

# Network Analysis & Systems

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UE18EC201: Network Analysis & Systems



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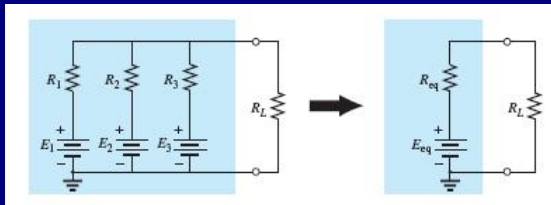


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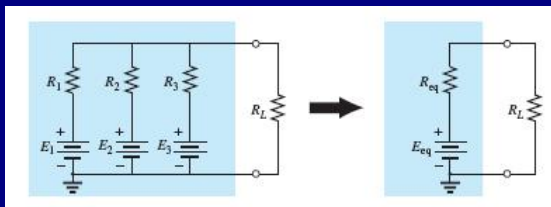
# Millman's Theorem (1)



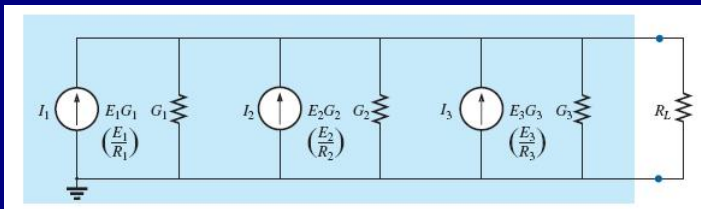
- Millman's theorem allows us to reduce any number of parallel voltage sources to one!
- The steps involved are:
  - 1 Convert all voltage sources to current sources.
  - 2 Combine all these current sources.
  - 3 Convert the resulting source to a voltage source.



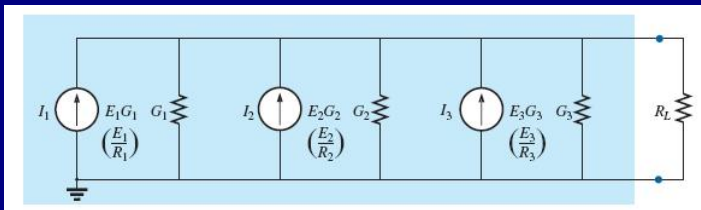
# Millman's Theorem (2)



Step 1:



# Millman's Theorem (3)



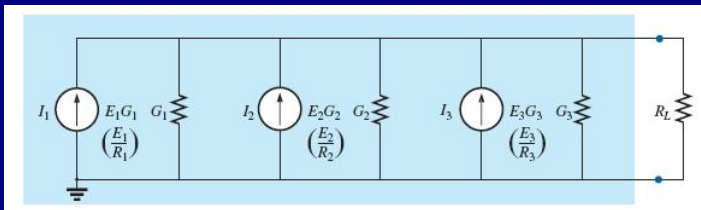
Step 2:

$$I_T = I_1 + I_2 + I_3$$

$$G_T = G_1 + G_2 + G_3$$



# Millman's Theorem (3)



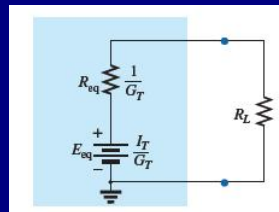
Step 2:

$$I_T = I_1 + I_2 + I_3$$

$$G_T = G_1 + G_2 + G_3$$

Step 3:

$$E_{eq} = \frac{I_T}{G_T} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3}$$



# Millman's Theorem (4)

More generally,

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

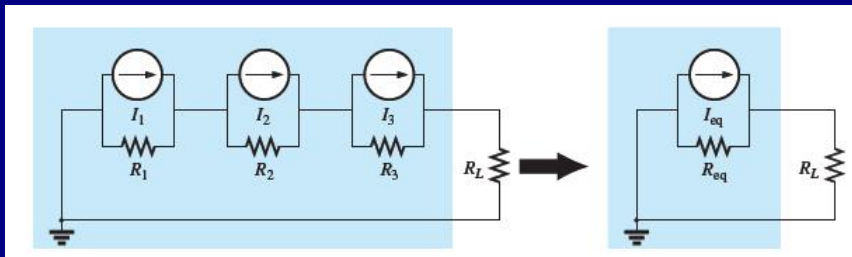
and

$$V_{eq} = \frac{\frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \cdots + \frac{V_n}{Z_n}}{\frac{1}{Z_{eq}}}$$





