15-09-19

3rd SEM-PESU

Problems on initial Conditions and Transient Behavior

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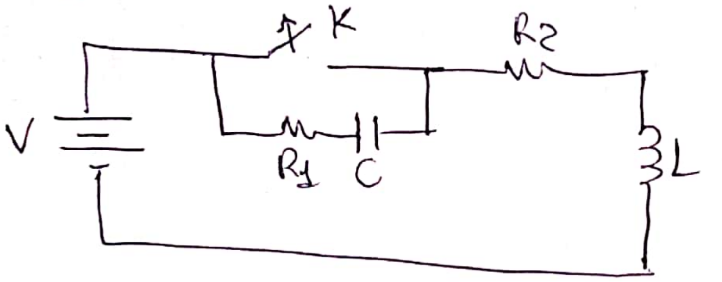
Network Analysis and Synthesis [UE18EC201]

(1) In the n/w shown the switch is opened at $t=0$ after n/w has attained steady state with switch closed. (a) Find an expression for voltage across switch at $t=0^+$; (b) If parameters are adjusted such that $i(0^+)=1A$ and $\frac{di}{dt}(0^+) = -1A/sec$. What is the value of derivative across the switch.

5.25



a) When K is closed 'L' acts like S.C. and 'C' acts like O.C.

$$\boxed{i(0^-) = \frac{V}{R_2} = i(0^+)} = 1 \quad -[1]$$

$$\boxed{V_C(0^-) = V_C(0^+) = 0V} \quad -[2]$$

b) When 'K' is opened at $t=0^+$:-

$$\boxed{V_K = R_1 i + \frac{1}{C}\int_{-\infty}^{t} i\, d\tau} \quad -[3]$$

$$\therefore V_K(0^+) = R_1 i(0^+) + \frac{1}{C}\int_{-\infty}^{0^+} i\, d\tau = \frac{R_1 V}{R_2} + V_C(0^+) = \boxed{\frac{V R_1}{R_2}} \quad -[4]$$

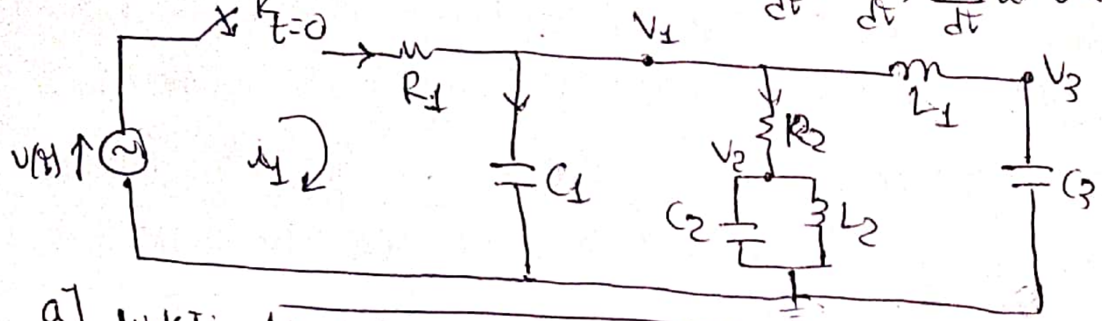
we have,

$$\boxed{V_K = R_1 i + \frac{1}{C}\int_{-\infty}^{t} i\, d\tau} \quad -[5] \rightarrow \text{Differentiating}$$

$$\boxed{\frac{dV_K}{dt} = R_1\frac{di}{dt} + \frac{1}{C} i} \quad -[6]$$

$$\therefore \boxed{\frac{dV_K(0^+)}{dt} = R_1\frac{di(0^+)}{dt} + \frac{1}{C} i(0^+) = +R_1(-1) + \frac{1}{C}(1) = \frac{1}{C} - R_1} \quad -[7]$$

ⓔ 5.27 In the network shown below, the switch 'K' is closed at $t=0$.
At $t=0^-$ all capacitor voltages and inductor currents are zero.
Find (a) V_1, V_2, V_3 at $t=0^+$ (b) $\frac{dV_1}{dt}, \frac{dV_2}{dt}, \frac{dV_3}{dt}$ at $t=0^+$



a] W.K.T:- $\boxed{i_{L_1}(0^+) = i_{L_2}(0^+) = i_{L_3}(0^+) = 0A}$ —[1]

b] At node V_1:-

$$\boxed{\frac{V(t)-V_1}{R_1} = C_1\frac{dV_1}{dt} + \frac{V_1-V_2}{R_2} + \frac{1}{L_1}\int_{-\infty}^{t} V\,d\tau} \quad -[1]$$

