Chapter 4

Stresses and Strains

Simple Stresses and Strains | Strsses in the members of a structure
Normal strain under Axial Loading | Stress-Strain diagram
True stress & True strain | Hooke's Law | Elastic Moduli
Poisson's Ratio | Numerical Problems

AJN

Stresses

- For us, engineers it is **how much force an object experiences**, and **how that force is spread over the object's area**

- Examples can be seen everywhere
  – The bench you are sitting on
  – Motorbikes and cars that you drive
  – Flyovers and bridges that you cross
  – Even the tall apartment building you live

Stresses – Some Examples

- Solid Vertical bar, supporting a weight
  - Each particle in the bar pushes on the particles immediately below it
- Your favourite drink. Coke or Pepsi?
  - Under pressure each particle gets pushed against by all the surrounding particles
  - The container walls bears the pressure of the gas inside as it pushes against the walls

Stresses

Stress analysis is important to:

- Make sure your apartment roof does not come crashing onto your head nor the head of the one below!
- The cars and bikes chassis does not break or crack just because you are a few grams too heavy!
- Chairs that you sit on does not crack under your weight

(It would pain!)

Stresses

- Materials must be strong enough to withstand stress
- If not, fatal, catastrophic and terrible accidents can happen
- Understanding stress and strength of materials is very important for our safety sake

Stress

Whenever a body is subjected to a force or a load, the average force some particles of the body exerts on adjacent particle separated by an imaginary surface is called Stress

- **Stress = Force per** unit **area**

$$\boxed{\sigma = \frac{P}{A}}$$

Generally denoted by **σ**, even though other symbols too are used.

- – **P** is the *force* or *load* applied (in Newtons **N**)
- – **A** is the "*cross-sectional*" area of the object

Stress

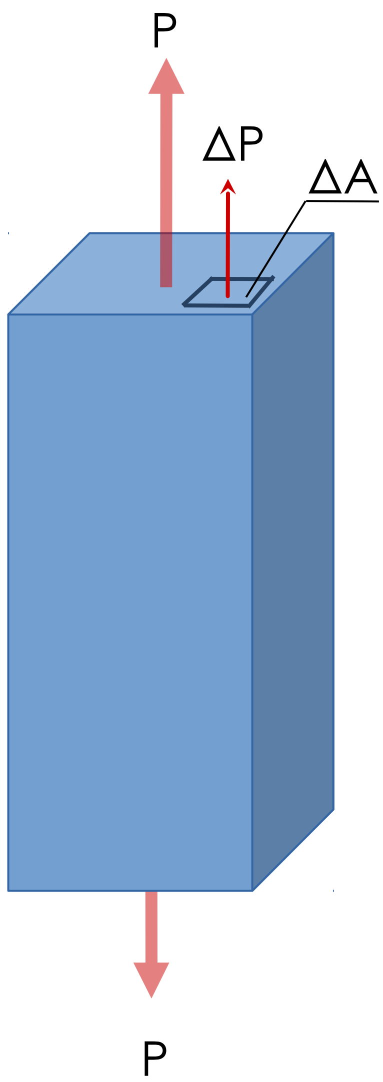
- When stress varies over the cross-section, we can write the stress at a point as

$$\sigma = \frac{\Delta P}{\Delta A}$$

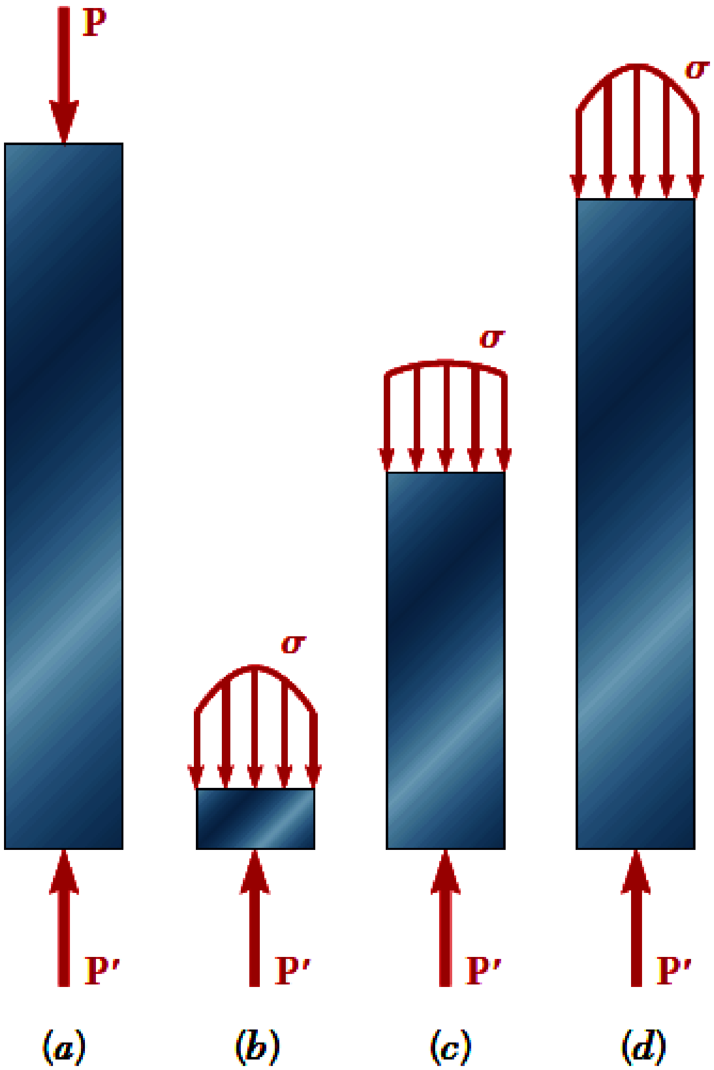
- We assume the force **P** is evenly distributed over the cross-section of the bar. In reality,

P= resultant force over the end of the bar

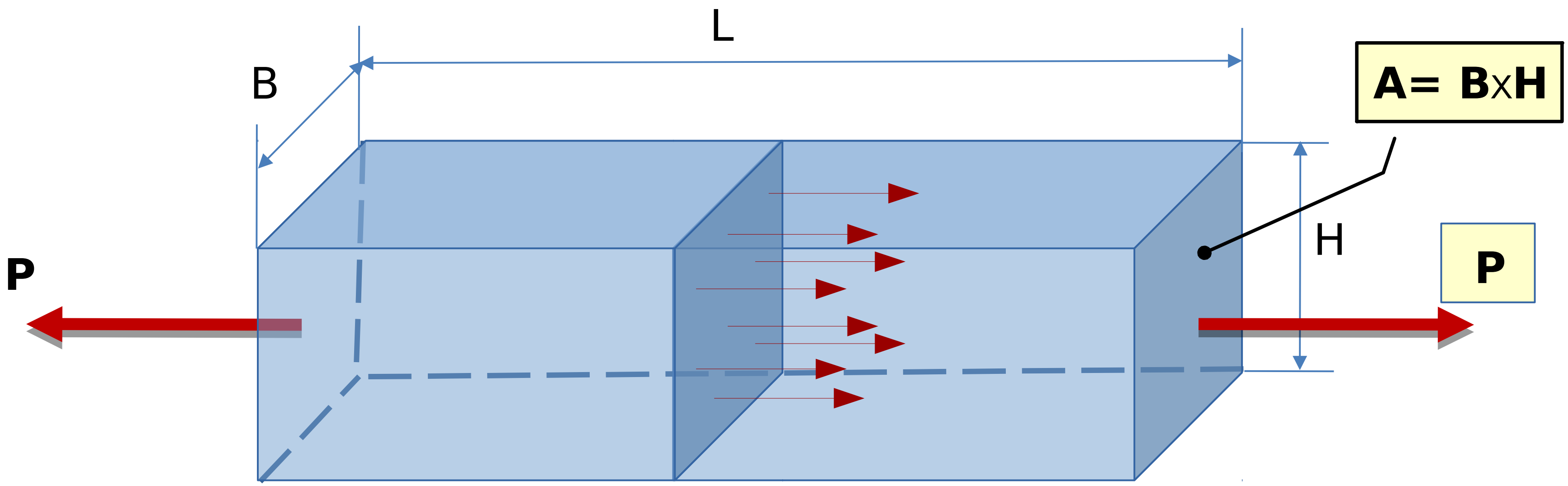
$$\int_A \sigma dA = P$$



Stress



A simple stress



$$\text{Stress, } \sigma = \frac{P}{A}$$

Tensile Stress

A simple stress…

Assuming, Breadth = Height = 1m

Load or Force, **P = 50 kN** Cross sectional Area, **A = 1 m²**

$$\text{Stress, } \sigma = \frac{P}{A}$$

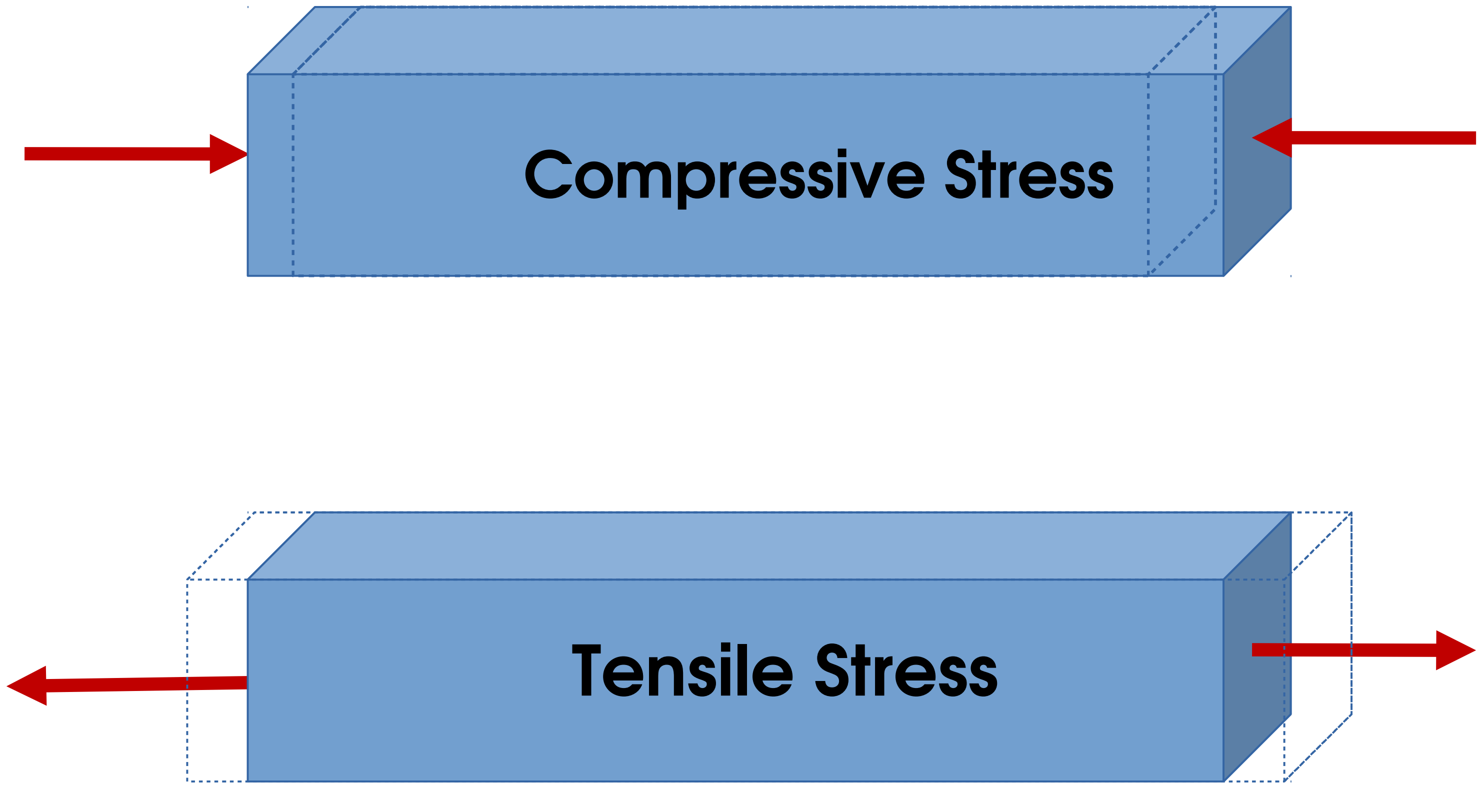
$$\text{Stress, } \sigma = \frac{P}{A} = \frac{50\ kN}{1\ m^2} = 50\,\frac{kN}{m^2} = 50{,}000\,\frac{N}{m^2}$$

Stresses

- Stress is dependent on the load, the cross sectional area and the object/material type
- Computing stress, works for all sorts objects experiencing all sorts of different forces
  - For example, if a rope breaks when a certain force is applied, using a bigger diameter rope will reduce the stress
- Stress on objects can be either

Compressive **Tensile** **Shear**

Simple stress…



Compressive & Tensile Stresses

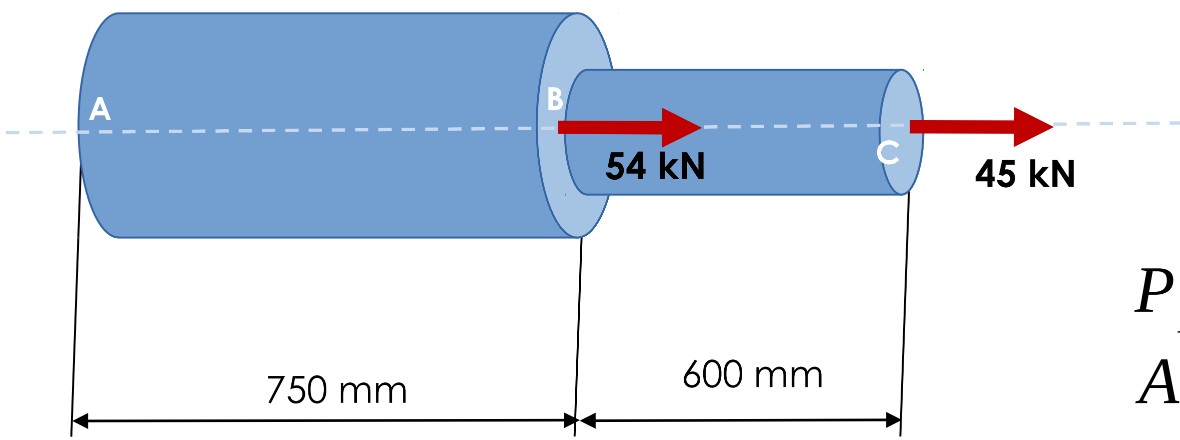
- Created by forces spread over an area
- Usually axial in nature; i.e acts along the axis of the object
- Tensile stress occurs when something is being pulled apart
  - Think of "tug-o-war" competition. The rope experiences <u>tensile stress</u>
- Compression occurs when something is being squeezed together
  - As you sit on a chair, its legs experiences <u>compressive stresses</u> as they are being *squeezed between you and the floor*

Remember!

- **+ve σ** indicates **Tensile stress**
- **-ve σ** indicates **Compressive stress**
  - Load or Force (**P**) is expressed in Newtons (**N**)
  - Area (**A**) is square meters (**m^2**)
  - Stress (**σ**) will be expressed in (**N/m^2**)
    - Unit for stress is also called as Pascal (**Pa**).
    - But **Pa** is an extremely small value, (10^5 Pa =1atm) we generally use kilopascal (**kPa**), megapascal (**MPa**) or gigapascal (**GPa**)

1 Kilo Pascal (kPa)	10^3 Pa	10^3 N/m^2
1 Mega pascal (MPa)	10^6 Pa	10^6 N/m^2
1 Giga Pascal (Gpa)	10^9 Pa	10^9 N/m^2

1 **Two solid cylinders are welded together at B. Knowing the diameters of AB (50mm) and BC (30 mm), find the normal stresses at the mid point of each rod respectively for the loading shown.**



We know,

$$\text{Stress, } \sigma = \frac{P}{A}$$

$$P_{AB} = \qquad P_{BC} =$$

$$A_{AB} = \qquad A_{BC} =$$

We know that in an axially loaded segment, the distribution of stresses is uniform. Hence, at the mid section, the stress measured **(σ)** would be the average stress **(σ_{avg})** and can be obtained using the above formula

$$\sigma_{AB_{mid}} = \qquad \sigma_{BC_{mid}} =$$

Answering…

$$A_{AB}=\frac{\pi}{4}(D_{AB})^2=\frac{\pi}{4}(0.050)^2=0.001963\ m^2$$

$$A_{BC}=\frac{\pi}{4}(D_{BC})^2=\frac{\pi}{4}(0.030)^2=0.000706\ m^2$$

$$\sigma_{BC}=\frac{P_{BC}}{A_{BC}}=\frac{45\times10^3}{0.000706}=63{,}739{,}376.77\ \frac{N}{m^2}=63.74\times10^6\frac{N}{m^2}=63.74\ \text{MPa}$$

$$\sigma_{AB}=\frac{(P_{AB}+P_{BC})}{A_{AB}}=\frac{(54+45)\times10^3}{0.001963}=\frac{99\times10^3}{0.001963}$$

$$=50{,}433{,}010.7\ \frac{N}{m^2}=\ 50.43\ \text{MPa}$$

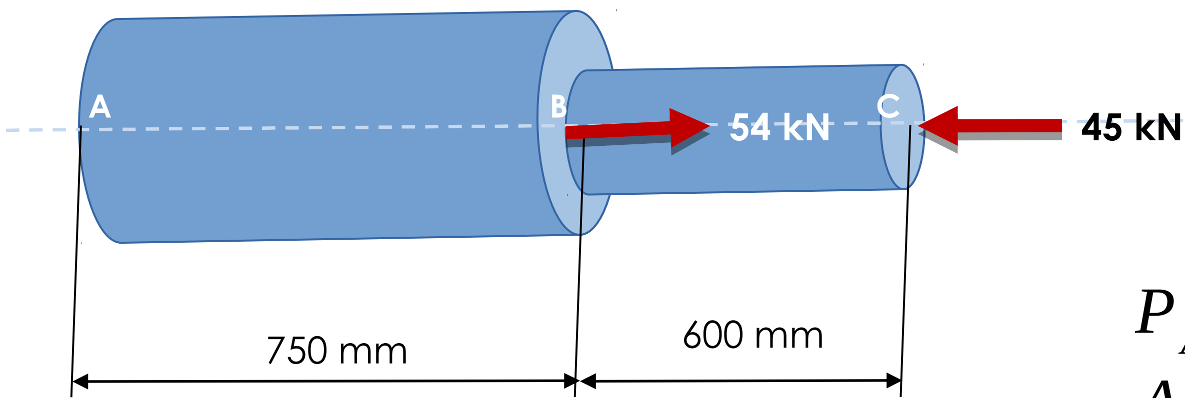
Try to keep the units of the linear dimension in meters (m) and that of the load or force in Newton (N).

This will help you get the answer in (N/m²) which is equivalent to Pascal (Pa).

