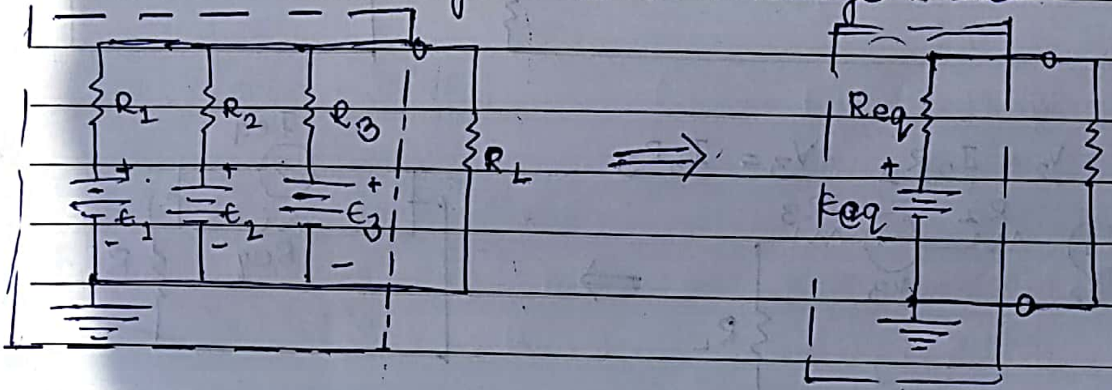
DATE | | NETWORK THEOREM 30th Sep' 19

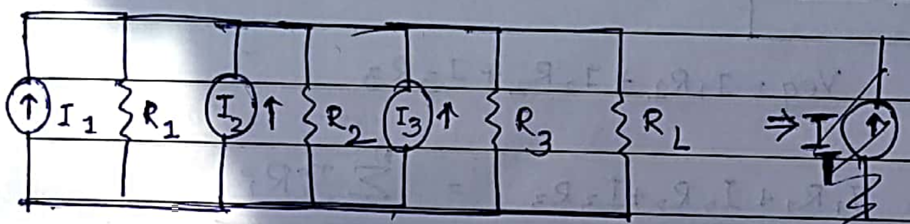
MILLMAN'S THEOREM : allow to reduce any no. of parallel voltage sources to one

STEPS :

* Convert all voltage source to current source
* Combine all these current source
* Convert the resulting source to a voltage source



$I_T = I_1 + I_2 + I_3$



$I_1 = \frac{E_1}{R_1} = E_1 G_1$

$I_2 = E_2 G_2$

$I_3 = E_3 G_3$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$

DC

$\boxed{G_{eq} = G_1 + G_2 + G_3}$

$E_{eq} = \frac{E_1G_1 + E_2G_2 + E_3G_3}{G_1 + G_2 + G_3}$

$\boxed{= \frac{\sum_{i=1}^{n=3} E_iG_i}{\sum_{i=1}^{n=3} G_i}}$

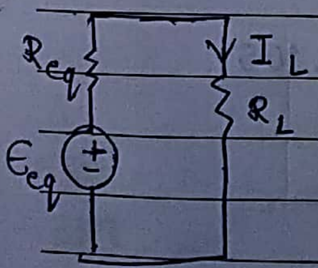
$E_{eq} = I_T \cdot R_{eq}$

$= \frac{I_T}{G_{eq}} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3}$

$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + G_3}$

AC ⇒ Admittance → Y

$\boxed{= \frac{1}{\sum_{i=1}^{n=3} G_i}}$



$\boxed{I_L = E_{eq} / R_{eq} + R_L}$

$\boxed{V_L = I_L R_L}$

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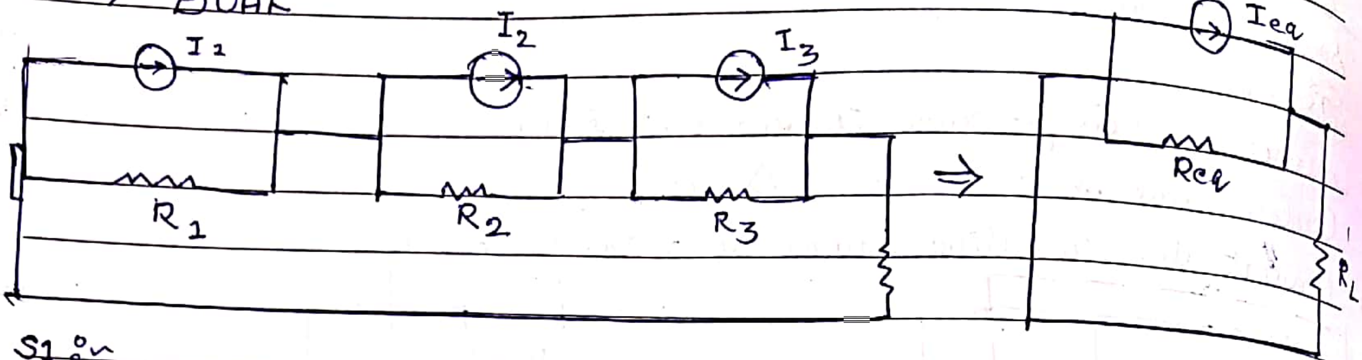
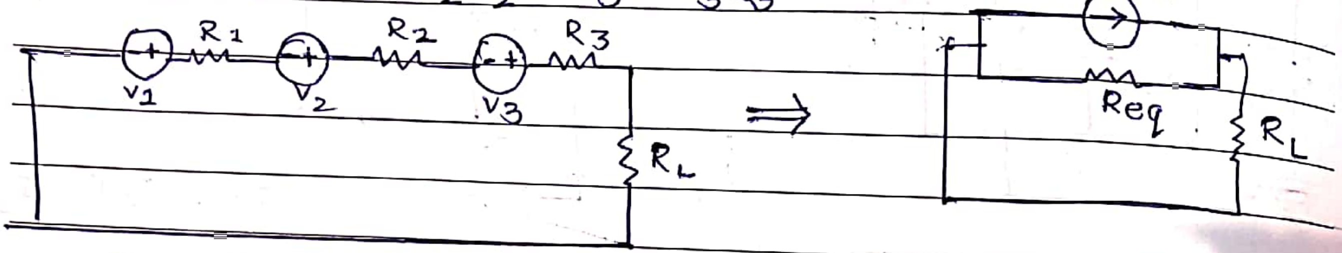
MILLMAN'S THEOREM

↳ DUAL



Sl^n:

$V_1 = I_1R_1$ $\quad V_2 = I_2R_2$ $\quad V_3 = I_3R_3$

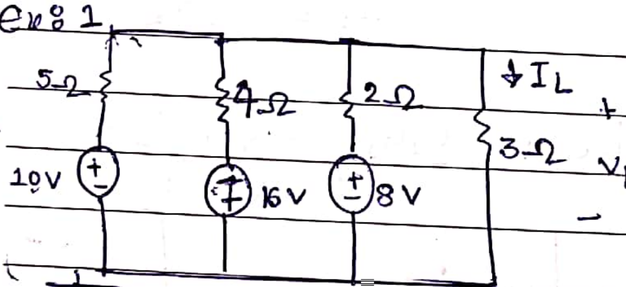


$V_{eq} = V_1 + V_2 + V_3$ $\qquad V_{eq} = I_1R_1 + I_2R_2 + I_3R_3$

$R_{eq} = R_1 + R_2 + R_3$

$$I_{eq} = \frac{V_{eq}}{R_{eq}} = \frac{I_1R_1 + I_2R_2 + I_3R_3}{R_1 + R_2 + R_3} = \frac{\sum_{i=1}^{n} I_iR_i}{\sum_{i=1}^{n} R_i}$$

Ex 1



$$E_{eq} = \frac{10(G_1) \mp 16(G_2) + 8(G_3)}{G_1 + G_2 + G_3}$$

$$= \cancel{\frac{50 \pm -64 + 16}{11}}$$

$$= \cancel{0.18}$$

$$= 10\left(\frac{1}{5}\right) - 16\left(\frac{1}{4}\right) + 8\left(\frac{1}{2}\right)$$

$$0.95$$

$$= \frac{\cancel{10 - 4 + 4}}{0.95} \quad 2 - 4 + 4$$

$$= 2.105$$

$$R_{eq} = \frac{1}{5} + \frac{1}{4} + \frac{1}{2}$$

$G_{eq} = 5 + 4 + 2$

$= 11$

$$R_{eq} = \frac{20}{19}$$

$$I_L = \frac{2.105}{\frac{20}{19} + 3} = \frac{2.105}{47/19} = 0.519$$

$V_L = 1.559\,V$

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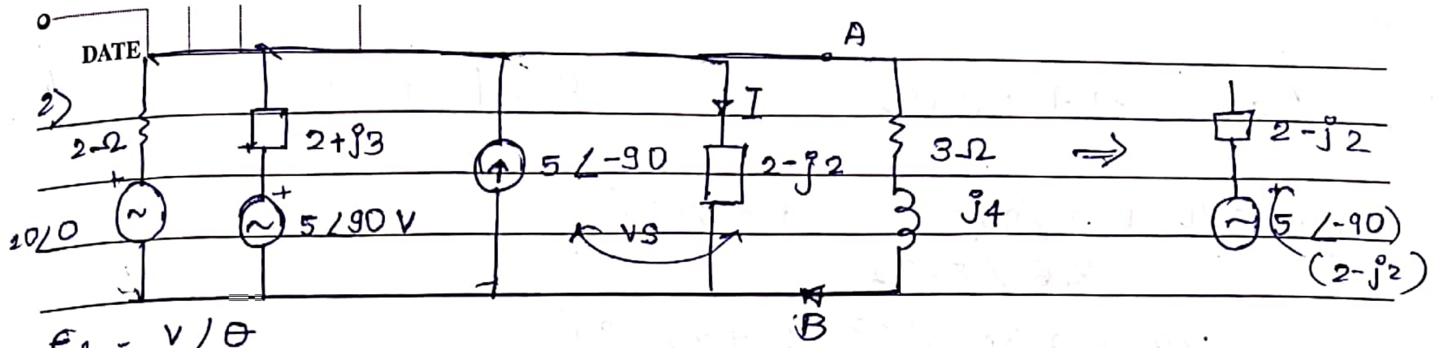
2)



$\epsilon_1 = V\angle\theta$

$= V[\cos\theta + j\sin\theta]$

$\epsilon_1 = V\angle-\theta$

$= V[\cos\theta - j\sin\theta]$

$$\epsilon_{eq} = \frac{\left(\frac{1}{2}\right)(10) + (0+j5)(2+j3) + (0-5j)(2-j2)}{\frac{1}{2} + \frac{1}{2+j3} + \frac{1}{2-j2}}$$

$\epsilon_{eq} = \epsilon_1 Y_1 + \epsilon_2 Y_2 + \epsilon_3 Y_3$

$= 5 + 10j + 15j^2$

$$R_{eq} = \frac{1}{Y_1 + Y_2 + Y_3}$$

$$= \frac{\frac{10\angle0}{2} + \frac{5\angle90^\circ}{2+j3} + \frac{5\angle-90^\circ(2-j2)}{2-j2}}{\frac{1}{2} + \frac{1}{2+j3} + \frac{1}{2-j2}}$$

$$\Rightarrow 5\angle0 + \frac{5\angle90}{3.60\angle56.3} + 5\angle-90^\circ$$

$$\Rightarrow 5\angle0 + 1.388\angle33.7 + 5\angle-90^\circ$$

$$Y_2 = \frac{1}{2+j3} \times \frac{2-j3}{2-j3} = \frac{2-j3}{(2)^2-(j3)^2} = \frac{2-j3}{4+9} = \frac{2-j3}{13}$$

$$Y_3 = \frac{1}{2-j2} \times \frac{2+j2}{2+j2} = \frac{2+j2}{(2)^2-(j2)^2} = \frac{2+j2}{4+4} = \frac{2+j2}{8}$$

$$\Rightarrow \frac{5 + 1.154 + 0.77j + (-j5)}{0.5 + \frac{2-j3}{13} + \frac{2+j2}{8}} = \frac{6.154 - 4.223j}{52 + 16 - 48j + 26 + 26j}$$

$$= \frac{640.016 - 439.192j}{94 - 22j} = \frac{776.215\angle-34.458}{96.54\angle-13.172}$$

$$Y_{eq} = \frac{1}{2} + \frac{1}{2+j3} + \frac{1}{2-j2}$$

$= 8.04\angle-21.286$

$= 7.5 - 2.91j$

$$= 0.5 + \frac{2-j3}{13} + \frac{2+j2}{8}$$

$= 0.5 + 0.153 - 0.23j + 0.25 + 0.125j = 0.903 - 0.15005j$

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$Y_{eq} = 0.904 + j0.02 \qquad 0.904 \angle 1.267$

$E_{eq} = \dfrac{5\angle 0 + \dfrac{5\angle 90^\circ}{3.6\angle 56.31} + 5\angle -90^\circ}{}$

$= \dfrac{5\angle 0 + 1.38\angle 33.69 + 5\angle -90^\circ}{0.904 + j0.02}$

$= \dfrac{5 + 1.148 + 0.765j + (-5j)}{0.904 + j0.02}$

$= \dfrac{6.148 - 4.235j}{0.904 + j0.02} = \dfrac{7.465\angle -34.56}{0.904\angle 1.267}$

$= 8.257\angle -35.827$

$= 6.69 - j4.8$

I_L ≠ E_{eq}

$Z_{eq} = 1.106 - j0.0236 \qquad 1.68\angle -48.8$

$I_L = \dfrac{E_{eq}}{Z_{eq} + R_L} = \dfrac{6.7 - j4.8}{1.106 - j0.0236 + 3 + j4}$

$= \dfrac{6.7 - j4.8}{4.106 + 3.976j}$

$= \dfrac{8.242\angle -35.618}{5.715\angle 44.078} = 1.44\angle -79.696$

$= 0.25 - 1.41j$ //

$V_L = I_L R_L = (0.25 - 1.41j)(3 + j4)$

$= (1.432\angle -79.945)(5\angle 53.13)$

$= 7.16\angle -26.815$

COMPLEX FREQUENCY.