At $t=0^+$:-

$$\frac{V(0^+)-V_1(0^+)}{R_1} = C_1\frac{d}{dt}V_1(0^+) + \frac{V_1(0^+)-V_2(0^+)}{R_2} + \underbrace{\frac{1}{L_1}\int_{-\infty}^{0^+}(V_1-V_3)\,d\tau}_{i_{L_1}(0^+)}$$

$$\therefore \frac{V(0^+)}{R_1} = C_1\frac{dV_1(0^+)}{dt} + \frac{0}{R_2} + 0$$

$$\therefore \boxed{\frac{dV_1(0^+)}{dt} = \frac{V(0^+)}{R_1C_1}} \quad -[2]$$

c] At node V_2:- $\boxed{\frac{V_1-V_2}{R_2} = C_2\frac{dV_2}{dt} + \frac{1}{L_2}\int_{-\infty}^{t} V_2\,d\tau}$ —[3]

$$\frac{V_1(0^+)-V_2(0^+)}{R_2} = C_2\frac{dV_2(0^+)}{dt} + \underbrace{\frac{1}{L_2}\int_{-\infty}^{0^+} V_2\,d\tau}_{i_{L_2}(0^+)}$$

$$\therefore \frac{0}{R_2} = C_2\frac{dV_2(0^+)}{dt} + 0$$

$$\therefore \boxed{\frac{dV_2(0^+)}{dt} = 0\ V/s} \quad -[4]$$

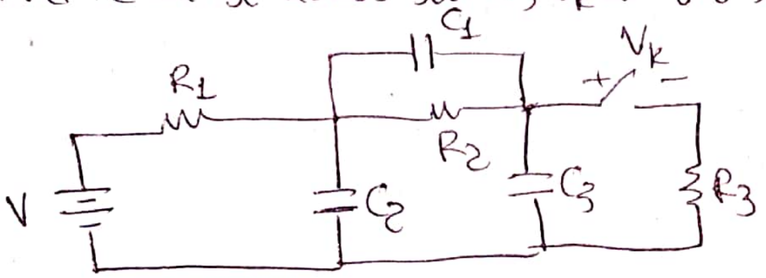
At node V_3:- $\boxed{\frac{1}{L_1}\int_{-\infty}^{t}(V_1 - V_3)\,d\tau = C_3\frac{dV_3}{dt}}$ —[5]

At $t=0^+$

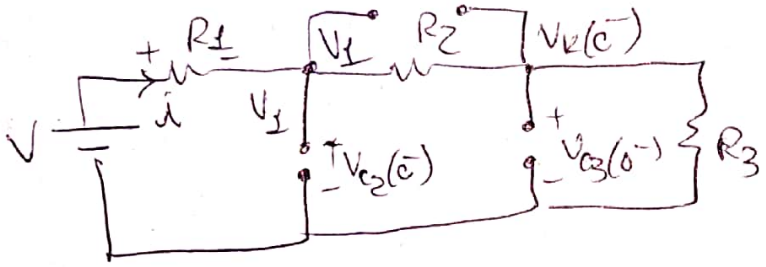
$$i_{L1}(0^+) = C_3 \frac{dV_3(0^+)}{dt}$$

$$0 = C_3 \frac{dV_3(0^+)}{dt} \qquad \therefore \boxed{\frac{dV_3(0^+)}{dt} = 0\ V/s}$$

(3) 5.28 In the n/w shown, steady state is reached, and at $t=0$, switch 'K' is opened. (a) Find the voltage across switch, V_k at $t=0^+$; (b) Find $\frac{dV_k}{dt}$ at $t=0^+$.



a] At $t=0^-$:-



At $t=0^-$ steady state is reached and all capacitors acts like O.C.

