) = C_2 V_{C_2}(0^-)$$

$$\frac{V_{C_1}(0^-)}{V_{C_2}(0^-)} = \frac{C_2}{C_1} \quad \text{— (2)}$$

(2) / in (1) ÷ eq2(1) both side by $V_{C2}(0^-)$

$$\frac{V_{C_1}(0^-)}{V_{C2}(0^-)} + 1 = \frac{\left(\frac{R_1}{R_1+R_2}\right)V}{V_{C2}(0^-)} \quad \text{— (3)}$$

substitute (2) in (3)

$$\frac{C_2}{C_1} + 1 = \frac{(R_1/R_1+R_2)V}{V_{C2}(0^-)}$$

$$\boxed{V_{C2}(0^-) = \frac{(R_1/R_1+R_2)V}{\left(\frac{C_2+C_1}{C_1}\right)} = \left(\frac{C_1}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V}$$

lly $$\boxed{V_{C1}(0^-) = \left(\frac{C_2}{C_1+C_2}\right)\cdot\left(\frac{R_1}{R_1+R_2}\right)V}$$

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At $t = 0^+$

KVL @ outer Loop (peripheral)

$$\cancel{V} = i_1(0^+)R_2 + V_{C_1}(0^+) + V_{C_2}(0^+) = V.$$

$$\cancel{\left(\frac{V}{R_1+R_2}\right)R_2 + \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V + \left(\frac{C_1}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V = V}$$

$$\cancel{i_1(0^+)R_2\, V\left(\frac{R_2}{R_1+R_2}\right) + \frac{R_1}{R_1+R_2}V\left(\frac{C_2}{C_1+C_2} + \frac{C_1}{C_1+C_2}\right) = V}$$

$$i_1(0^+)R_2 = V - V_{C_1}(0^+) - V_{C_2}(0^+)$$

$$= V - \left[\left(\frac{C_2}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V\right] - \left[\left(\frac{C_1}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V\right]$$

$$= V\left[1 - \frac{C_2R_1 + C_1R_1}{(C_1+C_2)(R_1+R_2)}\right]$$

$$= V\left[1 - \frac{R_1(\cancel{C_1+C_2})}{(\cancel{C_1+C_2})(R_1+R_2)}\right]$$

$$= V\left[1 - \frac{R_1}{R_1+R_2}\right]$$

$$i_1(0^+)R_2 = V\left[\frac{R_1+R_2-R_1}{R_1+R_2}\right]$$

$$\boxed{i_1(0^+) = \frac{V}{\cancel{R_2}}\left(\frac{\cancel{R_2}}{R_1+R_2}\right) = \frac{V}{R_1+R_2} = i(0^-)} = I_L(0^+)$$

Loop3:

$\cancel{(i_3(0^+) - i_1(0^+))}$

Current through inductor is $(i_1(0^+) - i_3(0^+)) = I_L(0^+)$

As we known current through inductor can't change instanteneously $\quad i_1(0^+) - i_3(0^+) = \frac{V}{R_1+R_2} \quad \because i_1(0^+) = i_1(0^-) = \frac{V}{R_1+R_2}$

Hence $i_3(0^+) = 0$.

Loop2: KVL @ loop 2.

$$(i_2(0^+) - i_1(0^+))R_1 + V_{C_1}(0^+) = 0$$

Voltage a/c ~~induct~~ capacitor can't change instanteneouly

$$i_2(0^+)R_1 - i_1(0^+)R_1 = -V_{C_1}(0^+)$$

$$= -\left[\left(\frac{C_2}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)\cdot V\right]$$

$$i_2(0^+)R_1 = i_1(0^+)R_1 - \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right)V.$$

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$$i_2(0^+)\,R_1 = \left(\frac{V}{R_1+R_2}\right) R_1 - \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{R_1}{R_1+R_2}\right) V$$

$$= \frac{V(R_1)}{R_1+R_2}\left[\cancel{R_1}1 - \frac{C_2\,\cancel{R_1}}{C_1+C_2}\right]$$

$$i_2(0^+)\,\cancel{R_1} = \frac{V\,\cancel{R_1}}{R_1+R_2}\left[\frac{C_1+\cancel{C_2}-\cancel{C_2}}{C_1+C_2}\right]$$

$$\boxed{i_2(0^+) = \left(\frac{V}{R_1+R_2}\right)\left(\frac{C_1}{C_1+C_2}\right)} \checkmark$$

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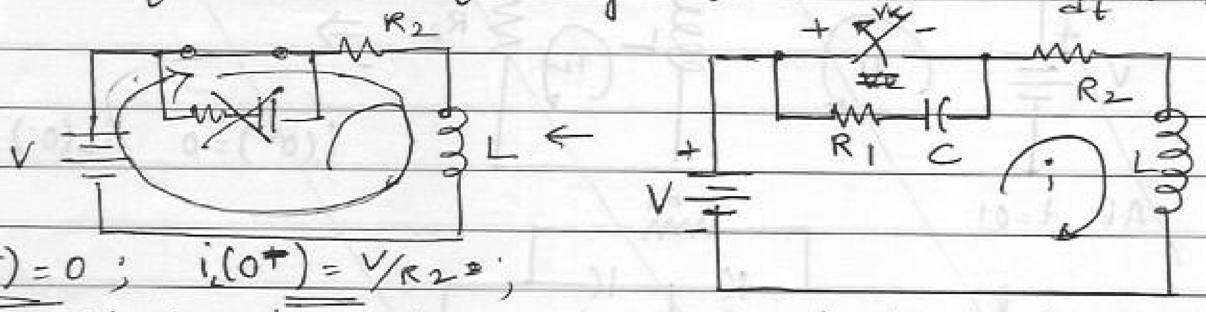
5.25) Switch k is opened at $t=0$ after the network has attained a steady state with switch closed.

a) find an expression for the voltage a/c switch K at $t=0^+$

b) If the parameters are adjusted such that $i(0^+)=1$ and $di/dt\,(0^+) = -1$. What is the value of derivative of voltage a/c the switch $\frac{dV_x}{dt}(0^+)$?



$V_c(0^-) = 0$; $i_L(0^+) = V/R_2$;

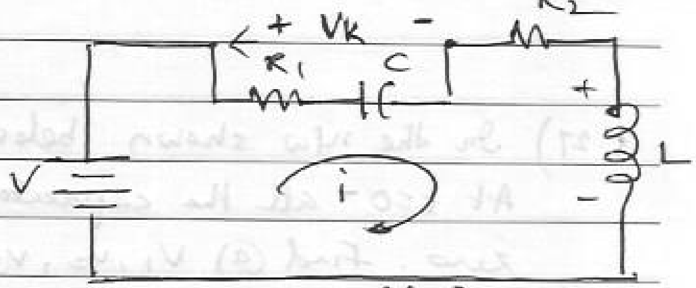
At $t=0^+$; As ckt has reached steady state

$i_L(0^+) = i_L(0^-) = \frac{V}{R_2}$ $\quad V_c(0^+) = V_c(0^-) = 0$.

$v_c(0^-) = 0$

& switch k is open

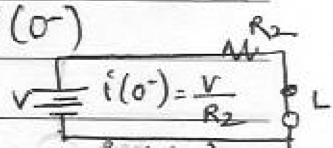
By KVL



$\cancel{iR_1 + V_k + iR_2 + L\frac{di}{dt} = V}$

Voltage a/c switch K.

$V_k = R_1 i(t) + \frac{1}{C}\int_{-\infty}^{t} i\, dt$ —— (i)



$V_k(0^+) = R_1 i(0^+) + \frac{1}{C}\int_{-\infty}^{0^+} i(0^+)\,dt$ $\quad$ ($\because V_c(0^-) = 0$

$= R_1 i(0^+) + V_c(0^+)$

$$\boxed{V_k(0^+) = R_1 \frac{V}{R_2}}$$

b) derivative of eq2 (1)

$\frac{di}{dt}(0^+) = 1$

$i(0^+) = 1$

$\frac{dV_x}{dt} = R_1 \cdot \frac{di(t)}{dt} + \frac{i}{C} = 0$

$\frac{dV_x(0^+)}{dt} = R_1 \frac{d}{dt} i(0^+) + \frac{i(0^+)}{C} = 0$

$= R_1(-1) + \frac{(1)}{C} = \frac{1}{C} - R_1$

$$\boxed{\frac{dV_x(0^+)}{dt} = \frac{1}{C} - R_1}$$

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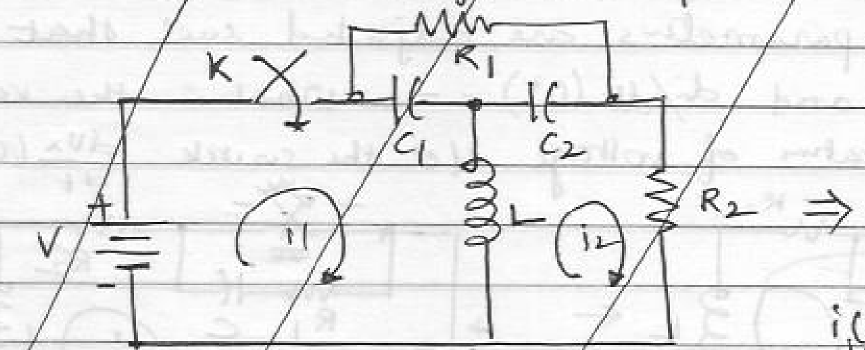
5.26) K is closed at $t=0$ connecting battery with unenergised system

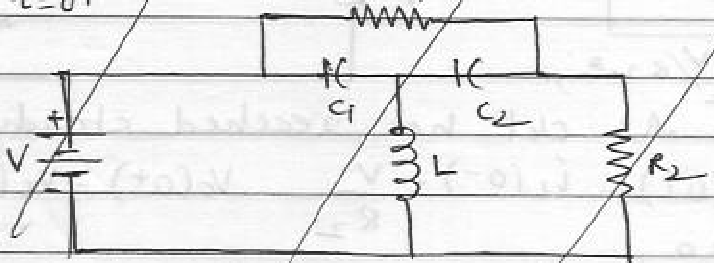
a) find voltage V_a at $t=0^+$

b) find voltage a/c capacitor C_1 at $t=\infty$

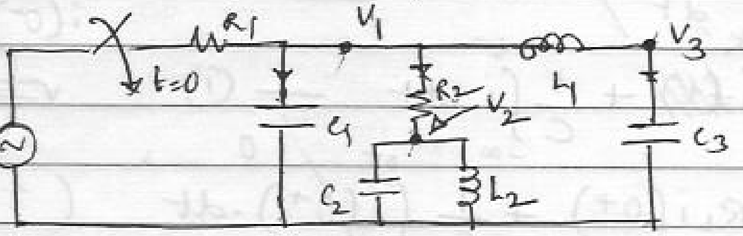


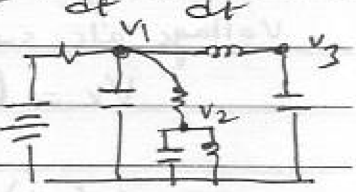
$i_1(0^-)=0 \quad i_2(0^-)=0$

At $t=0^+$



5.27) In the n/w shown below, the switch k is closed at $t=0$. At $t=0^-$ all the capacitor voltages and inductor currents are zero. Find (a) V_1, V_2, V_3 at $t=0^+$ b) $\frac{dv_1}{dt}, \frac{dv_2}{dt}, \frac{dv_3}{dt}$ at $t=0^+$





i) $i_{L_1}(0^-) = i_{L_2}(0^-) = 0$ & $V_{C_1}(0^-) = V_{C_2}(0^-) = V_{C_3}(0^-) = 0$

ii) At node V_1

$$\frac{V_1 - V(t)}{R_1} + C_1 \cdot \frac{dv_1}{dt} + \frac{V_1 - V_2}{R_2} + \frac{1}{L_1}\int_{-\infty}^{t} (V_1 - V_3)\cdot dt$$

