Nework Analysis & Systems

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UE18EC201: Network Analysis & Systems

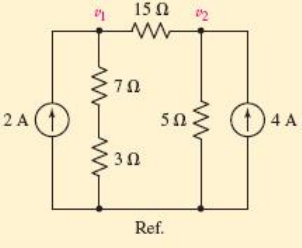
Network Analysis and Synthesis

Unit I: Basic Analysis — Nodal Analysis

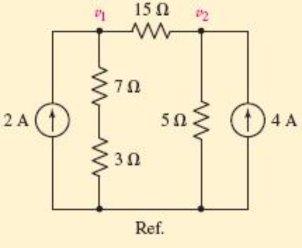
Nodal Analysis (1)

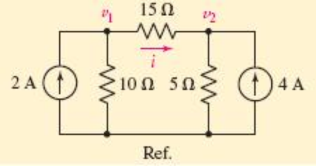
1. Identify and count the number of nodes: n.
2. Designate a reference or datum node. (Choose that node with the greatest number of branches connected to it.)
3. Label the remaining nodal voltages: $n - 1$.
4. Write KCL for each of these non-datum nodes.
5. Express any additional unknowns in terms of appropriate nodal voltages.
6. Arrange in the form of a matrix equation and solve for the nodal voltages.

Nodal Analysis: Example (2)



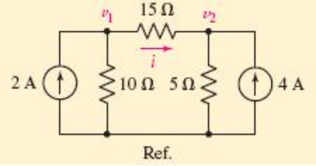
Nodal Analysis: Example (2)





Source: Hayt, Kemmerly and Durbin, 2012.

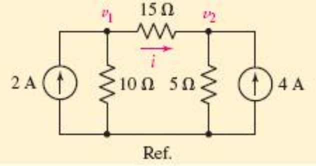
Nodal Analysis: Example (3)



Writing KCL equations, one gets

Node 1:

Nodal Analysis: Example (3)

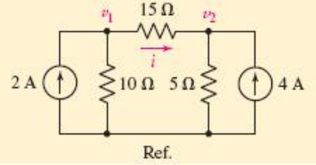


Writing KCL equations, one gets

$$\text{Node 1:} \quad \frac{v_1}{10} + \frac{v_1 - v_2}{15} - 2 = 0$$

Node 2:

Nodal Analysis: Example (3)

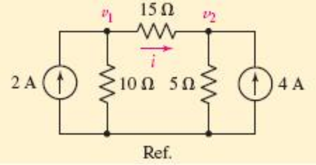


Writing KCL equations, one gets

$$\text{Node 1:} \quad \frac{v_1}{10} + \frac{v_1 - v_2}{15} - 2 = 0$$
$$\text{Node 2:} \quad \frac{v_2}{5} + \frac{v_2 - v_1}{15} - 4 = 0$$

After rearranging,

Nodal Analysis: Example (3)



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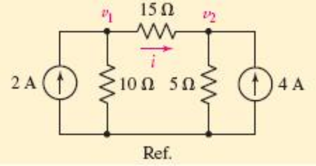
After rearranging,

$$\begin{aligned} 5v_1 - 2v_2 &= 60 \\ -v_1 + 4v_2 &= 60 \end{aligned}$$

Solving,

Nodal Analysis: Example (3)



Writing KCL equations, one gets

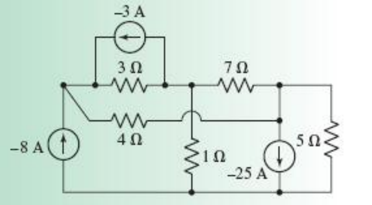
$$\text{Node 1:} \quad \frac{v_1}{10} + \frac{v_1 - v_2}{15} - 2 = 0$$
$$\text{Node 2:} \quad \frac{v_2}{5} + \frac{v_2 - v_1}{15} - 4 = 0$$

After rearranging,

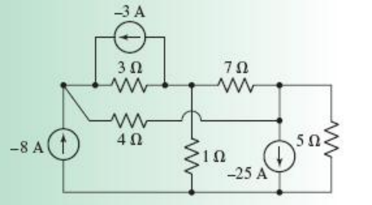
$$\begin{aligned} 5v_1 - 2v_2 &= 60 \\ -v_1 + 4v_2 &= 60 \end{aligned}$$

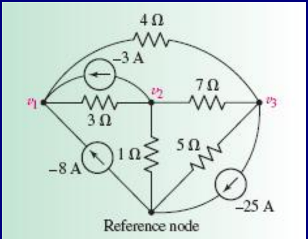
Solving, $v_1 = 20$ V and $v_2 = 20$ V, implying $i = 0$.

Nodal Analysis: Example (4)



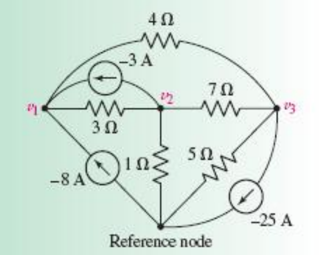
Nodal Analysis: Example (4)





Source: Hayt, Kemmerly and Durbin, 2012.

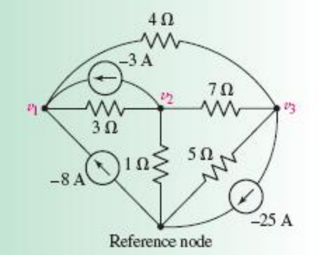
Nodal Analysis: Example (5)



Writing KCL equations, one gets

Node 1:

Nodal Analysis: Example (5)

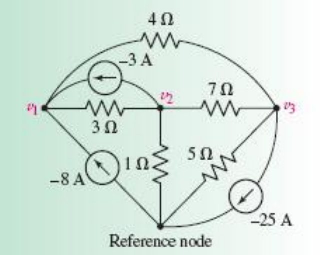


Writing KCL equations, one gets

Node 1: $\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$

Node 2:

Nodal Analysis: Example (5)



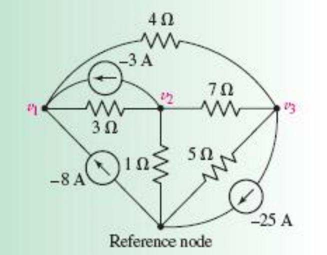
Writing KCL equations, one gets

Node 1: $\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$

Node 2: $\frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_2 - v_3}{7} - 3 = 0$

Node 3:

Nodal Analysis: Example (5)



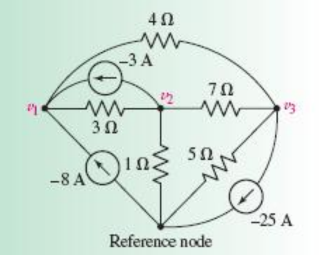
Writing KCL equations, one gets

Node 1: $\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$

Node 2: $\frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_2 - v_3}{7} - 3 = 0$

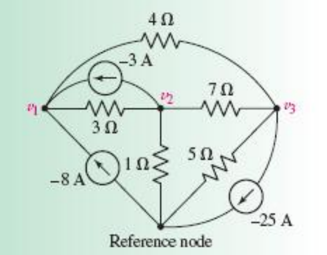
Node 3: $\frac{v_3 - v_1}{4} + \frac{v_3 - v_2}{7} + \frac{v_3}{5} - 25 = 0$