$V_1(0^-) = V_{C_2}(0^-)$:- $\boxed{i(0^-) = \frac{V}{R_1+R_2+R_3}}$ —[1]

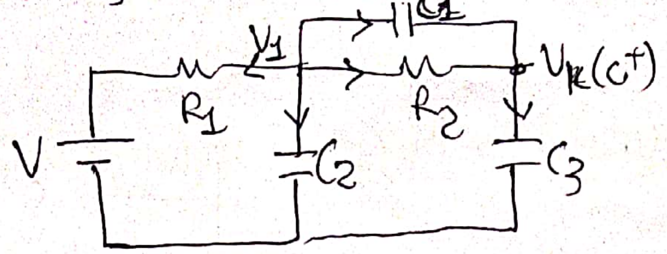
$+V_{C_2}(0^-) + R_1 i - V = 0 \qquad \therefore V_{C_2}(0^-) = V - R_1 i$

$$\therefore V_{C_2}(0^-) = V_1(0^-) = V_{C_2}(0^+) = V - \frac{V R_1}{R_1+R_2+R_3}$$

$$V_1 = \boxed{V_{C_2}(0^-) = \frac{V(R_2+R_3)}{R_1+R_2+R_3}} \text{ —[2]} = V_1$$

$$\boxed{V_{C_3}(0^-) = V_K(0^-) = V_K(0^+) = i(0^-)R_3 = \frac{V R_3}{R_1+R_2+R_3}} \text{ —[3]}$$

5] At $t=0^+$:- R_3 is not included in circuit.



Applying KCL at node V_1:-

$$\boxed{\frac{V_1 - V}{R_1} + C_2\frac{dV_1}{dt} + C_1\frac{d}{dt}(V_1 - V_k) + \frac{V_1 - V_k}{R_2} = 0} \quad -[4]$$

$$\therefore [C_1 + C_2]\frac{dV_1}{dt} = C_1\frac{dV_k}{dt} + \frac{V}{R_1} - \frac{V_1}{R_1} - \frac{V_1}{R_2} + \frac{V_k}{R_2}$$

$$\therefore [C_1 + C_2]\frac{dV_1}{dt} = C_1\frac{dV_k}{dt} + \frac{V}{R_1} - \frac{V(R_2+R_3)}{R_1(R_1+R_2+R_3)} - \underbrace{\frac{V(R_2+R_3)}{R_2(R_1+R_2+R_3)} + \frac{VR_3}{R_2(R_1+R_2+R_3)}}$$

$$\therefore [C_1 + C_2]\frac{dV_1}{dt} = C_1\frac{dV_k}{dt} + \frac{V}{R_1} - \frac{V(R_2+R_3)}{R_1(R_1+R_2+R_3)} - \frac{VR_2}{R_2(R_1+R_2+R_3)}$$

$$\therefore [C_1 + C_2]\frac{dV_1}{dt} = C_1\frac{dV_k}{dt} + \frac{1}{R_1}\left[\frac{VR_1}{R_1+R_2+R_3}\right] - \frac{V}{R_1+R_2+R_3}$$

$$\therefore \boxed{[C_1 + C_2]\frac{dV_1}{dt} = C_1\frac{dV_k}{dt}} \quad -[5]$$

ontinuation of problem 3(5.28)

Applying KCL at node V_k:-

$$\boxed{C_1\frac{d}{dt}(V_1-V_k)+\frac{V_1-V_k}{R_2}=C_3\frac{dV_k}{dt}} \quad -[6]$$

$$\therefore C_1\frac{dV_1}{dt}+\frac{1}{R_2}[V_1-V_k]=[C_1+C_3]\frac{dV_k}{dt}$$

$$\therefore C_1\frac{dV_1}{dt}+\frac{1}{R_2}\left[\frac{V(R_2+R_3)}{R_1+R_2+R_3}-\frac{VR_3}{R_1+R_2+R_3}\right]=[C_1+C_3]\frac{dV_k}{dt}$$

$$\therefore C_1\frac{dV_1}{dt}+\frac{1}{R_2}\left[\frac{VR_2}{R_1+R_2+R_3}\right]=[C_1+C_3]\frac{dV_k}{dt}$$

$$\therefore \boxed{C_1\frac{dV_1}{dt}+\frac{V}{R_1+R_2+R_3}=[C_1+C_3]\frac{dV_k}{dt}} \quad -[7]$$

