MAXIMA LAB

LAB 1

DIFFERENTIAL CALCULUS

EXERCISE PROBLEMS

1) Find the first and second derivative of $y = x^5$

- y:x^5;
- y1:diff(y,x,1);
- y2:diff(y,x,2);

Exercise problems: Lab 1

1) If $y = x^4 + \log(3x^2+5x-2)$ find y4

```
y: x^4 +  log (3*x^2+5*x-2);
y4: diff(y,x,4);
```

2) If $y = x^3 + \log\left(\frac{1}{x}\right)$ find y10

```
y:x^3 +log (1/x);
y10:diff(y,x,10);
```

3) If y = $3sin^{-1}(2x) - 5\,cos^{-1}(3x)$ then find y5

```
y:3*asin(2*x) - 5*acos(3*x);
y5:diff(y,x,5);
```

4) If y = $\sin(\sin(x))$, Prove that $\frac{d^2y}{dx^2} + tanx\frac{dy}{dx} + ycos^2x = 0$

```
y:sin(sin(x));
y1:diff(y,x,1);
y2:diff(y,x,2);
y3: sin(x)/cos(x);
Y:y2+ y3·y1+y·cos(x)^2;
```

5) If $y = \cos x \cos 2x \cos 3x$ find y_9

```
y:cos(x)*cos(2*x)*cos(3*x);
y9: diff(y,x,9);
```

6) Find the 24th derivative of $y = e^{3x}\cos^2(x)\sin x$

```
y: %e^(3*x) *cos(x)^2*sin(x);
y24:diff(y,x,24);
```

7) If $y = e^{2\sin(x)} + x^x + \frac{1}{x^2+81}$ find y3.Also find the maximum number of derivatives that can be displayed by maxima

```
y: x^x+ %e^(2·sin(x))+ 1/(x^2+81);
y3:diff(y,x,3);
y9:diff(y,x,9);
```

<< expression longer than allowed by the configuration setting!>>

The maximum number of derivatives that can be displayed is 8.

8) If $y=\frac{3x+2}{x^2-2x+5}$ prove that $n\geq 2$, $5y_n(0)=2ny_{n-1}(0)-n(n-1)y_{n-2}(0)$. Hence compute $y_2(0)$.

```
y:(3*x+2)/(x^2-2*x+5);
```

```
for n:2 thru 20 do (
    y2:diff(y,x,n),
   y3:diff(y,x,n-1),
   y4:diff(y,x,n-2),
    if ev(5*y2,x=0)=2*n*ev(y3,x=0)-n*(n-1)*ev(y4,x=0)
then
       flag:1
  else (
        flag:0))$
if flag=1 then
    disp("the theorem is true for any n≥2")
else
    disp("Not true")$
```

```
y1:diff(y,x,2);
ev(y1,x=0);
```

Or

```
at(y1,x=0);
```

9. Show that the angle of intersection of the curves $\mathrm{r} = \sin(\theta) + \cos(\theta)$and $r = 2\sin(\theta)$is $\frac{\pi}{4}$ using the angle between the radius vector and tangent for the curves.

```
r1:sin(%theta)+cos(%theta);
r2: 2*sin(%theta);
A1:trigreduce(r1/diff(r1,%theta));
A2:trigreduce(r2/diff(r2,%theta));
phi:trigsimp(trigreduce((A1-A2)/(1+A1*A2)));
atan(phi);
```

10)Show that the curves $r^n = a^n \cos(n\theta)$ and $r^n = b^n \sin(n\theta)$ intersect orthogonally.

```
r1:(a^n*cos(n*%theta))^(1/n);
r2:(b^n*sin(n*%theta))^(1/n);
A1:(r1/diff(r1,%theta));
A2:(r2/diff(r2,%theta));
product:A1*A2;
```

11) Find the Radius of curvature at the origin for

$$y^2 = x^2\left(\frac{3+x}{3-x}\right).$$

```
y: x*((3+x)/(3-x))^(1/2);
y1:diff(y,x);
y2:diff(y1,x);
roc: ((1+y1)^(3/2))/y2;
at(roc,x=0);
```

LAB 2
PARTIAL DIFFERENTIATION
EXERCISE PROBLEMS

1) If U(x,y)= $e^{\frac{x}{y}}$ **, find**, $U_y, U_{xx}, U_{yy}, U_{xy}, U_{yx}$

```
u:%e^(x/y);
ux: diff(u,x);
uy:diff(u,y);
uxx:diff(ux,x);
uyy:diff(uy,y);
uxy:diff(ux,y);
uyx:diff(uy,x);
```

Exercise problems: Lab 2

- 1) If U = sin^{-1}(xyz)
  find $U_x, U_y, U_{xx}, U_{yy}, U_{xy}, U_{yx}, U_z, U_{zz}, U_{zy}, U_{zx}, U_{xz}$, $U_{yz}, U_{xxx}, U_{yyy}, U_{zzz}, U_{xxy}, U_{yxy}$.

- u:asin(x*y*z);
- ux: diff(u,x);
- uy:diff(u,y);
- uxx:diff(ux,x);
- uyy:diff(uy,y);
- uxy:diff(ux,y);
- uyx:diff(uy,x);
- uz:diff(u,z);
- uzz:diff(uz,z);

- uzy:diff(uz,y);
- uzx:diff(uz,x);
- uxz:diff(ux,z);
- uyz:diff(uy,z);
- uxxx:diff(uxx,x);
- uyyy:diff(uyy,y);
- uzzz:diff(uzz,z);
- uxxy:diff(uxx,y);
- uyxy:diff(uyx,y);

2)Graph the level surface $f(x, y, z) = 4y^2$-2$z^3 + x^2$= 0 and also graph their partial derivatives $f_x, f_y, f_{xx}, f_{yy}, f_{xxx}, f_{yyy}$

- f: 4·y^2−2·z^3+x^2;

plot3d(((4·y^2+x^2)/2)^(1/3),[x,−5,5],[y,−5,5],[plot_format,gnuplot]);

- plot3d(fx:diff(f,x),[x,−10,10],[y,−10,10],[plot_format,gnuplot]);
- plot3d(fy:diff(f,y),[x,−10,10],[y,−10,10], [plot_format,gnuplot]);
- plot3d(fxx:diff(f,x,2),[x,-5,5],[y,-5,5], [plot_format,gnuplot]);
- plot3d(fyy:diff(f,y,2),[x,-5,5],[y,-5,5], [plot_format,gnuplot]);
- plot3d(fxxx:diff(f,x,3),[x,-5,5],[y,-5,5], [plot_format,gnuplot]);
- plot3d(fyyy:diff(f,y,3),[x,-5,5],[y,-5,5], [plot_format,gnuplot]);

3)Show that the function $z = e^{(x^2-y^2)}$ cos(2xy) satisfies the Laplace equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

- **z:(%e^(x^2-y^2))*cos(2*x*y);**
- **zxx:diff(z,x,2);**
- **zyy:diff(z,y,2);**
- **zxx+zyy;**

- **4) Verify that the given function u=cos(3t)sinx satisfies the wave equation** $3^2 u_{xx} = u_{tt}.$

- **u:cos(3*t)*sin(x);**
- **utt:diff(u,t,2);**
- **uxx:diff(u,x,2);**
- **is((3^2)*uxx=utt);**

- **5) If z=**$(1-2xy+y^2)^{-1/2}$**, verify** $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=y^2z^3$.

- **z:(1-2*x*y+y^2)^(-1/2);**
- **zx:diff(z,x);**
- **zy:diff(z,y);**
- **is ((x*zx)- (y*zy)=(y^2)*(z^3));**

LAB 3
EULER'S THEOREM
EXERCISE PROBLEMS

1)If $u = ax^2 + 2hxy + y^2$, Verify Euler's theorem.

- u:a*x^2+2*h*x*y+b*y^2;
- ux:diff(u,x);
- uy:diff(u,y);
- euler:x*ux+y*uy;
- eulersimply:ratsimp(euler);
- f:factor(eulersimply);
- is(f=2*u);

2) If $u = \frac{x}{y}\cos\left(\frac{x}{y}\right)$, Verify Euler's theorem.

- u: (x/y)*cos(x/y);
- ux:diff(u,x);
- uy:diff(u,y);
- euler:x*ux+y*uy;
- eulersimply:ratsimp(euler);
- f:factor(eulersimply);
- is(f=2*u);

3) If $u = log\left(\frac{x^3+x^2y-y^2x+2y^3}{x+y}\right)$,Prove that

$$xu_x + yu_y = 2, x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = -2.$$

- u:log((x^3+x^2*y-y^2*x+2*y^3)/(x+y));
- ux:diff(u,x);
- uy:diff(u,y);
- euler:x*ux+y*uy;
- eulersimply:ratsimp(euler);
- uxx:diff(ux,x);
- uxy:diff(ux,y);
- uyy:diff(uy,y);
- euler2: x^2*uxx+2*x*y*uxy+y^2*uyy;
- eulersimply:ratsimp(euler2);

4) Verify Eulers theorem for

f(x,y,z)=$3x^2yz + 5xy^2\mathrm{z} + 4z^4$.

- u:3*x^2*y*z+5*x*y^2*z+4*z^4;
- ux:diff(u,x);
- uy:diff(u,y);
- uz:diff(u,z);
- euler:x*ux+y*uy+z*uz;
- f:factor(euler);
- is(f= factor(4*u));

5)If $u = tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, show that $xu_x + yu_y = sin2u \; and \; x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = (1 - \; 4sin\, u)sin2u$.

- u: atan((x^3+y^3)/(x+y)); ux:diff(u,x);
- uy:diff(u,y);
- euler:x*ux+y*uy;
- r:ratsimp(euler);
- is(r=ratsimp(trigexpand(sin(2*u))));
- uxx: diff(ux,x);
- uyy:diff(uy,y);
- uxy:diff(ux,y);
- euler2:x^2*uxx+2*x*y*uxy+y^2*uyy;
- r2:ratsimp(euler2);
- is(r2=ratsimp(trigexpand(1-4*sin(u)^2)*sin(2*u)));

JACOBIANS
EXERCISE PROBLEMS

1)If u= $x^2 - 2\ y$
and v= x+y. Find the Jacobian.

- **u:x^2-2*y;**
- **v:x+y;**
- **j:jacobian([u,v],[x,y]);**
- **D: determinant(j);**

2)If x=cosu,y=cosu sinu,z=cosw sinv sin u. Find the Jacobian.

- x:cos(u);
- y:cos(u)*sin(u);
- z:cos(w)*sin(v)*sin(u);
- j:jacobian([x,y,z],[u,v,w]);
- d:determinant(j);

3)If u=xy/z,v=yz/x,w= zx/y , Show that J=4.

- u:x*y/z;
- v:y*z/x;
- w:z*x/y;
- j:jacobian([u,v,w],[x,y,z]);
- d:determinant(j);

4)If u= $x^2-2\,y^2$ **and v =**$2\,x^2-y^2$ **where x = rcos θ, y =rsinθ,Show that J= r^3 sin2θ.**

- **u:x^2-2*y^2;**
- **v:2*x^2-y^2;**
- **j:jacobian([u,v],[x,y]);**
- **d: determinant(j);**
- **subst(r*cos(%theta),x,d);**
- **subst(r*sin(%theta),y,%);**
- **trigreduce(%);**

5) If $x=r\cos\theta, y=r\sin\theta$ find J and J’. Verify JJ’=1.

- x:r*cos(%theta);
- y:r*sin(%theta);
- j:jacobian([x,y],[r,%theta]);
- d: determinant(j);
- d: trigsimp(d);

Contd……………………..

- r1: (x^2+y^2)^0.5;
- theta:atan(y/x);
- jdash:jacobian([r1,theta],[x,y]);
- ddash:determinant(jdash);
- ratsimp(ddash);
- subst(r*cos(%theta),x,%);
- subst(r*sin(%theta),y,%);
- jdash:trigsimp(%o17);
- d:r;
- d*jdash;
- %^2;

TAYLOR'S AND MACLAURIN'S SERIES
EXERCISE PROBLEMS

1) **Find the Taylor's expansion of e^xcosy about the point x=1,y=π/4.**

- **u:%e^x*cos(y);**
- **taylor(u,[x,1,3],[y,%pi/4,3]);**

2)Find the Taylor's expansion of $\sqrt{1+x+y^2}$ in powers of (x-1) and (y-0).

- **u:(1+x+y^2)^0.5;**
- **taylor(u,[x,1,3],[y,0,3]);**

3) Find the Maclaurin's Series expansion of $e^x log(1+y)$upto the first six terms

- u:%e^x*log(1+y);
- taylor(u,[x,0,6],[y,0,6]);

4) Find the Second order Maclaurin's series expansion of e^{x+2y}.

- u:%e^(x+2*y);
- taylor(u,[x,0,2],[y,0,2]);

LAB 5
INTEGRAL CALCULUS-REDUCTION FORMULAE
EXERCISE PROBELMS

1) Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^4 x dx$

- **integrate(cot(x)^4,x,%pi/4,%pi/2);**

2) **Evaluate** $\int_0^{\frac{\pi}{4}} \sec^{11} x dx$

- **integrate(sec(x)^11,x,0,%pi/4);**
- **ratsimp(%)**

3) EVALUATE $\int_0^{\frac{\pi}{2}} e^{2x} \cos^3 x dx$

- **integrate(%e^(2*x)*cos(x)^3,x,0,%pi/2);**

4) EVALUATE $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^{13} x dx$

- **integrate(sin(x)^7·cos(x)^13,x,0,%pi/2);**

5) Evaluate $\int \sin^7 x dx$

- integrate(sin(x)^7,x);

6) Evaluate $\int_0^{\frac{\pi}{4}} \tan^7 x dx$

integrate(tan(x)^7,x,0,%pi/4);

LAB-6

DOUBLE INTEGRAL

EXERCISE PROBLEMS

1) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)dxdy$

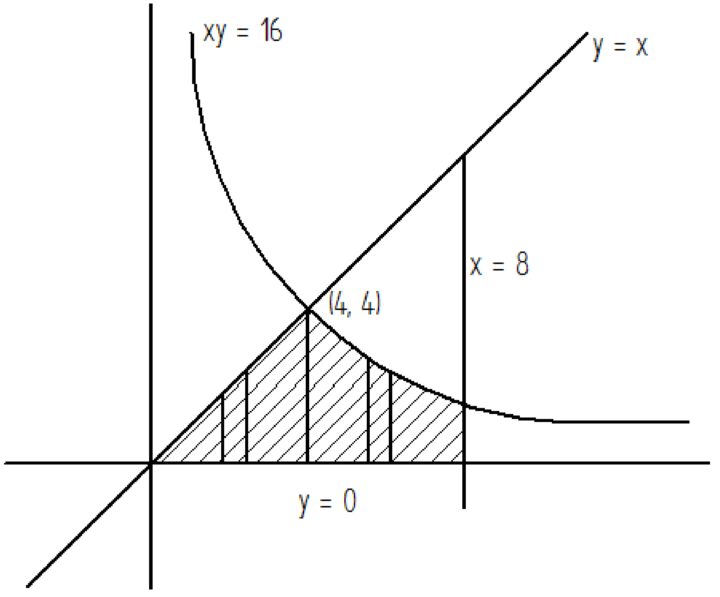
```
f(x,y):=x·(x^2+y^2);

integrate(f(x,y),y,0,x^2);

integrate(%,x,0,5);
```

2) EVALUATE $\int_0^1 \int_X^{\sqrt{X}} x(x^2 + y^2)\,dx\,dy$

```
f(x,y):=x*y;
Integrate(f(x,y),y,x,sqrt(x));
Integrate(%,x,0,1);
```

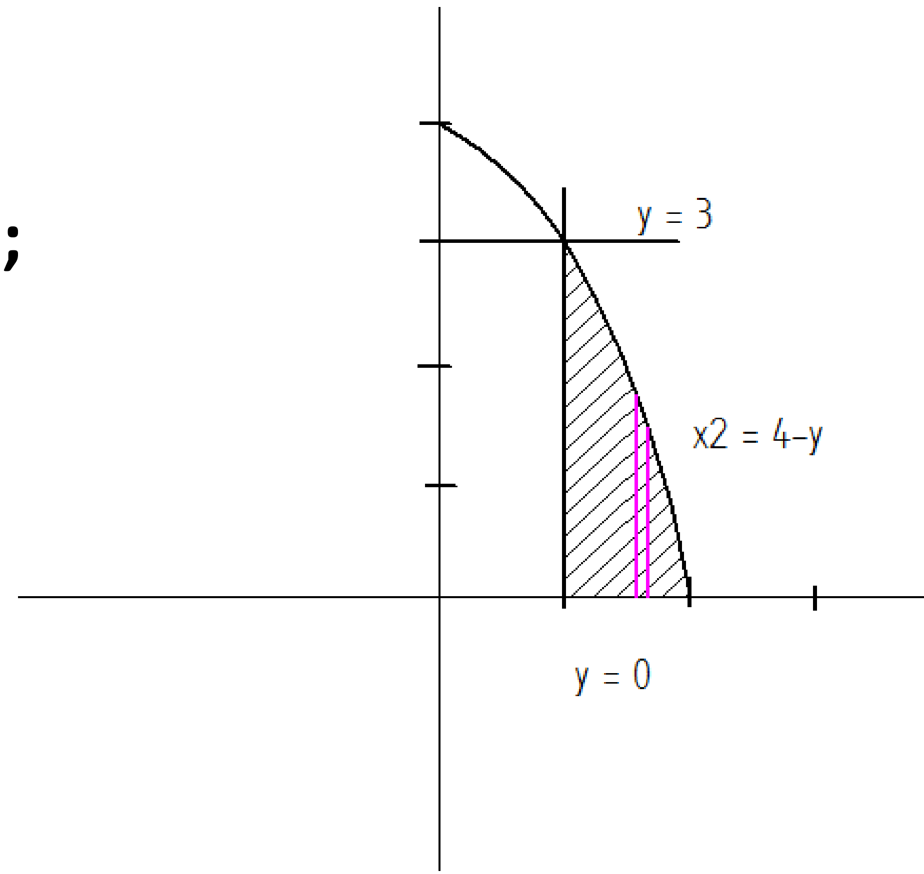
3) **Evaluate** $\iint_R x^2\,dxdy$ **by sketching the region in the first quadrant bounded by the lines x=y, y=0, x=8 and the curve xy=16.**

```
f(x,y):=x^2;
i1:integrate(f(x,y),y,0,x);
i1:integrate(%,x,0,4);
i2:integrate(f(x,y),y,0,16/x);
i2:integrate(%,x,4,8);
i:i1+i2;
```



4) Evaluate the following integral by changing the order of integration $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$

```
f(x,y):=x+y;
integrate(f(x,y),y,0,4−x^2);
integrate(%,x,1,2);
```



TRIPLE INTEGRALS

1. **Solve** $\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}(x^2+y^2+z^2)\,dx\,dy\,dz$

f(x,y,z):=x^2+y^2+z^2;

integrate(integrate(integrate(f(x,y,z),z,-a,a),y,-b,b),x,-c,c);

ratsimp(%);

2. Solve $\int_0^1 \int_{y^\wedge 2}^1 \int_0^{1-x} x \, dz dx dy$

```
f(x,y,z):=x;
integrate(integrate(integrate(f(x,y,z),z,0,1-
x),x,y^2,1),y,0,1);
```

3. Evaluate the triple integral of the function f(x,y,z)=x^2 over the region V enclosed by the plane x=0,y=0,z=0 and x+y+z=a.

```
f(x,y,z):=x^2;
integrate(integrate(integrate(f(x,y,z),z,0,a-x-y),y,0,a-x),x,0,a);
```

12) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$, $z=0$.

```
zmax:4-r*sin(theta);
v:integrate(integrate(integrate(r,z,0,zmax),r,0,2)
,%theta,0,2*%pi);
```

LAB 7-8
FIRST AND SECOND ORDER DIFFERENTIAL EQUATIONS
EXERCISE PROBLEMS

1) **Solve** $(x^2 - 1)\frac{dy}{dx} + 2xy = 1$ **.**

```
linode:(x^2-1)*(‘diff(y,x))+2*x*y=1;
ode2(linode,y,x);
```

2) Solve (x^2+1)dy+2xy dx= cotx dx

```
linode:(x^2+1)*(‘diff(y,x))+2*x*y-cot(x);
ode2(linode,y,x);
```

3. Solve $\frac{dy}{dx} + \frac{xy}{1-x^2} = x\sqrt{y}$

```
berode:('diff(y,x))+x*y/(1-x^2)-x*y^0.5;
ode2(berode,y,x);
```

4. Solve $\frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$

```
ode:('diff(y,x))-(x^3+y^3)/(x*y^2);
ode2(ode,y,x);
```

5. Solve $x^2 y \frac{dy}{dx} = xy^2 - e^{1/x^3}$

```
intfactorode: ‘diff(y,x,1)*(x^2*y)=(x*y^2-(%e)^(1/x^3));
ode2(intfactorode,y,x);
```

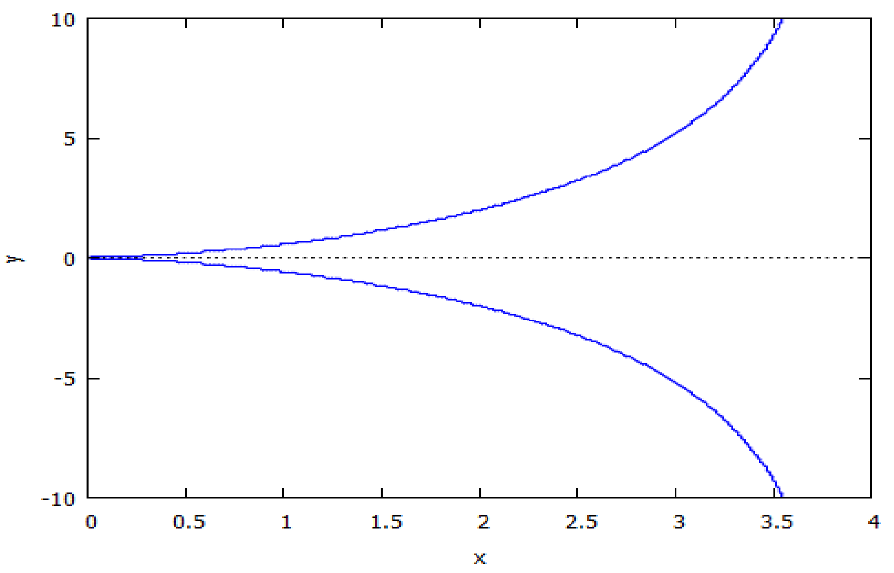
6. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx}$+2=0

```
ccode1:'diff(y,x,2)-3*'diff(y,x)+2=0;
ode2(ccode1,y,x);
```

LAB-9
Tracing of curves-Cartesian form

1) Plot the graph y^2*(4-x)=x^3

```
kill(all);
load(implicit_plot)$
implicit_plot(y^2*(4-x)=x^3,[x,0,4],[y,-10,10])$
```

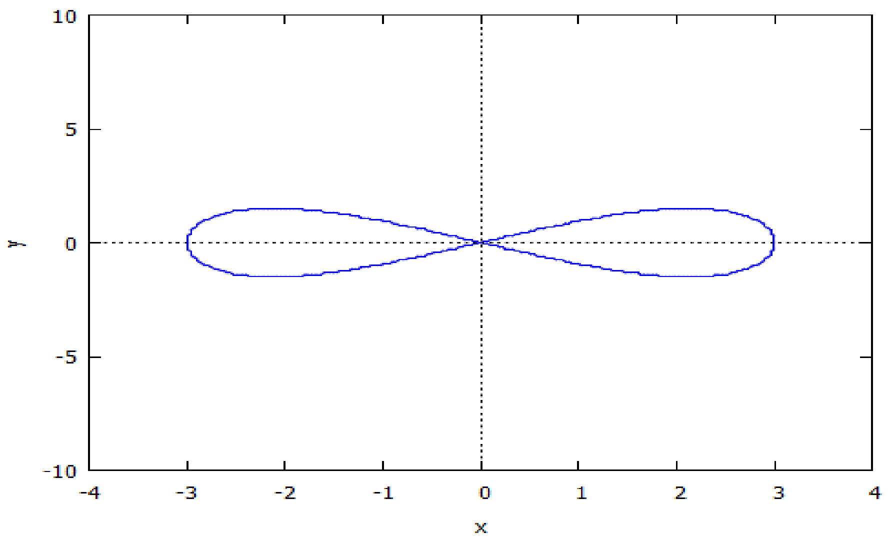


2)Plot the graph 9*y^2=x^2*(9-x^2)

kill(all);

load(implicit_plot)$

implicit_plot(9*y^2=x^2*(9-x^2),[x,-4,4],[y,-10,10])$

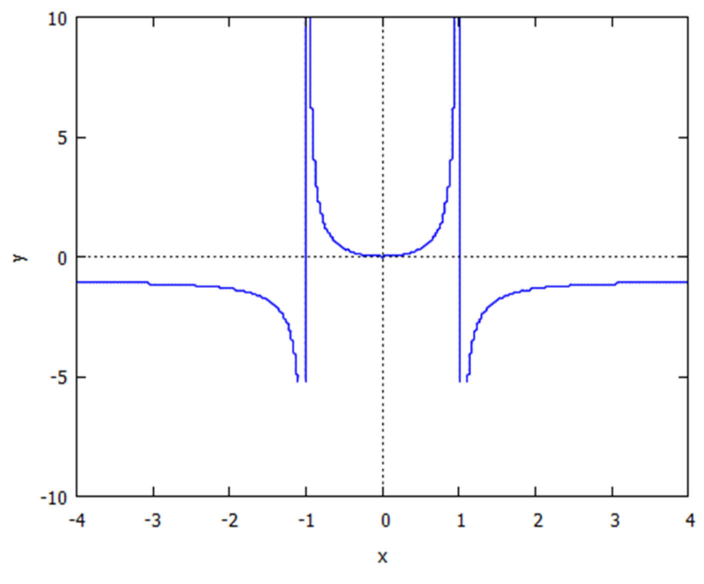


3.Plot the graph y=(x^2)/(1-x^2)

kill(all);

load(implicit_plot)$

implicit_plot(y=(x^2)/(1-x^2),[x,-4,4],[y,-10,10])$

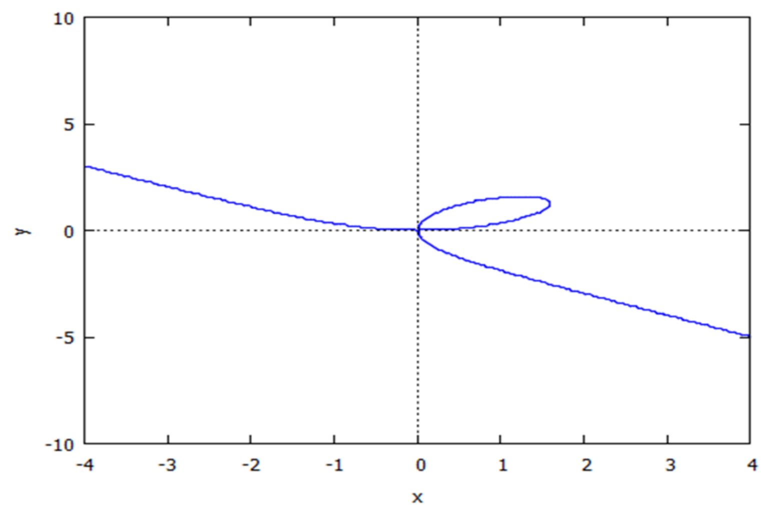


4.Plot the graph x^3+y^3=3*x*y

kill(all);

load(implicit_plot)$

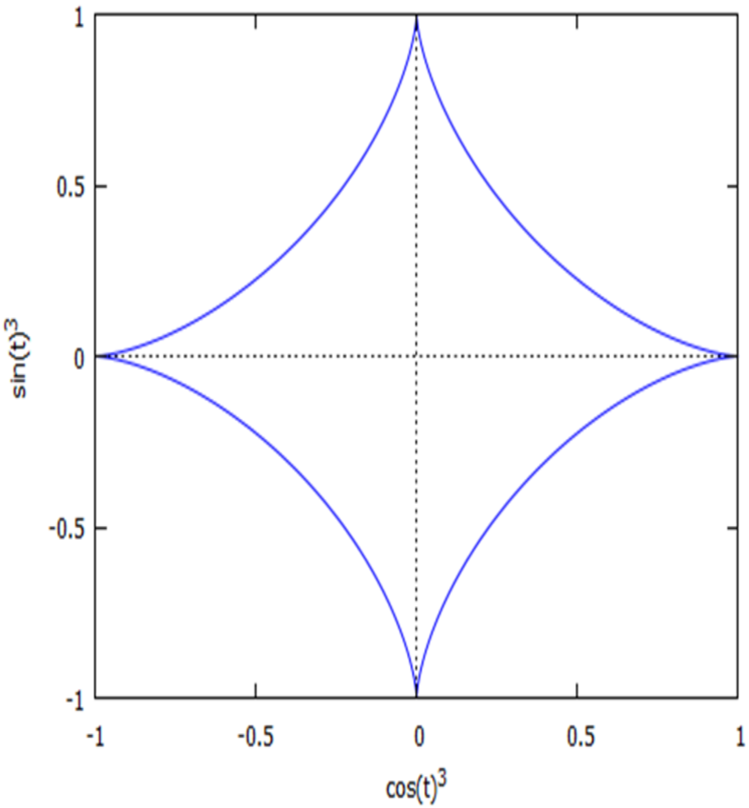
implicit_plot(x^3+y^3=3*x*y,[x,-4,4],[y,-10,10])$



Tracing of curves- parametric form

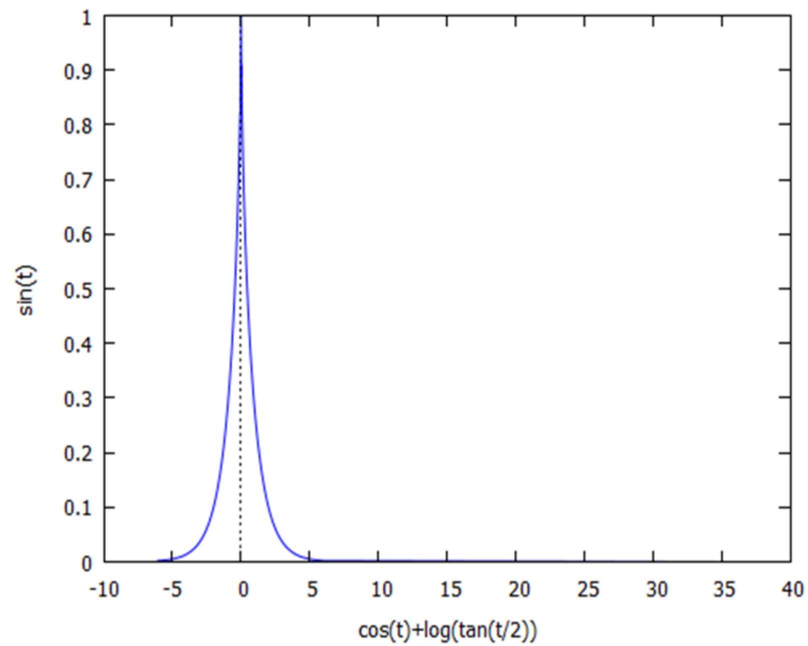
1) Plot the graph x=(cos(t))^3,y=(sin(t))^3

```
kill(all);
plot2d([parametric,(cos(t))^3,(sin(t))^3,[t,0,2*
%pi]])$
```



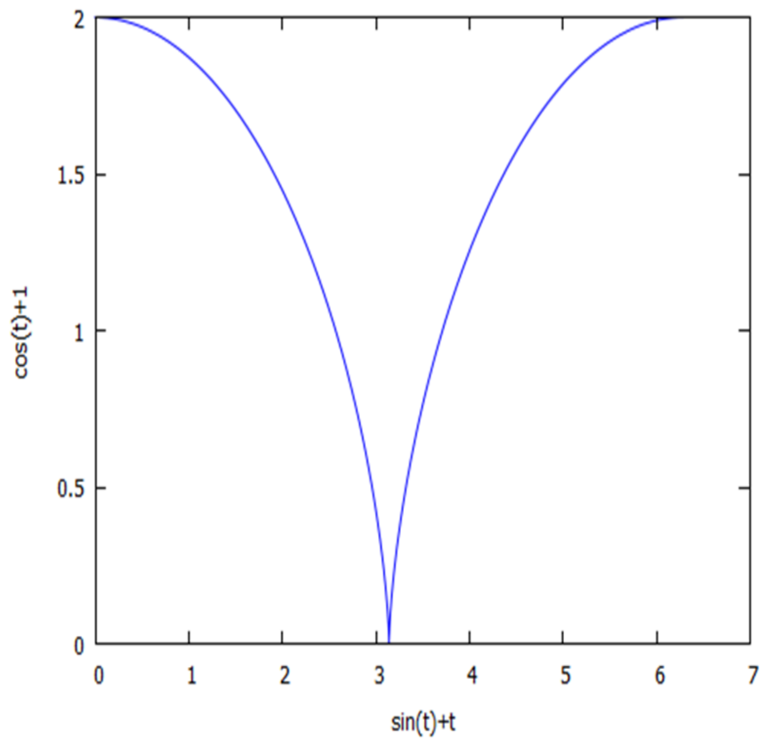
2)Plot the graph x=cos(t)+log(tan(t/2)),y=sin(t)

```
kill(all);
plot2d([parametric,cos(t)+log(tan(t/2)),sin(t),[t,
0,2*%pi]])$
```



3. Plot the graph x=t+sin(t),y=1+cos(t)

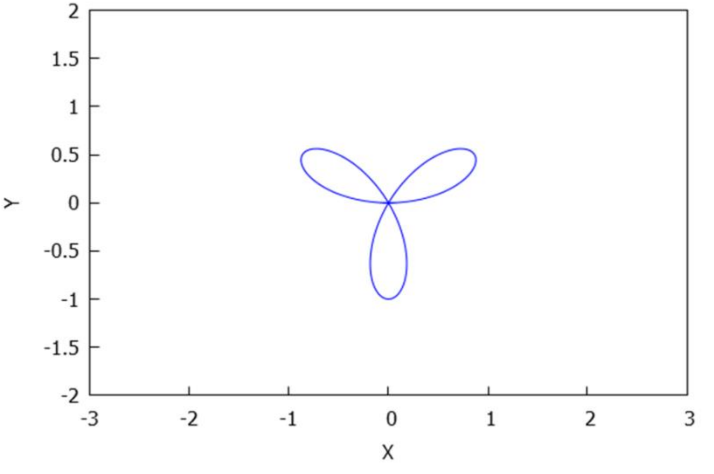
```
kill(all);
plot2d([parametric,t+sin(t),1+cos(t),[t,0,2*%pi]])$
```



LAB-10
Tracing of polar curves

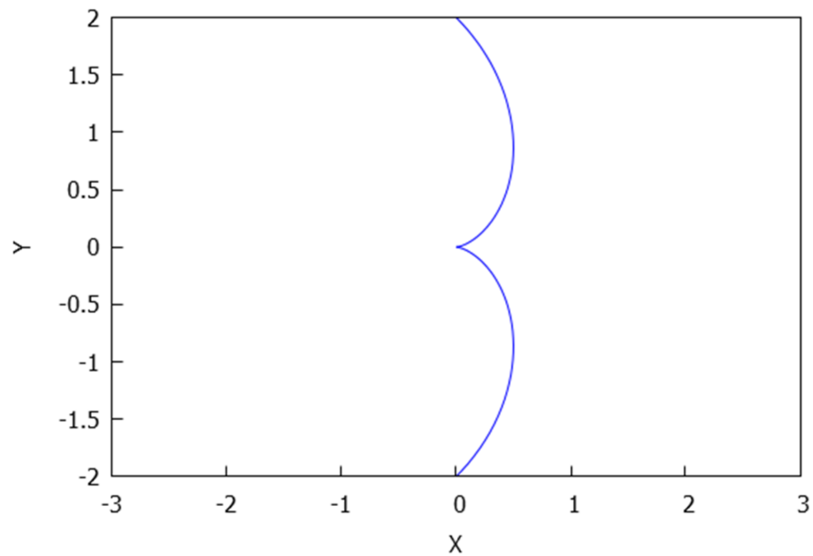
1) Plot $r = sin3\theta$

```
kill(all);
load("draw");
wxdraw2d(user_preamble="set grid polar",nticks=200,xrange=[-3,3],yrange=[-2,2],
polar(sin(3*%theta),%theta,0,2*%pi))$
```