Nodal Analysis: Example (6)



After rearranging,

Nodal Analysis: Example (6)



After rearranging,

$$\begin{aligned} 7v_1 - 4v_2 - 3v_3 &= -132 \\ -7v_1 + 31v_2 - 3v_3 &= 63 \\ -35v_1 - 20v_2 + 83v_3 &= 3500 \end{aligned}$$

Cramer's Rule (1)

Let

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

Or, more compactly,

$$Ax = b$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Cramer's Rule (2)

The solutions are

$$x_1 = \frac{D_1}{\Delta}, x_2 = \frac{D_2}{\Delta}, x_3 = \frac{D_3}{\Delta}$$

where

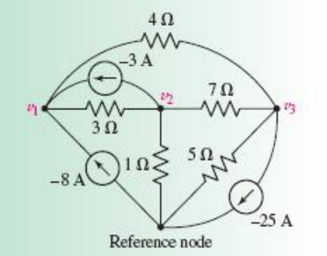
$$\Delta = \det A$$

and

$$D_1 = \det A_1, D_2 = \det A_2, D_3 = \det A_3$$

where A_i is formed from A after replacing its ith column with b.

Nodal Analysis: Example (7)

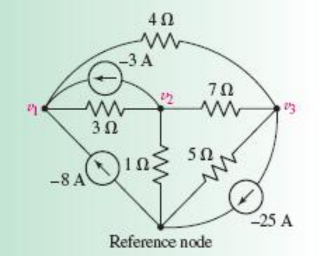


In matrix form:

$$A = \begin{pmatrix} 7 & -4 & -3 \\ -7 & 31 & -3 \\ -35 & -20 & 83 \end{pmatrix}, x = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad b = \begin{pmatrix} -132 \\ 63 \\ 3500 \end{pmatrix}$$

Solving,

Nodal Analysis: Example (7)



In matrix form:

$$A = \begin{pmatrix} 7 & -4 & -3 \\ -7 & 31 & -3 \\ -35 & -20 & 83 \end{pmatrix}, x = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad b = \begin{pmatrix} -132 \\ 63 \\ 3500 \end{pmatrix}$$

Solving, $v_1 = 5.4135$ V, $v_2 = 7.7368$ V, and $v_3 = 46.3158$ V.

Examples (1)

11. For the circuit of Fig. 4.37, determine the value of the voltage labeled v_1 and the current labeled i_1.

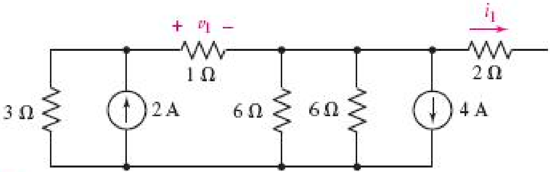


FIGURE 4.37

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

13. Using the bottom node as reference, determine the voltage across the 5 Ω resistor in the circuit of Fig. 4.39, and calculate the power dissipated by the 7 Ω resistor.

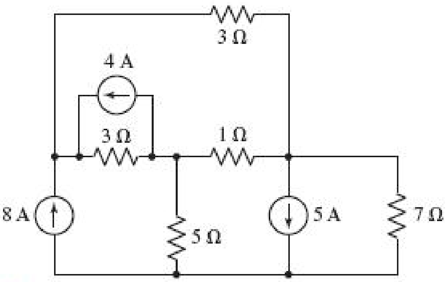
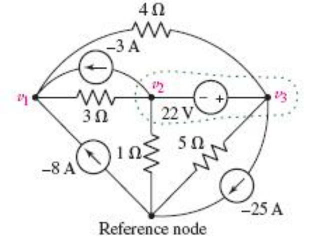


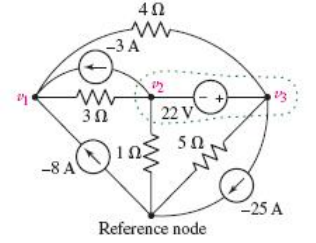
FIGURE 4.39

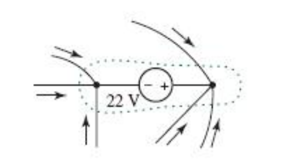
Source: Hayt, Kemmerly and Durbin, 2012.

Nodal Analysis: Supernode (8)



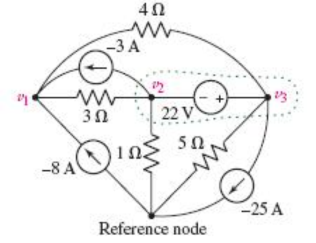
Nodal Analysis: Supernode (8)





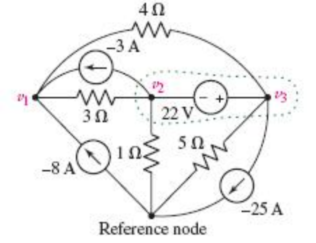
Source: Hayt, Kemmerly and Durbin, 2012.

Nodal Analysis: Supernode (9)



Node 1:

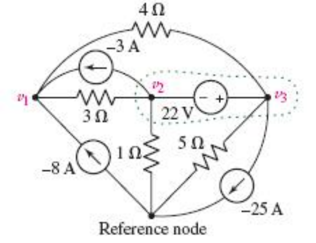
Nodal Analysis: Supernode (9)



Node 1: $\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$

Node 2:

Nodal Analysis: Supernode (9)

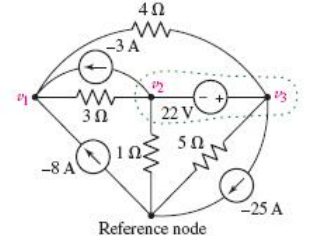


Node 1: $$\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$$

Node 2: $$\frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} - 28 = 0$$

There are only 2 equations in 3 unknowns.

Nodal Analysis: Supernode (9)



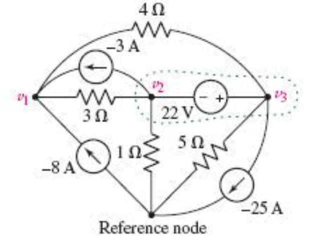
$$\text{Node 1:} \quad \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} + 8 + 3 = 0$$

$$\text{Node 2:} \quad \frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} - 28 = 0$$

There are only 2 equations in 3 unknowns. However, additionally,

$$v_2 - v_3 + 22 = 0$$

Nodal Analysis: Supernode (10)



Thus,

$$\begin{pmatrix} 7 & -4 & -3 \\ -35 & 80 & 27 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -132 \\ 1680 \\ -22 \end{pmatrix}$$

implying $v_1 = 1.0714$ V, $v_2 = 10.5$ V, $v_3 = 32.5$ V.

Nodal Analysis: Supernode (12)

1. Identify and count the number of nodes: n.
2. Designate a reference or datum node. (Choose that node with the greatest number of branches connected to it.)
3. Label the remaining nodal voltages: $n-1$.
4. If the circuit contains voltage sources, form a supernode about each one.
5. Write KCL for each of these non-datum nodes, and for each supernode that does not contain the datum node.
6. Relate the voltage across each voltage source to nodal voltages.
7. Express any additional unknowns in terms of appropriate nodal voltages.
8. Arrange in the form of a matrix equation and solve for the nodal voltages.