Substitute eq-[5] in eq-[7] for $\boxed{\frac{dV_1}{dt}}$:-

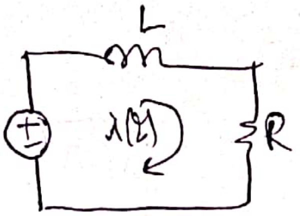
$$\therefore \frac{C_1^2}{C_1+C_3}\frac{dV_k}{dt}+\frac{V}{R_1+R_2+R_3}=[C_1+C_3]\frac{dV_k}{dt}$$

$$\therefore \frac{dV_k}{dt}\left[(C_1+C_3)-\frac{C_1^2}{(C_1+C_2)}\right]=\frac{V}{R_1+R_2+R_3}$$

$$\therefore \frac{dV_k}{dt}\left[\frac{C_1^2+C_1C_2+C_3C_1+C_2C_3-C_1^2}{C_1+C_2}\right]=\frac{V}{R_1+R_2+R_3}$$

$$\therefore \boxed{\frac{dV_k(0^+)}{dt}=\frac{V[C_1+C_2]}{[C_1C_2+C_2C_3+C_3C_1][R_1+R_2+R_3]}}$$

.23 A bolt of lightening having a waveform which is approximated as $v(t)=te^{-t}$ strikes a transmission line having resistance $R=0.1\Omega$ and inductance $L=0.1H$ (the line-to-line capacitance is assumed to be negligible). An equivalent n/w is shown in the accompanying diagram. What is the form of the current as a function of time. (This current will be in ampere per unit volt of lightening; likewise the time-base is normalized).



$$\boxed{v(t)=L\frac{di}{dt}+Ri} \quad [1]\text{-}a$$

$$\boxed{\frac{di}{dt}+\frac{R}{L}i=\frac{1}{L}v(t)} \quad [1]\text{-}b$$

$$\boxed{\frac{di}{dt}+i=10te^{-t}} \quad [1]\text{-}c$$

Direct method:- 1) Let, complementary soln:- $\boxed{i_c(t)=Ke^{st}}$ —[2]-a

$\frac{di}{dt}+i=0 \quad \therefore \quad Kse^{st}+Ke^{st}=0$

$\therefore \quad s+1=0 \quad \therefore \boxed{s=-1} \quad \boxed{i_c(t)=Ke^{-t}}$ —[2]-b

2) Particular solution:-

Given $v(t)=te^{-t}$

$i_p(t)=[A_0t+A_1]e^{-t}$

$i_p(t)=A_0te^{-t}+\boxed{A_1e^{-t}}$ → Same form as $i_c(t)$

↳ Change the form

$$\boxed{i_p(t)=A_0t^2e^{-t}+A_1te^{-t}=[A_0t+A_1]te^{-t}} \quad [3]$$

Substituting eq-(3) in eq-[1]-c:-

$$\therefore \frac{d}{dt}\{[A_0t^2e^{-t}+A_1te^{-t}]\}+[A_0t^2e^{-t}+A_1te^{-t}]=10te^{-t}$$

$$\therefore -A_0t^2e^{-t}+A_0 2te^{-t}+A_1e^{-t}-A_1te^{-t}+A_0t^2e^{-t}+A_1te^{-t}=10te^{-t}+0\cdot e^{-t}$$

$2A_0=10 \quad \boxed{A_0=5} \quad \boxed{A_1=0} \quad \therefore \boxed{i_p(t)=5t^2e^{-t}}A$ —[4]

Complete response:- $\boxed{i(t)=i_c(t)+i_p(t)}$ —[5]

$$\boxed{i(t)=Ke^{-t}+5t^2e^{-t}} \quad [6]$$

Initial condition $\boxed{i(0^+) = 0A = i(0^-)}$ → used to find the constant in e.c.

$\therefore i(0) = 0 = Ke^{-(0)} + 0 \quad \therefore \boxed{K=0}$

$\therefore \boxed{i(t) = 5t^2 e^{-t} A}$ —[7]

e) Laplace Transform method:- Taking L.T. of eq [1]-C :-

$$\boxed{SI(S) - i(0^-) + I(S) = \frac{10}{(S+1)^2}} \text{ —[8]}$$

$\therefore SI(S) + I(S) = \dfrac{10}{(S+1)^2} \quad \therefore \boxed{I(S) = \dfrac{10}{(S+1)^3}}$ —[9]

$\therefore i(t) = \dfrac{10}{2!} t^2 e^{-t} \quad \therefore \boxed{i(t) = 5t^2 e^{-t} A}$

