(1)

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Laplace transform:

$\mathcal{L}[K] = \mathcal{L}[K \cdot u(t)] = \dfrac{K}{s}$ LT deals with causal type function → (0^- to ∞)

$\mathcal{L}(\cos \omega_0 t \; u(t)) \rightarrow \dfrac{s}{s^2 + \omega_0^2}$ $\quad t \geq 0^-$

Properties of Laplace transform.

$f(t) \rightleftharpoons F(s)$

1) Linearity

$f_1(t) \longrightarrow F_1(s)$

and $f_2(t) \longrightarrow F_2(s)$ then

$\underbrace{c_1 f_1(t) + c_2 f_2(t)}_{\text{Linear comb}^{n} \text{ of } f_1(t) \,\&\, f_2(t)} \longrightarrow c_1 F_1(s) + c_2 F_2(s)$

2) Differentiation in time domain

a) $\mathcal{L}\left[\dfrac{df}{dt}\right]$

$$\int_{0^-}^{\infty} \frac{df}{dt} \cdot e^{-st} \cdot dt = f(t) \cdot e^{-st} \Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t)\,(-s) \cdot e^{-st} \cdot dt$$

$$= -f(0^-)\, e^{0^-} + s \int_{0^-}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

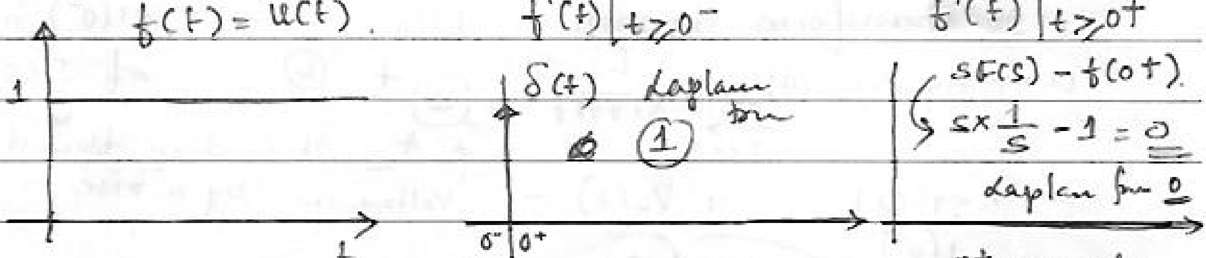
$$= -f(0^-) + sF(s)$$

$\mathcal{L}\left[f'(t)\Big|_{t \geq 0^-}\right] = sF(s) - f(0^-)$ —(i) (0^- to ∞)

$\mathcal{L}\left[f'(t)\Big|_{t \geq 0^+}\right] = sF(s) - f(0^+)$ —(ii) (0^+ to ∞)

Example:

$f(t) = u(t)$ $\qquad f'(t)\Big|_{t \geq 0^-}$ $\qquad f'(t)\Big|_{t \geq 0^+}$



0^+ onwards $u(t) = 1 \therefore f'(t) = 0$

$\mathcal{L}(u(t)) = 1/s$. $\quad$ ↗ $u(t)$

$sF(s) - f(0^-)$

$= s \cdot \frac{1}{s} - 0 = \underline{1}$ Jumping from 0 to 1 when t changes from 0^- to 0^+; after 0^+ derivative is zero.

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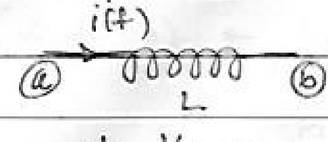
Application



Inductor having L henry carry current i(t) and voltage V_L a/c it.

$$V_L(t) = L \cdot \frac{di}{dt}$$

$$\mathcal{L}\left(V_L(t)\right) = \mathcal{L}\left[L \cdot \frac{di}{dt}\right]$$

Assumed $\mathcal{L}(i(t)) \Rightarrow I(s)$

$\mathcal{L}(v_L(t)) \Leftrightarrow V_L(s)$

$$V_L(s) = L\left[sI(s) - i(0^-)\right]$$

$$V_L(s) = \underbrace{LsI(s)}_{} - \underbrace{Li(0^-)}_{\text{Initial condn}} \quad -(1)$$

v_L & i_L – time dom

V_L & $I(s)$ – s-domain (LT)

$V_L(t)$ from 0^- to ∞

$$LsI(s) = V_L(s) + Li(0^-)$$

$$\left[I(s) = \left(\frac{V_L(s)}{Ls}\right) + \frac{i(0^-)}{s}\right] \quad -(2)$$

(1) & (2) Initial condn's of inductor need to be known.

Provided initial current $i(0^-) = 0$.

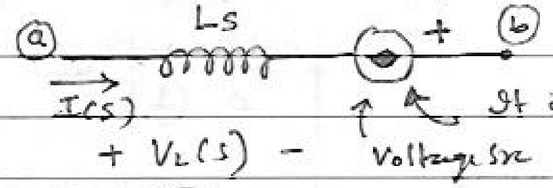
$$\left.\begin{aligned} V_L(s) &= (Ls)I(s) \\ I(s) &= \frac{V_L(s)}{(Ls)} \end{aligned}\right\} \quad -(3)$$

If a current $I(s)$ flows through a inductor, then $V_L(s) = (Ls)$ times $I(s)$ where (Ls) is regarded as generalised impedance of inductance in transform domain.

If a voltage a/c inductor is $V_L(s)$, then its current through inductor is $V_L(s)/(Ls)$ where $\left(\frac{1}{Ls}\right)$ is admittance of inductance in transform domain.

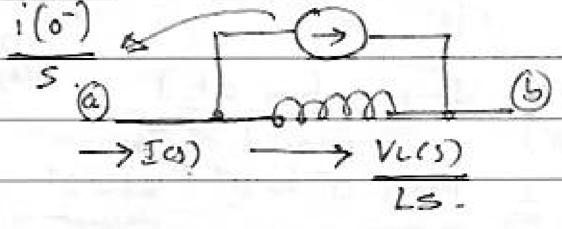
In transform domain

w.r.t eq^n (1)



$i(0^-)$ is independent of $I(s)$.

w.r.t eq^n (2)



(Ls) – generalised impedance of inductance

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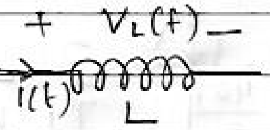
Transform Impedence and admittance

a) **Inductance** : Inductance of L Henry carry current $i(t)$ and voltage a/c if $V_L(t)$ [indued voltage].



W.K.T

$$V_L(t) = L \cdot \frac{di}{dt} \quad \text{—(1)}$$

Laplace transform of $V_L(t)$

$$\mathcal{L}(V_L(t)) = L\,\mathcal{L}\left(\frac{di}{dt}\right)$$

$$= L\left[sI(s) - I(0^-)\right]$$

$$\boxed{V_L(s) = LsI(s) - L\,I(0^-)} \quad \text{—(2)}$$

from (2)

$$LsI(s) = V_L(s) + L \cdot I(0^-)$$

$$I(s) = \frac{V_L(s) + L\,I(0^-)}{Ls}$$

$$\boxed{I(s) = \frac{V_L(s)}{Ls} + \frac{I(0^-)}{s} = I_1(s) + I(0^-)/s} \quad \text{—(3)}$$

where $I(s)$ LT of $i(t)$

In eq² (2) & (3) $\quad V_L(s)$ LT of $V_L(t)$ and

$i(0^-)$ is a constant, does not depend on Impedence $\quad i(0^-)$ - Initial current inductor.

With Initial cond² of Inductor known $V_L(s)$, $I_0(s)$ can be computed

Provided initial cond² are zero. ie $i(0^-) = 0$ eq² (2) (3) reduce

$$\left.\begin{aligned} V_L(s) &= LsI(s) \\ I(s) &= \frac{V_L(s)}{Ls} \end{aligned}\right\} \quad \text{—(4)}$$

If current $I(s)$ flows through inductor of L Henry, then, $V_L(s)$ is Ls times $I(s)$ where Ls is transform from impedence of inductance

||ly If $V_L(s)$ is voltage a/c inductor of L Henry then, $I(s)$ is $\left(\frac{1}{Ls}\right)$ times $V_L(s)$ where $\frac{1}{Ls}$ is transform form of admitance of inductance.

Impedence of Inductor $= Ls$ & Admitance of inductor $= \frac{1}{Ls}$

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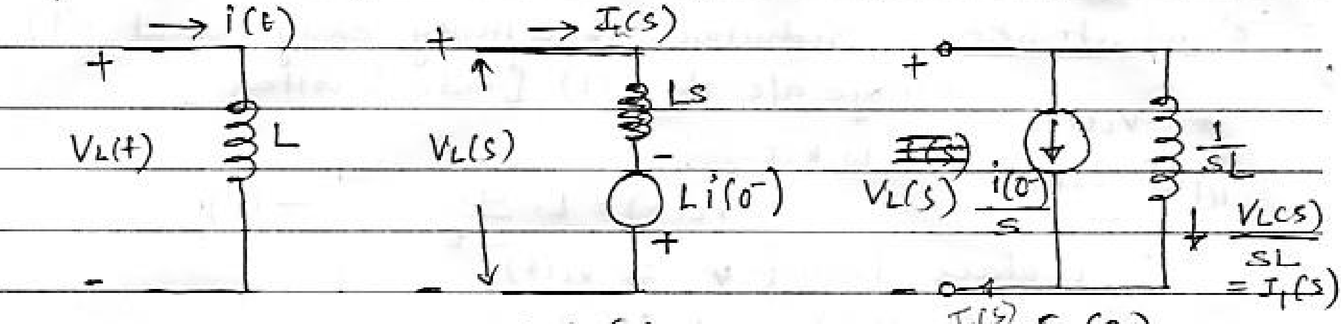
Eq.(2) and (3) representation using transform diagrams.



Eq² (2) Eq(3)

$$V_L(s) = I_L(s)\,Ls + L\cdot i(0^-) \qquad I_L(s) = \frac{V_L(s)}{Ls} + \frac{i(0^-)}{s}$$

b) <u>Capacitor</u> : A capacitor with capacitance of C Farads, having $V_c(t)$ as the voltage a/c it with charge $q(t)$

$$V_c(t) = \frac{q(t)}{c} = \frac{q(0^-) + \int_{0^-}^{t} i\,dt}{c}$$

$$= \frac{q(0^-)}{c} + \frac{1}{c}\int_{0^-}^{t} i\cdot d\tau$$

$$\left[V_c(t) = V_c(0^-) + \frac{1}{C}\int_0^t i\cdot dt \right] \quad -(1)$$

$V_c(0^-)$ — is a constant

Voltage due to initial charges on cap at $t=0^-$ (constant)

definite integral Extra charge deposited after $t=0^-$

Now LT on eq² (1)

$$V_c(s) = \frac{V_c(0^-)}{s} + \frac{1}{c}\left(\frac{I(s)}{s}\right)$$

$\frac{1}{cs}$ Impedence of Cap.

$$\boxed{V_c(s) = \frac{I(s)}{cs} + \frac{V_c(0^-)}{s}} \quad -(4)$$

Initial voltage

Now, $I(s)$ can be derived

$$\frac{I(s)}{cs} = V_c(s) - V_c(0^-)/s$$

$$I(s) = Cs\left[V_c(s) - \frac{V_c(0^-)}{s}\right]$$

$$\boxed{I(s) = Cs\,V_c(s) - \cancel{s}C\cdot V_c(0^-)} \quad -(5)$$

admitance of Capacitor

Initial current

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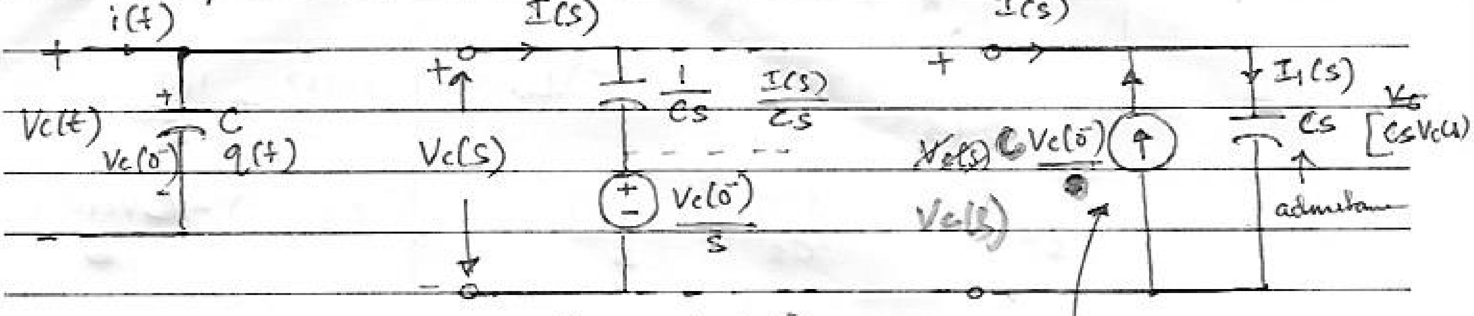
Eq² (2) and (3) can be pictorially represented as transform diagram

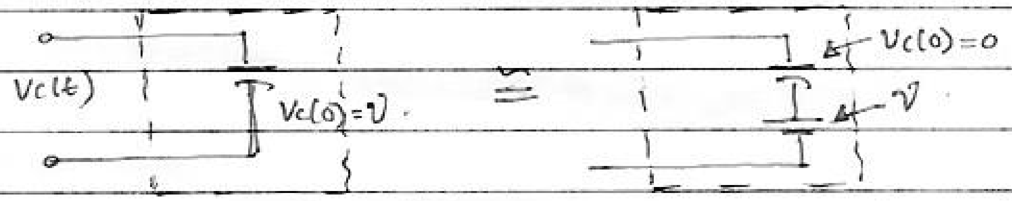


$$V_c(s) = \frac{I(s)}{Cs} + \frac{V_c(0^-)}{s}$$

Voltage a/c cap aft $t=0^-$ — Initial voltage a/c cap due to initial charg

(Cs) - Admittance of Capacitor

$(1/Cs)$ is impedence of cap.

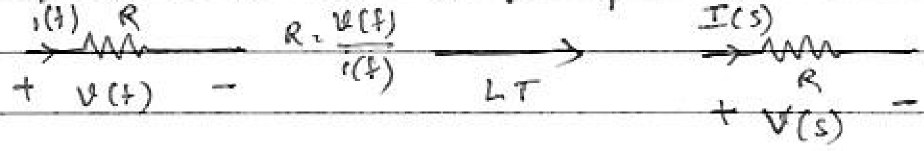


Capacitor with Initial voltage $V_c(0)=V$ is ≅ Capacitor with initial voltage $V_c(0)=0$ with a series connected voltage source $= V$.

→ In transform domain

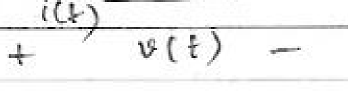
If 0^- conditions are known, substitution of 0^- conditions the response can be found directly as it takes care of transition from 0^- to 0^+ automatically. ∴ There is no need to know condⁿ at 0^+ if 0^- condⁿ are known.

c) Resistor : Resistor with resistance R with voltage $v(t)$ a/c it having $i(t)$ flowing through it



$R = \frac{v(t)}{i(t)}$ $\qquad$ $R = \frac{V(s)}{I(s)}$

$$\mathcal{L}\left.\frac{v(t)}{i(t)}\right|_{\text{zero initial cond}^n} = \frac{V(s)}{I(s)} = \begin{cases} R & \text{if } X = \text{Resistor} \\ Ls & \text{if } X = \text{Inductor} \\ \frac{1}{Cs} & \text{if } X = \text{Capacitor} \end{cases}$$



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Element	Impedance	Admittance	V(s)	I(s)
R	R	$\frac{1}{R}$	$I(s)R$.	$V_R(s)/R$.
L	Ls	$\frac{1}{Ls}$	$I(s)Ls - i(0^-).L$	$\frac{V_L(s)}{Ls} + \frac{i(0^-)}{s}$
C	$\frac{1}{Cs}$	Cs	$\frac{I(s)}{Cs} + \frac{V_c(0^-)}{s}$	$Cs\,V_c(s) - CV_c(0^-)$

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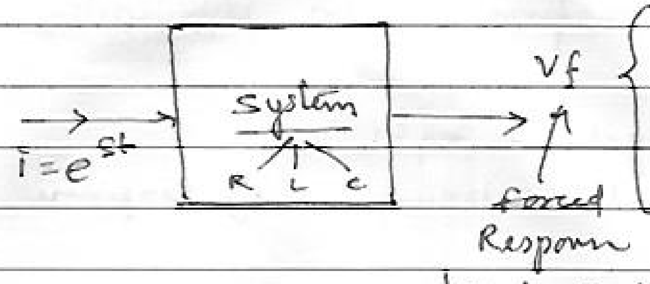
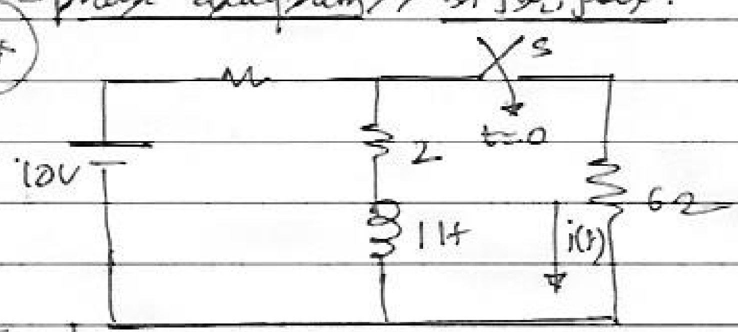


$V_f \begin{cases} R \cdot e^{st} & \text{if system is pure } R \\ Ls \cdot e^{st} & \text{if syst is pure } L \\ \frac{1}{Cs} e^{st} & \text{if " " " } C. \end{cases}$

due to current excitation.

- phase diagram for R, jwL, jwC.

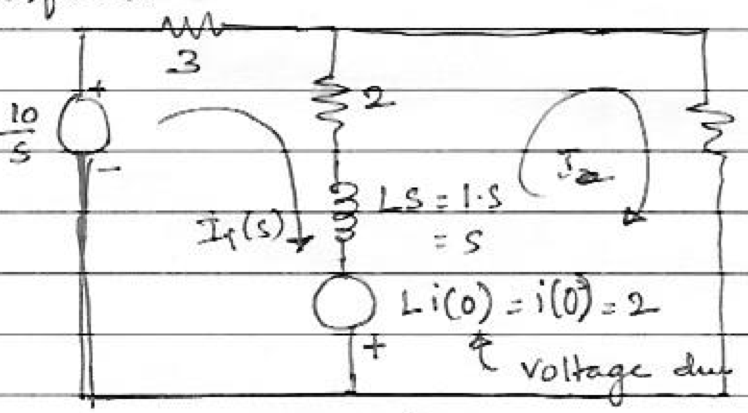
Ex



S - open for long time & closed at t = 0

$i(t)$ for $t \geq 0$.

Transform ckt



voltage due to initial current

Initial current when switch is open $= \frac{10}{3+2} = 2A$

$\cancel{3} I(s) + \cancel{2}[I_1(s) - I_2(s)$

$3 I_1(s) + (2+s)[I_1(s) - I_2(s)] - 2 - \frac{10}{s} = 0$

$3 I(s) + 2 I_1(s) - 2 I_2(s) + s I_1(s) - s I_2 = \frac{10}{s} + 2$

$\cancel{s I_1(s)}$

$$\left[(5+s) I_1(s) - (2+s) I_2(s) = \frac{10}{s} + 2 \right] \quad (1)$$

$(2+s)[I_2(s) - I_1(s)] + 6 I(s) + 2 = 0$

$$\left[(8+s) I(s) - (2+s) I_1(s) = -2 \right] \quad (2)$$

By solving

$I(s) = \frac{20/s + 4}{9(s-4)} = \frac{4s+20}{9s(s+4)}$

Using partial fraction $\frac{5/9}{s} - \frac{1/9}{s+4}$.

$$\boxed{i(t) = 5/9 - 1/9 \, e^{-4t}}$$

S - complex variable

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Step 1 - Setup the transform diagram with initial cond: replaced by sources.

Step 2 - Set up algebraic eq^ns.

Step 3 - Get the laplace transform of Response quantity to be found.

Step 4 - ILT to get Response in time domain

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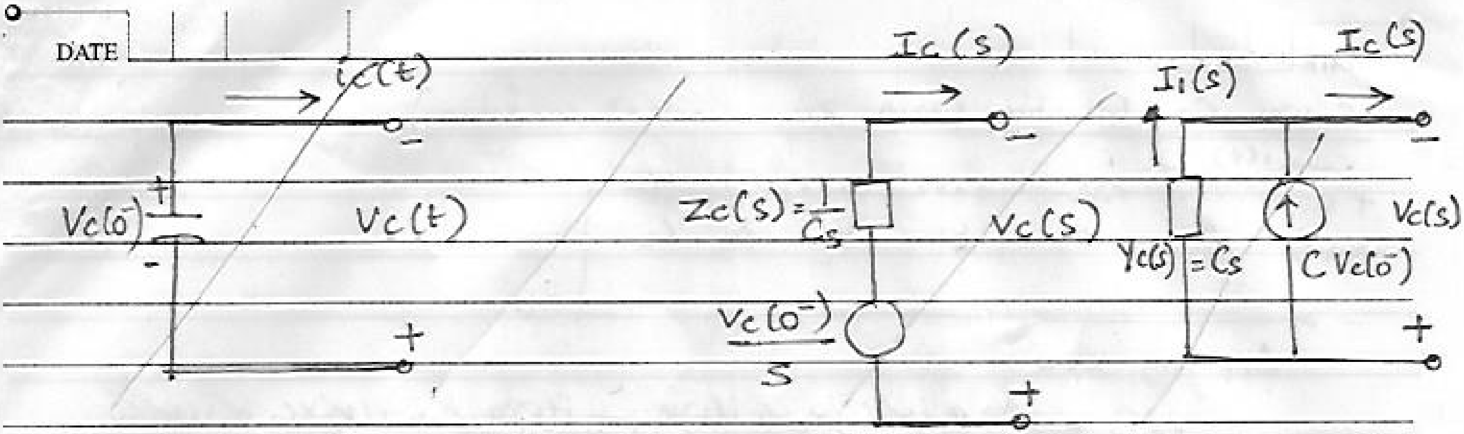
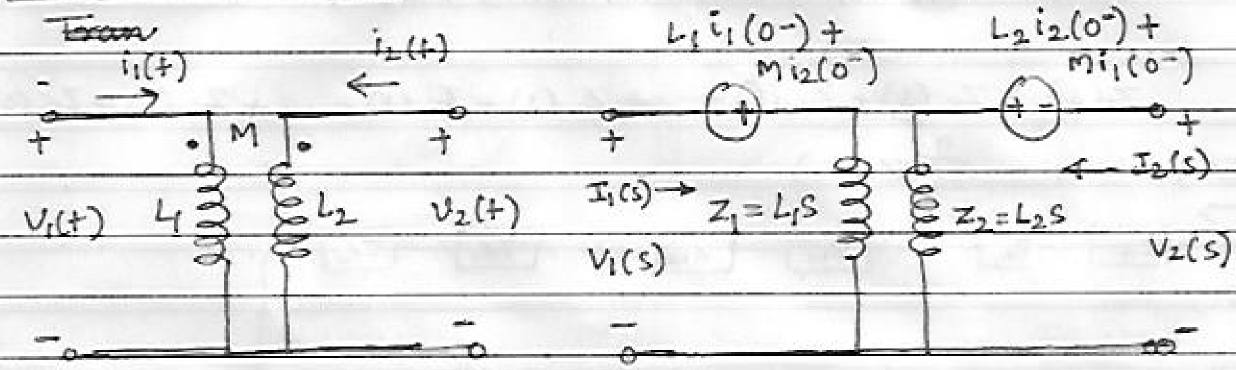


Mutual Coupled Coils



$$V_1(t) = L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt}$$

$$V_1(s) = L_1\left(sI_1(s) - i_1(0^-)\right) + M\left(sI_2(s) - i_2(0^-)\right)$$

$$= MsI_2(s) - Mi_2(0^-) + L_1 sI_1(s) - L_1 i_1(0^-)$$

lly

$$V_2(t) = L_2 \cdot \frac{di_2(t)}{dt} + m \cdot \frac{di_1(t)}{dt}$$

$$V_2(s) = L_2\left(sI_2(s) - i_2(0^-)\right) + m\left(sI_1(s) - i_1(0^-)\right)$$

$$= MsI_1(s) - mi_1(0^-) + L_2 sI_2(s) - L_2 i_2(0^-)$$

Remarks

- For single elements, under zero initial conditions, the transform impedence is the ratio of the transform of the element voltage to the transform of the element current

$$Z(s) = \frac{\mathcal{L}[V(t)]}{\mathcal{L}(i(t))} = \frac{V(s)}{I(s)}$$

- Under zero initial conditions, the reciprocal ratio transform admittance
- Initial conditions are represented by transform voltage sources or current sources.

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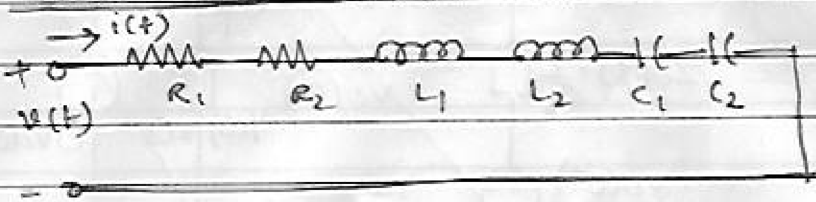
Series Combination (with zero initial condition)



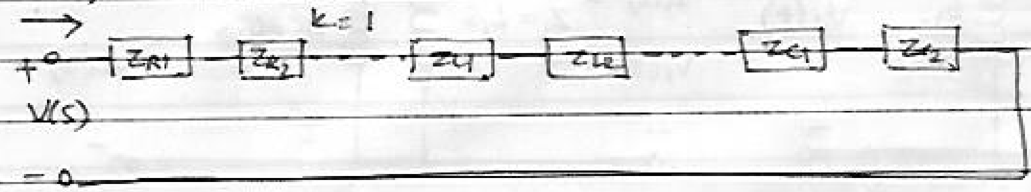
Applying KVL

$$V(t) = i(t)R_1 + i(t)R_2 + i(t)XL_1 + i(t)XL_2 + i(t)XC_1 + i(t)XC_2$$

$$V(s) = V_{R_1}(s) + V_{R_2}(s) + V_{L_1}(s) + V_{L_2}(s) + \cdots + V_{C_1}(s) + V_{C_2}(s)$$

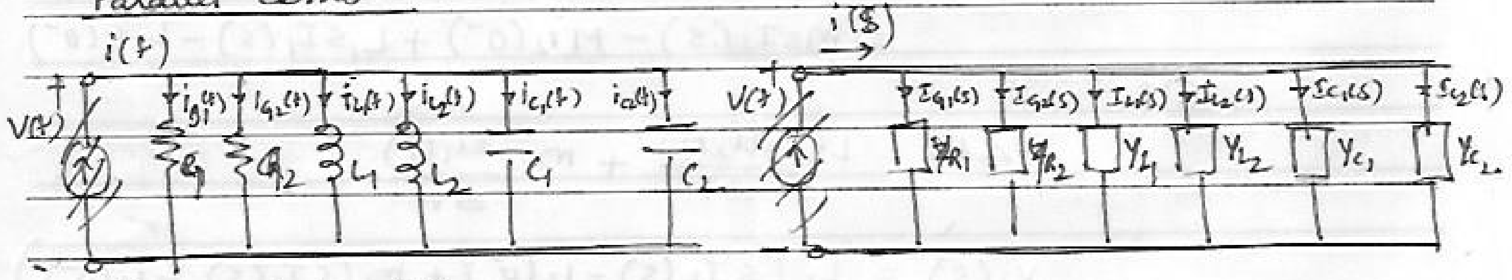
$$Z(s) = Z_{R_1}(s) + Z_{R_2}(s) + \cdots + Z_{L_1}(s) + Z_{L_2}(s) + \cdots + Z_{C_1}(s) + Z_{C_2}(s)$$

$$= \sum_{k=1}^{n} Z_k(s)$$



$$\therefore V(s) = I(s)\sum_{k=1}^{n} Z_k(s) = I(s)\left(Z_{R_1}(s) + Z_{R_2}(s) + \cdots + Z_{L_1}(s) + Z_{L_2}(s) + \cdots Z_{C_1}(s) + Z_{C_2}(s)\right)$$

Parallel comb^n



By applying KCL

$$i(t) = I_{G_1}(t) + I_{G_2}(t) + i_{L_1}(t) + i_{L_2}(t) + i_{C_1}(t) + i_{C_2}(t)$$

Corresponding transform eq^n

$$I(s) = I_{G_1}(s) + I_{G_2}(s) + I_{L_1}(s) + I_{L_2}(s) + I_{C_1}(s) + I_{C_2}(s)$$

$$= \frac{V(s)}{Z_{R_1}(s)} + \frac{V(s)}{Z_{R_2}(s)} + \frac{V(s)}{Z_{L_1}(s)} + \frac{V(s)}{Z_{L_2}(s)} + \frac{V(s)}{Z_{C_1}(s)} + \frac{V(s)}{Z_{C_2}(s)}$$

$$I(s) = V(s)\left[Y_{G_1}(s) + Y_{G_2}(s) + \cdots + Y_{L_1}(s) + Y_{L_2}(s) + \cdots + Y_{C_1}(s) + Y_{C_2}(s)\right]$$

$$Y(s) = Y_{G_1}(s) + Y_{G_2}(s) + \cdots + Y_{L_1}(s) + Y_{L_2}(s) + \cdots + Y_{C_1}(s) + Y_{C_2}(s)$$

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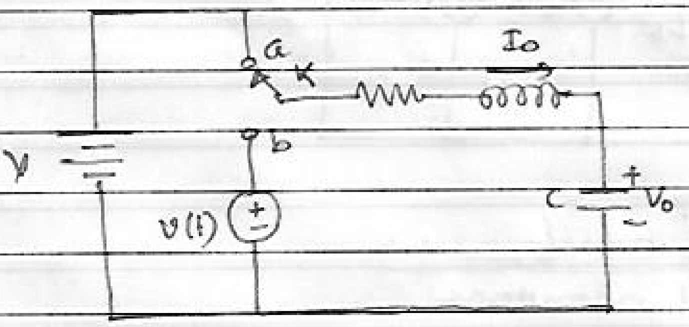
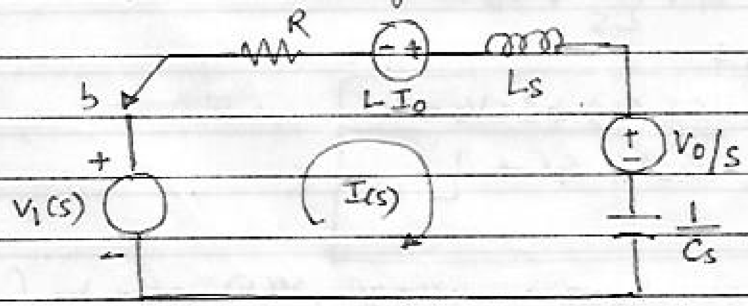
- An immittance is an impedence or an admittance.
- A network function relates currents or voltage at different parts of the n/w.
- Network functions are therefore immittance or voltage gain or current gain.
- A transfer function is a network function under zero ~~inf~~ initial cond^{n}.

Ex(1) : Switch K is ~~closed~~ held in position a until such time that a current I_0 flows in the inductor and the capacitor is charged to voltage V_0. At that instant, K is thrown to position b, connecting to a voltage source $v(t)$. Find $I(s)$ and so $i(t)$.



Equivalent transform n/w with the conditions given with transform impedences.



$$I(s) = \frac{\text{total transform voltage}}{\text{total impedence}} = \frac{V(s)}{Z(s)}$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V_1(s) + LI_0 - V_0/s}{R + Ls + \frac{1}{Cs}}$$ Xply Nr & Dr by s.

$$\boxed{I(s) = \frac{sV_1(s) + LI_0 s - V_0}{\left(Ls^2 + Rs + 1/C\right)}} \quad -(1)$$

By using partial fractions corresponding $i(t)$ by ILT.
This sol^{n} has been found without writing differential eq^{n} of system and automatically incorporates the required initial cond^{n}

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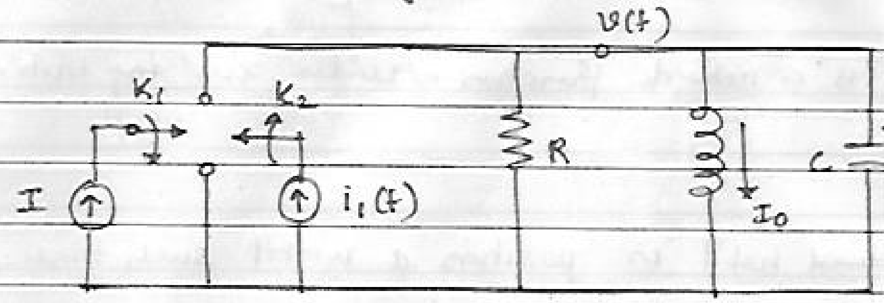
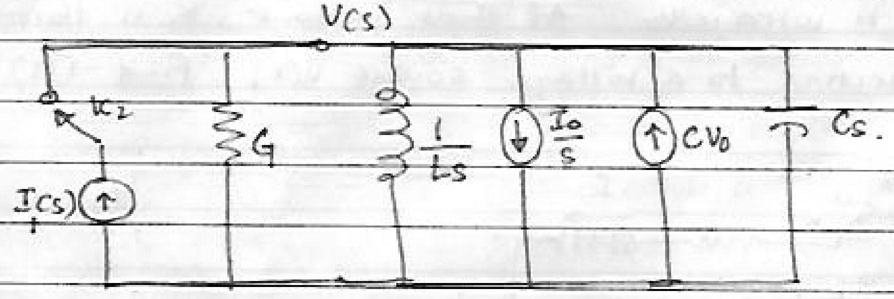
(2) Switch K_1 is opened at the instant when inductor current is I_0 and capacitor is charged to V_0. At the same instant, $t=0$, the switch K_2 is closed. It is required to find the transform of node voltage $V(s)$ so that $v(t)$ can be derived



Transformed n/w



$$V(s) = \frac{I_0(s)}{Y(s)} = \frac{\text{total transform current}}{\text{total admittance}}$$

$$= \frac{I_1(s) + CV_0 - I_0/s}{G + \frac{1}{Ls} + Cs}$$

Xply Nr & Dr by s

$$\boxed{V(s) = \frac{sI_1(s) + CsV_0 - I_0}{Cs^2 + Gs + \frac{1}{L}}}$$

The corresponding time domain voltage, $v(t)$ can be found by ILT.

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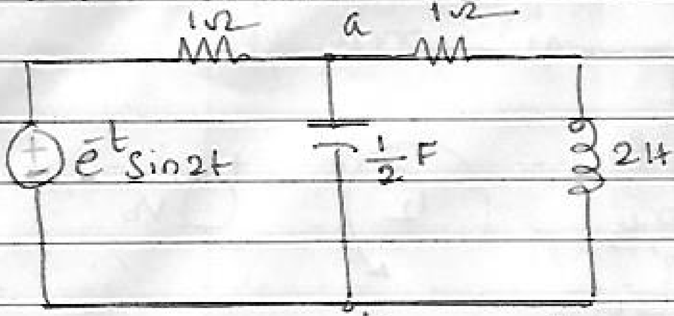
(3)



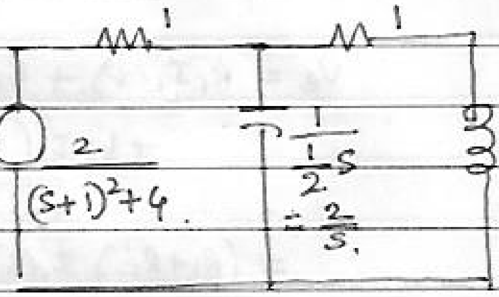
Assume network is initially relaxed (no current, no charge) and switch was closed at $t=0$. find $i(t)$ by finding $I(s)$.