Examples (1)

19. For the circuit shown in Fig. 4.45, determine a numerical value for the voltage labeled v_1.

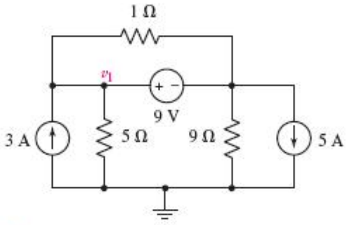


FIGURE 4.45

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

21. Employing supernode/nodal analysis techniques as appropriate, determine the power dissipated by the 1 Ω resistor in the circuit of Fig. 4.47.

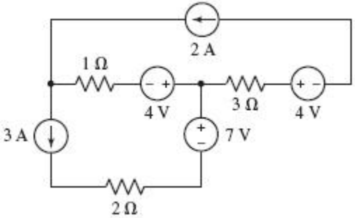


FIGURE 4.47

Source: Hayt, Kemmerly and Durbin, 2012.

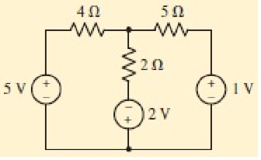
Network Analysis and Synthesis

Unit I: Basic Analysis — Mesh Analysis

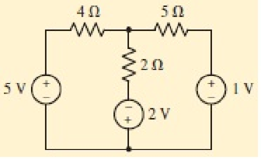
Mesh Analysis for Planar Circuits (1)

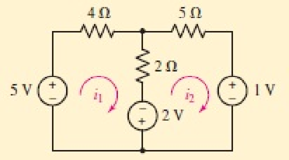
1. Identify and count the number of meshes nodes: m.
2. Label each of m mesh currents; define them clockwise.
3. Write KVL for each mesh.
4. Express any additional unknowns in terms of appropriate mesh currents.
5. Arrange in the form of a matrix equation and solve for the nodal voltages.

Mesh Analysis for Planar Circuits (2)



Mesh Analysis for Planar Circuits (2)

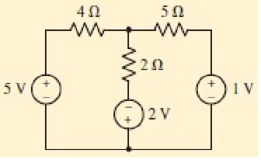


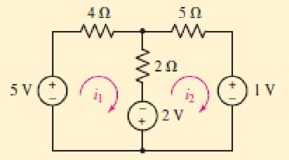


Clearly,

$$\text{Loop1: } 4i_1 + 2(i_1 - i_2) - 2 - 5 = 0$$

Mesh Analysis for Planar Circuits (2)



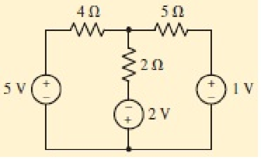


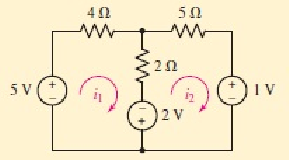
Clearly,

$$\begin{aligned} \text{Loop1: } 4i_1 + 2(i_1 - i_2) - 2 - 5 &= 0 \\ \text{Loop2: } 2(i_2 - i_1) + 5i_2 + 1 + 2 &= 0 \end{aligned}$$

Rearranging,

Mesh Analysis for Planar Circuits (2)





Clearly,

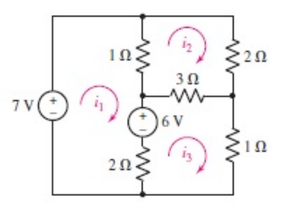
$$\begin{aligned} \text{Loop1: } 4i_1 + 2(i_1 - i_2) - 2 - 5 &= 0 \\ \text{Loop2: } 2(i_2 - i_1) + 5i_2 + 1 + 2 &= 0 \end{aligned}$$

Rearranging,

$$\begin{aligned} 6i_1 - 2i_2 &= 7 \\ -2i_1 + 7i_2 &= -3 \end{aligned}$$

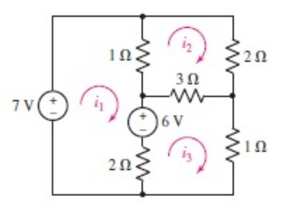
Solving, $i_1 = 1.132$ A and $i_2 = -0.1053$ A.

Mesh Analysis for Planar Circuits (3)



Thus,

Mesh Analysis for Planar Circuits (3)

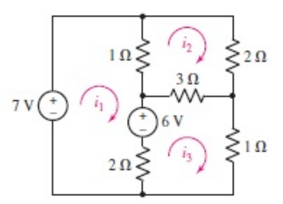


Thus,

$$\begin{aligned}(i_1 - i_2) + 6 + 2(i_1 - i_3) - 7 &= 0\\(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0\\3(i_3 - i_2) + i_3 + 2(i_3 - i_1) - 6 &= 0\end{aligned}$$

yielding

Mesh Analysis for Planar Circuits (3)

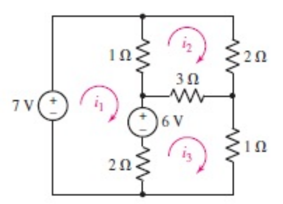


Thus,

$$\begin{aligned} (i_1 - i_2) + 6 + 2(i_1 - i_3) - 7 &= 0 \\ (i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0 \\ 3(i_3 - i_2) + i_3 + 2(i_3 - i_1) - 6 &= 0 \end{aligned}$$

yielding $3i_1 - i_2 - 2i_3 = 1$, $-i_1 + 6i_2 - 3i_3 = 0$, and $-2i_1 - 3i_2 + 6i_3 = 6$, resulting in

Mesh Analysis for Planar Circuits (3)



Thus,

$$\begin{aligned}(i_1 - i_2) + 6 + 2(i_1 - i_3) - 7 &= 0\\(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0\\3(i_3 - i_2) + i_3 + 2(i_3 - i_1) - 6 &= 0\end{aligned}$$

yielding $3i_1 - i_2 - 2i_3 = 1$, $-i_1 + 6i_2 - 3i_3 = 0$, and $-2i_1 - 3i_2 + 6i_3 = 6$, resulting in $i_1 = 3$ A, $i_2 = 2$ A and $i_3 = 3$ A.

Examples (1)

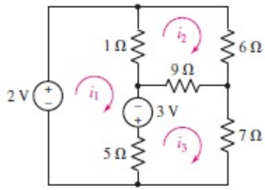


FIGURE 4.58

33. Calculate the power dissipated by each resistor in the circuit of Fig. 4.58.

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

37. Employing mesh analysis procedures, obtain a value for the current labeled *i* in the circuit represented by Fig. 4.62.

