Chapter 8: Feedback

8.1 General Considerations

8.2 Feedback Topologies

8.3 Effect of Feedback on Noise

8.4 Feedback Analysis Difficulties

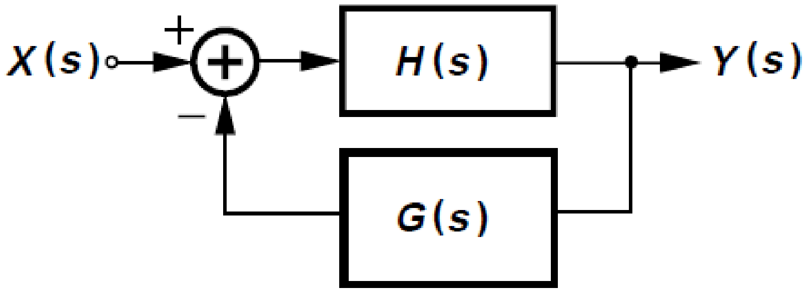
8.5 Effect of Loading

8.6 Bode's Analysis of Feedback Circuits

8.7 Loop Gain Calculation Issues

8.8 Alternative Interpretations of Bode's Method

General Considerations



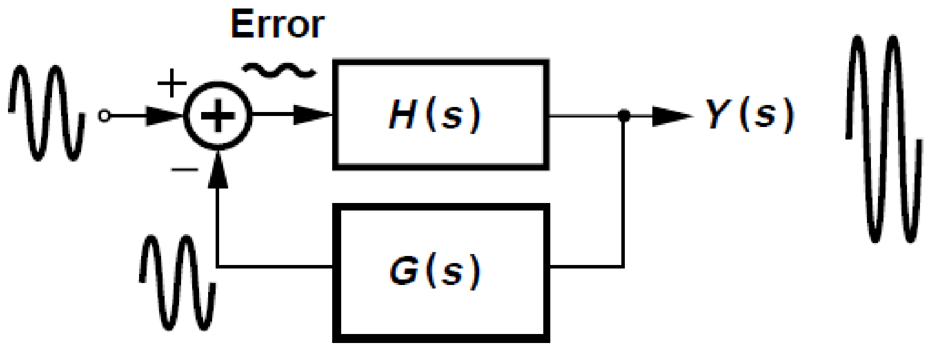
- Above figure shows a negative feedback system
- *H(s)* and *G(s)* are called the feedforward and forward networks respectively
- Feedback error is given by *X(s) – G(s)Y(s)*
- Thus

$$Y(s) = H(s)[X(s) - G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

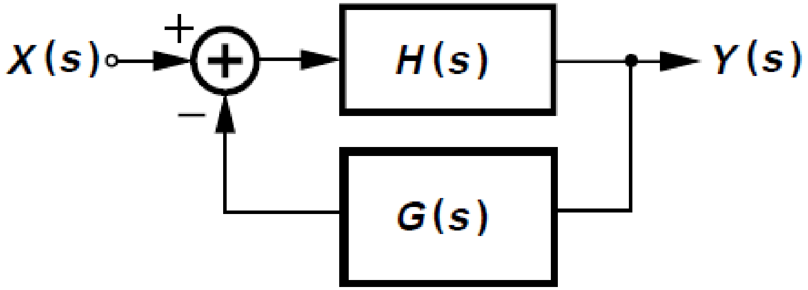
- *H(s)* is called the "open-loop" transfer function and *Y(s)/X(s)* is called the "closed-loop" transfer function

General Considerations



- In most cases, *H(s)* represents an amplifier and *G(s)* is a frequency-independent quantity
- In a well-designed negative feedback system, the error term is minimized, making the output of *G(s)* an “accurate” copy of the input and hence the output of the system a faithful (scaled) replica of the input
- *H(s)* is a “virtual ground” since the signal amplitude is small at this point
- In subsequent developments, *G(s)* is replaced by a frequency-independent quantity *β* called the *feedback factor*

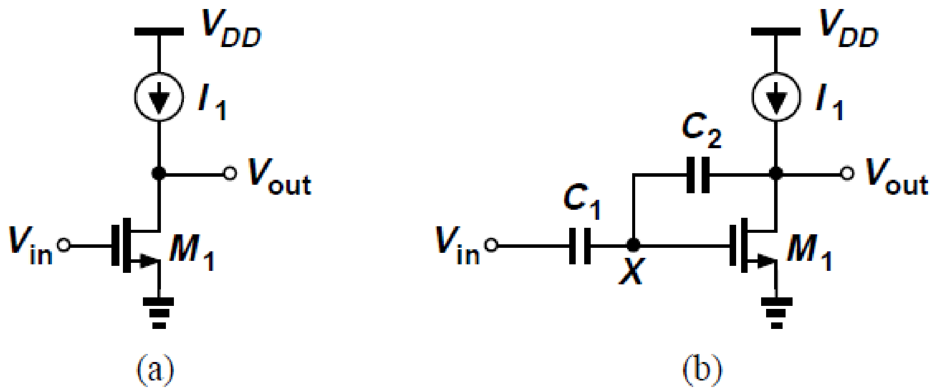
General Considerations



- Four elements of a feedback system
  - The feedforward amplifier
  - A means of sensing the output
  - The feedback network
  - A means of generating the feedback error, i.e., a subtractor (or an adder)
- These exist in every feedback system, though they may not be obvious in some cases

Properties of Feedback Circuits

- **Gain Desensitization:**

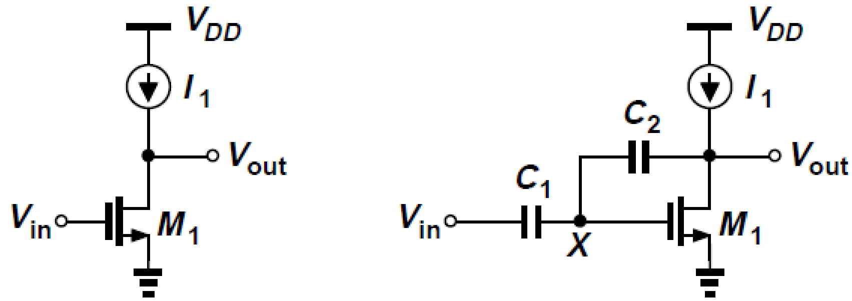


- **In Fig. (a) above, the CS stage has a gain of $g_{m1}r_{O1}$**
- **Gain is not well-defined since both g_{m1} and r_{O1} vary with process and temperature**
- **In the circuit of Fig. (b), the bias of M_1 is set by a means not shown, the overall voltage gain at low frequencies is given by**

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\left(1+\frac{1}{g_{m1}r_{O1}}\right)\frac{C_2}{C_1}+\frac{1}{g_{m1}r_{O1}}}$$

Properties of Feedback Circuits

- **<u>Gain Desensitization:</u>**

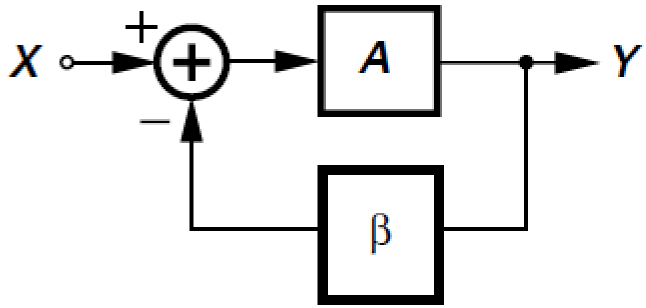


(a) (b)

- **If $g_{m1}r_{O1}$ is sufficiently large, then**

- **Compared to $g_{m1}r_{O1}$, this gain can be controlled with higher accuracy since it is a *ratio* of two capacitors, relatively unaffected by process and temperature variations if C_1 and C_2 are made of the same material**
- **Closed-loop gain is less sensitive to device parameters than the open-loop gain, hence called "gain desensitization"**

Properties of Feedback Circuits

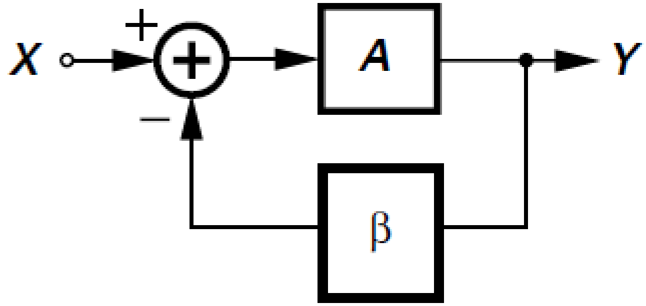


- **Frequency stability typically worsens as a result feedback**
- **For a more general case, gain desensitization is quantified by writing**

$$\frac{Y}{X} = \frac{A}{1+\beta A} \approx \frac{1}{\beta}\left(1-\frac{1}{\beta A}\right)$$

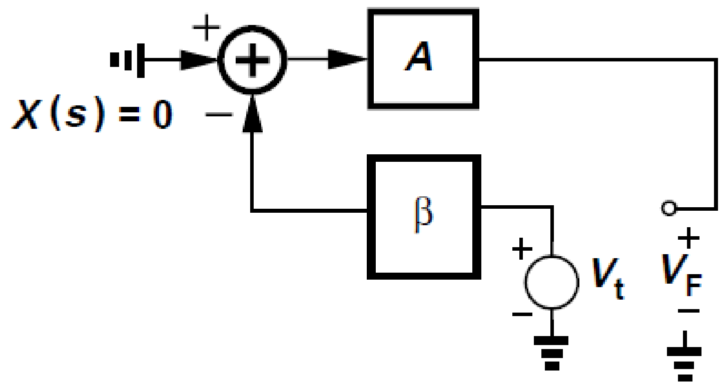
- **It is assumed *βA* >> 1; even if open-loop gain *A* varies by a factor of 2, *Y*/*X* varies by a small percentage since 1/(*βA*) << 1**

Properties of Feedback Circuits



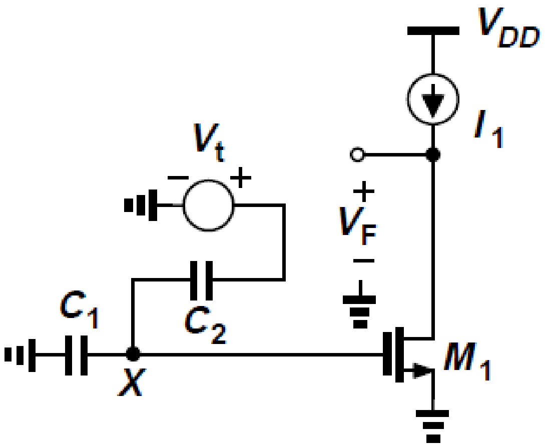
- **Called the "loop gain", the quantity *βA* is important in feedback systems**
- **The higher *βA* is, the less sensitive *Y/X* is to variations in *A*, but closed-loop gain is reduced, i.e., tradeoff between precision and closed-loop gain**
- **The output of the feedback network is equal to** $\beta Y = X \cdot \beta A/(1+\beta A)$ **approaching *X* as *βA* becomes much greater than unity**

Calculation of Loop Gain



- To calculate the loop gain:
  - Set the main input to (ac) zero
  - Inject a test signal in the "right" direction
  - Follow the signal around the loop and obtain the value that returns to the break point
  - Negative of the transfer function thus obtained is the loop gain
- Loop gain is a dimensionless quantity
- In above figure, $V_t\beta(-1)A = V_F$ and hence $V_F/V_t = -\beta A$

Calculation of Loop Gain: Example



- **Applying the given procedure to find the loop gain in the circuit above, we can write**

$$V_X = V_t C_2/(C_1 + C_2)$$

$$V_t \frac{C_2}{C_1 + C_2}(-g_{m1} r_{O1}) = V_F$$

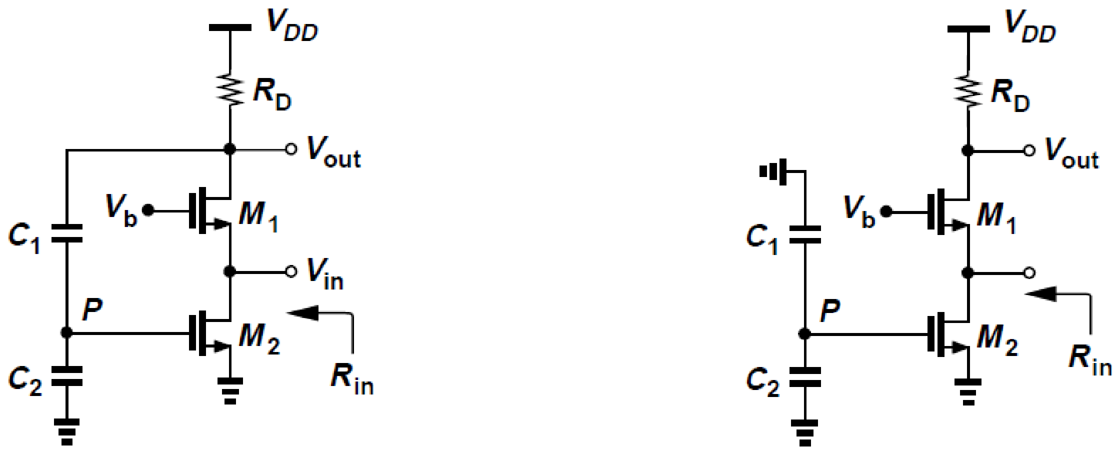
- **That is,**

$$\frac{V_F}{V_t} = -\frac{C_2}{C_1 + C_2} g_{m1} r_{O1}$$

- **The current drawn by *C_2* from the output is neglected**

Properties of Feedback Circuits

- **<u>Terminal Impedance Modification: Input Impedance</u>**



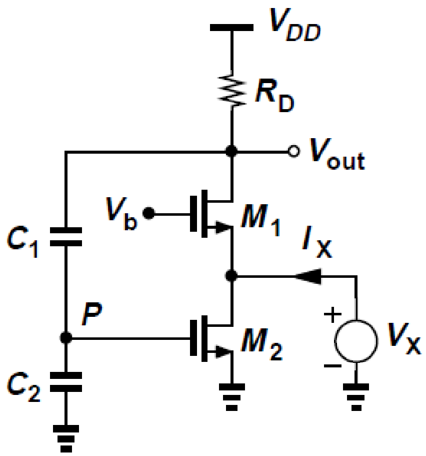
(a) (b)

- **In the circuit of Fig. (a), a capacitive voltage divider senses the output voltage of a CG stage and applies the result to the gate of current source M_2 and hence returning a signal to the input**
- **Neglecting channel-length modulation and the current drawn by C_1 and breaking the circuit as in Fig. (b), we can write**

$$R_{in,open} = \frac{1}{g_{m1} + g_{mb1}}$$

Properties of Feedback Circuits

- **Terminal Impedance Modification: Input Impedance**



(c)

- **For the closed-loop circuit of Fig. (c),**

$$V_{out} = (g_{m1} + g_{mb1})V_X R_D$$

$$V_P = V_{out}\frac{C_1}{C_1 + C_2}$$

- **Adding the small-signal drain currents of *M*₁ and *M*₂,**

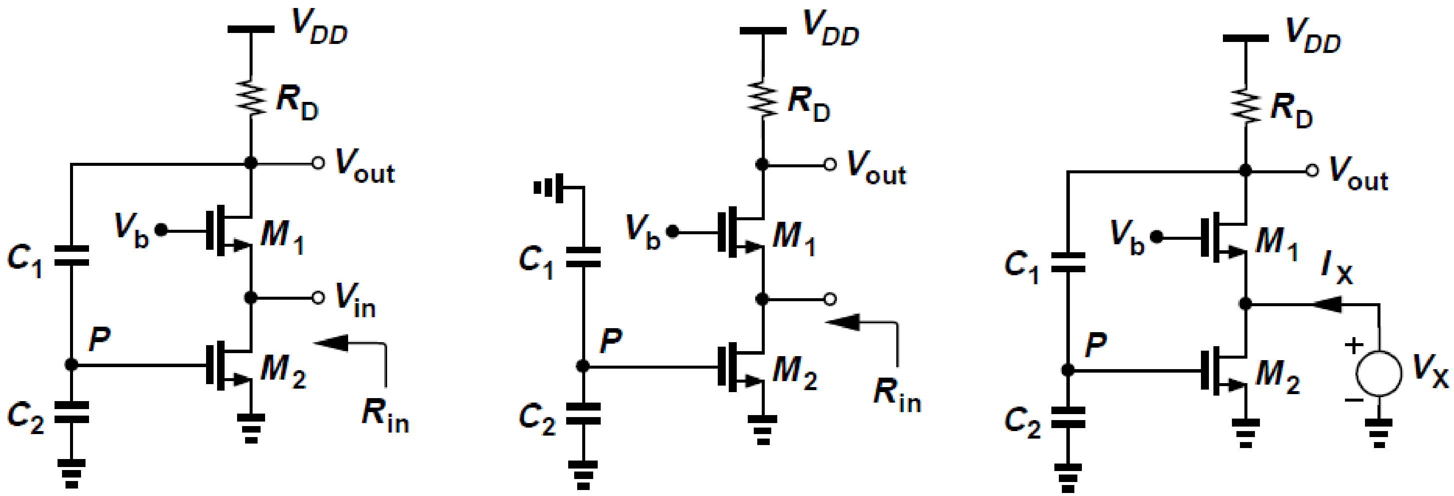
$$\begin{aligned} I_X &= (g_{m1} + g_{mb1})V_X + g_{m2}(g_{m1} + g_{mb1})\frac{C_1}{C_1 + C_2}R_D V_X \\ &= (g_{m1} + g_{mb1})\left(1 + g_{m2}R_D\frac{C_1}{C_1 + C_2}\right)V_X. \end{aligned}$$

- **It follows that**

$$\begin{aligned} R_{in,closed} &= V_X/I_X \\ &= \frac{1}{g_{m1} + g_{mb1}}\frac{1}{1 + g_{m2}R_D\dfrac{C_1}{C_1 + C_2}} \end{aligned}$$

Properties of Feedback Circuits

- **Terminal Impedance Modification: Input Impedance**

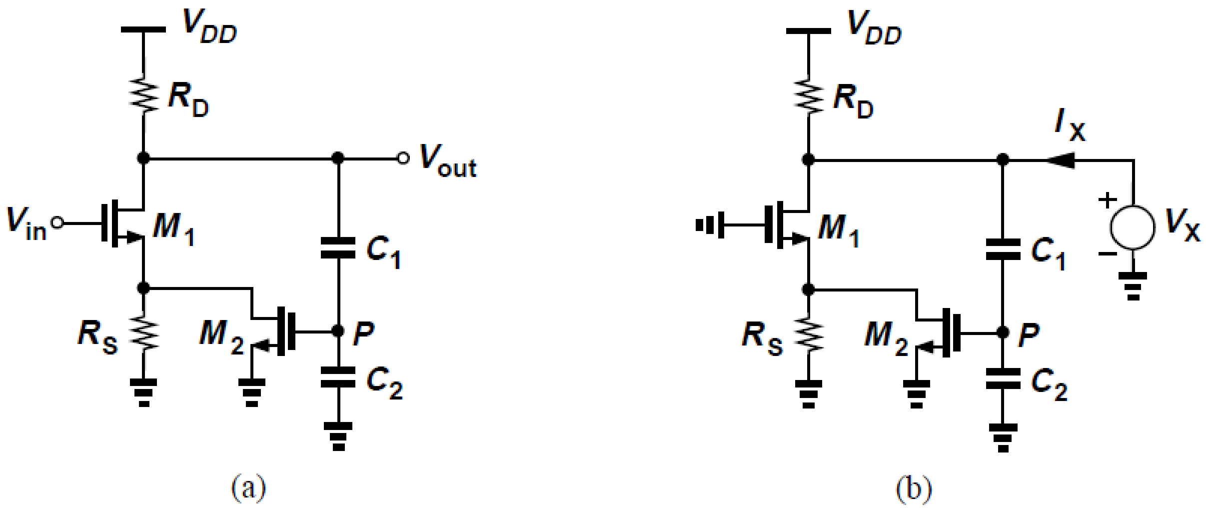


(a) (b) (c)

- **Feedback reduces the input impedance by a factor of** $1 + g_{m2}R_DC_1/(C_1 + C_2)$
- **It can be proved that** $g_{m2}R_DC_1/(C_1 + C_2)$ **is the loop gain**

Properties of Feedback Circuits

- **<u>Terminal Impedance Modification: Output Impedance</u>**

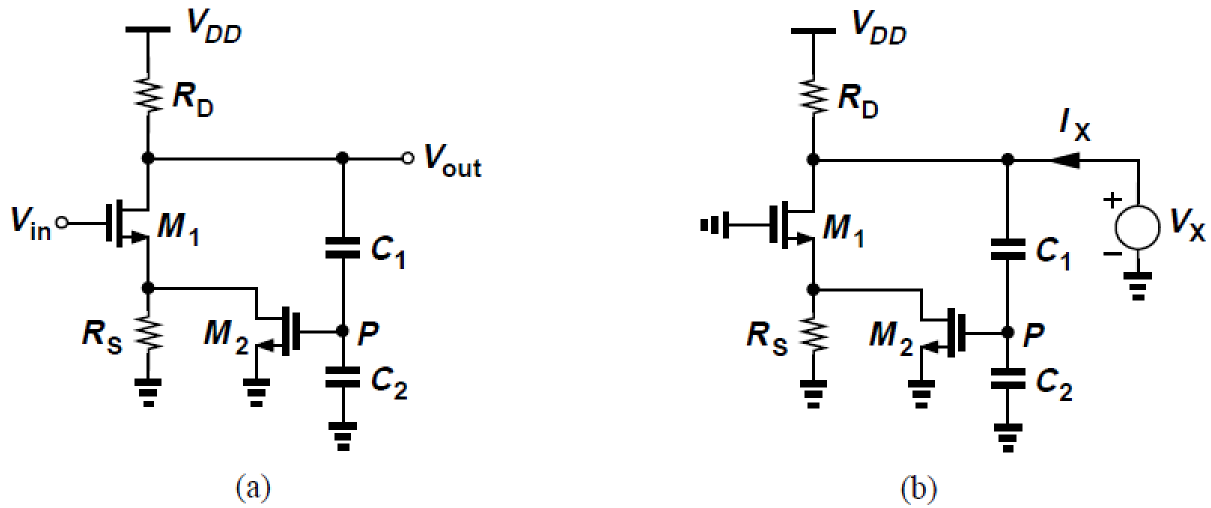


- **In the circuit of Fig. (a), M_1, R_S and R_D form a CS stage and C_1, C_2 and M_2 sense the output voltage, returning a current $[C_1/(C_1+C_2)]V_{out}g_{m2}$ to the source of M_1**
- **To find the output resistance at relatively low frequencies, the input is set to zero [Fig. (b)], so that**

$$I_{D1} = V_X \frac{C_1}{C_1+C_2} g_{m2} \frac{R_S}{R_S + \dfrac{1}{g_{m1}+g_{mb1}}}$$

Properties of Feedback Circuits

- **Terminal Impedance Modification: Output Impedance**



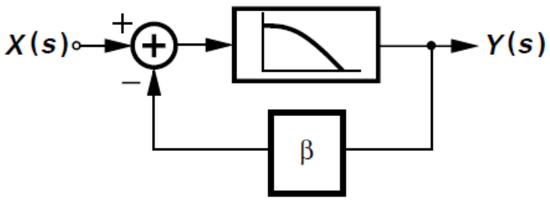
- **Since $I_X = V_X/R_D + I_{D1}$, we have**

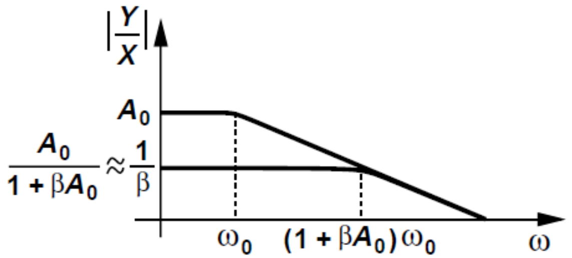
$$\frac{V_X}{I_X} = \frac{R_D}{1 + \dfrac{g_{m2}R_S(g_{m1}+g_{mb1})R_D}{(g_{m1}+g_{mb1})R_S+1}\dfrac{C_1}{C_1+C_2}}$$

- **This implies that negative feedback decreases the output impedance**
- **It can be verified that denominator is one plus the loop gain**

Properties of Feedback Circuits

- **<u>Bandwidth Modification:</u>**





- **Suppose the feedforward amplifier above has a one-pole transfer function**

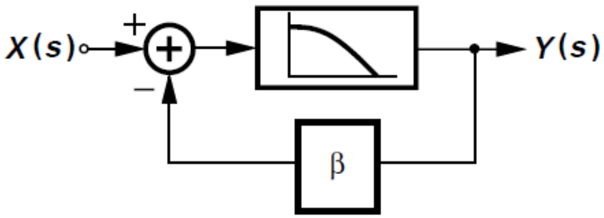
$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