At $t=0^+$

$$\frac{v_1(0^+) - V(0^+)}{R_1} + C_1 \frac{dv_1(0^+)}{dt} + \frac{\cancel{v_1(0^+) - v_2(0^+)}}{R_2} + \underbrace{\frac{1}{L_1}\int_{-\infty}^{t} \cancel{(v_1(0^+) - v_3(0^+))}\,dt}_{i_L(0^+)}$$

$$\frac{V(0^+)}{R_1} = C_1 \frac{dv_1(0^+)}{dt} + 0 + 0$$

$$\therefore \boxed{\frac{dv_1(0^+)}{dt} = \frac{1}{C_1}\cdot\frac{V(0^+)}{R_1}}$$

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iii) At node V_2 (KCL)

$$\frac{V_2-V_1}{R_2} + C_2 \cdot \frac{dv_2}{dt} + \frac{1}{L_2}\int_{-\infty}^{t} V_2 \cdot dt = 0.$$

At $t=0^+$

$$\frac{V_2(0^+) - \cancel{V_1(0^+)}^{\,0}}{\cancel{R_2}} + C \cdot \frac{dv_2(0^+)}{dt} + \underbrace{\frac{1}{L_2}\int_{-\infty}^{0^+} \cancel{V_2(0^+)}^{\,0} \cdot dt}_{I_{L_2}(0^+)} = 0$$

$$\frac{0}{R_2} + C \cdot \frac{dv_2(0^+)}{dt} + 0 = 0.$$

$$\therefore \boxed{\frac{dv_2(0^+)}{dt} = 0}$$ ✓

iv) ~~$\frac{V_3-V_1}{L}$~~ At node 3 (KCL)

$$\underbrace{\frac{1}{L_1}\int_{-\infty}^{t} (V_3 - V_1)\, dt}_{I_{L_1}(0^+)=0} + C_3 \cdot \frac{dv_3}{dt} \cdot \cancel{\frac{V_3-V_2}{}} = 0$$

At $t=0^+$

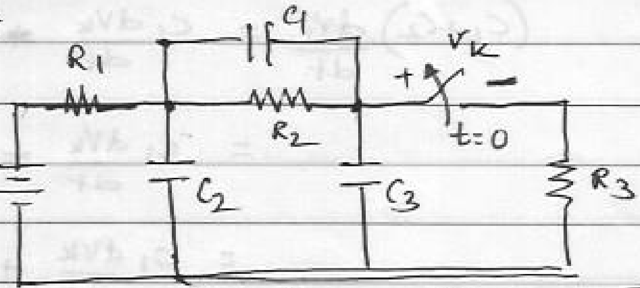
$$\frac{1}{L_1}\int_{-\infty}^{t} \cancel{V_3(0^+) - V_1(0^+)}^{\,0} \cdot dt + C_3 \cdot \frac{dv_3(0^+)}{dt} = 0$$

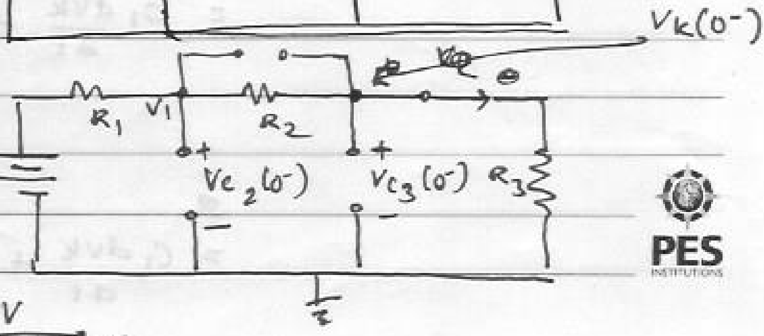
$$C_3 \cdot \frac{dv_3(0^+)}{dt} = 0$$

$$\boxed{\frac{dv_3}{dt}(0^+) = 0}$$ ✓

✓

5.28) In the n/w shown steady state is reaches and at $t=0$, switch K is opened a) find the voltage a/c switch, V_K at $t=0^+$
b) Find $\frac{V_K}{dt}$ at $t=0^+$





At $t=0^+$

$$i(0^-) = \frac{V}{R_1+R_2+R_3}$$

$$V_K(0^-) = 0.$$

$$\therefore \text{At } t=0^+ \quad i(0^+) = \frac{V}{R_1+R_2+R_3}$$

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Loop1: $V_{C_2}(0^-) + R_1 i - V = 0$

$$\therefore \left[V_{C_2}(0^-) = V - R_1 i\right]$$

$$= V - R_1\left(\frac{V}{R_1+R_2+R_3}\right)$$

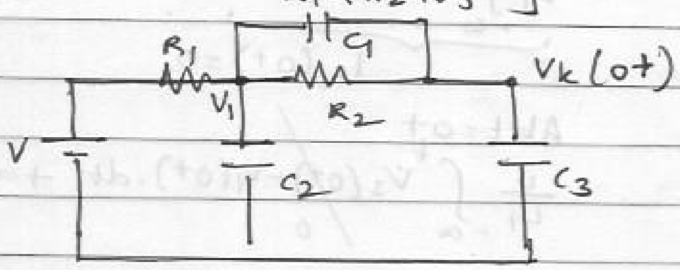
$$\left[V_{C_2}(0^-) = V - \frac{VR_1}{R_1+R_2+R_3}\right]$$

$$= \frac{V(R_1+R_2+R_3) - VR_1}{R_1+R_2+R_3}$$

$$\left[V_{C_2}(0^-) = \frac{V(R_2+R_3)}{R_1+R_2+R_3} = V_{C_2}(0^+) = \underline{\underline{V_1(0^-)}}\right] - (2)$$

$$\left[V_{C_3}(0^-) = \underline{\underline{V_k(0^-)}} = \cancel{V_k(0^+)} = i(0^-) \times R_3 = \frac{V \times R_3}{R_1+R_2+R_3}\right] - (3)$$

At; $t=0^+$



Applying KCL at node V_1

$$\frac{V_1-V}{R_1} + C_2 \cdot \frac{dV_1}{dt} + C_1\frac{d}{dt}(V_1-V_k) + \frac{V_1-V_k}{R_2} = 0$$

$$\frac{V_1-V}{R_1} + C_2 \cdot \frac{dV_1}{dt} + \frac{C_1 dV_1}{dt} - C_1 \cdot \frac{dV_k}{dt} + \frac{V_1-V_k}{R_2} = 0$$

$$(C_1+C_2)\frac{dV_1}{dt} + \frac{V_1-V}{R_1} - C_1 \cdot \frac{dV_k}{dt} + \frac{V_1-V_k}{R_2} = 0$$

$$(C_1+C_2)\frac{dV_1}{dt} = C_1\frac{dV_k}{dt} - \frac{(V_1-V)}{R_1} \cancel{+ C_1\frac{dV_k}{dt}} - \frac{(V_1-V_k)}{R_2}$$

$$= C_1\frac{dV_k}{dt} - \frac{V_1}{R_1} + \frac{V}{R_1} \cancel{+ C\frac{dV_k}{dt}} - \frac{V_1}{R_2} + \frac{V_k}{R_2}$$

$$= C_1\frac{dV_k}{dt} + \frac{V}{R_1} - \frac{V(R_2+R_3)}{R_1(R_1+R_2+R_3)} - \frac{V(R_2+R_3)}{R_2(R_1+R_2+R_3)} + \frac{V(R_3)}{R_2(R_1+R_2+R_3)}$$

$$= C_1\frac{dV_k}{dt} + \frac{V}{R_1} - \frac{V(R_2+R_3)}{R_1(R_1+R_2+R_3)} - \frac{V\cancel{R_2}}{\cancel{R_2}(R_1+R_2+R_3)}$$

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$$= C_1\frac{dV_k}{dt} + \frac{1}{R_1}\left[V - \frac{V(R_2+R_3)}{R_1+R_2+R_3}\right] - \frac{V}{R_1+R_2+R_3}$$

$$= \quad '' \quad + \frac{1}{R_1}\left[\frac{V(R_1+R_2+R_3) - V(R_2+R_3)}{(R_1+R_2+R_3)}\right] - \frac{V}{R_1+R_2+R_3}$$

$$= C_1\frac{dV_k}{dt} + \frac{1}{\cancel{R_1}}\left[\frac{V\cancel{R_1}}{(R_1+R_2+R_3)}\right] - \frac{V}{R_1+R_2+R_3}$$

$$\boxed{(C_1+C_2)\frac{dV_1}{dt} = C_1\frac{dV_k}{dt}}$$

$$\cancel{\frac{dV_k}{dt} = \left(\frac{C_1+C_2}{C_1}\right)\cdot\frac{dV_1}{dt}}$$

$$\left[\frac{dV_1}{dt} = \left(\frac{C_1}{C_1+C_2}\right)\frac{dV_k}{dt}\right] - (4)$$