Knowing this, you can convert it into kPa or MPa or GPa

The two loads are added (P_{AB}+P_{BC}), because at the weld junction, along with the load of 54 kN, the load effect of 45 kN is already present

2 **The same cylinders are taken again. But this time, the direction of one load is reversed. With this in mind, find the stresses at the mid sections of both the rods. Name the stresses and comment about the difference observed.**



We know,

$$\text{Stress, } \sigma = \frac{P}{A}$$

$P_{AB} =$ $\quad$ $P_{BC} =$

$A_{AB} =$ $\quad$ $A_{BC} =$

$\sigma_{AB_{mid}} =$ $\quad$ $\sigma_{BC_{mid}} =$

Answering…

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$$A_{BC}=\frac{\pi}{4}(D_{BC})^2=\frac{\pi}{4}(0.030)^2=0.000706\ m^2$$

In this problem, since the directions the forces act are opposite to each other, it is the difference of the forces that causes the stress at any section throughout the solid.

$$\sigma_{BC}=\frac{(P_{BC})}{(A_{BC})} = \frac{(-45)\times 10^3}{(0.000706)} = -63{,}739{,}376.77\ \frac{N}{m^2} = \boxed{-63.74\ \text{MPa}}$$

The negative symbol represents Compressive stress

$$\sigma_{AB}=\frac{(P_{AB}-P_{BC})}{(A_{AB})} = \frac{(54-45)\times 10^3}{(0.001963)} = 4{,}584{,}819.15\ \frac{N}{m^2} = 4.58\ MPa$$

$\sigma_{BC} = Compressive\ Stress$ $\sigma_{AB} = Tensile\ Stress$

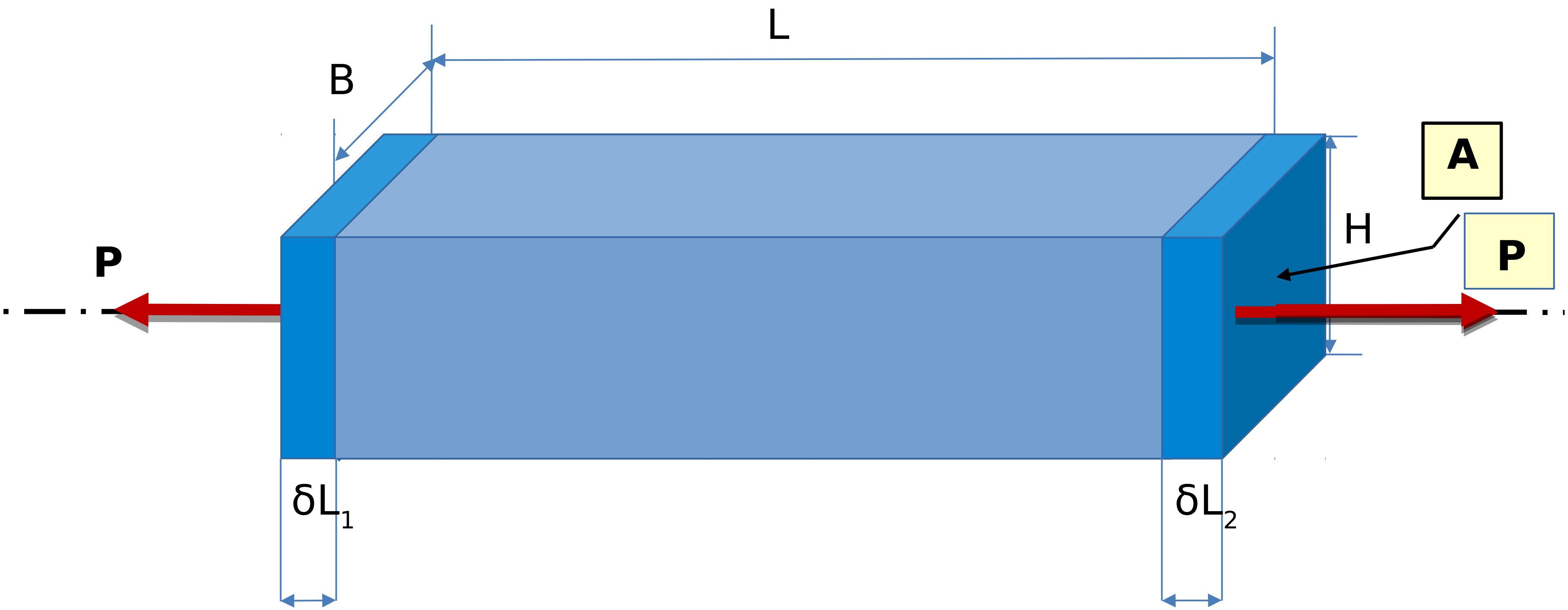
Strain

Remember: Strain is dimensionless!!

- **Strain** is a measurement of how an object reacts to stress.
- That is, the ***percentage change in an object's shape*** **when** a certain **stress is applied** to it.
- Think of how a rubber band elongates with a weight hung on it
- Change of length due to load applied
- ***Strain (ε)*** is defined as "***deformation per unit length under axial loading***"

$$\text{Strain, } \varepsilon = \frac{\text{Change in Length}}{\text{Original Length}} = \frac{\Delta L}{L} \; OR = \frac{\delta L}{L}$$

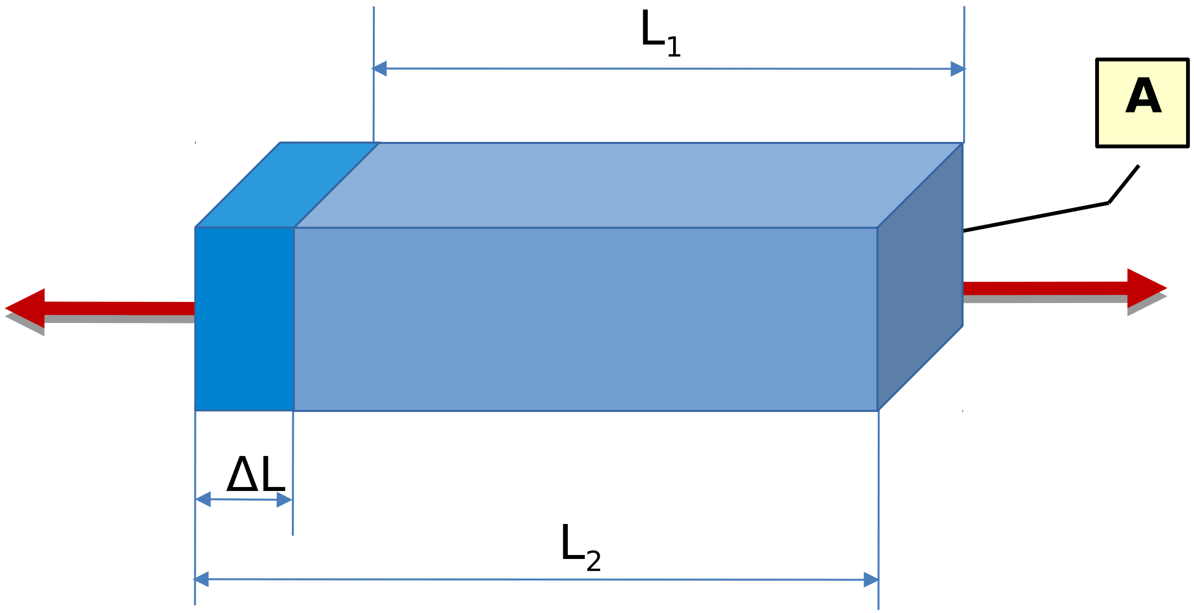
Simple strain



The small changes in length are, δL_1 and δL_2

$$\delta L_1 + \delta L_2 = \delta L$$

Strain



$$\text{Strain}, = \frac{\text{Change in Length}}{\text{Original Length}}$$

$$\boxed{\text{Strain}, \ \varepsilon = \frac{L_2 - L_1}{L_1} = \frac{\delta L}{L}}$$

Strain

Normal strain is dimensionless, but can be expressed in several ways

Let's say L = 100 mm and δ = 0.01 mm

$$\text{Strain, } \varepsilon = \frac{(L_2 - L_1)}{(L_1)} = \frac{(100.01 - 100)}{(100)} = \frac{(0.01)}{(100)} = 1 \times 10^{-4} = 100 \times 10^{-6}$$

Since 10^{-6} m is called 1 micrometre and which is represeneted by μ

$$\varepsilon = 100 \ \mu \quad (\textit{read as } 100 \textit{ micro strain})$$

3 **A 12 m long steel rod of 5 cm diametre, is bolted onto a wall at one end, while the other is pulled continuously using a mechanical force of around 100 kN. What is the stres and strain experienced by the rod if after 5 minutes of load application, the length of the rod becomes 12.35 m?**

We know, $\text{Stress, } \sigma = \dfrac{P}{A}$ & $\text{Strain, } \varepsilon = \dfrac{L_2 - L_1}{L_1} = \dfrac{\Delta L}{L}$

$$P = 100{,}000\ N \qquad L_1 = 12\ m \qquad L_2 = 12.35\ m$$

$$A = 0.001963\ m^2 \qquad \sigma = 50.96 \times 10^6 \frac{N}{m^2}$$

$$\text{Strain, } \varepsilon = \frac{L_2 - L_1}{L_1} = \frac{\Delta L}{L} = \frac{(12.35 - 12)}{12} = 0.02916 \approx 0.03$$

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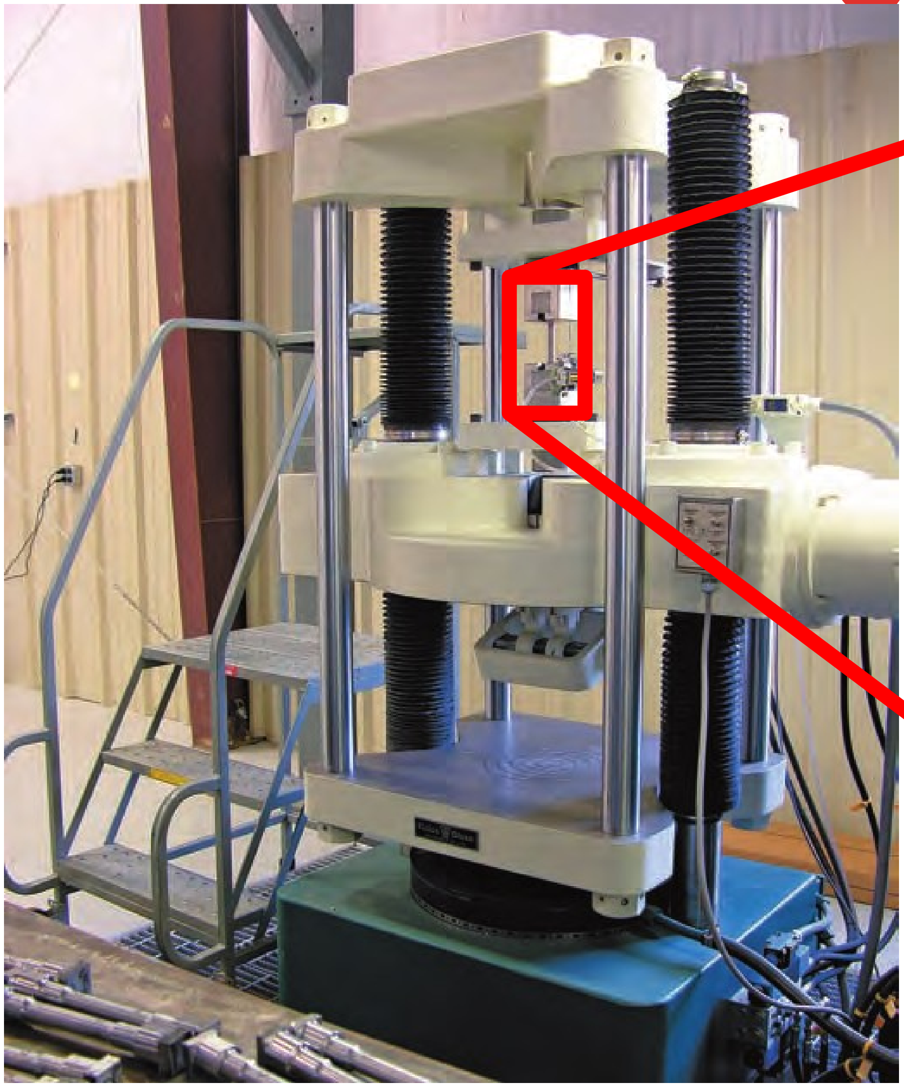
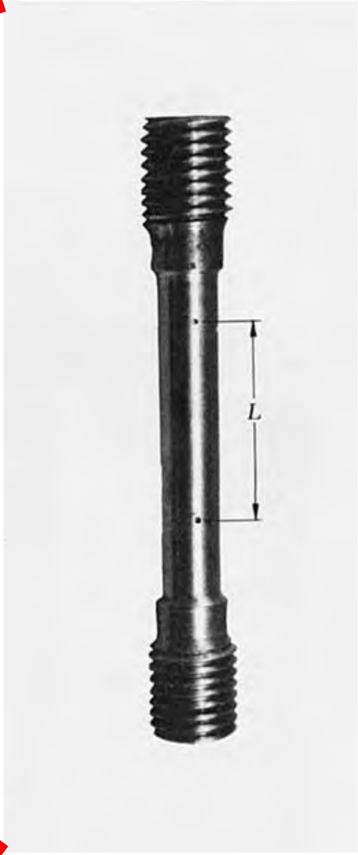
Stress-Strain Diagram

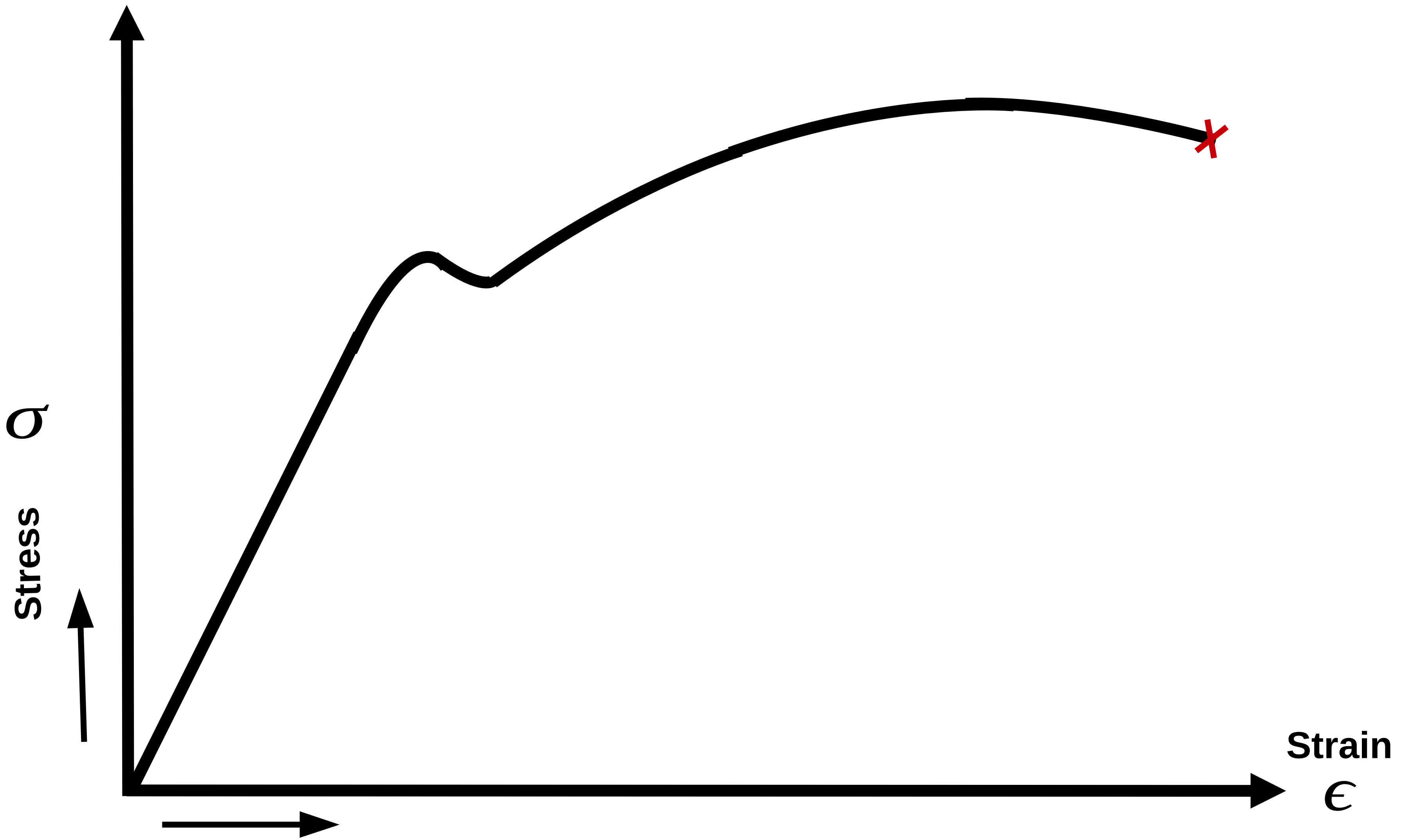
- Relationship between the stress and strain of any particular material is obtained over its stress–strain curve
- Unique for each material
- Found by recording amount of deformation (strain) at distinct intervals of tensile or compressive loading (stress)
- Reveal many material properties

Stress-Strain Diagram





Stress-Strain Diagram



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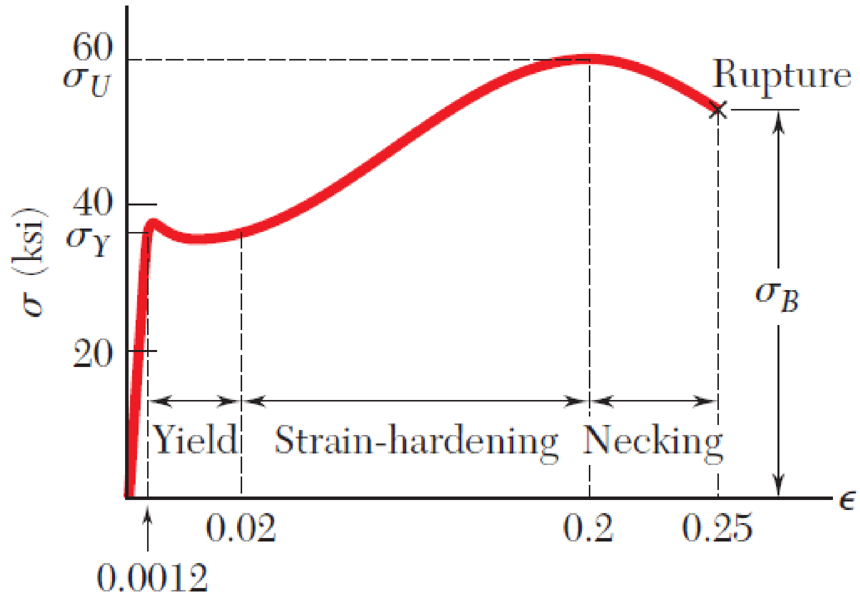
Stress-Strain Diagram

- Varies widely for various materials
- Test results usually depends on the temperature and the speed of loading
- Common characteristics helps divide materials into two broad categories
  - Ductile materials
  - Brittle materials

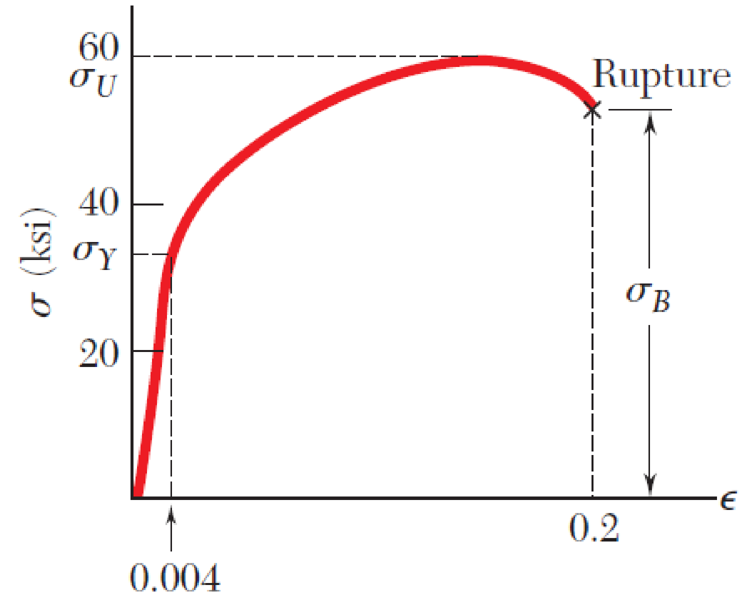
Ductile Vs Brittle

- **Ductility is** a solid material's **ability to deform under <u>tensile</u> stress**. Characterized by the material's ***ability to yield*** at normal temperatures.
  - eg. gold, copper, titanium, wrought iron, low carbon steels and brass
- **Brittleness** is the character of materials, when subjected to stress, it **breaks without significant deformation**
  - eg. Cast iron, concrete, high carbon steels, ceramics, and some polymers

