Curve Tracing:

1. The curve represented by the equation $a^2x^2 = y^3(2a - y)$ is

   a) symmetrical about $x-\text{axis}$ and passing through $(2a, 0)$

   b) symmetrical about both $x-\text{axis}$ and $y-\text{axis}$ and passing through origin

   c) symmetrical about $y-\text{axis}$ and passing through $(0, 2a)$

   d) symmetrical about both $x-\text{axis}$ and $y-\text{axis}$ and passing through $(2a, 0)$

2. The equation of tangents to the curve parallel to x-axis represented by the equation

   $y = \dfrac{8a^3}{x^2 + 4a^2}$ is

   a) y = 0

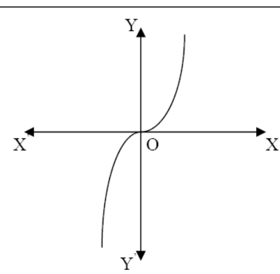
   b) y = -1

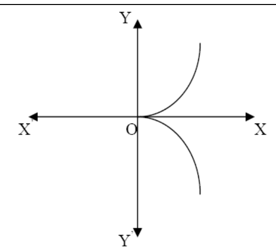
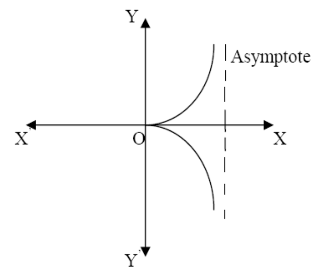
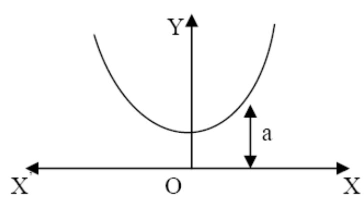
   c) y = 2a

   d) y = a

3. The region of absence for the curve represented by the equation $xy^2 = a^2(a - x)$ is

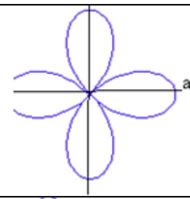
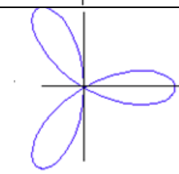
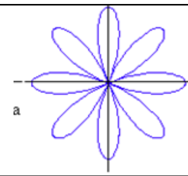
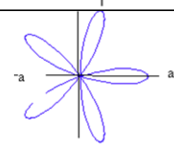
a)	$x > 0$ and $x < a$
b)	$x < 0$ and $x < a$
c)	$x < 0$ and $x > a$
d)	$x > 0$ and $x > a$

Question 4	The equation $y^2(2a - x) = x^3$ represents the curve
Option A	

Option B	
Option C	
Option D	

Question 5.	The equation of asymptotes parallel to y axis to the curve represented by the equation $\left(x^2-a^2\right)\left(y^2-b^2\right)=a^2b^2$
Option A	$x=a, x=-a$
Option B	$y=b, y=-b$
Option C	$x=b, x=-b$
Option D	$x=a, y=b$

Question 6.	**Which of the following curve represent the equation r = acos4θ**

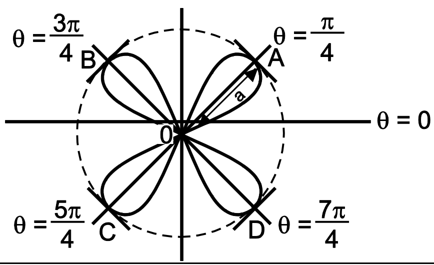
Option A	
Option B	
Option C	
Option D	

Question 7	The Three leaved rose $r = a\cos 3\theta$ has
Option A	Horizontal asymptote
Option B	Vertical asymptote
Option C	No asymptote since r is infinite for any θ
Option D	No horizontal or vertical asymptote

Question 8	The curve represented by the equation $r = \dfrac{2a}{1+\cos\theta}$ is
Option A	symmetrical about initial line and passing through pole
Option B	symmetrical about initial line and not passing through pole
Option C	symmetrical about $\theta = \dfrac{\pi}{2}$ and passing through pole
Option D	symmetrical about $\theta = \dfrac{\pi}{4}$ and passing through pole

