Network Analysis & Systems

Koshy George

Dept. of Elect. & Comm. Engineering,
PES University;
PES Centre for Intelligent Systems

UE18EC201: Network Analysis & Systems

Disclaimer (1)

- Errors and Omissions: The author assumes no responsibility or liability for any errors or omissions in the contents of this file. The information is provided on an "as is" basis with no guarantees of completeness, accuracy, usefulness or timeliness and without any warranties of any kind whatsoever, express or implied.
- Breach of Confidentiality: The information in this file are confidential and intended solely for the non-commercial use of the individual or entity to whom this file has been given, who accepts full responsibility for its use. You are not permitted to disseminate, distribute, or copy this file. If you are not the intended recipient you are notified that disclosing, copying, distributing or taking any action in reliance on the contents of this information is strictly prohibited.

Disclaimer (2)

- Fair Use: This file contains copyrighted material the use of which has not always been specifically authorised by the copyright owner.
- Copyright: No part of this file may be reproduced, stored in a retrieval system or transmitted in any form or by any means, mechanical, electronic, photocopying, recording or otherwise without the written permission of the author.

Overview of Syllabus & Lesson Plan (1)

Unit I (8+2 hours) **Conventions and Basic Analysis:**

- Reference directions; active element conventions.
- Star-Delta and source transformations.
- Kirchhoff's Laws; node and mesh analysis.
- DC and AC networks.
- Duality; dot convention for coupled circuits.

Ref. A:[1] Chapters 1–3.

Ref. B:[2] Chapters 2–5, 7.

[1] M. A. Van Valkenburg, *Network Analysis*, 3rd edn., Prentice Hall, 1975.

[2] W. H. Hayt, Jr., J. E. Kemmerly and S. M. Durbin, *Engineering Circuit Analysis*, 8th edn., Tata McGraw Hill, 2012.

Overview of Syllabus & Lesson Plan (2)

Unit II (11+4 hours) Transient Characteristics:

- Damping and time constants.
- First and Second Order Circuits:
  - Time-domain analysis.
  - Frequency-domain analysis.

Ref. A: Chapters 4–6; parts of 7-10.

Ref. B: Chapters 8–11, 13–15.

Unit III (8+4 hours) Network Theorems:

- Superposition theorem.
- Thevenin's and Norton's theorems.
- Maximum power transfer and reciprocity theorems.
- Millmann's and Tellegen's theorems.

Ref. A: Chapters 9 & 14.

Ref. B: Chapter 5.

Overview of Syllabus & Lesson Plan (3)

Unit IV (7+2 hours) Two Ports:

- Review of one-ports.
- z-parameters: open circuit analysis;
  y-parameters: short circuit analysis.
- h-parameters and t-parameters.
- Deriving two port network parameters from one another.
- Interconnection of two port networks.

Ref. A: Chapter 11.
Ref. B: Chapter 17.
Ref. C:[3] Chapter 9.

[3] F. F. Kuo, *Network Analysis and Synthesis*, 2nd edn., John Wiley, 2006.

Overview of Syllabus & Lesson Plan (4)

Unit V (8+2 hours) Network Synthesis:

- Hurwitz polynomials.
- Positive real functions.
- Elementary synthesis procedures.
- Properties of R-C impedance, R-L admittance, L-C immittance functions.
- Foster forms I and II.
- Cauer forms I and II.

Ref. C: Chapters 10 & 11.

Reference Books

A: M. A. Van Valkenburg, *Network Analysis*, 3rd edn., Prentice Hall, 1975.

B: W. H. Hayt, Jr., J. E. Kemmerly and S. M. Durbin, *Engineering Circuit Analysis*, 8th edn., Tata McGraw Hill, 2012.

C: F. F. Kuo, *Network Analysis and Synthesis*, 2nd edn., John Wiley, 2006.

D: R. L. Boylestad, *Introductory Circuit Analysis*, 10th edn., Prentice Hall, 2002.

E: John O'Malley, *Theory and Problems of Basic Circuit Analysis*, 2nd edn., McGraw Hill, 1992.

Advanced Reference Books

1. C. A. Desoer and E. S. Kuh, *Basic Circuit Theory*, McGraw Hill, 1969.
2. N. Balabanian, T. A Bickart and S. Seshu, *Electrical Network Theory*, John Wiley, 1969.
3. L. Zadeh and C. A. Desoer, *Linear Systems Theory*, McGraw Hill, 1963.
4. R. J. Schwarz and B. Friedland, *Linear Systems*, McGraw Hill, 1965.

Course Administration: In-Semester Assessment (1)

1. The maximum that can be earned in ISA is 40. This constitutes 40% of the total marks based on which Grades are to be assigned.
2. There shall be two computer-based tests (CBT) each for 40 Marks (duration 60 minutes). The marks obtained for each test shall be reduced to 15 Marks (i.e., 37.5% of ISA, totalling to 75% of ISA).
3. The tests are scheduled as per the Calendar of Events and administered centrally by the Office of the CoE.

Course Administration: In-Semester Assessment (2)

4. There shall be computer-aided and regular assignments (jointly termed hereinafter as Term Project) to be carried out in groups of strength at most 3, and evaluated continuously through the Semester. The total marks assigned to the Term Project shall be reduced to 10 marks (25% of ISA). The schedule for the term project shall be announced separately.
5. There shall be no marks for attendance.

Course Administration: End-Semester Assessment

1. The maximum that can be earned in ESA is 60. This constitutes 60% of the total marks based on which Grades are to be assigned.
2. There shall be an ESA for 100 marks (duration 180 minutes) based on the entire syllabus, and reduced to 60 marks.
3. The ESA is as per the Calendar of Events and administered by the Office of the CoE.

Final Grade: The final grades shall be awarded on based on the marks obtained for ISA and ESA.

Network Analysis and Synthesis

Unit I: Basic Analysis — Conventions

NAS: What? Why? Where?

- Systematic exposition of circuit theory.

NAS: What? Why? Where?

- Systematic exposition of circuit theory.
- Predict the behaviour (i.e., voltage or current) at any point of an electric network and at any instant of time when the voltage or current at some other point is known.

NAS: What? Why? Where?

- Systematic exposition of circuit theory.
- Predict the behaviour (i.e., voltage or current) at any point of an electric network and at any instant of time when the voltage or current at some other point is known.
- Physical reality vs. Mathematical idealisations.

NAS: What? Why? Where?

- Systematic exposition of circuit theory.
- Predict the behaviour (i.e., voltage or current) at any point of an electric network and at any instant of time when the voltage or current at some other point is known.
- Physical reality vs. Mathematical idealisations.
- The concepts have ramifications into Electronics, Communications, Control Systems and Signal Processing.

Conceptual Schemes

Definition

A conceptual scheme[1] is a theory that accounts for as many observations as possible.

[1] J. B. Conant, *Science and Common Sense*, Yale University Press, 1951.

Conceptual Schemes

Definition

A conceptual scheme[1] is a theory that accounts for as many observations as possible.

Examples:

- Atomic theory.
- Conservation of energy.
- Conservation of charge.
- Maxwell's equations.

[1] J. B. Conant, *Science and Common Sense*, Yale University Press, 1951.