$Z_{RL} = 1+2s \quad Y_{RL} = \frac{1}{1+2s}$

$Y_{ab} = Y_C + Y_{RL} \qquad \left(Z_C = \frac{1}{Cs} = \frac{1}{\frac{1}{2}s} \quad Y_C = \frac{1}{2}s \right)$

$= 0.5s + \left(\frac{1}{1+2s}\right) = \frac{1}{2}s + \frac{1}{1+2s} = \frac{s(1+2s)+2}{2(1+2s)}$

$Y_{ab}(s) = \frac{2s^2+s+2}{2(1+2s)}$



Z_{ab} is reciprocal

$Z_{ab}(s) = \frac{2(1+2s)}{2s^2+s+2}$

$Z_{total}(s) = Z_{ab}(s) + 1$

$= \frac{2(1+2s)}{2s^2+s+2} + 1 = \frac{2+4s+2s^2+s+2}{2s^2+s+2}$

$Z_{total}(s) = \frac{2s^2+5s+4}{2s^2+s+2}$ ✓

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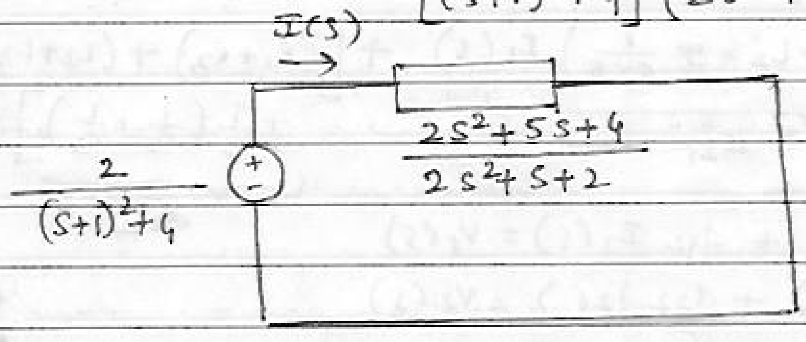
$I(s) = \frac{V(s)}{Z(s)} = V(s) \cdot Y(s)$

$V(s) = \frac{2}{(s+1)^2+(2)^2} = \frac{2}{(s+1)^2+4}$

$\therefore I(s) = \frac{2}{[(s+1)^2+4]} \times \frac{2s^2+s+2}{(2s^2+5s+4)}$



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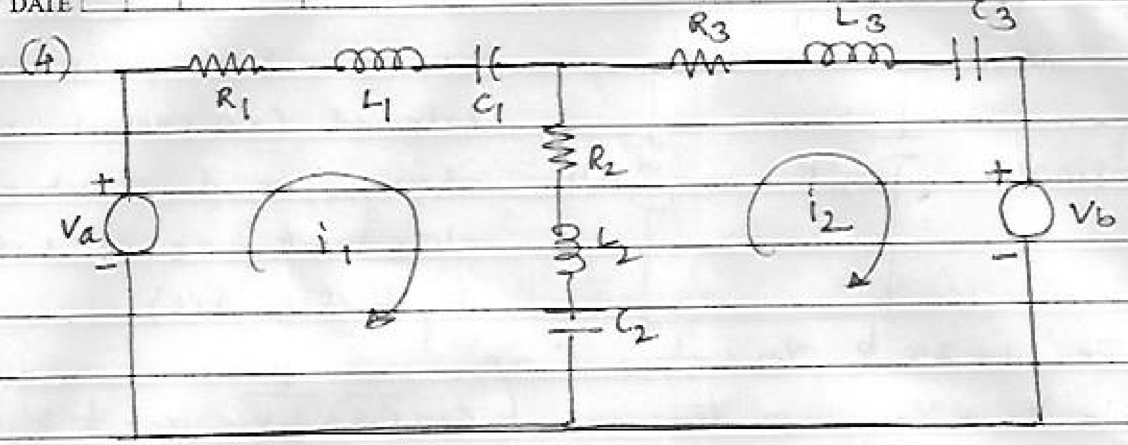

KVL @ loop1

$$V_a = R_1 I_1(s) + L_1 s I_1(s) + \frac{1}{C_1 s} I_1(s) + R_2 \left(I_1(s) - I_2(s)\right) + L_2 s \left(I_1(s) - I_2(s)\right) + \frac{1}{C_2 s}\left(I_1(s) - I_2(s)\right)$$

$$= (R_1 + R_2) I_1(s) + s(L_1 + L_2) I_1(s) + \frac{1}{s}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) I_1(s)$$

$$- R_2 I_2(s) - L_2 s I_2(s) - \frac{1}{C_2 s} I_2(s)$$

$$V_a = \underbrace{\left[(R_1 + R_2) + s(L_1 + L_2) + \frac{1}{s}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]}_{a_{11}} I_1(s)$$

$$+ \underbrace{\left[-R_2 \mp L_2 s \mp \frac{1}{C_2 s}\right]}_{a_{12}} I_2(s)$$

KVL @ loop2

$$-V_b = R_3 I_2(s) + L_3 s I_2(s) + \frac{1}{C_3 s} I_2(s) + \frac{1}{C_2 s}\left(I_2(s) - I_1(s)\right)$$

$$+ L_2 s \left(I_2(s) - I_1(s)\right) + R_2 \left(I_2(s) - I_1(s)\right)$$

$$-V_b = + \underbrace{\left(-R_2 \mp L_2 s \mp \frac{1}{C_2 s}\right)}_{a_{21}} I_1(s) + \underbrace{\left[(R_1 + R_3) + (L_2 + L_3)s + \frac{1}{s}\left(\frac{1}{C_2} + \frac{1}{C_3}\right)\right]}_{a_{22}} I_2(s)$$

$$a_{11}(s) I_1(s) + a_{12} I_2(s) = V_1(s)$$

$$a_{21}(s) I_1(s) + a_{22} I_2(s) = V_2(s)$$

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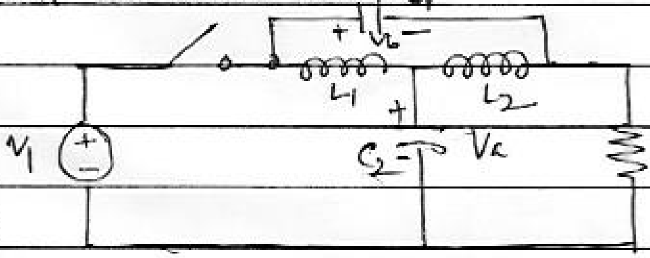
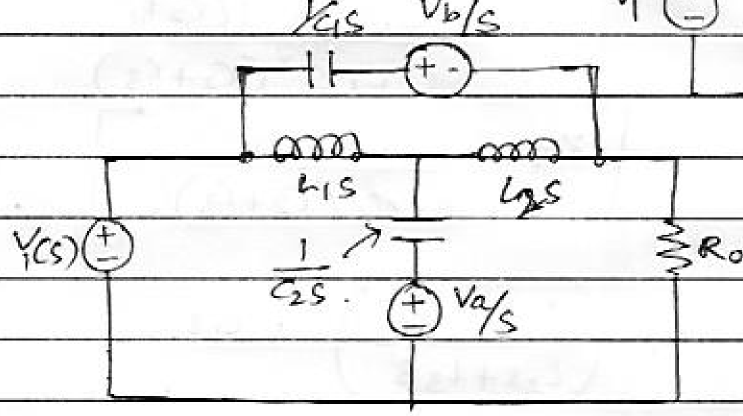
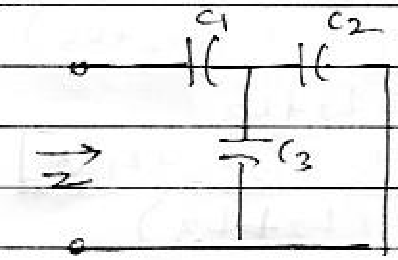
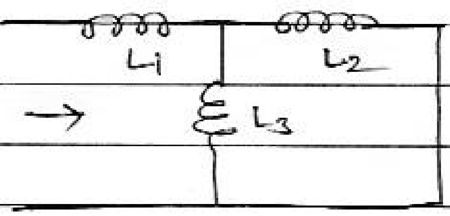
9.6) In the n/w of the figure, the switch K is closed at $t=0$, and at $t=0^-$ the indicated voltages are on the two capacitors. Draw the transform n/w for the analysis on the loop basis, representing all elements and all initial cond^ns

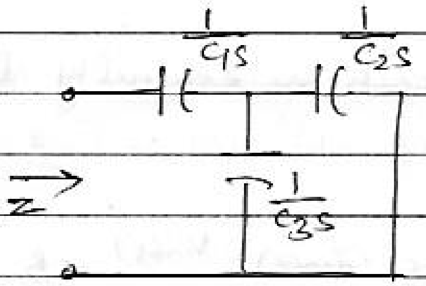




9.7) Determine the transform impedence for the two networks shown







$$\cancel{Z_{eq} = \frac{\left(\frac{1}{C_2s}+\frac{1}{C_3s}\right)\times \frac{1}{C_1s}}{\frac{1}{C_1s}+\frac{1}{C_2s}+\frac{1}{C_3}}}$$

$$\cancel{Z_{eq} = \frac{\frac{1}{C_1C_2s^2}+\frac{1}{C_1C_3s^2}}{\frac{1}{C_1s}+\frac{1}{C_2s}+\frac{1}{C_3s}}}$$

$$\frac{\frac{1}{C_2s}\times\frac{1}{C_3s}}{\frac{1}{C_2s}+\frac{1}{C_3s}} + \frac{1}{C_1s} = \frac{\frac{1}{C_2C_3s^2}}{\frac{C_2+C_3}{C_2C_3s}} + \frac{1}{C_1s}$$

$$= \frac{1}{s(C_2+C_3)} + \frac{1}{C_1s}.$$

$$= \frac{C_1s + s(C_2+C_3)}{C_1s^2(C_2+C_3)}$$

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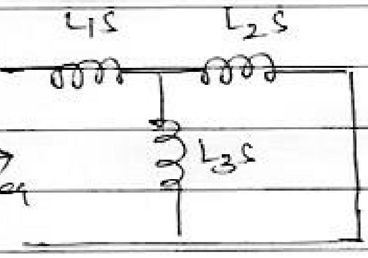
$$Z_{eq} = \left(\frac{\frac{1}{C_2 S} * \frac{1}{C_3 S}}{\frac{1}{C_2 S} + \frac{1}{C_3 S}}\right) + \frac{1}{C_1 S} = \frac{\frac{1}{\cancel{C_2 C_3} S^{\cancel{2}}}}{\frac{C_2 + C_3}{\cancel{C_2 C_3} \cancel{S}}} + \frac{1}{C_1 S}$$

$$= \frac{1}{S(C_2+C_3)} + \frac{1}{C_1 S}$$

$$= \frac{C_1 \cancel{S} + \cancel{S}(C_2+C_3)}{C_1 S^{\cancel{2}}(C_2+C_3)}$$

$$\boxed{Z_{eq} = \frac{C_1 + C_2 + C_3}{C_1 (C_2+C_3) S}}$$



$$Z_{eq} = \left(\frac{L_2 L_3 S^{\cancel{2}}}{L_2 \cancel{S} + L_3 \cancel{S}}\right) + L_1 S.$$

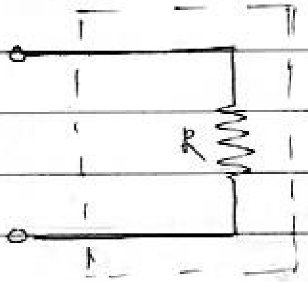
$$= \frac{L_2 L_3 S}{L_2 + L_3} + L_1 S$$

$$= \frac{S L_2 L_3 + S L_1 (L_2 + L_3)}{L_2 + L_3}.$$

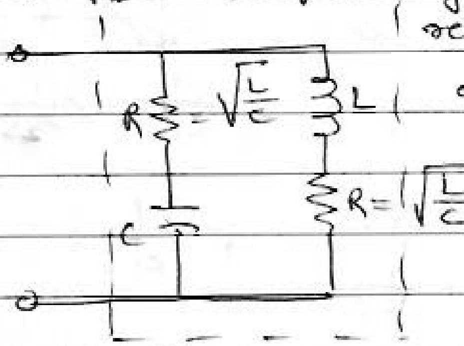
$$= \frac{S[L_1 L_2 + L_2 L_3 + L_1 L_3]}{(L_2 + L_3)}$$

9.12) Two block blocks with two terminals each are externally identical. It is known that one box contains n/w shown (a) and other contains n/w shown as in (b) with $R = \sqrt{\frac{L}{C}}$.

(a) i) show that the input impedence ; $Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = R$ for both networks. (ii) Investigate the possibility of distinguishing the purely resistive n/w. Any external cond^n may be made, initial of final cond^ns may be examined



(a)



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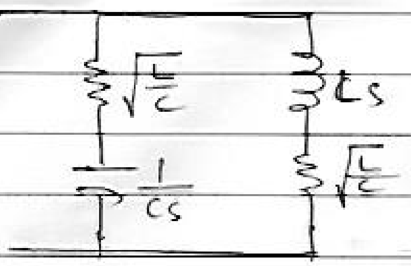


$$Z_{eq} = \frac{\left(Ls+\sqrt{\frac{L}{C}}\right) \times \left(\frac{1}{Cs}+\sqrt{\frac{L}{C}}\right)}{\left(Ls+\sqrt{\frac{L}{C}}\right) + \left(\frac{1}{Cs}+\sqrt{\frac{L}{C}}\right)}$$

$$= \frac{\left(Ls+\sqrt{\frac{L}{C}}\right)\left(\frac{1}{Cs}+\sqrt{\frac{L}{C}}\right)}{Ls+\frac{1}{Cs}+2\sqrt{\frac{L}{C}}}$$

$$= \frac{\frac{L\cancel{s}}{C\cancel{s}} + Ls\sqrt{\frac{L}{C}} + \frac{1}{Cs}\sqrt{\frac{L}{C}} + \frac{L}{C}}{Ls+\frac{1}{Cs}+2\sqrt{\frac{L}{C}}}$$

$$= \frac{2\frac{L}{C} + \sqrt{\frac{L}{C}}\left(Ls+\frac{1}{Cs}\right)}{Ls+\frac{1}{Cs}+2\sqrt{\frac{L}{C}}}$$

$$= \frac{2\sqrt{\frac{L}{C}}\sqrt{\frac{L}{C}} + \sqrt{\frac{L}{C}}\left(Ls+\frac{1}{Cs}\right)}{}$$

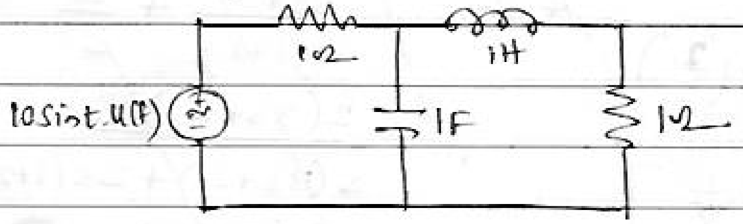
$$= \frac{\sqrt{\frac{L}{C}}\,\cancel{\left[Ls+\frac{1}{Cs}+2\sqrt{\frac{L}{C}}\right]}}{\cancel{\left(Ls+\frac{1}{Cs}+2\sqrt{\frac{L}{C}}\right)}}$$

$$= \sqrt{\frac{L}{C}}$$

$$= R$$

9.15) If the capacitors are uncharged and inductor current is zero at $t=0^-$, in the given n/w, show that the transform of generator current is

$$I_1(s) = \frac{10(s^2+s+1)}{(s^2+1)(s^2+2s+2)}$$



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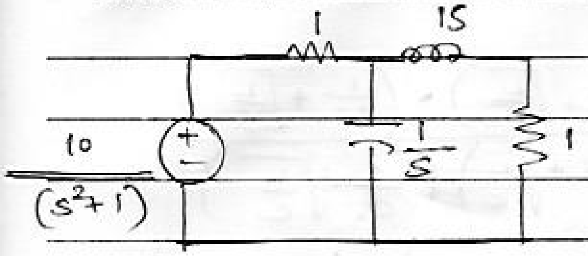
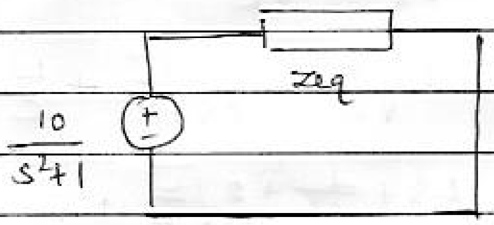


$$Zeq = \frac{(1+s)\times\frac{1}{s}}{(s+1)+\frac{1}{s}} + 1$$

$$= \frac{(s+1)}{(s^2+s+1)} + 1$$

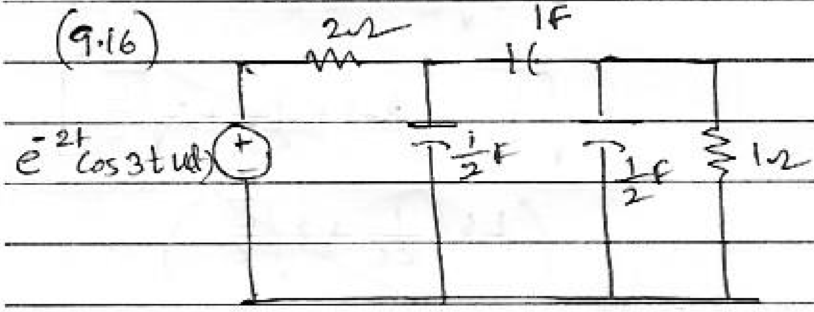
$$= \frac{(s+1)+(s^2+s+1)}{s^2+s+1}$$



$$Zeq = \frac{s^2+2s+2}{s^2+s+1}$$

$$\boxed{I(s) = \frac{10}{(s^2+1)} \times \frac{(s^2+s+1)}{(s^2+2s+2)}}$$

(9.16)



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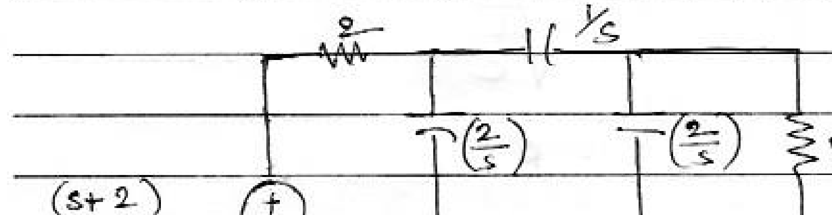
$$I(s) = \frac{s(s+2)(5s+6)}{(s^2+4s+13)(10s^2+18s+4)}$$

$\frac{2}{s}$ $\frac{2+s}{s}$ $\frac{2}{s}+1$



$$\cancel{Zeq = \frac{\left[\frac{(2/s)}{(2/s+1)} + \frac{1}{s}\right] \times \left(\frac{2}{s}\right)}{\left[\frac{(2/s)}{(2/s+1)} + \frac{1}{s}\right] + \left(\frac{2}{s}\right)} + 2}$$

$$\cancel{Zeq = \frac{\left[\left(\frac{2(s)+1(s+2)}{s(s+2)}\right) \times \frac{2}{s}\right]}{\left[\left(\frac{2s+(s+2)}{s(s+2)}\right) + \frac{2}{s}\right]} + 2}$$

$$\cancel{\frac{(2/s)}{(s+2)/s} + \frac{1}{s}}$$

$$\cancel{= \frac{2s+(s+2)}{s(s+2)}}$$

$$\cancel{= \frac{\frac{3s+2}{s(s+2)} \times \frac{2}{s}}{\frac{3s+2}{s(s+2)} + \frac{2}{s}}}$$

$$\cancel{= \frac{2(3s+2)}{s(3s+2)+2s(s+2)}}$$

$$\cancel{= \frac{2(3s+2)}{3s^2+2s+2s^2+4}}$$

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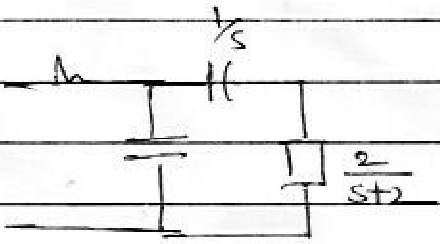
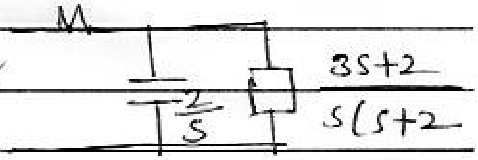
$$\to Z_1 \quad \frac{\frac{2}{s}\times 1}{\frac{2}{s}+1} = \frac{\left(\frac{2}{\cancel{s}}\right)}{\left(\frac{2+s}{\cancel{s}}\right)} = \frac{2}{s+2}$$



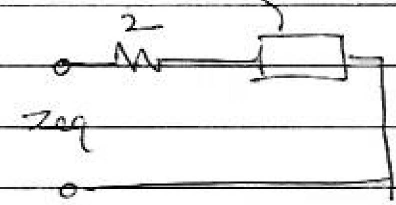
$$\to Z_2 \quad \frac{2}{(s+2)} + \frac{1}{s} = \frac{2s+s+2}{s(s+2)} = \frac{3s+2}{s(s+2)}$$



$$\to Z_3 \quad \frac{\frac{3s+2}{s(s+2)} \times \frac{2}{s}}{\frac{(3s+2)}{s(s+2)} + \frac{2}{s}} = \frac{(6s+4)/s^2(s+2)}{\frac{(3s+2)+2(s+2)}{s(s+2)}}$$

$$= \frac{6s+4}{\cancel{s^2}(\cancel{s+2})} \times \frac{\cancel{s}(\cancel{s+2})}{3s+2+2s+4}$$

$$= \boxed{\frac{6s+4}{s(5s+6)}}$$



$$\to Z_{eq} = \frac{6s+4}{s(5s+6)} + 2 = \frac{6s+4+2(5s+6)}{s(5s+6)}$$

$$Z_{eq} = \frac{6s+4+10s^2+12s}{s(5s+6)}$$

$$\boxed{Z_{eq} = \frac{10s^2+18s+4}{s(5s+6)}}$$

$$I(s) = \frac{(s+2)}{(s+2)^2+9} \times \frac{s(5s+6)}{10s^2+18s+4}$$

$$\boxed{I(s) = \frac{s(s+2)(5s+6)}{(s^2+4s+13)(10s^2+18s+4)}}$$

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For a L-mesh n/w with initial cond² sources, the sol² can be more generally written as

$$\sum_{j=1}^{L} a_{kj}(s)\, I_j(s) = V_k(s) \qquad k = 1, 2, \ldots L$$

Ex $L = 3$

$$\sum_{j=1}^{3} a_{kj}(s)\, I_j(s) = V_k(s) \qquad k = 1, 2, 3$$

$$\begin{array}{c} k=1 \\ k=2 \\ k=3 \end{array} \begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) \\ a_{31}(s) & a_{32}(s) & a_{33}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} \qquad — (1)$$

$L \rightarrow$ Number of loops

a_{kj} – Coefficient a_{kj} are impedences plus initial cond² sources.

Similarly for N-Node network with cond² sources, the sol² can be more generally written as

$$\sum_{j=1}^{N} b_{kj}(s)\, V_j(s) = I_k(s) \qquad k = 1, 2, \ldots N$$

Ex : $N = 3$

$$\sum_{j=1}^{3} b_{kj}(s)\, V_j(s) = I_k(s) \qquad k = 1, 2, 3$$

$$\begin{array}{c} k=1 \\ 2 \\ 3 \end{array} \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) \\ b_{31}(s) & b_{32}(s) & b_{33}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} \qquad — (2)$$

where N – Number of Nodes

b_{kj} – Coefficient b_{kj} are admittance plus initial condition sources

a_{kj}, b_{kj} are coefficients with initial cond² because C & L may be holding energy (inductor having current $I(0)$ & Cap having $V_c(0^-)$) which will be moved to RHS side ie Response Vector

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$\therefore$ Eq(1) can be redefined compactly as

$$[Z][I] = [V] + [V_0] = [V']$$ for Mesh analysis

$\uparrow$ Initial cond^n voltages

where $[Z]$ - Impedence matrix

$[I]$ - Loop current matrix $I_1, I_2, \ldots I_L$

$[V']$ - Sum of voltage sources (~~Independent~~ + (sources + Initial cond^n voltages)

$V_1 + V_{01}, V_2 + V_{02}, V \ldots V_L + V_{0L}$

$L = 3$

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s) & Z_{13}(s) \\ Z_{21}(s) & Z_{22}(s) & Z_{23}(s) \\ Z_{31}(s) & Z_{32}(s) & Z_{33}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_1'(s) \\ V_2'(s) \\ V_3'(s) \end{bmatrix}$$

$V_1'(s) = V_1(s) + V_{01}(s)$

$V_2'(s) = V_2(s) + V_{02}(s)$

$V_3'(s) = V_3(s) + V_{03}(s)$

$V_j'(s) = V_j(s) + V_{0j}(s)$ $\uparrow$ Initial cond^n

$\|^{ly}$ Eq^n(2) can be redefined compactly as

$$[Y][V] = [I] + [I_0] = [I']$$ for Node Analysis

where $[Y]$ - Admittance matrix

$[V]$ - Node voltage ~~matrix~~ vector $V_1, V_2, \ldots V_N$

$[I']$ - Sum of current sources (sources + Initial cond^n sou)

$N = 3$

$$\begin{bmatrix} Y_{11}(s) & Y_{12}(s) & Y_{13}(s) \\ Y_{21}(s) & Y_{22}(s) & Y_{23}(s) \\ Y_{31}(s) & Y_{32}(s) & Y_{33}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} I_1'(s) \\ I_2'(s) \\ I_3'(s) \end{bmatrix}$$

$I_1'(s) = I_1(s) + I_{01}(s)$

$I_2'(s) = I_2(s) + I_{02}(s)$

$I_3'(s) = I_3(s) + I_{03}(s)$

$$I_j'(s) = I_j(s) + I_{0j}(s)$$

$\uparrow$ transform of all current sources @ Node j

$\uparrow$ transf of all initial cond^n at Node j

~~If Δ - Determinant of $[Z]$, $L = 3$ the transform eq^n~~

$\rightarrow V_j'(s) = V_j(s) + V_{0j}(s)$

where $V_j'(s)$ — Voltage transform

$V_j(s)$ — transform all sources in loop j

$V_{0j}(s)$ — transform of all initial cond^n loop j.

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Now,

$$[Z][I] = [V'] \cancel{\neq [V']} \qquad , \qquad [Y][V] = [I']$$
$$= [V] + [V_0] \qquad\qquad\qquad = [I] + [I_0]$$

Let us consider mesh (L) = 3

$$\begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1' \\ V_2' \\ V_3' \end{pmatrix} \quad \text{—— (1)}$$

Each current is obtained by determining response to a voltage and then summing

$$\begin{pmatrix} Z_{11} & V_1 & Z_{13} \\ Z_{21} & V_2 & Z_{23} \\ Z_{31} & V_3 & Z_{13} \end{pmatrix} \qquad \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1' \\ 0 \\ 0 \end{pmatrix} \qquad \underbrace{\frac{I_{11}'}{\Delta_{11}}, \frac{I_{21}'}{\Delta_{12}}, \frac{I_{31}'}{\Delta_{13}}}_{\text{Partial responses}}$$

for I_2

$$\Delta_{12} = \begin{pmatrix} Z_{21} & Z_{23} \\ Z_{31} & Z_{33} \end{pmatrix} \qquad \begin{pmatrix} & & \\ & " & \\ & & \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ V_2' \\ 0 \end{pmatrix} \qquad \underbrace{\frac{I_{12}'}{\Delta_{21}} \; \frac{I_{22}'}{\Delta_{22}} \; \frac{I_{32}'}{\Delta_{23}}}$$

$$\Delta_{22} = \begin{pmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{pmatrix} \qquad \begin{pmatrix} & & \\ & " & \\ & & \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_3' \end{pmatrix} \qquad \underbrace{\frac{I_{13}'}{\Delta_{31}} \; \frac{I_{23}'}{\Delta_{32}} \; \frac{I_{33}'}{\Delta_{33}}}$$

$$\Delta_{32} = \begin{pmatrix} Z_{11} & Z_{13} \\ Z_{21} & Z_{23} \end{pmatrix}$$

(or)

Ex: $I_2(s) = I_{21}'(s) + I_{22}'(s) + I_{23}'(s)$

In eq^n (2),

$$\boxed{I_2(s) = \frac{\Delta_{12}}{\Delta_Z} V_1'(s) + \frac{\Delta_{22}}{\Delta_Z} V_2'(s) + \frac{\Delta_{32}}{\Delta_Z} V_3'(s)} \quad -(2)$$

$V_j'(s) = V_j(s) + V_{oj}(s)$ is summation of voltages, $V_j(s)$ being the transform of all the sources in loop j and $V_{oj}(s)$ being the transform voltage representation of all initial conditions in loop j. This summation, of course, must be made by taking into account the polarity of the various sources w.r.t to reference direction of the loop. We have assumed that any current source in the n/w is transformed into voltage source before eq^n were formulated

$$\cancel{\Delta_{12} = \begin{pmatrix} Z_{11} & V_1' & Z_{13} \\ Z_{21} & 0 & Z_{23} \\ Z_{31} & 0 & Z_{33} \end{pmatrix}} \qquad \cancel{\Delta_{22} = \begin{pmatrix} Z_{11} & 0 & Z_{13} \\ Z_{21} & V_2' & Z_{23} \\ Z_{31} & 0 & Z_{33} \end{pmatrix}} \qquad \cancel{\Delta_{32} = \begin{pmatrix} Z_{11} & 0 & Z_{13} \\ Z_{21} & 0 & Z_{23} \\ Z_{31} & V_3' & Z_{33} \end{pmatrix}}$$

Note: All the current sources are transformed into voltage sources

Eq^n (2), We observe that, each of the transform sources one at a time and then add the partial responses to get $I_2(s)$. This is in essence, the superposition principle (or) principle of linearity

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Two aspects of superposition are important in network analysis.

i) A given response in a network resulting from a number of independent sources (including initial cond² sources) may be computed by summing the response to each individual sources with all other sources ~~reduced~~ to made inoperative (reduced to zero voltage or zero current)
This statements described additivity property of linear n/w.

2) If all sources are multiplied by a constant, the response is multiplied by the same constant. This statement describes the property of homogeneity in a linear n/w.
Eq² (2) is evident for additivity & homogenity

$$I_2(s) = \frac{1}{\Delta_2}\left[\Delta_{12} V_1'(s) + \Delta_{22} V_2'(s) + \Delta_{32} V_3'(s)\right]$$

∴ Superposition is combined property of additivity and homogeneity, especially in n/w with initial conditions.

Procedure to find response in a element in n/w using Superposition principle

a) Name the terminals of branch/element when response to be found as A,B

b) Consider a independent source in n/w and make all other independent sources inoperative (deactivate)
 - Voltage sources are replaced by short circuit (S C) so that voltage strength is zero
 - Current sources are replaced by open circuit (O.C) so that current strength is zero.

 Dependent sources (controlled sources) must be left intact.

c) Using KVL / KCL / network reduction principle find the response due to active source.

d) ~~Rpe~~ Repeat ~~b, c~~ step (b), (c) for all other independent sources one by one

e) Sum the responses obtained by individual independent sources to get total response

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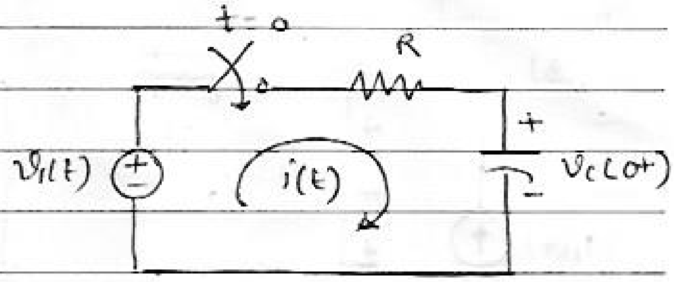
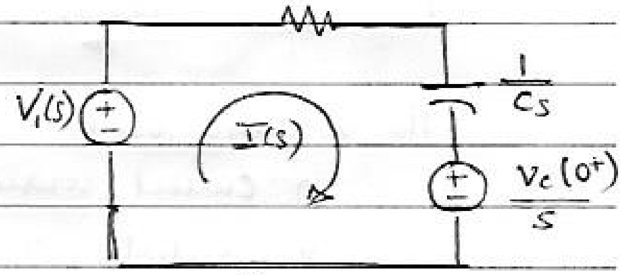
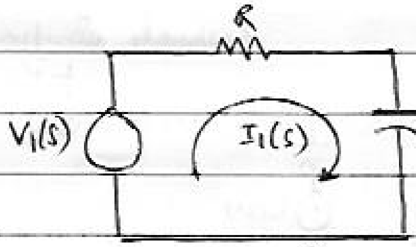
(2) Find $i(t)$ in the n/w using superposition thm. K is closed at $t=0$, where cap is initially has $V_c(0^+)$





Step 1:

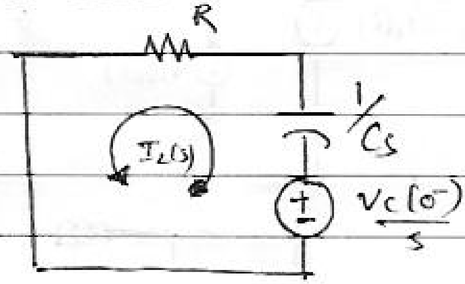
$$I_1(s) = \frac{V_1(s)}{R + 1/Cs} \quad -(1)$$



Step 2

$$R\,I_2(s) + I_2(s) \times \frac{1}{Cs} = -\frac{V_c(0^-)}{s}$$

$$I_2(s) = \frac{+V_0(0^+)/s}{R + 1/Cs} \quad -(2)$$



Sum of $I_1(s)$ & $I_2(s)$

$$I(s) = I_1(s) + I_2(s)$$

$$= \frac{V_1(s)}{R + 1/Cs} - \frac{V_c(0^+)/s}{R + 1/Cs}$$

$$\boxed{I(s) = \frac{V_1(s) - V_c(0^+)/s}{R + 1/Cs}} \quad -(3)$$

Taking ILT $i(t)$ can be found

- The Numerator shows that the response is due to the superimposition of the response due to each of the sources in network.
- Let $V_1(s) = V_0/s$ and $V_c(0) = V_0$ – Two sources are equal and opposite so $i(t) = 0$

If, instead $V_1(s) = 2V_0/s$ & $V_c(0) = V_0$ – then the superimposed responses added to

$$i(t) = \frac{V_0}{R} \cdot e^{-t/RC}$$

– homogeneity aspect of superposition

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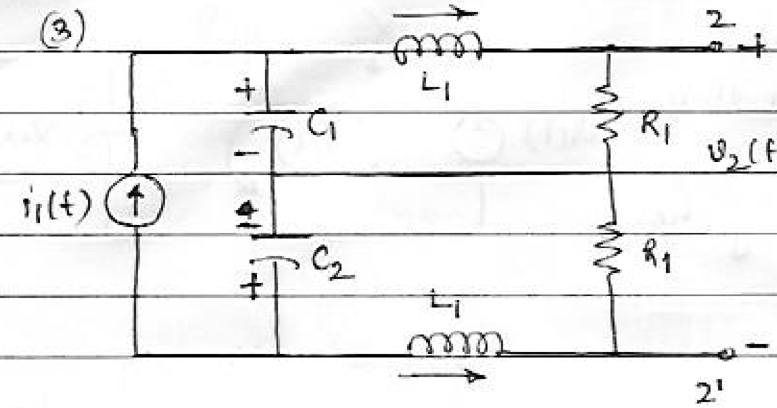
(3)

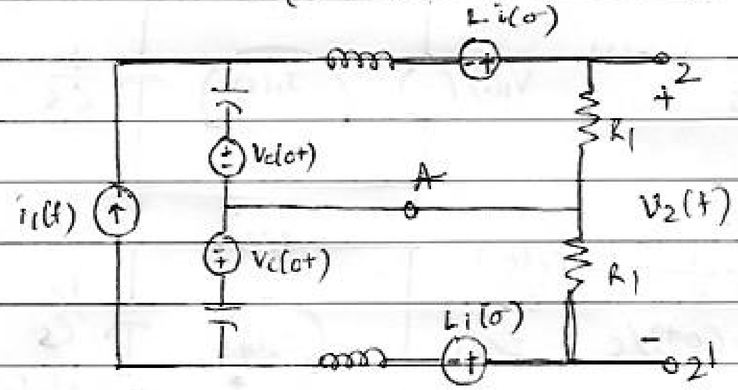


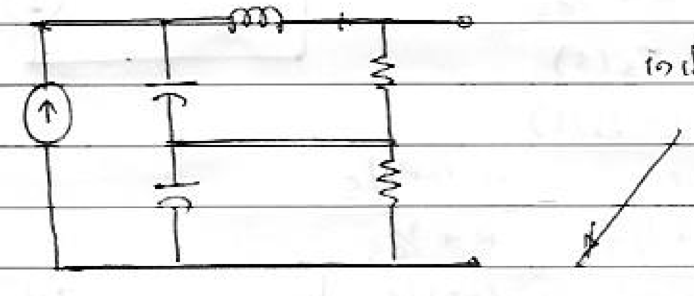
Initial charge on cap is same & current through inductor is same.

The response $v_2(t)$ is due to

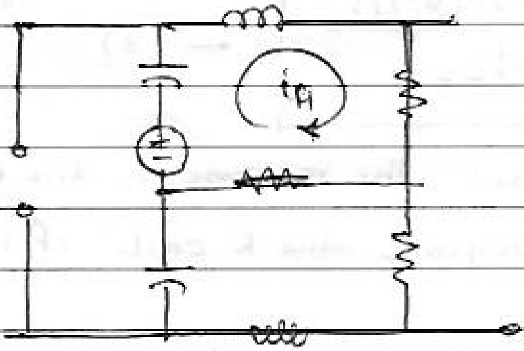
i) Current source $i_1(t)$ and

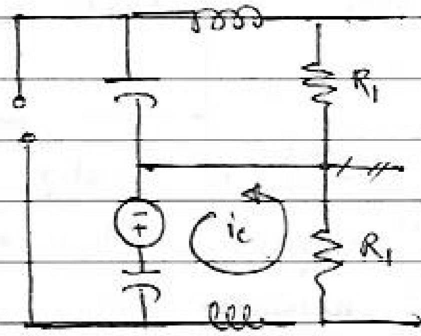
ii) Initial voltage a/c cap and current through inductor (which Initial cond^{n}) gives.





In this response due to initial cond^{n} will always cancel, and the response will depend only on the excitation from $i_1(t)$.





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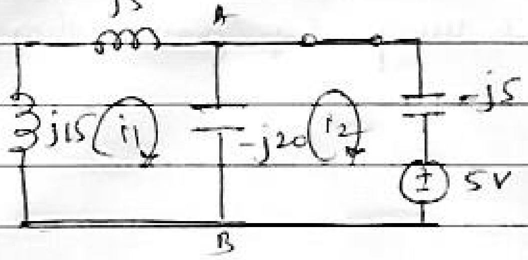


$j20 i_1 - j20(i_1 - i_2) = 0$

$+j20 i_2 = 0 \quad \therefore i_2 = 0.$

$-j20(i_2 - i_1) - j5 i_2 + 5 = 0$

$+j20 i_1 - j25 i_2 = -5$

$i_1 = \dfrac{-5}{j20} = +j0.25$

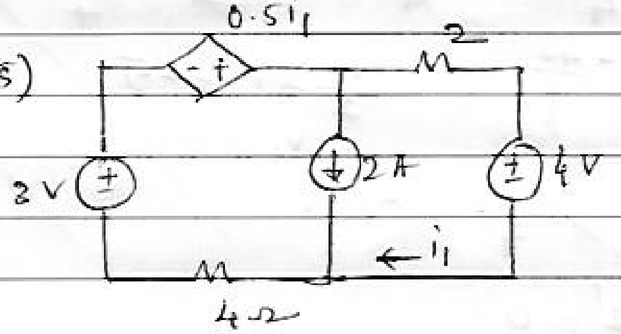
$V_{AB} = (i_1 - i_2) \times -j20$

$= j0.25 \times -j20$

$= 5V$

$V_{AB\,total} = 7.5 - 20 + 5 = \underline{\underline{-7.5\,V}}$ ✓

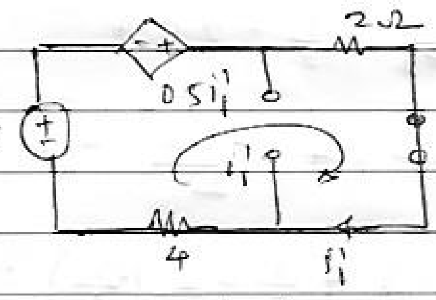
5) Find i_1 using superposition

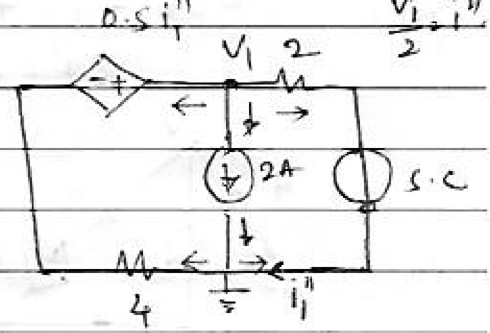


$\frac{V_1}{2} = i_1''$

$\frac{V_1}{4} = \frac{i_1''}{2}$

$\frac{V_1}{2} = i_1''$





$-3 - 0.5 i_1' + 6 i_1' = 0$

$5.5 i_1' = 3 \quad \boxed{i_1' = \cancel{2A}\ 0.545\,A}$

$\dfrac{V_1 - 0.5 i_1''}{4} + 2 + \dfrac{V_1}{2} = 0$

$\cancel{\dfrac{V_1}{4} - \dfrac{0.5 i_1''}{4} + 2 + \dfrac{V_1}{2} = 0}$

$\cancel{-(2 + i_1'') - \dfrac{0.5}{4} i_1'' + 2 + i_1'' = 0}$

$\cancel{0.875 i_1'' = -4}$

$\dfrac{V_1}{4} - \dfrac{0.5}{4} i_1'' + 2 + i_1'' = 0$

$\left(\because \dfrac{V_1}{2} = i_1''\right)$

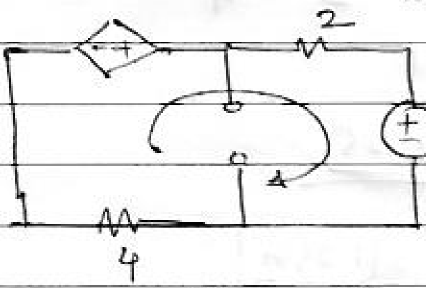
$\therefore \dfrac{i_1''}{2} - \dfrac{0.5}{4} i_1'' + 2 + i_1'' = 0$

$\therefore 1.375 i_1'' = -2$

$\boxed{i_1'' = -1.45}$

$\frac{V_1}{4} + \frac{V_1}{2}$

$\frac{3V_1}{4}$



$+4 + 4 i_1''' - 0.5 i_1''' + 2 i_1''' = 0$

$5.5 i_1''' = -4$

$i_1''' = -0.727$

$\therefore i_1 = 0.545 - 1.45 - 0.727$

$\boxed{i_1 = -1.632}$

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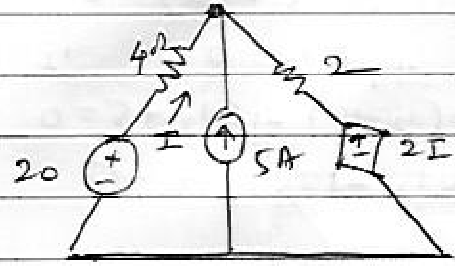
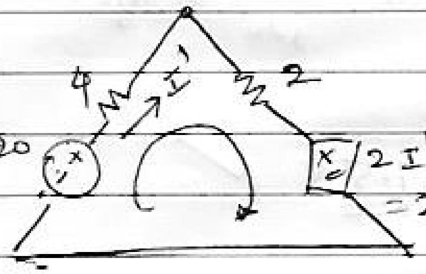
(7) For ckt in fig find current I using superposition theorem





$2I' = 2\times\left(\frac{V_1-20}{4}\right)$

$\frac{V_1}{2} - \frac{40}{4}$

$$\cancel{\frac{V_1-20}{4} + \frac{V_1 - 2\left(\frac{V_1-20}{4}\right)}{2} = 0}$$

$$\cancel{\frac{V_1}{4} - 5 + \frac{V_1}{2} - \frac{V_1}{4} + \frac{40}{4} = 0}$$

$$\cancel{\frac{V_1}{2} = -5}$$

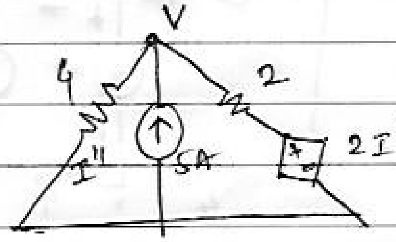
$$\cancel{V_1 = -10\ V}$$

$$\therefore \cancel{\frac{-10-20}{4} = \frac{-30}{4} = -7.5}$$

By KVL

$$-20 + 4I' + 2I' + 2I' = 0$$

$$8I' = 20 \qquad I' = \frac{20}{8} = \frac{5}{2} = 2.5\ A$$



$$\frac{V}{4} - 5 + \frac{V - 2I''}{2} = 0$$

$$\frac{V}{4} - 5 + \frac{V}{2} - \frac{2}{2}\left(\frac{-V}{4}\right) = 0$$

$$\frac{V}{4} - 5 + \frac{V}{2} + \frac{V}{4} = 0$$

$$\boxed{V = 5}$$

$$I'' = -\left(\frac{V}{4}\right) = -\left(\frac{5}{4}\right) = -1.25$$

$$I = I' + I'' = 2.5 - 1.25 = \boxed{1.25A}$$

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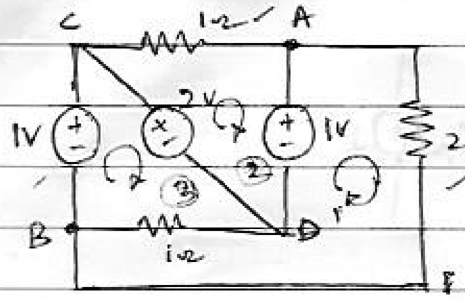
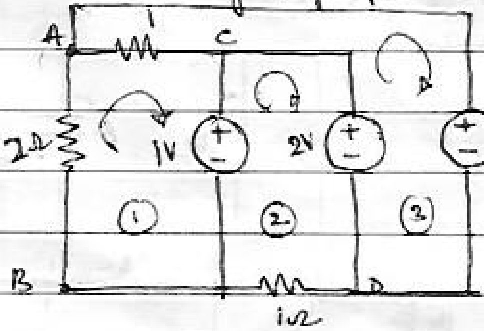
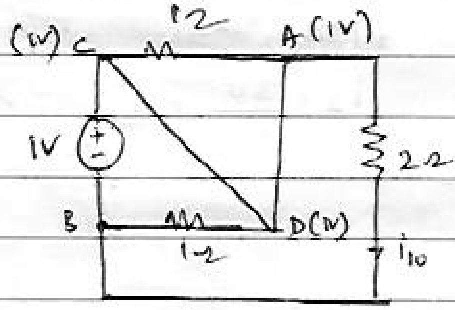
(3) Determine the current in 2Ω resistor using superposition thrm.