KCL at V_k

$$C_1\frac{d(V_k - V_1)}{dt} + \frac{V_k - V_1}{R_2} + C_3\cdot\frac{dV_k}{dt} = 0$$

$$C_1\frac{dV_k}{dt} - C_1\cdot\frac{dV_1}{dt} + \frac{V_k}{R_2} - \frac{V_1}{R_2} + C_3\frac{dV_k}{dt} = 0$$

$$(C_1+C_3)\frac{dV_k}{dt} = C_1\frac{dV_1}{dt} + \frac{V_1}{R_2} - \frac{V_k}{R_2}$$

$$= C_1\cdot\frac{dV_1}{dt} + \frac{1}{R_2}\left[V_1 - V_k\right]$$

$$= C_1\frac{dV_1}{dt} + \frac{1}{R_2}\left[\frac{V(R_2+R_3)}{R_1+R_2+R_3} - \frac{VR_3}{R_1+R_2+R_3}\right]$$

$$= C_1\frac{dV_1}{dt} + \frac{1}{\cancel{R_2}}\left[\frac{V\cancel{R_2}}{R_1+R_2+R_3}\right]$$

$$(C_1+C_3)\frac{dV_k}{dt} = C_1\frac{dV_1}{dt} + \frac{V}{R_1+R_2+R_3} - (5)$$

(4) in (5)

$$= \left(\frac{C_1^2}{C_1+C_2}\right)\frac{dV_k}{dt} + \frac{V}{R_1+R_2+R_3}$$

$$(C_1+C_3)\frac{dV_k}{dt} - \frac{C_1^2}{(C_1+C_2)}\frac{dV_k}{dt} = \frac{V}{R_1+R_2+R_3}$$

$$\frac{dV_k}{dt}\left[(C_1+C_3) - \frac{C_1^2}{C_1+C_2}\right] = \frac{V}{R_1+R_2+R_3}$$

$$\frac{dV_k}{dt}\left[\frac{\cancel{C_1^2}+C_1C_2+C_1C_3+C_2C_3-\cancel{C_1^2}}{(C_1+C_2)}\right] = \frac{V}{R_1+R_2+R_3}$$

$$\boxed{\frac{dV_k}{dt} = \frac{V(C_1+C_2)}{(C_1C_2+C_1C_3+C_2C_3)(R_1+R_2+R_3)}}$$

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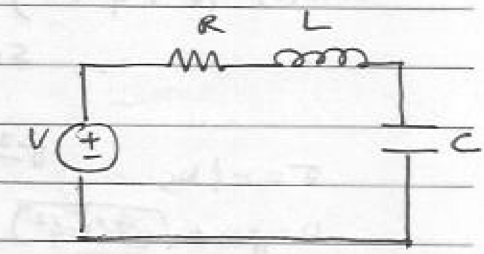
6.29) Consider a RLC ckt, which is excited by a voltage source

a) Determine the characteristic eq² corresponds to differential eq² for $i(t)$

b) Suppose L & C are fixed in value but R varies from 0 to ∞ what will be locus of the roots of the characteristic eq² ?

c) Plot the roots of characteristic eq² in s-plane if $L = 1H$, $C = 1\mu F$ & R has the values (i) 500Ω (ii) $1k\Omega$ iii) $3k\Omega$ iv) $5k\Omega$

Sol:



a) $$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i \cdot d\tau = V$$

by differential

$$L \cdot \frac{d^2 i}{dt^2} + R \cdot \frac{di}{dt} + \frac{1}{C} i = 0$$

$$\left(\frac{d^2 i}{dt^2} + \frac{R}{L} \cdot \frac{di}{dt} + \frac{1}{LC} i\right) = 0$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) i = 0$$ where $i = k \cdot e^{st}$ form for homogen

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$s_1, s_2 = -\alpha \pm \sqrt{(\alpha)^2 - \omega_s^2}$$ — Roots may be
i) Real, Not equal
ii) Real & equal
iii) Complex Conjugal

$$\therefore i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

&

$$\alpha = \frac{R}{2L} \quad \omega_s = \frac{1}{\sqrt{LC}} \quad \xi = \frac{R}{L}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{\alpha}{\omega_s} \quad \alpha = \xi \cdot \omega_s$$

$$\therefore s_1, s_2 = -\xi\omega_s \pm \sqrt{(\xi\omega_s)^2 - \omega_s^2}$$

$$\boxed{s_1 s_2 = -\xi\omega_s \pm \omega_s\sqrt{\xi - 1}}$$

$\xi = \frac{R}{L}\sqrt{\frac{C}{L}}$ if R varies from 0 to ∞
ξ varies from 0 to ∞.

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Case(i); When $\boxed{R=0,\ \xi=0}$

$$\therefore\ s_1, s_2 = -\xi\omega_s \pm \omega_s\sqrt{\xi^2-1}$$

$$= 0 + \omega_s\sqrt{-1}$$

$$\boxed{s_1, s_2 = \pm j\omega_s} \quad \text{---(i)}$$

Roots are ~~Imaginary~~ complex conjugate & purely imaginary, & Response is Oscillatory & Roots lies on imaginary axis

case(2) $(0 < \xi < 1)$

$$s_1, s_2 = -\xi\omega_s \pm \omega_s\sqrt{\xi^2-1} = \cancel{\sigma \pm j\omega}$$

$$= -\xi\omega_s \pm \omega_s\sqrt{-(1-\xi^2)} = -\xi\omega_s \pm j\omega_s\sqrt{1-\xi^2}$$

$$= \sigma \pm j\omega$$

$\sigma = -\xi\omega_s$

$\omega = j\omega_s \pm \sqrt{(1-\xi^2)}$

$\sigma^2 = \xi^2\omega_s^2$

$\omega^2 = \omega_s^2(1-\xi^2)$

$$\cancel{\sigma^2 + \omega_s^2 = \xi^2\omega_s^2 \pm \omega_s^2(\xi^2-1)}$$

$$= \cancel{\xi^2\omega_s^2 \pm \omega_s^2\xi^2 - \omega_s^2}$$

$$\sigma^2 + \omega^2 = \xi^2\omega_s^2 + \omega_s^2(1-\xi^2)$$

$$= \xi^2\cancel{\omega_s^2} + \omega_s^2 - \omega_s^2\xi^2$$

$$\boxed{\sigma^2 + \omega^2 = \omega_s^2} \checkmark$$

$\sigma^2 + x^2 = r^2$

Roots move on a Circle, described by a circle of radius ω_s.

(case iii) $\xi > 1$;

$$s_1, s_2 = -\xi\omega_s \pm \omega_s\sqrt{\xi^2-1} \qquad \xi > 1 \quad \xi^2 >> 1$$

$$= -\xi\omega_s \pm \omega_s\sqrt{\xi^2} \qquad \therefore 1 \text{ is neglected.}$$

$$s_1 s_2 = -\xi\omega_s \pm \omega_s\xi$$

$$s_1 = -\xi\omega_s + \xi\omega_s \qquad s_2 = -\xi\omega_s - \xi\omega_s$$

$$= \underline{0} \qquad\qquad = \underline{-2\xi\omega_s}$$

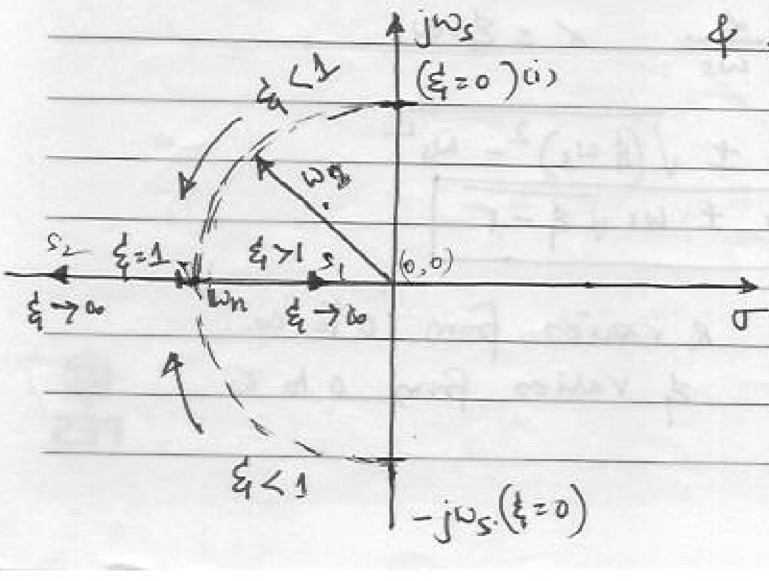
As $\xi \to \infty$, $s_2 \to -\infty$ on one end

& $s_1 = 0$ on another end

case iv) $\xi = 1$

$$s_1, s_2 = -\omega_s \pm \omega_s\sqrt{1-1}$$

$$= \underline{\underline{-\omega_s}}$$



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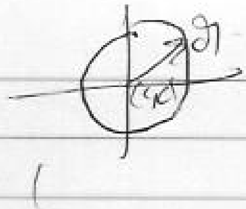
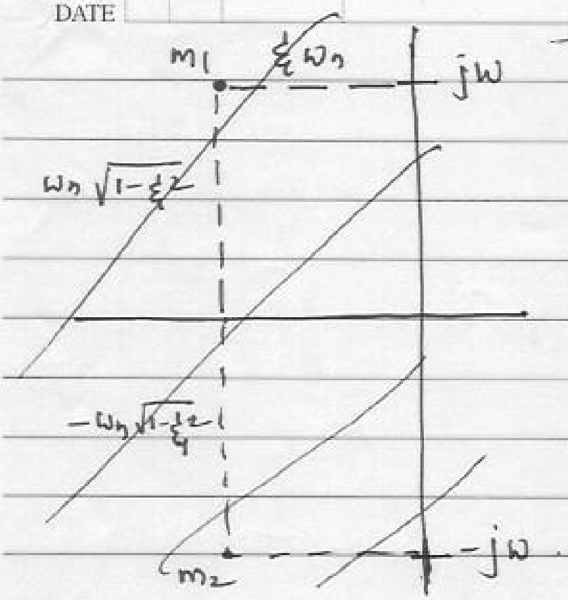


$x^2+y^2=r^2$

$(x-a)^2+(y-c)$



$s_1, s_2 = -\xi \omega_s \pm \omega_s\sqrt{1-\xi^2}$

$0 < \xi < 1$

3) Location of Roots

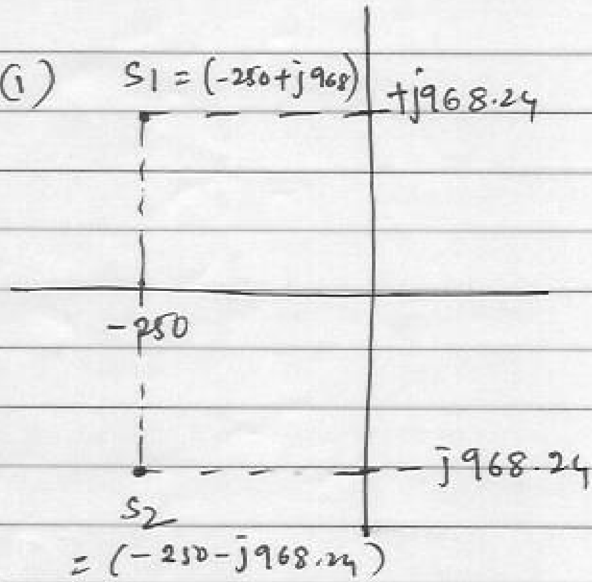
1) $R = 500\Omega$, $L = 1H$, $C = 10\mu F$

$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 10\times 10^{-6}}} = \underline{\underline{1000 Hz}}$