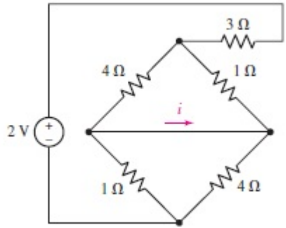
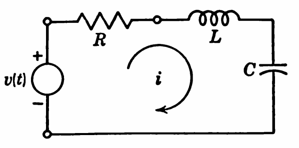


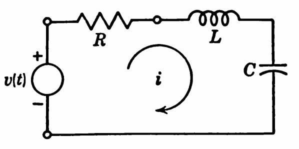
FIGURE 4.62

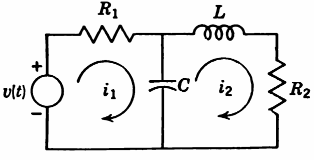
Source: Hayt, Kemmerly and Durbin, 2012.

Examples (3)



Examples (3)





Source: Van Valkenburg, 1975.

Examples (4)

Example 3

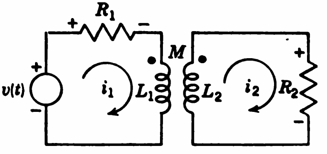
Examples (4)

Example 3

Example 4

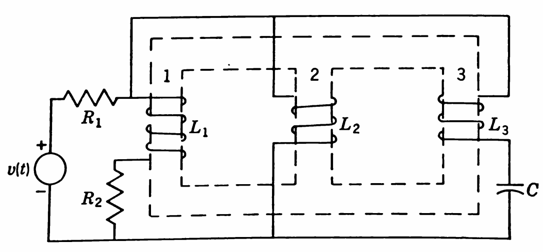
Source: Van Valkenburg, 1975.

Examples (5)

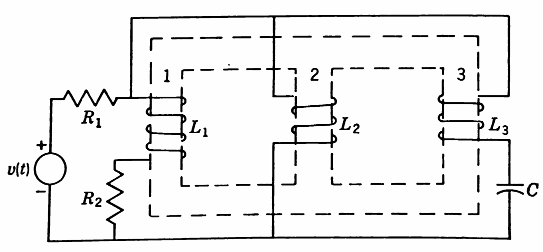


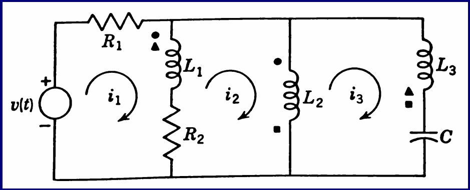
Source: Van Valkenburg, 1975.

Examples (6)



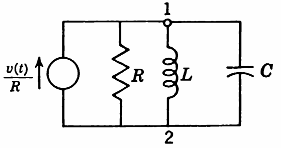
Examples (6)



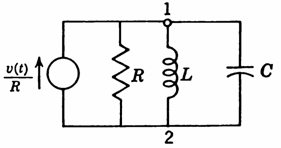


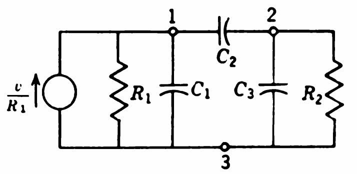
Source: Van Valkenburg, 1975.

Examples (7)



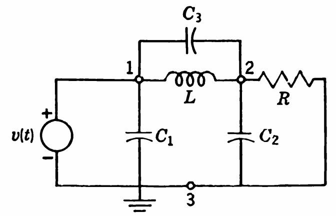
Examples (7)





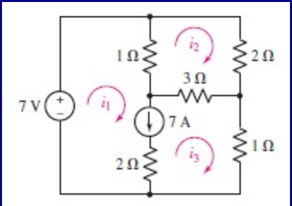
Source: Van Valkenburg, 1975.

Examples (8)

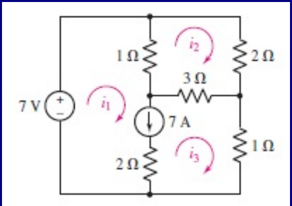


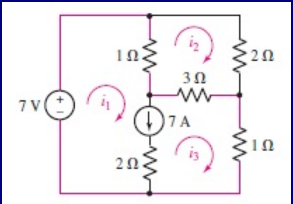
Source: Van Valkenburg, 1975.

Mesh Analysis for Planar Circuits: Supermesh (3)



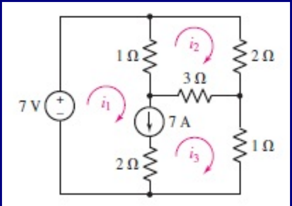
Mesh Analysis for Planar Circuits: Supermesh (3)

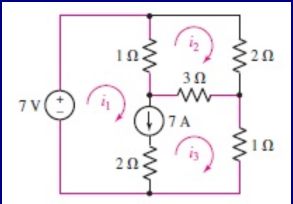




Source: Hayt, Kemmerly and Durbin, 2012.

Mesh Analysis for Planar Circuits: Supermesh (3)



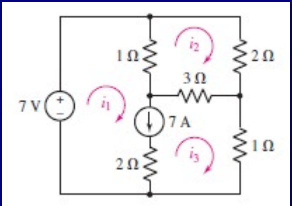


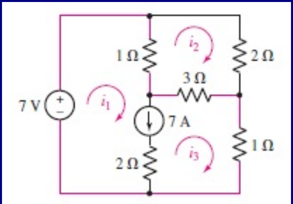
Source: Hayt, Kemmerly and Durbin, 2012.

Thus,

$$
\begin{aligned}
(i_1 - i_2) + 3(i_3 - i_2) + i_3 - 7 &= 0 \\
(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0
\end{aligned}
$$

Mesh Analysis for Planar Circuits: Supermesh (3)





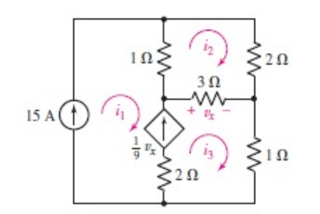
Source: Hayt, Kemmerly and Durbin, 2012.

Thus,

$$
\begin{aligned}
(i_1 - i_2) + 3(i_3 - i_2) + i_3 - 7 &= 0 \\
(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0 \\
i_1 - i_3 &= 7
\end{aligned}
$$

Solving these equations, $i_1 = 9$ A, $i_2 = 2.5$ A and $i_3 = 2$ A.

Mesh Analysis for Planar Circuits: Supermesh (4)



Source: Hayt, Kemmerly and Durbin, 2012.

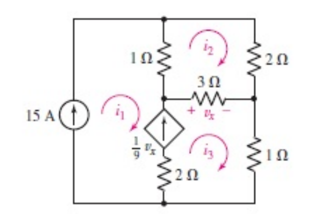
Clearly,

$$i_3 - i_1 = \frac{v_x}{9}, \quad v_x = 3(i_3 - i_2), \quad i_1 = 15$$

Thus,

$$i_2 + 2i_3 = 45$$

Mesh Analysis for Planar Circuits: Supermesh (4)



Source: Hayt, Kemmerly and Durbin, 2012.

Clearly,

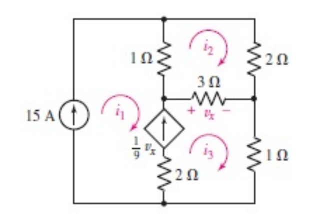
$$i_3 - i_1 = \frac{v_x}{9}, \quad v_x = 3(i_3 - i_2), \quad i_1 = 15$$

Thus,

$$i_2 + 2i_3 = 45$$

Moreover, for the second loop, $6i_2 - 3i_3 = 15$.

