Network Analysis & Systems

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UE18EC201: Network Analysis & Systems

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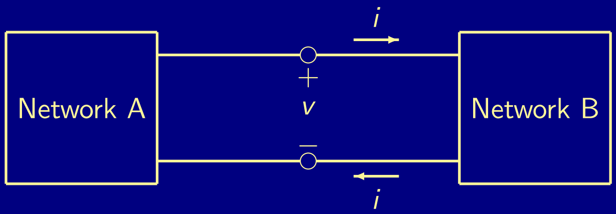
Network Analysis and Synthesis

Unit III: Network Theorems

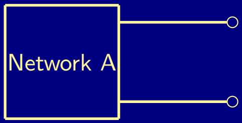
Network Theorems — Thévenin and Norton (1)



- Network B is the focus of interest.
- Network A to be replaced by an equivalent network s.t. the current i and the voltage v remain invariant.

Network Theorems — Thévenin and Norton (2)

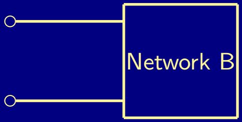
Assumptions on Network A:



- Linear elements.
- Independent or dependent sources.
- Initial conditions on passive elements.
- No magnetic or controlled-source coupling to Network B.

Network Theorems — Thévenin and Norton (3)

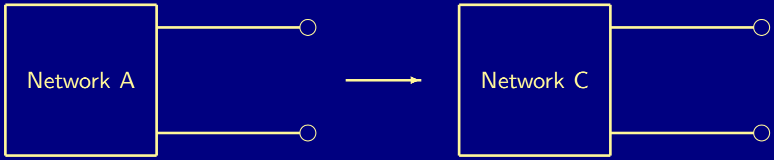
Assumptions on Network B:



- Linear, nonlinear, or time-varying elements.
- Independent or dependent sources.
- Initial conditions on passive elements.
- No magnetic or controlled-source coupling to Network A.

Network Theorems — Thévenin and Norton (4)

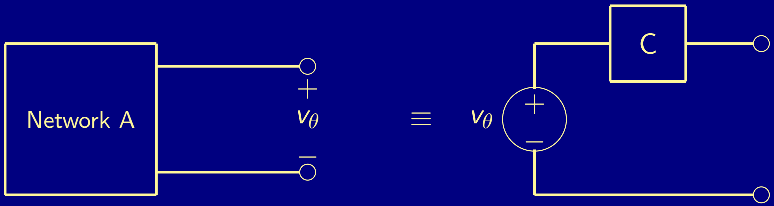
Step A: Derive Network C from Network A



- Set all initial conditions to zero: $v_C = 0$ and $i_L = 0$.
- Turn off all independent sources: $v = 0$ (i.e., s.c.) for voltage sources and $i = 0$ (i.e., o.c.) for current sources.
- Controlled sources continue to operate.
- Determine the driving-point impedance or admittance.

Network Theorems — Thévenin and Norton (5)

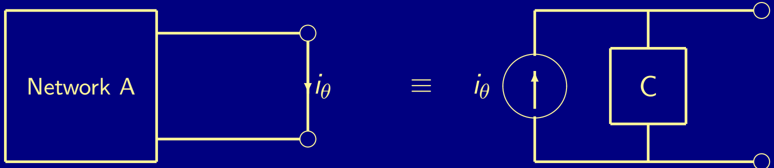
Thévenin Equivalent Network:



- v_θ is the voltage at the open terminals of network A with network B removed.
- v_θ is connected in series with network C.

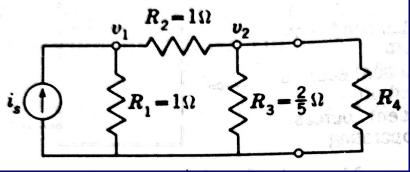
Network Theorems — Thévenin and Norton (6)

Norton Equivalent Network:



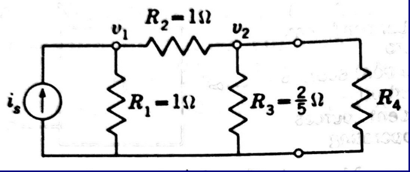
- i_θ is the current in the shorted terminals of network A with network B removed.
- i_θ is connected in parallel with network C.

Network Theorems — Thévenin and Norton (7)



Source: Van Valkenburg, 1975.

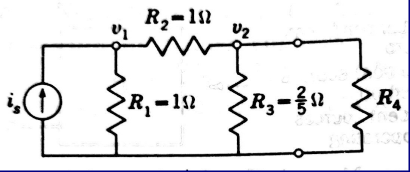
Network Theorems — Thévenin and Norton (7)



Source: Van Valkenburg, 1975.