$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{500}{2}\sqrt{\frac{1\times 10^{-6}}{1}}$

$\xi = 0.25$. $\therefore \xi < 1$

(1)



$s_1, s_2 = -\xi\omega_s \pm j\omega_s\sqrt{1-\xi^2}$

$= -0.25\times 1000 \pm j\,1000\sqrt{1-0.25^2}$

$= -250 \pm j\,968.245$

2) $\underline{R = 1000\Omega}$

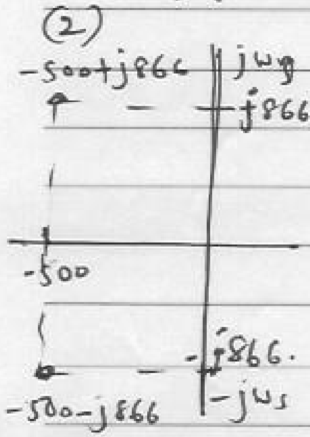
$\omega_n = 1000 Hz$

$\xi = \frac{1000}{2}\sqrt{\frac{1\times 10^{-6}}{1}}$

$= 500\times 1\times 10^{-3}$

$\xi = \underline{\underline{0.5}}$ $\boxed{\xi < 1}$

(2)



$s_1, s_2 = -0.5\times 1000 \pm j\,1000\sqrt{1-0.5^2}$

$\boxed{s_1 s_2 = -500 \pm j\,866.025}$

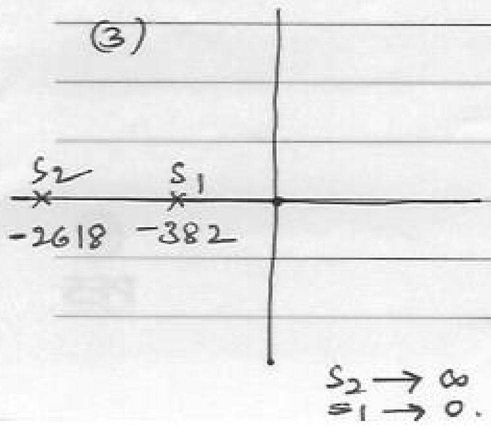
3) $R = 3000$.

$\xi = \frac{3000}{2}\sqrt{\frac{1\times 10^{-6}}{1}} = 1500\times 1\times 10^{-3}$

$= \underline{\underline{1.5}}$

$\boxed{\xi > 1}$

(3)



$s_1 \cdot s_2 = -\xi\omega_s \pm \omega_n\sqrt{\xi^2-1}$

$= -1.5\times 1000 \pm 1000\sqrt{1.5^2-1}$

$s_1 s_2 = -1500 \pm 1118$

$s_1, s_2 = -\cancel{318}\; -382, -2618$

4) $\underline{R = 5000}$

$\xi = 2.5$ $\xi > 1$

$s_1, s_2 = -2500 \pm 2291.28 = -208.72, -4791.25$

$s_2 \to \infty$

$s_1 \to 0$.

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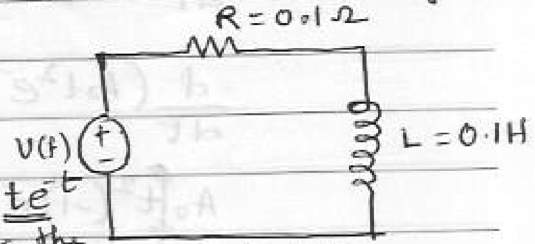
6.23 (1) A bolt $\frac{di}{dt}$ (discharge of lightening) of lightening having a waveform which approximated as $v(t) = t \cdot e^{-t}$ strikes a transmission line having resistance of $R = 0.1\Omega$ and inductance $L = 0.1H$ (the line to line capacitance negligible)

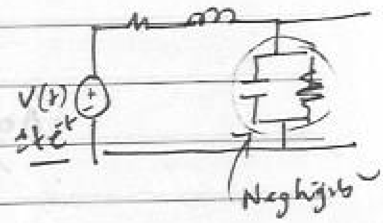


What is the form of the current as a function of time?

(This current will be Amp/unit volt of lightening, Hence the time base is linear)

sol^n:

sol^n:



KVL $\quad L\frac{di}{dt} + Ri = te^{-t}$ — (1)

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{t}{L}e^{-t}$$

$$\frac{di}{dt} + \left(\frac{0.1}{0.1}\right)i = \frac{t}{0.1}e^{-t}$$

$$\boxed{\frac{di}{dt} + i = 10t \cdot e^{-t}} \quad -(2)$$

Characteristic eq^n can be written as

$$si + i = 10te^{-t}$$

$$\boxed{(s+1)\,i = 10te^{-t}} \quad -(3)$$

The response of the n/w.

$$\boxed{i(t) = i_c(t) + i_p(t)}$$

(i) Complementary sol^n $i_c(t)$ of the form $k \cdot e^{st}$

$$(s+1)\,i = 0.$$

$\therefore\ s = -1$ and $i_c(t) = K \cdot e^{st}$

$$= K \cdot e^{-t}$$

$$\left[\,i_c(t) = K \cdot e^{-t}\,\right] \quad -(4)$$

(ii) Particular sol^n $i_p(t)$

In this, forcing factor is $(10 \cdot t)e^{-t}$ ie $(a_1 t^n)e^{rt}$ form

$$\therefore\ i_p(t) = (A_0 t + A_1)\,e^{-t}$$

$$\boxed{i_p(t) = A_0 te^{-t} + \underbrace{A_1 e^{-t}}} \quad -(5)$$

↳ It is of $i_c(t)$ form.

$\therefore$ change the form by multiplying by a linear factor t to eq^n (5)

$$i_p(t) = (A_0 te^{-t} + A_1 e^{-t}) \times t = \boxed{A_0 t^2 e^{-t} + A_1 t \cdot e^{-t}} \quad -(6)$$

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Substituting $i_p(t)$ - eq^n (6) in eq^2 (2)

$$\frac{d(i_p(t))}{dt} + i_p(t) = 10te^{-t}$$

$$\frac{d}{dt}\left(A_0t^2e^{-t} + A_1te^{-t}\right) = 10te^{-t}$$

$$A_0 t^2(-1)e^{-t} + A_0e^{-t}2(t) + A_1t(-1)e^{-t} + A_1e^{-t}(1) + 10te^{-t}$$

$$-A_0t^2e^{-t} + A_02te^{-t} - A_1te^{-t} + A_1e^{-t} = 10te^{-t}$$

$$+ A_0t^2e^{-t} + A_1te^{-t} = 10te^{-t}$$

$$-\cancel{A_0t^2e^{-t}} + A_02te^{-t} - \cancel{A_1te^{-t}} + A_1e^{-t}$$

$$+ \cancel{A_0t^2e^{-t}} + \cancel{A_1te^{-t}} = 10(te^{-t}) + 0e^{-t}$$

Equaling coefficients

$2A_0 = 10$ $\boxed{A_1 = 0}$

$\boxed{A_0 = 5}$

$$\therefore i_p(t) = A_0t^2e^{-t} + A_1te^{-t}$$

$$= 5t^2e^{-t} + 0te^{-t} = 5t^2e^{-t}$$

$$\boxed{i(t) = \underbrace{Ke^{-t}}_{i_c(t)} + \underbrace{5t^2e^{-t}}_{i_p(t)}} \quad -(7) \qquad \boxed{i(t) = 5t^2e^{-t}}$$

Using initial condition ie $i(0^-) = 0 = i(0^+)$, K is found

$t = 0^+$ in eq^2 (7) $i(0^+) = Ke^0 + 50$

$i(0^+) = K$.

$\boxed{K = 0}$

$$\therefore i(t) = 0 \times e^{-t} + 5t^2e^{-t}$$

$$\boxed{i(t) = 5t^2e^{-t}} \quad -(8)$$

This indicates that there is No complementary sol^n; i(t) has only particular sol^n.

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method 2 : $\frac{di}{dt} + i = 10te^{-t}$ — (1)

LT $sI(s) - i(0^-) + I(s) = \frac{10}{(s+1)^2}$

$sI(s) + I(s) = \frac{10}{(s+1)^2}$

$I(s)(s+1) = \frac{10}{(s+1)^2}$

$I(s) = \frac{10}{(s+1)^3}$ —— (2)

Using ILT

$i_p(t) = \frac{10}{2!} t^2 e^{-t} = \underline{\underline{5t^2e^{-t}}}$

$\frac{di}{dt} + i = 0$

$SI(s) - i(0^-) + I(s) = 0$

$(s+1)I(s) = i(0^-)$

$I_c(s) = 0.$

ILT $\boxed{i_c(t) = 0}$

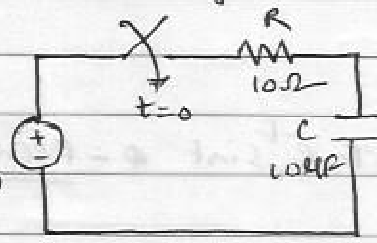
$10 \cancel{t} \times e^{-t} \times \cancel{t} e^{-t}$

$10 \times t \cdot e^{-2t}$

$\int_{0^-}^{0^+}$

6.24) In the n/w switch K is closed at t=0 with capacitor initially unenergised. For the numerical values given find i(t)

$\underline{10 \cdot e^{-t}} \sin t \; u(t)$



sol : KVL to the loop.