DATE

The soln to DE had terms of the form

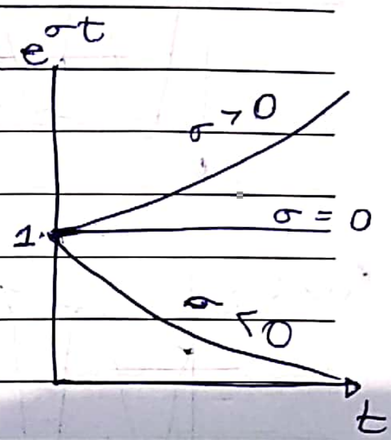
$$r(t) = k_n e^{s_n t}$$

- Complex frequency : $s_n = \sigma_n + j\omega_n$
- radian / angular frequency : ω_n rad/sec

$$\omega_n = 2\pi f_n = \frac{2\pi}{T_n}$$

- NEPER frequency : σ_n neper per sec

The unit for the natural logarithm is the neper



Case 1 : $s_n = \sigma_n + j0$ (neper frequency only)

$\sigma > 0$: Increases exponentially

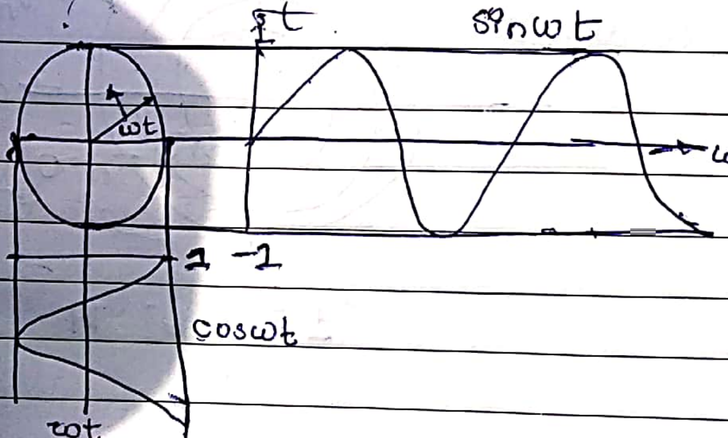
$\sigma < 0$: Decreases or decay exponentially

$\sigma = 0$: Direct current / voltage

Case 2 : $s_n = 0 + j\omega_n$ (radian frequency only)

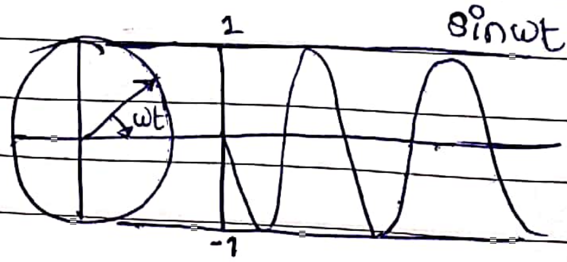
$$k_n e^{\pm j\omega_n t} = k_n (\cos\omega_n t \pm j\sin\omega_n t)$$

- ⋆ $e^{\pm j\omega_n t}$ is known as unit rotating phasor
- ⋆ phasor has magnitude & phase wrt a reference
- ⋆ The unit phasor $e^{j\omega_n t}$ rotates counterclockwise considered (+ve)
- ⋆ $e^{-j\omega_n t}$ → rotate in the clock wise direcⁿ (-ve)



$e^{j\omega_n t}$

$e^{-j\omega_n t}$



$\cos\omega t$ will be same as $e^{j\omega_n t}$

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case 3: $s_n = \sigma_n + j\omega_n$

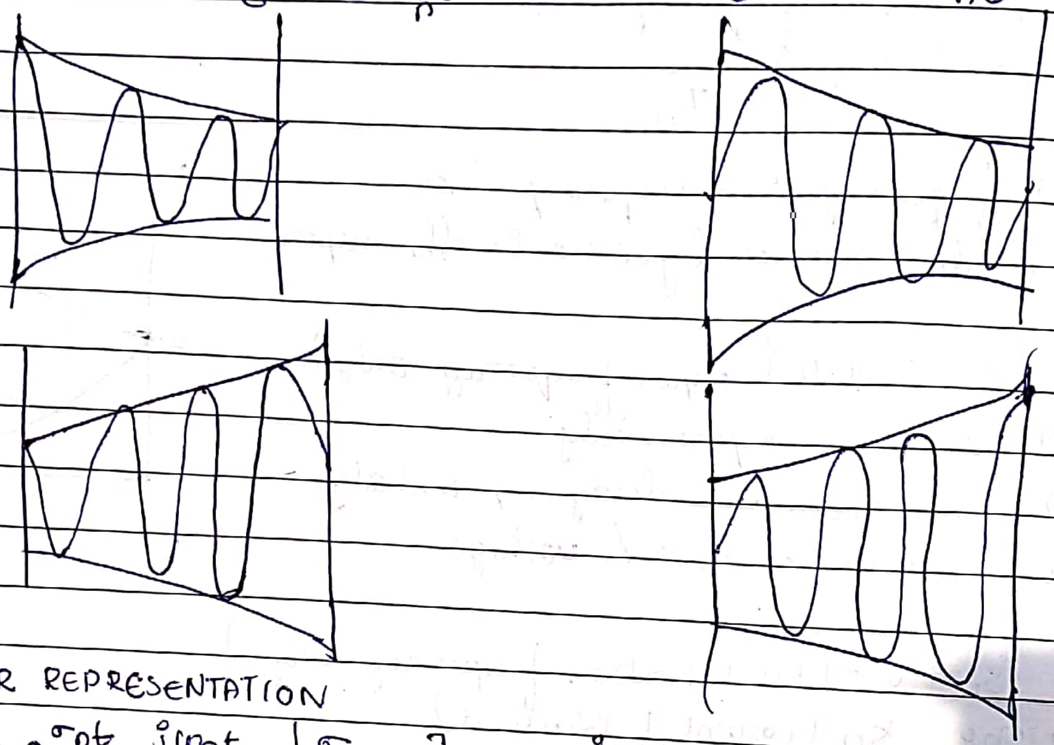
$$k_n e^{s_n t} = k_n e^{\sigma_n t} e^{\pm j\omega_n t} = K_n e^{\sigma_n t}\left(\cos\omega_n t \pm j\sin\omega_n t\right)$$

for +ve rotation, for $k_n = 1$

$$\text{Re } e^{s_n t} = e^{\sigma_n t}\cos\omega_n t \qquad e^{s_n t} = e^{\sigma_n t}\sin\omega_n t$$

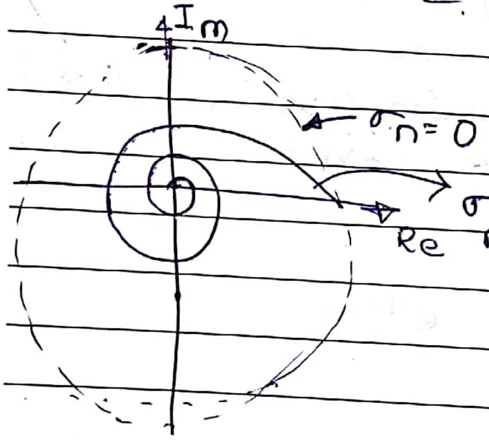
$\sigma_n < 0$:

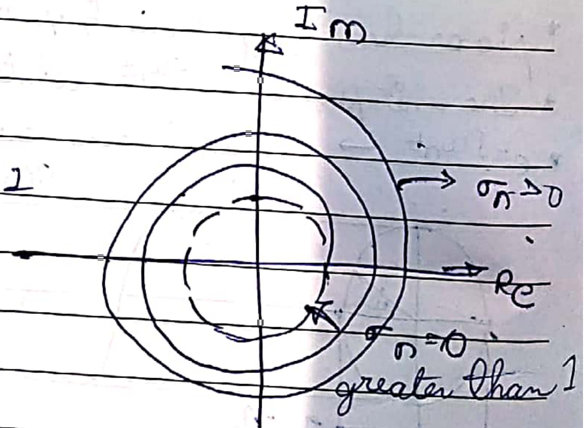
$\sigma_n > 0$:



PHASOR REPRESENTATION

$$y(t) = e^{\sigma_n t} e^{j\omega_n t} \quad \left[\sigma_n = 0\right] = e^{j\omega_n t}$$





1st Oct' 19

NETWORK FUNCTIONS :-

$$\frac{dy}{dt} + \frac{1}{T} y(t) = \epsilon$$

$$sY(s) - y(0^-) + \frac{1}{T} Y(s) = \epsilon(s)$$

$$Y(s)\left[s + \frac{1}{T}\right] - y(0^-) = \epsilon(s)$$

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$$Y(s) = \frac{E(s) + Y(0^-)}{s + 1/T}$$

$$Y(s) = \frac{Y(0^-)}{s + 1/T} + \frac{E(s)}{s + 1/T}$$

$$Y(t) = Y(0^-)e^{-t/T} + E(s)e^{-t/T}$$

Assume

$$Y(0^-) = 0$$

$$Y(s) = \left[\frac{1}{s + 1/T}\right] E(s)$$

$$Y(s) = F(s)\,E(s)$$

$$F(s) = \frac{Y(s)}{E(s)}$$

↓

Network Function — ratio of 2 Laplace Transform

||^{ly} for 2nd order ckt, under zero initial cond^{ns}

$$Y(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} E(s) = F(s)\,E(s)$$

$$E(s) \longrightarrow \boxed{F(s)} \longrightarrow Y(s)$$

$E(s) \to$ Excitation

$Y(s) \to$ response

$F(s) \to$ Transform admittance

* If the excitation is a voltage & the response is a current then $F(s)$ is an Admittance

* If the excitation is a current & the response is a voltage then $F(s)$ is Impedance

* Together they are reffered to as Immitance.

Resistance :

$V_R(t) = R\,i_R(t) \to (1)$ or $V_R(s) = R\,I_R(s)$

$i_R(t) = \frac{1}{R} v_R(t) = G V_R(t)$ (2) ; $I_R(s) = \frac{V_R(s)}{R} = G V_R(s)$

$Z_R(s) \triangleq \frac{V_R(s)}{I_R(s)} = R$	$Y_R(s) \triangleq \frac{I_R(s)}{V_R(s)} = G$

* $Z_R(s)$ is called the Transform Impedance & $Y_R(s)$ is Transform Admittance

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Taking LT of (1)

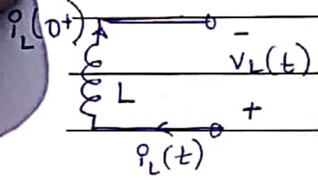
* Resistance won't get change in Laplace domain

$$V_R(s) = R\, I_R(s)$$

$$\frac{V_R(s)}{I_R(s)} = R = Z_R(s)$$

2) INDUCTANCE (L):-

$$v_L(t) = L\frac{di}{dt} \rightarrow (1)$$



Taking LT

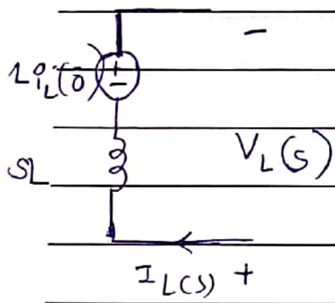
$$V_L(s) = L\left[s I_L(s) - I_L(0^-)\right]$$

$$V_L(s) = L s I_L(s) - L\, I_L(0^-)$$

$$\underbrace{V_L(s) + L I_L(0^-)}_{} = L s I_L(s)$$

$$V_1(s) = s L I_L(s)$$

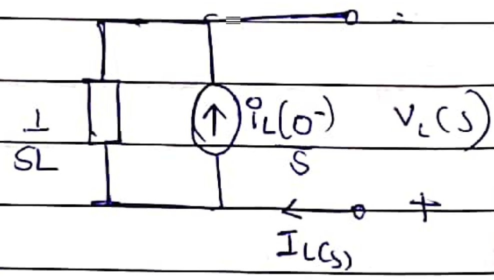
$$\boxed{\frac{V_1(s)}{I_L(s)} = \underbrace{sL}_{Z_L(s)} \rightarrow \text{Transform Impedance}}$$



b) $$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau = \underbrace{\frac{1}{L}\int_{-\infty}^{0} v_L(\tau)\,d\tau}_{i_L(0^-)} + \frac{1}{L}\int_{0}^{t} v_L(\tau)\,d\tau$$

$$i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0}^{t} v_L(\tau)\,d\tau$$

$$I_L(s) = \frac{i_L(0^-)}{s} + \frac{1}{Ls} V_L(s)$$



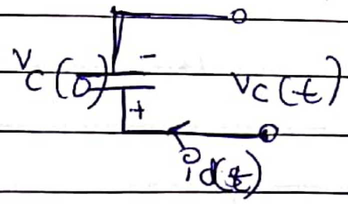
$$\underbrace{I_L(s) - \frac{i_L(0^-)}{s}}_{I_1(s)} = \frac{1}{SL} V_L(s)$$

$$I_1(s) = \frac{1}{SL} V_L(s)$$

$$\boxed{\frac{I_1(s)}{V_1(s)} = \frac{1}{SL} = Y_L(s) \rightarrow \text{Transform admittance}}$$