# Millman's Theorem — Dual (5)



$$R_{eq} = R_1 + R_2 + R_3$$

$$I_{eq} = \frac{I_1 R_1 + I_2 R_2 + I_3 R_3}{R_{eq}}$$



# Millman's Theorem — Dual (6)

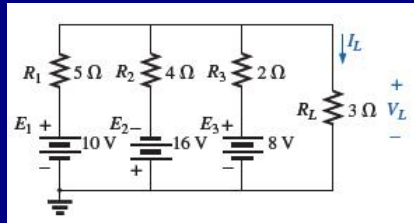
More generally,

$$Z_{eq} = Z_1 + Z_2 + \cdots Z_n$$

and

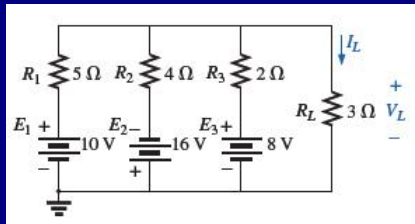
$$I_{eq} = \frac{I_1 Z_1 + I_2 Z_2 + \cdots + I_n Z_n}{Z_{eq}}$$





$$\frac{1}{R_{eq}} =$$

# Examples (1)



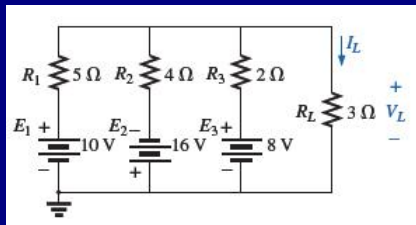
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 0.95 \text{ S}$$

Therefore,

$$E_{eq} =$$



## Examples (1)



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 0.95 \text{ S}$$

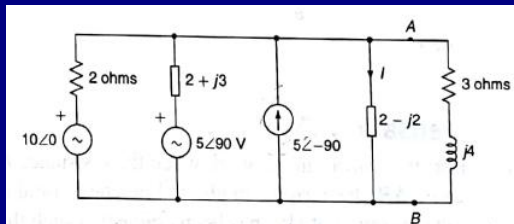
Therefore,

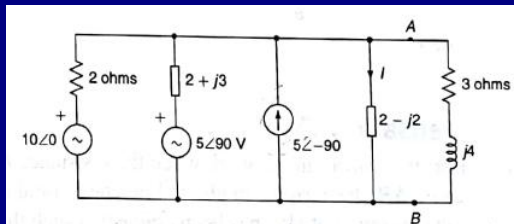
$$E_{eq} = \left( \frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3} \right) R_{eq} = \frac{2}{0.95} = 2.11 \text{ V}$$

Therefore, a voltage of 2.11 V in series with a  $1.05 \Omega$ .



## Examples (2)





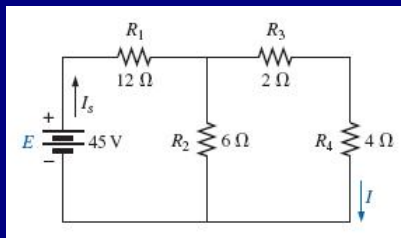
- Equivalent source:

$$\frac{1}{Z_{eq}} = \frac{1}{2} + \frac{1}{2+j3} + \frac{1}{2-j2} = 0.9038 + j0.0192$$

$$V = \left( \frac{10\angle 0^\circ}{2} + \frac{5\angle 90^\circ}{2+j3} + 5\angle -90^\circ \right) / (0.9038 + j0.0192)$$
$$= 6.7059 - j4.8235.$$



# Examples (3)

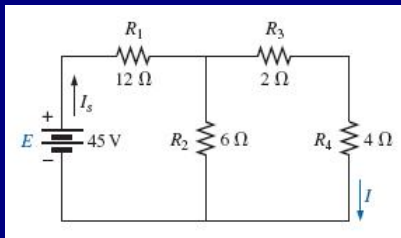


■  $R_{eq} =$





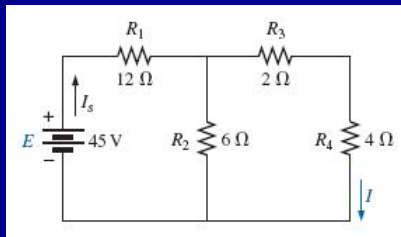
## Examples (3)



- $R_{eq} = 15\Omega$ .
- $I =$



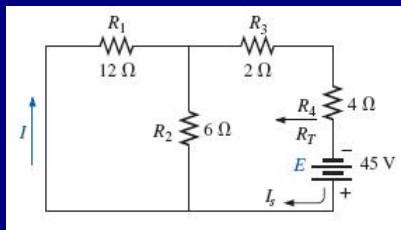
# Examples (3)



- $R_{eq} = 15\ \Omega$ .
- $I = 1.5\ \text{A}$ .