$Ri + \frac{1}{C}\int_{-\infty}^{t} i \cdot d\tau = 10e^{-t} \cdot \sin t \; u(t)$ — (1)

$R\frac{di}{dt} + \frac{1}{C} i = 10\left[e^{-t} \cdot \cos t + \sin t (-1) e^{-t}\right]$

$\left[\frac{di}{dt} + \frac{1}{RC} i = \frac{10}{R}\left[e^{-t} \cos t - e^{-t} \sin t\right]\right]$ — (2)

Substitute R & C

~~Partial~~ then , $\frac{di}{dt} + 10^4 i = \underbrace{\left(e^{-t} \cos t - e^{-t} \sin t\right)}_{\text{RHS.}}$ — (2)

(i) Complementary sol : $i_c(t) = K \cdot e^{st}$

$\frac{di}{dt} + \frac{1}{RC} i = 0.$ $\quad si + \left(\frac{1}{RC}\right) i = 0$ — $\underline{\underline{\text{general}}}$

$\left(s + \frac{1}{RC}\right) = 0$

$s = -\frac{1}{RC} = -\frac{1}{10 \times 10 \times 10^{-6}}$

$\boxed{s = -10^{+4}}$

$\therefore \boxed{i_c(t) = K \cdot e^{-10^{+4} t}}$

(ii) Particular sol: $i_p(t)$ = w.r.t to forcing function

$10 \cdot e^{-t} \cdot \sin t$ $\qquad \cancel{10}\, \cancel{e^{-t} \cos t} - \cancel{10}\, e^{-t} \cdot \sin t$

$10\, t^0 e^{-t} \sin t$

$\cancel{i(t) = e^{-t}\left(A_1 \cos \omega t + B_1 \sin \omega t\right) - e^{-t}\left(A_2 \cos \omega t + B_2 \sin \omega t\right)}$

$\cancel{= e^{-t}\left(\frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}\right) \cos\left(\omega t - \tan^{-1}(\omega RC)\right)}$

$\cancel{- e^{-t}\left(A_2 \cos \omega t + \right.}$

$$\boxed{i_p(t) = F_1 e^{-t} \cos t + F_2\, e^{-t} \cdot \sin t} \quad -(4)$$

(4) in (2)

$$\frac{d}{dt}\left(F_1 e^{-t} \cos t + F_2 e^{-t} \sin t\right) + \frac{1}{RC}\left(F_1 e^{-t} \cos t + F_2 e^{-t} \sin t\right)$$
$$= \cancel{10} e^{-t} \cos t - \cancel{10} e^{-t} \sin t$$

$$F_1 e^{-t} \cdot (-\sin t) + F_1 \cos t \cdot (-1) e^{-t} + F_2 e^{-t} \cos t + F_2 \sin t (-1) \cdot e^{-t}$$
$$+ \frac{1}{RC}\left(F_1 e^{-t} \cos t + F_2 e^{-t} \sin t\right)$$
$$= \cancel{10} e^{-t} \cos t - \cancel{10} e^{-t} \sin t$$

$$-F_1 e^{-t} \sin t - F_1 e^{-t} \cos t + F_2 e^{-t} \cos t - F_2 e^{-t} \sin t + 10^4 \left(F_1 e^{-t} \cos t + F_2 e^{-t} \sin t\right) = \cancel{10} e^{-t} \cos t - \cancel{10} e^{-t} \sin t$$

$$-F_1 e^{-t} \sin t + 9999 F_1 e^{-t} \cos t + F_2 e^{-t} \cos t + 9999 F_2 e^{-t} \sin t$$
$$= \cancel{10} e^{-t} \cos t - \cancel{10} e^{-t} \sin t$$

$$e^{-t} \sin t \left(9999 F_2 - F_1\right) + e^{-t} \cos t \left(9999 F_1 + F_2\right)$$
$$= \cancel{10} e^{-t} \cos t - \cancel{10} e^{-t} \sin t$$

$$9999 F_2 - F_1 = -1$$
$$9999 F_1 + F_2 = 1$$

$$\left.\begin{aligned} -F_1 + 9999 F_2 &= -1 \\ 9999 F_1 + F_2 &= 1 \end{aligned}\right\} - (A)$$

$F_1 = 1 \times 10^{-4}$

$F_2 = -9.999 \times 10^{-5}$

$= -10^{-4}$

$$\therefore \boxed{i_p(t) = 10^{-4} e^{-t} \cos t - 10^{-4} \cdot e^{-t} \cdot \sin t}$$

$$i(t) = i_c(t) + i_p(t)$$

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$$i(t) = K \cdot e^{-10^4 t} + 10^{-4} e^{-t} \cos t - 10^{-4} e^{-t} \sin t$$

$t = 0^+$ By substituting initial condⁿ K is found.

$$i(0^+) = K e^{0} + 10^{-4} e^{0} \cos(0) - 10^{-4} e^{0} \sin(0)$$

$$= K + 10^{-4} - 10^{-4}(0)$$

$$0 = K + 10^{-4}$$

Final response ∴ $K = -10^{-4}$.

$$\therefore \boxed{i(t) = -10^{-4} e^{-10^4 t} + 10^{-4} e^{-t} \cos t - 10^{-4} e^{-t} \sin t}$$

6.33) A switch is closed at $t=0$, connecting a battery of voltage V with series RL ckt.

a) Show that, energy in the resistor as function of time is
$$\omega_R = \frac{V^2}{R}\left(t + \frac{2L}{R} e^{-tR/L} - \frac{L}{2R} e^{-2\frac{R}{L}t} - \frac{3L}{2R}\right) \text{ Joule}$$

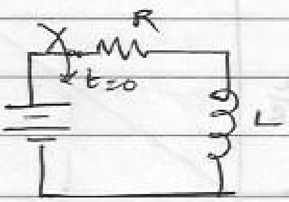
b) find an expression for energy in magnetic field as a function of time

c) Sketch ω_R and ω_L as function of time. Show the steady state asymptotes, that is, the values that ω_R, ω_L approach $t \to \infty$

d) Find the total energy supplied by the voltage source in steady state as function of time.

a) $i(0^-) = 0 = i(0^+)$

$i(\infty) = \frac{V}{R}$ ⟨∵ L is SC



$$\cancel{\omega_R = \int_0^t i^2(\tau) \cdot R \cdot dt = \int_0^t \left(\frac{V}{R}\right)^2 \times R}$$

$$\left[i(t) = \frac{V}{R}\left(1 - e^{-t\frac{R}{L}}\right) \right] \quad -(1)$$

$$\omega_R = \int_0^t i^2(\tau) \cdot R \cdot dt = \int_0^t \left(\frac{V}{R}\right)^2 \left(1 - e^{-t\frac{R}{L}}\right)^2 \cdot R \cdot dt$$

$$= \frac{V^2}{R^2} \times R \int_0^t \left(1 - e^{-\frac{R}{L}t}\right)^2 \cdot dt$$

$$W_R = \frac{V^2}{R}\int_0^t \left(1 + e^{-2t\frac{R}{L}} - 2\times 1\times e^{-t\frac{R}{L}}\right).dt$$

$$= \frac{V^2}{R}\left[t\Big|_0^t - \frac{e^{-2R/L\,t}}{\left(\frac{2R}{L}\right)}\Bigg|_0^t + \frac{2e^{-\frac{R}{L}t}}{\left(\frac{R}{L}\right)}\Bigg|_0^t \right]$$

$$= \frac{V^2}{R}\left[t - \left(\frac{L}{2R}e^{-2\frac{R}{L}t} - \frac{L}{2R}e^0\right) + \frac{2L}{R}\left(e^{-\frac{R}{L}t} - e^0\right)\right]$$

$$= \frac{V^2}{2}\left[t - \frac{L}{2R}e^{-2\frac{R}{L}t} + \frac{L}{2R} + \frac{2L}{R}.e^{-\frac{R}{L}t} - \frac{2L}{R}\right]$$

$$= \frac{V^2}{2}\left[t + \frac{2L}{R}e^{-R/L\,t} - \frac{L}{2R}e^{-2\frac{R}{L}t} + \frac{L}{2R} - \frac{2L}{R}\right]$$

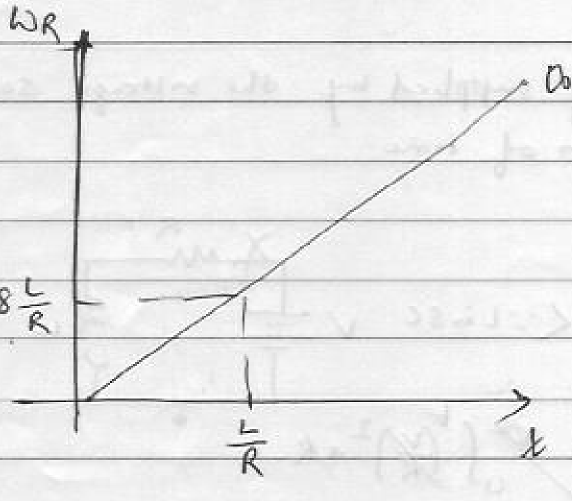
$$W_R = \frac{V^2}{2}\left[t + \frac{2L}{R}e^{-\frac{R}{L}t} - \frac{L}{2R}e^{-2\frac{R}{L}t} - \frac{3L}{2R}\right] \text{ Joules}$$

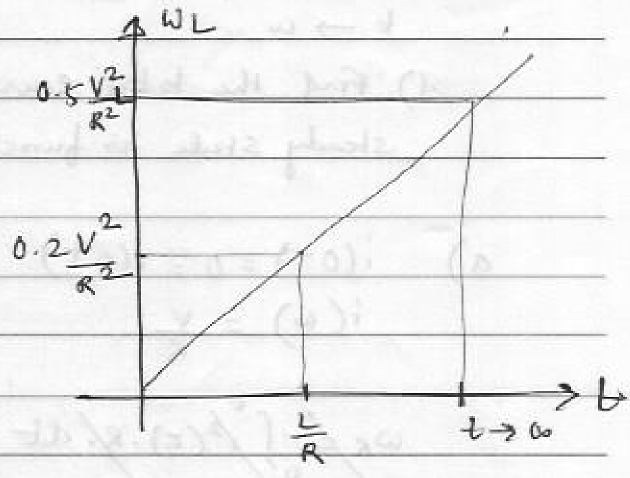
b) Energy in magnetic f

$$W_L = \frac{1}{2}L\, i^2(t)$$

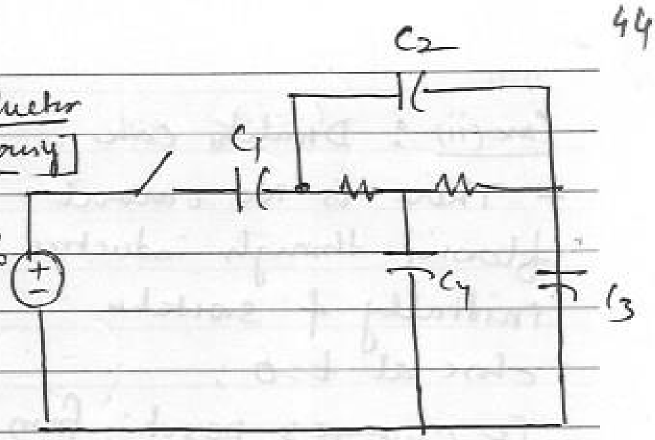
$$= \frac{1}{2}\times L\times\frac{V^2}{R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2$$

c)





d)

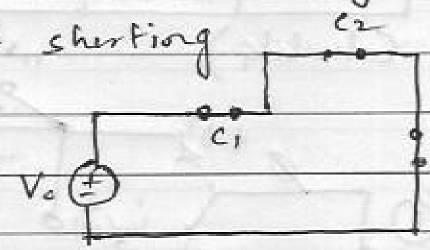


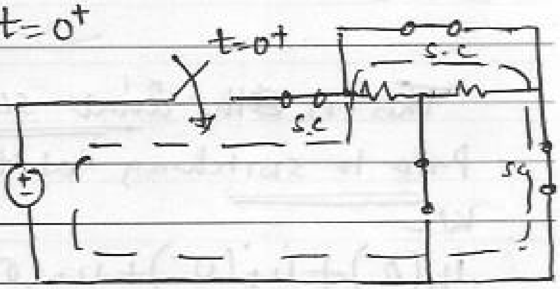
* Special cases [capacitor voltage & inductor current changes instanteneaouy]

- Capacitors are not initially charged and switch k is closed at $t=0$
- W.K.T capacitor acts like as short circuit

∴ It results into voltage source shorting

$t=0^+$





∴ the voltage source produces an infinite current over an infinitesimally short interval of time.

