EXAMPLE 13.11 *Obtain the open circuit parameters and loop equations of the network shown in Fig. E13.14.*

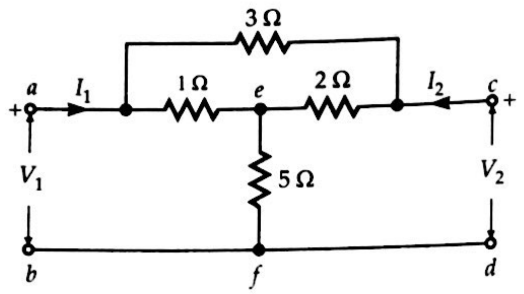

Fig. E13.14

SOLUTION. Let us first assume the output terminals (c-d) are open circuited and V_1 is applied at a-b terminals (input port). The circuit configuration has been shown in Fig. E13.15. The equivalent resistance looking from the terminals a-b is given as

$$R_{eq1} = [(3+2)\,||\,1] + 5 = \frac{35}{6}\,\Omega$$

$$\therefore \quad V_1 = I_1 \times R_{eq1} = \frac{35}{6} I_1.$$

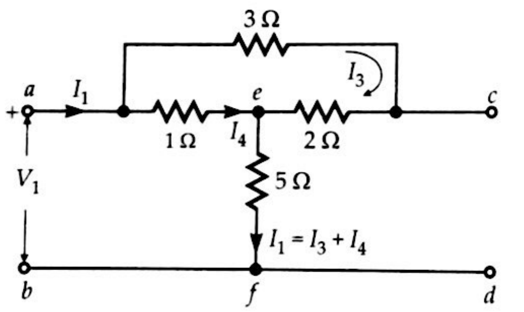

Fig. E13.15

$$\text{Hence } Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{35}{6}\,\Omega.$$

$$\text{Again, } \quad I_3 = I_1 \frac{1}{1+3+2} = \frac{1}{6} I_1$$

Thus drop V_2 across terminal c-d is given by

$$I_3 \times 2 + I_1 \times 5 = V_2$$

i.e., $$\frac{I_1}{3} + I_1 \times 5 = V_2 \qquad \left[\because \frac{I_1}{6} = I_3\right]$$

or $$\left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_{21} = \frac{16}{3}\,\Omega$$

Let us now open terminals a-b (input port) applying the voltage V_2 at terminals c-d (output port). The circuit has been shown in Fig. E13.16.

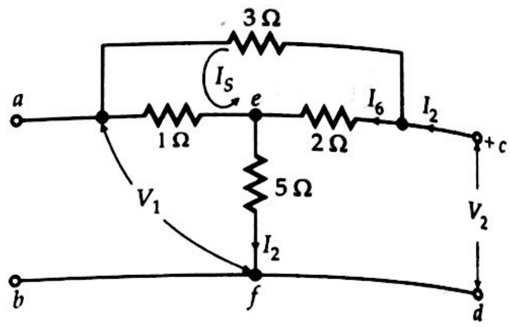

Fig. E13.16

The equivalent resistance of the network looking from output port (terminal c-d) is given by

$$R_{eq2} = [(3+1)\,||\,2] + 5 = \frac{19}{3}\,\Omega$$

$$\therefore \quad V_2 = I_2 \times R_{eq2} = \frac{10}{3} I_2$$

i.e. $$\left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_{22} = \frac{10}{3}\,\Omega.$$

$$\text{Again, } \quad I_5 = I_2 \frac{2}{2+3+1} = \frac{I_2}{3}\text{ A}$$

$$\text{Hence, } \quad V_1 = \left(\frac{I_2}{3}\right) \times 1 + 5I_2 = \frac{16}{3} I_2$$

or $$\left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_{12} = \frac{16}{3}\,\Omega.$$

$$\therefore \quad \mathbf{Z_{11}} = \frac{35}{6}\,\Omega;\ \mathbf{Z_{12}} = \frac{16}{3}\,\Omega$$

$$\mathbf{Z_{21}} = \frac{16}{3}\,\Omega;\ \mathbf{Z_{22}} = \frac{19}{3}\,\Omega.$$

EXAMPLE 13.12 *A small signal equivalent circuit of CB mode of BJT is shown in Fig. E13.17. Assuming the voltage sources to be independent, find the Z-parameters and draw the equivalent circuit.*