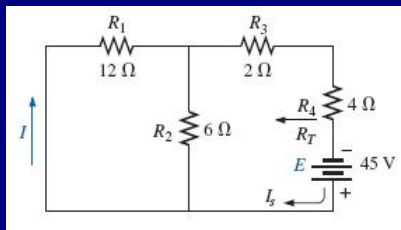
## Examples (4)



■  $R_{eq} =$



# Examples (4)

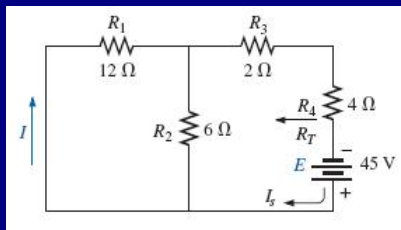


■  $R_{eq} = 10\ \Omega.$

■  $I =$



## Examples (4)

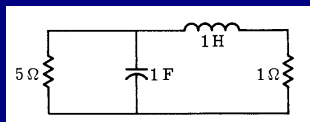


- $R_{eq} = 10\ \Omega$ .
- $I = 1.5\text{ A}$ .



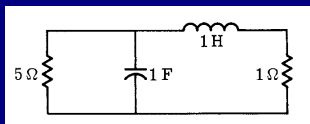
# Examples (5)

Reciprocity Theorem:

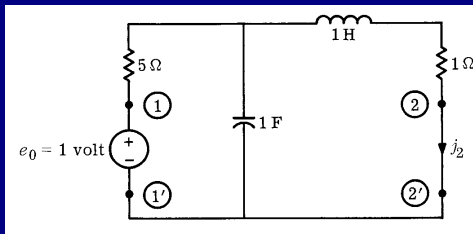


# Examples (5)

Reciprocity Theorem:

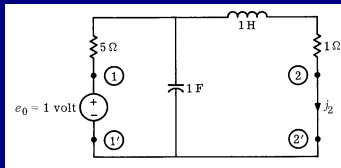


Reciprocity under DC conditions:



# Examples (6)

Reciprocity under DC conditions:



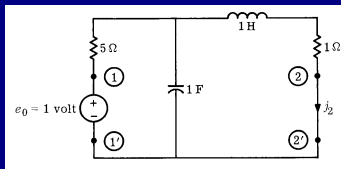
Current  $j_2 =$





# Examples (6)

Reciprocity under DC conditions:

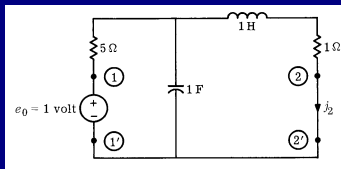


Current  $j_2 = 1/6\text{ A}$ .

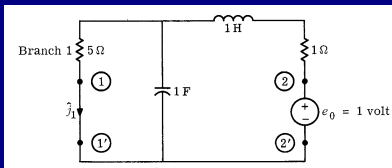


# Examples (6)

Reciprocity under DC conditions:



Current  $j_2 = 1/6$  A.

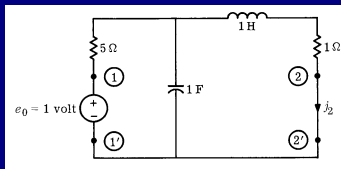


Current  $\hat{j}_1 =$

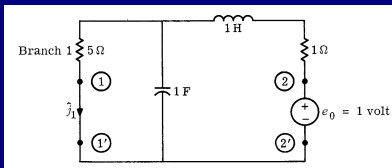


# Examples (6)

Reciprocity under DC conditions:



Current  $j_2 = 1/6\text{ A}$ .

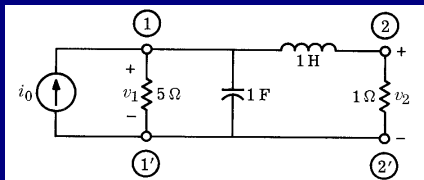


Current  $\hat{j}_1 = 1/6\text{ A}$ .