- The charge transfered

$$q = \int_{0^-}^{0^+} i(\tau)\,d\tau \text{ is however finite}$$

and is such that

$$V_0 = v_{C_1}(0^+) + v_{C_2}(0^+) + v_{C_3}(0^+)$$

$$V_0 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

(All initially loop which in series ∴ q is same)

[Thus infinite current in the interval $[0^-, 0^+]$ deposits sufficient charges on the cap that KVL satisfied]

The infinite current in the interval $[0^-, 0^+]$ is mathematically modelled using Dirac Delta impulse. Under this condition, the voltage ale 4 cap changes instanteneasly.

A similar analysis holds even if the cap are charged prior to switching

Note: Ideal model of cap is assumed, In practice there is always some stray resistance which prevents infinite current flowing

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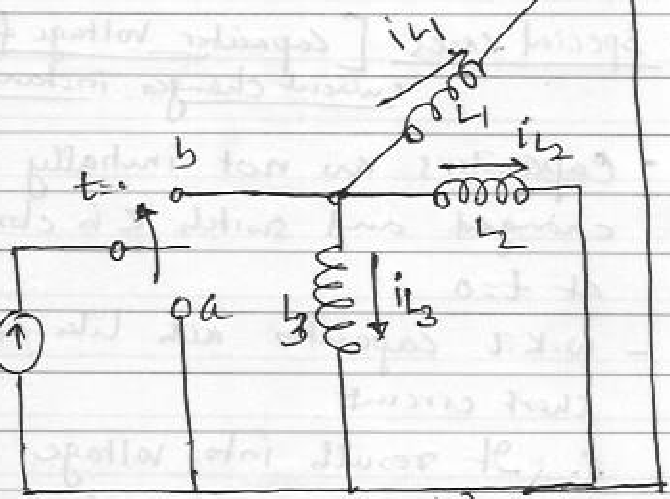
Case (ii) : Duality case

- There is No current flowing through inductors initially & switch close at $t=0$; ie changes position from a to b

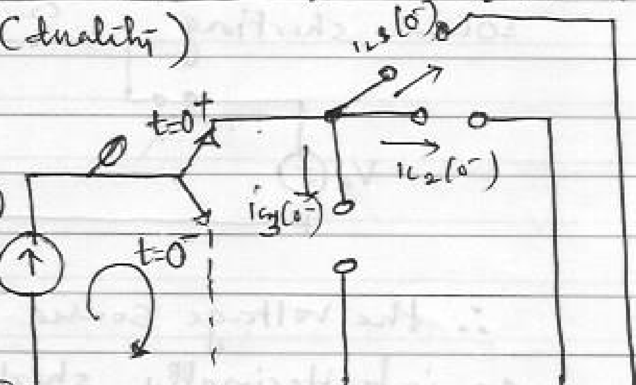


This is the dual scenario (duality)

- Prior to switching at $t=0^-$

KCL

$$i_{L_1}(0^-)+i_{L_2}(0^-)+i_{L_3}(0^-)=0 \quad -(1)$$

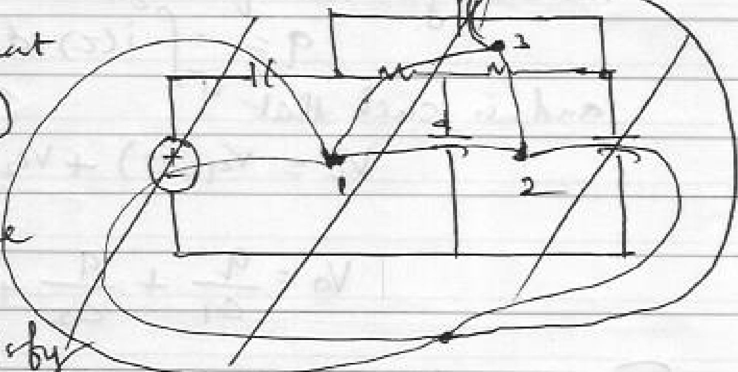


- Immediately after switch $t=0^+$, KCL

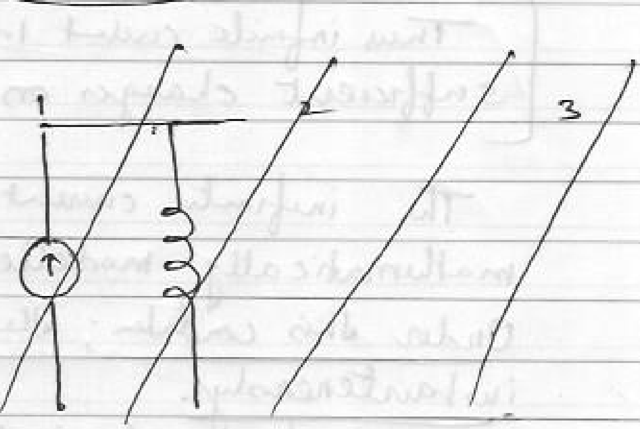
$$\boxed{i_L(0^+)+i_{L_2}(0^+)+i_{L_3}(0^+)=I_0} \quad -(2)$$

From (2), it is clear that $i_L(0^+)$, $i_{L_2}(0^+)$, $i_{L_3}(0^+)$ cannot be zero

$\therefore$ Current through the inductor must change instantaneously to satisfy both the eq 2



- In this case, A infinite voltage is generated due to ~~is~~ switching action that produces sufficient ~~flux~~ finite flux for the above to satisfy



$$\phi=\int_{0^-}^{0^+} v(t)\cdot dt \Longrightarrow \frac{\phi}{L_1}+\frac{\phi}{L_2}+\frac{\phi}{L_3}=I_0$$

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- The infinite voltage in the interval $[0^-, 0^+]$ is mathematically modelled using the Dirac Delta impulse.
- Under this cond^n, the current through an inductor changes instantaneously.
- A similar analysis holds even if the inductors are carrying prior to switching
- Note, ideal model of inductor is assumed. In practice, there is always some stray resistance which prevents infinite voltage to be generated.

*** Note:** Relationship between quantity $\boxed{Q = \frac{1}{2\xi}}$

Overdamped : $\xi > 1$ or $Q < 0.5$ (or) $R > R_{cr}$ ← $R_{critical}$

Critically damp : $\xi = 1$. or $Q = 0.5$
$R = R_{cr}$

Underdamped : $\xi < 1$, $Q > 0.5$ (or) $R < R_{cr}$.

* RL ckt driven by $V\sin(\omega t + \theta)$.

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \sin(\omega t + \theta)$$

Using the procedure

$$i_p(t) = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\therefore\ i(t) = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right) + k e^{-\frac{R}{L}t}$$

If switch is closed at $t = 0$, $i(0^+) = 0$.

$$\therefore\ k = -\frac{V}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

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$$k = -\frac{V}{\sqrt{R^2+\omega^2L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

If θ is chosen s.t

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

then $k=0$

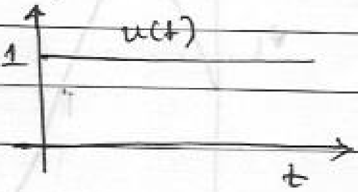
- There is no transient response
- A similar phenomenon can be shown to exist for RC n/w

Factor in $v(t)$	Neccessary choice for particular integral
V	A
$a_1 t^n$	$B_0t^n + B_1t^{n-1} + \cdots + B_{n-1}t + B_n$.
$a_2 e^{rt}$	$C.e^{rt}$
$a_3 \cos\omega t$, $a_4 \sin\omega t$	$D\cos\omega t + E\sin\omega t$
$a_5 t^n e^{rt}\cos\omega t$, $a_6 t^n e^{rt}\sin\omega t$	$(F_1t^n + \cdots + F_{n-1}t + F_n)e^{rt}\cos\omega t + (G_1t^n + \cdots + G_n)e^{rt}\sin\omega t$.

* Waveform Synthesis :

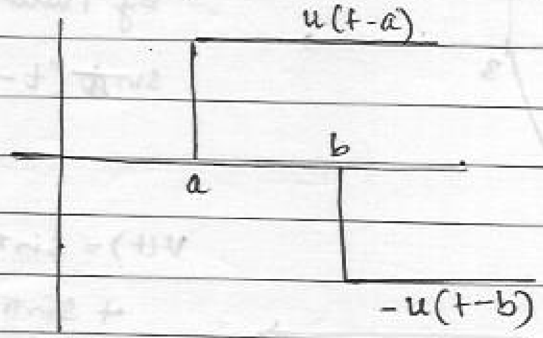
A unit step function

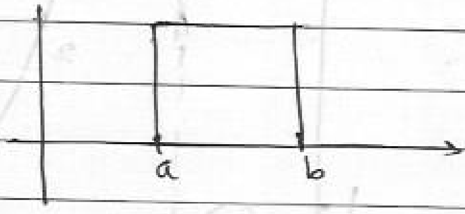
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



A function which changed abruptly from zero to unit value at t=0

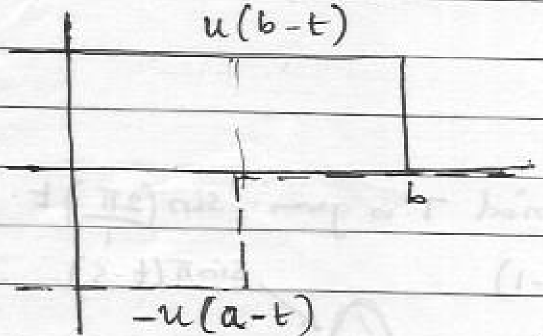
(i) A pulse function in terms of step function.

