

# ***Chapter 5: Current Mirrors and Biasing Techniques***

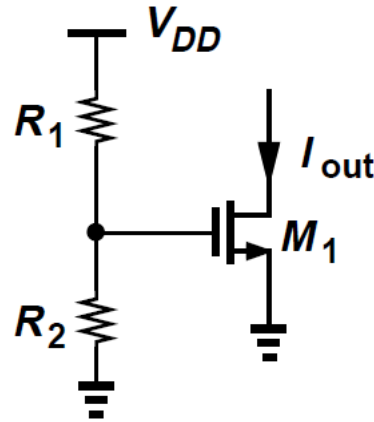
**5.1 Basic Current Mirrors**

**5.2 Cascode Current Mirrors**

**5.3 Active Current Mirrors**

**5.4 Biasing Techniques**

# Basic Current Mirrors

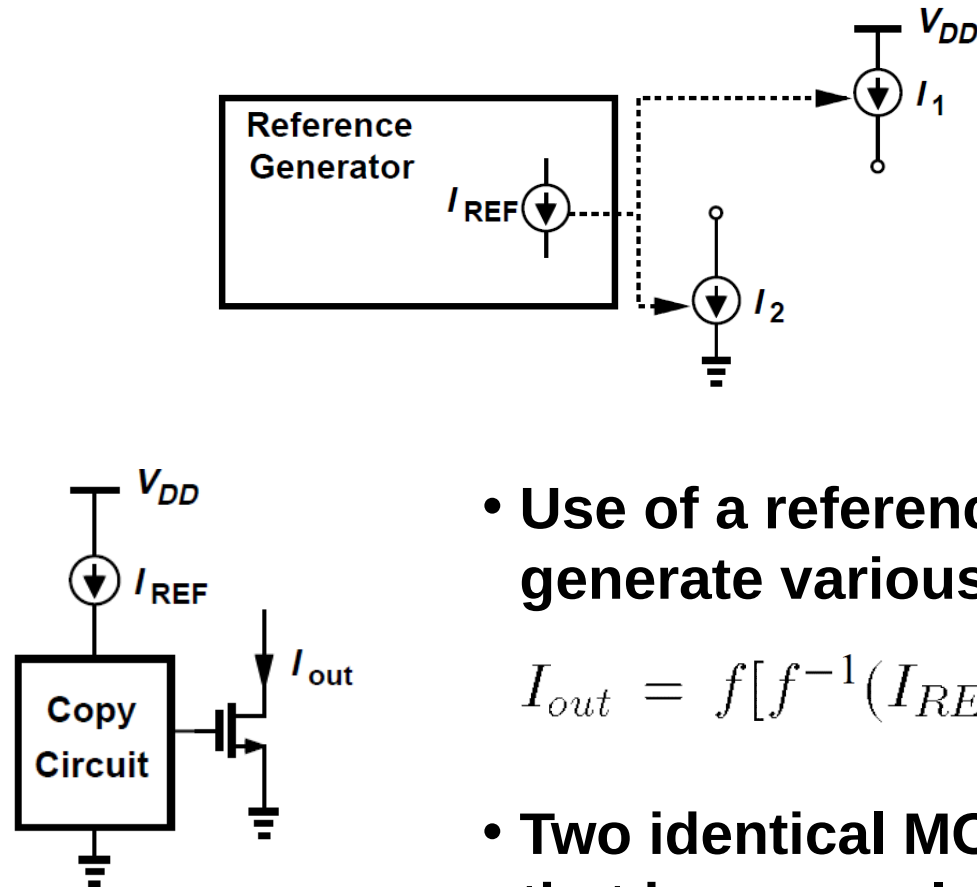


- Assuming  $M_1$  is in saturation, we can write

$$I_{out} \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left( \frac{R_2}{R_1 + R_2} V_{DD} - V_{TH} \right)^2.$$

- The threshold voltage may vary by 50 to 100 mV from wafer to wafer
- Both  $\mu_n$  and  $V_{TH}$  exhibit temperature dependence
- We must seek other methods of biasing MOS current sources.

# Conceptual means of copying currents

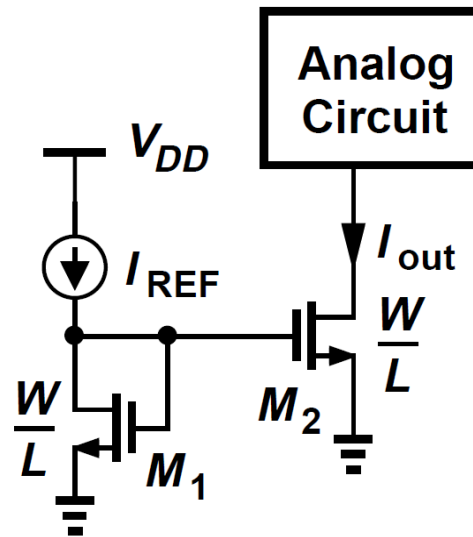


- Use of a reference to generate various currents.

$$I_{out} = f[f^{-1}(I_{REF})] = I_{REF}$$

- Two identical MOS devices that have equal gate-source voltages and operate in saturation carry equal currents

# Effect of Channel-Length Modulation



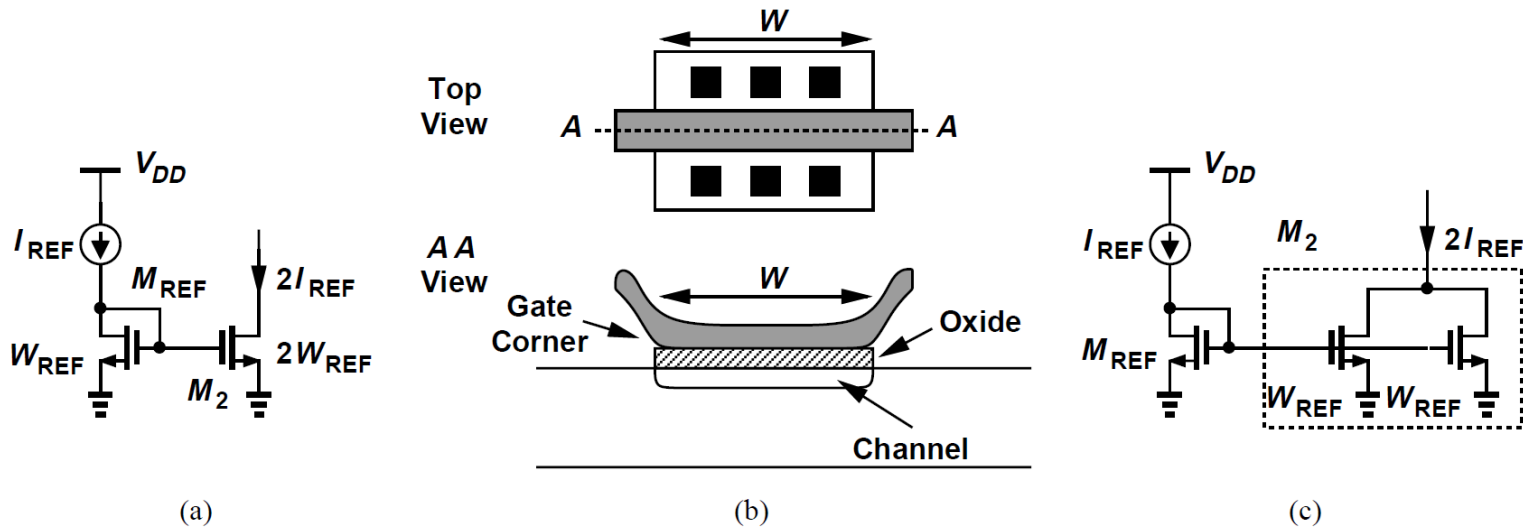
- Neglecting channel-length modulation, we can write

$$I_{REF} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2$$
$$I_{out} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 (V_{GS} - V_{TH})^2,$$

$$I_{out} = \frac{(W/L)_2}{(W/L)_1} I_{REF}.$$

- Allows precise copying of the current with no dependence on process and temperature

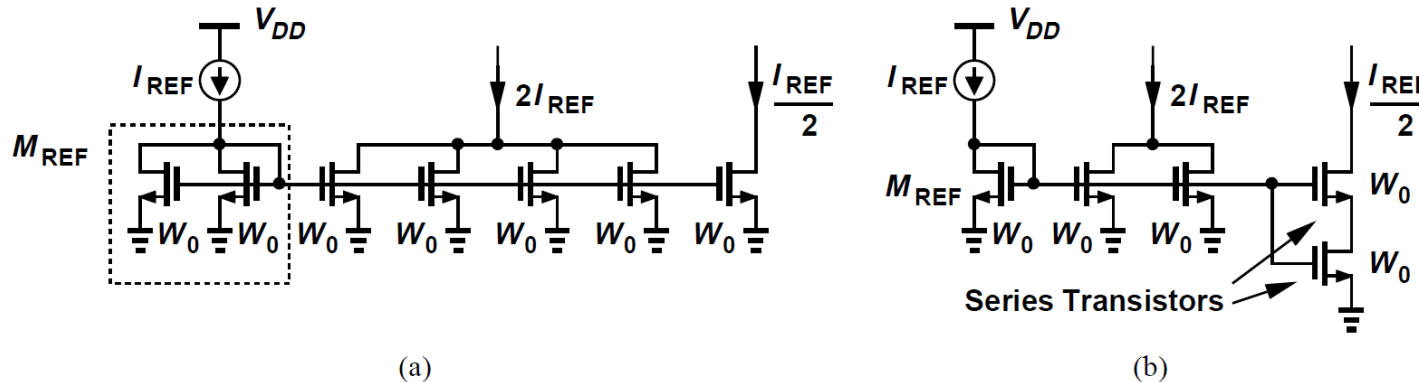
# Sizing issues



- Current mirrors usually employ the same length for all of the transistors.
- Current ratioing is achieved by only scaling the width of transistors.
- Direct scaling of the width also faces difficulties.
- We thus prefer to employ a “unit” transistor and create copies by repeating such a device.

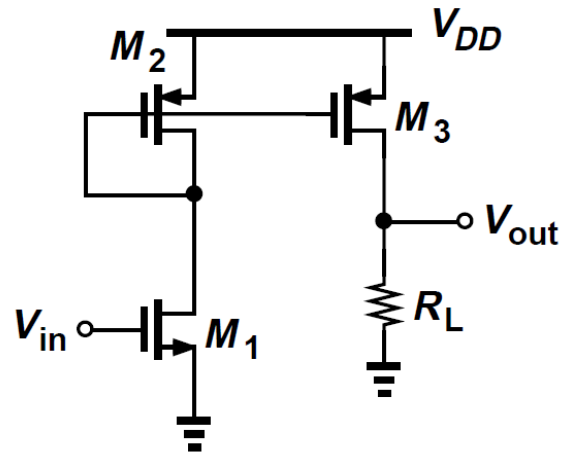
# Sizing Issues

- How do we generate a current equal to  $I_{REF}/2$  from  $I_{REF}$ ?



- (a) half-width device, and (b) series transistors
- Approach (b) preserves an effective length of  $(L_{drawn}-2L_D)$  for each unit, yielding an equivalent length of  $2(L_{drawn} - 2L_D)$
- Current mirrors can process signals as well, example next slide.

