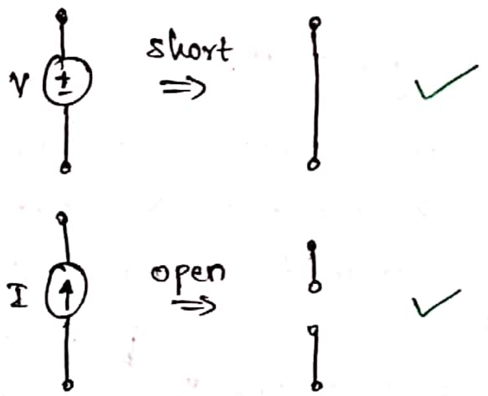
NAS: NETWORK THEORMS

Superposition Theorms

In any linear resistive network, the voltage accross or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the seperate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

→



Thevenin's theorm

Steps

(i) Find thevenin's resistance R_{th} by removing all voltage soures and open circuting current sources. (don't consider the load resistor R_L)

(ii) Find thevenin's voltage by plugging in the voltages, i.e measure the open ciruit voltage V_{th}.

(iii) Use the R_{th} and V_{th} to find current through load resistor R_L

Norton's theorm

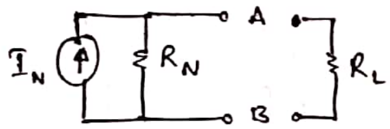
Steps

(i) Short the load resistor

(ii) Measure the short circuit currents, its the Norton current (I_N)

(iii) Measure the open circuit resistance, i.e Norton Resistance (R_N)

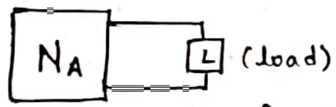
(iv) Now, redraw the circuit, with R_N parallel with I_N



(v) Now, calculate load current I_L using current divider rule.

Maximum power transfer theorm

- Consider the general network,



let resistor in N_A be Z_s and load resistor is R_L

case ①: Z_s is non-complex (pure)
R_L is non-complex (pure)

i.e $Z_s = R_s$

- For maximum power transfer

$$\boxed{R_L = R_s}$$

- $$\boxed{P_{max} = \frac{V^2}{4R_L}}$$

case ②: Z_s is complex and R_L is pure load

i.e $Z_s = R_s + jX_s$

- For maximum power transfor

$$\boxed{R_L = \sqrt{R_s^2 + X_s^2}}$$ i.e $R_L = |Z_s|$

- $$\boxed{P_{max} = \frac{V^2}{4\sqrt{R_s^2 + X_s^2}}}$$

case ③ : Z_s is complex and R_L is complex.

i.e $Z_s = R_s + jX_s$

$R_L = R_L + jX_L$

⊙ For maximum power transfer

$$\boxed{\begin{array}{l} R_L = Z_s^* \\ \text{i.e } R_L + jX_L = R_s - jX_s \end{array}}$$

$R_L = R_s$

$X_L = -X_s$

⊙ $$\boxed{P_{max} = \frac{V^2}{4\sqrt{R_s^2 + X_s^2}}}$$

⊛ Reciprocity theorm

" In a linear network, if the position of the excitation and response are interchanged, their ratio remain unchanged "

⇒ conditions in order to apply this theorm

(i) The circuit must have a single source

(ii) Initial conditions are assumed to be zero or absent in the circuit

(iii) Dependent sources are excluded even if they are linear

(iv) when the position of source and response are interchanged, their directions should be marked same as in the original circuit.