Mesh Analysis for Planar Circuits: Supermesh (4)



Source: Hayt, Kemmerly and Durbin, 2012.

Clearly,

$$i_3 - i_1 = \frac{v_x}{9}, \quad v_x = 3(i_3 - i_2), \quad i_1 = 15$$

Thus,

$$i_2 + 2i_3 = 45$$

Moreover, for the second loop, $6i_2 - 3i_3 = 15$. Solving these equations, $i_2 = 11$ A and $i_3 = 17$ A.

Examples (1)

43. Through appropriate application of the supermesh technique, obtain a numerical value for the mesh current i_3 in the circuit of Fig. 4.68, and calculate the power dissipated by the 1 Ω resistor.

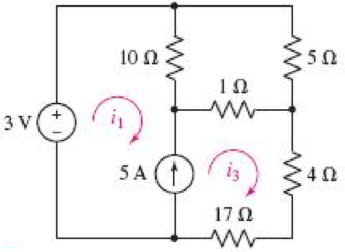


FIGURE 4.68

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

49. Define three clockwise mesh currents for the circuit of Fig. 4.74, and employ the supermesh technique to obtain a numerical value for each.

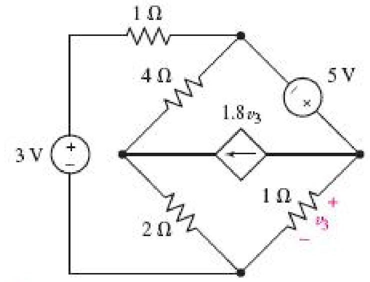


FIGURE 4.74

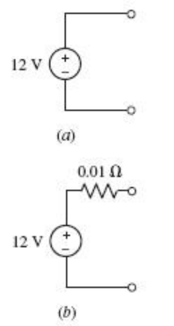
Source: Hayt, Kemmerly and Durbin, 2012.

Network Analysis and Synthesis

Unit I: Basic Analysis — Transformations

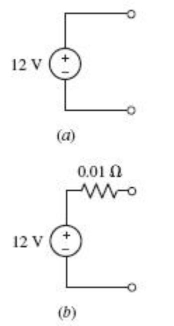
Source Transformations (1)

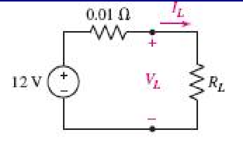
Practical Sources:



Source Transformations (1)

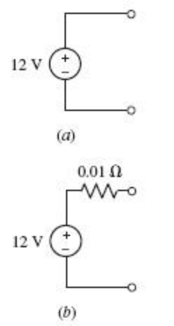
Practical Sources:

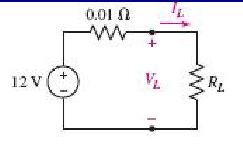


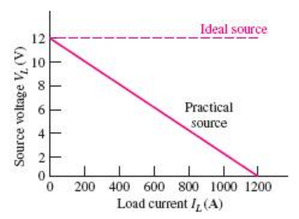


Source Transformations (1)

Practical Sources:

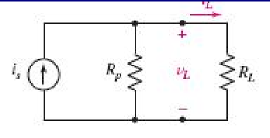


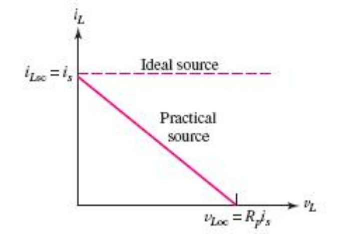




Source Transformations (2)

Practical Sources:





Source Transformations (3)

Definition

Two networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals.

Source Transformations (3)

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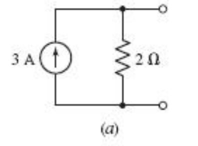
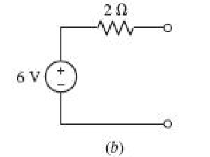
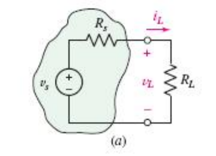
Two networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals.

Source Transformations:

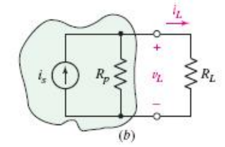




Source Transformations (4)

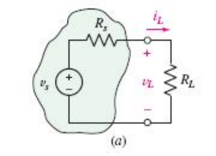


$$v_L = v_s \frac{R_L}{R_s + R_L}$$

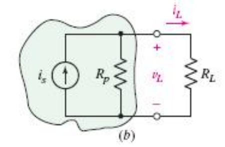


$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) R_L$$

Source Transformations (4)



$$v_L = v_s \frac{R_L}{R_s + R_L}$$

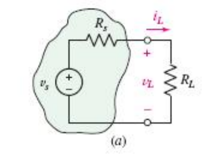


$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) R_L$$

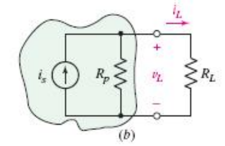
- The two networks are equivalent at the specified terminals if

$$R_p = R_s, \quad v_s = R_p i_s = R_s i_s$$

Source Transformations (4)



$$v_L = v_s \frac{R_L}{R_s + R_L}$$



$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) R_L$$

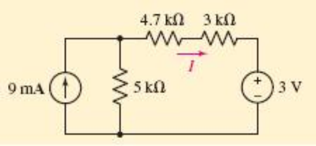
- The two networks are equivalent at the specified terminals if

$$R_p = R_s, \quad v_s = R_p i_s = R_s i_s$$

- Caution: Equivalence is only w.r.t. current-voltage relationships!

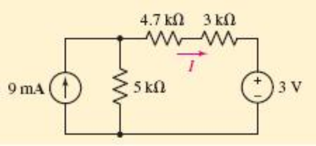
Source Transformations (5)

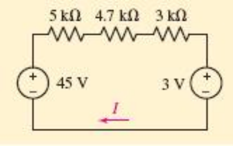
Example:



Source Transformations (5)

Example:

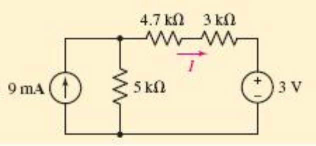


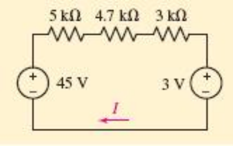


Applying KVL,

Source Transformations (5)

Example:



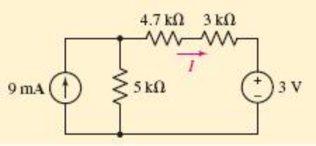


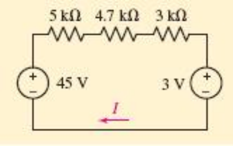
Applying KVL,

$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

Source Transformations (5)

Example:





Applying KVL,

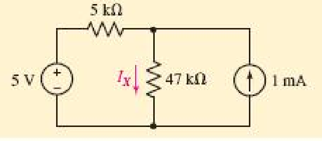
$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

Therefore,

$$I = 3.307\text{mA}$$

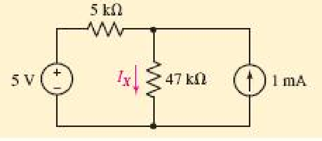
Source Transformations (6)

Example 1:
Find the current I_x.