# Example



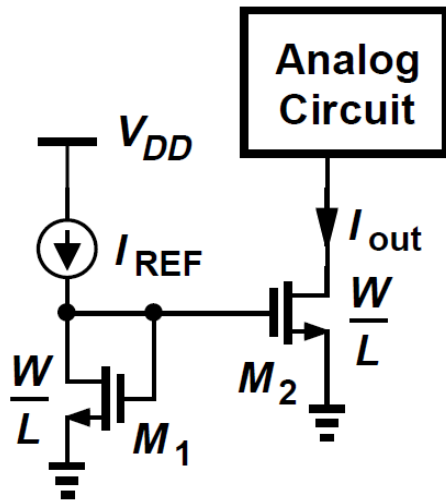
- Calculate the small-signal voltage gain of the circuit shown in Figure.

$$I_{D2} = I_{D1}$$

$$I_{D3} = I_{D2}(W/L)_3/(W/L)_2$$

- Gain=  $g_{m1}R_L(W/L)_3/(W/L)_2$

# Cascode Current Mirrors



$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS1})$$

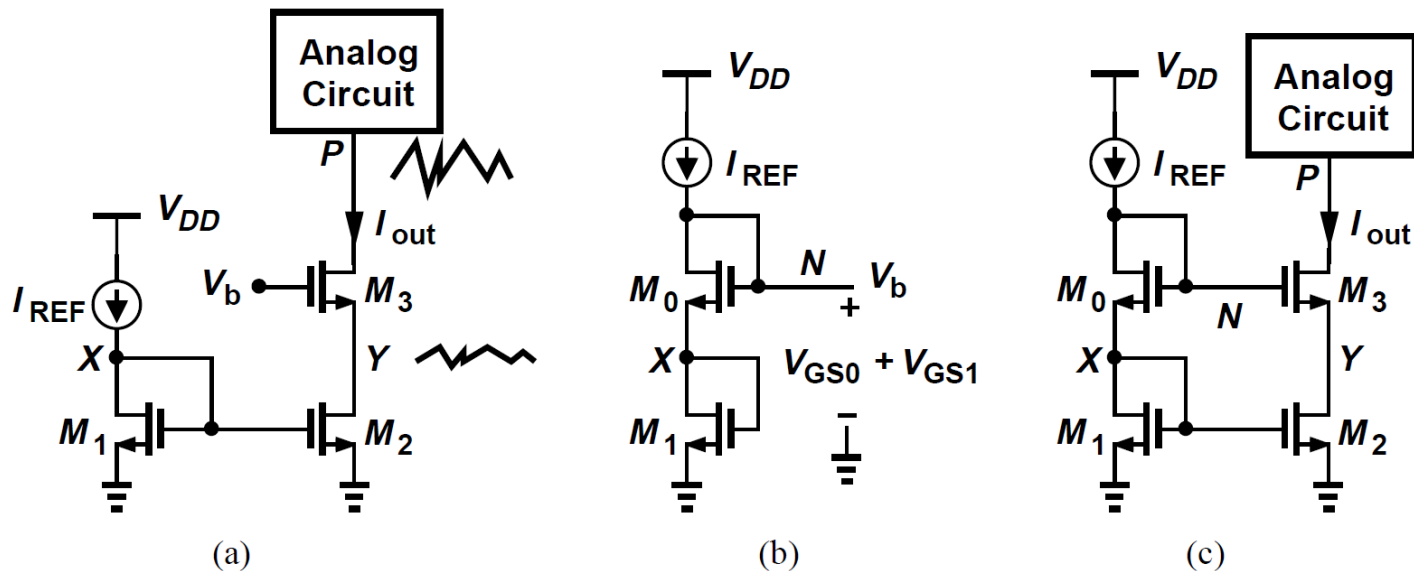
$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS2})$$

$$\frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1} \cdot \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}.$$

- While  $V_{DS1} = V_{GS1} = V_{GS2}$ ,  $V_{DS2}$  may not equal  $V_{GS2}$
- We can (a) force  $V_{DS2}$  to be equal to  $V_{DS1}$ , or (b) force  $V_{DS1}$  to be equal to  $V_{DS2}$ .

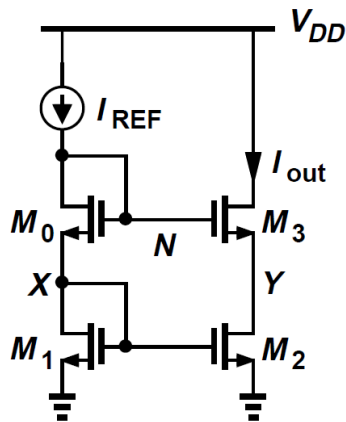


# First Approach

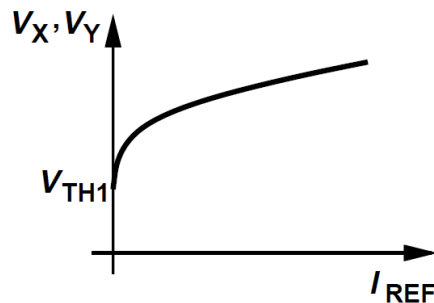


- A cascode device can shield a current source, thereby reducing the voltage variations across it.
- But, how do we ensure that  $V_{DS2} = V_{DS1}$ ?
- We must generate  $V_b$  such that  $V_b - V_{GS3} = V_{DS1}(= V_{GS1})$

# Example



(a)



(b)

- sketch  $V_X$  and  $V_Y$  as a function of  $I_{REF}$ . If  $I_{REF}$  requires 0.5 V to operate as a current source, what is its maximum value?

$$V_Y = V_X \approx \sqrt{2I_{REF}/[\mu_n C_{ox}(W/L)_1]} + V_{TH1}$$

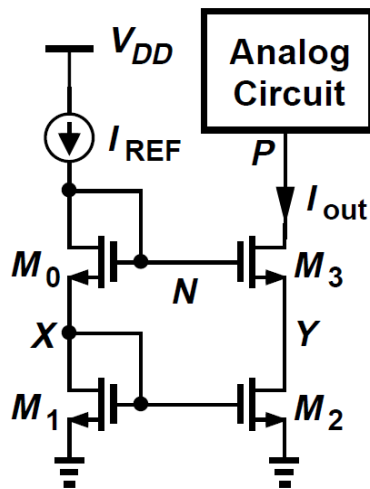
$$V_N = V_{GS0} + V_{GS1}$$

$$= \sqrt{\frac{2I_{REF}}{\mu_n C_{ox}}} \left[ \sqrt{\left(\frac{L}{W}\right)_0} + \sqrt{\left(\frac{L}{W}\right)_1} \right] + V_{TH0} + V_{TH1}$$

$$V_{DD} - \sqrt{\frac{2I_{REF}}{\mu_n C_{ox}}} \left[ \sqrt{\left(\frac{L}{W}\right)_0} + \sqrt{\left(\frac{L}{W}\right)_1} \right] - V_{TH0} - V_{TH1} = 0.5 \text{ V}$$

$$I_{REF,max} = \frac{\mu_n C_{ox} (V_{DD} - 0.5 \text{ V} - V_{TH0} - V_{TH1})^2}{2 (\sqrt{(L/W)_0} + \sqrt{(L/W)_1})^2}$$

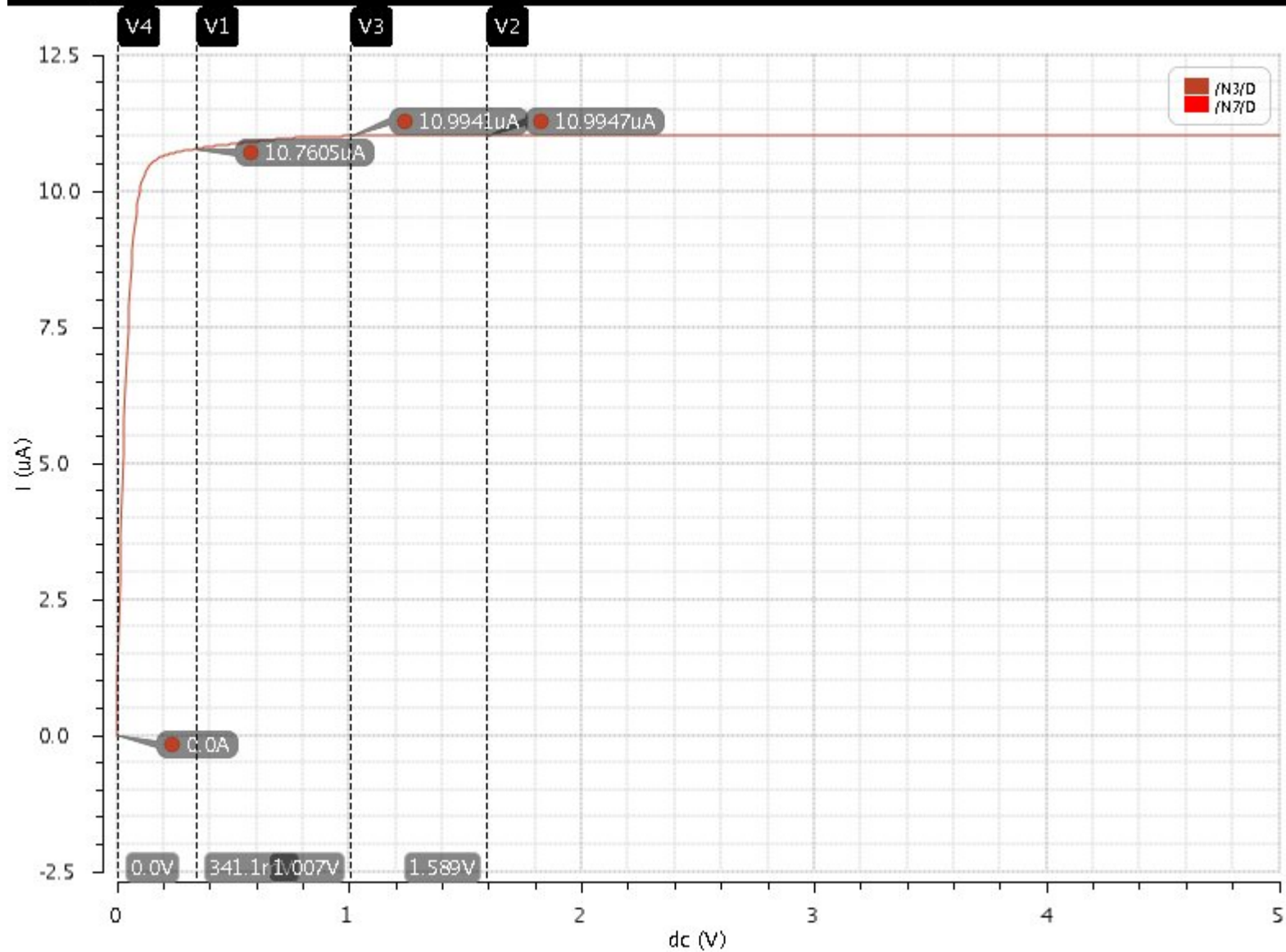
# Example



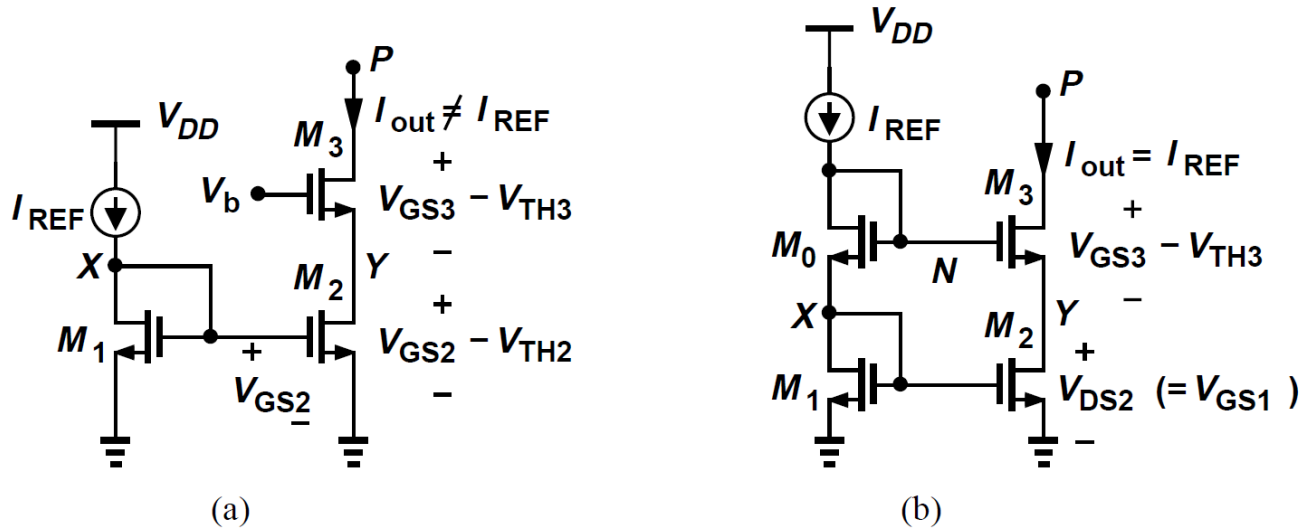
- the minimum allowable voltage at node P is equal to

$$\begin{aligned}V_N - V_{TH} &= V_{GS0} + V_{GS1} - V_{TH} \\ &= (V_{GS0} - V_{TH}) + (V_{GS1} - V_{TH}) + V_{TH}\end{aligned}$$

- The cascode mirror “wastes” one threshold voltage in the headroom.
- Because  $V_{DS2} = V_{GS2}$ , whereas  $V_{DS2}$  could be as low as  $V_{GS2} - V_{TH}$  while maintaining M2 in saturation.