# Examples (7)

Reciprocity under sinusoidal steady-state conditions:

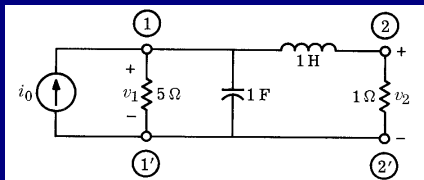


$$i_0(t) = 2 \cos(2t + \pi/6)$$



# Examples (7)

Reciprocity under sinusoidal steady-state conditions:



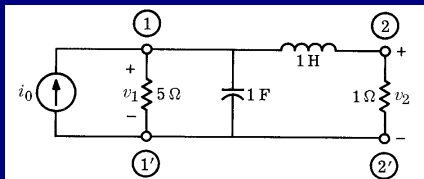
$$i_0(t) = 2 \cos(2t + \pi/6)$$

$$v_2(t) =$$



# Examples (7)

Reciprocity under sinusoidal steady-state conditions:



$$i_0(t) = 2 \cos(2t + \pi/6)$$

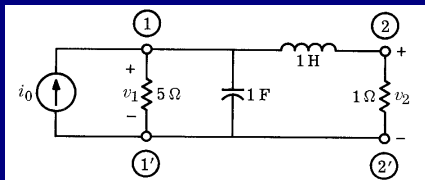
$$v_2(t) = 0.542 e^{j250} =$$

$$0.542 \cos(2t + 250^\circ) \text{ V.}$$



## Examples (7)

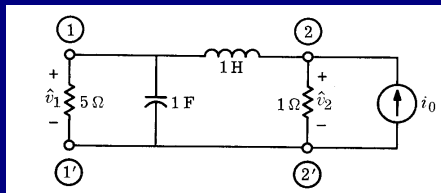
Reciprocity under sinusoidal steady-state conditions:



$$i_0(t) = 2 \cos(2t + \pi/6)$$

$$v_2(t) = 0.542 e^{j250} =$$

$$0.542 \cos(2t + 250^\circ) \text{ V.}$$

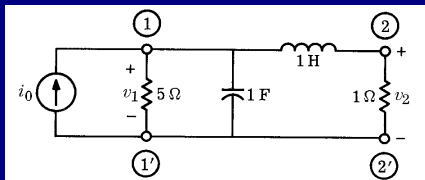


$$\hat{v}_1 =$$



## Examples (7)

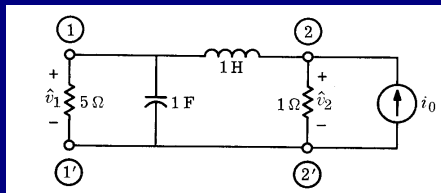
Reciprocity under sinusoidal steady-state conditions:



$$i_0(t) = 2 \cos(2t + \pi/6)$$

$$v_2(t) = 0.542 e^{j250} =$$

$$0.542 \cos(2t + 250^\circ) \text{ V.}$$



$$\hat{v}_1 = 0.542 e^{j250} =$$

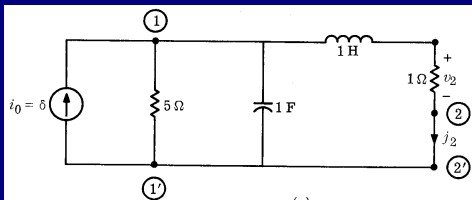
$$0.542 \cos(2t + 250^\circ) \text{ V.}$$





# Examples (7)

Reciprocity under transient conditions:

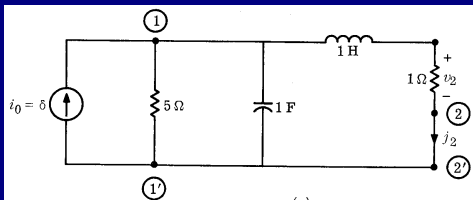


$$j_2 =$$



# Examples (7)

Reciprocity under transient conditions:

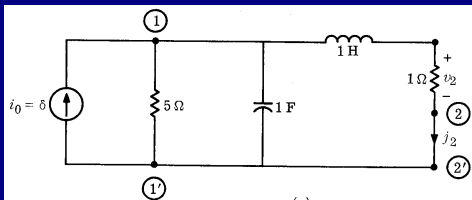


$$j_2 = 1.09e^{-0.6t} \sin 0.916t 1(t) \text{ A.}$$

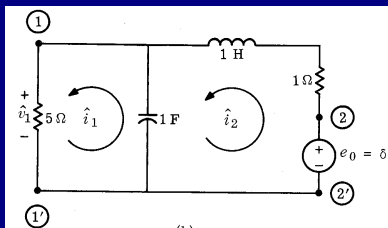


## Examples (7)

Reciprocity under transient conditions:



$$j_2 = 1.09e^{-0.6t} \sin 0.916t \mathbf{1}(t) \text{ A.}$$

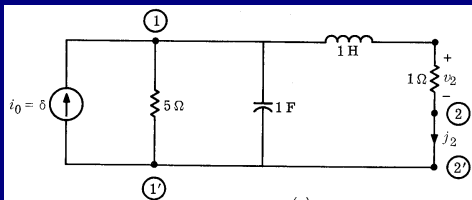


$$\hat{j}_1 =$$

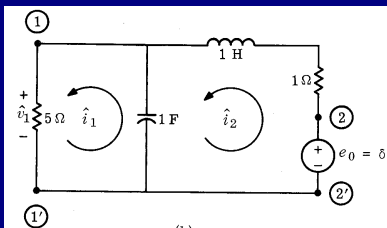


## Examples (7)

Reciprocity under transient conditions:



$$j_2 = 1.09e^{-0.6t} \sin 0.916t \mathbf{1}(t) \text{ A.}$$

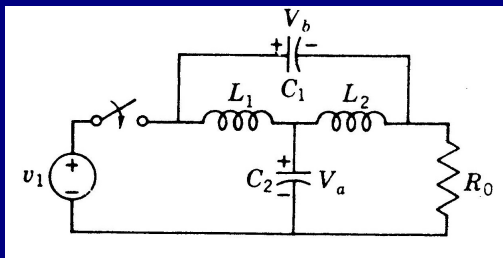


$$\hat{j}_1 = 1.09e^{-0.6t} \sin 0.916t \mathbf{1}(t) \text{ A.}$$



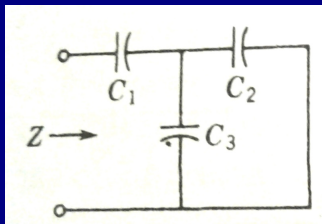
# Examples (8)

Prob. 9-6, Van Valkenburg:



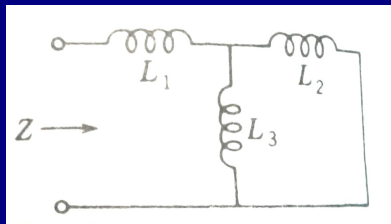
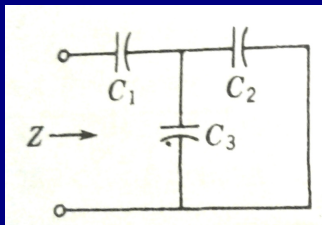
## Examples (9)

Prob. 9-7, Van Valkenburg:



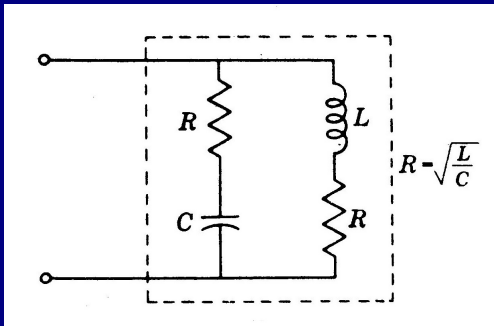
# Examples (9)

Prob. 9-7, Van Valkenburg:



# Examples (10)

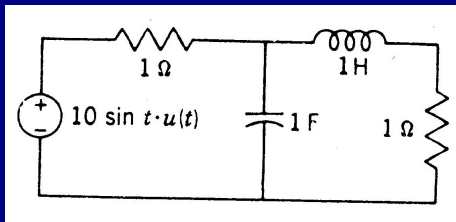
Prob. 9-12, Van Valkenburg:





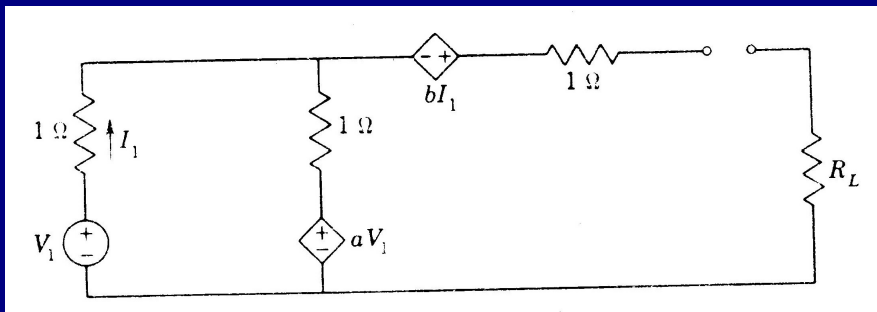
# Examples (11)