Conceptual Schemes

Definition

A conceptual scheme[1] is a theory that accounts for as many observations as possible.

Examples:

- Atomic theory.
- Conservation of energy.
- Conservation of charge.
- Maxwell's equations.
- Circuit theory.

[1] J. B. Conant, *Science and Common Sense*, Yale University Press, 1951.

Circuit Theory

- A physical circuit is a system of interconnected apparatus — components, loads, sources of energy, connecting wires, etc.
  - The function of the circuit is to transfer and transform energy.
  - Energy is transferred from a point of supply (source) to a point of transformation or conversion called load (sink).
  - Energy transfer is accomplished by the transfer of charge.
- The analysis of such circuits in terms of voltages and currents is via the conceptual scheme Circuit Theory.

References, Conventions and Notations (1)

- Behaviour of a circuit or network can be sufficiently described by voltage and current.
- These are functions that map the set of positive real numbers to the vector space of real numbers:
  - Voltage: $v : \mathbb{R}_+ \longrightarrow \mathbb{R}$.
  - Current[2]: $i : \mathbb{R}_+ \longrightarrow \mathbb{R}$.
- The symbols $v(\cdot)$ and $i(\cdot)$ refer to the entire function of time.
- The symbols $v(t)$ and $i(t)$ respectively refer to the value of the voltage or the current at the specific instant of time t.

[2] The symbol arises from the French word *intensité*.
[3] Seshu and Balabanian, 1959.

References, Conventions and Notations (1)

- Behaviour of a circuit or network can be sufficiently described by voltage and current.
- These are functions that map the set of positive real numbers to the vector space of real numbers:
  - Voltage: $v : \mathbb{R}_+ \longrightarrow \mathbb{R}$.
  - Current[2]: $i : \mathbb{R}_+ \longrightarrow \mathbb{R}$.
- The symbols $v(\cdot)$ and $i(\cdot)$ refer to the entire function of time.
- The symbols $v(t)$ and $i(t)$ respectively refer to the value of the voltage or the current at the specific instant of time t.
- Careless notation is a consequence of, and in turn leads to, sloppy and improper patterns of thinking[3].

[2] The symbol arises from the French word *intensité*.
[3] Seshu and Balabanian, 1959.

References, Conventions and Notations (2)

- Current is the phenomenon of transferring charge from one point of a circuit to another:

$$i(t) = \frac{dq}{dt} A \left[= \frac{\text{coulomb}^4}{\text{second}} \right]$$

where A is the symbol for *amperes*[5].

Coulomb: 

Ampère: 

[4] In honour of Charles-Augustin de Coulomb, French military engineer and physicist, 1736–1806.

[5] In honour of André-Marie Ampère, French physicist and mathematician, 1775–1836.

References, Conventions and Notations (3)

- 1 ampere corresponds to 6.24×10^{18} electrons moving across a cross-section in 1 second. (Why?)
- Current is the time rate of flow of free electrons.

[6] In honour of Alessandro Giuseppe Antonio Anastasio Volta, Italian physicist and chemist, 1745-1827.

References, Conventions and Notations (3)

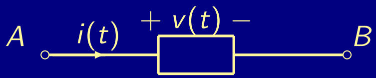
- 1 ampere corresponds to 6.24×10^{18} electrons moving across a cross-section in 1 second. (Why?)
- Current is the time rate of flow of free electrons.
- The work per unit charge is defined as voltage:

$$v = \frac{dw}{dq} \left[= \frac{\text{joules}}{\text{coulomb}} \right]$$

where V is the symbol for *volts*[6].

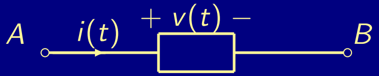
[6] In honour of Alessandro Giuseppe Antonio Anastasio Volta, Italian physicist and chemist, 1745-1827.

References, Conventions and Notations (4)



- A two-terminal lumped element is a one-port.
- Mathematically it is referred to as a branch (graph theory).
- The standard reference directions are as shown:
  - Voltage by plus and minus symbols.
  - Current by an arrow.
- The electrical potentials at A and B are measured w.r.t. to some fixed but arbitrary reference.

References, Conventions and Notations (5)

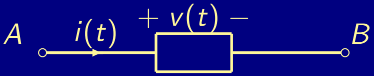


- The branch voltage at any instant t is

$$v(t) = v_A(t) - v_B(t)$$

- At any instant t, the branch voltage $v(t) > 0$ whenever the electrical potential at A at time t is greater than the electrical potential at B at time t.

References, Conventions and Notations (5)

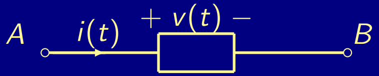


- The branch voltage at any instant t is

$$v(t) = v_A(t) - v_B(t)$$

- At any instant t, the branch voltage $v(t) > 0$ whenever the electrical potential at A at time t is greater than the electrical potential at B at time t.
- The branch current at time t is positive whenever at time t a net flow of positive charges enters the branch at node A and leaves it at node B.

References, Conventions and Notations (5)

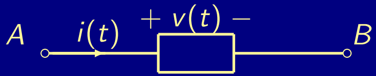


- The branch voltage at any instant t is

$$v(t) = v_A(t) - v_B(t)$$

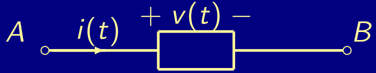
- At any instant t, the branch voltage $v(t) > 0$ whenever the electrical potential at A at time t is greater than the electrical potential at B at time t.
- The branch current at time t is positive whenever at time t a net flow of positive charges enters the branch at node A and leaves it at node B.
- We are not referring to the flow of electrons!

References, Conventions and Notations (6)



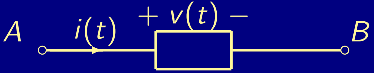
- It is possible to assign independent references for voltage and current.

References, Conventions and Notations (6)



- It is possible to assign independent references for voltage and current.
- However, we shall use associated reference directions as shown.

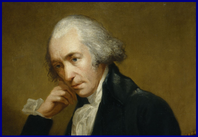
References, Conventions and Notations (6)



- It is possible to assign independent references for voltage and current.
- However, we shall use associated reference directions as shown.

- With these associated reference directions, the power *delivered to the branch* at time t is

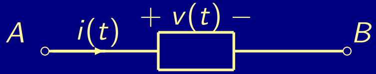
$$p(t) = v(t)i(t) = \frac{dw}{dt} W \left[= \frac{\text{joule}}{\text{second}} \right]$$



where W is the symbol for *watts*[a].

[a] In honour of James Watt, Scottish mechanical engineer and chemist, 1736–1819.