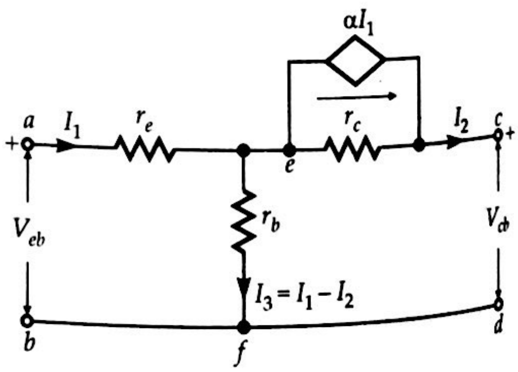

Fig. E13.17

SOLUTION. Applying KVL to the loops,

EXAMPLE 13.17 *An π attenuator has been shown in Fig. E13.25. Find the Y-parameters and draw the equivalent Y-parameter circuit.*

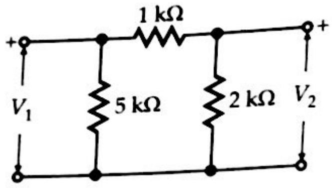

Fig. E13.25

SOLUTION. The values of the arm impedences of the π network are given in ohms. In form of admittances, Fig. E13.25 is redrawn as in Fig. E13.26.

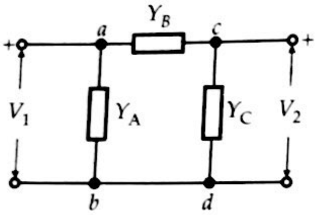

Fig. E13.26

$$\therefore \quad Y_A = \frac{1}{5\,k\Omega} = 0.2 \times 10^{-3} \text{ mho}$$

$$Y_B = \frac{1}{1\,k\Omega} = 10^{-3} \text{ mho}$$

$$Y_C = \frac{1}{2\,k\Omega} = 0.5 \times 10^{-3} \text{ mho.}$$

The Y-parameters are

$$\mathbf{Y_{11} = Y_A + Y_B = 1.2 \times 10^{-3} \text{ mho}}$$
$$\mathbf{Y_{12} = -10^{-3} \text{ mho}}$$
$$\mathbf{Y_{21} = -10^{-3} \text{ mho}}$$
$$\mathbf{Y_{22} = Y_B + Y_C = 1.5 \times 10^{-3} \text{ mho.}}$$

The Y-parameter equivalent circuit has been shown in **Fig. E13.27.**

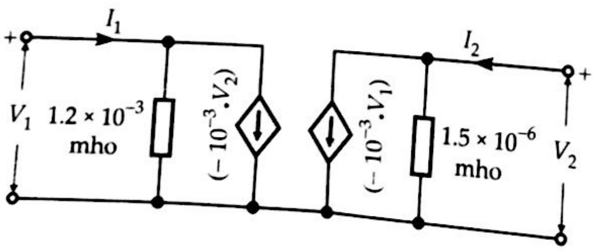

Fig. E13.27

EXAMPLE 13.18 *On short circuit test, the currents and voltages were determined experimentally for an unknown two port network as*

$$\left.\begin{array}{l} I_1 = 1\,mA \\ I_2 = -0.5\,mA \\ V_1 = 25\,V \end{array}\right|_{\text{at } V_2 = 0} \quad \text{and} \quad \left.\begin{array}{l} I_1 = -1\,mA \\ I_2 = -10\,mA \\ V_2 = 50\,V \end{array}\right|_{\text{at } V_1 = 0}$$

Determine the Y-parameters and draw the Y-parameter model.

SOLUTION. From the text,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1 \times 10^{-3}}{25} = 40\,\mu \text{ mho}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{0.5 \times 10^{-3}}{25} = -20\,\mu \text{ mho}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1 \times 10^{-3}}{50} = -20\,\mu \text{ mho}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = -\frac{10 \times 10^{-3}}{50} = -200\,\mu \text{ mho}$$