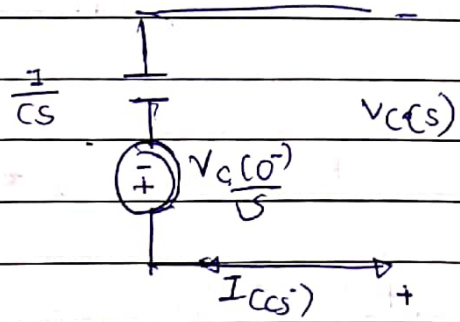
DATE

3) CAPACITANCE :-



$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)\,d\tau \rightarrow (1)$$

$$= \underbrace{\frac{1}{C}\int_{-\infty}^{0} i_c(\tau)\,d\tau}_{V_c(0^-)} + \frac{1}{C}\int_{0}^{t} i_c(\tau)\,dt$$



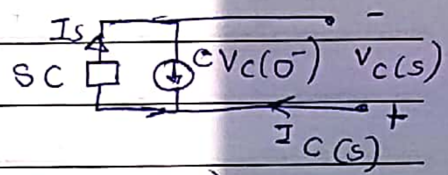
$$V_c(s) = \frac{V_c(0^-)}{S} + \frac{1}{CS} I_c(s) \rightarrow (2)$$

$$\underbrace{V_c(s) - \frac{V_c(0^-)}{S}}_{} = \frac{1}{CS} I_c(s)$$

$$V_1(s) = \frac{1}{CS} I_c(s)$$

$$\boxed{= \frac{V_1(s)}{I_c(s)} = \frac{1}{CS} \quad \text{Tranform Impedance}}$$

b) $i_c(t) = C\frac{dV_c}{dt}$



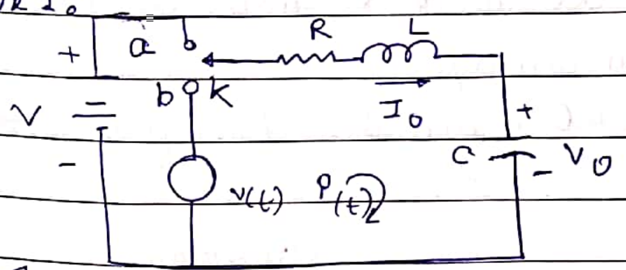
$$I_c(s) = C\left[sV_c(s) - V_c(0^-)\right] = CsV_c(s) - CV_c(0^-)$$

$$I_c(s) = CsV_c(s) - CV_c(0^-)$$

$$\underbrace{I_c(s) + CV_c(0^-)}_{I_1} = CsV_c(s^-)$$

$$\boxed{\frac{I_1}{V_c(s)} = CS \quad \text{Tranform Admittance}}$$

Ex 1:



Initial current is I_0 & voltage is V_0

Find the Transform $I(s)$

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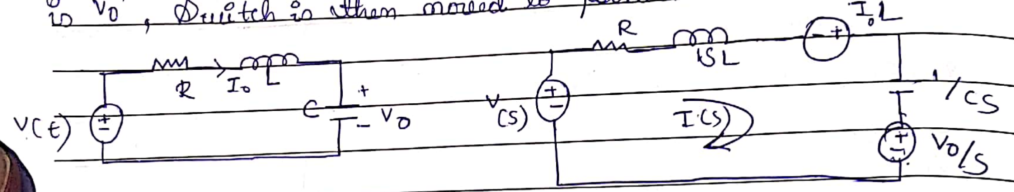
Switch is in 'A' pos until initial current through 'L' is 'I_0' & voltage is 'V_0', Switch is then moved to position 'B'



$$V(s) = RI(s) + sLI(s) - LI_0 + \frac{1}{CS} I(s) + \frac{V_0}{S}$$

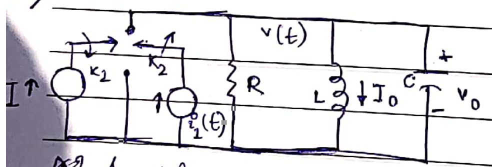
$$\underbrace{V(s) + LI_0 - V_0/s}_{V_1(s)} = I_s \underbrace{\left[R + sL + 1/CS \right]}_{Z(s)}$$

$$I(s) = \frac{V_1(s)}{Z(s)}$$

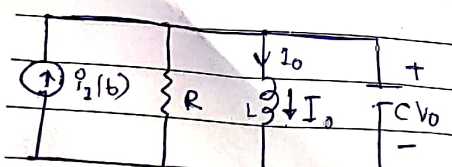
$$I(s) = \frac{V(s) + LI_0 - V_0/s}{R + sL + 1/CS}$$
$$= \frac{SV(s) + sLI_0 - V_0}{LS^2 + RS + 1/C}$$

2)

5th Oct '19



* Switch K_1 gets connected to the ckt at $t = 0^-$ with initial current I_0 in the inductor & initial voltage V_0 across capacitor
* Switch K_2 gets connected to ckt at $t = 0$. find the Transform for the node voltage $v(t)$?



Series . Impedance
parallel - Admittance

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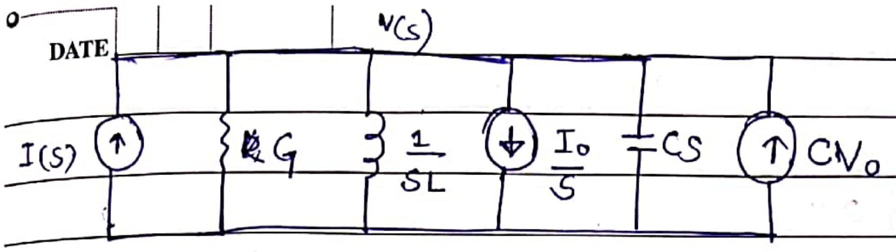


$$V(s) = \frac{I(s)}{Y(s)}$$

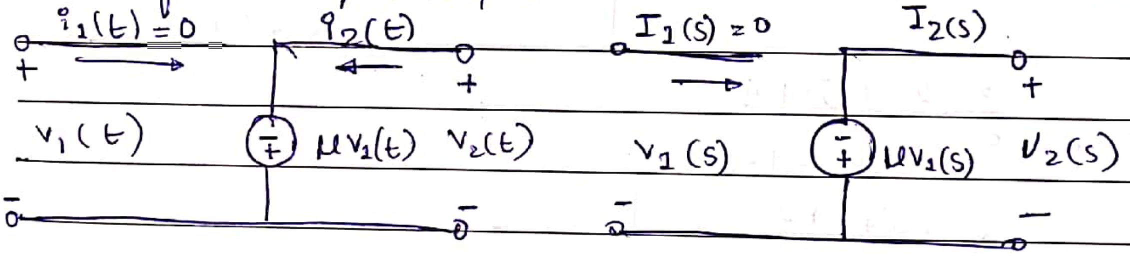
$$I(s) = I_1(s) + CV_0 - \frac{I_0}{s}$$

$$Y(s) = G + \cancel{\frac{I_0}{s}}\ \frac{1}{sL} + \cancel{CV_0}\ CS$$

$$V(s) = \frac{I_1(s) + CV_0 - \frac{I_0}{s}}{G + \frac{1}{sL} + CS}$$

$$\boxed{V(s) = \frac{sI_1(s) + CsV_0 - I_0}{s^2 C + \frac{1}{L} + Gs}}$$

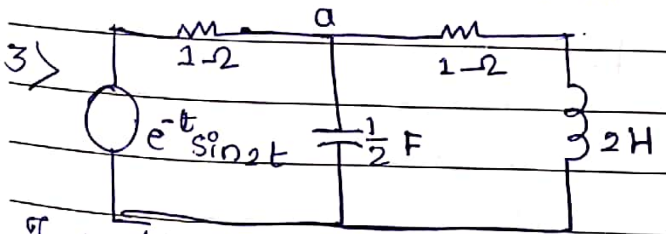
Transform Impedance & Admittance



Network funs are :- an immitance or voltage gain / current gain

Initial condition : No voltage across capacitor, no current across inductor at Zero time is a network function.

3)



Find the current supplied by source in the laplace domain I(S) ?

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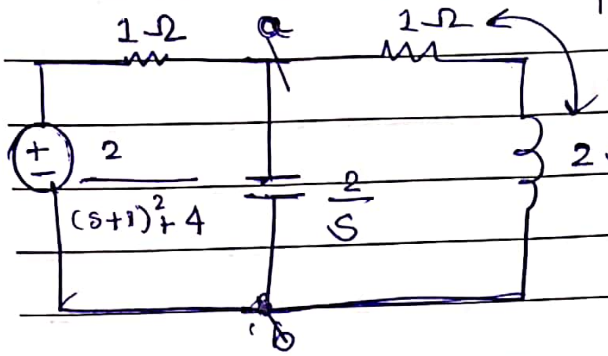
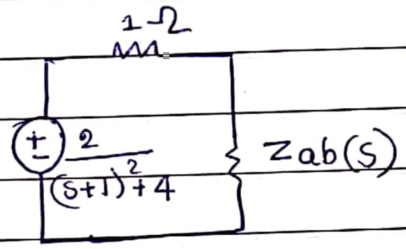


$$I(s) = \frac{V(s)}{Z(s)}$$

$$Y_{ab}(s) = \frac{2}{S} * \frac{S}{2} + \frac{1}{1+2S} = \frac{S+2S^2+2}{2+4S}$$

$$Z_{ab}(s) = \frac{2(2s+1)}{2s^2+s+2}$$



$$Z(s) = 1 + Z_{ab}s$$

$$1 + \frac{2(2s+1)}{2s^2+s+2} = \frac{2s^2+s+2+4s+2}{2s^2+s+2}$$

$$\boxed{Z(s) = \frac{2s^2+5s+4}{2s^2+s+2}}$$

$$I(s) = \frac{2}{(s+1)^2+4} \div \frac{2s^2+5s+4}{2s^2+s+2}$$

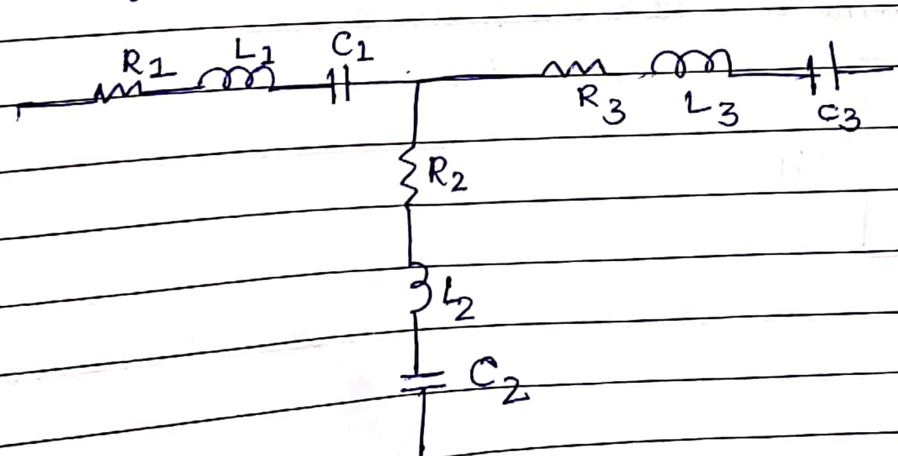
$$\boxed{I(s) = \frac{2}{(s+1)^2+4} \times \frac{2s^2+s+2}{2s^2+5s+4}} \quad +S$$

$s^2+1+2s+4$

s^2+2s+5

Assignment :- Find $I(t)$?

4)



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$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_a(s) \\ -V_b(s) \end{bmatrix}$$

parallel → Admittance

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$z_1 I_1(s) + z_{12} I_2(s) = V_a(s)$$

$$z_2 I_1(s) + z_{22} I_2(s) = -V_b(s)$$

$$z_{11} = \left(R_1 + sL_1 + \frac{1}{cs}\right) \qquad z_{12} = \left(R_2 + sL_2 + \frac{1}{C_2 s}\right)$$

$$z_{12} = z_{21}$$

$$z_{22} = R_3 + sL_3 + \frac{1}{C_3 s}$$

$$\sum_{i=1}^{L} z_k(s) I_j(s) = V_k(s) \quad //$$

In general $(z)(I) = (V) + (V_0) = (V')$ / $(Y)(V) = (I) + (I_0) = (I')$

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V'_1(s) \\ V'_2(s) \\ V'_3(s) \end{bmatrix}$$

To find $I_1(s) = ?$ find $I_{11}(s)$ by shorting $V'_2(s)$ & $V'_3(s) = 0$

$I_{12}(s) \rightarrow V'_1(s)$ & $V'_3(s) = 0$

$I_{13}(s) \rightarrow V'_2(s)$ & $V'_1(s) = 0$

$$\boxed{\text{This theorem is known as Superposition theorem}}$$

↓

Principle of Linearity

$$I'_1(s) = I_{11}(s) + I_{12}(s) + I_{13}(s)$$

$$I = [I_1(s)\ I_2(s) \ldots\ldots I_L(s)]^T$$

* Each current (or voltage) is obtained by determining the individual responses to a voltage (or current) & then summing up the responses

DATE | | | RECIPROCITY THEOREM :- 9th Oct '19

1) No initial conditions

$[z][I] = \cancel{[V_0]} + [V] = [V']$

$[z][I] = [V]$

2) $[z]$ is Symmetric

$z_{12} = z_{21}$

$z_{13} = z_{31}$

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{12} & z_{22} & z_{23} \\ z_{13} & z_{23} & z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

To find I_1.

