Network Analysis & Systems

Koshy George

Dept. of Elect. & Comm. Engineering,
PES University;
PES Centre for Intelligent Systems

UE18EC201: Network Analysis & Systems

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Network Analysis and Synthesis

Part V: Network Synthesis

Partial Fraction Expansion (1)

$$H(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \triangleq \frac{B(s)}{A(s)}$$

Case I: The poles are distinct (i.e., the roots are simple):

$$A(s) = \prod_{i=1}^{n} (s - p_i)$$

Then, the Heaviside's expansion theorem states that

$$H(s) = \sum_{i=1}^{n} \frac{k_i}{s - p_i}$$

where the residues are determined from

$$k_i = (s - p_i) H(s)\Big|_{s=p_i}$$

Partial Fraction Expansion (2)

$$H(s) = \sum_{i=1}^{n} \frac{k_i}{s - p_i}, \quad k_i = (s - p_i)H(s)\Big|_{s=p_i}$$

- The inverse Laplace transform is then

$$h(t) = \sum_{i=1}^{n} k_i e^{p_i t}$$

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Example:

$$\frac{s+4}{2s^2 + 5s + 3} =$$

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Example:

$$\frac{s+4}{2s^2+5s+3} = \frac{1}{2}\left(\frac{6}{s+1} - \frac{5}{s+3/2}\right)$$

Partial Fraction Expansion (3)

- If a pole p_i is complex with residue k_i, then the residue of the complex conjugate $\bar{p}_i$ is $\bar{k}_i$. Example:

$$\frac{s}{s^2 + 2s + 5} =$$

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$$\frac{s}{s^2+2s+5} = \frac{1}{4}\left(\frac{2+j1}{s+1-j2} + \frac{2-j1}{s+1+j2}\right)$$

Therefore,

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\} =$$

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Therefore,

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\} &= \frac{1}{4}\left((2+j1)e^{(-1+j2)t} + (2-j1)e^{(-1-j2)t}\right) \\ &= 0.5e^{-t}\left(2\cos 2t - \sin 2t\right)\end{aligned}$$

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Tip: Completion of squares:

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+5}\right\} &= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+2^2}\right\} \\ &= 0.5e^{-t}\left(2\cos 2t - \sin 2t\right)\end{aligned}$$

Partial Fraction Expansion (3)

- Euclidean Theorem: Given two polynomials $a(s)$ and $b(s)$ there exists polynomials $q(s)$ and $r(s)$ s.t.

$$a(s) = b(s)q(s) + r(s)$$

where $\deg r(s) < \deg b(s)$. Here $q(s)$ is the quotient and $r(s)$ the remainder.

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- (Remainder Theorem:) In particular let $b(s) = s - \lambda$. Then, the degree of $r(s)$ is 0, i.e., $r(s)$ is a constant R. Hence,

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  Clearly, $a(\lambda) = R$.

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$$a(s) = (s - \lambda)q(s) + R$$

  Clearly, $a(\lambda) = R$.
- Note: $s - \lambda$ is a root of $a(s)$ if, and only if, $R = 0$.

Partial Fraction Expansion (4)

$$
\begin{aligned}
H(s) &= \frac{s^4 - 4s^3 + 7s^2 - 24s + 36}{(s^2+2s+5)(s^2+4s+13)(s+4)} \\
&= \frac{k_1}{s+1-j2} + \frac{\bar{k}_1}{s+1+j2} + \frac{k_2}{s+2-j3} + \frac{\bar{k}_2}{s+2+j3} + \frac{k_3}{s+4}
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Computation of k_1:

$$B(s) = (s^2+2s+5)\underbrace{(s^2-6s+14)}_{q_1(s)} + \underbrace{(-22s-34)}_{r_1(s)}$$

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$$

$$
(s^2+4s+13)(s+4) = (s^2+2s+5)\underbrace{(s+6)}_{q_2(s)} + \underbrace{(12s+22)}_{r_2(s)}
$$

Partial Fraction Expansion (5)

Computation of k_1 (residue of $p_1(s) = s + 1 - j2$):

- Let $p_2(s) = s + 1 + j2$.
- Therefore, using the remainder theorem,

$$k_1 = \frac{r_1(-1 + j2)}{r_2(-1 + j2)p_2(-1 + j2)} = -0.05621 + j0.43491$$

- Clearly, the residue at p_2 is $-0.05621 - j0.43491$.