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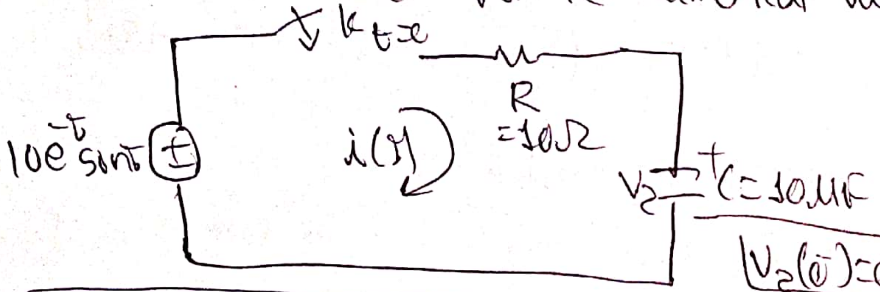
(5) In the n/w shown switch 'k' is closed at t=0, with capacitor initially uncharged. For the numerical values given find i(t).

6-2e



$\boxed{V_2(0^-) = 0V}$

$$\boxed{10e^{-t}\sin(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i\, d\tau} \text{ —[1]}$$

$$R\frac{di}{dt} + \frac{1}{C} i = 10\cos t\, e^{-t} - 10e^{-t}\sin t$$

$$\therefore \boxed{\frac{di}{dt} + 10^4 i = e^{-t}\cos t - e^{-t}\sin t} \text{ —[2]}$$

a) Complementary soln:- $\boxed{i_c(t) = Ke^{St}}$ —[3]-a $\quad S + 10^4 = 0 \quad \boxed{S = -10^4}$

$\therefore \boxed{i_c(t) = Ke^{-10^4 t}}$ —[3]-b

Particular solution:- $\boxed{i_p(t) = F_1 e^{-t}\cos t + F_2 e^{-t}\sin t}$ —[4]

Substituting eq-[4] in eq-[2]:-

$$10^4 F_1 e^{-t}\cos t + 10^4 F_2 e^{-t}\sin t - F_1 e^{-t}\cos t - F_1 e^{-t}\sin t - F_2 e^{-t}\sin t$$
$$+ F_2 e^{-t}\cos t = e^{-t}\cos t - e^{-t}\sin t$$

$$e^{-t}\cos t[9999F_1 + F_2] + [-F_1 + 9999F_2] = e^{-t}\cos t - e^{-t}\sin t$$

$\therefore$ $\boxed{9999F_1 + F_2 = 1}$ —[5]-a

$\boxed{-F_1 + 9999F_2 = -1}$ —[5]-b $\times 9999$

$$\therefore \quad 9999F_1 + F_2 = 1$$
$$-9999F_1 + 99.98\times10^6 F_2 = -9999$$
$$\overline{\quad 99.98\times10^6 F_2 = -9999 \quad}$$

$\therefore$ $\boxed{F_2 = -10^{-4}}$

$\boxed{F_1 = 10^{-4}}$

$\therefore$ $\boxed{i(t) = i_c(t) + i_p(t)}$ —[6]

$\therefore$ $\boxed{i(t) = Ke^{-10^4 t} + 10^{-4}e^{-t}\cos t - 10^{-4}e^{-t}\sin t}$ —[7]

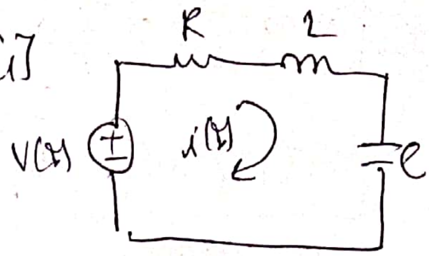
~~i(0)=0~~ W.K.T:- $\boxed{i(0) = 0}$ —[8]-a

$\therefore$ $i(0) = 0 = K + 10^{-4}$ $\quad\therefore$ $\boxed{K = -10^{-4}}$ —[8]-b

$\therefore$ $\boxed{i(t) = -10^{-4}e^{-10^4 t} + 10^{-4}e^{-t}\cos t - 10^{-4}e^{-t}\sin t}$ —[9]

6. 6.29 Consider, a series R-L-C ckt. which is excited by a voltage
(a) Determine the characteristic equn. corresponding to differential equn. $i(t)$; (b) Suppose, L and C are fixed in value but 'R' varies from 0 to ∞.
What will be the locus of the roots of the characteristic equn.? (c) Plot the roots of characteristic equn. in s-plane if $L=1H$; $C=1\mu F$ and 'R' has the values (i) 500 Ω; (ii) 1 kΩ; (iii) 3 kΩ; (iv) 5 kΩ

(a) $$\boxed{v(t) = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt} \quad -[1]$$



Diff. eq-[1]:-

$$\therefore L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dv(t)}{dt}$$

$$\therefore \boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\,i = \frac{1}{L}\frac{dv(t)}{dt}} \quad -[2]$$

Characteristic equn. is given for homogeneous equn.

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0} \quad -[3]$$

Put, $i = Ke^{st}$ we get the characteristic equn:- $\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$ —[4]

$$\boxed{s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}} \quad -[5]$$

Put, $2\xi\omega_n = \frac{R}{L}$ and $\omega_n = \frac{1}{\sqrt{LC}}$

we define, $\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$ —[6]

$\therefore \cancel{s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\ }}$

$$\therefore \boxed{s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}} \quad -[6]$$

When R varies from $0 \to \infty \Rightarrow \xi$ varies from $0 \to \infty$.

when $R=0 \Rightarrow \xi=0$ $s_{1,2}=\pm j\omega_n$ roots are complex conjugate and purely imaginary. The response is oscillatory. The roots lie on imaginary axis $j\omega$.

case-ii] when $\xi \to 0<\xi<1$ $s_{1,2}=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}=\sigma \pm j\omega$

$\sigma^2+\omega^2=\xi^2\omega_n^2+\omega_n^2(1-\xi^2)=\omega_n^2$. The roots move on a locus, described by a circle of radius ω_n.

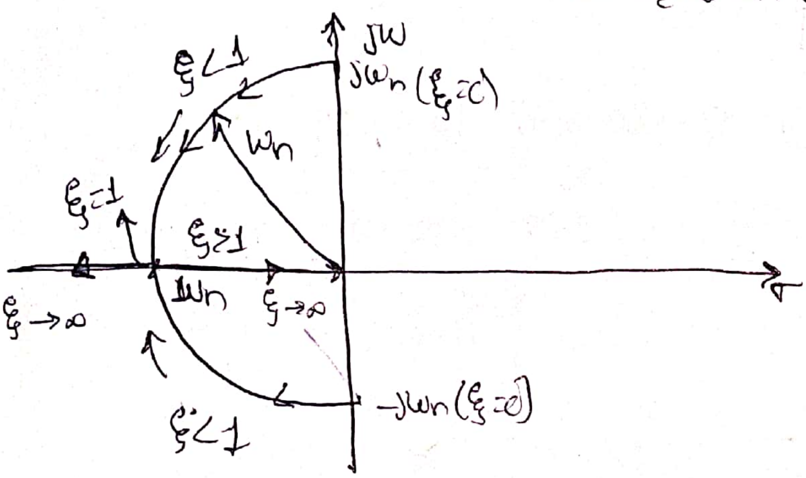
case-iii] when $\xi=1$:- $\boxed{s_{1,2}=-\omega_n}$ The roots are real, negative and equal where, $\sigma=-\omega_n$ on real line.

case-iv] when $\xi>1$:- $s_{1,2}=-\xi\omega_n \pm \omega_n\sqrt{\xi^2}$ as $\xi^2>>1$

$\therefore s_{1,2}=-\xi\omega_n \pm \omega_n\xi$ $\boxed{s_1=-2\xi\omega_n}$ $\boxed{s_2=0}$

the roots move such that,

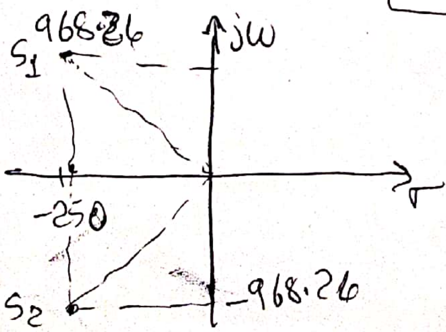
As $\xi\to\infty$, $s_1\to-\infty$ on one end $s_2=0$ on another end.



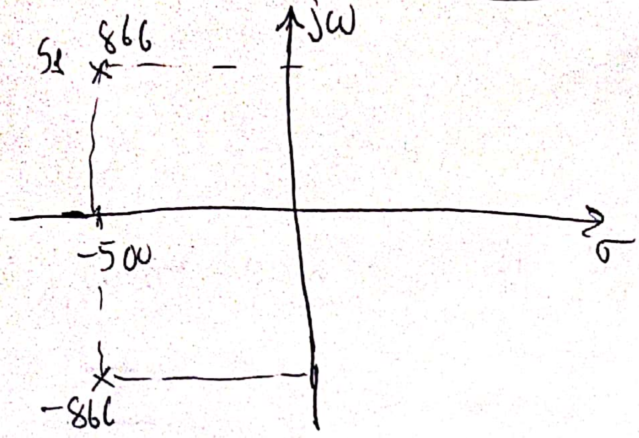
(c) Location of roots:- i] $\boxed{R=500\Omega}$ $\boxed{L=1H}$ $\boxed{C=1\mu F}$ $\boxed{\omega_n=\frac{1}{\sqrt{LC}}=1000Hz}$

$\xi=\frac{R}{2}\sqrt{\frac{C}{L}}=\frac{500}{2}\times10^{-3}=0.25$ $\boxed{\xi<1}$ $\to$ Underdamped

$s_{1,2}=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}=\boxed{-250\pm j968.24}$

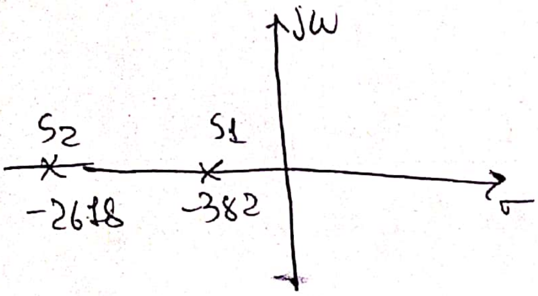


ii] $\boxed{R = 1000\,\Omega}$ $\boxed{\xi = 0.5}$ $\boxed{\xi < 1}$ $\therefore$ $\boxed{s_{1,2} = -500 \pm j\,866}$



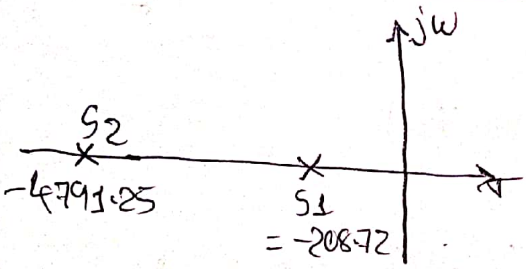
iii] $\boxed{R = 3000\,\Omega}$ $\xi = \frac{3000}{2} \times 10^{-3} = \boxed{1.5}$ $\boxed{\xi > 1}$ $\boxed{s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$

$\boxed{s_{1,2} = -1500 \pm 1118 = -382, -2618}$

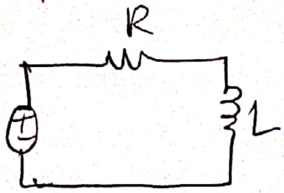


iv] $\boxed{R = 5000\,\Omega}$ $\xi = \frac{5000}{2} \times 10^{-3} = \boxed{2.5}$ $\boxed{\xi > 1}$

$\boxed{s_{1,2} = -2500 \pm 2291.28 = -208.72, -4791.25}$



33. A switch is closed at $t=0$, connecting a battery of voltage V with a series RL-circuit. (a) Show that, energy in the resistor ~~in time~~ as a function of time is $w_R = \frac{V^2}{R}\left(t + \frac{2L}{R}e^{-tR/L} - \frac{L}{2R}e^{-2R/L\,t} - \frac{3L}{2R}\right)$ Joules;
(b) Find an expression for energy in magnetic field as a function of time; (c) Sketch w_R and w_L as a function of time. Show the steady state asymptotes, that is the values that w_R and w_L approach as $t \to \infty$;
(d) Find the total energy supplied by the voltage source in steady state/ as function of time.



$\therefore \cancel{i(t) = i(\infty)+[i(0)-i}$

$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-tR/L}} \quad -[1]$$

$$\boxed{i(0) = 0A} -[2] \qquad \boxed{i(\infty) = \frac{V}{R}A} -[3]$$

$$\therefore \boxed{i(t) = \frac{V}{R}\left[1 - e^{-tR/L}\right]A} -[4]$$

(a) $$w_R = \int_0^t i^2(t) R\,dt = \frac{V^2}{R^2} \times R \int_0^t \left[1 - e^{-tR/L}\right]^2 dt$$

$$\therefore w_R = \frac{V^2}{R}\int_0^t \left[1 + e^{-2R/L\,t} - 2e^{-tR/L}\right]dt$$

$$\therefore w_R = \frac{V^2}{R}\left[t - \frac{e^{-2R/L\,t}}{2R} \times L\Big|_0^t + 2\,\frac{e^{-tR/L}}{R} \times L\Big|_0^t\right]$$

$$\therefore w_R = \frac{V^2}{R}\left[t - \frac{L}{2R}\left[e^{-\frac{2R}{L}t} - 1\right] + \frac{2L}{R}\left[e^{-tR/L} - 1\right]\right]$$

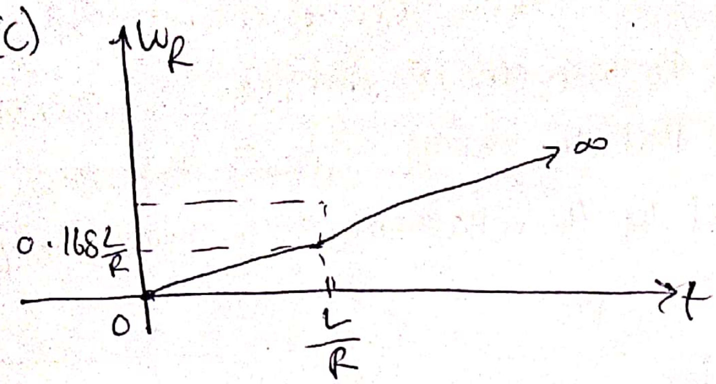
$$\therefore w_R = \frac{V^2}{R}\left[t + \frac{2L}{R}e^{-tR/L} - \frac{L}{2R}e^{-t2R/L} + \frac{L}{2R} - \frac{2L}{R}\right]$$

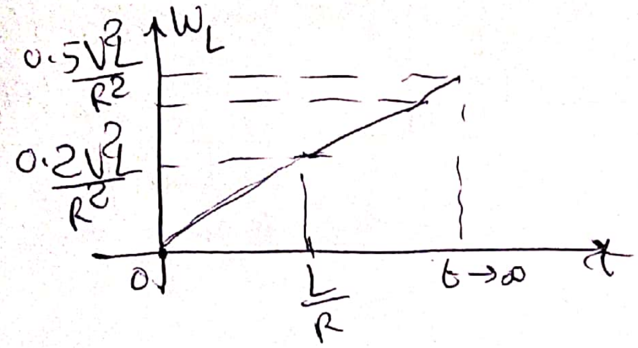
$$\therefore \boxed{w_R = \frac{V^2}{R}\left[t + \frac{2L}{R}e^{-tR/L} - \frac{L}{2R}e^{-t2R/L} - \frac{3L}{2R}\right] \text{Joules}}$$

(b) $W_L = \frac{1}{2} L i_L^2(t)$

$$\therefore \boxed{W_L = \frac{1}{2} \times L \times \frac{V^2}{R^2}\left[1 - e^{-tR/L}\right]^2}$$

(c)





(d) In steady state:- $W_s = \int_0^t V \cdot \frac{V}{R} dt = \boxed{\frac{V^2}{R} t}$

As a function of time:- $W_s = \int_0^t V i(t)\, dt = \frac{V^2}{R} \int_0^t \left[1 - e^{-tR/L}\right] dt$

$$\therefore W_s = \frac{V^2}{R}\left[t + \frac{e^{-R/L\, t}}{R} L \Big|_0^t\right]$$

$$\therefore W_s = \frac{V^2}{R}\left[t + \frac{L}{R}\left[e^{-R/L\, t} - 1\right]\right]$$

$$\therefore \boxed{W_s = \frac{V^2}{R}\left[t - \frac{L}{R}\left(1 - e^{-R/L\, t}\right)\right]}$$