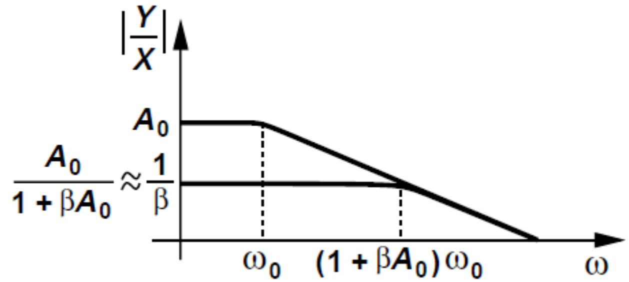
- **A_0 is the low-frequency gain and ω_0 is the 3-dB bandwidth**
- **Transfer function of the closed-loop system is**

$$\frac{Y}{X}(s) = \frac{\frac{A_0}{1+\frac{s}{\omega_0}}}{1+\beta\frac{A_0}{1+\frac{s}{\omega_0}}} = \frac{A_0}{1+\beta A_0+\frac{s}{\omega_0}} = \frac{\frac{A_0}{1+\beta A_0}}{1+\frac{s}{(1+\beta A_0)\omega_0}}$$

Properties of Feedback Circuits

- **Bandwidth Modification:**

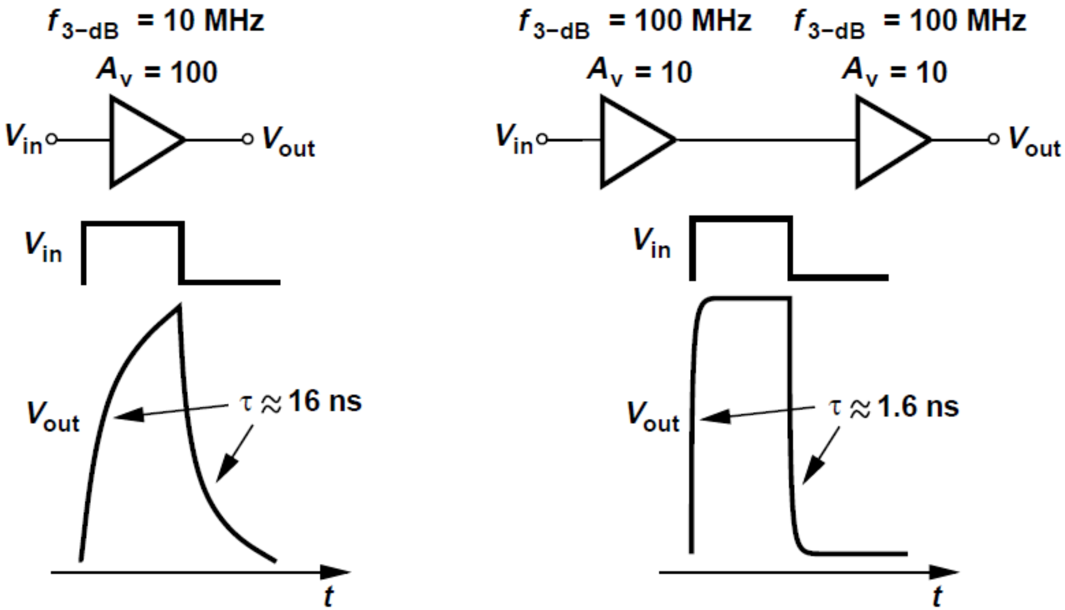




- **The closed-loop gain at low frequencies is reduced by a factor of $1+\beta A_0$ and the 3-dB bandwidth is increased by the same factor, revealing a pole at $(1+\beta A_0)\omega_0$**
- **If *A* is large enough, closed-loop gain remains approximately equal to 1/*β* even if *A* experiences substantial variations**
- **At high frequencies, A drops so that *βA* is comparable to unity and closed-loop gain falls below 1/*β***

Properties of Feedback Circuits

- **Bandwidth Modification:**
- **Gain-bandwidth product of a one-pole system is $A_0\omega_0$ and does not change with feedback**

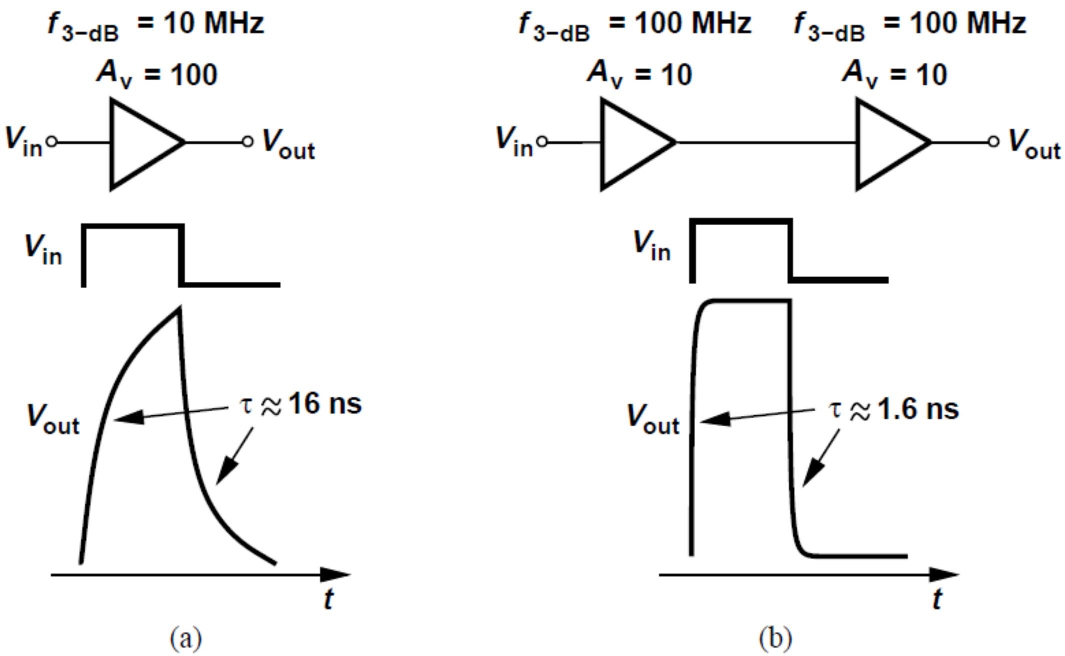


(a) (b)

- **For a single-pole amplifier with open loop gain of 100 and 3-dB bandwidth of 10 MHz, the response to a 20 MHz square wave exhibits long rise and fall times [Fig. (a)] with a time constant** $1/(2\pi f_{3-\text{dB}}) \approx 16\ \text{ns}$

Properties of Feedback Circuits

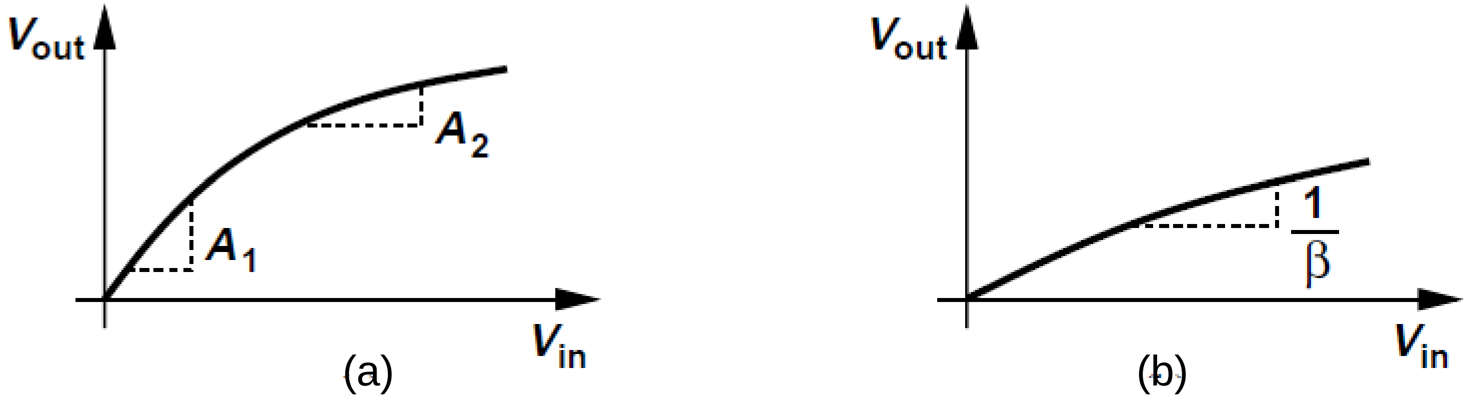
- **Bandwidth Modification:**



- **If feedback is applied to the amplifier such that the gain and bandwidth are modified to 10 and 100 MHz respectively, two such amplifiers cascaded in series yield a much faster response [Fig. (b)], at the cost of double the power consumption**

Properties of Feedback Circuits

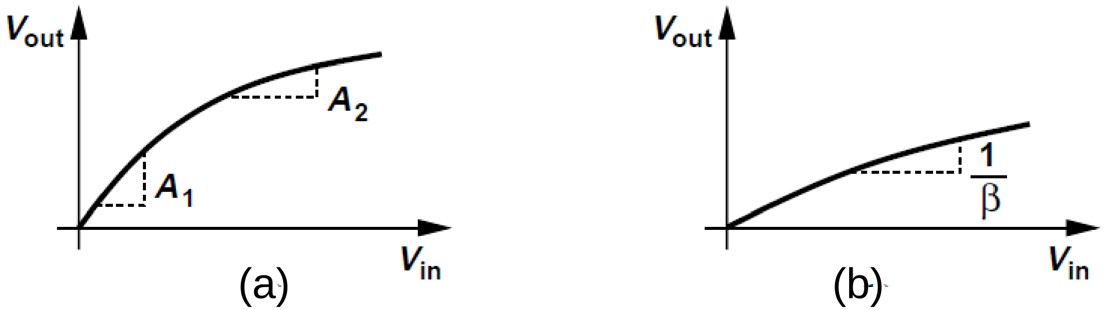
- <u>**Nonlinearity Reduction:**</u>
- **Negative feedback reduces nonlinearity in analog circuits**



- **A nonlinear characteristic departs from a straight line, i.e., its *slope* (or small-signal gain) varies [Fig. (a)]**
- **A closed-loop feedback system incorporating such an amplifier exhibits less gain variation and higher linearity [Fig. (b)]**

Properties of Feedback Circuits

- **<u>Nonlinearity Reduction:</u>**



- **In Fig. (a), open-loop gain ratios between regions 1 and 2 is**

$$r_{open} = \frac{A_2}{A_1}$$

- **Assuming $A_2 = A_1 - \Delta A$, we can write**

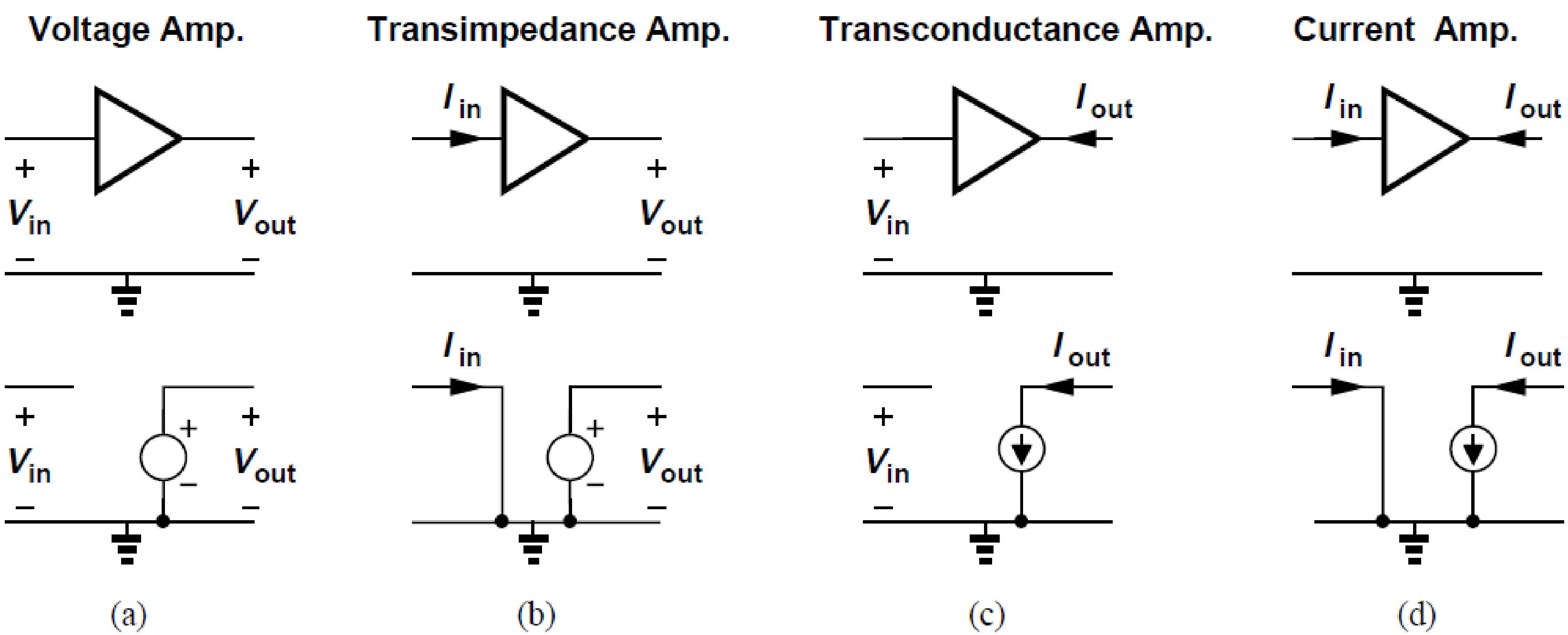
$$r_{open} = 1 - \frac{\Delta A}{A_1}$$

- **For the amplifier in negative feedback [Fig. (b)], the closed-loop gain ratio is much closer to 1 if the loop gain $1 + \beta A_2$, is large**

$$r_{closed} = \frac{\dfrac{A_2}{1+\beta A_2}}{\dfrac{A_1}{1+\beta A_1}} \approx 1 - \frac{\Delta A}{1+\beta A_2}\frac{1}{A_1}$$

Types of Amplifiers

- **Four possible amplifier configurations depending on whether the input and output signals are voltage or current quantities**

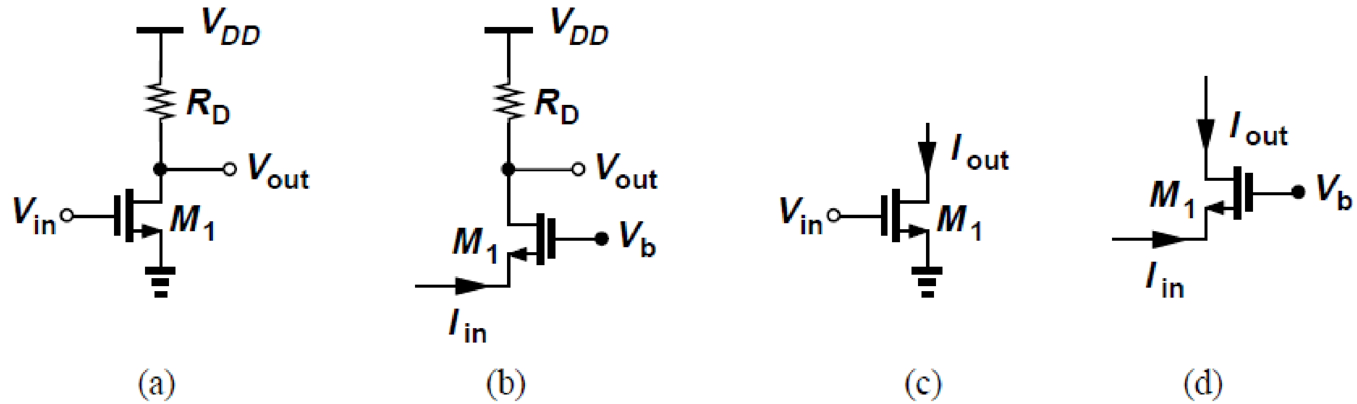


- **Figs. (a) – (d) show the four amplifier types with the corresponding idealized models**

Types of Amplifiers

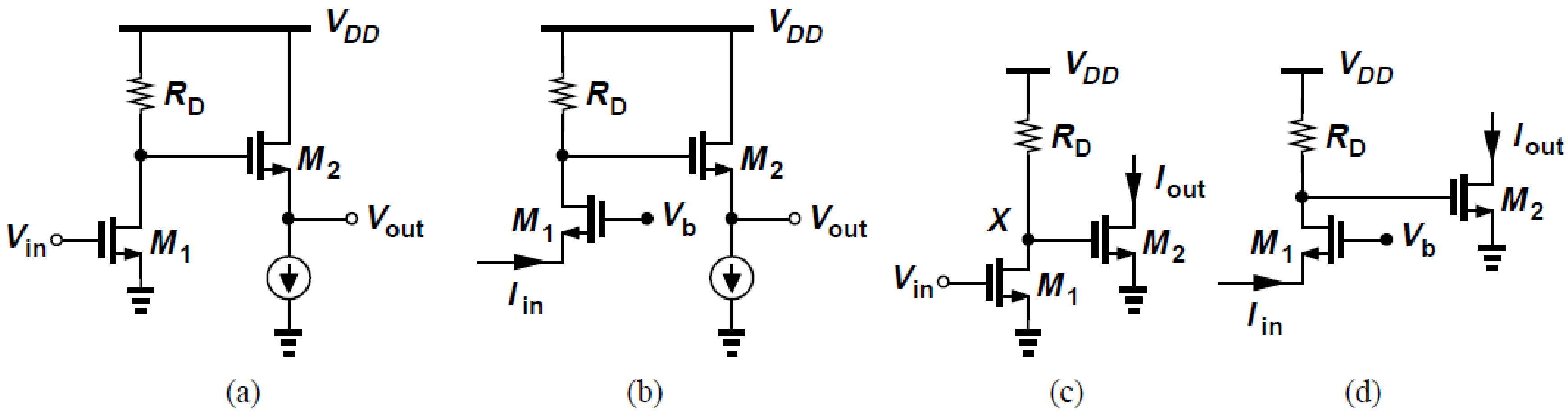
- **The four configurations have quite different properties**
- **Circuits sensing a voltage must exhibit a high input impedance whereas those sensing a current must provide a low input impedance**
- **Circuits generating a voltage must exhibit a low output impedance while those generating a current must provide a high output impedance**
- **Gains of transimpedance and transconductance amplifiers have dimensions of resistance and conductance, respectively**
- **Sign conventions must be followed, taking into account the directions of I_{in} and I_{out} in transimpedance and transconductance amplifiers**

Types of Amplifiers



- **In Fig. (a), a common-source stage senses and produces voltages**
- **In Fig. (b), a common-gate stage serves as a transimpedance amplifier, converting the source current to a voltage at the drain**
- **In Fig. (c), a common-source transistor operates as a transconductance amplifier (or *V/I* converter), generating an output current in response to an input voltage**
- **In Fig. (d), a common-gate device senses and produces currents**

Types of Amplifiers

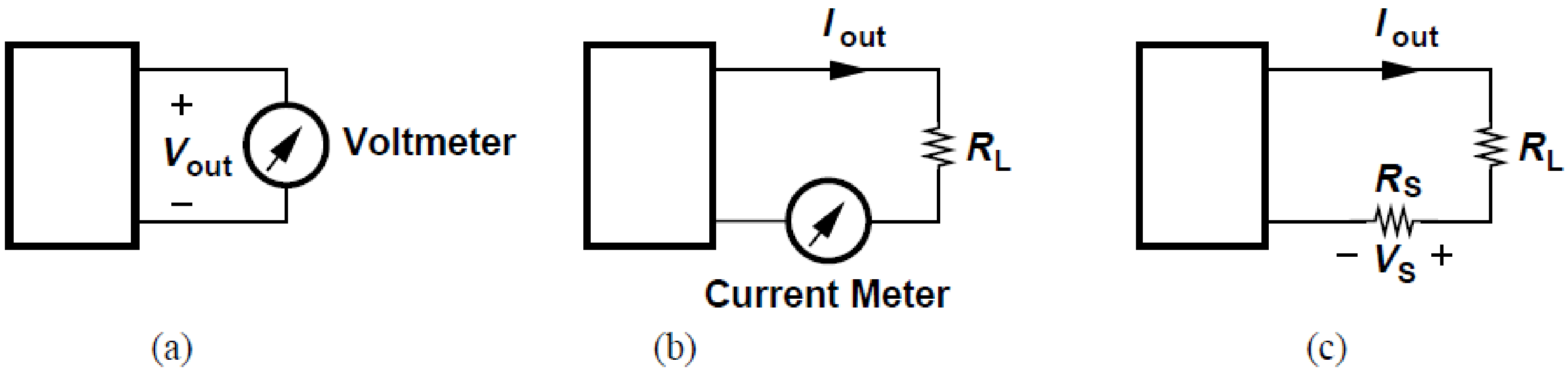


- **Figs. (a) – (d) depict modifications to previous amplifier configurations to alter the output impedance or increase the gain**

Sense and Return Mechanisms

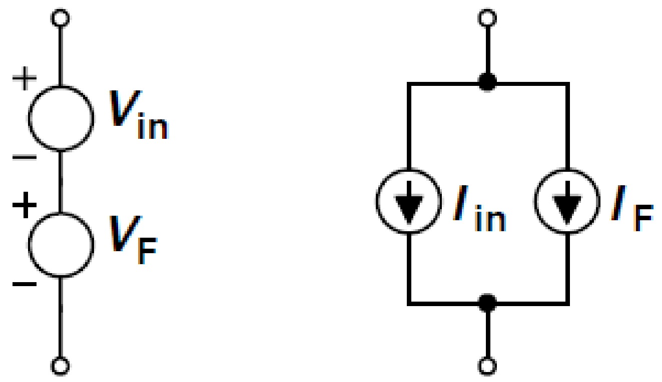
- Placing a circuit in a feedback loop requires sensing an output signal and returning a fraction of it to the summing node at the input
- Four types of feedback
  - Voltage-Voltage
  - Voltage-Current
  - Current-Current
  - Current-Voltage
- First term is the quantity sensed at the output, and the second term is the type of signal returned to the input

Sense and Return Mechanisms



- **To sense a voltage, we place a voltmeter in parallel with the corresponding port [Fig. (a)], ideally introducing no loading, also called "shunt feedback"**
- **To sense a current, a current meter is inserted in series with the signal [Fig. (b)], ideally exhibiting zero resistance, also called "series feedback"**
- **In practice, the current meter is replaced by a small resistor [Fig. (c)], with the voltage drop as a measure of the output current**

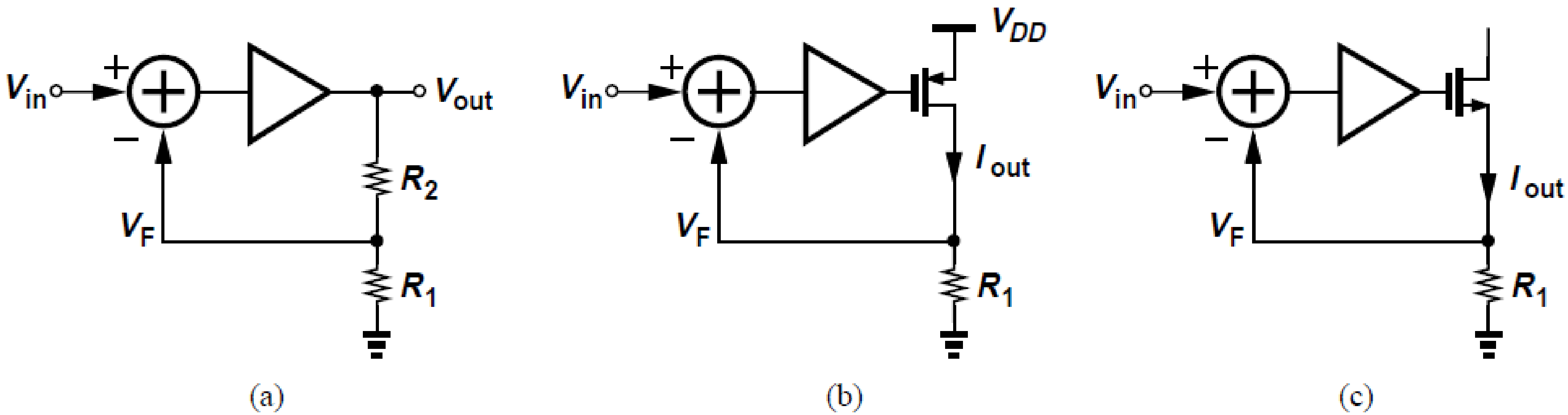
Sense and Return Mechanisms



(a) (b)