Partial Fraction Expansion (6)

Computation of k_2:

$$B(s) = (s^2 + 4s + 13)\underbrace{(s^2 - 8s + 26)}_{q_1(s)} + \underbrace{(-24s - 302)}_{r_1(s)}$$

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- Let $p_4(s) = s + 2 + j3$.
- Therefore, using the remainder theorem,

$$k_2 = \frac{r_1(-2 + j3)}{r_2(-2 + j3)p_4(-2 + j3)} = -1.68047 + j0.20020$$

Partial Fraction Expansion (7)

Computation of k_3:

$$B(s) = (s+4)\underbrace{\left(s^3 - 8s^2 + 39s - 180\right)}_{q_1(s)} + \underbrace{756}_{r_1(s)}$$

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Therefore, using the remainder theorem,

$$k_3 = \frac{756}{169} = 4.47337$$

Partial Fraction Expansion (8)

Residues at simple poles:

- Recall:

$$k_1 = (s - p_1)H(s)\Big|_{s=p_1} =$$

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$$k_1 = (s - p_1)H(s)\Big|_{s=p_1} = \frac{(s - p_1)B(s)}{(s - p_1)A_1(s)}\Bigg|_{s=p_1} = \frac{B(p_1)}{A_1(p_1)}$$

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- Alternatively, as $A'(s) = A_1(s) + (s - p_1)A_1'(s)$,

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- Alternatively,

$$k_1 = \left.\left(\frac{d}{ds}\left(\frac{1}{H(s)}\right)\right)^{-1}\right|_{s=p_1}$$

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- Alternatively,

$$k_1 = \left(\frac{d}{ds}\left(\frac{1}{H(s)}\right)\right)^{-1}\Bigg|_{s=p_1} = \frac{B^2(s)}{B(s)A'(s) - A(s)B'(s)}\Bigg|_{s=p_1}$$

Partial Fraction Expansion (9)

Case II: The poles are not distinct (i.e., there exists multiple roots): Suppose that, for example, p_1 is repeated q times, and the remaining poles are distinct:

$$A(s) = (s - p_1)^q \prod_{i=2}^{n-q} (s - p_i)$$

Then, the Heaviside's expansion theorem states that

$$H(s) = \sum_{j=1}^{q} \frac{k_{1j}}{(s - p_i)^j} + \sum_{i=2}^{n-q} \frac{k_i}{s - p_i}$$

The residues $k_2, \ldots, k_{n-q}$ are determined from

$$k_i = (s - p_i)H(s)\big|_{s=p_i}, \quad i = 2, 3, \ldots, n - q$$

Partial Fraction Expansion (10)

$$H(s) = \sum_{j=1}^{q} \frac{k_{1j}}{(s-p_i)^j} + \sum_{i=2}^{n-q} \frac{k_i}{s-p_i}$$

The quantities $k_{11}, \ldots, k_{1q}$ are obtained as follows:

$$k_{1j} = \frac{1}{(q-j)!} \left(\frac{d^{q-j}}{ds^{q-j}} (s-s_1)^q H(s) \right) \Bigg|_{s=s_1}, \quad j = 1, 2, \ldots, q$$

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Example:

$$\frac{s+2}{s(s+1)^2(s+3)} =$$

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Example:

$$\frac{s+2}{s(s+1)^2(s+3)} = \frac{2/3}{s} - \frac{1/2}{(s+1)^2} - \frac{3/4}{s+1} + \frac{1/12}{s+3}$$

Partial Fraction Expansion (11)

$$\frac{1}{3s^2(s^2+4)} \quad =$$

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$$\frac{1}{3s^2(s^2+4)} = \frac{1}{3}\left(\frac{k_{11}}{s} + \frac{k_{12}}{s^2} + \frac{\alpha s + \beta}{s^2+4}\right)$$

Partial Fraction Expansion (11)

