Questions on self learning components

- 1. Find the nth derivative of $\sin^4 x$
- 2. Find the nth derivative of $\frac{x^4}{(x-1)(x-2)}$
- 3. Find the nth derivative of $e^{3x} \cos x \sin^2 x$

4.If
$$y = \tan^{-1} \frac{\sqrt{x^2 + 1} - 1}{x}$$
, show that $y_n = \frac{1}{2} (-1)^{n-1} (n-1)! \sin(\theta) \sin^n(\theta)$

5. If
$$y\sqrt{(1+x^2)} = \log(x+\sqrt{x^2+1})$$
, prove that $(1+x^2) y_{n+2} + (2n+3) xy_{n+1} + (n+1)^2 y_n = 0$

6.If,
$$y = \cos\left(m\sin^{-1}x\right)$$
 show that $(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$. Also find $y_n(0)$.

7. Evaluate
$$\int_{0}^{2a} \frac{x^3}{\sqrt{(2 ax - x^2)}} dx$$

8. Evaluate
$$\int_{0}^{a} x^{2} (a^{2} - x^{2})^{3/2} dx$$

9. Evaluate
$$\int_{0}^{\infty} \frac{x^3}{(a^2 + x^2)^5} dx$$

10.Evaluate
$$\int_{0}^{\pi/4} \tan^{6} x \, dx$$