8.1 (p263)

From Eqn. 3, we know we may write the current through the resistor as

$$i(t) = I_0 e^{-Rt/L} = 6e^{-1000t/500\times10^{-9}}.$$

$$Thus,\ i(0.5ns) = 6e^{-1000/500} = \underline{812.0\ \text{mA}}$$

8.2 (p267)

For $t < 0$, 10 V appears across the 4 Ω resistor, so a dc current of 10/4 A = 2.5 A flows through the inductor (which acts as a short circuit).

For $t > 0$, the battery is removed so we write the simple KVL equation:

$10i_L + v = 0$ where $v = 5\dfrac{di_L}{dt}$. Thus, $10i_L + 5\dfrac{di_L}{dt} = 0$, which has the characteristic equation $10 + 5s = 0$, with solution $s = -2$.

Thus, we can represent the circuit with the equation $i_L(t) = i_L(0)e^{st} = 2.5e^{-2t}$ A.

Finally, $v = 5\dfrac{di_L}{dt} = 5(2.5)(-2)e^{-2t} = \underline{-25e^{-2t}\ \text{V}}.$

8.3 (p270)

$i(t) = I_o e^{-t/\tau}$ and $i(0) = I_o$

(a) $\dfrac{i(2\tau)}{i(\tau)} = \dfrac{e^{-2}}{e^{-1}} = \underline{0.3679}$

(b) $\dfrac{i(0.5\tau)}{i(0)} = e^{-0.5} = \underline{0.6065}$

(c) $\dfrac{i(t)}{i(0)} = e^{-t/\tau} = 0.2$, so $\dfrac{t}{\tau} = -\ell n\, 0.2 = \underline{1.609}$

(d) if $i(0) - i(t) = i(0)\,\ell n 2$

Then $i(t) = i(0) - i(0)\,\ell n 2 = i(0)\left[1 - \ell n 2\right] = i(0)e^{-t/\tau}$

Thus, $1 - \ell n 2 = e^{-t/\tau}$ and so $\dfrac{t}{\tau} = -\ell n(1 - \ell n 2)$

$= \underline{1.181}$

8.4 (p275)

Before the switch is thrown, the 80 Ω resistor is connected only by one of its terminals and therefore may be ignored. ($i = 0$)

With no current flow permitted through the capacitor (it is assumed any transients have long since died out), $v(0) = 50$ V. We know $v(0) = 50$ V since the capacitor voltage cannot change in zero time.

After the switch is thrown, the only remaining circuit is a simple source-free RC circuit.

With $\tau = RC = 160\ \mu\text{s}$,

$$v(t) = v(0)e^{-t/\tau} \quad \text{so} \quad v(\tau) = 50\,\mathrm{e}^{-1} = \underline{18.39\text{ V}}$$

8.5 (p278)

For $t < 0$: $i_2(0^-) = 0,\ i_1(0^-) = 2\times\dfrac{2}{2+8} = 0.4\text{ A},$

and $i_L(0^-) = 2 - 0.4 = 1.6\text{ A}$

For $t > 0$, 100% of the 2-A source contributes to i_2. The 8–Ω resistor is shorted out so $\underline{i_1 = 0}$.

Thus, $i_2(t) = 2 - i_L(t)$

where

$$i_L(t) = i_L(0^+)e^{-t/\tau} \qquad \text{and } \tau = \frac{L}{R} = \frac{0.4}{2} = 0.2\text{ s}$$
$$i_L(0^+) = i_L(0^-) = 1.6\text{ A}$$

so $i_L(0.15) = 1.6\exp\left(\dfrac{-0.15}{0.2}\right) = \underline{755.6\text{ mA}}$

and $i_2(0.15) = \underline{1.244\text{ A}}$

8.6 (p280)

For $t < 0$, $v_C(0^-) = 120\times\dfrac{1250}{1250+250} = \underline{100\text{ V}}$

$$v(0^-) = \left[120\times\frac{2000\,//\,500}{600 + 2000\,//\,500}\right]\times\frac{400}{400+100} = \underline{38.4\text{ V}}$$

At $t = 0^+$, the capacitor voltage cannot differ from its value at $t = 0^-$.