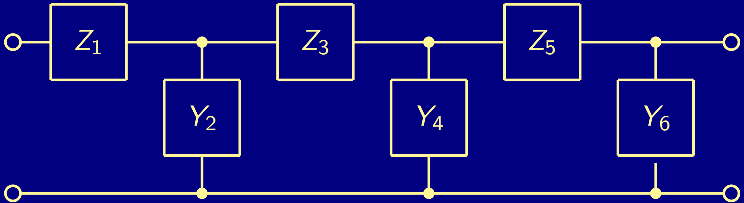
$$\begin{aligned}\frac{1}{3s^2(s^2+4)} &= \frac{1}{3}\left(\frac{k_{11}}{s}+\frac{k_{12}}{s^2}+\frac{\alpha s+\beta}{s^2+4}\right)\\ &= \frac{1}{3}\left(\frac{1/4}{s^2}+\frac{-1/4}{s^2+4}\right)\end{aligned}$$

Note that

$$\begin{aligned}1 &= k_{11}s(s^2+4)+k_{12}(s^2+4)+(\alpha s+\beta)s^2\\ k_{12} &= s^2H(s)\Big|_{s=0}=\frac{1}{12}\end{aligned}$$

and compare coefficients.

Continued Fraction Expansion (1)



$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

- This is an example of continued fraction and Z is the impedance of the ladder network.

Continued Fraction Expansion (2)

Given a network function $H(s)$, it may be expressed as the following continued fraction expansion:

$$H(s) = b_1(s) + \cfrac{a_2(s)}{b_2(s) + \frac{a_3(s)}{b_3(s)+\cdots}}$$

Continued Fraction Expansion (2)

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Example:

$$\frac{s^4 + 3s^2 + 1}{s^3 + 2s} = s + \cfrac{1}{s + \frac{1}{s+\frac{1}{s}}}$$

Continued Fraction Expansion (3)

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$$\begin{array}{lllllll}
s^3+2s) & s^4+3s^2+1 & (s & & & & \\
 & s^4+2s^2 & & & & & \\
\hline
 & s^2+1) & s^3+2s & (s & & & \\
 & & s^3+s & & & & \\
\hline
 & & s) & s^2+1 & (s & & \\
 & & & s^2 & & & \\
\hline
 & & & 1) & s & (s & \\
 & & & & s & & \\
\hline
 & & & & 0 & & \\
\hline
\end{array}$$

Continued Fraction Expansion (4)

$$H(s) = \frac{B(s)}{A(s)}$$

- Determine $q_1(s)$ and $r_1(s)$ s.t. $B(s) = A(s)q_1(s) + r_1(s)$.

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- and, so on.

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- and, so on.
- With each step the degree of the remainder polynomial decreases, and so the algorithm must stop after a finite number of steps.

Continued Fraction Expansion (5)

$$\frac{s^4 + 3s^2 + 1}{s^3 + 2s} = q_1 + \frac{1}{q_2 + \frac{1}{q_3 + \frac{1}{q_4}}}$$

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$$3s^5 + 15s^3 + 12s = r_1(s)\underbrace{s}_{q_2} + \underbrace{2s^3 + 8s}_{r_2}$$

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$$3s^5 + 15s^3 + 12s = r_1(s)\underbrace{s}_{q_2} + \underbrace{2s^3 + 8s}_{r_2}$$

$$r_1(s) = r_2(s)\underbrace{\frac{3s}{2}}_{q_3} + \underbrace{s^2 + 4}_{r_3}$$

Continued Fraction Expansion (5)

$$H(s) = \frac{s^6 + 8s^4 + 17s^2 + 4}{3s^5 + 15s^3 + 12s}$$

$$s^6 + 8s^4 + 17s^2 + 4 = (3s^5 + 15s^3 + 12s)\underbrace{\frac{s}{3}}_{q_1} + \underbrace{3s^4 + 13s^2 + 4}_{r_1}$$

$$3s^5 + 15s^3 + 12s = r_1(s)\underbrace{s}_{q_2} + \underbrace{2s^3 + 8s}_{r_2}$$

$$r_1(s) = r_2(s)\underbrace{\frac{3s}{2}}_{q_3} + \underbrace{s^2 + 4}_{r_3}$$

$$r_2(s) = r_3(s)\underbrace{2s}_{q_4} + \underbrace{0}_{r_4}$$

Continued Fraction Expansion (6)

$$\frac{s^6+8s^4+17s^2+4}{3s^5+15s^3+12s} = q_1+\cfrac{1}{q_2+\cfrac{1}{q_3+\cfrac{1}{q_4}}}$$

$$= \frac{s}{3}+\cfrac{1}{s+\cfrac{1}{\frac{3s}{2}+\frac{1}{2s}}}$$