Prob. 9-15, Van Valkenburg:



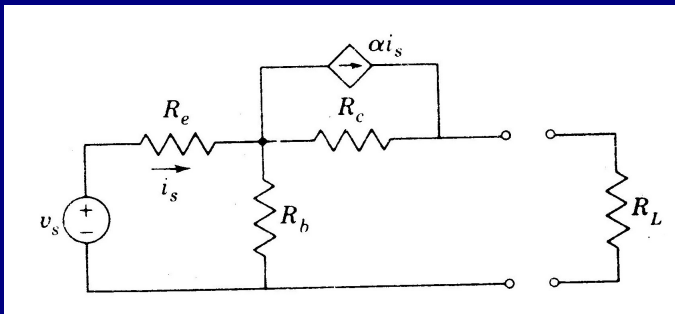
## Examples (12)

Prob. 9-17, Van Valkenburg:



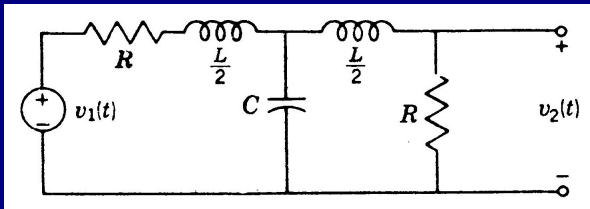
# Examples (13)

Prob. 9-20, Van Valkenburg:

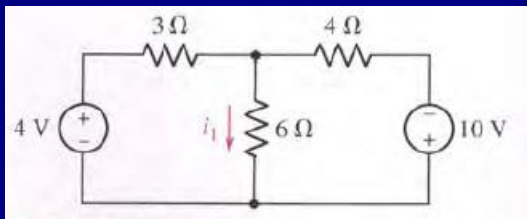


# Examples (14)

Prob. 9-28, Van Valkenburg:



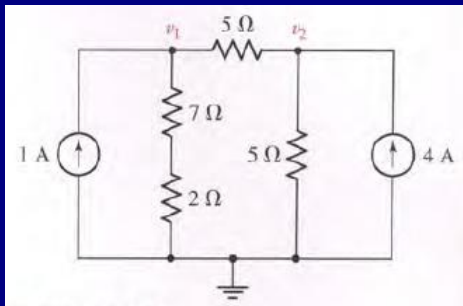
## Examples (15)



- Find the contribution of the 4 V source to the current labelled  $i_1$ .
- Find the contribution of the 10 V source to the current labelled  $i_1$ .
- Determine  $i_1$ .



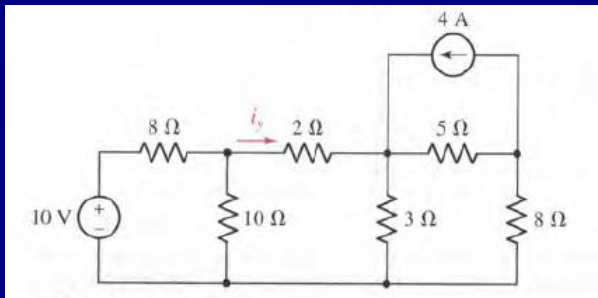
## Examples (16)



- Determine the contribution of the 1 A source to  $v_1$ .
- Calculate the total current flowing through the 7 Ω resistor.



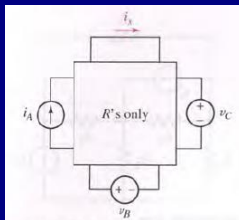
## Examples (17)



Use the principle of superposition to determine the current labelled  $i_y$ .



## Examples (18)

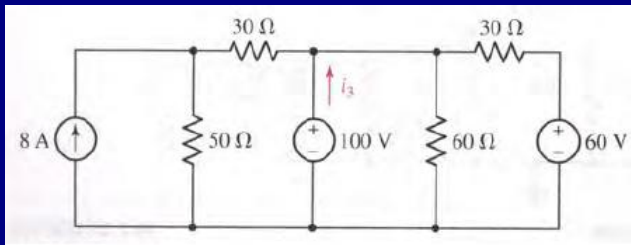


- With sources  $i_A$  and  $v_B$  on and  $v_C = 0$ ,  $i_x = 20$  A.
- With sources  $i_A$  and  $v_C$  on and  $v_B = 0$ ,  $i_x = -5$  A.
- With all three sources on,  $i_x = 12$  A.
- Find  $i_x$  if the only source operating is (a)  $i_A$ , (b)  $v_B$ , (c)  $v_C$ .
- Find  $i_x$  if  $i_A$  and  $v_C$  are doubled in magnitude and  $v_B$  is reversed.





