REVISION: UNIT: I

To Explore Numericals:

★ COMPTON EFFECT:

① Compton shift, $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1-\cos\theta)$

$= \lambda_c (1-\cos\theta)$ | $\lambda_c = 2.42\times10^{-12}$ m Compton wavelength

Wavelength of incident x-rays } $\lambda_i = \lambda_f - \Delta\lambda$

Wavelength of scattered x-rays } $\lambda_f = \lambda_i + \Delta\lambda$

② Energy lost by the photon, $= E_i - E_f$

$= h\nu_i - h\nu_f = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$

= Energy gained by the electron.

③ Momentum lost by the photon $= \frac{h}{\lambda_i} - \frac{h}{\lambda_f}$

= momentum gained by the electron.

④ Energy lost by the x-ray photon is maximum when, $\theta = 180^\circ$. (i.e, head on collision of the x-ray photon with the electron. The scattered electrons move along the direction of incidence of the photon and energy change is maximum).

* de BROGLIE's HYPOTHESIS

① $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$; $E = \frac{1}{2}mv^2$ (K.E)

$E = \frac{3}{2}kT$ (Thermal Energy)

$E = eV$ (Energy due to an accelerating voltage)

② Ratio of de Broglie wavelength of 'e' and ~~photon~~ proton, $m_e = 9.11 \times 10^{-31}$ kg

$m_{proton} = 1.67 \times 10^{-27}$ kg

$$\frac{\lambda_e}{\lambda_{proton}} = \frac{m_{proton}}{m_e}$$

③ If electron and proton are accelerated by same voltage, V, then $\frac{\lambda_{electron}}{\lambda_{proton}} = \frac{\sqrt{m_{proton}}}{\sqrt{m_{electron}}}$

④ Ratio of momentum and energy of 'photon' and 'electron' with same wavelength, $P_{ph} = \frac{h}{\lambda}$ $P_e = \frac{h}{\lambda}$

$$\frac{P_{ph}}{P_e} = 1$$

Energy of photon, $E_{ph} = \frac{hc}{\lambda}$ Energy of electron $= \frac{p^2}{2m}$

$= \frac{h^2}{2m\lambda^2}$

$$\therefore \frac{E_{ph}}{E_{electron}} = \frac{\frac{hc}{\lambda}}{\frac{h^2}{2m\lambda^2}} = \frac{2m\lambda c}{h}$$

* WAVE PACKETS

① $V_g = \frac{d\omega}{dk} = V_{particle}$ $V_{phase} = \frac{\omega}{k}$

$y = 10 \sin(30t - 40x) \cdot \cos(0.3t - 0.5x)$

Find V_g and V_{phase}.

If $V_{ph} = V_g/2$ what is the relation b/w V_{ph} & λ.

$V_{ph} = 2V_g$ what is the relation b/w V_{ph} & λ.

* HEISENBERG's UNCERTAINTY PRINCIPLE :

① Conjugate pairs, $\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \geq \frac{\hbar}{2}$

$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

$\Delta \theta \cdot \Delta L \geq \frac{\hbar}{2}$

$\Delta \theta$ = Uncertainty in angular position.

ΔL = Uncertainty in angular momentum.

② Momentum, $p = \hbar k$, $k = \frac{2\pi}{\lambda}$ (propagation const.)

③ $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

$\Delta x \cdot \Delta\left(\frac{h}{\lambda}\right) \geq \frac{h}{4\pi}$

$\Delta\left(\frac{1}{\lambda}\right) = \frac{-1}{\lambda^2} \cdot \Delta \lambda$

$\left|\Delta\left(\frac{1}{\lambda}\right)\right| = \frac{\Delta \lambda}{\lambda^2}$

$\Delta x \cdot h \cdot \frac{\Delta \lambda}{\lambda^2} \geq \frac{h}{4\pi}$

$\Delta x \cdot \Delta \lambda \geq \left|\frac{\lambda^2}{4\pi}\right|$

④ $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

$p = mv$, $\Delta p = m \Delta v$

$\therefore \Delta x \cdot m \Delta v \geq \frac{h}{4\pi}$

⑤ For a spectral line of width, $\Delta \lambda$

$\Delta t \geq \frac{h}{4\pi \cdot \Delta E}$

$\Delta E = \Delta\left(\frac{hc}{\lambda}\right)$

$= hc \cdot \frac{-1}{\lambda^2} \cdot \Delta \lambda$

$|\Delta E| = \frac{hc\, \Delta \lambda}{\lambda^2}$

$\Delta t \geq \frac{h}{\frac{4\pi \cdot hc\, \Delta \lambda}{\lambda^2}}$

$\Delta t \geq \frac{\lambda^2}{4\pi \cdot c \cdot \Delta \lambda}$

* WAVE FUNCTION:

① ψ is acceptable — FCS both ψ and derivatives
$\psi \Rightarrow 0$ as $x \Rightarrow \infty$
Normalisable.

eg: $\psi = Ae^{x}$ is not acceptable since $\psi \Rightarrow \infty$ as $x \Rightarrow \infty$

$\psi = \left(\frac{e^{ax}}{x-a}\right)$ is not acceptable as ψ is discontinuous at $x = a$.

$\psi = e^{x^2}$ is acceptable for limited intervals of x
(small values of x)