Question 9	The number of loops in the Folium of Descartes $x^3 + y^3 = 3axy$ are
Option A	2
Option B	1

Option C	3
Option D	5

Question 10	The following figure represents the curve whose equation is 
Option A	$r = a\cos 3\theta$
Option B	$r = a\sin 2\theta$
Option C	$r = a\sin 3\theta$
Option D	$r = a\left(1+\cos\theta\right)$

Answers:

1-c	2-c	3-c	4-c	5-b
6-c	7-d	8-b	9-b	10-b

Double and Triple integral:

1. The value of $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{(1+y^3)} dydx$ is.........
2. The integral $\int_{-2}^{1} \int_{x^2+4x}^{3x+2} dydx$ after changing the order of integration is1. $\int_0^5 \int_{\frac{y-2}{3}}^{\sqrt{y+4}-2} dxdy$ 2. $\int_{-4}^5 \int_{\frac{y-2}{3}}^{\sqrt{y+4}-2} dxdy$ 3. $\int_0^5 \int_0^{\sqrt{y+4}-2} dxdy$ 4. $none\ of\ these$

3. Value of $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$ is

   1. $\frac{1}{24}$ 2. $\frac{1}{48}$ 3. 1 4. 0

4. Match the following

1. Jacobian of the transformation $x = rcos\theta, y = rsin\theta$	• r^2
• To change Cartesian coordinates (x, y, z) to spherical coordinates $(r, \theta, \emptyset)$: $dxdydz$ is replaced by	• $rdrd\theta$
• To change Cartesian coordinates (x, y, z) to cylindrical coordinates (r, θ, z) : $dxdydz$ is replaced by	• r
• Jacobian of the transformation $x = rsin\theta, y = rcos\theta$	• $rdrd\theta d\phi$
	• $r^2 sin\theta drd\theta d\phi$
	• $rdrd\theta dz$
	• -r
	• $r^2 sin\theta drd\theta dz$

5. The range of azimuthal angle ϕ in the spherical polar coordinates is

   a. [0,2π] b. [0,π] c. [0,π/2] d. [-π, +π]

6. The equation to a surface in spherical coordinates is given by θ=π/3. The surface is

   a. A sector of a circle b. A cone making an angle of π/3 with the z-axis c. A vertical plane making an angle of π/3 with the z-axis d. A vertical plane making an angle of π/3 with the x-axis.

7. Expressed in spherical coordinate system the equation $x^2 + y^2 + z^2 = 4z$ becomes

   $a. r = 4cos\theta sin\emptyset$ $b. r = 4sin\theta cos\emptyset$ $c. r = 4cos\theta$ $d. r = 4sin\theta$

8. The value of the integral $\int_0^a \int_{\frac{y^2}{a}}^{2a-y} xydxdy$ is........

9. The integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)dxdy$ after changing into polar coordinates is.......

10. The integral $\int_0^1 \int_x^{\sqrt{x}} xydxdy$ after changing the order of integration is......

Answers:

1. The value of $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{(1+y^3)} dydx$ is......$\frac{1}{3} log5$...

2. The integral $\int_{-2}^1 \int_{x^2+4x}^{3x+2} dydx$ after changing the order of integration

   is1. $\int_0^5 \int_{\frac{y-2}{3}}^{\sqrt{y+4}-2} dxdy$ 2. $\int_{-4}^5 \int_{\frac{y-2}{3}}^{\sqrt{y+4}-2} dxdy$ 3. $\int_0^5 \int_0^{\sqrt{y+4}-2} dxdy$ 4. $none\ of\ these$