Thus, $v_C(0^+) = 100$ V

and with the source removed,

$$v(0^+) = v_C(0^+) \times \left[\frac{2000//500}{850+2000//500}\right] \times \frac{400}{100+400} = 25.6 \text{ V} \qquad [1]$$

$$\tau = R_{eq} \cdot C = [1250//(850+2000//500)] \times 4 \times 10^{-6}$$
$$= 2.5 \text{ ms}$$

$$v_C(1.3 \text{ ms}) = 100e^{-1.3/2.5} = 59.45 \text{ V}$$

Replacing $v_C(0^+)$ in Eq. [1] with v_C (1.3 ms) yields $v(1.3 \text{ ms}) = \frac{59.45}{100} \times 25.6 = 15.22 \text{ V}$

8.7 (p282)

(a) We approach problem by nodal analysis, choosing the bottom node as our reference:

$$0 = 0.5v_1 + v_1 - v_C \qquad \text{or} \quad 3v_1 - 2v_C = 0 \quad [1]$$

and

$$-1.5v_1 = \frac{v_1 - v_C}{1} + 0.002\frac{dv_C}{dt} \quad [2]$$

Simplifying Eqn 2 and noting that from Eqn 1 we know $v_1 = \frac{2}{3}v_C$, we can write a single differential equation that describes the circuit: $0.002\frac{dv_C}{dt} + \left(1 + \frac{4}{3}\right)v_C = 0$. This has the characteristic equation 0.002s + 4/3 = 0, which has solution s = -2000/3.
Thus, we may write the capacitor voltage as $v_C(t) = v_C(0)e^{st} = 11e^{-2000t/3}$ V.
(b) Since the voltage decays, rather than grows, with time, the circuit is stable.

8.8 (p286)

(a) 3 – 0 + 0.8 = 3.8

(b) [4] (0) = 0

(c) 2 sin 0.8 π = 1.176

8.9 (p289)

(a) $t = 0^-$, so only 60 V is across the RL circuit. Thus $|v_L(0^-)| = \underline{0}$ and $|i_L(0^-)| = \frac{60}{10} = \underline{6\text{ A}}$

(b) At $t = 0^+$, the source voltage changes to 60 – 40 = 20 V. The inductor current cannot change, so $|i_L(0^+)| = \underline{6\text{ A}}$. The current through the resistor is 6 A, so the voltage dropped across the inductor is 20 – 10 (6) = – 40 V. Thus, $|v_L(0^+)| = \underline{40\text{ V}}$

(c) At $t = \infty$, the source voltage is 20 V but all transients have died out. Thus, $|i_L(\infty)| = \frac{20}{10} = \underline{2\text{ A}}$ and $|v_L(\infty)| = \underline{0}$. The direction of i_L has not changed.

(d) For $t > 0$, $|i_L(t)| = \left| i_L(\infty) + \left[i_L(0^+) - i_L(\infty) \right] e^{-t/\tau} \right|$

where $|i_L(t)| = 2\text{ A}$ and $\tau = L/R = \frac{50\times10^{-3}}{10} = 5\text{ ms}$.

Thus, $i_L(3\text{ ms}) = 2 + (6-2)e^{-3/5} = \underline{4.195\text{ A}}$

We then find that $|v_L(3\text{ ms})| = \left| 20 - 4.195(10) \right| = \underline{21.95\text{ V}}$

8.10 (p293)

(a) $v_S(0^-) = 0$ so $i_L(0^-) = \underline{0}$

(b) $i_L(0^+) = i_L(0^-)$ so $i_L(0^+) = \underline{0}$

τ = R/L = 4/200 = 1/50 s
i_f = 20/200 = 0.1 A (treating inductor as a short circuit)

We expect a solution of the form $i(t) = i_n(t) + i_f$, where i_f is given above and $i_n(t) = Ke^{-t/\tau}$. Thus, $i(t) = Ke^{-50t} + 0.1$. Since i(0) = 0, K = -0.1 A = -100 mA. Thus, $i(t) = 100\left(1 - e^{-50t}\right)$ mA.