$I_1 = \frac{\Delta I_1}{\Delta}$

$\Delta \rightarrow$ Determinant of matrix

$\Delta I_1 \rightarrow$ replace 1st column row by the V_1 & V_2 & V_3

$$\Delta I_1 = \begin{vmatrix} V_1 & z_{12} & z_{13} \\ V_2 & z_{22} & z_{23} \\ V_3 & z_{23} & z_{33} \end{vmatrix}$$

$\Delta_{11} = \begin{bmatrix} 22 & 23 \\ 23 & 33 \end{bmatrix}$

$$I_1 = V_1 \frac{\Delta_{11}}{\Delta} + V_2 \frac{\Delta_{12}}{\Delta} + \frac{\Delta_{13}}{\Delta} V_3 \rightarrow (1)$$

To find $I_3 = \frac{\Delta I_3}{\Delta}$ 3rd column shld be replaced with V_1, V_2, V_3

$$\Delta I_3 = \begin{vmatrix} z_{11} & z_{12} & V_1 \\ z_{12} & z_{22} & V_2 \\ z_{13} & z_{23} & V_3 \end{vmatrix}$$

$$I_3 = \frac{V_1 \Delta_{13}}{\Delta} + V_2 \frac{\Delta_{23}}{\Delta} + V_3 \frac{\Delta_{33}}{\Delta} \rightarrow (2)$$

Assuming V_1 & $V_2 = 0$ in 1st equation

$$I_1 = V_3 \frac{\Delta_{13}}{\Delta} \rightarrow (3)$$

Assuming V_2 & $V_3 = 0$ in (2) eqn

$$I_3 = V_1 \frac{\Delta_{13}}{\Delta} \rightarrow (4)$$

We take this condition in such a way get the symmetric equations in the both equations

PES UNIVERSITY

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from (3) $\frac{I_1}{V_3} = \frac{\Delta_{13}}{\Delta} \longrightarrow 5a$

from (4) $\frac{I_3}{V_1} = \frac{\Delta_{13}}{\Delta} \longrightarrow 5b$

5a & 5b 's RHS are equal so LHS are also equal

$\boxed{\frac{I_1}{V_3} = \frac{I_3}{V_1}}$ // This holds good only when $\boxed{\frac{I_1 = I_3}{V_1 = V_3}}$

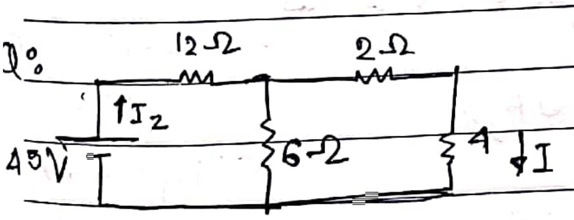
Generally $\frac{I_k}{V_j} = \frac{I_j}{V_k}$ //

* V_j is the only source in network. This results in the current I_k in k^{th} loop
* If we shifted voltage source to k^{th} loop then current flowing in the j^{th} loop is I_j.
* The ratio of the response transform to the excitation transform is invariant to an interchange of the position in the network of the excitation & the respons

CONDITIONS FOR CKT TO BE RECIPROCAL

* for a network to be reciprocal, there should not be any dependent sources there should be only R, L, C & Transformers as a element
* Only one voltage source should be there.
* The impedances should be symmetric
* No Initial conditions

1)



Find I ?

$$\begin{bmatrix} 18 & -6 \\ -6 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \end{bmatrix}$$

$I_1 = 3A$ //

$I_2 = 1.5A$ //

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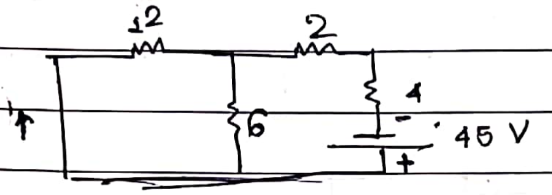
When u are asked to find current replace that voltage source in the same branch where we asked to find the current

To reduce voltage to Zero, that that, if current source is there open ckt that

$I = 1.5A$

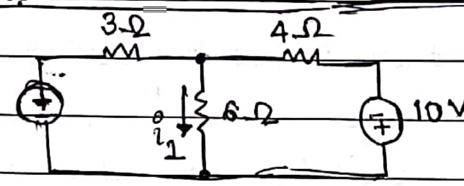


$$= \begin{bmatrix} 18 & -6 \\ -6 & 12 \end{bmatrix} \begin{bmatrix} I \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 45 \end{bmatrix}$$

$I = 1.5$

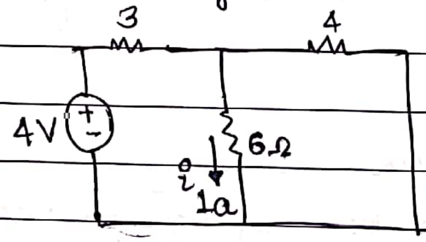
SUPERPOSITION PROBLEMS :-

1



=

(a) Due to only 4V source



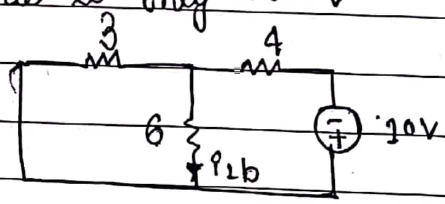
$$= \begin{bmatrix} 9 & -6 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} i_{1a} \\ i_{2a} \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \times$$

$i_{1a} = \cancel{0.74A}$ // $= +0.259A$

$i_{1b} = 0.29$ $i_{1a} =$

0.74
~~0.44~~

(b) Due to only 10 V



$$= \begin{bmatrix} 9 & -6 \\ \cancel{10} & \cancel{6} \end{bmatrix} \begin{bmatrix} i_{1a} \\ i_{1b} \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \times$$

$-6 \quad 10$

$-90i_1 + 60i_2 = -40$
$36i_1 - 60i_2 = 0$

$i_b = -0.553A$ //

$-54i_1 = -40$

$i_t = i_a + i_b = -0.294A$

$4 - 3i_1 - 6(i_1 - i_2) = 0$
$-4i_2 - 6(i_2 - i_1) = 0$

$4 - 3i_1 - 6i_1 = 0$

$4 - 9i_1 = 0$

$4 - 3i_1 - 6i_1 + 6i_2 = 0 \Rightarrow 4 - 9i_1 + 6i_2 = 0$

$-4i_2 - 6i_2 + 6i_1 = 0$ $-10i_2 + 6i_1 = 0$

$-9i_1 + 6i_2 = -4$
$6i_1 - 10i_2 = 0$

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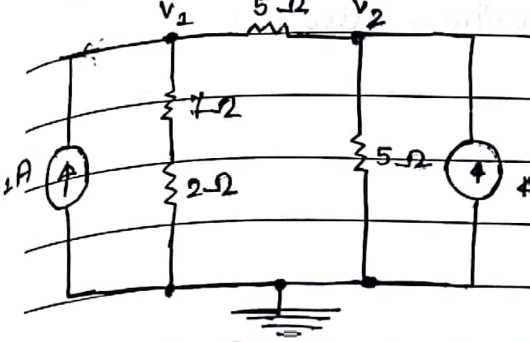
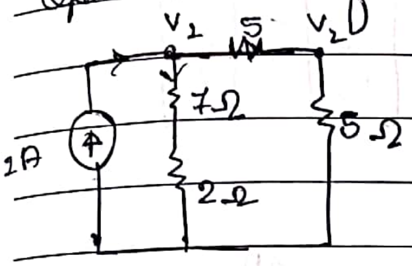


Determine the contribution of the 1A source to V_1.

Calculate the total current flowing through the 7 Ω resistor

Open the one of the source (4A source).



current through 7 Ω = i_{1a}

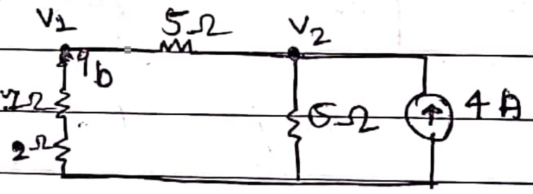
$$1 = \frac{V_1}{9} + \frac{V_1}{10}$$

$$1 = \frac{10V_1 + 9V_1}{90} \qquad 90 = 19V_1$$

$$V_1 = 4.736\ V$$

$$i_{1a} = \frac{V_1}{9} = 0.526A.$$

Open the 1A



$$4 = \frac{V_1}{}$$

$$i_{1b} = \frac{4 \times 5}{9+10} = \frac{20}{19} = 1.052A$$

$$i_t = 1.052 + 0.526 = 1.579A$$

① $4 - 3i_1 - 6(i_1 - i_2) = 0 \Rightarrow 4 - 9i_1 + 6i_2 = 0$

$-4i_2 - 6(i_2 - i_1) = 0 \Rightarrow -10i_2 + 6i_1 = 0$

$-9i_1 + 6i_2 = -4 \quad \times 10 \Rightarrow -54i_1 + 36i_2 = -24$

$6i_1 - 10i_2 = 0 \quad \times 6 \quad 60i_1$

$-90i_1 + 60i_2 = -40$

$36i_1 - 60i_2 = 0$

$-54i_1 = -40$

$i_1 = 0.74$

$-6.6 + 6i_2 = -4$

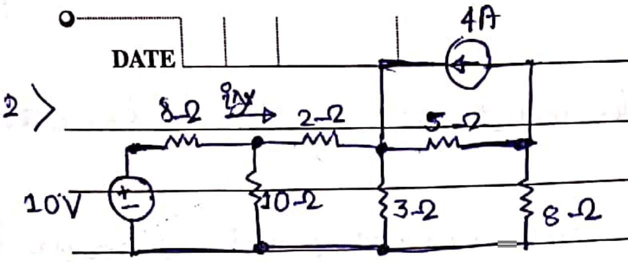
$6i_2 = 2.667$

$i_2 = 0.44 = -0.3$

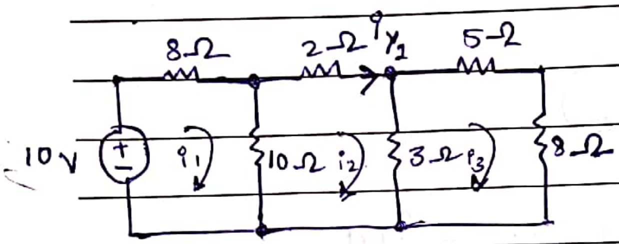
6.74

12th Oct' 19

2) Use Superposition theorem.



Due to 10 V



$$\begin{bmatrix} 18 & -10 & 0 \\ -10 & 15 & -3 \\ 0 & -3 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$i_{y_2} = 0.625$

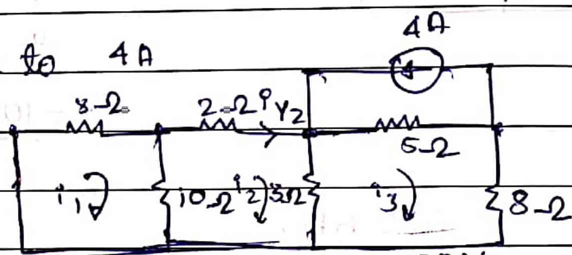
$\frac{\Delta I_2}{\Delta} = I_2 = i_{y_1}$

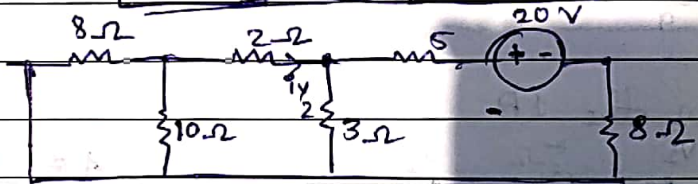
$$= \begin{vmatrix} 18 & \cancel{10} & 0 \\ -10 & \cancel{0} & -3 \\ 0 & 0 & 16 \end{vmatrix} =$$

$\Rightarrow$ ~~8~~

~~18 -10 0~~

Due to 4A





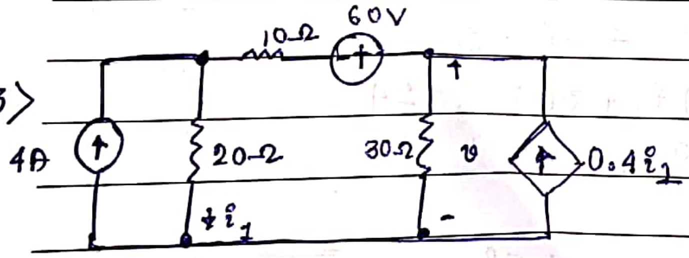
$$\begin{bmatrix} 18 & -10 & 0 \\ -10 & 15 & -3 \\ 0 & -3 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20 \end{bmatrix}$$

$i_{y_2} = -0.4\cancel{57}22$

$= i_{y_1} + i_{y_2}$

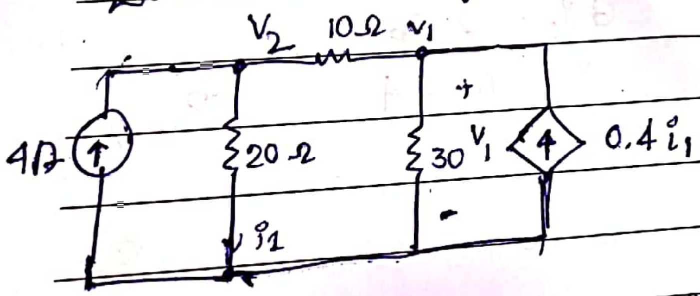
$i_y = 0.203$ //

3)



Use Superposition theorem for finding 'v'

Due to 4A only :-



KCL at node V_2 :-

$i_1 = \frac{V_2}{20}$

$4 = \frac{V_2}{20} + \frac{V_2 - V_1}{10}$

$4 = \frac{V_2}{20} + \frac{V_2}{10} - \frac{V_1}{10}$

$4 = \frac{3V_2}{20} - \frac{V_1}{10}$ ($\times 1$, $\times 2$)

$80 = 3V_2 - 2V_1$

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KCL at node V_2 $= \frac{V_1}{30} - \frac{V_2 - V_1}{10} + 0.4\left(\frac{V_2}{20}\right)$