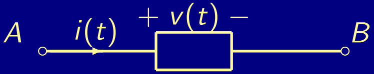
References, Conventions and Notations (7)



The power *delivered to the branch* at time t is

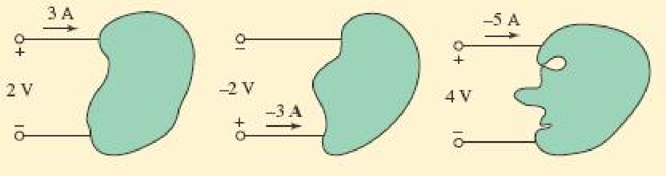
$$p(t) = v(t)i(t)$$

References, Conventions and Notations (7)



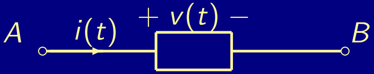
The power *delivered to the branch* at time t is

$$p(t) = v(t)i(t)$$



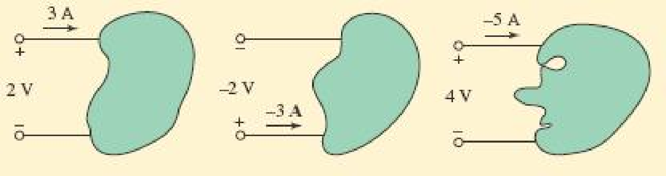
Determine the power delivered to (or, power absorbed by) each network.

References, Conventions and Notations (7)



The power *delivered to the branch* at time t is

$$p(t) = v(t)i(t)$$



Determine the power delivered to (or, power absorbed by) each network.

Ans: 6W, 6W, -20W

Examples

19. A new type of device appears to accumulate charge according to the expression $q(t) = 10t^2 - 22t$ mC (t in s). (a) In the interval $0 \le t < 5$ s, at what time does the current flowing into the device equal zero? (b) Sketch $q(t)$ and $i(t)$ over the interval $0 \le t < 5$ s.

Examples

19. A new type of device appears to accumulate charge according to the expression $q(t) = 10t^2 - 22t$ mC (t in s). (a) In the interval $0 \leq t < 5$ s, at what time does the current flowing into the device equal zero? (b) Sketch $q(t)$ and $i(t)$ over the interval $0 \leq t < 5$ s.

21. The current waveform depicted in Fig. 2.27 is characterized by a period of 8 s. (a) What is the average value of the current over a single period? (b) If $q(0) = 0$, sketch $q(t)$, $0 < t < 20$ s.

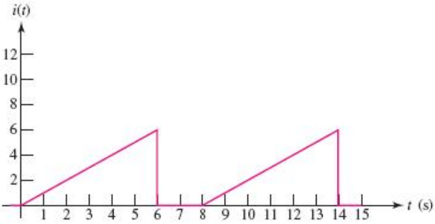


FIGURE 2.27 An example of a time-varying current.

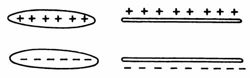
Source: Hayt, Kemmerly and Durbin, 2012.

Network Analysis and Synthesis

Unit I: Basic Analysis — Components

Passive Component: Capacitor (1)

Parallel plate capacitor with area a and separation d:



- Coulomb's Law for point charges: $F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}$.
- Electrical (force per unit charge): $E = \frac{F}{q}$.
- Ignoring fringing effects, by Gauss' law, $q = \epsilon \oint E \cdot da = \epsilon E a$.
- The voltage across the plates $v = \int E \cdot dl = Ed$.
- Therefore,

$$q = \frac{\epsilon a}{d} v \triangleq Cv$$

where C is called capacitance.

Passive Components: Capacitor (2)

- $q = Cv$: The unit of capacitance is *farad*[7]; the symbol is F.
- Capacitance is the ability of a device to store energy in the form of an electric field.
- Inverse capacitance is called elastance with unit *daraf*.
- Thus,

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) = C\frac{dv}{dt}$$

if C does not vary with time; otherwise, $i = C\frac{dv}{dt} + v\frac{dC}{dt}$.

[7] In honour of Michael Faraday, British chemist, 1791–1867.

Passive Components: Capacitor (3)

- Note that

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau = q_0 + \int_{0}^{t} i(\tau)d\tau$$

Passive Components: Capacitor (3)

- Note that

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau = q_0 + \int_{0}^{t} i(\tau)d\tau$$

- We exclude the possibility of infinite current.
- Therefore,

$$\lim_{t \longrightarrow 0} \int_{0}^{t} i(\tau)d\tau = 0$$

Equivalently, the charge cannot change instantaneously.

Passive Components: Capacitor (3)

- Note that

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau = q_0 + \int_{0}^{t} i(\tau)d\tau$$

- We exclude the possibility of infinite current.
- Therefore,

$$\lim_{t \longrightarrow 0} \int_{0}^{t} i(\tau)d\tau = 0$$

  Equivalently, the charge cannot change instantaneously.
- Conservation of charge:

$$C_1 v_1 = C_2 v_2$$

Passive Components: Capacitor (3)

- Note that

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau = q_0 + \int_{0}^{t} i(\tau)d\tau$$

- We exclude the possibility of infinite current.
- Therefore,

$$\lim_{t \longrightarrow 0} \int_{0}^{t} i(\tau)d\tau = 0$$

  Equivalently, the charge cannot change instantaneously.
- Conservation of charge:

$$C_1 v_1 = C_2 v_2$$

- If C does not change with time, the voltage across a capacitance cannot change instantaneously.

Passive Components: Capacitor (4)

- Recall that the current through a capacitor is $i = C\frac{dv}{dt}$.
- Therefore,

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Cv(t)\frac{dv(t)}{dt} \end{aligned}$$

Passive Components: Capacitor (4)

- Recall that the current through a capacitor is $i = C\frac{dv}{dt}$.
- Therefore,

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Cv(t)\frac{dv(t)}{dt} \end{aligned}$$

- Thus, the change in the stored energy is

Passive Components: Capacitor (4)

- Recall that the current through a capacitor is $i = C\frac{dv}{dt}$.
- Therefore,

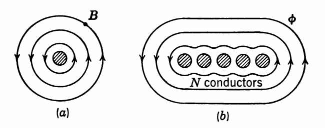
$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Cv(t)\frac{dv(t)}{dt} \end{aligned}$$

- Thus, the change in the stored energy is

$$\begin{aligned} \int_{t_0}^{t_1} p(\tau)d\tau &= C\int_{t_0}^{t_1} v(\tau)\frac{dv(\tau)}{d\tau}d\tau \\ &= \frac{1}{2}C\left(v^2(t_1) - v^2(t_0)\right) \end{aligned}$$

- If the stored energy at time t_0 is zero, then energy stored in the capacitor is $w = \frac{1}{2}Cv^2$ J.

Passive Components: Inductor (5)



- A current carrying conductor exerts a force.
- The magnetic flux density $B = \frac{\mu}{4\pi} \int \frac{i dl \times r}{r^3}$.
- For a coil with N conductors, the magnetic flux $\phi = N \oint B \cdot dl \triangleq Li$, where L is the inductance of the coil.
- The unit of inductance is *henry*[8]; the symbol is H.

[8] In honour of Joseph Henry, American physicist, 1797–1878.

Passive Components: Inductor (6)

- Inductance is the measure of the ability of a device to store energy in the form of an magnetic field.
- A changing magnetic field in one circuit induces a voltage in another:

$$v = \frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

if L does not vary with time; otherwise, $i = L\frac{di}{dt} + i\frac{dL}{dt}$.

Passive Components: Inductor (6)

- Inductance is the measure of the ability of a device to store energy in the form of an magnetic field.
- A changing magnetic field in one circuit induces a voltage in another:

$$v = \frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

if L does not vary with time; otherwise, $i = L\frac{di}{dt} + i\frac{dL}{dt}$.

