

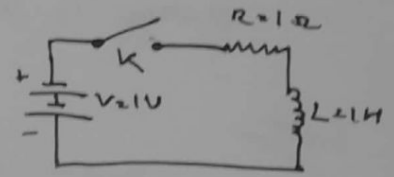
- 24 In the circuit shown,  $K$  is closed at  $t=0$ . The current flowing in the circuit is given by the equation  $i = (1 - e^{-t})$  amp,  $t > 0$ . At a certain time the current has a value of  $0.63$  amp.
- At what rate is the current changing?
  - What is the value of total flux linkage?
  - What is rate of change of flux linkage?
  - What is the voltage across inductor?
  - How much energy is stored in magnetic field of inductor?
  - What is the voltage across the resistor?
  - Rate at which energy is being stored.
  - At what rate energy is being dissipated?
  - At what rate battery is supplying energy.

Soln

Given  $i(t) = 0.63 = 1 - e^{-t}$

$\therefore e^{-t} = 0.37$

$t = -\ln 0.37 = 0.994 \text{ sec}$



a) rate of change of current,  $\frac{di}{dt} = e^{-t} = e^{-0.994} = 0.37 \text{ A/sec}$

b)  $\phi = L i(t) = 1(1 - e^{-t}) = 0.67 \text{ weber (at } t = 0.994 \text{ sec)}$

c) rate of change of flux linkage,  $\frac{d\phi}{dt} = e^{-t} = 0.37 \text{ V}$

d) Using KVL:  $-V + iR + V_L = 0$   
 $V_L = 1 - 1(0.67) = 0.37 \text{ V}$

e) energy stored by inductor,  $E = \frac{1}{2} L I^2 = \frac{1}{2} \times 1 \times 0.63^2 = 0.1984 \text{ J}$

f) Voltage across resistor,  $V_R = iR = 0.63 \times 1 = 0.63 \text{ V}$

g) Rate at which energy is being stored,  $P_L = \frac{E}{t} = \frac{0.198}{0.994} = 0.199 \text{ J/s}$

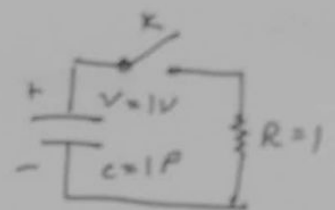
h) Rate at which energy is being dissipated,  $P_R = \frac{I^2 R}{t} = \frac{(0.63)^2}{0.994} = 0.39 \text{ W}$

i) Rate at which battery is supplying energy,  $P_B = VI = 1 \times 0.63 = 0.63 \text{ W}$

1-25 In the circuit shown the capacitor is charged to a voltage of 1V and at  $t=0$  the switch  $K$  is closed. The current in the circuit is known to be of form  $i(t) = e^{-t}$  amp  $t \geq 0$  at a certain time the current has a value of 0.37 A. a) At what rate voltage across the capacitor is changing b) what is the value of charge on the capacitor c) what is the rate of change of product  $CV$ ? d) What is the voltage across capacitor? e) How much energy is stored in electric field of capacitor? f) What is voltage across resistor? g) At what rate is energy being taken from the electric field of the capacitor? h) At what rate energy is being dissipated as heat.

Soln

Given  $V = 1V$   $C = 1F$   
 $i(t) = e^{-t}$  at  $t = 0.994 \text{ sec}$



a)  $i = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{i}{C} \Rightarrow \frac{0.37}{1} = 0.37 \text{ V/sec}$

b)  $C = \frac{Q}{V}$  charge of capacitor  $Q = CV = 1 \times 1 = 1 \text{ coulomb}$

c)  $\frac{d(CV)}{dt} = \frac{dQ}{dt} = i(t) = e^{-t} \Rightarrow 0.37$

d) Using KVL:-

$$-V_C + iR = 0$$

$$V_C = 0.37 \times 1 = 0.37 \text{ V}$$

e) energy stored in the capacitor  $E = \frac{1}{2} CV_C^2 = \frac{1}{2} \times 1 \times 0.37^2 = 0.0684 \text{ J}$

f) voltage across the resistor  $V_R = iR = 0.37 \text{ V}$

g) Rate of energy being taken from capacitor  $P = \frac{dE}{dt} = \frac{0.0684}{0.994} = 0.0688$

h) Rate of energy being dissipated as heat  $P_r = I^2 R = 0.37^2 = 0.1369 \text{ W}$