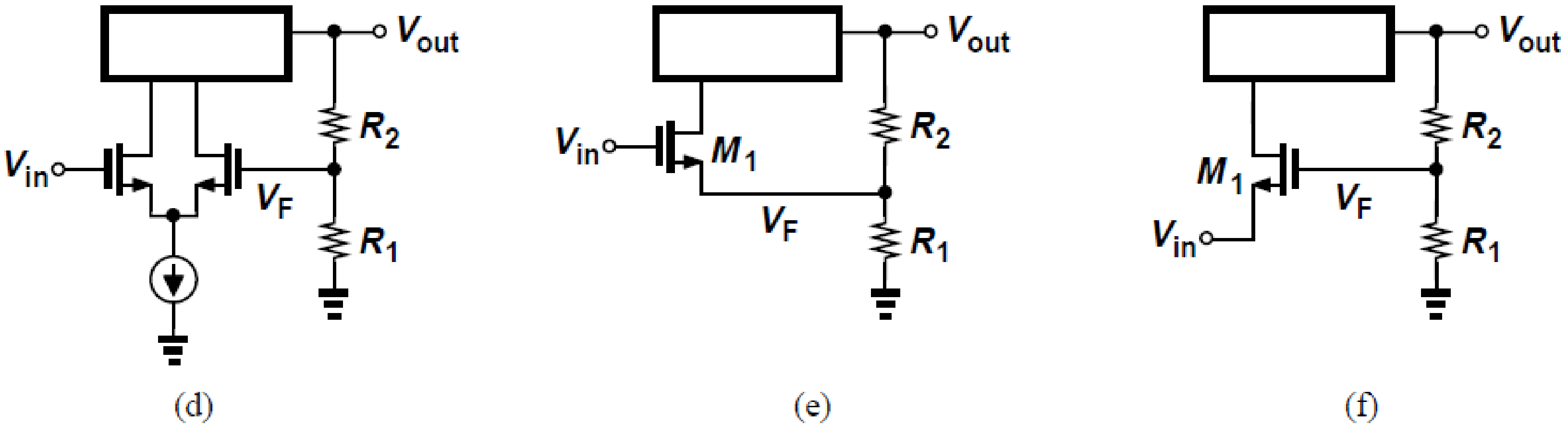
- Addition of the feedback signal and the input signal can be performed in the voltage or current domains
- Voltages are added in series [Fig. (a)]
- Currents are added in parallel [Fig. (b)]

Sense and Return Mechanisms



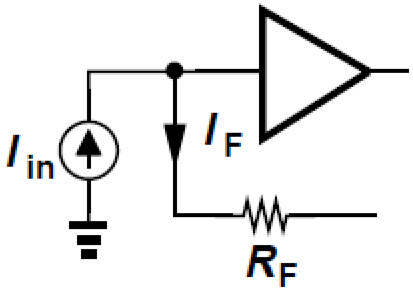
- **A voltage can be sensed by a resistive (or capacitive) divider in parallel with the port [Fig. (a)]**
- **A current can be sensed by placing a small resistor in series with the wire and sensing the voltage across it [Figs. (b) and (c)]**

Sense and Return Mechanisms

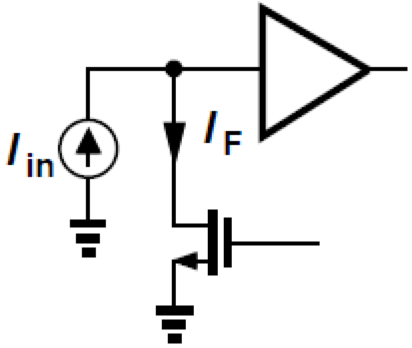


- To subtract two voltages, a differential pair can be used [Fig. (d)]
- A single transistor can also perform voltage subtraction [Figs. (e) and (f)] since I_{D1} is a function of $V_{in} – V_F$

Sense and Return Mechanisms



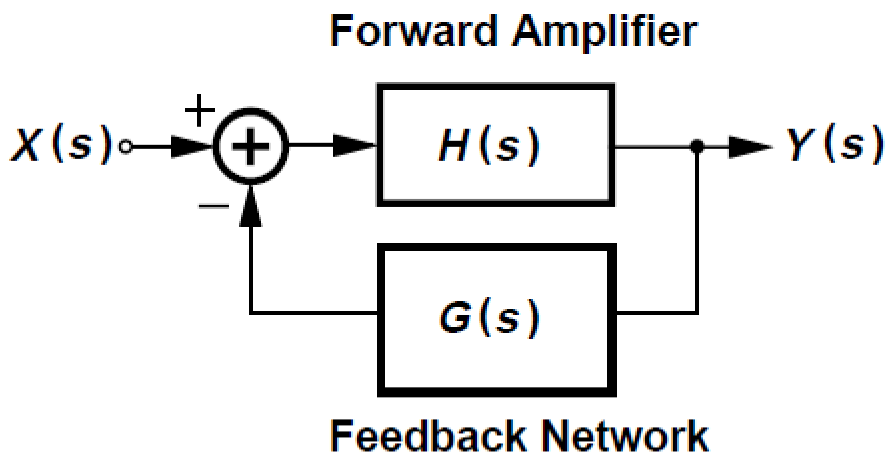
(g)



(h)

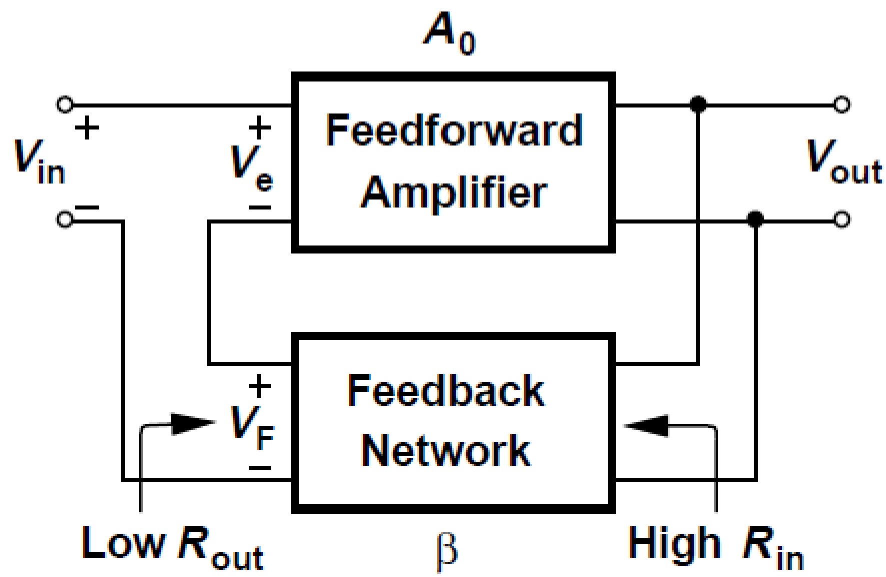
- **Current subtraction can be performed as shown in Figs. (g) and (h)**
- **For voltage subtraction, the input and feedback signals are applied to two distinct nodes**
- **For current subtraction, the input and feedback signals are applied to a single node**

Feedback Topologies



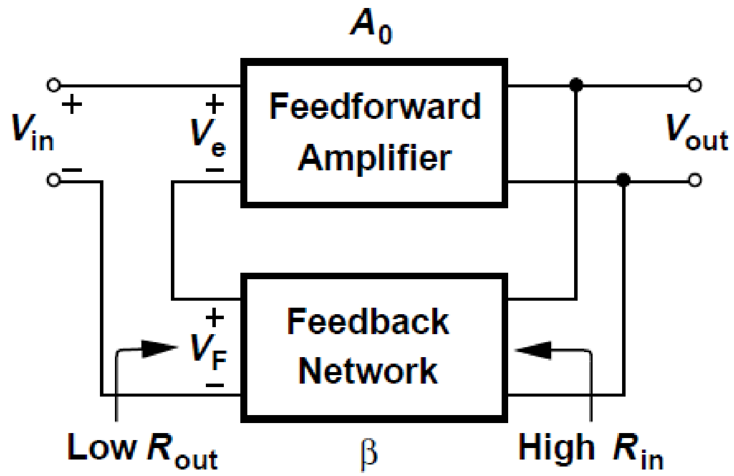
- In the above figure, *X* and *Y* can be a current or a voltage quantity
- Main amplifier is called “feedforward” or simply “forward” amplifier around which feedback is applied
- Four “canonical” topologies result from placing each of the four amplifier types in negative feedback

Voltage-Voltage Feedback



- This topology senses the output voltage and returns the feedback signal as a voltage
- Feedback network is connected in *parallel* with the output and in *series* with the input
- An ideal feedback network in this case has infinite input impedance (ideal voltmeter) and zero output impedance (ideal voltage source)

Voltage-Voltage Feedback

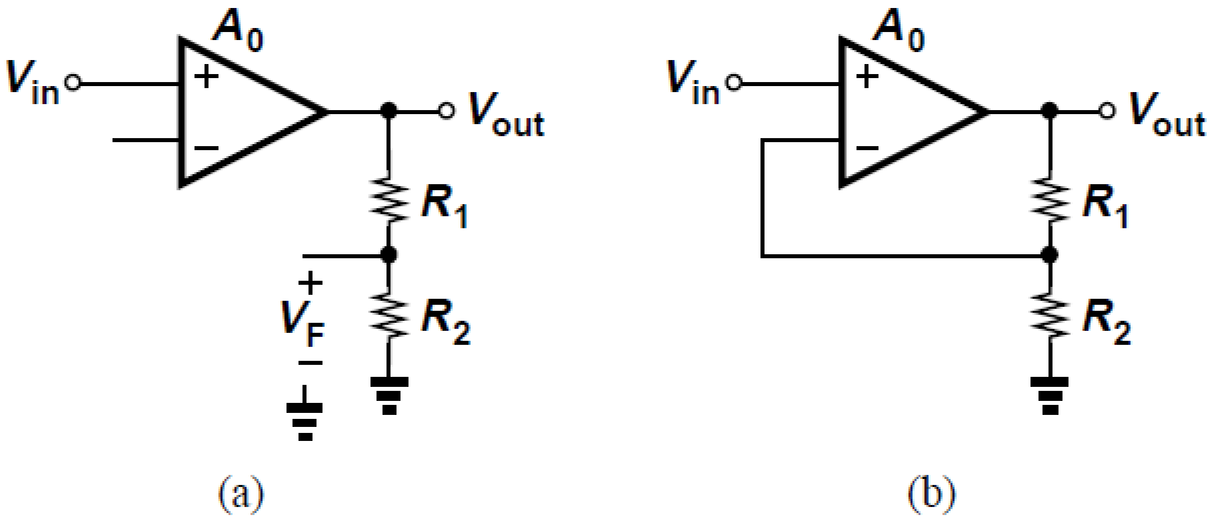


- Also called "series-shunt" feedback; first term refers to the *input* connection and second to the *output* connection
- We can write $V_F = \beta V_{out}$, $V_e = V_{in} - V_F$, $V_{out} = A_0(V_{in} - \beta V_{out})$, and hence

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \beta A_0}$$

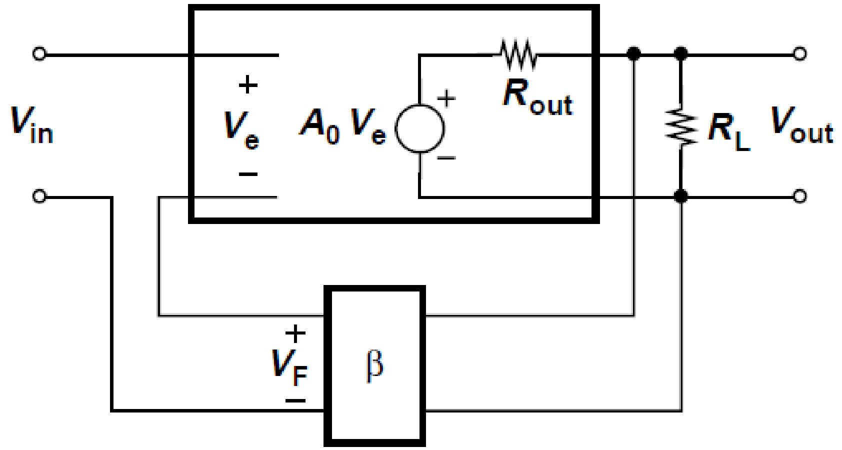
- βA_0 is the loop gain and the overall gain has dropped by $1+ \beta A_0$

Voltage-Voltage Feedback



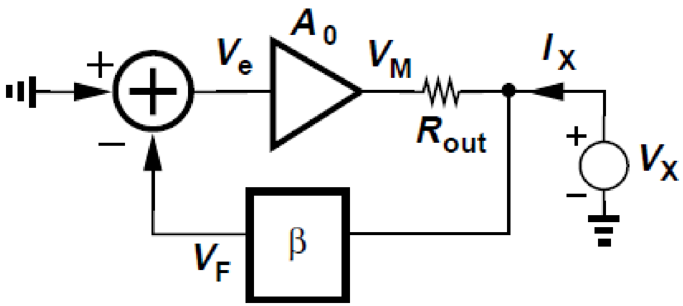
- **As an example of voltage-voltage feedback, a differential voltage amplifier with single-ended output can be used as the forward amplifier and a resistive divider as the feedback network [Fig. (a)]**
- **The sensed voltage V_F is placed in series with the input to perform subtraction of voltages**

Voltage-Voltage Feedback: Output Resistance



- If output is loaded by resistor R_L, in open-loop configuration, output decreases in proportion to $R_L/(R_L+R_{out})$
- In closed-loop V_{out} is maintained as a constant replica of V_{in} regardless of R_L as long as loop gain is much greater than unity
- Circuit "stabilizes" output voltage despite load variations, behaves as a voltage source and exhibits low output impedance

Voltage-Voltage Feedback: Output Resistance

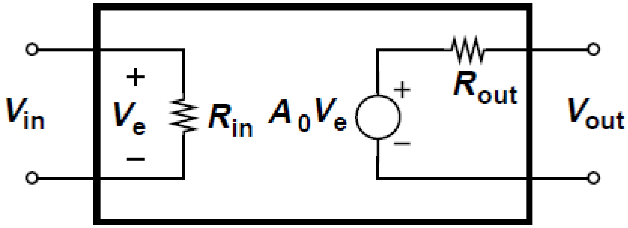


- In the above model, R_{out} represents the output impedance of the feedforward amplifier
- Setting input to zero and applying a voltage at the output, we write $V_F = \beta V_X$, $V_e = \beta V_X$, $V_M = \beta A_0 V_X$ and hence $I_X = [V_X - (-\beta A_0 V_X)]/R_{out}$ (if current drawn by feedback network is neglected)
- It follows that

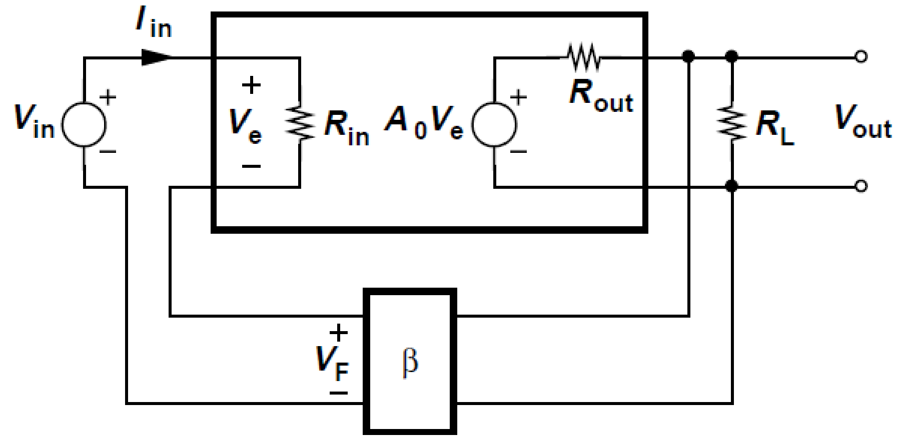
$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + \beta A_0}$$

- Output impedance and gain are lowered by same factor

Voltage-Voltage Feedback: Input Resistance



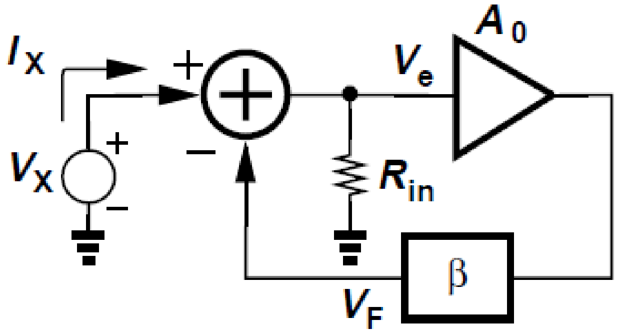
(a)



(b)

- **Voltage-voltage feedback also modifies input impedance**
- **In Fig. (a) [open-loop], R_{in} of the forward amplifier sustains the entire V_{in}, whereas only a fraction in Fig. (b) [closed-loop]**
- **I_{in} is less in the feedback topology compared to open-loop system, suggesting increase in the input impedance**

Voltage-Voltage Feedback: Input Resistance

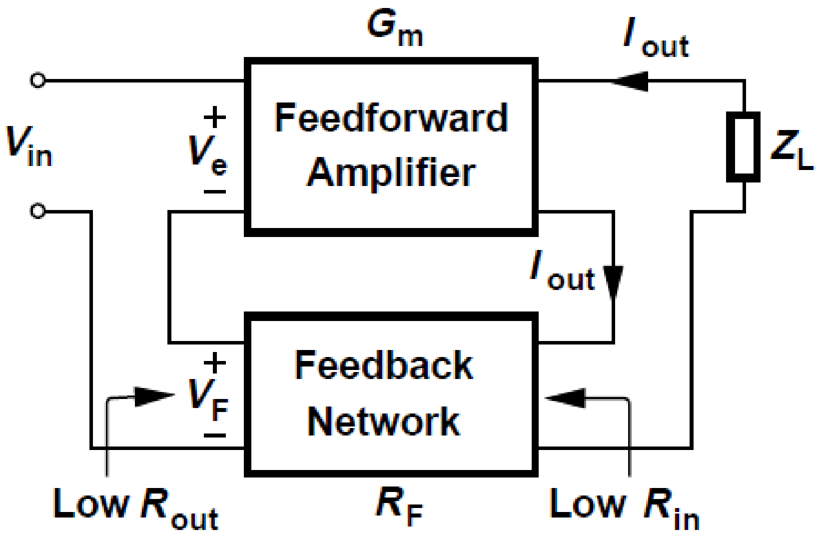


- In the above model, $V_e = I_X R_{in}$ and $V_F = \beta A_0 I_X R_{in}$
- Thus, we have $V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in}$
- Hence, $I_X R_{in} = V_X - \beta A_0 I_X R_{in}$ and

$$\frac{V_X}{I_X} = R_{in}(1 + \beta A_0)$$

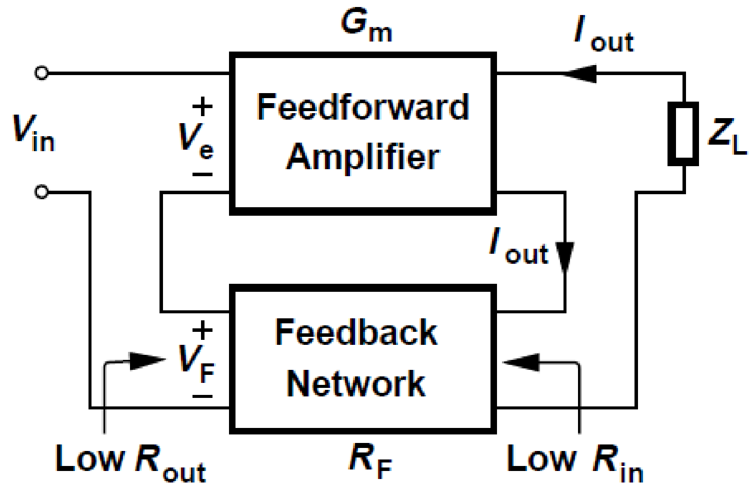
- Input impedance increases by the factor $1+\beta A_0$, bringing the circuit closer to an ideal voltage amplifier
- Voltage-voltage feedback decreases output impedance and increases input impedance, useful as a buffer stage

Current-Voltage Feedback



- **This topology senses the output current and returns a voltage as the feedback signal**
- **The current is sensed by measuring the voltage drop across a (small) resistor placed in series with the output**
- **Feedback factor β has the dimension of resistance and is hence denoted by R_F**

Current-Voltage Feedback



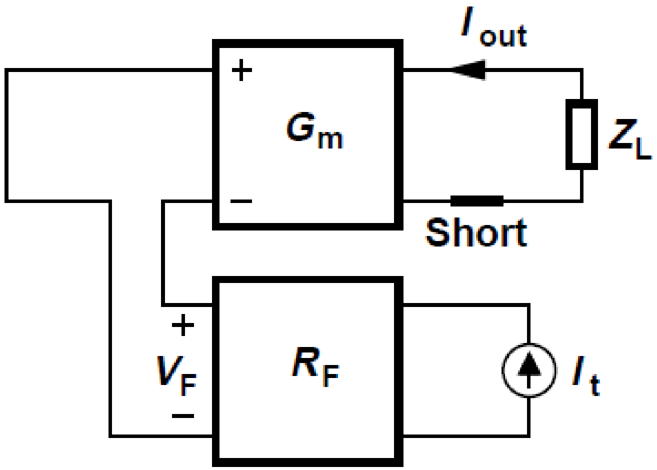
- A G_m stage must be terminated by a finite impedance to ensure it can deliver its output current
- If $Z_L = \infty$, an ideal G_m stage would sustain an infinite output voltage

- We write $V_F = R_F I_{out}$, $V_e = V_{in} - R_F I_{out}$ and hence $I_{out} = G_m(V_{in} - R_F I_{out})$
- It follows that

$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

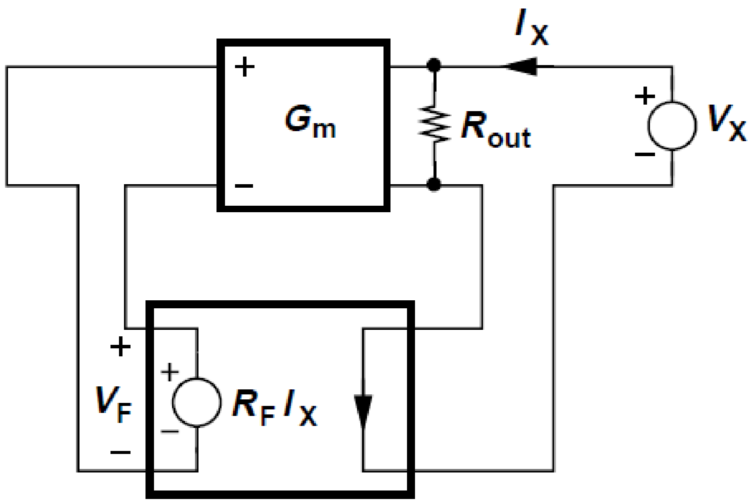
- Ideal feedback network in this case exhibits zero input and output impedances

Current-Voltage Feedback: Loop Gain



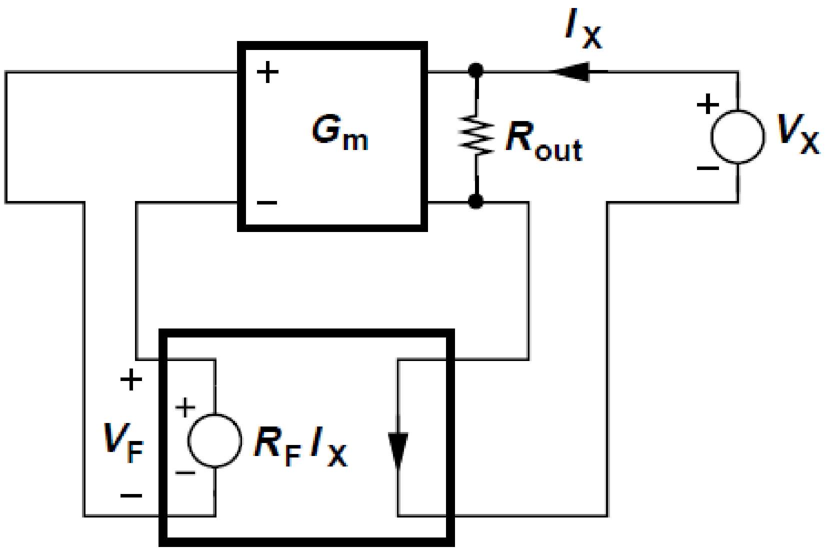
- To calculate the loop gain, the input is set to zero and the loop is broken by disconnecting the feedback network from the output and replacing it with a short at the output (if the feedback network is ideal)
- Test signal It is injected, producing $V_F = R_F I_t$ and hence $I_{out} = -G_m R_F I_t$
- Thus, loop gain is $G_m R_F$ and transconductance of the amplifier is reduced by $1+G_m R_F$ when feedback is applied

Current-Voltage Feedback: Output Resistance



- Sensing the current at the output increases the output impedance
- System delivers the same current waveform as the load varies, approaching an ideal current source which exhibits a high output impedance
- In the above figure, R_{out} represents the finite output impedance of the feedforward amplifier
- Feedback network produces V_F proportional to I_X, i.e., $V_F = R_F I_X$