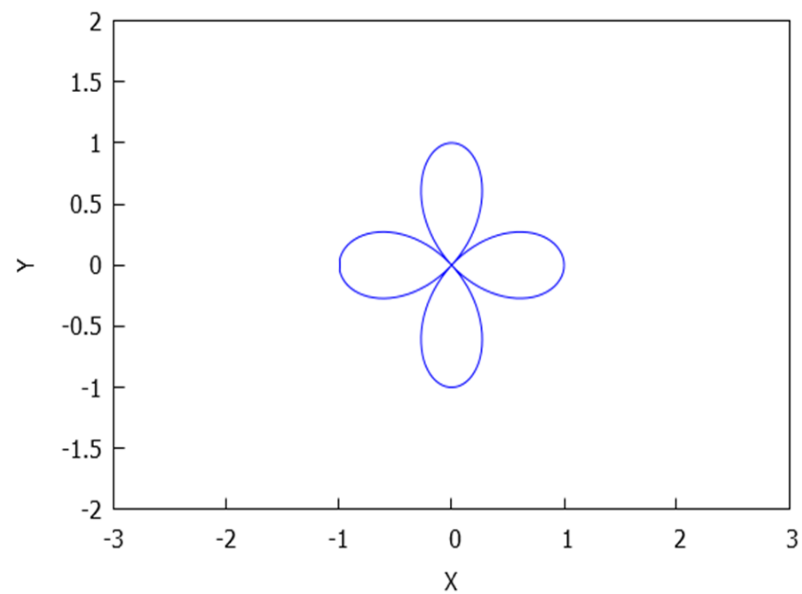
2) Trace the cardioid $r = 2(1 - cos\theta)$.

```
kill(all);
load("draw");
wxdraw2d(user_preamble="set grid
polar",nticks=200,xrange=[-3,3],yrange=[-2,2],
polar(2*(1-cos(%theta)),%theta,0,2*%pi))$
```



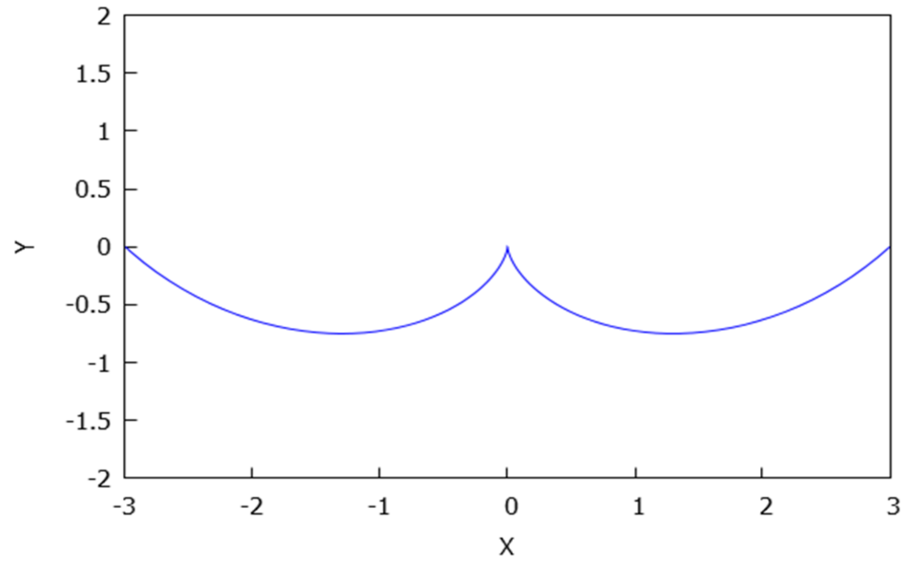
3) Trace the petal rose $r = cos2\theta$

```
kill(all);
load("draw");   (press shift+enter)
wxdraw2d(user_preamble="set grid polar",nticks=200,xrange=[-3,3],yrange=[-2,2],
 polar(cos(2*%theta),%theta,0,2*%pi))$
```



4)PLOT $r = 3(1 + sin\theta)$

- kill(all);
- load("draw"); (press shift+enter)
- wxdraw2d(user_preamble="set grid polar",nticks=200,xrange=[-3,3],yrange=[-2,2],
- polar(3*(1+sin(%theta)),%theta,0,2*%pi))$



5) Trace the curve $r^2 cos2\theta = 4$

- kill(all);
- load("draw"); (press shift+enter)

wxdraw2d(user_preamble="set grid polar",nticks=200,xrange=[-3,3],yrange=[-2,2],

polar(2/(cos(2*%theta))^(1/2),%theta,0,2*%pi))$