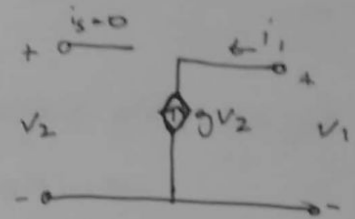
1 For the controlled source shown in figure, prepare a plot similar to that given

Sol From the above figure we can see that

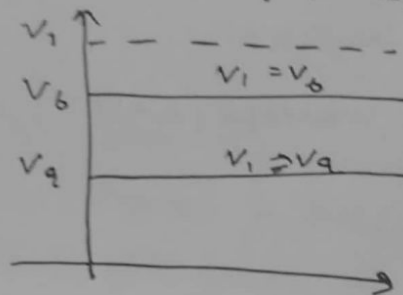
$$V_2 = gV_1$$

when  $V_1 = V_a$   
 $V_2 = gV_a$

> when  $V_1 = V_b$   
 $V_2 = gV_b$



hence we can plot the graph as follows:-



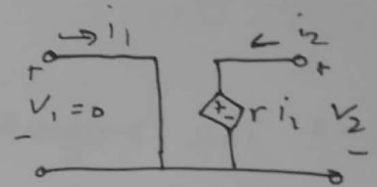
2-2 Repeat problem 2-1 for the controlled source given in below figure

Sol From the above figure we can see that

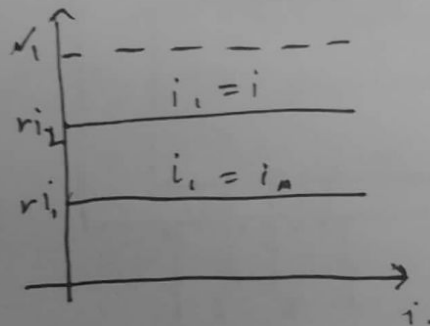
$$V_2 = ri_1$$

when  $i_1 = i_a$   
 $V_2 = ri_a$

when  $i_1 = i_b$   
 $V_2 = ri_b$



hence we can plot the graph as follows:-

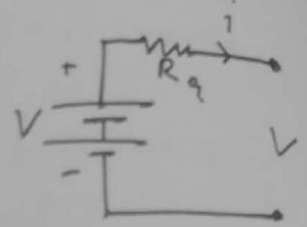


2-3 The circuit of the figure is a model for a battery of emf  $\mathcal{E}$  terminal voltage  $V$  & internal resistance  $R_b$ . For this circuit is a function of  $i$ . Identify features of the plot such as slopes, intercepts on.

