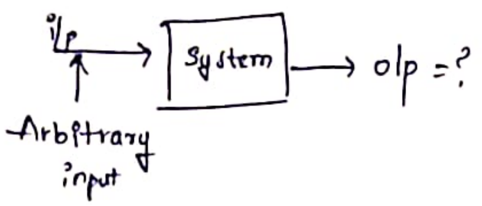
5/9/19

Module-2 : Time-Domain Analysis, (2.1 to 2.4)

signals ⟷ system

Synthesis (I/p, o/p known, System unknown)   Analysis (Input, System known Output unknown)



Q1: What is the o/p of the system to any arbitrary i/p?
(i) How to Express x(n) in terms of $\delta(n)$?

→ Time-Domain Analysis:

Assume system is linear & Time-Invariant :- LTI System.

→ Impulse Response:

o/p of a system when the i/p is an Impulse function

Assume the impulse response of the system is known.

Impulse Response is the Identity ~~to~~ a system. It is unique.

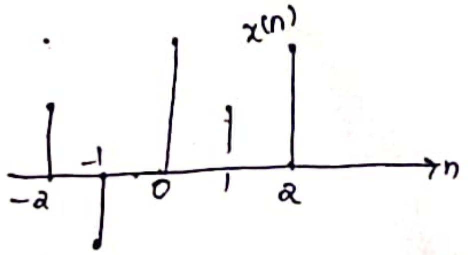
~~Eg. If~~ .   ~~$\delta(n)=0, n\neq 0$~~

~~$=1, n=1$~~

Q: The ultimate Q/A that should be finally answered is Q1?

→ Discrete-time LTI System

* The representation of DTs in terms of impulses.



If u want to find δ at -1

$$x(-1)\,\delta(n+1) = \begin{cases} x(-1), & n=-1 \\ 0, & n\neq -1 \end{cases}$$

$$x(0)\,\delta(n) = \begin{cases} x(0), & n=0 \\ 0, & n\neq 0 \end{cases}$$

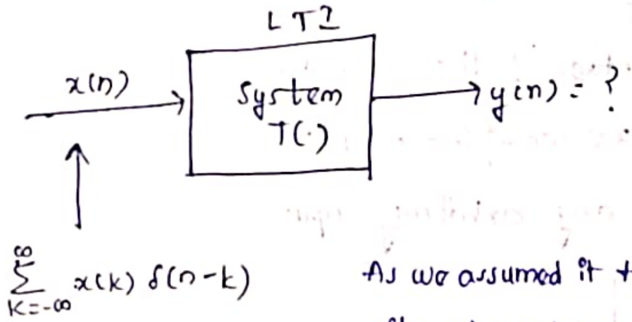
$$x(1)\,\delta(n-1) = \begin{cases} x(1), & n=1 \\ 0, & n \neq 1 \end{cases}$$

$$x(n) = \cdots\cdots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + \cdots$$

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)}$$

Eg:

Q.



As we assumed it to be linear its allowed to take transform inside (just like laplace transform) $\therefore \left[\because T[ax_1 + bx_1] = Tax_1 + Tbx_1\right]$

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} T\left[x(k)\cdot\delta(n-k)\right] \rightarrow \text{superposition + Laplace + Homogeneity}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\cdot T\left[\delta(n-k)\right] \rightarrow \text{Time invariance}$$

Let, $h(n)$ denote the impulse response of the system

i.e., $T[\delta(n)] = h(n)$

$T[\delta(n-n_0)] = h(n-n_0)$

$T[\delta(n-k)] = h(n-k)$ → System is T.I

$$\therefore y(n) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k)$$

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8. CONVOLUTION THEOREM

$x(n) \rightarrow$ [$h(n)$ system] $\rightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

Mixing of two signals is called as "convolution of signals"

$= x(n) * h(n)$

Convolution Sum

$\rightarrow$ convolution of the signal

(here we are mixing $x(n)$ & $h(n)$ to find the $y(n)$ output)

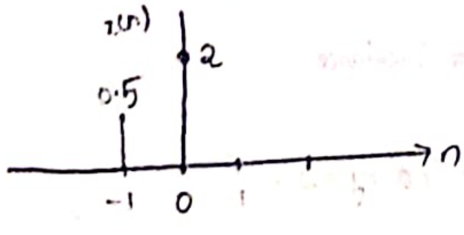
advantage of the convolution is we can find output for any arbitrary Input

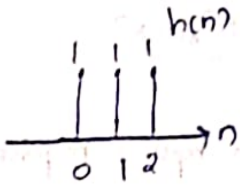
$$\boxed{\begin{aligned} y(n) &= x(n) * h(n) \\ y(n) &= \sum_{k=-\infty}^{\infty} x(k)\,h(n-k) \end{aligned}}$$

$\rightarrow$ property of time invariance

Q3: calculate the output for a system (of an LTI) with impulse response $h(n) = \{1,1,1\}$

$x(n) = \{0.5, \underset{\uparrow}{2}\}$





Method-1 : Express $x(n)$ in terms of $\delta(n)$

$x(n) =$ ~~δ(n+1) δ(~~ $x(-1)\,\delta(n+1) + x(0)\,\delta(n)$

$0.5\,\delta(n+1) + 2\,\delta(n)$

$x(n) \rightarrow$ [$T(\cdot)$] $\rightarrow y(n)$

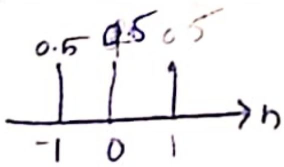
$y(n) = T[x(n)]$

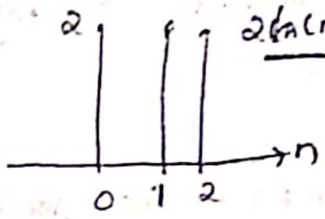
$= T\,[0.5\,\delta(n+1) + 2\,\delta(n)]$

$= 0.5\,T\,[\delta(n+1)] + 2\,T\,[\delta(n)]$

$y(n) = 0.5\,h(n+1) + 2h(n)$

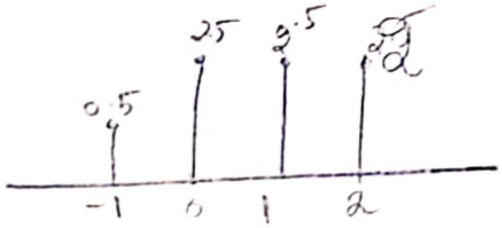
$0.5\,h(n+1)$ + $2h(n)$





$0.5\,h(n+1) + 2h(n) = y(n)$

$y(n) =$



$y(n) = \{$

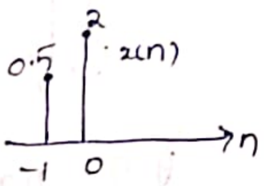
Method-2 convolution sum

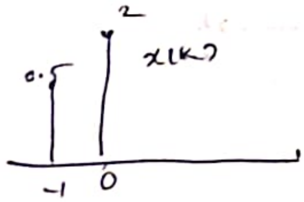
$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k) = y(n)$$

Ex 2.2 $x(n) = \{0.5, 2\}$

$h(n) = \{1, 1, 1\}$

step-1 find & plot $x(k)$

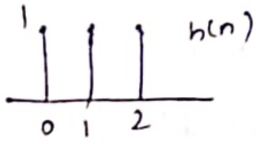


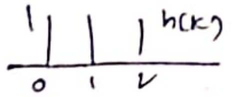


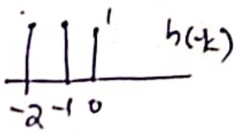
$h(-k+n)$

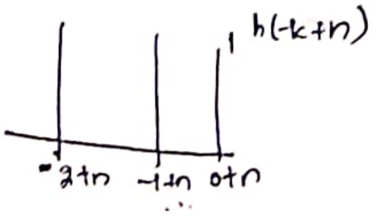
Step-2: to find & plot $h(n-k)$

$h(n)$
↓
$h(k)$
↓
$h(-k)$
↓
$h(-k+n)$

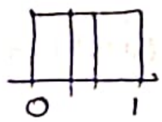




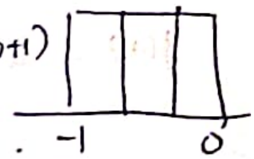




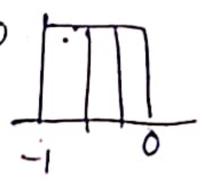
$x(n) \rightarrow$



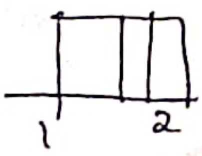
, $x(n+1)$



$x(-n) \Rightarrow$



$x(n-1)$



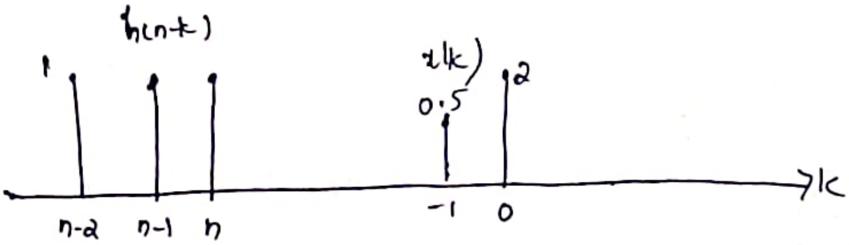
$x(-n+1) \Rightarrow$ for $x(n+1)$ it's shifting towards left by 1 unit

for $x(-n+1)$ it's shifting toward right by 1 unit

Note:

The signal $x(n+1)$ is shifted left by 1 unit whereas signal $x(-n+1)$ is shifted right by one unit i.e., shifting gets reversed for signal $x(-n)$ in comparision with $x(n)$

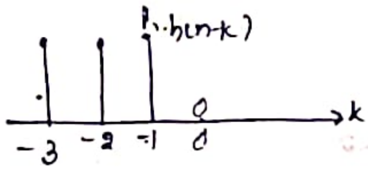
Step-3 plot $x(k)$ & $h(n-k)$ on the same k-axis

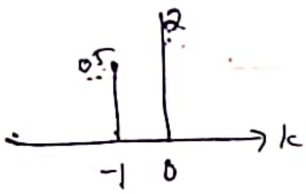


(i) $n < -1$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = 0$$

(ii) $n = -1$

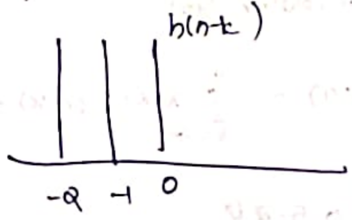


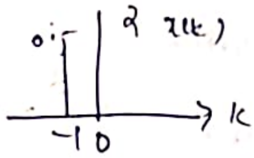


$y(-1) = 2(0) + (0.5)(1) + 0(1) + 0(1)$

$= 0.5$

(iii) $n = 0$





$y(0) = 1(0.5) + 2(1)$

$= 2.5$

(iv) $n = 1$

$y(1) = 2.5$

(v) $n = 2$

$y(2) = 2$

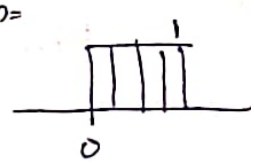
(vi) $n > 2$

$y(n) = 0$

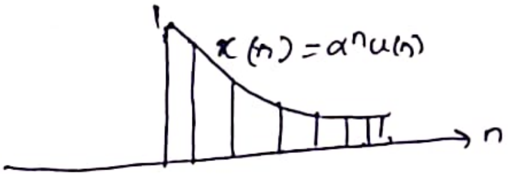
$$y(n) = \{0.5, 2.5, 2.5, 2\}$$
$\uparrow$