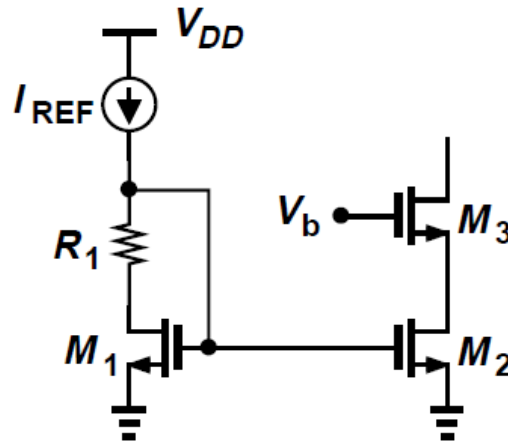


# Approach summary



- In Fig(a),  $V_b$  is chosen to allow the lowest possible value of  $V_P$  but the output current does not accurately track  $I_{REF}$ .
- In Fig(b), a higher accuracy is achieved, but the minimum level at P is higher by one threshold voltage.

# Second Approach

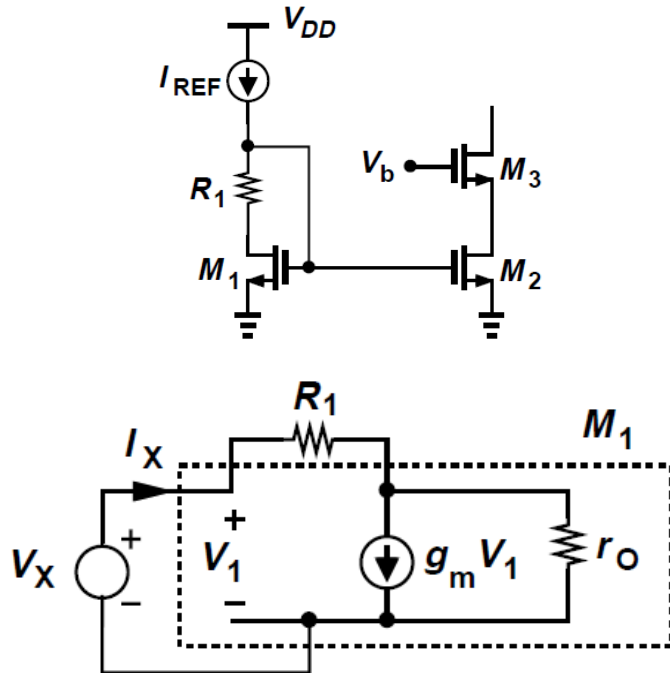


- Consider the branch shown in Fig. 5.16(b)
- As a candidate and write  $V_b = V_{GS5} + R_6 I_6$ .

$$R_1 I_{REF} \approx V_{TH1}$$

$$V_b = V_{GS3} + (V_{GS1} - V_{TH1})$$

# Small signal Model

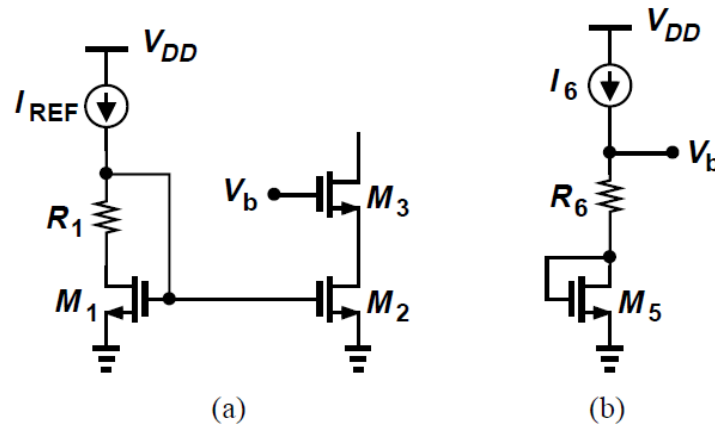


$$\frac{V_X - I_X R_1}{r_O} + g_m V_X = I_X.$$

$$\frac{V_X}{I_X} = \frac{R_1 + r_O}{1 + g_m r_O},$$

- Reduces to  $1/g_m$  in the absence of channel-length modulation.
- Thus, from a small-signal point of view, the combination is close to a diode-connected device.
- But
  - (1) It may be difficult to guarantee that  $R_1 I_{REF} \approx V_{TH1}$
  - (2) The generation of
 
$$V_b = V_{GS3} + (V_{GS1} - V_{TH1})$$
 is not straightforward.

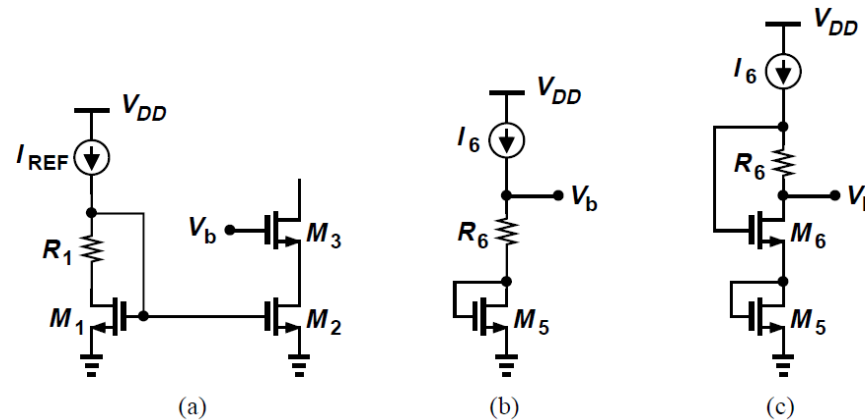
# Generate $V_b$



- Consider the branch shown in Fig(b) as a candidate and write  $V_b = V_{GS5} + R_6 I_6$ .
- $V_{GS5} = V_{GS3}$
- However, the condition  $R_6 I_6 = V_{GS1} - V_{TH1} = V_{GS1} - R_1 I_{REF}$  is hard to meet.



# Generate $V_b$

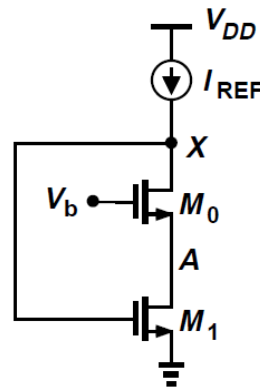


$$V_{GS5} = V_{GS3}$$

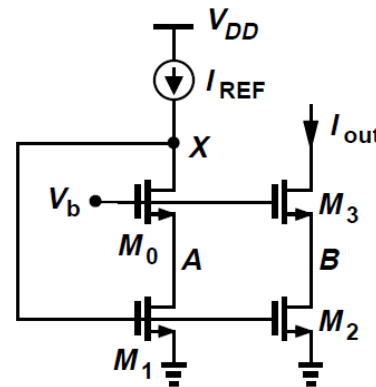
$$\begin{aligned} V_{GS6} - R_6 I_6 &= V_{GS1} - V_{TH1} \\ &= V_{GS1} - R_1 I_{REF} \end{aligned}$$

- It is now possible to ensure that  $V_{GS6}$  and  $V_{GS1}$  track each other.
- For example, we may simply choose  $I_6 = I_{REF}$ ,  $R_6 = R_1$ , and  $(W/L)_6 = (W/L)_1$

# Another circuit topology



(a)



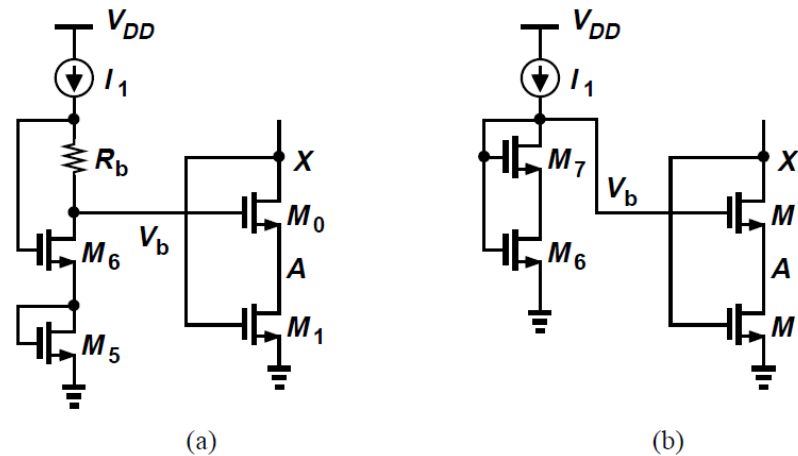
(b)