By applying KVL in the above loop we get:-

$$-V + iR_b + V = 0$$

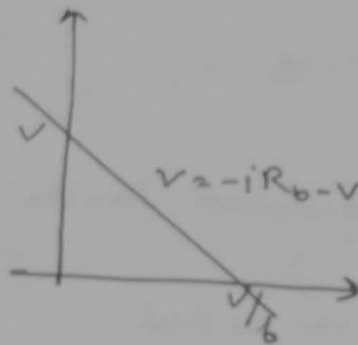
$$V = -iR_b + V$$



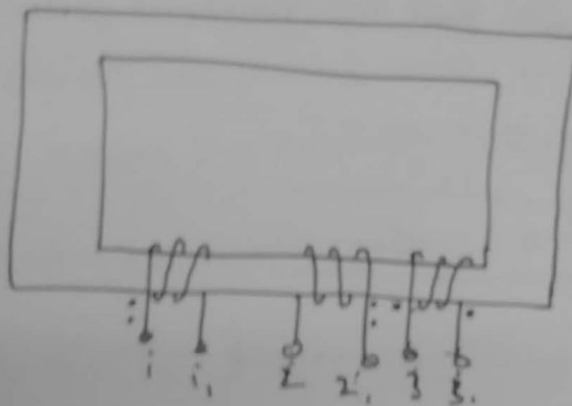
The above equation is in the form

$$y = mx + c$$

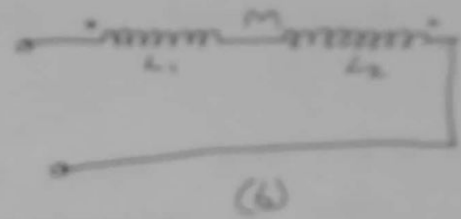
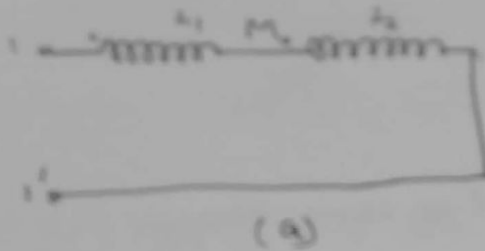
$\therefore$  the slope  $m = -R_b$  & the intercept,  $c = V$   
Therefore the graph is



2-4 The magnetic system shown has three windings mark 1-1', 2-2' and 3-3'. Using different forms of dots establish polarity marking



Each schematic in figure shows two inductors with coupling but with different dot marking. For each of the two systems determine the equivalent inductance of the system at terminals 1-1' by combining inductances



Ans

a) In fig (a) when the current enters the dotted terminal of  $L_1$  & then leaves & then enters through  $L_2$  the inductance of the flux gets adds up due same polarity.

$$e_1 = L_1 \frac{di}{dt} + M \frac{di}{dt} = (L_1 + M) \frac{di}{dt}$$

similarly,  $e_2 = (L_2 + M) \frac{di}{dt}$

$$e = e_1 + e_2 = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$\frac{e}{\frac{di}{dt}} = L = L_1 + L_2 + 2M$$

$$\text{eqn :- } L_1 + L_2 + 2M.$$

b) In fig (b) when current enters the dotted end of  $L_1$  & leaves it will enter the non dotted terminals resulting in opposite polarity & in turn subtracting flux

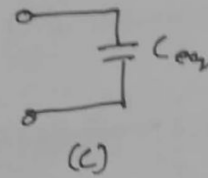
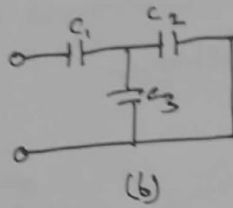
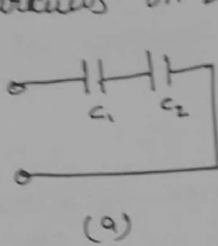
$$e_1 = L_1 \frac{di}{dt} - M \frac{di}{dt} = (L_1 - M) \frac{di}{dt}$$

$$e_2 = (L_2 - M) \frac{di}{dt}$$

$$e = e_1 + e_2 = (L_1 + L_2 - 2M) \frac{di}{dt}$$

$$\text{eqn. } L = L_1 + L_2 - 2M$$

3-1 What must be relationship between  $C_{eq}$  &  $C_1$  &  $C_2$  in circuits in 2(a) & (b) equivalent repeat for circuit in (c)



Soln a) In the circuit (a) current is same through  $C_1$  &  $C_2$  &  $V_0$  can be given by.

$$V_1 = \frac{1}{C_1} \int i dt \quad \& \quad V_2 = \frac{1}{C_2} \int i dt$$

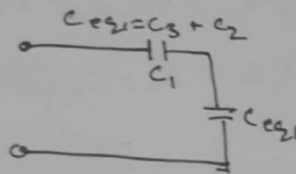
$\therefore$   $V$  is the voltage across  $C_{eq}$  :-

$$V = V_1 + V_2$$

$$\frac{1}{C_{eq}} \int i dt = \frac{1}{C_1} \int i dt + \frac{1}{C_2} \int i dt$$