- Find $v_\theta = v_2$: Apply nodal analysis to obtain

Network Theorems — Thévenin and Norton (7)



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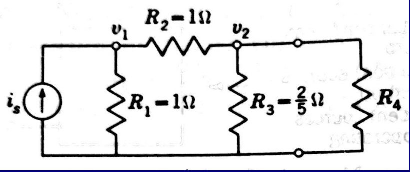
- Find $v_\theta = v_2$: Apply nodal analysis to obtain $v_2 = i_s/6$.
- Find R_θ after open circuiting the current source:

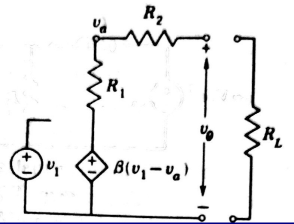
Network Theorems — Thévenin and Norton (7)



Source: Van Valkenburg, 1975.

- Find $v_\theta = v_2$: Apply nodal analysis to obtain $v_2 = i_s/6$.
- Find R_θ after open circuiting the current source: $R_\theta = 1/3$.

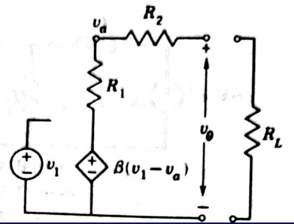
Network Theorems — Thévenin and Norton (8)



- Find v_θ:

Source: Van Valkenburg, 1975.

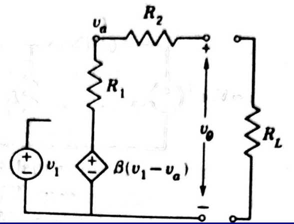
Network Theorems — Thévenin and Norton (8)



- Find v_θ: Clearly, $v_\theta = v_a = \beta(v_1 - v_a)$. Therefore,

Source: Van Valkenburg, 1975.

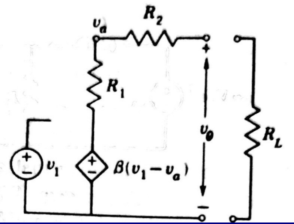
Network Theorems — Thévenin and Norton (8)



Source: Van Valkenburg, 1975.

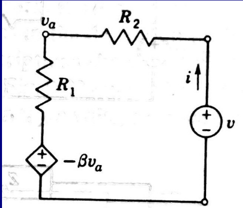
- Find v_θ: Clearly, $v_\theta = v_a = \beta(v_1 - v_a)$. Therefore, $v_\theta = \frac{\beta}{1+\beta} v_1$.
- Find R_θ:

Network Theorems — Thévenin and Norton (8)



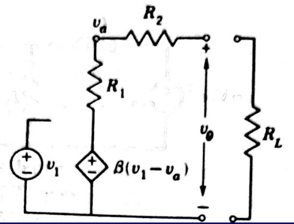
- Find v_θ: Clearly, $v_\theta = v_a = \beta(v_1 - v_a)$. Therefore, $v_\theta = \frac{\beta}{1+\beta} v_1$.
- Find R_θ: Set $v_1 = 0$ and connect a voltage source v to the output terminals.

Source: Van Valkenburg, 1975.



- For the loop,

Network Theorems — Thévenin and Norton (8)



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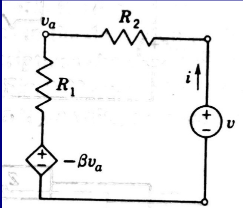
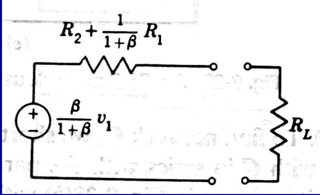
- Find v_θ: Clearly, $v_\theta = v_a = \beta(v_1 - v_a)$. Therefore, $v_\theta = \frac{\beta}{1+\beta} v_1$.
- Find R_θ: Set $v_1 = 0$ and connect a voltage source v to the output terminals.