3. Value of $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$ is

   2. $\frac{1}{24}$ 2. $\frac{1}{48}$ 3. 1 4. 0

4. Match the following

2. Jacobian of the transformation $x = rcos\theta, y = rsin\theta$	• r^2
• To change Cartesian coordinates (x, y, z) to spherical coordinates $(r, \theta, \emptyset)$: $dxdydz$ is replaced by	• $rdrd\theta$
• To change Cartesian coordinates (x, y, z) to cylindrical coordinates (r, θ, z) : $dxdydz$ is replaced by	• r
• Jacobian of the transformation $x = rsin\theta, y = rcos\theta$	• $rdrd\theta d\phi$
	• $r^2 sin\theta drd\theta d\phi$
	• $rdrd\theta dz$
	• -r
	• $r^2 sin\theta drd\theta dz$

5. The range of azimuthal angle ϕ in the spherical polar coordinates is
   b. [0,2π] b. [0,π] c. [0,π/2] d. [-π, +π]
6. The equation to a surface in spherical coordinates is given by θ=π/3. The surface is
   b. A sector of a circle b. A cone making an angle of π/3 with the z-axis c. A vertical plane making an angle of π/3 with the z-axis d. A vertical plane making an angle of π/3 with the x-axis.
7. Expressed in spherical coordinate system the equation $x^2 + y^2 + z^2 = 4z$ becomes
   $a. r = 4cos\theta sin\emptyset$ $b. r = 4sin\theta cos\emptyset$ $c. r = 4cos\theta$ $d. r = 4sin\theta$
8. The value of the integral $\int_0^a \int_{\frac{y^2}{a}}^{2a-y} xydxdy$ is........$\frac{19}{24}a^4$
9. The integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)dxdy$ after changing into polar coordinates is...$\int_0^a \int_0^{\frac{\pi}{2}} r^3 d\theta dr$....
10. The integral $\int_0^1 \int_x^{\sqrt{x}} xydxdy$ after changing the order of integration is......$\int_0^1 \int_{y^2}^{y} xydydx$

Application of Double and Triple Integral:

1. To change cartesian co-ordinates (x, y, x) to spherical polar co-ordinates $(r, \theta, \emptyset)$; $dxdydz$ is replaced by
   a) $x = rsin\theta cos\emptyset; y = rsin\theta sin\emptyset; z = rcos\theta$
   b) $x = rsin\theta cos\emptyset; y = rsin\theta cos\emptyset; z = rcos\theta$
   c) $x = rsin\theta sin\emptyset; y = rsin\theta sin\emptyset; z = rcos\theta$
   d) $x = rsin\theta cos\emptyset; y = rsin\theta sin\emptyset; z = rsin\theta$
2. $\iint (x + y)^2 \, dxdy$ over the area bounded by the ellipse is
   a) $\pi ab(a^2 + b^2)$ b) $\frac{1}{4}\pi ab(a^2 + b^2)$ c) $\frac{1}{4}\pi ab(a^3 + b^3)$ d) None
3. $\iint x^2 y^3 dxdy$ taken over the rectangle $0 \le x \le 1; 0 \le y \le 3$ is

a) 27/2 b)25/7 c)27/4 d)None

4. $\iint xy(x+y)dxdy$ over the area between $y+x^2; y=x$ is

   a) 3/356 b) 3/240 c) 3/256 d)None

5. Area of the ellipse by using double integrals is

   a) πab b)πab^2 c)$\pi a^2 b$ d)None

6. $\iint dxdy$ over the area bounded by $x=0; y=0; x^2+y^2=1; 5y-3$ is

   a) $\frac{6}{25}+\frac{1}{2}sin\frac{3}{5}$ b) 25/2 c)25 d) None

7. $\iint ydxdy$ over the region bounded by the parabolas $y^2=4x; x^2=4y$ is

   a) 48/5 b) 47/2 c) 40/3 d)None

8. $\iint(x^2+y^2)dxdy$ in the positive quadrant for which $x+y\le 1$ is

   a) ½ b) 1/3 c) 1/6 d) None

9. The area between the parabola $y^2=4x; x^2=4y$ is

   a) 16/3 b) 15/2 c) 14/3 d) None

10. The value of $\int_0^2\int_0^{x^2} e^{\frac{y}{x}}dydx$ is a) e^2-1 b) e^2+1 c) e^2-2 d) None

Answers:

1	a
2	b
3	c
4	c
5	a
6	a
7	a
8	c
9	a
10	a

Differential Equations:

Pointers:

- If M(x,y) and N(x,y) are homogeneous functions of the same degree then $(Mx+Ny)^{-1}$ is an I.F of Mdx+Ndy=0, Provided Mx+Ny not equal to zero.
  In case Mx+Ny =0 then **$1/x^2$ or $1/y^2$ 0r 1/xy** are Integrating factors.