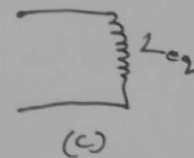
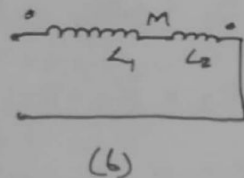
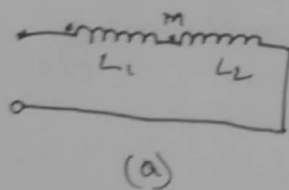
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

b) In the figure (b) we can see current through each capacitor is different, but  $C_2$  &  $C_3$  are in parallel with each other.



$$C_2 \& C_{eq1} \text{ are in series } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{eq1}} \\ = \frac{1}{C_1} + \frac{1}{C_2 + C_3}$$

3-2 What must be relationship between  $L_{eq}$  &  $L_1$  &  $L_2$  &  $M$  for the circuits of Fig (a) and (b) to be equivalent with (c)?



Soln a) In fig (a) current enters through dotted terminal of  $L_1$  & then enters dotted terminal of  $L_2$  :-

$$\therefore e = e_1 + e_2 = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2M \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + 2M$$

5 a) In fig (b) where the current enters dotted terminal at  $L_1$  but undotted terminal at  $L_2$  :-

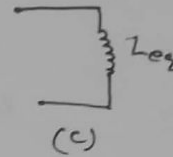
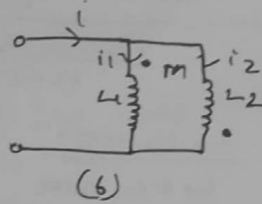
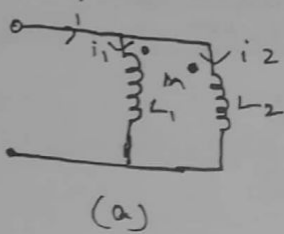
$\therefore$

$$e = e_1 + e_2$$

$$e = \frac{L_1 di_1}{dt} + \frac{L_2 di_1}{dt} - \frac{2m di_1}{dt}$$

$$L_{eq} = L_1 + L_2 - 2m$$

3-3 Repeat Prob 3-2 for below three circuits



a) In fig (a) :-  $e_1 = L_1 \frac{di_1}{dt} + m \frac{di_2}{dt}$   
 $= L_1 \frac{di_1}{dt} + m \frac{d(i - i_1)}{dt} \quad \therefore i = i_1 + i_2$

$$e_2 = L_2 \frac{di_2}{dt} + m \frac{di_1}{dt}$$

$$= L_2 \frac{d(i - i_1)}{dt} + m \frac{di_1}{dt}$$

$$e = e_1 + e_2 \Rightarrow L_1 \frac{di_1}{dt} + L_2 \frac{di_1}{dt} + L_2 \frac{di}{dt} - m \frac{di_1}{dt} + m \frac{di}{dt} - m \frac{di_1}{dt}$$

$$\Rightarrow (L_1 + L_2) \frac{di_1}{dt} + (L_2 + m) \frac{di}{dt} = 0$$

$$\Rightarrow (L_1 + L_2 - 2m) \frac{di_1}{dt} = m(L_2 - m) \frac{di}{dt}$$

$$= \frac{di_1 (L_1 + L_2 - 2m)}{L_2 - m} = \frac{di}{dt}$$

b) In fig (b) :-  $e_1 = L_1 \frac{di_1}{dt} - m \frac{di_2}{dt} \Rightarrow L_1 \frac{di_1}{dt} - m \frac{d(i - i_1)}{dt}$

$$e_2 = L_2 \frac{di_2}{dt} - m \frac{di_1}{dt} \Rightarrow L_2 \frac{d(i - i_1)}{dt} - m \frac{di_1}{dt}$$

$$e_1 = e_2$$

$$L_1 \frac{di_1}{dt} - m \frac{di}{dt} + m \frac{di_1}{dt} = L_2 \frac{di}{dt} - L_2 \frac{di_1}{dt} - m \frac{di_1}{dt}$$

$$\Rightarrow \left( \frac{L_1 + L_2 + 2m}{m + L_2} \right) \frac{di_1}{dt} = \frac{di}{dt}$$