The governing equations are then

$$I_1 = 40 \times 10^{-6}\, V_1 + (-20 \times 10^{-6})\, V_2$$

and $$I_2 = (-20 \times 10^{-6})\, V_1 + (-200 \times 10^{-6})\, V_2$$

The equivalent circuit of the Y-parameter model is drawn in **Fig. E13.28.**

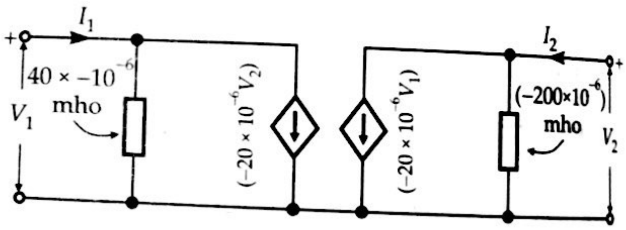

Fig. E13.28

EXAMPLE 13.19 *Find Y-parameters of network shown in Fig. E13.29 from Z-parameters.*

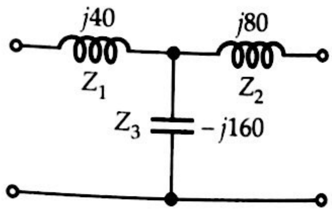

Fig. E13.29

SOLUTION.

$$Z_{11} = Z_1 + Z_3 = -j\,120\,;$$
$$Z_{12} = Z_3 = -j\,160$$
$$Z_{21} = Z_3 = -j\,160\,;$$
$$Z_{22} = Z_2 + Z_3 = -j\,80$$

Here,

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{-j\,80}{(-j\,120)(-j\,80) - (-j\,160)^2}$$

$$= \frac{-j\,80}{16{,}000} = \frac{-j}{200}\ \textbf{mho.}$$

$$Y_{12} = Y_{21} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{j\,160}{16{,}000} = \frac{j}{100}\ \textbf{mho.}$$

$$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{-j\,120}{16{,}000} = \frac{-j}{133.33}\ \textbf{mho.}$$

EXAMPLE 13.20 *Find the parameters Y_a, Y_b and Y_c of the equivalent π network shown in Fig. E13.30 to represent a*

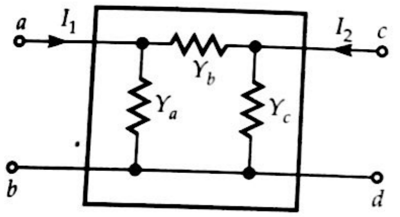

Fig. E13.30

2-port network for which following measure- ments were taken :

(*a*) *With terminal pairs c-d short circuited, a voltage of 10 ∠0° V applied at terminal-pairs a-b resulting in $I_1 = 2.50\ \angle 0°$ A and $I_2 = -0.50\ \angle 0°$ A.*

(*b*) *With the terminal-pair a-b short-circuited, the same applied voltage at terminal-pair c-d the current at output port resulted is $I_2 = (1.50\ \angle 0°$ A).*

SOLUTION. With terminal-pair *c-d* short-circuited

$$V_1 = I_1 / (Y_a + Y_b)$$

or $$Y_a + Y_b = \frac{I_1}{V_1} = \frac{2.50\ \angle 0°}{10} = 0.25\ \text{mho.}$$

Also $$-I_2 / Y_b = V_1$$

or $$Y_b = \frac{-I_2}{V_1} = \frac{0.50\ \angle 0°}{10}$$

$$= 0.05\ \text{mho};$$

$$\therefore \quad \mathbf{Y_a = 0.20\ mho\ ;\ Y_b = 0.05\ mho}$$

With terminal-pair *a-b* short circuited

$$V_2 = I_2 / (Y_b + Y_c)$$

or $$Y_b + Y_c = \frac{I_2}{V_2} = \frac{1.50}{10} = 0.15\ \text{mho}$$

$$\therefore \quad \mathbf{Y_c = 0.10\ mho.}$$

EXAMPLE 13.21 *In a π network, the series arm impedance is $0.05 \times 10^{-3} \angle -90°$ mho and shunt arm impedences are $0.1 \times 10^{-3} \angle 0°$ and 0.2×10^{-3} 90° mho. Find the Y-parameters.*