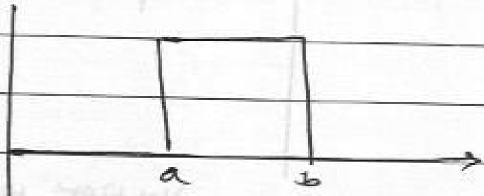




$v(t) = u(t-a) + (-u(t-b))$

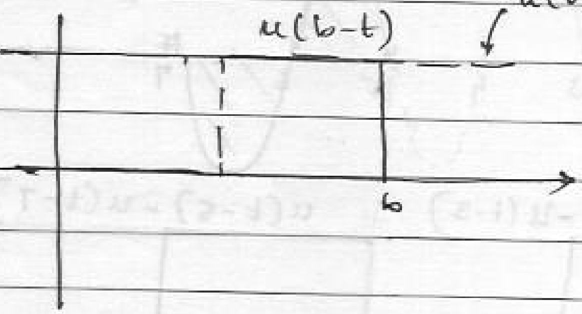
$v(t) = u(t-a) - u(t-b)$

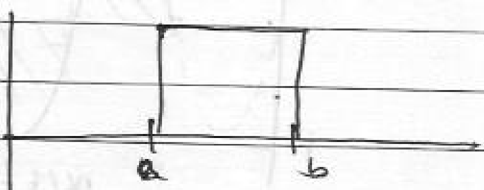




$u(b-t) + (-u(a-t))$

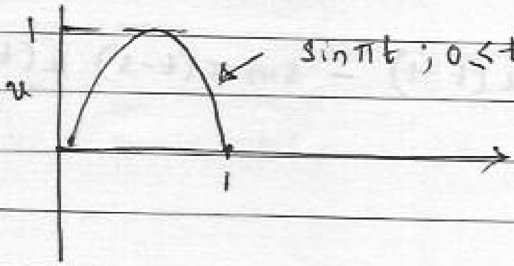
$\boxed{v(t) = u(b-t) - u(a-b)}$

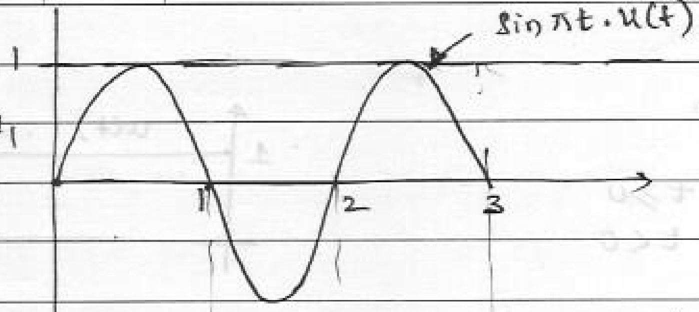




$\boxed{v(t) = u(t-a) \times u(b-t)}$

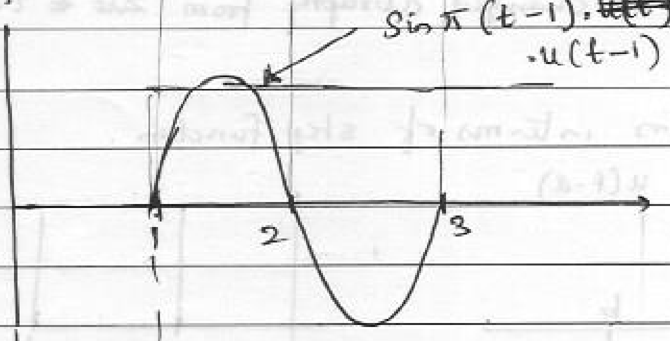
(ii) A half cycle of sine wave





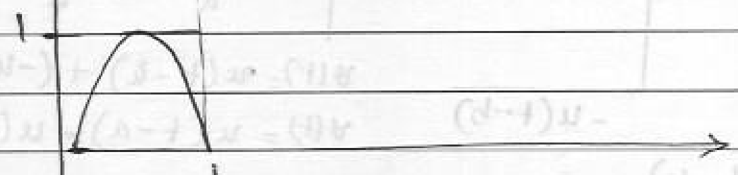
- Sinewave defined for positive time, and zero for all (-)ve time.

$\sin\pi t \cdot u(t)$.



- ~~Sinwave~~ is shifted version of sin πt u(t) by 1 unit times

$\sin\pi(t-1) \times u(t-1)$

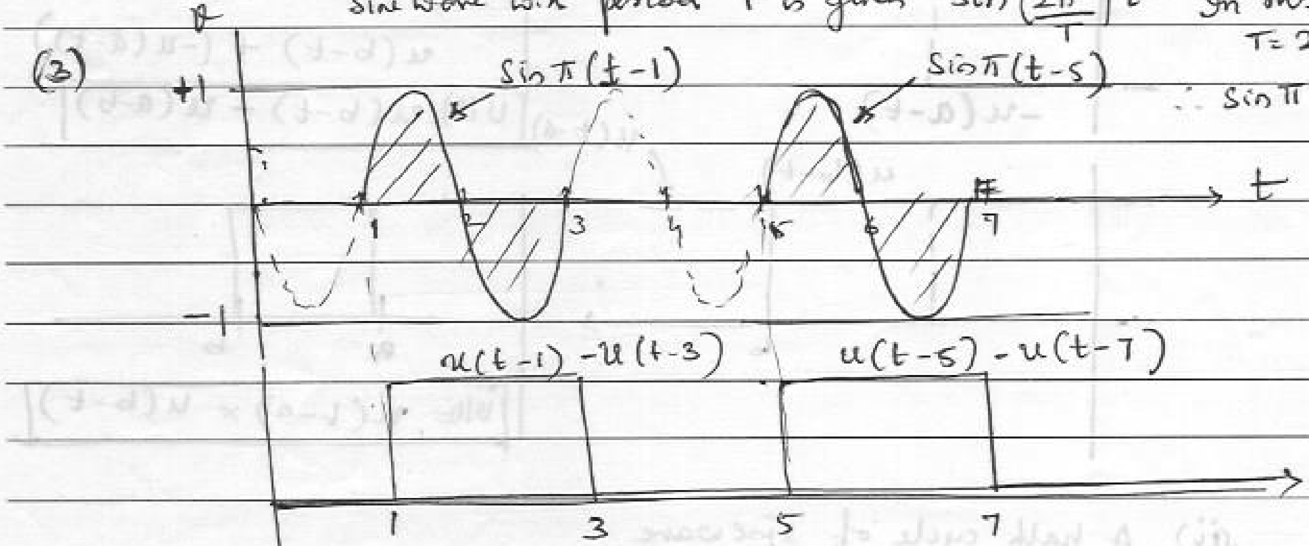


$v(t) = \sin\pi t\, u(t) + \sin\pi(t-1) \cdot u(t-1)$.

Sine wave with period T is given $\sin\left(\frac{2\pi}{T}\right)t$. In this $T = 2$

$\therefore \sin\pi t$

(3)



$\sin\pi(t-1) \cdot u(t-1) - \sin\pi(t-3) \cdot u(t-3)$

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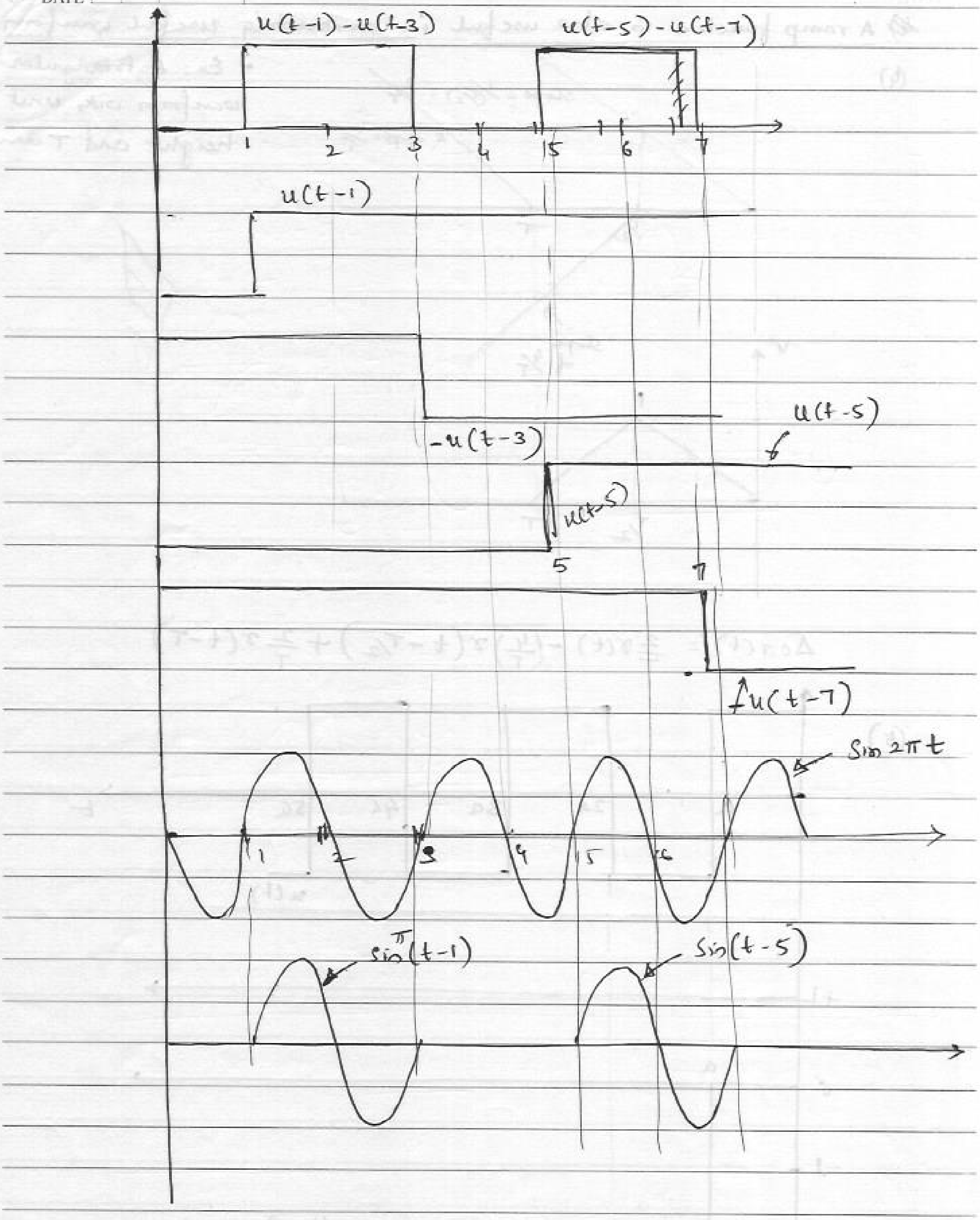

~~sin(t-~~

$$u(t) = \left[\sin\pi(t-1)\cdot u(t-1)\right] - \left[\sin\pi(t-3)\cdot u(t-3)\right] \quad \text{—(1)}$$
$$+ \left[\sin\pi(t.5)\cdot u(t-5)\right] - \left[\sin\pi(t-7)\cdot u(t-7)\right]$$