$$\begin{pmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \begin{pmatrix} +1 \\ 2 \\ -1 \end{pmatrix}$$

$i_{10} = \frac{1}{2} = 0.5$

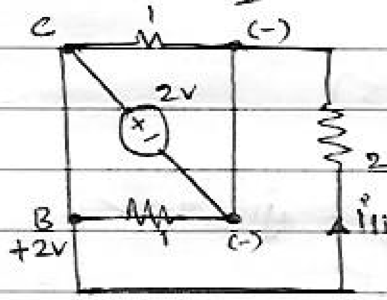
$i_{11} = 0.5$ $i_{21} = 0$ $i_{31} = +0.5$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$i_{2A} = 0$ $i_{22} = +1$ $i_{23} = 0$ $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

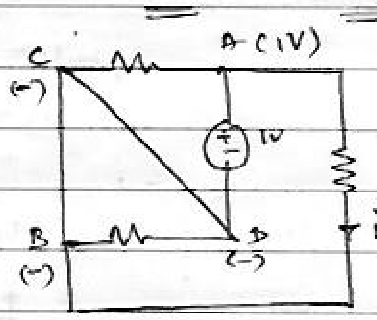
$i_{31} = -0.5$ i_{32} 0 $i_{33} = -3/2$ $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$i_1 = 0A$ $\frac{1}{2} \times -1$ $i_3 = \frac{-1}{2} - \frac{3}{2} = \frac{-1-3}{2} = \frac{4}{2}$

$i_1 = 0A$ $i_2 = +1$ $= -1A$



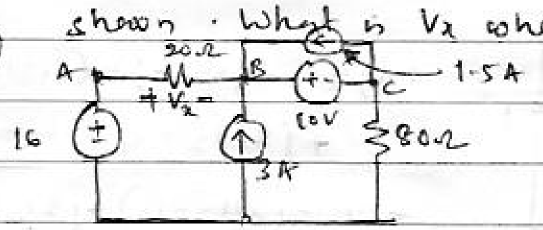
$i_{A1} = \frac{2}{2} = 1A$

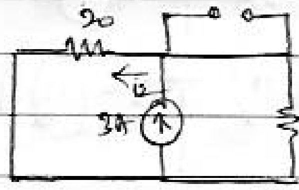


$i_{13} = \frac{1}{2} = 0.5$

$i_{total}(i_1) = 0.5 + 0.5 - 1 = \underline{0A}$

(4) Find components of V_x caused by each sources acting alone on ckt shown. What is V_x when all sources are acting

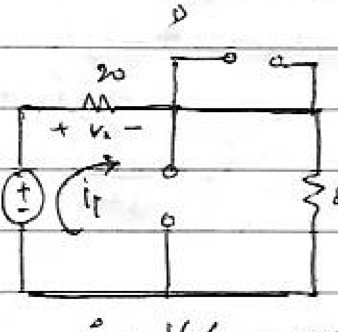




$i_2 = \left(\frac{80}{80+20}\right) \times 3A$

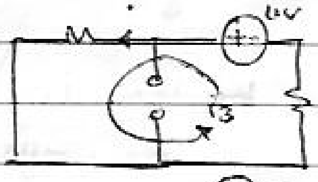
$i_2 = 2.4A$

$V_x = -(2.4 \times 20) = -48$



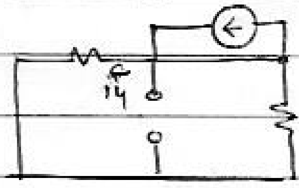
$V_x = 3.2 - 48 - 2$

$= -46.8V$



$i_3 = \frac{10}{100} = 0.1$

$V_{x_3} = -(0.1 \times 20) = -2V$



$i_4 = 0$

$V_{x_4} = 0$

$i_t = 16/100 = \cancel{0.2}\ 0.16$

$V_{x_1} = 0.16 \times 20 = \cancel{4V}\ 3.2V$

$\therefore$ ~~$V_x = 4 - 9.6 - 2 = -7.6V$~~

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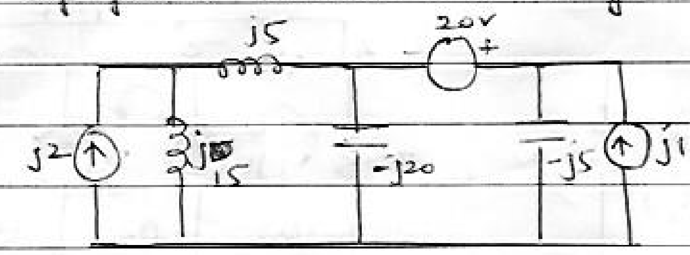
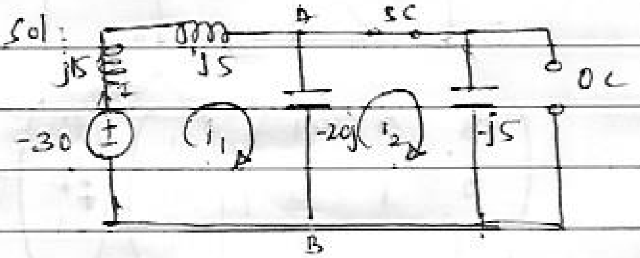
(5) Apply superposition theorem. find voltage a/c capacitor of reactance 20Ω



Sol:



$-(+30) + j15 i_1 + j5 i_1 - j20(i_1 - i_2) = 0$

$+j20 i_2 = -30$

$i_2 = \frac{30}{j20} = +j1.5$ ✓

$-j5(i_2) - 20j(i_2 - i_1) = 0$

$-j5 i_2 - 20j i_2 + 20j i_1 = 0$

$-j25 i_2 + j20 i_1 = 0$

$j20 i_1 = j25 i_2$

$= j25(+j1.5)$

$j20 i_1 = -37.5$

$i_1 = -37.5/j20 = +j1.875$ ✓

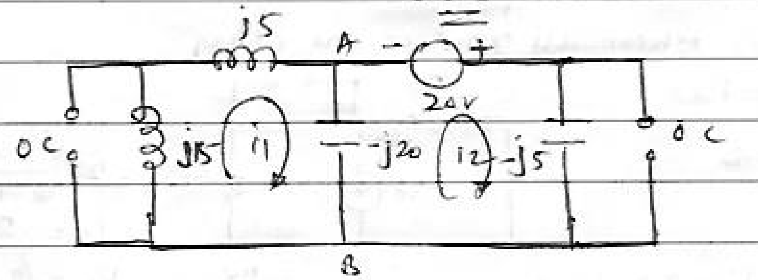
$i_{AB} = i_1 - i_2 = +j1.875 - (+j1.5)$

$= +j0.375$

$V_{AB} = +j0.375 \times -j20$

$= -j^2 7.5$

$= +7.5\ V$ ✓



$+j15 i_1 + j5 i_1 - j20(i_1 - i_2) = 0$

$+j20 i_2 = 0 \quad \underline{i_2 = 0}$

$-20 - j20(i_2 - i_1) - j5 i_2 = 0$

$+j20 i_1 - 25j i_2 = 20$

$j20 i_1 = \cancel{25j i_2} + 20$

$i_1 = \frac{20}{j20} = -1jA$

$i_{AB} = i_1 - i_2 = -1jA$

$V_{AB} = -1j \times -j20 = +j^2 20$

$= \underline{-20V}$

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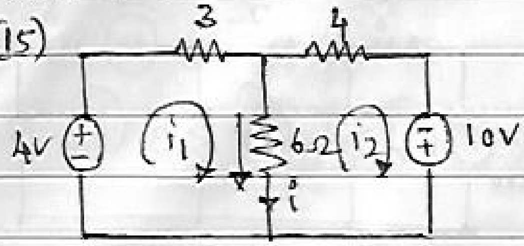
(15)



$$\begin{pmatrix} 9 & -6 \\ -6 & 10 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

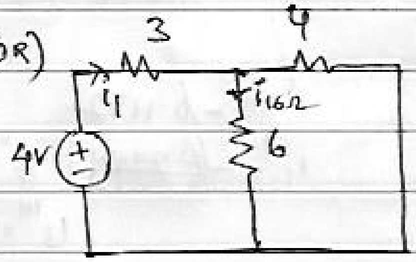
$i_1' = 0.740 \quad i_2' = 0.445$

$$\begin{pmatrix} 9 & -6 \\ -6 & 10 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix} \qquad i_1'' = 1.112 \quad i_2'' = 1.666$$

$i_1 = 1.852\,A \qquad i_2 = 2.111$

$\underline{\underline{i = i_1 - i_2 = -0.259}}$

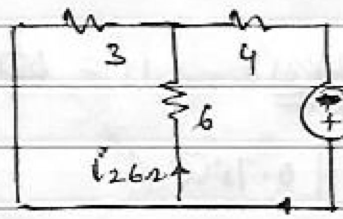
(OR)



$R_{eq} = \frac{4\times6}{10} + 3 = 2.4 + 3 = 5.4$

$i_1 = 4/5.4 = 0.740$

$i_{16\Omega} = \left(\frac{4}{4+6}\right) \times 0.740 = 0.296$



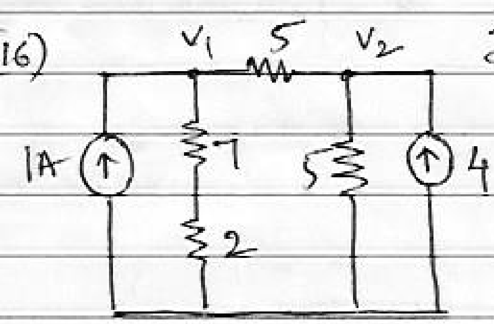
$R_{eq} = \frac{3\times6}{9} + 4 = 6\Omega$

$i_2 = +10/6 = 1.6667$

$i_{26\Omega} = \frac{3}{3+6} \times 1.667 = 0.5556\,A$

$i_a = i_{16\Omega} - i_{26\Omega} = +0.296 - 0.5556 = -0.259\,A$

(16) find current through 7Ω resistor due to two current sources using super position



$$\begin{pmatrix} \left(\frac{1}{5}+\frac{1}{9}\right) & -\left(\frac{1}{5}\right) \\ -\left(\frac{1}{5}\right) & \left(\frac{1}{5}+\frac{1}{5}\right) \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$V_1' = \cancel{24.319}\ 4.7348$

$V_2' = \cancel{21.2159}\ 2.367$

$$\begin{pmatrix} .3112 & -0.2 \\ -0.2 & 0.4 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Current through 7Ω resistor

$= \frac{14.2045}{9}$

$\underline{= 1.5782\,A}$

$$\begin{pmatrix} 0.3112 & -0.2 \\ -0.2 & 0.4 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$V_1'' = \cancel{4.863} + 9.4696$

$V_2'' = \cancel{7.568}\ 14.734$

$V_1 = 4.7348 + 9.4696 = 14.2045$ ✓

$V_2 = 2.367 + 14.734 = 17.1027$ ✓

$\therefore V_1 = -12.4311$

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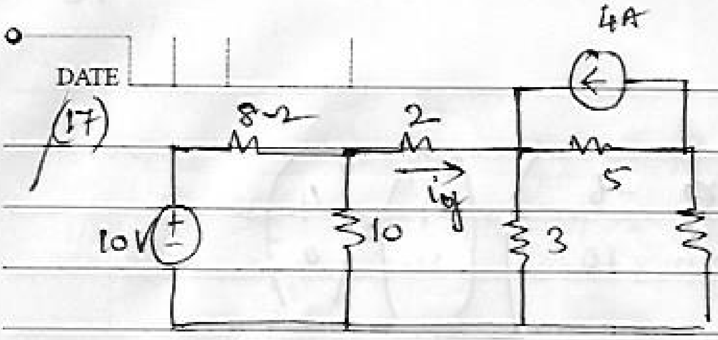
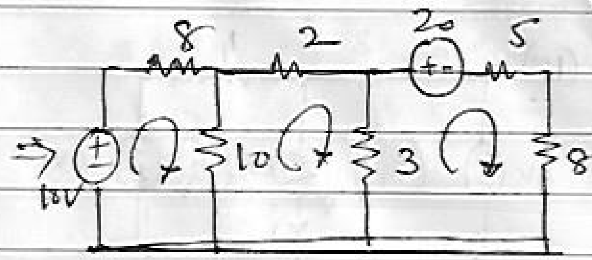
(17)





$$\begin{pmatrix} 18 & -10 & 0 \\ -10 & 15 & -3 \\ 0 & -3 & 16 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ -20 \end{pmatrix}$$

$i_1' = 0.9030$
$i_2' = 0.6254$
$i_3' = 0.1172$

$$\begin{pmatrix} 18 & -10 & 0 \\ -10 & 15 & -3 \\ 0 & -3 & 16 \end{pmatrix}\begin{pmatrix} i_1' \\ i_2' \\ i_3' \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

$i_1'' = i_2'' = i_3'' = 0$

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} i_1'' \\ i_2'' \\ i_3'' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$i_1''' = -0.2345$
$i_2''' = 0.4222$
$i_3''' = -1.3291$

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} i_1''' \\ i_2''' \\ i_3''' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -20 \end{pmatrix}$$

$i_1' = \cancel{0.6684}\ \cancel{0.9125}$ $\quad i_2' = \cancel{0.2032}\ \cancel{0.64258}$
$i_1' = 0.9030$ $\quad i_3' = \cancel{0.1711}\ \cancel{-1.211}$
$i_2' = 0.6254$
$i_3' = 0.1172$
$i_1'' = 0 \quad i_2'' = 0 \quad i_3'' = 0$

$i_1''' = -\cancel{0.2281}$ -0.1172 $\quad i_2''' = \cancel{0.410}$ -0.2111
$i_3''' = \cancel{-1.2927}$ -0.6645

$i_1 = \cancel{0.6844}$ 0.7858 $\quad i_2 = 0.28288 \quad i_3 = -1.1216$

$i_1 = 0.9030 - 0.2345$
$= 0.6685$

$\boxed{i_0 = i_2 = \cancel{0.28284}\ A}$ 0.7858

$i_y = 0.2033\ A$

$\boxed{i_y = i_2 = 0.6254 - 0.4222 = 0.2032}$

$i_1 = 0.6684 \quad i_2 = 0.2032 \quad i_3 = -1.21188$

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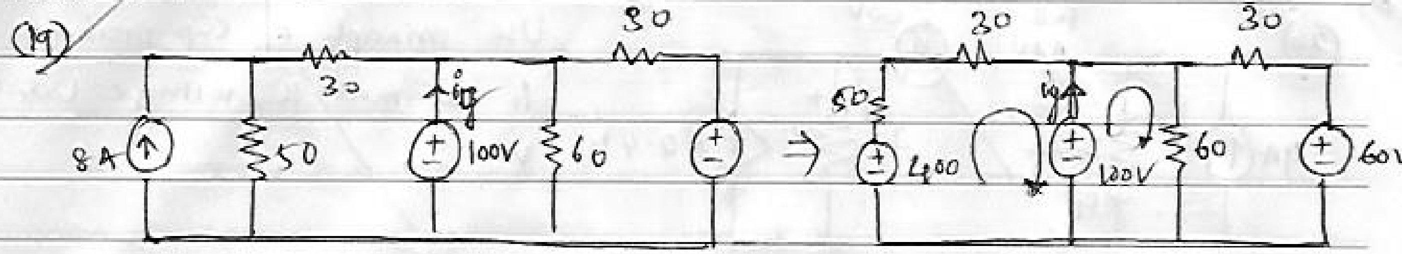
(19)



$$\begin{pmatrix} 80 & 0 & 0 \\ 0 & 60 & -60 \\ 0 & -60 & 90 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ -60 \end{pmatrix}$$

$i_1 = 3.75$

$i_2 = 3$

$i_3 = 1.333$

$$\begin{pmatrix} 80 & 0 & 0 \\ 0 & 60 & -60 \\ 0 & -60 & 90 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 300 \\ 0 \\ 0 \end{pmatrix} \qquad i_1' = 3.75 \quad i_2' = 0 \quad i_3' = 0$$

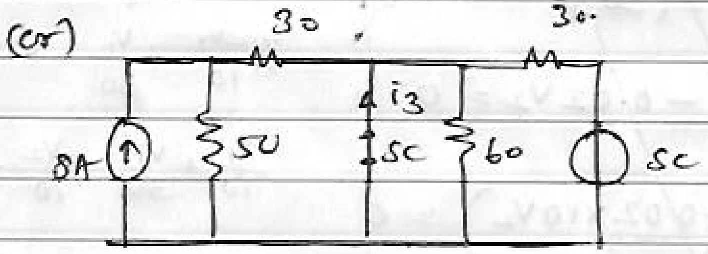
$$\begin{pmatrix} & " & \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 100 \\ 0 \end{pmatrix} \qquad i_1'' = 0, \quad i_2'' = 5 \quad i_3'' = 3.334$$

$$\begin{pmatrix} & & \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -60 \end{pmatrix} \qquad i_1''' = 0 \quad i_2''' = -2 \quad i_3''' = -2$$

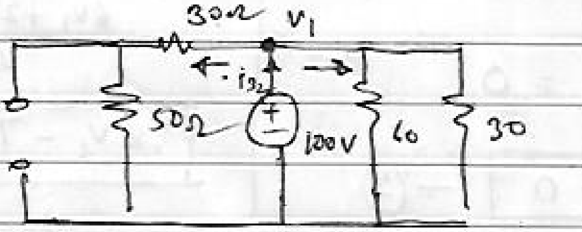
$$i_1 = 3.75 \qquad i_2 = 3 \qquad i_3 = 1.334.$$

$$i_y = i_2 - i_1 = 3 - 3.75 = \underline{\underline{-0.75\,A}}$$

(or)



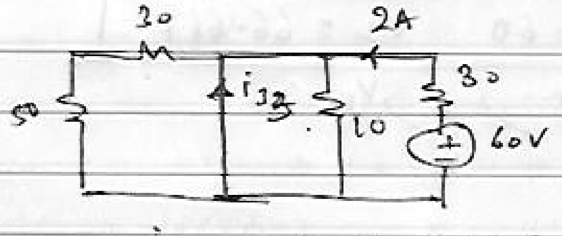
$$i_{31} = -\left(\frac{50 \times 8}{80}\right) = -5A$$



$$\frac{V_1}{80} + \frac{V_1}{10} + \frac{V_1}{30} = i_{32}$$

$$\frac{100}{80} + \frac{100}{10} + \frac{100}{30} = i_{32}$$

$$i_{32} = 6.25\,A$$



$$\cancel{i_{33} = \frac{60}{30} = 2A}$$

$$i_{33} = -\left(\frac{60}{30}\right) = -2A$$

$$i_3 = -5 - 2 + 6.25 = -0.75A$$

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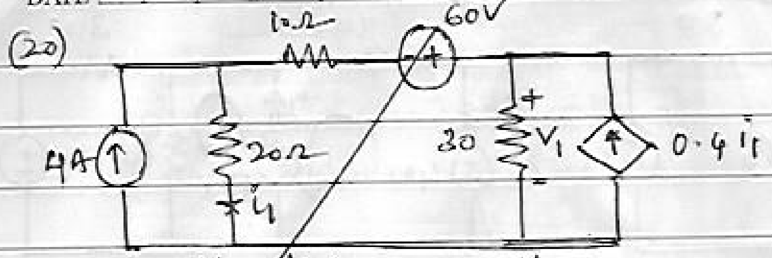
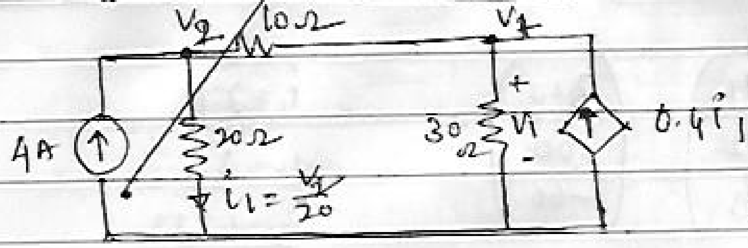
(20)



Use principle of superposition to determine the voltage labelled V_1.



$$-4 + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_1}{20} + \frac{V_1}{10} - \frac{V_2}{10} = 4$$

$$V_1 + 2V_1 - 2V_2 = 4 \times 20$$

$$3V_1 - 2V_2 = 80 \quad — (1)$$

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{20} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{30} \end{bmatrix} \begin{bmatrix} V_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} 4 \\ \ \end{bmatrix}$$

$$\frac{V_2}{30} + \frac{V_2 - V_1}{10} + 0.4 i_1 = 0$$

$$\frac{V_2 + 3V_2 - 3V_1}{} = 0.4\left(\frac{V_1}{20}\right) \qquad \because i_1 = V_1/20$$

$$-3V_1 - \frac{0.4 V_1}{20} + 4V_2 = 0$$

$$\boxed{\frac{V_2}{20} + \frac{V_2 - V_1}{10} = 4 \Rightarrow V_2 + 2V_2 - 2V_1 = 4 \times 20 \\ -2V_1 + 3V_2 = 80 \ - (1).}$$

$$\frac{V_1 - V_2}{10} + \frac{V_1}{30} \pm 0.4\left(\frac{V_2}{20}\right) = 0$$

$$\frac{V_1}{10} - \frac{V_2}{10} - \frac{V_1}{30} - 0.02 V_2 = 0$$

$$\frac{3V_1 - V_1}{30} - \left(\frac{V_2 + 0.02 \times 10 V_2}{10}\right) = 0$$

$$\frac{2V_1}{30} - \frac{1.2 V_2}{10} = 0$$

$$\boxed{2V_1 - 3.6 V_2 = 0} \ — (2)$$

$$\frac{V_1 - V_2}{10} + \frac{V_1}{30} - 0.4 \times \frac{V_2}{20} = 0$$

$$\frac{V_1}{10} + \frac{V_1}{30} - \frac{V_2}{10} - \frac{0.4 V_2}{20} = 0$$

$$\frac{6V_1 + 2V_1 - 6V_2 - 3 \times 0.4 V_2}{60} = 0$$

$$\boxed{8V_1 - 7.2 V_2 = 0} \ - (2)$$

$$\boxed{V_1 = 60 \quad V_2 = 66.667}$$

$$\frac{V_1}{30} = \frac{60}{30} \Rightarrow 2A \quad \therefore V_1 =$$

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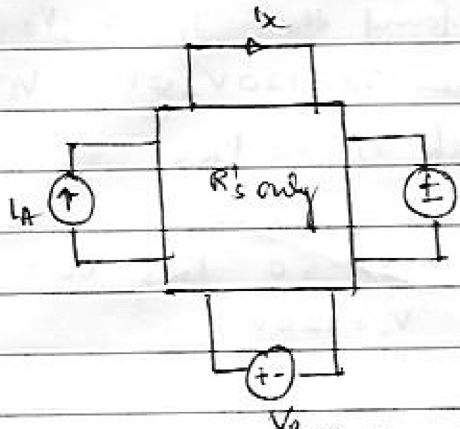
(18)



With source i_A and V_B on and $V_C = 0$

$i_x = 20$.

With sources i_A and V_C on, $V_B = 0$ $i_x = -5A$

with all sources ON, $i_x = 12A$

A) find i_x if only source operating is a) i_A b) V_B c) V_C.

B) find i_x if i_A and V_C are doubled in magnitude and V_B is reversed.

Sol(18) i) $\cancel{i_A(i_x) + V_B(i_x) = 20}$ $\quad i_x(i_A) + i_x(V_B) = 20$ — ①

ii) $\cancel{i_A(i_x) + V_C(i_x) = -5}$ $\quad i_x(i_A) + i_x(V_C) = -5$ — ②

iii) $\cancel{i_A(i_x) + V_B(i_x) + V_C(i_x) = 12A}$ $\quad i_x(i_A) + i_x(V_B) + i_x(V_C) = 12$ — ③

~~Find ix~~

$\boxed{i_x(i_A) = 3 \quad i_x(V_B) = 17 \quad i_x(V_C) = -8}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 20 \\ -5 \\ 12 \end{bmatrix}$$

By solving

$i_x(i_A) = 3 \quad i_x(V_B) = 17$

$i_x(V_C) = -8A$

~~if i_A is operating $\quad i_A(i_x) = 12$~~

~~$3 i_x = 12 \quad i_x = 12/3 = 4A$~~

~~if V_B is operating $\quad V_B(i_x) = 12 \quad i_x = 12/V_B = 12/17 = 0.705A$~~

~~if V_C ,, $\quad V_C(i_x) = 12 \quad i_x = 12/V_C = 12/-8 = -1.5$~~

B) $\cancel{2 i_x(i_A) + V_B(i_x) + 2 i_x(V_C) = 2(3) - 17 + 2(-8) = 0}$

$\cancel{i_x(V_B)}$ $\quad \cancel{= 6 - 17 + 16 = -27}$

$\cancel{2 i_x(i_A) - i_x(V_B) + 2 i_x(V_C) =}$

~~$i_x \alpha i_A \quad i_x = K_1 i_A$~~ / 20

$2 i_x(i_A) - i_x(V_B) + 2 i_x(V_C) = 2(3) - 17 + 2(-8)$

$= -27$

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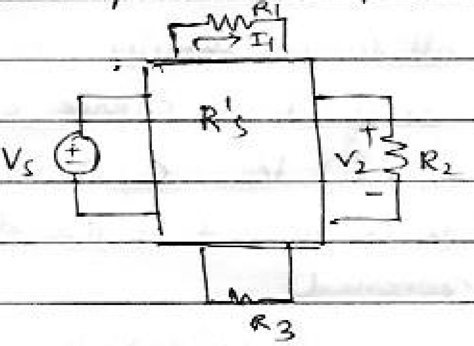
1) for linear ckt shown in fig it is found that $I_1 = 3A$, $V_2 = 50V$ and power delivered to $R_3 = 60W$, when $V_S = 120V$. when V_S is change to 105V find new values of I_1, V_2, P_{R3}.



$I_1 = 3A$ $V_2 = 50$ $P_{R3} = 60$ Watts.

$V_S = 120V$

If the n/w consists of linear elements the response that is current or voltage is directly propotional to voltage or current source

$I_1 \propto V_S$, $I_1 = K_1 V_S$ $3 = K_1(120)$ $K_1 = \frac{3}{120} = \frac{1}{40}$.

$V_2 \propto V_S$ $V_2 = K_2 V_S$ $50 = K_2(120)$ $K_2 = \frac{50}{120} = \frac{5}{12}$

$P_{R3} = I_3^2 R_3 = \left(\frac{V_S}{R_3}\right)^2 R_3 = \frac{V_S^2}{R_3}$.

$\therefore P_{R3} \propto V_S^2$ $P_{R3} = K_3 V_S^2$

$60 = K_3 (120)^2$

$\frac{60}{(120)^2}$ $K_3 = \frac{1}{240}$

$\therefore$ When V_S is changed to 105V.

$I_1 = K_1(V_S) = (1/40)\, 105 = 2.625$ A

$V_2 = K_2(V_S) = (5/12)\, 105 = 43.75$ V

$P_{R3} = K_3 V_S^2 = \left(\frac{1}{240}\right) 105^2 = \boxed{45.9375 \text{ W}}$

52.5

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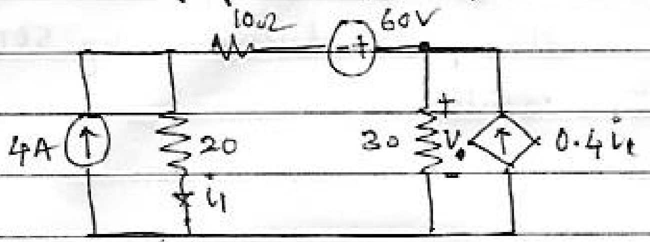
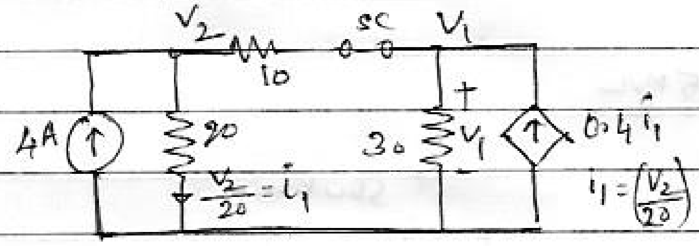
(20) Use superposition principle find voltage labelled





Node 2

$$\frac{V_2}{20} + \frac{V_2 - V_1}{10} - 4 = 0$$

$$V_2 + 2V_2 - 2V_1 = 80$$

$$-2V_1 + 3V_2 = 80 \quad \text{—(1)}$$

$$\frac{V_1}{30} + \frac{(V_1 - V_2)}{10} - 0.4\, i_1 = 0$$

$$\frac{V_1}{30} + \frac{V_1}{10} - \frac{V_2}{10} - 0.4\frac{V_2}{20} = 0$$

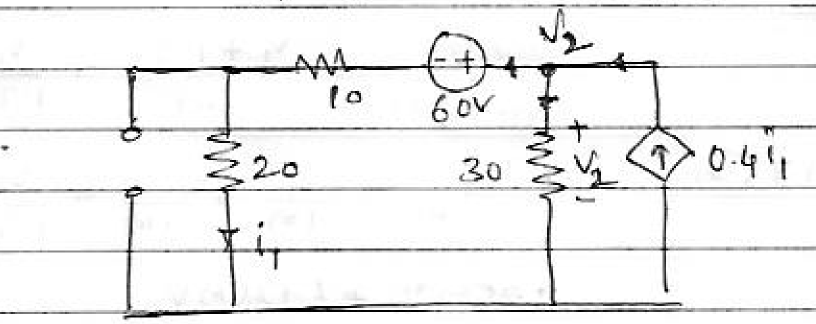
$$\frac{2V_1 + 6V_1 - 6V_2 - 0.4(3)V_2}{60} = 0$$

$$\boxed{8V_1 - 7.2V_2 = 0} \quad \text{—(2)}$$

$$\therefore \boxed{V_1 = 60}$$

$$V_2 = 66.667$$

$V_1 = 60V$ due to 4A current source



$$\frac{V_2 - 60}{30} + \frac{V_2}{30} - 0.4\left(\frac{V_2 - 60}{30}\right) = 0$$

$$2V_2 - 0.4V_2 - 60 + 0.4 \times 60 = 0$$

$$1.6V_2 = 60 - 0.4 \times 60$$

$$\boxed{V_2 = 22.5}$$

$$\boxed{V = V_1 + V_2 = 60 + 22.5 = 82.5V}$$

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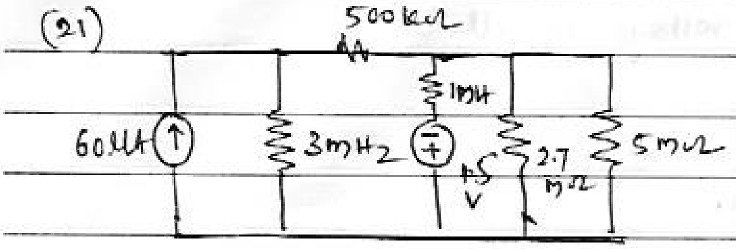
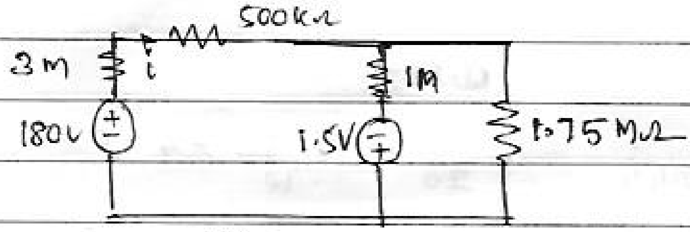
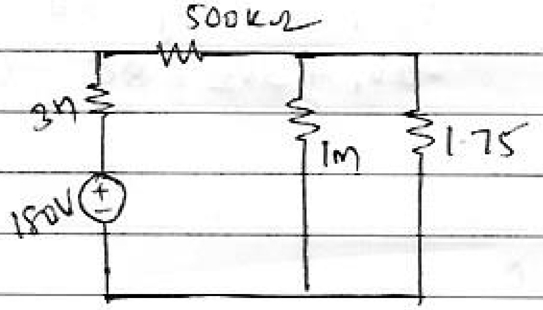
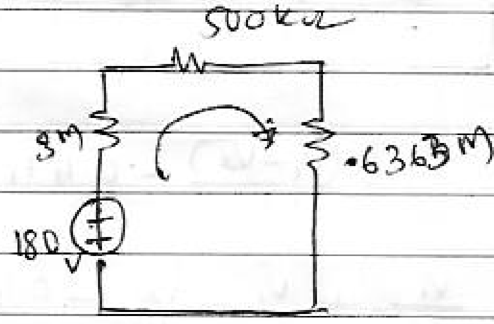
(21)



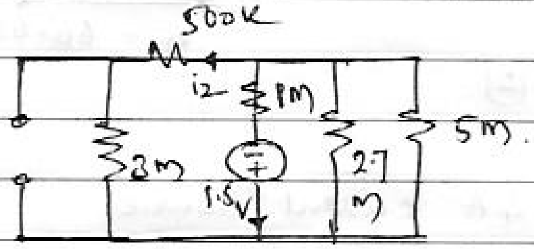
Use principle of super-position to find the power dissipated by 500KΩ resistor



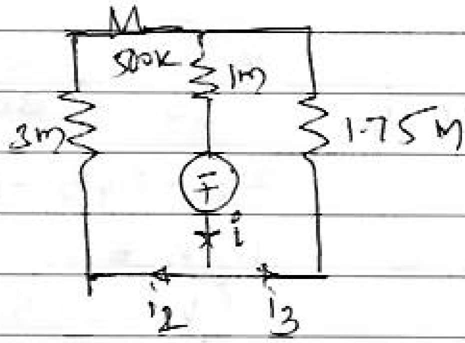




$i_1 = 0.0435 = \underline{43.51\mu A}$



$i_2 = 0.6998\ \mu A$



$$\frac{V_1}{500K+3M} + \frac{V_1+1.5V}{1M} + \frac{V_1}{1.75} = 0$$

$$\frac{V_1}{3.5M} + \frac{V_1}{1M} + \frac{1.5}{1M} + \frac{V_1}{1.75M} = 0$$

$1.75M\,V_1 + 6.125M\,V_1$

$- 1.5 \times 6.125M + V_1 \times 3.5M = 0$

$11.375\,M V_1 = -9.1875M$

$V_1 = -0.80769$

$i_2 = -0.237\,\mu A$

$i = i_1 - i_2 = 43.51\mu A - (-0.237\mu)$

$\boxed{i = 43.747\,\mu A}$

$P_{500k} = i^2 R = (43.747 \times 10^{-6})^2 \times 500K$

$= 9.56 \times 10^{-4}$

$\boxed{P_{500k} = 956\,\mu W}$

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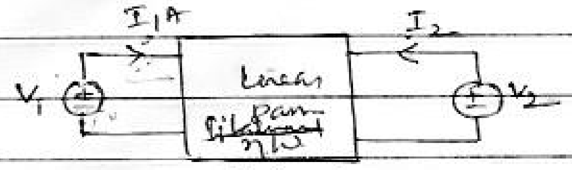
(x) complete the table given



Bilateral

Ex No	V_1	V_2	I_1	I_2
1	50	100	-1	27
2	100	50	7	24
3	200	0		
4			20	0
5			10	30

$I_1' \propto V_1$ $\qquad I_2' \propto V_1$ $\qquad I_1'' \propto V_2$ $\qquad I_2'' \propto V_2$

$I_1' = K_1 V_1$ $\qquad I_2' = K_2 V_1$ $\qquad I_1'' = K_3 V_2$ $\qquad I_2'' = K_4 V_2$

$I_1 = I_1' - I_1'' = K_1 V_1 - K_3 V_2$ — (1)

$I_2 = I_2'' - I_2' = K_4 V_2 - K_2 V_1 = -K_2 V_1 + K_4 V_2$ — (2)

$K_1 V_1 - K_3 V_2 = I_1$ — (1)

$-K_2 V_1 + K_4 V_2 = I_2$ — (2)

Case (1) & (2) $V_1 = 50$ $\quad V_2 = 100$ $\quad I_1 = -1$ $\quad$ ~~$I_2 = 27$~~ $\quad I_1 = 7$

Using eq^2 (1)

$50 K_1 - 100 K_3 = -1$

$100 K_1 - 50 K_3 = 7$ $\Big\}$ Using eq^2 (1)

$\boxed{K_1 = 0.1 \quad K_3 = 0.06}$ $\qquad 0.1 V_1 - 0.06 V_2 = I_1$

Case (1) & (2) for find K_2, K_4 using eq^2 (2)

$-K_2 V_1 + K_4 V_2 = I_2$

$-K_2 50 + K_4 100 = 27$

$-K_2 100 + K_4 50 = 24$

$\boxed{K_2 = -0.14 \quad K_4 = 0.2}$

(5) $V_1 = 133.8$ $V_2 = 56.33$

$\|^{ly}$

(5) $I_1 = 10$ & $I_2 = 30$

$V_1 = 133.8$

$V_2 = 56.33V$

(3) when $V_1 = 200$ $\quad V_2 = 0$.

$I_1 = K_1 V_1 - K_3 V_2 = 0.1 \times 200 + 0.14 \times 0 = 20A$

$I_2 = -K_2 V_1 + K_4 V_2 = +0.14 \times 200 + 0.2 \times 0 = 28A$

(4) when $I_1 = 20$ $\quad I_2 = 0$; $\quad 20 = 0.1 V_1 - 0.06 V_2$

$0 = +0.14 V_1 + 0.2 V_2$

$V_1 = 140$ $\quad V_2 = -98.59$

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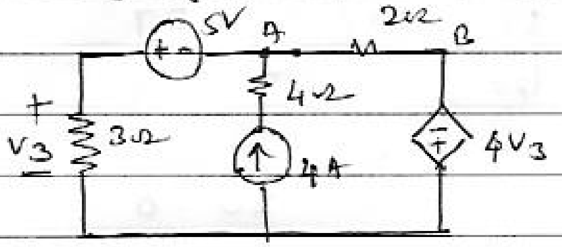
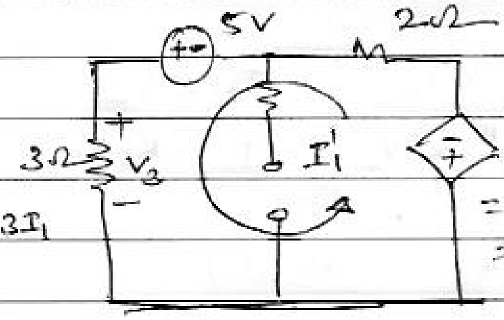
(x) Determine current through 2Ω resistor of n/w shown in fig. using superposition.



Consider 5V



$V_3 = 3I_1$

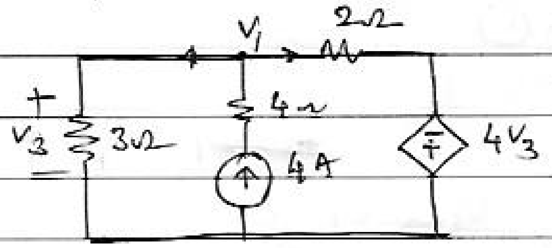
$= 4(3I_1)$
$= 12I_1$

$+12I_1 + 2I_1 - 5 + 3I_1 = 0$.

$17I_1 = 5$

$I_1' = 0.294$

Consider 4A source



$$\frac{V_1}{3} - 4 + \frac{V_1 + 4V_3}{2} = 0$$

$$\frac{V_1}{3} + \frac{V_1}{2} + \frac{4}{2}\left(3 \times \frac{V_1}{3}\right) = 4$$

$\because V_3 = i \times 3\Omega$

$= \left(\frac{V_1}{3}\right) \times 3 = V_1$

$$\therefore \frac{V_1}{3} + \frac{V_1}{2} + 2V_1 = 4$$

$2V_1 + 3V_1 + 12V_1 = 4 \times 6$.