- For the DE f(xy)ydx + g(xy)xdy=0 ,$(Mx-Ny)^{-1}$ is an I.F , Provided Mx-Ny not equal to zero. In case Mx-Ny=0, then $\frac{M}{N}=\frac{y}{x}$ and the given diff equation reduces to xdy+ydx=0 with **xy=C** as its solution.

1. The integrating factor of the differential equation $x\log x\frac{dy}{dx}+y=\log x^2$ is

   A) $\log x^2$ B) logx C)xlogx D)$x\log x^2$
2. ydx-xdy=0 can be reduced to exact, if divided by
   a) x^2+y^2 b)y^2 c)xy d)All of these
3. For the differential equation $\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{d^2y}{dx^2}\right)^6+y=x^4$, the order and degree respectively are

   a)2,6 b)3,2 c)2,4 d) None of these
4. The general solution of the differential equation (x-y)dx+(y-x)dy=0 is

   a) $\frac{x^2}{2}-y-\frac{y^2}{2}=c$ b) $\frac{x^2}{2}-y+\frac{y^2}{2}=c$ c) $\frac{x^2}{2}-xy+\frac{y^2}{2}=c$ d)None of these
5. Integrating factor for the differential equation $\frac{dy}{dx}+\frac{2x}{y}=y^2$

   a) y^2 b) e^{x^2} c) e^{2y} d) e^{y^2}
6. The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx}+xy=\sinh^{-1}x$

   a) $\frac{1}{\sqrt{1-x^2}}$ b) $\sqrt{1-x^2}$ c) $\sqrt{1+x^2}$ d) $\frac{x}{\sqrt{1+x^2}}$
7. If M(x,y)dx+N(x,y)dy=0 is said to be exact then the condition is

   a) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ c) $\frac{\partial M}{\partial y}\rangle\frac{\partial N}{\partial x}$ d)M=N
8. The integrating factor for $(x+2y^3)\frac{dy}{dx}=y$

   a)logy b)e^y c)1/y d)y+1
9. The differential equation of the form Mdx+Ndy=0, for which $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{2}{x}$ then the integrating factor is

a)2x b)x^2 c)2logx d) e^{x^2}

10. The family of straight lines passing through the origin is represented by the differential equation:

a) ydx+xdy=0 b)xdy-ydx=0 c)xdx+ydy=0 d)ydy-xdx=0

Answers:

1-b	2-b	3-b	4-c	5-a
6-c	7-b	8-c	9-b	10-b

Misc.Questions:

1) A thin plate covers the triangular region bounded by the x-axis and the lines x = 1 and y = 2x in the first quadrant. The plate's density at the point (x, y) is δ(x, y) = 6x + 6y + 6. Find the plate's moments of inertia about the coordinates axes and the origin.

Ans: $I_x=12$, $I_y=39/5$, $I_0= 99/5$.

2) Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}(x^2+y^2)^{3/2}\,dydx$ by changing into polar coordinates

Ans:π/5

3) Using the triple integral find the volume of the solid within the cylinder $x^2+y^2=9$ and between the planes z=1 and x+z=5

Ans: 36π

4) Find the centroid of the semicircular region in the xy plane bounded by the x-axis and the curve

$$y = \sqrt{a^2 - x^2}$$

Ans: Centroid(0, 4a/3π)

5) The integrating factor of the differential equation $y^2\,dx+(3xy-1)dy=0$ is

a) y^2 b)2y c)y^3 4)3x

Ans: y^3