MODULE 4

TWO PORT NETWORKS

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-port parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other.

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A Typical one port or two terminal network is shown in figure 1.1. For example resistor, capacitor and inductor are one port network.

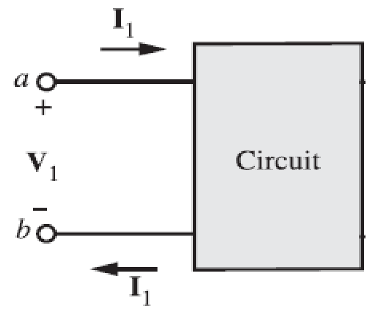

Fig.1.1

Fig. 1.2 represents a two-port network.A four terminal network is called a two-port network when the current entering one terminal of a pair exits the other terminal in the pair. For example, I1 enters terminal 'a' and exit terminal 'b' of the input terminal pair 'a-b'. Example for four-terminal or two-port circuits are op amps, transistors, and transformers.

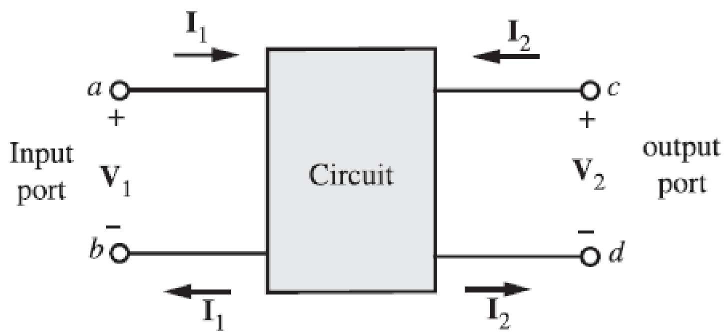

Fig.1.2

To characterize a two-port network requires that we relate the terminal quantities V_1, V_2, I_1 *and* I_2.The various terms that relate these voltages and currents are called parameters. Our goal is to derive four sets of these parameters.

1.3 Open circuit Impedance Parameter (z Parameter):

Let us assume the two port network shown in figure is a linear network then using superposition theorem, we can write the input and output voltages as the sum of two components, one due to I_1 and other due to I_2:

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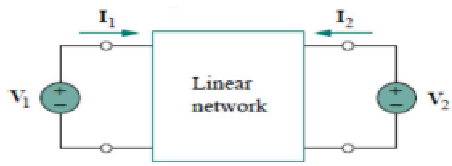


$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

the z terms are called the z parameters, and have units of ohms. The values of the parameters can be evaluated by setting $I_1 = 0$ or $I_2 = 0$.

The **z** parameters are defined as follows:

Thus

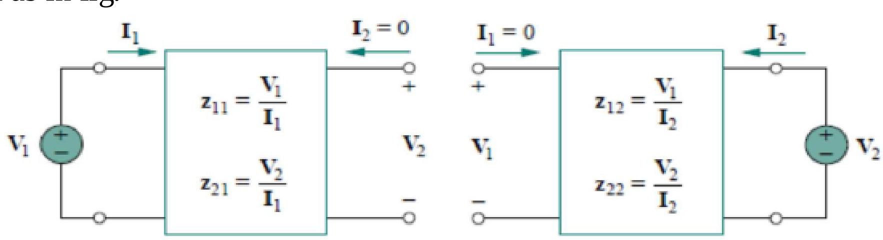
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

In the preceding equations, letting $I_1 = 0$ or $I_2 = 0$ is equivalent to open-circuiting the input or output port. Hence, the **z** parameters are called open-circuit impedance parameters.

Here z_{11} is defined as the open-circuit input impedance, z_{22} is called the open-circuit output impedance, and z_{12} and z_{21} are called the open-circuit transfer impedances.

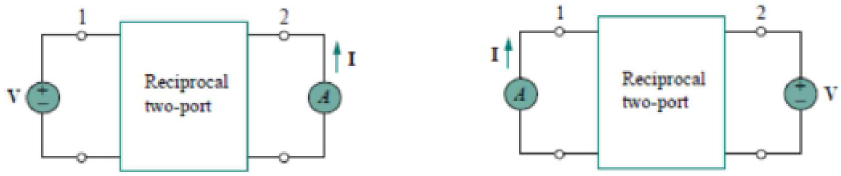
If $z_{12} = z_{21}$, the network is said to be **reciprocal network**. Also, if $z_{11} = z_{22}$ then the network is called a **symmetrical network.**

We obtain z_{11} and z_{21} by connecting a voltage V_1 (or a current source I_1) to port 1 with port 2 open-circuited as in fig.



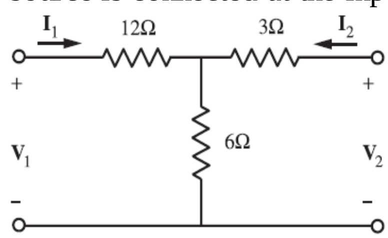
Similarly z_{12} and z_{22} by connecting a voltage V_2 (or a current source I_2) to port 2 with port 1 open-circuited as in fig.

A two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

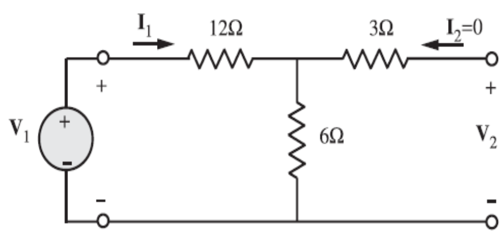


Example 8.1

Determine the z parameters for the circuit in the following figure and then compute the current in a 4Ω load if a $24\angle 0^0$ V source is connected at the input port.



To find z_{11} and z_{21}, the output terminals are open circuited. Also connect a voltage source V_1 to the input terminals. This gives a circuit diagram as shown in Fig



Applying KVL to the left-mesh, we get

$$12I_1 + 6I_1 = V_1$$

$$\Rightarrow \quad V_1 = 18I_1$$

Hence $z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = 18\Omega$

Applying KVL to the right-mesh, we get

$$-V_2 + 3\times 0 + 6I_1 = 0$$

$$\Rightarrow \quad V_2 = 6I_1$$

Hence $z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = 6\ \Omega$

To find z_{12} and z_{22}, the input terminals are open circuited. Also connect a voltage source V_2 to the output terminals. This gives a circuit diagram as shown in Fig.

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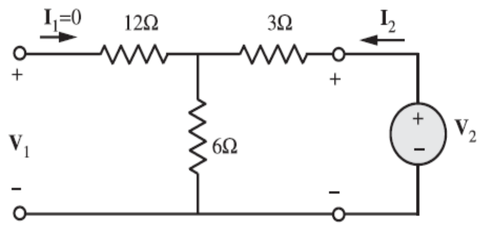