Ductile

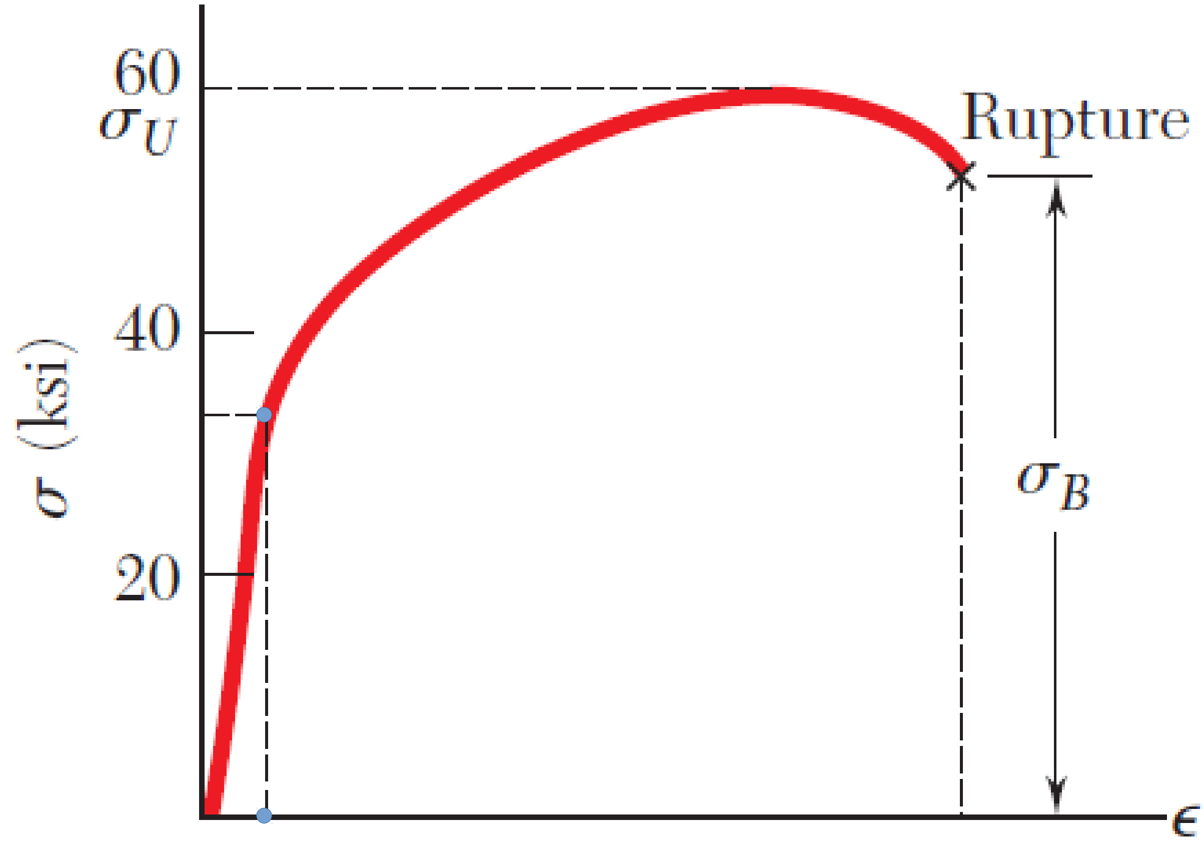


(a) Low-carbon steel



(b) Aluminum alloy

Ductile

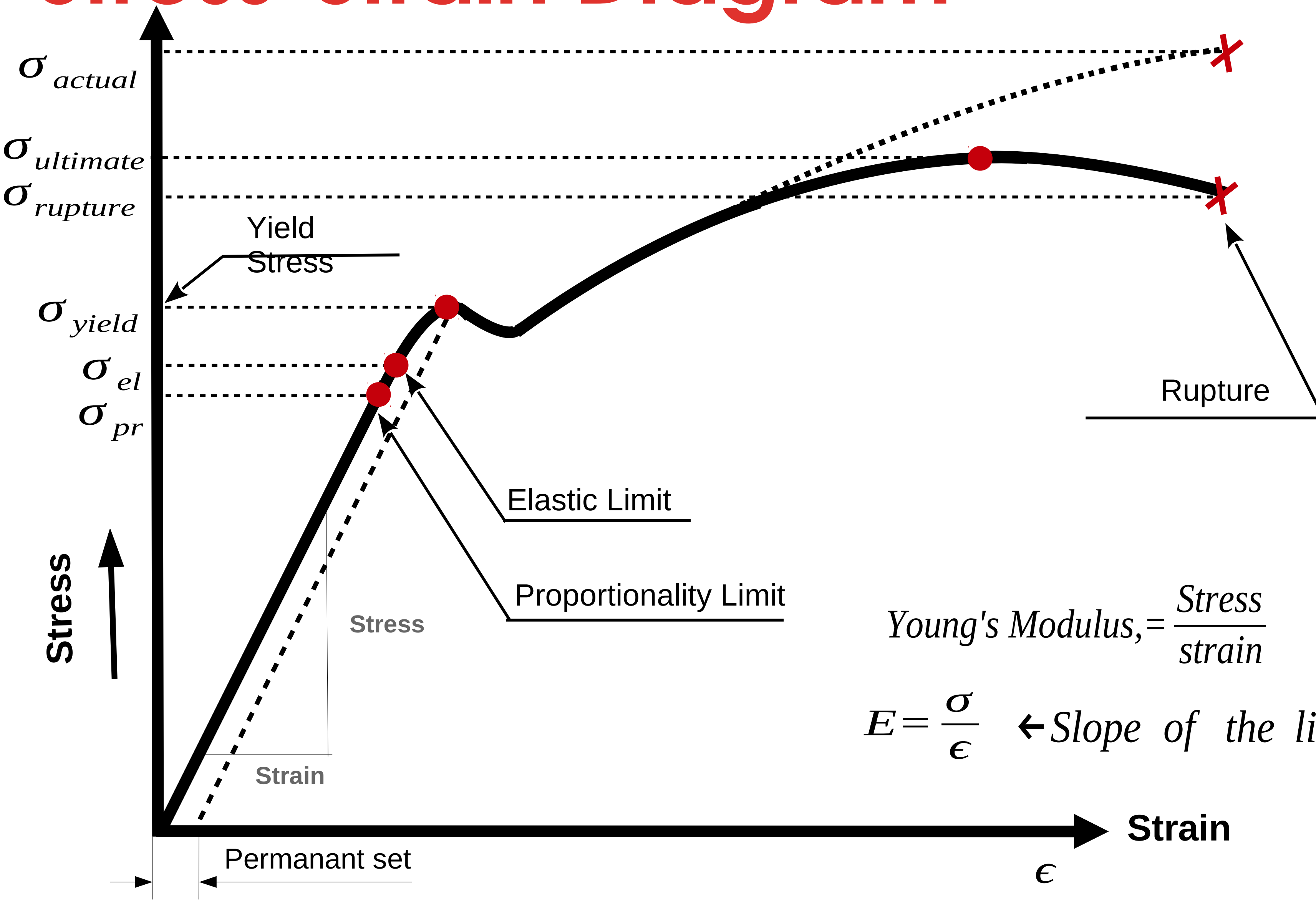


Yield point strain = Usually 2% of the total strain

Ductile

- Ductile materials, are characterized by their ability to yield at normal temperatures
- Subjected to an increasing load, length increases linearly with the load very slowly
- Initial part of Stress-strain diagram is a straight line with steep slope
- After critical value $\boldsymbol{\sigma}_y$ has been reached, specimen undergoes large deformation with small increase in load
- Deformation caused by slippage of material along oblique surfaces (due to shearing stresses)

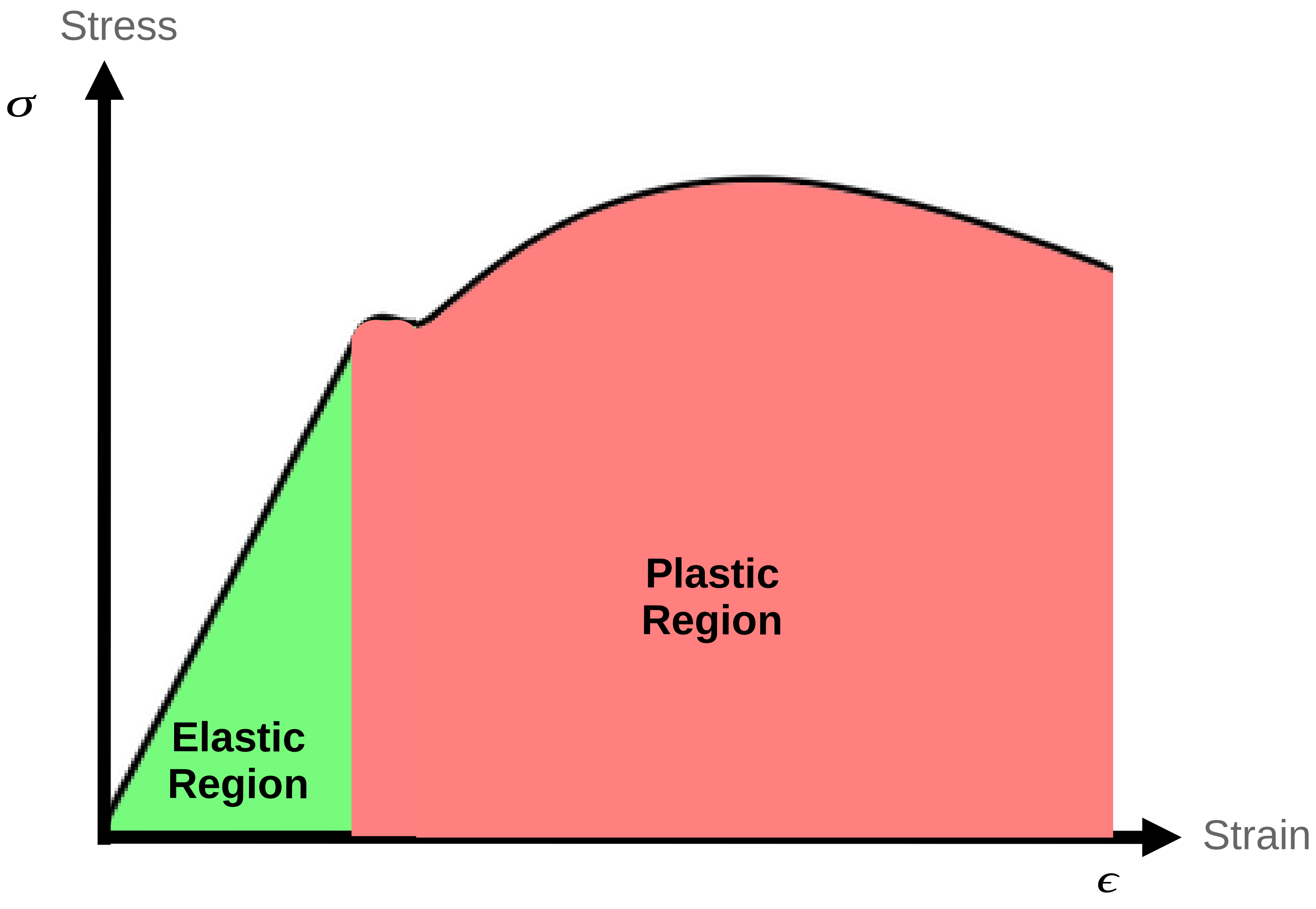
Stress-Strain Diagram



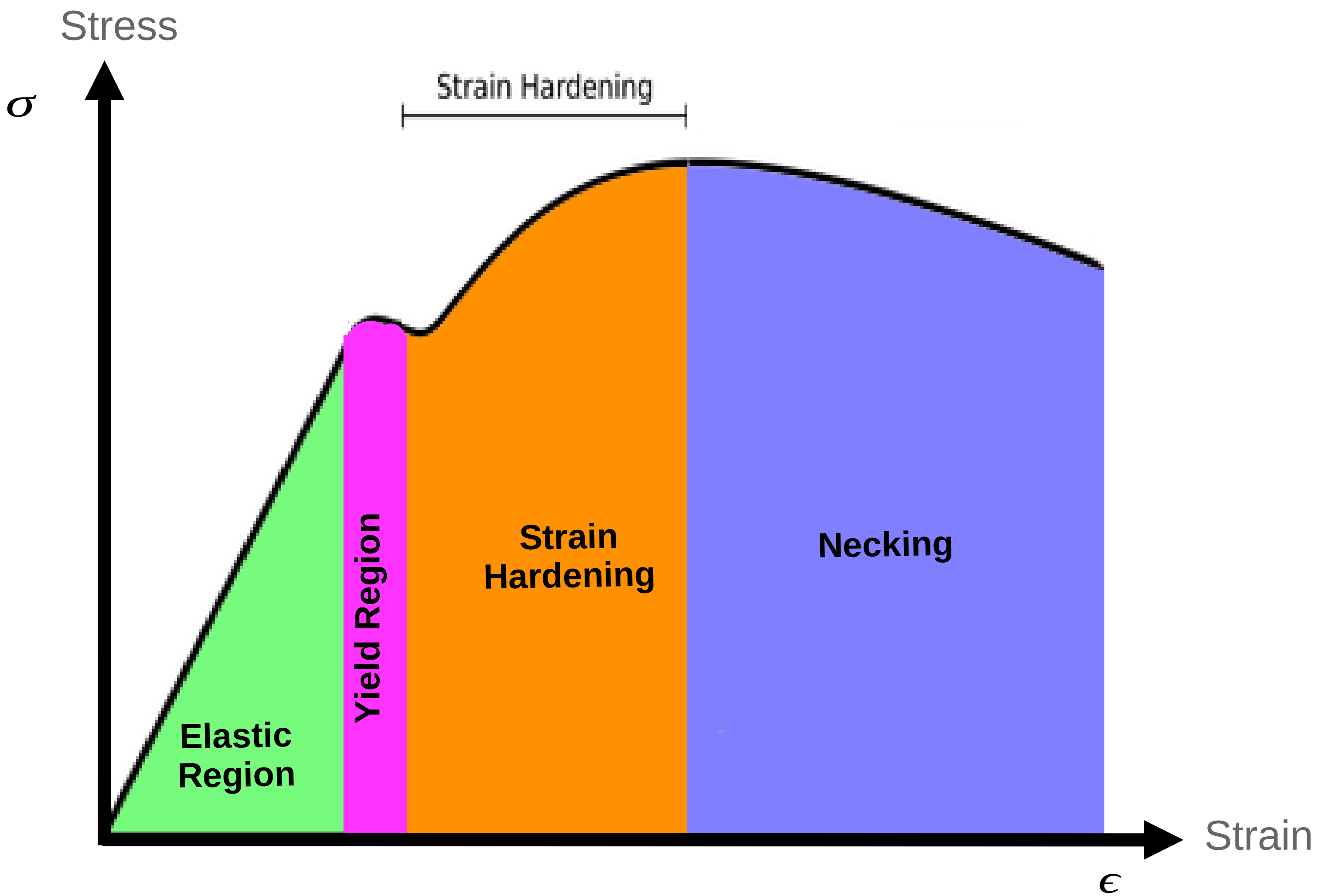
$$\text{Young's Modulus,} = \frac{\text{Stress}}{\text{strain}}$$

$$E = \frac{\sigma}{\epsilon} \leftarrow \text{Slope of the line}$$

Stress-Strain Diagram



Stress-Strain Diagram



Stress-Strain Diagram

- Proportional Limit
- Elastic Limit
- Yield Point
- Ultimate Stress Point
- Breaking/ Rupture Point
- Actual Stress

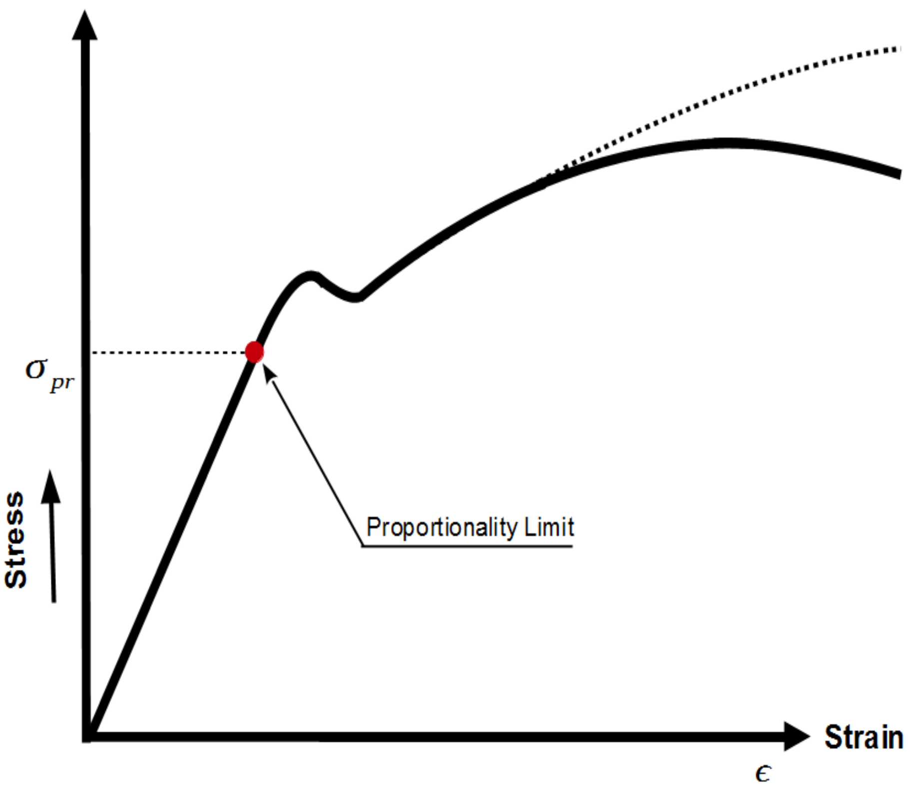
Stress-Strain Diagram

Proportional Limit: Proportional limit, marks the end of elastic behaviour. **Hooke's Law is violated after this point.**

That in which the stress is proportional to the strain or equivalently that in which the load is proportional to the displacement.

Can be called the **proportional limit** point or **limit of proportionality**

Stress up to this point is known as proportional limit stress



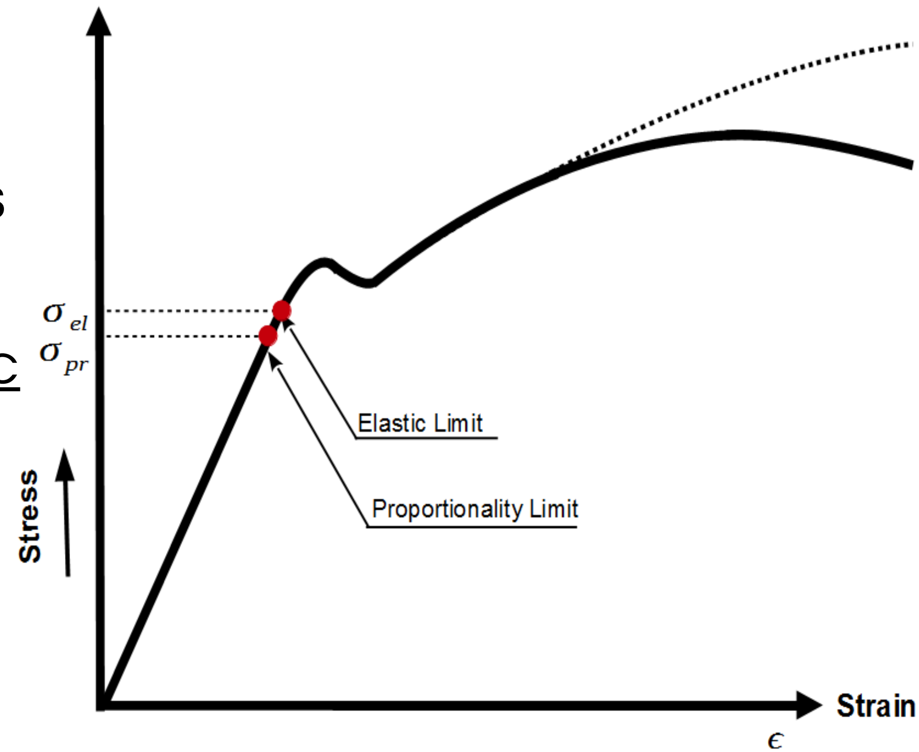
Stress-Strain Diagram

Elastic Limit: The maximum stress or force per unit area within a solid material that can arise before the onset of permanent deformation.

In this diagram, 2nd point gives the elastic limit.

When stresses up to the elastic limit are removed, the material resumes its original size and shape i.e, **no permanent deformation**

Hooke's law may not be satisfied between $\boldsymbol{\sigma}_{Pr}$ & $\boldsymbol{\sigma}_{el}$

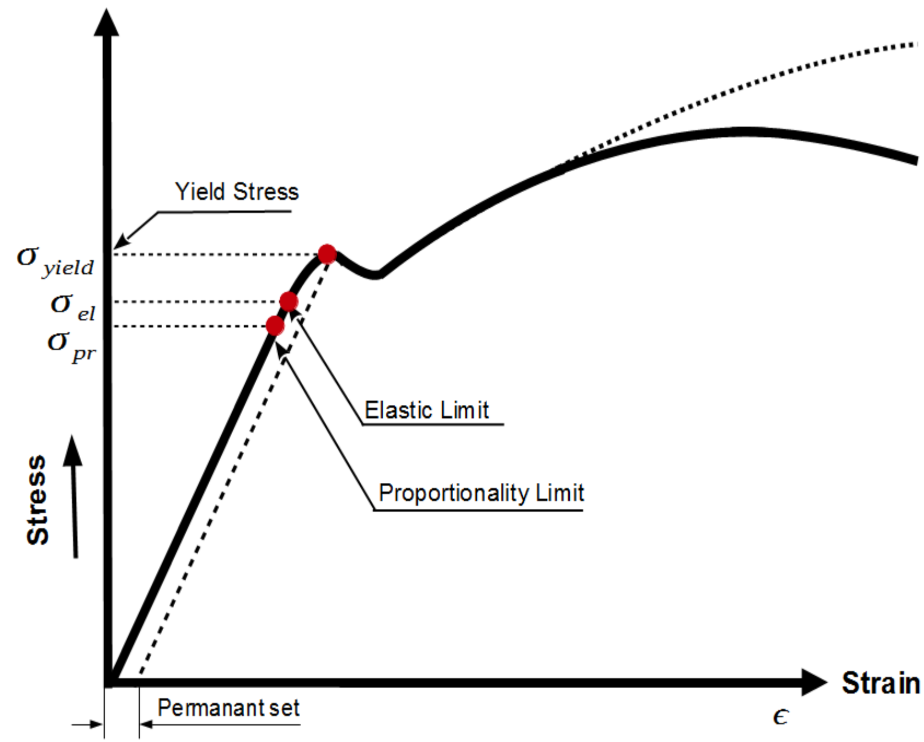


Stress-Strain Diagram

Yield Stress: A yield strength or yield point of a material is defined **as the stress at which a material begins to deform plastically**.