SOLUTION. For the π network, representing in *Y*-parameter form,

$$Y_{11} = Y_A + Y_B\ ;\ Y_{12} = Y_{21} = -Y_B\ ;\ Y_{22} = Y_B + Y_C$$

In this problem,

$$Y_A = 0.1 \times 10^{-3}\ \angle 0° = 0.1 \times 10^{-3}\ \text{mho}$$

$$Y_B = 0.05 \times 10^{-3}\ \angle -90° = -j\,0.05 \times 10^{-3}\ \text{mho}$$

$$Y_C = 0.2 \times 10^{-3}\ \angle 90° = +j\,0.2 \times 10^{-3}\ \text{mho};$$

$$\therefore \quad \mathbf{Y_{11} = (0.1 - j\,0.05) \times 10^{-3}\ mho}$$

$$\mathbf{Y_{12} = Y_{21} = j\,0.05 \times 10^{-3}\ mho}$$

$$\mathbf{Y_{22} = j\,0.15 \times 10^{-3}\ mho.}$$

EXAMPLE 13.22 *Following short circuit currents and voltages are obtained experimentally for a two port network :*

(*a*) *With output short-circuited,*

$$I_1 = 5\ mA;\ I_2 = -0.3\ mA;\ V_1 = 25\ V.$$

(*b*) *With input short-circuited,*

$$I_1 = -5\ mA;\ I_2 = 10\ mA;\ V_2 = 30\ V.$$

Determine Y-parameters.

SOLUTION. From the text, we know, for *Y*-parameter representation,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0};\ Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0};\ Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

Thus, from the h-parameter representation,

$$0 = -h_{11} I_1 + h_{12} V_2$$

or,
$$\frac{I_1}{V_2} = \frac{h_{12}}{h_{11}} \quad \text{...(13.56)}$$

Assuming $V_1 = V_2$, following the principle of reciprocity, the left hand sides of equations (13.55) and (13.56) will be same. This is possible provided,

$$h_{21} = -h_{12} \quad \text{...(13.57)}$$

Thus, $h_{21} = -h_{12}$ exhibits the *condition of reciprocity*.

For establishing the condition of symmetry, let us incorporate the concept of Z-parameter network again.

By definition, when the output is shorted, $I_2 = 0$ and then

$$Z_{11} = \frac{V_1}{I_1} = \frac{h_{11} I_1 + h_{12} V_2}{I_1} = h_{11} + h_{12}\frac{V_2}{I_1}$$

$$= h_{11} - h_{12}\frac{h_{21}}{h_{22}}$$

$$\left[\because \text{ from } I_2 = h_{21} I_1 + h_{22} V_2, \quad \text{for } I_2 = 0, \frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}\right]$$

$$= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} \quad \text{...(13.58)}$$

where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$.

Again, with input ports shorted, $I_1 = 0$. This gives, for the Z parameter representation,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{V_2}{h_{22} V_2}$$

[in the expression, $I_2 = h_{21} I_1 + h_{22} V_2$, for $I_1 = 0, I_2 = h_{22} V_2$]

$$\therefore \quad Z_{22} = \frac{1}{h_{22}} \quad \text{....(13.59)}$$

The condition of symmetry ($Z_{11} = Z_{22}$) leads to $\Delta h = 1$ such that equations (13.58) and (13.59) are identical. Thus the *condition of symmetry* is given by

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

Table 13.1 Conditions for Reciprocity and Symmetry in Terms of Various Parameters

Parameter	*Condition for reciprocity*	*Condition for symmetry*
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
$ABCD$	$AD - BC = 1$	$A = D$

EXAMPLE 13.27 *Find Z-parameters for the lattice network shown in Fig. E13.36.*

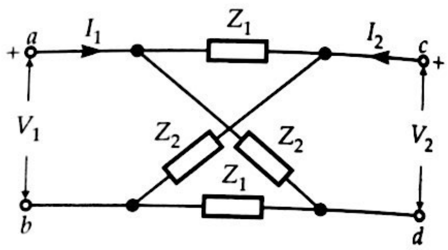

Fig. E13.36

SOLUTION. In the first step, the output terminals are open-circuited with V_1 at the input. The network is redrawn with I_1 as the input current (Fig. E13.37). The equivalent impedance of the network looking back from the input terminals is

$$(Z_1 + Z_2) \| (Z_2 + Z_1) \text{ i.e., } \left(\frac{Z_1 + Z_2}{2}\right).$$

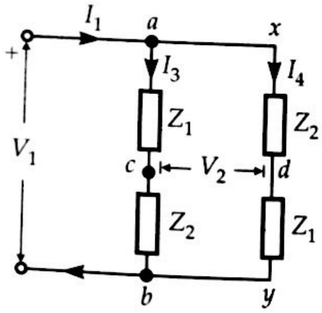

Fig. E13.37

Hence, $V_1 = I_1\left(\frac{Z_1 + Z_2}{2}\right)$

or
$$\frac{V_1}{I_1} = \left. Z_{11}\right|_{I_2 = 0} = \frac{Z_1 + Z_2}{2}$$

Now,
$$V_2 = V_c - V_d = (V_1 - I_3 Z_1) - (V_1 - I_4 Z_2)$$
$$= I_4 Z_2 - I_3 Z_1$$