$17V_1 = 24$

$V_1 = 1.414$

$$\frac{V_1 + 4V_1}{2} = I''$$

$5V_1/2 = I''$ $\qquad I_1'' = 5(1.414)/2 = 3.529$

$I = -I_1' + I_1'' = -0.294 + 3.529$

$\boxed{I = 3.235 \text{ from A to B}}$

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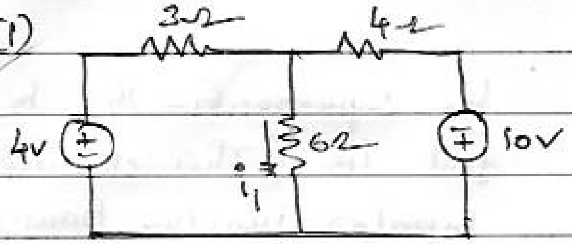
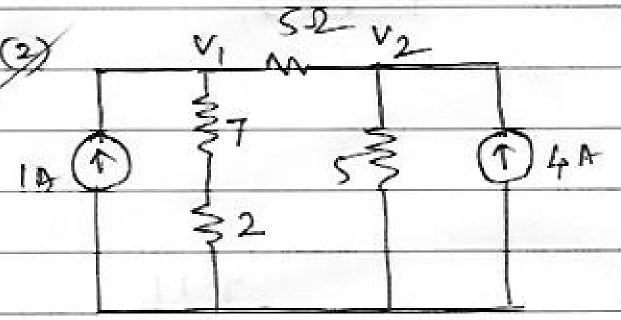
(1)



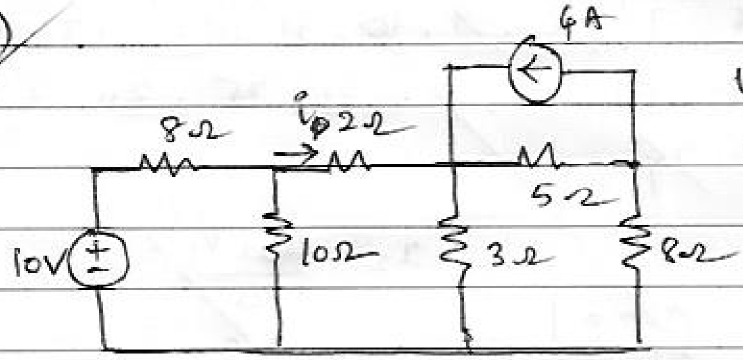
Find contribution of the 4V and 10V source to the current labelled i_1

(2)



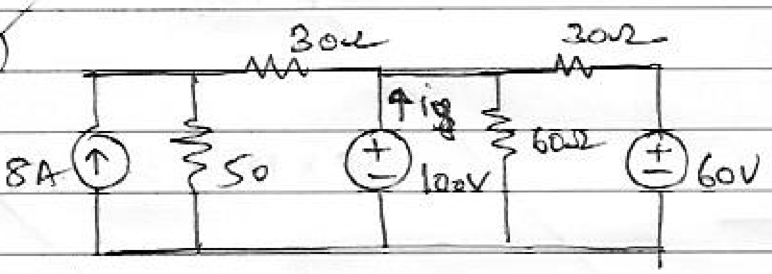
Determine contribution of 1A source ~~and~~ ~~eq~~ to V_1 and Calculate total current through the 7Ω resistor

(3)



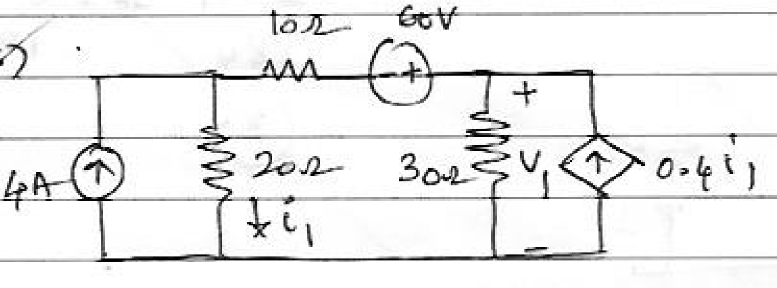
Use principle of superposition to determine the current labelled i_y

(4)



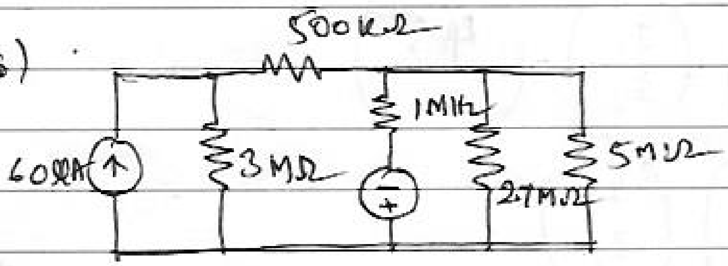
Use superposition to determine the current labelled i_3.

(5)



Use principle of superposition to determine the voltage labelled V.

(6)



Use principle of superposition to find the power — disscipated by 500Ω resistor

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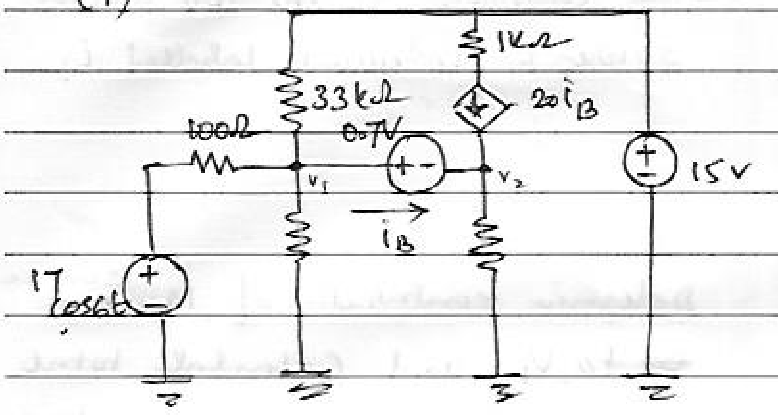
(7)



Use superposition thm to find i_B (This ckt model is bipolar junction transistor amplifier).

Verify

9-17.

~~1~~, ~~2~~, ~~3~~, ~~4~~, ~~5~~, $\boxed{6,\ 7}$ ~~8~~, ~~9~~, ~~10~~, ~~11~~, $\boxed{12}$, 13, 14
~~15~~, ~~16~~, ~~17~~, ~~18~~, ~~19~~, ~~20~~, ~~21~~, 22, ~~23~~, 24, 25, 26, 27,
28, 29, 30, 31, 32, ~~33~~, ~~34~~.

$\boxed{9\text{-}17}$, $\boxed{9\text{-}20}$ $\boxed{9\text{-}28}$

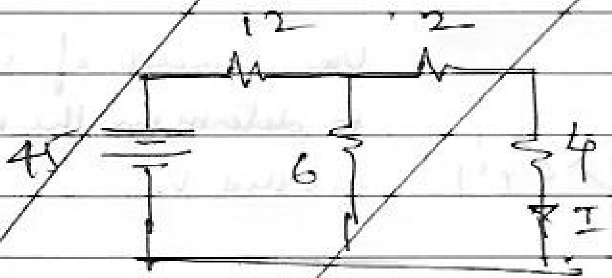
$$I_2 = \frac{\Delta_{21} V_1}{\Delta}$$

$$= \frac{-6}{180} \times 45$$

9-17
9-20
9-28 } Theorems

$18 \times 12 - 36 = 180$

$$\frac{12 \times V_1}{180} \quad - \frac{6 V_2}{180}.$$



(3)

$$\begin{pmatrix} 18 & -6 \\ -6 & 12 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 45 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 45 & -6 \\ 0 & 12 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

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Suppose

$$[Z][I] = [V] \quad -(1) \qquad [V] \rightarrow \text{Independent source excluding Initial cond: sources}$$

~~and~~

$$\begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

where $[Z]$ is symmetric matrix – ie Network having R, L, C and transformer but not dependent sources.

$$\begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{23} \\ Z_{13} & Z_{23} & Z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \qquad \Delta = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{23} \\ Z_{13} & Z_{23} & Z_{33} \end{vmatrix}$$

while Cramers & Expansion by minors.

$$I_k = \frac{D_k}{\Delta} = \frac{V_1 \Delta_{1k} + V_2 \Delta_{2k} + V_3 \Delta_{3k}}{\Delta} \quad \text{where } k = 1, 2, \ldots \text{ loops}$$

$$k=1 \quad I_1 = \frac{D_1}{\Delta} = \frac{V_1 \Delta_{11} + V_2 \Delta_{21} + V_3 \Delta_{31}}{\Delta} = \frac{\Delta_{11}}{\Delta} V_1 + \frac{\Delta_{21}}{\Delta} V_2 + \frac{\Delta_{31}}{\Delta} V_3 \quad -(1)$$

$$D_1 = \begin{pmatrix} V_1 & Z_{12} & Z_{13} \\ V_2 & Z_{22} & Z_{23} \\ V_3 & Z_{23} & Z_{33} \end{pmatrix} \quad \text{when } \Delta_{11} = \begin{pmatrix} Z_{22} & Z_{23} \\ Z_{23} & Z_{33} \end{pmatrix} \quad \Delta_{21} = \begin{pmatrix} Z_{12} & Z_{13} \\ Z_{23} & Z_{33} \end{pmatrix} \quad \Delta_{31} = \begin{pmatrix} Z_{12} & Z_{13} \\ Z_{22} & Z_{23} \end{pmatrix}$$

$$k=2 \quad I_2 = \frac{D_2}{\Delta} = \frac{\Delta_{12}}{\Delta} V_1 + \frac{\Delta_{22}}{\Delta} V_2 + \frac{\Delta_{32}}{\Delta} V_3 \quad -(2)$$

$$D_2 = \begin{pmatrix} Z_{11} & V_1 & Z_{13} \\ Z_{12} & V_2 & Z_{23} \\ Z_{13} & V_3 & Z_{33} \end{pmatrix} \quad \text{where } \Delta_{12} = \begin{pmatrix} Z_{12} & Z_{23} \\ Z_{13} & Z_{33} \end{pmatrix} \quad \Delta_{22} = \begin{pmatrix} Z_{11} & Z_{13} \\ Z_{13} & Z_{33} \end{pmatrix} \quad \Delta_{32} = \begin{pmatrix} Z_{11} & Z_{13} \\ Z_{12} & Z_{23} \end{pmatrix}$$

$$k=3 \quad I_3 = \frac{D_3}{\Delta} = \frac{\Delta_{13}}{\Delta} V_1 + \frac{\Delta_{23}}{\Delta} V_2 + \frac{\Delta_{33}}{\Delta} V_3 \quad -(3)$$

$$D_3 = \begin{pmatrix} Z_{11} & Z_{12} & V_1 \\ Z_{12} & Z_{22} & V_2 \\ Z_{13} & Z_{23} & V_3 \end{pmatrix} \quad \text{when } \Delta_{13} = \begin{pmatrix} Z_{12} & Z_{22} \\ Z_{13} & Z_{23} \end{pmatrix} \quad \Delta_{23} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{13} & Z_{23} \end{pmatrix} \quad \Delta_{33} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{pmatrix}$$

Now if $V_1 = V_2 = 0$ in eq² (1) then

$$I_1 = \frac{\Delta_{31}}{\Delta} V_3.$$

|||ly if $V_2 = V_3 = 0$ in eq² (3) then $I_3 = \dfrac{\Delta_{13}}{\Delta} V_1$.

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ie $I_1 = \frac{\Delta_{31}}{\Delta} V_3$ and $I_3 = \frac{\Delta_{13}}{\Delta} V_1$

where $\Delta_{31} = \begin{vmatrix} z_{12} & z_{13} \\ z_{22} & z_{23} \end{vmatrix}$ $\Delta_{13} = \begin{vmatrix} z_{12} & z_{22} \\ z_{13} & z_{23} \end{vmatrix}$

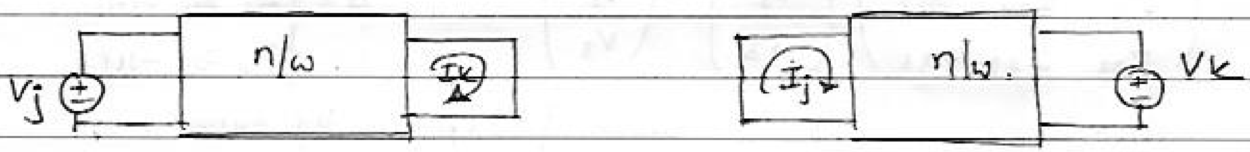
$= z_{12}z_{23} - z_{13}z_{22}$ $= z_{12}z_{23} - z_{13}z_{22}$

$\therefore \Delta_{31} = \Delta_{13}$.

$\therefore \frac{I_1}{V_3} = \frac{\Delta_{13}}{\Delta}$ and $\frac{I_3}{V_1} = \frac{\Delta_{13}}{\Delta}$

In general, for a n/w with ~~K-loops~~ L-loops., L = 1, . . . k.

$$\boxed{\frac{I_k}{V_j} = \frac{I_j}{V_k}}$$ ~~iff $V_j = V_k$~~.



- Suppose V_j is the only source in the n/w. This results in the current I_k in the k^{th} loop.
- If the source is now moved to the k^{th} loop, then the current in the j^{th} loop is $I_j = I_k$.
- This is the principle of Reciprocity.
- The ratio of the response transform to excitation transform is invarient to an interchanged of the position in the n/w of the exitation and response.
- When the above property holds good, the n/w is reciprocal otherwise the n/w is said to be non reciprocal.

Cond^ns of the n/w to be reciprocal

1) The impedence matrix should be ~~reciprocal~~ symmetric
ie n/w having R, L, C and transformer as linear elements but not dependent sources. - Network should not have dependent source (Controlled sources)

2) There should be only one source of excitation in the n/w.
$\therefore$ there should be no initial cond^n transformed sources, meaning the n/w elements must be initially relaxed or in zero state.

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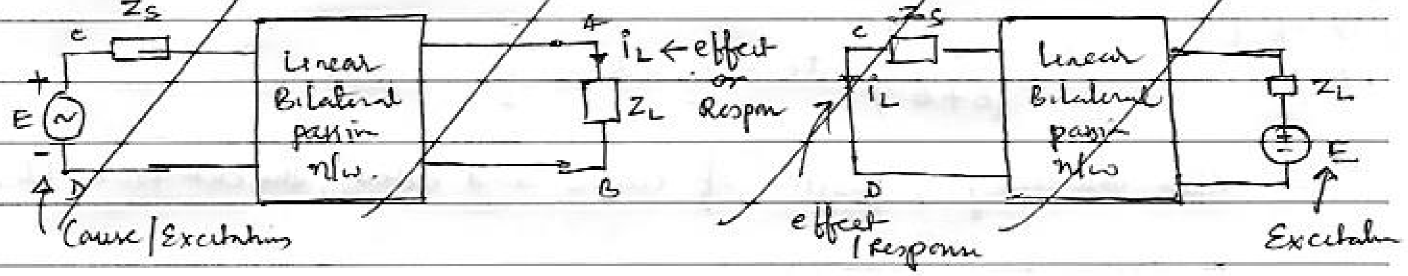
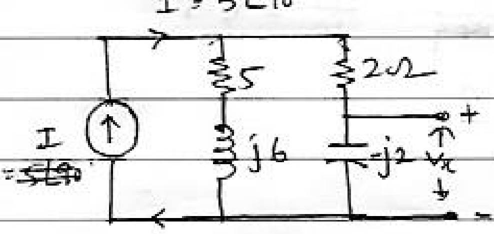
* Reciprocity theorem.

The theorem states that in a linear bilateral single source n/w the ratio of cause to effect or effect to cause remain constant even if position of cause and effect are interchange

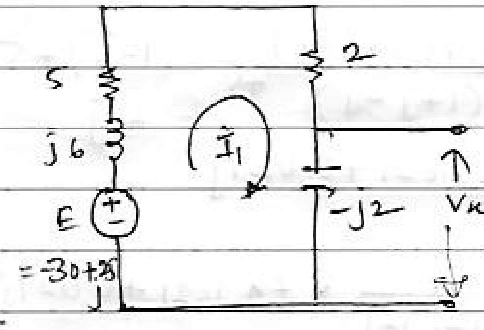


Ex(1) In the ckt shown in fig find Vx and prove reciprocity.



$I = 5\angle 90°$

$E = I \times (5+j6) = j5 \times (5+j6) = -30+25j$



$I_1 = \frac{E}{Z} = \frac{-30+25j}{(7+4j)} = -1.692 + 4.538j$

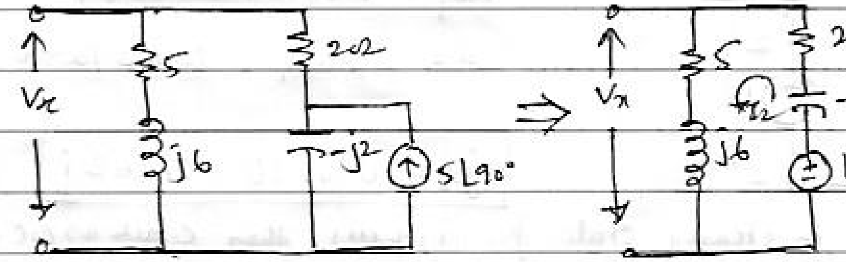
$V_x = I_1 \times -j2$

$= (-1.692 + 4.538j) \times -j2$

$= 9.076 + 3.384j$

$= 9.686 \angle 20.45$

To verify reciprocity theorem of cause and effect are interchanged After interchanging the positions ckt look like



$I_2 = \frac{10}{7+4j}$

$= 1.076 + 0.6153j$

$= \cancel{0.39 + 0.624}j$

$V_x = I_2 \times (5+j6)$

$= \cancel{1.575 + 2.6521}j$

$= 1.69 + 9.838j$

$V_x = 9.076 + 3.384j = 9.686 \angle 20.45$

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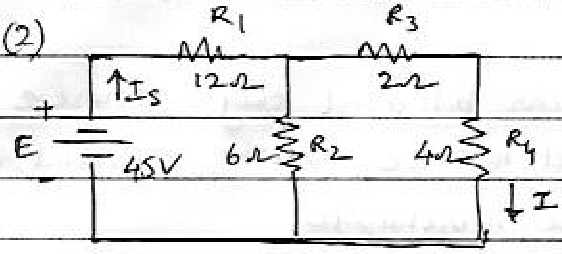
(2)



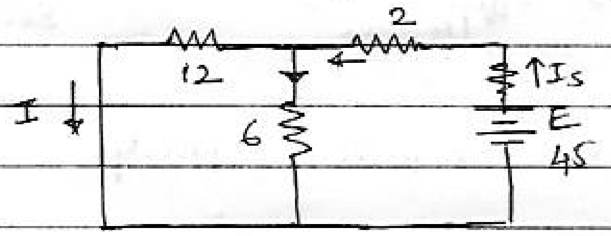
$R_{eq} = \left(\frac{6\times 6}{6+6}\right) + 12 = 15\,\Omega$

$I_s = \frac{E}{R_{eq}} = 45/15 = 3\,A$

$\begin{pmatrix} 18 & -6 \\ -6 & 12 \end{pmatrix}$

$I = \left(\frac{6}{6+6}\right) \times I_s = \frac{6}{12} \times 3 = \frac{3}{2} = 1.5\,A$

After interchanging position of cause and effect, the ckt looks like

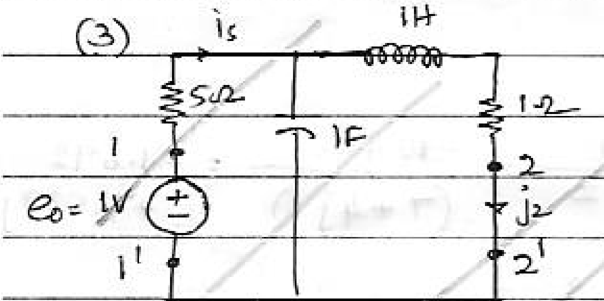


$R_{eq} = \left(\frac{12\times 6}{12+6}\right) + 2 + 4 = 10\,\Omega$

$I_s = \frac{E}{R_{eq}} = \frac{45}{10} = 4.5$

$\underline{I = \left(\frac{6}{6+12}\right) \times I_s = 1.5\,A}$

(3)



(i) $X_L = \omega L = 1\times 1 = 1\,\Omega$

$X_L = j$

$X_C = \omega C = 1\times 1 = 1$

$X_C = -j$

$z_{eq} = \frac{(1+j)\times(-j)}{(1+j-j)} + 5 = (1-j)+5 = 6-j$

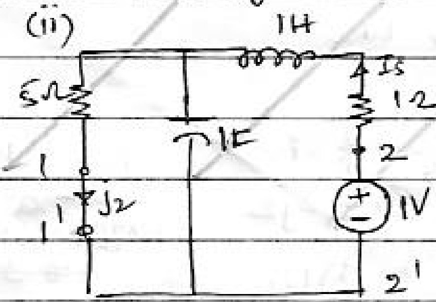
$I_s = 0.1621 + 0.027j$

$j_2 = \frac{-j}{(1+j)-j} \times (0.1621 + 0.027j)$

$\boxed{j_2 = 0.027 - 0.162j}$

After interchanging

(ii)



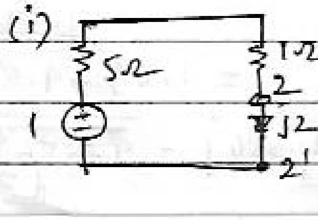
$z_{eq} = 1.192 + 0.0384j$

$I_s = V/z_{eq} = 0.8378 - 0.027j$

$\boxed{j_2 = 0.0270 - 0.162j}$

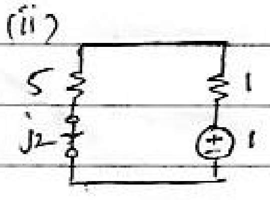
If L & C are in steady state for (i) & (ii), then C acts as O.C & L acts as S.C

(i)



$\underline{j_2 = \frac{1}{6}\,A}$

(ii)



$\underline{j_2 = \frac{1}{6}\,A}$

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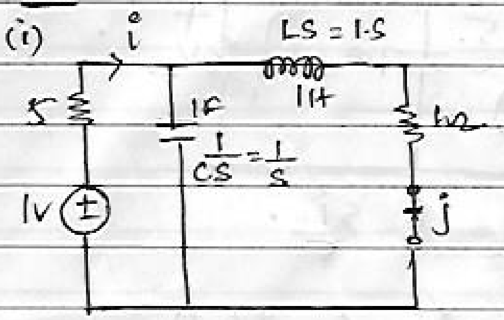
OR

(i)



$$Zeq = \left[\frac{(1+s)\times\left(\frac{1}{s}\right)}{\left(1+s+\frac{1}{s}\right)}\right] + 5$$

$$= \left[\frac{\left(\frac{1}{s}+1\right)}{\left(\frac{s+s^2+1}{s}\right)}\right] + 5 = \left[\left(\frac{1+s}{s}\right)\times\left(\frac{s}{s^2+s+1}\right)\right] 5$$

$$Zeq = \left(\frac{1+s}{s^2+s+1}\right) + 5 = \frac{(1+s)+5(s^2+s+1)}{s^2+s+1}$$

$$= \frac{5s^2+6s+6}{s^2+s+1}$$

$$i = \frac{1}{Zeq} = \frac{s^2+s+1}{5s^2+6s+6}$$

$$j_2 = \frac{\left(\frac{1}{s}\right)}{\left(\frac{1}{s}+1+s\right)} \times i$$

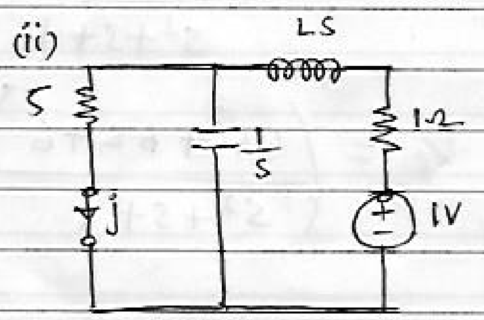
$$= \left(\frac{\left(\frac{1}{s}\right)}{\left(1+s+s^2\right)}\right) \times \frac{s^2+s+1}{5s^2+6s+6}$$

$$\cancel{j = \frac{5s^2+6s+6}{s}} \qquad \cancel{\text{if } s=0 \quad j = 0+0+6}$$

$$= \left(\frac{1}{s}\right)\times\left(\frac{s}{s^2+s+1}\right)\times\frac{s^2+s+1}{5s^2+6s+6}$$

$$j = \frac{1}{5s^2+6s+6} \qquad \text{if } s=0 \quad j = \frac{1}{6}$$

(ii)



$$\boxed{j = \frac{1}{6}}$$

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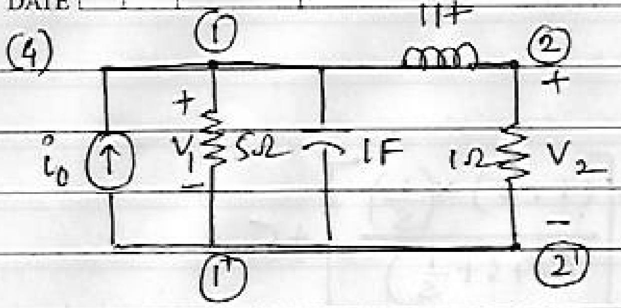
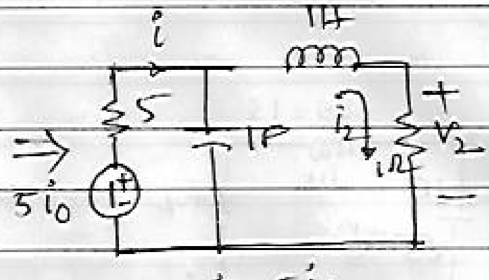
(4)



$\Rightarrow$



$$i = \frac{5i_0}{5}$$

$$i_2 = \frac{(-j)}{(1-j+j)} \times \frac{5i_0}{5} = -j(i_0)$$

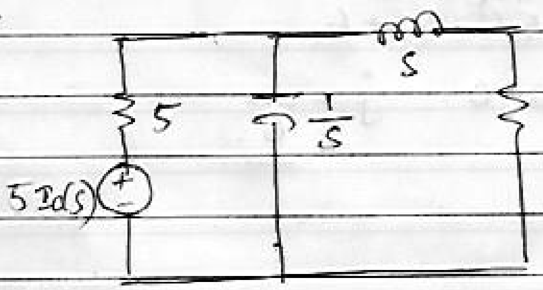
$$Z_{eq} = \frac{(1+Ls) \times \frac{1}{Cs}}{\left(1+Ls+\frac{1}{Cs}\right)} + 5$$

$$= \frac{Cs + Ls\left(\frac{1}{Cs} + \frac{L}{C}\right)}{\frac{Cs + LCs^2 + 1}{Cs}} + 5$$

$$= \frac{1+Ls}{Cs} \times \frac{Cs}{Cs + LCs^2 + 1} + 5$$

$$= \frac{1+Ls+5(Cs+LCs+1)}{Cs+LCs^2+1}$$

$$= \frac{5Cs + 5LCs + 5 + 1 + Ls}{LCs^2 + Cs + 1}$$



$$\frac{(s+1) \times \frac{1}{s}}{s+1+\frac{1}{s}} = \frac{(s+1)}{(s^2+s+1)} + 5$$

$$= \frac{s+1+5(s^2+s+1)}{s^2+s+1}$$

$$Z_{eq} = \left(\frac{5s^2+6s+6}{s^2+s+1}\right)$$

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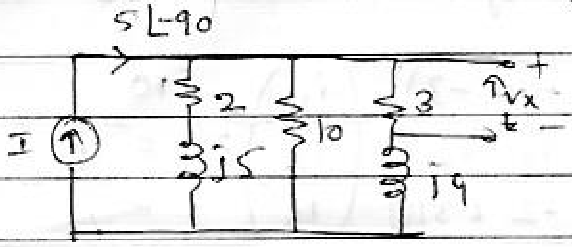
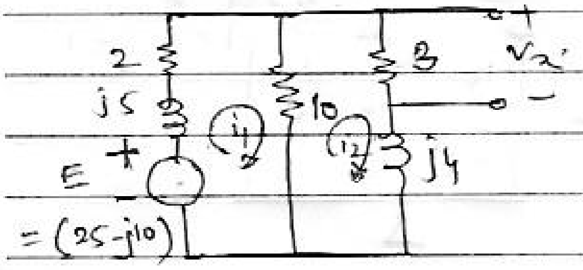
2) In n/w shown find voltage Vx and verify reciprocity thm.





$$\begin{bmatrix} 12+j5 & -10 \\ -10 & 13+j4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25-j10 \\ 0 \end{bmatrix}$$

$\Delta = 36+113j$

$$\begin{bmatrix} 12+j5 & 25-j10 \\ -10 & 0 \end{bmatrix}$$

$I_2 = \left[\frac{\Delta_{12}}{\Delta}\right] \times V_1$

$\cancel{= \frac{13+j4}{(12+j5)\times(13+j4) - 100}} = \cancel{\frac{13+j4}{36+113i}}$

$= 0.0654 - .0942j$

$\cancel{I_2 = \frac{-10}{\Delta} \times V_1} = \cancel{-}0.1635$

$-0.5 - 6.76i$

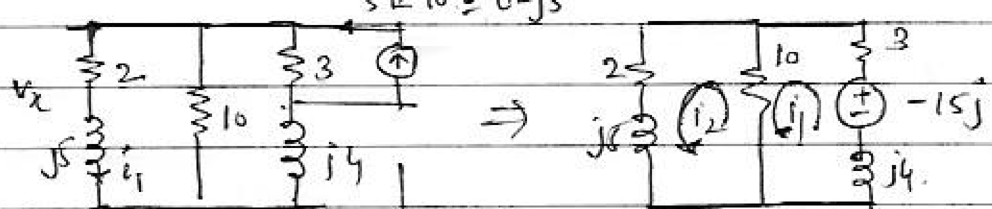
$(12+j5)i_1 - 10i_2 = 25-j10$ — ①

$(-10i_1 + (13+j4)i_2 = 0$ — ②

$$I_2 = \frac{D_2}{\Delta} = \frac{\begin{vmatrix} 12+j5 & 25-j10 \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 12+j5 & -10 \\ -10 & 13+j4 \end{vmatrix}} = \frac{250-100j}{36+113j}$$

$\boxed{I_2 = -0.1635 - j2.26}$ $\quad V_x = 3i_2 = 6.81\angle{-94.13°}$

$5\angle{-90} = 0-j5$



$$i_2 = \frac{D_2}{\Delta} = \frac{\begin{vmatrix} 13+j4 & -15j \\ -10 & 0 \end{vmatrix}}{36+113j}$$

$= -1.2 - 0.38j$

$$\begin{bmatrix} 13+j4 & -10 \\ -10 & 12+j5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -15j \\ 0 \end{bmatrix}$$

$\therefore V_x = i_2 \times (2+j5) = -0.5 - 6.76i$
$= 6.81\angle{-94.13}$

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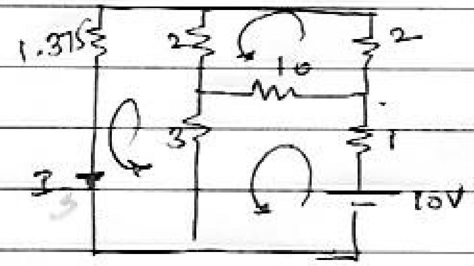
(3) In the ckt find current I and verify reciprocity



$$\begin{pmatrix} 14 & -10 & -3 \\ -10 & 14 & -2 \\ -3 & -2 & 6.375 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

$$I_3 = I = \frac{D_3}{\Delta} = \cancel{\frac{\Delta_{13} V_1}{\Delta}} = \cancel{\begin{vmatrix} -10 & 14 \\ -3 & -2 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} 14 & -10 & 10 \\ -10 & 14 & 0 \\ -3 & -2 & 0 \end{vmatrix}}{\Delta}$$

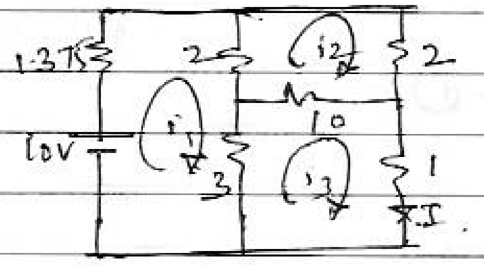
$$\Delta = 14(14 \times 6.375 - 4) + 10(-10 \times 6.375 - 6) - 3(20 + 14 \times 3) = 310$$

$$= \frac{14(0) + 10(0) + 10(20+42)}{\cancel{620}\ \ 310}$$

$$= \frac{620}{310} = 2A.$$

$$\boxed{I = 2A}$$

By interchanging the positions



$$\begin{pmatrix} 6.375 & -2 & -3 \\ -2 & 14 & -10 \\ -3 & -10 & 14 \end{pmatrix} \begin{pmatrix} \ \\ \ \\ \ \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

$$I_2 = I = \frac{\begin{vmatrix} 6.375 & -2 & 10 \\ -2 & 14 & 0 \\ -3 & -10 & 0 \end{vmatrix}}{\Delta}$$

$$= \frac{6.375(0) + 2(0) + 10(20 + 14 \times 3)}{310}$$

$$= \underline{\underline{2A}}$$

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Millman's Theorem

→ n voltage sources $E_1, E_2, E_3 \ldots E_n$ with ~~their~~ series impedence connected in parallel may be replaced by single voltage source E with ~~internal~~ impedence Z.

ie Any number of parallel voltage sources reduced to one ‡ voltage source.

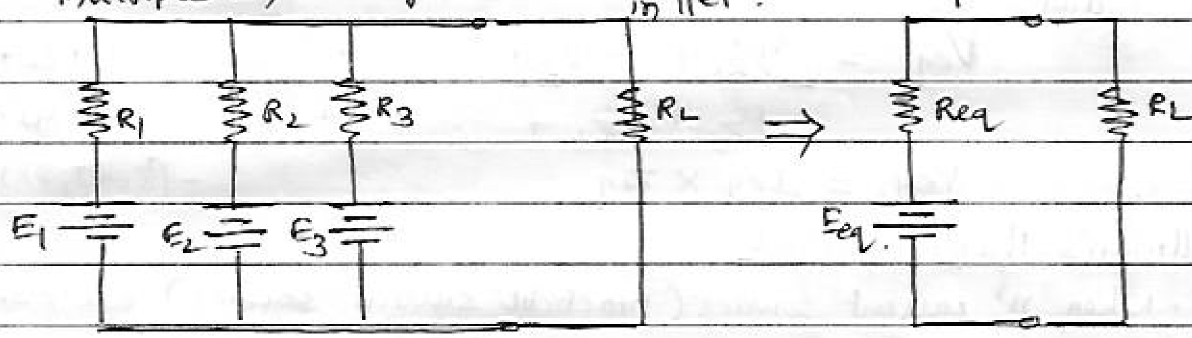
→ Steps involved

i) Convert voltage source to current sources

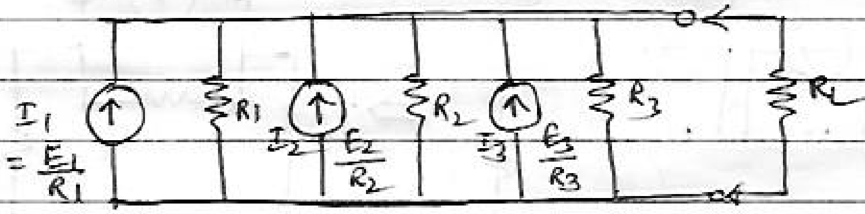
ii) Combine all these current source and impedences.

iii) Convert resulting current source to voltage source.

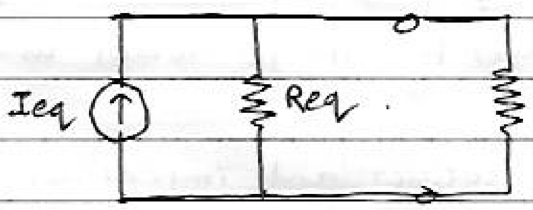
Multiple (3) voltage sources connected in ||el.   Equivalent ckt



Step 1:



step 2:



$$I_{eq} = I_1 + I_2 + I_3$$

$$= \frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}$$

$$\boxed{I_{eq} = E_1G_1 + E_2G_2 + E_3G_3} \quad -(1)$$

$$I_{eq} = \sum_{i=0}^{n} E_i G_i \text{ (or) } \sum_{i=0}^{n} \frac{E_i}{R_i}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$G_T = G_1 + G_2 + G_3 \quad -(2)$$

$$\left.\begin{aligned} \frac{1}{R_{eq}} &= \sum_{i=0}^{n} \frac{1}{R_i} \\ G_{eq} &= \sum_{i=0}^{n} G_i \end{aligned}\right\} -(3)$$

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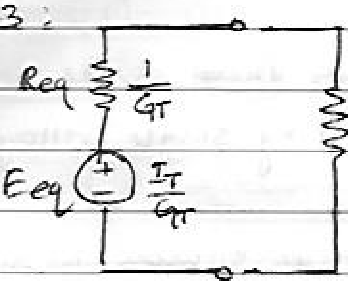
Step 3:



$$E_{eq} = \frac{\sum_{i=0}^{n} I_i}{\sum_{i=0}^{n} G_i}$$

$n = 3$

$$E_{eq} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{I_1 + I_2 + I_3}{G_T}$$

More generally,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \cdots + \frac{1}{Z_n}.$$

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

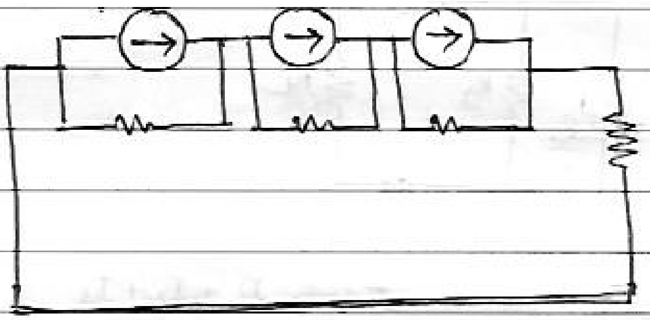
and

$$V_{eq} = \frac{V_1/Z_1 + V_2/Z_2 + \cdots + V_n/Z_n}{1/Z_1 + 1/Z_2 + \cdots + 1/Z_n} = \frac{I_1 + I_2 + \cdots I_n}{G_1 + G_2 + \cdots + G_n}.$$

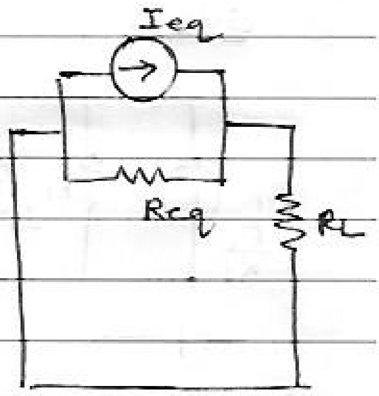
$$V_{eq} = I_{eq} \times Z_{eq} \qquad = (I_1 + I_2 + \cdots I_n)\, Z_{eq}.$$

Millman's theorem - Dual

- When 'n' current sources (practical current sources) are connected in series, they can be replaced by single current source with impedance Z in parallel.



$\Rightarrow$



Step 1: Convert all current sources to voltage sources using source transformation.

Step 2: Combine all voltage sources and impedances to get Eeq & Zeq

Step 3: Convert Eeq & Zeq to equivalent current source Ieq and Zeq.