$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 + 2m}$$

$$e = e_1$$

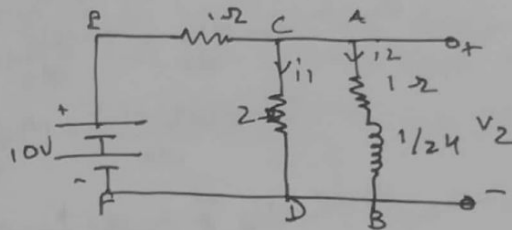
$$L_{eq} \frac{di}{dt} = L_1 \frac{di_1}{dt} - m \frac{di_2}{dt}$$

$$L_{eq} = \frac{L_1 \frac{di_1}{dt} - m \frac{di_2}{dt}}{\frac{di}{dt}}$$

$$L_{eq} = \frac{L_1 \frac{di_1}{dt} - m \frac{di_1}{dt} \left( \frac{L_1 + L_2 + 2m}{L_2 + m} \right)}{\left( \frac{L_1 + L_2 + 2m}{L_2 + m} \right) \frac{di_1}{dt}}$$

$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 + 2m}$$

- 3-6 The problem may be solved using the two Kirchhoff laws and voltage current relationships for the elements. Refer to Fig. A + time. to after the switch K was closed, it is found that  $V_2$  Determine the values of  $i_2(t_0)$  and  $\frac{di_2(t_0)}{dt}$



Soln

Given  $V_2 = +5V$

Let  $V_{AB}$  be the voltage across AB

$$V_{AB} = Ri_2 + L \frac{di_2}{dt}$$

$$= i_2 + \frac{1}{2} \frac{di_2}{dt}$$

Since  $V_2$  is in parallel with AB we can say that

$$V_{AB} = V_2$$

$$i_2 + \frac{1}{2} \frac{di_2}{dt} = 5$$

$$\frac{1}{2} \frac{di_2}{dt} = 5 - i_2$$

$$\frac{di_2}{dt} = 10 - 2i_2$$

$$\frac{di_2}{dt} = \frac{2(5 - i_2)}{1}$$



$$\int_0^t \frac{di_2}{5-i_2} = \int_0^t 2 dt$$

$$\ln(5-i_2) - \ln(5) = 2t$$

$$\ln(5-i_2) = 2t + \ln 5$$

$$5-i_2 = e^{2t + \ln(5)}$$

$$i_2 = 5 - 5e^{-2t}$$

$$\text{Let } V_{CD} = 10 - i_1$$

We know that  $V_{CD}$  is parallel with  $V_2$

$$10 - i_1 = 5$$

$$i_1 = 5$$

by applying KVL in loop CDEF

$$10 - 5 \times 1 = 2i_1$$

$$5 = 2i_1$$

$$i_1 = 2.5$$

by applying KCL at node C

$$i = i_1 + i_2$$

$$i_2(t_0) = 7.5 \text{ A}$$

$$\textcircled{1} \Rightarrow 5 - 5e^{-2t} = 2.5$$

$$5e^{-2t} = 2.5$$

$$t = 0.3465 \text{ sec}$$

$$\frac{di(t_0)}{dt} = \frac{-10e^{-2t}}{1}$$

$$= -5 \text{ A/sec.}$$

3-7 This problem is similar to 3-6. In the circuit given in fig :- if it is given that  $v_2(t_0) = 2 \text{ V}$  &  $(dv_2/dt)(t_0) = -10 \text{ V/sec}$ , where  $t_0$  is the time at which the switch  $K$  was closed. Determine the value of  $C$

