Network Analysis & Systems

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UE18EC201: Network Analysis & Systems

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Network Analysis and Synthesis

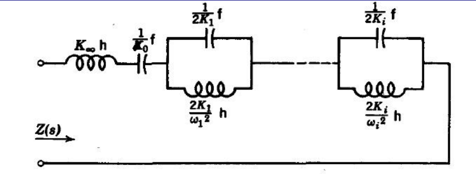
Part V: Network Synthesis

RC Impedance Functions (1)

Recall: The PFE of an LC-immittance function

$$
\begin{aligned}
H(s) &= \frac{K(s^2+\omega_1^2)(s^2+\omega_3^2)\cdots(s^2+\omega_i^2)\cdots}{s(s^2+\omega_2^2)(s^2+\omega_4^2)\cdots(s^2+\omega_j^2)\cdots} \\
&= \frac{k_0}{s} + \frac{2k_2 s}{s^2+\omega_2^2} + \frac{2k_4 s}{s^2+\omega_4^2} + \cdots + k_\infty s
\end{aligned}
$$

Foster Form I realisation:



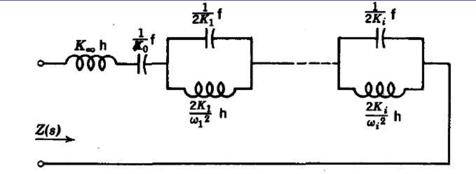
RC Impedance Functions (1)

Recall: The PFE of an *LC*-immittance function

$$H(s) = \frac{K(s^2+\omega_1^2)(s^2+\omega_3^2)\cdots(s^2+\omega_i^2)\cdots}{s(s^2+\omega_2^2)(s^2+\omega_4^2)\cdots(s^2+\omega_j^2)\cdots}$$

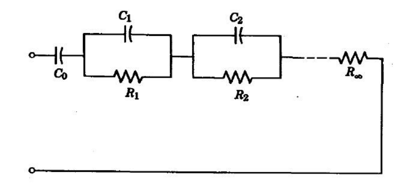
$$= \frac{k_0}{s} + \frac{2k_2 s}{s^2+\omega_2^2} + \frac{2k_4 s}{s^2+\omega_4^2} + \cdots + k_\infty s$$

Foster Form I realisation:



- Replace all inductances by resistances

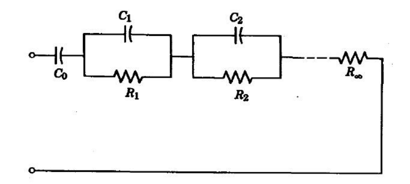
RC Impedance Functions (2)



Therefore,

$$Z(s) =$$

RC Impedance Functions (2)

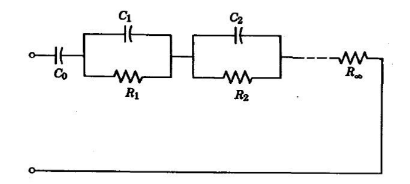


Therefore,

$$Z(s) = \frac{K_0}{s} + K_\infty + \frac{K_1}{s+\sigma_1} + \frac{K_1}{s+\sigma_2} + \cdots$$

where $C_0 = 1/K_0$, $R_\infty = K_\infty$, $C_1 = 1/K_1$, $R_1 = K_1/\sigma_1$, and so on.

RC Impedance Functions (2)



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where $C_0 = 1/K_0$, $R_\infty = K_\infty$, $C_1 = 1/K_1$, $R_1 = K_1/\sigma_1$, and so on.

- Clearly, the poles of an RC-impedance are on the negative real axis.
- The residues are real and positive.

RC Impedance Functions (3)

- From the Foster II Form for an *RC*-admittance function, it follows that the poles are also on the negative real axis.
- Therefore, the poles and zeros of an *RC*-immittance function are on the negative real-axis.

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- Moreover,

$$\frac{dZ(\sigma)}{d\sigma} =$$

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$$\frac{dZ(\sigma)}{d\sigma} = -\frac{K_0}{\sigma^2} - \frac{K_1}{(\sigma+\sigma_1)^2} - \frac{K_1}{(\sigma+\sigma_2)^2} - \cdots \leq 0$$

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- At $\sigma = 0$, the capacitor C_0, if it exists, is a o.c. Therefore, $Z(s)$ has a pole at $\sigma = 0$. If C_0 does not exist, then

$$Z(0) = R_1 + R_2 + \cdots + R_\infty$$

RC Impedance Functions (4)

- At $\sigma = \infty$, all capacitors are s.c. If R_∞ exists, then $Z(\infty) = R_\infty$; otherwise, $Z(\infty) = 0$.
- To summarise,

$$Z(0) = \begin{cases} \infty, & C_0 \text{ present} \\ \sum R_i, & C_0 \text{ missing} \end{cases}$$

$$Z(\infty) = \begin{cases} 0, & R_\infty \text{ missing} \\ R_\infty, & R_\infty \text{ present} \end{cases}$$

Therefore, $Z(0) \geq Z(\infty)$.