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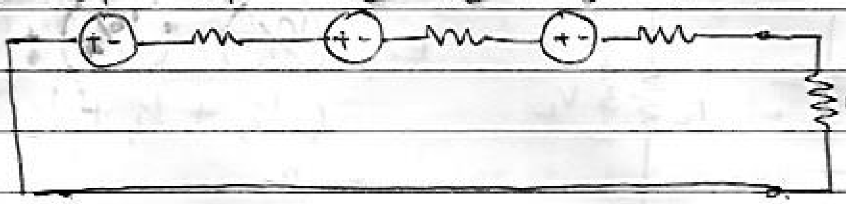
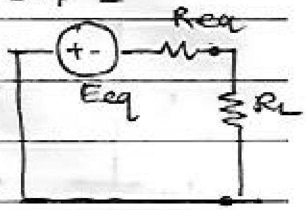
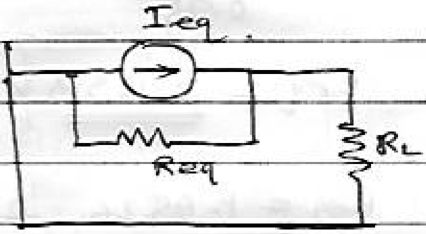
Step 1



Step 2



Step 3 :



$$E_{eq} = E_1 + E_2 + E \cdots E_n$$
$$= I_1R_1 + I_2R_2 + \cdots I_nR_n$$
$$R_{eq} = R_1 + R_2 + \cdots R_n.$$

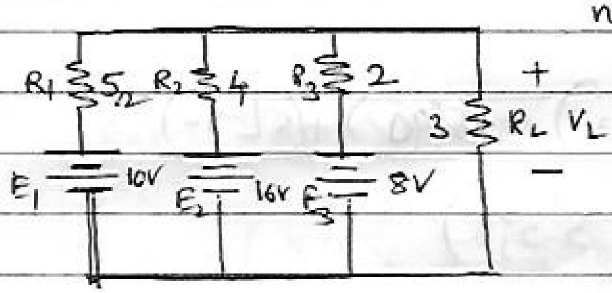
$$I_{eq} = \frac{E_{eq}}{R_{eq}} = \frac{I_1R_1 + I_2R_2 + \cdots + I_nR_n}{R_1 + R_2 + \cdots + R_n.}$$

More generally

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_n.$$

$$I_{eq} = \frac{E_1 + E_2 + \cdots + E_n}{Z_1 + Z_2 + \cdots + Z_n} = \frac{I_1Z_1 + I_2Z_2 + \cdots + I_nZ_n}{Z_{eq}.}$$

Example(1)



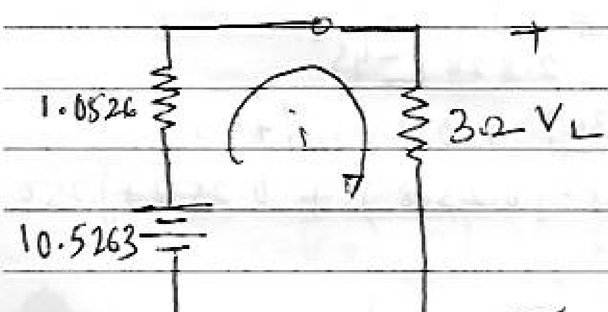
$n = 3$

$$E_{eq} = \frac{\sum_{i=0}^{3} I_i}{\sum_{i=0}^{3} G_i} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3}$$

$$E_{eq} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$E_{eq} = \frac{(10/5) + (16/4) + (8/2)}{(1/5 + 1/4 + 1/2)} = \frac{10}{0.95} = 10.5263.$$

$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{4} + \frac{1}{2} = 0.95 \qquad R_{eq} = \frac{1}{0.95} = 1.0526$$



$$i = \frac{V}{R_{eq} + R_L} = \frac{10.5263}{1.0526 + 3} = 2.5974$$

$$V_L = iR_L = 2.5974 \times 3$$
$$= \underline{\underline{7.7922V}}$$

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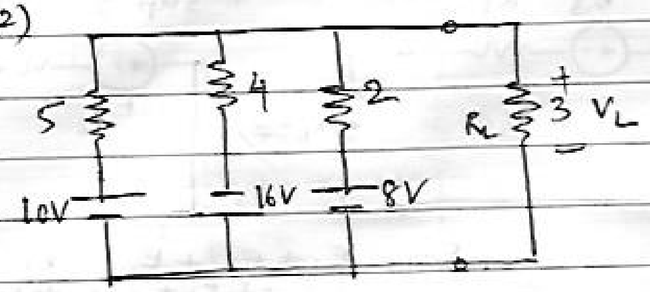
(2)



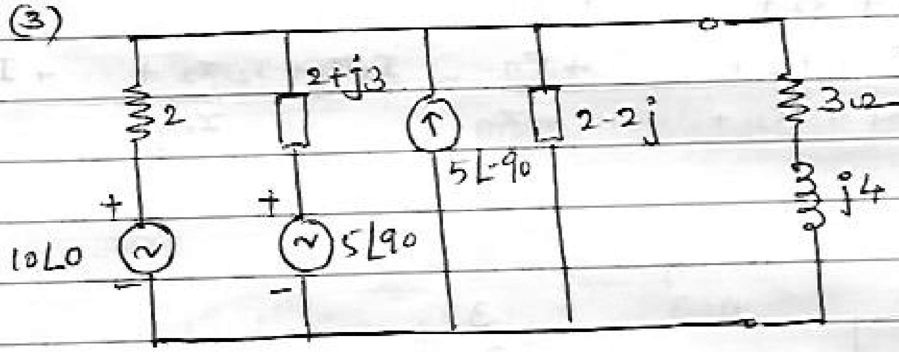
$$E_{eq} = \frac{\left(\frac{10}{5}\right) - \left(\frac{16}{4}\right) + \left(\frac{8}{2}\right)}{\left(\frac{1}{5} + \frac{1}{4} + \frac{1}{2}\right)}$$

$$= \frac{2}{0.95}$$

$$E_{eq} = \underline{\underline{2.1053\ V}}$$

$\cancel{R_{eq}} = \quad \frac{1}{R_{eq}} = 0.95 \qquad R_{eq} = \underline{\underline{1.0526\ \Omega}}$

(3)



$$E_{eq} = \frac{\left(\frac{10\angle 0^\circ}{2}\right) + \left(\frac{5\angle 90}{2+j3}\right) + \cancel{(5\angle 90)} + (5\angle -90)}{\left(\frac{1}{2} + \frac{1}{2+j3} + \frac{1}{2-2j}\right)}$$

$1.3889\angle 33.6910 \quad = 5\angle 0^\circ$

$$I_{eq} = (5+j0) + \frac{5\angle 90}{3.6\angle 56.309} + 5\angle -90$$

$$= 5+j0 + 1.1556+j0.770 + 0-j5$$

$$= (6.1556 - j4.23)$$

$$= 7.4688\angle -34.496$$

$$\frac{1}{Z_{eq}} = 0.5 + \frac{1}{3.605\angle 56.309} + \frac{1}{2.828\angle -45}$$

$$= 0.5\angle 0 + 0.2774\angle -56.309 + 0.3536\angle +45$$

$$= (0.5+j0) + (0.1538 - j0.2308) + 0.250 + j.250$$

$$= 0.9038 + j0.0192$$

$$\frac{1}{Z_{eq}} = 0.904\angle 1.21698 \qquad Z_{eq} = \frac{1}{0.904\angle 1.21698}$$

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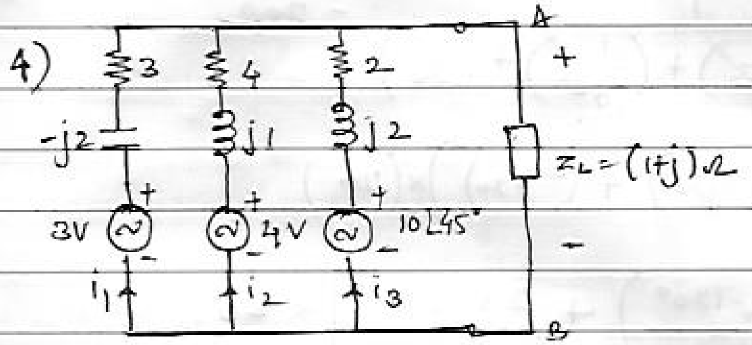
$$E_{eq} = \frac{I_{eq}}{(1/Z_{eq})} = I_{eq} \times Z_{eq} = 7.4688\angle -34.496 \times \frac{1}{0.904\angle 1.21698}$$

$$= 8.2619\angle -35.7130$$

$$\left[E_{eq} = 6.708 - j4.8226\right]$$

4)



$$E_{eq} = \frac{\sum_{i=0}^{3} \frac{E_i}{Z_i}}{\sum_{i=0}^{3} \frac{1}{Z_i}}$$

$$Z_{eq} = \frac{1}{\sum_{i=0}^{3} \frac{1}{Z_i}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

$$Z_{eq} = \frac{1}{\frac{1}{3-j2} + \frac{1}{4+j} + \frac{1}{2+2j}}$$

$$Z_{eq} = \frac{1}{0.71606 - j0.1549} = 1.334 + j0.2887$$

$$E_{eq} = \left(\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}\right) Z_{eq} \qquad 10\angle 45^\circ = 7.071 + j7.071$$

$$= \left(\frac{3}{3-j2} + \frac{4}{4+j} + \frac{10\angle 45^\circ}{2+j2}\right)(1.334 + j0.2887)$$

$$\left[E_{eq} = 6.689 + j1.794\right] \qquad 5.168 + j\frac{50}{221}$$

$$I_L = \frac{E_{eq}}{Z_{eq} + Z_L} = \frac{6.689 + j1.794}{(1.334 + j0.2887) + (1+j)}$$

$$I_L = (2.521 - j0.6236) \text{ Amp.}$$

$$V_{AB} = I_L \times Z_L = (2.521 - j0.6236)(1+j) = 3.1446 + j1.897 \text{ V.}$$

$$i_1 = -\left(\frac{V_{AB} - 3}{(3-j2)}\right) = -(-0.258 + j0.460) = 0.258 - j0.460 \qquad i_2 = -\left(\frac{V_{AB} - 4}{(4+j)}\right) = (0.078 - 0.49j)\text{ A}$$

$$i_3 = -\left(\frac{V_{AB} - 10\angle 45^\circ}{(2+j2)}\right) = (2.26 + 0.32j)\text{ A}$$

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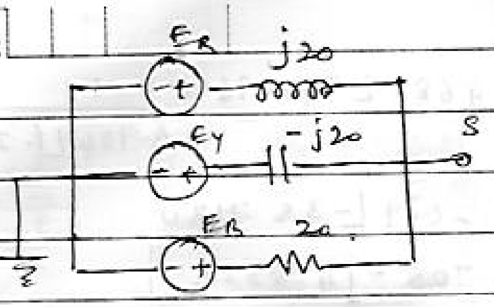
5)



$E_R = 230V$

$E_Y = 236\angle -120°$

$E_B = 230\angle 120$

find V_s.

$$\frac{V}{Z_{eq}}, \quad Z_{eq} = \frac{1}{\left(\cancel{\frac{1}{j20}}\right) + \left(\cancel{\frac{1}{-j20}}\right) + \left(\frac{1}{20}\right)} = 20\Omega$$

$$E_{eq} = \left(\left(E_R/j20\right) + \left(E_Y/-j20\right) + \left(E_Z/20\right)\right) \times (20)$$

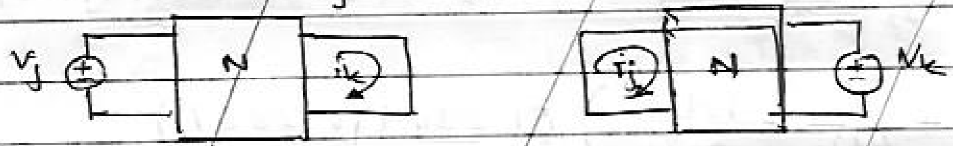
$$= \left[\left(\frac{230}{j20}\right) + \left(\frac{230\angle -120°}{-j20}\right) + \frac{230\angle 120}{20}\right] \times 20$$

$$V_S = E_{eq} = (84.18 - 145.8j)\ V = 168.37\angle -60°$$

* Reciprocity

More generally, for a network with L-loops

$$\frac{I_k}{V_j} = \frac{I_j}{V_k}$$



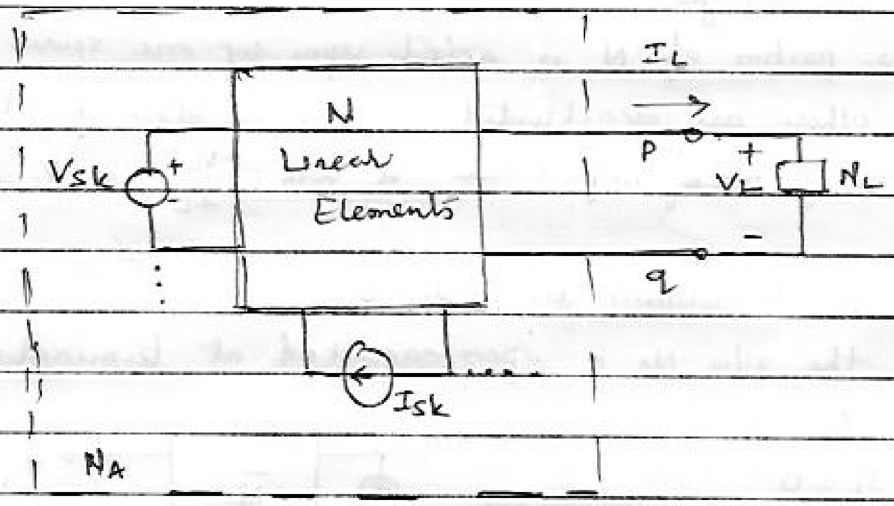
- Suppose V_j is the only source in the n/w. This results in the current I_k in the k^{th} loop.
- If this source is now moved to the k^{th} loop, then the current in the j^{th} loop is $I_j = I_k$.
- This is the principle of reciprocity.
- The ratio of the response transform to the excitation transform is invarient to an interchange of position in the network of the excitation and the response.
- When this property holds, we say that the n/w a reciprocal otherwise the network is non-reciprocal.

DATE * Thevenin's theorem

A 2 terminal n/w N_A containing linear elements and independent sources is equivalent to simpler network containing an independent voltage source in series with 2-terminal network having an impedance excluding the load.

The source voltage is the open circuit voltage of N_A. ie load Z_L disconnected.

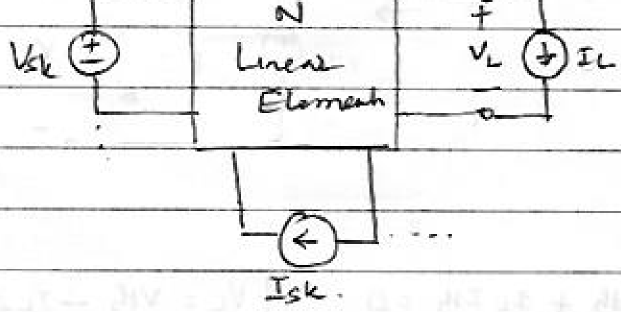
The impedance (series impedance) is effective impedance of N_A measured at the terminal after deactivating all independent sources ~~ie Deactivating by in~~ N_A. ie Voltage sources are S.C and current sources are O.C



The network N_A consists of Network N of linear elements with several independent voltage sources (V_{sk}) and current sources I_{sk} acting on N. The two terminal of network N_A is connected to network N_L (externally load n/w is connected). Due to N_A, there is a V_L a/c N_L and I_L flowing through N_L.

Now, by substitution method N_L can be replaced by current source I_L (I_L is current flowing through N_L)

– Elements can be replaced by either a Voltage source or current sources



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Using superposition principle to computing V_L ~~th~~ by superposing the effects of all independent sources & current source I_L

V_L is computed $\therefore V_L = A_0 I_L + A_1 V_{S1} + A_2 V_{S2} + \cdots + A_3 I_{S1} + A_4 I_{S2}$ — (1)

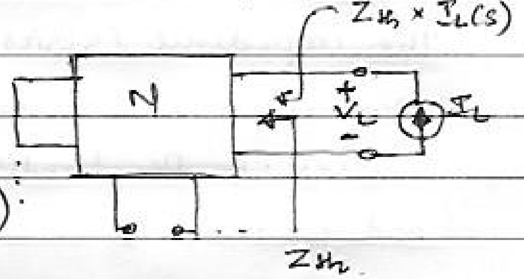
$V_L = A_0 I_L + B_0$ — (2)

Now the effective impedence of N_A can be computed by deactivating all the independent sources in the n/w except I_L source.

ie $B_0 = 0$.

$\therefore V_L = A_0 I_L$

$\boxed{A_0 = \frac{V_L}{I_L} = Z_{th}}$ (Effective impedence).



Now, ~~source~~ voltage

In above ckt, Linear portion of N is acted upon by one source I_L and all others are deactivated. $\therefore$ I_L is driving the one port n/w $\therefore$ driving ~~force~~ point impedence is $Z_{th} = \frac{V_L}{I_L}$ ie $V_L = I_L \times Z_{th}$

Now, in eq² (1) to obtain B_0, $I_L = 0$.

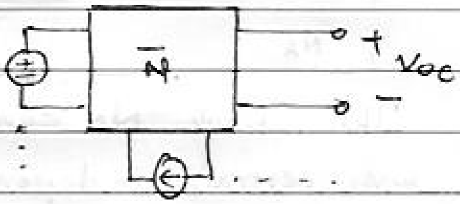
$\therefore$ the n/w N_A is open circuited at terminals pq

$\therefore$ $V_L = A_0 I_L + B_0$.

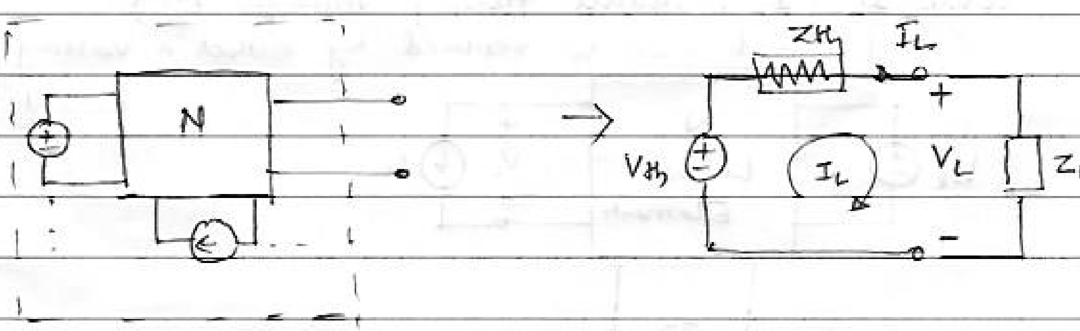
$I_L = 0$

$\boxed{V_L = B_0 = V_{OC} = V_{th}}$

Open ckt Voltage.



Superimposing the two cases discussed above gives the thevnins equivalent of network N_A having open ckt Voltage V_{th} in series Effective impedence Z_{th}



$V_L - V_{th} + I_L Z_{th} = 0 \quad \therefore \boxed{V_L = V_{th} - I_L Z_{th}}$

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Conditions of Network N_A and N_L.

- Network N_A consists of Linear elements(N) which may be either passive elements (R, L, C) which are reciprocal and dependent sources which are non-reciprocal.
- Network N_A consist of independent sources (either current/voltage sources and initial condn of passive components).
- No coupling between N_A and N_L
  ie No magnetic (or) controlled sources in N_A depending on current/voltage in N_L & vice versa.

Similarly, Network N_L

- N_L can be linear or non linear, ~~a/too~~ or time varying element.
- Independent / dependent sources.
- Initial condn for passive components.
- No coupling with N_L. (magnetically / controlled sources)

Steps to find Effective impedence (Z_{th})

i) Set all the initial conditions to zero ie $V_C = 0$, $I_L = 0$ and turn off all independent sources by S.C voltage srces and O.C current sources.

ii) Controlled ckt in Network are kept intact

iii) Determine the driving point impedence ~~of~~ (or) admittance

Steps to find Open ckt voltage $V_{OC} = V_{th}$

i) Open terminal of network N_A with network N_L removed. (ie load removed)

ii) Find open ckt voltage $V_{oc} = V_{th}$ due to all the sources - acting in the network.

iii) Replace N_A with $V_{OC} = V_{th}$ and series effective impedence Z_{th}.

iv) Connect the N_L (load impedence) and calculate the load current I_L.

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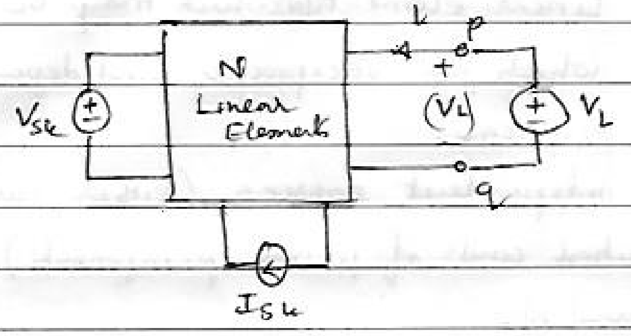
Norton's theorem



$$i = C_0 V_L + C_1 V_{S1} + C_2 V_{S2} + C_3 I_{S1} + C_3 I_{S2} \quad \text{—(1)}$$

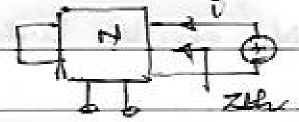
$$i = C_0 V_L + D_0 \quad \text{—(2)}$$

If $D_0 = 0$ ie All independent sources made inactive

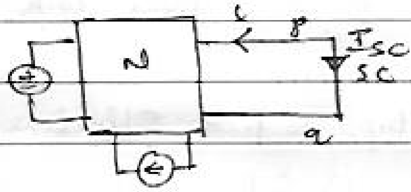
$i = C_0 V_L$

$\left[C_0 = i / V_L \right] = G_{eq}$ (admittance)

Nortons



Now, By removing V_L and SC the terminal p, q, the current i can be computed



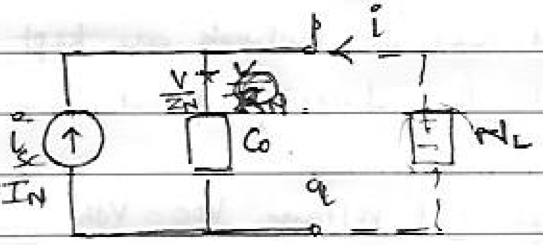
(ii) If $V_L = 0$, SC the p, q

eqn (2) is

$i = 0 + D_0$

$\therefore \boxed{D_0 = i} = -I_{SC} = I_N$.

$\therefore$ i =



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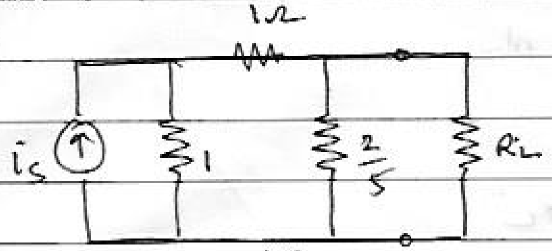
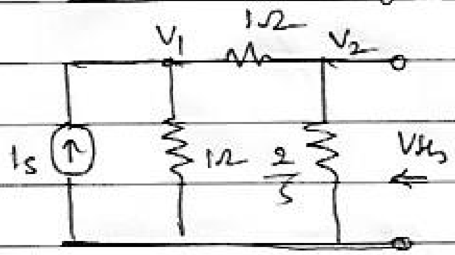
(1)



find V_{th} and R_{th}.



Using Node analysis.

$$\text{At } N_1, \quad \frac{V_1}{1} + \frac{(V_1 - V_2)}{1} - I_s = 0$$

$$2V_1 - V_2 = I_s \quad -(1)$$

$$\text{At } N_2, \quad \frac{V_2}{(2/5)} + \frac{(V_2 - V_1)}{1} = 0$$

$$7/2\, V_2 - V_1 = 0 \quad -(2)$$

from (1)

$$2V_1 = I_s + V_2$$

$$V_1 = (I_s + V_2)/2 \quad -(3)$$

(3) in (2)

$$7/2\, V_2 - \frac{(I_s + V_2)}{2} = 0$$

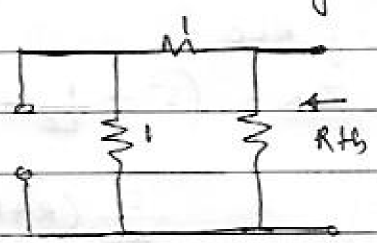
$$7/2\, V_2 - \frac{I_s}{2} - \frac{V_2}{2} = 0$$

$$\frac{6}{2} V_2 = \frac{I_s}{2}$$

$$V_2 = \frac{I_s}{\cancel{2}} \times \frac{\cancel{2}}{6} = I_s/6$$

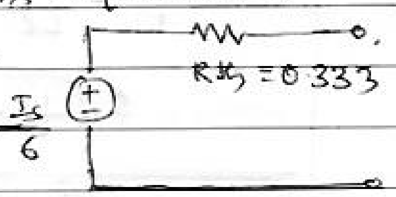
$$\boxed{V_2 = I_s/6} = V_0 = V_{th}$$

$Z^{th} (R^{th})$ found by S.C voltage & O.C current source



$$R_{th} = \frac{2 \times 2/5}{2 + 2/5} = \frac{1}{3} = 0.333\,\Omega$$

Thevenin's equivalent is



$R_{th} = 0.333$

$\frac{I_s}{6}$

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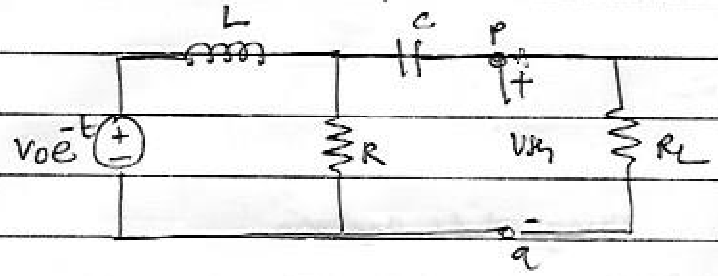
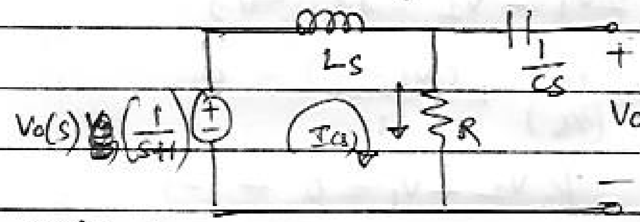
2) find thevenins eq^n ckt for the n/w.



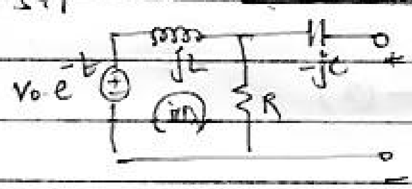
Compute V_{th} by O.C pq



$$I(s) = \left[\frac{V_0(s)}{(s+1)}\right] \div \left[R + Ls\right]$$

$$= \frac{V_0(s)}{(s+1)(Ls+R)}$$

$\mathcal{L}(e^{-t}) = \frac{1}{s+1}$ (or) time domain



$$i(t) = \frac{V_0 e^{-t}}{R+jL}$$

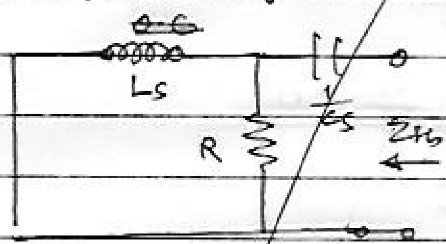
$$\therefore V_{oc} = \left(\frac{V_0 e^{-t}}{R+jL}\right) R$$

$$V_{oc} = I(s) \times R$$

$$\boxed{V_{oc} = \frac{V_0(s) R}{(R+Ls)(s+1)}}$$

R_{th}, s.c voltage sour, initial cond^n as zero.

L relaxed acts as O.C

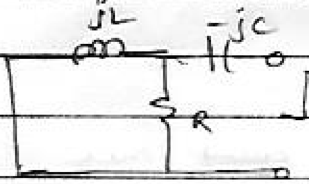


$$Z_{th} = \frac{R \times Ls}{R+Ls} + \frac{1}{cs}$$

$$= \frac{RLs \times cs + (R+Ls)}{cs(R+Ls)}$$

$$= \frac{RLcs^2 + R + Ls}{cs(R+Ls)}$$

Z_{th} is time domain



$$z = \frac{R \times jL}{R+jL} + (-jc)$$

$$= \frac{jRL - jc(R+jL)}{(R+jL)}$$

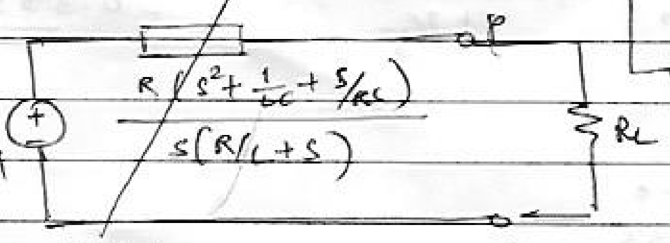
$$= \frac{jRL - jRC + LC}{(R+jL)}$$

Divide Nr & Dr by RLC

$$Z_{th} = \frac{\left(s^2 + \frac{1}{LC} + \frac{s}{RC}\right)}{\frac{s}{RL}(R+Ls)}$$

$$= \frac{\left(s^2 + \frac{1}{LC} + \frac{s}{RC}\right)}{s\left(\frac{1}{L} + \frac{s}{R}\right)} \times \frac{R}{R}$$

$$\boxed{Z_{th} = \frac{R\left(s^2 + \frac{1}{LC} + \frac{s}{RC}\right)}{s\left(R/L + s\right)}}$$



$$V_{th} = \frac{V_0(s) R}{(s+1)(R+Ls)}$$

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$e^{-t} = \frac{V_0}{s+a}$

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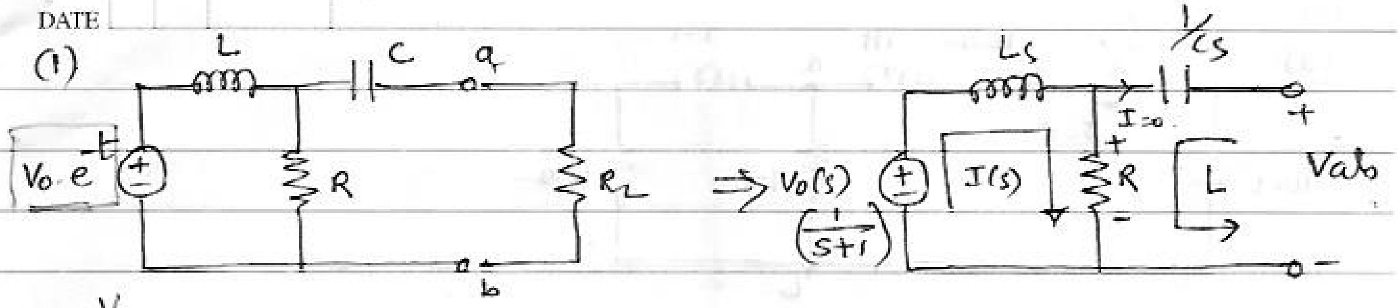
(1)



$\frac{V_0}{s+a}$

$\left(\frac{1}{s+1}\right)V_0$

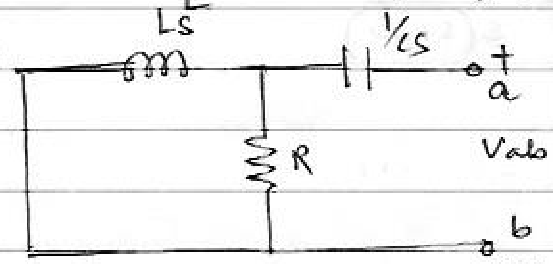
$$I(s) = \frac{V_0(s)}{(s+1)(R+Ls)} \quad -①$$

KVL to loop L

$$-V_{ab} + \frac{V_0(s)}{(s+1)(R+Ls)} \cdot R$$

$$\boxed{V_{ab} = \frac{V_0(s)}{(s+1)(R+Ls)} R} \quad -(2)$$

R_{th}



$$Z_{eq} = \frac{R \times Ls}{R+Ls} + \frac{1}{Cs}$$

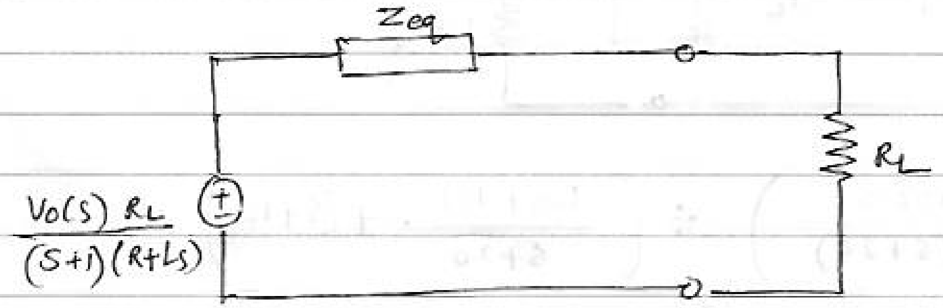
$$= \frac{(RLs)Cs + 1(R+Ls)}{Cs(R+Ls)}$$

$$Z_{eq} = \frac{RLCs^2 + R + Ls}{Cs(R+Ls)}$$

$\div$ Nr, Dr by RLC

$$Z_{eq} = \frac{\left(s^2 + \frac{1}{LC} + \frac{s}{RC}\right)}{\frac{s}{RL}(R+Ls)}$$

$$Z_{eq} = \frac{(s^2 + 1/LC + s/RC)}{s(1/L + s/R)} \times \frac{R}{R} = \frac{R(s^2 + 1/LC + s/RC)}{s(R/L + s)}$$



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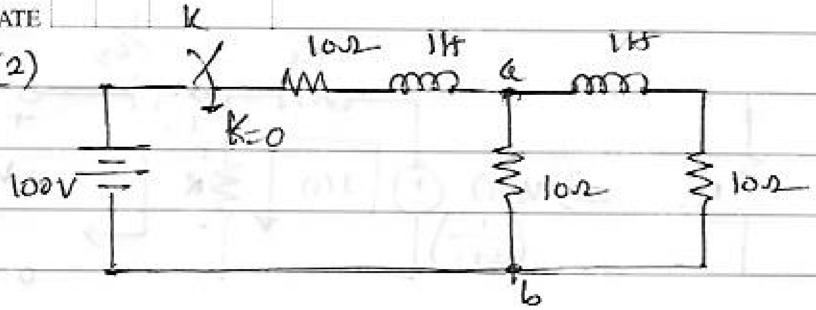
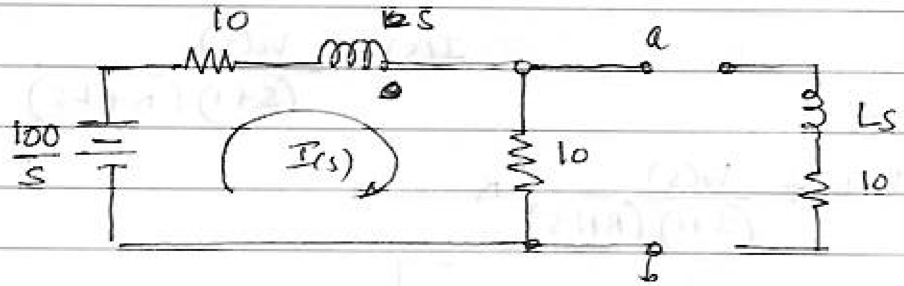
(2)

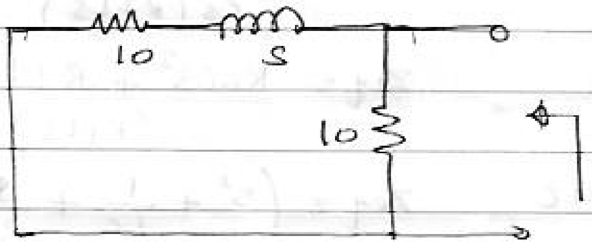




$$I(S) = \frac{100}{(20+S)S} = \frac{100}{S(S+20)}$$

$$V_{th} = I(S) \times 10$$

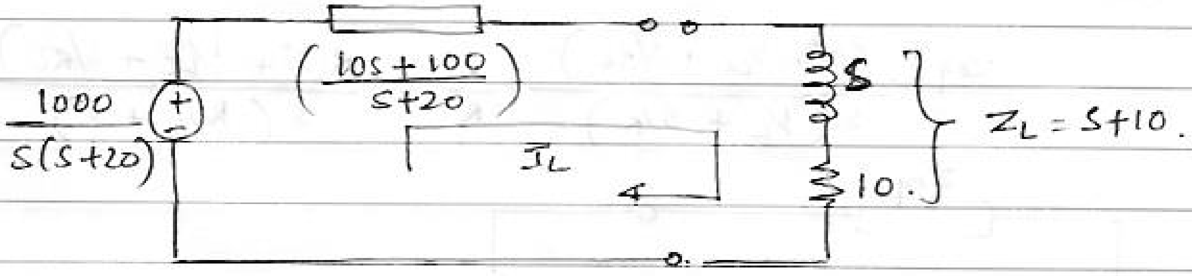
$$\boxed{V_{th} = \frac{1000}{S(S+20)}} \quad \text{--- (1)}$$



$$Z_{th} = \frac{\cancel{10}(S+10) \times 10}{S+10+10}$$

$$Z_{th} = \left(\frac{10S+100}{S+20}\right)$$

Thevenins equivalent



$$I_L = \left(\frac{1000}{S(S+20)}\right) \div \left(\frac{10S+100}{S+20} + (S+10)\right)$$

$$= \frac{1000}{S(S+20)} \times \frac{(S+20)}{10S+100+S^2+20S+10S+200.}$$

S^2+20S $\cancel{S^2+20S}$

$$\boxed{I_L = \frac{1000\ (S+20)}{2S^2+60S+300}}$$

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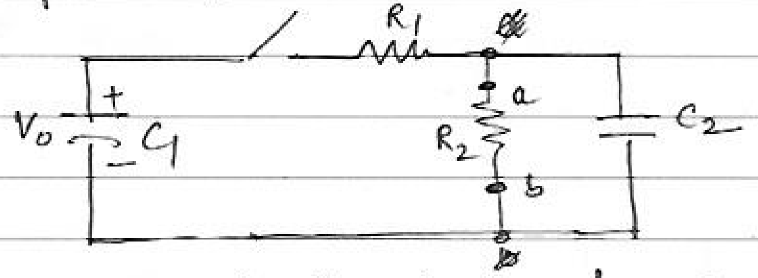
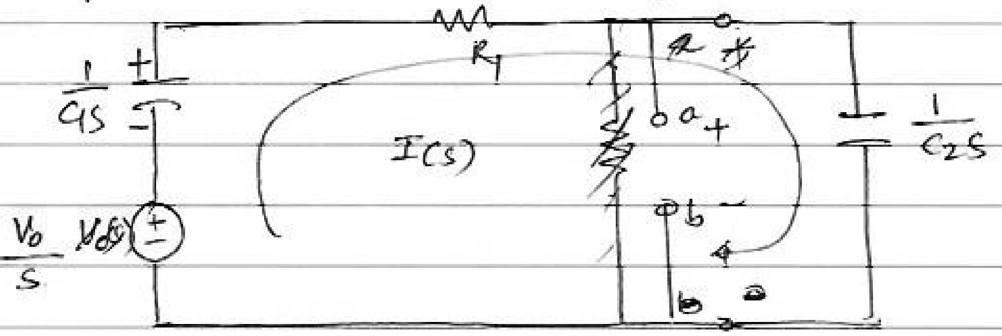
(4) Find current in the resistor R_2. Initial cond^n ~~afe~~ i.e voltage of capacitor $C_1 = V_0$ and $C_2 = 0V$



Open ckt (a, b) and transformed ckt is

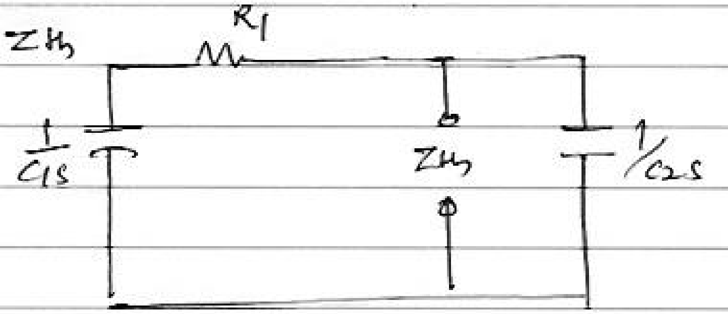


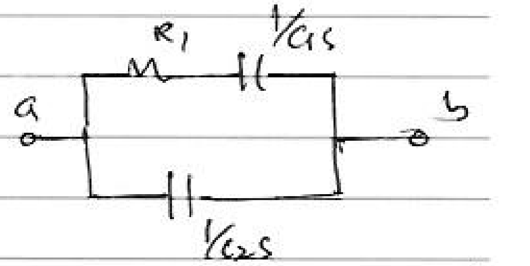
$$I(s) = \frac{(V_0/s)}{R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$V_{ab} = I(s) \times \frac{1}{C_2 s} = \frac{(V_0/s)}{R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}} \times \frac{1}{C_2 s}$$

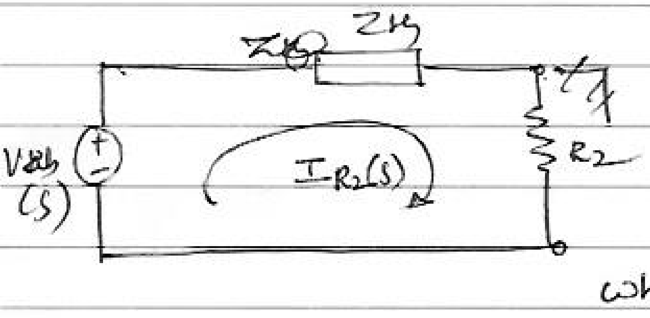
$$\boxed{V_{ab} = \frac{V_0/C_2 s^2}{R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}}$$

Z_{th}





$$Z_{th} = \frac{\left(R_1 + \frac{1}{C_1 s}\right) \times \frac{1}{C_2 s}}{R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$



$$I_{R_2}(s) = \frac{V_{ab}(s)}{Z_{th} + R_2}$$

where $Z_{th} = \frac{\left(R_1 + \frac{1}{C_1 s}\right) \frac{1}{C_2 s}}{R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$