$\Rightarrow \frac{V_1}{30} = \frac{V_2}{10} - \frac{V_1}{10} + \frac{V_2}{50}$

$\Rightarrow \frac{2}{15}V_1 = \frac{3V_2}{25}$

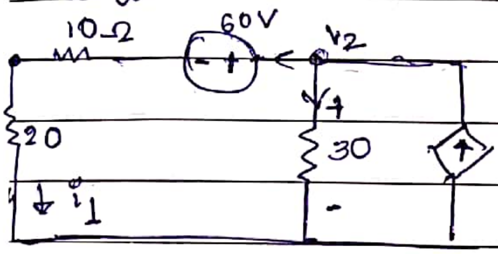
$\therefore V_1 = \frac{3}{25} \times \frac{15}{2} V_2 = \boxed{\frac{9}{10}V_2 - V_1}$

$80 = 3V_2 - 2\left(\frac{9}{10}V_2\right) \Rightarrow 80 = 3V_2 - \frac{9}{5}V_2 \Rightarrow 80 = \frac{6}{5}V_2$

$V_2 = \cancel{676}\ 66.667$ ×

$V_1 = 60.0003$

Due to 60 V



$i_1 = \frac{V_2 - 60}{30}$

$\Rightarrow \frac{V_2 - 60}{30} + \frac{V_2}{30} = 0.4 i_1$

$0.4\left(\frac{V_2}{30} - \frac{60}{30}\right)$

$\Rightarrow \frac{V_2}{30} - 2 + \frac{V_2}{30} = \frac{0.4 V_2}{30} - \frac{4}{5}$

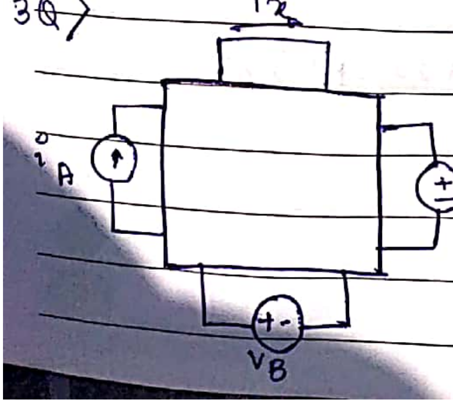
$\Rightarrow \frac{V_2}{15} - \frac{0.4 V_2}{30} = 2 - \frac{4}{5}$

$\Rightarrow \frac{4}{75}V_2 = \frac{6}{5}$

$V_2 = \frac{6}{5} \times \frac{75}{4} = \frac{45}{2} = 22.5$

$V = V_1 + V_2 = 82.514\,V$

3Q)



i) With sources i_A & V_B on & $V_C = 0$, $i_x = 20A$

ii) With source i_A & V_C on & $V_B = 0$, $i_x = -5A$

iii) With all 3 sources on, $i_x = 12A$

iv) Find i_x if the only source operating is

(a) i_A (b) V_B (c) V_C

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Find i_x if i_A & v_C are doubled in magnitude & v_B is reversed.

$i_x(i_A) + i_x(v_B) = 20A \rightarrow (1)$

$i(x)\, i_A + i_x(v_C) = -5A \rightarrow (2)$

$i_x(i_A) + i_x(v_B) + i_x(v_C) = 12 \rightarrow (3)$

$\Rightarrow \quad 20 + i_x(v_C) = 12$

$\Rightarrow \quad i_x(v_C) = -8$ //

$i_x(i_A) - 8 = -5$

$i_x(i_A) = 3A$ //

$\Rightarrow \quad 3 + i_x(v_B) - 8 = 12$

$\Rightarrow \quad i_x(v_B) = 12 + 8 - 3 \quad = 17$

i_A & v_C are doubled

$i_x(2i_A) = 6A$

$i_x(2v_C) = -16V$ &.

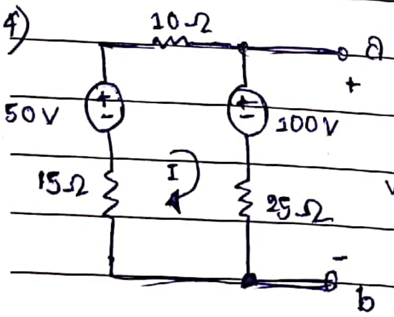
$i_x(-v_B) = -17V$

$i_x(i_A) + i_x(v_B) + i_x(v_C) = \cancel{12}$

$6 - 17 - 16$

$= -27$

Q: 4)



Find the Thevenin equivalent at terminals a & b. How much power would be delivered to a resistor connected to a & b if Rab equals (a) 50Ω (b) 12.5 Ω

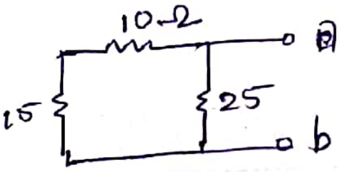
(ans) V_{th} = $+V_{ab} - 100 - 25I - V_b = 0$ $\quad = -100 + 25 = +75$ //

$V_{ab} = 100 + 25I$

$-15I + 50 - 10I - 100 - 25I$

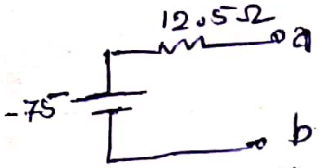
$= -50I - 50$

$= 50I = 50 \quad I = -1$



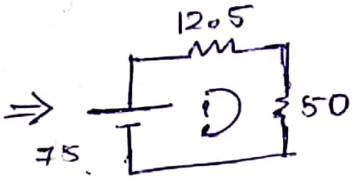
$= 25^{-1} + 25^{-1}$

$= 12.5\,\Omega$ //



Rab = 50 Ω

$\Rightarrow$



$75 = 12.5i + 50i$

$75 = 62.5i$

$i = 1.2$

$P = I^2 R$

$= 72$ //

$P = VI$

$Rab = 12.5$

$75 = 25i$

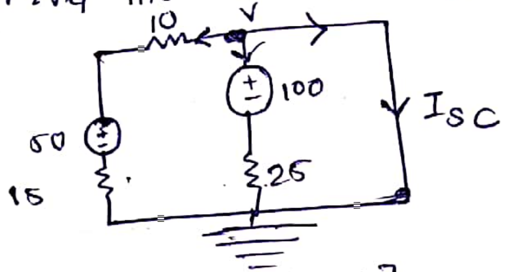
$i = 3$

$P = (3)^2 \times \cancel{50}\ 12.5$

$= 112.5$

Norton.

Find the corresponding



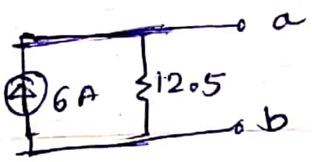
[v = 0 gnd]

By nodal analysis

$\frac{v-50}{25} + \frac{v-100}{25} + I_{sc} = 0$

$= \frac{V}{10} + \frac{V}{25} - 5 - 4 = I_{sc} \Rightarrow \frac{7}{50}V - 9 = I_{sc}$

$= I_{sc} = +6$

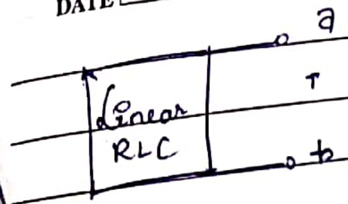


Scanned by CamScanner

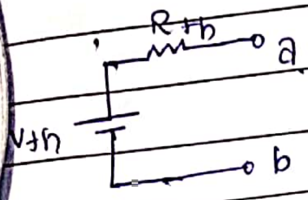
12^{th} Oct'19

THEVENIN'S & NORTION'S THEOREM

DATE

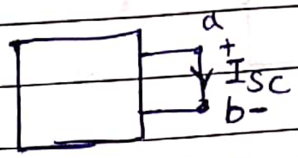


1) remove load
2) find the voltage across the AB = $V_{ab} = V_{th}$
3) Zero the all independent sources & find the eq resistance across ab



NORTION :

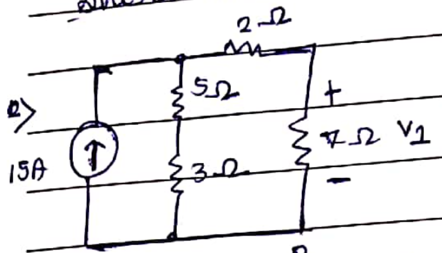
short the the ab terminal



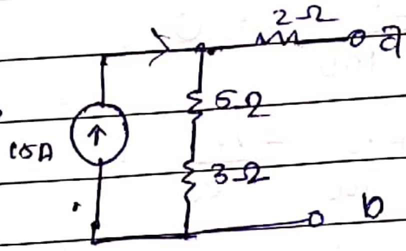
$I_{SC} = I_N$

$I_N = \frac{V_{th}}{R_{th}}$

2)



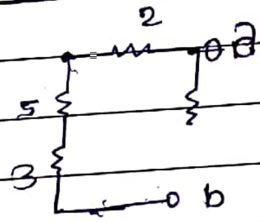
$\Rightarrow$



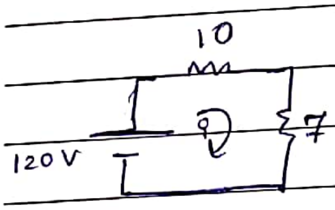
$V_{th} = (15)(8)$

$= 120$

Rth



= $R_{th} = 10$

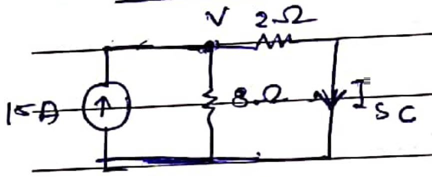


$i = \frac{120}{17}$ i

$i = 7.058$

$V_{(7\Omega)} = 49.41$

Norton



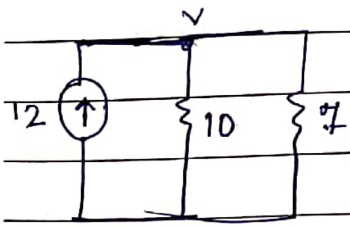
$15 = \frac{V}{8} + \frac{V}{2} = \frac{120}{\cancel{240}} = 5V$

$V = \cancel{48}\ 24$

$I_{SC} = \frac{24}{2} = 12$

$\frac{15 \times 8}{10}$

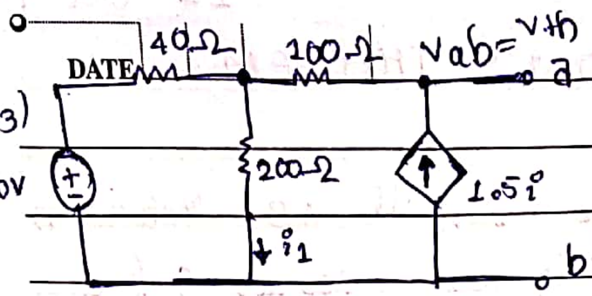
$R_{th} = 10$



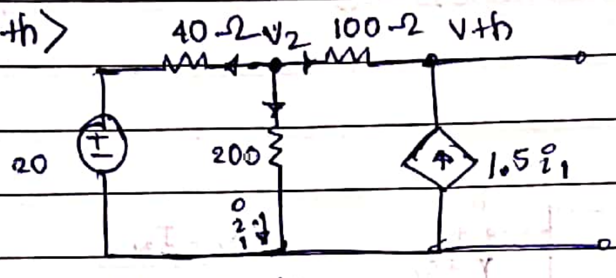
$12 = \frac{V}{10}^{\times 7} + \frac{V}{7}^{\times 10}$

$V = 49.41$

3)



Vth>



$i_1 = \frac{V_2}{200}$

KCl at node V_2

$$\frac{V_2 - 20}{40} + \frac{V_2}{200} + \frac{V_2 - V_{th}}{100} = 0$$

$$\frac{V_2}{40} - \frac{1}{2} + \frac{V_2}{200} + \frac{V_2}{100} - \frac{V_{th}}{100} = 0$$

$$\Rightarrow \frac{3V_2}{100} \quad \boxed{\frac{V_2}{25} - \frac{V_{th}}{100} = 0.5}$$

KCL at node Vab

$$\frac{V_2 - V_{ab}}{100} + 1.5\left(\frac{V_2}{200}\right) = 0$$

$$\Rightarrow \frac{V_2}{100} + \frac{1.5\,V_2}{200} - \frac{V_{ab}}{100} = 0$$

$$\Rightarrow \frac{7}{400} V_2 = \frac{V_{ab}}{100}$$

$$V_2 = \frac{400}{7 \times 100} V_{ab} = \frac{4}{7} V_{ab} = 22.22$$

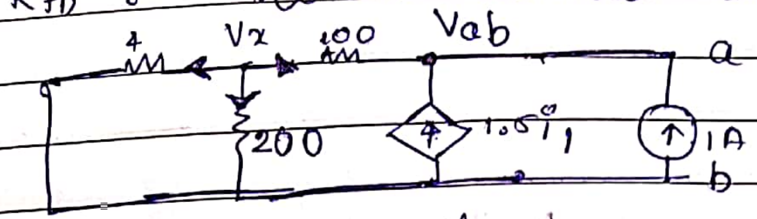
$$\frac{4 V_{ab}}{175} - \frac{1}{100} V_{ab} = 0.5$$

$$\frac{9}{700} V_{ab} = 0.5$$

$$V_{ab} = 38.889 \,//$$

R_{th} in Cannot deactivate the Dependent

Special Case //



apply arbitrary voltage or current value find the current & voltage across it respectively assuming it to 1V & 1A.