$$V_{CD} \text{ the potential} = 3 - 2i$$

$$V_{CD} = V_2 \text{ parallel}$$

$$3 - 2i = 2$$

$$2i = 1$$

$$i = 0.5 \text{ A}$$

Let  $V_R$  be potential drop across  $R = 1 \Omega$

$$V_R = V_2$$

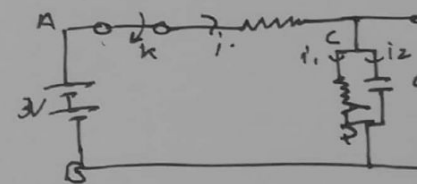
$$1 \times I_1 = V_2 = 2$$

$$i_1 = 2 \text{ A}$$

by using KCL in node C

$$i = i_1 + i_2$$

$$i_2 = 0.6 - 2 = -1.4 \text{ A}$$



We know that for a capacitor

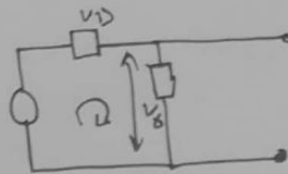
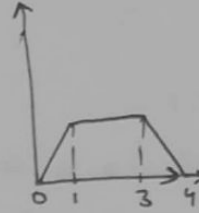
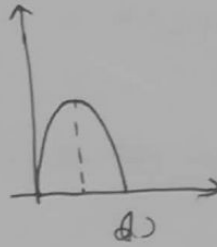
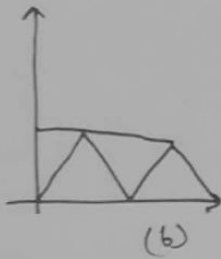
$$i_2 = C \frac{dV}{dt}$$

$$-1.5 = C(-10)$$

$$\Rightarrow C = 0.15 \text{ F}$$

3-12

$$R = 2 \Omega \quad C = 1 \text{ F}$$



Since  $V_b$  is in parallel with  $V_2$  we can say that

$$V_b = V_2$$

By KVL

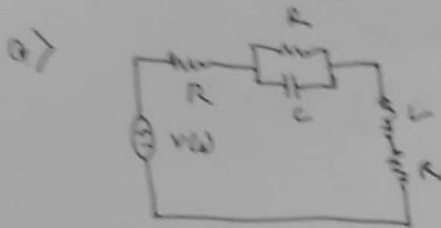
$$V_1 + V_A + V_B = 0$$

$$V_1 - iR + V_B$$

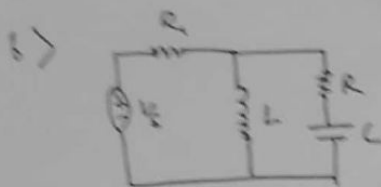
$$V_2 = V_1 - 2i - V_2$$

b)

For each of the following circuit shown in Fig. determine the number of independent loop currents and the number of independent node-to-node voltages that may be used in writing equations for Kirchhoff laws.



Number of independent loop current - 3  
Number of independent node to node voltage - 4



Number of independent loop current - 3  
Number of independent node to node voltage - 3

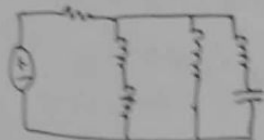


Number of independent loop current - 3  
Number of independent node to node voltage - 2



Number of independent loop current - 4  
Number of independent node to node voltage - 4

3-24 The circuit of Fig. 3-24 but with different loop current variables chosen. Using specified currents, write the Kirchhoff voltage law equations for the circuit.



by applying KVL loop 1

$$R_2(i_1 - i_2) - v(t) + R_1 i_1 + L_1 \frac{d(i_1 - i_2)}{dt} + M_{12} \frac{d(i_2 - i_3)}{dt} - M_{13} \frac{di_3}{dt} = 0$$

by apply KVL loop 2

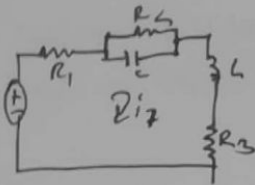
$$+ M_{13} \frac{di_3}{dt} - M_{12} \frac{d(i_2 - i_3)}{dt} + R_2(i_2 - i_1) + L_2 \frac{d(i_2 - i_1)}{dt} + L_3 \frac{d(i_2 - i_3)}{dt} + M_{23} \frac{d(i_1 - i_3)}{dt} = 0$$

by applying KVL in loop 3

$$M_{23} \frac{di_3}{dt} - M_{21} \frac{d(i_1 - i_2)}{dt} + L_3 \frac{d(i_2 - i_3)}{dt} + L_4 \frac{di_3}{dt} + \frac{1}{C} \int i_3 dt - M_{31} \frac{d(i_1 - i_2)}{dt} = 0$$