Current-Voltage Feedback: Output Resistance

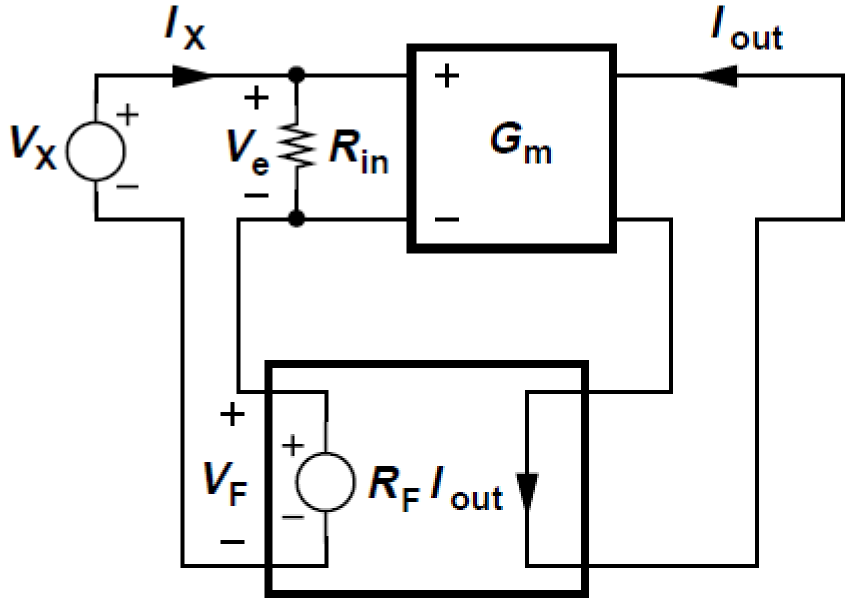


- **The current generated by G_m equals $-R_F I_X G_m$**
- **As a result, $-R_F I_X G_m = I_X - V_X/R_{out}$, yielding**

$$\frac{V_X}{I_X} = R_{out}(1 + G_m R_F)$$

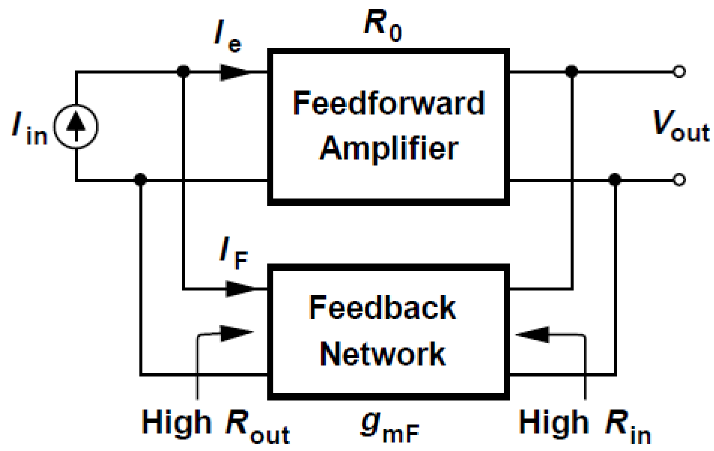
- **The output impedance therefore increases by a factor of $1+G_m R_F$**

Current-Voltage Feedback: Input Resistance



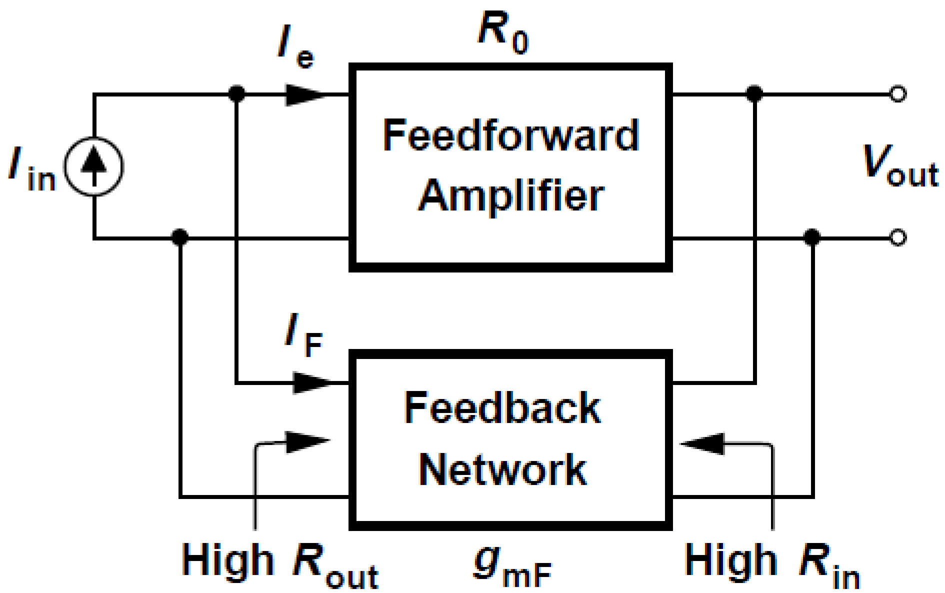
- Current-voltage feedback increases the input impedance by a factor of one plus the loop gain
- As shown in the above figure, we have $I_X R_{in} G_m = I_{out}$
- Thus, $V_e = V_X - G_m R_F I_X I_{in}$ and
- Current-voltage feedback increases both the input and output impedances while decreasing the feedforward transconductance

Voltage-Current Feedback



- In this type of feedback, the output voltage is sensed and a proportional current is returned to the input summing point
- Feedforward path incorporates a transimpedance amplifier with gain R_0 and the feedback factor g_{mF} has a dimension of conductance
- Feedback network ideally exhibits infinite input and output impedances
- Also called “shunt-shunt” feedback

Voltage-Current Feedback

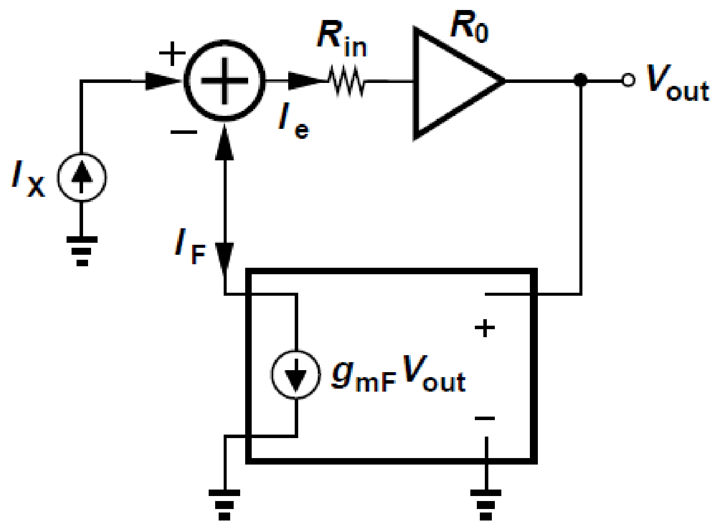


- Since $I_F = g_{mF}V_{out}$ and $I_e = I_{in} - I_F$, we have $V_{out} = R_0 I_e = R_0(I_{in} - g_{mF}V_{out})$
- It follows that

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + g_{mF}R_0}$$

- This feedback lowers the transimpedance by a factor of one plus the loop gain

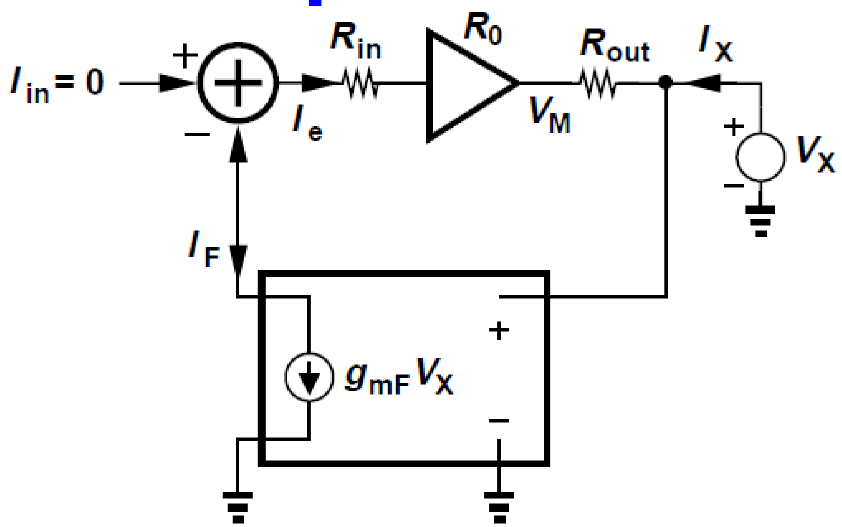
Voltage-Current Feedback: Output Impedance



- Voltage-current feedback decreases the output impedance
- Input resistance R_{in} of R_0 appears in series with the input port
- We write $I_F = I_X - V_X/R_{in}$ and $(V_X/R_{in})R_0 g_{mF} = I_F$
- Thus,

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + g_{mF} R_0}$$

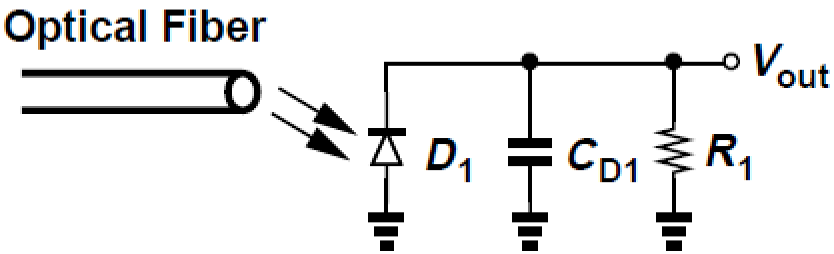
Voltage-Current Feedback: Input Impedance



- **Voltage-current feedback decreases the input impedance too**
- **From the figure, we have $I_F = V_X g_{mF}$, $I_e = -I_F$, and $V_M = -R_0 g_{mF} V_X$**
- **Neglecting the input current of the feedback network, $I_X = (V_X - V_M)/R_{out} = (V_X + g_{mF} R_0 V_X)/R_{out}$**
- **Thus,**

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + g_{mF} R_0}$$

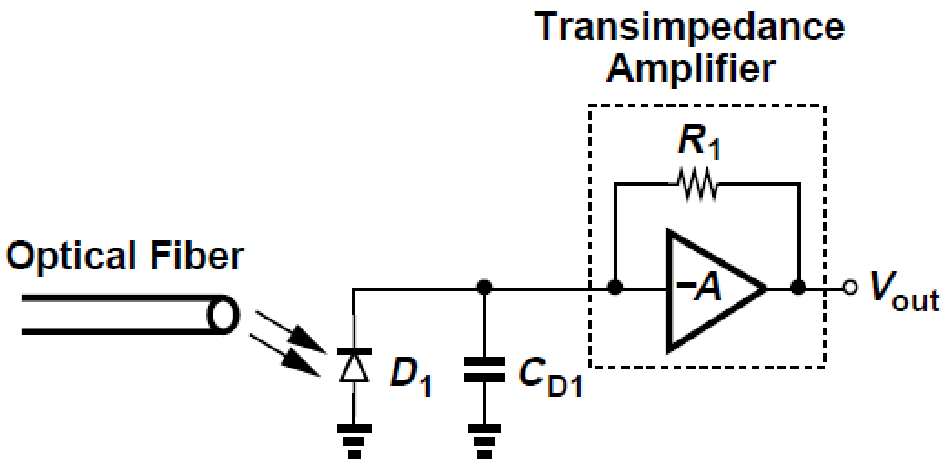
Voltage-Current Feedback: Applications



(a)

- Amplifiers with low input impedance are used in fiber optic receivers, where light received through a fiber is converted to a current by a reverse-biased photodiode
- This current is converted to a voltage for processing by subsequent stages
- Fig. (a) show this conversion using a resistor at the cost of bandwidth due to large junction capacitance C_{D1} of the diode

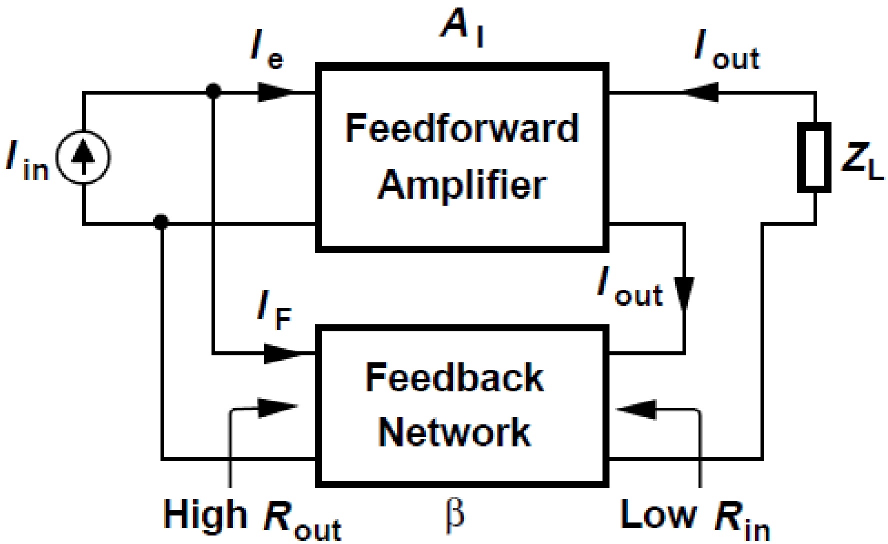
Voltage-Current Feedback: Applications



(b)

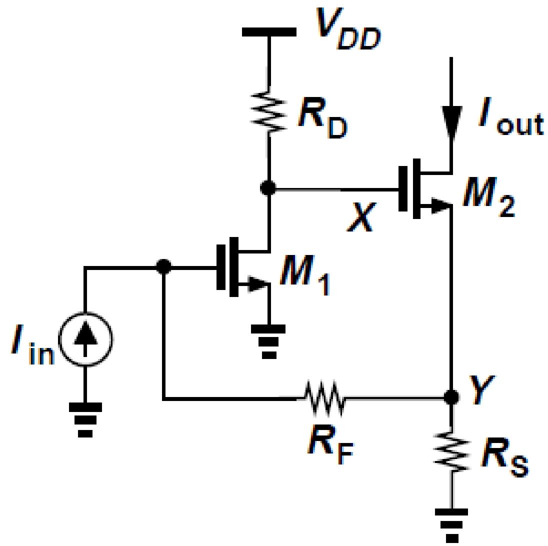
- To improve performance, the feedback topology of Fig. (b) is employed, where R_1 is placed around the voltage amplifier A to form a "transimpedance amplifier" (TIA)
- The input impedance is $R_1/(1+A)$ and output voltage is approximately $R_1 I_{D1}$
- Bandwidth thus increases from $1/(2\pi R_1 C_{D1})$ to $(1+A)/(2\pi R_1 C_{D1})$ if A itself is a wideband amplifier

Current-Current Feedback



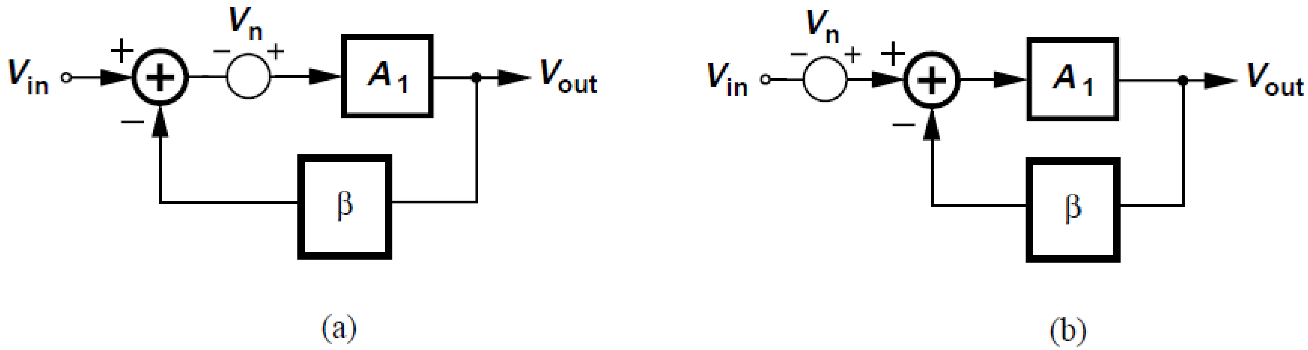
- **Output voltage is sensed and a proportional current is returned**
- **Feedforward amplifier is characterized by a current gain A_I and feedback network by a current ratio β**
- **It can be proved that the closed-loop current gain is equal to $A_I/(1+\beta A_I)$, the input impedance is divided by $1+\beta A_I$, and the output impedance is multiplied by $1+\beta A_I$**

Current-Current Feedback: Example



- Above figure shows an example of current-current feedback
- Since the source and drain currents of M_1 are equal (at low frequencies), resistor R_S is inserted in the source network to monitor the output current
- Resistor R_F senses the output voltage and returns a current to the input

Effect of Feedback on Noise

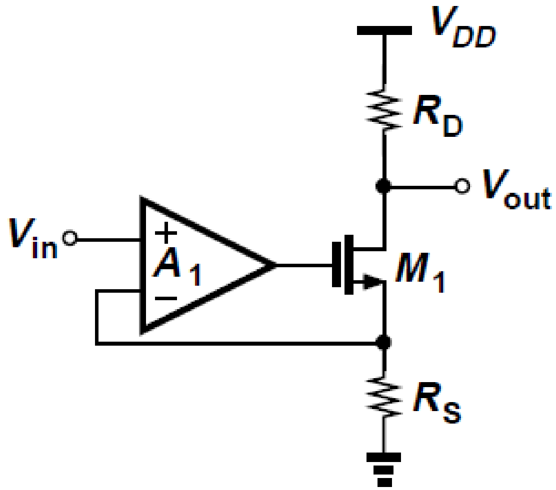


- **Feedback does not improve noise performance of circuits**
- **In Fig. (a), the open-loop amplifier A_1 is characterized by only an input-referred noise voltage and the feedback network is assumed to be noiseless**
- **We have $(V_{in} - \beta V_{out} + V_n)A_1 = V_{out}$, and hence**

$$V_{out} = (V_{in} + V_n)\frac{A_1}{1 + \beta A_1}$$

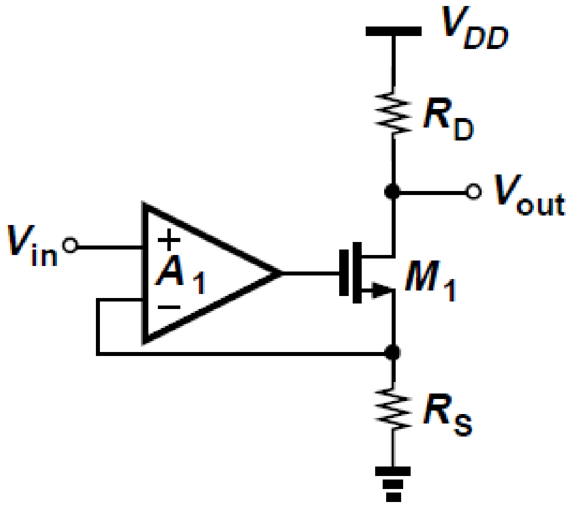
- **Circuit can be modified as in Fig. (b), input-referred noise is still V_n**

Effect of Feedback on Noise



- Output of interest may not always be the quantity sensed by the feedback network
- In above circuit, output is at the drain of M_1 whereas the feedback network senses source voltage of M_1
- Here, input-referred noise of the closed-loop circuit is not equal to that of the open-loop circuit even if the feedback network is noiseless

Effect of Feedback on Noise



- Consider only the noise of R_D, $V_{n,RD}$ in this circuit
- Closed-loop voltage gain of the circuit is

$$-A_1 g_m R_D / [1 + (1 + A_1) g_m R_S]$$

- Input-referred noise voltage due to R_D is if $\lambda = \gamma = 0$

$$|V_{n,in,closed}| = \frac{|V_{n,RD}|}{A_1 R_D}\left[\frac{1}{g_m} + (1 + A_1) R_S\right]$$

- Input-referred noise of the open-loop circuit is

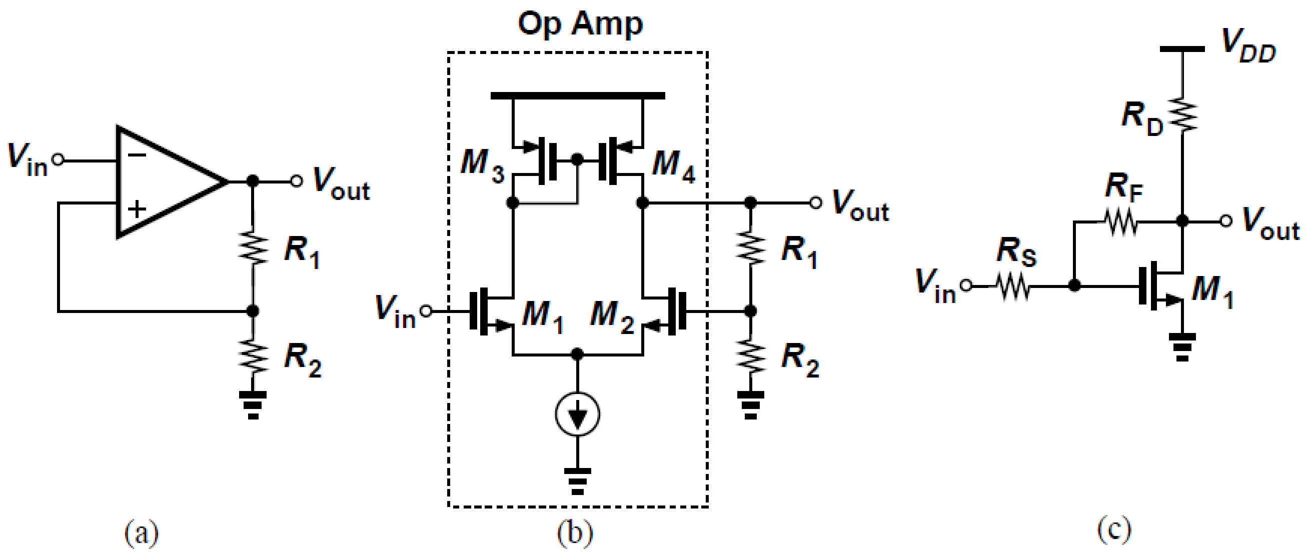
$$|V_{n,in,open}| = \frac{|V_{n,RD}|}{A_1 R_D}\left[\frac{1}{g_m} + R_S\right]$$

- As $A_1 \to \infty$, $|V_{n,in,closed}| \to |V_{n,RD}| R_S / R_D$,whereas $|V_{n,in,open}| \to 0$

Feedback Analysis Difficulties

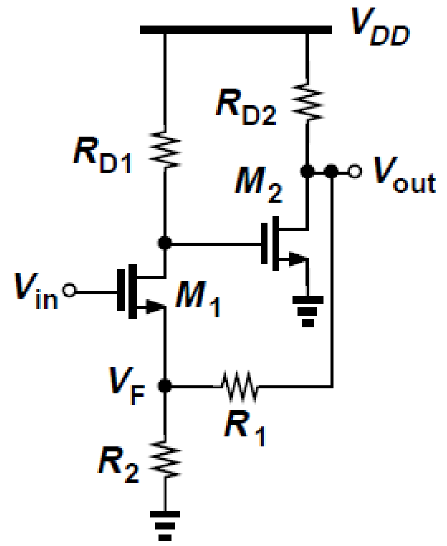
- **Analysis approach used proceeds as follows:**
  - **Break the loop and obtain the open-loop gain and input and output impedances**
  - **Determine the loop gain, βA_0 and hence the closed-loop parameters from their open-loop counterparts**
  - **Use the loop gain to study properties such as stability, etc.**
- **The simplifying assumptions made may not hold in all circuits**
- **Five difficulties arising in the analysis of feedback circuits are discussed subsequently**

Feedback Analysis Difficulties: (1)



- In the non-inverting amplifier of Fig. (a) and its simple implementation in Fig. (b), the feedback branch consisting of R_1 and R_2 may draw significant *signal* current from the output, reducing its *open-loop* gain
- In the circuit of Fig. (c), the open-loop gain of the forward CS stage falls if R_F is not very large
- In all cases, the "output" loading results from non-ideal input impedance of the feedback network