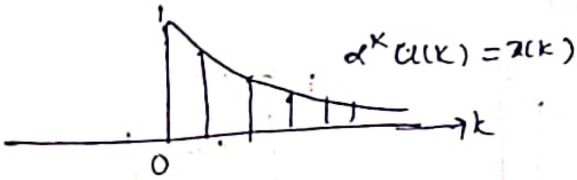
Exa.3 $x(n) = \alpha^n u(n)$; $0 \le \alpha \le 1$

$h(n) = u(n)$



Sol^n (i) plot $x(k)$

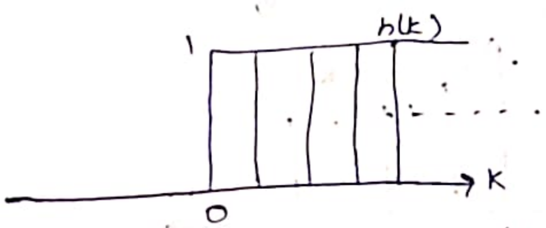




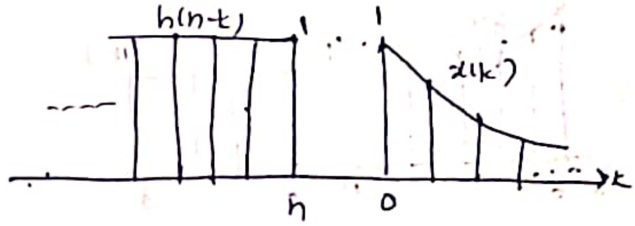
$x(0)\,\delta(n)$.
$1\,\delta(n)$

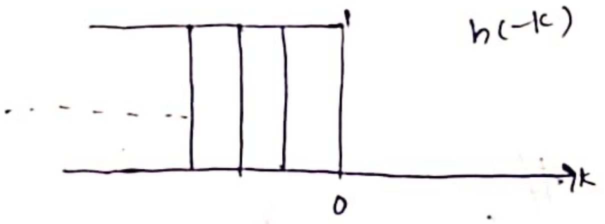
$x(k)\,\delta(n-k)$

(ii) plot $h(n-k)$



(iii) plot $x(k)$ & $h(n-k)$ in the same k-axis

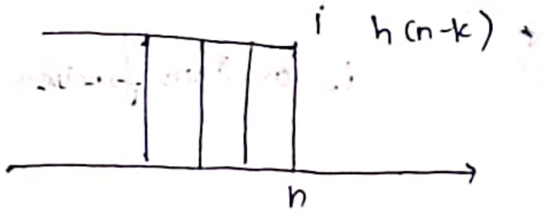




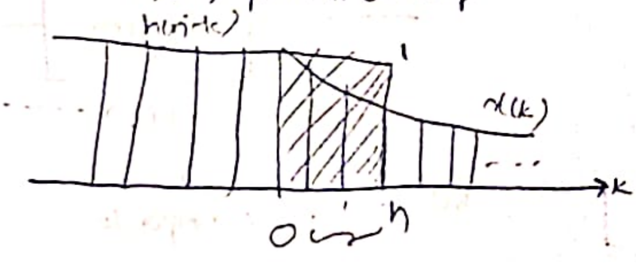
(iv) Find $y(n)$

(i) $n < 0$; No overlap

$\Rightarrow y(n) = 0$



(ii) $n \geq 0$; partial overlap



$$y(n) = \sum_{k=0}^{n} x(k)\,h(n-k)$$

$$y(n) = \sum_{k=0}^{n}$$

$$y(n) = \sum_{k=0}^{n} \alpha^k \cdot 1$$

$$y(n) = \frac{1-\alpha^{n+1}}{1-\alpha}$$

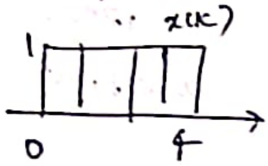
$$\Rightarrow y(n) = \begin{cases} 0, & n < 0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha} & n \geq 0 \end{cases}$$

Ex 24

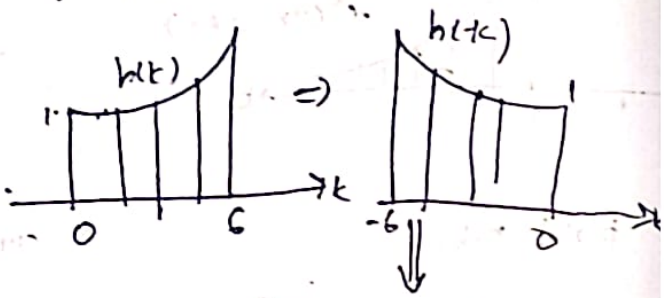
$$x(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$h(n) = \begin{cases} \alpha^n, & 0 \le n \le 6 \quad ; \alpha > 1 \\ 0, & \text{otherwise.} \end{cases}$$

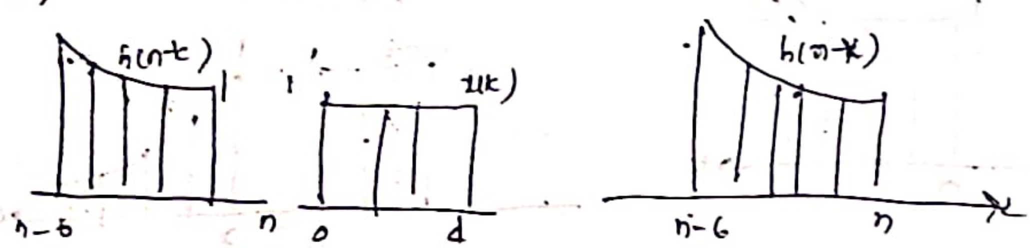
i) $x(k)$



ii) $h(n-k)$



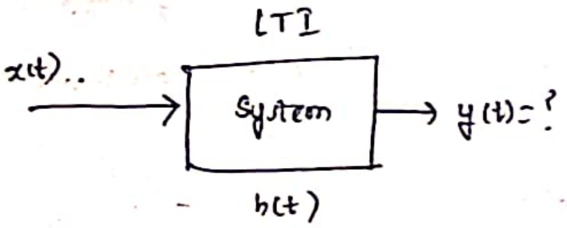
(iii)



7/9/19

Continuous Time-Domain Analysis:

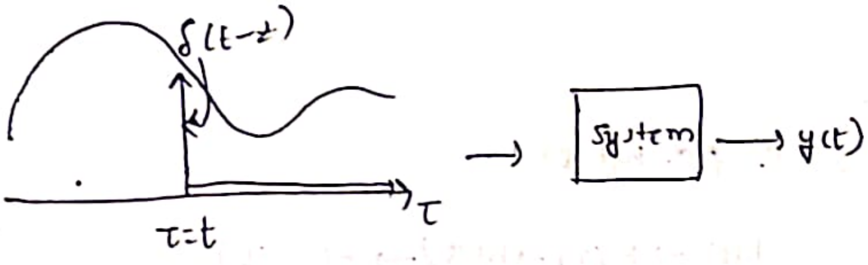
(" Impulse function is an Even funct



$h(t)$ = Impulse response

i.e., $T[\delta(t)] = h(t)$

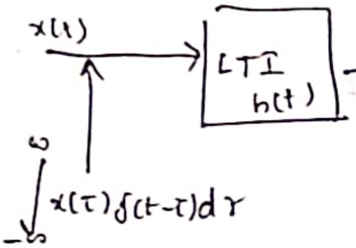
Q: How to Express $x(t)$ in terms of impulse function?



$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

The i/p $x(t)$ can be expressed as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$



$$y(t) = T[x(t)]$$

$$= T\left[\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right]$$

$$= \int_{-\infty}^{\infty} x(\tau)T[\delta(t-\tau)]d\tau$$ → due to linearity

$$T[\delta(t)] = h(t)$$

$$T[\delta(t-\tau)] = h(t-\tau)$$

$$\therefore \; y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$
$= x(t) * h(t)$	$= x(n) * h(n)$
convolution Integral	convolution sum

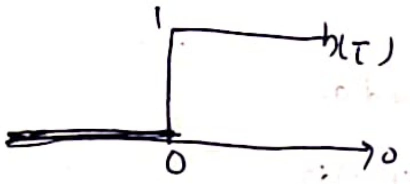
Ex2-6 $\quad x(t) = e^{-at} u(t), \; a > 0$

$h(t) = u(t)$

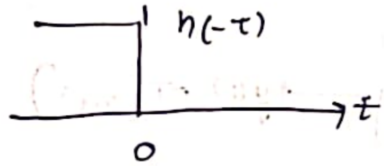
Sol (i) plot $x(\tau)$

(ii) plot $h(t-\tau)$.

$h(t) \rightarrow h(\tau) \rightarrow h(-\tau) \rightarrow h(-\tau+t)$

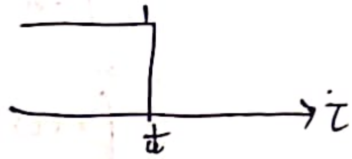


(iii) · $h(t-\tau)$



(iv)

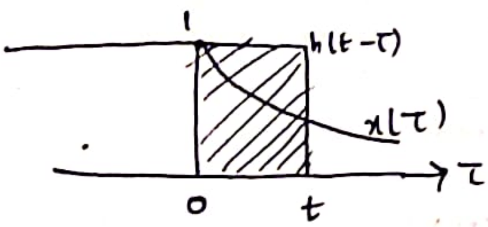
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$



case (i): $t \le 0$

No overlap $\Rightarrow y(t) = 0$

case (ii) : $t \geq 0$ partial overlap



$$\rightarrow \quad y(t) = \int_0^t x(\tau) h(t-\tau) d\tau$$

$$= \int_0^t e^{-a\tau} \cdot 1 \, d\tau$$

$$= \frac{1 - e^{-at}}{a}$$

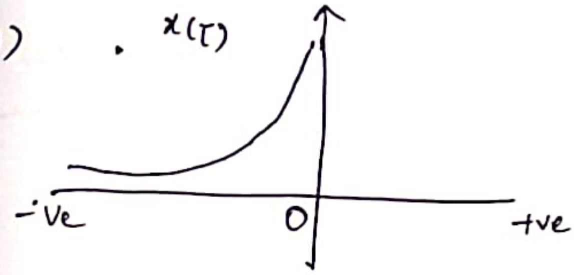
$$y(t) = \begin{cases} 0, & t \le 0 \\ \frac{1}{a}(1 - e^{-at}), & t \geq 0 \end{cases}$$

$$y(t) = \frac{1}{a}(1 - e^{-at}) \, u(t)$$

Ex 2.8 : $x(t) = e^{2t} u(-t)$

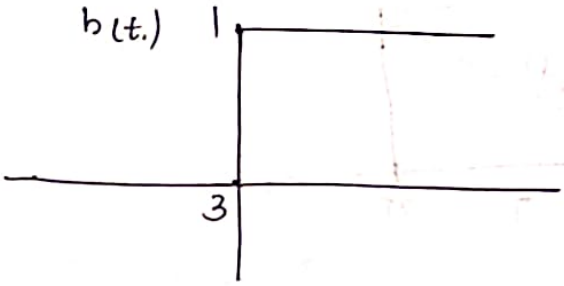
$h(t) = u(t-3)$

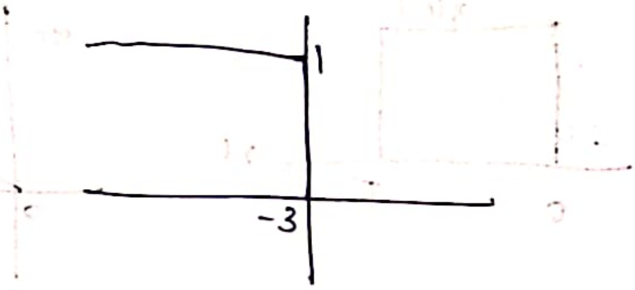
(i)

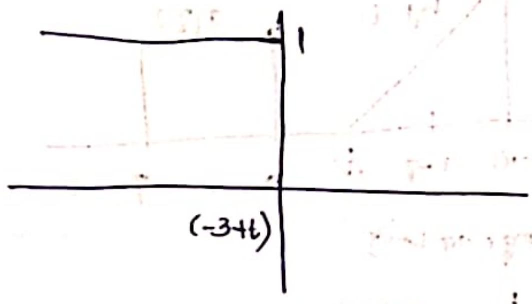


(ii) plot $h(t-\tau)$

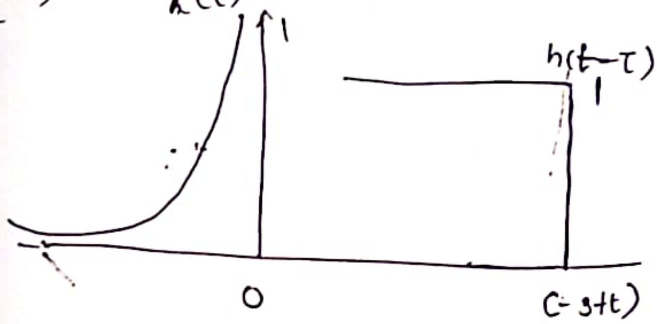
$h(t) \rightarrow h(\tau) \rightarrow h(-\tau) \rightarrow h(-\tau+t)$



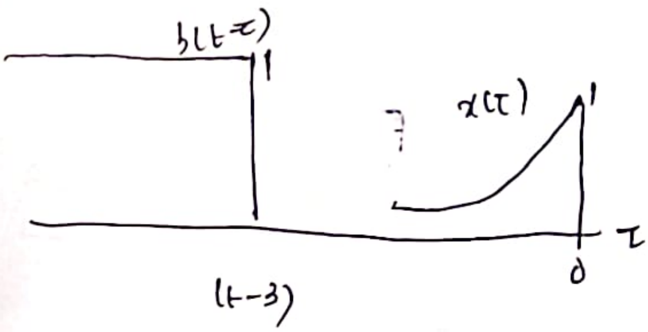




(iii)



case(i) :- (or)



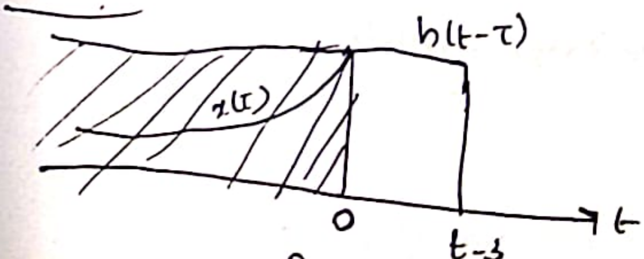
case(i) $t-3 \leq 0$

from $t-3$ to $-\infty$ it will overlap

$$y(t) = \int_{-\infty}^{t-3} x(\tau) h(t-\tau) d\tau$$

$$= \int_{-\infty}^{t-3} e^{2\tau} d\tau = y(t) = \frac{1}{2} e^{2(t-3)}$$

case (ii) :- $t-3 \geq 0$



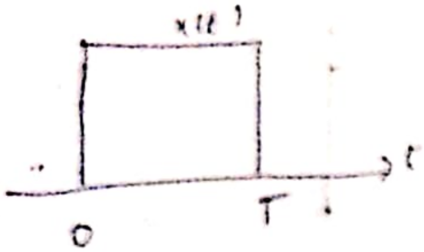
$$y(t) = \int_{-\infty}^{0} x(\tau) h(t-\tau) d\tau$$

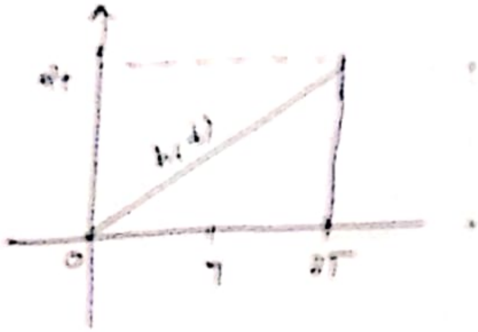
$$y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)} & t \leq 3 \\ \frac{1}{2} & t \geq 3 \end{cases}$$

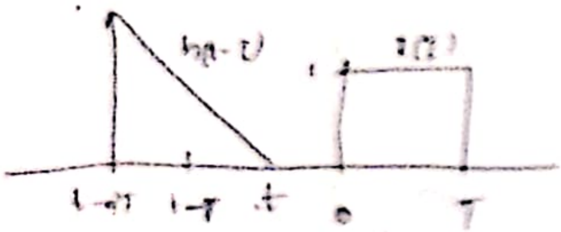
Q27

$$x(t) = \begin{cases} 1, & 0<t<T \\ 0, & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} t, & 0\le t\le 2T \\ 0, & \text{otherwise} \end{cases}$$







overlap range & their limits

(i) $t<0$ → No overlap

(ii) $0<t<T$ → $\int_0^t$

(iii) $t-2T<0$ (or) $t>T$ → $\int_0^T$

(iv) $0<t-2T<T$ → $\int_{t-2T}^{T}$

(v) $t-2T>T$ → No overlap

9/9/18

Properties of convolution

1) commutative:-

$$x(n) * h(n) = h(n) * x(n)$$

Proof: LHS: $x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

Let, $n-k = r \Rightarrow k = n-r$

$k = -\infty \Rightarrow r = \infty$

$k = \infty \Rightarrow r = -\infty$

$$x(n) * h(n) = \sum_{r=\infty}^{-\infty} x(n-r)h(r)$$

$$= \sum_{r=-\infty}^{\infty} h(r)x(n-r)$$

$$= h(n) * x(n)$$

$\Rightarrow$ * operator satisfies the commutative property

continuous time signals

$$x(t) * h(t) = h(t) * x(t)$$

$$\downarrow$$

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

2) Associative property

$$[x_1(n) * x_2(n)] * h(n) = x_1(n) * [x_2(n) * h(n)]$$

Proof: Let, $f_1(n) = x_1(n) * x_2(n)$

$f_2(n) = x_2(n) * h(n)$

~~LHS~~ $f_1(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} f_1(k)h(n-k)$$

$T(n) = \sum_{k=-\infty} x_1(k)\, x_2(n-k)$

LHS :

$$f_1(n) * h(n) = \sum_{m=-\infty}^{\infty} f_1(m)\, h(n-m)$$

$$= \sum_{m=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x_1(k)\, x_2(m-k) \right] h(n-m)$$

Let, $r = m-k$

$m = k+r$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \left[\sum_{k=-\infty}^{\infty} x_2(r)\, h(n-k-r) \right]$$

$$f_2(n) = \sum_{r=-\infty}^{\infty} x_2(r)\, h(n-r)$$

$$f_2(n-k) = \sum_{r=-\infty}^{\infty} x_2(r)\, h(n-k-r)$$

$$\Rightarrow = \sum_{k=-\infty}^{\infty} x_1(k)\, f_2(n-k)$$

$$= x_1(n) * f_2(n)$$

$$= x_1(n) * [x_2(n) * h(n)]$$

$= RHS$

1. Prove :

$$[x_1(t) * x_2(t)] * h(t) = x_1(t) * [x_2(t) * h(t)]$$

3. Distributive property

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

Proof LHS: $x(n) * [h_1(n) + h_2(n)] = \sum_{k=-\infty}^{\infty} x(k) [h_1(n-k) + h_2(n-k)]$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h_1(n-k) + \sum_{k=-\infty}^{\infty} x(k)\, h_2(n-k)$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

$= RHS$

Prove $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

4. $x(n) * \delta(n) = \delta(n) * x(n) = x(n)$

5. $x(n) * \delta(n-k) = x(n-k)$

6. $\delta(n-k) * \delta(n-m) = \delta(n-(m+k))$

7. $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

8. $u(n-1) * \delta(n)$

4. $x(t) * \delta(t) = x(t)$

5. $x(t) * \delta(t-t_0) = x(t_0)$

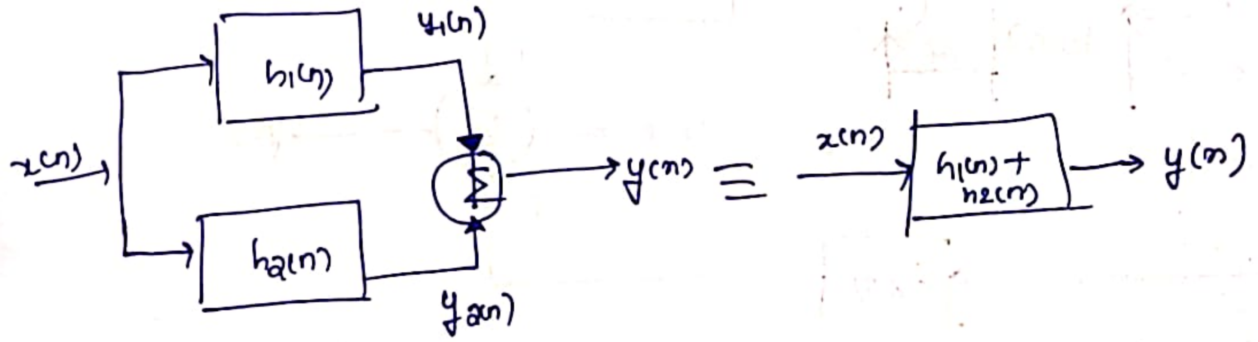
6. $x(t) * u(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Q) Convolute the two continuous time signals $x_1(t) = e^{-2t}u(t)$, $x_2(t) = u(t+2)$

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

$$x_2(t) * x_1(t) = \int_{-\infty}^{\infty} x_2(\tau) x_1(t-\tau) d\tau$$

Parallel connection

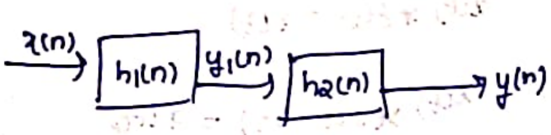


$y(n) = y_1(n) + y_2(n)$

$= x(n) * h_1(n) + x(n) * h_2(n)$

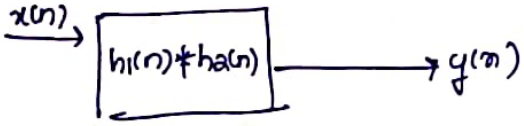
$y(n) = x(n) * [h_1(n) + h_2(n)]$

Cascade Connection



$y(n) = y_1(n) * h_2(n)$

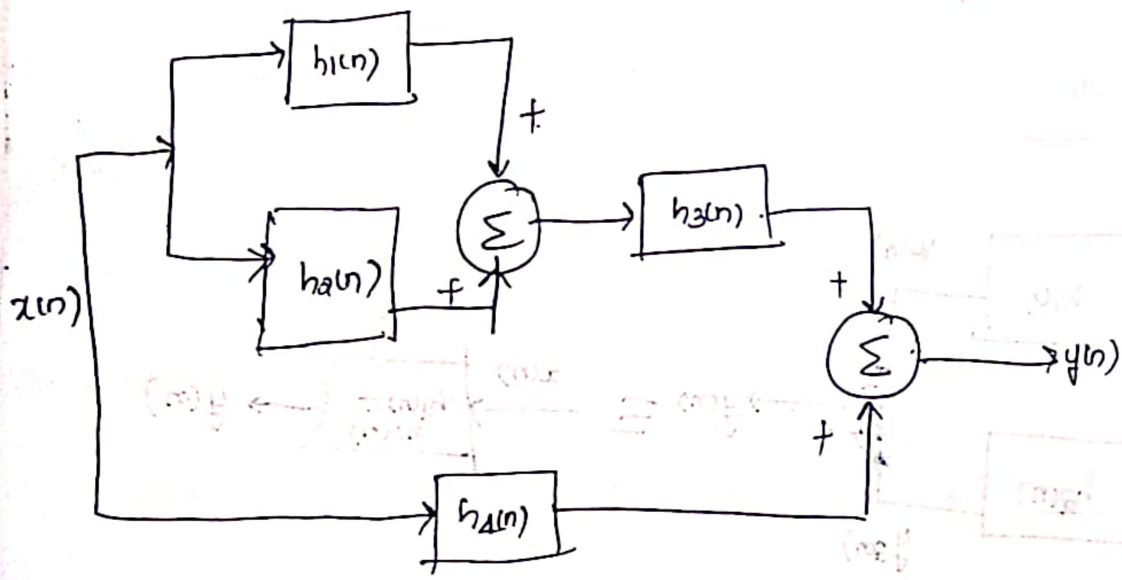
$y(n) = x(n) * [h_1(n) * h_2(n)]$



$\cancel{y(n) = y_1(n) * h_2(n)}$

$\cancel{y(n) = x(n) * \{h_1(n) * h_2(n)\}}$

1) consider the interconnection of LIT systems. The impulse response of each system is given as



find the overall Impulse Response of the system.

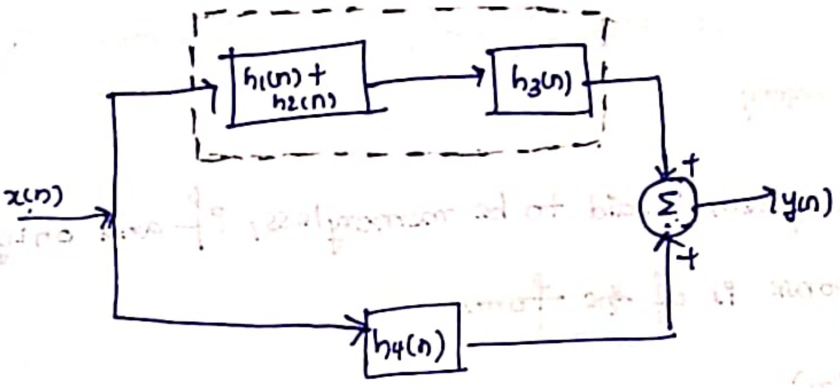
$h_1(n) = u(n)$

$h_2(n) = u(n+2) - u(n)$

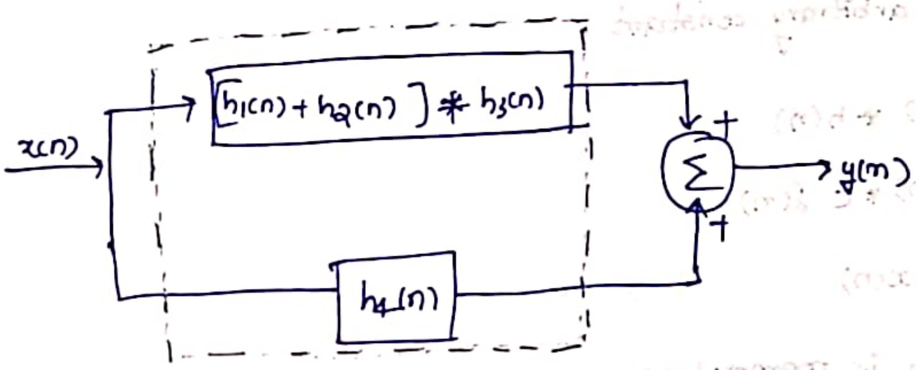
$h_3(n) = \delta(n-2)$

$h_4(n) = \alpha^n u(n)$

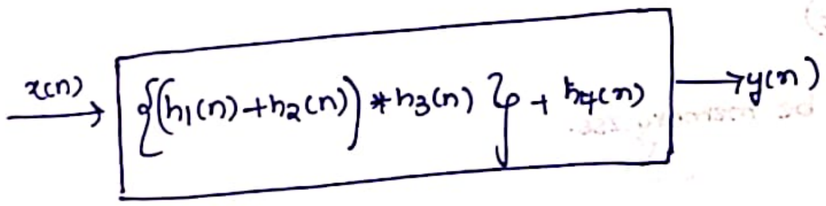
step 4) :-



(ii)



(iii)



the overall impulse response is,

$$h(n) = \{(h_1(n) + h_2(n)) * h_3(n)\} + h_4(n)$$

$$x(n) * \delta(n-k) = x(n-k)$$

$$= [u(n+2) * \delta(n-2)] + \alpha^n u(n)$$

$$u(n+2) * \delta(n-2) = u[n+2-2]$$

$$= u(n)$$

$$= u(n) + \alpha^n u(n)$$

$$\boxed{h(n) = (1+\alpha^n)u(n)}$$

Properties of system

1) Systems with or without memory:

<u>Theorem</u>: A discrete LTI system is said to be memoryless, if and only if its impulse response is of the form

$$h(n) = c\,\delta(n)$$

$c \rightarrow$ arbitrary constant

<u>Proof</u>: $y(n) = x(n) * h(n)$

$= x(n) * c\,\delta(n)$

$y(n) = c\,x(n)$

$\Rightarrow$ the system is memoryless

for CTS, $h(t) = c\,\delta(t)$

for system to be memoryless.

2) Causality:

<u>Theorem</u>: A LTI DTS is call causal, if $h(n) = 0, n < 0$ / $h(t) = 0, t < 0$ (CTS)

<u>proof</u>: $y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$h(n) = 0, \quad n < 0$

$h(n-k) = 0, \quad n-k < 0$

$k > n$

$$y(n) = \sum_{k=-\infty}^{n} x(k)\,h(n-k)$$

from the above eq^n it is clear that output $y(n)$ at any time (n) depends on the input $x(n)$ which is the present and the past inputs & hence the system is causal.

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3) Stability:

Theorem: A LTI system is said to be BIBO stable if and only if its impulse response is absolutely summable

i.e., $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$

Proof: $y(n) = x(n) * h(n)$

$= h(n) * x(n)$

$|y(n)| = \left| \sum_{K=-\infty}^{\infty} h(K)\, x(n-k) \right|$

$|y(n)| \leq \sum_{K=-\infty}^{\infty} |h(K)|\, |x(n-k)|$

if $|x(n)| \leq M$ then $|x(n-k)| \leq M$

$\Rightarrow |y(n)| \leq M \sum_{K=-\infty}^{\infty} |h(K)|$

$\Rightarrow$ for $y(n)$ to be bounded, $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$

Similarly for CTS, the system is BIBO stable if & only if

$\int_{-\infty}^{\infty} |h(\tau)| \, d\tau \leq \infty$

Absolutely Integrable

4) Invertibility

Theorem: discrete Time LTI system with impulse response $h(n)$ is said to be invertible if & only if

$h(n) * h^{-1}(n) = \delta(n)$

$[h(t) * h^{-1}(t) = \delta(t)]$

$$x(n) \to \boxed{h(n)} \xrightarrow{y_1(n)} \boxed{h^{-1}(n)} \to y(n)$$

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$y(n) = y_1(n) * h^{-1}(n)$

$= x(n) * [h(n) * h^{-1}(n)]$

$= x(n) * \delta(n)$

$= x(n)$ //

12/9/19

Investigate the causality and stability of the following systems.

1) a) $h(n) = 2^n u(n-1)$

b) $h(n) = (0.5)^{|n|}$

c) $h(t) = e^{-2|t|}$

d) $h(t) = e^{at} u(t-1)$

Properties of systems ($h(n) / h(t)$)

	Discrete time system	Continuous time system
1. causal (if mod is there its non-causal)	$h(n) = 0$ for $n < 0$	$h(t) = 0$ for $t \le 0$
2. stability	$\sum_{K=-\infty}^{\infty} \lvert h(K) \rvert < \infty$	$\int_{-\infty}^{\infty} \lvert h(\tau) \rvert d\tau < \infty$
3. Memory	$h(n) = c\,\delta(n)$	$h(t) = c\,\delta(t)$
4. Invertibility	$h(n) * h^{-1}(n) = \delta(n)$	$h(t) * h^{-1}(t) = \delta(t)$

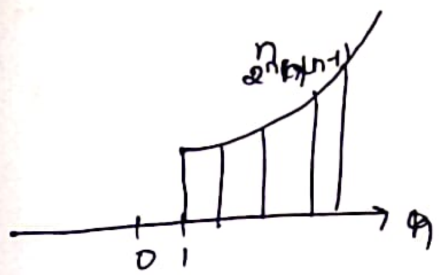
(a) $h(n) = 2^n u(n-1)$

Memory: since $h(n) \neq c\,\delta(n)$ the has memory

causality:- $2^n u(n-1)$

$\alpha^n \Rightarrow \alpha > 1 \uparrow$ Exp

$\Rightarrow \alpha < 1 \downarrow$ Exp



System is causal. since $h(n)=0,\ n<0$

(system is unstable since $h(n)$ is not absolutely summable)

stability : $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left| 2^n u(n-1) \right| = \sum_{n=1}^{\infty} 2^n = 2^1+2^2+2^3+\dots\infty$$

~~system is~~

Invertibility:

(b) $h(n) = (0.5)^{|n|}$

$$|n| = \begin{cases} -n & ; n<0 \\ n & ; n\geq 0 \end{cases}$$

(i) System has memory since, its not in the delta form.

(ii) $h(n) \neq 0$ for $n<0$, so, system is not causal

(iii) $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} 0.5^{|n|} \Rightarrow = \sum_{n=-\infty}^{-1} 0.5^{-n} + \sum_{n=0}^{\infty} 0.5^{n}$$

let $n=-m$, for the first term on RHS

$$= \sum_{m=\infty}^{1} 0.5^{m} + \sum_{n=0}^{\infty} 0.5^{n}$$

$$= \sum_{m=1}^{\infty} 0.5^{m} + \sum_{n=0}^{\infty} 0.5^{n} \Rightarrow \frac{0.5}{1-0.5} + \frac{1}{1-0.5}$$

$\Rightarrow 3 < \infty$

system is stable

(c) $h(t) = e^{-2|t|}$ → impulse

i) Memory : ~~dies~~

$h(t) \neq c\delta(t)$ → system has memory

ii) Non-causal

iii) $\int_{-\infty}^{\infty} |h(\tau)| d\tau = \int_{-\infty}^{\infty} e^{-2|\tau|} d\tau$

$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty = \int_{-\infty}^{-1} e^{2\tau} d\tau \int_{0}^{\infty} e^{-2\tau} d\tau$

$= 1 \Rightarrow$ stable

(iv) Invertibility

(d) $h(t) = e^{2t} u(t-1)$

do ~~has~~ Memory

causal ~~since, depending on past~~ $\because h(t) \neq 0$, for $n<0$; system is causal

Unstable ~~since, its an delta funct^n~~

$\int_{-\infty}^{\infty} h(\tau) | d\tau$

Invertibility

Eg: $h(t) = \delta(t+2) * \delta(t-2) = \delta(t)$

$h(t) * h^{-1}(t) = \delta(t)$

$\delta(t-m) * \delta(t-n) = \delta(t-(n+m))$

Step Response

$$x(n) = u(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$y(n) = x(n) * h(n)$$
$$= u(n) * h(n)$$
$$= h(n) * u(n)$$

$$s(n) = \sum_{k=-\infty}^{\infty} h(k)\,u(n-k) = \sum_{k=-\infty}^{n} h(k)$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u(n-k) = \begin{cases} 1 & n-k \geq 0 \rightarrow k \leq n \\ 0 & n-k < 0 \\ & k > n \end{cases}$$

$$\boxed{\begin{array}{l} s(n) = \displaystyle\sum_{k=-\infty}^{n} h(k) \\ \text{similarly } s(t) = \displaystyle\int_{\tau=-\infty}^{t} h(\tau)\,d\tau \end{array}}$$

2) find step response for LTI system when impulse response is:-

a) $\left(\frac{1}{2}\right)^n u(n) = h(n)$

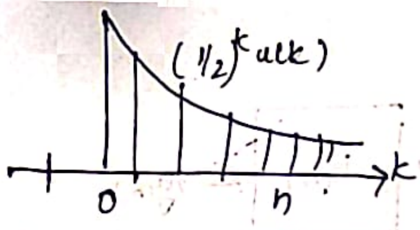
$s(n) = 0$; $n < 0$

$$s(n) = \sum_{k=0}^{n} (1/2)^k, \quad n \geq 0$$

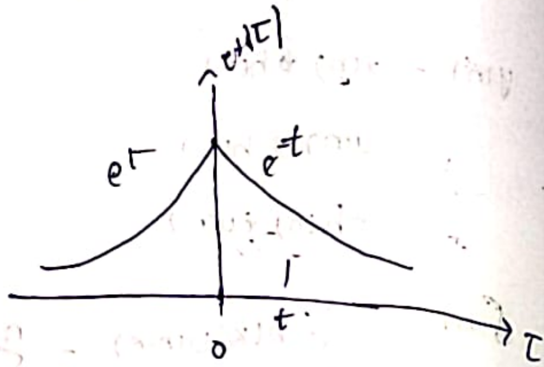
$$= \frac{(1/2)^{n+1} - 1}{(1/2 - 1)}, \quad n \geq 0$$

$$s(n) = \sum_{k=-\infty}^{n} h(k) = \sum_{k=-\infty}^{n} (1/2)^k u(k)$$

$$s(n) = \sum_{k=0}^{n} (1/2)^k$$



(b) $x(t) = e^{-|t|}$



$$e^{-|t|} = \begin{cases} e^{t} & t \le 0 \\ e^{-t} & t \geq 0 \end{cases}$$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$s(t) = \int_{-\infty}^{t} e^{\tau} d\tau \quad , \quad t<0$$

$$= \int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau \quad , t \geq 0$$

$$= (e^{\tau})_{-\infty}^{0} + \left(\frac{e^{-\tau}}{-1}\right)_0^{t} = (1-0) - (e^{-t} - 1)$$

$$= 2 - e^{-t} \quad , t \geq 0$$

$$s(t) = \begin{cases} 0 & t<0 \\ 2 - e^{-t} & t \geq 0 \end{cases}$$

13/9/19

Find the step response for

(a) (i) $h(n) = \frac{1}{3} \sum_{k=0}^{2} \delta(n-k)$ (ii) $h(n) = \delta(n) + \delta(n-1)$

(i) $s(n) = \sum_{k=-\infty}^{n} h(k) \longrightarrow$ Running sum

$$h(n) = \frac{1}{3}\left[\delta(n) + \delta(n-1) + \delta(n-2)\right]$$

i) $n<0$: $s(n) = 0$

ii) $n=0$; $s(n) = 1/3$

iii) $n=1$; $s(n) = \sum_{k=-\infty}^{n} h(k) = 2/3$

iv) $n=2$; $s(n) = 1$

v) $n>2$, $s(n) = 1$

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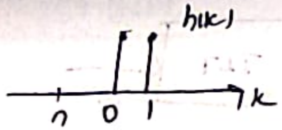


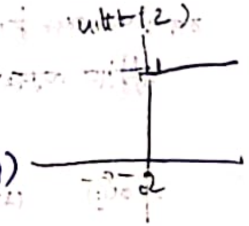
i) $n<0$: $s(n)=0$

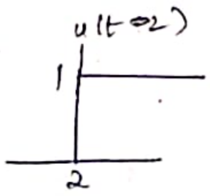
ii) $n=0$: $s(n)=1$

iii) $n \geq 1$: $s(n) = \sum_{k=-\infty}^{n} h(k) = 2$

A linear time invariant has an impulse response given by

$$h(t) = \begin{cases} 0 & t \leq -4 \\ e^{-|t|} & -4 \leq t \leq 4 \\ 0 & t > 4 \end{cases}$$

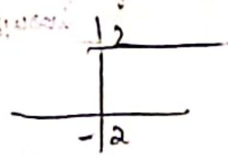


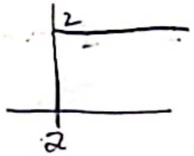


$x(t) = 2u(t+2) - 2u(t-2)$

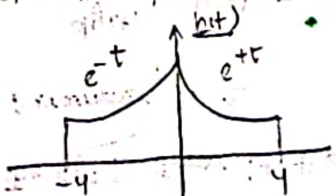
$y(t) = ?$

$2u(t+2)$ $\quad$ $2u(t-2)$

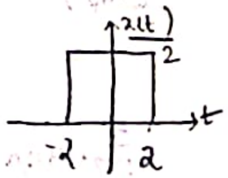




to plot h(t) & s(t)



$x(t) = 2u(t+2) - 2u(t-2)$



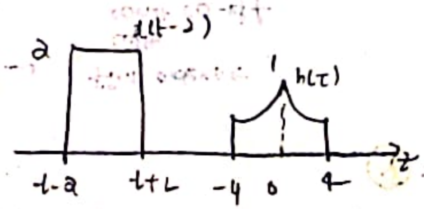
$y(t) = x(t) * h(t)$

$= h(t) * x(t)$

$= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$

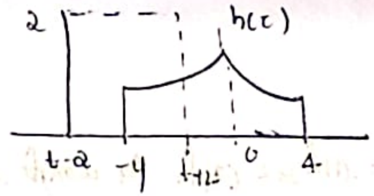
$h(t) \to h(\tau)$

$x(t) \to x(\tau) \to x(-\tau) \to x(-\tau+t)$



case (i) $\quad t+2 < -4$

no overlap, $y(t) = 0$

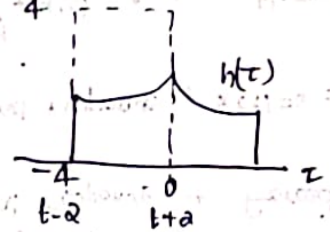


case (ii) $\quad -4 < t+2 < 0$

$$y(t) = \int_{-4}^{t+2} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-4}^{t+2} e^{\tau} \cdot 2\, d\tau = 2e^{t+2} - 2e^{-4}$$

case (iii)



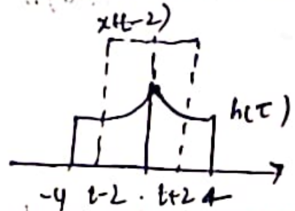
$$y(t) = \int_{-4}^{0} 2e^{\tau} d\tau$$

$$= 2 - 2e^{-4}$$

case (iv) $\quad -2 < t < 2$

$\uparrow$

$0 < t+2 < 4$



$$y(t) = \int_{t-2}^{t+2} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{t-2}^{0} e^{\tau}\, 2\, d\tau + \int_{0}^{t+2} e^{-\tau}\, 2\, d\tau$$

case (v); $t=2$

$$y(t) = \int_{0}^{4} e^{-\tau}\, 2\, d\tau$$

case (vi) $\quad 2 < t < 6$

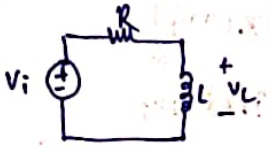
$$y(t) = \int_{t-2}^{4} e^{-\tau}\, 2\, d\tau$$

case (vii) $\quad t > 6$

$y(t) = 0$

Linear constant co-eff Diff Eqⁿs

$$\underbrace{\frac{dy(t)}{dt} + y(t)}_{\text{outputs}} = \underbrace{x(t)}_{\text{input (dependent on time)}} \leftarrow \text{I/P} \quad \rightarrow \text{first order DE with const co-eff}$$



$$\Rightarrow L\frac{di}{dt} + IR = V_i(t) + \frac{dv_i}{dt}$$

Linearity of DE :

The Eqⁿ is linear if the power of derivative $\left(\frac{dy(t)}{dt}\right)$ & output $y(t)$ parameter is 1

$$\left(\frac{dy(t)}{dt}\right)^1 + (y(t))^1 = x(t) \rightarrow \text{linear}$$

$$\left(\frac{d^2y(t)}{dt^2}\right)^1 + \left(\frac{dy(t)}{dt}\right)^1 + (y(t))^1 = x(t) \rightarrow \text{linear}$$

ODE (✓)
Linearity (✓)
constant co-eff (✓)

→ partial diff: if the variable depends on more than 2 dependent variables.

Solution of DE

$$x(t) \rightarrow \boxed{LTI} \rightarrow y(t) \quad \underline{\text{Response}}$$

$$x(t) \rightarrow \boxed{LTI} \rightarrow y(t) \quad \text{Response}$$

find response for initial conditions = 0
↳ $\underline{\text{zero-state response}}$
Forced response

find response for $x(t) = 0$
↳ zero i/p response / Natural Response
Bcoz i/p into not forced/ driven by anything.
↳ zero ~~sta~~ i/p response
Natural response

$$\boxed{\text{Total response} = \text{Natural response} + \text{forced response}}$$

If the input of the system is zero & no initial conditions then ~~output~~ total output response is zero.

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

$$y(t) = \underbrace{\text{Natural response}}_{(y_H(t))} + \underbrace{\text{forced response}}_{y(p(t))}$$

$y(p(t)) \rightarrow p \rightarrow$ particular integral

to find Natural response, $x(t) = 0 \Rightarrow y_H(t)$

$$\frac{dy(t)}{dt} + y(t) = 0 \leftarrow \text{Homogenous Eq}^n$$

$$y(t) = y_h(t) + y_p(t)$$

Q
(*) find the total response of the system given by $y'(t) + 5y(t) = u(t)$, $\underline{\text{I.C}}$: $y(0)$

$\underline{\text{Sol}^n}$ The total response

$$y_T(t) = y_h(t) + y_p(t)$$

To find $y_h(t)$:

$\Rightarrow y'(t) + 5y(t) = 0$

Auxillary Eq^n → Let, $y(t) = Ae^{st}$

$y'(t) = Ase^{st}$

$Ase^{st} + 5Ae^{st} = 0 \Rightarrow Ase^{st}(s+5) = 0$

$Ae^{st} \neq 0$, $s+5 = 0$ → Auxilary Eq^n

$s = -5$ → Roots of the Auxillary Eq^n

Sl.No	Roots of characteristic Eq^n	Form of Natural Response
1.	Real or distinct : s_1, s_2	$A_1e^{s_1t} + A_2e^{s_2t}$
2.	Complex conjugate $a \pm j\omega$ Eg: $s_{1,2} = -1 \pm 2j$	$e^{at}[k_1\cos\omega t \pm k_2\sin\omega t]$ → $e^{-t}[k_1\cos 2t \pm k_2\sin 2t]$
3.	Real, repeated, s^{p+1} $s_{1,2} = 3, 3$	$e^{st}[k_0 + k_1t + \dots + k_pt^p]$ → $e^{3t}[k_0 + k]$

→ Unless the initial condition are known we can find the output as well the unique response of the system.

Hence, the initial conditions should be given which are drives the system.

$y_h(t) = Ae^{-5t}$ → unique?

A → arbitrary const.

$y_h(0) = 2 = Ae^{-5(0)} = A$

$\boxed{A = 2}$

$$\boxed{y_h(t) = 2e^{-5t}}$$

(i) Zero-Input Response (i/p ⇒ 0)

$y_h(t) = 2e^{-5t}$

$y(0) = 2$

(ii) Zero-state Response $y_f(t)$

Particular Integer $y_p(t)$

Sl.No	Input	structure of particular response
1.	C_0	C_1 (another constant)
2.	e^{at} ; $a \neq s$: (roots)	Ce^{at}
3.	$\cos(\omega t + \beta)$	$c_1\cos\omega t + c_2\sin\omega t$
4.	$e^{at}[\cos(\omega t + \beta)]$	$e^{at}[c_1\cos\omega t + c_2\sin\omega t]$
5.	t	$c_0 + c_1t$
6.	t^p	$c_0 + c_1t + c_2t^2 + \dots + c_pt^p$
7.	te^{at}	$e^{at}[c_0 + c_1t]$

a1) Zero-state Response $y_f(t)$

Particular Integer $y_p(t)$

Let, $y_p(t) = k$

Substitute $y_p(t)$ into the DE

$y_p'(t) = 0$

$0 + 5k = 1, \; t \geq 0$

$k = 1/5$

$y_p(t) = 1/5$

$y_f(t) = \underbrace{y_h(t)}_{\text{i.c's}=0} + y_p(t)$

$y_f(t) = Ae^{-5t} + 1/5$

i.c's $= 0$, $y(0) = 0$

$y(0) = A + 1/5 = 0$

$A = -1/5$

$$\boxed{y_f(t) = -1/5 e^{-5t} + 1/5}$$

The total response

$y_T(t)$ = Natural response + force Response

$= y_N(t) + y_f(t)$

$$\boxed{y_T(t) = 2e^{-5t} - 1/5 e^{-5t} + 1/5}$$

The solution of D.E gives the total response

Method - I Total response = Natural Response + Forced Response

$y_T(t) = y_N(t) + y_F(t)$

(i) To find $y_N(t)$

(a) to generate the characteristic eq^n

(b) Depending on the roots of the characteristic Eq^n, $y_N(t)$ takes a certain form which involves constants

(c) Using i.c's given, find the constants and get back $y_N(t)$

(ii) to find $y_F(t)$

$y_F(t) = y_h(t) + y_p(t)$

where $y_h(t)$ = Response of homogeneous D.E with i.c's = 0
$y_p(t)$ = Particular Response.

(a) to find $y_p(t)$

* Depending on the form of the i/p, $y_p(t)$ takes a certain form
* substitute $y_p(t)$ into D.E to find constant

(b) then substitute $y_h(t)$ & $y_p(t)$ to find $y_F(t)$

(iii) Find $y_T(t) = y_N(t) + y_F(t)$

Q: $\frac{d^2 y(t)}{dt} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$ i/p $x(t) = e^{-t}u(t)$;

$y(0) = 3$, $y'(0) = 0$

find (i) $y_N(t)$ (ii) $y_F(t)$ (iii) $y_T(t)$

to find $y_N(t)$

let $y(t) = Ae^{st}$ $\Rightarrow Ae^{st}[s^2+5s+6] = 0$

$y'(t) = Ase^{st}$ $s^2+5s+6 = 0$

$y''(t) = As^2e^{st}$ $s_1 = -2, s_2 = -3$

$y_N(t) = A_1e^{-2t} + A_2e^{-3t}$

$y'_N(t) = -2A_1e^{-2t} - 3A_2e^{-3t}$

Apply i.c's to find A_1 & A_2

$y(0) = 3 \Rightarrow A_1 + A_2$

$A_1 + A_2 = 3$

$y'(0) = 0 \Rightarrow -2A_1 - 3A_2 = 0$

$A_1 = 9, A_2 = -6$

$$\boxed{y_N(t) = 9e^{-2t} - 6e^{-3t}}$$

ii) to find $y_F(t)$

$x(t) = e^{-t}u(t)$

$e^{at} \longrightarrow ce^{at}$

$e^{-t} \longrightarrow ce^{-t} \Rightarrow y_p(t)$

$ce^{-t} - 5ce^{-t} + 6ce^{-t} = -e^{-t} + 4e^{-t}$

$c = 1.5$

$y_p(t) = 1.5e^{-t}$

$y_p(t) = y_h(t) + y_p(t)$

$y_p(t) = A_1e^{-2t} + A_2e^{-3t} + 1.5e^{-t}$

I.c's $= 0$

$y'_p(t) = -2A_1e^{-2t} - 3A_2e^{-3t} - 1.5e^{-t}$

$y(t)$

$\downarrow$

$y(0) = 0$, $y'(0) = 0$

$A_1 + A_2 + 1.5 = 0$

$-2A_1 - 3A_2 - 1.5 = 0$

$\Rightarrow$ $A_1 = -3$

$A_2 = 1.5$

$$\boxed{y_p(t) = -3e^{-2t} + 1.5e^{-3t} + 1.5e^{-t}} \rightarrow (2)$$

(1+2) $y_T(t) = y_N(t) + y_p(t)$

(1) + (2)

Difference Equation

Total response = Natural Response (Zero - i/p response) + forced Response

$y_N(t)$ $\qquad y_F(t)$.

CT $\rightarrow y_T(t)$

DT $\rightarrow y_T(n) = y_N(n) + y_F(n)$

$y_N(n) \downarrow$ ic's, $x_1 = 0$

$y_F(n) \downarrow y_h(n) + y_p(n)$, $\downarrow$ ic's = 0

S.NO	Roots of characteristic Eqn	structure of natural response
1.	Real & distinct z_1, z_2	$k_1 z_1^n + k_2 z_2^n$
2.	Complex conjugate $ze^{j\Omega}$ $s = a + jb$ $z = \sqrt{a^2+b^2}$, $\Omega = \tan^{-1}(b/a)$	$z^n [k_1 \cos \Omega n + k_2 \sin \Omega n]$
3.	Real repeated z^{r+1}	$z^n [k_0 + k_1 n + \dots + k_r n^r]$

1/19

Find total response of LTI system described by the difference Eqn:- $y(n) + 4y(n-1) + 3y(n-2) = u(n)$, $y(-1) = 0$, $y(-2) = 1$

Total response = Natural + Forced Response
(zero-i/p response) (zero-state response)

$\therefore y_T(n) = y_N(n) + y_F(n) \quad \rightarrow (1)$

(1) To find $y_N(n)$ i/p = 0 ⇒ $x(n) = 0$

$$y_N(n) + 4y_N(n-1) + 3y_N(n-2) = 0$$

Let, $y_N(n) = z^n$, $y_N(n-1) = z^{n-1}$

$$z^n + 4z^{n-1} + 3z^{n-2} = 0$$

$$z^{n-2}[z^2 + 4z + 3] = 0$$

⇒ characteristic eqⁿ ⇒ $z^2 + 4z + 3 = 0$

$$z^2 + z + 3z + 3 = 0$$

$$(z+1)(z+3) \Rightarrow z = -1, -3$$

roots are real & distinct

$$y_N(n) = k_1 z_1^n + k_2 z_2^n$$

$$y_N(n) = k_1(-1)^n + k_2(-3)^n$$

Apply i.c's to find k_1 & k_2

(i) $y_N(-1) = k_1(-1)^{-1} + k_2(-3)^{-1}$

$$= -k_1 - \frac{k_2}{3} = 0$$

(ii) $y_N(-2) = k_1(-1)^{-2} + k_2(-3)^{-2} = 1$

$$= k_1 + \frac{k_2}{9} = 1$$

solving, $\boxed{k_1 = 1.5, \; k_2 = -4.5}$

$$y_N(n) = 1.5(-1)^n - 4.5(-3)^n$$

(2) To find $y_F(n)$

$$y_F(n) = y_h(n) + y_p(n)$$

$$= k_1(-1)^n + k_2(-3)^n + y_p(n)$$

$y_h(n)$ is found by letting i.c's = 0

S.No	Input signal	Form of the particular response
1.	c_0 (constant)	c_1 (another const)
2.	z^n	$c\,z^n$
3.	$\cos(n\Omega+\beta)$	$c_1\cos n\Omega + c_2\sin n\Omega$
4.	$z^n\cos(n\Omega+\beta)$	$z^n[c_1\cos n\Omega + c_2\sin n\Omega]$
5.	n	$c_0 + c_1 n$
6.	n^p	$c_0 + c_1 n + c_2 n^2 + \dots c_p n^p$
7.	$n\,z^n$	$z^n[c_0 + c_1 n]$
8.	$n^p z^n$	$z^n(c_0 + c_1 n + \dots + c_p n^p)$

Note (#): If input $x(n) = z^n$ where z is also a root of the characteristic eqⁿ repeated m times then the particular response must be multiplied by n^m.

for Eg: input $x(n) = (0.2)^n$ & for same system roots are 0.2 & 0.3. the particular response $c(0.2)^n n^{(1)}$

(i) to find $y_p(n)$, $x(n) = u(n)$

Let, $y_p(n) = c$; $n \geq 0$

$$y_p(n) + 4y_p(n-1) + 3y_p(n-2) = u(n)$$

$$c + 4c + 3c = 1$$

$$c = 1/8$$

$$y_p(1) = 1/8$$

$$y_F(n) = k_1(-1)^n + k_2(-3)^n + 1/8$$

Letting i.c's = 0, to find k_1 & k_2

$y(-1) = 0$ $\quad y(-2) = 0$

Note:- while calculating forced response take i.c's as zero though i.c's Equal to something.

$y(-1) = 0$; $y(-2) = 0$

$y_F(-1) = k_1 (-1)^{-1} + k_2 (-3)^{-1} + \frac{1}{8} = 0$

$-k_1 - \frac{k_2}{3} + 1/8 = 0$

$y_F(-2) = k_1 + \frac{k_2}{9} + \frac{1}{8} = 0$

$k_1 = -0.25, \quad k_2 = 1.125$

$$\boxed{y_F(n) = -0.25(-1)^n + 1.125(-3)^n + 1/8} \rightarrow ②$$

$y_T(n) = y_N(n) + y_P(n)$

① + ②

$= 1.5(-1)^n + 4.5(-3)^n - 0.25(-1)^n + 1.125(-3)^n + 1/8$

$$\boxed{y_T(n) = 1.25(-1)^n - 3.375 \cdot (-3)^n + 1/8}$$

Q: find the total response for LTI system described by

$y(n) - 0.9y(n-1) + 0.2y(n-2) = (0.5)^n$, $y(-1) = 1$, $y(-2) = -4$

$y_T(n) = y_N(n) + y_F(n) \rightarrow ①$

(i) To find $y_N(n)$ i/p = 0 ⇒ x(n) = 0

$y_N(n) - 0.9 y_N(n-1) + 0.2 y_N(n-2) = 0$

$y_{Nn} = z^n$

$z^n - 0.9z^{n-1} + 0.2z^{n-2} = 0$

$z^{n-2}[z^2 - 0.9z + 0.2] = 0$,

$z^2 - 0.9z + 0.2 \Rightarrow z = 0.5$ or $z = 0.4$

roots are real & distinct

$y_N(n) = k_1 z_1^n + k_2 z_2^n$

$y_N(n) = k_1(0.5)^n + k_2(0.4)^n$

Apply i.c's to find k_1, k_2

$y(-1) = k_1(0.5)^{-1} + k_2(0.4)^{-1} = 1$

$\frac{k_1}{0.5} + \frac{k_2}{0.4} = 1$

$y(-2) \Rightarrow k_2(0.5)^2 + k_2(0.4)^2 = -4$

$\frac{k_1}{(0.5)^2} + \frac{k_2}{(0.4)^2} = -4$

$$\boxed{\begin{aligned} k_1 &= -6.5 \\ k_2 &= -4.5 \end{aligned}}$$

$y_N(n) = 6.5(0.5)^n - 4.8(0.4)^n$

(a) to find $y_F(n)$

$y_F(n) = y_h(n) + y_p(n)$

$= k_1(0.5)^n + k_2(0.4)^n + y_p(n)$

\# Note

Here we can also take $(0.4)^n$ but in Q/A $(0.5)^n$ is repeated so, take $(0.5)^n$

find $y_p(n) \Rightarrow$ $y_p(n) = c(0.5)^n$

$y_p(n) - 0.9y_p(n-1) + 0.2y_p(n-2) = (0.5)^n$

$= 1$

$c(0.5)^n - 0.9[c(0.5)^{n-1}] + 0.2[c(0.5)^{n-2}] = (0.5)^n$

$c(0.5)^{n-2}\left[\frac{1}{(0.5)^2} - \frac{0.9}{(0.5)} + 0.2\right] = (0.5)^n$

$c(0.5)^n = 1/0.3$

$\frac{c(0.5)^n}{(0.5)^2}\left[\frac{12}{5}\right] = (0.5)^n \Rightarrow c = \frac{(0.5)^2\times 5}{12}$

$c = 0.104$

find $y_p(n)$: $z^n \to$ m-times $\to n^m$

$\boxed{y_p(n) = cn(0.5)^n}$

$y_p(n) - 0.9y_p(n-1) + 0.2y_p(n-2) = (0.5)^n$

$cn(0.5)^n - 0.9\times c(n-1)(0.5)^{n-1} + c(0.2)(n-2)(0.5)^{n-2} = (0.5)^n$

$cn(0.5)^n - \frac{0.9c(0.5)^n(n-1)}{0.5} + \frac{0.2}{(0.5)^2}\times c(n-2)(0.5)^n = (0.5)^n$

$c\times n - 1.8c[n-1] + 0.8c[n-2] = 1$

$cn - 1.8cn + 1.8c + 0.8cn - 1.6c = 1$

$1.8cn - 1.8cn + 0.2c = 1$

$\boxed{c = 5}$

$\boxed{y_p(n) = 5n(0.5)^n}$

$y_F(n) = k_1(0.5)^n + k_2(0.4)^n + 5n(0.5)^n$

Apply I.C's $= 0$, to find k_1, k_2

$y_F(-1) = k_1(0.5)^{-1} + k_2(0.4)^{-1} + 5(-1)(0.5)^{-1} = 0$

$y_F(-2) = k_1(0.5)^{-2} + k_2(0.4)^{-2} + 5(-2)(0.5)^{-2} = 0$

$\boxed{k_1 = -15,\ k_2 = 16}$

$$\boxed{y_F(n) = -15(0.5)^n + 16(0.4)^n + 5n(0.5)^n} \rightarrow ②$$

3) $y_T(n) = y_N(n) + y_F(n)$

$$= 6.5(0.5)^n - 4.8(0.4)^n + 16(0.4)^n - 15(0.5)^n + 5n(0.5)^n$$

Q: $y''(t) + 4y'(t) + 3y(t) = 36t\,u(t)$, $y'(0) = 1$, $y(0) = 0$

$y(t) = Ae^{st}$ $\Rightarrow s^2Ae^{st} + 4As e^{st} + 3Ae^{st} = 0$

$y'(t) = Ase^{st}$ $\quad Ae^{st}[s^2 + 4s + 3] = 0$

$y''(t) = As^2e^{st}$ $\quad s = -1, -3$

$y_N(t) = A_1e^{-t} + A_2e^{-3t}$

$y'_N(t) = -A_1e^{-t} - 3A_2e^{-3t}$

Apply I.C. to find A_1, A_2

$y(0) = 0 \Rightarrow A_1 + A_2 = 0$

$y'(0) = 1 \Rightarrow -A_1 - 3A_2 = 1$

$A_1 = +0.5$

$A_2 = -0.5 \Rightarrow \boxed{y_N(t) = 0.5e^{-t} - 0.5e^{-3t}}$

ii) to find $y_F(t)$

$x(t) = 36t\,u(t)$

$y_F(t) = y_h(t) + y_p(t)$

$= k_1e^{-t} + k_2e^{-3t} + y_p(t)$

$At \to C_0 + C_1t$

$te^{at} \to e^{at}(C_0 \ldots$

$t^{(A)} \to C_0 + C_1 \ldots$

$y_p(t) = (C_0 + C_1t)$

$y'_p(t) = C_1$

$y''_p(t) = 0$

$\Rightarrow 0 + 4[C_1] + 3[C_0 + C_1t] = 36t + 0$

Equate the coefficient of powers of t

$\Rightarrow 4C_1 + 3C_0 = 0 \quad (t^0)$

$3C_1 = 36 \quad (t^1)$

$C_1 = 12, \quad C_0 = -16$

$y_p(t) = -16 + 12t$

$y_F(t) = k_1e^{-t} + k_2e^{-3t} + 12t - 16 \Rightarrow y_F(0) = k_1 + k_2 - 16 = 0$

$y'_F(t) = -k_1e^{-t} - 3k_2e^{-3t} + 12 \Rightarrow y'_F(0) = -k_1 - 3k_2 + 12 = 0$

$k_1 = 18$

$k_2 = -2$

$y_F(t) = 18e^{-t} - 2e^{-3t} + 12t - 16$ — ③

3) $y_T(t) = y_N(t) + y_F(t)$

$y_T(t) = 0.5e^{-t} + 0.5e^{-3t} + 18e^{-t} - 2e^{-3t} + 12t - 16$

Scanned by CamScanner

Block diagram representat^n of first order systems

$y(n) + ay(n-1) = bx(n)$

$y(n) = -ay(n-1) + bx(n)$

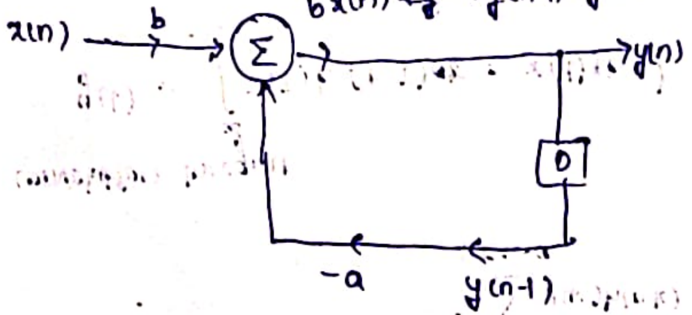
$x(n) \xrightarrow{a} a\,x(n)$

$y(n) \rightarrow \boxed{D} \rightarrow y(n-1)$

→ Differentiater which shifts the waveform

Block diagram

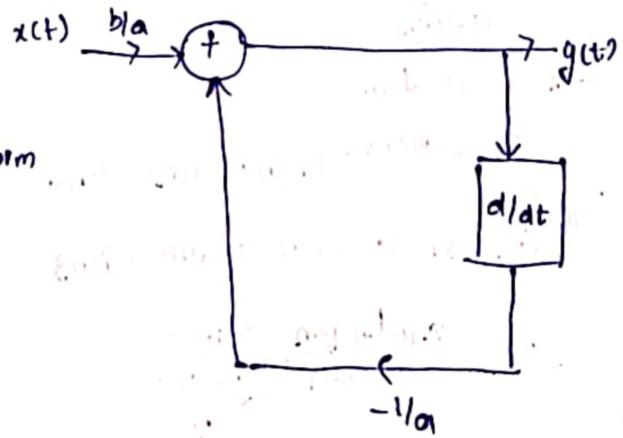
$bx(n) + -ay(n-1) = y(n)$

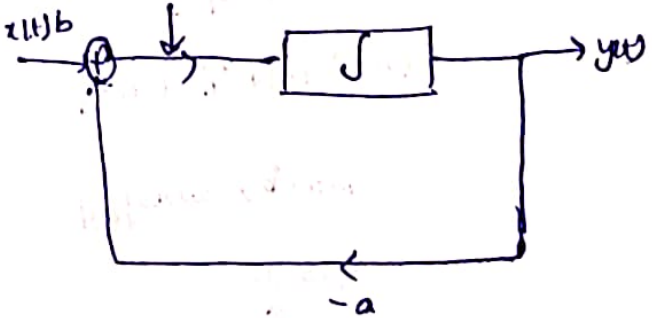


Eg: $\frac{dy(t)}{dt} + ay(t) = bx(t) \Rightarrow \frac{dy(t)}{dt} = bx(t) - ay(t)$

$y(t) = \frac{b}{a}x(t) - \frac{1}{a}\frac{dy(t)}{dt}$

$y(t) = \int_{-\infty}^{\infty} [bx(\tau) - ay(\tau)]\,d\tau$





Designing Integral block is Easy than differentiation block

Review of chapter-2 [Examples]

1) LTI System → convolution sum

$x(n) \longrightarrow \boxed{\begin{matrix}LTI\\h(n)\end{matrix}} \longrightarrow y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

2) convolution Integral

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = x(t) * h(t)$$

3) properties of LTI System

- Commutative property
- Associative " "
- Distributive " "

} Discrete & continuous

(i) Memoryless ; $h(n) = c\delta(n)$

(ii) Causality : $h(n) = 0 ; n < 0$

(iii) Stability : $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

(iv) Invertibility : $h(n) * h^{-1}(n) = \delta(n)$

Unit step response

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

$$\delta(t) = \frac{du(t)}{dt}$$

↓

$\frac{d\,s(t)}{dt} \rightarrow h(t)$ Unit Impulse

$\delta(n) = u(n) - u(n-1) \Rightarrow$ relat^n b/t^n signals

$h(n) = s(n) - s(n-1) \Rightarrow$ relat^n b/t^n response of the signals

4) Linear constant - co-efficient Differential Eq^n

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

3) total response = $\underset{\text{(zero i/p)}}{y_N(t)} + \underset{\text{(zero state)}}{y_F(t)}$

Total response $= y_N(t) + y_F(t)$

$$y_T(t) = \underset{\substack{\downarrow\\ \text{given i.c's}}}{y_N(t)} + \underset{\substack{\downarrow\\ \text{set ic=0}}}{(y_h(t) + y_p(t))}$$

Method-II

$y_T(t) =$ complementary funct^n + parameter Particular Integral

$$= \underset{\substack{\downarrow\\ \text{Given ic's}}}{y_h(t)} + y_p(t)$$

Difference Equation

$$y_T(n) = y_N(n) + y_F(n)$$

$$= \underset{\substack{\downarrow\\ \text{Given i.c's}}}{y_N(n)} + \underset{\substack{\downarrow\\ \text{i.c's=0}}}{y_h(n) + y_p(n)}$$

5) Block diagram representation

first-order eq^n

Review of chap-1

1) classification of signals

- 5 types

(i) periodic; T_0, N_0

- Energy → $E < \infty$

Power → $P < \infty$

Even/odd

2) operation on signals

→ Ind variable → Timeshifting, scaling, Reversal

→ Dep variable → Amplitude

3) Types of signals

Exponential → CT → e^{at}

Exponential → DT → α^n

complex E.S → $e^{j\omega_0 t}$ → $T_0 = \frac{2\pi}{\omega_0}$

→ $e^{j\omega_0 n}$ → $N_0 = m\frac{2\pi}{\omega_0}$

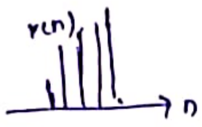
$\delta(t) = \frac{du(t)}{dt}$

$\delta(n) = u(n) - u(n-1)$

→ Ramp signal $r(t)$

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

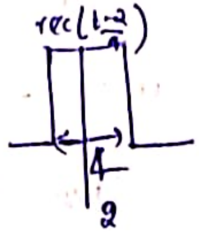
$$r(n) = \begin{cases} n; & n \geq 0 \\ 0; & n < 0 \end{cases}$$



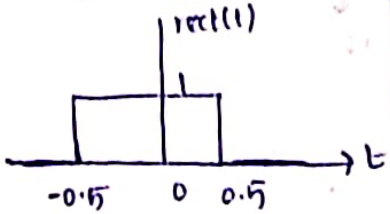
→ pulse, rectangular function

$$\text{rect}(t) = \begin{cases} 1, & |t| < 0.5 \\ 0, & \text{elsewhere} \end{cases}$$

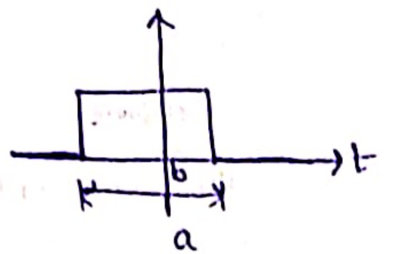
$\text{rec}\left(\frac{t-2}{4}\right)$ ⇒ Means, $\xleftarrow{4}$ and shift by 2 units



⇒

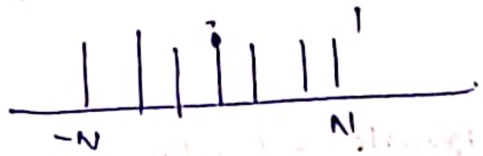


$\text{rec}\left(\frac{t-b}{a}\right)$ a = width



$$\text{rect}\left(\frac{n}{2N}\right) = \begin{cases} 1, & |n| < N \\ 0, & \text{elsewhere} \end{cases}$$

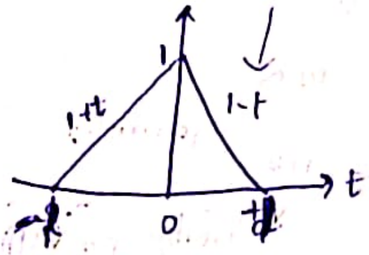
(N+1) samples



Exercise HW-2

Triangular function

$$tri(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & \text{elsewhere} \end{cases}$$

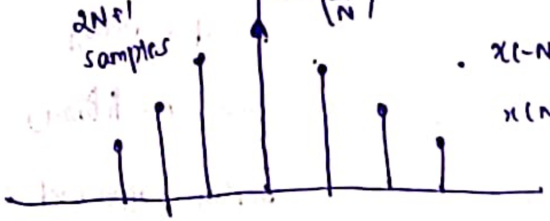


$tri\left(\frac{t-b}{a}\right) \rightarrow$ width 2a centered at b

$$tri\left(\frac{n}{N}\right) = \begin{cases} 1-\frac{|n|}{N}, & |n|<N \\ 0, & \text{elsewhere} \end{cases}$$

$tri\left(\frac{n}{N}\right)$

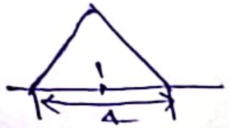
2N+1 samples



$x(-N)=0$

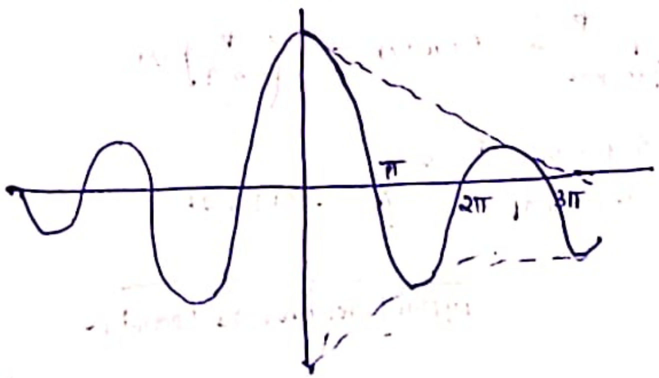
$x(N)=0$

$\rightarrow tri\left(\frac{t-1}{2}\right)$



→ **sinc function** (sinc over argument)

$$\text{sinc}\, x = \frac{\sin x}{x} = \text{sinc}(x)$$



Systems → LTI

properties:
1. Linearity
2. T.I
3. Stable
4. Causal
5. Memory
6. Invertible

$y(n) = x(n) + x(n-1)$

Sampling → to convert continuous to discrete signals

CTS → DS → digital

↑ analog; quantize

CTS → DTS

└ samples

$A \rightarrow p\cos(t/6) \rightarrow \pi/2 \rightarrow T_0$

$\;\;\; \cos(n/6) \rightarrow$ aperiodic

Eg :- Let $x(t) = e^{j\omega_0 t}$ with fundamental period, $T_0 = \frac{2\pi}{\omega_0}$, consider the discrete time sequence $x(n)$. obtain by uniform sampling of $x(t)$ with sampling interval T_s. find the condition on the value of T_s so, that $x(n)$ is periodic.

A) $x(t) = e^{j\omega_0 t}$, $T_0 = \frac{2\pi}{\omega_0}$

T_s, $x(n) = x(nT_s) = e^{j\omega_0 n T_s}$, $x(t) = x(nT_s)$

Soln If $x(n)$ is periodic, st. $x(n) = x(n+N_0)$

$$e^{j\omega_0 n T_s} = e^{j\omega_0 (n+N_0) T_s} = e^{j\omega_0 n T_s} \cdot e^{j\omega_0 N_0 T_s}$$

Thus, $x(n)$ is periodic if the ratio $\frac{T_s}{T_0}$ of sampling interval & fundamental period of $x(t)$ is a rational number

$\Rightarrow$ for $x(n)$ to be periodic

$$e^{j\omega_0 N_0 T_s} = 1 \Rightarrow \omega_0 N_0 T_s = m 2\pi$$

$$\frac{\cancel{2\pi}}{T_0} N_0 T_s = m \cancel{2\pi} \Rightarrow \boxed{\frac{T_s}{T_0} = \frac{m}{N_0} = \text{rational number}}$$

Eg :- consider a sinusoidal signal $x(t) = \cos 15t$

a) find the value of sampling interval T_s such that $x(n) = x(nT_s)$ is a periodic sequence

b) find the fundamental period of $x(n)$ if $T_s = 0.1\pi$ sec

a) $\frac{T_s}{T_0}$ = rational, $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{15}$

$$\frac{T_s}{T_0} = \frac{m}{N_0} \Rightarrow T_s = T_0 \frac{m}{N_0}$$

$$\boxed{T_s = \frac{2\pi}{15} \cdot \frac{m}{N_0}}$$

b) $T_s = \frac{\pi}{10}$ sec

$$\frac{T_s}{T_0} = \frac{m}{N_0} \Rightarrow N_0 = m \frac{T_0}{T_s} = \frac{m}{N_0} = \frac{15}{2\pi} \times \frac{\pi}{10}$$

$$N_0 = \frac{4}{3} m$$

so at $m = 3$, we get an integer

$\therefore \boxed{N_0 = 4}$ is the fundamental period of $x(n)$