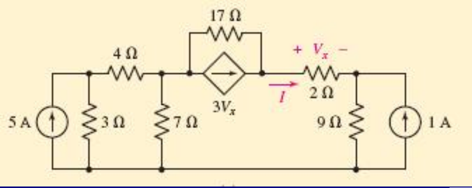
Source Transformations (6)

Example 1:
Find the current I_x.



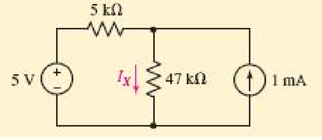
Ans: $I_x = 192\mu$A.

Example 2:
Find the current I.



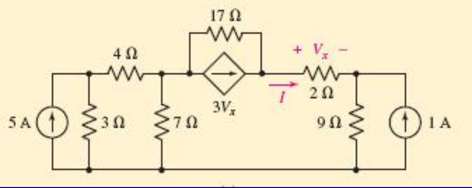
Source Transformations (6)

Example 1:
Find the current I_x.



Ans: $I_x = 192\mu\text{A}$.

Example 2:
Find the current I.



Ans: $I = 21.28\text{mA}$.

Examples (1)

15. Determine the current labeled I in the circuit of Fig. 5.60 by first performing source transformations and parallel-series combinations as required to reduce the circuit to only two elements.

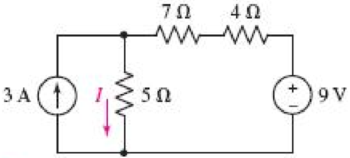


FIGURE 5.60

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

17. (*a*) Determine the current labeled *i* in the circuit of Fig. 5.61 after first transforming the circuit such that it contains only resistors and voltage sources. (*b*) Simulate each circuit to verify the same current flows in both cases.

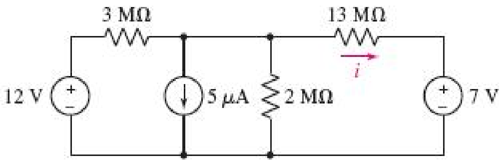
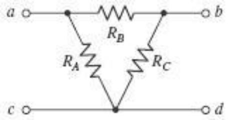
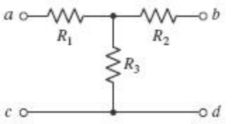


FIGURE 5.61

Source: Hayt, Kemmerly and Durbin, 2012.

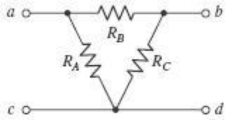
Star-Delta Transformation (1)

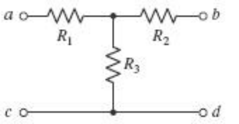




- Also known as delta-wye transformation.
- The first circuit is a delta network, arising from a Π-network.
- The second circuit is a star network, or arising from a T-network.

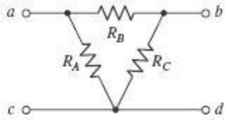
Star-Delta Transformation (1)

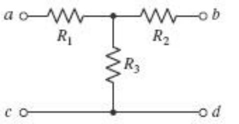




- Also known as delta-wye transformation.
- The first circuit is a delta network, arising from a Π-network.
- The second circuit is a star network, or arising from a T-network.
- Recall: Two networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals.

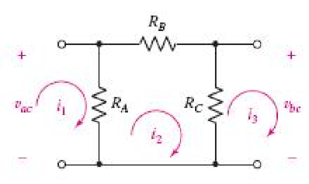
Star-Delta Transformation (1)

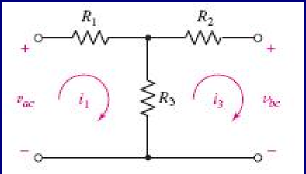




- Also known as delta-wye transformation.
- The first circuit is a delta network, arising from a Π-network.
- The second circuit is a star network, or arising from a T-network.
- Recall: Two networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals.
- Objective: Obtain equivalent networks.

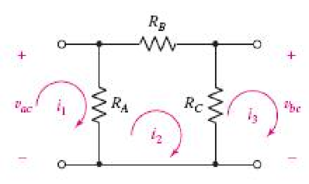
Star-Delta Transformation (2)

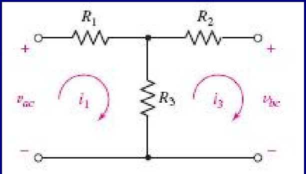




Using loop analysis,

Star-Delta Transformation (2)





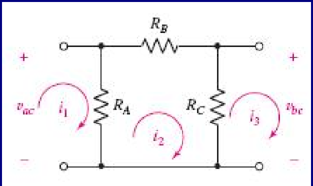
Using loop analysis,

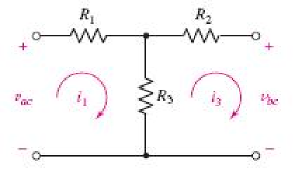
$$
\begin{aligned}
v_{ac} &= R_A i_1 - R_A i_2 && (1)\\
0 &= -R_A i_1 + (R_A + R_B + R_C) i_2 - R_C i_3 && (2)\\
v_{bc} &= R_C i_2 - R_C i_3 && (3)
\end{aligned}
$$

and

$$
\begin{aligned}
v_{ac} &= (R_1 + R_3) i_1 - R_3 i_3 && (4)\\
v_{bc} &= R_3 i_1 - (R_2 + R_3) i_3 && (5)
\end{aligned}
$$

Star-Delta Transformation (3)





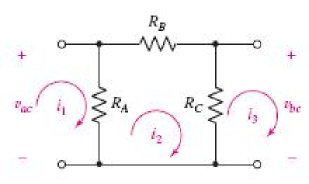
To convert a delta-network to a star-network:

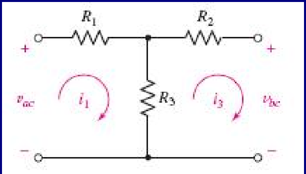
$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_C R_A}{R_A + R_B + R_C}$$

Star-Delta Transformation (4)





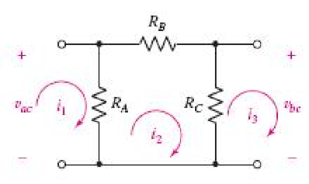
To convert a star-network to a delta-network:

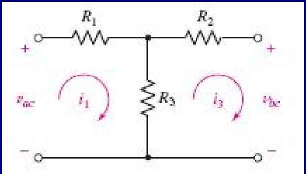
$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

Star-Delta Transformation (4)





To convert a star-network to a delta-network:

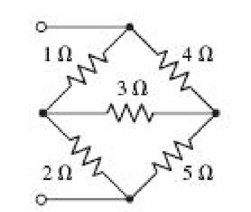
$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

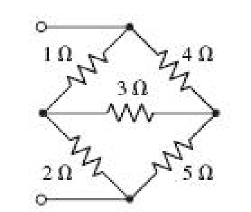
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

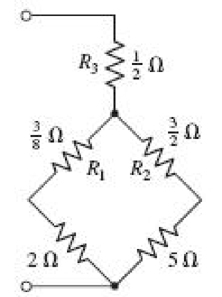
- Holds good for impedance-networks.