⊛ Millman's theorm

- Applicable for multiple voltage source conneted in parallel.
- It is used to convert multiple voltage source in parallel into a single voltage source.

steps

(i) Convert all voltage source into current source $I = V/R$

(ii) Combine these current sources (I_{eq})

(iii) Find the resultant resistance (R_{eq})

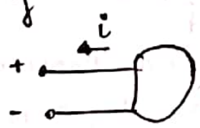
(iv) Convert resulting current source into voltage soure, series with Req ($I_{eq} R_{eq} = V_{eq}$)

⊛ Tellegen's theorm

" In any electrical network which satisfies Kirchoff's laws, the summation of instantaneous power in all the branches is equal to zero "

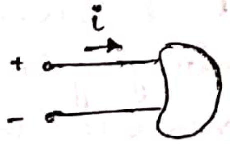
Note ⊙ Power is +ve if current is coming out of +ve terminal

P = +ve if



⊙ Power is -ve if current is entering into the +ve terminal

P = -ve if



⊛ Energy (Work) and Power

In general, $\boxed{W = \int_{t_1}^{t_2} v(t)\, i(t)\, dt}$

$$\boxed{P = \frac{dw}{dt} = v(t)\, i(t)}$$

⇒ For a resistor, with $i = I_m \sin\omega t$

$$\boxed{W_R = \frac{R I_m^2}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right)}$$

$$\boxed{P_R = \frac{R I_m^2}{2}(1 - \cos 2\omega t)}$$

⇒ For a inductor, with $i = I_m \sin\omega t$

$$\boxed{W_L = \frac{L I_m^2}{4}(1 - \cos 2\omega t)}$$

$$\boxed{P_L = \frac{L I_m^2}{2}\, \omega \sin 2\omega t}$$

⇒ For a capacitor, with $i = I_m \sin\omega t$, $v = V_m \sin\omega t$

$$\boxed{W_c = \frac{C V_m^2}{4}(1 - \cos 2\omega t)}$$

$$\boxed{P_c = \frac{C V_m^2}{2}\, \omega \sin 2\omega t}$$

⊡ For non-sinusoidal but periodic current $i(t)$

→ $$P_{avg} = \frac{1}{T}\int_{t_0}^{t_0+T} R\, i^2(t)\, dt$$

⇒ $$I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\, dt} = I_{rms}$$

⇒ $$V_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\, dt} = V_{rms}$$

⊛ Additional points.

⇒ Complex power

If $V = V_m e^{i\theta_v} = V_m \angle\theta_v$

$I = I_m e^{i\theta_i} = I_m \angle\theta_i$

$\therefore Z = \frac{V}{I} = \frac{V_m}{I_m}\angle\theta_v - \theta_i$

⇒ $\boxed{Z = |Z| \angle\theta}$

Here power factor is $\boxed{P.F = \cos\theta}$

→ phasor power or apparent power (S)

$$\boxed{S = \frac{1}{2} V I^*}$$

→ Reactive power (Q)

$$\boxed{Q = V_{rms} I_{rms} \sin\theta}$$

Note : When $V = V_m \sin\omega t$,

→ P.F is leading if $I = I_m \sin(\omega t + \theta)$

→ P.F is lagging if $I = I_m \sin(\omega t - \theta)$

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NAS : Two Port Networks

* TABLE - 1

⇒ Two port parameters

Name	Functions: Express (Dep)	Functions: In terms of (Indep)	Equations
1) Z-parameters (open-circuit impedance)	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
2) Y-parameters (short-circuit impedence)	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
3) T-parameters	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
4) Inverse T-parameters	V_2, I_2	V_1, I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
5) H-parameters	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
6) Inverse H-parameters	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

* TABLE - 2

⇒ Conditions for Passive Networks and Electrical Symmetry.