KCL at Vx

$$\frac{V_x}{40} + \frac{V_x}{200} + \frac{V_x - V_{ab}}{100} = 0$$

$$\boxed{\frac{V_x}{25} - \frac{V_{ab}}{100} = 0}$$

$$\boxed{i_1 = \frac{V_x}{200}}$$

$$\frac{V_x}{\cancel{25}_{1}} = \frac{V_{ab}}{\cancel{100}_{4}} \qquad V_{ab} = 4V_x$$

KCL at Vab

$$\frac{V_x - V_{ab}}{100} + 1.5 i_1 + 1 = 0$$

$$= \frac{V_x}{100} - \frac{V_{ab}}{100} + \frac{3}{400} V_x + 1 = 0$$

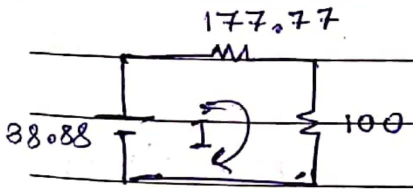
$$= \frac{7}{400} V_x - \frac{V_{ab}}{100} + 1 = 0$$

$$\frac{7}{1600} V_{ab} - \frac{V_{ab}}{100} + 1 = 0$$

$$\frac{-9}{1600} V_{ab} = +1$$

$$9 V_{ab} = 1600$$

$$V_{ab} = 177.77 // = R_{th}$$



$$38.88 = 277.77 I$$

$$I = 0.1399$$

$$P = 1.96 W //$$

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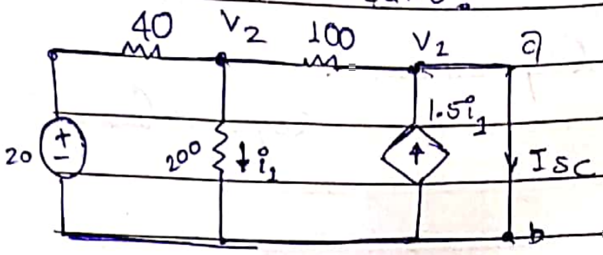
Norton's:



KCL at node V_2

$i_1 = \frac{V_2}{200}$

KCL at node V_1:

$\frac{V_2 - V_1}{100} + 1.5i_1 = I_{sc}$

$\frac{V_2 - 20}{40} + \frac{V_2 - 0}{200} + \frac{V_2 - V_1}{100} = 0$

$5V_2 - 100 + V_2 + 2V_2 - 2V_1 = 0$

$8V_2 - 2V_1 = 100$

$-4V_2 - V_1 = 50$

$V_1 = 0 \Rightarrow V_2 = 12.5$

$\frac{V_2 - V_1}{100} + \frac{1.5V_2}{200} = I_{sc}$

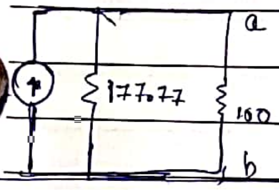
$2V_2 - 2V_1 + 1.5V_2 = 200I_{sc}$

$3.5V_2 = 200I_{sc}$

$I_N = I_{sc} = 0.2187A$

$R_{th} = R_N = 177.77\Omega$

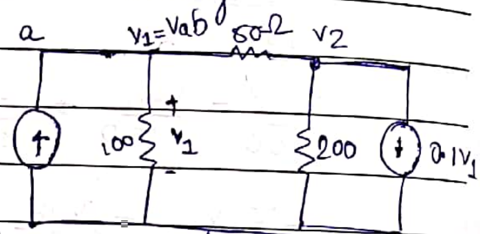
Eq ckt:



when no independent sources in the ckt

$V_{th} = 0 = V_n$ always

@ R_N:



$V_{ab} = 1 = \frac{V_1}{100} + \frac{V_1 - V_2}{50}$

$100 = V_1 + 2V_1 - 2V_2$

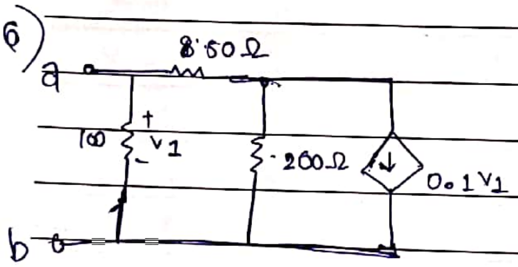
$100 = 3V_1 - 2V_2$

node at V_2:

$\frac{V_1 - V_2}{50} = \frac{V_2}{200} + 0.1V_1$

$\Rightarrow 16V_1 + 5V_2 = 0$

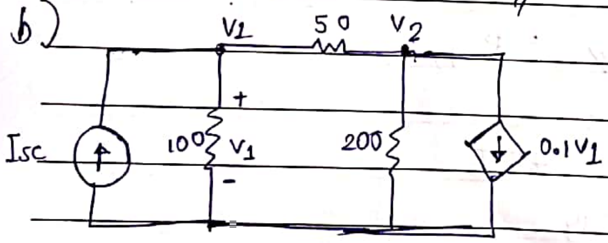
6)



$V_1 = 10.638$; $V_2 = -34.04$

$R_{th} = R_N = 10.638$ //

b)



at V_1

$I_{sc} = \frac{V_1}{100} + \frac{V_1 - V_2}{50}$

$I_{sc} = \frac{3V_1 - 2V_2}{100}$, $= \frac{3V_1 - 2\left(\frac{-16V_1}{5}\right)}{100}$

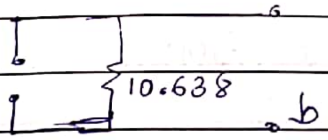
at V_2: $\frac{V_1 - V_2}{50} = \frac{V_2}{200} + 0.1V_1$

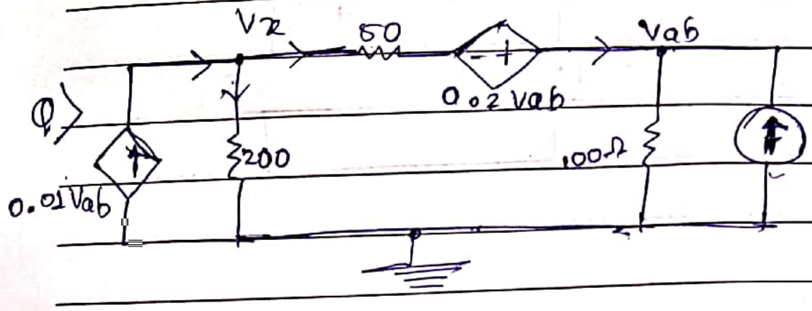
$16V_1 + 5V_2 = 0$

$= \frac{3V_1 + \frac{32V_1}{5}}{100}$ $= \frac{47V_1}{500} = I_{sc}$

$V_1 = 0 \Rightarrow I_{sc}$ zero

Equivalent ckt:





(Vth = 0 ∴ no independent source)

KCL at node Vab

$$\frac{0.2V_{ab} + V_x - V_{ab}}{50} + 1 = \frac{V_{ab}}{100}$$

$$\frac{V_{ab}}{100} = 1 + \frac{V_x - 0.8V_{ab}}{50}$$

$$V_{ab} = 100 + 2V_x - 1.6V_{ab}$$

$$2.6\,V_{ab} = 100 + 2V_x$$

$$V_x + 50 = 1.3\,V_{ab}$$

$$\frac{5.2\,V_{ab}}{5} + 50 = 1.3\,V_{ab}$$

$$50 = (1.3 - 1.04)\,V_{ab}$$

$$50 = 0.26\,V_{ab}$$

KCL at node V_x:

$$\frac{V_x}{200} + \frac{V_x - 0.2V_{ab} - V_{ab}}{50} = 0.01\,V_{ab}$$

$$V_x + 4V_x - 3.2V_{ab} = 2V_{ab}$$

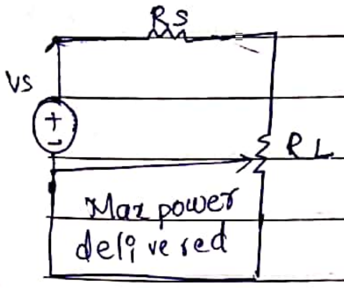
$$5V_x = 5.2\,V_{ab}$$

$$V_{ab} = 192.307\,V$$

$$R_{th} = 192.307\,\Omega$$

MAX POWER TRANSFORM THEOREM:

Purely Resistive Load:-



$$I_L = \frac{V_s}{R_s + R_L} \quad ; \quad P_L = I_L^2 R_L$$

$$P_L = \left(\frac{V_s}{R_s + R_L}\right)^2 R_L$$

$$\frac{dP_L}{dR_L} = 0 \Rightarrow \frac{V_s^2 (R_s + R_L)^2 - V_s^2 R_L \cdot 2\,(R_s + R_L)}{(R_s + R_L)^4}$$

$$= \cancel{V_s^2}(R_s + R_L)^{\cancel{2}} = \cancel{V_s^2} R_L (2) \cancel{(R_s + R_L)}$$

$$R_s + R_L = 2R_L$$

$$R_s = R_L$$

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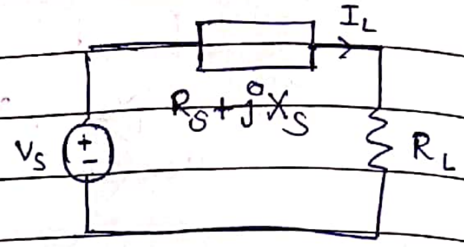
2) Purely Resistive Load with Source Impedance:



$$I_L = \frac{V_S}{(R_S + R_L) + jX_S}$$

$$|I_L| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + X_S^2}}$$

$$P_L = |I_L|^2 R_L$$

$$= \frac{V_S^2 R_L}{\left(\sqrt{(R_S+R_L)^2+X_S^2}\right)^2} = \frac{V_S^2 R_L}{(R_S+R_L)^2+X_S^2}$$

$$\frac{dP_L}{dR_L} = 0 \Rightarrow \frac{V_S^2\left[(R_S+R_L)^2 + X_S^2\right] - V_S^2 R_L\left[2(R_S+R_L)\right]}{\left[(R_S+R_L)^2+(X_S)^2\right]^2}$$

$$V_S^2\left[(R_S+R_L)^2 + X_S^2\right] = V_S^2 R_L\, 2(R_S+R_L)$$

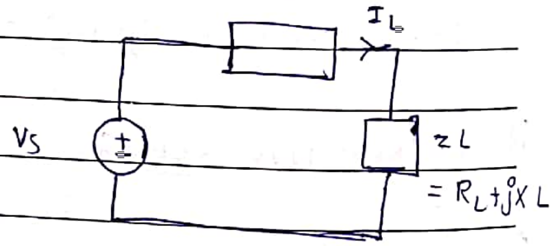
$$R_S^2 + R_L^2 + 2R_SR_L + X_S^2 = 2R_SR_L + 2R_L^2$$

$$R_S^2 = R_L^2 - X_S^2$$

$$R_S^2 + X_S^2 = R_L^2$$

$$R_L = |R_S + jX_S|$$

(c) Variable Load Imp with Source Impedance



$$I_L = \frac{V_S}{(R_S+R_L)+j(X_S+X_L)}$$

$$|I_L| = \frac{V_S}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}}$$

$$P_L = |I_L|^2 R_L$$

$$= \frac{V_S^2 R_L}{(R_S+R_L)^2+(X_S+X_L)^2}$$

Assume $R_L = K$: ($\because$ R_L & X_L both variables)

$$\frac{dP_L}{dX_L} = 0 \Rightarrow \frac{\left(V_S^2\left[(R_S+R_L)^2+(X_S+X_L)^2\right] - V_S^2R_L(2)(R_S+R_L)\right)}{\left[(R_S+R_L)^2+(X_S+X_L)^2\right]^2}$$

$$\Rightarrow R_L 2(R_S+R_L) = (R_S+R_L)^2 + (X_S+X_L)^2$$

$$2R_LR_S + R_L^2 = R_S^2 + R_L^2 + 2R_SR_L + (X_S+X_L)^2$$

$$R_L^2 = R_S^2 + (X_S+X_L)^2$$

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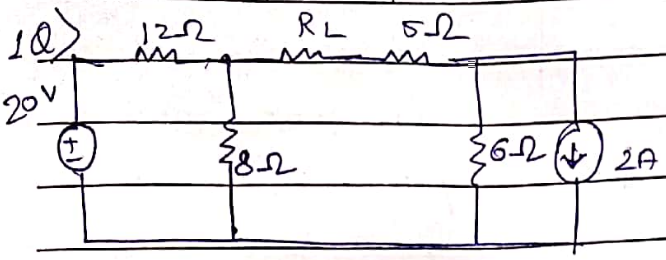
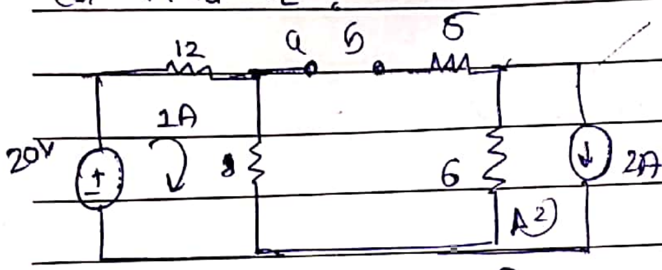
1Q) Max power dissipated in R_L = ?



(a) Find R_L = ?

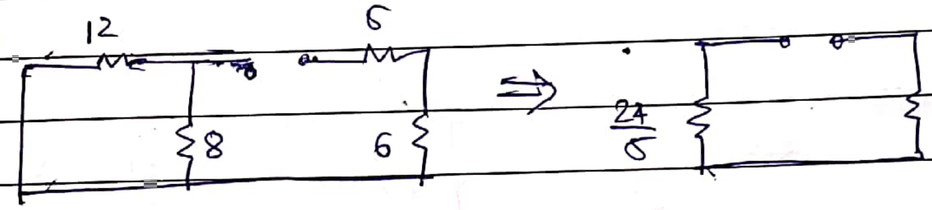


$V_{ab} = V_{th}$

$V_a - 8(1) - 6(2) - V_b$

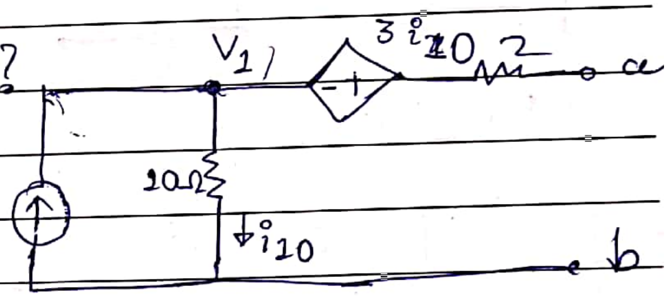
$V_{th} = V_a - V_b = 8 + 12 = 20\,V$

R_{th}



$R_{th} = 15.8$

2) Thevenin equivalent = ? Max power = ?