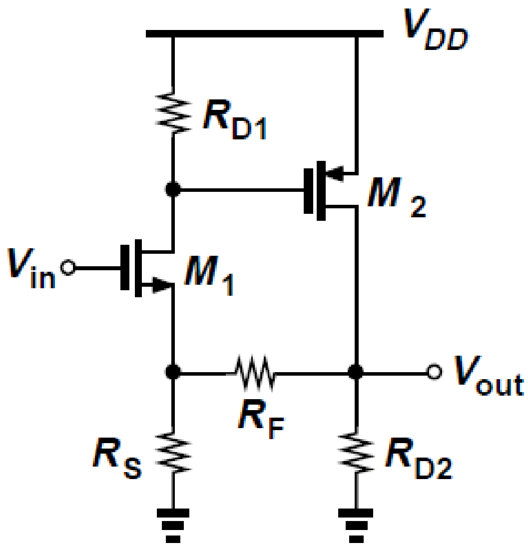
Feedback Analysis Difficulties: (1)



(d)

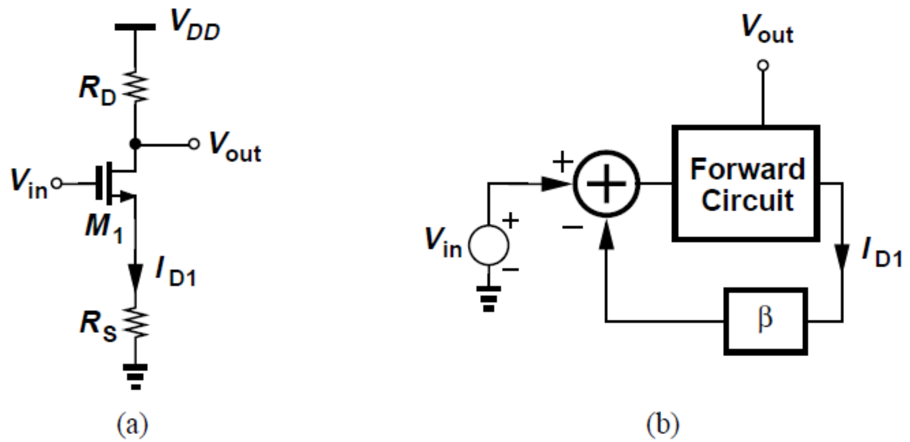
- In the circuit of Fig. (d), R_1 and R_2 sense V_{out} and return a voltage to the source of M_1
- Since the output impedance of the feedback network may not be sufficiently small, we surmise that M_1 is degenerated considerably even as far as the open-loop forward amplifier is concerned
- This is a case of “input loading” due to non-ideal output impedance of the feedback network

Feedback Analysis Difficulties: (2)



- **Some circuits cannot be clearly decomposed into a forward amplifier and a feedback network**
- **In the above two-stage network, it is unclear whether R_{D2} belongs to the feedforward amplifier or the feedback network**
- **The former may be chosen, reasoning that M_2 needs a load to operate as a voltage amplifier, although this choice is arbitrary**

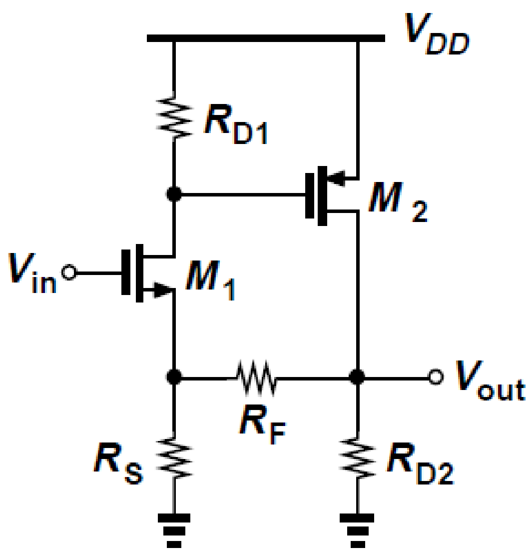
Feedback Analysis Difficulties: (3)



- Some circuits do not readily map to the four canonical topologies
- A simple degenerated CS stage does not contain feedback because the source resistance measures the drain current, converts it to a voltage, and subtracts the result from the input [Fig. (a)]
- It is not immediately clear which feedback topology represents this arrangement because the sensed quantity, I_{D1} is different from the output of interest, V_{out} [Fig. (b)]

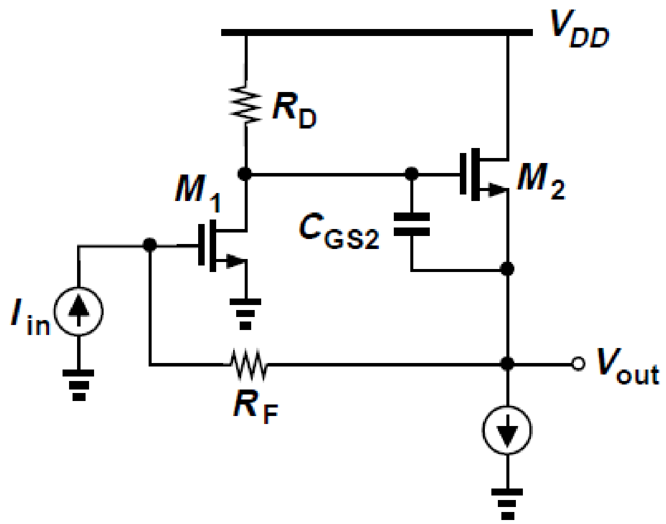
Feedback Analysis Difficulties: (4)

- **General feedback system thus far assumes unilateral stages, i.e., signal propagation in only one direction around the loop**
- **In practice, the loop may contain bilateral circuits, allowing signals to flow from the input, through the feedback network, to the output**
- **In the circuit below, the input travels through R_F and alters V_{out}**



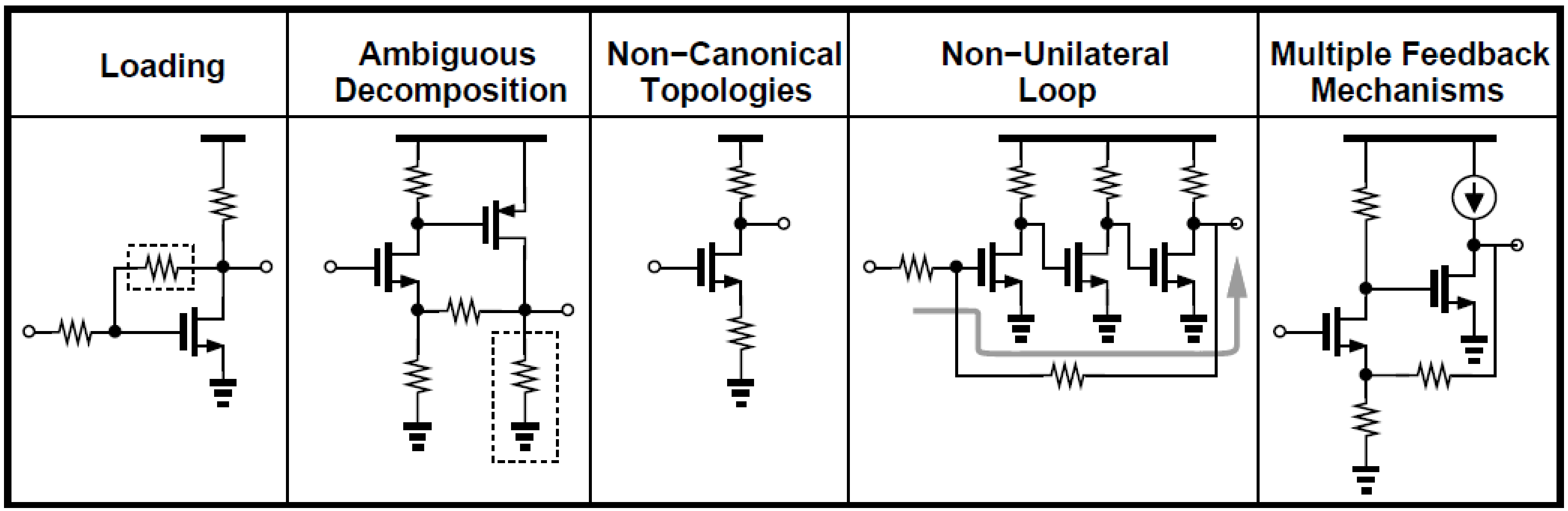
Feedback Analysis Difficulties: (5)

- Some circuits contain multiple feedback mechanisms (loosely called "multiloop circuits")
- In the topology below, for example, R_F provides feedback around the circuit, and C_{GS2} around M_2
- It might be said that the source follower itself contains degeneration and hence feedback
- It is not exactly clear which loop should be broken and the meaning of "loop gain"



Feedback Analysis Difficulties: Summary

- **The five difficulties in the analysis of feedback circuits are summarized below**



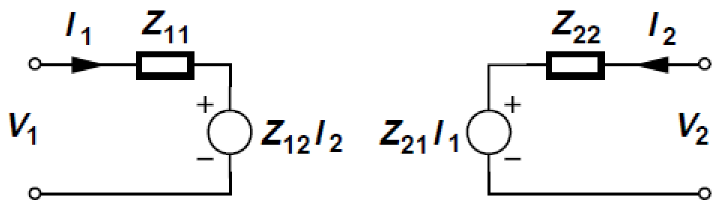
Feedback Analysis Methods

- We introduce two methods of feedback circuit analysis
  - Two-port method
  - Bode's method
- The details of the two methods are outlined below

Two−Port Method	Bode's Method
• Computes open−loop and closed−loop quantities and the loop gain. • Includes loading effects. • Neglects feedforward through feedback network. • Can be applied recursively to multiple feedback mechanisms. • Does not apply to non−canonical topologies.	• Computes closed−loop quantities without breaking the loop. • Applies to any topology. • Provides loop gain only if one feedback mechanism is present.

Review of Two-Port Network Models

- **A two-port linear (and time-invariant) network can be represented by any one of four two-port network models**
- **The “Z model” in Fig. (a) consists of input and output impedances in series with current-dependent voltage sources**



(a)

- **The Z model is described by two equations**

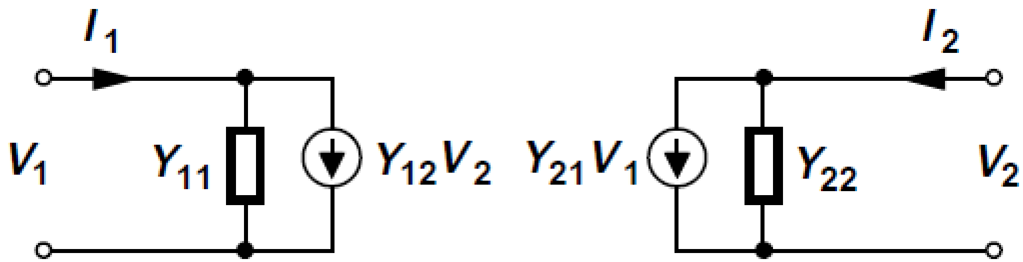
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

- **Each Z parameter has a dimension of impedance and is obtained by leaving one port open, e.g., $Z_{11} = V_1/I_1$ when $I_2 = 0$**

Review of Two-Port Network Models

- **The "Y model" in Fig. (b) comprises input and output admittances in parallel with voltage-dependent current sources**



(b)

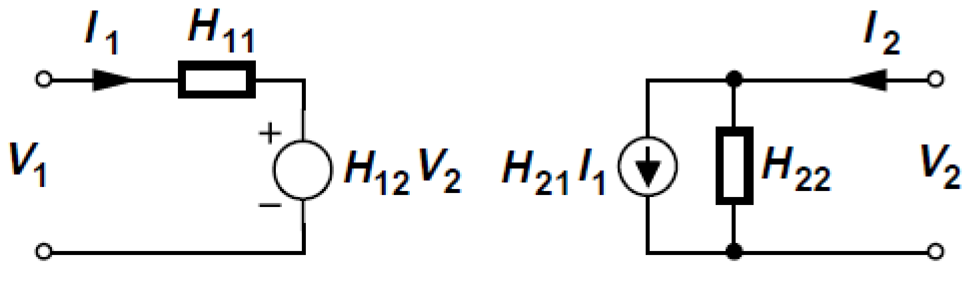
- **The Y model is described by**

$$
\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 \\
I_2 &= Y_{21}V_1 + Y_{22}V_2
\end{aligned}
$$

- **Each Y parameter is calculated by shorting one port, e.g., $Y_{11} = I_1/V_1$ when $V_2 = 0$**

Review of Two-Port Network Models

- **The “H model” in Fig. (c) incorporates a combination of impedances and admittances and voltage and current sources**



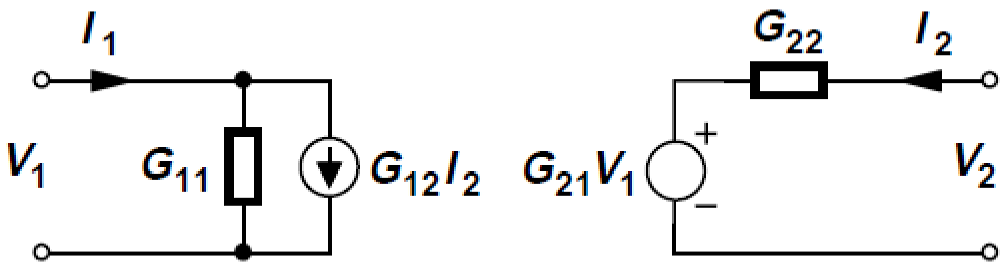
(c)

- **The H model is described by**

$$\begin{aligned} V_1 &= H_{11}I_1 + H_{12}V_2 \\ I_2 &= H_{21}I_1 + H_{22}V_2 \end{aligned}$$

Review of Two-Port Network Models

- The "G model" in Fig. (d) is also a "hybrid model" and is characterized by a combination of impedances and admittances and voltage and current sources



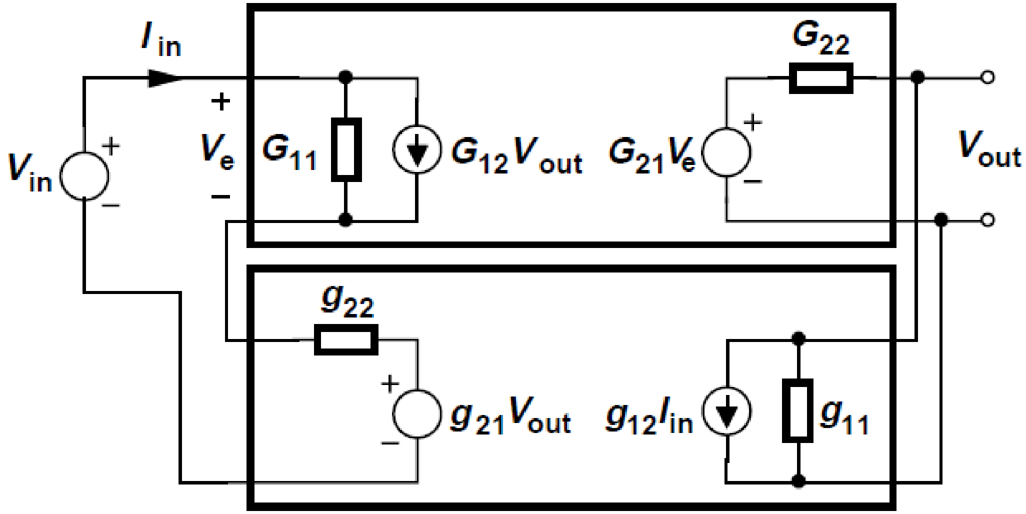
(d)

- The G model is described by

$$
\begin{aligned}
I_1 &= G_{11}V_1 + G_{12}I_2 \\
V_2 &= G_{21}V_1 + G_{22}I_2
\end{aligned}
$$

Loading in Voltage-Voltage Feedback

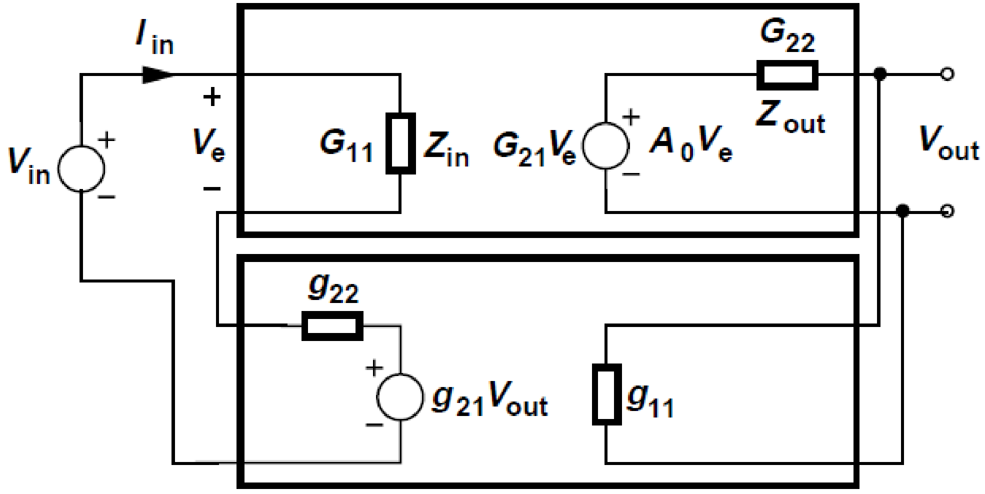
- The Z and H models fail to represent voltage amplifiers if the input current is very small – as in a simple CS stage, therefore the G model is chosen
- Fig. (a) shows the complete equivalent circuit, with the forward and feedback network parameters denoted by upper-case and lower-case letters, respectively



(a)

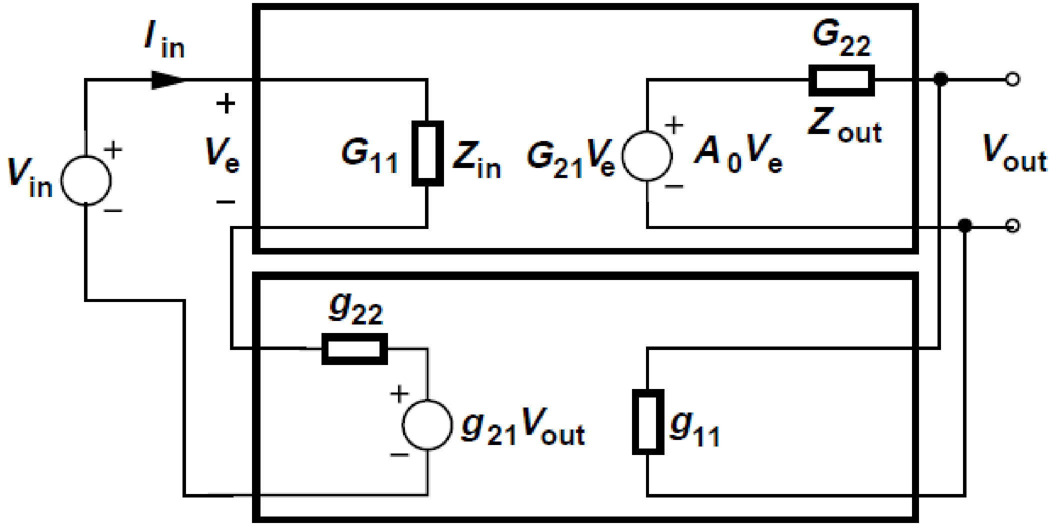
Loading in Voltage-Voltage Feedback

- The analysis is simplified by neglecting two quantities:
  - The amplifier's internal feedback, $G_{12}V_{out}$
  - The "forward" propagation of the input signal through the feedback network, $g_{12}I_{in}$
- The loop is "unilateralized"
- Fig. (b) shows the resulting circuit with intuitive amplifier notations



(b)

Loading in Voltage-Voltage Feedback



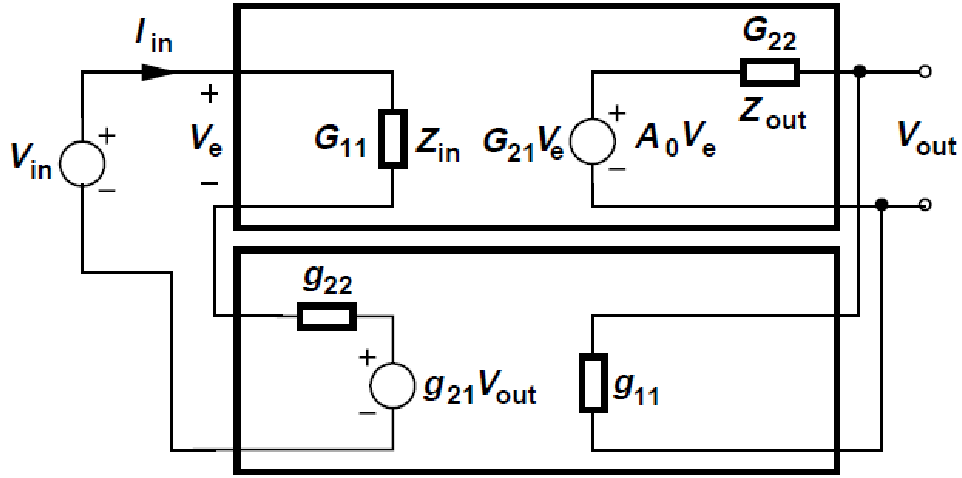
(b)

- **The closed-loop voltage gain is directly computed recognizing that *g_{11}* is an admittance and *g_{22}* is an impedance, and by writing a KVL around the input network and a KCL at the output node**

$$V_{in} = V_e + g_{22}\frac{V_e}{Z_{in}} + g_{21}V_{out}$$

$$g_{11}V_{out} + \frac{V_{out} - A_0V_e}{Z_{out}} = 0.$$

Loading in Voltage-Voltage Feedback



(b)

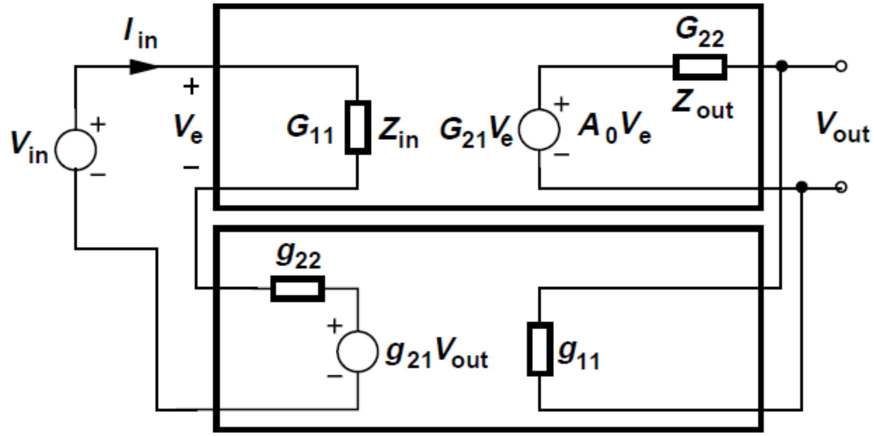
- **Eliminating V_e,**

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})+g_{21}A_0}$$

- **Expressing this in the form of $A_{v,open}/(1+\beta A_{v,open})$,**

$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}}{1+g_{21}\frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}}$$

Loading in Voltage-Voltage Feedback



(b)

- **We can thus write,**

$$A_{v,open} = \frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}$$

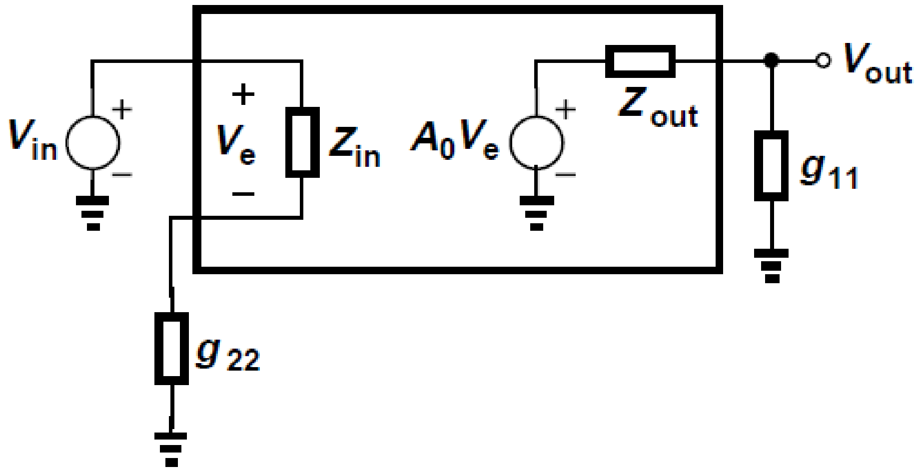
$$\beta = g_{21}.$$

- **The equivalent open-loop gain contains a factor A0, i.e., the original amplifier's voltage gain (before immersion in feedback)** $1 + g_{22}/Z_{in}$

- $1 + g_{11}Z_{out}$ **This gain is attenuated by two factors, and ssss**

Loading in Voltage-Voltage Feedback

- **The loaded forward amplifier is as shown below, excluding the two generators $G_{12}V_{out}$ and $g_{12}I_{in}$**



- **Allows a quick and intuitive understanding not possible from direct analysis**