Applying KVL to the left-mesh, we get

$V_1 = 12 \times 0 + 6I_2$

$V_1 = 6I_2$

$z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = 6\ \Omega$

Applying KVL to the right-mesh, we get

$-V_2 + 3I_2 + 6I_2 = 0$

$V_2 = 9I_2$

$z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = 9\ \Omega$

The equations for the two-port network are, therefore

$V_1 = 18I_1 + 6I_2 \quad (1)$

$V_2 = 6I_1 + 9I_2 \quad (2)$

The terminal voltages for the network shown in Fig.8.2 are

$V_1 = 24\angle 0^0 \qquad (3)$

$V_2 = -4I_2 \qquad (4)$

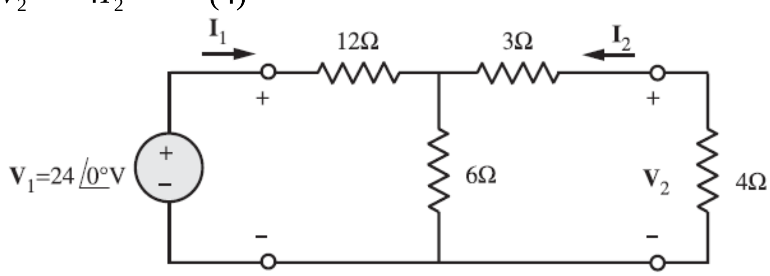


Fig.8.2

Combining equations (1) and (2) with equations (3) and (4) yields

$24\angle 0^0 = 18I_1 + 6I_2$

$0 = 6I_1 + 13I_2$

On Solving, we get $I_2 = -0.73\angle 0^0$ A

1.4 Admittance Parameter(y Parameter):

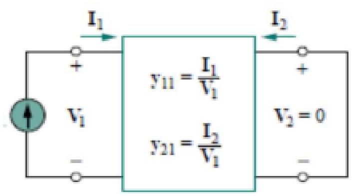
The terminal currents can be expressed in terms of the terminal voltages: The y terms are known as the admittance parameters (or, simply, y parameters) and have units of siemens.

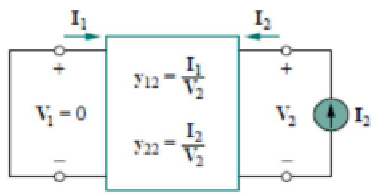
$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



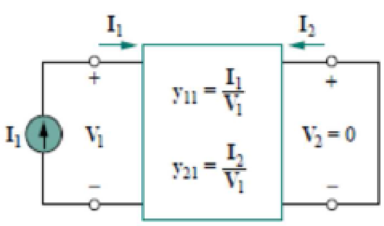


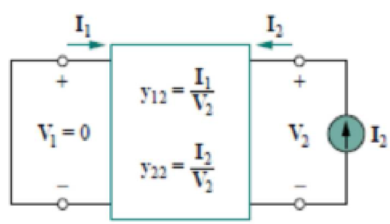
The y terms are called the y parameters, and have units of seimens. The values of the parameters can be evaluated by setting $V_1 = 0$ or $V_2 = 0$.

The y parameters are defined as follows:

Thus

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$





In the preceding equations, letting $V_1 = 0$ or $V_2 = 0$ is equivalent to short-circuiting the input or output port. Hence, the y parameters are called short-circuit admittance parameters.

If $y_{12} = y_{21}$, the network is said to be **reciprocal network**. Also, if $y_{11} = y_{22}$ then it is called a **symmetrical network.**

A reciprocal network ($y_{12} = y_{21}$) can be modeled by the equivalent circuit in Fig.8.3

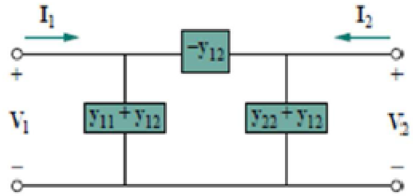


Fig.8.3

Example 8.4

Determine the admittance parameters of the T network shown in Fig.8.4

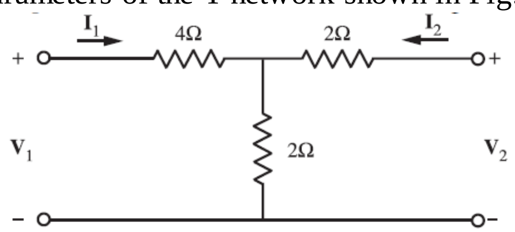
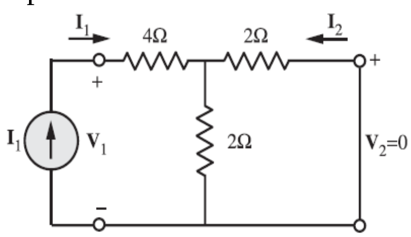


Fig.8.4

To find y_{11} and y_{21}, we have to short the output terminals and connect a current source I_1 to the input terminals. The circuit so obtained is shown in Fig.



$$I_1 = \frac{V_1}{4 + \frac{2 \times 2}{2+2}} = \frac{V_1}{5}$$

Hence $y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{5} S$

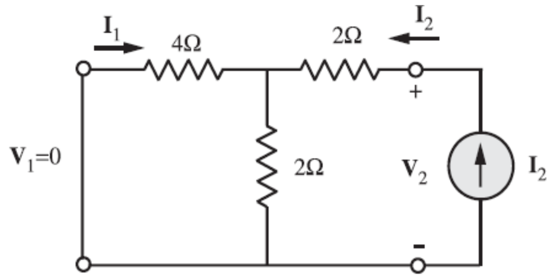
Using the principle of current division,

$$-I_2 = \frac{I_1 \times 2}{2+2} = \frac{I_1}{2}$$

Appliying KVL to first loop => $V_1 = 5I_1 = -10I_2$

Hence $y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{10} S$

To find y_{12} and y_{22}, we have to short-circuit the input terminals and connect a current source I_2 to the output terminals. The circuit so obtained is shown in Fig.



$$I_2 = \frac{V_2}{2+\dfrac{4\times 2}{4+2}} = \frac{3V_2}{10}$$

Hence $y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0} = \dfrac{3}{10}S$

$$-I_1 = \frac{I_2 \times 2}{2+4}$$

$$-I_1 = \frac{I_2}{3}$$

Applying KVL to loop 2

$$V_2 = 4I_2 + 2I_1 = -10I_1$$

Hence $y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0} = -\dfrac{1}{10}S$

It may be noted that, $y_{12} = y_{21}$ therefore the network is reciprocal.
Thus, in matrix form we have

$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{5} & -\dfrac{1}{10} \\ -\dfrac{1}{10} & \dfrac{3}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Example 8.5

For the network shown in Fig.8,5 determine the y parameters.

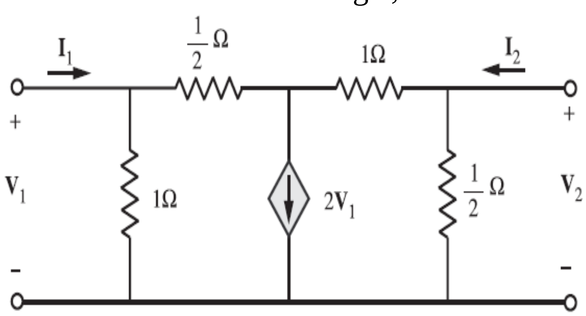


Fig.8.5

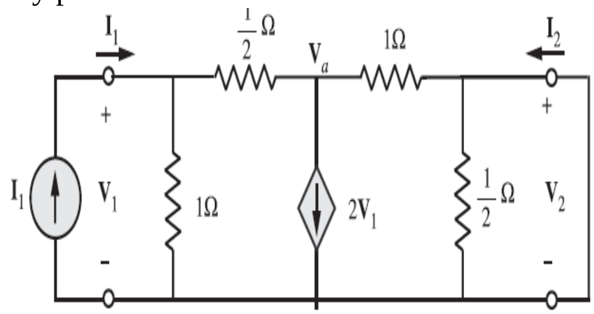


Fig.8.6

To find y_{11} and y_{21}short the output terminals and connect a current source I_1 to the input terminals. The resulting circuit diagram is as shown in Fig. 8.6

KCL at node V_1 :

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_a}{\frac{1}{2}}$$

$$\Rightarrow \quad 3V_1 - 2V_a = I_1$$

KCL at node V_a :

$$\frac{V_a - V_1}{\frac{1}{2}} + \frac{V_a - 0}{1} + 2V_1 = 0$$

$$\Rightarrow \quad V_a = 0$$

Substituting this in above equation => $3V_1 = I_1$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 3S$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = 0S$$