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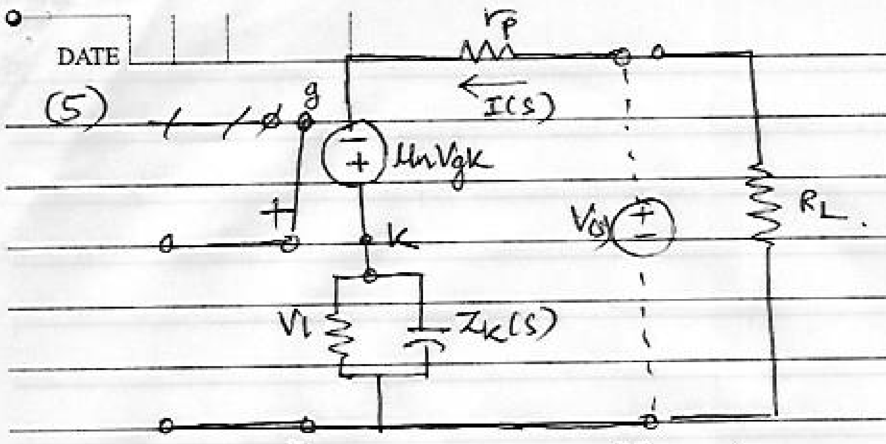
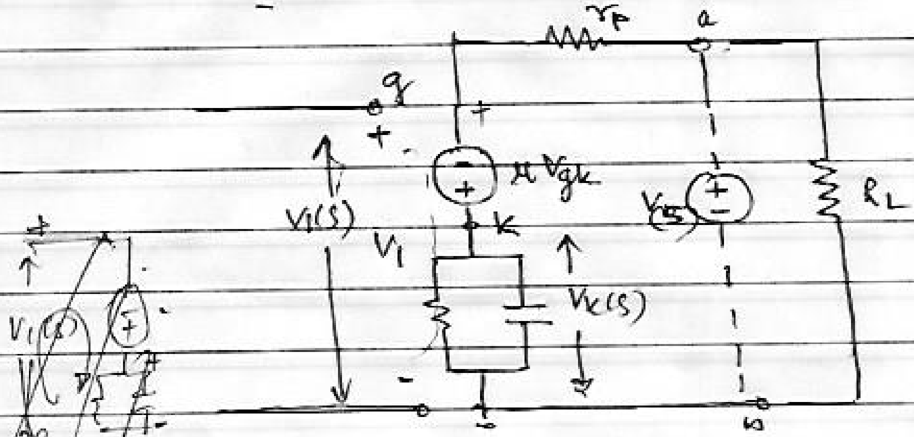
(5)



The n/w contains a controlled source which depends on the voltage $V_{gk}(s)$



$$V_{gk}(s) = V_1(s) - V_k(s)$$

$$\cancel{-\mu_n V_{gk}(s) + V_k(s) - V_1(s) = 0}$$

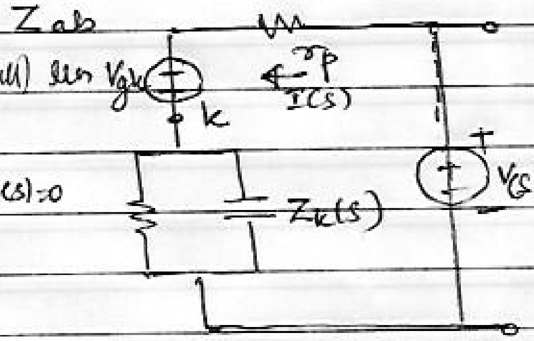
$$\cancel{V_1(s) + \mu_n V_{gk} - V_k(s) = 0}$$

$$\cancel{\mu_n V_{gk}(s) =}$$

Zab

$-V_1(s) - \mu_n V_{gk}(s)\ \mu_n V_{gk}$

$+V_k(s)$



$V_1(s) = 0$

$Z_{ab} \neq r_p/\cancel{4}$

$I(s) = V(s)$

$V(s) \leftarrow$ Voltage Source inserted

$$\cancel{V(s) = I(s)\, r_p + I(s)\, \mu_n Z_k(s) + I(s)\, Z_k(s)}$$

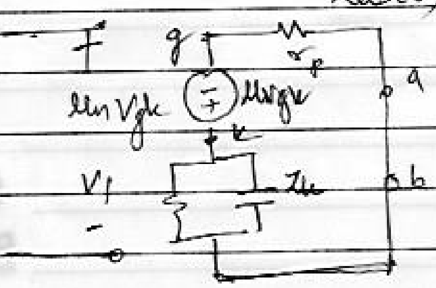
$$V(s) = I(s) \cdot r_p - \mu_n\left(V_1(s) - V_k(s)\right) + V_k(s)$$

$$= I(s) \cdot r_p - \mu_n\left(0 - I(s) \cdot Z_k(s)\right) + I(s) \cdot Z_k(s)$$

$$= I(s) \cdot r_p + \mu_n I(s) \cdot Z_k(s) + I(s) \cdot Z_k(s)$$

$$Z_o(s) = Z_{ab}(s) = \frac{V(s)}{I(s)} = r_p + \mu_n Z_k(s) + Z_k(s)$$

$$Z_{ab}(s) = Z_o(s) = r_p + Z_k(s)\left(1 + \mu\right)$$



$$\cancel{I_o(s) \cdot Z_k + I_o(s) \cdot \mu V_{gk} + I_o(s) \cdot r_p = 0}$$

$$I_o(s) = V_1(s)$$

$$\boxed{I_o(s) = \frac{\mu V_1(s)}{r_p + (1+\mu) Z_k(s)}}$$

$$Z_o = r_p + (1+\mu) Z_k$$

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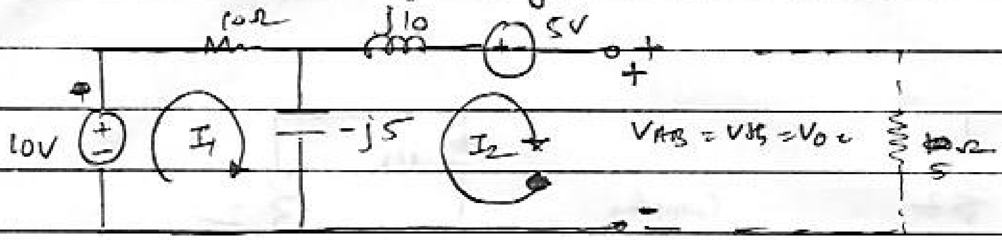
(1) Find current in 5Ω resistor connected a/c terminals A and B in ckt shown in fig using thevenin's thm



KVL for current I_1

$$-10 + 10I_1 - j5(I_1 - I_2) = 0$$

$$(10 - j5)I_1 + j5I_2 = 10 \quad -(1) \qquad I_1 = \frac{10 - j5I_2}{(10 - j5)}$$

$$-j5(I_2 - I_1) + j10I_2 + 5V + V_{AB} = 0 \qquad V_{AB} = I_2 \times 5\Omega$$

$$+j5I_1 + j5I_2 = -V_{AB} - 5V \quad -(2)$$

$$+j5I_1 + j5I_2 + I_2 \times 5 = -5V$$

$$V_{AB} = -5V - j5I_1 - j5I_2$$

$$\cancel{= -5 - j5\frac{(10 - j5I_2)}{(10 - j5)} - j5I_2 = 0}$$

$$+j5I_1 + (5 + j5)I_2 = -5V \quad -(2)$$

$$10i_1 + (5 + 10j)I_2 = 5$$

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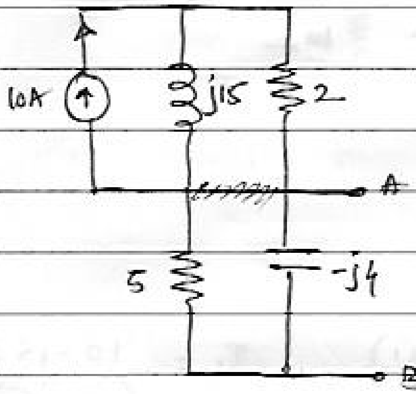
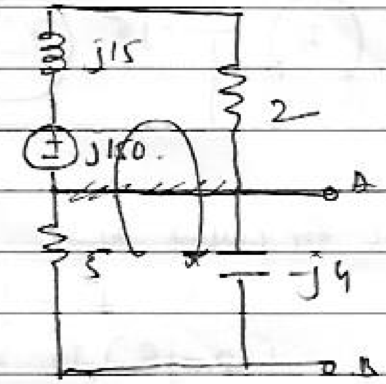
(2) find thevenins eq. ck for the n/w shown in fig. What imp must be connected between A & B for max power transfer. What is max power



Converting current source to volt



By KVL

$$5I_1 - j150 + j15 I_1 + 2I_1 - j4 I_1 = 0$$

$$I_1(7 + j11) = -j150$$

$$I_1 = +j150/(7+j11)$$

$$I_1 = 9.7 + 6.17j$$

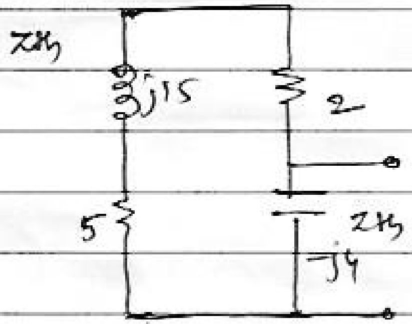
$$I_1 = 11.5 \angle 32.47^\circ$$

$$V_{th} = -j4 \times (9.7 + 6.17j)$$

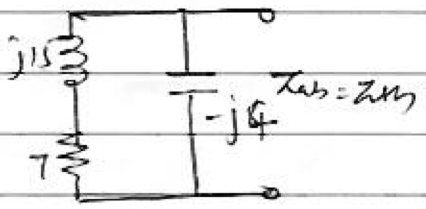
$$= 24.68 - 38.8j$$

$$\boxed{V_{th} = 45.98 \angle -57.54}$$

Z_{th}



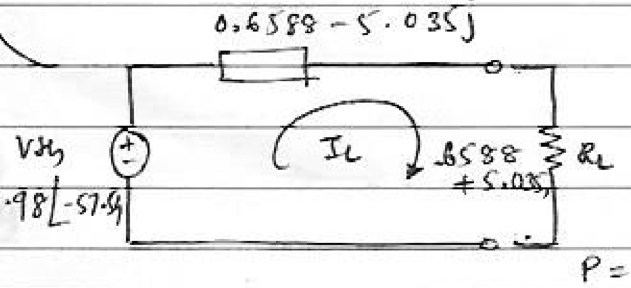
$\Rightarrow$



$$I_L = \frac{V_{th}}{Z_{th} + R_L} = 18.731 - 29.45j$$

$$Z_{th} = \frac{(7+15j) \times (-j4)}{(7+11j)}$$

$$= 0.6588 - 5.035j$$



for max power

$$R_L = Z_{th}^*$$

$$R_L = 0.6588 + 5.035j$$

$$P = |I_L|^2 \times R_L = |18.731 - 29.45j|^2 \times R_L$$

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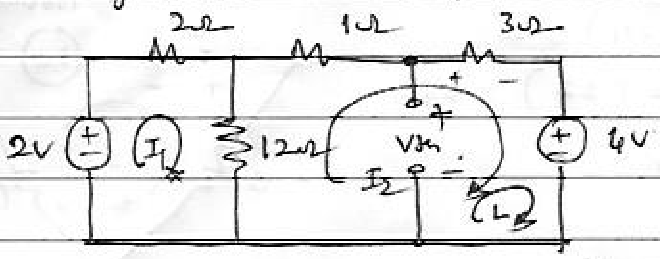
(3) Using thevenins thm find current is 2Ω resistor



KVL @ I_1

$-2 + 2I_1 + 12(I_1 - I_2) = 0.$

$\boxed{14I_1 - 12I_2 = 2}$ — ①

@ I_2 $\quad 4I_2 + 4 + 12(I_2 - I_1) = 0.$

$\boxed{-12I_1 + 16I_2 = -4}$ — ②

$I_1 = -0.2\,A \qquad I_2 = -0.4\,A$

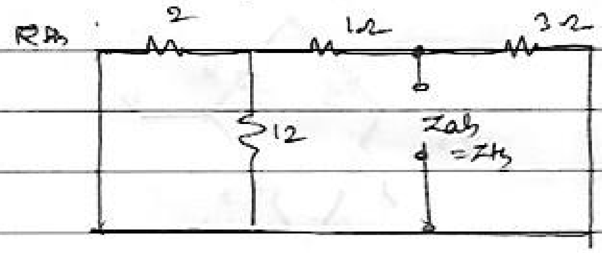
∴ ~~Vth~~ KVL at loop L.

$-V_{th} + 3(-0.4) + 4 = 0$

$V_{th} = 4 - 1.2 = 2.8\,V$

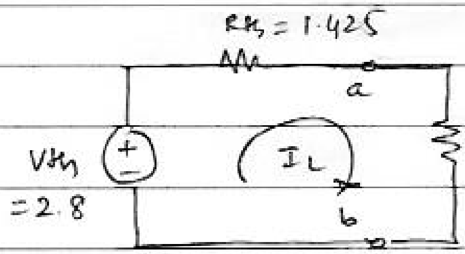
$\boxed{V_{th} = 2.8V}$

$V_{th} = 3 \times 0.4$



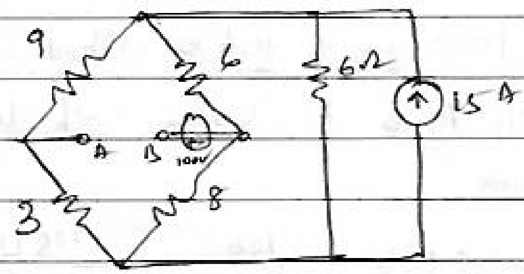
$\frac{12 \times 2}{14} + 1 = 2.714\,\Omega$

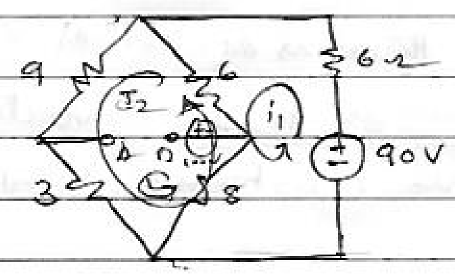
$Z_{th} = \frac{2.714 \times 3}{2.714 + 3} = 1.425\,\Omega$



$I_L = \frac{2.8}{2 + 1.425} = 0.8175.$

(4) Obtain thevenins eqn w.r.t to terminals A, B.





KVL 1, 2

@1 $\quad -90 + 6i_1 + 6(i_1 - i_2) + 8(i_1 - i_2)$

$20i_1 - 14i_2 = 90$ — ①

@2 $\quad 9i_2 + 3i_2 + 8(i_2 - i_1) + 6(i_2 - i_1) = 0$

$-14i_1 + 26i_2 = 0$ — ②

$i_1 = 7.222$

$i_2 = 3.888$

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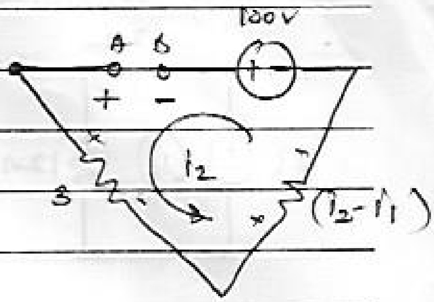
Applying KVL to loop L

$$-V_{AB} - 100 + 3i_2 - 8(i_2 - i_1) = 0$$

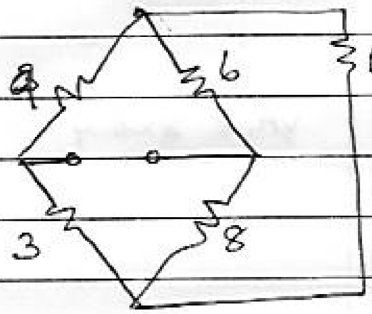
$$-100 + 3\times 3.888 + 8(3.888 - 7.22) = V_{AB}$$

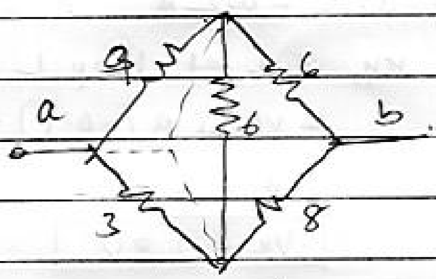
$$-100 + 11.664 + 26.656 = V_{AB}$$

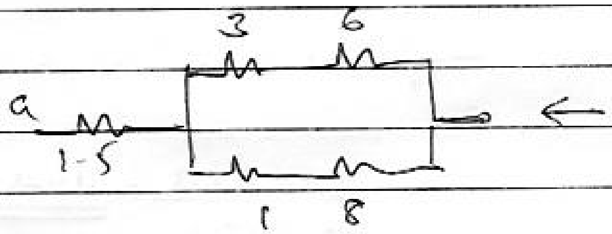
$$V_{AB} = \underline{\underline{-115\,V}}$$

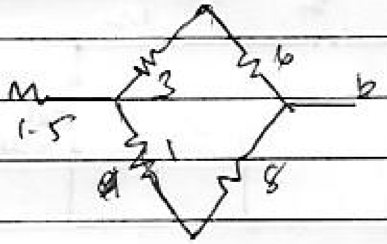


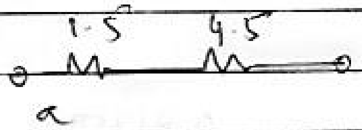
R_{th} (Thevenin's Resistance)

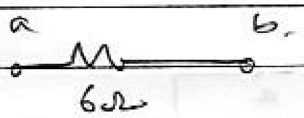


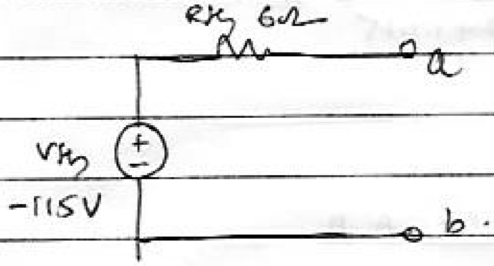






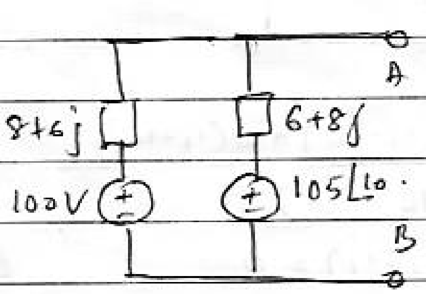






← Thevenin's equivalent ckt

5) Obtain Thevenin's eq. ckt as seen from point A & B. What is the load impedance to be connected a/c A&B for I_{max}. If load impedance is pure resistance what P_{max}



$$E_{eq} = \frac{\sum E_i/z_i}{\sum \frac{1}{z_i}} = \frac{\frac{100}{8+6j} + \frac{105\angle 10}{6+8j}}{\frac{1}{8+6j} + \frac{1}{6+8j}}$$

$$E_{eq} = 102.96 + 8.86j$$

$$Z_{eq} = 3.57 + 3.57j.$$

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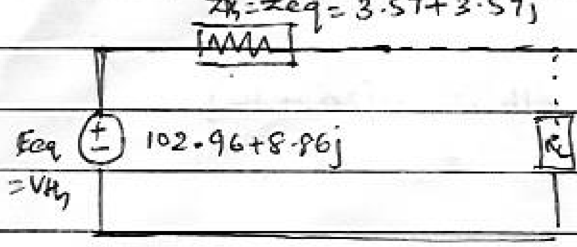


for max power transfer

$R_L = \cancel{Z_{th}}\ Z_{th}^*$

$\boxed{R_L = 3.57 - 3.57j}$

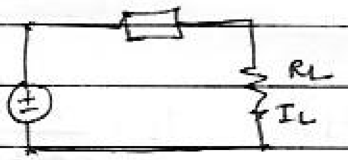
$\therefore I_L = \dfrac{102.96 + 8.86j}{Z_{th} + R_L}$

$= \dfrac{102.96 + 8.86j}{2(3.57)}$

$= 14.42 + 1.24j$.

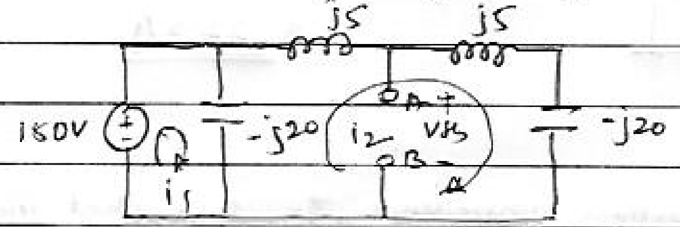
(i) $P_{L\,max} = \dfrac{|V|^2}{R} = \dfrac{\cancel{102.96^2 + (8.86)^2}}{} = 747.5W$. ?

(ii) Load is pure resistive



$R_L = |Z_{th}| = \sqrt{(3.57)^2 + (3.57)^2}$
$= 5.05$
$=$

6) find thevenin's eq. w.r.t terminals A & B of ckt. Find current in the load imp of $(10 - j7.5)\Omega$ connected a/c AB



$\cancel{-150 - j20 \times I_1 = 0}$

$\cancel{-j20 I_1 = 150}$ $\quad I_1 =$

$\cancel{I_1 = 7.5 j}$

$\cancel{-j10 \times i_2}$

$-150 - j20(i_1 - i_2) = 0$

$-150 - j20 i + j20 i_2 = 0$

$-20j\, i_1 + 20j\, i_2 = 150 \quad -(1)$

$-j20(i_2 - i_1) - j10 i_2 = 0$

$-j30 i_2 + j20 i_1 = 0$

$-j30 i_2 = -j20 i_1$

$i_2 = 0.6667 i_1$

$-20j\, i_1 + 20j(0.6667 i_1) = 150$

$i_1 = \dfrac{150}{-(20/3)i} = \underline{\underline{22.5 i}}$

$\underline{\underline{i_2 = 15.0075 i}}$

$V_{th} = i_2 \times (j5 - j20)$

$= 15.0075 i \times -j15$

$\boxed{V_{th} = 225.11 \text{ Volts}}$

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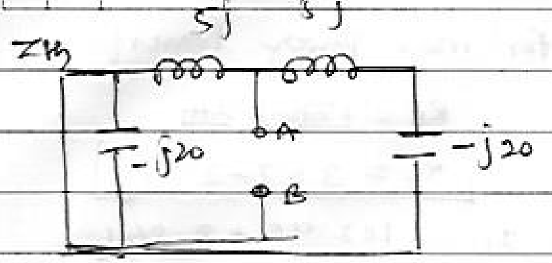
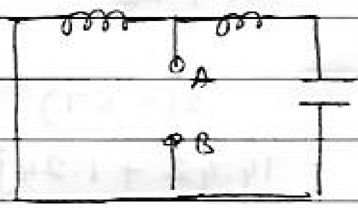
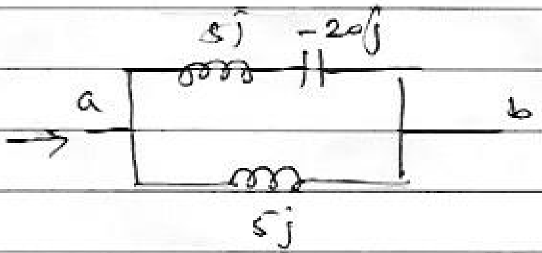


$Z_{th} = -j26 + 10j$

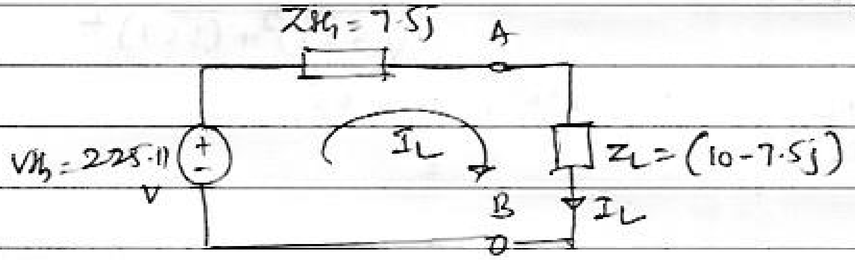
$=$



$\rightarrow$



$$Z_{th} = \frac{-15j \times 5j}{-15j + 5j} = 7.5j$$



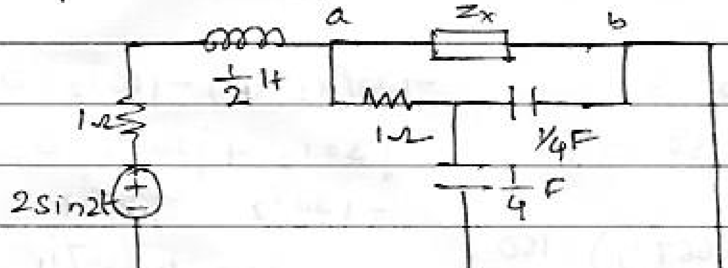
$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

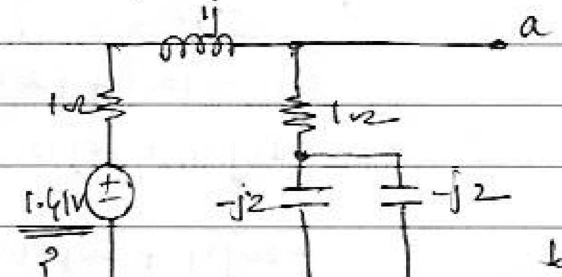
$$= \frac{225.11}{10 - 7.5j + 7.5j}$$

$\boxed{I_L = 22.75}$

$= \underline{22.5A}$

(8) 7) for the n/w shown in fig determine impedence Z_x such that max power is transfered from source to load of imp Z_x. Find Pmax





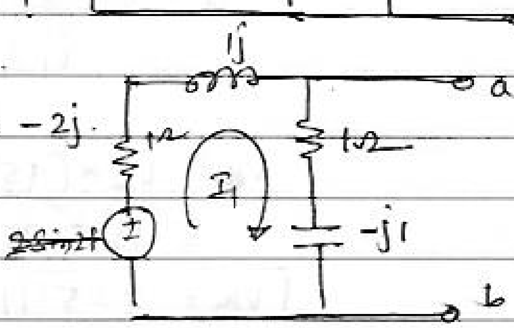
$\omega = 2 \quad \therefore \; X_L = j\omega L = j2 \times \frac{1}{2} = j1.$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j2 \times \frac{1}{4}} = -2j.$$

$\omega = 2\pi f$

$\frac{2}{2\pi} = f$

$$\frac{-j2 \times -j2}{-(j2) + (-j2)}$$



KVL: $1.41 - 2i_1 = 0$

$2i_1 = 1.41 \quad \therefore \; i_1 = 0.705$

$\boxed{V_{AB} = i_1 \times (1 - j) = 0.705 - 0.705j}$

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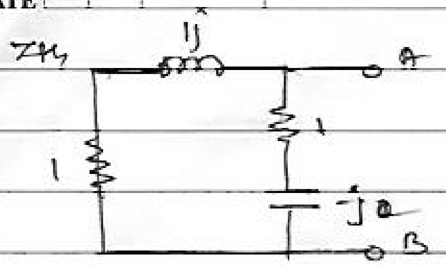
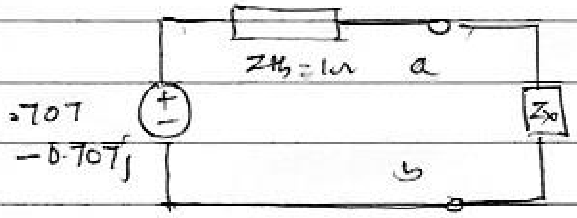


$$Z_{th} = \frac{(1+j)(1-j)}{1+j+1-j} = 1\Omega$$



P_{max} $Z_{th} = Z_x = 1\Omega$

$$\therefore \quad I_L = \frac{V_{th}}{1+1} = 0.35(1-j)\,A$$
$$= 0.5\angle -45^\circ$$

$$P_{max} = |I_L|^2 Z_x = 0.28\,W$$

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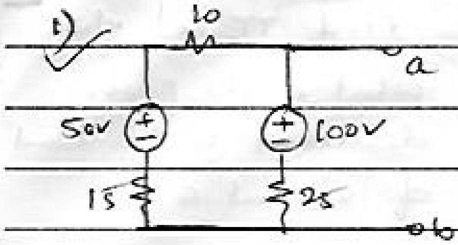
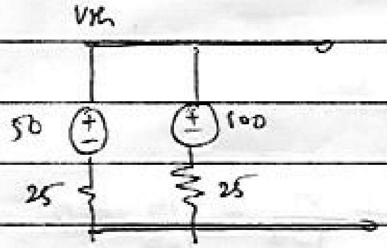
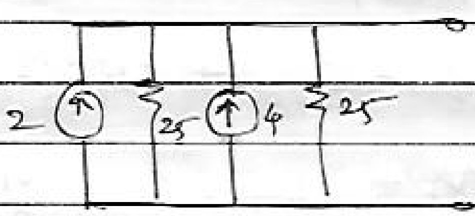
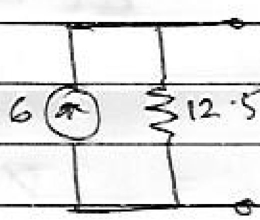
1) 

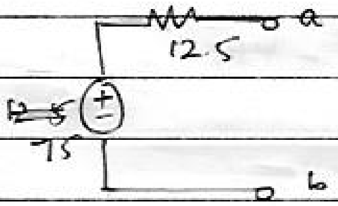
find thevenin equivalent at terminal a & b. How much power is delivered to a resistor connected to a and b if Rab equals

a) 50Ω b) 12.5Ω

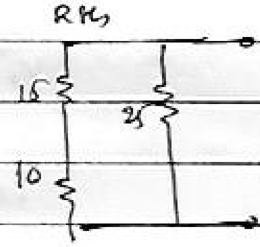




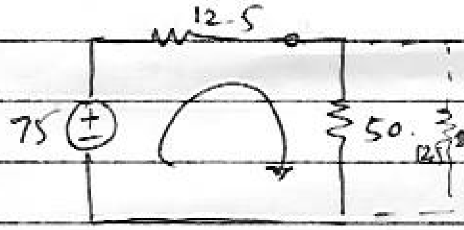




$V_{th} = 75$



$$\frac{25 \times 25}{25+25} = 12.5$$



if $R = 50$

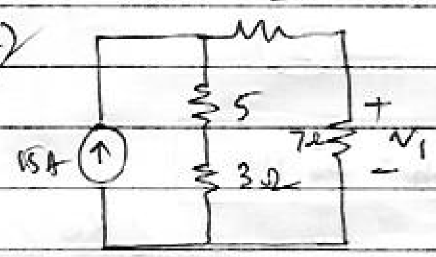
$I = 75/62.5 = 1.2$

~~if R = 50~~ $P = i^2R = 72$ watts.

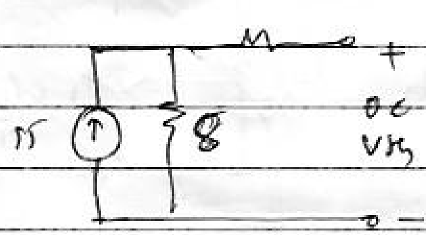
if $R = 12.5$

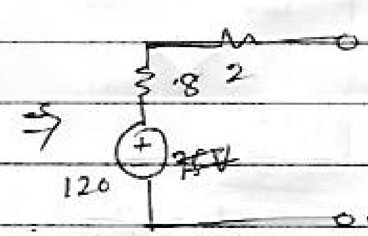
$i = 75/25 = 3\text{ A}$

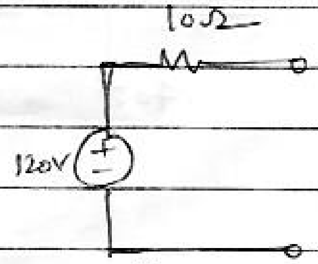
$P = i^2R = 3^2 \times 12.5 = 112.5$ watts.

(2) 

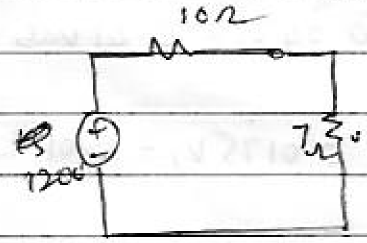
Find thevenins equivalent of the n/w connected to 7Ω resistors. find the corresponding Nortons equivalent. Compute V_1 using both ckts.



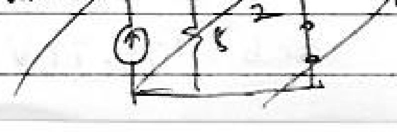




Thevenins equivalent



$$I = \frac{15}{17} = 0.8823 \qquad V_1 = \underline{6.1764}$$

Norton 

~~$R_{eq} = 1.6$~~

~~$\frac{1}{R_{eq}} = 0.625$~~

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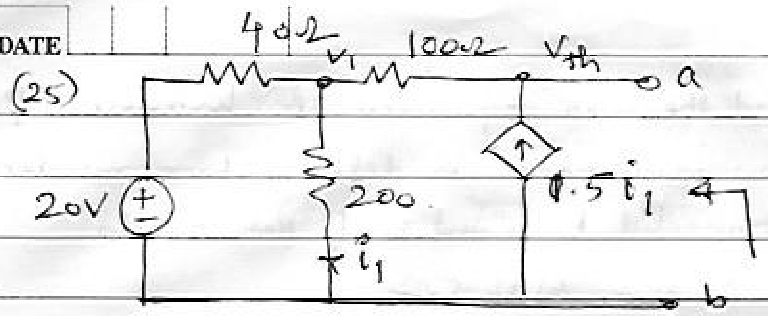
(25)



Find TE of n/w what is the power delivered to the load of 10Ω at a, b

KCL at Node 1

$$\frac{V_1-20}{40} + \frac{V_1}{200} + \frac{V_1 - V_{th}}{100} = 0.$$

$$\cancel{0.04V_1} \frac{V_1}{40} - \frac{20}{40} + \frac{V_1}{200} + \frac{V_1}{100} - \frac{V_{th}}{100} = 0.$$

$$V_1\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{100}\right) - \frac{V_{th}}{100} = \frac{20}{40}.$$

$$.04V_1 - 0.01\,V_{th} = 0.5.$$

$$\boxed{4V_1 - V_{th} = 50} \quad -(1)$$

Node 2 $\dfrac{V_{th}-V_1}{100} - 1.5\,i_1 = 0$

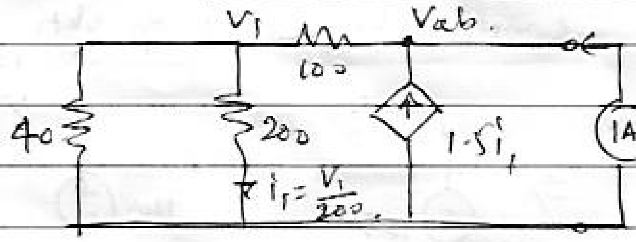
$$\frac{V_{th}}{100} - \frac{V_1}{100} - 1.5\left(\frac{V_1}{200}\right) = 0 \quad < i_1 = \frac{V_1}{200}$$

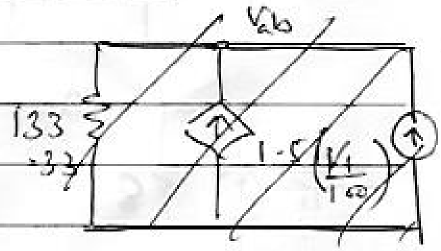
$$0.01V_{th} - 0.0175\,V_1 = 0$$

$$\boxed{-1.75V_1 + V_{th} = 0} \quad -(2)$$

$$\boxed{V_1 = 22.22V \qquad V_{th} = 38.88}$$

R_{th}





$$i_1 = \frac{V_1}{100}.$$

$$\frac{V_1}{40} + \frac{V_1}{200} + \frac{V_1 - V_{ab}}{100} = 0. \qquad 0.04V_1 - 0.01V_{ab} = 0 \quad -(1)$$

$$\frac{V_{ab}-V_1}{100} - 1.5\left(\frac{V_1}{200}\right) + 1 = 0 \qquad 0.0175V_1 - 0.01V_{ab} = 1 \quad -(2)$$

$$V_{ab} = 177.77\,V \qquad R_{th} = \frac{V_{ab}}{1A} = 177.77\,\Omega$$

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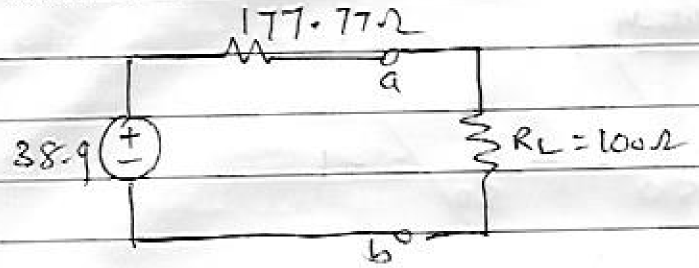
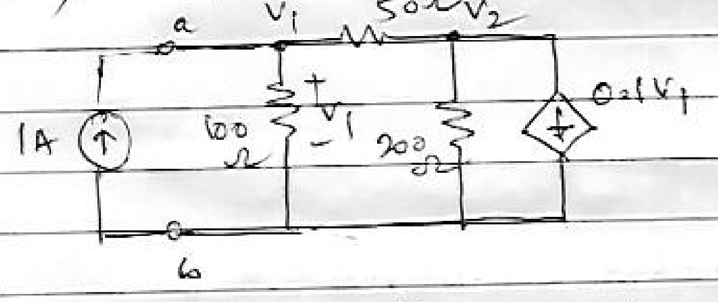


$$P_{ab} = \left[\frac{38.91}{277.77}\right]^2 \times 100.$$

$$= 1.96\,W.$$

26) Find TNE eq² ckt



At Node 1

$$\frac{V_1}{100} + \frac{V_1 - V_2}{50} = 1.$$

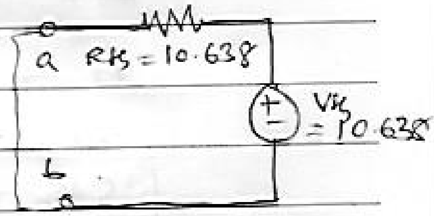
$$V_1 + 2V_1 - 2V_2 = 100.$$

$$\boxed{3V_1 - 2V_2 = 100} \quad (1)$$

$$\frac{V_2}{200} + \frac{V_2 - V_1}{50} + 0.1V_1 = 0$$

$$V_2 + 4V_2 - 4V_1 + 0.1 \times 200\,V_1 = 0$$

$$\boxed{16V_1 + 5V_2 = 0} \quad (2)$$



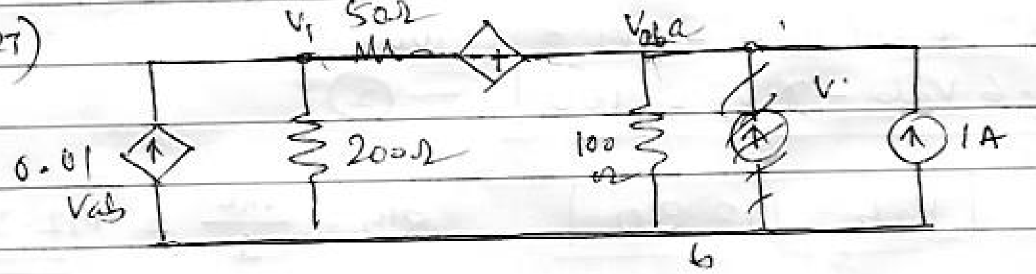
$$\boxed{V_1 = 10.638} \qquad V_2 = -34.042.$$

?

$\cancel{V_{th} = 10.638}$ $\qquad R_{th} = V_1/1A = 10.638.$

$\cancel{I_N = 1A}$ $\qquad$ As there is no ~~d~~ independent sources $I_N = 0$, $V_{th} = 0$.

27)



$$\frac{V_2}{100} + \frac{V_1}{200} + \frac{V_1 - V_{ab} + 0.2V_{ab}}{50} - 0.1\,V_{ab} = 0.$$

$$\frac{V_1}{200} + \frac{V_1}{50} - \frac{0.8V_{ab}}{50} - 0.1\,V_{ab} = 0.$$

$$5V_1 - 3.2\,V_{ab} - 20V_{ab} \quad 5V_1 - 4.8\,V_{ab} = 0.1 \times 200\,V_{ab}$$

$$5V_1 - 23.2\,V_{ab} = 0 \quad \boxed{5V_1 - 24.8\,V_{ab} = 0}$$

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$$\frac{V_{ab}}{100} + \frac{V_{ab} - 0.2V_{ab} - V_1}{50} + 1 = 0$$

$$\frac{V_{ab}}{100} + \frac{\cancel{V_{ab}}\ 0.8V_{ab}}{50} - \frac{V_1}{50} = -1$$

$$V_{ab} + 1.6V_{ab} - 2V_1 = -100$$

$$\boxed{-2V_1 + 2.6V_{ab} = -100}$$

Node V_1

$$\frac{V_1}{200} + \frac{4(V_1 + 0.2V_{ab} - V_{ab})}{50} - 0.01V_{ab} = 0$$

$$V_1 + 4V_1 - 0.8 \times 4V_{ab} - 0.01 \times 200\, V_{ab} = 0$$

$$5V_1 - 3.2V_{ab} - 2V_{ab} = 0 \qquad (V_1 = 23.2\ \text{volt})$$

$$\boxed{5V_1 - 5.2V_{ab} = 0} \quad \text{— (1)}$$

Node V_{ab}

$$\frac{V_{ab}}{100} + \frac{V_{ab} - 0.2V_{ab} - V_1}{50} - 1 = 0$$

$$\frac{V_{ab}}{100} + \frac{0.8V_{ab}}{50} - \frac{V_1}{50} = 1$$

$$V_{ab} + 1.6V_{ab} - 4V_1 = 1 \times 100$$

$$\boxed{2.6V_{ab} - 2V_1 = 100} \quad \text{— (2)}$$

$$\boxed{V_{ab} = 192.30} \qquad R_{th} = \frac{V_{ab}}{1} = 192.30\,\Omega$$

$V_2 = 0.1497$

$R = 0.1497$

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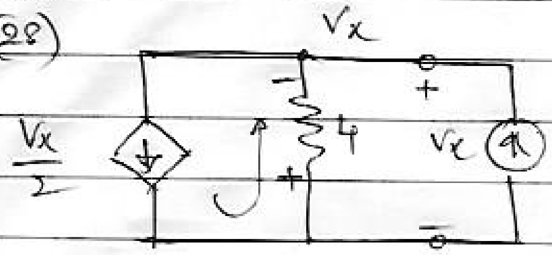
(28)

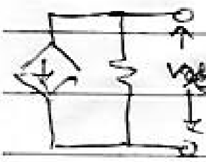


$$\frac{V_x}{2} + \frac{V_x}{4} - 1 = 0$$

$$2V_x + V_x - 4 = 0$$

$$3V_x = 4 \qquad V_x = \frac{4}{3} = 1.33$$

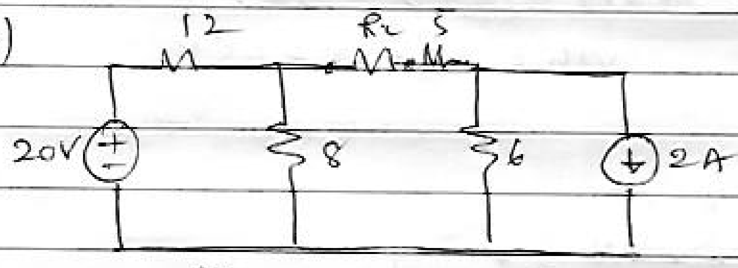
$$V_x = 1.33 \qquad R_{th} = \frac{V_x}{1} = 1.33\ \Omega$$



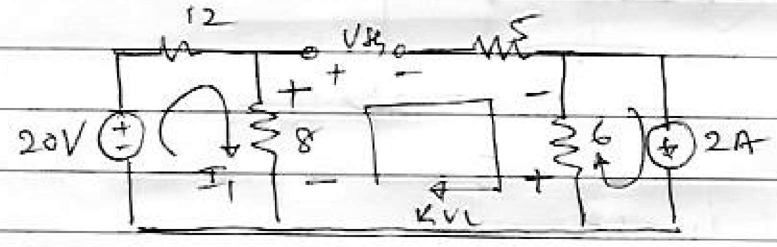
$3V_x = 0 \qquad V_x = 0$

$V_{th} = \underline{0\ V}$

(29)



What is the maximum power that could be dissipated in R_L

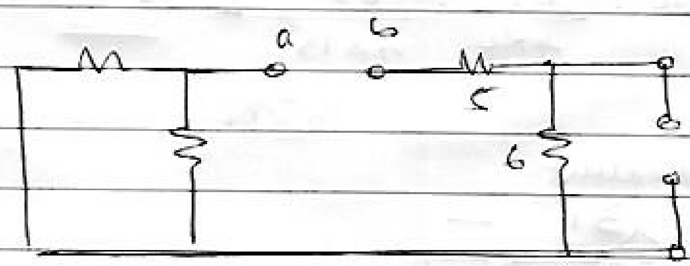


$$-8I_1 + V_{th} + 5\times 0 + 6\times 2 = 0$$

$$-8(20/20) + V_{th} - 12 = 0$$

$$V_{th} = 12 + 8 = \underline{20\ V}$$

R_{th}

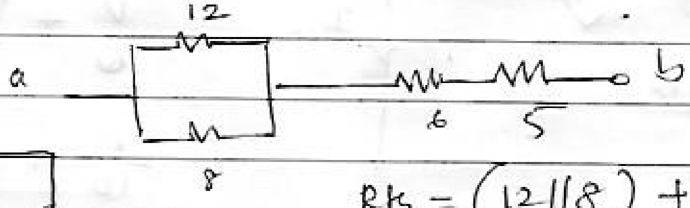


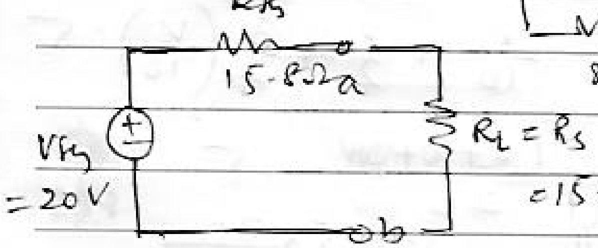
P_L is max when $R_L = R_S$

$$\therefore P_L = I_L^2 \times R_L$$

$$= \frac{V_{th}^2}{4R_L}$$

$$= \frac{20^2}{4\times 15.8}$$





$$R_{th} = (12 \| 8) + 11$$

$$= \underline{15.8\ \Omega}$$

$$\boxed{P_L = 6.349\ W}$$

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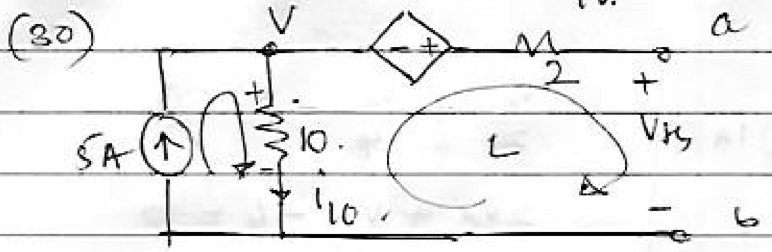
(30)

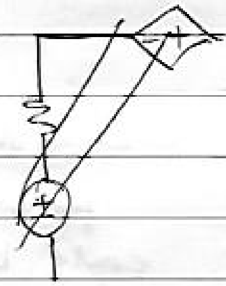


$i_{10} = 5A$

~~$i_{10} = 5 \times 10 =$ $i_{10} = 5A$~~

~~KVL loop L~~

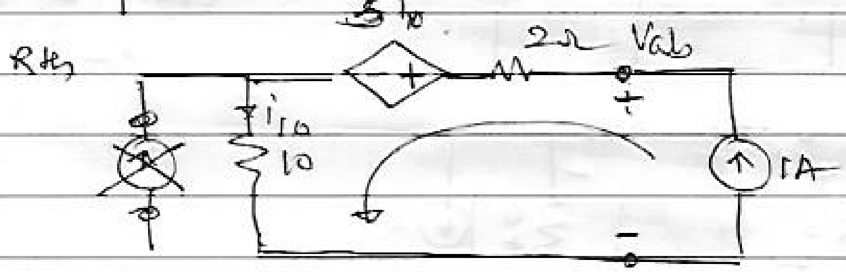
~~$+V_{th} - 10 \times 5 - 3 \times 5 +$~~



$+V_{th} - 10 \times 5 - 3 \times 5 = 0$

$V_{th} = 50 + 15 = \underline{\underline{65\,V}}$

R_{th}



$i_{10} = 1A$

$-V_{ab} + 2 + 3(1) + 10(1) = 0$

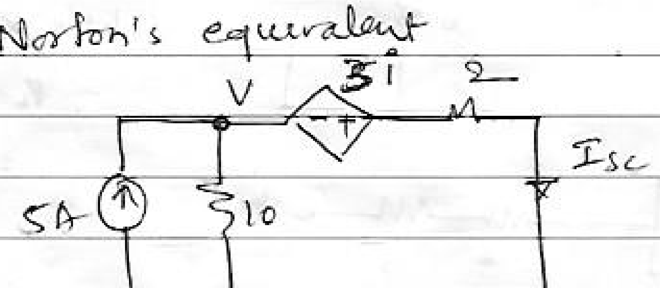
$V_{ab} = 15\,V$

$\therefore R_{th} = \frac{15}{1} = \underline{\underline{15\,\Omega}}$

$P_{Lmax} = \frac{V_{th}^2}{4R_L} = \frac{65^2}{4 \times 15} = \underline{\underline{70.41}}$ watts

~~$I_L^2 R_L =$~~

* Norton's equivalent



$\frac{V}{10} + \frac{V + 3i}{2} - 5 = 0.$

$\frac{V}{10} + \frac{V}{2} + \frac{3}{2}\left(\frac{V}{10}\right) = 5$

$\frac{(2 + 10 + 3)V}{20} = 5$

$V = 5 \times \frac{20}{15}$

$\therefore V = 6.6667$

$\boxed{I_{SC} = \left[V + 3\left(\frac{V}{10}\right)\right] \div 2 = 4.334}$

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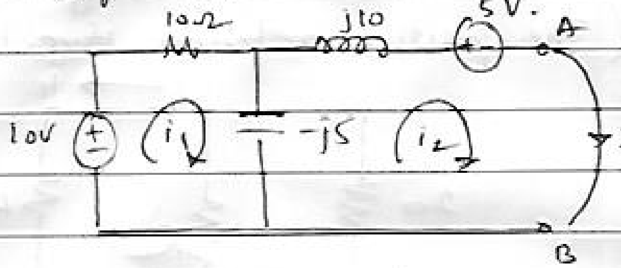
\# What current in 5Ω resistor connected between A & B in ckt. in fig using Norton's

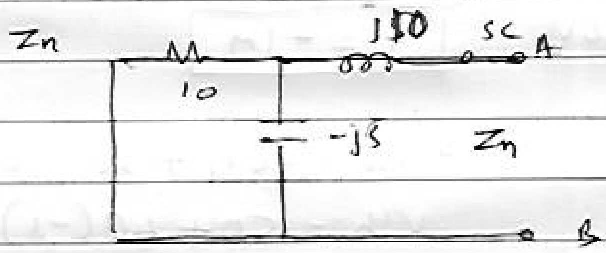


$$\begin{pmatrix} 10-j5 & +j5 \\ +j5 & j5 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix} \qquad \Delta = j50 + 25 + 25 = 50 + j50$$

$$\begin{pmatrix} 10-j5 & 10 \\ j5 & -5 \end{pmatrix} \qquad \Delta_2 = -50 - j25.$$

$$\therefore I_2 = \Delta_2/\Delta = (-0.75 + j0.25)\,A = I_N.$$

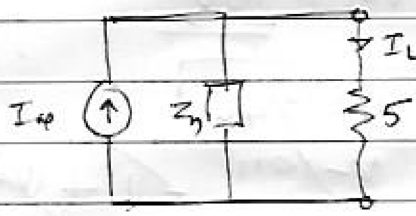
Z_n



$$Z_{eq} = \frac{10 \times -j5}{10-j5} + j10$$

$$= \frac{-j50 + j10(10-j5)}{10-j5}$$

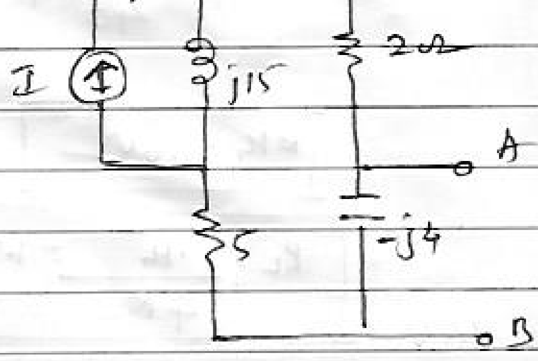
$$= \underline{\underline{2+6j}}$$

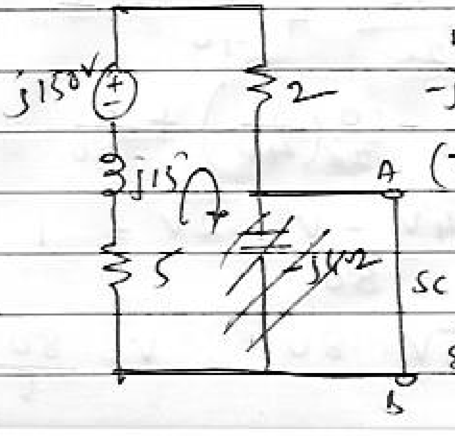


$$I_L = \frac{Z_n \times I_N}{Z_n + 5} = \frac{(2+6j) \times (-0.75 + j0.25)}{7+6j}.$$

$$I_L = -0.53 - 0.117j.$$

② find Norton's equivalent ckt. What impedance must be connected between A & B for max power





KVL

$-j150 + 7i_1 + 15j\, i_1 = 0$

$(7+15j)\, i_1 = j150$

$$i_1 = \frac{j150}{(7+15j)}$$

$I_1 = 8.21 + 3.83j.$

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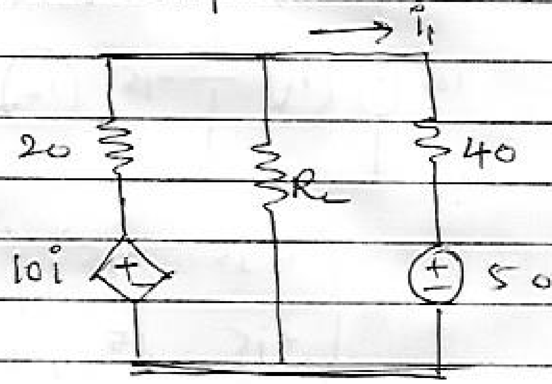
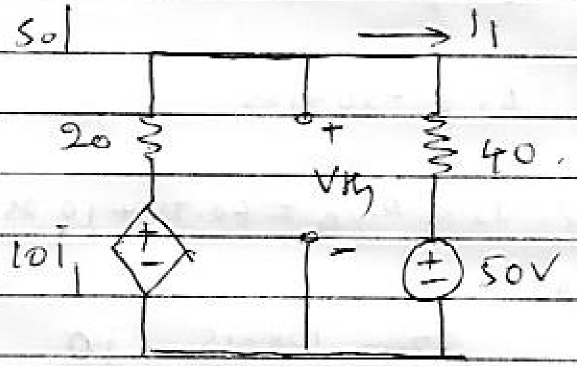
(31) Determine that value of R_L to which maximum power can be delivered. Calculate the voltage of the corresponding R_L. (The positive reference direction is at the top)





KVL

$-10i + 20i_1 + 40i_1 + 50 = 0$

$50i_1 + 50 = 0$

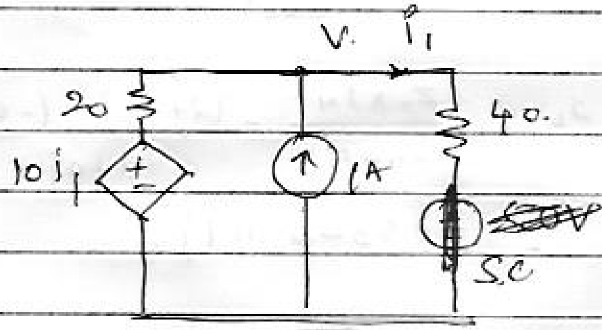
$\boxed{i_1 = -1A}$

$-V_{th} + 40i_1 + 50 = 0$

$V_{th} = 50 + 40(-1) = 0$

$= 50 - 40$

$V_{th} = \underline{\underline{10V}}$



$\boxed{i_1 = V/40}$

$\cancel{\frac{V - 10i_1}{20} - 1 + \frac{V}{40} = 0}$

$\boxed{i_1 = \frac{V}{40}}$

$\cancel{\frac{V}{20} - 10\left(\frac{V}{40}\right) + \frac{V}{40} = 1}$

$\cancel{\frac{2V - 10V + V}{40} = 1}$

$\frac{V - 10i}{20} + \frac{V}{40} = 1.$

$\frac{V}{20} - \frac{10}{20}\left(\frac{V}{40}\right) + \frac{V}{40} = 1$

$\boxed{R_{th} = 16\Omega}$

$\frac{4V - V + 2V}{80} = 1$

$\boxed{P_L = \frac{V_{th}^2}{4R_L} = 1.5625 \text{ Watts}}$

$5V = 80 \qquad V = \frac{80}{5} = 16V.$

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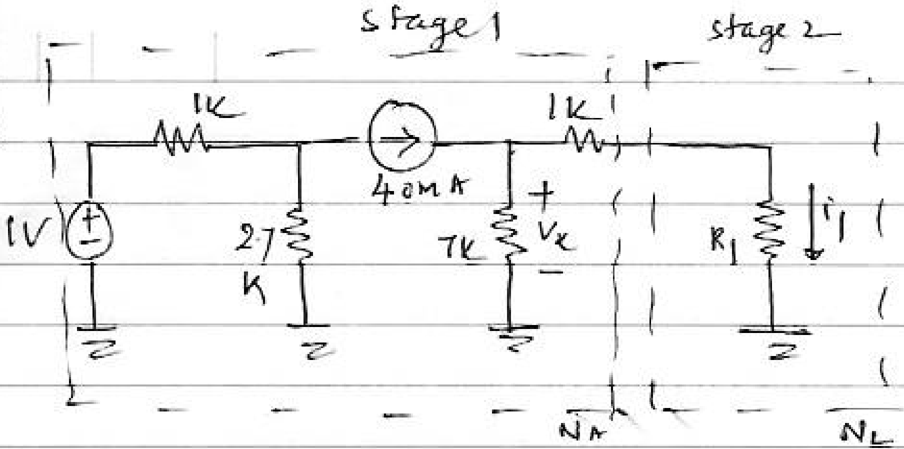
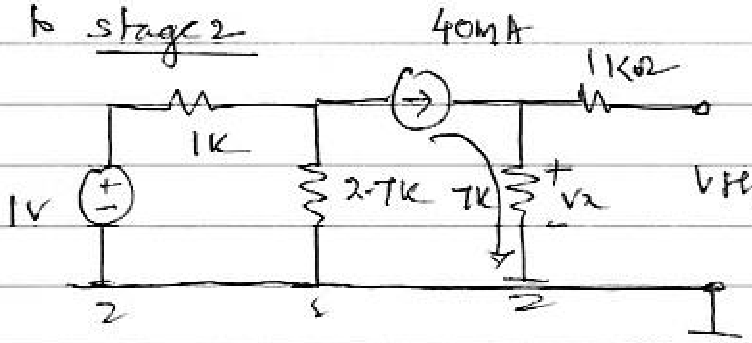
(32)

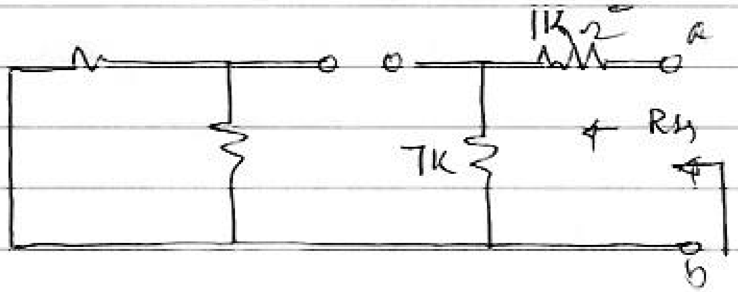


Select R_1 so that max power is transferred from stage1 to stage 2



$$V_x = 40\times10^{-3}\times7\times10^{3}$$

$$\boxed{V_{th} = V_x = 280V}$$



$R_{th} = 7K+1K\Omega$

$= 8K\Omega$

$R_L = R_{th} = 8K\Omega$ for max power

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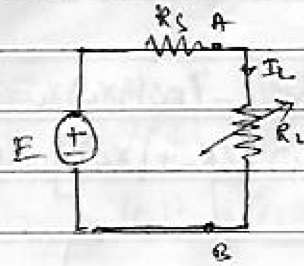
* Maximum Power transfer theorem.

Case (1): When source impedence and load impedence are resistive.



Apply KVL

$$-E + I_L R_S + I_L R_L = 0$$

$$I_L R_S + I_L R_L = E.$$

$$I_L = \left(\frac{E}{R_S + R_L}\right) \quad -(1)$$

Power received by the load resistance $P_L = |I_L|^2 \cdot R_L$ watts.

$$P_L = \frac{|E|^2}{(R_S+R_L)^2} \cdot R_L \quad -(2)$$

Power received by load resistance is maximum at some value of R_L (0 to ∞)

ie It maximum when

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left(\frac{|E|^2}{(R_S+R_L)^2} \cdot R_L\right) = 0$$

$$|E|^2 \left[\frac{(R_S+R_L)^2 \frac{dR_L}{dR_L} - R_L \frac{d}{dR_L}(R_S+R_L)^2}{((R_S+R_L)^2)^2}\right] = 0$$

$$|E|^2 \left[\frac{(R_S+R_L)^{\cancel{2}\,1} - R_L\, 2\cancel{(R_S+R_L)}(1)}{(R_S+R_L)^{\cancel{4}\,3}}\right] = 0$$

$$|E|^2 \cdot \left[\frac{R_S+R_L-2R_L}{(R_S+R_L)^3}\right] = 0.$$

$$R_S + R_L - 2R_L = 0 \Rightarrow R_S - R_L = 0.$$

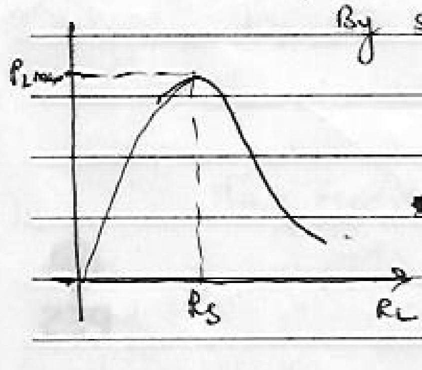
$$\therefore \boxed{R_S = R_L} \quad -(3)$$

For power received by load resistance to be maximum, load resistance must be equal to source resistance ie $R_L = R_S$.

By substituting (3) in (2).

$$P_L = \frac{|E|^2}{(R_L+R_L)^2}(R_L) = \frac{|E|^2 R_L}{4R_L^2} = \frac{|E|^2}{4R_L} \text{ watts.}$$

$$\boxed{P_L = \frac{|E|^2}{4R_L}}$$



Variation of P_L w.r.t R_L. It is max when $R_L = R_S$.

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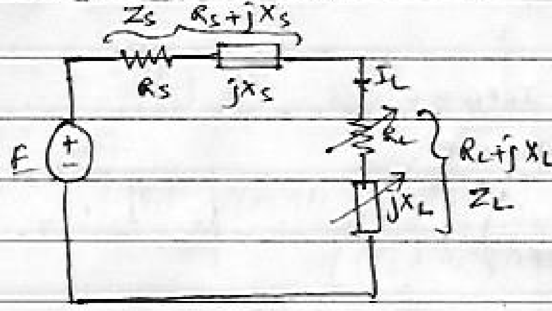
Case (ii): The source impedence and load impedence are complex & whose resistance and reactance are variable independently.



Applying KVL

$E - (R_s + jX_s) I_L - (R_L + jX_L) I_L = 0$

$\therefore I_L \left[(R_s + jX_s) + (R_L + jX_L) \right] = E$

$$\left[I_L = \frac{E}{[(R_s+R_L) + j(X_s+X_L)]} \right] \quad \text{—(1)}$$

$$|I_L| = \frac{|E|}{\sqrt{(R_s+R_L)^2 + (X_s+X_L)^2}}$$

Load power $P_L = |I_L|^2 \cdot R_L$

$$P_L = \frac{|E|^2}{(R_s+R_L)^2 + (X_s+X_L)^2} \times R_L \quad \text{— (2)}$$

Keeping load resistance constant & varying X_L. Then power received by load is maximum $\frac{dP_L}{dX_L} = 0$.

$$\frac{d}{dX_L}\left[\frac{|E|^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2} \right] = 0$$

$$|E|^2 R_L \left[\frac{(R_s+R_L)^2 + (X_s+X_L)^2 (0) - 1 \cdot (0 + 2(X_s+X_L))}{\left((R_s+R_L)^2 + (X_s+X_L)^2\right)^2} \right] = 0$$

$-2(X_s + X_L) = 0$

$-2X_s = 2X_L$

$\therefore \boxed{X_L = -X_s} \quad \text{— (3)}$

It means when $X_L = -X_s$, maximum power is received by load

Substituting (3) in (2)

$$P_L = \frac{|E|^2}{(R_s+R_L)^2 + (X_s - X_s)^2} \times R_L$$

$$\boxed{P_L = \frac{|E|^2 R_L}{(R_s+R_L)^2}} \quad \text{—(4)}$$

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Keeping X_L constant at $-X_S$ and Varying R_L. then power received by load is max $\frac{dP_L}{dR_L}=0$

$$\frac{d}{dR_L}\left[\frac{|E|^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2}\right]=0$$

$$|E|^2\left[\frac{(R_S+R_L)^{\cancel{2}}\cdot 1 - R_L\,2\,\cancel{(R_S+R_L)}}{(R_S+R_L)^{\cancel{4}3}}\right]=0 \quad \because \cancel{X_S} \quad X_L=-X_S.$$

$$|E|^2\left[\frac{R_S+R_L-2R_L}{(R_S+R_L)^3}\right]=0$$

$$R_S+R_L-2R_L=0 \qquad \therefore \quad R_S-R_L=0.$$

$$\boxed{R_L=R_S} \quad -(5)$$

We have Load impedence

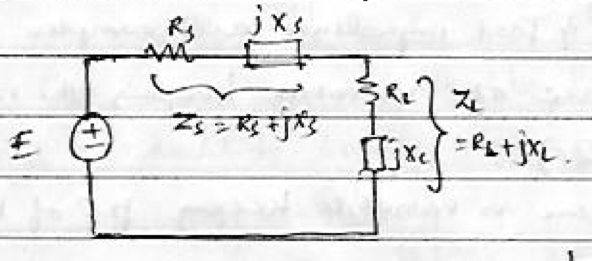
$$Z_L=R_L+jX_L$$

By substituting (4) & (5) [max power cond^n]

$$\left.\begin{aligned} Z_L &= R_S-jX_S. \\ \therefore\ Z_L &= Z_S^{*}. \end{aligned}\right\} -(6)$$

It means, when source impedence and load impedence are complex, ~~conj~~ Maximum power is received by load when load imp and source impedence are complex conjugate.

Case (iii) - When source & load impedence are complex but only load resistance is varying.



By KVL

$$E=I_L\left((R_S+R_L)+j(X_L+X_S)\right)$$

$$I_L=\frac{E}{(R_S+R_L)+j(X_L+X_S)}$$

$$|I_L|=\frac{|E|}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}} \quad -(1)$$

Power received

$$P_L=\frac{|E_L|^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2} \quad -(2)$$

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When R_L is varied from (0 to ∞) Power received by load is max when

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left[\frac{|E|^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2}\right] = 0$$

$$|E|^2\left[\frac{(R_S+R_L)^2+(X_S+X_L)^2\cdot 1 - R_L\left(2(R_S+R_L)\right)}{\left((R_S+R_L)^2+(X_S+X_L)^2\right)^2}\right]$$

$$(R_S+R_L)^2+(X_S+X_L)^2 - 2R_LR_S - 2R_L^2 = 0.$$

$$R_S^2+R_L^2+\cancel{2R_SR_L}+(X_S+X_L)^2 - \cancel{2R_LR_S} - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 + (X_S+X_L)^2 = 0.$$

$$R_L^2 = R_S^2 + (X_S+X_L)^2.$$

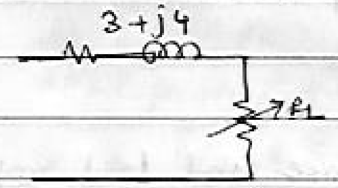
$$\boxed{R_L = \sqrt{R_S^2+(X_S+X_L)^2}}$$

fixed impedance $R_S + j(X_S+X_L)$

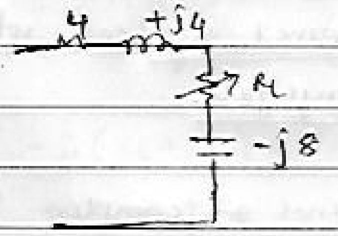
Magnitude of this $\sqrt{R_S^2+(X_S+X_L)^2}$

Power received by load is maximum when R_L is equal to magnitude of fixed impedance

Ex:



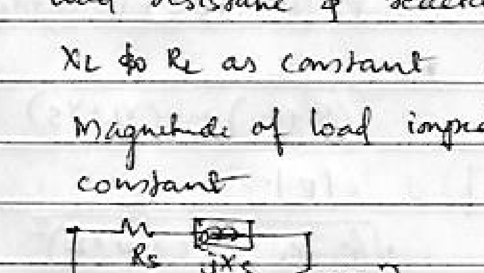
$R_L = \sqrt{(3)^2+(4)^2} = 5\,\Omega.$

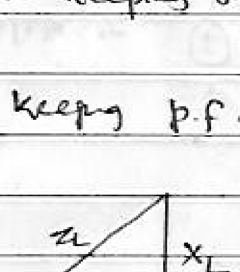


$$R_L = \sqrt{(4)^2+(X_S+X_L)^2}$$
$$= \sqrt{16+(4-8)^2} = \sqrt{16+16} = \sqrt{32}\,\Omega$$

Case (iv): When source impedance & load impedance are complex with load resistance & reactance are variable keeping the ratio of X_L to R_L as constant (or)

Magnitude of load impedance is variable keeping p.f of load as the constant





$\cos\phi_L = \frac{R_L}{|Z_L|}$ $R_L = |Z_L|\cos\phi_L$

$\tan\phi = \frac{X_L}{R_L}$. $\quad$ III^{ly} $X_L = |Z_L|\sin\phi_L$

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By KVL

$$E = [(R_s + jX_s) + (R_L + jX_L)] I_L$$

$$I_L = \frac{E}{(R_s+R_L) + j(X_s+X_L)}$$

$$|I_L| = \frac{|E|}{\sqrt{(R_s+R_L)^2 + (X_s+X_L)^2}}$$

$$P_L = |I_L|^2 R_L = \frac{|E|^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2}$$

$$P_L = \frac{|E|^2 |Z_L| \cdot \cos\phi_L}{(R_s + |Z_L|\cos\phi_L)^2 + (X_s + |Z_L| \cdot \sin\phi_L)^2}$$

Z_L is varied from 0 to ∞.

When power received by load is max. $\frac{dP_L}{dZ_L} = 0$.

$$\therefore \frac{d}{dZ_L}\left(\frac{|E|^2 \cdot |Z_L| \cos\phi_L}{(R_s + |Z_L|\cos\phi_L)^2 + (X_s + |Z_L|\sin\phi_L)^2}\right) = 0.$$

$$|E|^2 \left[\frac{D_r(\cos\phi_L) - N_u(2(\ )\cos\phi_L + 2(\ )\sin\phi_L)}{((R_s + Z_L\cos\phi_L)^2 + (X_s + |Z_L|\sin\phi_L)^2)^4}\right] = 0$$

$$D_r \cos\phi_L = N_u [2(\ )\cos\phi_L + 2(\ )\sin\phi_L].$$

$$R_s^2 + X_s^2 + |Z_L|^2 = 2|Z_L|^2$$

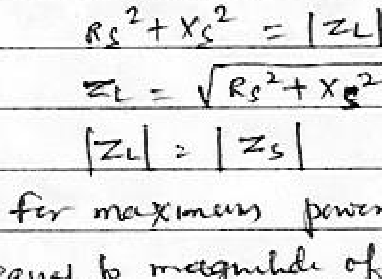
$$R_s^2 + X_s^2 = |Z_L|^2$$

$$Z_L = \sqrt{R_s^2 + X_s^2}$$

$$|Z_L| = |Z_s|$$

for maximum power transfer magnitude of load imp must be equal to magnitude of source impedance.

Ex: In the ckt shown below find the value of R_L & X_L if the load resistance & reactance variable with ratio $X_L/R_L = 0.2$



As $X_L/R_L = 0.2$ [Ratioed]

~~for~~.

$$|Z_L| = |Z_s|$$

$$= \sqrt{(5)^2 + (3)^2} = \sqrt{34} = 5.83$$

$\therefore R_L = |Z_L| \cdot \cos\phi_L$ $\quad \tan\phi_L = \frac{X_L}{R_L} = 0.2 \quad \therefore \phi_L = 11.3^\circ$

$= 5.83\cos(11.3) = 5.71$ & $X_L = |Z_L|\sin\phi_L = 1.41$ $\quad Z_L = 5.71 + j1.41$

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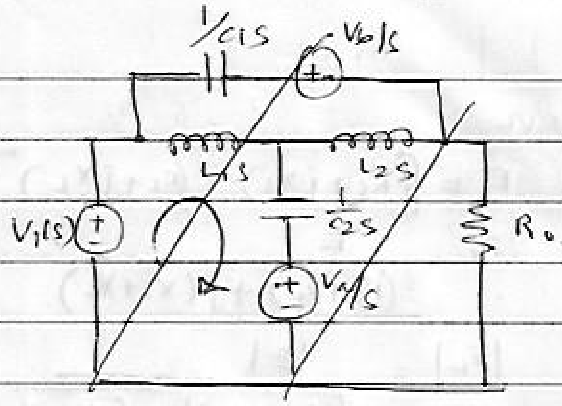

$K_1 + \frac{1}{C_2}$

$I_1(s)\, L_1 s + \frac{1}{C_2 s}$ (1

$\left(L_1 s + \frac{1}{C_2 s}\right)$

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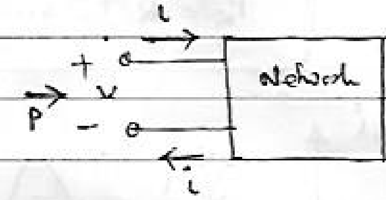
Energy and Power

- Consider a one port network with reference direction for v & i to define power
- The energy absorbed by the n/w from time t_1 to t_2 is



$$w = \int_{t_1}^{t_2} v(\tau)\, i(\tau)\, d\tau \text{ Joules} \quad -(1)$$

The rate at which energy is absorbed is the power (ie instantaneous power)

$$p(t) = \frac{dw}{dt} = \frac{d}{dt}\left(\int_{t_1}^{t_2} v(\tau)\, i(\tau)\, d\tau\right) = v(t)\cdot i(t) \text{ Watts} \quad -(2)$$

Convention : For the voltage & current reference shown in fig, A positive p indicate flow of energy into the n/w. If either the voltage / current reference is ~~changed~~, reversed, so is the reference of energy

(i) Let us consider n/w with single linear elements like R, L and C.

a) Resistor (R) :

w.k.t $v = iR$ and energy absorbed is

$$w_R = \int_{t_1}^{t_2} v(t)\cdot i(t)\, dt = \int_{t_1}^{t_2} i^2(t)\cdot R \cdot dt = R\int_{t_1}^{t_2} i^2(t)\cdot dt$$

$$= \frac{1}{R}\int_{t_1}^{t_2} v^2(t)\, dt \quad -(3)$$

and Instantaneous power (rate at which energy is absorbed)

$$P_R = R\, i^2(t) = \frac{1}{R} v^2(t) \quad -(4)$$

For a sinusoidal current, $i = I_m \sin\omega t$ with $t_1 = 0$ ~~and~~, energy absorbed (eq: (3)) can be given by

$$w_R = \int_0^{t_2} i^2(t)\cdot R \cdot dt = R\int_0^{t_2} I_m^2 \sin^2\omega t \cdot dt$$

$$= I_m^2 R \int_0^{t_2} \cancel{\sin}\left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$\boxed{w_R = \frac{I_m^2 R}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right) T} \quad -(5)$$

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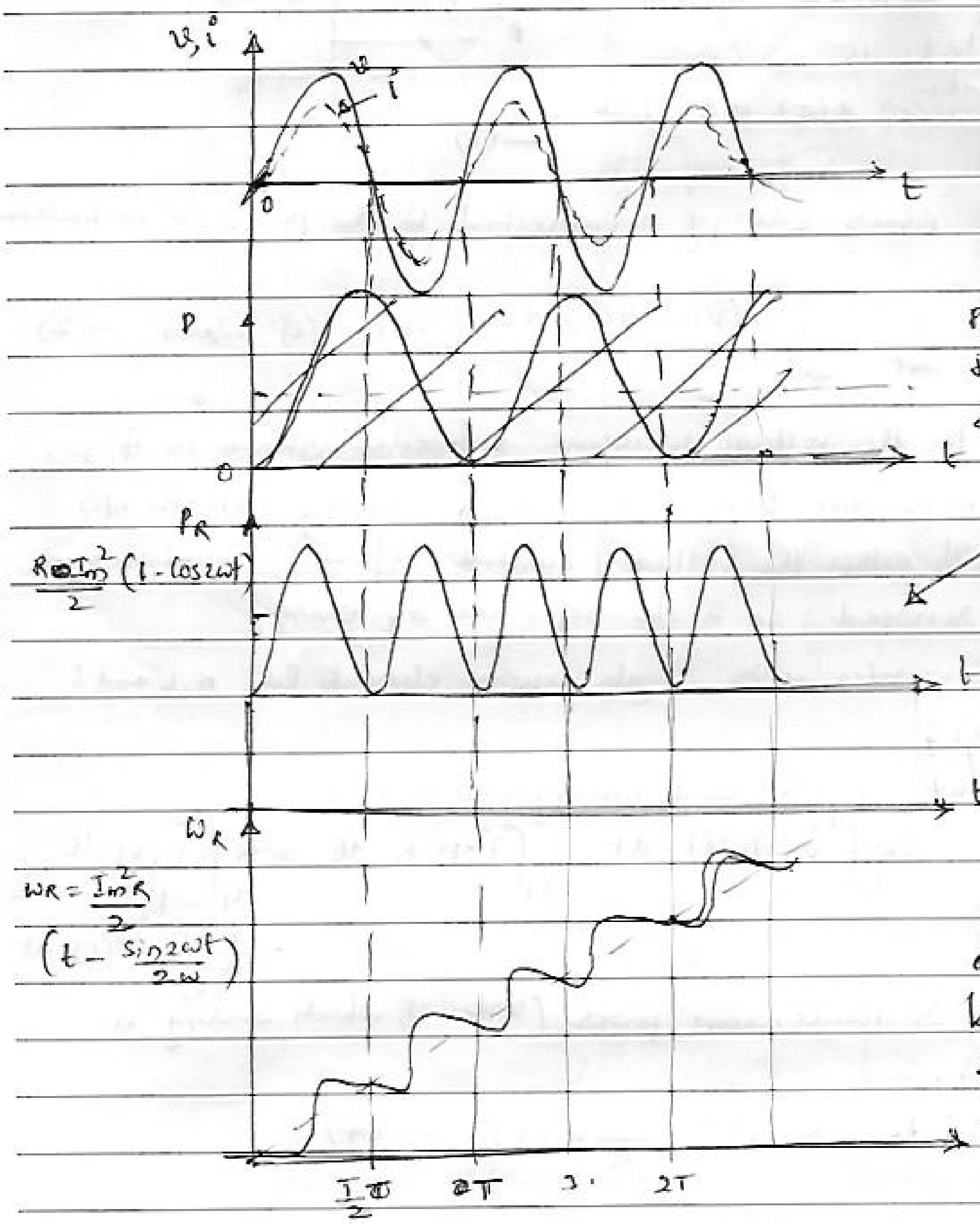
Instantaneous power eq^n (4) can be written as

$$\boxed{P_R = R\cdot I_m^2 \sin^2\omega t = \frac{R\, I_m^2}{2}\left(1-\cos 2\omega t\right)} \text{ watts} \quad -(6)$$



P_R varies at twice the freq of V_R, i_R & P_R is always (+)ve.

$P_{av,R} = \frac{R}{2} I_m^2 = R\left(\frac{I_m}{\sqrt{2}}\right.$

Energy W_R is always (+)ve and increases to very large value with increasing time.

$\Delta W_R = R\int_0^T i^2(t)\, dt = \frac{1}{2} R I_m^2 T$

Energy absorbed per cycle

Conclusion:

- Both power and energy varies at twice the freq of voltage or current
- Power and energy are always positive
- Energy increases with time

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b) ~~for~~ an inductor (L)

W.K.T $V = L \cdot \frac{di}{dt}$, the energy which enters inductor for storage (energy absorbed) is - from eq^n (1)

$$W_L = \int_{t_1}^{t_2} (\cancel{L \cdot \frac{di}{dt}})\, v(t) i(t)\, dt$$

$$= \int_{t_1}^{t_2} \left(L \cdot \frac{di}{\cancel{dt}}\right) i \cdot \cancel{dt} = \int_{t_1}^{t_2} L \cdot i \cdot di = L \frac{i^2}{2} \Big|_{t_1}^{t_2}$$

$$\boxed{W_L = \frac{L}{2}\left(i^2(t_2) - i^2(t_1)\right)} \text{ J.} \quad — (7)$$

$i(t_2)$ and $i(t_1)$ are current at t_2 & t_1

If we let $i(t_1) = 0$ at t_1 and if $i(t_2) = I_m \sin \omega t$, then

$$\boxed{W_L = \frac{L}{2}\left(I_m^2 \sin^2 \omega t\right) = \frac{L I_m^2}{2} \frac{(1 - \cos 2\omega t)}{2}} \quad - (8)$$

and the power

$$P_L(t) = \frac{d}{dt} \int L \frac{di}{dt} i \cdot dt = L i \frac{di}{dt} \quad — (9)$$

If $i(t) = I_m \sin \omega t \quad t \geq 0$

$$P_L(t) = L I_m \sin \omega t \frac{d}{dt}(I_m \sin \omega t)$$

$$= L I_m^2 \sin \omega t \cos \omega t\, (\omega)$$

$$= L I_m^2 \omega \sin \omega t \cos \omega t$$

$$\boxed{P_L(t) = \frac{L I_m^2}{2} \omega \sin 2\omega t} \text{ Watts} \quad — (10)$$

From v, i, P_L, and W_L it is clear that

- v & i i.e v leads i by 90°
- P_L & W_L are varying at twice the freq of V_L & i_L but the maximum value of W_L remain ~~cons~~ same for each cycle
- P_L is (+)ve aswell as (-)ve and W is always (+)ve.