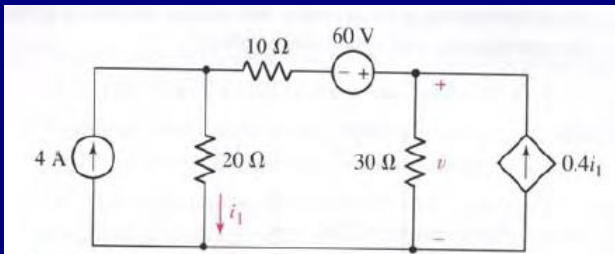
# Examples (19)



Use the principle of superposition to determine the current labelled  $i_3$ .



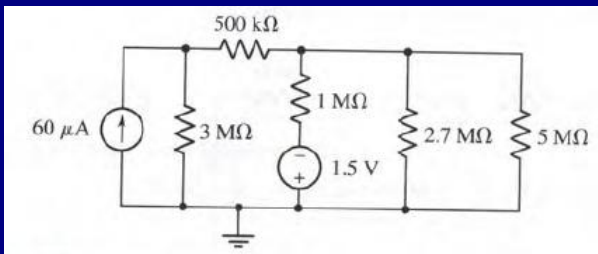
## Examples (20)



Use the principle of superposition to determine the voltage labelled  $v$ .



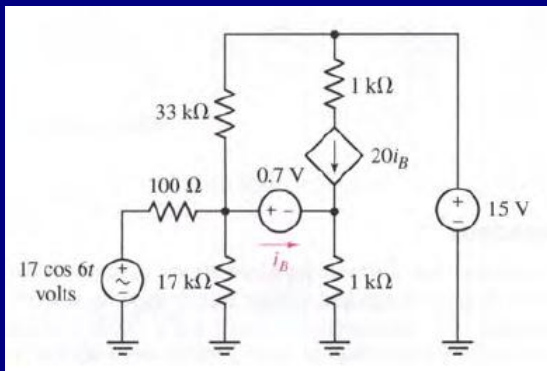
## Examples (21)



Use the principle of superposition to find the power dissipated by the  $500\ \text{k}\Omega$  resistor.



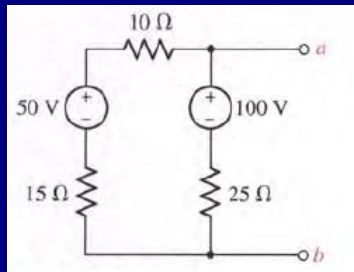
## Examples (22)



Use the principle of superposition to find  $i_B$ . (This circuit models a bipolar junction transistor amplifier.)



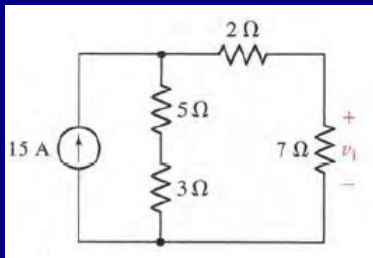
## Examples (23)



Find the Thévenin equivalent at terminals  $a$  and  $b$ . How much power would be delivered to a resistor connected to  $a$  and  $b$  if  $R_{ab}$  equals (a)  $50\ \Omega$ , (b)  $12.5\ \Omega$ .



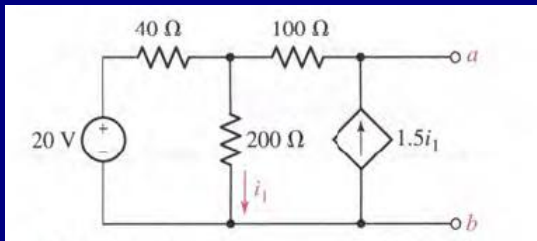
# Examples (24)



- Find the Thévenin equivalent of the network connected to the 7 Ω resistor.
- Find the corresponding Norton equivalent.
- Compute  $v_1$  using both equivalent circuits.



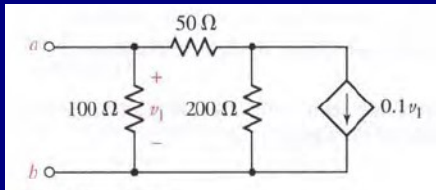
## Examples (25)



- Find the Thévenin equivalent of the network.
- What is the power delivered to a load of  $100\ \Omega$  at  $a$  and  $b$ .