To find y_{22} and y_{12} short-circuit the input terminals and connect a current source I_2 to the output terminals. The resulting circuit diagram is shown in Fig. 8.6

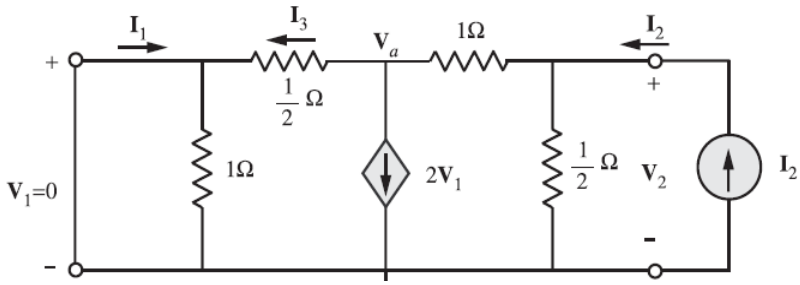


Fig.8.6

KCL at node V2: $\dfrac{V_2}{\frac{1}{2}} + \dfrac{V_2 - V_a}{1} = I_2$

$\Rightarrow \quad 3V_2 - V_a = I_2 \ldots(5)$

$\Rightarrow$

KCL at node Va:

$$\frac{V_a - 0}{\frac{1}{2}} + \frac{V_a - V_2}{1} + 0 = 0$$

$\Rightarrow \quad 3V_a - V_2 = 0$

$$V_a = \frac{1}{3}V_2 \ldots\ldots\ldots(6)$$

Substituting equation (6) in equation (5) yields

$$3V_2 - \frac{V_2}{3} = I_2$$

$\Rightarrow \quad \dfrac{8}{3}V_2 = I_2$

Hence

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_2=0} = \frac{8}{3}S$$

We have $\quad V_a = \dfrac{1}{3}V_2$

Also $I_1 + I_3 = 0$

$\Rightarrow \quad I_1 = -I_3$

$$= -\frac{V_a}{\frac{1}{2}} = -2V_a \ldots\ldots\ldots(7)$$

Making use of equation (7) in (6) yields

$$\frac{I_1}{2} = \frac{1}{3}V_2$$

Hence $y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{2}{3}S$

1.6 Transmission Parameters:

The transmission parameters are defined by the equations:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

A, **B**, **C** and **D** parameters represent the open-circuit voltage ratio, the negative short-circuit transfer impedance, the open-circuit transfer admittance, and the negative short-circuit current ratio, respectively.

Example 8.7

Determine the transmission parameters in the s domain for the network shown in Fig.8.7

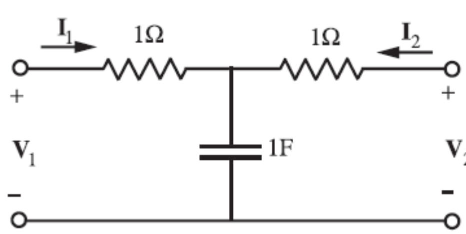


Fig.8.7

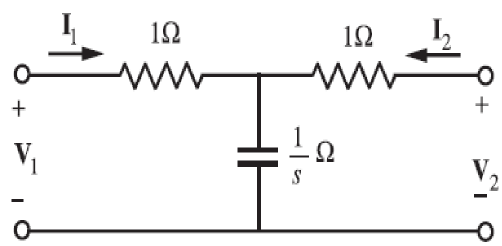
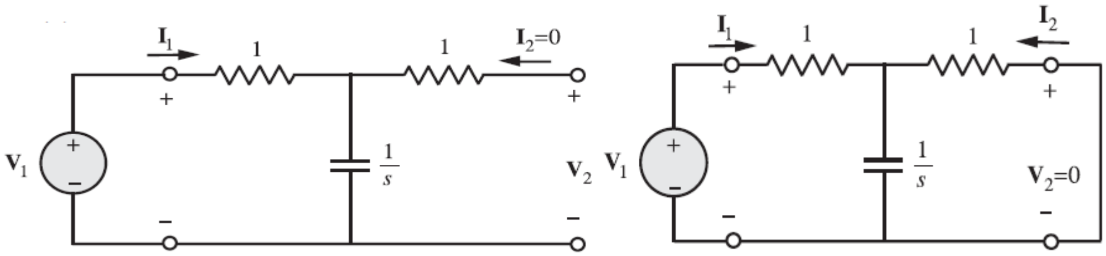


Fig.8.8

The s domain equivalent circuit with the assumption that all the initial conditions are zero is shown in Fig. 8.8

To find the parameters **A** and **C**, open-circuit the output port and connect a voltage source V_1 at the input port as shown in Fig. below (Left side)



$$I_1 = \frac{V_1}{1+\frac{1}{s}} = \frac{sV_1}{s+1}$$

$$V_2 = \frac{1}{s} I_1 = \frac{V_1}{s+1}$$

Therefore $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = s+1$

And $C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = s$

To find the parameters B and D, short-circuit the output port and connect a voltage source V_1 to the input port as shown in Fig. above (Right)

Applying the current division formula

$$I_2 = -I_1\left(\frac{1}{s+1}\right)$$

Hence $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = (s+1)$

The total impedance as seen by the source V_1 is

$$Z = 1 + \frac{1\times\frac{1}{s}}{1+\frac{1}{s}} = \frac{s+2}{s+1} = \frac{V_1}{I_1}$$

$$I_1 = -I_2(s+1) = \frac{V_1(s+1)}{(s+2)}$$

Hence $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = (s+2)$

Example 8.9
Find the transmission parameters for the network shown in Fig.8.9

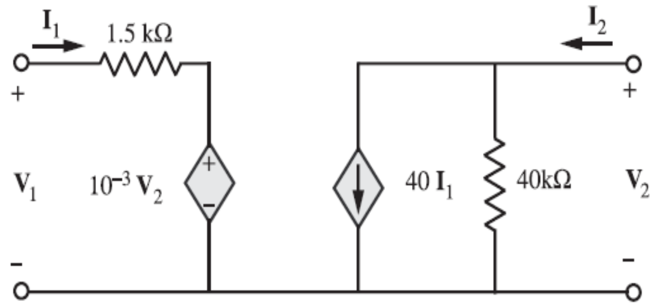


Fig.8.9

To find the parameters **A** and **C**, open the output port and connect a voltage source V1 to the input port as shown in Fig.8.10

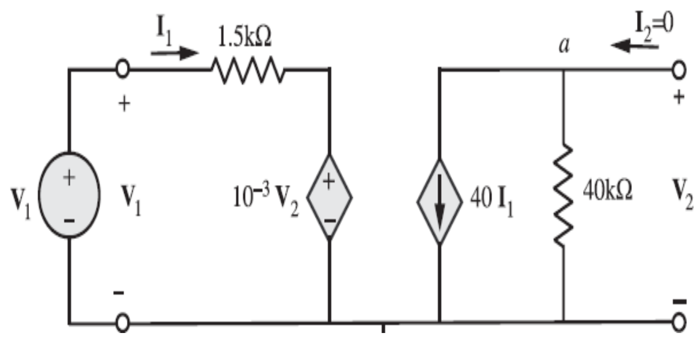


Fig.8.10

Applying KVL to the input loop, we get
$V_1 = 1.5\times10^3 I_1 + 10^{-3}V_2$
Also KCL at node a gives

$$40I_1 + \frac{V_2}{40\times10^3} = 0$$

$$\Rightarrow \quad I_1 = \frac{-V_2}{160\times10^3} = -6.25\times10^{-6}V_2$$

Substitute the value of I_1 in the preceding loop equation we get

$$V_1 = 1.5\times10^3(-6.25\times10^{-6}V_2) + 10^{-3}V_2$$

$$\Rightarrow \quad V_1 = -8.375\times10^{-3}V_2$$

Hence $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = -8.375\times10^{-3}$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = -6.25\times10^{-6}$$