- Note that

$$\phi(t) = \int_{-\infty}^{t} v(\tau)d\tau = \phi_0 + \int_0^t v(\tau)d\tau$$

- Therefore, $\lim_{t\longrightarrow 0}\int_0^t v(\tau)d\tau = 0$. Equivalently, the flux linkages cannot change instantaneously.

Passive Components: Inductor (6)

- Inductance is the measure of the ability of a device to store energy in the form of an magnetic field.
- A changing magnetic field in one circuit induces a voltage in another:

$$v = \frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

  if L does not vary with time; otherwise, $i = L\frac{di}{dt} + i\frac{dL}{dt}$.
- Note that

$$\phi(t) = \int_{-\infty}^{t} v(\tau)d\tau = \phi_0 + \int_0^t v(\tau)d\tau$$

- Therefore, $\lim_{t \longrightarrow 0} \int_0^t v(\tau)d\tau = 0$. Equivalently, the flux linkages cannot change instantaneously.
- Conservation of flux linkages: $L_1 i_1 = L_2 i_2$.

Passive Components: Inductor (6)

- Inductance is the measure of the ability of a device to store energy in the form of an magnetic field.
- A changing magnetic field in one circuit induces a voltage in another:
$$v = \frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$
if L does not vary with time; otherwise, $i = L\frac{di}{dt} + i\frac{dL}{dt}$.
- Note that
$$\phi(t) = \int_{-\infty}^{t} v(\tau)d\tau = \phi_0 + \int_0^t v(\tau)d\tau$$
- Therefore, $\lim_{t\longrightarrow 0}\int_0^t v(\tau)d\tau = 0$. Equivalently, the flux linkages cannot change instantaneously.
- Conservation of flux linkages: $L_1 i_1 = L_2 i_2$.
- If L does not change with time, the current through an inductance cannot change instantaneously.

Passive Components: Inductor (7)

- Recall that the voltage across an inductor is $v = L\frac{di}{dt}$.
- Therefore,

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Li(t)\frac{di(t)}{dt} \end{aligned}$$

Passive Components: Inductor (7)

- Recall that the voltage across an inductor is $v = L\frac{di}{dt}$.
- Therefore,

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Li(t)\frac{di(t)}{dt} \end{aligned}$$

- Thus, the change in the stored energy is

Passive Components: Inductor (7)

- Recall that the voltage across an inductor is $v = L\frac{di}{dt}$.
- Therefore,

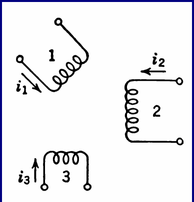
$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Li(t)\frac{di(t)}{dt} \end{aligned}$$

- Thus, the change in the stored energy is

$$\begin{aligned} \int_{t_0}^{t_1} p(\tau)d\tau &= L\int_{t_0}^{t_1} i(\tau)\frac{di(\tau)}{d\tau}d\tau \\ &= \frac{1}{2}L\left(i^2(t_1) - i^2(t_0)\right) \end{aligned}$$

- If the stored energy at time t_0 is zero, then energy stored in the inductor is $w = \frac{1}{2}Li^2$ J.

Passive Components: Mutual Inductance (8)



- Magnetically coupled circuits.
- The currents flowing through one circuit affects own and other circuits.
- For example, the current i_1 results in the flux linkages $\phi_1 = L_1 i_1$ in circuit 1, but as well flux linkages $M_{21} i_1$ and $M_{31} i_1$ in circuits 2 and 3, respectively.
- Here, M_{21} and M_{22} are called mutual inductances.

Passive Components: Mutual Inductance (9)

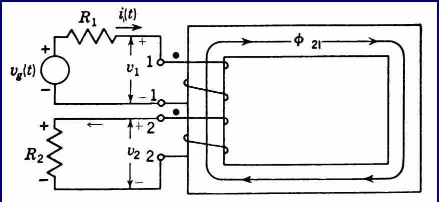
- Assuming the current directions and winding senses are such that the flux linkages are additive,

$$\begin{aligned}
\phi_1 &= L_1 i_1 + M_{12} i_2 + M_{13} i_3 \\
\phi_2 &= M_{21} i_1 + L_2 i_2 + M_{23} i_3 \\
\phi_3 &= M_{31} i_1 + M_{32} i_2 + L_3 i_3
\end{aligned}$$

- Thus,

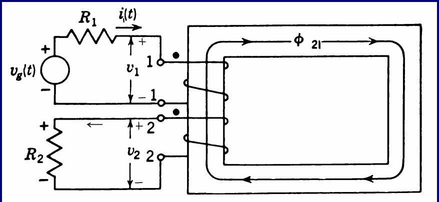
$$\begin{aligned}
v_1 &= L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} \\
v_2 &= M_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M_{23} \frac{di_3}{dt} \\
v_3 &= M_{31} \frac{di_1}{dt} + M_{32} \frac{di_2}{dt} + L_3 \frac{di_3}{dt}
\end{aligned}$$

Passive Components: Dot Convention (10)



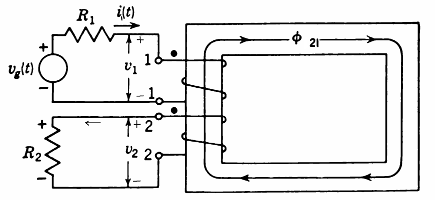
- Transformer: Changing current in one coil induces a voltage in other coils.
- The dot convention helps in ascertaining the voltage directions.

Passive Components: Dot Convention (10)



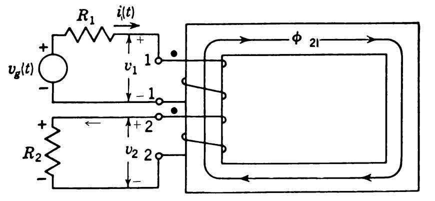
- Transformer: Changing current in one coil induces a voltage in other coils.
- The dot convention helps in ascertaining the voltage directions.
- In the winding 1-1, the current $i_1(t)$ is in the specified direction when a time-varying voltage $v_g(t)$ is connected.
- The positive end of the widing is indicated with a dot.

Passive Components: Dot Convention (11)



- If current flows into the dot of the 1-1 winding, the specified direction of the flux is ascertained by the right-hand rule.

Passive Components: Dot Convention (11)



- If current flows into the dot of the 1-1 winding, the specified direction of the flux is ascertained by the right-hand rule.
- The direction of the induced voltage is obtained by Lenz's law.
- The current caused by this induced voltage is such that it opposes the change in the flux that produced this voltage.
- That is, the current caused by the induced voltage results in the flux ϕ_{12} that opposes ϕ_{21}; i.e., current flows out from that end of the winding marked by the dot.