- For the loop, $v = iR_2 + v_a = iR_2 + iR_1 - \beta v_a$.
- Therefore,

$$v = i\left(R_2 + R_1\left(1 - \frac{\beta}{1+\beta}\right)\right)$$

Network Theorems — Thévenin and Norton (9)



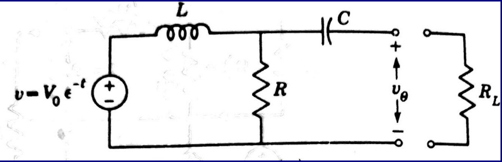
Thus,

$$v_\theta = \frac{\beta}{1+\beta} v_1$$

and

$$R_\theta = R_2 + R_1 \left(1 - \frac{\beta}{1+\beta}\right) = R_2 + R_1 \frac{1}{1+\beta}$$

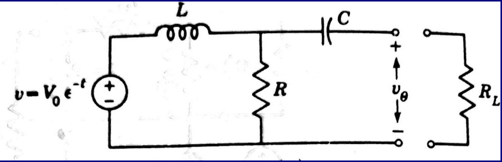
Network Theorems — Thévenin and Norton (10)



Source: Van Valkenburg, 1975.

- Find $V_\theta(s)$:

Network Theorems — Thévenin and Norton (10)



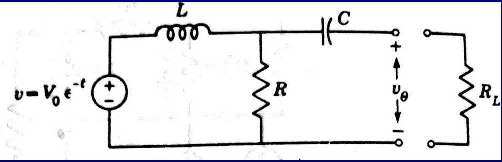
Source: Van Valkenburg, 1975.

- Find V_θ(s): Clearly,

$$V_\theta(s) = \frac{R}{Ls + R} V(s) = \frac{V_0 R}{(Ls + R)(s + 1)}$$

- Find Z_θ:

Network Theorems — Thévenin and Norton (10)



Source: Van Valkenburg, 1975.

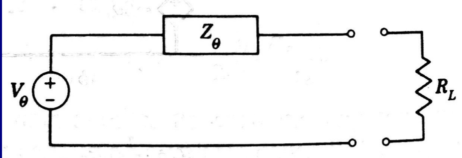
- Find $V_\theta(s)$: Clearly,

$$V_\theta(s) = \frac{R}{Ls+R}V(s) = \frac{V_0 R}{(Ls+R)(s+1)}$$

- Find Z_θ:

$$Z_\theta(s) = \frac{1}{Cs} + \frac{1}{\frac{1}{R}+\frac{1}{Ls}} = \frac{R(s^2+(1/RC)s+(1/LC))}{s(s+(R/L))}$$

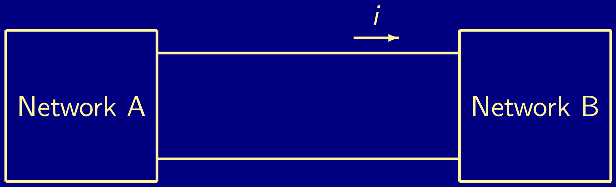
Network Theorems — Thévenin and Norton (11)



$$V_\theta(s) = \frac{V_0 R}{(Ls+R)(s+1)}$$

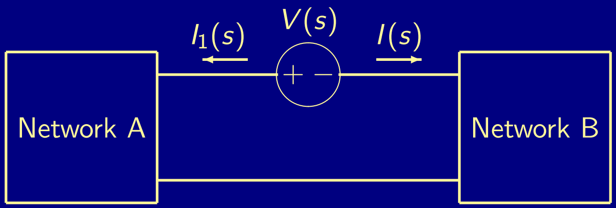
$$Z_\theta(s) = \frac{R(s^2 + (1/RC)s + (1/LC))}{s(s+(R/L))}$$

Network Theorems — Thévenin and Norton (12)



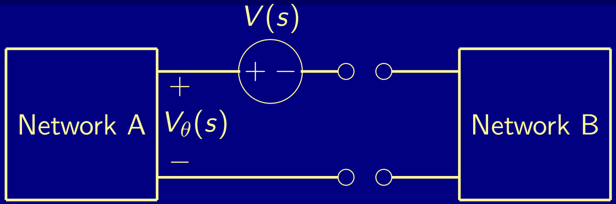
- The current $i(t)$ (or, $I(s)$) flows into Network B.

Network Theorems — Thévenin and Norton (13)



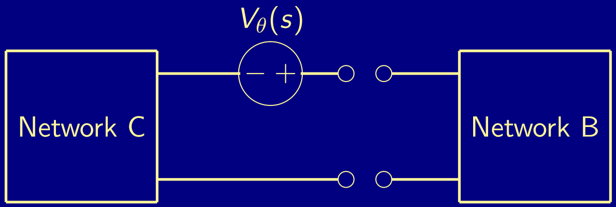
- Include a voltage source $V(s)$ so that the resulting current $I_1(s)$ is such that $I_1(s) = -I(s)$.
- Therefore, the net current flowing into network B is zero.