[V_c and V_d being the potentials at point c and d]

However,

$$I_3 = I_1 \frac{(Z_2 + Z_1)}{(Z_1 + Z_2) + (Z_2 + Z_1)} = \frac{I_1}{2}$$

[impedances in branch ab and xy are same hence $I_3 = I_4 = \frac{I_1}{2}$]

Also, $I_4 = \frac{I_1}{2}$

Hence $V_2 = \frac{I_1}{2} \times Z_2 - \frac{I_1}{2} \times Z_1 = \frac{Z_2 - Z_1}{2} \cdot I_1$

$$\therefore \quad \frac{V_2}{I_1} = Z_{21} \Big|_{I_2 = 0} = \frac{Z_2 - Z_1}{2}$$

As the given network is symmetrical Z_{12} must be equal to Z_{21} and Z_{22} should be equal to Z_{11}.

Hence $\mathbf{Z_{11} = Z_{22} = \frac{Z_1 + Z_2}{2}}$

and $\quad \mathbf{Z_{12} = Z_{21} = \frac{Z_2 - Z_1}{2}}$.

EXAMPLE 13.28 *Obtain Z parameter of the following network [Fig. E13.38(a) and Fig. 13.38(b)].*

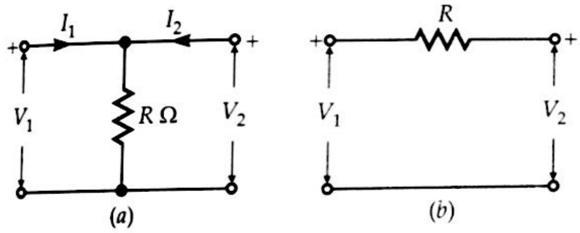

Fig. E13.38

SOLUTION. In Fig. 13.38(a), with output open-circuited, $I_2 = 0$.

$$\therefore \quad V_1 = I_1 R \text{ giving } \frac{V_1}{I_1} = Z_{11}\ (I_2 = 0) = R$$

Also, $I_1 R = V_2 \quad \therefore \quad \frac{V_2}{I_1}\Big|_{I_2 = 0} = Z_{21} = R$

Again with input open-circuited, $I_1 = 0$

$$\therefore \quad V_2 = I_2 R \text{ giving } \frac{V_2}{I_2} = Z_{22} = R\,(\text{at } t_1 = 0)$$

and $\quad I_2 R = V_1$ giving $\frac{V_1}{I_2}\Big|_{I_1 = 0} = Z_{12} = R$

$\therefore \quad \mathbf{Z_{11} = R;\ Z_{12} = R}$

$\quad\ \ \mathbf{Z_{21} = R;\ Z_{22} = R.}$

In **Fig. E13.38(*b*)** since I_1 and I_2 are not independent, **Z parameters cannot be determined.**

[As soon as V_1 will be applied I_1 will flow causing I_2 to flow as $(-I_1)$. Then I_1 and I_2 are not independent].

EXAMPLE 13.29 *Obtain Z parameter for Fig. E13.39.*

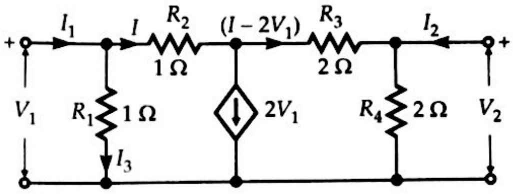

Fig. E13.39

SOLUTION. Let us first assume

$I_2 = 0$ [output is open-circulated. Refer to Fig. E13.40]