Passive Components: Dot Convention (12)

Fact

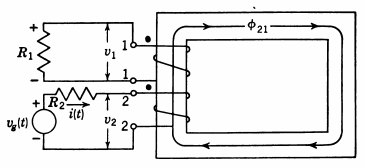
1. *An increasing current into the dotted terminal on one winding induces a voltage in the second winding which is positive at the dotted terminal.*
2. *An increasing current out of a dotted terminal induces a voltage in the second winding which is positive at the undotted terminal.*

Passive Components: Dot Convention (12)

Fact

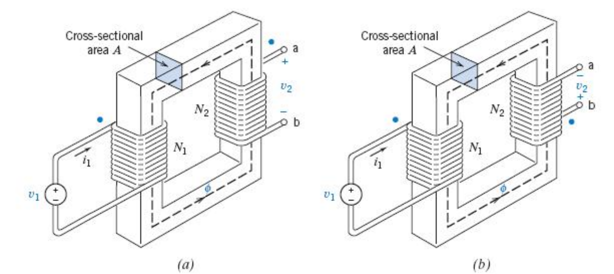
1. *An increasing current into the dotted terminal on one winding induces a voltage in the second winding which is positive at the dotted terminal.*
2. *An increasing current out of a dotted terminal induces a voltage in the second winding which is positive at the undotted terminal.*

Example:

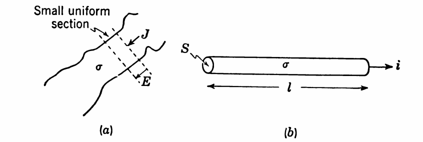


Passive Components: Dot Convention (13)

Example:

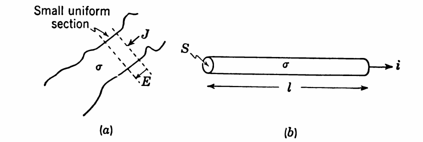


Passive Components: Resistor (14)



- Electrical energy is transformed into thermal energy due to collisions of electrons.
- This loss of electrical energy per unit charge is interpreted as a drop in potential across the device.

Passive Components: Resistor (14)



- Electrical energy is transformed into thermal energy due to collisions of electrons.
- This loss of electrical energy per unit charge is interpreted as a drop in potential across the device.
- Ohm's law: The current density J is proportional to the electric field E: $J = \sigma E$, where σ is the conductivity.
- The current $i = \int_S J \cdot dS = JS$ for a conductor with uniform cross-section.
- Moreover, $v = \int E \cdot dl = El$.

Passive Components: Resistor (15)

- Therefore,

$$v = \frac{l}{\sigma S} i \triangleq Ri$$

where R is called the resistance.
- The unit for resistance is *ohm*[9].
- The reciprocal of resistance is called conductance with unit *mho*.

[9] In honour of Georg Simon Ohm, German physicist and mathematician, 1789–1854. It took 30 years for others to accept his ideas!

Passive Components: Resistor (16)

- Recall that the voltage across a resistor is $v(t) = Ri(t)$.
- Therefore, the power dissipated is

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Ri^2(t) \\ &= \frac{1}{R}v^2(t) \end{aligned}$$

Passive Components: Resistor (16)

- Recall that the voltage across a resistor is $v(t) = Ri(t)$.
- Therefore, the power dissipated is

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Ri^2(t) \\ &= \frac{1}{R}v^2(t) \end{aligned}$$

- Thus, the electrical energy transformed to thermal energy is

$$\int_{t_0}^{t_1} p(\tau)d\tau = R\int_{t_0}^{t_1} i^2(\tau)d\tau$$

Passive Components: Resistor (16)

- Recall that the voltage across a resistor is $v(t) = Ri(t)$.
- Therefore, the power dissipated is

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Ri^2(t) \\ &= \frac{1}{R}v^2(t) \end{aligned}$$

- Thus, the electrical energy transformed to thermal energy is

$$\begin{aligned} \int_{t_0}^{t_1} p(\tau)d\tau &= R\int_{t_0}^{t_1} i^2(\tau)d\tau \\ &= RI^2(t_1 - t_0) \end{aligned}$$

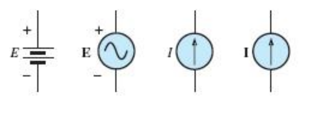
if the current is a constant.

Active Components: Sources (1)

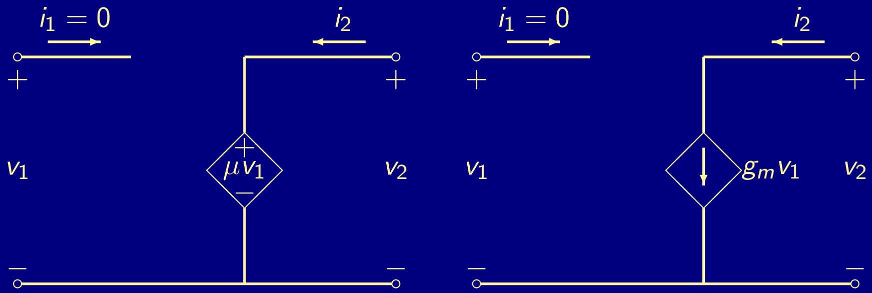
Definition

1. A source is a voltage or current generator capable of supplying energy to a circuit.
2. An independent source is one that is not dependent on other circuit variables.
3. An ideal voltage source is independent of the current through it.
4. An ideal current source is independent of the voltage across it.

Independent sources:

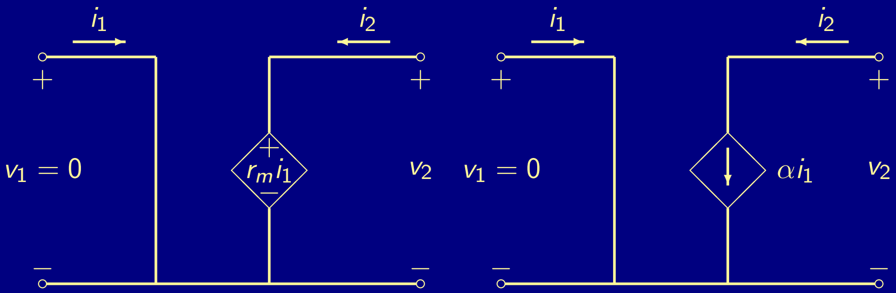


Active Components: Dependent Sources (2)



1. Voltage-controlled voltage source (VCVS).
2. Voltage-controlled current source (VCCS).

Active Components: Dependent Sources (3)



1. Current-controlled voltage source (CCVS).
2. Current-controlled current source (CCCS).

Examples (1)

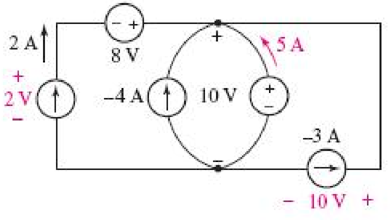


FIGURE 2.31

31. Some of the ideal sources in the circuit of Fig. 2.31 are supplying positive power, and others are absorbing positive power. Determine which are which, and show that the algebraic sum of the power absorbed by each element (taking care to preserve signs) is equal to zero.

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

33. Refer to the circuit represented in Fig. 2.33, while noting that the same current flows through each element. The voltage-controlled dependent source provides a current which is 5 times as large as the voltage V_x. (a) For $V_R = 10$ V and $V_x = 2$ V, determine the power absorbed by each element. (*b*) Is element *A* likely a passive or active source? Explain.