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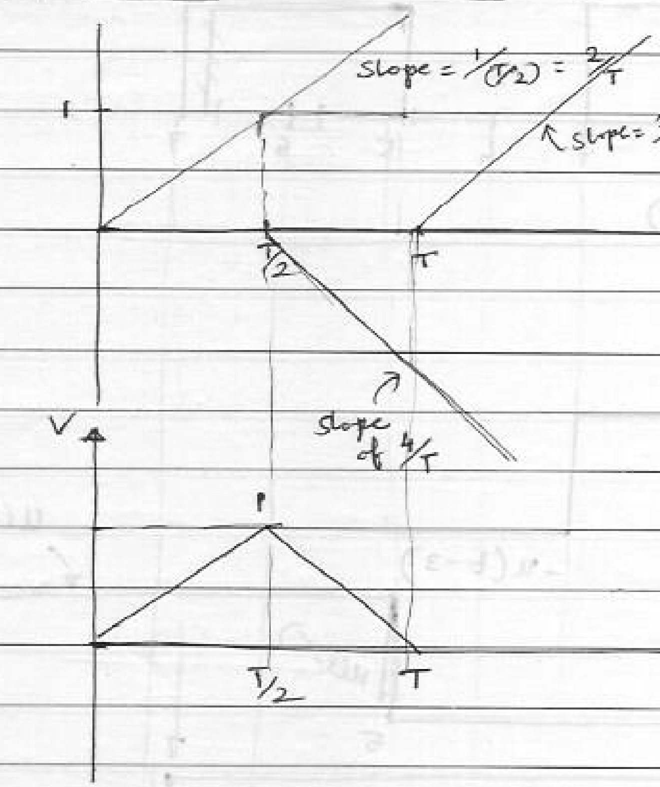
A ramp function is also useful in constructing useful waveform

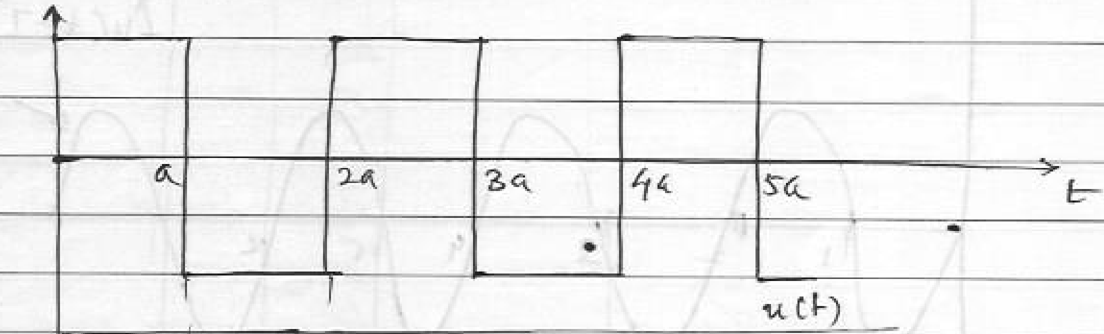
(4)

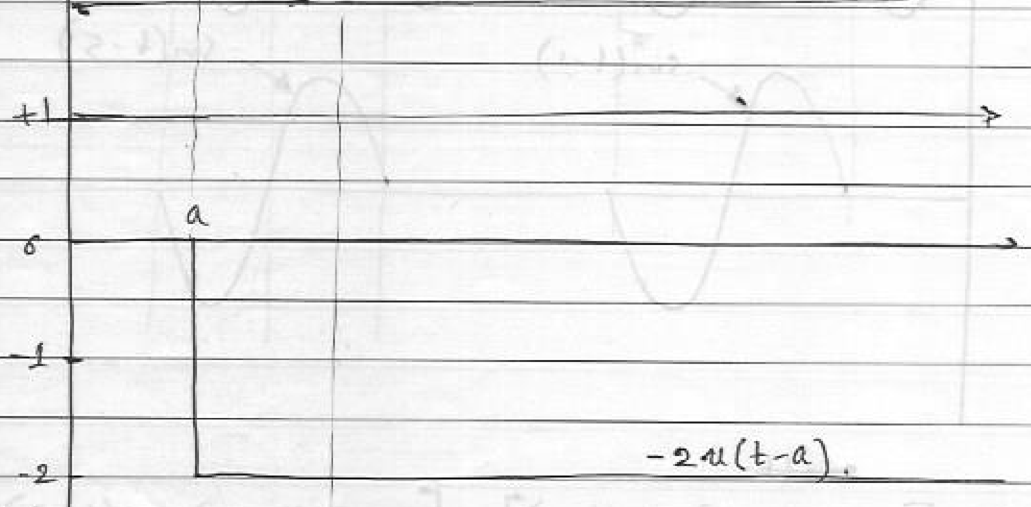
- Ex: A triangular waveform with unit height and T duration

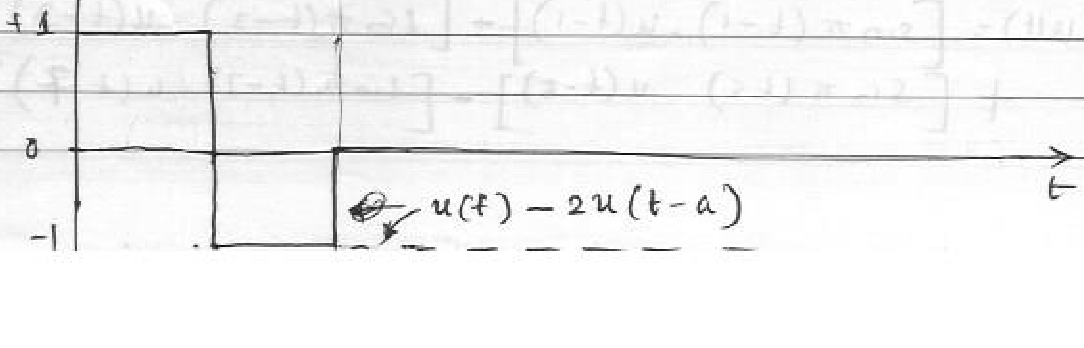


$$\Delta_{0,T}(t) = \frac{2}{T}r(t) - \left(\frac{4}{T}\right)r\left(t - T/2\right) + \frac{2}{T}r(t-T)$$

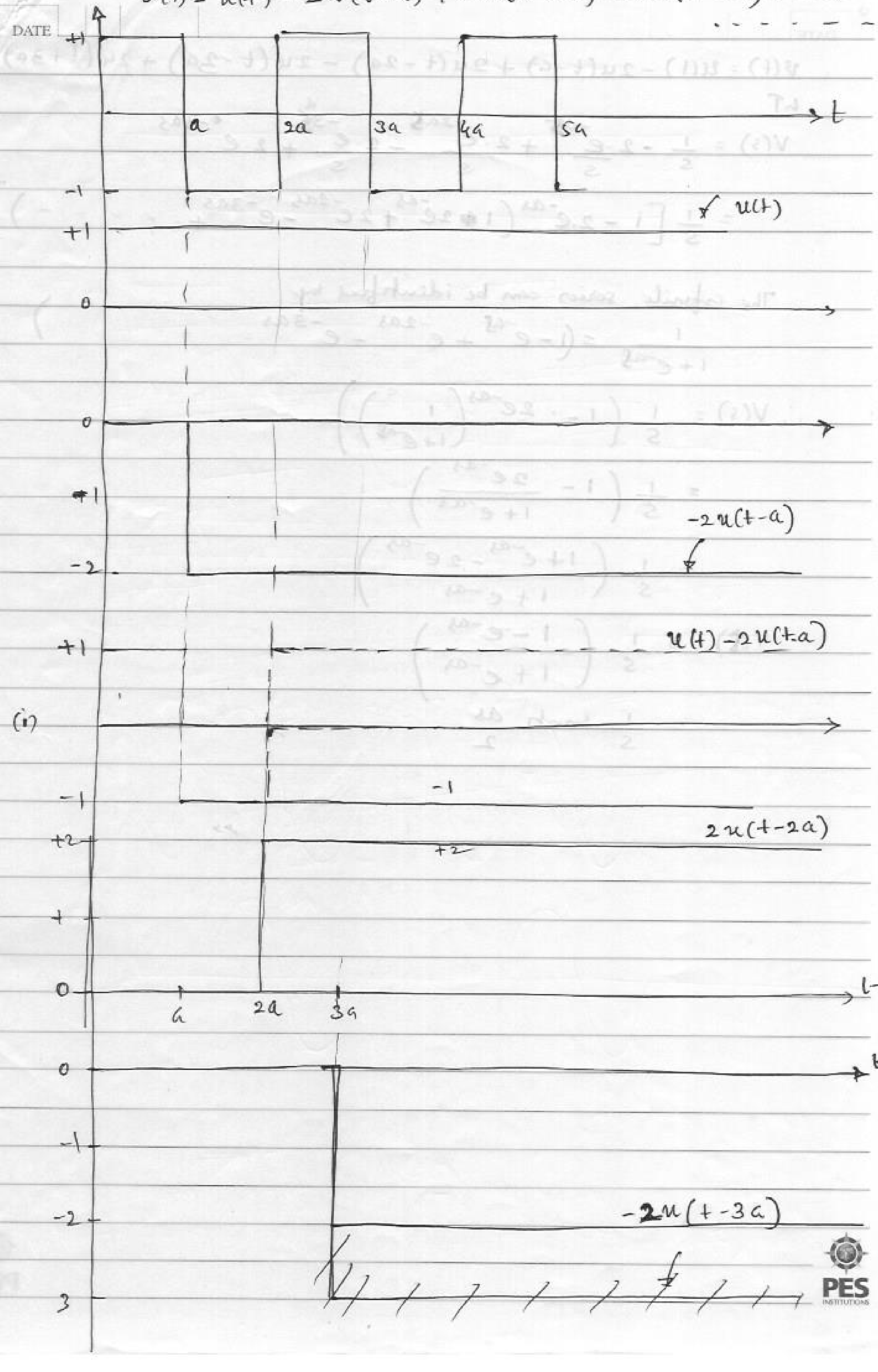
(5)







$v(t) = u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + 2u(t+3a)$
$\ldots\ldots$ 48



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$v(t) = u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + 2u$

. LT

$$V(s) = \frac{1}{s} - 2\cdot\frac{e^{-as}}{s} + 2\cdot\frac{e^{-2as}}{s} - 2\cdot\frac{e^{-3as}}{s} + 2\cdot e^{-3as}$$

$$= \frac{1}{s}\left[1 - 2\cdot e^{-as}\left(1 - 2e^{-as} + 2e^{-2as} - e^{-3as} + \cdots\right)\right]$$

The infinite series can be identified by

$$\frac{1}{1+e^{-as}} = \left(1 - e^{-as} + e^{-2as} - e^{-3as} \cdots\right)$$

$$\therefore V(s) = \frac{1}{s}\left(1 - 2e^{-as}\left(\frac{1}{1+e^{-as}}\right)\right)$$

$$= \frac{1}{s}\left(1 - \frac{2e^{-as}}{1+e^{-as}}\right)$$

$$= \frac{1}{s}\left(\frac{1+e^{-as} - 2e^{-as}}{1+e^{-as}}\right)$$

$$V(s) = \frac{1}{s}\left(\frac{1-e^{-as}}{1+e^{-as}}\right)$$

$$= \frac{1}{s}\tanh\frac{as}{2}$$