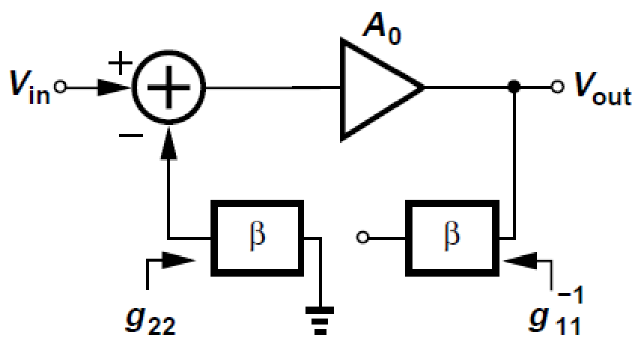
Loading in Voltage-Voltage Feedback

- **g_{11} and g_{22} are computed as follows:**

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I2=0}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V1=0}$$

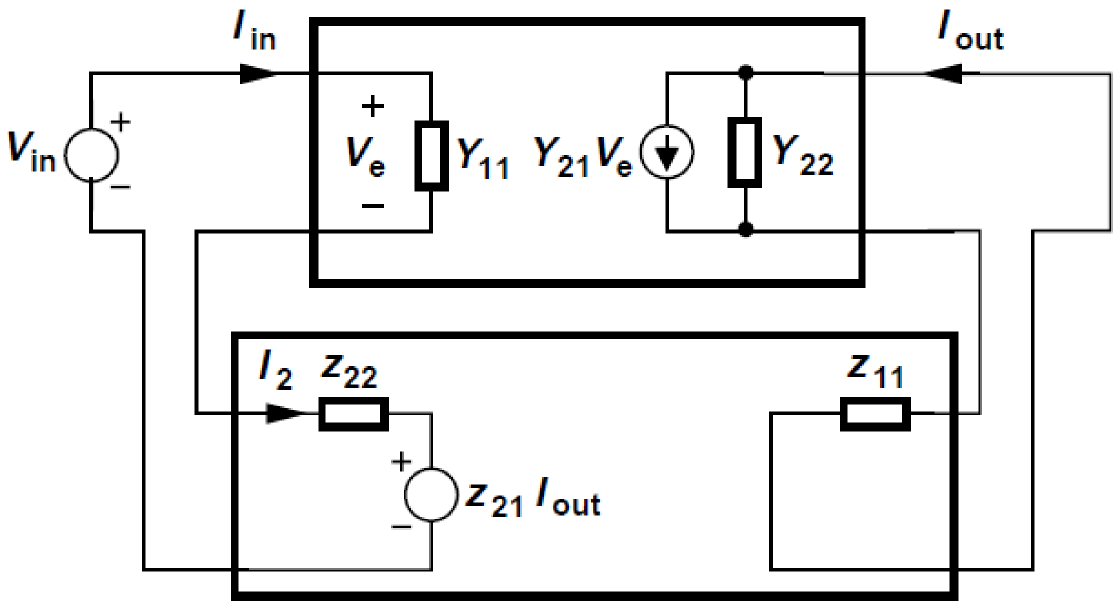
- **As shown below, g_{11} is obtained by leaving the output of the feedback network open whereas g_{22} is calculated by shorting the input of the feedback network**



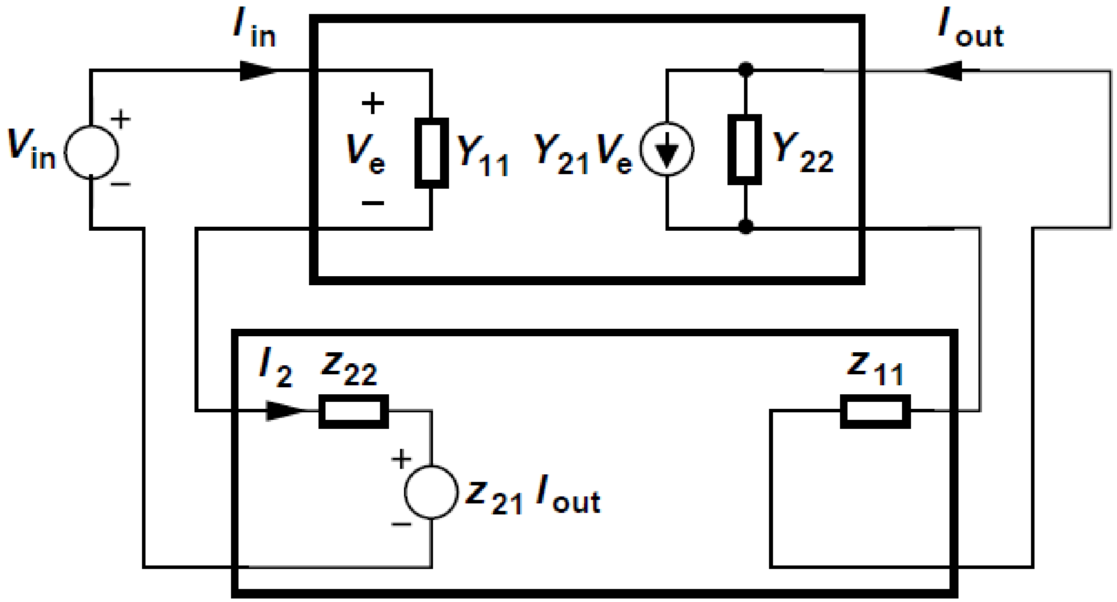
- **Loop gain is simply the loaded open-loop gain multiplied by g_{21}**
- **Open loop input and output impedances are scaled by $1 + g_{21}A_{v,open}$ to yield closed-loop values**

Loading in Current-Voltage Feedback

- **In this case, the feedback network appears in series with the output to sense the current**
- **Forward amplifier and feedback network are represented by Y and Z models respectively, neglecting the generators $Y_{12}V_{out}$ and $z_{12}I_{in}$, as shown below:**



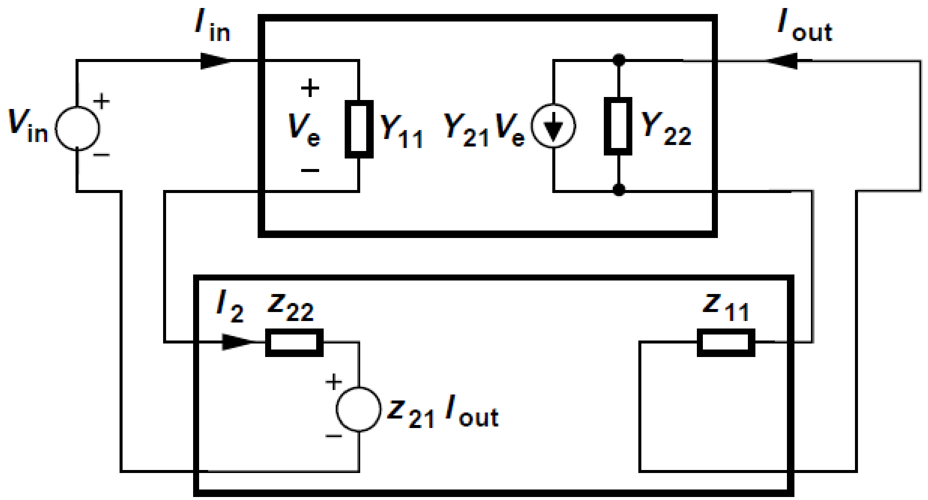
Loading in Current-Voltage Feedback



- To compute the closed-loop gain I_{out}/V_{in}, and obtain open-loop parameters in the presence of loading, we note that $I_{in} = Y_{11}V_e$ and $I_2 = I_{in}$ and write two KVLs:

$$
\begin{aligned}
V_{in} &= V_e + Y_{11}V_e z_{22} + z_{21}I_{out} \\
-I_{out}z_{11} &= \frac{I_{out} - Y_{21}V_e}{Y_{22}}.
\end{aligned}
$$

Loading in Current-Voltage Feedback



- **Eliminating V_e, we get**

$$\frac{I_{out}}{V_{in}} = \frac{\dfrac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}}{1 + z_{21}\dfrac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}}$$

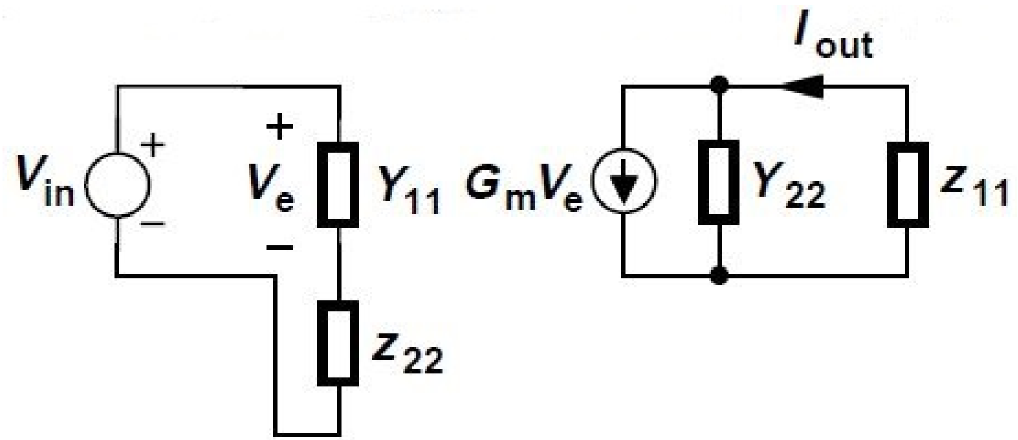
- **The loaded open-loop gain and feedback factor can be seen to be**

$$G_{m,open} = \frac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}$$

$$\beta = z_{21}.$$

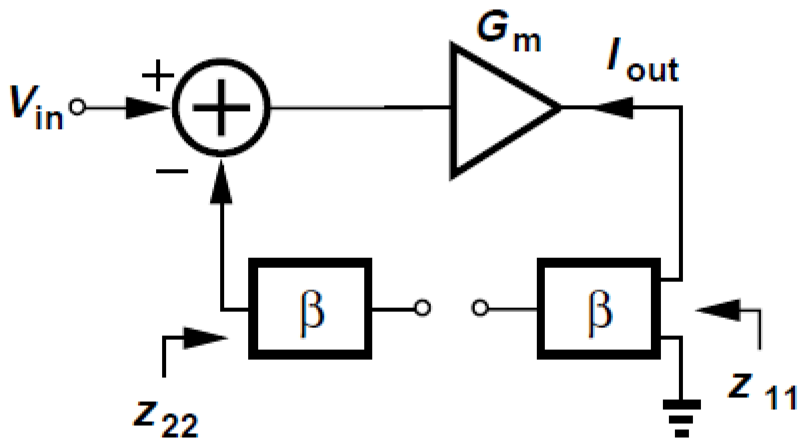
Loading in Current-Voltage Feedback

- Y_{21}, the transconductance gain of the original amplifier is attenuated by $(1+z_{22}Y_{11})^{-1}$ and $(1+z_{11}Y_{22})^{-1}$, which respectively correspond to voltage division at the input and current division at the output
- The loaded open-loop amplifier can be pictured as below



Loading in Current-Voltage Feedback

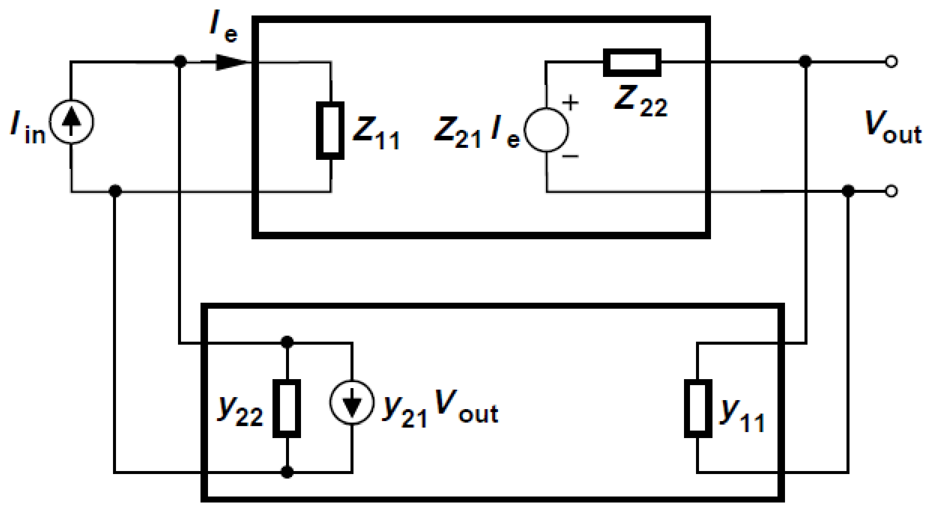
- **Since $z_{22} = V_2/I_2$ with $I_1 = 0$ and $z_{11} = V_1/I_1$ with $I_2 = 0$, the conceptual picture below shows how to properly break the feedback**



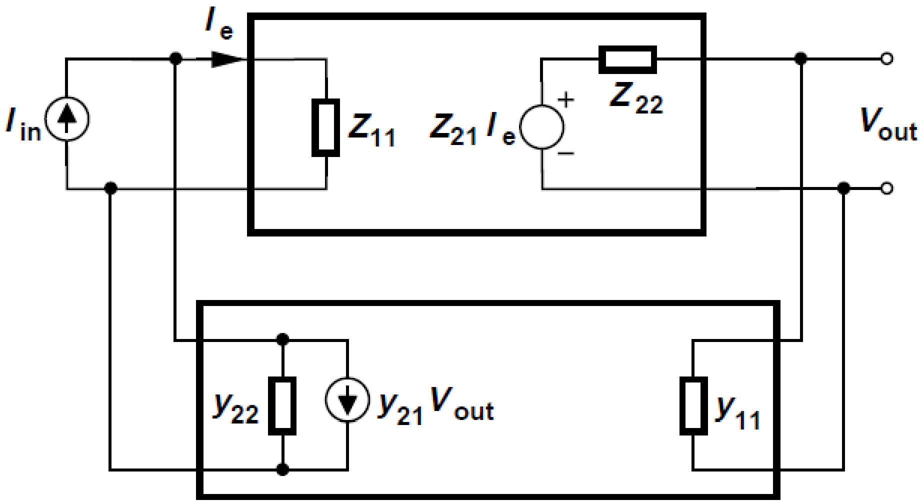
- **The loop gain is $z_{21}G_{m,open}$**

Loading in Voltage-Current Feedback

- In this configuration, the forward (transimpedance) amplifier generates an output voltage in response to the input current and can thus be represented by a Z model
- Feedback network lends itself to a Y model since it senses the output voltage and returns a proportional current
- The equivalent circuit below ignores the effect of Z_{12} and y_{12}



Loading in Voltage-Current Feedback



- **We compute the closed-loop gain, V_{out}/I_{in}, by writing two equations**

$$I_{in} = I_e + I_e Z_{11} y_{22} + y_{21} V_{out}$$

$$y_{11} V_{out} + \frac{V_{out} - Z_{21} I_e}{Z_{22}} = 0.$$

- **Eliminating I_e, we get**

$$\frac{V_{out}}{I_{in}} = \frac{\dfrac{Z_{21}}{(1 + y_{22} Z_{11})(1 + y_{11} Z_{22})}}{1 + y_{21} \dfrac{Z_{21}}{(1 + y_{22} Z_{11})(1 + y_{11} Z_{22})}}$$

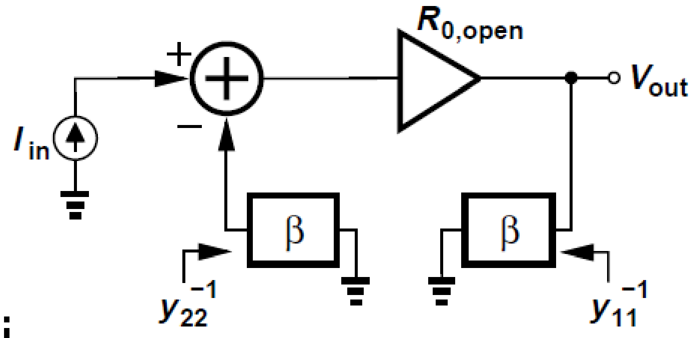
Loading in Voltage-Current Feedback

- **Thus, the equivalent open-loop gain and feedback factor are given by**

$$R_{0,open} = \frac{Z_{21}}{(1 + y_{22}Z_{11})(1 + y_{11}Z_{22})}$$

$$\beta = y_{21}.$$

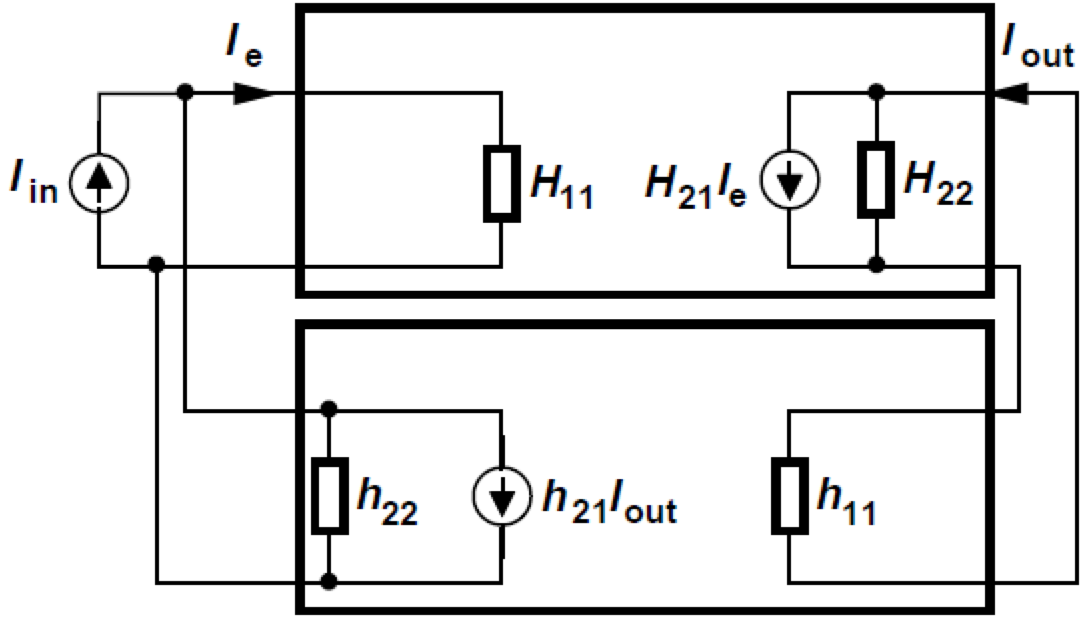
- **Interpreting the attenuation factors in $R_{0,open}$ as current division at the input and voltage division at the output, we arrive at the conceptual view shown below**



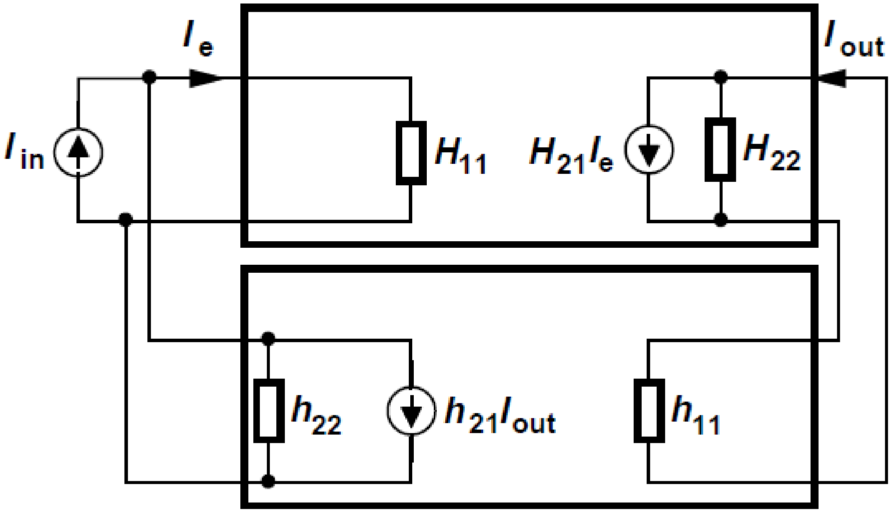
- **The loop gain is given by $y_{21}R_{0,open}$**

Loading in Current-Current Feedback

- **The forward amplifier in this case generates an output current in response to the input current and can be represented by an H model and so can the feedback network**
- **The equivalent circuit with the H_{12} and h_{12} generators is shown below**



Loading in Current-Current Feedback



- **We can write**

$$I_{in} = I_e H_{11} h_{22} + h_{21} I_{out} + I_e$$

$$I_{out} = -I_{out} h_{11} H_{22} + H_{21} I_e$$

- **Eliminating I_e, we get the closed-loop gain I_{out}/I_{in}**

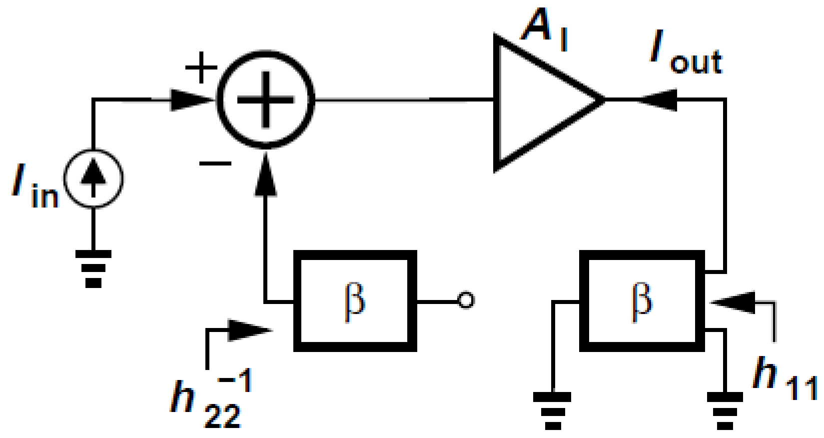
$$\frac{I_{out}}{I_{in}} = \frac{\dfrac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})}}{1+h_{21}\dfrac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})}}$$

Loading in Current-Current Feedback

- **As with previous topologies, we define the equivalent open-loop current gain and the feedback factor as**

$$\begin{aligned} A_{I,open} &= \frac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})} \\ \beta &= h_{21}. \end{aligned}$$

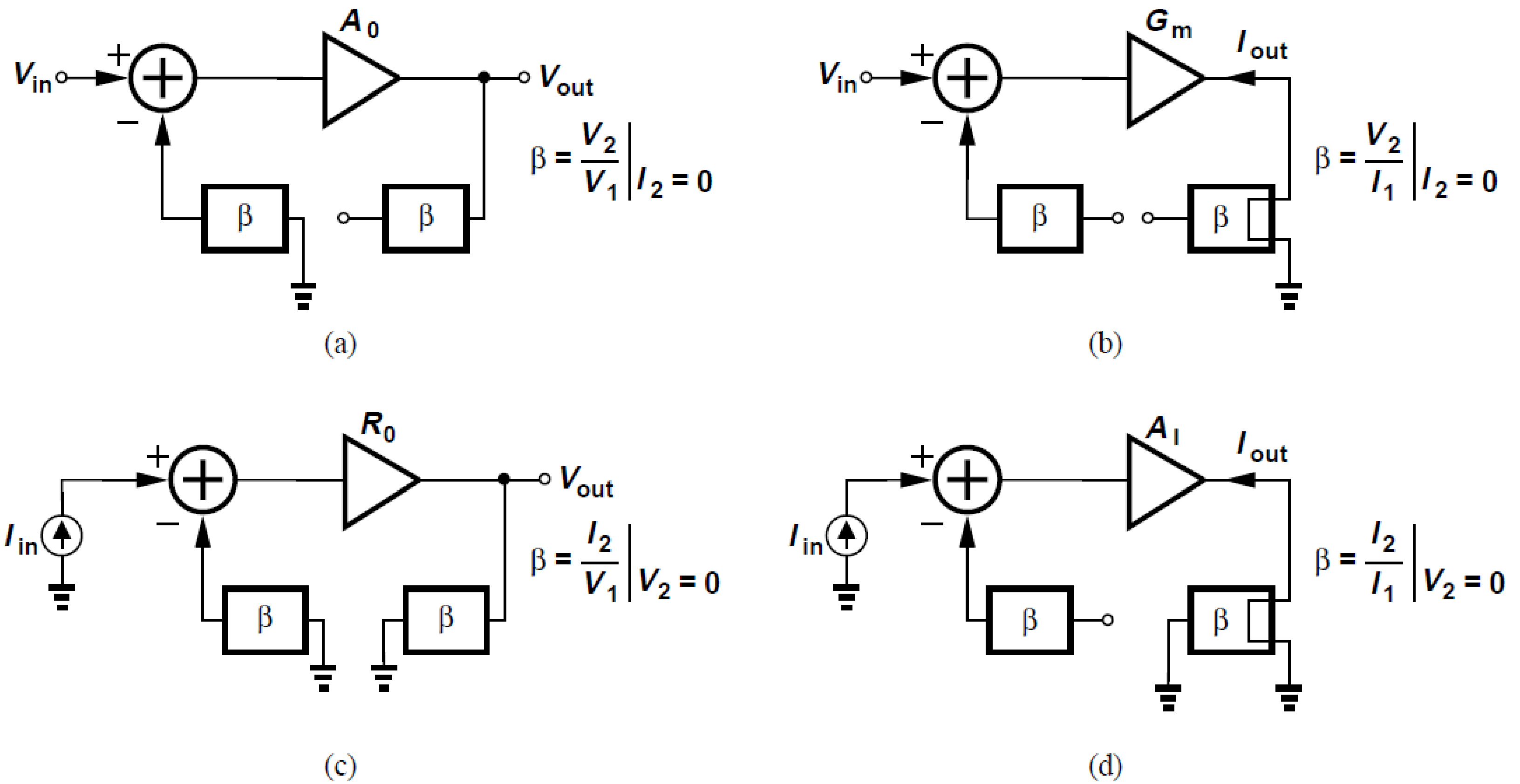
- **The conceptual view of the broken loop is shown below**



- **The loop gain is equal to $h_{21}A_{I,open}$**

Summary of Loading Effects

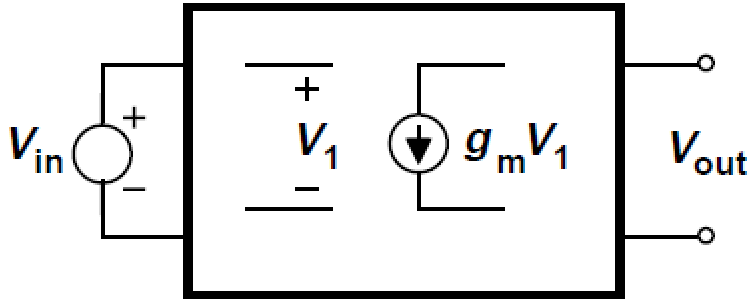
- **Figs. (a) – (d) summarize the loading effects in all four topologies**



Summary of Loading Effects

- **The analysis of loading is carried out in three steps:**
  1) **Open the loop with proper loading and calculate the open-loop gain, A_{OL}, and the open-loop input and output impedances**
  2) **Determine the feedback ratio β, and hence the loop gain, βA_{OL}**
  3) **Calculate the closed-loop gain and input and output impedances by scaling the open-loop values by a factor of $1+\beta A_{OL}$**
- **In the equations defining β, the subscripts 1 and 2 refer to the input and output ports of the feedback network, respectively**

Bode's Analysis of Feedback Circuits: Observations

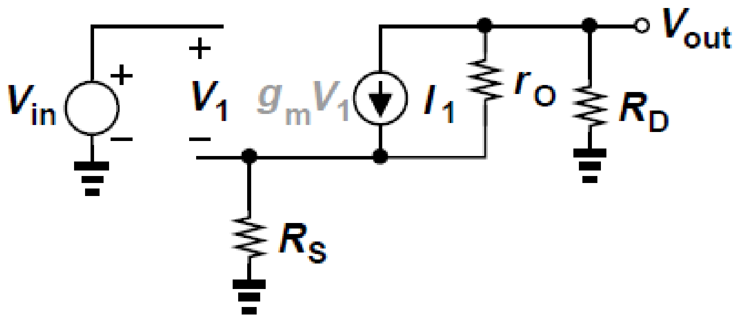


(a)

- Consider the general circuit in Fig. (a), where one transistor is explicitly shown in its ideal form
- From previous analysis, V_{out} can eventually be expressed as A_vV_{in} or $H(s)V_{in}$
- If the dependent current source is denoted by I_1 and we do not make the substitution $I_1 = g_mV_1$, then V_{out} is obtained as a function of both V_{in} and I_1:

$$V_{out} = AV_{in} + BI_1$$

Bode's Analysis of Feedback Circuits: Observations



(b)

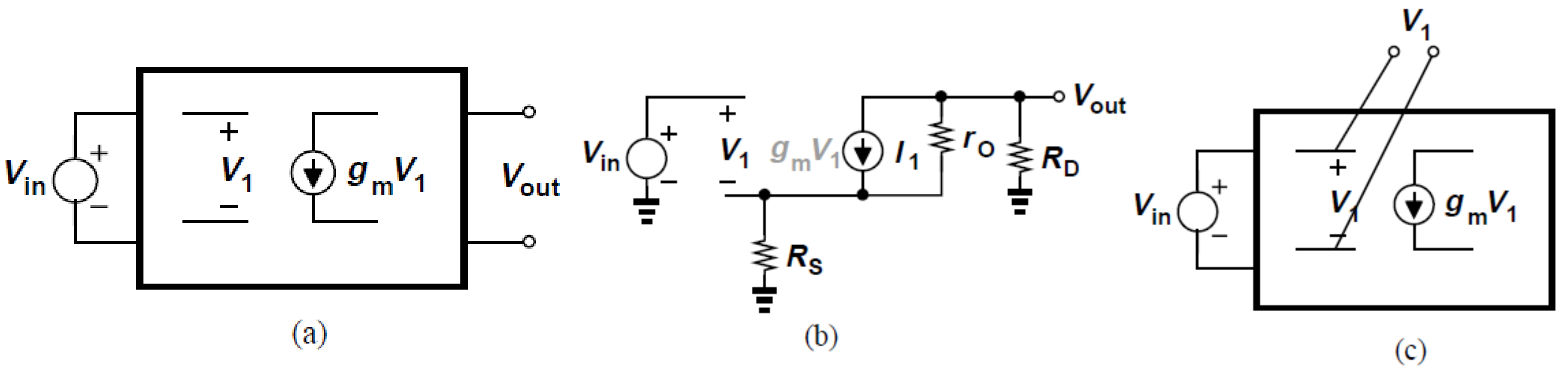
- As an example, in the degenerated CS stage of Fig. (b), we note that the current flowing upward through R_D (and downward through R_S) is $-V_{out}/R_D$ and hence the voltage drop across r_O is $(-V_{out}/R_D - I_1)r_O$
- KVL around the output network gives

$$V_{out} = \left(-\frac{V_{out}}{R_D} - I_1\right) r_O - \frac{V_{out}}{R_D} R_S$$

$$V_{out} = \frac{-r_O}{1 + \dfrac{r_O + R_S}{R_D}} I_1$$

- In this case, $A = 0$ and $B = -r_O R_D/(R_D + r_O + R_S)$

Bode's Analysis of Feedback Circuits: Observations



- **Next, consider V_1 as the signal of interest, i.e., we wish to compute V_1 as a function of V_{in} in the form of $A_v V_{in}$ or $H(s)V_{in}$**
- **We can pretend that V_1 is the "output", as in Fig. (c)**
- **In a similar manner, V_1 can be written, if we temporarily forget that $I_1 = g_m V_1$, as**

$$V_1 = CV_{in} + DI_1$$

- **KVL around the o... es**

$$V_1 = V_{in} - \frac{r_O R_S}{R_D + r_O + R_S} I_1$$

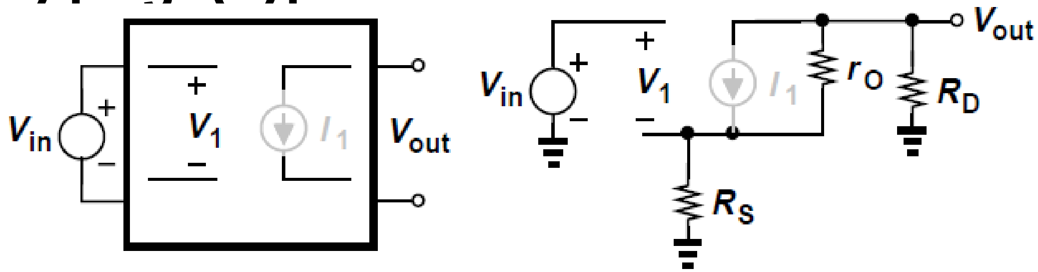
- **Hence,** $C = 1$ **and** $D = -r_O R_S/(R_D + r_O + R_S)$

Interpretation of Coefficients

- A is given by

$$A = \frac{V_{out}}{V_{in}} \text{ with } I_1 = 0$$

- A is obtained as the voltage gain of the circuit if the dependent current source is set to zero, by setting $g_m = 0$
- V_{out} in this case can be considered the "feedthrough" of the input signal (in the absence of the ideal transistor) [Fig. (a)]



(a)

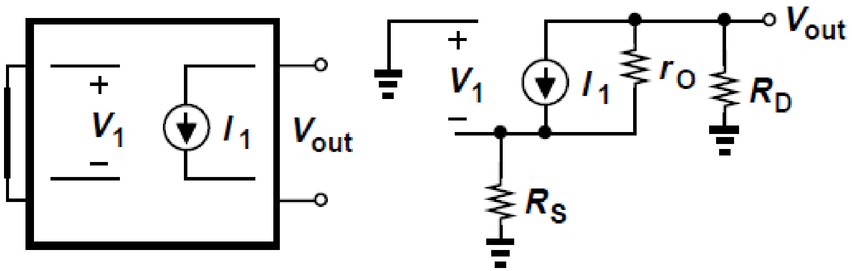
- In the CS example, $V_{out} = 0$ if $I_1 = 0$ because no current flows through R_S, r_O, and R_D, i.e., $A = 0$

Interpretation of Coefficients

- **As for the *B* coefficient, we have**

$$B = \frac{V_{out}}{I_1} \text{ with } V_{in} = 0$$

- **We set the input to zero and compute V_{out} as a result of I_1 [Fig. (b)], pretending that I_1 is an independent source**



(b)

- **In the CS example,**

$$\left(-\frac{V_{out}}{R_D} - I_1\right) r_O - \frac{V_{out}}{R_D} R_S = V_{out} \qquad V_{out} = \frac{-r_O R_D}{R_D + r_O + R_S} I_1$$

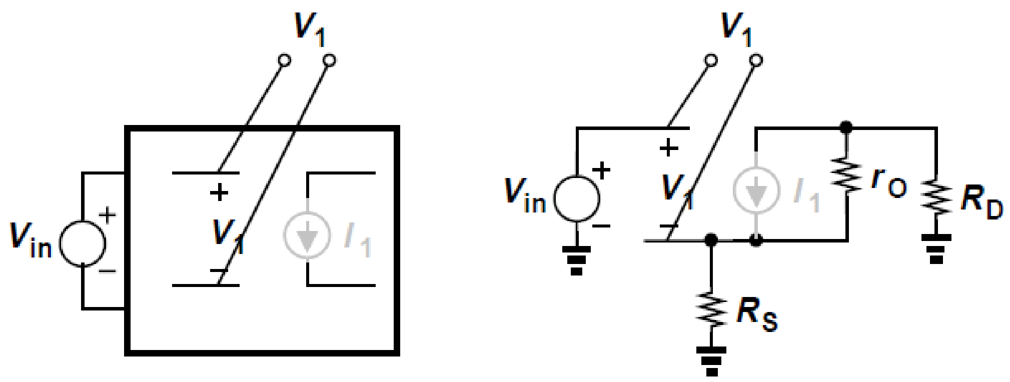
- **Thus,** $B = -r_O R_D/(R_D + r_O + R_S)$

Interpretation of Coefficients

- **The C coefficient is interpreted as**

$$C = \frac{V_1}{V_{in}} \text{ with } I_1 = 0$$

- **This is the transfer function from the input to V_1 with the transistor's g_m set to zero [Fig. (c)]**



(c)

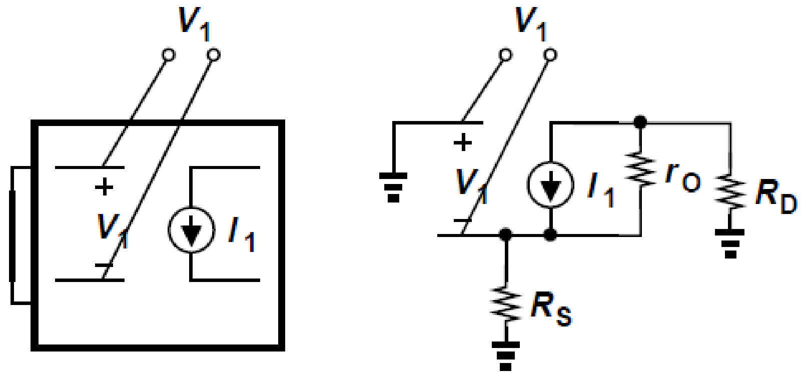
- **In the CS circuit, no current flows through R_S under this condition, yielding $V_1 = V_{in}$ and $C = 1$**

Interpretation of Coefficients

- **Lastly, the D coefficient is obtained as**

$$D = \frac{V_1}{I_1} \text{ with } V_{in} = 0$$

- **As shown in Fig. (d), this represents the transfer function from I_1 to V_1 with the input at zero**



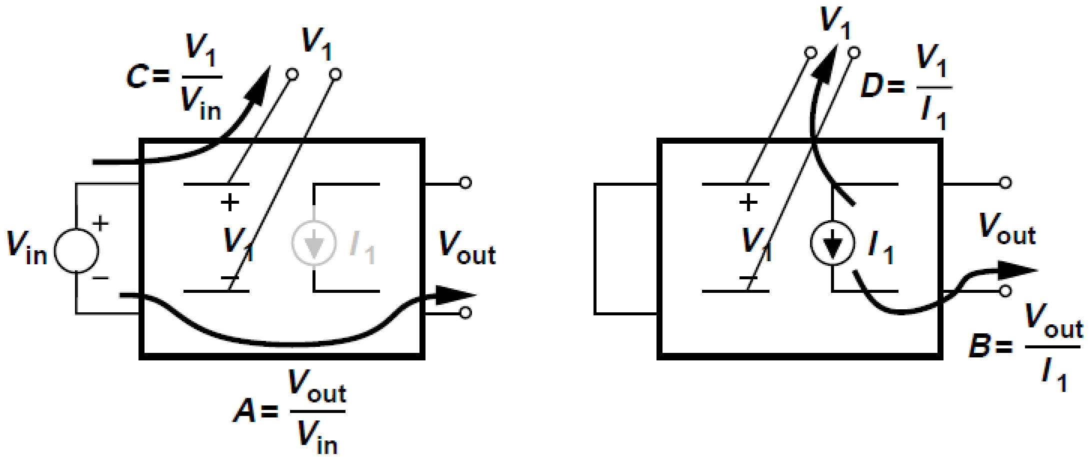
(d)

- **In the CS example, under the above condition,**

$$-V_1 - \left(\frac{V_1}{R_S} + I_1\right) r_O = \frac{V_1}{R_S} R_D \qquad V_1 = -\frac{r_O R_S}{R_D + r_O + R_S} I_1$$

- **Hence,** $D = -r_O R_S / (R_D + r_O + R_S)$

Interpretation of Coefficients: Summary



(a) (b)

- In summary, the *A-D* coefficients are computed as shown in Figs. (a) and (b)
- We disable the transistor by setting its g_m to zero and obtain *A* and *C* as feedthroughs from V_{in} to V_{out} and to V_1 respectively
- We set the input to zero and calculate *B* and *D* as the gain from I_1 to V_{out} and to V_1 respectively
- The former step finds responses to V_{in} with $g_m = 0$ and the latter to I_1 with $V_{in} = 0$

Bode's Analysis

- V_{out}/V_{in} is expressed in terms of *A-D* coefficients
- Since

$$
\begin{aligned}
V_{out} &= AV_{in} + BI_1 \\
V_1 &= CV_{in} + DI_1
\end{aligned}
$$

and in the actual circuit, $I_1 = g_m V_1$, we have

$$V_1 = \frac{C}{1 - g_m D} V_{in}$$

- The closed-loop gain is therefore equal to

$$\frac{V_{out}}{V_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

- The first term represents the input-output feedthrough when $g_m = 0$
- We can also write

$$\frac{V_{out}}{V_{in}} = \frac{A + g_m(BC - AD)}{1 - g_m D}$$

Bode's Analysis: Observations

$$\frac{V_{out}}{V_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

- **If $A = 0$, then closed-loop gain equation yields $V_{out}/V_{in} = g_m BC/(1-g_m D)$, which resembles the generic feedback equation $A_0/(1+\beta A_0)$**
- **$g_m BC$ is loosely called the "open-loop" gain**

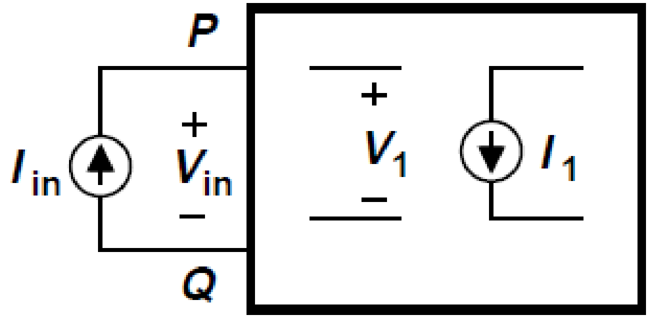
Bode's Analysis: Return Ratio and Loop Gain

$$\frac{V_{out}}{V_{in}} = \frac{A + g_m(BC - AD)}{1 - g_m D}$$

- **The closed-loop gain expression above may suggest that *$1 - g_m D = 1 +$ loop gain* and hence *loop gain $= -g_m D$***
- **In both cases, we set the main input to zero, break the loop by replacing the dependent source with an independent one, and compute the returned quantity**
- **In Bode's original treatment, the term "return ratio" *(RR)* is used to refer to $-g_m D$ and is ascribed to a given dependent source in the circuit**
- ***RR* appears to be the same as the true loop gain even if the loop cannot be completely broken**
- ***RR* is equal to the loop gain if the circuit contains only one feedback mechanism and the loop traverses the transistor of interest**

Blackman's Impedance Theorem

- Blackman's theorem determines the impedance seen at any port of a general circuit
  - Can be proved using Bode's approach



(a)

- In the general circuit of Fig. (a), the impedance between nodes *P* and *Q* is of interest
- One of the transistors is explicitly shown by the voltage-dependent current source I_1

Blackman's Impedance Theorem

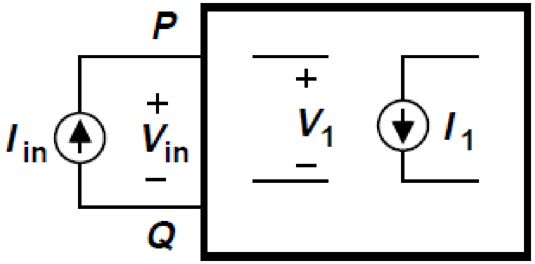
- Let us pretend that I_{in} is the input signal and V_{in} the output signal so that we can utilize Bode's results:

$$\begin{aligned} V_{in} &= AI_{in} + BI_1 \\ V_1 &= CI_{in} + DI_1 \end{aligned}$$

- It follows that

$$Z_{in} = \frac{V_{in}}{I_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

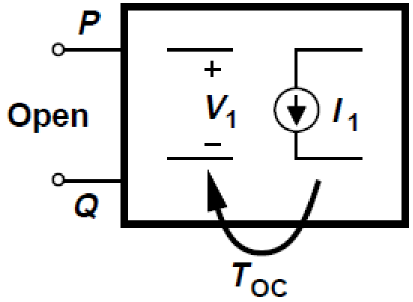
where g_m denotes the transconductance of the transistor modeled in Fig. (a)



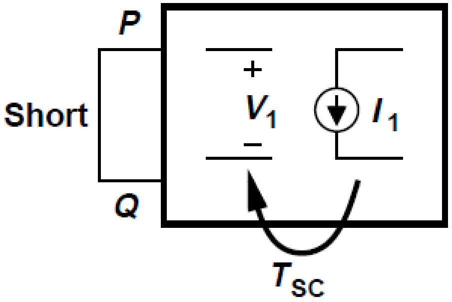
(a)

Blackman's Impedance Theorem

- Recognizing that $V_1/I_1 = D$ if $I_{in} = 0$, we call $-g_m D$ the "open-circuit loop gain" (because the port of interest is left *open*) and denote it by T_{oc} [Fig. (b)]



(b)



(c)

- If $V_{in} = 0$, then $I_{in} = (-B/A)I_1$ and hence

$$\frac{V_1}{I_1} = \frac{AD - BC}{A}$$

- We call $-g_m$ times this quantity the "short-circuit" loop gain (because $V_{in} = 0$) and denote it by T_{sc} [Fig. (c)]

Blackman's Impedance Theorem

- Both T_{oc} and T_{sc} can be viewed as return ratios associated with I_1 for two circuit topologies

$$T_{oc} = -g_m \frac{V_1}{I_1}\Big|_{Iin=0}$$
$$T_{sc} = -g_m \frac{V_1}{I_1}\Big|_{Vin=0}$$

- In the third step, we use T_{oc} and T_{sc} to rewrite

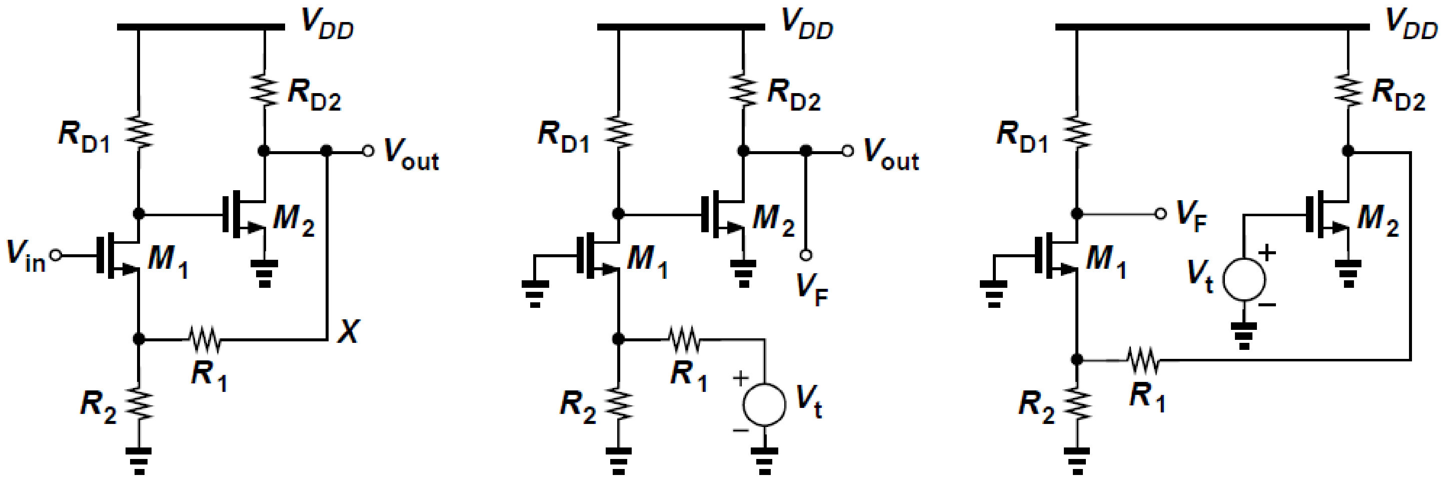
$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{A - g_m(BC - AD)}{1 - g_m D} = A\frac{1 + T_{sc}}{1 + T_{oc}}.$$

- *A* can be roughly viewed as the "open-loop" impedance without the transistor in the feedback loop
- In addition, if $|T_{sc}| \ll 1$ then $Z_{in} \approx A/(1+T_{oc})$ and if $|T_{oc}| \ll 1$, then $Z_{in} \approx A(1+T_{sc})$
- Closed-loop impedance cannot be expressed as Z_{in} multiplied or divided by *(1 + loop gain)*

Loop Gain Calculation Issues

- **Loop gain plays a central role in feedback systems**
- **If poles and zeros in the loop are considered, then the loop gain [called "loop transmission" *T(s)* in this case] reveals circuit's stability properties**
- **Loop gain calculation proceeds as**
  - **Break the loop at some point, apply a test signal, follow it around the loop (in the proper direction), and obtain the returned signal**
- **This elicits two questions:**
  1) **Can the loop be broken at any arbitrary point?**
  2) **Should the test signal be a voltage or current?**
- **In such a test, the actual input and output disappear**

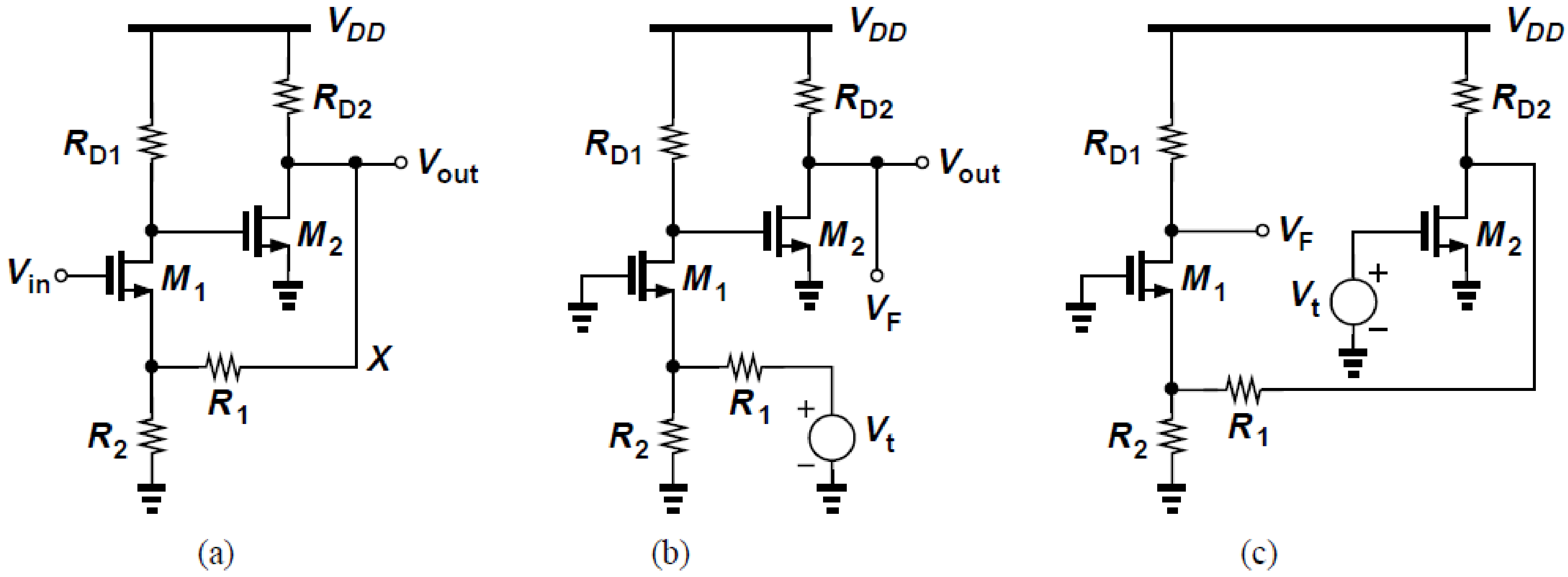
Loop Gain Calculation Issues



(a) (b) (c)

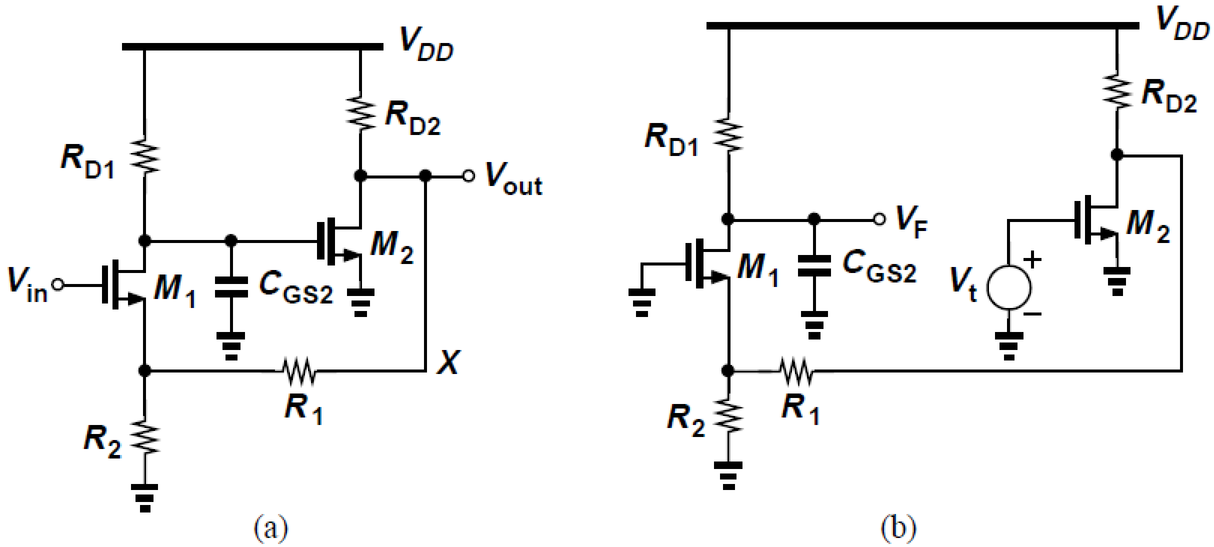
- In the two-stage amplifier of Fig. (a), resistive divider consisting of R_1 and R_2 senses output voltage and returns a fraction to source of M_1
- As shown in Fig. (b), we set V_{in} to zero, break the loop at node X, apply a test signal to the right terminal of R_1 and measure the resulting V_F
- In circuit of Fig. (a), R_1 draws an ac current from R_{D2} but in Fig. (b), it does not
- Gain of second CS stage has been altered

Loop Gain Calculation Issues



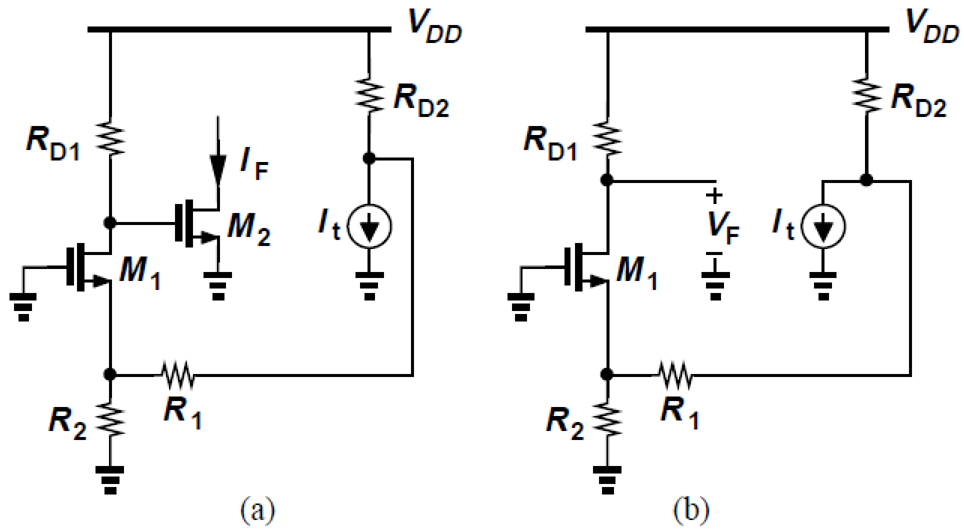
- It is best to break the loop at the *gate* of a MOSFET
- We can break the loop at the gate of M_2 [Fig. (c)] and thus not alter the gain associated with first stage at low frequencies

Loop Gain Calculation Issues



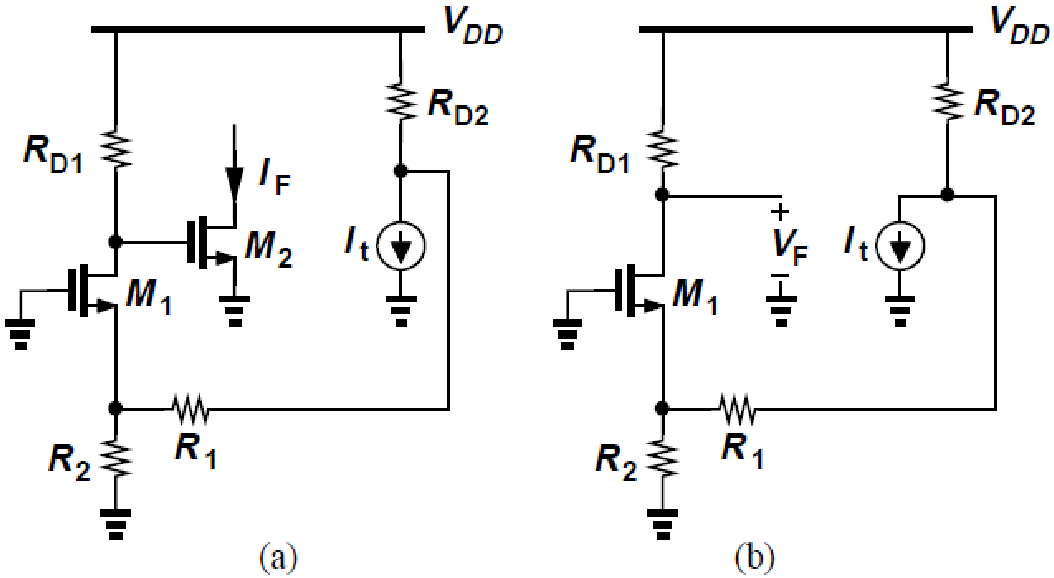
- **To include C_{GS} of M_2 [Fig. (a)], we break the loop after C_{GS2} [Fig. (b)] to ensure that the load seen by M_1 remains unchanged**
- **It is always possible to break the loop at the gate of a MOSFET**
- **For the feedback to be negative, the signal must be sensed by at least one gate in the loop because only the common-source topology inverts signals**

Loop Gain Calculation Issues



- **Can we apply a test current instead of a test voltage?**
- **We can break the loop at the drain of M_2, inject a current I_t, and measure the current returned by M_2 [Fig. (a)]**
- **If drain of M_2 is tied to ac ground, this node does not experience voltage excursions as in closed-loop circuit – when r_{O2} is taken into account**
- **In general, cannot inject I_t without altering some aspects of the circuit**

Loop Gain Calculation Issues



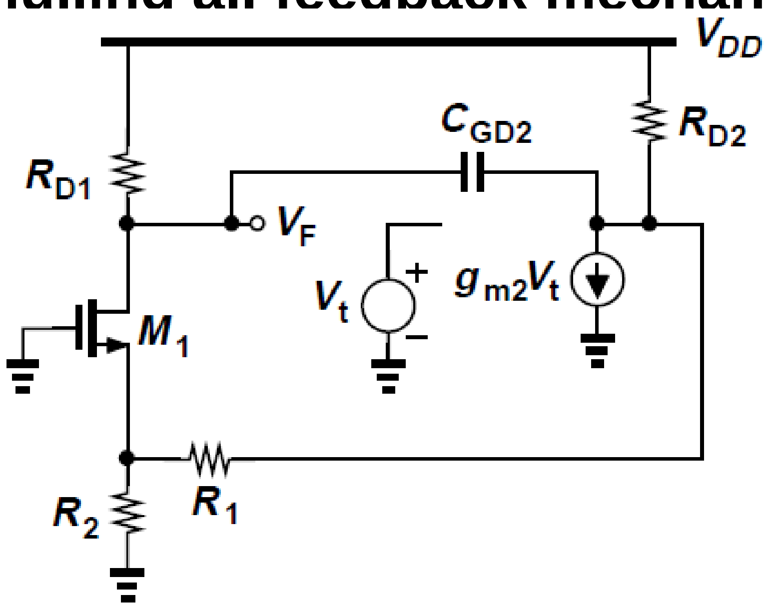
- If controlled current source of M_2 is replaced with an independent current source It, and compute the returned V_{GS} as V_F [Fig. (b)]
- Since in the original circuit, the dependent source and V_{GS2} were related by a factor of g_{m2}, the loop gain can be written as $(- V_F/I_t) \times g_{m2}$
- This approach is feasible even if M_2 is degenerated
- This result is the same as return ratio of M_2

Loop Gain Calculation Issues

- **In summary, the "best" place to break a feedback loop is**
  - **The gate-source of a MOSFET if voltage injection is desired**
  - **The dependent current source of a MOSFET if current injection is desired (provided that the returned quantity is VGS of the MOSFET)**
- **These two methods are related because they differ only by a factor of g_m**

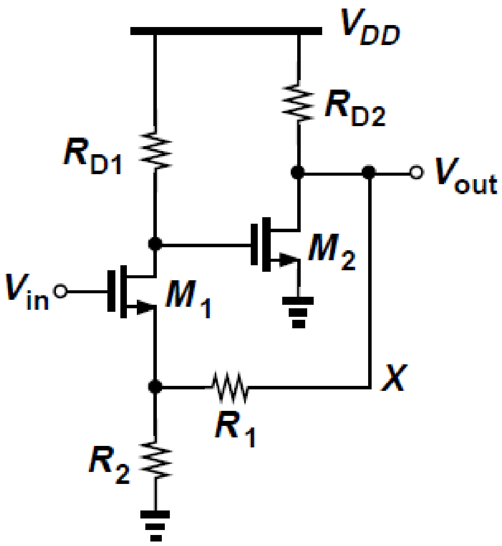
Loop Gain Calculation Issues

- **If we include C_{GD2} in previous circuit and inject a test voltage or current, C_{GD2} does not allow a "clean break"**
- **As shown below, even though gate-source voltage is provided by the independent source V_t, C_{GD2} creates "local" feedback from the drain of M_2 to its gate, raising the question whether loop gain should be obtained by nulling all feedback mechanisms**



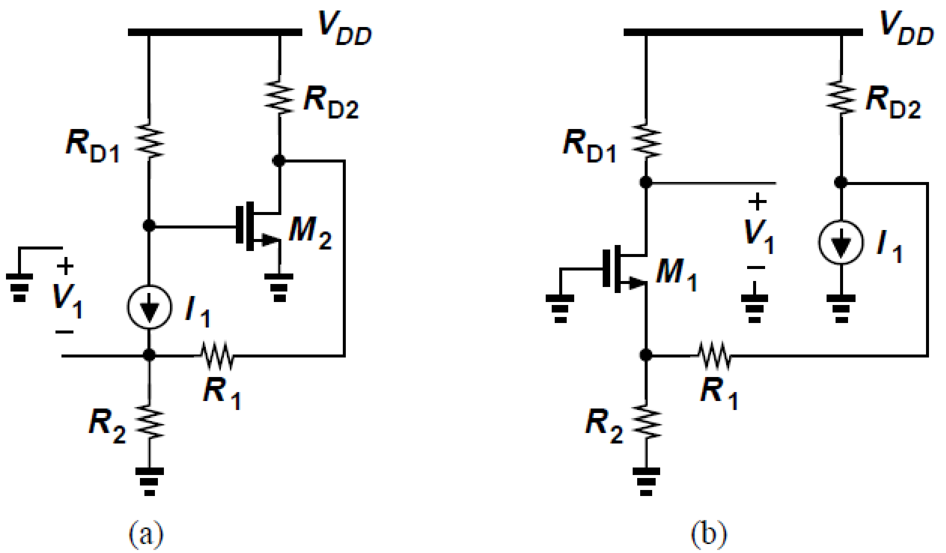
Difficulties with Return Ratio

- We may view the return ratio associated with a given dependent source as the loop gain
- Circuits containing more than one feedback mechanism exhibit different return ratios for different ratios
- In circuit of Fig. (a) below, R_1 and R_2 provide both "global" and "local" feedback (by degenerating M_1)



(a)

Difficulties with Return Ratio



- **Using equivalent circuits of Figs. (a) and (b), it can be shown that return ratios for M_1 and M_2 are given by**

$$\text{Return Ratio}|_{M1} = \frac{g_{m1}R_2(R_1 + R_{D2} + g_{m2}R_{D2}R_{D1})}{R_1 + R_2 + R_{D2}}$$

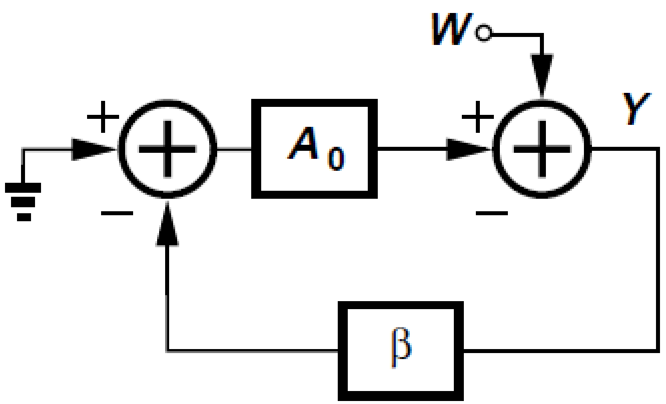
$$\text{Return Ratio}|_{M2} = \frac{g_{m1}g_{m2}R_2R_{D1}R_{D2}}{(1 + g_{m1}R_2)(R_1 + R_{D2}) + R_2}$$

- **Different return ratios obtained because disabling M_1**

Difficulties with Return Ratio

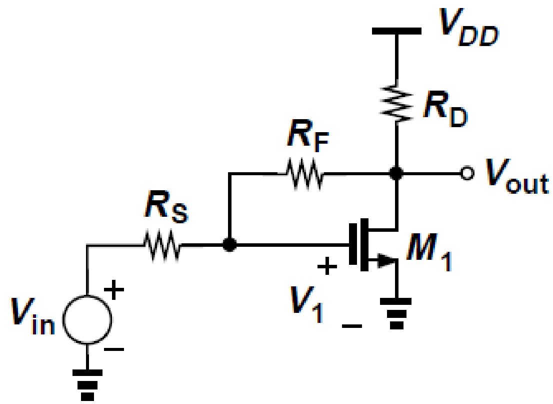
- **Another method of loop gain calculation is to inject a signal without breaking the loop as shown in figure below and write *$Y/W = 1/(1 + \beta A_0)$* and hence**

$$\text{Loop Gain} = \left(\frac{Y}{W}\right)^{-1} - 1$$

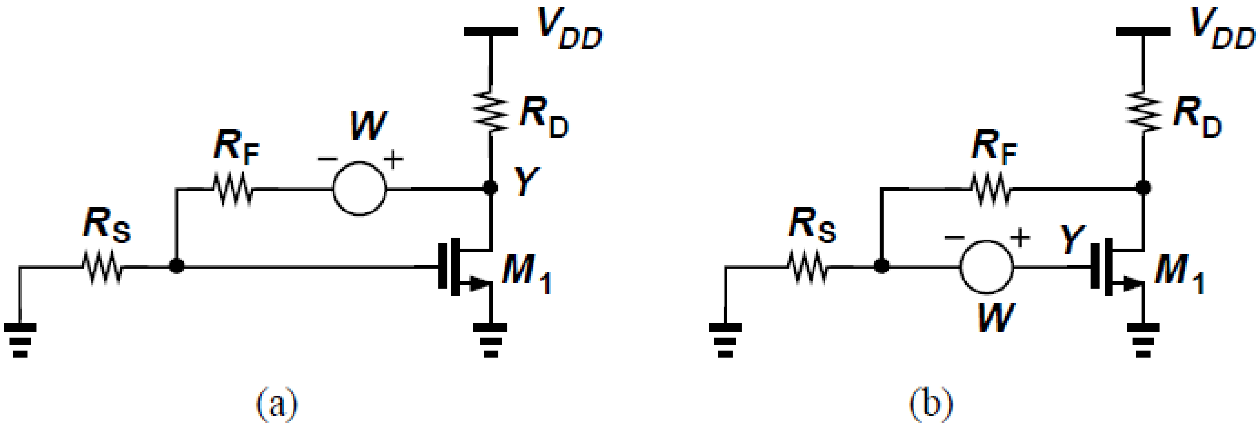


- **This method assumes a unilateral loop, yielding different loop gains for different injection points if the loop is not unilateral**

Difficulties with Return Ratio



- As an example, above circuit can be excited as in Figs. (a) or (b), producing different values for $(Y/W)^{-1} - 1$



Alternative Interpretations of Bode's Method

- <u>**Asymptotic Gain Form:**</u>
- **From Bode's results,** $V_{out}/V_{in} = A + g_m BC/(1 - g_m D)$ **and** $V_{out}/V_{in} = A$ if $g_m = 0$ **(the dependent source is disabled) and** $V_{out}/V_{in} = A - BC/D$ if $g_m \to \infty$ **(the dependent source is "very strong")**
- **We denote these values of V_{out}/V_{in} by H_0 and H_∞ respectively, and $-g_m D$ by T**
- **H_0 can be considered as the direct feedthrough and H_∞ as the "ideal gain". i.e., if the dependent source were infinitely strong (or if the loop gain were infinite)**
- **It follows that**

$$
\begin{aligned}
\frac{V_{out}}{V_{in}} &= H_0 + \frac{g_m BC}{1+T} \\
&= H_0 \frac{1+T}{1+T} + \frac{g_m BC}{1+T} \\
&= \frac{H_0}{1+T} + \frac{T(H_0 + g_m BC/T)}{1+T}
\end{aligned}
$$

Alternative Interpretations of Bode's Method

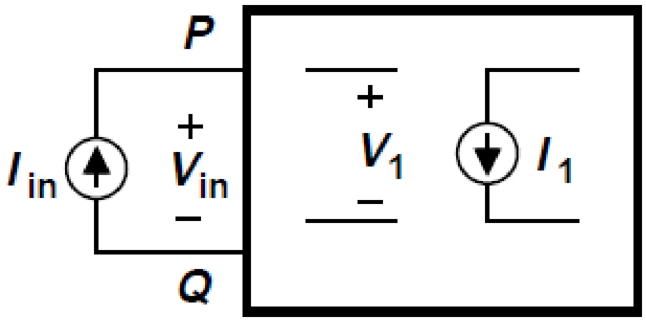
- **Asymptotic Gain Form (contd.):**
- **Since** $H_0 + g_m BC/T = A - g_m BC/(g_m D) = A - BC/D = H_\infty$ **we have,**

$$\frac{V_{out}}{V_{in}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

- **Called the "asymptotic gain equation", this form reveals that the gain consists of an ideal value multiplied by *T/(1 + T)* and a direct feedthrough multiplied by *1/(1 + T)***
- **Calculations are simpler here if we recognize from** $V_1 = CV_{in} + DI_1$ and $I_1 = g_m V_1$ **that** $V_1 = CV_{in}/(1 - g_m D) \rightarrow 0$ if $g_m \rightarrow \infty$ (provided that $V_{in} < \infty$).
- **This is similar to how a virtual ground is created if the loop gain is large**

Alternative Interpretations of Bode's Method

- **Double Null Method:**
- **From Blackman's Impedance Theorem, we recognize that [refer Fig. (a)]**
  - T_{oc} **is the return ratio with** $I_{in} = 0$**, i.e.,** T_{oc} **denotes the RR with the input set to zero**
  - T_{sc} **is the RR with** $V_{in} = 0$**, i.e.,** T_{sc} **represents the RR with the output forced to zero**



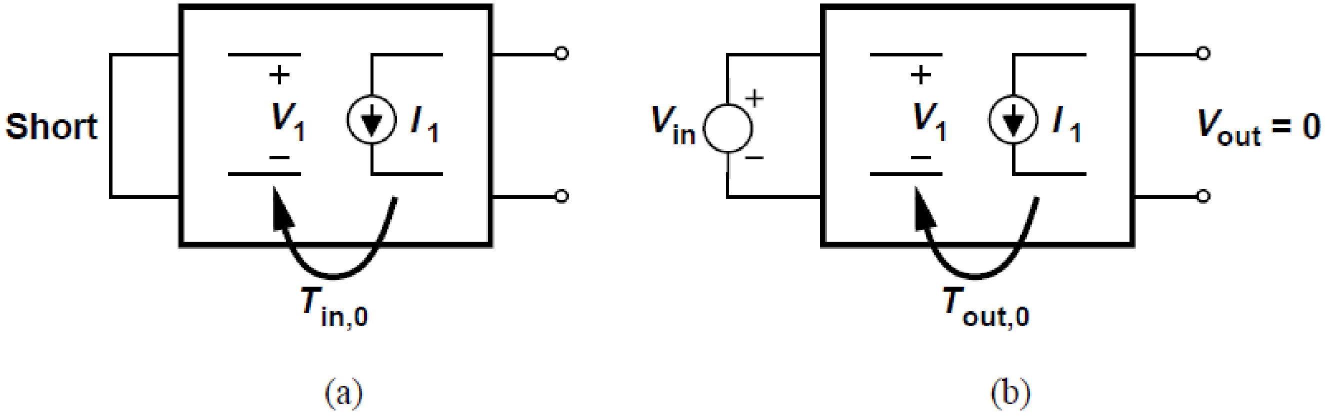
(a)

Alternative Interpretations of Bode's Method

- <u>Double Null Method (contd.):</u>
- Making a slight change in our notation, we postulate that the transfer function of a given circuit can be written as

$$\frac{V_{out}}{V_{in}} = A\frac{1 + T_{out,0}}{1 + T_{in,0}}$$

- Where $A = V_{out}/V_{in}$ with the dependent source set to zero, and Tout,0 and Tin,0 respectively denote the return ratios for $V_{out} = 0$ and $V_{in} = 0$



Alternative Interpretations of Bode's Method

- **<u>Double Null Method (proof):</u>**
- **Beginning from**

$$V_{out} = AV_{in} + BI_1$$

$$V_1 = CV_{in} + DI_1$$

- **We observe that if**

$V_{in} = 0$, then $V_1/I_1 = D$ and hence $T_{in,0} = -g_m D$

- **On the other hand, if**

$V_{out} = 0$, then $V_{in} = (-B/A)I_1$ and hence $V_1/I_1 = (AD - BC)/A$
i.e., $T_{out,0} = -g_m(AD - BC)/A$

- **Combining these results yields**

$$\frac{V_{out}}{V_{in}} = A\frac{1 + T_{out,0}}{1 + T_{in,0}}$$

- **Division by *A* in these calculations assumes *A ≠ 0***