To find the parameters **B** and **D**, refer the circuit shown in Fig.8.11

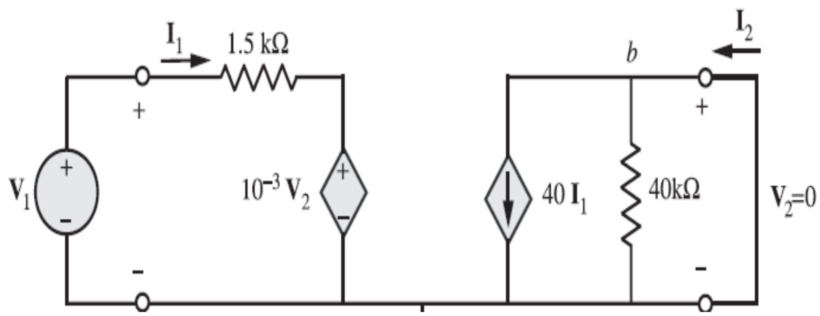


Fig.8.11

Applying KCL at node b, we find

$$40I_1 + 0 = I_2$$

$$I_2 = 40I_1$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{-1}{40}$$

Applying KVL to the input loop, we get

$$V_1 = 1.5\times10^3 I_1$$

$$V_1 = 1.5\times10^3 \times \frac{I_2}{40}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{-1.5\times10^3}{40} = -37.5\Omega$$

1.7 Hybrid Parameters (h parameters):

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h]\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

The parameters h_{11}, h_{12}, h_{21} and h_{22} represent the short circuit input impedance, the open circuit reverse voltage gain, the short-circuit forward current gain, and the open-circuit output admittance respectively. Because of this mix of parameters, they are called **hybrid parameters.**

Example 8.12

For the network shown in Fig.8.12 determine the h parameters.

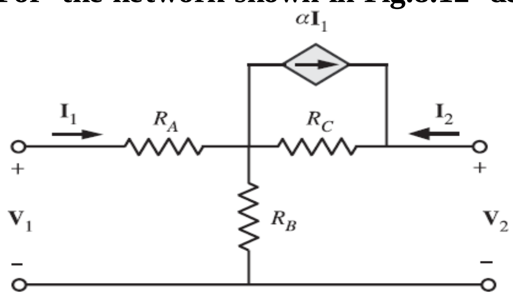
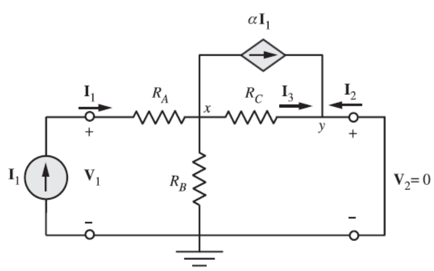


Fig.8.12

To find h_{11} and h_{21} short-circuit the output terminals so that $V_2 = 0$ .Also connect a current Source I_1 to the input port as in Fig.



Applying KCL at node x

$$-I_1 + \frac{V_x}{R_B} + \frac{V_x - 0}{R_C} + \alpha I_1 = 0$$

$$\Rightarrow \quad I_1[\alpha - 1] = -V_x\left[\frac{1}{R_B} + \frac{1}{R_C}\right]$$

$$\Rightarrow \quad V_x = \frac{(1-\alpha)I_1 R_B R_C}{R_B + R_C}$$

Hence, $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$

$$= \left.\frac{V_x + I_1 R_A}{I_1}\right|_{V_2=0}$$

$$= \frac{(1-\alpha) I_1 R_B R_C}{(R_B + R_C) I_1} + R_A$$

$$= \frac{(1-\alpha) R_B R_C}{(R_B + R_C)} + R_A$$

KCL at node y

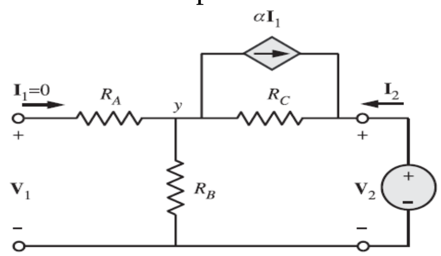
$$\alpha I_1 + I_2 + I_3 = 0$$

$$\alpha I_1 + I_2 + \frac{V_x - 0}{R_C} = 0$$

Substitute for V_x in the above equation and simplifying results in

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{(\alpha R_C + R_B)}{(R_C + R_B)}$$

To find h_{12} and h_{22} open-circuit the input port so that I_1. Also, connect a voltage source V_2 between the output terminals as shown in Fig.



KCL at node y:

$$\frac{V_1}{R_B} + \frac{V_1 - V_2}{R_C} + \alpha I_1 = 0$$

Since $I_1 = 0$ we get

$$\frac{V_1}{R_B} + \frac{V_1}{R_C} - \frac{V_2}{R_C} = 0$$

$$V_1\left[\frac{1}{R_B} + \frac{1}{R_C}\right] = \frac{V_2}{R_C}$$

$$\Rightarrow \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_B}{R_B + R_C}$$

Applying KVL to the output mesh, we get

$$-V_2 + R_C(\alpha I_1 + I_2) + R_B I_2 = 0$$

Since $I \;\; = 0$ we get

$R_C I_2 + R_B I_2 = V_2$

Hence, $h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0} = \dfrac{1}{R_C + R_B}$

Example 8.13

Determine the **h** parameters of the circuit shown in Fig.8.13

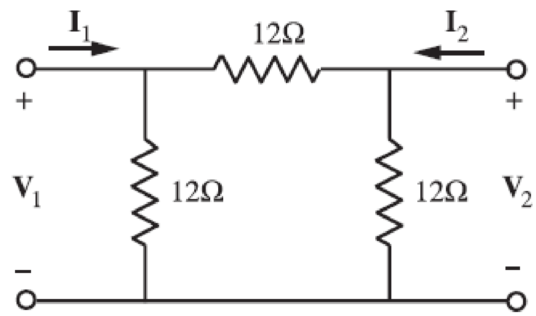


Fig.8.13

Performing Δ to Y transformation, the network shown in Fig.8.13 takes the form as shown in Fig.8.14

Since all the resistors are of same value $R_Y = \dfrac{1}{3} R_\Delta$

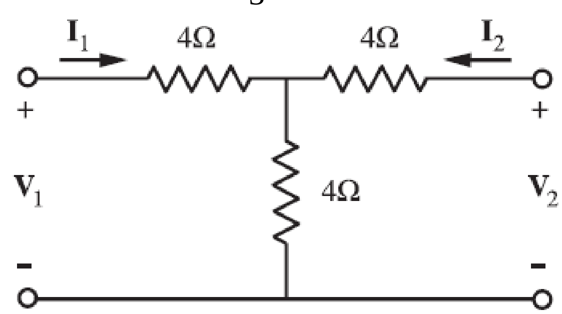


Fig.8.14

To find h_{11} and h_{21}, short-circuit the output port and connect a current source I_1 to the input port as in Fig. 8.15