$i_{10} = 5A$ (∵ open ckt)

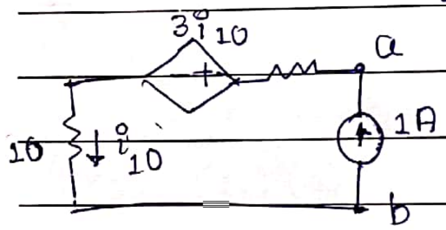
Node V_1

$5 = 5 + \frac{V_1 + 3i_{10} - V_a}{2}$

$V_1 + 15 - V_a = 0$

$(10)(5) + 15 = V_a$

$V_a = 65$



$V_a - 2(1) - 3i_{10} - 10i_{10} = V_b$ 20

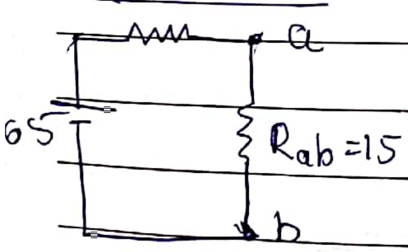
$V_{ab} = V_{th} = 2 + 13i_{10}$

$= 2 + 13(1)$

$V_{ab} = 15$

$R_{ab} = 15.$

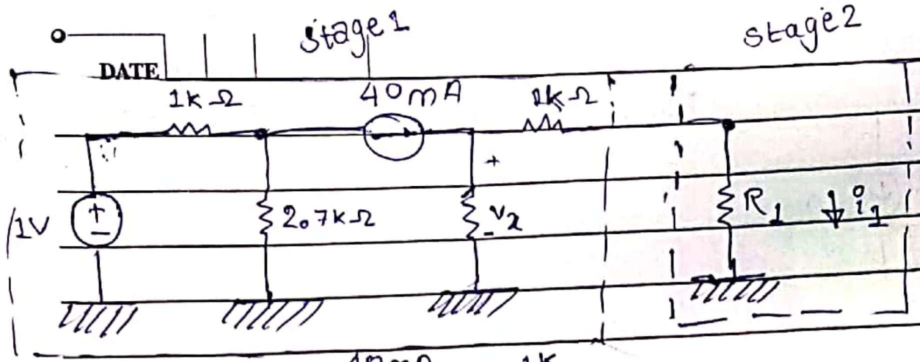
Thevenins :



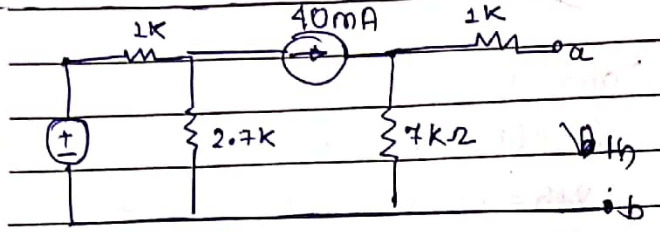
$R_{ab} = R_{th} = 15$

$P = I^2 R_L$

$= 15 \times \left[\frac{13}{6}\right]^2 = 70.42W$

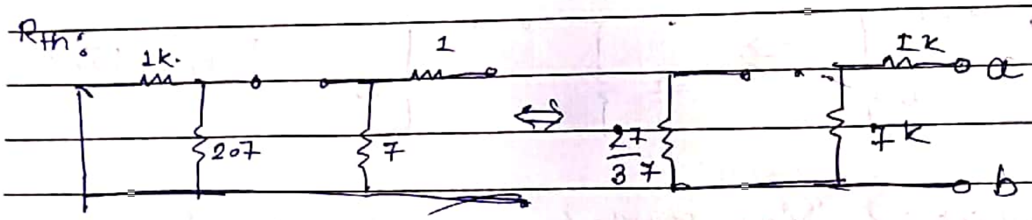


Select R_1 so that max power transferred from stage 1 to stage 2.



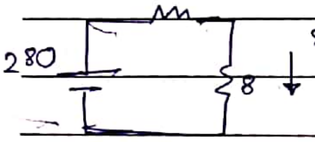
$V_{th} = 280V$

R_{th}:



$R_{ab} = 8\Omega$

$R_L = R_{ab} = 8k\Omega$

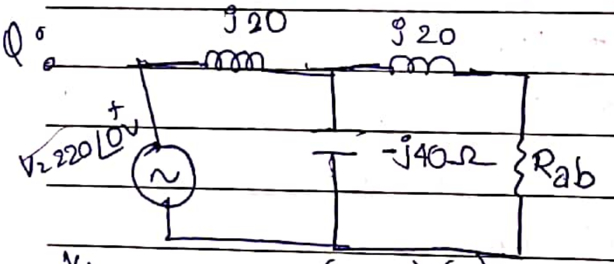


$i = \frac{280}{16} = 17.5$

$P_8 = i^2 (8)\, W$

$= 2450 kW$

Q:



$V = 220 \angle 0°$

$= 220 + j(0)$

$V_{th} = ? \Rightarrow (-j40)(i)$

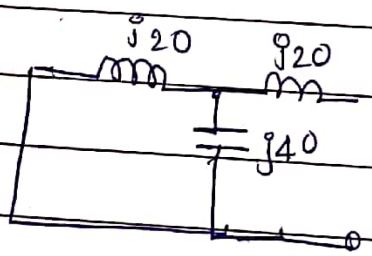
$\frac{i}{i} = \frac{j20 - j40}{220}$

$V_{th} = -j(40)(11j) = \frac{-j(20)}{220}$

$= 40 \times 11 = 440V$

$\frac{i}{i} = \frac{-j}{11}$ $\quad i = \frac{11 \times j}{j \times j} \Rightarrow i = 11j$

Z_{th}:



$Z_{th} = \frac{j20 \times (-j40)}{j20 + (-j40)} = \frac{800 \times j}{j(-20) \times j}$

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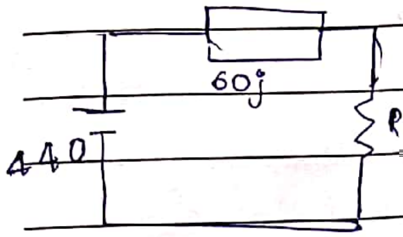
$Z_{th} = 40j + j20 = 60j$

$R_L = \sqrt{60^2} = 60\Omega$

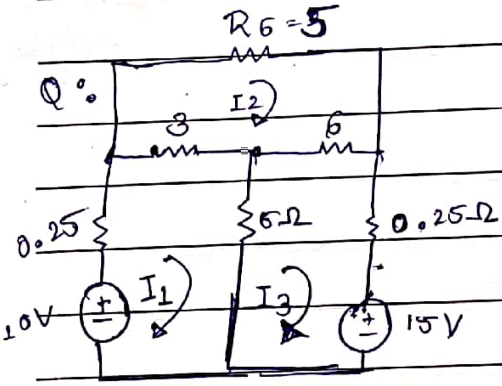


$I_L = \frac{440}{60+60j} = \frac{440}{60(1+j)}$ $|I_L| = \frac{44}{6\sqrt{2}}$

$P_L = \left(\frac{44}{6}\right)^2 \left(\frac{1}{2}\right)(60)$

$= 1613.33\,W$

Q :



absorbed

i) power ~~delivered~~ by 2 voltage sources

ii) power dissipated by all resistors

iii) total power = 0

$$\begin{bmatrix} 8.25 & -3 & -5 \\ -3 & 14 & -6 \\ -5 & -6 & 11.25 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -15 \end{bmatrix}$$

$i_1 = -0.283A$, $= I_{R_1}$

$i_2 = -0.889A$.

$i_3 = -\cancel{1.933A} = 0.609A$

i) (a) $P_{E_1} = (+10)(+I_1) = (-10)(-0.283) = \cancel{+}2.83$

ii) $P_{E_2} = (-15)(-1.933) = 28.995$ (supplied) (absorbed → -28.995).

iii) $P_{R_1} = -(I_1)^2(R_1) = \cancel{+}0.02$

$P_{E_1} + P_{E_2} = 26.16$

$-P_{R_2} = -(I_1 - I_2)^2(3) = -8.167\,W$

$P_{R_3} = (I_3)^2(5) = \cancel{18.68W}$. 13.6125

$P_{R_4} = (I_3 - I_2)^2(6) = +6.54\,W$ ✓

$P_{R_5} = (I_3)^2(0.25) = +0.934\,W$ ✓

$P_{R_6} = (I_2)^2(5) = -3.9516$

Total power = $2.83 + 28.995 - 0.02 - 8.167 - 18.68 - 6.54 - 0.934 - 3.9516.$

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$I_1 = -0.2829\ A = I_{R_1}$ $\qquad$ $I_{R_4} = -1.044\ A$

$I_{R_2} = 0.609\ A$ $\qquad$ $I_{R_5} = I_2 = -1.933\ A$

$I_{R_3} = 1.653\ A$ $\qquad$ $I_3 = I_{R_6} = -0.889\ A$

In any ckt the algebraic sum of power supplied (instantaneous or complex power) & power absorbed by passive elements is Zero. This is Tellegen's Theorem.

$P_{E_1} = 10 I_1 = -2.829\ W$

$P_{E_2} = 15 I_2 = -28.995\ W$ (abs)

$= 28.995\ W$

$P_E = P_{E_1} + P_{E_2} = 26.166\ W$ //

$P_{R_1} = 0.02\ W$ $\qquad$ $P_{R_5} = 0.934\ W$

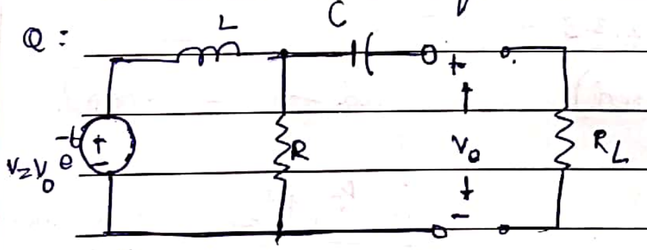
$P_{R_2} = 1.101\ W$ $\qquad$ $P_{R_6} = 3.95\ W$ $\qquad$ $P_E - P_R = 0$

$P_{R_3} = 13.6125\ W$ $\qquad$ $P_E = P_R$

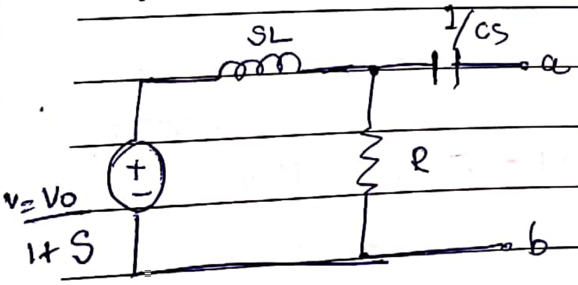
$P_{R_4} = 6.539\ W$ $\qquad$ $P_R = 26.147\ W$ //

Find Thevenin's equivalent for ckt shown

Q:



If L & C are there convert into s domain



$$I(s) = \frac{V_0/(s+1)}{SL+R} = \frac{V_0}{(s+1)(SL+R)} = \frac{V_0/L}{(s+1)(s+R/L)}$$

$$V_0 = R\,I(s) = \frac{V_0 R}{L(s+1)(s+R/L)} = V_{th}$$

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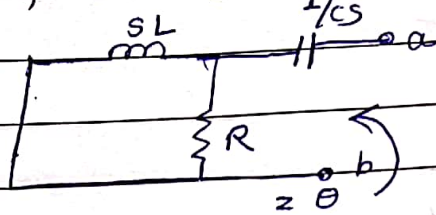
R_{th}, (Short the source)



$$Z_\theta = (SL \| R) + \frac{1}{CS}$$

$$= \frac{SLR}{SL+R} + \frac{1}{CS}$$

$$= \frac{CS^2LR + SL + R}{CS^2L + CSR}$$

$$= \frac{SCLR + L + R/S}{CSL + CR}$$

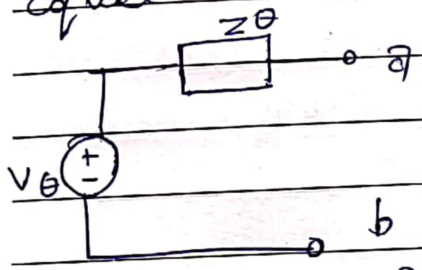
$$= \frac{SCR + 1 + R/SL}{CS + CR/L}$$

$$= \frac{R\left(SC + 1/R + 1/SL\right)}{C\,(S + R/L)}$$

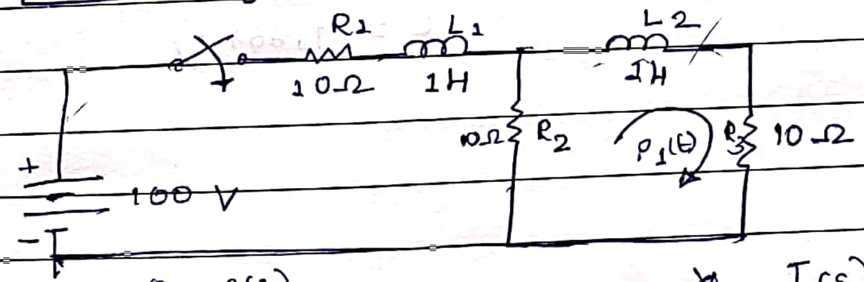
Divide throughout by LRC

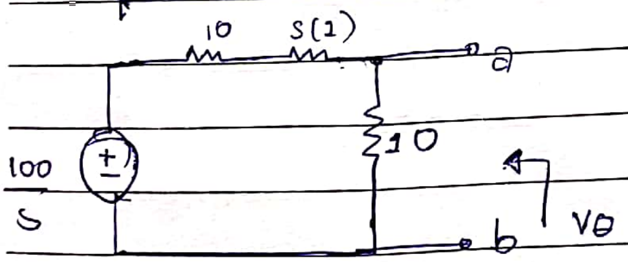
$$\frac{S^2 + S/RC + 1/LC}{S^2/R + S/L}$$