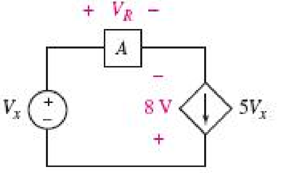


FIGURE 2.33

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (3)

35. The circuit depicted in Fig. 2.34 contains a dependent current source; the magnitude and direction of the current it supplies are directly determined by the voltage labeled v_1. Note that therefore $i_2 = -3v_1$. Determine the voltage v_1 if $v_2 = 33i_2$ and $i_2 = 100$ mA.

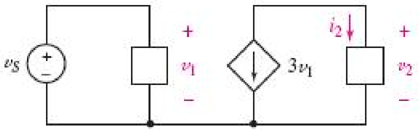


FIGURE 2.34

Source: Hayt, Kemmerly and Durbin, 2012.

Network Analysis and Synthesis

Unit I: Basic Analysis — Physics vs. Mathematical Tractability

Physical Devices and Approximations (1)

$$x(t) \longrightarrow \boxed{f(\cdot)} \longrightarrow y(t) = f(x(t))$$

- All the earlier relationships may be pictorially represented as shown above. For example:
  - Capacitance: $i(t) = C\frac{dv(t)}{dt}$ implies $x(t) = v(t)$ and $y(t) = i(t)$.
  - Inductance: $i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$ implies $x(t) = v(t)$ and $y(t) = i(t)$.
  - Resistance: $v(t) = Ri(t)$ implies $x(t) = i(t)$ and $y(t) = v(t)$.

Physical Devices and Approximations (1)

$$x(t) \longrightarrow \boxed{f(\cdot)} \longrightarrow y(t) = f(x(t))$$

- All the earlier relationships may be pictorially represented as shown above. For example:
  - Capacitance: $i(t) = C\frac{dv(t)}{dt}$ implies $x(t) = v(t)$ and $y(t) = i(t)$.
  - Inductance: $i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$ implies $x(t) = v(t)$ and $y(t) = i(t)$.
  - Resistance: $v(t) = Ri(t)$ implies $x(t) = i(t)$ and $y(t) = v(t)$.
- These relationships are based on approximations!

Physical Devices and Approximations (2)

$$x(t) \longrightarrow \boxed{f(\cdot)} \longrightarrow \quad y(t) = f(x(t))$$

Definition

We say $f(\cdot)$ is linear (or, it satisfies superposition principle) if it satisfies the following properties:

Additivity

$$f(x_1(t) + x_2(t)) = f(x_1(t)) + f(x_2(t))$$

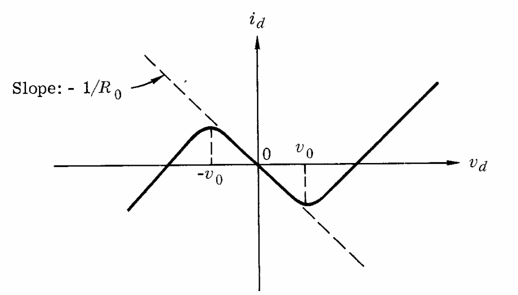
Homogeneity For any scalar α,

$$f(\alpha x(t)) = \alpha f(x(t))$$

Otherwise, $f(\cdot)$ is said to be nonlinear.

Physical Devices and Approximations (3)

Tunnel Diode: Example of a nonlinear device



Physical Devices and Approximations (4)

$$x(t) \longrightarrow \boxed{f(\cdot)} \longrightarrow y(t) = f(x(t))$$

Definition

We say $f(\cdot)$ is time-invariant if, for any $T > 0$,

$$f(x(t-T)) = y(t-T), \quad \forall t \geq T$$

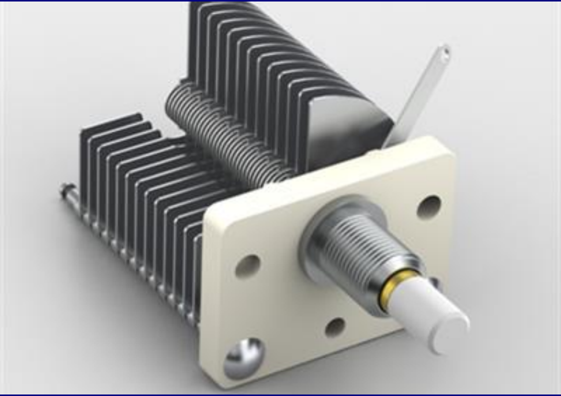
given that

$$f(x(t)) = y(t)$$

Otherwise, it is said to be time-varying.

Physical Devices and Approximations (5)

Example: Varying Capacitor:



Physical Devices and Approximations (6)

Definition

A component is said to be lumped if its physical dimensions are relatively smaller than the wavelengths of the signals during the normal course of operation. Otherwise, they are said to be distributed.

Physical Devices and Approximations (6)

Definition

A component is said to be lumped if its physical dimensions are relatively smaller than the wavelengths of the signals during the normal course of operation. Otherwise, they are said to be distributed.

Examples:

- In an audio circuit, if the highest frequency is 25 kHz, the corresponding wavelength is 12 km. This is much larger than the size of the circuit.

Physical Devices and Approximations (6)

Definition

A component is said to be lumped if its physical dimensions are relatively smaller than the wavelengths of the signals during the normal course of operation. Otherwise, they are said to be distributed.

Examples:

- In an audio circuit, if the highest frequency is 25 kHz, the corresponding wavelength is 12 km. This is much larger than the size of the circuit.
- In a computer operating at 1 GHz, the corresponding wavelength is 0.3m. This is comparable to the physical dimension.

Physical Devices and Approximations (6)

Definition

A component is said to be lumped if its physical dimensions are relatively smaller than the wavelengths of the signals during the normal course of operation. Otherwise, they are said to be distributed.

Examples:

- In an audio circuit, if the highest frequency is 25 kHz, the corresponding wavelength is 12 km. This is much larger than the size of the circuit.
- In a computer operating at 1 GHz, the corresponding wavelength is 0.3m. This is comparable to the physical dimension.
- In a microwave circuit, the wavelengths are between 10 cm and 1mm. Therefore, a cavity resonator is no longer a lumped device.

Physical Devices and Approximations (7)

Assumption

All passive components (in this course) are assumed to be lumped, linear and time-invariant, unless otherwise stated.

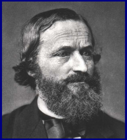
Network Analysis and Synthesis

Unit I: Basic Analysis — Kirchoff's Laws

Kirchhoff's Laws (1)

Network equations are formulated from the following postulates:

- Conservation of energy: Kirchhoff's[10] Voltage Law (KVL).
- Conservation of charge: Kirchhoff's Current Law (KCL).

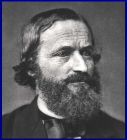


[10] Gustav Robert Kirchchoff, German physicist, 1824–1887. Proposed these laws at the age of 23.

Kirchhoff's Laws (1)

Network equations are formulated from the following postulates:

- Conservation of energy: Kirchhoff's[10] Voltage Law (KVL).
- Conservation of charge: Kirchhoff's Current Law (KCL).



Law

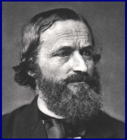
KVL *For any lumped circuit, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loop is zero.*

[10] Gustav Robert Kirchchoff, German physicist, 1824–1887. Proposed these laws at the age of 23.