- In this case

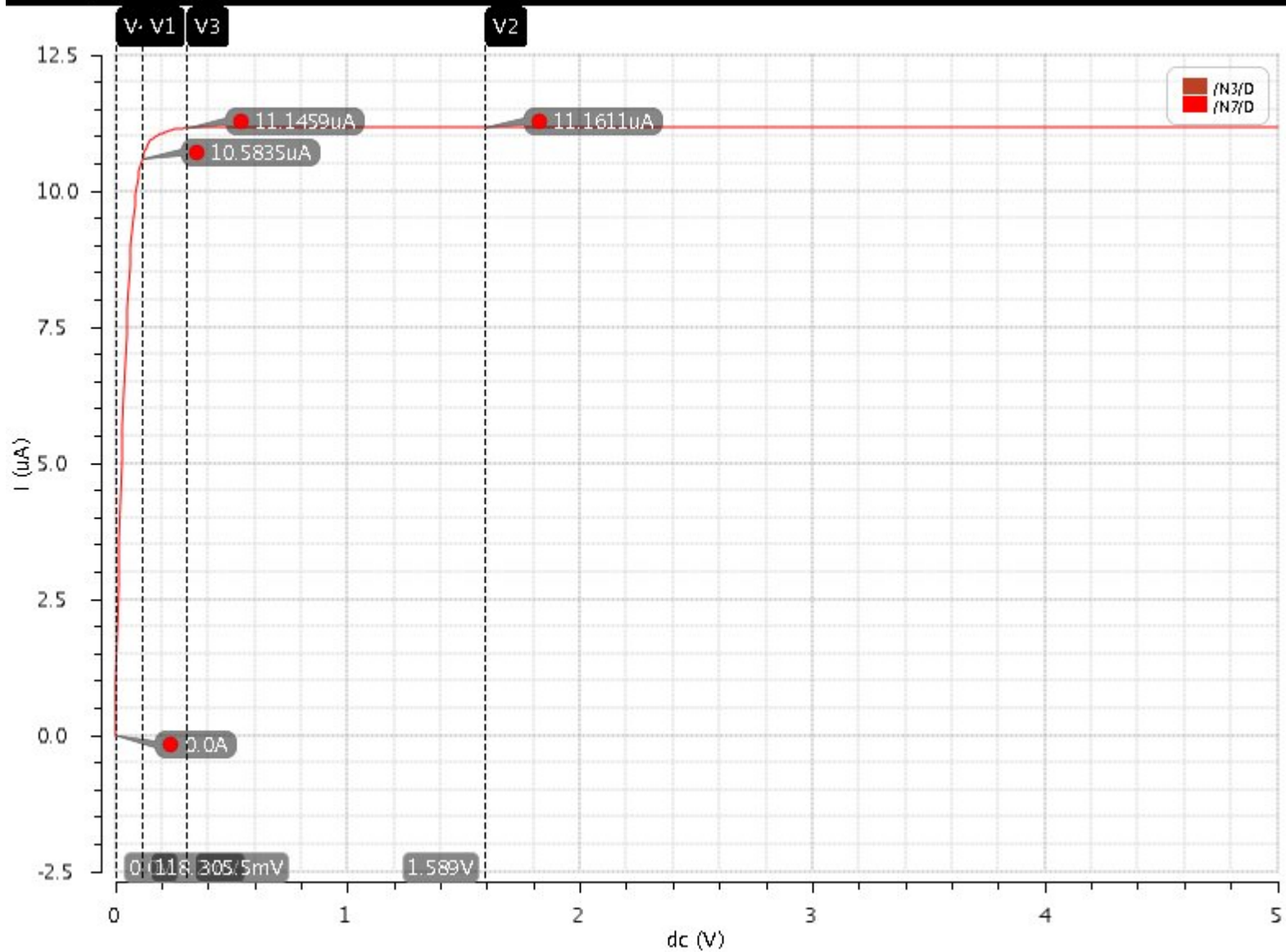
$$V_{DS1} = V_b - V_{GS0}$$

- **Must have**  $V_b - V_{TH0} \leq V_X (= V_{GS1})$  **for M0 to be saturated** and  $V_{GS1} - V_{TH1} \leq V_A (= V_b - V_{GS0})$  **for M1 to be saturated.**
- **A solution exists if**  $V_{GS0} + (V_{GS1} - V_{TH1}) < V_{GS1} + V_{TH0}$
- **We must therefore size M0 to ensure its overdrive is well below  $V_{TH1}$ .**

# How to generate $V_b$

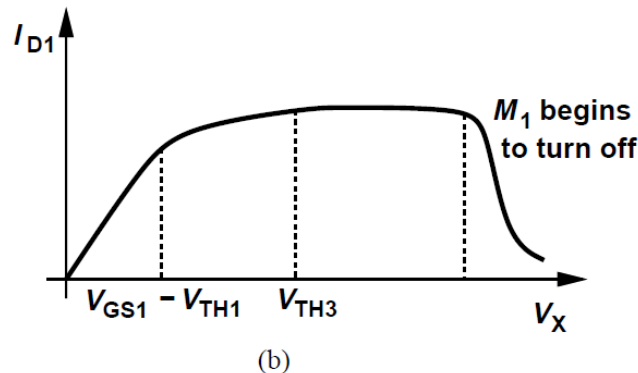
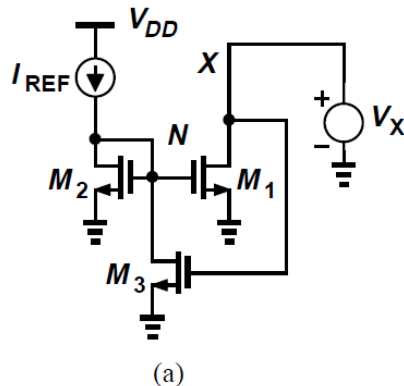


- In figure (a)  $V_{DS6} = V_{GS6} - R_b I_1 \approx V_{GS1} - V_{TH1}$
- Some inaccuracy nevertheless arises because  $M_5$  does not suffer from body effect whereas  $M_0$  does.
- Also, the magnitude of  $R_b I_1$  is not well-controlled.
- A simpler alternative is shown in Fig(b)



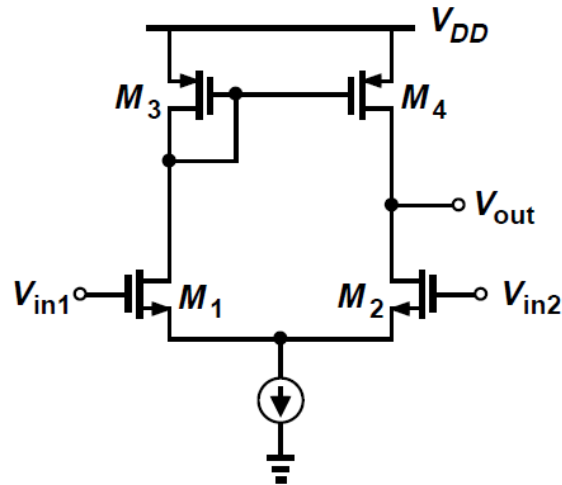


# Example



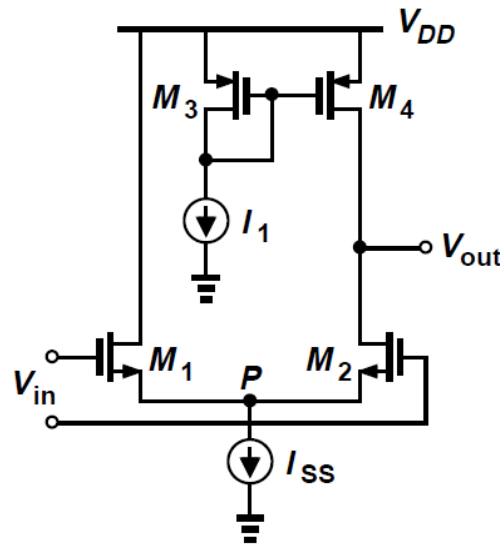
- An alternative current mirror exhibiting a high output impedance.
- Small signal: If we choose  $g_{m3}g_{m1}/g_{m2} \approx r_{O1}^{-1}$  the net change in  $I_{D1}$  is small.
- Figure b for large signal.
- The above circuit does pose its own voltage headroom limitation:  $V_X$  must exceed  $V_{TH3}$ .

# Active Current Mirrors



- A five-transistor “operational transconductance amplifier” (OTA).
- Note that the output is single-ended, hence the circuit is sometimes used to convert differential signals to a single-ended output.

# Quick Calculation



- We may simply discard one output of a differential pair as shown in Fig.
- What is the small-signal gain?

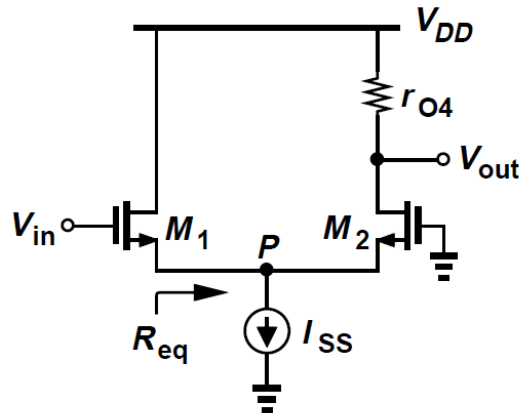
$$|A_v| = G_m R_{out}$$

$$|A_v| = \frac{g_{m1}}{2} [(2r_{O2}) || r_{O4}].$$



# Second Approach

- We calculate  $V_P / V_{in}$  and  $V_{out}/V_P$

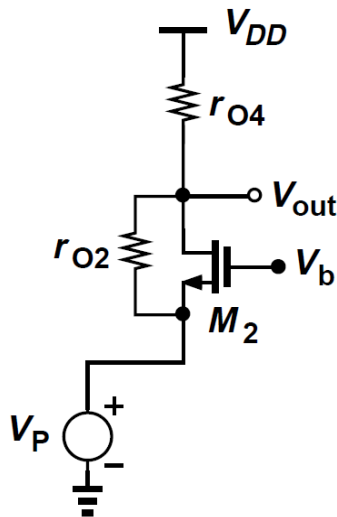


$$\frac{V_P}{V_{in}} = \frac{R_{eq} || r_{O1}}{R_{eq} || r_{O1} + \frac{1}{g_{m1}}}$$

$$R_{eq} = \frac{r_{O2} + r_{O4}}{1 + g_{m2}r_{O2}}$$

$$\frac{V_P}{V_{in}} = \frac{g_{m1}r_{O1}(r_{O2} + r_{O4})}{(1 + g_{m1}r_{O1})(r_{O2} + r_{O4}) + (1 + g_{m2}r_{O2})r_{O1}}$$

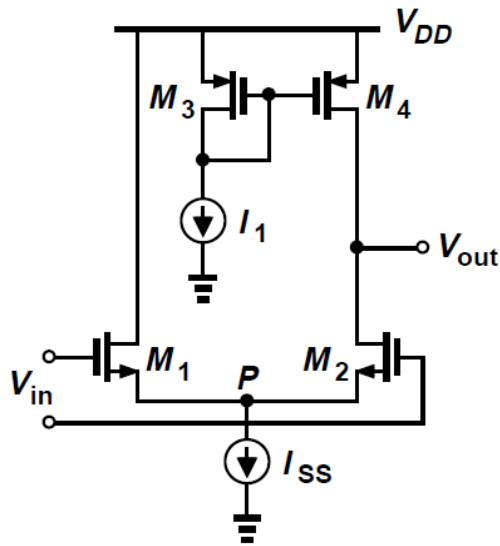
- Caculate  $V_{out}/V_P$



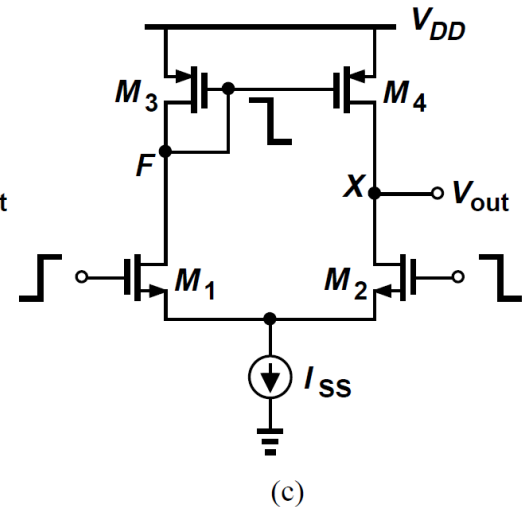
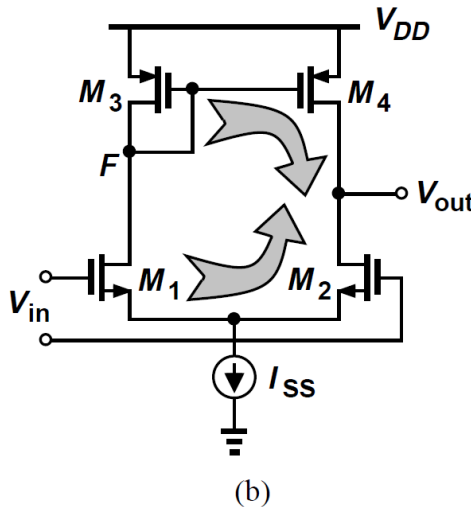
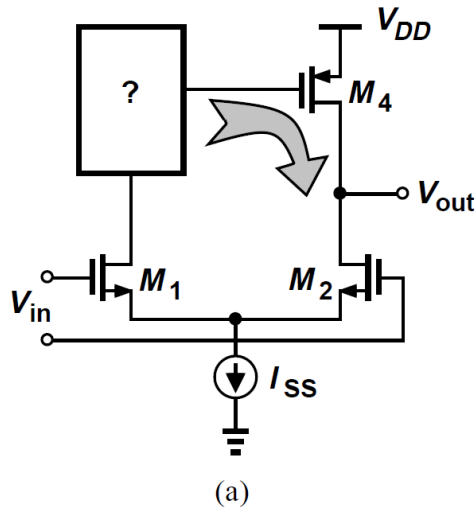
$$\frac{V_{out}}{V_P} = \frac{(1 + g_{m2}r_{O2})r_{O4}}{r_{O2} + r_{O4}}.$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{g_{m2}r_{O2}r_{O4}}{2r_{O2} + r_{O4}} \\ &= \frac{g_{m2}}{2} [(2r_{O2}) || r_{O4}] \end{aligned}$$

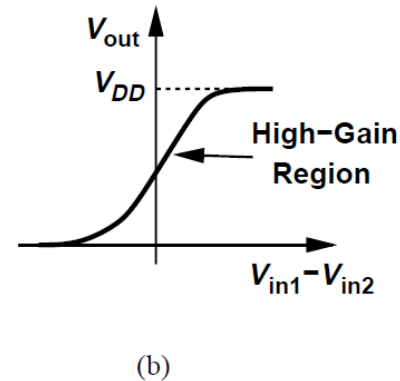
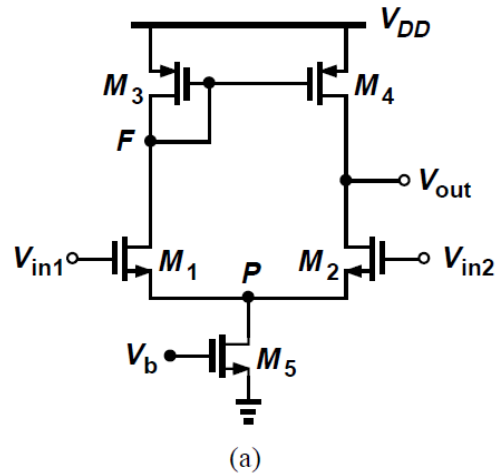
# Differential Pair with Active Load



- The small-signal draincurrent of M1 is “wasted.”
- It is desirable to utilize this current with proper polarity at the output.
- This can be accomplished by the five-transistor OTA.
- M3 enhances the gain.
- The five-transistor OTA is also called a differential pair with active load.