Network Theorems — Thévenin and Norton (13)



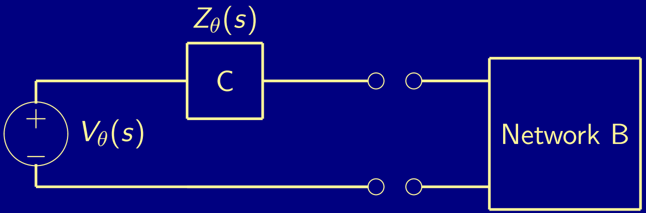
- Since the net current flowing into network B is zero, the two networks can be broken without affecting the conditions in network A.
- The voltage at the broken terminals is clearly zero.
- Also, the network B can be shorted.
- By KVL, $V_\theta(s) = V(s)$.
- That is, the open circuit voltage at the terminals is $V(s)$.

Network Theorems — Thévenin and Norton (14)



- All independent sources are reduced to zero.
- Dependent sources are not changed.
- Thus network A is transformed into network C.
- The polarity of the source $V_\theta(s)$ is reversed so that when connected, the current $I(s)$ will flow into network B.

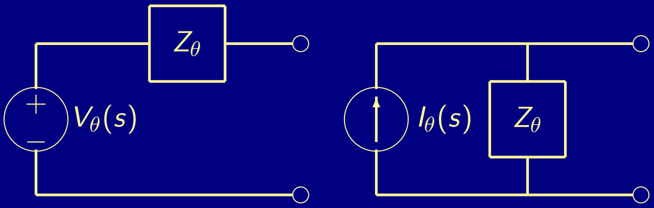
Network Theorems — Thévenin and Norton (15)



- Impedance of C is $Z_\theta(s)$; the voltage source is $V_\theta(s)$.
- If A has only independent sources, then $Z_\theta(s)$ is the impedance of the passive network; otherwise, $Z_\theta(s)$ must be determined for the active network.
- This is the Thévenin equivalent network.
- If $Z_B(s)$ is the impedance of B, then the current is

$$I(s) = \frac{V_\theta(s)}{Z_\theta(s) + Z_B(s)}$$

Network Theorems — Thévenin and Norton (16)



- Perform source transformation to obtain the current source

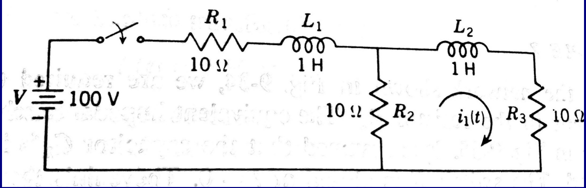
$$I_\theta(s) = \frac{V_\theta(s)}{Z_\theta(s)}$$

- This is the Norton equivalent network.
- The voltage of network B is

$$V_B(s) = \frac{I_\theta(s)}{Y_\theta(s) + Y_B(s)}$$

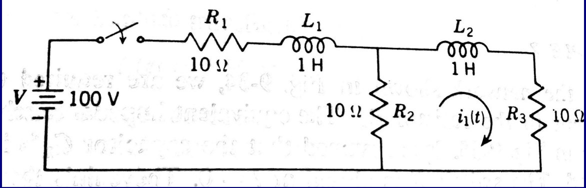
Examples (1)

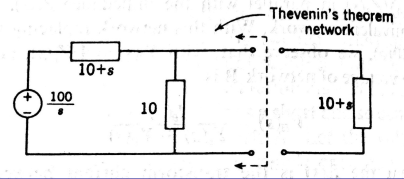
Example 6, Van Valkenburg, pp. 267–268:



Examples (1)

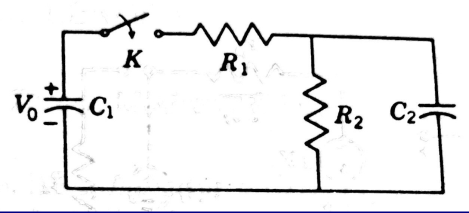
Example 6, Van Valkenburg, pp. 267–268:





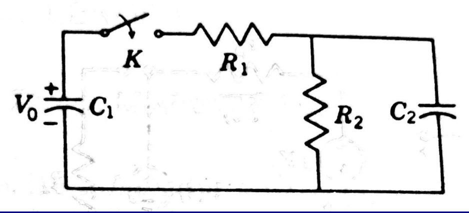
Examples (2)

Example 7, Van Valkenburg, pp. 268–269:



Examples (2)

Example 7, Van Valkenburg, pp. 268–269:



Examples (3)

Example 8, Van Valkenburg, p. 270: