HW-2

Exercise problems from Ch-2

2.1, 2.2, 2.5, 2.9

2.11, 2.14, 2.15, 2.16

2.17, 2.18, 2.23, 2.29,

2.32, 2.33

Signals & Systems

→ signal - which carries info.

if data becomes deterministic (always true) then its not an info.

Info - when data is random

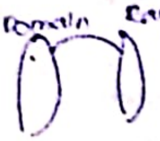
Signal contains info. when its random

Sinusoidal wave gives info. only for the 1st period.

signal → process → to analyse

* Mathematical representation.

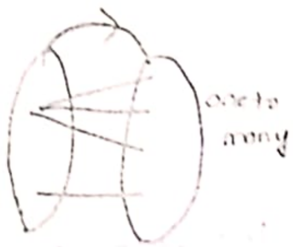
→ function - relation b/n one no. to another no.



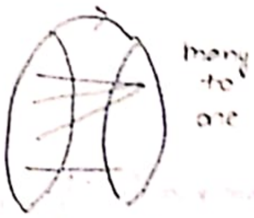
Laplace transform

onto function - Every element in domain should have an image from range

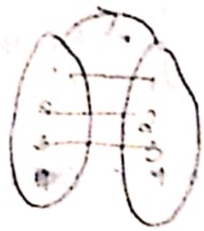
one to one - Every element of domain is matched to one element of range



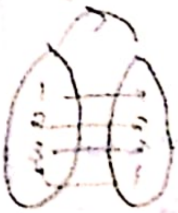
$f(x) = \sqrt{x}$ invalid



$f(x) = x^2$



onto



one to one

(R) A function is invertible if its both onto & one-one

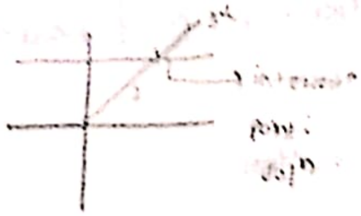
Ex: consider $\sqrt{3} = \pm i$ (complex no)

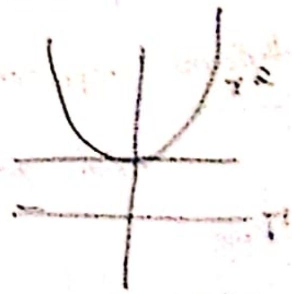
let, $x = \sqrt{-1}$ as. $1 = \sqrt{-1}$

$x^2 = -1$ ⇒ cannot be graphically solved

Where $3x = 6$ ⇒ graphically solved

can be solved algebraically also





No intersection

so no soln

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1) $f(x)$ ~ single valued funct^n – one independent Variable

$f(x,y,z)$ ~ Multi-valued funct^n – Two (or) more independent Variable

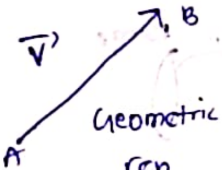
Signals $v(t) \to$ 1D signal – speech signal

$v(t_1,t_2) \to$ 2D signal – Image signal

* for Every signal to be processed we need a system

Signals

Vectors



$\vec{v}$ Geometric rep

$\vec{v} = 4\hat{i}+5\hat{j}+6\hat{k}$ – Algebraic rep

$\vec{v} = (4,5,6)$

Other ways of representing a vector – limits vector to 3d

Actual way of representing a Vector is by column matrix.

$$\vec{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \to \text{By def}$$

Mathematically elements can be in column matrix. This means the Vector can be higher than 3 dimension.

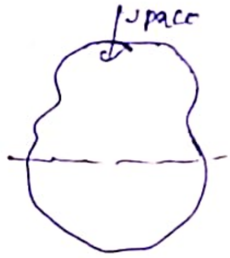
$\vec{v} = (4)$ ~ singleton matrix ~ 1D vector.

All 1 Dimensional Vectors are called scalars

value of vector is scalar

Axiomatic representat^n

Axioms are facts – no proof needed



space

Eg: Real no. line

Rules

1) Closed under vector addition

Consider $a, b \in$ space then $a+b=c$

$c \in$ space

2) Closed under scalar multiplication $\Rightarrow$ ~ $\alpha a \in$ space when α is a scalar & $a \in$ space

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if these 2 rules are satisfied then the space is called vector space

Eg. of Vector space

→ Real no. line

→ Re set of all matrices – Matrix space

→ Set of all functions – Function space

Examples for non-vector space

→ set of all +ve integers

(*) Define a Vector – An element of a vector space

function space is a vector space

A function is an element of ~~vector~~ function space

A vector is an element of vector space

So :- function is a vector

SIGNAL

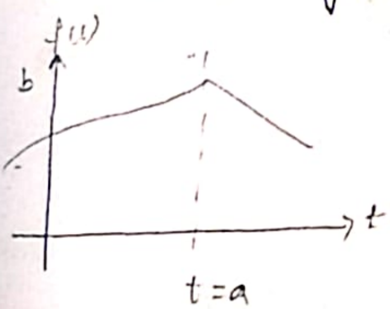
signal is a function. function vector. so, signal is a vector

Signal ⟶ function ⟶ Vector

classification of signals

1) continuous time signals
2) descrete time signals
3) Analog signals
4) Digital signals.

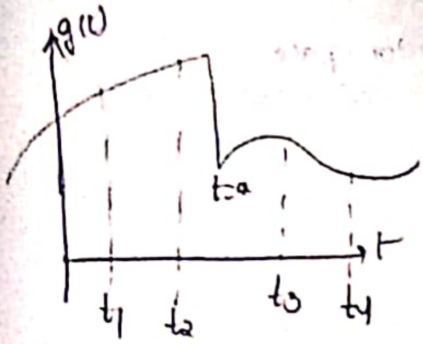
continuous time signals – signal should have values for all t



for any func^n to be continuous $\boxed{\text{LH Limit} = \text{RH Limit}}$

$\lim_{t \to a} f(t) = f(a)$. From both sides the value becomes b

Therefore continuous function

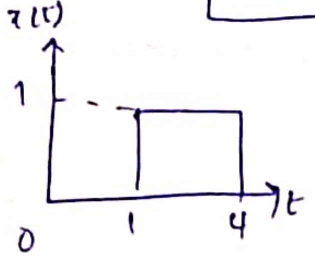


But g(t) is not continuous function for the entire range t

But, continuous at t_1, t_2 to t_2 & t_3 to t_4

But, the signal is continuous cauz at t=a it has a value.

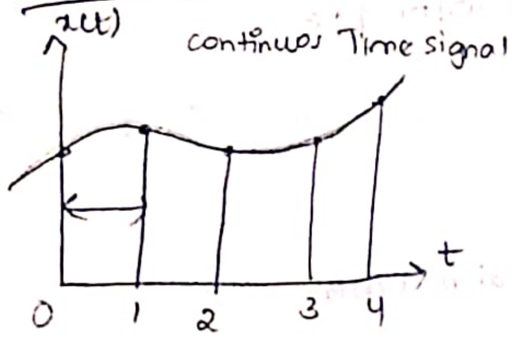
Representationⁿ :- $\boxed{x(t)}$

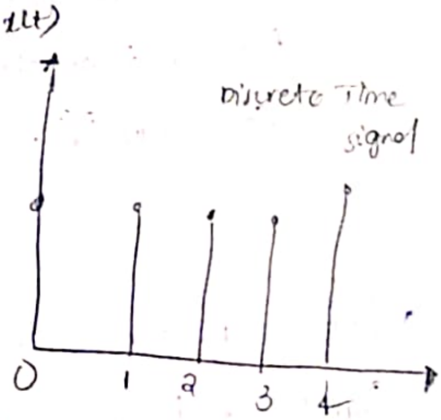


Graphical rep / waveform

$$x(t) = \begin{cases} 1 & 1 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

2) Discrete Time signal :





Sampling Interval = T_s

can be uniform / nonuniform

This is done when sometimes channel is unable to hold the complete signal

Continuous Time signal $\xrightarrow{\text{sample}}$ Discrete Time signal

For continuous time signal domain is set of real numbers $\mathbb{R}$

For discrete time signal domain is set of integers $\mathbb{Z}$

Notation - $x(nT_s)$ where n is an integer

$x(t) \big|_{t=nT_s} \Rightarrow \boxed{x(n)}$ as T_s is fixed (or) x[n]

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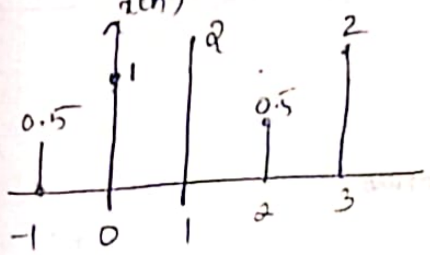
Representation



Graphical rep / waveform

$x(n) = \{x(-1), x(0), x(1), x(2), x(3)\}$

$x(n) = \{0.5, 1, 2, 0.5, 2\}$

If only this given consider 1st term corresponds to $n=0$

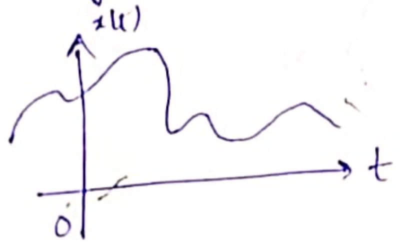
DTS also called Discrete time sequence

Else if $n=0$ to be mentioned then its the one with arrow

$x(n) = \{0.5, 1, \underset{\uparrow}{2}, 1, 2, 3\}$ so, $n=0$ is 2

Analog signals

Range ∈ Infinite set

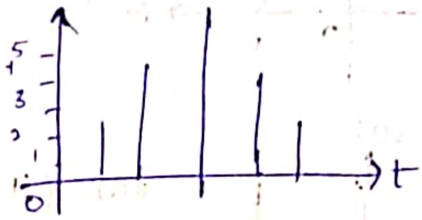


Naturally all signals are analog

* prone to noise

Digital signals

Range ∈ Finite set



signals used in devices are digital

* No noise
* Understanding & processing is Easy

Analog → Digital

step-1 - Sampling

step-2 - Quantization

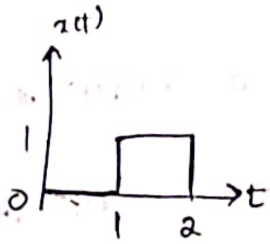
Operations on signals

Operations on independent Variable

1) scaling 2) shifting 3) Time reversal

1) scaling

$x(t) = \begin{cases} 1 & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$



$x(2t)$ ~ scaling the time axis 2 times

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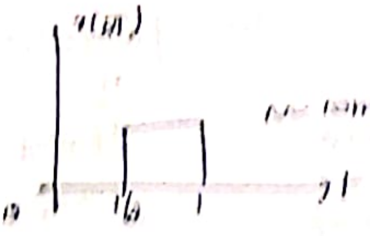
x(at) is scaling the time axis by a

$x(2t) = \begin{cases} 1 & 1 \le 2t \le 2 \\ 0 & \text{otherwise} \end{cases} \Rightarrow x(2t) = \begin{cases} 1 & 1/2 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

$x(t) \rightarrow x(at)$



is compressed

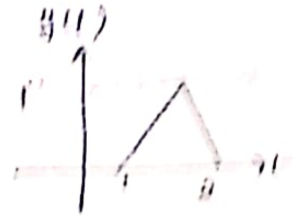
$a > 1 \rightarrow x(t)$ gets compressed

$0 < a < 1 \rightarrow x(t)$ expands

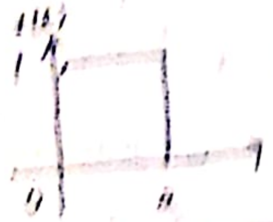
4) Time shifting

a)

shifting



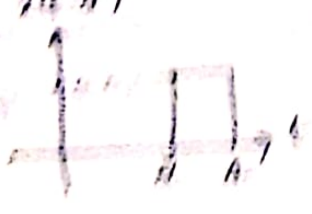
b)



$x(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & \text{elsewhere} \end{cases}$

shifts towards right

towards left



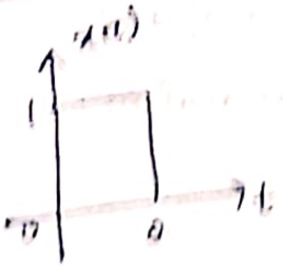
$x(t-t_0) = \begin{cases} 1 & t_0 \le t \le t_0 + 2 \\ 0 & \text{elsewhere} \end{cases}$

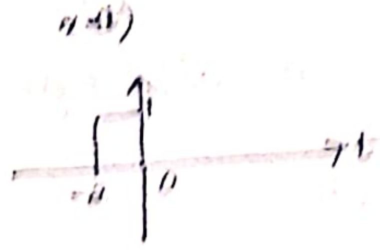
$x(t+t_0) = \begin{cases} 1 & -t_0 \le t \le 2 - t_0 \\ 0 & \text{elsewhere} \end{cases}$

$x(t-t_0) \rightarrow$ for $t_0 < 0 \rightarrow x(t)$ shifts left $\rightarrow$ advanced version of $x(t)$

for $t_0 > 0 \rightarrow x(t)$ shifts right $\rightarrow$ delayed version of $x(t)$

3) Time reversal





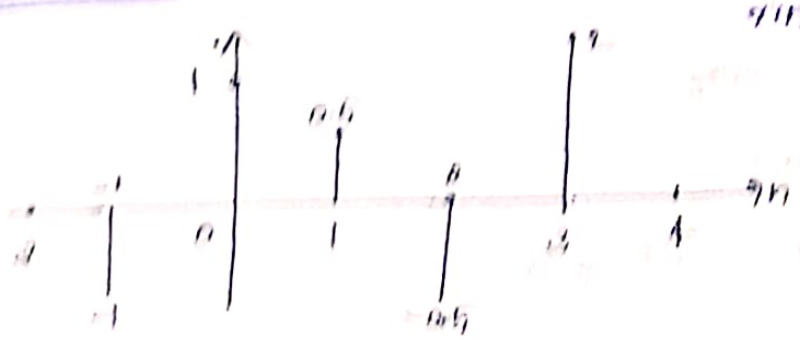
$x(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$

$x(-t) = \begin{cases} 1 & 0 < -t < T \Rightarrow -T < t < 0 \\ 0 & \text{otherwise} \end{cases}$

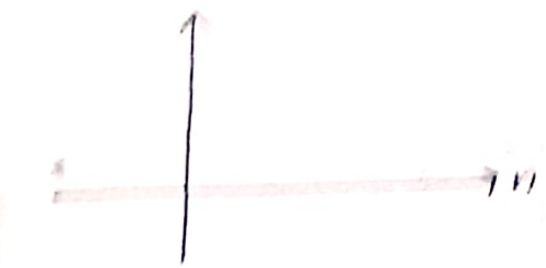
Note: The operations on a independent variable for a discrete time change

1) a) Time scaling: $x(n) \rightarrow x(kn)$

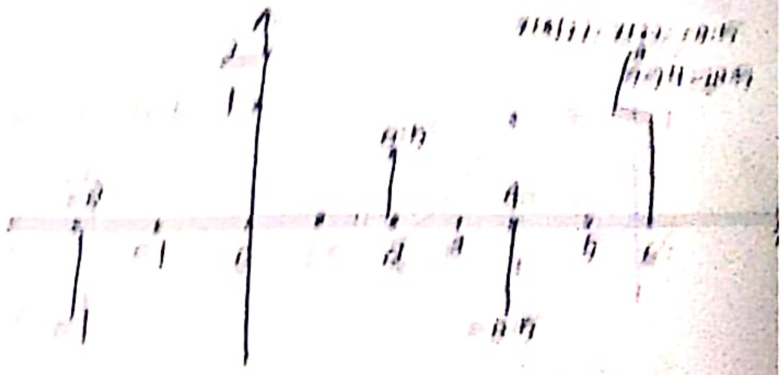
$x(n) = \{$[illegible]$\}$



$x(2n)$



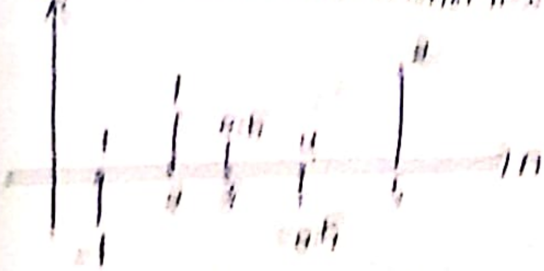
$x(n/2)$ [illegible]

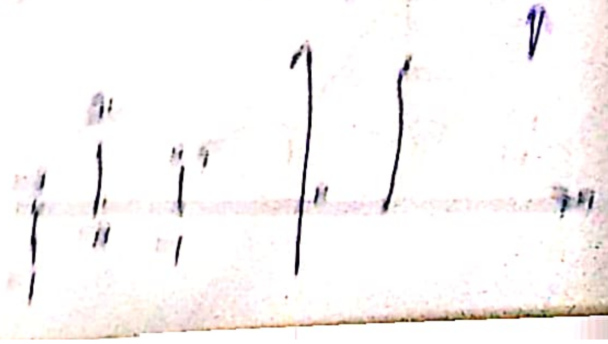


b) Time shifting

$x(n) = \{$[illegible]$\}$

$x(n-2)$ [illegible]





for

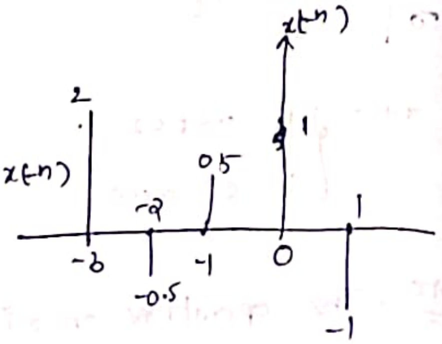
$x(n-n_0) \Rightarrow$ for $n_0<0 \Rightarrow x(n)$ shifts left $\rightarrow$ advanced version of $x(t)$

$\Rightarrow$ for $n_0>0 \Rightarrow x(n)$ shifts right $\rightarrow$ delayed version of $x(t)$

3) Time reversal

$x(n) = \{-1, \underset{\uparrow}{1}, 0.5, -0.5, 2\} \longrightarrow x(-n) = \{2, -0.5, 0.5, \underset{\uparrow}{1}, -1\}$

$x(t) \longrightarrow x(-2t+4)$



doing all activities together

Precedence rule $\rightarrow$ shifting first

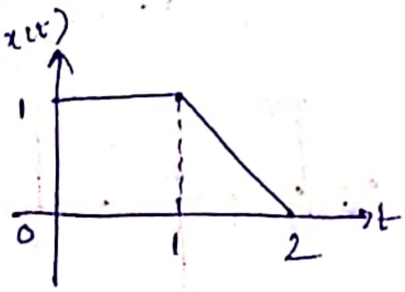
$\rightarrow$ Then scaling

$x(t) \longrightarrow x(at+b)$

i) $x(t) \longrightarrow x(t+b) \rightarrow$ shifting first

ii) $x(t+b) \longrightarrow x(at+b) \rightarrow$ scaling (scaling happens only on t not on b.)

Eg:- 1.1, 1.2, 1.3



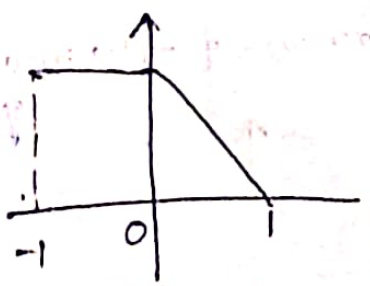
find 1) $x(t+1)$

2) $x(-t+1)$

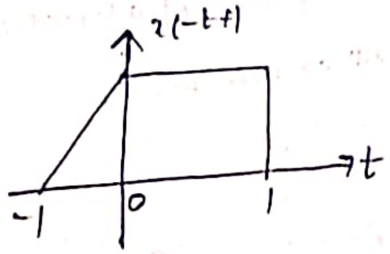
3) $x(2t)$

4) $x(2t+1)$

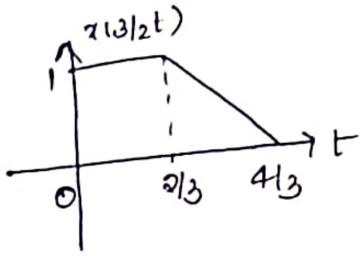
(i) $x(t+1)$



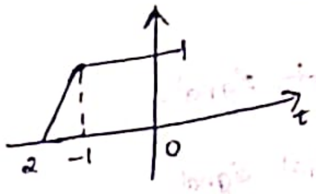
(ii) $x(-t+1)$



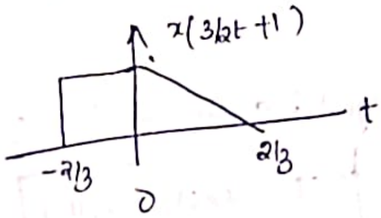
(iii) $x(3/2)t$



$x(-t)$



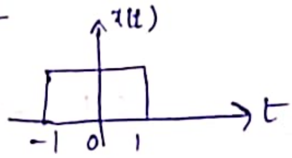
(iv) $x(3/2\,t + 1)$



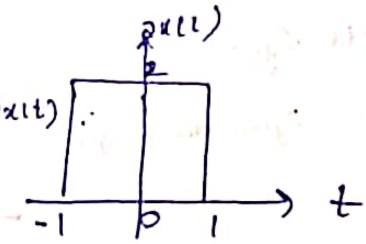
Assignment : chap1 → Ex → 1.4 & 1.5

Operations on dependent Variable

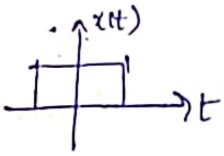
1) Amplitude scaling



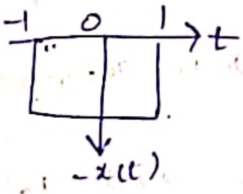
⇒ $2x(t)$



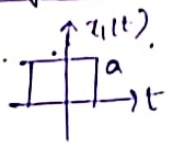
2) Amplitude reversal

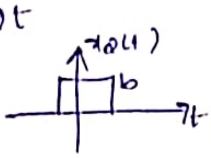


⇒

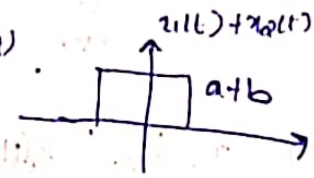


3) Addition of signals





$x_1(t)+x_2(t)$ ⇒



4) Multiplication of signals

$$z(t) = y(t) \cdot x(t)$$

5) Diff of signal

$$\frac{dx(t)}{dt}$$

6) Integration of signal

$$\int_{\tau=0}^{t} x(\tau)\, d\tau$$

Aperiodic signal

A signal which is not periodic

Energy signal

$$\boxed{\text{Energy} = \int \cancel{x^2(t)\,dt} \quad \int [x(t)]^2\,dt}$$

The total energy of the continuous time signal over the interval $t_1 \le t \le t_2$ is defined as

$$\boxed{E = \int_{t_1}^{t_2} x^2(t)\,dt} \rightarrow \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

for discrete time signal

$$\boxed{E = \sum_{n=n_1}^{n} [x(n)]^2} \rightarrow \sum_{n=-\infty}^{\infty} |x(n)|^2$$

for periodic signals ($-\infty$ to ∞) energy is also infinite

Power signal

Signals whose energies are infinite, i.e., in case of periodic signals (we consider avg. power of the signal) are called power signals

* Time averaged power

Continuous time signal $\Rightarrow$ $\boxed{P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt}$

Discrete time signal $\Rightarrow$ $\boxed{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}$

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Prob $x_1(t) = x(t + kT_1)$

$x_2(t) = x(t + lT_2)$

$x_1(t) + x_2(t) \Rightarrow$ periodic ?

A) let, $x(t) = x_1(t) + x_2(t)$

for $x(t)$ to be periodic with T,

then $x(t) = x(t+T)$

$\Rightarrow x(t) = x_1(t) + x_2(t)$

$x(t+T) = x_1(t+kT_1) + x_2(t + lT_2)$

for $x(t)$ to be periodic, the condition is; $\Rightarrow kT_1 = lT_2 = T$

$$\boxed{\frac{T_1}{T_2} = \frac{l}{k}}$$

→ The ratio T_1/T_2 must be a rational for $x(t)$ to be periodic

→ $x(t)$ is periodic with T which is LCM of T_1 & T_2.

for DTS,

$x(n) = x_1(n) + x_2(n)$

When, $x_1(n) = x_1(n + mN_1)$

$x_2(n) = x_2(n + lN_2)$

then $x(n)$ is periodic,

if $mN_1 = lN_2$

$$\boxed{\frac{N_1}{N_2} = \frac{l}{m}}$$

Types of SIGNALS / std test signals

Eg:- To check whether the building can withstand an earthquake:

We Modulate the particular building to all required conditions, then

We simulate the environment for the particular input to check the output

1) Modulation

↓

2) simulation

Modulation:- which governs the particular situation

Eqⁿˢ required

simulation:- Replicate the (input) problem & check for the output.

1) Exponential Signal
2) Sinusoidal
3) Unit step signal
4) Unit Impulse Signal
5) Ramp Signal

CT & DT

1) Continuous time Exponential signal

Real: $x(t) = Ce^{at}$

$a>0 \Rightarrow x(t)$ is a growing Exponential

$a<0 \Rightarrow x(t)$ is a decaying Exponential

$a=0 \Rightarrow x(t)$ is constant

Complex Exponential

1) Show that the complex exponential function $x(t) = e^{j\omega_0 t}$ is periodic & find its fundamental period.

$x(t) = e^{j\omega_0 t}$ = periodic?

Let, $x(t)$ be periodic with T

$\Rightarrow x(t+T) = x(t)$

$\Rightarrow e^{j\omega_0(t+T)} = e^{j\omega_0 t}$

(or)

$e^{j\omega_0 t}\, e^{j\omega_0 T} = e^{j\omega_0 t} \rightarrow (1)$

For (1) to be true

$e^{j\omega_0 T} = 1 \Rightarrow$ $\omega_0 T = n2\pi$

$\boxed{T = \frac{n2\pi}{\omega_0}}$

Fundamental Period

$\boxed{T_0 = \frac{2\pi}{|\omega_0|}}$

2) show that the sinusoidal signal $x(t) = \cos(\omega_0 t + \phi)$ is periodic & find its fundamental period?

$x(t) = \cos(\omega_0 t + \phi)$

Let, $x(t)$ be periodic with T

$x(t+T) = \cos(\omega_0(t+T) + \phi)$

$x(t) = x(t+T)$ if its periodic with T

$\cos(\omega_0 t + \phi) = \cos(\omega_0(t+T) + \phi)$

$\cancel{\omega_0 t} = \omega_0 T = m2\pi$

$T = m\frac{2\pi}{\omega_0}$

$$\boxed{T_0 = \frac{2\pi}{|\omega_0|}}$$

3) show that the discrete exponential signal is periodic & find the condition of periodicity

$x(n) = e^{j\Omega_0 n}$

Let, $x(n) = x(n+N)$

$e^{j\Omega_0 n} = e^{j\Omega_0(n+N)}$

$e^{j\Omega_0 n} = e^{j\Omega_0 n} e^{j\Omega_0 N}$

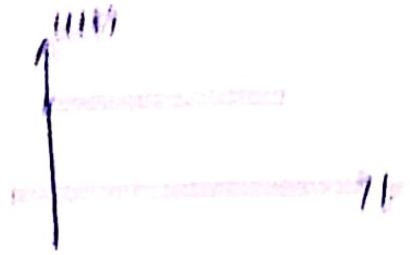
$e^{j\Omega_0 N} = 1$

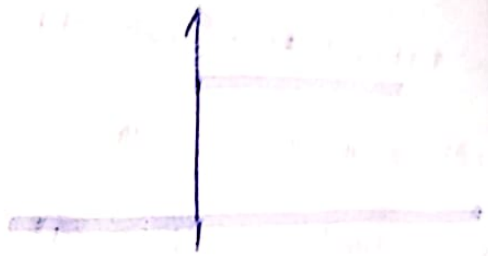
$\Omega_0 N = m2\pi \Rightarrow \boxed{\frac{\Omega_0}{2\pi} = \frac{m}{N}} \Rightarrow$ condition for $x(n)$ to be periodic

$N \rightarrow$ periodicity

Unit step signal u(t)

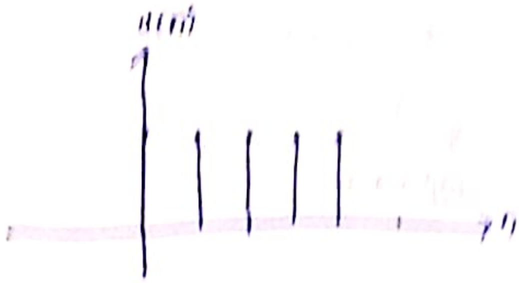
CT : $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

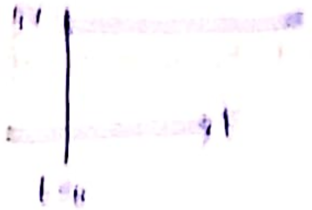




DT : u(n)

$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



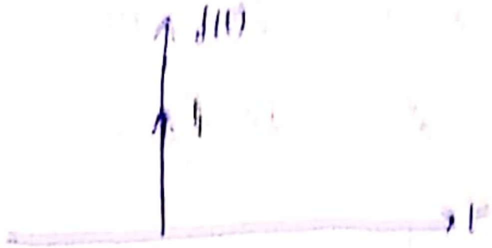


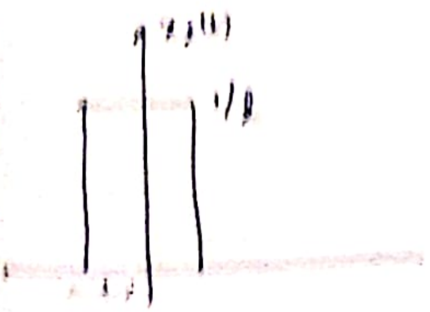
Unit Impulse function δ(t)

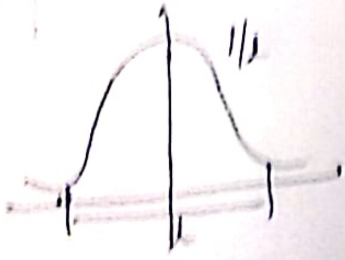
$\delta(t) = 0, \ t \neq 0$

$\int \delta(t)\,dt = 1, \ t = 0$

Area of the signal = 1

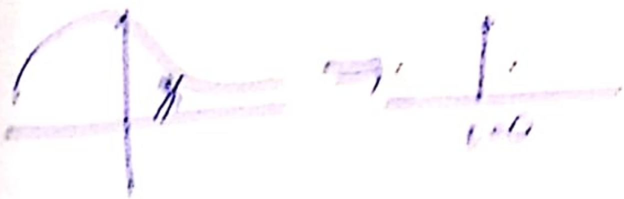






$\lim_{\Delta \to 0} x_\Delta(t) = \delta(t)$

Impulse function is the limit of an equivalence class of functions



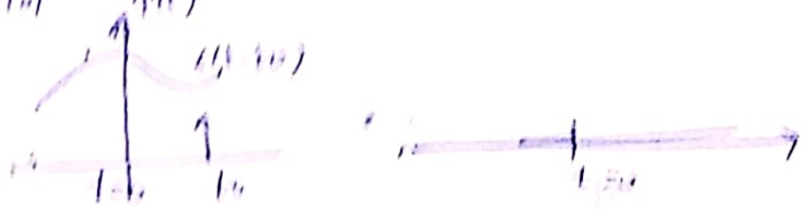
Multiplication of a function by an impulse

Consider a signal $\phi(t)$

Now consider $\phi(t)\delta(t) = \phi(0)\delta(t)$ — (1)

$\phi(t)\delta(t-t_0) = \phi(t_0)\delta(t-t_0)$ — (2)

we are giving a signal to make a continuous signal to a discrete signal using unit impulse function



From (1),

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \int_{-\infty}^{\infty} \phi(0)\delta(t)\,dt \Rightarrow \phi(0)\int_{-\infty}^{\infty} \delta(t)\,dt$$

$$= \phi(0)\cdot 1$$

Sampling Property

From (2),

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} \phi(t_0)\delta(t-t_0)\,dt$$

$$= \phi(t_0)$$

Note!! Impulse function is a generalized function

consider : $\int_{-\infty}^{\infty} \frac{du(t)}{dt}\phi(t)\,dt \longrightarrow (2)$

let, $u = \phi(t)$, $dv = \frac{du(t)}{dt}\,dt$

du

$\frac{du}{dt} = \frac{d\phi(t)}{dt}$, $v = u(t)$

$\int u\,dv = uv\Big] - \int v\,du$

$= \phi(t)\,u(t)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} u(t)\cdot\frac{d\phi(t)}{dt}\,dt$

integrate $\delta(t)$ to get step function

$= \phi(\infty)\,u(\infty) - \phi(-\infty)\,u(-\infty) - \int_0^{\infty} \frac{d\phi(t)}{dt}\,dt$

diff step function to get $\delta(t)$

$= \phi(\infty) - \phi(t)\Big]_0^{\infty}$

$= \phi(\infty) - \phi(\infty) + \phi(0)$

$= \phi(0)$

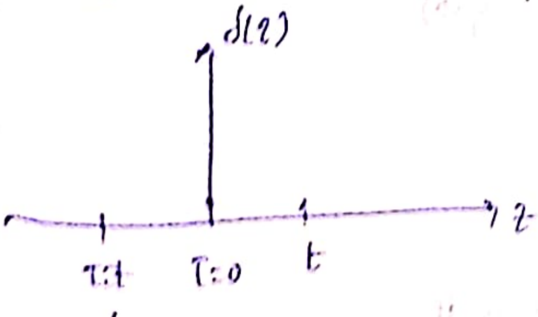
from ① & ②

$\Rightarrow \boxed{\frac{du(t)}{dt} = \delta(t)} \Rightarrow u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

0 $t<0$

1 $t \geq 0$

$\tau < 0$



$\phi \to$ from diff

28/2/2

1.1.2

1.2 → Exponential & Sinusoidal Signals

eg

CTS

$\cancel{x(t) = C}$

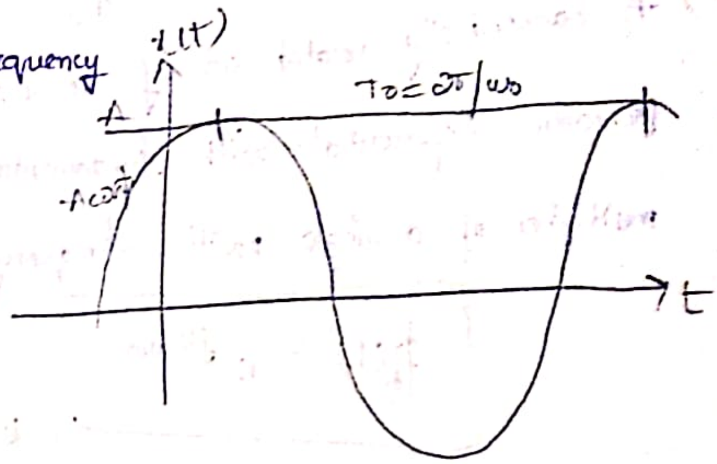
$x(t) = Ce^{at}$

$x(t) = 1$, $x(t) = e^{j\omega_0 t}$ $\longrightarrow$ $x(t) = A\cos(\omega_0 t + \phi)$ with some phase diff

$T_0 \to$ undefined $\quad T_0 = \dfrac{2\pi}{|\omega_0|}$

$\omega_0 = 0$

↑ fundamental frequency



Energy over one period

$$E_0 = \int_0^{T_0} |x(t)|^2 dt$$

$$= \int_0^{T_0} |e^{j\omega_0 t}|^2 dt$$

$|e^{j\omega t}| = \cos t + j\sin t$

$= \sqrt{\cos^2 t + \sin^2 t} = 1$

$E = t_0 \to \infty$

$$P_{avg} = \frac{E}{T_0} = 1$$

Note (H):

- All periodic signals are power signals

→ The periodic complex exponentials is useful to understand sets of harmonically related complex i.e., set of periodic exponentials all of which are periodic with a common period T_0

→ $x(t) = e^{j\omega t}$

$x(t + T_0) = e^{j\omega(t + T_0)}$

$= e^{j\omega T_0} \cdot e^{j\omega t} = e^{j\omega t} \cdot 1$

$e^{j\omega T_0} = 1$

$\omega T_0 = k2\pi$, $k = 0, \pm1, \pm2, \ldots$ (1)

If we define, $\omega_0 = \frac{2\pi}{T_0} \rightarrow$ (2)

$\hookrightarrow$ fundamental frequency

If (1) has to be satisfied, it should be an integer multiple of

$\omega_0 \Rightarrow \omega = k\omega_0 \Rightarrow \; = k\frac{2\pi}{T_0} \Rightarrow \omega T_0 = k2\pi$

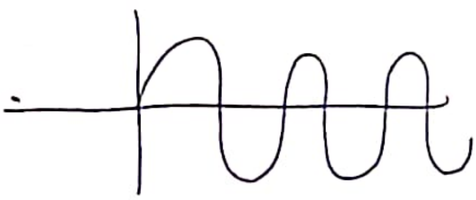
$\rightarrow$ A harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency ω_0

$$\boxed{\phi_k(t) = e^{jk\omega_0 t}} \quad , \; k = 0, \pm 1, \pm 2, \ldots$$

$$T_0 = \frac{2\pi}{|k\omega_0|}$$

$\rightarrow$

(1) $e^{j(\omega_0 t + \phi)}$

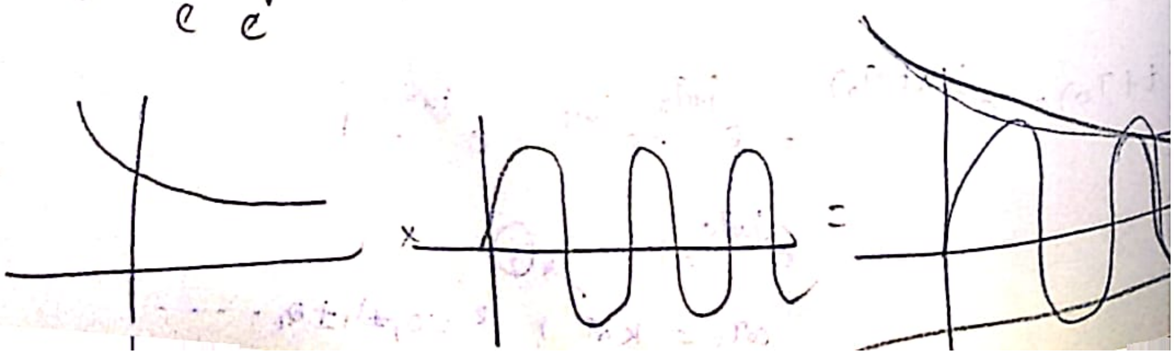


If we xly e^{rt} how the waveform will result?

(2) $e^{\overset{\text{r is real}}{r}t} \rightarrow$ and its exp $\rightarrow$

Now

(1)+(2) $e^{-2t} e^{j(\omega_0 t + \phi)}$



→ here ; it works as a damping factor after adding two signals together.

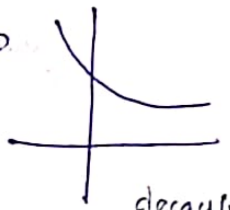
This damping happens in the RLC capacitor.

Discrete time complex Exponentials & sinusoidal signals

CT : $x(t) = Ce^{at}$, $x(n) = C\alpha^n$, $\alpha = e^{\beta}$

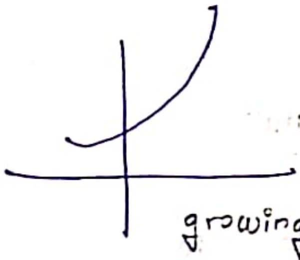
$\Rightarrow x(n) = Ce^{\beta n}$

Real $x(n)$ if $a<0$



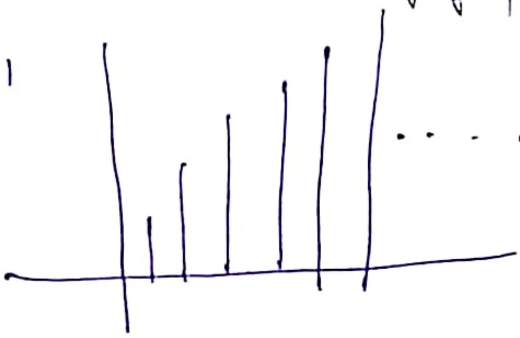
decaying Exp

if $a>0$

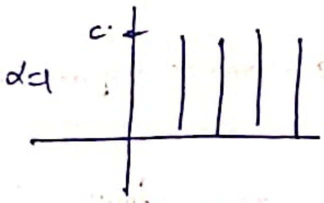


growing. Exp

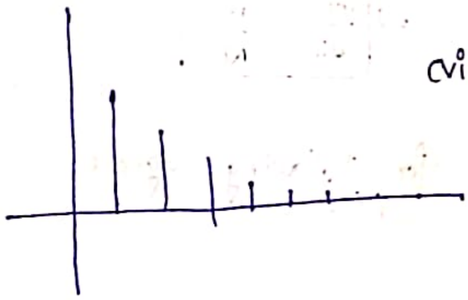
(i) if $\alpha>1$



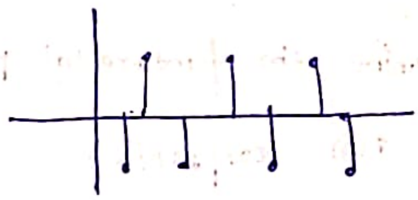
(v) if $\alpha=1$ $\Rightarrow x(n) = c(1)$ $= \text{const}$



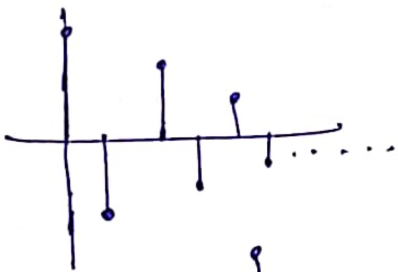
(ii) if $0<\alpha<1$



(vi) if $\alpha=-1$ $\Rightarrow x(n) = c(-1)^n$



(iii) $-1<\alpha<0$



(vii) $\alpha=0$

(iv) $\alpha<-1$

Periodicity of complex Exponential

1) $e^{j\omega_0 t}$:- $\omega_0 = \pi$, $\omega_0 = 2\pi$

$\omega_0 = 1$, $\omega_0 = 2$

$T_0 = \frac{2\pi}{|\omega_0|}$

$\omega_0 \to 0 - \pi$

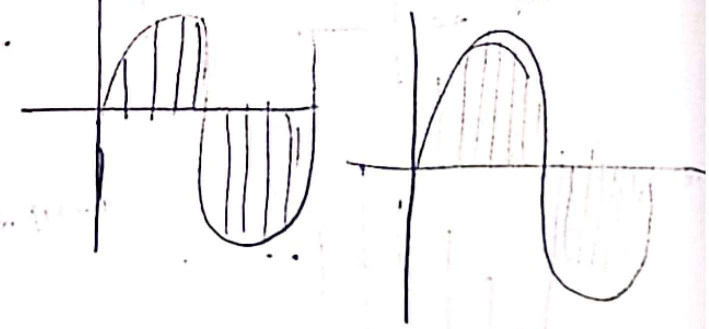
$e^{j\omega_0 n} \cdot e^{j(\omega_0 + 2\pi)n}$

$= e^{j\omega_0 n} e^{j2\pi m}$

$= e^{j\omega_0 n}$

II) periodicity :-

$\frac{\omega_0}{2\pi} = \frac{m}{N}$



Discrete time - complex Exponential

$x(n) = e^{j\omega_0 n}$ $\Rightarrow$ $\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$

Determine the fundamental period for following signals

(i) $x(n) = \cos(2\pi n/12)$

(ii) $x(n) = \cos(8\pi n/31)$

(iii) $x(n) = \cos(n/6)$

(i) $\cos(2\pi n/12)$

Continuous Time signal is periodic for any ω_0

$x(t) = \cos(2\pi t/12) = \cos(\omega_0 t)$ $\Rightarrow$ $\omega_0 = \frac{2\pi}{12}$

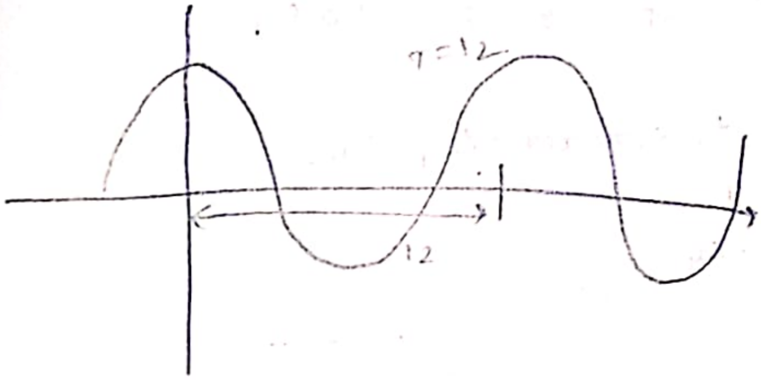
w.k.t $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi/12} = 12$ takes 12sec to complete 1 cycle.

But, in discrete time signal $\frac{\omega_0}{2\pi}$ should be rational

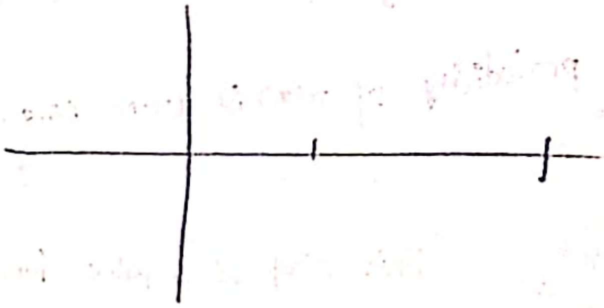
$$x(n) = \cos\left(2\pi n/12\right) = \cos(\omega_0 n)$$

for $x(n)$ to be periodic, the ratio $\frac{\omega_0}{2\pi}$ has to be rational

$$\frac{\omega_0}{2\pi} = \frac{\cancel{2\pi}/12}{\cancel{2\pi}} = \boxed{\frac{1}{12} = \frac{m}{N}} \Rightarrow N = 12 \qquad N = \text{periodicity}$$



But, for discrete,



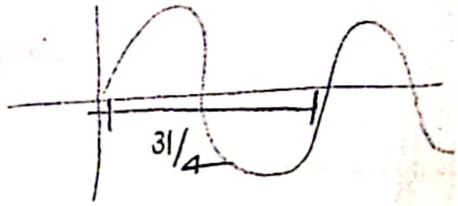
(ii) $x(n) = \cos(8\pi n/31)$

Fundamental period

CTS $x(t) = \cos(8\pi t/31) \Rightarrow \omega_0 = 8\pi/31$, $T_0 = \frac{2\pi}{\omega_0} = \frac{\cancel{2\pi}\times 31}{\cancel{8\pi}\,4}$

for DTS $\Rightarrow x(n) = \cos(8\pi n/31)$

$$\frac{\omega_0}{2\pi} = \frac{\overset{4}{\cancel{8\pi}}/31}{\cancel{2\pi}} = \boxed{\frac{4}{31} = \frac{m}{N}} \qquad N = 31$$



Discrete signals takes only integers. Here, The to fundamental period $31/4$ is not an integer. That's why the period is not same in discrete & continuous.

(iii) $x(n) = \cos(n/6)$

CTS $\Rightarrow$ $x(t) = \cos(t/6) \Rightarrow \omega_0 = 1/6$, $T_0 = \frac{2\pi}{\omega_0} = 2\pi(6) = 12\pi$

DTS $\Rightarrow$ $x(n) = \cos(n/6)$

$$\frac{\omega_0}{2\pi} = \frac{1}{6(2\pi)} = \frac{1}{12\pi} \Rightarrow \boxed{\frac{1}{12\pi} \neq \frac{m}{N}}$$

Not rational

We can sample this signal But, not in Equal interval of time.

so, A-periodic

4) Determine the fundamental period of a DTS $x(n) = e^{\ldots} + \ldots$

A) $x(n) = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$

$= x_1(n) + x_2(n)$

$x_1(n) = e^{j\omega_1 n} \Rightarrow \omega_1 = 2\pi/3$, $\frac{\omega_1}{2\pi} = \frac{2\pi/3}{2\pi} = \frac{1}{3} \Rightarrow \boxed{N_1 = 3}$

$x_2(n) = e^{j\omega_2 n} \Rightarrow \omega_2 = 3\pi/4$; $\frac{\omega_2}{2\pi} = \frac{3\pi/4}{2\pi} = \frac{3}{8} = \frac{m}{N} \Rightarrow \boxed{N_2 = 8}$

check! $\frac{N_1}{N_2} = \frac{3}{8} \Rightarrow$ rational no $\Rightarrow$ to sum $x(n)$ is periodic

periodicity of $x(n)$ is LCM $(3,8) = 24$

1.4 Unit step & impulse functions

continuous $\to$ tells area

Discrete $\to$ tells abt amplitude

CT : $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

Dirac-Delta function

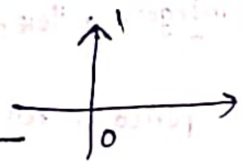
$\delta(t) = 0$, $t \neq 0$

$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$, $t = 0$

Sampling property . $\int_{-\infty}^{\infty} \phi(t)\,\delta(t - t_0)\,dt = \phi(t)\big|_{t=t_0}$

$= \phi(t_0)$

$\delta(t) = \frac{du(t)}{dt}$ (or) $u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\,d\tau$

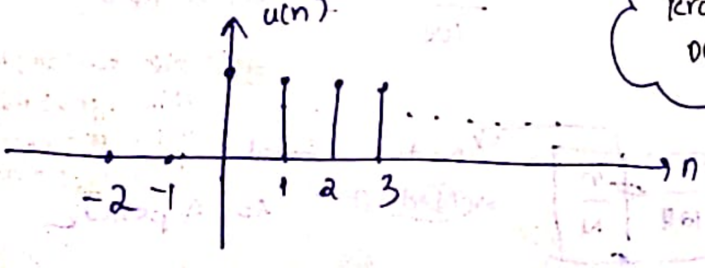


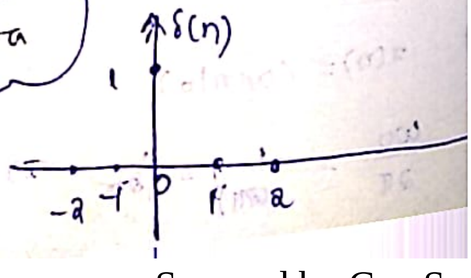
DT :- $u(n) = \begin{cases} 1 & , n \geq 0 \\ 0 & , n < 0 \end{cases}$

$\delta(n) = 0$, $n \neq 0$

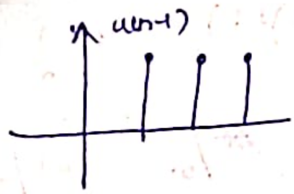
$\delta(n) = 1$, $n = 0$

Kronecker delta





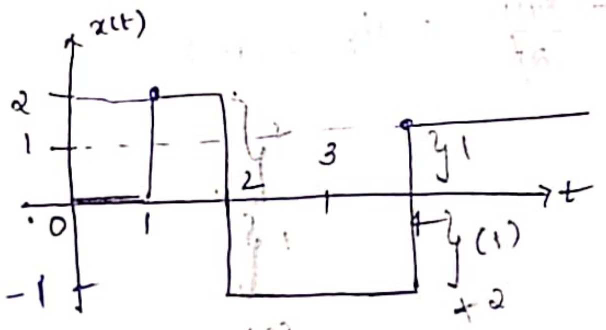
$$\boxed{u(n) - u(n-1) = \delta(n)}$$



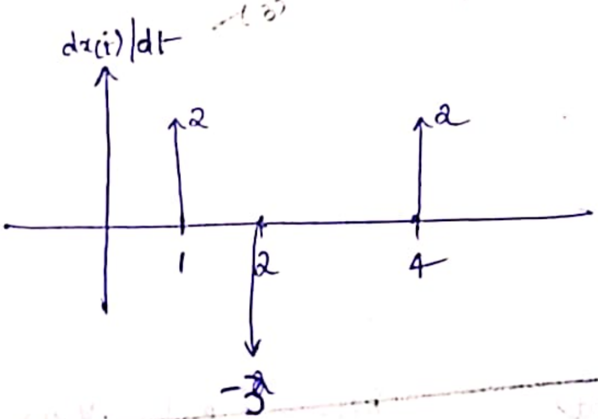
Derivative in continuos becomes subtraction in descrete domain $\left[\begin{array}{l}\delta(t) = \frac{du(t)}{dt} \\ \delta(n) = u(n) - u(n-1)\end{array}\right]$

Integration in continuos becomes addition in descrete domain $\left[\begin{array}{l}u(t) = \int_{\tau=-\infty}^{t} \delta(\tau) d\tau \\ u(n) = \delta(n) + u(n-1)\end{array}\right]$

$$\boxed{\begin{array}{l}\delta(n) = u(n) - u(n-1) \\ u(n) = \sum_{m=-\infty}^{n} \delta(m)\end{array}}$$



find $dx(t)/dt$ (Area)



Q) Determine the fundamental period

1. $x(t) = \cos(t + \pi/4)$ $\quad \omega t + \pi/4 \quad$ ~~$T_0 = 2\pi/\omega = 2\pi/\pi/4$~~ $\quad t = 1 \Rightarrow T_0 = 2\pi/\omega = 2\pi/1$

2. $x(t) = \sin\left(\frac{2\pi}{3}t\right) \Rightarrow \sin\left(\frac{2\pi}{3}t\right) = \cos\left(\left(\frac{\pi}{2} - \left(\frac{2\pi}{3}\right)\right)t\right) = \cos\left(\frac{3\pi - 4\pi}{6}\right)t$

$= \cos\left(-\frac{\pi}{6}\right)t$

$= \cos \pi/6\, t$

$T_0 = 2\pi/\omega = \frac{2\pi \times 6}{\pi} = 12$

3. $x(n) = \cos^2\left(\frac{\pi}{8}n\right) \quad \frac{\cos\left(2\left(\frac{\pi}{8}\right)+1\right)}{2} = \cancel{\cos\left(\frac{\pi}{8}+1\right)/2}$

$\omega =$

$\cos 2A = 2\cos^2(A) - 1$

$\cos^2 A = \left(\frac{\cos 2A + 1}{2}\right)$

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$$\frac{1}{2}\left(\cos\left(\cancel{2}(\pi/8)\right)+1\right) \Rightarrow \frac{1}{2}\left[\cos(\pi/4+1)\right]$$

$$T_0 = 2\pi/\omega = 2\pi/\pi/4 = 8$$

4) $x(n) = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n$

$$= \cos\frac{\pi}{3}n + \cos\left[\frac{\pi}{2} - \pi/4\right]n$$

$$= \cos\pi/3\,n + \cos\left[\frac{2\pi}{8}\right]n = \cos\pi/3\,n + \cos\pi/4\,n$$

$x_1(n) = \cos(\pi/3)n \Rightarrow \frac{\omega_0}{2\pi} = \frac{\cancel{\pi}/3}{2\cancel{\pi}} = 1/6 \Rightarrow N=6$

$x_2(n) = \cos(\pi/4)n$

$\Rightarrow \frac{\omega_0}{2\pi} = \frac{\cancel{\pi}/4}{2\cancel{\pi}} = 1/8 \Rightarrow N=8$

LCM of $(6,8) \Rightarrow 24$

$N=24$

29/8/19

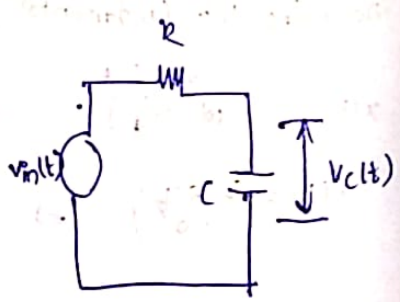
SYTEM

To process a signal we need a system.

$x(t) \longrightarrow \boxed{T} \longrightarrow T\{x(t)\} = y(t)$

system → Transformation of signals

$i(t) \longrightarrow \boxed{R} \longrightarrow V_R(t)$



$V_{in}(t) \longrightarrow \boxed{RC} \longrightarrow V_c(t)$

Functional

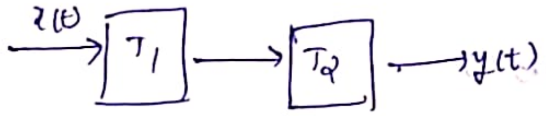
function ——— Value

$$\int_{-\infty}^{\infty} \phi(t)\,\delta(t)\,dt = \phi(t)\Big|_{t=0}$$

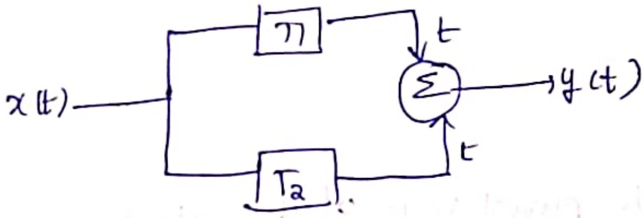
$$= \phi(0)$$

Interconnection of systems

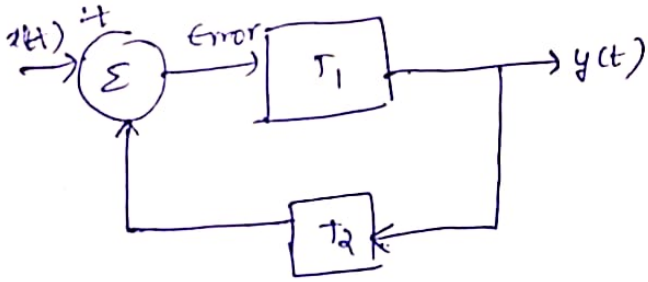
- cascade



- parallel



feedback system   why do we need feedback?



Properties of the system

1. Linearity

$f(x) \xrightarrow{\text{is linear if}}$ (i) superposition $\Rightarrow f(x+y) = f(x) + f(y)$.

(ii) Homogeneity $\Rightarrow f(\alpha x) = \alpha f(x)$

A system described by its T $T\{\cdot\}$ is linear if the following relation is observed

$$\boxed{T\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 T_1\{x_1(n)\} + a_2 T_2\{x_2(n)\}} \longrightarrow (1)$$

Eg:- $x(t) \longrightarrow \boxed{T} \longrightarrow y(t) = x^2(t)$

$$T\{x(t)\} = x^2(t) = y(t)$$

sol^n

LHS of ①

$$T\left[a_1 x_1(t) + a_2 x_2(t)\right] = \left[a_1 x_1(t) + a_2 x_2(t)\right]^2$$

$$= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2 x_1(t) x_2(t)$$

RHS of ①

$$a_1 T[x_1(t)] = a_1 x_1^2(t)$$

$$a_2 T[x_2(t)] = a_2 x_2^2(t)$$

$$\Rightarrow a_1 x_1^2(t) + a_2 x_2^2(t)$$

LHS ≠ RHS So, system is not linear

2) **Causality:-**

A system is said to be causal if the present value of the output signals depends only on the present value, (or) past value (or) the combination of present & past values of inputs

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n)$$

$y(n) = x(n) + x(n-1) + x(n-2) + x(n-3) \rightarrow$ causal (present + past)

$y(n) = x(n) + x(n-1) + x(n+2) + x(n-3) \rightarrow$ non-causal

$y(n) = x(n-1) \rightarrow$ causal (past)

$y(n) = x(n) \rightarrow$ causal (present)

$y(n) = x(n+1) \rightarrow$ Non-causal (future)

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3) Time - Invariance :-

If a time delay (or) time advance of the input ~~leads~~ signal leads to an identical time shift in the output signal

LHS RHS

$\rightarrow T[x(n-n_0)] = y(n-n_0)$

$T[x(t-t_0)] = y(t-t_0)$

$T[x(n)] = nx(n) = y(n)$ | $y(n-n_0) = (n-n_0)\, n, (n-n_0)$

$x(n-n_0) = nx(n-n_0)$

4) Memory :-

~~A system has memory if its output~~

A system is memory-less if the present value of the output depends only on present values of the input.

if output depends only on input (i.e. Memory less)

$y(t) = kx(t)$

Resistor —WW— (Memory less) $\Rightarrow$ $y(t) = x(t) + x(t-1)$

$V(t) = R\,I(t)$

Capacitor —||— (has Memory)

$$V(t) = \frac{1}{C}\int_{\tau=-\infty}^{t} i(\tau)\,d\tau = \frac{1}{C}\underbrace{\int_{-\infty}^{0}}_{\text{past}} + \underbrace{\int_{0}^{t}}_{\text{present}}$$

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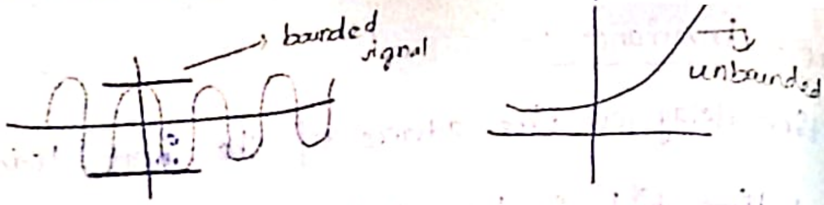
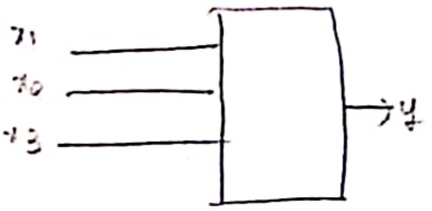
5) **stability:-**



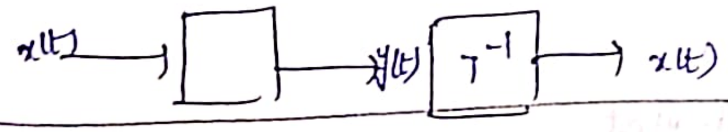
A system is said to be bounded input and bounded output system if for every bounded input signal the output is bounded.



$y = x_1 + x_2 + x_3$

6) **Invertibility:-**

A system is said to be invertible if the input to the system may be uniquely determined from the output.



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Problems

1) for a system described by $T[x(n)] = a x(n) + b$ check for the following properties

(i) Linearity →

(ii) Causality

(iii) Time-Invariance

(iv) stability

(V) Memory

1. $T[x(n)] = a\, x(n) + b$

i) Linearity

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

LHS:- $T[a_1 x_1(n) + a_2 x_2(n)] = a[a_1 x_1(n) + a_2 x_2(n)] + b$

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RHS : $a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a[a_1 x_1(n) + b] + a_2 [\cancel{a_2 x_2} \cdot a x_2(n) + b]$

Since $\Rightarrow$ LHS $\neq$ RHS $\Rightarrow$ the system is not linear

ii) <u>Causality</u>

Its given,

$T[x(n)] = y(n) = a x(n) + b$

At, ~~x = x0~~ $n = n_0$,

$y(n_0) = a x(n_0) + b$

Since, the present value of o/p depends only on present value of i/p, the system is casual

iii) <u>Time-Invariance</u>

$y(n) = a x(n) + b$

$T[x(n-n_0)] = y(n-n_0)$

<u>LHS</u>: $T[x(n-n_0)] = a x(n-n_0) + b$

<u>RHS</u>: $y(n-n_0) = a x(n-n_0) + b$

Since, LHS = RHS, the system is time-Invariant

iv) <u>Stability : BIBO Stability</u>

$y(n) = a x(n) + b \longrightarrow (1)$

let $|x(n)| < M$ $\rightarrow$ amplitude of the signal is less than M

$|y(n)| = |a x(n) + b|$

$\leq |a x(n)| + |b|$

$\leq |a|\,|x(n)| + |b|$

$\leq aM + b$

$|y(n)| \leq \infty$

Since, the o/p is bounded, for bounded i/p $\Rightarrow$ the system is BIBO stability

v) <u>Memory</u>

Since, the present o/p depends only on the present i/p, the system is Memoryless.

2) $T[x(n)] = x(n) + n$

i) <u>Linearity</u>

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

<u>LHS</u>:- $T[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(n) + a_2 x_2(n) + n$

<u>RHS</u>:- $a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 [x_1(n) + n] + a_2 [x_2(n) + n]$

Since, LHS $\neq$ RHS its not linear

ii) <u>Causality</u>

Its given

$T[x(n)] = y(n) = x(n) + n$

At $n = n_0$,

$y(n_0) = x(n_0) + n_0$

since, the system depends on present & past its causal

iii) <u>Time-Invariance</u>

$y(n) = x(n) + n$

$T[x(n-n_0)] = y(n-n_0)$

<u>LHS</u>:- $T[x(n-n_0)] = x(n-n_0) + n$ (takes input & adds n to that) it doesn't replace n with $n-n_0$

<u>RHS</u>:- $y(n-n_0) = x(n-n_0) + (n-n_0)$

since, LHS $\neq$ RHS, ~~the system is time-Invariant~~

Time varient

iv) <u>stability : BIBO stability</u>.

$y(n) = x(n) + n \longrightarrow (1)$

let, $|x(n)| < M$

$|y(n)| = |x(n) + n|$

$\leq |x(n)| + n \uparrow$

As, $n \to \infty$, $|y(n)|$ is not bounded

$\Rightarrow$ not stable

~~since, o/p is b~~

v) Memory

since, the present o/p depends only on the present i/p its memory less

) $y(t) = T[x(t)]$

$$= \begin{cases} 0 & , t<0 \\ x(t)+x(t-1) & , t \geq 0 \end{cases}$$

i) Linearity :- $T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(n)] + a_2T[x_2(n)]$

LHS :- $T[a_1x_1(t) + a_2x_2(t)] =$ ~~$a_1x_1(t) + a_2x_2(t) + a_1x(t)$~~.

~~$a_1x_1(t) + a_2x_2(t) + a_1x(t) +$~~

$= a_1x(t) + a_1x(t-1) + a_2x(t) + a_2x(t-1)$

RHS :- $a_1T[x_1(t)] + a_2T[x_2(t)]$

$= a_1[x_1(t) + x(t-1)] + a_2[x(t) + x(t-1)]$

LHS = RHS ; so, linear

ii) Causality :-

$T[x(t)] = y(t) = x(t) + x(t-1)$

At , $n = n_0$

$y(t_0) = x(t_0) + x(t_0 - 1)$ it depends on (present + past) so, its casual

iii) Time - Invariance.

$y(t) = x(t) + x(t-1)$

$T[x(t-t_0)] = y(t-t_0)$

LHS :- $T[x(t-t_0)] = x(n-n_0) + x(n-n_0-1)$

RHS :- $y(n-n_0) = x(n-n_0) + x(n-n_0-1)$

LHS = RHS , Time in-Variant

$T(x(t-t_0)) = y(t-t_0)$

LHS ; $T[x(t-t_0)] = 0, \; t<0$

$= x(t-t_0) + x(t-t_0-1), \; t \geq$

$y(t) = 0, \; t<0$

$x(t) + x(t-1), \; t \geq 0$

RHS :- $y(t-t_0) = 0, \; t<t_0$

$x(t-t_0) + x(t-t_0-1), \; t \geq t_0$

Ranges aren't equal so, time variant

iv) Stability : BIBO stability

$y(n) = x(t) + x(t-1)$ —①

let, ~~x(n)~~ $|x(n)| < M$

$|y(n)| = |x(t) + x(t-1)|$

Both are bounded

so, it's stable

$y(n) \leq |x(t)| + |x(t-1)|$

v) Memory :-

It has memory because, it depends on present + past.