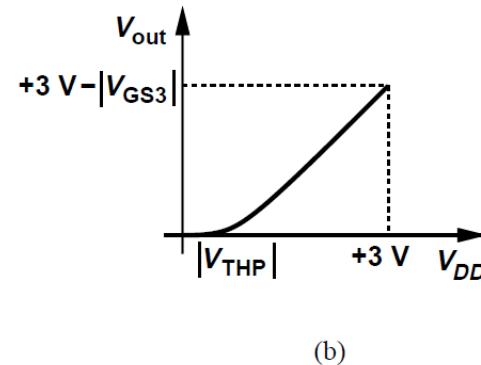
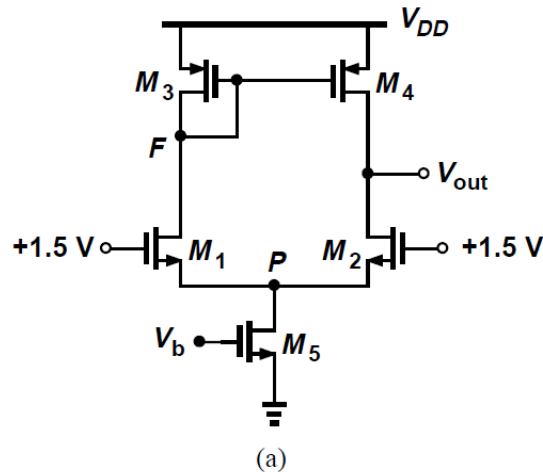


# Large-Signal Analysis



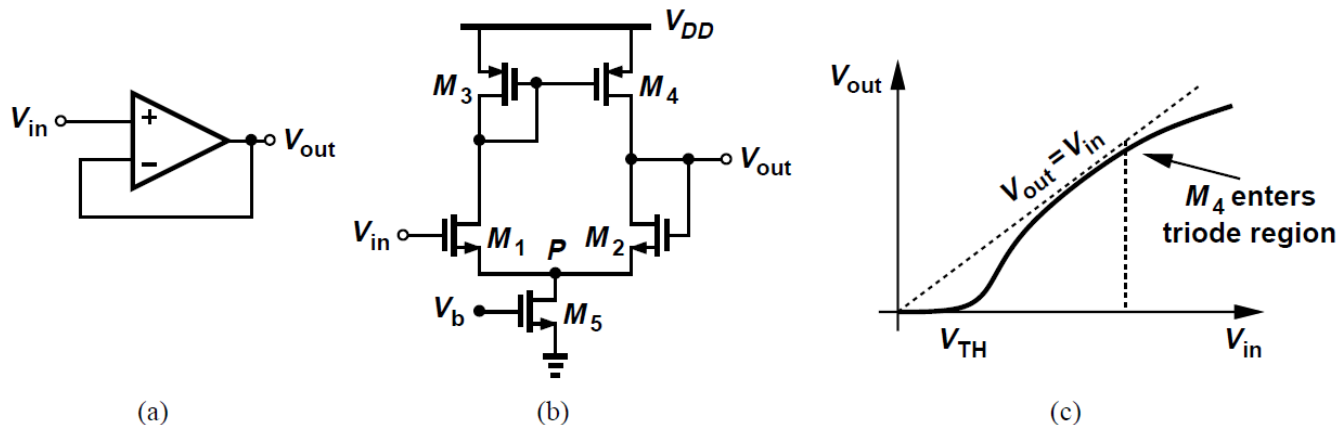
- If  $V_{in1}$  is much more negative than  $V_{in2}$ ,  $V_{out} = 0$ .
- As  $V_{in1}$  approaches  $V_{in2}$ , The output voltage then depends on the difference between  $I_{D4}$  and  $I_{D2}$ . For a small difference between  $V_{in1}$  and  $V_{in2}$ , both  $M_2$  and  $M_4$  are saturated, providing a high gain.
- As  $V_{in1}$  becomes more positive than  $V_{in2}$ , allowing  $V_{out}$  to rise and eventually driving  $M_4$  into the triode region .

# Example



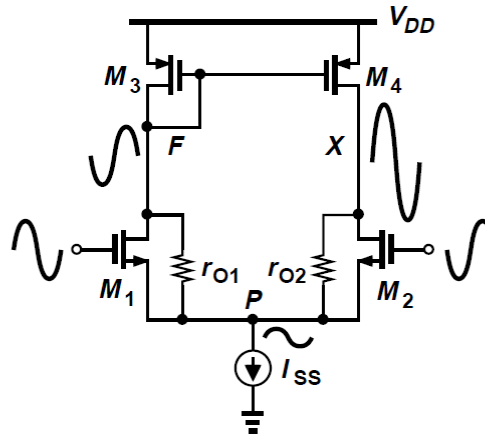
- For  $V_{DD} = 3 \text{ V}$ , symmetry requires that  $V_{out} = V_F$ .
- As  $V_F$  and  $V_{out}$  fall below  $+1.5 \text{ V} - V_{TH}$ ,  $M_1$  and  $M_2$  enter the triode region, but their drain currents are constant if  $M_5$  is saturated.
- Eventually  $M_5$  into the triode region. Thereafter, the bias current of all of the transistors drops, lowering the rate at which  $V_{out}$  decreases.

# Example



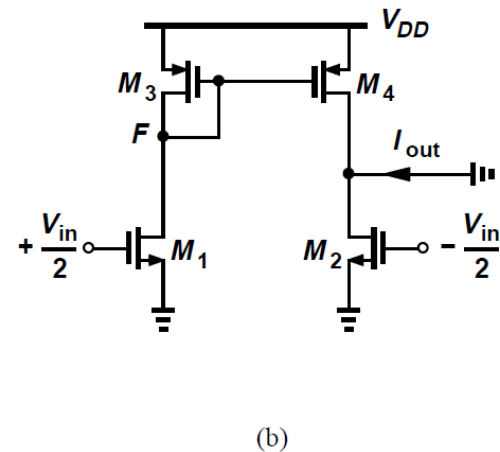
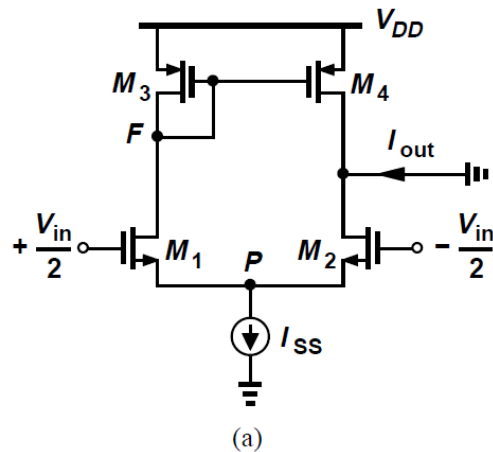
- Sketch the large-signal input-output characteristic of the unity-gain buffer shown in Fig if the op amp is realized as a five-transistor OTA.
- We begin with  $V_{in} = 0$  and note that  $M_1, M_3$ , and  $M_4$  are off.
- As  $V_{in}$  rises,  $V_{out} \approx V_{in}$ . This unity-gain action continues as  $V_{in}$  increases.
- For a sufficiently high  $V_{in}$ :  $M_1$  and  $M_4$  went to triode region.

# Small-Signal Analysis



- With small differential inputs, the voltage swings at nodes  $F$  and  $X$  are vastly different.
- The effects of  $V_F$  and  $V_X$  at node  $P$  (through  $r_{O1}$  and  $r_{O2}$ , respectively) do not cancel each other and this node cannot be considered a virtual ground.

# Approximate Analysis



- Node P can be approximated by a virtual ground.

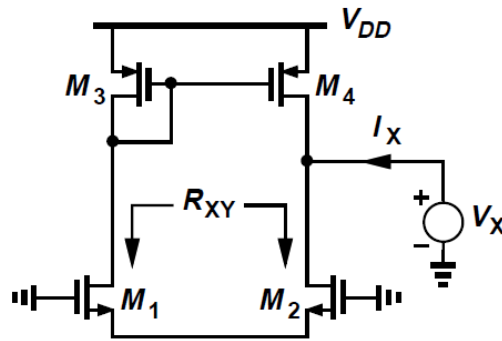
$$I_{D1} = |I_{D3}| = |I_{D4}| = g_{m1,2} V_{in}/2$$

$$I_{D2} = -g_{m1,2} V_{in}/2$$

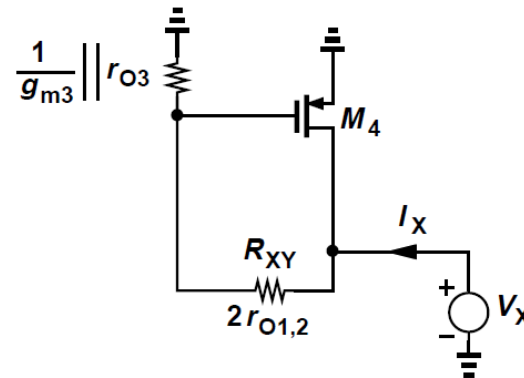
$$I_{out} = -g_{m1,2} V_{in}$$

$$|G_m| = g_{m1,2}$$

# Calculation of Rout



(a)



(b)

- Any current flowing into M1 must flow out of M2, and the role of the two transistors can be represented by a resistor  $R_{XY} = 2r_{O1,2}$
- The current drawn from  $V_X$  by  $R_{XY}$  is mirrored by M3 onto M4 with unity gain.

$$I_X = \frac{V_X}{2r_{O1,2} + \frac{1}{g_{m3}} \parallel r_{O3}} \left[ 1 + \left( \frac{1}{g_{m3}} \parallel r_{O3} \right) g_{m4} \right] + \frac{V_X}{r_{O4}}$$