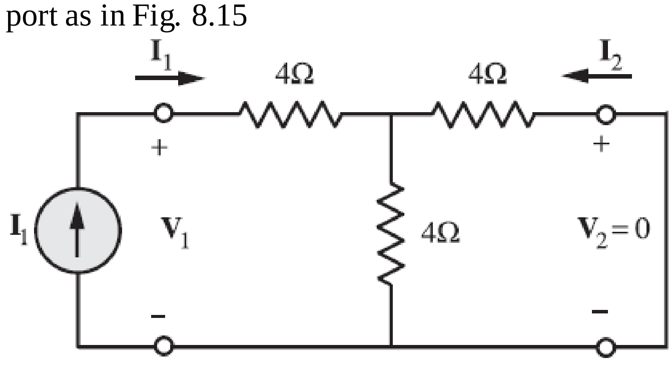


Fig.8.15

$$V_1 = I_1[4+(4||4)]$$

$$V_1 = 6I_1$$

Hence $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 6\Omega$

Using the principle of current division

$$-I_2 = \frac{I_1}{4+4}\times 4$$

$$\Rightarrow \quad -I_2 = \frac{I_1}{2}$$

$$\Rightarrow \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{1}{2}$$

To find h_{12} and h_{22}, open-circuit the input port and connect a voltage source V_2 to the output port as shown in Fig. 8.16

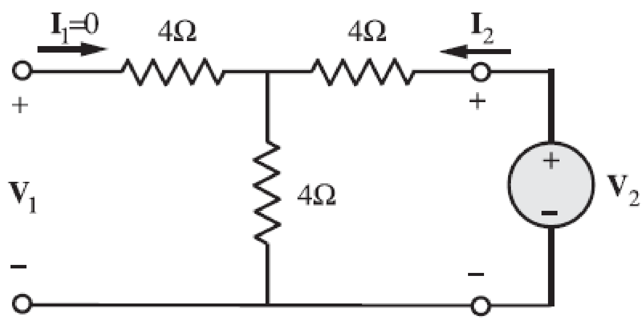


Fig.8.16

Using the principle of voltage division, we get

$$V_1 = \frac{V_2}{4+4}\times 4$$

$$\Rightarrow \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

$$V_2 = (4+4)I_2 = 8I_2$$

Hence, $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{8}S$

1.8 Relationship between the two port parameters:

If all the two-port parameters for a network exist, it is possible to relate one set of parameters to another, since these parameters interrelate the variables V_1, I_1, V_2 and I_2.

1.9 Relation between the z parameters and y parameters:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Or $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

And y Parameters are related by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Equating above two equations we see that

$$[y] = [z^{-1}]$$

To find z^{-1}

$$z^{-1} = \frac{adjo\,\mathrm{int}([z])}{\Delta z} \text{ where } \Delta z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix} = z_{11}z_{22} - z_{12}z_{21}$$

And $adjo\,\mathrm{int}([z]) = \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$

Hence, $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{z_{22}}{\Delta z} & \dfrac{-z_{12}}{\Delta z} \\ \dfrac{-z_{21}}{\Delta z} & \dfrac{z_{11}}{\Delta z} \end{bmatrix}$

1.10 Relation between z and h parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

From second equation

$$I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2$$

Substitute this in 1st equation

$$V_1 = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} I_1 + \frac{z_{12}}{z_{22}} V_2$$

Writing above equation in matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta z}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ -\dfrac{z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Compare this with h parameter equation

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta z}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ -\dfrac{z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{bmatrix}$$

Similarly all parameters can be related as shown in Table 1.11

Two-Port Parameter Conversion Table

Desired Parameters	Given Parameters [z]	[y]	[h]	[t]
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
[t]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_t = AD - BC$

Table 1.11

Example 8.17

Determine the y parameters for a two-port network if the z parameters are:

$$z=\begin{bmatrix}10 & 5\\ 5 & 9\end{bmatrix}$$

$$\Delta z = 10\times 9-5\times 5=65$$

$$\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}=\begin{bmatrix}\frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z}\\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z}\end{bmatrix}=\begin{bmatrix}\frac{9}{65} & \frac{-5}{65}\\ \frac{-5}{65} & \frac{10}{65}\end{bmatrix}$$

Example 8.18

Following are the hybrid parameters for a network:

$h=\begin{bmatrix}5 & 2\\ 3 & 6\end{bmatrix}$ **Determine the y parameters for the network.**

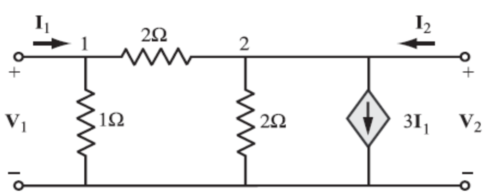
Solution

$$\Delta h = 5\times 6-3\times 2=24$$

$$\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}=\begin{bmatrix}\frac{1}{h_{11}} & \frac{-h_{22}}{h_{11}}\\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}}\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & -\frac{6}{5}\\ \frac{3}{5} & \frac{24}{5}\end{bmatrix}$$

Example 8.19

Determine the y and z parameters for a two-port network.



Applying KCL to the node 1, $1.5V_1-0.5V_2=I_1$

Applying KCL to the node 2, $-0.5V_1+V_2=I_2-3I_1$

Writing above equations in the matrix form

$$\begin{bmatrix}1.5 & -0.5\\ -0.5 & 1\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 1\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}$$

$$\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}1.5 & -0.5\\ -0.5 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0\\ -3 & 1\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}$$

$$= \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Therefore $[z] = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$

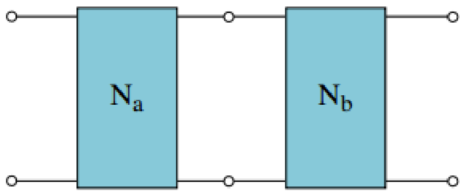
Hence $[y] = [z]^{-1} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$

1.12 Interconnection of Two port network:

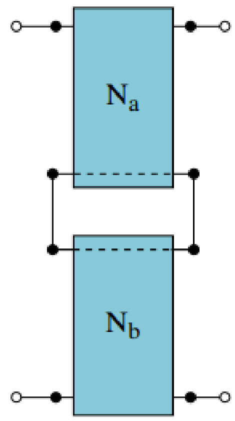
A large, complex network may be divided into subnetworks for the purposes of analysis and design. The subnetworks are modeled as two-port networks, interconnected to form the original network. The two-port networks may therefore be regarded as building blocks that can be interconnected to form a complex network. The interconnection can be in series, in parallel, or in cascade. Although the interconnected network can be described by any of the six parameter sets, a certain set of parameters may have a definite advantage. For example, when the networks are in series, their individual z parameters add up to give the z parameters of the larger network. When they are in parallel, their individual y parameters add up to give the y parameters of the larger network. When they are cascaded, their individual transmission parameters can be multiplied together to get the transmission parameters of the larger network.

The basic interconnections to be considered are: parallel, series and cascade.

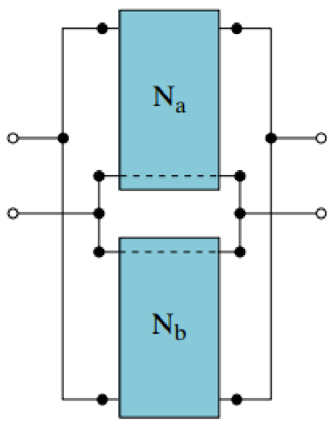
PARALLEL: Voltages are the same.Current of interconnection is the sum of currents.
SERIES: Currents are the same. Voltage of interconnection is the sum of voltages.
CASCADE: Output of first subsystem acts as input for the second.