Eqckt :



Q :





$$I(s) = \frac{100/S}{10 + S + 10} = \frac{100/S}{S+20}$$

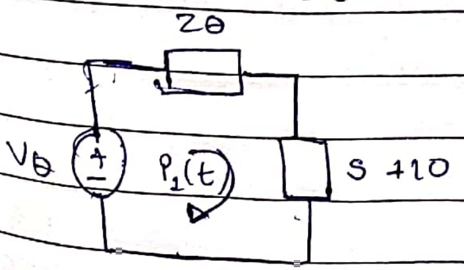
$$= \frac{100}{S(S+20)}$$

$$V_\theta = 10\, I(s) = \frac{1000}{S(S+20)} \quad //$$

$$Z_\theta = 10 + S \,\|\, 10$$

$$\frac{1}{Z_\theta} = \frac{1}{10+S} + \frac{1}{10} \Rightarrow \frac{10 + 10 + S}{100 + 10S} = \frac{20+S}{100+10S}$$

$$Z_\theta = \frac{100 + 10S}{20 + S} \quad //$$



$$I_1(s) = \frac{\dfrac{1000}{S(S+20)}}{\dfrac{100+10S}{20+S} + S + 10}$$

$$= 100 + 10S + 20S + 200 + S^2 + 10S$$

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$$= \frac{1000}{s(\cancel{s+20})} \times \frac{\cancel{300+40s+s^2}\ \ 20+\cancel{s}}{300+40s+s^2}$$

$$= \frac{1000}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}$$

~~100 Put s=0~~

$$1000 = A(s+10)(s+30) + B(s)\cancel{(s+30)} + C(\cancel{s})(s+10)$$

Put $s = 0$

$1000 = A(10)(30)$

$1000 = A(300)$

$A = \frac{10}{3}$

Put $s+30 = 0$

$1000 = C(-30)(-20)$

$1000 = C(600)$

$\frac{5}{3} = C$

$C = 1.667$ //

$s+10 = 0$

$s = -10$

Put $s+10 = 0$

$1000 = B(-10)(20)$

$1000 = B(-200)$

$B = -5$

$$I(s) = \frac{10}{3s} - \frac{5}{s+10} + \frac{1.667}{s+30}$$

$$i_2(t) \Rightarrow \frac{10}{3} - 5e^{-10t} + 1.667e^{-30t}\ A\ //$$

A°

- Linear elements
- Independent & dependent
- Initial conditions passive elements
- b

B°

- Linear, nonlinear, time varying
- Inde / Dependent
- Initial conditions on passive elem
- No magnetic / controlled-source coupling to network A

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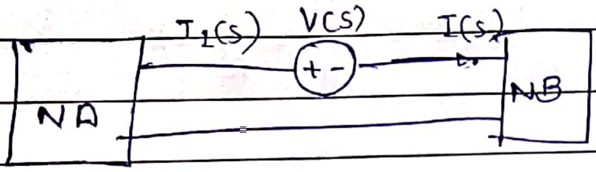
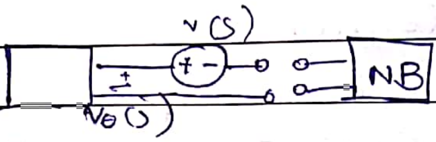


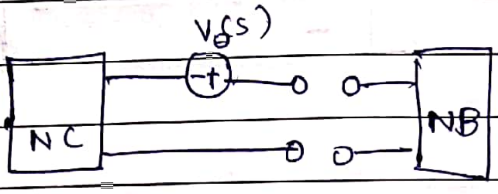
* $I(s) = -I_1(s)$

Current is Zero

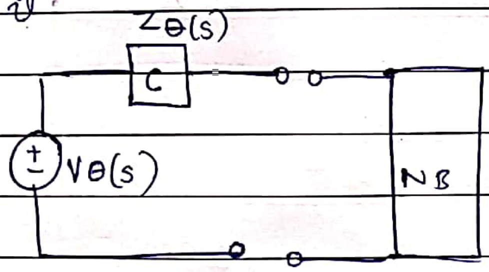


* The voltage at broken terminal is Zero

By KVL $V_\theta(s) = V(s)$



* all independent sources are reduced to 0
* Dependent sources are not changed
* NA is transformed into Network C
* $V_\theta(s)$ polarity is reversed so that when connected to B, flow through it.



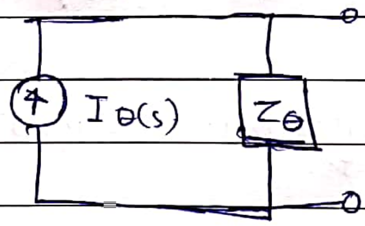
~~Thev~~ Thevenin eq network

* If $Z_B(s)$ is the impedance of B, then the current is

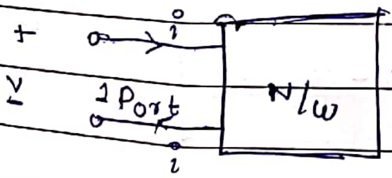
$$I(s) = \frac{V_\theta(s)}{Z_\theta(s) + Z_B(s)}$$

Voltage of network B

$$V_B(s) = \frac{I_\theta(s)}{Y_\theta(s) + Y_B(s)}$$



ENERGY & POWER



$p(t) = v(t).i(t)$ Instantaneous power (absorbed)

$W(t) = \int_{t_1}^{t_2} p(\tau)\,d\tau \rightarrow$ Energy absorbed

(2) Resistor: $V = IR \rightarrow$ (1)

(a) Instantaneous power $= v(t)\,i(t)$

$= i^2(t)R \; / \; \frac{v^2(t)}{R}$ absorbed

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b) $W_R(t) \rightarrow$ energy absorbed $= \int_{t_1}^{t_2} i^2(t) R\, dt = \int_{t_1}^{t_2} \frac{v^2(t)}{R} dt$

c) For Sinusoid :- $i(t) = I_m \sin\omega t$

Instantaneous power $= P(t) = v(t).i(t)$

$= i^2(t) R$

$= I_m^2 R \sin^2\omega t = \boxed{\frac{I_m^2 R}{2}[1-\cos 2\omega t]}$

at t

Energy $= \int_{t_0}^{t} p(\tau) d\tau$

$= \int_0^t \frac{R I_m^2}{2}[1-\cos 2\omega \tau] d\tau = \frac{R I_m^2}{2} \int_0^t (1-\cos 2\omega\tau) d\tau$

$\boxed{= \frac{R I_m^2}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right) \longrightarrow 2}$

d) Average power $= P_{av_R} = \frac{1}{nT} \int_0^{nT} p(\tau) d\tau$

$= \frac{1}{nT}\int_0^{nT} \frac{R I_m^2}{2}[1-\cos 2\omega t] dt = \frac{R I_m^2}{2\cancel{nT}}\left[\cancel{nT} - \cancel{\frac{\sin 2\omega t}{2\omega}}^{\,0}\right]_0^{nT}$

$= \sin 2\omega nT = \sin 2 \times \frac{2\pi}{T} \times n \times T = \sin 4\pi = 0$ for any value of n

$= \boxed{\frac{I_m^2 R}{2}}$

e) Average Energy / cycle :- $= \int_0^T \frac{R I_m^2}{2}[1-\cos 2\omega t]$

$= \frac{R I_m^2}{2}\left[T - \cancel{\frac{\sin 2\omega T}{2\omega}}^{\,0}\right] = \boxed{\frac{I_m^2 R T}{2}}$

always power absorbed

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② **INDUCTANCE :-** $v = L\dfrac{dI}{dt} \rightarrow ①$

a) Instantaneous power $P_L(t) = V(t).I(t)$

$$= \boxed{i(t).\,L\frac{di}{dt}}$$

b) Energy : $W_L(t) = \int_{t_1}^{t_2} P_L(\tau)\,d\tau = \int_{t_1}^{t_2} i(t)\,L\frac{di}{d\tau}\,d\tau = L\left.\frac{i^2(t)}{2}\right]_{t_1}^{t_2}$

$$= \boxed{\frac{L}{2}\left[i^2(t_2) - i^2(t_1)\right]}$$

(c) For a Sinusoid : $i(t) = I_m \sin\omega t$

Inst power $= L\,i(t)\dfrac{di(t)}{dt} = L\,I_m \sin\omega t\,(I_m\omega\cos\omega t)$

$= L I_m^2\,\omega \sin\omega t \cos\omega t$

$$\boxed{P_L(t) = \frac{L I_m^2\,\omega \sin(2\omega t)}{2}}$$

(d) Inst Energy $w_L(t) = \dfrac{1}{2} L i^2(t)$

$= \dfrac{1}{2} L I_m^2 \sin^2\omega t$

$$= \boxed{\frac{1}{4} L I_m^2\left[1 - \cos 2\omega t\right]}$$

(e) Average Power (L) : $P_{avg} = \dfrac{1}{nT}\displaystyle\int_0^{nT} P_L(\tau)\,d\tau$

$= \dfrac{1}{nT}\displaystyle\int_0^{nT} \frac{L I_m^2\,\omega\,\cancel{\sin(2\omega t)}}{2}$ $\longrightarrow$ periodic signal, integrating goes to Zero

$\cos 2\times\frac{2\pi}{T}\times nT$

$$\boxed{P_{avg\,L} = 0}$$

(f) Average Energy / cycle = Zero.

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3) CAPACITANCE : $i(t) = C \frac{dv(t)}{dt}$ $\rightarrow$ (1)

a) Inst Power $P_c(t) = i(t)\, v(t)$

$$\boxed{= v(t)\, C \frac{dv(t)}{dt}}$$

b) Energy $w_c(t) = \int_{t_1}^{t_2} P_c(\tau)\, d\tau$

$$= \int_{t_1}^{t_2} v(t)\, C \frac{dv(t)}{\cancel{d\tau}} \cancel{d\tau}$$

$$= C \left. \frac{v^2(t)}{2} \right|_{t_1}^{t_2} = \boxed{\frac{1}{2} C \left[v^2(t_2) - v^2(t_1) \right]}$$

(c) For a Sinusoid:

$v(t) = V_m \sin \omega t$

(d) Inst. power : $P_c(t) = v(t)\, i(t)$

$$= \cancel{v(t)}\; C V_m \sin \omega t \; V_m \omega \cos \omega t$$

$$= C V_m^2 \omega \sin(2\omega t)$$

$$= \frac{1}{2} C V_m^2 \omega \sin(2\omega t)$$

(e) Energy : $\frac{1}{2} C v^2(t) = \frac{1}{2} C V_m^2 \sin^2 \omega t = \boxed{\frac{1}{4} C V_m^2 \left[1 - \cos 2\omega t \right]}$

(f) Average Power:

$$P_{avg\,C} = \frac{1}{nT} \int_0^{nT} P_c(\tau)\, d\tau = \frac{1}{nT} \int_0^{nT} \frac{1}{2} C V_m^2 \omega \sin 2\omega t \, dt$$

$$= 0$$

(g) Average Energy $= 0$ //

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EFFECTIVE / RMS VALUES :

The effective / RMS value of a periodic current is that steady DC current that will produce same power in a resistor as produced on an average by the periodic current for the same time.

Let the steady current = I

$$p = I^2 R$$

Now, $P_{av}\ R = \frac{1}{2} I_m^2 \cancel{R} = I^2 \cancel{R}$

$$\boxed{I = \frac{I_m}{\sqrt{2}} = I_{rms} = I_{eff}}$$

Illy $\boxed{V_{rms} = V_{eff} = \frac{V_m}{\sqrt{2}}}$

Average Power $= I_{rms}^2 R$
$= \frac{V_{rms}^2}{R}$

$P_{av} = I_{eff} \times V_{eff}$

The RMS value of a non sinusoidal but periodic current of period 'T' is given by

$$P_{av} = \frac{1}{nT} \int_0^{nT} R\, i^2(t)\, dt$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt} \qquad V_{eff} = \sqrt{\frac{1}{T} \int_0^T v^2(t)\, dt}$$

POWER FACTOR :

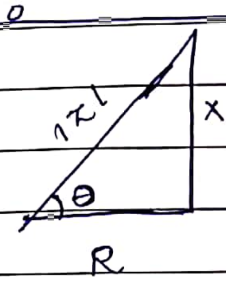
$P_{av} = I_{rms}^2 R = I_{eff} V_{eff}$

$Z(j\omega) = R + jX(j\omega)$

$Z(s) = R + jX(s)$

$P_{av} = I_{rms}^2\ Re\{Z(j\omega)\}$

$I_{rms}^2 |Z| \cos\theta$



$\frac{R}{|Z|} = \cos\theta$

$R = |Z| \cos\theta$

$\theta = \tan^{-1}\left(\frac{X j(\omega)}{R}\right)$

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* Power factor is the cosine of $\angle^{le}$ of impedance
* Power factor is also the cosine of the angle between V & I.

Ex: If $V = V_m \sin \omega t$

$i = I_m \sin(\omega t + \theta)$ leads v //

If i leads 'v' power factor is leading