- For  $2r_{O1,2} \gg (1/g_{m3}) \parallel r_{O3}$

$$R_{out} \approx r_{O2} \parallel r_{O4}$$



# Exact Analysis

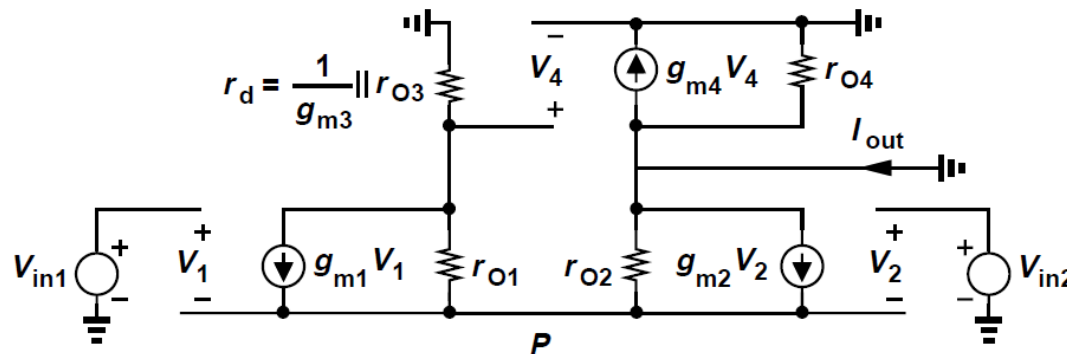


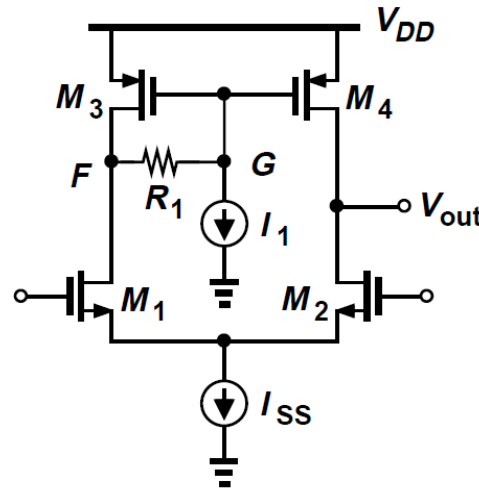
Figure 5.34. Equivalent circuit of five-transistor OTA

$$\begin{aligned}
 G_m R_{out} &= -g_{m1} r_{O1} r_{O4} \frac{2g_{m3} r_{O3} + 1}{(2g_{m3} r_{O3} + 1) r_{O4} + 2r_{O1}(1 + g_{m3} r_{O3}) + r_{O3}} \\
 &= -\frac{g_{m1} r_{O1} r_{O4}}{r_{O1} + r_{O3}} \cdot \frac{2g_{m3} r_{O3} + 1}{2(g_{m3} r_{O3} + 1)}.
 \end{aligned}$$

$$|A_v| = g_{m1}(r_{O1} || r_{O4}) \frac{2g_{m4} r_{O4} + 1}{2(g_{m4} r_{O4} + 1)}$$

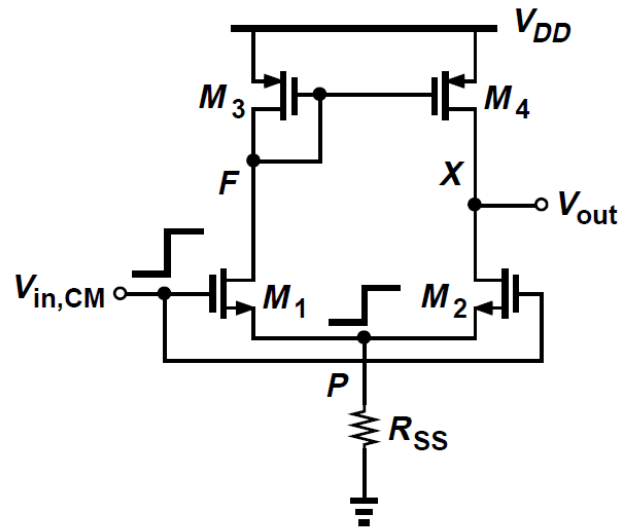
- We can view this result as our approximate solution,  $g_{m1}(r_{O1} || r_{O4})$ , multiplied by a “correction” factor that is less than unity.

# Headroom Issues



- The five-transistor OTA does not easily lend itself to low-voltage operation.
- The value of  $I_1$  must be much less than  $I_{SS}/2$ .
- Insert a resistor in series with the gate and draw a constant current from it.

# Common-Mode Properties

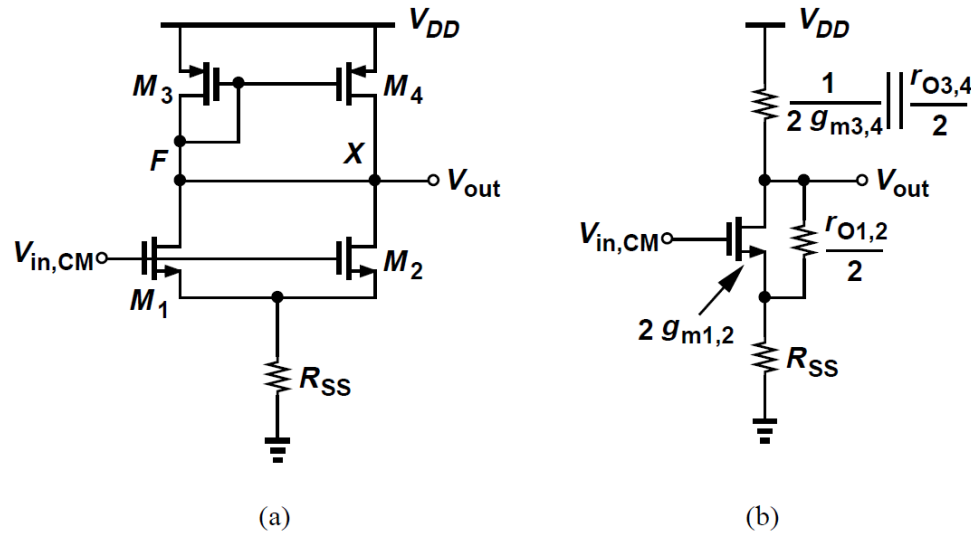


$$A_{CM} = \frac{\Delta V_{out}}{\Delta V_{in,CM}}.$$

$$A_{CM} \approx \frac{-\frac{1}{2g_{m3,4}} \parallel \frac{r_{O3,4}}{2}}{\frac{1}{2g_{m1,2}} + R_{SS}}$$

$$= \frac{-1}{1 + 2g_{m1,2}R_{SS}} \frac{g_{m1,2}}{g_{m3,4}}$$

# CMRR

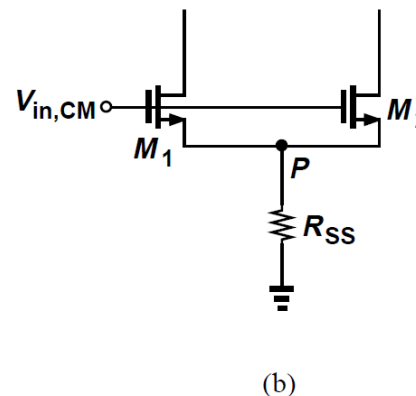
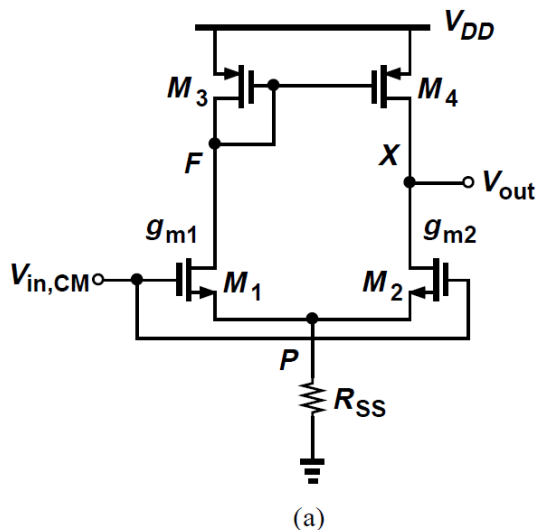


(a) Simplified circuit of Fig. 5.37, (b) equivalent circuit of (a).

$$\begin{aligned}
 CMRR &= \left| \frac{A_{DM}}{A_{CM}} \right| \\
 &= g_{m1,2}(r_{O1,2} \parallel r_{O3,4}) \frac{g_{m3,4}(1 + 2g_{m1,2}R_{SS})}{g_{m1,2}} \\
 &= (1 + 2g_{m1,2}R_{SS})g_{m3,4}(r_{O1,2} \parallel r_{O3,4}).
 \end{aligned}$$

- **Even with perfect symmetry, the output signal is corrupted by input CM variations.**

# Effect of Mismatches

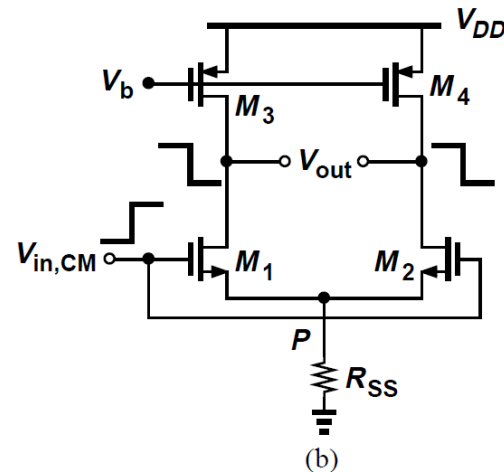
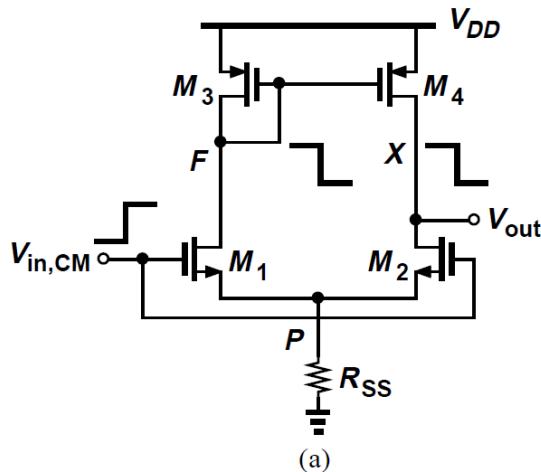


- we consider the case where the input transistors exhibit slightly different transconductances .
- How does  $V_{out}$  depend on  $V_{in,CM}$ ?

$$\frac{\Delta V_{out}}{\Delta V_{in,CM}} \approx \frac{(g_{m1} - g_{m2})r_{O3} - g_{m2}/g_{m3}}{1 + (g_{m1} + g_{m2})R_{SS}}.$$

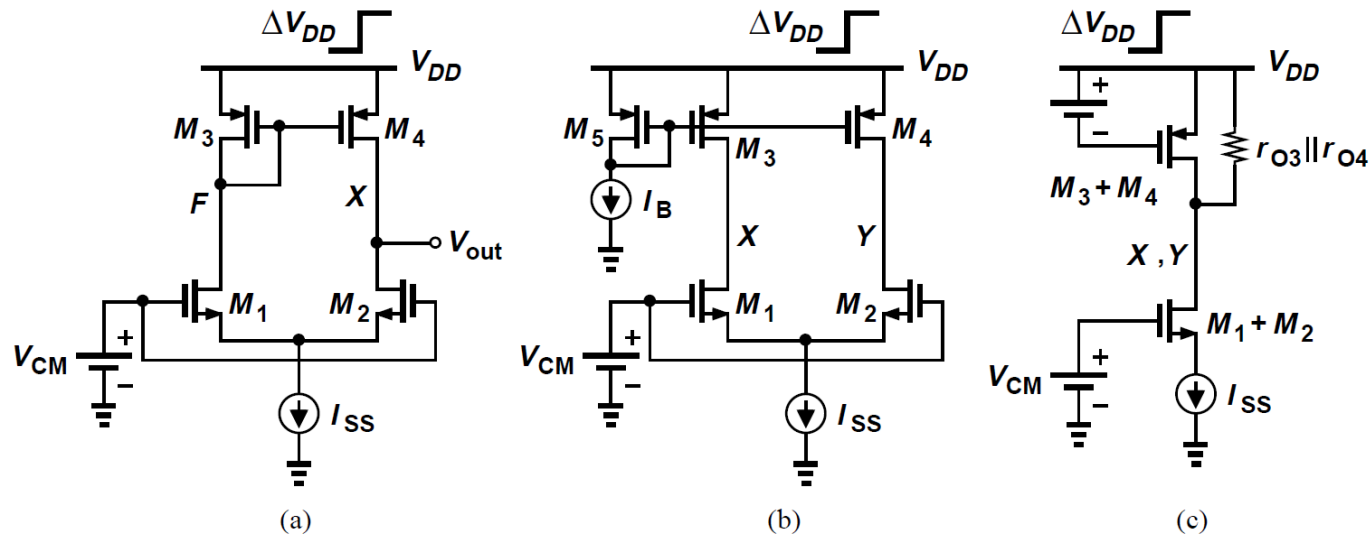
- This result contains the additional term  $(g_{m1} - g_{m2})r_{O3}$

# Other Properties



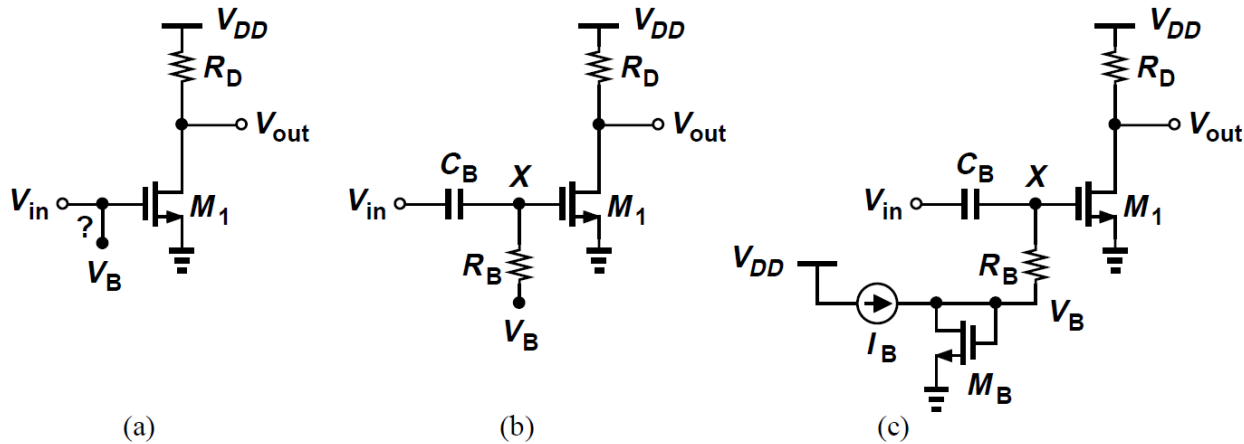
- A finite CMRR even with perfectly matched transistors.
- The supply rejection of this OTA is inferior.
- Change  $V_{DD}$  by a small amount, how much does  $V_F$  change?

# Other Properties



- For (a), The gain from  $V_{DD}$  to  $V_{out}$  is about unity.
- Now consider the fully-differential topology in Fig(b).
- In this case, too, the output voltages change by  $\Delta V_{DD}$  but their difference remains intact.
- This circuit requires common-mode feedback.

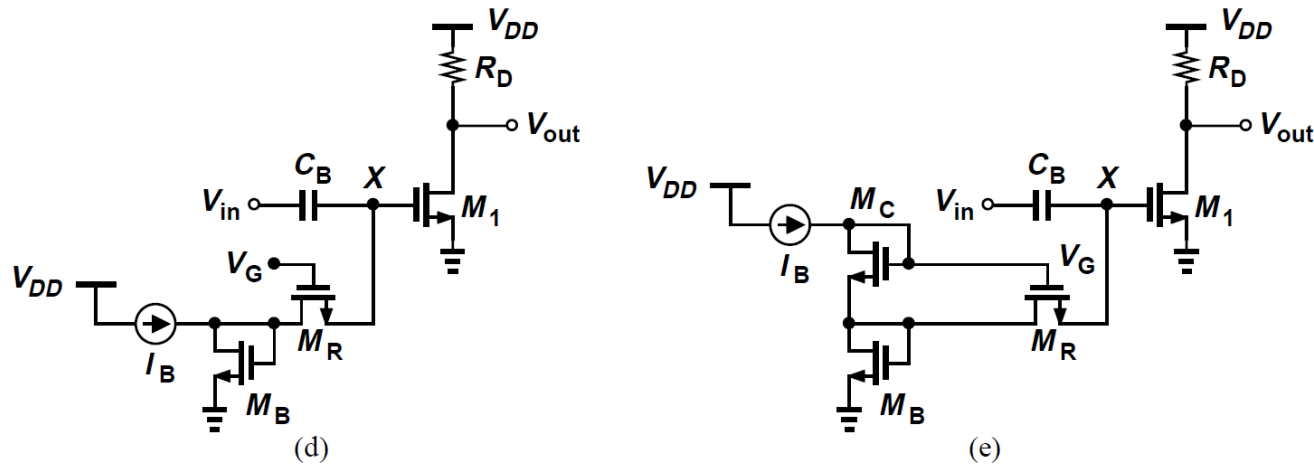
# Biasing Techniques



- **Simple CS Stage**
- **How do we ensure that  $V_B$  does not “fight”  $V_{in}$ ?**
- **Couple  $V_{in}$  capacitively and establish a high impedance for  $V_B$ .**
- **Node  $X$  in Fig (b) must have a dc path to a voltage.**
- **The bias voltage must be generated by a diode-connected device**
- **Typically select  $I_B$  about one-tenth to one-fifth of  $I_{D1}$  so as to minimize the power.**

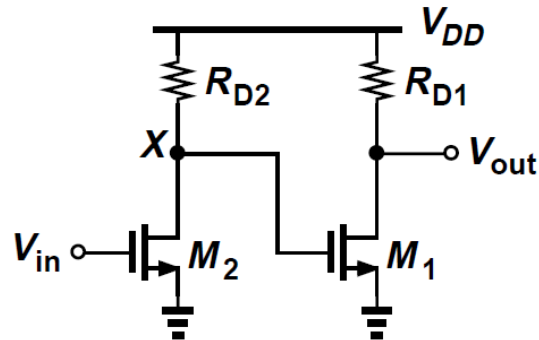


# Biasing Using a MOSFET



- The capacitor and the resistor may occupy a large chip area.
- The capacitor introduces its own parasitics.
- In applications requiring a large RC product, one can replace  $R_B$  with a long, narrow MOSFET.
- But how do we guarantee that  $M_R$  does not turn off?
- The overdrive of  $M_R$  must be well controlled. This difference can be created by means of a diode-connected device.

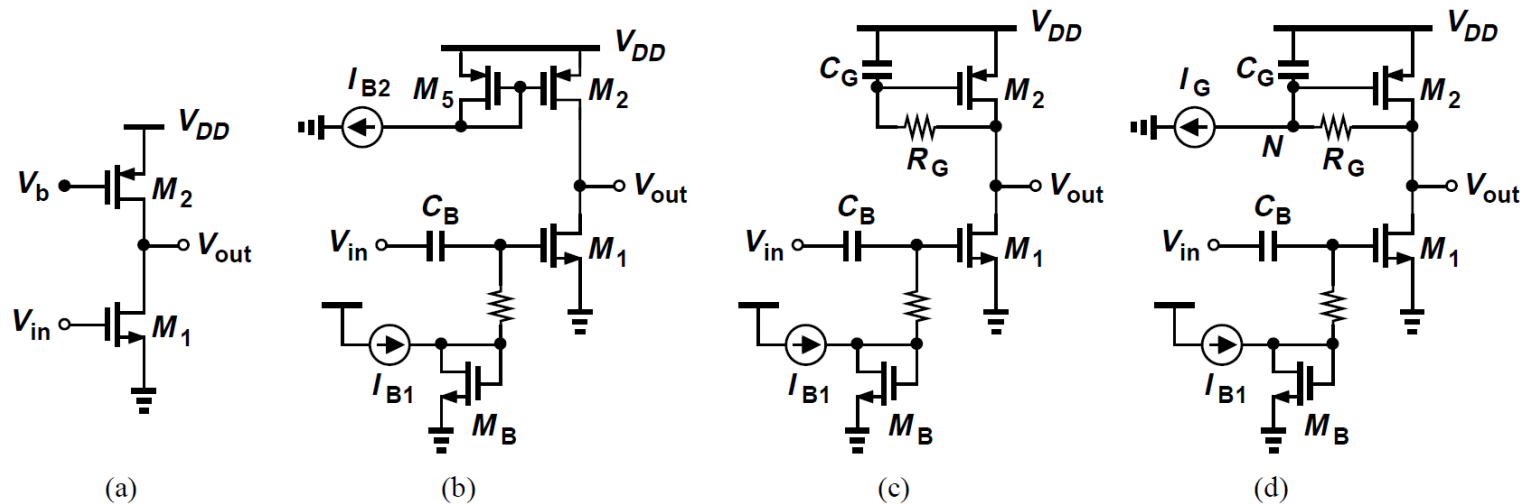
# Direct Coupling



Direct coupling between two stages.

- Possible to remove the input coupling capacitor and provide the bias voltage from the preceding stage?
- The bias conditions of  $M_1$  are influenced by those of  $M_2$ .
- The PVT variations are amplified.
- One can employ direct coupling between two stages if each has a low gain.

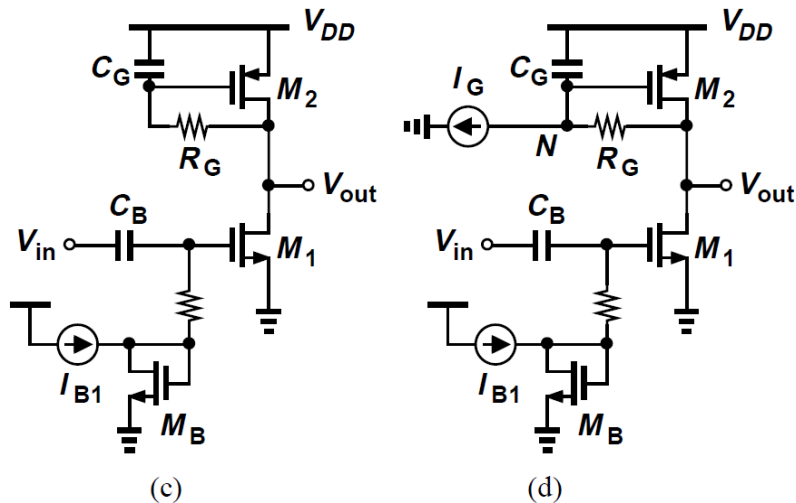
# CS stage with Current-Source Load



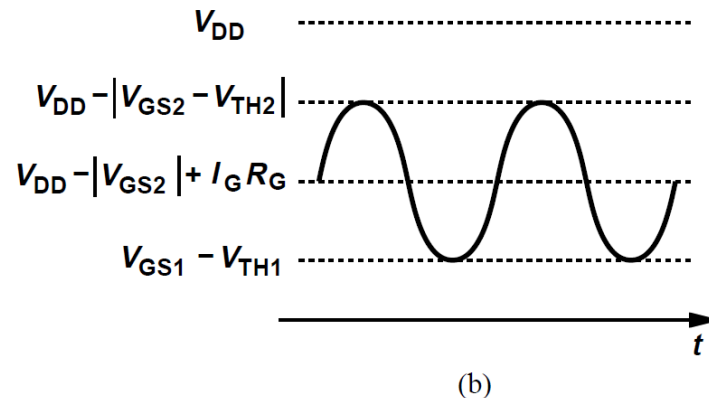
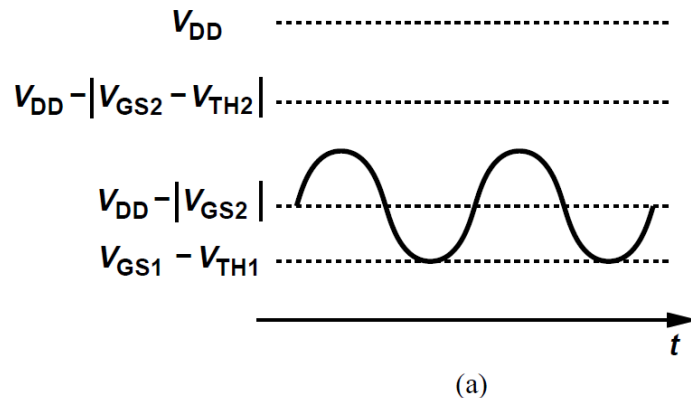
- If the copied currents in Fig (b) are not exactly equal, each transistor wants to impose its own current.
- To resolve this issue, we modify the circuit as shown in Fig (c).

- Select  $A_v = -g_{m1}(r_{O1} || r_{O2} || R_G)$
- We can  $R_G \gg r_{O1} || r_{O2}$  instant current of  $I_G$  from  $R_G$ , so that  $V_{out}$  is higher.

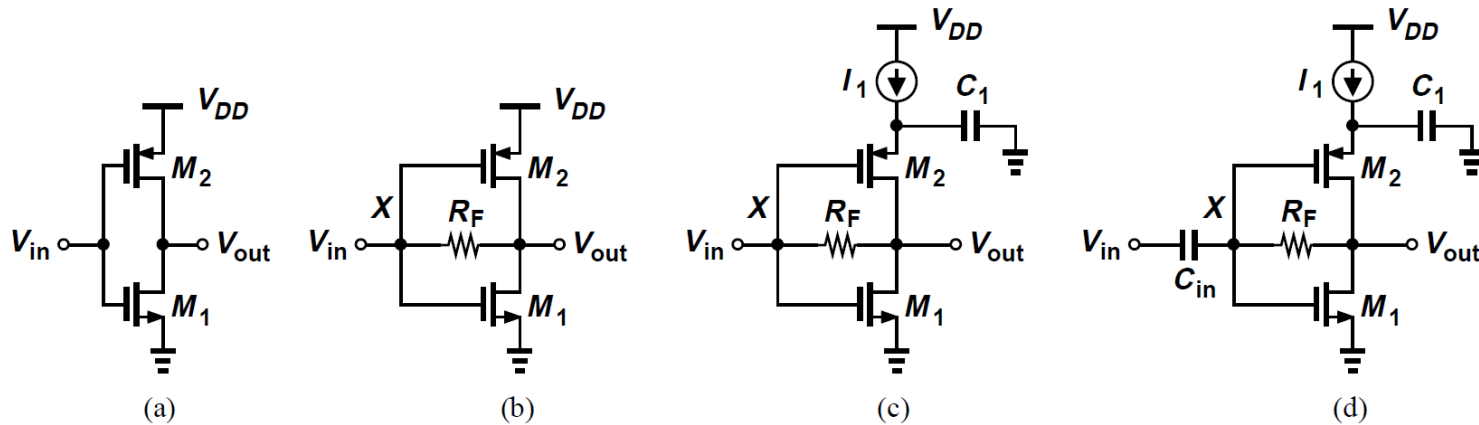
# Example



- Compare the maximum allowable voltage swings.
- In Fig (c), the up-swing cannot reach its maximum.
- In Fig. 5.45(d), on the other hand,  $I_G R_G$  can shift the operating point such that the down-swing and the up-swing are approximately equal.

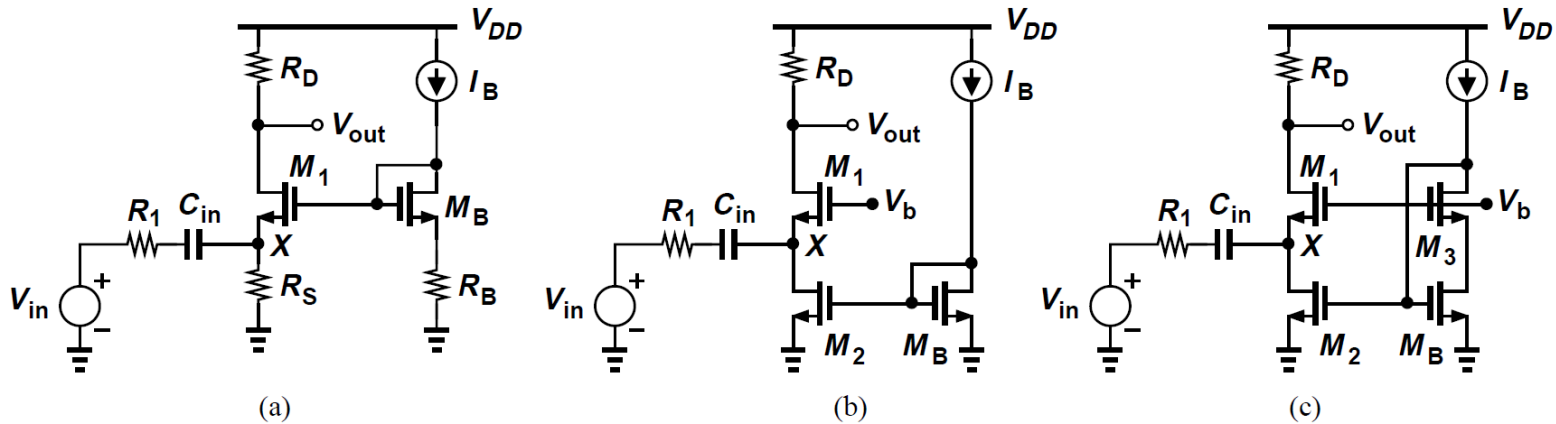


# Complementary CS Stage



- Shown in Fig. (b), each transistor is configured as a diode-connect device and guaranteed to operate in saturation.
- Self-biased topology.
- To define the bias current accurately, we modify the circuit as shown in Fig. (c).
- Since the bias voltage at node  $X$  must track  $V_{out}$ , the input must be capacitively coupled.

# CG Biasing

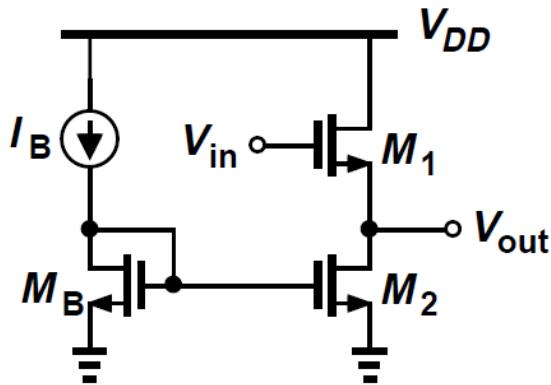


- The circuit of Fig. (a) faces difficulties in low-voltage design.

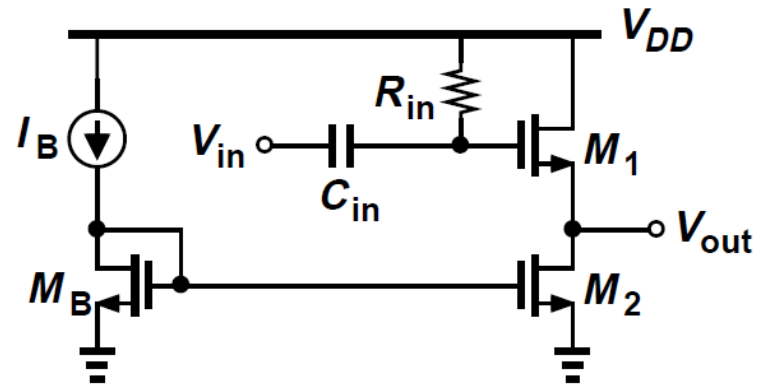
$$\frac{V_X}{V_{in}} = \frac{\frac{1}{g_{m1} + g_{mb1}} || R_S}{\frac{1}{g_{m1} + g_{mb1}} || R_S + R_1}$$

- $R_S$  may reach or even exceed  $R_D$ .
- Replace  $R_S$  with a current source.

# Source Follower Biasing



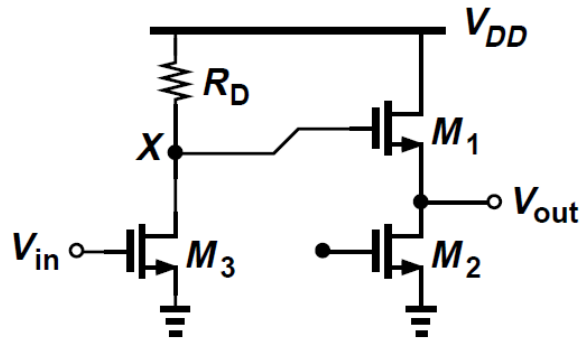
(a)



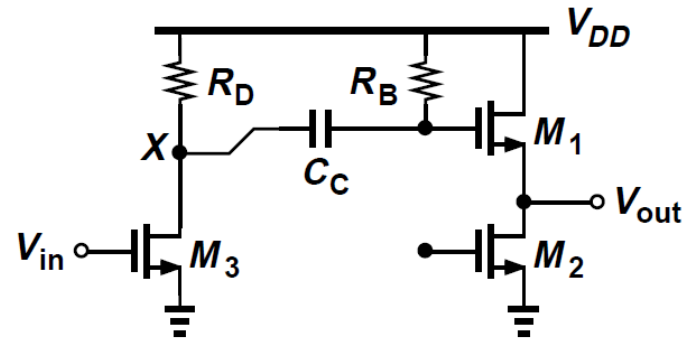
(b)

- In applications where the input dc voltage may vary considerably, capacitive coupling can be used.
- Study the performance with and without capacitive coupling between the two stages.

# Use of SF



(a)

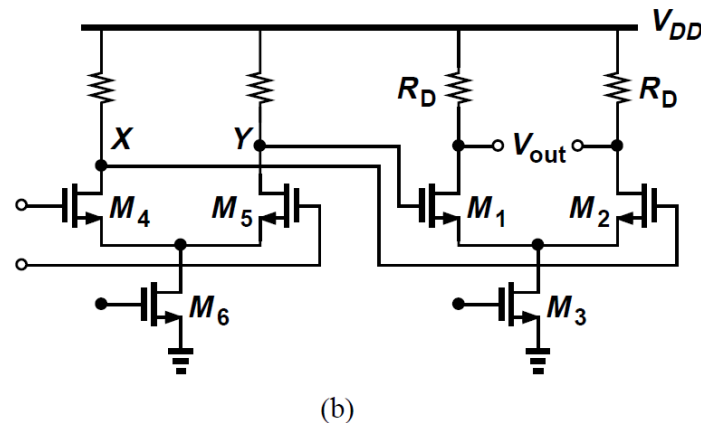
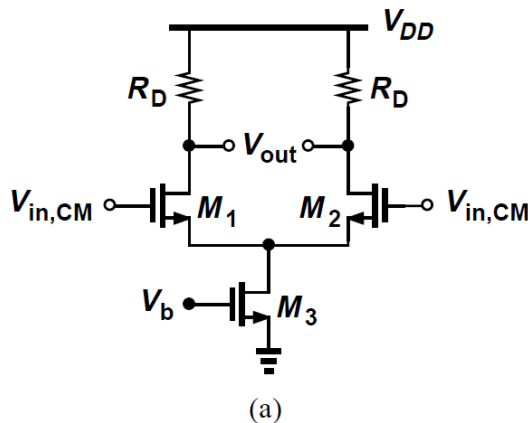


(b)

- In Fig (a), the minimum drain voltage of M3 is given by  $V_{GS1} + V_{DS2min}$ , leaving little for the allowable voltage drop across  $R_D$ .
- Fig (b), on the other hand, the first stage's gain can be independently maximized.



# Differential Pair Biasing



- Since the bias currents of  $M_1$  and  $M_2$  in Fig. (a) are relatively insensitive to their gate voltages, we can directly connect their gates to the preceding stage.
- If the bias value of  $V_X$  and  $V_Y$  is chosen equal to two overdrives above ground, then it is an excessively low common-mode level for the second stage.
- May resort to capacitive coupling in some cases.