RC Impedance Functions (5)

$$Z(s) = \frac{(s+\sigma_2)(s+\sigma_4)}{(s+\sigma_1)(s+\sigma_3)}$$

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- Also, the poles and zeros interlace on the negative real axis.

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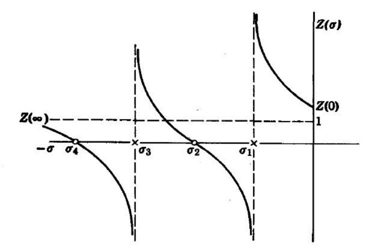

RC Impedance Functions (6)

To summarise:

- The poles and zeros lie on the negative real axis.
- The poles and zeros interlace.
- The singularity nearest to, or at, the origin must be a pole.
- The singularity nearest to, or at, $-\infty$ must be a zero.
- The residues of the poles must be real and positive.

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- $$Z(s) = \frac{(s+1)(s+2)}{s(s+3)}$$

RC Impedance Functions (7)

Consider the network function

$$H(s) =$$

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$$H(s) = \frac{K_0}{s} + K_\infty + \frac{K_1}{s+\sigma_1} + \frac{K_1}{s+\sigma_2} + \cdots$$

If $H(s)$ is an admittance, then

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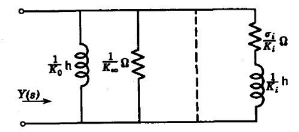

- Thus, an RC-impedance $Z_{RC}(s)$ can also be realised as an RL-admittance $Y_{RL}(s)$.
- Clearly, all the properties of RL-admittances are the same as that of RC-impedances

RC Impedance or *RL* Admittance Functions (1)

- Foster one-ports are synthesised by PFE.
- The first step is to remove $\min \operatorname{Re} Z(j\omega) = Z(\infty)$.
- For a strictly proper network function, $Z(\infty) = 0$.
- If the degree of the numerator is equal to the degree of the denominator, $Z(\infty)$ is obtained by dividing the denominator into the numerator. The quotient is $Z(\infty)$.

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- For a strictly proper network function, $Z(\infty) = 0$.
- If the degree of the numerator is equal to the degree of the denominator, $Z(\infty)$ is obtained by dividing the denominator into the numerator. The quotient is $Z(\infty)$.
- The same remarks hold for *RL* admittance functions $Y(j\omega)$.

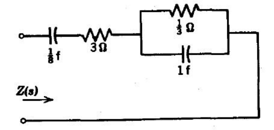
RC Impedance or RL Admittance Functions (2)

$$H(s) = \frac{3(s+2)(s+4)}{s(s+3)} =$$

RC Impedance or *RL* Admittance Functions (2)

$$H(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{8}{s} + \frac{1}{s+3} + 3$$

As an *RC* impedance (Foster Form I):



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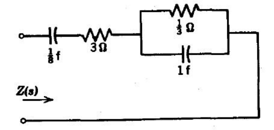
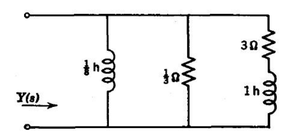


As an *RL* admittance (Foster Form II):



RC Impedance or *RL* Admittance Functions (3)

- Alternatively, once $\min \operatorname{Re} Z(j\omega) = Z(\infty)$ is removed, a zero at $s = \infty$ is created for the remainder. Upon inversion, it has a pole at $s = \infty$, which can be removed. This process can be repeated.
- This is merely CFE.
- The quotients represent the elements of a ladder network: Cauer Form I

RC Impedance or *RL* Admittance Functions (4)

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$$
\begin{array}{l}
s^2 + 3s\,\overline{)\,3s^2 + 18s + 24}\,(3 \\
\qquad\quad 3s^2 + 9s \\
\qquad\quad \overline{\qquad\quad 9s + 24\,)\,s^2 + 3s\,(\tfrac{1}{9}s} \\
\qquad\qquad\qquad\qquad\quad s^2 + \tfrac{8}{3}s \\
\qquad\qquad\qquad\qquad\quad \overline{\qquad \tfrac{1}{3}s\,)\,9s + 24\,(27} \\
\qquad\qquad\qquad\qquad\qquad\qquad 9s \\
\qquad\qquad\qquad\qquad\qquad\qquad \overline{\qquad 24\,)\,\tfrac{1}{3}s\,(s/72} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \tfrac{1}{3}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad =
\end{array}
$$

RC Impedance or *RL* Admittance Functions (5)

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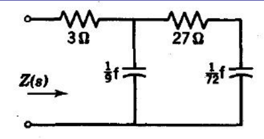

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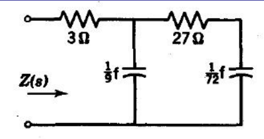
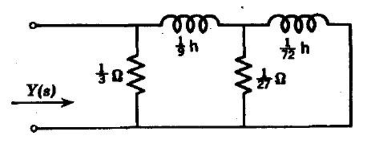


$H(s)$ as an admittance (Cauer Form I):



RC Admittance and RL Impedance Functions (1)

Consider the network function

$$H(s) = \frac{K_0}{s} + K_\infty + \frac{K_1 s}{s+\sigma_1} + \frac{K_1 s}{s+\sigma_2} + \cdots$$

- Observe that the terms (third term onwards) is multiplied by a factor s. Otherwise, they cannot be realised as RL-impedance or RC-admittance.

RC Admittance and *RL* Impedance Functions (1)

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- Observe that the terms (third term onwards) is multiplied by a factor s. Otherwise, they cannot be realised as *RL*-impedance or *RC*-admittance.

The properties of *RC* Admittance and *RL* Impedance Functions are:

- The poles and zeros lie on the negative real axis.
- The poles and zeros interlace.
- The singularity nearest to, or at, the origin must be a zero.
- The singularity nearest to, or at, $-\infty$ must be a pole.
- The residues of the poles must be real and negative.

RC Admittance and *RL* Impedance Functions (2)

Since the residue is negative, the following artifice is used:

$$\frac{H(s)}{s} = \frac{K_0}{s} + K_\infty + \frac{K_1}{s+\sigma_1} + \frac{K_1}{s+\sigma_2} + \cdots$$

where K_0, K_∞, K_1, ..., ≥ 0.

- That is, the properties of $Z_{RL}(s)/s$ are the same as that of an *RC* impedance.

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$$H(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)} =$$

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$$H(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)} = 2 - \frac{1/2}{s+2} - \frac{15/2}{s+6}$$

In contrast, a PFE of

$$\frac{H(s)}{s} =$$

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In contrast, a PFE of

$$\frac{H(s)}{s} = \frac{1/2}{s} + \frac{1/4}{s+2} + \frac{5/4}{s+6}$$

RC Admittance and *RL* Impedance Functions (3)

Therefore, $H(s) = \frac{1}{2} + \frac{s/4}{s+2} + \frac{5s/4}{s+6}$.
$H(s)$ as an admittance (Foster Form I):

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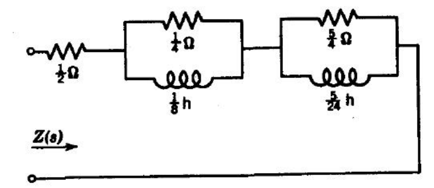

$H(s)$ as an impedance (Foster Form II):

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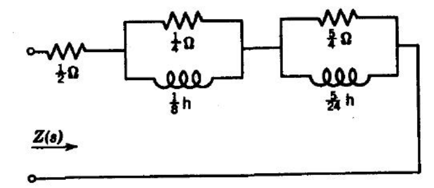

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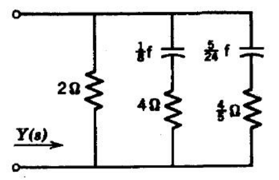

RC Admittance and *RL* Impedance Functions (4)

- To synthesise a Cauer one-port, one must remove $Z(0) = \min \operatorname{Re} Z(j\omega)$. The remainder has a zero at $s = 0$.
- Therefore, the inverse has a pole at $s = 0$, which is then removed.
- The process is repeated.

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- Therefore, the inverse has a pole at $s = 0$, which is then removed.
- The process is repeated.
- This is a CFE with the polynomials arranged in ascending order.

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$$
\begin{array}{l}
12 + 8s + s^2 \overline{)6 + 8s + 2s^2} \left(\tfrac{1}{2} \right. \\
\qquad\quad 6 + 4s + \tfrac{1}{2}s^2 \\
\qquad\quad \overline{\qquad\quad 4s + \tfrac{3}{2}s^2 \overline{)12 + 8s + s^2} \left(3/s \right.} \\
\qquad\qquad\qquad\qquad 12 + \tfrac{9}{2}s \\
\qquad\qquad\qquad\qquad \overline{\quad \tfrac{7}{2}s + s^2 \overline{)4s + \tfrac{3}{2}s^2} \left(\tfrac{8}{7} \right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad 4s + \tfrac{8}{7}s^2 \\
\qquad\qquad\qquad\qquad\qquad\qquad \overline{\qquad \tfrac{5}{14}s^2 \overline{)\tfrac{7}{2}s + s^2} \left(49/5s \right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \tfrac{7}{2}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \overline{\qquad s^2 \overline{)\tfrac{5}{14}s^2} \left(\tfrac{5}{14} \right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{\underline{\tfrac{5}{14}s^2}}
\end{array}
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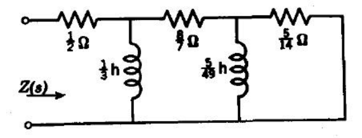