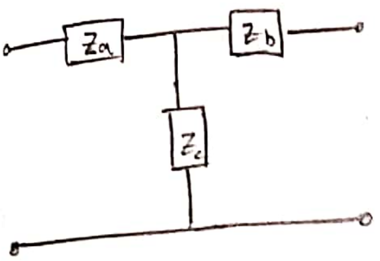
Parameter	Condition for Passive Ntk.	Condition for Electrical Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$\Delta T = AD - BC = 1$	$A = D$
T^{-1}	$\Delta T^{-1} = A'D' - B'C' = 1$	$A' = D'$
H	$h_{12} = -h_{21}$	$\Delta h = h_{11} h_{22} - h_{21} h_{12} = 1$
H^{-1}	$g_{12} = -g_{21}$	$\Delta g = g_{11} g_{22} - g_{21} g_{12} = 1$

*TABLE - 3 : PARAMETER CONVERSION TABLE

	[Z]	[Y]	[T]	[T^{-1}]	[H]	[G]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta T'}{C'} & \frac{A'}{C'} \end{matrix}$	$\begin{matrix} \frac{\Delta H}{H_{2}} & \frac{H_{12}}{H_{22}} \\ \frac{-H_{21}}{H_{22}} & \frac{1}{H_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{G_{11}} & \frac{-G_{12}}{G_{11}} \\ \frac{G_{21}}{G_{11}} & \frac{\Delta G}{G_{11}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A'}{B'} & \frac{-1}{B'} \\ \frac{-\Delta T'}{B'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} \frac{1}{H_{11}} & \frac{-H_{12}}{H_{11}} \\ \frac{H_{21}}{H_{11}} & \frac{\Delta H}{H_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta G}{G_{22}} & \frac{G_{12}}{G_{22}} \\ \frac{-G_{21}}{G_{22}} & \frac{1}{G_{22}} \end{matrix}$
[T]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{D'}{\Delta T'} & \frac{B'}{\Delta T'} \\ \frac{C'}{\Delta T'} & \frac{A'}{\Delta T'} \end{matrix}$	$\begin{matrix} \frac{-\Delta H}{H_{21}} & \frac{-H_{11}}{H_{21}} \\ \frac{-H_{22}}{H_{21}} & \frac{-1}{H_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{G_{21}} & \frac{G_{22}}{G_{21}} \\ \frac{G_{11}}{G_{21}} & \frac{\Delta G}{G_{21}} \end{matrix}$
[T^{-1}]	$\begin{matrix} \frac{Z_{22}}{Z_{12}} & \frac{\Delta Z}{Z_{12}} \\ \frac{1}{Z_{12}} & \frac{Z_{11}}{Z_{12}} \end{matrix}$	$\begin{matrix} \frac{-Y_{11}}{Y_{12}} & \frac{-1}{Y_{12}} \\ \frac{-\Delta Y}{Y_{12}} & \frac{-Y_{22}}{Y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta T} & \frac{B}{\Delta T} \\ \frac{C}{\Delta T} & \frac{A}{\Delta T} \end{matrix}$	$\begin{matrix} A' & B' \\ C' & D' \end{matrix}$	$\begin{matrix} \frac{1}{H_{12}} & \frac{H_{11}}{H_{12}} \\ \frac{H_{22}}{H_{12}} & \frac{\Delta H}{H_{12}} \end{matrix}$	$\begin{matrix} \frac{-\Delta G}{G_{12}} & \frac{-G_{22}}{G_{12}} \\ -\frac{G_{11}}{G_{12}} & \frac{1}{G_{12}} \end{matrix}$
[H]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B'}{A'} & \frac{1}{A'} \\ \frac{-\Delta T'}{A'} & \frac{C'}{A'} \end{matrix}$	$\begin{matrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{matrix}$	$\begin{matrix} \frac{G_{22}}{\Delta G} & \frac{G_{12}}{\Delta G} \\ \frac{-G_{21}}{\Delta G} & \frac{G_{11}}{\Delta G} \end{matrix}$
[G]	$\begin{matrix} \frac{1}{Z_{11}} & \frac{-Z_{12}}{Z_{11}} \\ \frac{Z_{21}}{Z_{11}} & \frac{\Delta Z}{Z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta Y}{Y_{22}} & \frac{Y_{12}}{Y_{22}} \\ \frac{-Y_{21}}{Y_{22}} & \frac{1}{Y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & \frac{-\Delta T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C'}{D'} & \frac{-1}{D'} \\ \frac{\Delta T'}{D'} & \frac{B'}{D'} \end{matrix}$	$\begin{matrix} \frac{H_{22}}{\Delta H} & \frac{-H_{12}}{\Delta H} \\ \frac{-H_{21}}{\Delta H} & \frac{H_{11}}{\Delta H} \end{matrix}$	$\begin{matrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{matrix}$