Star-Delta Transformation (5)

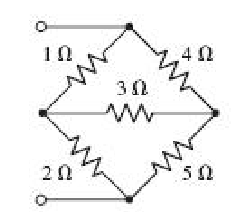


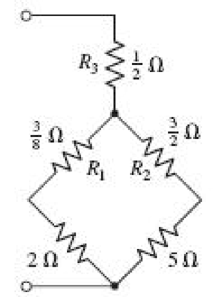
Star-Delta Transformation (5)





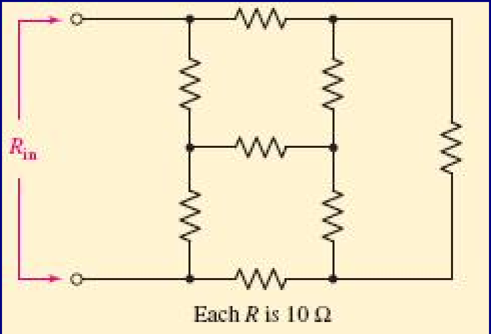
Star-Delta Transformation (5)



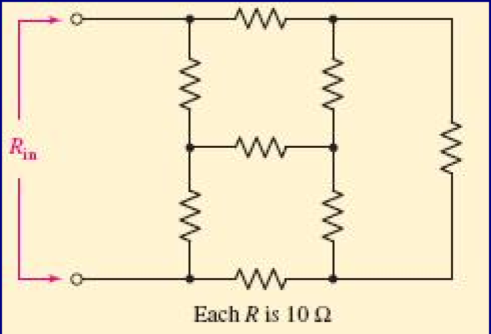


Ans: $\frac{159}{71}\Omega$.

Star-Delta Transformation (6)



Star-Delta Transformation (6)



Ans: 11.43Ω.

Examples (1)

57. Convert the Y- (or "T-") connected networks in Fig. 5.96 to Δ-connected networks.

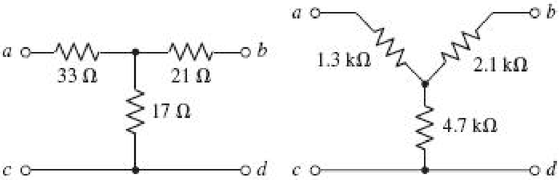


FIGURE 5.96

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

59. For the network of Fig. 5.98, select a value of R such that the network has an equivalent resistance of $70.6\ \Omega$.

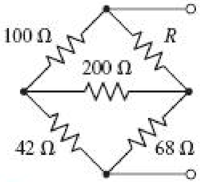


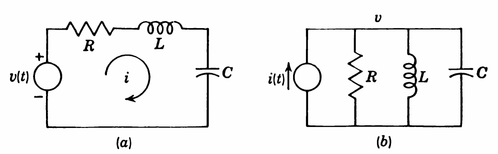
FIGURE 5.98

Source: Hayt, Kemmerly and Durbin, 2012.

Network Analysis and Synthesis

Unit I: Basic Analysis — Duality

Principle of Duality (1)

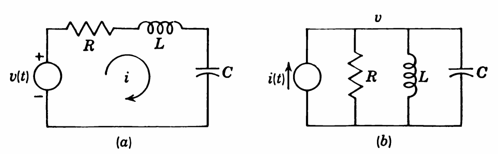


From KVL and KCL, respectively,

$$
\begin{aligned}
L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i d\tau &= v(t) \\
C\frac{dv}{dt} + \frac{1}{R}i + \frac{1}{L}\int_{-\infty}^{t} v d\tau &= i(t)
\end{aligned}
$$

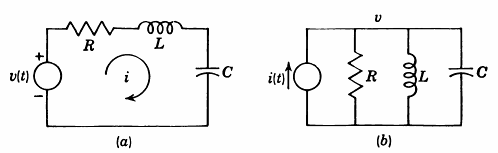
- These equations differ only in the coefficients. The solution of one is the same as the other.
- These networks are called duals.
- The roles of voltages and currents have been interchanged.

Principle of Duality (2)



- These are not equivalent networks!

Principle of Duality (2)

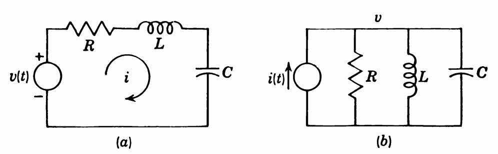


- These are not equivalent networks!

Analogous quantities:

$$
\begin{array}{cc}
Ri & \frac{1}{R}v \\
L\frac{di}{dt} & C\frac{dv}{dt} \\
\frac{1}{C}\int_{-\infty}^{t} i\,d\tau & \frac{1}{L}\int_{-\infty}^{t} v\,d\tau
\end{array}
$$

Principle of Duality (3)



Dual quantities:

R	$\frac{1}{R}$
L	C
Loop current i	Node-pair voltage v
Charge q	Flux linkages ψ
Loop	Node-pair
Short circuit	Open circuit

Principle of Duality (4)

To obtain a dual network,

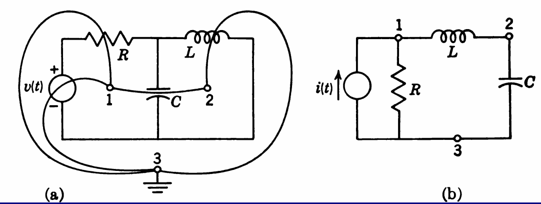
- Associate a node with every loop.
- Place an extra node, the datum node, outside the network.
- Draw lines from node-to-node through the elements, traversing only one element at a time. For each element traversed, connect the dual element between the nodes.

Principle of Duality (4)

To obtain a dual network,

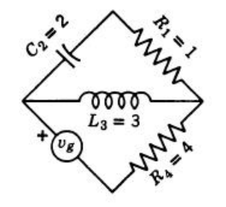
- Associate a node with every loop.
- Place an extra node, the datum node, outside the network.
- Draw lines from node-to-node through the elements, traversing only one element at a time. For each element traversed, connect the dual element between the nodes.

Example:



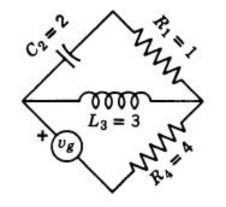
Principle of Duality (5)

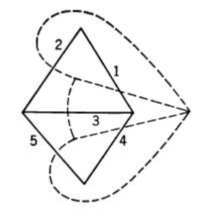
Example:



Principle of Duality (5)

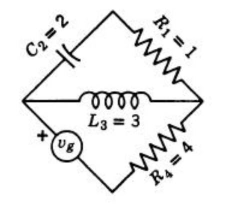
Example:

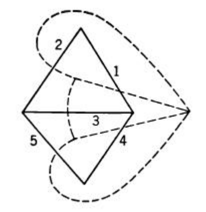


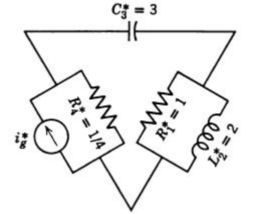


Principle of Duality (5)

Example:







Principle of Duality (6)

Fact

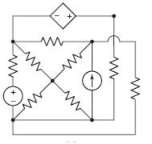
If $\mathcal{N}$ be any network and S be any true statement concerning the behaviour of $\mathcal{N}$. Suppose that $\mathcal{N}'$ is a dual of $\mathcal{N}$. Let the statement S' be derived from S by replacing every quantity by its dual. Then S' is a true statement concerning the behaviour of $\mathcal{N}'$

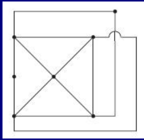
Example: KVL and KCL are dual principles.