Kirchhoff's Laws (1)

Network equations are formulated from the following postulates:

- Conservation of energy: Kirchhoff's[10] Voltage Law (KVL).
- Conservation of charge: Kirchhoff's Current Law (KCL).



Law

KVL *For any lumped circuit, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loop is zero.*

Query: What is a loop?

[10] Gustav Robert Kirchchoff, German physicist, 1824–1887. Proposed these laws at the age of 23.

Kirchhoff's Laws (2)

Law

KCL *For any lumped circuit, for any of its cut sets, and at any time, the algebraic sum of all the branch currents traversing the cut-set branches is zero.*

KCL *For any lumped circuit, for any of its nodes, and at any time, the algebraic sum of all the branch currents leaving the node is zero.*

KCL *For any lumped circuit, for any of its nodes, and at any time, the algebraic sum of all the branch currents entering the node is zero.*

Kirchhoff's Laws (2)

Law

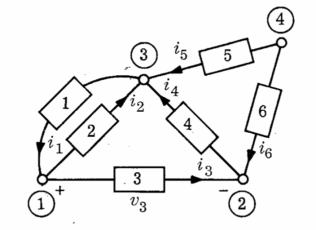
KCL *For any lumped circuit, for any of its cut sets, and at any time, the algebraic sum of all the branch currents traversing the cut-set branches is zero.*

KCL *For any lumped circuit, for any of its nodes, and at any time, the algebraic sum of all the branch currents leaving the node is zero.*

KCL *For any lumped circuit, for any of its nodes, and at any time, the algebraic sum of all the branch currents entering the node is zero.*

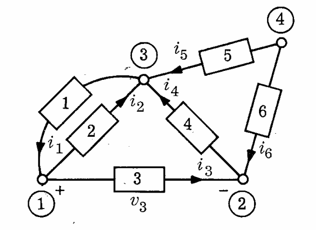
Queries: What is a node? What is a cut-set?

Kirchhoff's Laws: KCL (3)



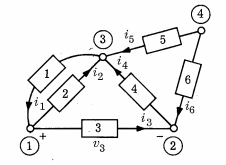
- In a lumped circuit, the two-terminal elements are called branches, and the terminals of these elements are called nodes.
- The example has six branches and 4 nodes.

Kirchhoff's Laws: KCL (3)



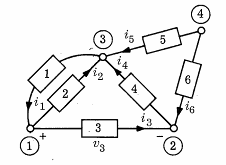
- In a lumped circuit, the two-terminal elements are called branches, and the terminals of these elements are called nodes.
- The example has six branches and 4 nodes.
- The voltage across a branch is called branch voltage, and the current through a branch is called branch current.
- In the example, i_1 is the current through branch 1, and v_3 is the branch voltage associated with branch 3.

Kirchhoff's Laws: KCL (4)



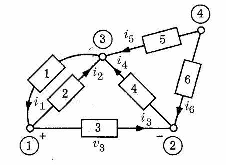
- Associated reference directions for each branch can be assigned arbitrarily.

Kirchhoff's Laws: KCL (4)



- Associated reference directions for each branch can be assigned arbitrarily.
- Applying KCL to node 2,

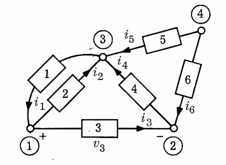
Kirchhoff's Laws: KCL (4)



- Associated reference directions for each branch can be assigned arbitrarily.
- Applying KCL to node 2,

$$-i_3(t) + i_4(t) - i_6(t) = 0 \quad \forall\, t$$

Kirchhoff's Laws: KCL (4)

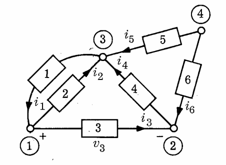


- Associated reference directions for each branch can be assigned arbitrarily.
- Applying KCL to node 2,

$$-i_3(t) + i_4(t) - i_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KCL to node 3,

Kirchhoff's Laws: KCL (4)



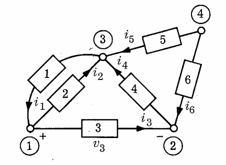
- Associated reference directions for each branch can be assigned arbitrarily.
- Applying KCL to node 2,

$$-i_3(t) + i_4(t) - i_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KCL to node 3,

$$i_1(t) - i_2(t) - i_4(t) - i_5(t) = 0 \quad \forall\, t$$

Kirchhoff's Laws: KCL (4)



- Associated reference directions for each branch can be assigned arbitrarily.
- Applying KCL to node 2,

$$-i_3(t) + i_4(t) - i_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KCL to node 3,

$$i_1(t) - i_2(t) - i_4(t) - i_5(t) = 0 \quad \forall\, t$$

- These equations are called node equations.

Kirchhoff's Laws: KCL (5)

Comments:

- The node equations are linear homogeneous algebraic equations in the branch currents, and the coefficients are constants.
- KCL is independent of the nature of the elements as long as they are lumped.
- KCL implies that charges cannot accumulate at any node — conservation of charge at every node.

Examples (1)

7. Referring to the single node diagram of Fig. 3.49, compute:

(a) i_B, if $i_A = 1$ A, $i_D = -2$ A, $i_C = 3$ A, and $i_E = 0$;

(b) i_E, if $i_A = -1$ A, $i_B = -1$ A, $i_C = -1$ A, and $i_D = -1$ A.

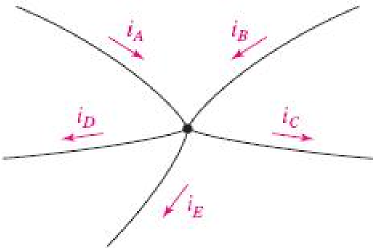


FIGURE 3.49

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

9. In the circuit shown in Fig. 3.51, the resistor values are unknown, but the 2 V source is known to be supplying a current of 7 A to the rest of the circuit. Calculate the current labeled i_2.

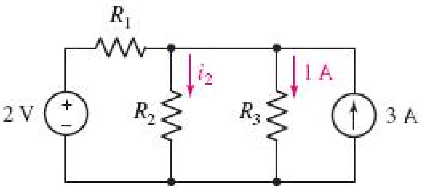


FIGURE 3.51

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (3)

11. In the circuit depicted in Fig. 3.53, i_x is determined to be 1.5 A, and the 9 V source supplies a current of 7.6 A (that is, a current of 7.6 A leaves the positive reference terminal of the 9 V source). Determine the value of resistor R_A.

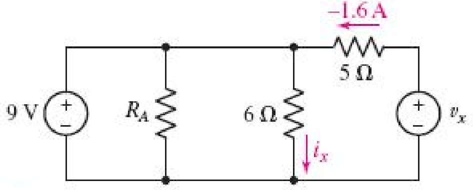
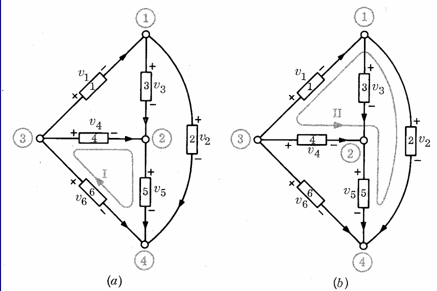


FIGURE 3.53

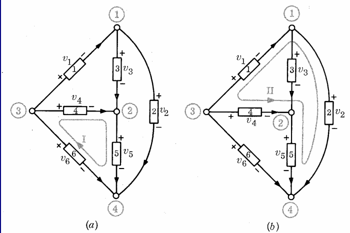
Source: Hayt, Kemmerly and Durbin, 2012.

Kirchhoff's Laws: KVL (6)



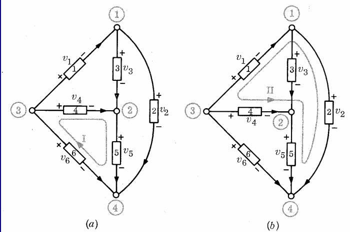
- A path is formed by starting at one node, traversing one or more branches in succession, and ending at another node.
- A closed path is a path whose starting node is the same as the ending node.
- A loop is a closed path.

Kirchhoff's Laws: KVL (7)



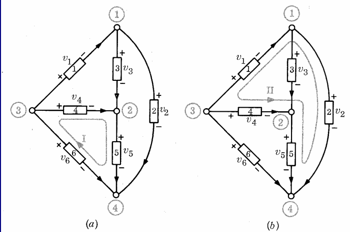
- The example illustrates 2 loops.
- A reference direction is assigned arbitrarily.
- A postive sign is assigned to those branch voltages whose reference directions agree with that of the loop.
- A negative sign is assigned to those branch voltages whose reference directions do not agree with that of the loop.

Kirchhoff's Laws: KVL (8)



- Applying KVL to loop I,

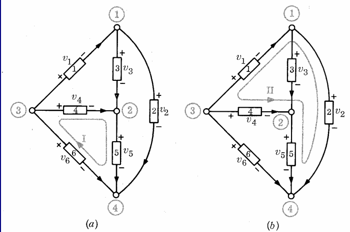
Kirchhoff's Laws: KVL (8)



- Applying KVL to loop I,

$$v_4(t) + v_5(t) - v_6(t) = 0 \quad \forall\, t$$

Kirchhoff's Laws: KVL (8)

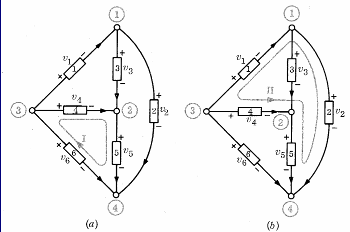


- Applying KVL to loop I,

$$v_4(t) + v_5(t) - v_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KVL to loop II,

Kirchhoff's Laws: KVL (8)



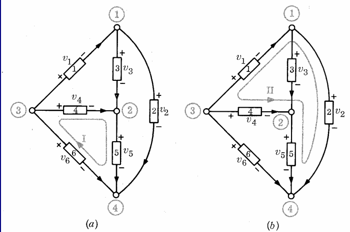
- Applying KVL to loop I,

$$v_4(t) + v_5(t) - v_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KVL to loop II,

$$-v_1(t) + v_4(t) + v_5(t) - v_2(t) = 0 \quad \forall\, t$$

Kirchhoff's Laws: KVL (8)



- Applying KVL to loop I,

$$v_4(t) + v_5(t) - v_6(t) = 0 \quad \forall\, t$$

- Similarly, applying KVL to loop II,

$$-v_1(t) + v_4(t) + v_5(t) - v_2(t) = 0 \quad \forall\, t$$

- These equations are called loop equations.

Kirchhoff's Laws: KVL (9)

Comments:

- The loop equations are linear homogeneous algebraic equations in the branch voltages, and the coefficients are constants.
- KVL is independent of the nature of the elements as long as they are lumped.
- KVL implies conservation of energy across any loop.

Examples (1)

19. In the circuit of Fig. 3.60, it is determined that $v_1 = 3$ V and $v_3 = 1.5$ V. Calculate v_R and v_2.

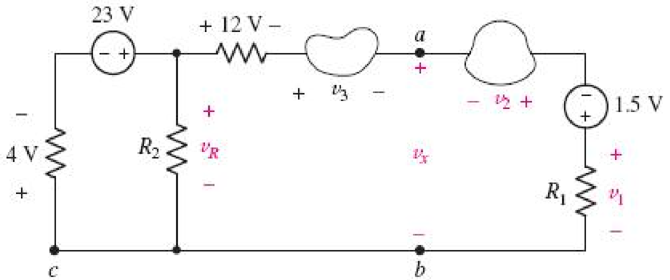


FIGURE 3.60

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (2)

21. Determine the value of v_x as labeled in the circuit of Fig. 3.61.

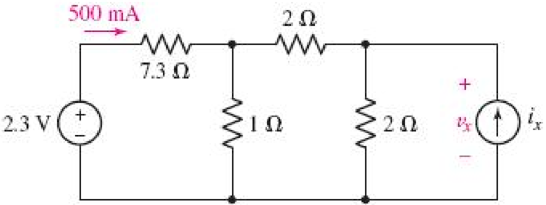


FIGURE 3.61

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (3)

23. (*a*) Determine a numerical value for each current and voltage (i_1, v_1, etc.) in the circuit of Fig. 3.63. (*b*) Calculate the power absorbed by each element and verify that they sum to zero.

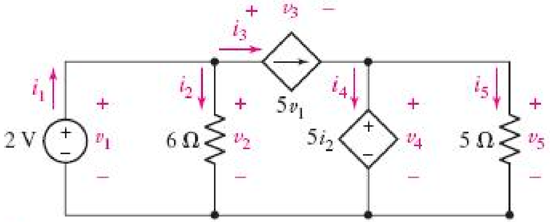
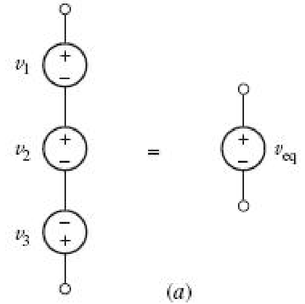


FIGURE 3.63

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (4)

35. Determine the numerical value for v_{eq} in Fig. 3.74a, if (a) $v_1 = 0$, $v_2 = -3$ V, and $v_3 = +3$ V; (b) $v_1 = v_2 = v_3 = 1$ V; (c) $v_1 = -9$ V, $v_2 = 4.5$ V, $v_3 = 1$ V.



(a)

Source: Hayt, Kemmerly and Durbin, 2012.

Examples (5)

39. (*a*) For the circuit of Fig. 3.77, determine the value for the voltage labeled v, after first simplifying the circuit to a single current source in parallel with two resistors. (*b*) Verify that the power supplied by your equivalent source is equal to the sum of the supplied powers of the individual sources in the original circuit.

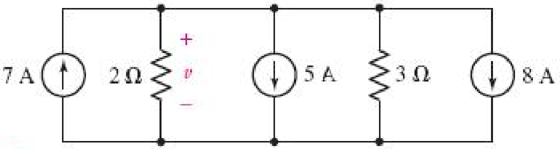


FIGURE 3.77

Source: Hayt, Kemmerly and Durbin, 2012.