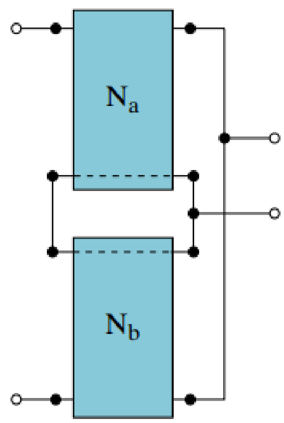
(a) Cascade



(b) Series



(c) Parallel



(d) Series-Parallel

1.13 Networks in parallel:

Two two-port networks are in parallel when their port voltages are equal and the port currents of the larger network are the sums of the individual port currents. In addition, each circuit must have a common reference and when the networks are connected together, they must all have their common references tied together.

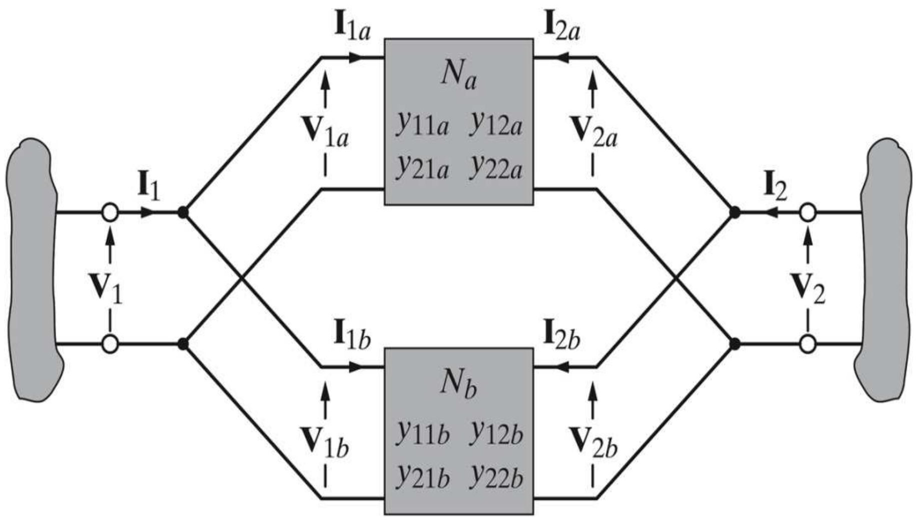


Fig.8.20

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I = YV$$

$$I_a = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}, V_a = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}, \; Y_a = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix}$$

$$=> I_a = Y_a V_a$$

In a similar manner

$$I_b = Y_b V_b$$

$$I_1 = I_{1a} + I_{1b}, \; I_2 = I_{2a} + I_{2b}$$

$$V_1 = V_{1a} = V_{1b}, \; V_2 = V_{2a} = V_{2b}$$

$$\Rightarrow \begin{cases} I = I_a + I_b \\ V = V_a = V_b \end{cases}$$

$$\Rightarrow I = Y_a V_a + Y_b V_b = (Y_a + Y_b) V$$

$$Y = Y_a + Y_b$$

Hence if two port sub networks are connected in parallel then the resultant network y parameters can be calculated by adding individual y parameters of sub networks.

1.14 Neworks in series

The networks are regarded as being in series because their input currents are the same and their voltages add. In addition, each network has a common reference, and when the circuits are placed in series, the common reference points of each circuit are connected together.

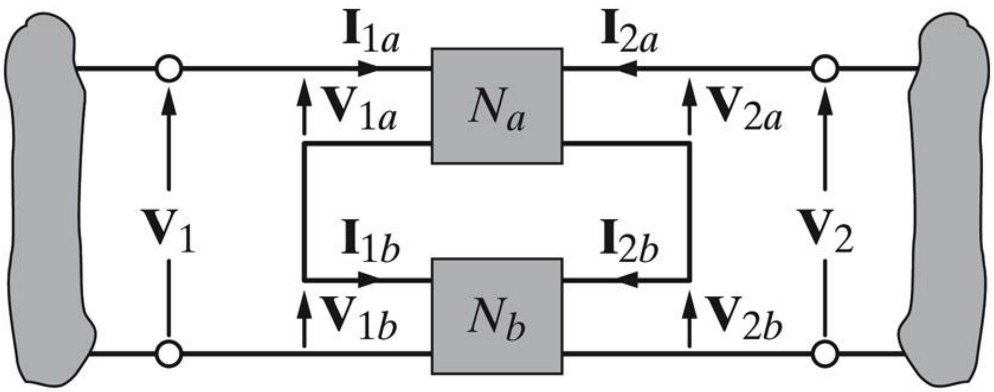


Fig.8.21

Description of subnetworks

$V_a = Z_a I_a, \quad V_b = Z_b I_b$

Where $Z_a = \begin{bmatrix} z_{11a} & z_{12a} \\ z_{21a} & z_{22a} \end{bmatrix}, \quad Z_b = \begin{bmatrix} z_{11b} & z_{12b} \\ z_{21b} & z_{22b} \end{bmatrix}$

Since by definition $I_a = I_b = I$ and $V_1 = V_{1a} + V_{1b}$

⇨ $V_1 = (Z_a + Z_b)I$

⇨ $Z = Z_a + Z_b$

1.15 Networks in cascade:

Output of first subsystem acts as input for the second

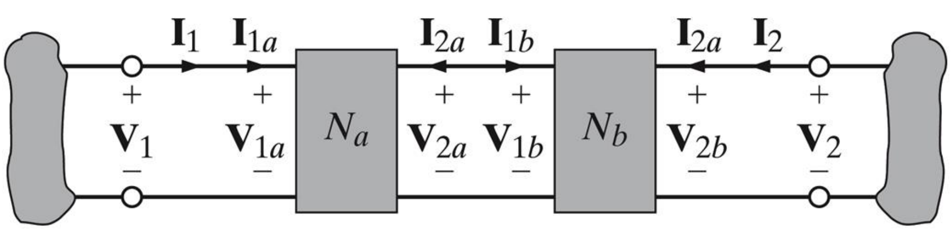


Fig.8.22

Interconnection constraints

$V_{2a} = V_{1b} \quad I_{2a} = -I_b$

$V_1 = V_{1a} \quad V_1 = V_{1a}$ and

$I_1 = I_{1a} \quad I_2 = I_{2b}$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Hence $\begin{bmatrix} V_1 \\ I \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C & D \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I \end{bmatrix}$

It is this property that makes the transmission parameters so useful. Keep in mind that the multiplication of the matrices must be in the order in which the networks *Na* and *Nb* are cascaded.

Example 8.23:Comptute the y parameter of the network shown in Fig.Q.8.23

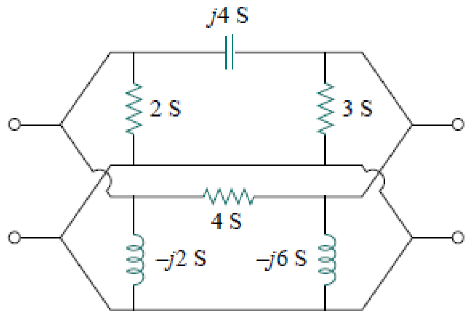
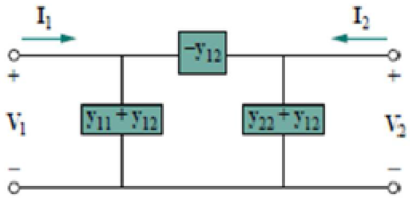


Fig.Q.8.23

Let us refer to the upper network as *Na* and the lower one as *Nb*. The two networks are connected in parallel. Comparing *Na* and *Nb* with the circuit shown below.



we obtain

$$y_{12a} = -j4 = y_{21a} \qquad y_{11a} + y_{12a} = 2 \qquad y_{22a} + y_{12a} = 3$$

Or

$$y_{12a} = -j4 = y_{21a} \qquad y_{11a} = 2 + j4 \qquad y_{22a} = 3 + j4$$

i.e. $Y_a = \begin{bmatrix} 2+j4 & -j4 \\ -j4 & 3+j4 \end{bmatrix} S$

And

$$y_{12b} = -4 = y_{21b} \qquad y_{11b} + y_{12b} = -j2 \qquad y_{22b} + y_{12b} = -j6$$

or

$$y_{12b} = -4 = y_{21b} \qquad y_{11b} = 4 - j2 \qquad y_{22b} = 4 - j6$$

i.e. $Y_b = \begin{bmatrix} 4-j2 & -4 \\ -4 & 4-j6 \end{bmatrix} S$

Therefore, overall y parameters are

$$[Y]=[Y_a]+[Y_b]=\begin{bmatrix} 6+j2 & -4-j4 \\ -4-j4 & 7-j2 \end{bmatrix} S$$

1.16 Network Functions for One Port and Two Port Network

1.17 Driving Point Functions:

The impedance or admittance found at a given port is called a driving point impedance(or admittance).