# Examples (26)

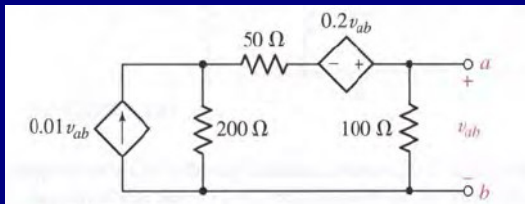


Find the Norton equivalent of the network.





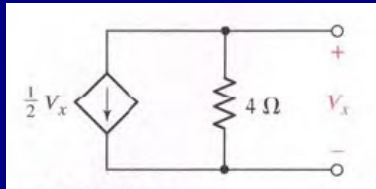
## Examples (27)



Find the Thévenin equivalent of the network.



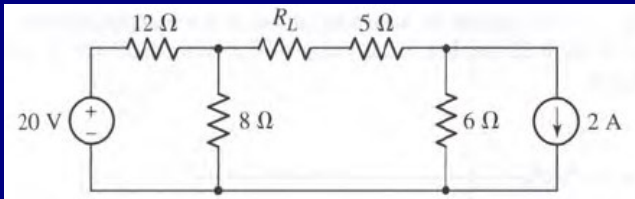
# Examples (28)



Find the Thévenin and Norton equivalents of the network.



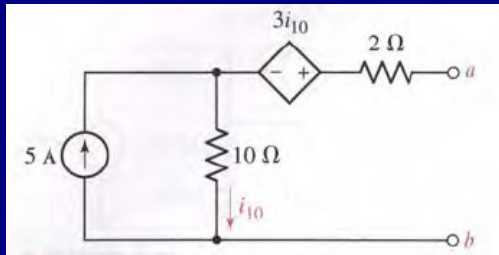
## Examples (29)



What is the maximum power that could be dissipated in  $R_L$ ?



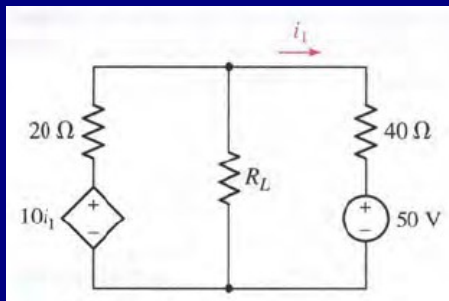
## Examples (30)



Find the Thévenin equivalent of the network. Determine the maximum power that can be drawn from it.



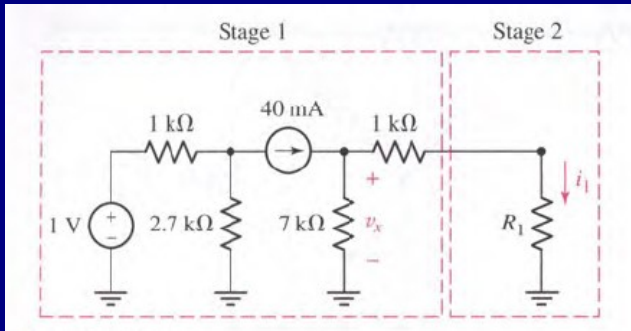
## Examples (31)



Determine that value of  $R_L$  to which a maximum power can be delivered. Calculate the voltage across the corresponding  $R_L$ .  
(The positive reference direction is at the top.)



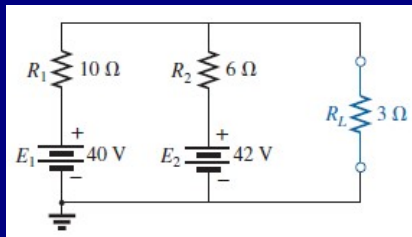
## Examples (32)



Select  $R_1$  so that maximum power is transferred from Stage 1 to Stage 2.



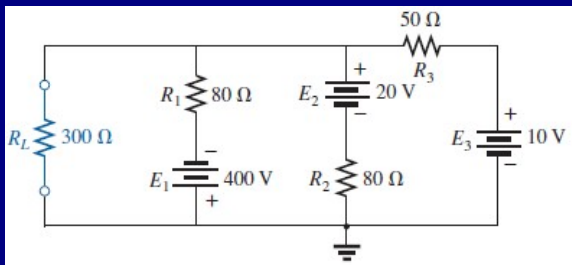
## Examples (33)



Using Millman's theorem find the current through and voltage across the resistor  $R_L$ .



## Examples (34)



Using Millman's theorem find the current through and voltage across the resistor  $R_L$ .