Network Analysis and Synthesis

Unit I: Basic Analysis — Graph Theory

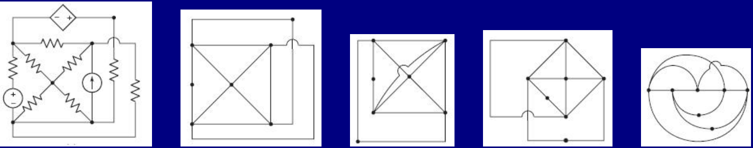
Introduction to Graph Theory (1)





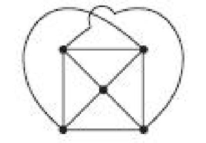
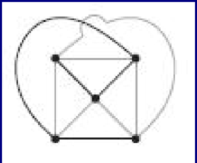
- In a lumped circuit, the two-terminal elements are called branches, and the terminals of these elements are called nodes.
- The example has 12 branches and 7 nodes.
- A path is formed by starting at one node, traversing one or more branches in succession, and ending at another node.
- A closed path is a path whose starting node is the same as the ending node.
- A loop is a closed path.
- A mesh is a loop which does not contain any other loops within it.

Introduction to Graph Theory (2)



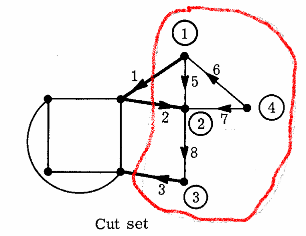
- The topological properties are not changed when the graph is twisted, folded, stretched, or tied in knots, as long as no parts of the graph are cut apart or joined together!
- A graph that can be drawn on a plane so that no branch passes over or under any other branch is called a planar graph; otherwise, it is called a non-planar graph.

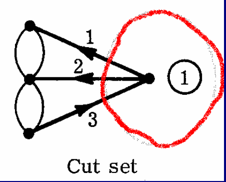
Introduction to Graph Theory (3)

- A connected graph is one which consists of only one part.
- A tree of a connected graph is a connected subgraph which contains all of the nodes of the graph but does not contain any loops.
- Trees are not unique.
- The complement of a tree is called a co-tree.
- The branches of a co-tree are called chords or links.

Introduction to Graph Theory (3)





- A set of branches of a connected graph is called a cut set if
  1. the removal of all the branches of the set causes the remaining graph to have separate parts and
  2. the removal of all but any one of the branches of the set leaves the remaining graph connected.
- KCL:

$$i_1 - i_2 + i_3 = 0, \quad i_1 + i_2 - i_3 = 0$$

Kirchhoff's Laws Revisited (1)

- Recall that
$$Ax = y$$
can be solved provided there are m equations in the m unknowns; i.e., the $m \times m$ matrix A should be non-singular.
- That is, the m equations ought to be linearly independent.

Kirchhoff's Laws Revisited (1)

- Recall that
$$Ax = y$$
can be solved provided there are m equations in the m unknowns; i.e., the $m \times m$ matrix A should be non-singular.
- That is, the m equations ought to be linearly independent.
- Therefore, how many network (loop plus node) equations should we write to get independent equations?

Kirchhoff's Laws Revisited (2)

Consider a network with b branches and n nodes.

- There are b branch voltages and b branch currents $\Longrightarrow 2b$ equations?

Kirchhoff's Laws Revisited (2)

Consider a network with b branches and n nodes.

- There are b branch voltages and b branch currents $\Longrightarrow 2b$ equations?
- However, the b branch voltages and b branch currents are related $\Longrightarrow b$ equations.

Kirchhoff's Laws Revisited (2)

Consider a network with b branches and n nodes.

- There are b branch voltages and b branch currents $\Longrightarrow 2b$ equations?
- However, the b branch voltages and b branch currents are related $\Longrightarrow b$ equations.
- One of the n nodes is considered the reference node or datum or ground.
- Therefore, one can write $n-1$ node equations using KCL.
- Hence, one requires only $b-(n-1)$ equations using KVL.

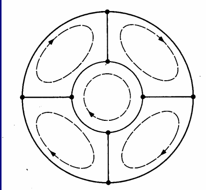
Kirchhoff's Laws Revisited (2)

Consider a network with b branches and n nodes.

- There are b branch voltages and b branch currents $\Longrightarrow 2b$ equations?
- However, the b branch voltages and b branch currents are related $\Longrightarrow b$ equations.
- One of the n nodes is considered the reference node or datum or ground.
- Therefore, one can write $n - 1$ node equations using KCL.
- Hence, one requires only $b - (n - 1)$ equations using KVL.
- Are all these b equations linearly independent?

Kirchhoff's Laws Revisited (3)

- Suppose we know the node-to-datum voltages. Then, we can determine all the branch voltages!
- Therefore, the number of independent voltages is $n-1$.
- Observation: $n-1 < b$.
- Also, the number of independent branch currents is $b-n+1$[1].
- These currents are the loop currents.



[1] Established by Kirchchoff in 1847!

Examples (1)

56. Replace the dependent voltage source in the circuit of Fig. 4.79 with a dependent current source oriented such that the arrow points upward. The controlling expression $0.1\,v_1$ remains unchanged. The value V_2 is zero. (*a*) Determine the total number of simultaneous equations required to obtain the power dissipated by the 40 Ω resistor if nodal analysis is employed. (*b*) Is mesh analysis preferred instead? *Explain*.

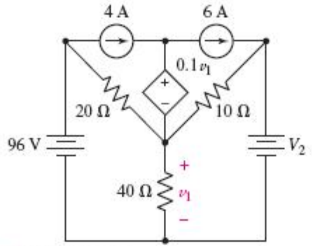


FIGURE 4.79

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

57. After studying the circuit of Fig. 4.80, determine the total number of simultaneous equations that must be solved to determine voltages v_1 and v_3 using (*a*) nodal analysis; (*b*) mesh analysis.

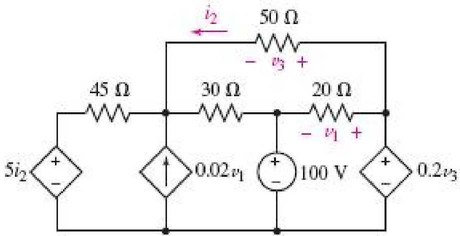


FIGURE 4.80

Source: Hayt, Kemmerly and Durbin, 2012.