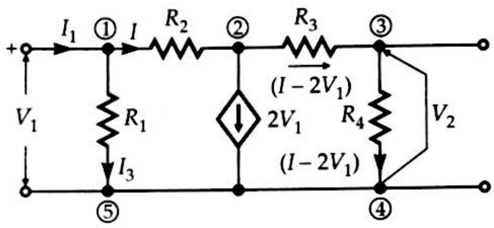

Fig. E13.40

At *node (1)*,

$$I_1 = I_3 + I$$

or $\quad I = I_1 - I_3 = I_1 - \frac{V_1}{R_1} = \left(I_1 - \frac{V_1}{1} \right)$

$$= (I_1 - V_1) \quad \ldots(i)$$

However,

$$I = \frac{\text{Voltage at node (1)} - \text{Voltage at node (2)}}{R_2}$$

$$= \frac{V_1 - (I - 2V_1)4}{1} \quad \ldots(ii)$$

Comparing (i) and (ii),

$$(I_1 - V_1) = \frac{V_1 - (I - 2V_1)4}{1}$$

$$= \frac{V_1 - [(I_1 - V_1) - 2V_1]4}{1}$$

or $\quad I_1 - V_1 = \frac{V_1 - [4I_1 - 4V_1 - 8V_1]}{1}$

Applying KVL in the loops, the loop equation is given by

$$V_1 = I_1 + (I_1 + I_2)2$$
$$= 3I_1 + 2I_2 \qquad ...(v)$$

and $\quad (I_3 - I_1)2 + I_3 = 0$

or $\quad -(I_1 + I_2)2 - I_2 = 0 \qquad (\because I_3 = -I_2)$

or $\quad 2I_1 + 3I_2 = 0$

$$\therefore \quad I_1 = -\frac{3}{2} I_2 . \qquad ...(vi)$$

Utilising equation, in (v),

$$V_1 = -\frac{9}{2} I_2 + 2I_2 = -\frac{5}{2} I_2$$

$$\therefore \quad \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{5}{2} \text{ ohm} = B.$$

From (vi) $\quad \dfrac{I_1}{-I_2} = \dfrac{3}{2} = D.$

Hence the transmission parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Here $AD - BC = 1$ and $A \neq D$

Thus the circuit is reciprocal but not symmetrical.

EXAMPLE 13.37 *Find ABCD parameters of the following lattice network (Fig. E 13.54).*

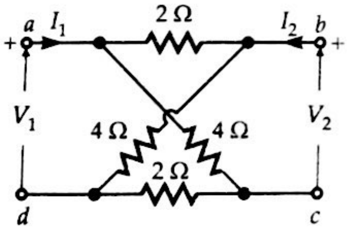

Fig. E13.54

SOLUTION. Here with output port open-circuited, (refer to Fig. E13.55)

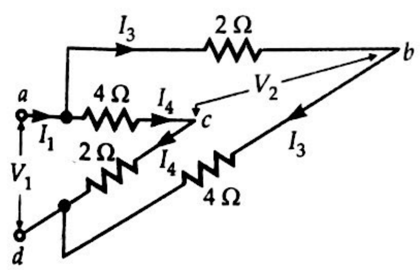

Fig. E13.55

$$V_1 = I_1 \{(2+4) \,||\, (4+2)\} = I_1 \times 3$$

$$\therefore \quad \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_{11} = 3\,\Omega.$$

Again, the voltage between nodes b and c being V_2,

$$V_2 = (V_1 - 2I_3) - (V_1 - 4I_4) = 4I_4 - 2I_3 \qquad ...(i)$$

However,

$$V_1 = 4I_4 + 2I_4 = 2I_3 + 4I_3 \quad \text{or} \quad I_3 = I_4 \qquad ...(ii)$$

Thus we get,

$$V_2 = 4I_4 - 2I_3 = 2I_4 \; [\text{using } (ii) \text{ in } (i)]$$

But $\quad I_3 = I_4$

$\therefore \quad I_1 = 2I_3 = 2I_4 \qquad [\because I_1 = I_3 + I_4]$

Hence $\quad V_2 = 2I_4 = I_1$

$$\therefore \quad \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1\,\Omega = Z_{21}$$

Again open-circuiting the input port and applying voltage V_2 at output port, in a similar way

$$Z_{12} = 1\,\Omega \text{ and } Z_{22} = 3\,\Omega$$

thus $\quad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$

ABCD parameters may then be determined as follows (See Art. 13.11.7) :

$$A = \frac{Z_{11}}{Z_{21}} = \frac{3}{1} = \mathbf{3}, \quad B = \frac{\Delta Z}{Z_{21}} = \mathbf{8\,\Omega}$$

$$C = \frac{1}{Z_{21}} = \frac{1}{1} = \mathbf{1 \text{ mho}}, \quad D = \frac{Z_{22}}{Z_{21}} = \mathbf{3}$$

and $\quad AD - BC = 1.$

Also, $\quad A = D.$

Thus the circuit is reciprocal and also symmetrical.

13.11 INTER-RELATIONSHIPS BETWEEN PARAMETERS OF TWO PORT NETWORK

13.11.1 *Z*-parameters in Terms of *Y*-parameters

Z being the impedance and Y being the admittance,

$$[Z] = [Y]^{-1}$$

or $\quad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$

CHAPTER 13

However, assuming zero initial condition, the equations of $V_1(s)$ and $V_2(s)$ reduce to

$$V_1(s) = L_1\, sI_1(s) + Ms\, I_2(s) \quad ...(13.150)$$
$$V_2(s) = Ms\, I_1(s) + L_2\, sI_2(s)$$

The diagrammatic representation is then shown in Fig. 13.39.

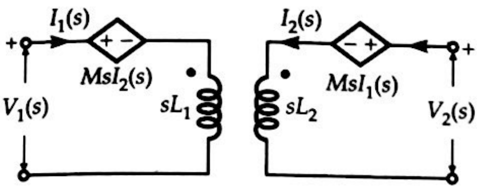

Fig. 13.39 Diagrammatic representation of the circuit model of a transformer.

13.20 MODELING OF NETWORK COMPONENTS

1. Transformer

The governing equations for an ideal transformer are given by (as shown in equation 13.147)

$$V_2 = nV_1$$
$$I_2 = -I_1/n.$$

For our convenience we can rewrite these two equations as

$$V_1 = \frac{1}{n}.V_2$$
$$I_2 = \left(-\frac{1}{n}\right).I_1$$

In matrix form, these equations are shown as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1/n \\ -1/n & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad ...(13.151)$$

Equation (13.151) exhibits the modeling form in terms of h-parameter concept.

We can also write the governing equations of an ideal transformer in the following forms (as shown in Art 13.19)

$$V_1(s) = sL_1 . I_1(s) + sM . I_2(s)$$
$$V_2(s) = sM . I_1(s) + sL_2 . I_2(s)$$

Thus, in Z-parameter form, these equations can be shown as

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \quad ...(13.152)$$

The governing equations can further be reoriented in following fashion :

$$V_1 = \frac{1}{n}.V_2 + 0.(-I_2)$$

and $\quad I_1 = 0.V_2 + n(-I_2)$

Thus, in $ABCD$ parameter form we can write

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad ...(13.153)$$

It may be noted here that the ideal transformer being a lossless device (explained in Art 13.19) it is neither storing nor dissipating energy and hence it is a passive device (however, practical transformers being composed of resistances in winding, they cannot be treated as passive device since the practical transformer dissipates energy). Also, it may be observed from above that with $L_1 = L_2$, in Z parameter form $Z_{11} = Z_{22}$ for the ideal transformer. Also from $ABCD$ parameter representation, it is evident that $AD - BC = 1$, for ideal transformer, making it a reciprocal device.

Thus, the ideal transformer model exhibits that it is a passive element and has both symmetry and reciprocity.

CHAPTER 13

2. Gyrator

The *gyrator* is an inverting circuit element commonly used in electronic circuits. We will show the model of gyrator as a two port network and its properties rather describing the hardware of a gyrator. A gyrator is shown as a two port device as exhibited in Fig. 13.40.

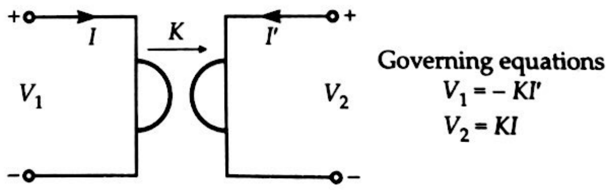

Fig. 13.40 Representation of a gyrator.

Let us assume first that the gyrator is terminated by a resistor R (Fig. 13.41).

Here, $\quad V_2 = -I'R, \quad ...(13.154a)$

$$V_1 = -KI' \quad ...(13.154b)$$

(since the gyrator is characterised by $V_1 = -KI'$ and $V_2 = KI$)

$$\text{Voltage gain} = \frac{h_{21}\ R_L}{h_{12}\ h_{21}\ R_L - h_{11}\ (1 + h_{22}\ R_L)}$$

$$= \frac{40 \times 10 \times 10^3}{40 \times 350 \times 10^{-6} \times 10 \times 10^3 - 1000\,(1 + 30 \times 10^{-6} \times 10^4)}$$

$$= \frac{4 \times 10^5}{140 - 1000\,(1 + 0.3)} = \mathbf{-3.4 \times 10^2.}$$

[–ve voltage gain means 180° phase shift between V_2 and V_1].

Current gain

$$= \frac{h_{21}}{1 + h_{22}\ R_L} = \frac{40}{1 + 30 \times 10^{-6} \times 10^4}$$

$$= \frac{40}{1 + 0.3} = \mathbf{30.76.}$$

EXAMPLE 13.57 *Obtain Z-parameters of an ideal transformer.*

SOLUTION. From the text, the equations for the circuit model of the ideal transformer are given by

$$V_1\ (s) = sL_1\ I_1\ (s) + sMI_2\ (s)$$
$$V_2\ (s) = sMI_1\ (s) + sL_2\ I_2\ (s)$$

Comparing these two equations with the standard form of Z-parameter network,

$$\mathbf{Z_{11} = sL_1\ ;\ Z_{12} = sM}$$
$$\mathbf{Z_{21} = sM\ ;\ Z_{22} = sL_2.}$$

EXAMPLE 13.58 *An ideal transformer with turns ratio n : 1 is shown in Fig. E13.83. Check whether the circuit model is reciprocal or not.*

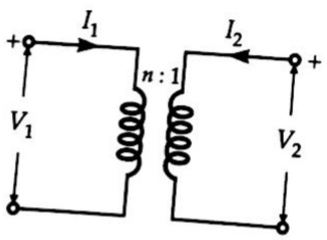

Fig. E13.83

SOLUTION. Here, $V_1 = \frac{1}{n} . V_2$

$I_1 = -n\, I_2$ [refer to the text]

These equations can be configured as

$$V_1 = \frac{1}{n} V_2 + 0(-I_2)$$

$$I_1 = 0 . V_2 + n(-I_2)$$

Comparing these equations with the sta[ndard] equations of transmission parameters,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Here, $\quad AD - BC = \frac{1}{n} \times n - 0 = 1$

$\therefore$ **The ideal transformer is reciprocal.**

EXAMPLE 13.59 *Find the transmision parameter matri[x] Fig. E13.84.*

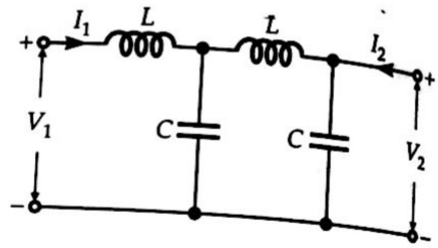

Fig. E13.84

SOLUTION. Let us first transform the given network of Fig. E13.84 into s-domain. The Laplace configuration of the network is shown in Fig. E13.85.

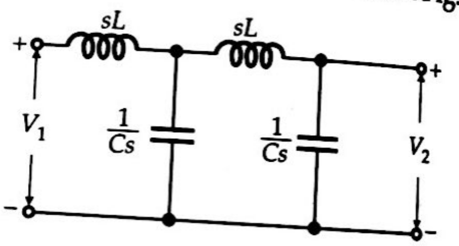

Fig. E13.85

It may be observed that this circuit is a cascaded combination of the two networks as shown in Fig. E13.86.

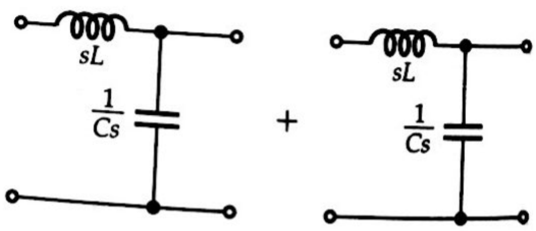

Fig. E13.86

Again each of the two cascaded networks can be assumed to be cascade connection of one series element with one shunt element.

Here, $V_1 = ZI_1 + V_2$ and $I_1 = -I_2$ for the series network [Fig. E13.87(*b*)]

i.e., $$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$