1.Driving Point Impedance:

$Z_{11}(s) = V_1(s)/I_1(s)$

2. Driving Point Admittance:

$Y_{11}(s) = I_1(s)/V_1(s) = 1/Z_{11}(s)$

1.18 Transfer Function:

The transfer function relates the transform of a quantity at one port to the transform of another quantity at another port.Thus transfer functions which relate voltages and currents have following possible forms:

The ratio of one voltage to another voltage, or the **voltage transfer ratio.**

The ratio of one current to another current ,or the **current transfer ratio**.

The ratio of one current to another voltage or one voltage to another voltage.

Transfer function for the two port network:

Denominator	Numerator	
	$V_2(s)$	$I_2(s)$
$V_1(s)$	$G_{12}(s)$	$Y_{12}(s)$
$I_1(s)$	$Z_{12}(s)$	$\alpha_{12}(s)$

Example 8.24
Find driving point impedance Z(s) for the network shown in Fig.Q.8.24

Solution
The driving point impedance for RLC series circuit is

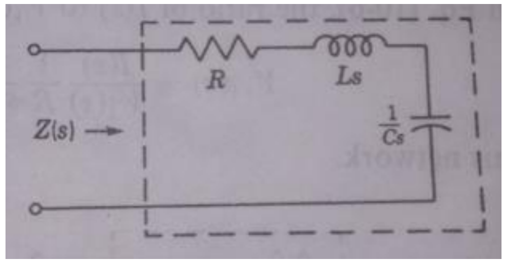


Fig.Q.8.24

$$Z(s) = R + Ls + \frac{1}{sC} = \frac{s^2 LC + sRC + 1}{sC}$$

Example 8.25
Find driving point impedance Z(s) for the network shown in Fig.Q.8.25

Solution
The driving point impedance for series RL network shunted by a capacitor is

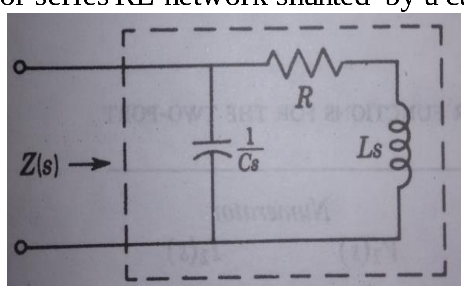


Fig.8.25

$$Z(s) = \frac{1}{sC + 1/(R + sL)} = \frac{1}{C} \frac{s + R/L}{s^2 + (R/L)s + 1/LC}$$

and driving point admittance function $Y(s) = \frac{1}{Z(s)}$

Example 8.26

For the network shown find the voltage ratio $G_{12}(s)=\dfrac{V_2(s)}{V_1(s)}$ **and** $Y_{11}(s)=\dfrac{I(s)}{V_1(s)}$ **by considering no current in the output terminals.**

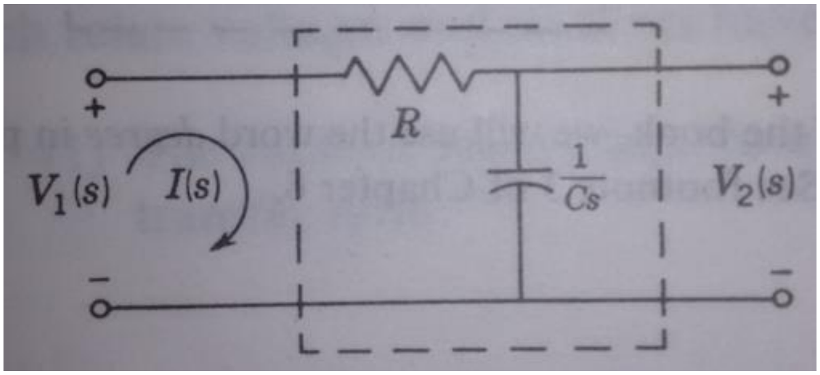


Fig.8.26

Solution

The network acts as a voltage divider .

$$RI(s)+\frac{1}{sC}I(s)=V_1(s)$$

and

$$\frac{1}{sC}I(s)=V_2(s)$$

Therefore the ratio of these equations is

$$G_{12}(s)=\frac{V_2(s)}{V_1(s)}=\frac{(1/sC)I(s)}{(R+1/sC)I(s)}=\frac{1/RC}{s+1/RC}$$

$$Y_{11}(s)=\frac{I(s)}{V_1(s)}=\frac{1}{R+1/sC}=\frac{1}{R}\frac{s}{s+1/RC}$$

Example 8.27

For the network shown in Fig.8.27 find $G_{12}(s)$

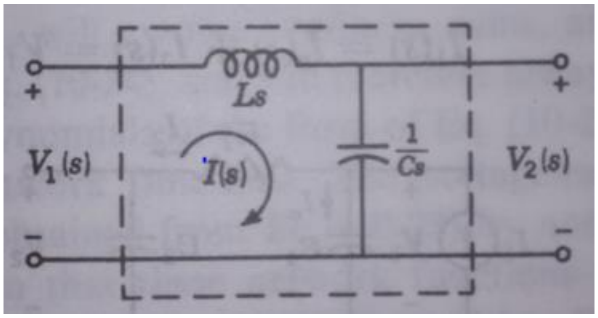


Fig.8.27

Solution

$$G_{12}(s)=\frac{V_2(s)}{V_1(s)}=\frac{V_C}{V_L+V_C}=\frac{1/sC}{sL+1/sC}=\frac{1}{s^2LC+1}=\frac{1/LC}{s^2+1/LC}$$

Where V_L and V_C are the voltages across Inductor and capacitor respectively.

Example 8.28

For the network shown in Fig. 8.28 find $G_{12}(s)$

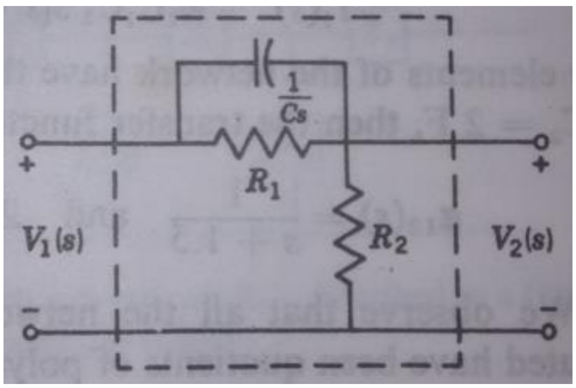


Fig.8.28

Solution

R_1 and $1/sC$ can be combined into an equivalent impedance having the value

$$Z_{eq}(s)=\frac{1}{sC+1/R_1}=\frac{R_1}{sR_1C+1}$$

$$G_{12}(s)=\frac{V_2(s)}{V_1(s)}=\frac{R_2}{R_2+Z_{eq}(s)}$$

$$G_{12}(s)=\frac{sR_1R_2C+R_2}{sR_1R_2C+R_1+R_2}=\frac{s+1/R_1C}{s+(R_1+R_2)/R_1R_2C}$$

Exercise: Q.101

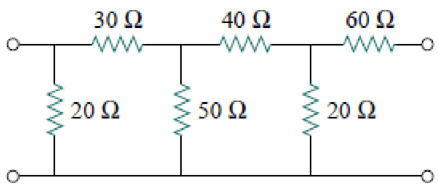


Fig.Q.101

Ans: $[T]=\begin{bmatrix} 29.25 & 2200\Omega \\ 0.425S & 32 \end{bmatrix}$

Q.102 **for the bridge circuit shown in Fig.Q.102. Find the transmission parameters.**

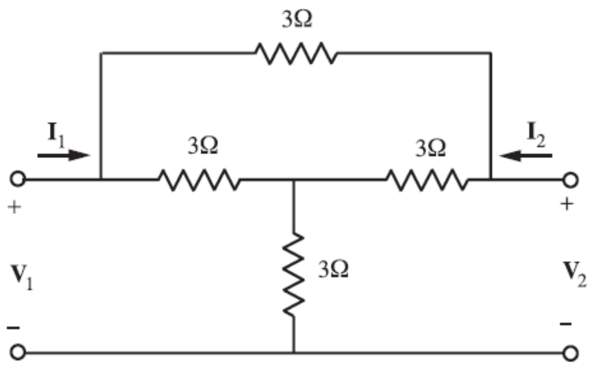


Fig.Q.102

Q.103 The network of Fig.Q.103 contains both a dependent current source and a dependent voltage source. Determine y and z parameters.

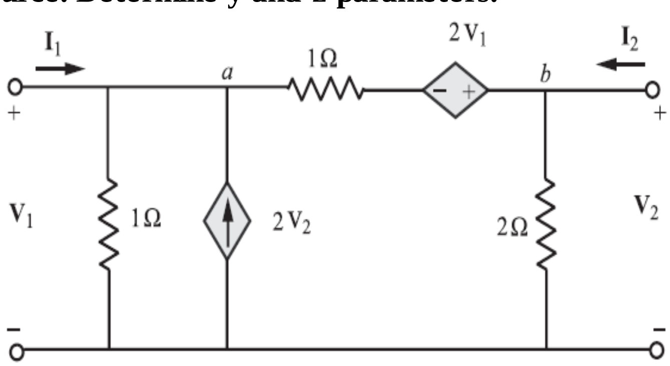


Fig.Q.103

Q.104 Find the h parameters for the network shown in Fig.Q.104. Keep the result in s domain.

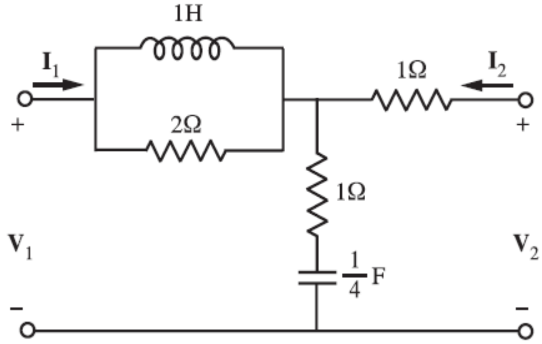


Fig.Q.104

Ans : $h_{11} = \dfrac{5s+4}{2(s+2)}$ $\quad h_{12} = \dfrac{s+4}{2(s+2)}$ $\quad h_{21} = \dfrac{-(s+4)}{2(s+2)}$ $\quad h_{22} = \dfrac{s}{2(s+2)}$

105. For the network shown in Fig.Q.105, determine the h parameters

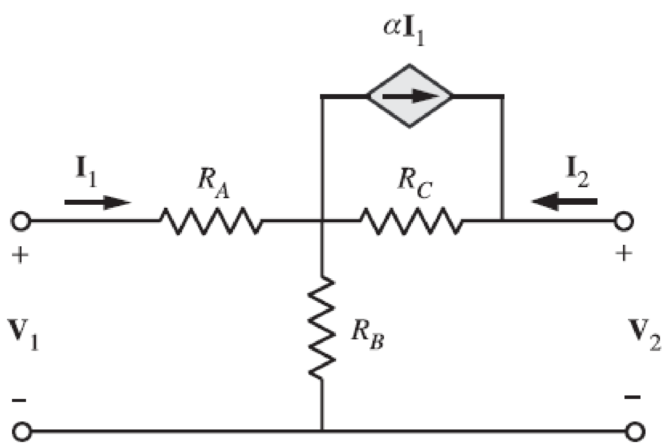


Fig.Q.105

Ans:

$h_{11} = \dfrac{(1-\alpha)R_B R_C}{R_B + R_C} + R_A$ $\quad h_{21} = \dfrac{-(\alpha R_C + R_B)}{R_B + R_C}$ $\quad h_{12} = \dfrac{R_B}{R_B + R_C}$ $\quad h_{22} = \dfrac{1}{R_B + R_C}$

Q.106 For what value of 'a' is the circuit shown in Fig.Q.106 is reciprocal? Also find h parameters.

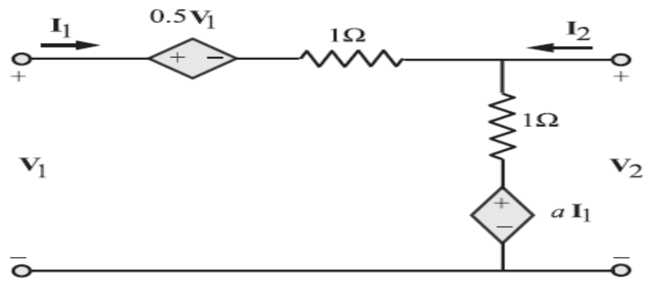


Fig.Q.106

Ans: a=2 and $[h]=\begin{bmatrix} 2 & 2 \\ 2 & 0.5 \end{bmatrix}$

Q107. What is the value of n for the network shown in Fig.Q.107 to be reciprocal? Also find **y**12 and **y**21 for that n

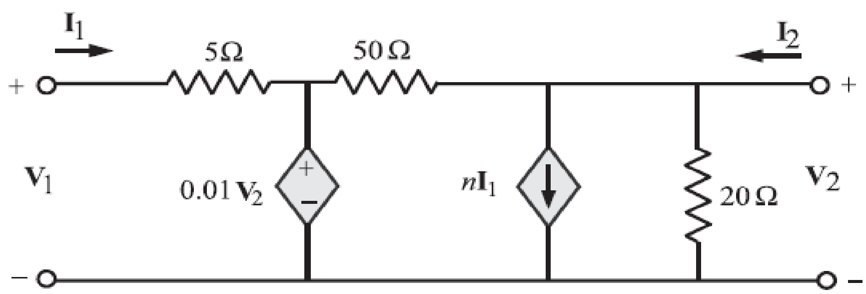


Fig.Q.107

Ans: n= -0.01 and **y**12 = **y**21=-0.002

Summary

1. Various electronic devices such as transistors, transformers etc can be modelled using various two port network parameters such as impedance, admittance, transmission and hybrid parameters.

2. Relation between various two port network parameters studied and verified for different two port parameters.

3. Complex two port network in cascade, parallel or any other form can be modeled using simple two port network parameters.

4. Various driving point & transfer functions are defined and determined for the two port networks.