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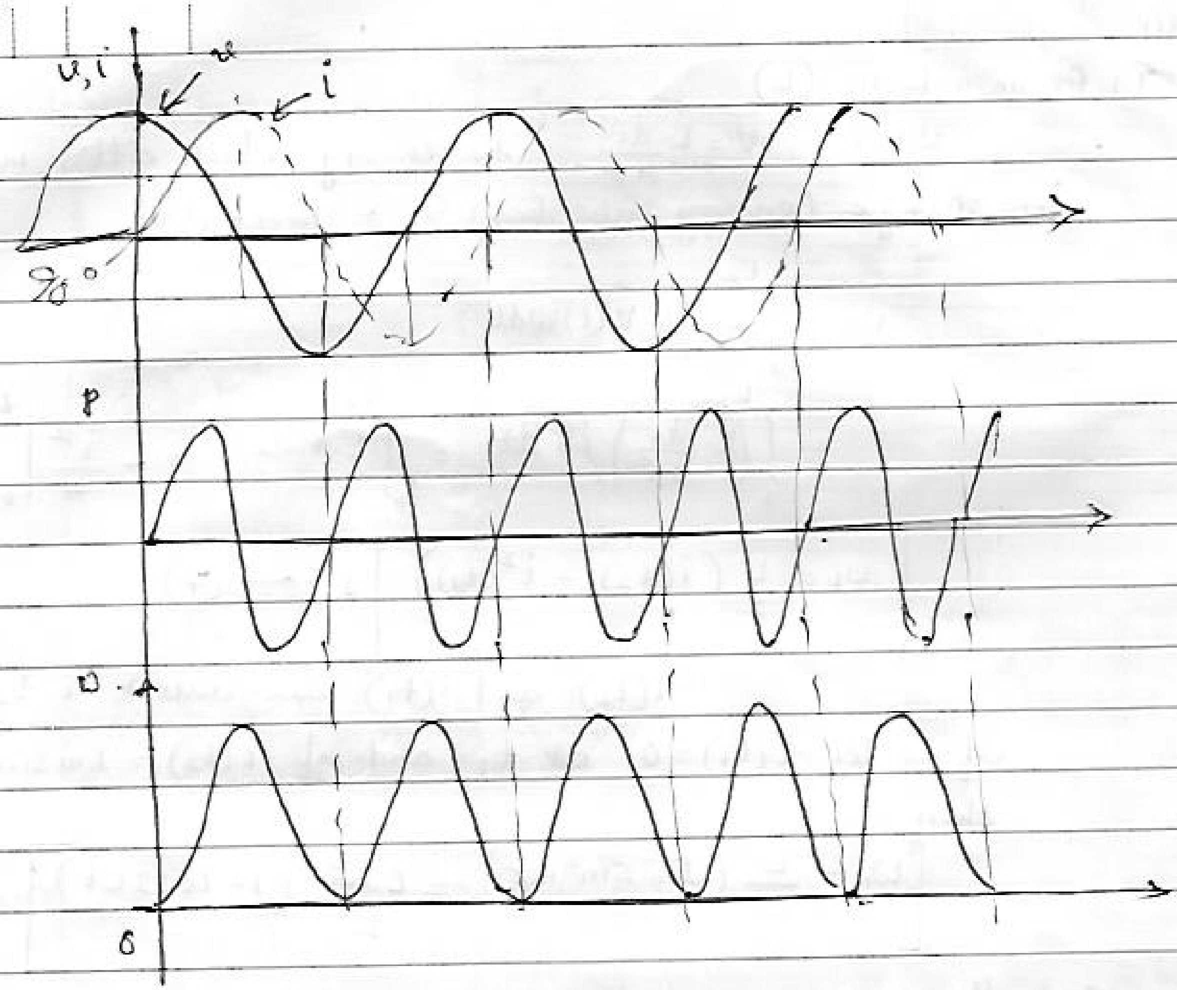

c) For Capacitor,

$$i = C\frac{dv}{dt} \quad -(i)$$

Energy absorbed is

$$\omega_C = C\int_{t_1}^{t_2} v(t)\,\frac{dv(t)}{\cancel{dt}}\,\cancel{dt} = C\int_{t_1}^{t_2} v\,dv$$

$$\left[\omega_C = \frac{1}{2}C\left(V^2(t_2) - \cancel{V^2(t_1)}\right)\right]$$

0

$t>0$.

and $\left[P_C(t) = C.V.\frac{dv}{dt}\right] \quad -(12)$

~~For~~ ~~instantaneous~~ sinusoidal voltage $v(t) = V_m \sin\omega t,\ t \geq 0$

$$\omega_C(t) = \frac{1}{2}CV_m^2\sin^2\omega t = \frac{1}{4}CV_m^2(1-\cos 2\omega t)$$

$$P_C(t) = CV_m^2\frac{d}{dt}(V_m\sin\omega t) = C.V_m^2\sin\omega t\cos\omega t\,(\omega)$$

(sinωt written above)

$$\boxed{P_C(t) = CV_m^2\omega\,\frac{\sin 2\omega t}{2}} \quad -(13)$$

Only v & i phase shift change wrt inductor

W_C, P_C behaves similarly as inductor

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Average Power

Inductor: If T is the period

$$P_{av,L} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} L I_m^2 \omega \sin 2\omega t \, dt = 0.$$

Similarly for capacitor

$$P_{av,C} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} C V_m^2 \omega . \sin 2\omega t \, dt = 0.$$

for Resistor;

$$P_{av,R} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} R I_m^2 (1 - \cos 2\omega t) . dt = \left(\frac{1}{2} R I_m^2\right)$$

$$= R\left(\frac{I_m}{\sqrt{2}}\right)^2$$

$P_{av,L}$ and $P_{av,C}$ is equal to zero for ~~a~~ each cycle ~~to~~ and for a resister $P_{av,R} = \frac{R}{2} I_m^2 = R\left(\frac{I_m}{\sqrt{2}}\right)^2$

Average Energy

- for each cycle, the energy suppled to a capacitor or inductor is zero ie $\underset{1 cycle}{W_{av,L}} = \underset{1 cycle}{W_{av,C}} = 0$

- for Resistor, the energy absorbed per cycle

$$\Delta W_R = R\int_0^T i^2(t) . dt = \frac{1}{2} R . I_m^2 T.$$

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* Effective (or) RMS (Root Mean Square) Value

- Effective value of a periodic current is defined as constant value of current which will produce the same power in a resistor as is produced on the avg by the periodic current

- for constant current I,

$$P = I^2 R \quad \text{--- (1)}$$

for sinusoidal steady state, the avg power

$$P_{av,R} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} R I_m^2 (1-\cos 2\omega t)\, dt = \frac{1}{2} R I_m^2$$

According defn of Effect / RMS value

$$P = P_{av,R}$$

$$I^2 R = R\cdot\left(\frac{I_m}{\sqrt{2}}\right)^2$$

$$\therefore \boxed{I_{eff} = \frac{I_m}{\sqrt{2}}} \quad \text{--- (2)}$$

$\therefore P = V^2/R$ ~~unit~~ for d.c.

$$V^2/R = \frac{R}{2}(I_m)^2 \times \frac{R}{R} = \frac{(I_m R)^2}{2R} = \frac{V_m^2}{R2} = \frac{1}{R}\left(\frac{V_m}{\sqrt{2}}\right)^2$$

$$\therefore \boxed{V_{eff} = \frac{V_m}{\sqrt{2}}} \quad \text{--- (3)}$$

finally $P_{av} = I_{eff}^2 R = I_{eff} V_{eff}$

For Non sinusoidal but periodic current $i(t)$ of period T

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} R i^2(t)\cdot dt$$

$$\therefore \quad I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\cdot dt} = I_{rms}$$

$$V_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\cdot dt} = V_{rms}$$

$P_{av} = I_{rms} V_{rms}$

Avg Power

T is period

Inductor $\quad P_{av,L} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} L I_m^2 \sin 2\omega t \cdot dt = 0$

Capacitor $\quad P_{av,C} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} C V_m^2 \omega \sin 2\omega t \cdot dt = 0$

Resistor $\quad P_{av,R} = \frac{1}{nT}\int_0^{nT} \frac{1}{2} R I_m^2 (1-\cos 2\omega t) \cdot dt$

$$= \frac{1}{2} R I_m^2$$

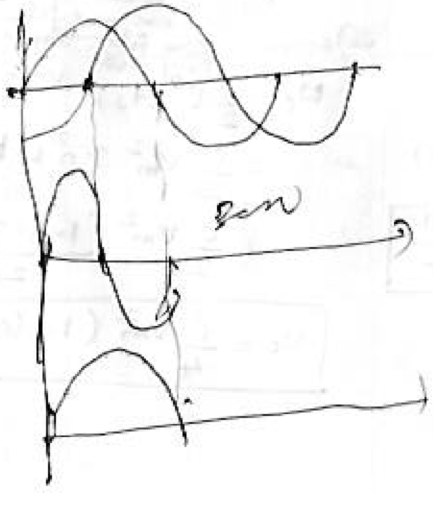
$$P_{av,R} = R \cdot \left(\frac{I_m}{\sqrt{2}}\right)^2$$

Avg Energy

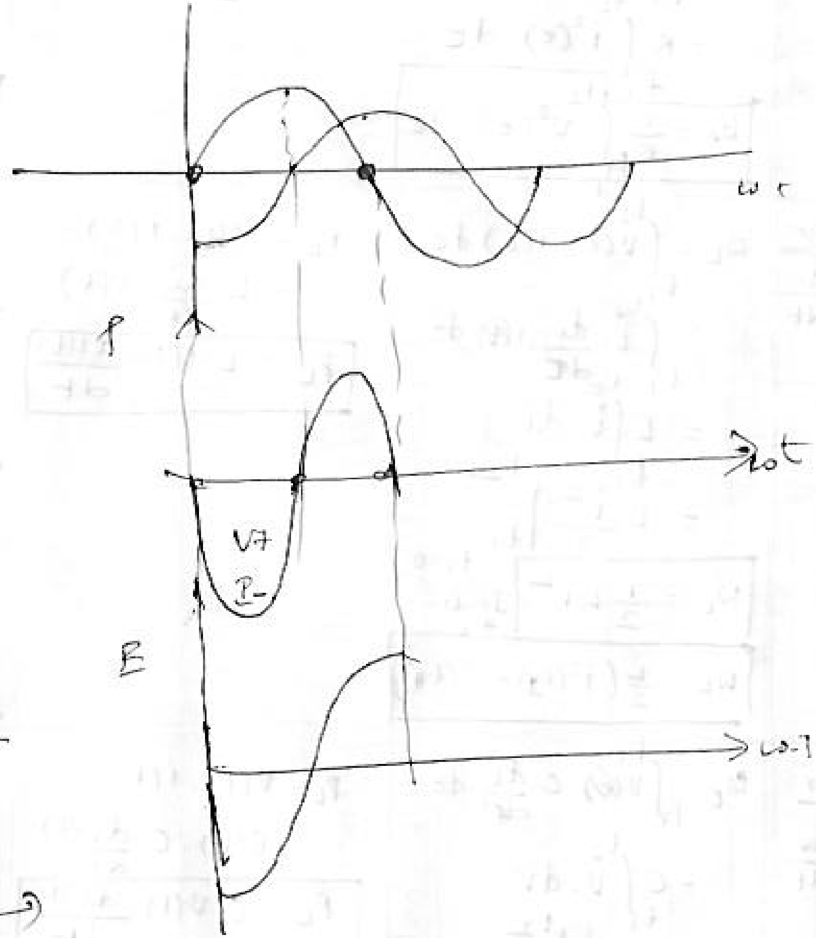
Inductor & capacitor $\quad \boxed{W_{av,L} = W_{av,C} = 0}$

R $\quad \Delta W_R = R \int_0^T i^2(t) \cdot dt$

$$\boxed{\Delta W_R = \frac{1}{2} R I_m^2 T}$$



RMS / Effective Power



Resistor (R) — $v(t) = i(t).R.$

Power $(P_R) = v(t).i(t)$

$$\boxed{P_R = i^2(t).R = \frac{v^2(t)}{R}} \quad \text{— (i)}$$

(i) If excitation is sinusoidal.

$$i(t) = I_m \sin\omega t.$$

$$P_R = (I_m^2 . \sin^2\omega t).R$$

$$= I_m^2 R . \sin^2\omega t$$

$$= I_m^2 R \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$\boxed{P_R = \frac{I_m^2 R}{2}(1-\cos 2\omega t)}$$

Avg power for one cycle

$$P_{av} = \frac{\frac{I_m^2 R}{2}\int_0^T (1-\cos 2\omega t)}{T}$$

$$= \frac{\cancel{\frac{I_m^2 R}{2}}}{T} \qquad T = \pi$$

$$= \frac{\frac{I_m^2 R}{2}\left[t\Big|_0^T + \frac{\sin 2\omega t}{2\omega}\Big|_0^T\right]}{T}$$

$$\boxed{P_{av} = \frac{I_m^2 R}{2}}$$

$V_{rms} = \frac{V_m}{\sqrt{2}}$ $\quad I_{rms} = \frac{I_m}{\sqrt{2}}$

$$\boxed{P_{av} = \frac{I_m (V_m)}{2} = V_{rms} . I_{rms}.}$$

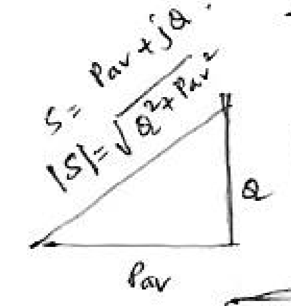
(ii) Non sinusoidal & periodic

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(\tau) R . d\tau$$

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(\tau)\, d\tau}.$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(\tau) . d\tau}$$

$$\boxed{P_{av} = V_{rms} \times V_{rms}}$$



Avg Power and Complex power

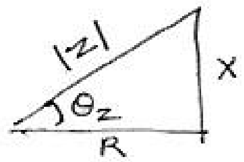
$$P_{av} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R} \text{ watts.}$$

In terms of rms values

$$P_{av} = \frac{I_m^2 R}{2} = \left(\frac{I_m}{\sqrt{2}}\right)^2 . R = I_{rms}^2 R.$$

$$P_{av} = \frac{V_m^2}{2R} = \frac{1}{R}\left(\frac{V_m}{\sqrt{2}}\right)^2 = \frac{V_{rms}^2}{R}.$$

$Z(j\omega) = R + jX.$



$R = Re(Z(j\omega))$ $\longrightarrow$ $\cos\theta = \frac{R}{|Z|}$

~~R = Re Z(jω)~~ $\quad R = |Z| . \cos\theta$ — (a)

$$\sin\theta = \frac{X}{|Z|}$$

$X = |Z| \sin\theta.$ — (b)

$$P_{av} = I_{rms}^2 R$$

$$\boxed{P_{av} = I_{rms}^2 |Z| . \cos\theta_z} \quad \text{— (I)}$$

~~R = Re Z jω~~

$$\boxed{P_{av} = V_{rms} . I_{rms} . \cos\theta_z}$$

Power factor (pf) — cosine angle of the impedance.

(or) phase difference between v & i

P.f — leading | lagging

I leads V $\quad$ V leads I.

Imaginary part of power

$Q = \cancel{I_{rms}, V_{rms}} |Z|$

$$\boxed{Q = I_{rms}^2 |Z| . \sin\theta}$$

$$\boxed{Q = I_{rms} V_{rms} . \sin\theta_z}$$

$S = P_{av} + jQ = V_{rms} I_{rms} \cos\theta_z$ ...

Ex : $V = V_m e^{j\theta_v}$ $\quad I = I_m e^{j\theta_i}$

$= V_m \angle\theta_v \quad = I_m \angle\theta_i$

~~Pav~~ = $\boxed{Z = \dfrac{V_m \angle\theta_v}{I_m \angle\theta_i} = \dfrac{V_m}{I_m} \angle\theta_v - \theta_i}$ when $pf = \theta_v - \theta_i$

$\to$ (or) $\dfrac{V_{rms}}{I_{rms}} \angle\theta_v - \theta_i = |Z| \angle\theta$.

<u>complete power S</u> = $V_{rm} I_{rms} \cdot \angle\theta_v - \theta_i$

$$S = P_{av} + jQ = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P_{av}} + j\underbrace{V_{rms} I_{rms} \cdot \sin(\theta_v - \theta_i)}_{Q}$$

$\to$ Reactive power

<u>Note</u> :- ① If reactive power is (−)ve it means,

i) pf angle is negative..

ii) pf is leading and.

iii) Reactance is/ Capacitor is nature

for:- $S = P_{av} + j\underline{Q}$

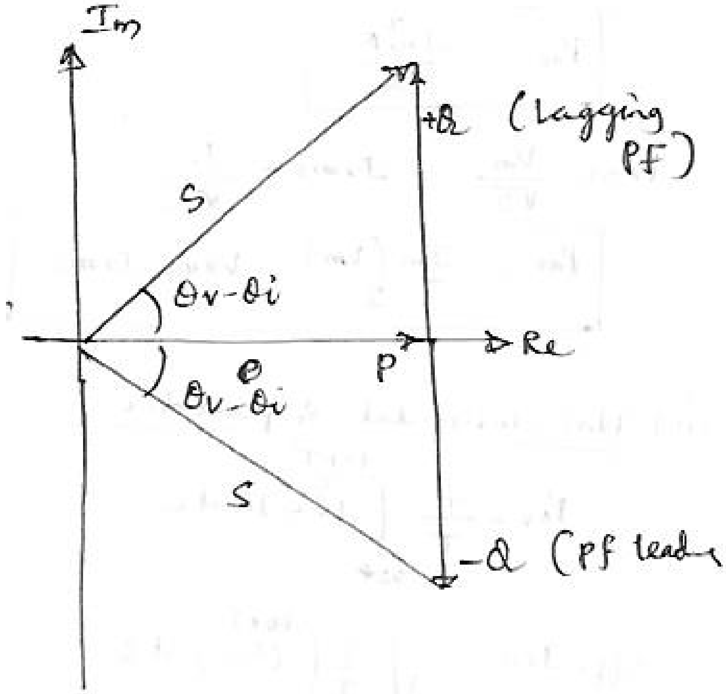
$= P_{av} - jQ$

② If Q is (+)ve.

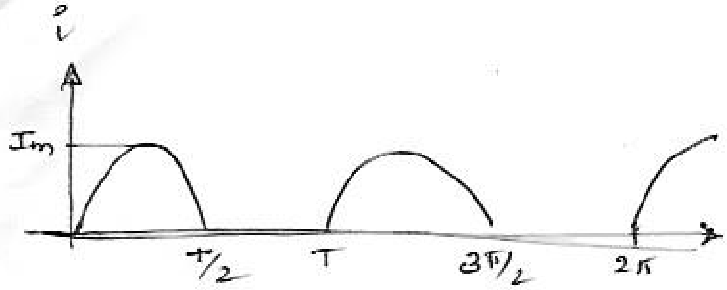
i) pf angle is (+)ve

ii) pf is lagging

iii) Reactance is Inductor in nature



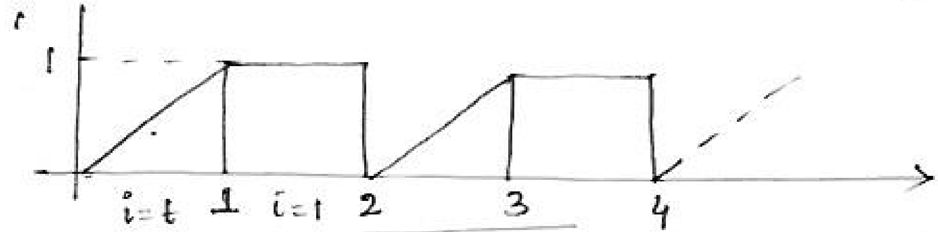
1)



$I_{rms} = \frac{I_m}{2}$.

Period Waveform describe a

2)



$i(t) = \begin{cases} t & 0<t<1 \\ 1 & 1<t\leq 2 \end{cases}$

$T=2$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(\tau)\cdot d\tau}$$

$$= \sqrt{\frac{1}{2}\left(\int_0^1 t^2\cdot dt + \int_1^2 (1)\, d\tau\right)}$$

$$\cancel{= \sqrt{\frac{1}{2}\left[\frac{t^3}{3}\Big|_0^1 + 1\,\Big|_1^2\right]}}$$

$$\cancel{= \sqrt{\frac{1}{2}\left[\frac{1}{3} + \left(\frac{2}{2} - 1\right)\right]} = \sqrt{\frac{1}{2}\times\frac{1}{3} +}}$$

$$= \sqrt{\frac{1}{2}\left[\frac{t^3}{3}\Big|_0^1 + t\Big|_1^2\right]} = \sqrt{\frac{1}{2}\left[\left(\frac{1}{3}-0\right) + (2-1)\right]}$$

$$= \sqrt{\frac{1}{2}\left(\frac{1}{3}+1\right)}$$

$I_{rms} = \underline{0.816}$

* Resistor (R) — ~~v(t)=~~ $i(t)\cdot R$ $\boxed{v(t) = i(t)\cdot R}$

$P(t) = \cancel{v(t)}\, i^2(t)\cdot R$

$W_R = \int_{t_1}^{t_2} i^2(\tau)\cdot R \cdot d\tau$

i) Energy $W_R = R\int_{t_1}^{t_2} i^2(\tau)\, d\tau$ — $P(t) = i^2(t)\, R = \dfrac{v^2(t)}{R}$

$i(t) = I_m \sin\omega t$

$i = I_m \sin\omega t \quad t \geq 0$

$$W_R = R\int_0^{t_2} I_m^2 \sin^2\omega t \cdot dt$$

$$= I_m^2 R \int_0^{t_2} \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$= \frac{I_m^2 R}{2}\int_0^{t_2} (1-\cos 2\omega t)\, dt$$

$$= \frac{I_m^2 R}{2}\left[t\Big|_0^{t_2} + \frac{\sin 2\omega t}{2\omega}\Big|_0^{t_2}\right]$$

$$\boxed{W_R = \frac{I_m^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]}$$

$P(t) = I_m^2 \sin^2\omega t\, R$

$= I_m^2 R\left[\dfrac{1-\cos 2\omega t}{2}\right]$

$$\boxed{P(t) = \frac{I_m^2 R}{2}\left[1-\cos 2\omega t\right]}$$

Avg Energy and Power $[0 \text{ to } T]$ — one cycle

W_{av}

$$\Delta W_R = \frac{I_m^2 R}{2}\int_0^{T}\left(t - \frac{\sin 2\omega t}{2\omega}\right) dt$$

$$= \frac{I_m^2 R}{2}$$

$$\Delta W_R = \frac{I_m^2 R}{2}\int_0^T (1-\cos 2\omega t)\cdot dt$$

$$= \frac{I_m^2 R}{2}\left[t\Big|_0^T + \frac{\sin 2\omega t}{2\omega}\Big|_0^T\right]$$

$$\boxed{\Delta W_R = \frac{I_m^2 R}{2} T}$$

One cycle $(0-T)$

$$\cancel{P_{av}} = \frac{I_m^2 R}{2}\int_0^T (1-\cos 2\omega t)$$

$$= \frac{I_m^2 R}{2}$$

$$P_{av} = \frac{1}{nT}\int_0^{nT} \frac{I_m^2 R n}{2}(1-\cos 2\omega t)$$

$$= \frac{I_m^2 R \times T}{T\, 2}$$

$$\boxed{P_{av} = \frac{I_m^2 R}{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Periodic — Non-Sinusoidal

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} R\, i^2(t)\, dt$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\cdot dt} \qquad V_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\, d\tau}$$

(1) ~~Network shown~~

$i(t) = 4\cos(100\pi t + 10^\circ)$ A

$v(t) = 120\cos(100\pi t - 20)$ V. find the apparent power & power factor of the load. Determine the element values that form the series connected load

i) ~~pf~~ $\angle = \theta_v - \theta_i = -20 - 10 = -30$. $p.f = \cos(\theta_v - \theta_i) = \cos(-30)$
$= \sqrt{3}/2$

~~ii)~~ It is leading as I leads V.

ii) $S = V_{rms} I_{rms} \angle \theta_v - \theta_i$

$= \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} \angle -20 - 10$

$\boxed{S = 240\angle -30}$ $\boxed{S = 207.84 - j120}$ ✓

$\uparrow P_{av}$ $\uparrow Q$

Q is (-)ve ∴ Reactance is ~~+ve~~ capacitive in nature

$Z = \frac{V}{I} = \cancel{\frac{120\angle -20}{4\angle 10}} \neq \frac{V_m}{I_m} \angle \theta_v - \theta_i = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$

$= |Z| \angle \theta_v - \theta_i$

$Z = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$ $= \frac{120}{4} \angle -20 - 10 = 30\angle -30$.

$= 25.98 - j15\Omega$

$= 120$ $X = [X_L \& X_C]$

R & C/L

~~$X_C = j15$~~

~~$\frac{1}{2\pi f C} = -15$~~

~~$C = -\frac{1}{2\pi f 15}$~~ =

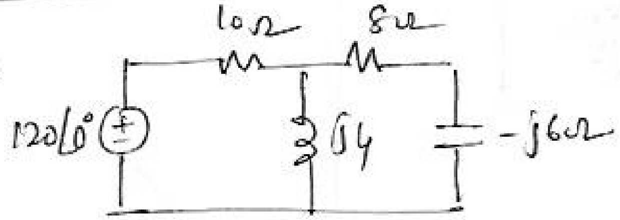
$X_C = -15$

$\frac{1}{2\pi f C} = -15$

$C = \left(\frac{1}{2\pi f 15}\right) = \frac{1}{100\pi \times 15}$

$2\pi f = 100\pi$

(2)



pf =

$I = 8.846 - j3.328$

$= 9.451\angle -20.58$

$Z = \frac{(8 - j6) \times j4}{8 - 2j} + 10$

$\boxed{I \text{ Lags } V}$

$\boxed{Z = 11.88 + j4.41}$ ~~0.470~~

$\theta_v - \theta_i = 0 - (-20) = \underline{\underline{20^\circ}}$

$P_{av} = V_{rms} I_{rms} \cos\theta = 1.062$ kW

$\boxed{\cos(20) = 0.939}$ 936

$\cancel{P_{av} = 60\sqrt{2} \times 6.68 \; \cos\theta}$

$\cancel{= 60\sqrt{2} \times 6.68 \times (1.889)} \times 0.939 = \cancel{6327}$

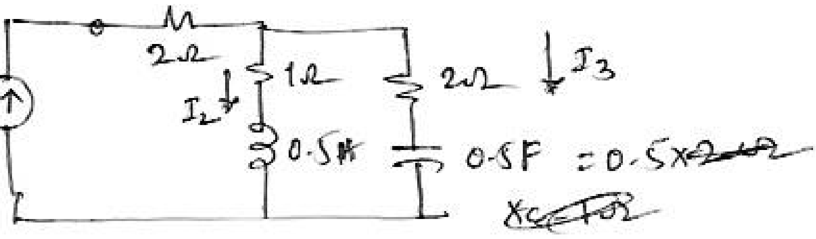
$P_{av} = V_{rms} I_{rms} \cdot \cos\theta$

$P_{av} = \cancel{60\sqrt{2}}\ 120 \times 9.45 \times 0.93$ $\qquad \frac{V^2}{Z}$

$= 1064.8$ watts $= 1.064$ KW

$\underline{1.061 \text{ KW}}$

(3) The n/w shown, current driven up source is $i_1 = 5\sqrt{2}\sin 2t$ and in steady state. find P_{av} & Q



$I_1 = 5\sqrt{2}\ \sin 2t$ $\qquad X_L = 0.5 \times \omega = 0.5 \times 2 = 1$

$Z = 3 + j0.333 = 3 + j\frac{1}{3}$ $\qquad X_C = \frac{1}{\omega C} = \frac{1}{2 \times 0.5} = 1$

$P_{av} = V_{rms} I_{rms} \cos\theta.$ $\qquad 3 + 0.333j$

$= I_{rms}^2 |Z| \cos\theta.$

$= I_{rms}^2 R.$

$P_{av} = I_{rms}^2 R = \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 \times 3 = 5^2 \times 3 = 75\,W.$

$Q = I_{rms}^2 \, Im(Z) = 5^2 \times (0.333) = 8.3325$

$= \frac{25}{3}$

$\boxed{S = P_{av} + jQ = 75 + j8.333}$ $\qquad |S| = \underline{75.4}$

~~Q. ϕ = Vrms Irms~~

~~$P_{av} = V_{rms} I_{rms} \cdot \cos(0) = V_{rms} I_{rms}$~~

~~$\frac{75}{I_{rms}} = V_{rms}$~~

~~$|S| = V_{rms} I_{rms} \angle 0.$~~

~~$75.4 = V_{rms} I_{rms}$~~

~~$V_{rms} = \frac{75.4}{5} = \underline{15.1}$~~

$S = \frac{1}{2} V I^*$ $V = V_m \angle \theta_v$ $I = I_m \angle \theta_i$

$= \frac{1}{2} V_m I_m \angle \theta_v - \theta_i = V_{rms} I_{rms} \angle \theta_v - \theta_i$

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Tellegen's theorem:

- This theorem is applicable to all types of networks containing
  - Both active and passive elements
  - Linear or non linear and
  - Sources may be time-varying (or) time invariant.
- It states that the summation of all instantaneous or complex powers delivered by the sources and absorbed or | and stored by the passive elements of the network is zero.
- Resistors present in the n/w absorb and dissipate power, whereas capacitor and inductor store power in them.

R_1 R_2 R_3

R

$I^2 R$

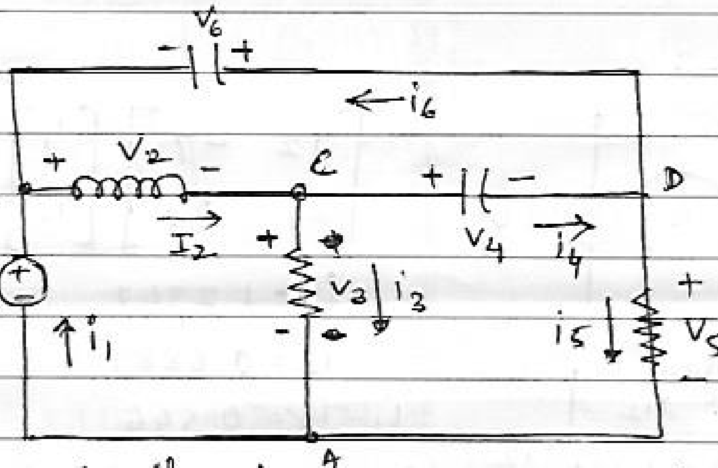
$= R = 2, 10, 5\Omega$

$V_{rms} I_{rms}$

$P = V_{rms} I_{rms} \cdot \cos\phi$

$P = VI$

$= V\angle 0 \; I\angle 0$



The arbitrarily chosen voltages and currents satisfy eq^n

$$\sum_{k=1}^{b} V_k i_k = 0 \qquad — (1)$$

if Kirchoff's voltage and current laws have been satisfied.

Let us complete the following table for above ckt and verify ~~check~~ Tellegen's theorem

	1	2	3	4	5	6
V_k	4	2	__	3	__	__
I_k	2	2	4	__	__	__
P_k						$\Sigma P_k = ?$

Using KVL @ loop ABCA, ACDA, ABDA

@ ABCA: $-V_1 + V_2 + V_3 = 0 \quad \therefore V_3 = V_1 - V_2 = 4 - 2 = 2V$

@ ACDA: $-V_3 + V_4 + V_5 = 0 \quad \therefore V_5 = V_3 - V_4 = 2 - 3 = -1V$

@ ABDA: $-V_1 - V_6 + V_5 = 0 \quad V_6 = -V_1 + V_5 = -4 - 1 = -5$

$V_{rms} I_{rms} \cos\phi + j V_{rms} I_{rms} \sin\phi$

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Using KCL, @ node B, C, and D

@ node B : $i_6 = i_1 + i_2 \quad \therefore i_6 = 2+2 = 4A$

@ node C : $i_2 = i_3 + i_4 \quad i_4 = i_2 - i_3 = 2-4 = -2A$

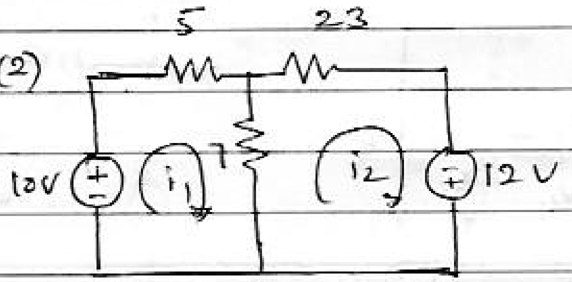
@ node D : $i_4 = i_5 + i_6 \quad i_5 = i_4 - i_6 = -2-4 = -6A$

$k \rightarrow$	1	2	3	4	5	6
V_k	4	2	$\boxed{2}$	3	$\boxed{-1}$	$\boxed{-5}$
I_k	2	2	4	$\boxed{-2}$	$\boxed{-6}$	$\boxed{4}$
P_k	8	4	8	-6	6	-20

$\sum P_k$

$$\sum_{k=1}^{b} V_k i_k = 8+4+8-6+6-20 = 26-26 = \underline{\underline{0}}$$

(2)

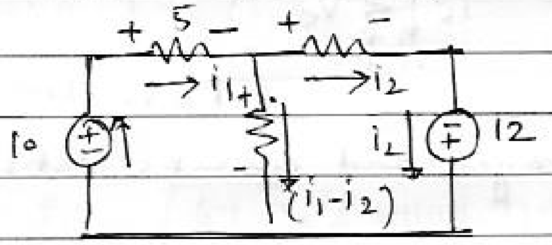


$$\begin{bmatrix} 12 & -7 \\ -7 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$$

$i_1 = 1.2347$

$i_2 = 0.6881$

$i_1 - i_2 = 0.546$



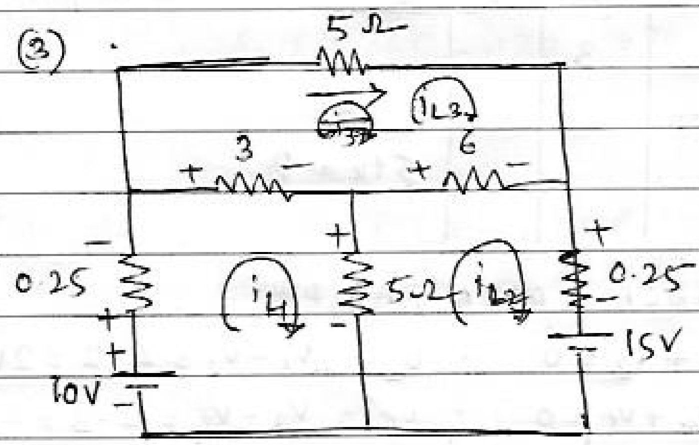
$P_k = P_{generated} + P_{absorbed}$

$= (10 \times 1.2347) + (12 \times 0.6881)$

$+ [-5 \times 1.2347 - 23 \times 0.6881 - 7 \times 0.546]$

$= 20.59 - 20.59 = \underline{\underline{0}}$

(3)



$$\begin{bmatrix} 8.25 & -5 & -3 \\ -5 & 11.25 & -6 \\ -3 & -6 & 14 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -15 \\ 0 \end{bmatrix}$$

$i_1 = -0.2829$

$i_2 = -1.9333$

$i_3 = -6.8891$

~~$i_{3\Omega} = i_1 - i_3 = 0.8062$~~ i

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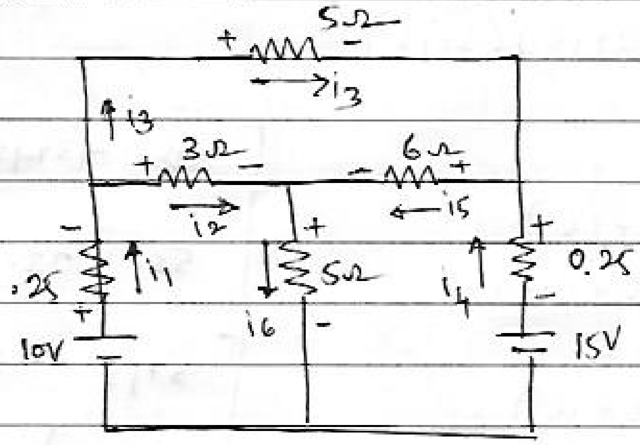


$i_1 = -0.2829$

$i_2 = i_{L_1} - i_{L_3}$

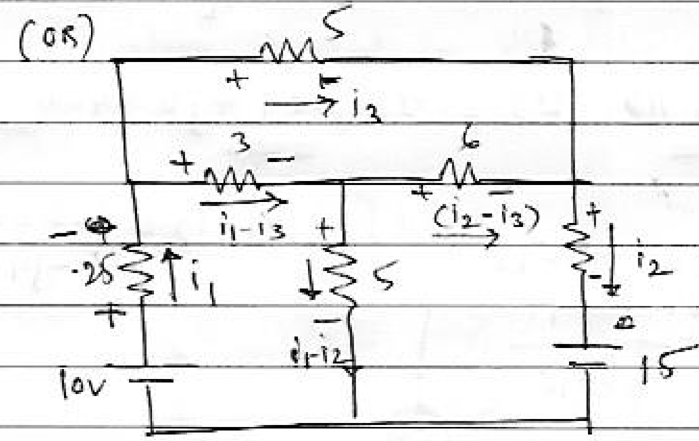
$i_3 = i_{L_3} = -0.8891$

$i_4 = -i_{L_2} = +1.9333$

$i_5 = i_{L_3} - i_{L_2} = 1.0442$

$i_6 = i_{L_1} - i_{L_2} = 1.6504.$

(OR)



$i_1 - i_3 = 0.6062$

$i_2 - i_3 = \cancel{1.6503}\ 1.0442$

$i_1 - i_2 = 1.6563.$

~~P~~ $-\cancel{0.06}323,\ \cancel{-3.1325}$

$0.09186 + 3.271 +$

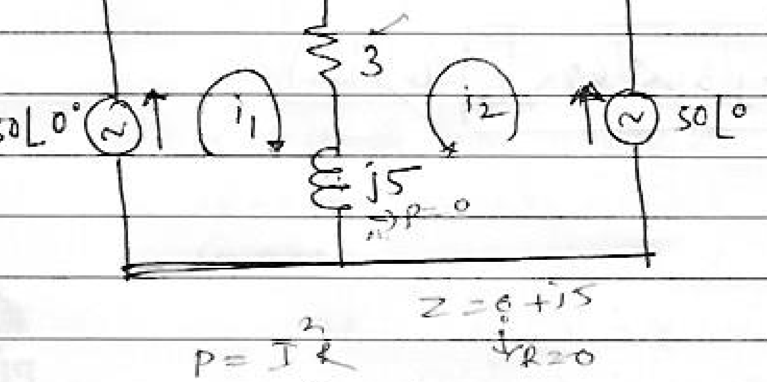
$P_{ab} = 0.0200 + 1.1024 + 13.6175 + 6.54 + 0.9344 + 3.952$

$= \underline{26.166}\ \cancel{+2.829} = \cancel{28.995}$

$\underline{P_{del}}$ by src $= \cancel{2.829}\ \cancel{+}\ 28.9995 - 2.829 = \underline{26.170}$

~~Pt = Pabs~~ $\underline{\text{Pdelivered}} = \underline{\text{Pabsorbed}}$

(3)



~~for~~ the ckt shown, verify tellegen's thm

$$\begin{bmatrix} 5+j5 & -(3+j5) \\ -(3+j5) & 33-j3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 5\angle 0 \end{bmatrix}$$

$Z = 6 + j5$

$R = 0$

$P = I^2 R$

$= I^2 \times 31$

DATE

$\Delta = (5+5j)\times(33-j3) - (3+j5)(3+j5)$

$\Delta = 196 + j120$

$$\begin{bmatrix} 50 & -(3+j5) \\ 50 & 33-j3 \end{bmatrix}$$

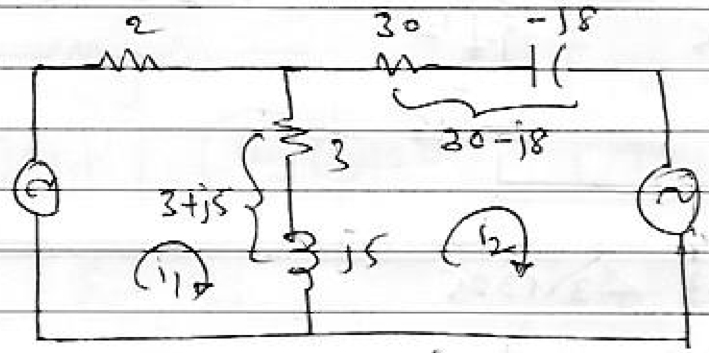
$50\times(33-j3) + 50(3+j5)$
$= 1800 + 100i$

$$\begin{bmatrix} 5+j5 & 50 \\ -(3+j5) & 50 \end{bmatrix}$$

$50\times(5+j5) + 50(3+j5)$
$= 400 + 500j$

$\underline{\underline{I_1 = 6.9069 - j3.718}}$ $\quad$ $\underline{\underline{I_2 = 2.6204 + j0.9466}}$

$i_1 - i_2 = 4.2865 - j4.6644$



$P_{ab} = (67.763 - j102.719) + (189.794 - j136.890)$
$+ 218.80 + j101.064$
$= (476.357 - j138.544)$

$P_{del} = (345.345 + j185.9) + (131.02 + 47.33j)$
$= (476.365 - j138.57)$

$\cancel{P_{ab} = P_{del} = \boxed{124.833 - j345.625}}$ (textbook)