(c) $i_L(8\text{ ms}) = \underline{32.97\text{ mA}}$ (note time units are NOT seconds; convert first)

(d) $i_L(15\text{ ms}) = \underline{52.76\text{ mA}}$

8.11 (p295)

(a) $i_R(0^-) = \underline{0}$

(b) $i_L(0^-) = 0$ so $i_L(0^+) = 0$

Thus, all of the source current is shunted through the 60–Ω resistor; hence,
$i_R(0^+) = \underline{10 \text{ mA}}$

(c) $i_R(\infty) = 10 \times \dfrac{40}{40+60} = \underline{4 \text{ mA}}$

(d) $\tau = \dfrac{L}{R_{eq}} = \dfrac{0.1}{40+60} = 1 \text{ ms}$

$$\begin{aligned} i_R(t) &= i_R(\infty) + \left[i_R(0^+) - i(\infty)\right]e^{-t/\tau} \\ &= 4 + [10-4]e^{-10^3 t} \text{ mA} \end{aligned}$$

so $i_R(1.5 \text{ ms}) = \underline{5.339 \text{ mA}}$

8.12 (p298)

(a) At $t = 0^-$, only the current source is on, so

$$v_C(0^-) = 1 \times [25//(20+80)] = \underline{20 \text{ V}}$$

(b) $v_C(0^+) = v_C(0^-)$, so $v_C(0^+) = \underline{20 \text{ V}}$

(c) At $t = \infty$, both sources are on, so

$$\begin{aligned} v_C(\infty) &= 1 \times [25//(20+80)] + 10 \times \frac{(100)}{125} \\ &= 20 + 8 = \underline{28 \text{ V}} \end{aligned}$$

(d) $v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]e^{-t/\tau}$

where $\tau = R_{eq} C$
$R_{eq} = 25//100 = 20$ kΩ, so $\tau = 100$ ms

Thus, $$\begin{aligned} v_C(80 \text{ ms}) &= 28 + [20-28]e^{-80/100} \\ &= \underline{24.41 \text{ V}} \end{aligned}$$

8.13 (p300)

For $t < 0$, all voltages and currents are zero, so $v(0^+) = v(0^-) = 0$.

For $t > 0$, we begin by pulling off the capacitor and performing a Thévenin transformation on the remaining circuit, yielding a final circuit consisting of a voltage source $23.5\cos 3t$ V in series with 14.7 Ω in series with the 22 μF capacitor.

Writing a simple KVL equation, $-23.5\cos 3t + 14.7 i_C + v = 0$ and noting that $i_C = C\frac{dv}{dt} = 22\times 10^{-6}\frac{dv}{dt}$, we can describe the voltage across the capacitor using

$$-23.5\cos 3t + (14.7)22\times 10^{-6}\frac{dv}{dt} + v = 0$$

or

$$\frac{dv}{dt} + 3092v = 72.67\times 10^{-3}\cos 3t$$

Anticipating that $v(t) = v_f(t) + v_n(t)$ with $v_n(t) = Ke^{-t/\tau} = Ke^{-3092t}$ and

$$v_f(t) = e^{-t/\tau}\int e^{t/\tau}\left(72.67\times 10^{-3}\cos 3t\right)dt = e^{-3092t}\int e^{3092t}\left(72.67\times 10^{-3}\cos 3t\right)dt$$

Performing the indicated integration, we find that $v_f(t) = 23.5\cos 3t + 0.0228\sin 3t$ V.
Thus, $v(t) = 23.5\cos 3t + 0.0228\sin 3t + Ke^{-3092t}$. Since v(0) = 0, K = -23.5 V and
$v(t) = 23.5\cos 3t + 0.0228\sin 3t - 23.5e^{-3092t}$ V.

8.14 (p305)

(a) The first term charges up the inductor, governed by the circuit time constant of L/R = 100 ms. Since the second term does not activate until $t = 2$ s, a full 20 time constants later, the voltage across the resistor reaches 3 V at around $t = 0.5$ s and stays there until 2 s. This results in a current which increases to 3 A. At 2 s, the second term effectively turns off the source, and the inductor begins to release its stored energy. It takes roughly 5 time constants for all of the energy to be dissipated (through the resistor), but since the third term does not activate until $t = 3$ s, the inductor current has plenty of time to reach zero. At $t = 4$ s, the process repeats.

(b) The first two terms are identical to part (a), so we expect the inductor to charge up over approximately 500 ms, then reach a "dc" current of 3 A until $t = 2$ s, when the source is removed and the inductor begins to discharge. However, the source turns on again at $t = 2.1$ s, only one time constant after the discharge cycle begins, so the inductor cannot fully discharge. The fourth term does not activate until 20 time constants later, so that the inductor can fully charge again. Thus, the inductor never fully discharges with this pulse train.