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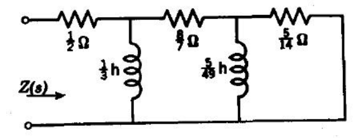

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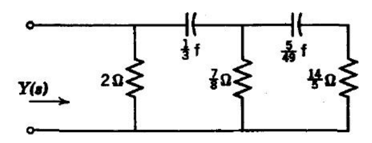

Special *RLC* Functions (1)

Consider

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- This function is neither LC, RC nor RL.

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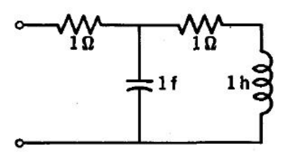
$$
\begin{array}{l}
s^2 + s + 1\,\overline{)\,s^2 + 2s + 2}\,(1 \leftarrow Z \\
\qquad\qquad\;\; \underline{s^2 + \;\; s + 1} \\
\qquad\qquad\qquad s + 1\,\overline{)\,s^2 + s + 1}\,(s \leftarrow Y \\
\qquad\qquad\qquad\qquad\;\; \underline{s^2 + s} \\
\qquad\qquad\qquad\qquad\qquad\;\; 1\,\overline{)\,s + 1}\,(s + 1 \leftarrow Z \\
\qquad\qquad\qquad\qquad\qquad\quad\;\; \underline{\underline{s + 1}}
\end{array}
$$

Special *RLC* Functions (2)

Thus, a realisation of

$$Z(s) = \frac{s^2 + 2s + 2}{s^2 + s + 1}$$

is



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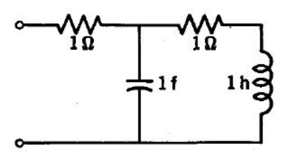

Consider

$$Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)}$$

- The poles and zeros do not interlace.

Special *RLC* Functions (3)

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cannot be used. However,

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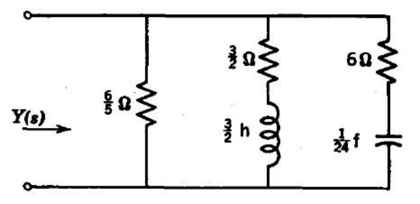


Special *RLC* Functions (4)

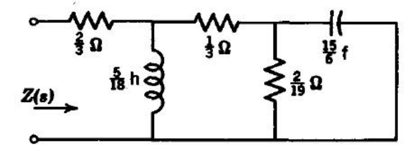
$$Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)}$$

- Sometimes, a mixed CFE can help:

$$
\begin{array}{l}
6 + 5s + s^2 \overline{)\,4 + 5s + s^2}\,\left(\tfrac{2}{3}\right. \\
\qquad\qquad\quad 4 + \tfrac{10}{3}s + \tfrac{2}{3}s^2 \\
\qquad\qquad\quad \overline{\qquad\quad \tfrac{5}{3}s + \tfrac{1}{3}s^2)\, 6 + 5s + s^2 \left(18/5s\right.} \\
\qquad\qquad\qquad\qquad\qquad\quad 6 + \tfrac{6}{5}s \\
\qquad\qquad\qquad\qquad\qquad\quad \overline{\quad \tfrac{19}{5}s + s^2)\, \tfrac{1}{3}s^2 + \tfrac{5}{3}s \left(\tfrac{1}{3}\right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \tfrac{1}{3}s^2 + \tfrac{19}{15}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \overline{\qquad \tfrac{6}{15}s)\, \tfrac{19}{5}s + s^2 \left(\tfrac{19}{2}\right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \tfrac{19}{5}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \overline{\qquad\quad s^2)\, \tfrac{6}{15}s \left(6/15s\right.} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{\underline{\tfrac{6}{15}s}}
\end{array}
$$

Special *RLC* Functions (5)

- Observe a reversal of the order of polynomials involved.
- Realisation:



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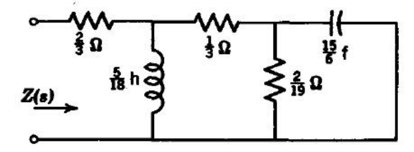

- Moral: For a PR function, if it is not synthesisable using only two kinds of elements, still try a PFE or a CFE. It may still work out with three kinds of elements.
- Appears (with the current level of knowledge that has been gathered) to be more of an art than science.