Prior to the yield point the material will deform elastically and returns to its original shape when stress is removed.

Once the yield point is passed, some fraction of the deformation will be permanent and non-reversible even when load is removed completely

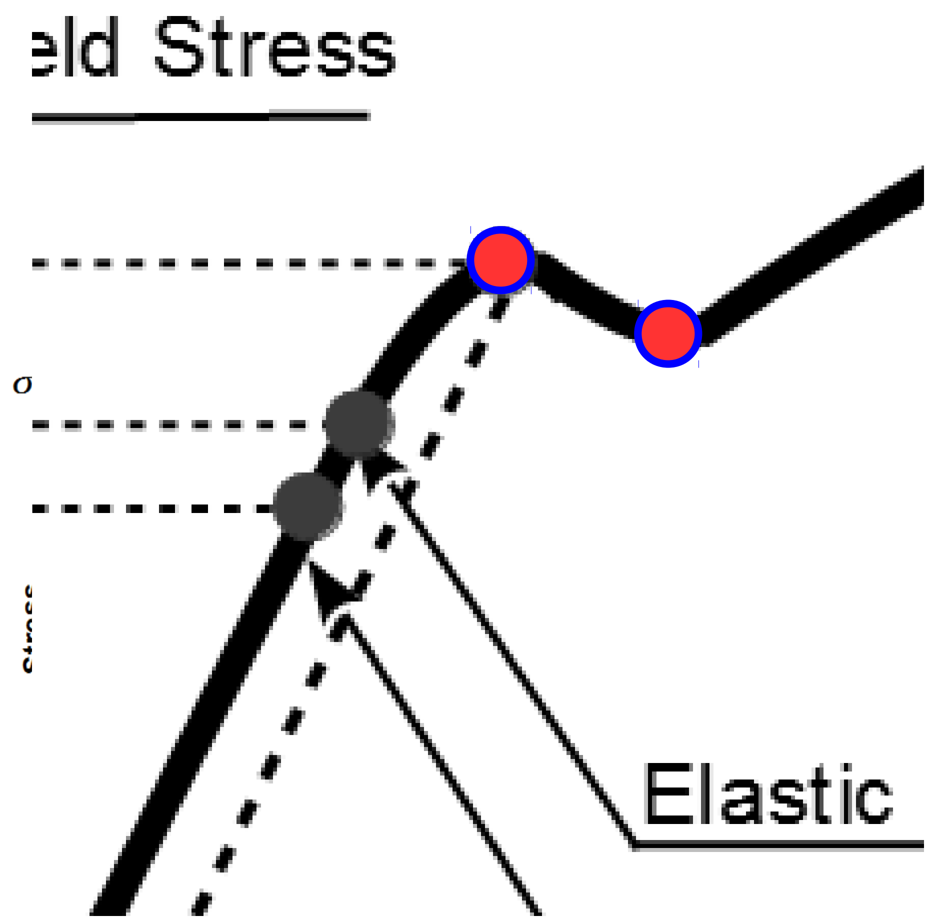


Stress-Strain Diagram

Elongation of the specimen after it has started to yield can be as high as 200 times its deformation before yield

The upper yield point, corresponds to the load reached just before yielding starts,

Lower yield point, which corresponds to the load required to maintain yield

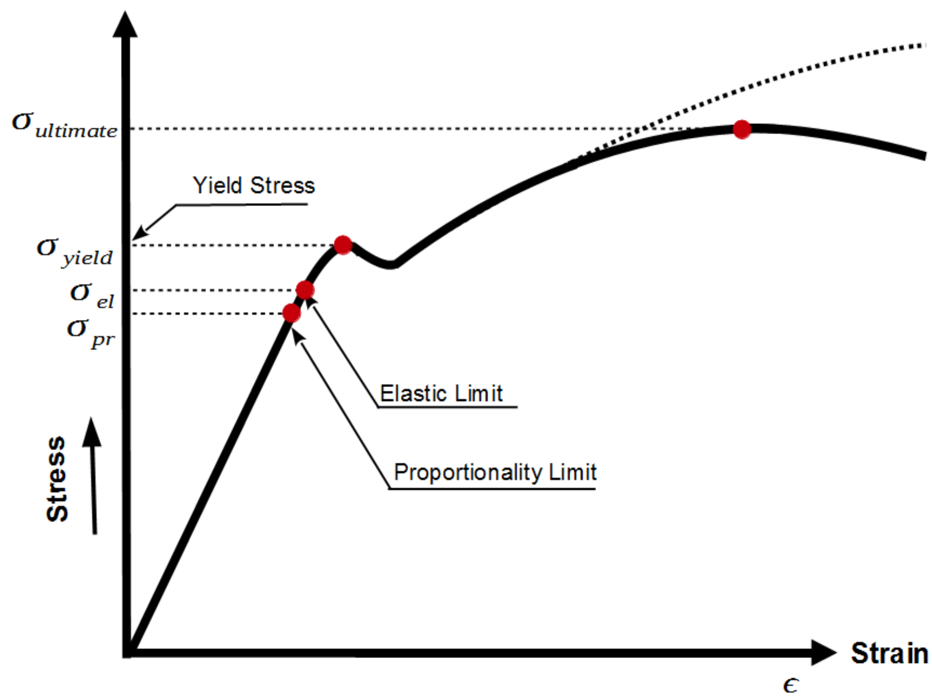


Stress-Strain Diagram

Ultimate Stress: Ultimate stress point is **the maximum strength that material have to bear stress before breaking**.

It can also be defined as the ultimate stress corresponding to the peak point on the stress strain graph.

On the graph the top most point is the ultimate stress point. After point U material have very minute or zero strength to face further stress.



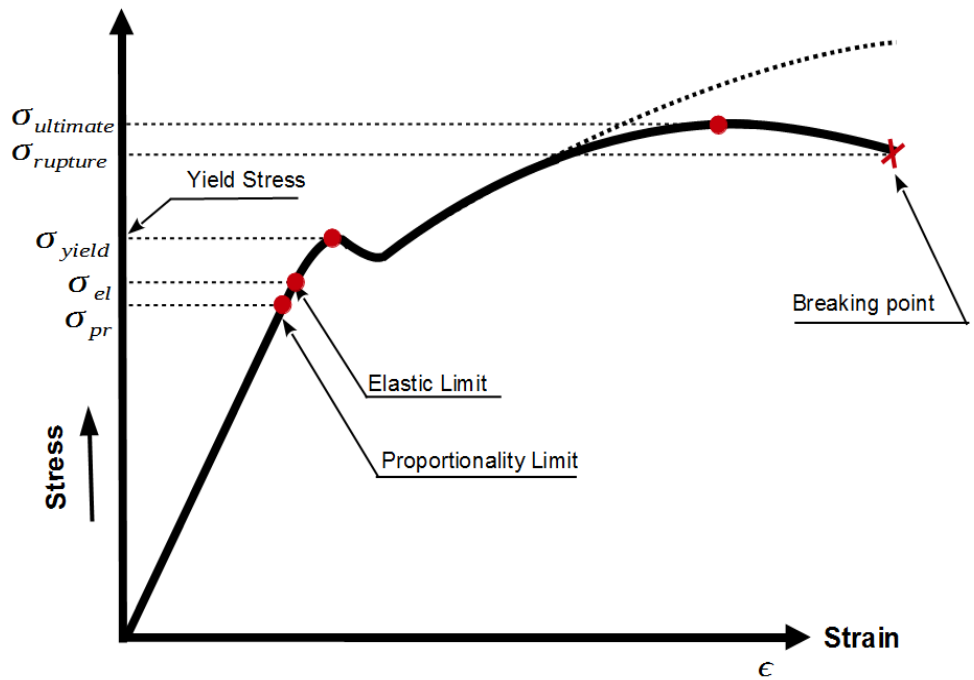
Stress-Strain Diagram

Breaking Stress: The **stress at which the material breaks.**

The stress associated with this point known as breaking strength or rupture strength.

On the stress strain curve, the end point marked ✗ is the breaking stress point.

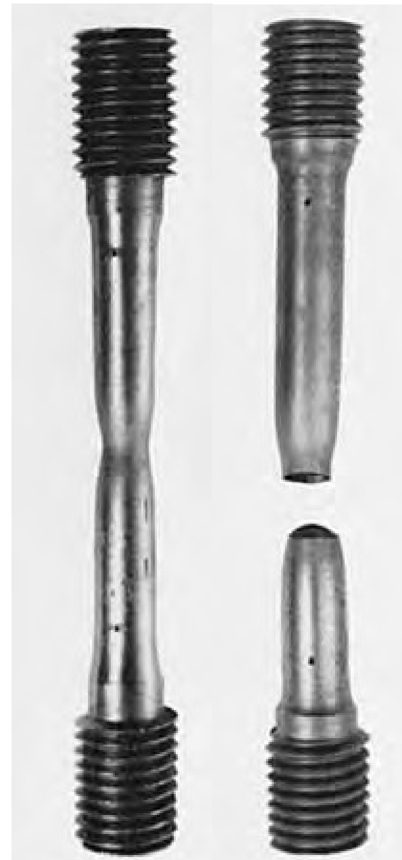
It is usually less than or equal to the ultimate stress.



Stress-Strain Diagram

Necking

- Starts after a maximum value of the load that relates to **Ultimate stress** has been reached
- Diameter of a portion of the specimen begins to decrease, because of local instability
- This phenomenon is known as **necking**
- After necking lower loads are sufficient to keep specimen elongating further, until rupture



Ductile material

Brittle material

True Stress & True Strain

- For plotting stress strain diagrams ***original cross-sectional area*** of the specimen is accurately determined as ***A_0***
- Stress is obtained by dividing load ***P*** by **A_0**
- It is the cross sectional area measured before any deformation
- Stress plotted in earlier diagrams represent engineering stress

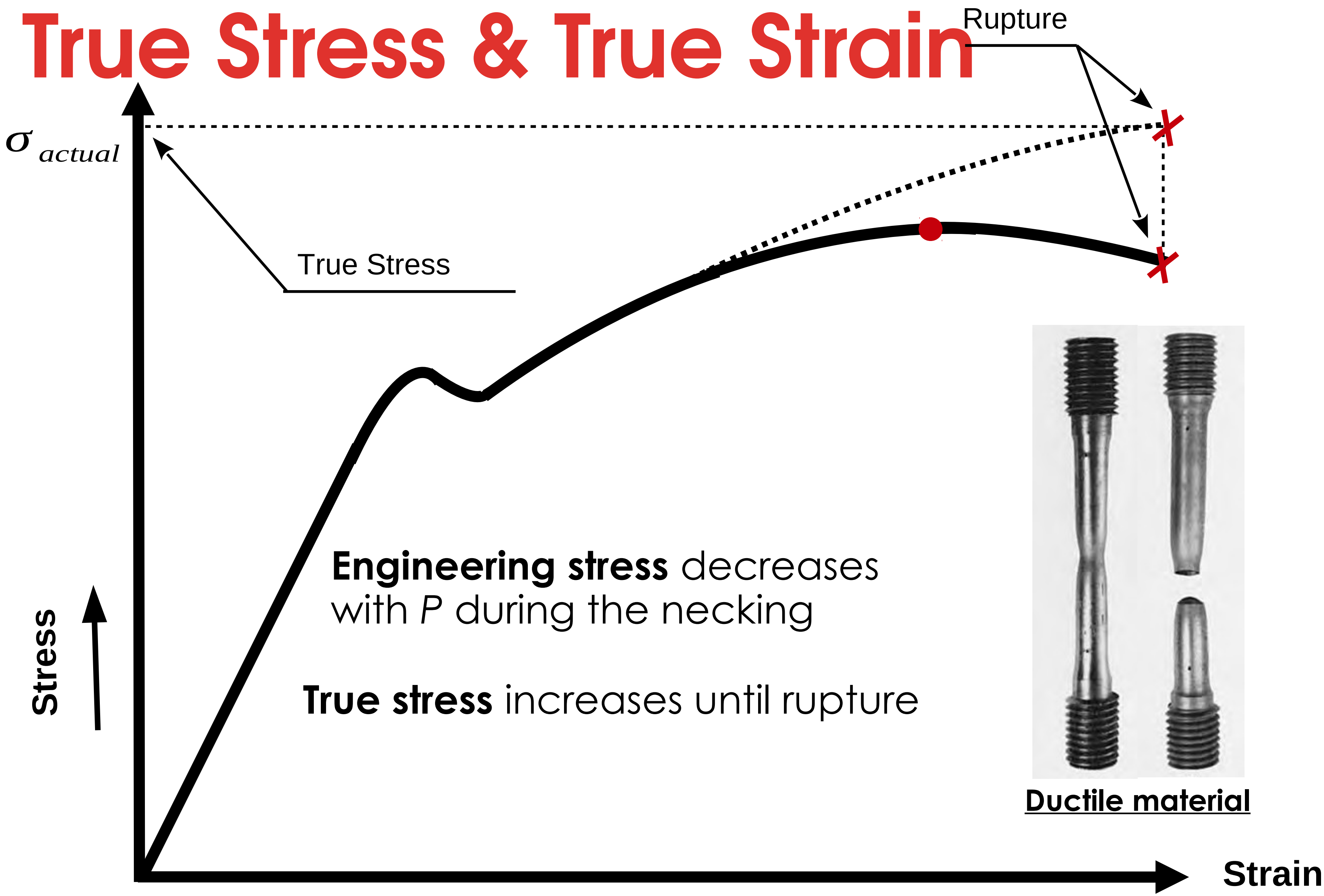
Engineering stress, $\boxed{\sigma = \dfrac{P}{A_0}}$

- But **cross sectional area of specimen decreases as *P* increases**
- True stress got by dividing ***P*** by the **instantaneous c.s area *A*** of deformed specimen

True stress, $\boxed{\sigma_T = \dfrac{P}{A}}$

- While **engineering stress** $(\sigma \propto P)$, **decreases** with *P* during the necking **true stress** $\sigma \propto P$ and $\sigma \propto (\dfrac{1}{A})$, increases until rupture

True Stress & True Strain



Engineering stress decreases with *P* during the necking

True stress increases until rupture

True Stress & True Strain

- The diagram obtained by plotting true stress versus true strain **reflects more accurately the behavior of the material**
- Our responsibility: To determine whether the load **P** produces an acceptable stress and an acceptable deformation
- Diagram based on engineering stress & strain is used
- Involves data namely the **original cross-sectional area** and length of the undeformed member
- Design calculations cannot be based on Instantaneous rate of deformation or change in dimension

True Stress & True Strain

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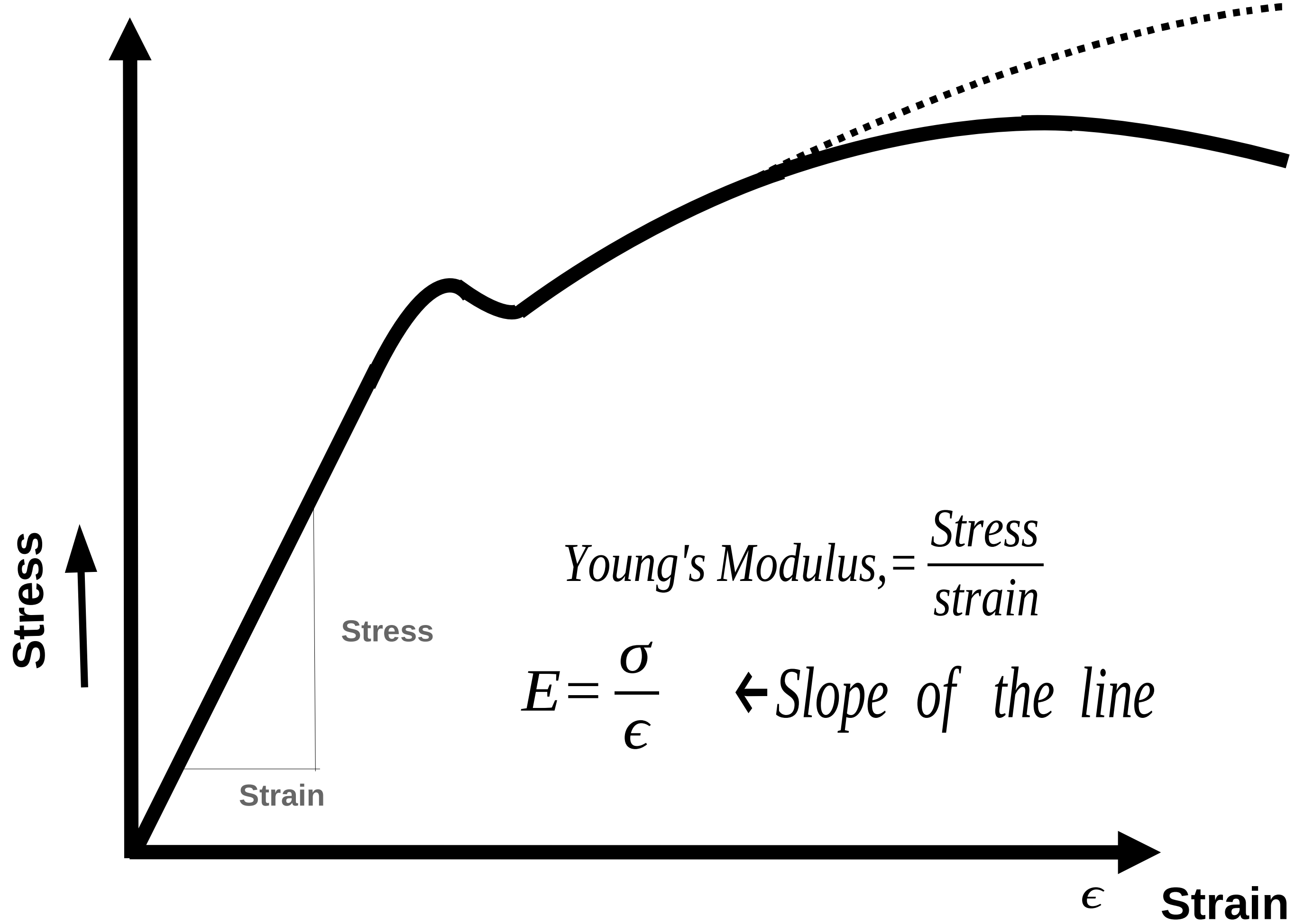
Hooke's Law



$$Young's\ Modulus, = \frac{Stress}{strain}$$

$$E = \frac{\sigma}{\epsilon} \quad \leftarrow Slope\ of\ the\ line$$

Hooke's Law

- Structures we design are expected to undergo only moderate deformations
- Preferably within the straight line portion of the stress strain diagram i.e within the elastic limit
- Robert Hooke (1635–1703), an English scientist brought about a an observation that

Within elastic limits,
Stress is directly proportional to Strain

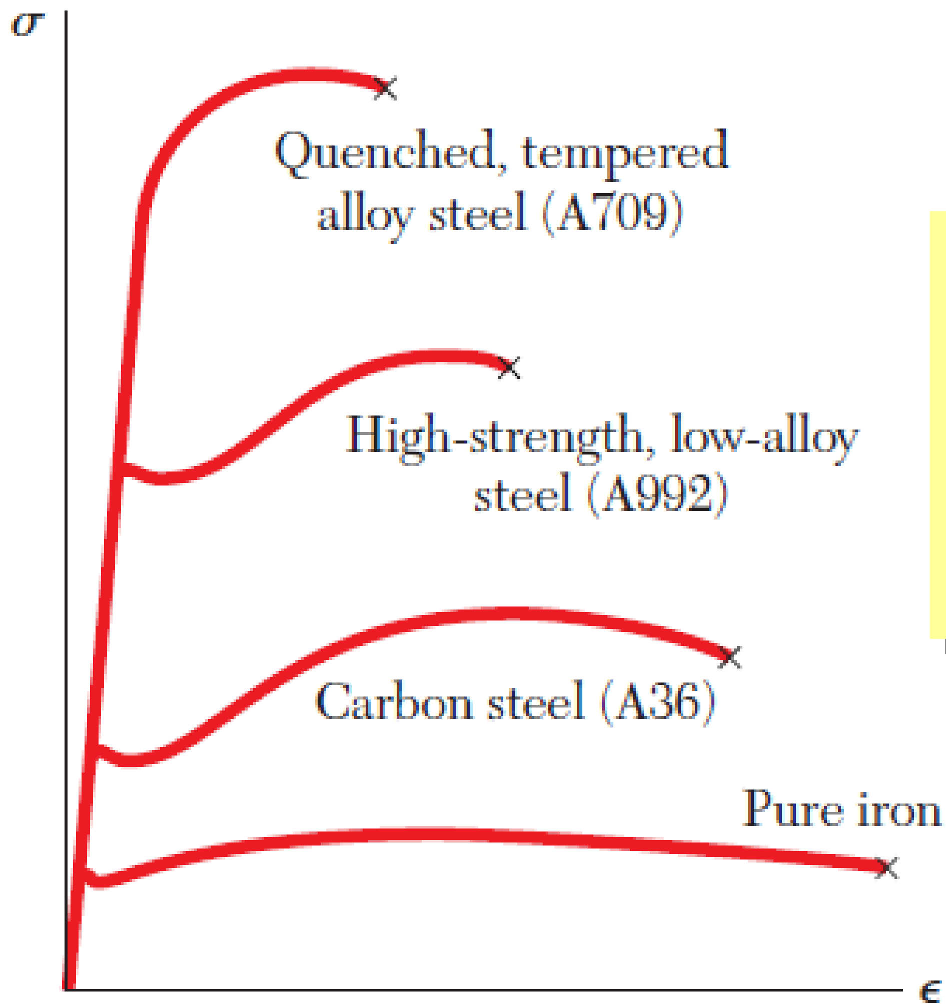
$$\sigma \propto \varepsilon$$

$$\boxed{\sigma = E\,\varepsilon}$$

Elastic Modulus $\sigma \propto \varepsilon \quad \Rightarrow \quad \sigma = E\varepsilon$

- The coefficient **E** is called the **modulus of elasticity** or also **Young's modulus**
- Hence **modulus *E*** is expressed in the same units as the stress, namely in **Pa** or one of its SI multiples
- Largest value of stress for which Hooke's law can be used for material is known as ***proportional limit***
- For ductile materials *having well-defined yield point*, proportional limit almost coincides with the yield point
- Properties of metals, such as strength, ductility etc can be changed using heat treatment; but **Young's Modulus, E** remains same
- Independent of the direction of loading

Elastic Moduli



Remember,

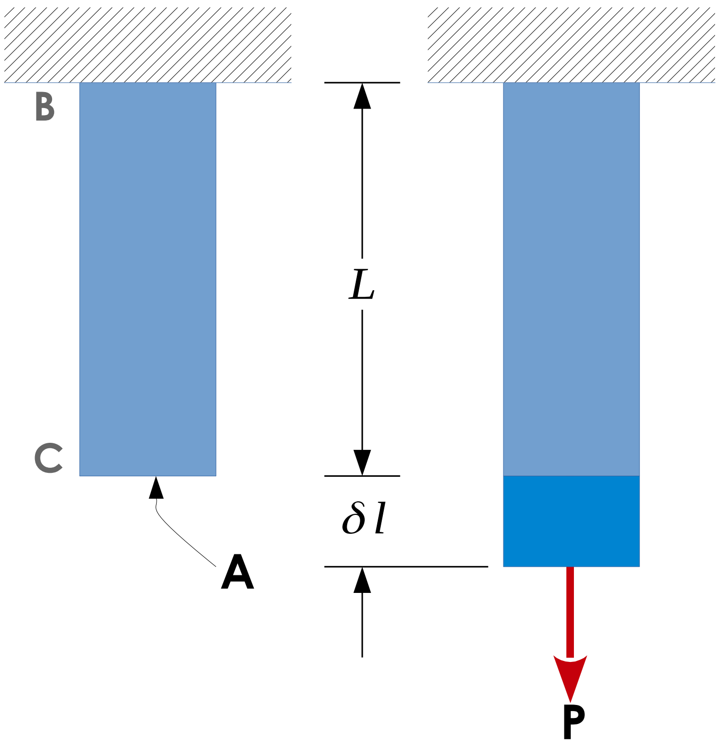
E=Stress/ Strain, is the slope of the line. Here, it can be seen it is constant for all the steels

Factor of Safety (FoS)

- **Maximum load**, a structural member is **allowed to carry is smaller** than ultimate load
- This smaller load is referred to as **allowable load**, working load or design load
- **Only a fraction of ultimate-load** capacity is utilized when the allowable load **is applied**
- Remaining, kept in reserve to assure safety of performance
- The ratio of **ultimate load** to **allowable load** is used to define the factor of safety

$$\mathbf{FoS} = \frac{\textit{ultimate load}}{\textit{allowable load}} = \frac{\textit{ultimate stress}}{\textit{allowable stress}}$$

Deformations – Axial Loading



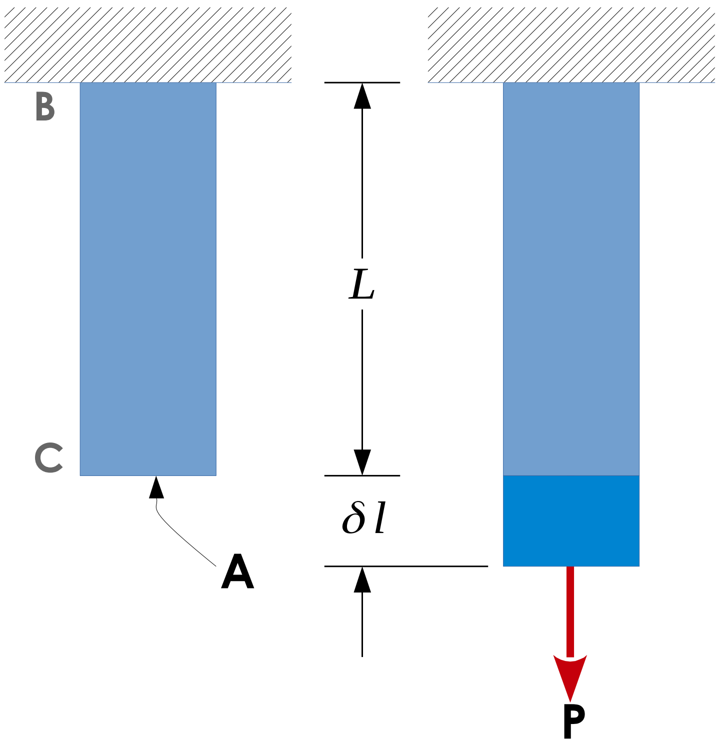
Deformations – Axial Loading

we know ,

$$\sigma = \frac{P}{A} \qquad \varepsilon = \frac{\delta L}{L}$$

$$\frac{\sigma}{\varepsilon} = E \quad \Rightarrow \quad \sigma = E\,\varepsilon$$

$$\varepsilon = \frac{\sigma}{E} \quad \Rightarrow \quad \varepsilon = \frac{P}{A\,E}$$



Substituting in $\varepsilon \times L = \delta L$

$$\boxed{\delta L = \frac{PL}{A\,E}}$$

Deformations – Axial Loading

- This equation is used only if the rod has
  - **Constant Young's Modulus (E)** i.e homogenous
  - **Uniform** cross-sectional area **A**
  - Is **loaded at the ends**
- If loaded at other points, or is of different materials, it must be divided into component parts that individually satisfy the above conditions
- The deformation of the whole rod is then a sum of the deformation of individual parts

$$\boxed{\sum_i \frac{P_i L_i}{A_i E_i}}$$

4 **An 80m long wire of 5mm diameter is made of a steel with E = 200 GPa and an Ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine,**

1. **Largest allowable tension in the wire**
2. **Corresponding elongation of the wire**

Given,

$$L_1 = 80\text{m} \quad \varphi = 5\text{mm} = 5\times 10^{-3} m \Rightarrow A = 0.000019625\, m^2$$

$$E = 200\, GPa \qquad \sigma_U = 400\, MPa \qquad F.S = 3.2$$

To find, Largest allowable tension in the wire = Allowable Tensile stress

$$3.2 = \frac{\sigma_U}{\sigma_{all}} = \frac{400}{\sigma_{all}} \qquad \textit{Factor of Safety},\ F.S = \frac{\textit{Ultimate Stress}}{\textit{Allowable Stress}} \qquad \delta l = \frac{PL}{AE}$$

$$\sigma_{all} = \frac{\sigma_U}{3.2} = 125\, MPa$$

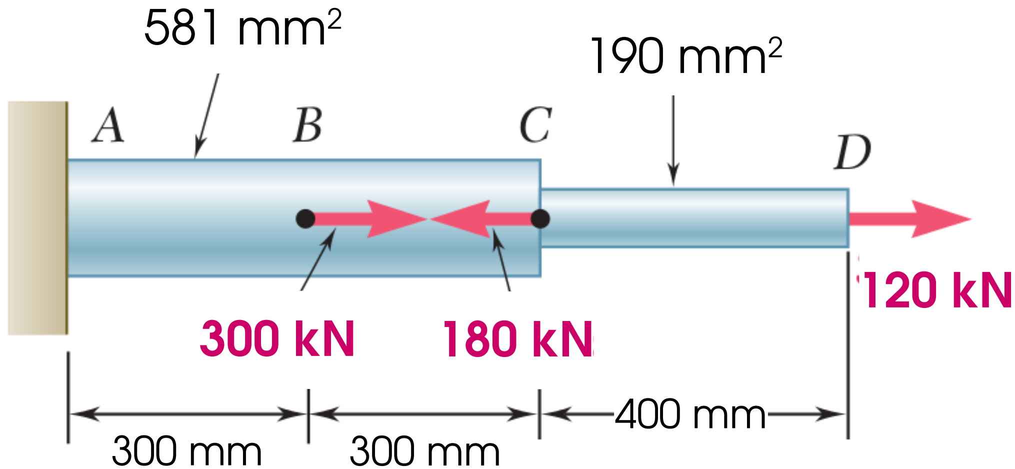
$$\delta l = \frac{(2.453\times 10^3\, N)\times 80\, m}{0.000019625\, m^2 \times 200\times 10^9\, \frac{N}{m^2}}$$

$$\sigma_{all} = \frac{P}{A} \Rightarrow P = \left(125\times 10^6\, \frac{N}{m^2}\right)\times 0.000019625\, m^2$$

$$P = 2453.125\, N$$

$$\delta l = 0.05\, m = 50\, mm$$

5 **Determine the deformation of the steel rod shown under the given loads (E= 200 GPa).**



Given,

$$L_1 = L_2 = 300\,m \ \& \ L_3 = 400\,m \qquad A_1 = A_2 = 580\,mm^2 \ \& \ A_3 = 190\,mm^2$$

$$P_1 = 240\,kN = 240\times10^3\,N$$

$$P_2 = -60\,kN = -60\times10^3\,N$$

$$P_3 = 120\,kN = 120\times10^3\,N$$

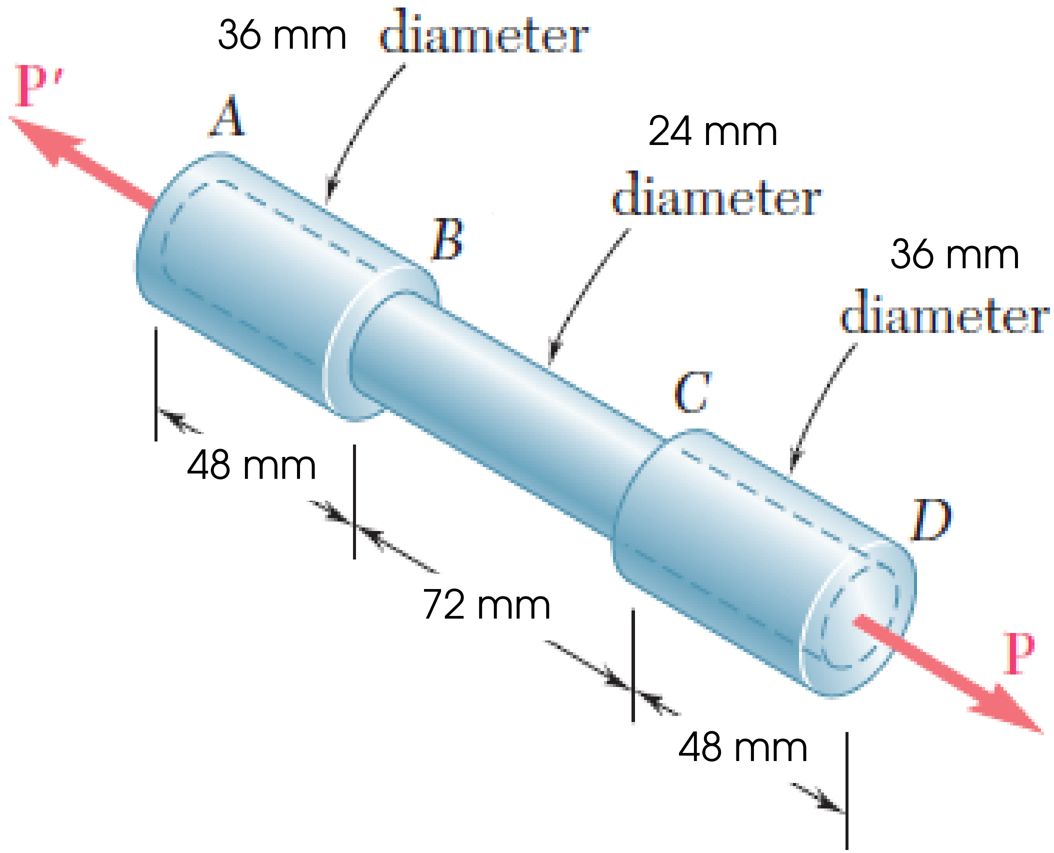
We just saw, $\delta L = \sum_i \frac{P_i L_i}{A_i E_i}$

$$\delta L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\left(\frac{P_1 L_1}{A_1}\right)+\left(\frac{P_1 L_1}{A_1}\right)+\left(\frac{P_1 L_1}{A_1}\right)\right)$$

$$\delta L = \frac{1}{200\times10^9}\left(\left(\frac{(240\times10^3)(0.3)}{580\times10^{-6}}\right)+\left(\frac{(-60\times10^3)(0.3)}{580\times10^{-6}}\right)+\frac{(120\times10^3)(0.4)}{190\times10^{-6}}\right)$$

$$\delta L = \frac{0.3457\times10^9}{200\times10^9} = 1.73\times10^{-3}\,m = \delta L = 1.73\,mm$$

6 The specimen shown is made from a 24 mm diameter cylindrical steel rod with two 36 mm outer diameter sleeves bonded to the rod as shown. Knowing that E = 200 GPa, determine
(*a*) the load P so that the total deformation is 0.04 mm
(*b*) the corresponding deformation of the central portion *BC*.



Given,

$$\varphi_{rod} = 24\,mm = 0.024\,m \qquad \varphi_{(sleeve1)} = \varphi_{(sleeve2)} = 36\,mm = 0.036\,m$$

$$A_{rod} = \frac{\pi}{4}(0.024)^2 = 4.524\times10^{-4}\,m^2 \qquad E = 200\,GPa = 200\times10^9\,\frac{N}{m^2}$$

$$A_{(sleeve\,1)} = A_{(sleeve\,2)} = \frac{\pi}{4}(0.036)^2 = 1.018\times10^{-3}\,m^2$$

$$\delta L = 0.04\,mm = 4\times10^{-5}\,m$$

$$\text{We know, } \delta L = \sum_i \frac{P_i L_i}{A_i E_i} \qquad \delta L = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P\,L_3}{A_3 E_3}$$

$$\delta L = \frac{P}{E}\left[\left(\frac{L_{sleeve1}}{A_{sleeve1}}\right) + \left(\frac{L_{rod}}{A_{rod}}\right) + \left(\frac{L_{sleeve3}}{A_{sleeve3}}\right)\right]$$

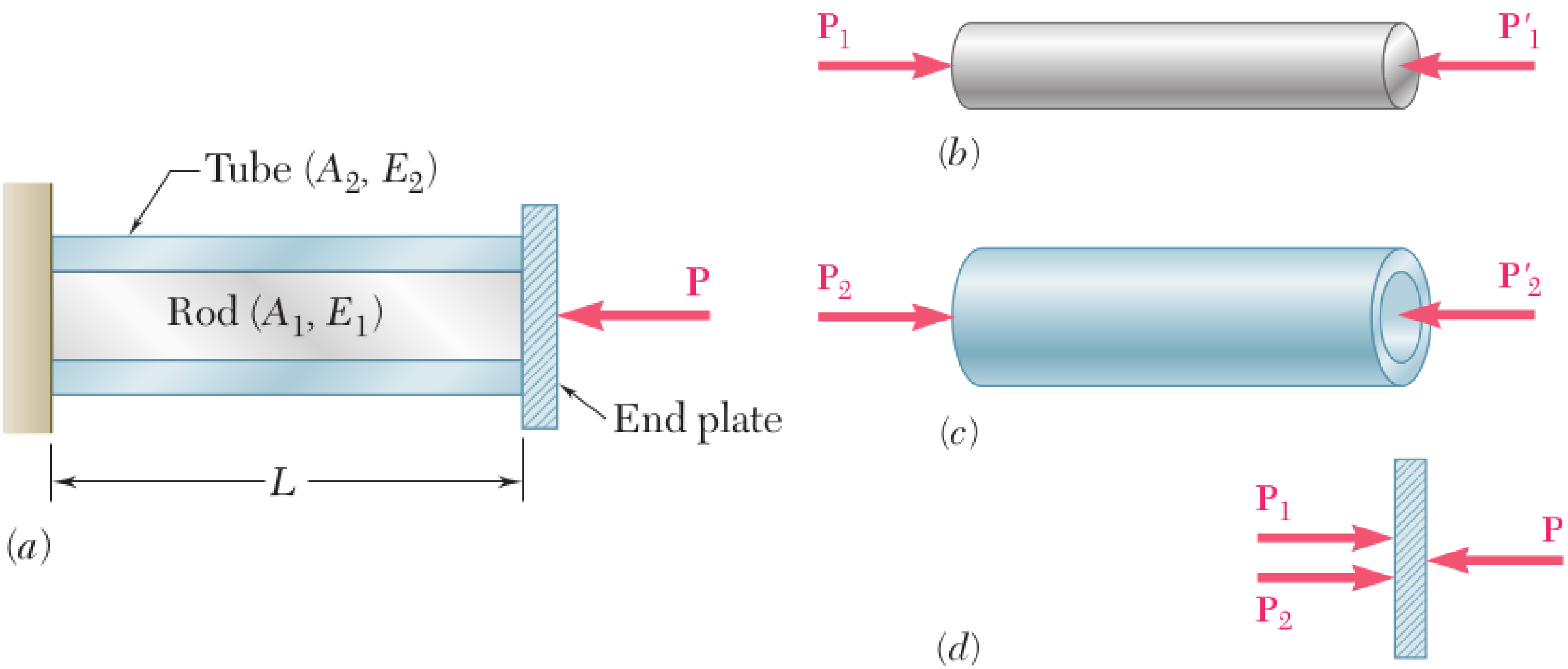
$$4\times10^{-5} = \frac{P}{200\times10^9}\left[\frac{0.048}{1.018\times10^{-3}} + \frac{0.072}{4.524\times10^{-4}} + \frac{0.048}{1.018\times10^{-3}}\right]$$

$$P = 31564.04\,N = \mathbf{31.56\,kN} \qquad \delta L_{rod} = 0.0025\times10^{-3}\,m = \mathbf{0.0025\,mm}$$

Statically Indeterminate Problems

- Internal forces cannot be determined from statics alone
- Reactions cannot be determined by drawing free-body diagrams & writing equilibrium eqns
- Equilibrium equations complemented by relations involving geometrical deformations
- Problems of this type are said to be statically indeterminate, since statics alone is not enough to solve them

Statically Indeterminate Problems



Staticalliy Indeterminate Problems

- Due to load applied on the end plate, load experienced by plate and rod could be different
- But **total load is the sum of loads** on rod and tube

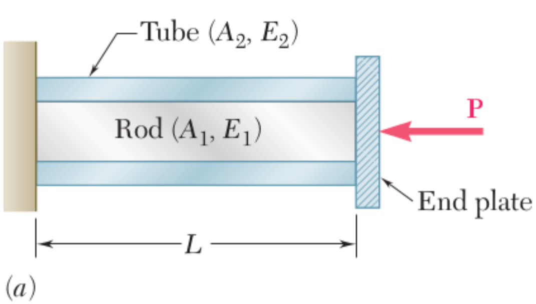
$$P = P_1 + P_2$$

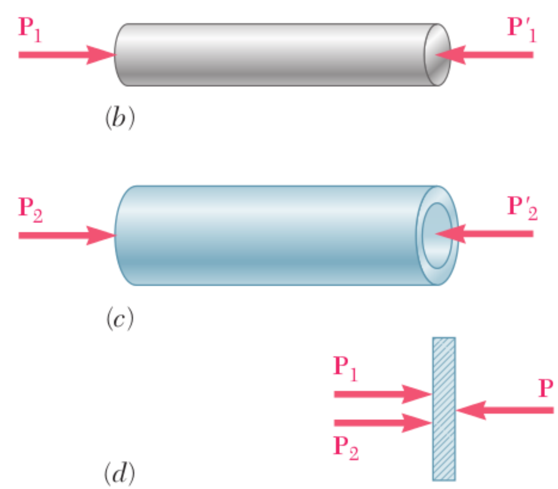
- **Deformation** experienced by both are the **same**

$$\delta = \delta_1 = \delta_2$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$$





7 **Two gage marks are placed 250 mm apart on 12 mm dia aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.**

$$\delta l = L_{orig} - L_{elong} = 250.18 - 250 \; mm = 0.18 \; mm = 0.18 \times 10^{-3} m$$

$$\varepsilon = \frac{\delta l}{L_{orig}} = \frac{0.18 \times 10^{-3} m}{250 \times 10^{-3} m} = 7.2 \times 10^{-4}$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(12 \times 10^{-3})^2 = 113.097 \times 10^{-6} m^2$$

$$\sigma = \frac{P}{A} = \frac{6 \times 10^3}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \frac{N}{m^2} = 53.052 \, MPa$$

$$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{7.2 \times 10^{-4}} = 73.683 \times 10^9 \boxed{= 73.683 \, GPa}$$

8 **A polystyrene rod of length 300 mm and diameter 12 mm is subjected to a 3 kN tensile load. Knowing that E=3.1 GPa, determine**

(a) the elongation of the rod

(b) the normal stress in the rod

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(12\times10^{-3}\right)^2 = 113.097\times10^{-6} m^2$$

$$\delta = \frac{PL}{AE} = \frac{\left(3\times10^{3}\right)\left(300\times10-3\right)}{\left(113.097\times10^{-6}\right)\left(3.1\times10^{9}\right)} = 2.56\times10^{-3} m = 2.56\, mm$$

$$\sigma = \frac{P}{A} = \frac{3\times10^{3}}{113.097\times10^{-6}} = 26.525\times10^{6}\frac{N}{m^2} = 26.525\, MPa$$

a) elongation $= 2.56\, mm$

b) Normal Stress in rod $= 26.525\, MPa$

9 **A cast iron tube is used to support a compressive load. Knowing that E=69 Gpa and that the maximum allowable change in length is 0.025%, determine**

(b) maximum normal stress in the tube

(b) the minimum wall thickness for a load of 7.2 kN if the outside diametre of the tube is 50 mm

$$E = 69\,GPa = 69\times10^9\ \frac{N}{m^2} \qquad \varepsilon = \frac{\delta l}{L} = \frac{\frac{0.025}{100}L}{L} = 0.00025$$

$$\sigma = E\times\varepsilon = (69\times10^9)(0.00025) = 17.25\times10^6\ \frac{N}{mm^2} = 17.25\,MPa$$

$$\sigma = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma} = \frac{7.2\times10^3}{17.25\times10^{-6}} = 417.39\times10^{-6}m^2$$

$$A = \frac{\pi}{4}(D_o^{\,2} - D_i^{\,2}) \Rightarrow D_i^{\,2} = D_o^{\,2} - \frac{4A}{\pi} \qquad D_i = \sqrt{(50\times10^{-3})^2 - \frac{4(417.39\times10^{-6})}{\pi}}$$

$$= \sqrt{0.00197\,m^2} = 44.368\ mm$$

$$\text{Thickness}\quad t = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(50 - 44.36) = 2.82\,mm$$

a) Normal stress $= 17.25\,MPa$

b) Wall thickness $= 2.82\,mm$

10 **A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that E=105 Gpa and that the maximum allowable stress is 180 Mpa, determine**

(a) the smallest possible diameter for the rod

(b) corresponding maximum length of the rod

$$\sigma = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma} = \frac{4\times10^{3}}{180\times10^{6}} = 22.22\times10^{-6}$$

$$A = \frac{\pi}{4}D^2 \Rightarrow D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(22.22\times10^{-6})}{3.14}} = 5.322\times10^{-3}m$$

$$\delta = \frac{PL}{AE} = L = \frac{AE\delta}{P} = \frac{(22.22\times10^{-6})(105\times10^{9})(3\times10^{-3})}{(4\times10^{3})}$$

$$L = 1.750\,mm$$

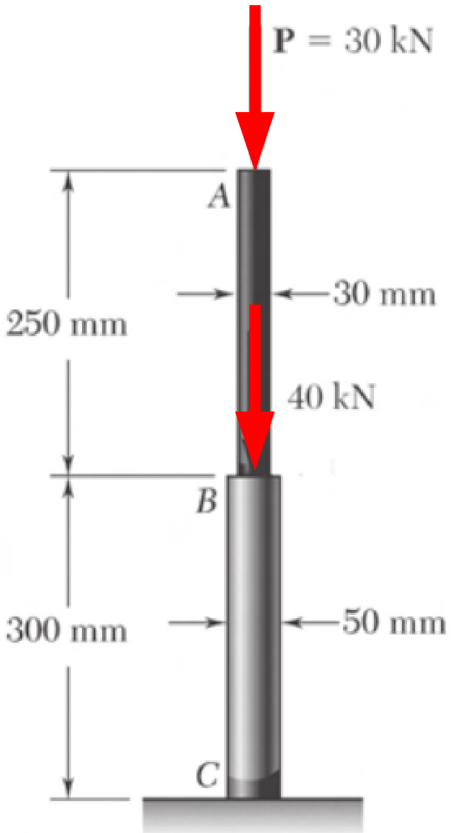
a) Smallest dia $= 5.32\,mm$

b) Max length of rod $= 5.32\,MPa$

11 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel (E=200 Gpa) and rod BC of brass (E=105 Gpa). Determine

(a) the total deformation or the composite rod

(b) the deflection of point B



Rod AB:

$$Load_{AB} = -P_{AB} = -30\times10^{3}\ N$$

$$L_{AB} = 0.025\,m = 250\times10^{-3}\,m,\quad E = 200\,GPa = 200\times10^{9}\ \frac{N}{m^2}$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(30\times10^{-3})^2 = 706.85\times10^{-6}m^2$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{Ab}E_{AB}} = \frac{-(30\times10^3)(250\times10^{-3})}{(200\times10^9)(706.85\times10^{-6})} = -53.0525\times10^{-6}m$$

Rod BC:

$$Load_{BC} = -P_{BC} = (30+40)\,kN = -70\times10^{3}\ N$$

$$L_{BC} = 0.300\,m = 300\times10^{-3}\,m,\quad E = 105\,GPa = 105\times10^{9}\ \frac{N}{m^2}$$

$$A_{BC} = \frac{\pi}{4}D^2 = \frac{\pi}{4}(50\times10^{-3})^2 = 1962.5\times10^{-6}m^2$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E_{BC}} = \frac{-(70\times10^3)(300\times10^{-3})}{(105\times10^9)(1962.5\times10^{-6})} = -101.859\times10^{-6}m$$

$$\delta_{tot} = \delta_{AB}+\delta_{BC} = -(154.9)\times10^{-6} = -0.1549\,mm$$

a) Deformation of rod = $-0.1549\,mm$

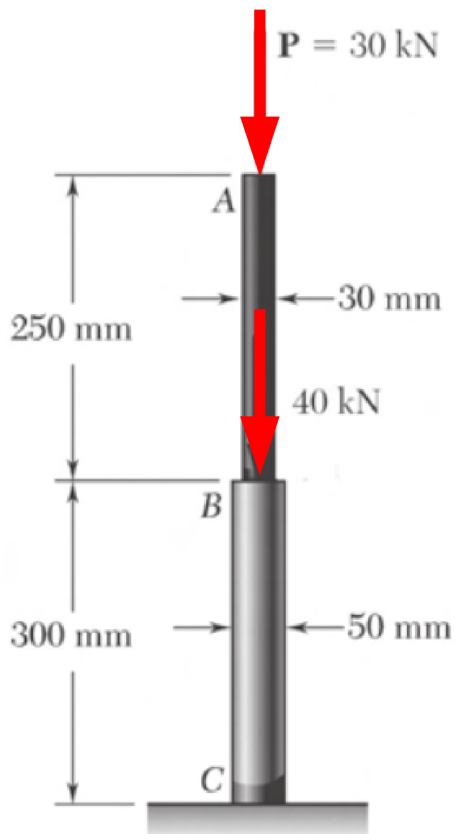
b) deflection of pt B = $\delta_{BC}\,0.1019\,mm$

12

For the previous question, also determine

(a) the load P for which the total deformation is -0.2 mm

(b) corresponding deflection of point B if the load at B is 40 kN



Rod AB: $Load_{AB} = -P_{AB}$

$$L_{AB} = 0.025\,m = 250\times10^{-3}\,m, \quad E = 200\,GPa = 200\times10^{9}\,\frac{N}{m^2}$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(30\times10^{-3})^2 = 706.85\times10^{-6}\,m^2$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{Ab}E_{AB}} = \frac{-(P)(250\times10^{-3})}{(200\times10^{9})(706.85\times10^{-6})} = -1.76841\times P$$

Rod BC: $Load_{BC} = -P_{BC} = -(P_{AB}+40)\,kN$

$$L_{BC} = 0.300\,m = 300\times10^{-3}\,m, \quad E = 105\,GPa = 105\times10^{9}\,\frac{N}{m^2}$$

$$A_{BC} = \frac{\pi}{4}D^2 = \frac{\pi}{4}(50\times10^{-3})^2 = 1962.5\times10^{-6}\,m^2$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E_{BC}} = \frac{(-(P+40)\times10^{3})(300\times10^{-3})}{(1050\times10^{9})(1962.5\times10^{-6})}$$

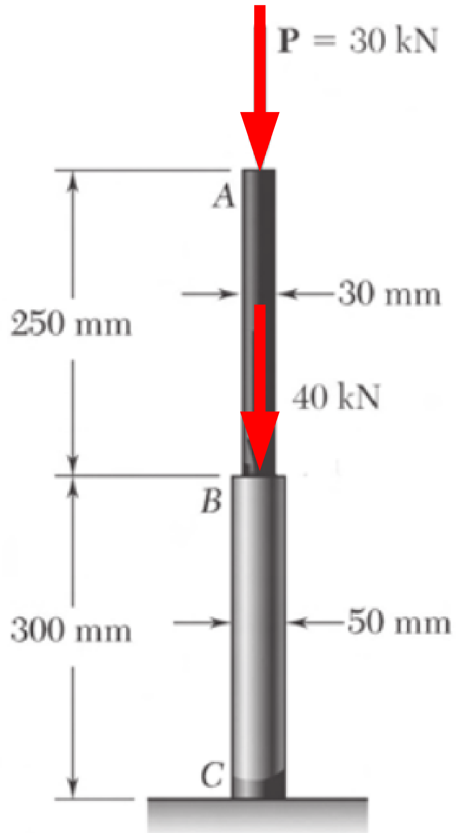
$$= (-1.45513\times10^{-9})(P) - 58.205\times10^{-6}$$

12

For the previous question, also determine

(a) the load P for which the total deformation is -0.2 mm

(b) corresponding deflection of point B



Total Deformation:

$$\delta_{tot} = \delta_{AB} + \delta_{BC}$$

$$-0.2\times10^{-3} = -1.76841\times P$$

$$+\left(-1.45513\times10^{-9}\right)\left(P\right)-58.205\times10^{-6}$$

$$P = 44.987\times10^{3}\ N \quad = 44.987\ kN$$

Deflection of point B

$$\delta_{B} = \delta_{BC}$$

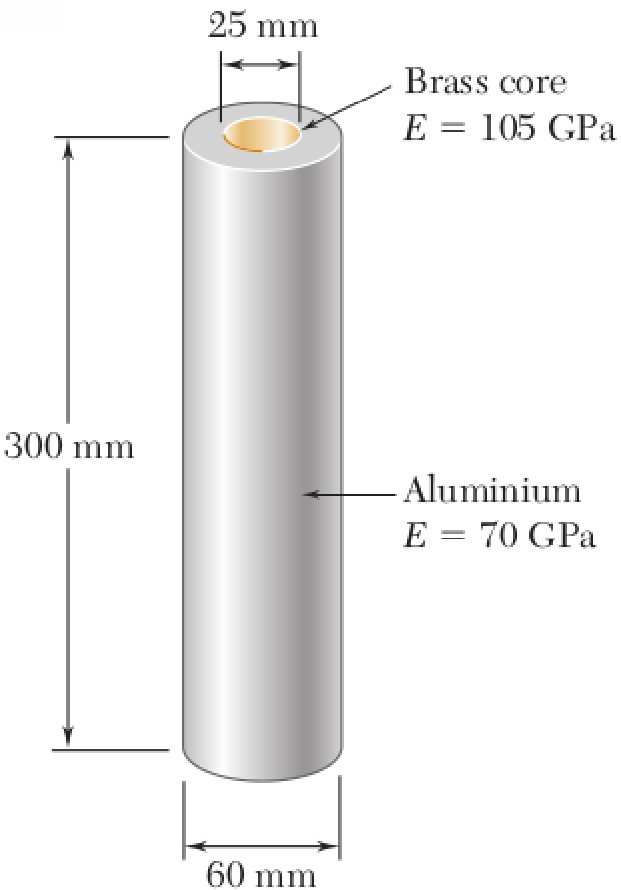
$$= \left(-1.45513\times10^{-9}\right)\left(44.987\times10^{3}\right)-58.205\times10^{-6}$$

$$= 0.1222\times10^{-3}\ m \quad = 0.122\ mm$$

13 **An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine**

(a) the normal stress in the aluminium shell

(b) the corresponding deformation of the assembly



Aluminium Rod	Brass Rod

$$E = 70\,GPa = 70\times10^{9}\,\frac{N}{m^2}$$
$$L = 300\,mm = 300\times10^{-3}\,m$$
$$\Phi_{al} = 60\,mm = 60\times10^{-3}\,m$$
$$A_{al} = 3600\,mm^2 = 36\times10^{-4}\,m$$

$$E = 105\,GPa = 105\times10^{9}\,\frac{N}{m^2}$$
$$L = 300\,mm = 300\times10^{-3}\,m$$
$$\Phi_{al} = 25\,mm = 25\times10^{-3}\,m$$
$$A_{al} = 625\,mm^2 = 6.25\times10^{-4}\,m$$

We know that for compound bars,

$$\boxed{1)\ P = P_1 + P_2}$$

Load is shared between both members

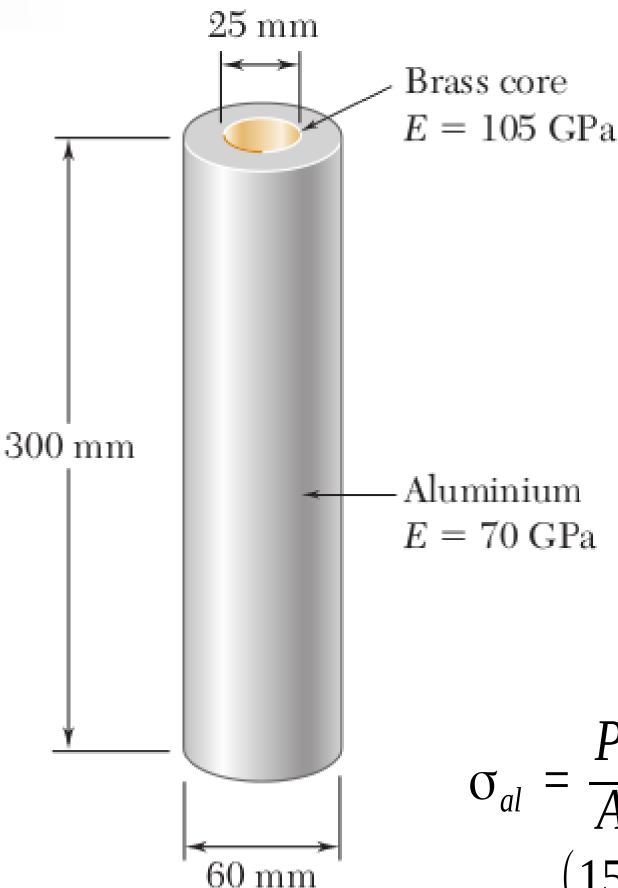
$$\boxed{2)\ \delta L = \delta L_1 + \delta L_2}$$

Total deflection is sum of individual deflections

13 **An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine**

(a) the normal stress in the aluminium shell

(b) the corresponding deformation of the assembly



Here,

$$200\times10^{3} = P_{al} + P_{br} \quad \text{...from point (1)}$$

$$\frac{P_{al}L}{A_{al}E_{al}} = \frac{P_{br}L}{A_{br}E_{br}} \quad \text{...from point (2)}$$

$$= \frac{P_{al}}{P_{br}} = \frac{A_{al}E_{al}}{A_{br}E_{br}} = \frac{P_{al}}{P_{br}} = \frac{(29.75\times10^{-4})(70\times10^{9})}{(6.25\times10^{-4})(105\times10^{9})} = 3.17$$

$$\Rightarrow \quad P_{al} = 3.17P_{br}$$

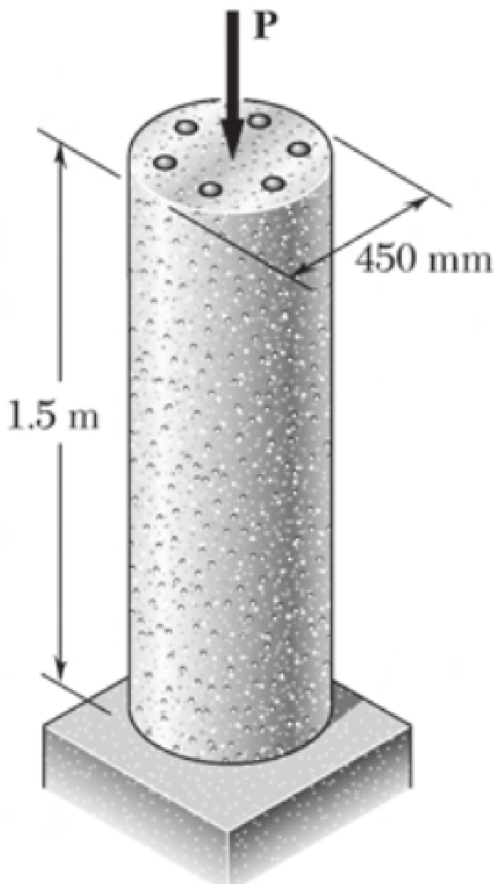
$$200\times10^{3} = 3.17P_{br} + P_{br} \Rightarrow \quad P_{br} = \frac{200\times10^{3}}{4.17} = 47{,}961.63\,N$$

$$P_{al} = 3.17\times47961.63 \Rightarrow \quad P_{al} = 152.04\,kN$$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} \quad \sigma_{al} = \frac{151.92\times10^{3}}{36\times10^{-4}} = 42.2\times10^{6}\frac{N}{m^{2}} \quad = 42.2\,MPa$$

$$\delta = \frac{(151.92\times10^{3})(300\times10^{-3})}{\frac{\pi}{4}(29.75\times10^{-4})(70\times10^{9})} = 2.78\times10^{-4}m \quad = 0..28\,mm$$

14 **The 1.5 m concrete post is reinforced with six steel bars, each with 28 mm diameter. Knowing that E_s = 200 Gpa and E_c = 25 Gpa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post**



P_c = axial force on concrete $\quad$ P_s = axial force on steel bars

$$L = 1.5\ m \qquad \Phi_C = 450\times10^{-3}\ m \quad \Phi_s = 28\times10^{-3}\ m$$

$$\delta_c = \frac{P_c L_c}{A_c E_c} \quad \Rightarrow \quad P_c = \frac{\delta_c A_c E_c}{L_c}$$

$$\delta_s = \frac{P_s L_c}{A_s E_s} \quad \Rightarrow \quad P_s = \frac{\delta_s A_s E_s}{L_s}$$

Since
$$P = P_c + P_s$$
$$\delta_c = \delta_s = \delta_L$$

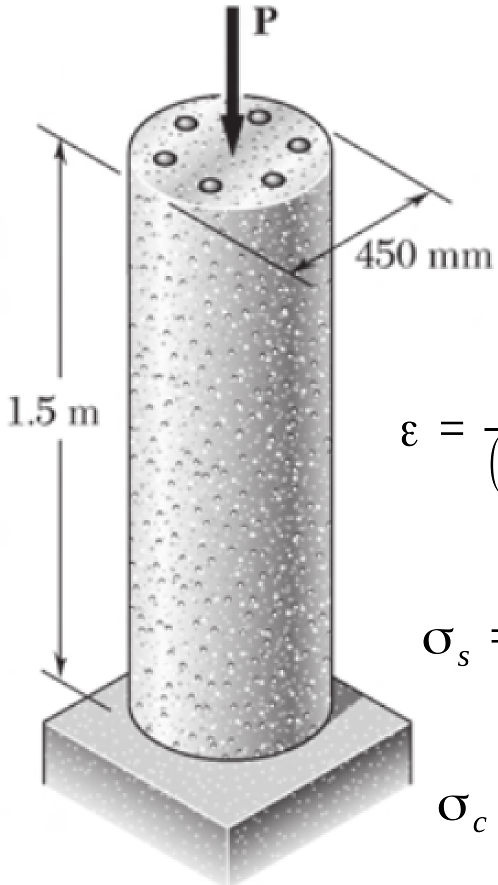
$$P = \frac{\delta L}{L}\left(E_c A_c + E_s A_s\right)$$

$$\frac{\delta L}{L} = \frac{P}{\left(E_c A_c + E_s A_s\right)}$$

$$A_s = \frac{\pi}{4} D^2 \times (6) = \frac{\pi}{4}(28\times10^{-3})^2\times(6) = 3.69\times10^{-3}\ m$$

$$A_c = \frac{\pi}{4} D^2 - 6\,A_s = \frac{\pi}{4}(450\times10^{-3})^2 - 6\,A_s = 136.90\times10^{-3}\ m$$

14 **The 1.5 m concrete post is reinforced with six steel bars, each with 28 mm diameter. Knowing that E_s = 200 Gpa and E_c = 25 Gpa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post**



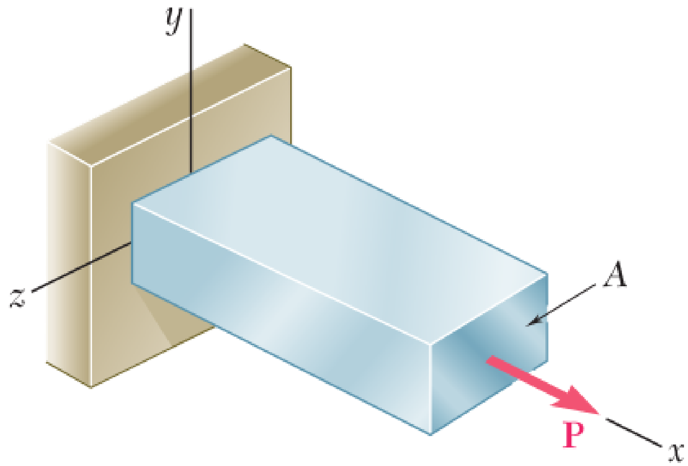
$$\varepsilon = \frac{\delta L}{L} \quad \& \quad \frac{\delta L}{L} = \frac{P}{(E_c A_c + E_s A_s)}$$

$$\varepsilon = \frac{P}{(E_c A_c + E_s A_s)}$$

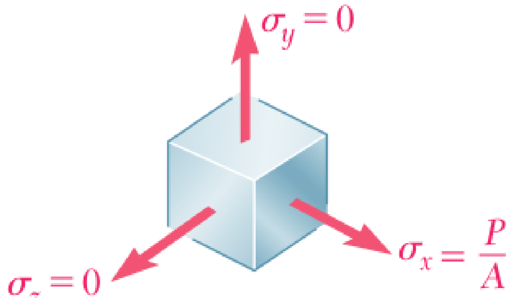
$$\varepsilon = \frac{1550\times10^3}{\left((25\times10^9)(136.9\times10^{-3}) + (200\times10^9)(3.69\times10^{-3})\right)} = 4.53\times10^{-4}$$

$$\sigma_s = E_s\varepsilon = (200\times10^9)(4.53\times10^{-4}) = 90.6\times10^6\,\frac{N}{m^2} \quad = 90.6\,MPa$$

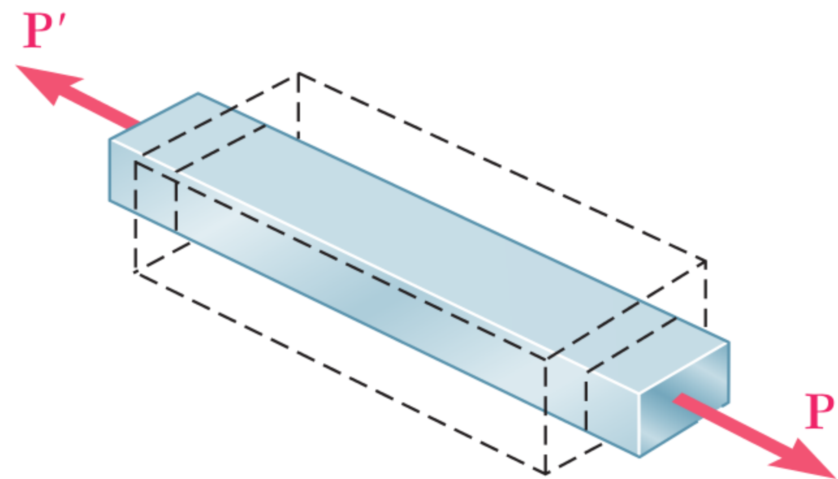
$$\sigma_c = E_c\varepsilon = (25\times10^9)(4.53\times10^{-4}) = 11.33\times10^6\,\frac{N}{m^2} \quad = 11.33\,MPa$$



(a)



(b)



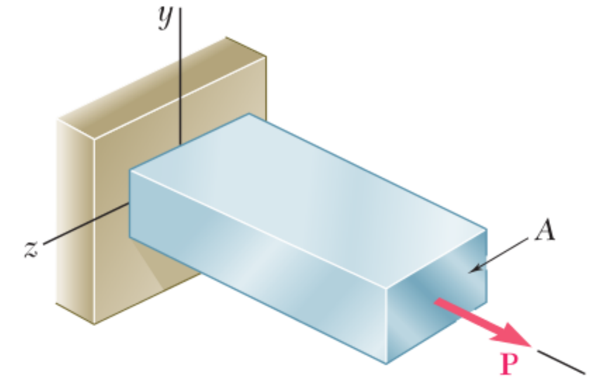
Poisson's Ratio

- Within elastic limits, homogeneous slender bars obey Hooke's Law
- If load is along *x-axis,*

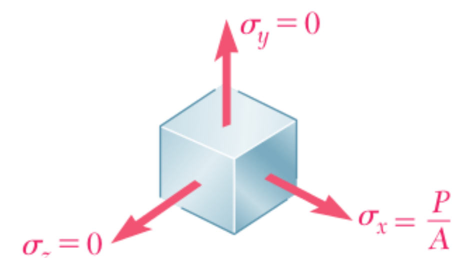
$$\sigma_x = \frac{P}{A} \quad \text{and} \quad \varepsilon_x = \frac{\sigma_x}{E}$$

- When load acts along an x-axis, increase or decrease in length i.e, deformation happens along all three axis
- Decrease in breadth (z-axis) and thickness or height (y-axis) noticed

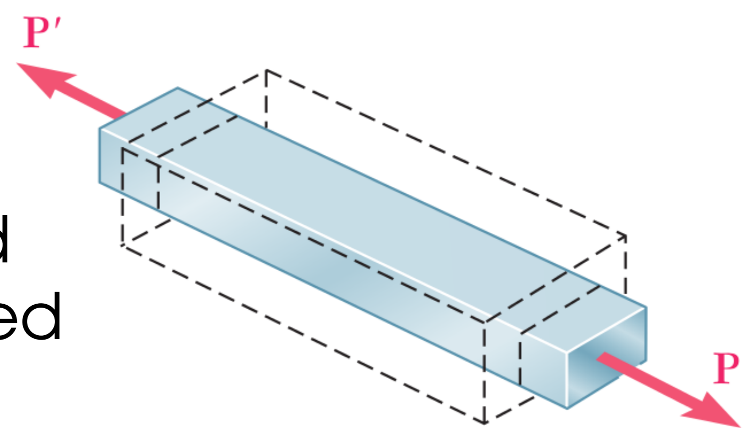
$$i.e., \ \sigma_y = \sigma_z \neq 0$$



(a)



(b)



Poisson's Ratio

- Hence, there is strain in all directions and it must have the **same value for any transverse direction**

$$\varepsilon_y = \varepsilon_z$$

- Strain (due to **lateral contraction**) is known as **lateral strain**
- Strain along the transverse axis is always the opposite to the one along the axis
- Ratio between the lateral and axial strain is known as Poisson's Ratio

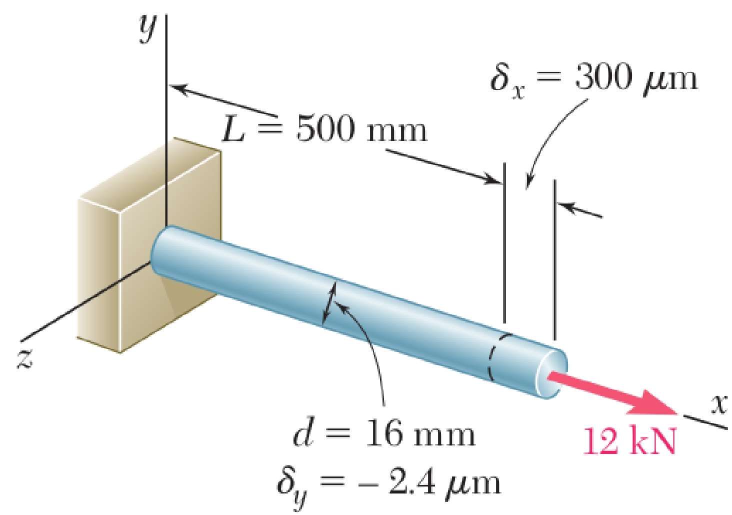
$$\nu = -\frac{lateral\ strain}{axial\ strain} \quad \Rightarrow \quad \nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Note: The 'minus' sign is used to obtain a positive value for the Poisson's ratio. All materials have their axial strain and lateral strain opposite to each other

15 **A 500-mm-long, 16-mm-diameter rod made of a homogenous, isotropic material is observed to increase in length by 300 μm, and to decrease in diameter by 2.4 mm when subjected to an axial 12-kN load. Determine the modulus of elasticity and Poisson's ratio of the material.**

Given,

P = 12 kN L = 500 mm D = 16 mm δ_x = 300 mm δ_y = 2.4 mm



$$A = \frac{\pi \times D^2}{4} = \frac{\pi(16\times10^{-3})}{4} = 201\times10^{-6}\,m^2$$

$$\sigma_x = \frac{P}{A} = \frac{12\times10^3\,N}{201\times10^{-6}\,m^2} = 59.7\,MPa$$

$$\varepsilon_x = \frac{\delta_x}{L} = \frac{300\ \mu m}{500\,mm} = 600\times10^{-6}$$

$$\varepsilon_y = \frac{\delta_y}{D} = \frac{-2.4\ \mu m}{16\,mm} = -150\times10^{-6}$$

$$\sigma_x = E\,\varepsilon_x \quad \Rightarrow \quad E = \frac{59.7\ MPa}{600\times10^{-6}} = 99.5\,GPa$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{-150\times10^{-6}}{600\times10^{-6}} = 0.25$$

Generalized Hooke's Law

- Till now, only loads along a single axis were considered and hence,

$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0 \quad \text{and} \quad \sigma_z = 0$$

- Structures subjected to loads acting in the three coordinate axes experience normal stress in each

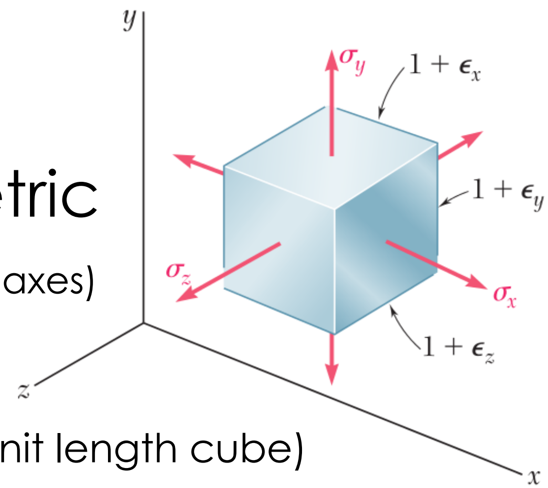
$$\sigma_x = \frac{P_x}{A_x}, \quad \sigma_y = \frac{P_y}{A_y} \quad \text{and} \quad \sigma_z = \frac{P_z}{A_z}$$

- All different from zero. This condition is referred to as **multiaxial loading**

Bulk Modulus

- When loads are applied and deformations happen, there is change in volume

$$v = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$



- Change in volume due to the volumetric strain (due to the contractions/elongations along the lateral axes)

$$e = v - 1 = (\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

(where, **e** is the change in volume and **1** is the volume of the unit length cube)

- Originally a unit volume, the quantity **e** represents **the change in volume per unit volume**

- This is called **dilatation** of the material

Bulk Modulus

- From, Generalised Hooke's Law,

$$e = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$e = (\sigma_x + \sigma_y + \sigma_z)\,\frac{1-2\nu}{E}$$

Remember,

Pressure, P is not force. It is equivalent to F/A. That is, it is equivalent to stress

P = σ

- If each of the stress element is due to a hydrostatic pressure ***-p***

$$e = \frac{-3p(1-2\nu)}{E}$$

Where, $K = \dfrac{E}{3(1-2\nu)}$ or $e = -\dfrac{p}{K}$

- Same unit as of Young's modulus (E), that is in **Pa**

Bounds of Poisson Ratio

- Material subjected to a hydrostatic pressure only decreases in volume
- Dilatation **e is negative**; bulk modulus **k is positive**

$$e = -\frac{p}{K} \quad \Rightarrow \quad K = \frac{-p}{-e}$$

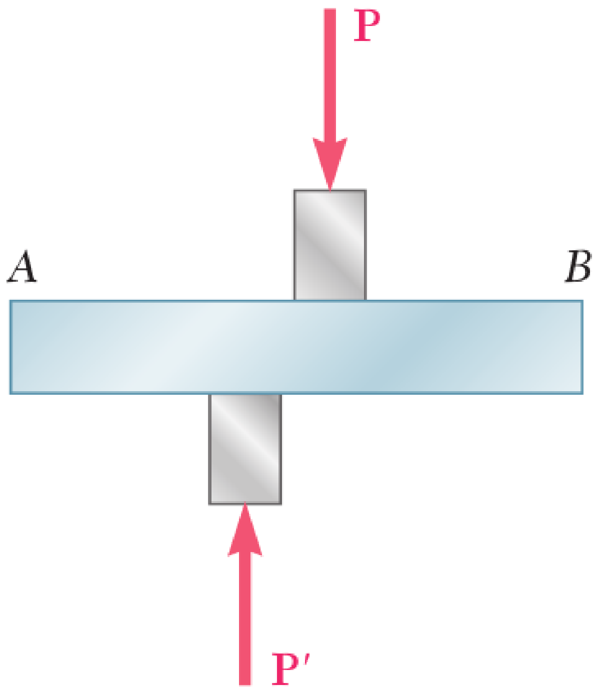
- So, from equation $K = \dfrac{E}{3(1-2\nu)}$ it is clear that,

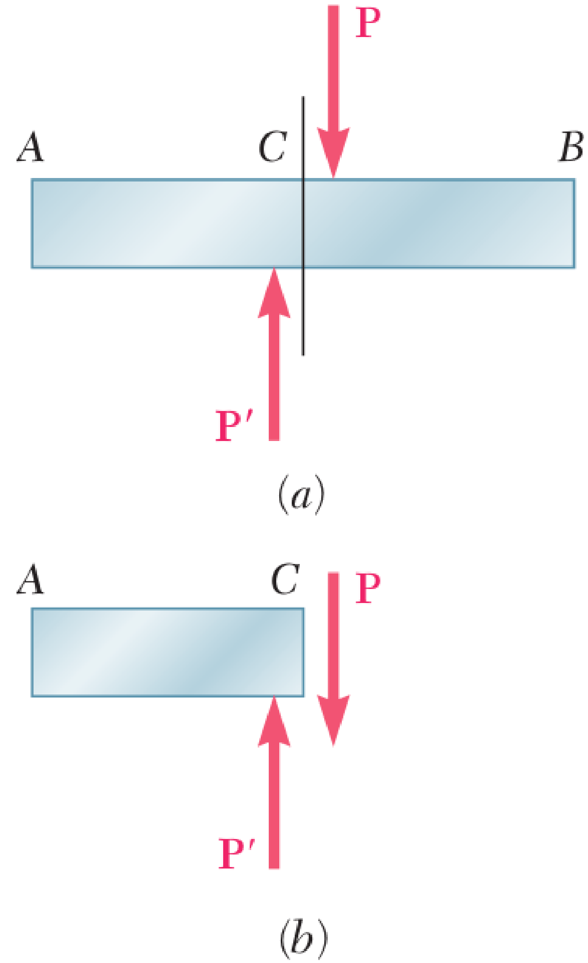
$$(1-2\nu) > 0 \quad \Rightarrow \quad \nu < \frac{1}{2}$$

- But since ν is positive for all engineering materials, the bounds of Poisson Ratio is

$$0 < \nu < \frac{1}{2}$$

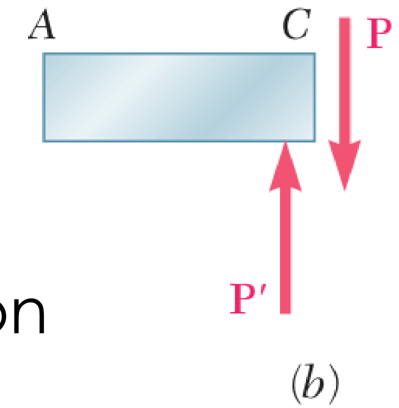
Shear Stresses





Shear stress is where the stress is parallel to the surface of the material as opposed to normal stress

Shear Stresses



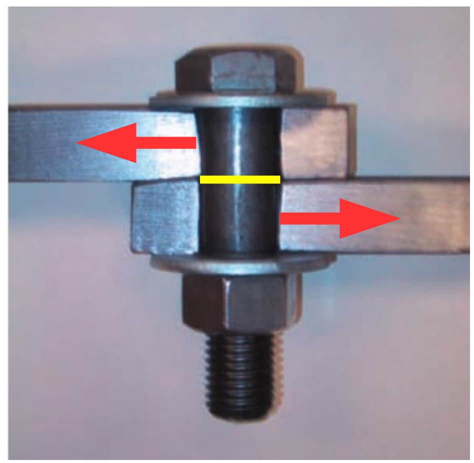
- Internal forces exist in the plane of the section and their resultant is equal to **P**
- These elementary **internal forces** are called **shearing forces**, and **magnitude P** of their resultant is **the shear** in the section

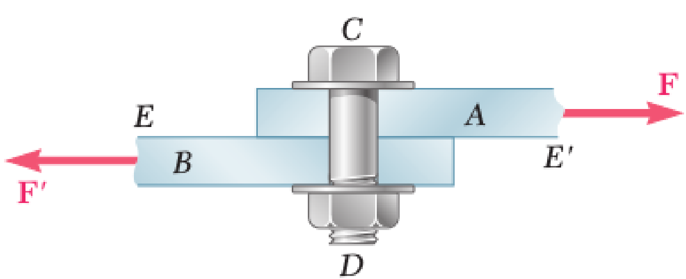
$$\tau = \boxed{\tau_{avg} = \frac{P}{A}}$$

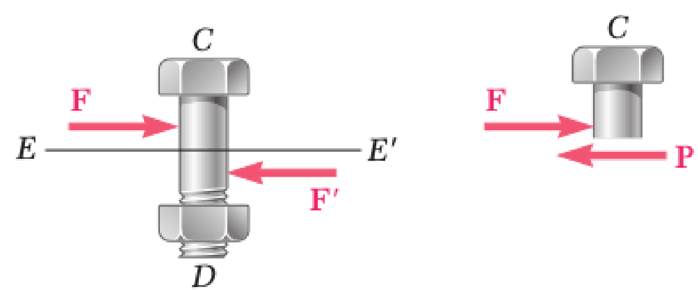
- Note that it is an **average value** of shearing stress over the entire section that is obtained
- **Not uniform**. Actual value varies from zero at surface of member to max value much larger than τ_{avg}

Shear Stresses

- Observed commonly in bolts, pins, and rivets used to connect various structural members and machine components

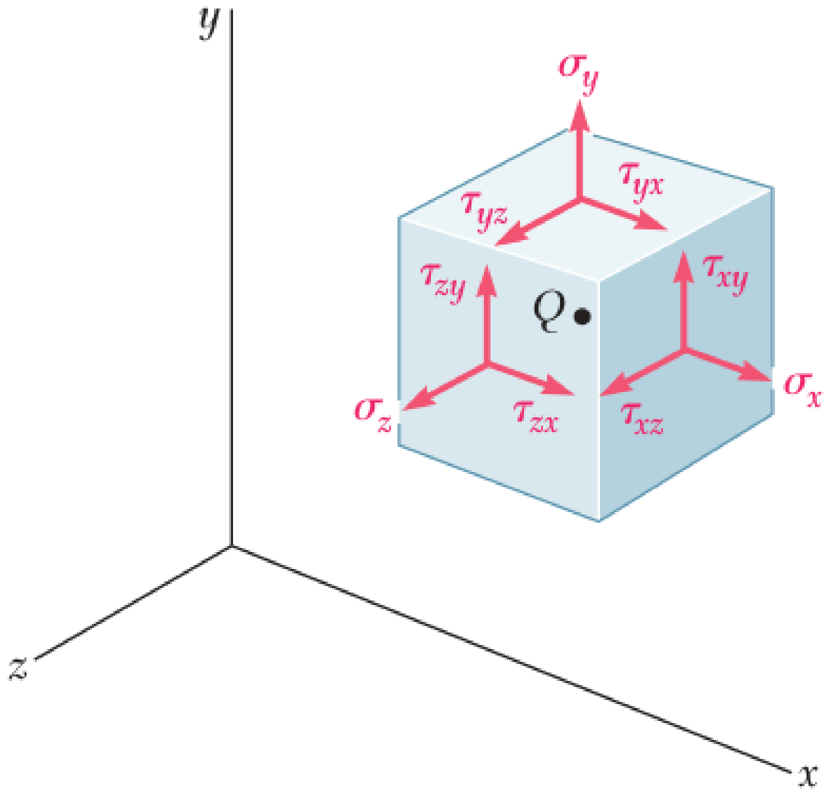




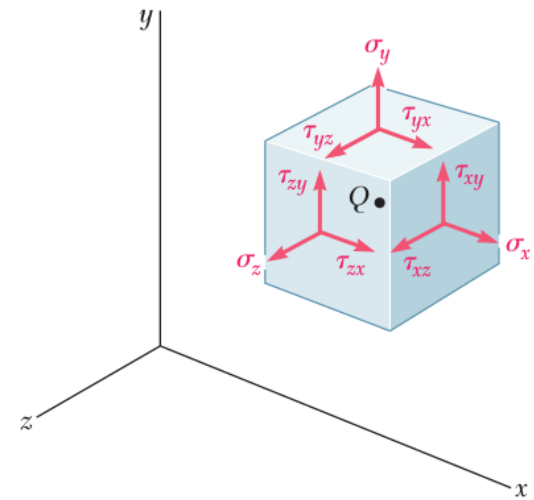


(a) (b)

Shear Strain

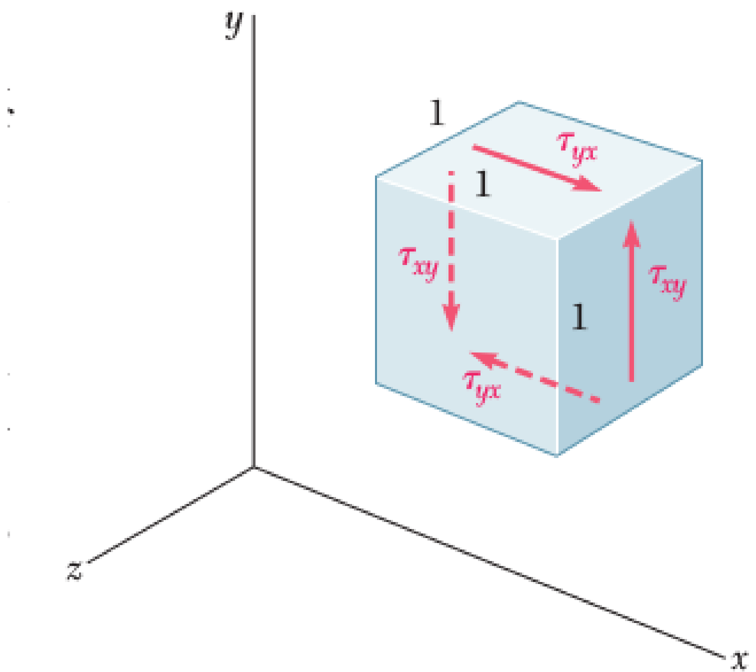


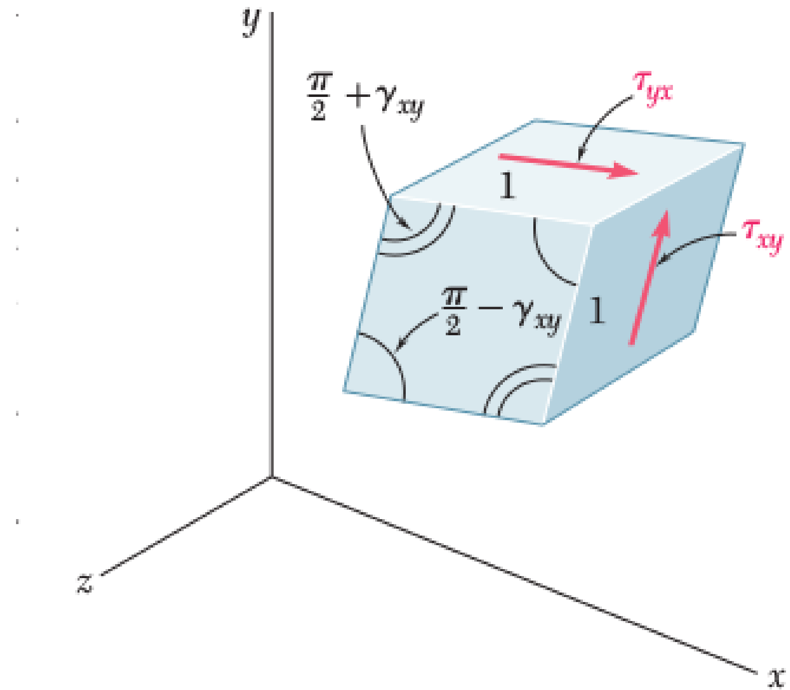
Shear Strain



- Earlier, while considering multiaxial loading, we did not assume shear stress
- These stresses have no direct effect on the normal strains and, as long as all the deformations involved remain small
- Shearing stresses will tend to deform a cubic element of material into an oblique parallelepiped – **Shearing strain develops**

Shear Strain



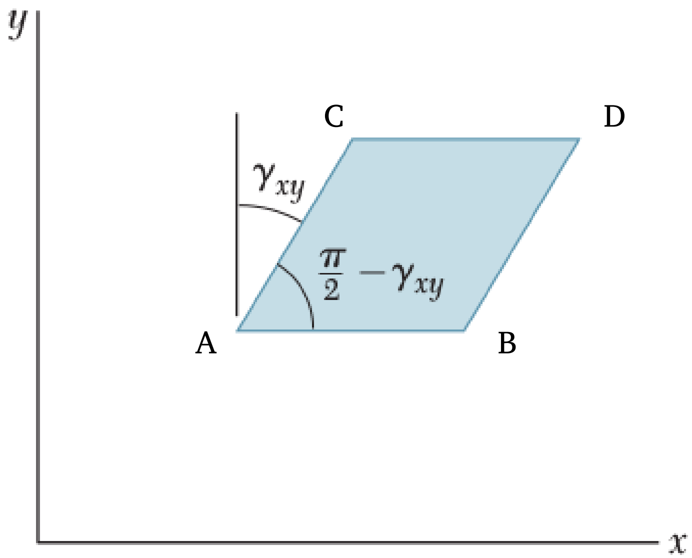


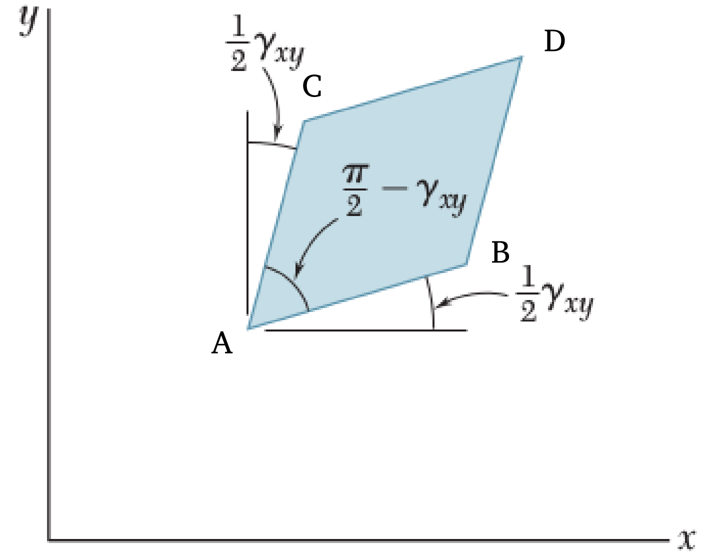
Let us consider two shear forces

τ_{yx} - Offset along y axis
Acting along x axis

τ_{xy} - Offset along x axis
Acting along y axis

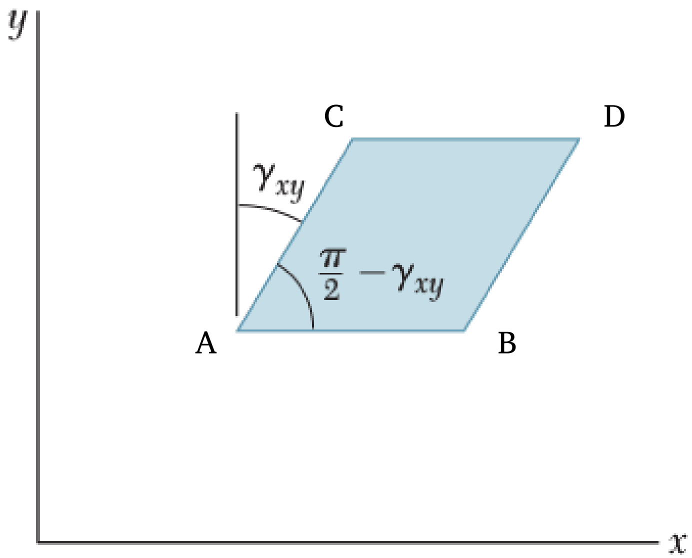
Shear Strain

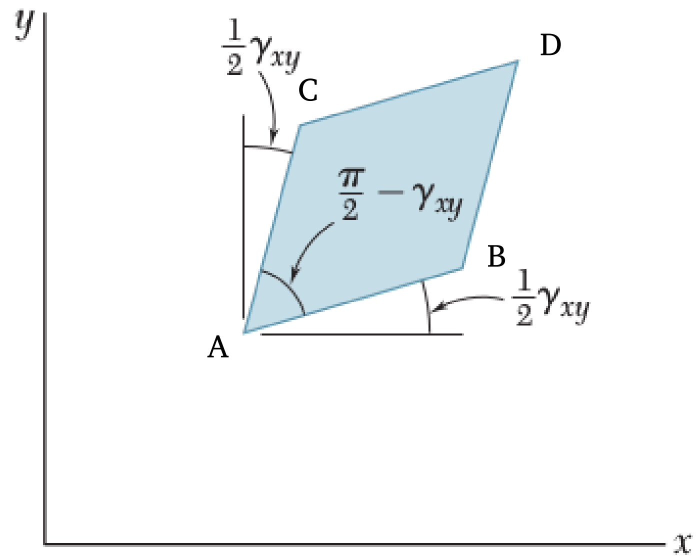




- Two angles formed by four faces under stress are reduced while the other two are increased
- The small angle γ_{xy} (expressed in radians) defines the shearing strain corresponding to the x and y directions.

Shear Strain





- The engineering shear strain γ can also be defined as the change in angle between lines AC and AB.

Shear Strain

- Plotting successive values of τ_{xy} against the values of γ_{yx} shearing stress-strain diagram is obtained
- For shearing stress not exceeding the proportional limit in shear, we can write for any homogeneous isotropic material

$$\tau_{xy} = G\gamma_{xy}$$

- This is known as **Hooke's law for shearing stress and strain**, and the constant **G** is called the **modulus of rigidity or shear modulus**
- G is expressed in pascals or in psi.
- G of any material is **less than one-half**, but more than one-third of E of that material.

Relations between E, K, & G

$$K = \frac{E}{3(1-2\nu)}$$

$$\frac{E}{2G} = 1+\nu$$

used to determine one of the constants E, ν, or G from the other two

Some more worked-out problems

17 **Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.**

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt.

The spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium

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$P_b = P_s$

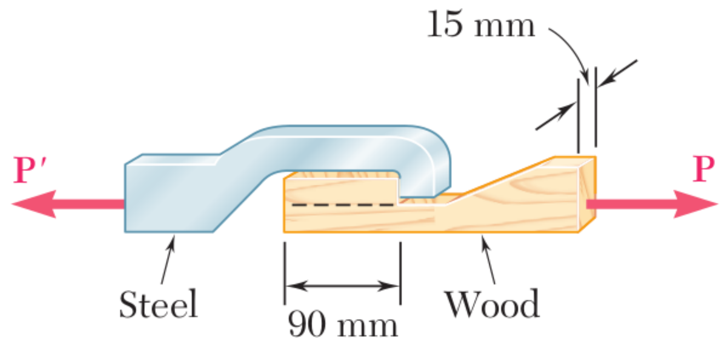
For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s, $\dfrac{\pi}{4}\sigma_b d_b^2 = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm}$$

18 **When the force P reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.**



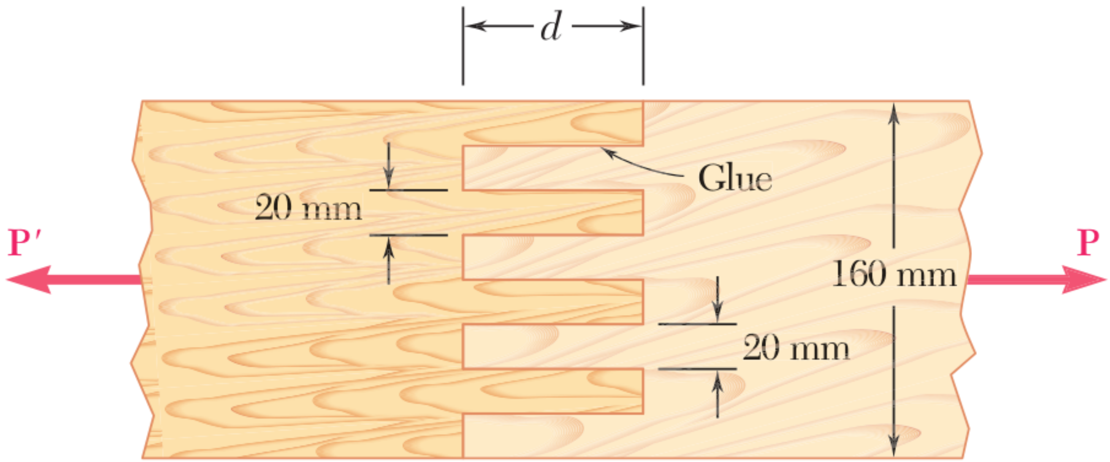
Area being sheared: $A = 90\text{ mm} \times 15\text{ mm} = 1350\text{ mm}^2 = 1350 \times 10^{-6}\text{m}^2$

Force: $P = 8 \times 10^3\text{ N}$

Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6\text{ Pa}$

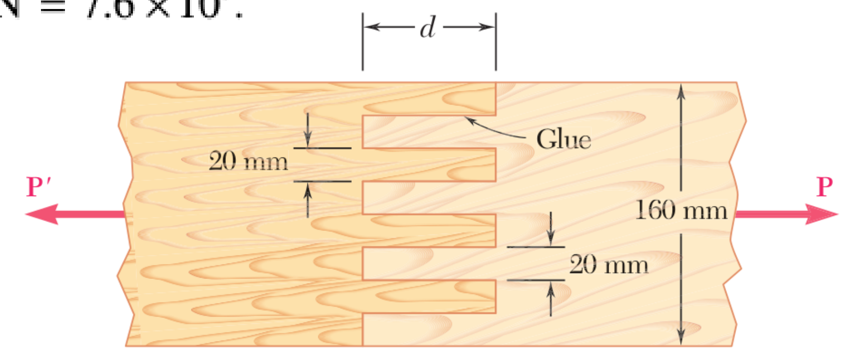
$\tau = 5.93\text{ MPa}$

19 **Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length '*d*' of the cuts if the joint is to withstand an axial load of magnitude P = 7.6 kN.**



19 **Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length '*d*' of the cuts if the joint is to withstand an axial load of magnitude P = 7.6 kN.**

Seven surfaces carry the total load $P = 7.6\ \text{kN} = 7.6 \times 10^3$.



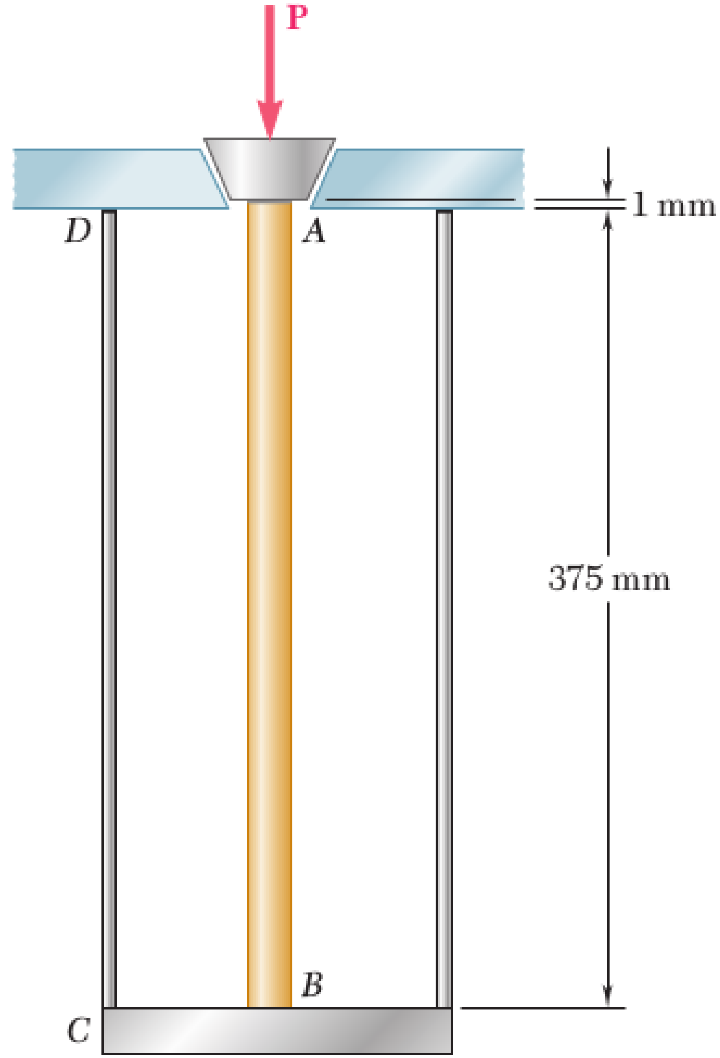
Let $t = 22$ mm.

Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} \qquad = 1.32404 \times 10^{-3}\text{m}^2 \qquad = 1.32404 \times 10^3\text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} \qquad d = 60.2\ \text{mm}$$

20 The brass tube AB (E = 105 GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder (E = 72 GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force P that must be applied to the cylinder.



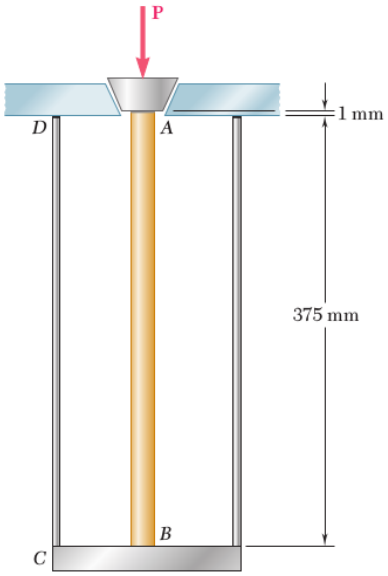
20 The brass tube AB (E = 105 GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is….

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$



Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9}) P$$

$$P = 21.547 \times 10^3 \text{ N} \qquad P = 21.5 \text{ kN}$$