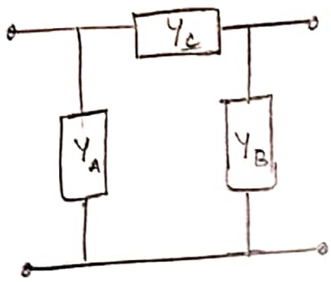
* Important points.

⊙ 'T' Equivalent of z-parameters.



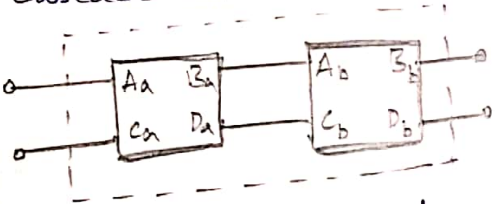
$$z_{11} = Z_a + Z_c$$
$$z_{22} = Z_b + Z_c$$
$$z_{12} = Z_c$$
$$z_{21} = Z_c$$

⊙ 'π' Equivalent of Y-parameters



$$Y_{11} = Y_A + Y_C$$
$$Y_{22} = Y_B + Y_C$$
$$Y_{12} = -Y_C$$
$$Y_{21} = -Y_C$$

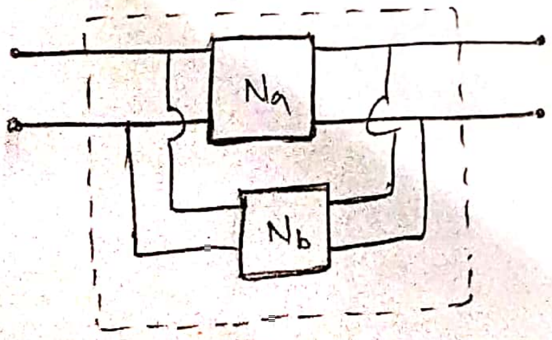
⊙ Cascade connected two networks



Equivalent parameter matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

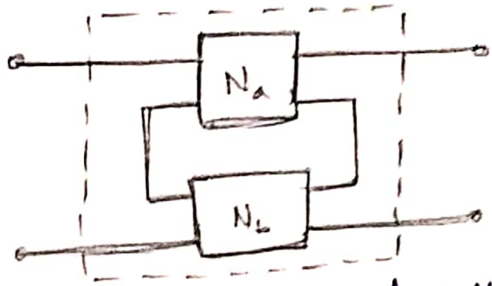
⊙ Parallel connected two networks



Equivalent parameter matrix

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11a}+Y_{11b} & Y_{12a}+Y_{12b} \\ Y_{21a}+Y_{21b} & Y_{22a}+Y_{22b} \end{bmatrix}$$

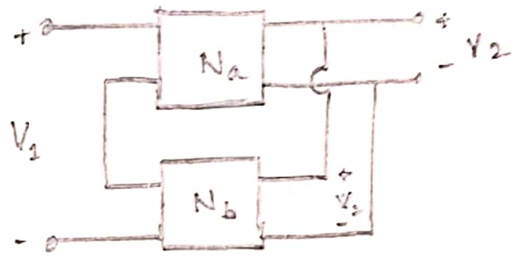
⊙ Series connected two-networks



Equivalent parameter matrix

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11a}+z_{11b} & z_{21a}+z_{21b} \\ z_{12a}+z_{12b} & z_{22a}+z_{22b} \end{bmatrix}$$

⊙ Input series and output parallel two networks.



Equivalent parameter matrix

$$[H] = [H_a] + [H_b]$$

i.e

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} + \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix}$$