3-21

q)



Applying KVL

$$R_2 i_1 = \frac{1}{C} \int (i_1 - i_2) dt$$

Applying KVL in loop 2 we get

$$V(t) - R_1 i_2 - \frac{1}{C} \int (i_2 - i_1) dt - L \frac{di_2}{dt} - R_3 i_2 = 0$$

6)



Applying KVL in loop 1 we get

$$V(t) - i_1 R_1 - L \frac{d(i_1 - i_2)}{dt} = 0$$

Applying KVL in loop 2

$$L \frac{d(i_2 - i_1)}{dt} + R_2 i_2 + \frac{1}{C} \int i_2 dt = 0$$

c)

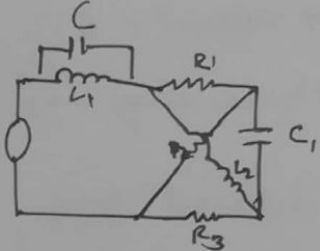


Applying KVL

$$V(t) - L \frac{di_1}{dt} - R(i_1 - i_2) = 0$$

Applying loop 2

$$R(i_2 - i_1) + \frac{1}{C} \int i_2 dt = 0$$



KVL

Loop 1:  $V(t) - L_1 \frac{d(i_1 - i_4)}{dt} - R_1(i_1 - i_2) - \frac{1}{C_1} \int (i_1 - i_2 - i_3) dt - R_3(i_1 - i_3) = 0$

2:  $R_1(i_2 - i_1) + \frac{1}{C_2} \int (i_2 - i_1 - i_3) dt + L_2 \frac{di_2}{dt} = 0$

3:  $R_2 i_3 + R_3(i_3 - i_1) + \frac{1}{C_2} \int (i_3 - i_1 - i_2) dt = 0$

4:  $L_1 \frac{d(i_4 - i_1)}{dt} + \frac{1}{C_1} \int i_4 dt = 0$

for the circuit shown in Fig



$$\text{node 4: } \frac{v_4}{R} + C \frac{d(v_4 - v_2)}{dt} + \frac{v_4 - v_1}{R} = \frac{v_4 - v_3}{R}$$

$$2(v_4 - v_2) + 2v_4 + \frac{1}{2} \frac{d(v_4 - v_3)}{dt} + 2(v_4 - v_1) = 0$$

$$\text{node 3: } C \frac{d(v_3 - v_1)}{dt} + i_2 + \frac{dv_3}{dt} + C \frac{d(v_3 - v_4)}{dt} = 0$$

$$i_2 + \frac{1}{2} \frac{d(v_3 - v_1)}{dt} + \frac{1}{2} \frac{dv_3}{dt} + \frac{1}{2} \frac{d(v_3 - v_4)}{dt} = 0$$

$$\text{node 2: } -i_2 + \frac{v_2}{R_6} + \frac{v_2 - v_1}{R} = 0 + \frac{v_2 - v_4}{R} = 0$$

$$\text{node 1: } \frac{v_1 - v_4}{R} + \frac{v_1 - v_2}{R} = C \frac{d(v_1 - v_3)}{dt} = 0$$

$$4v_1 - 2v_2 + \frac{1}{2} \frac{d(v_1 - v_3)}{dt} - 2v_4 = 0$$

node 1, 2, 3, 4 is node equation.

3-58 Consider the circuit shown in Fig



$$v_1 = iR_1$$

$$v_2 = iR_2$$

by KVL we get

$$V = v_1 + v_2$$