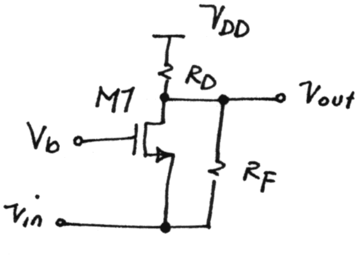
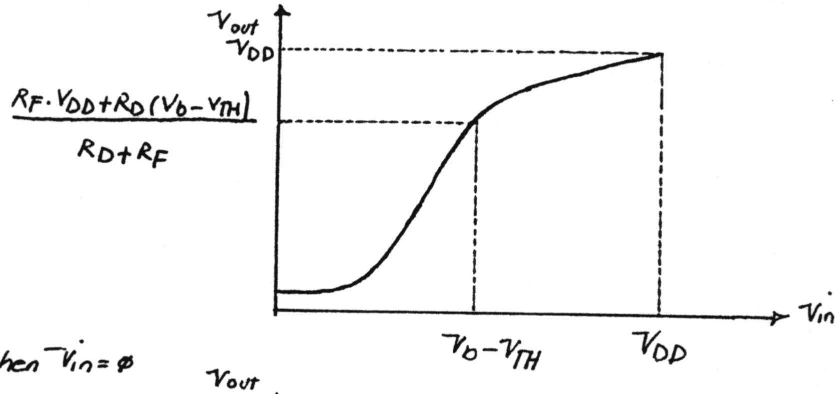
Chapter 3

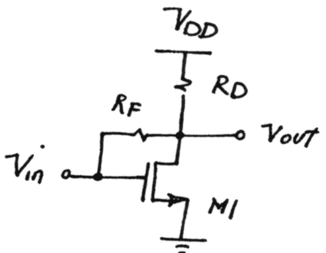
3.1.



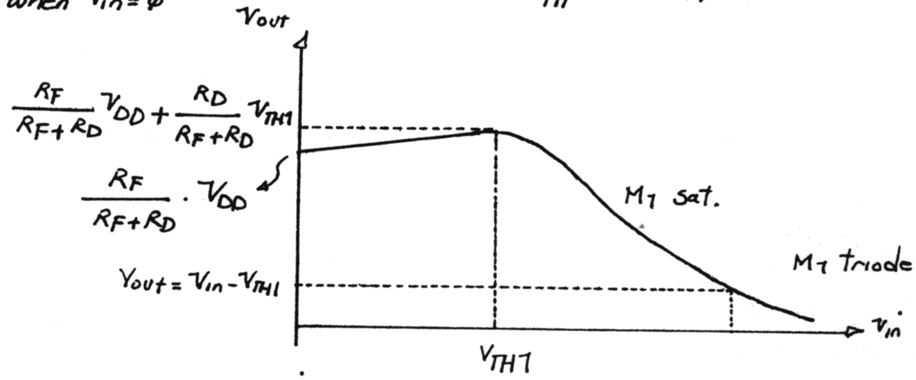
(a)



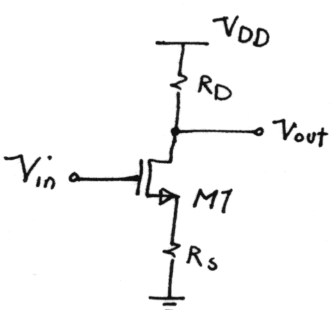
We assume that M_1 is saturated when $V_{in} = \phi$



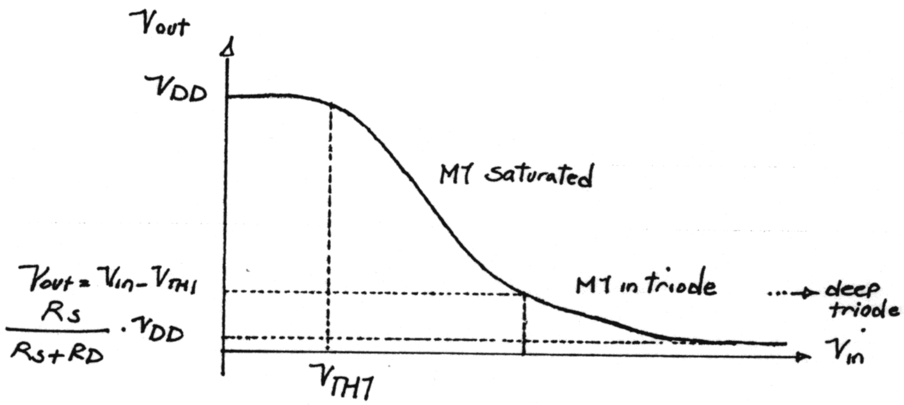
(b)

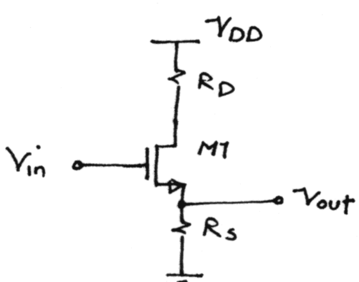


if $V_{in} < V_{TH1} \rightarrow V_{out} = \frac{R_F}{R_F+R_D} V_{DD} + \frac{R_D}{R_F+R_D} V_{in}$

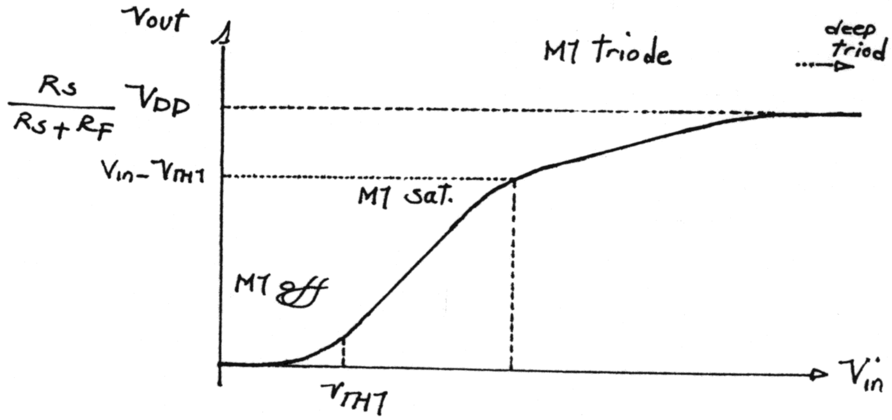


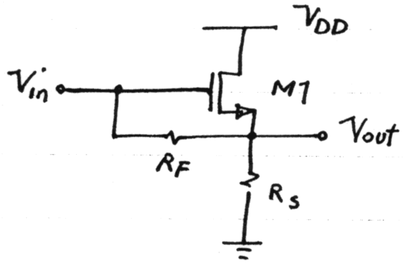
(c)



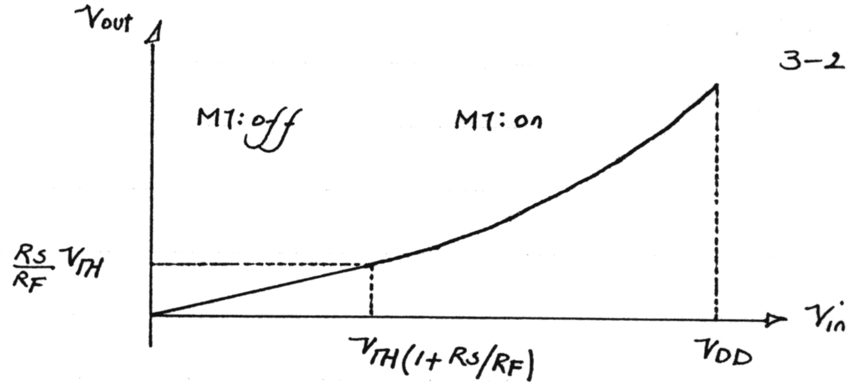


(d)



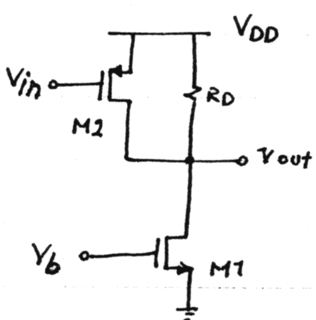


(e)

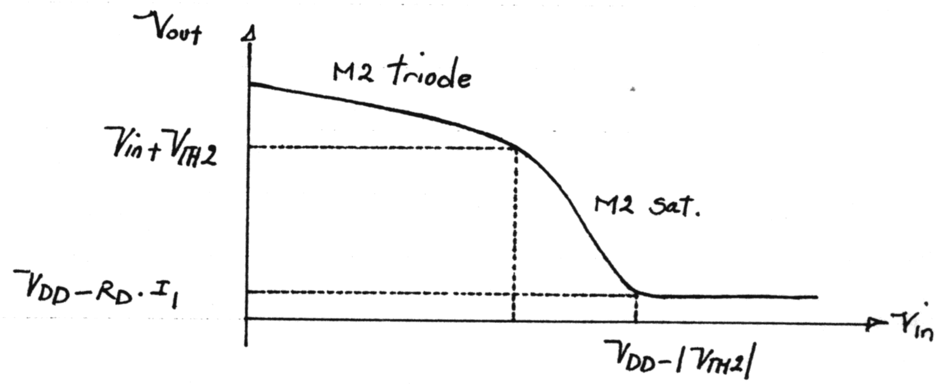


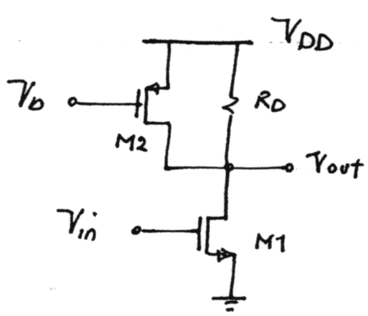
if $V_{in} < (1+R_S/R_F)V_{TH} \rightarrow V_o = \frac{R_S}{R_S+R_F} V_{in}$

3.2.

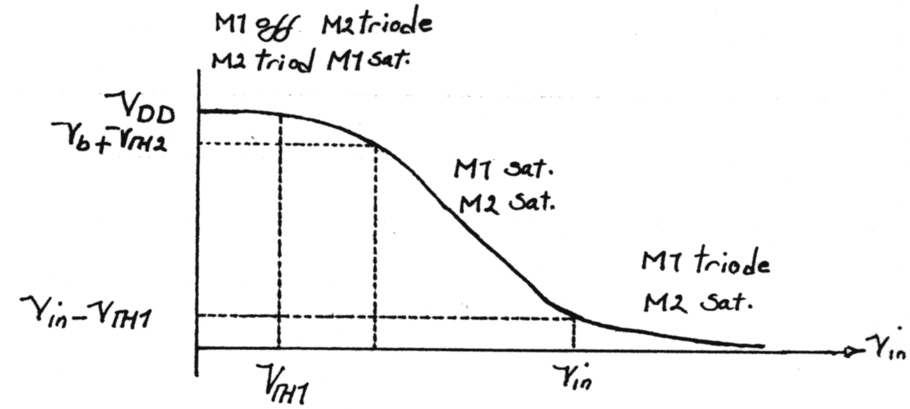


(b)

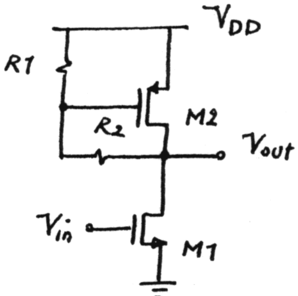




(a)

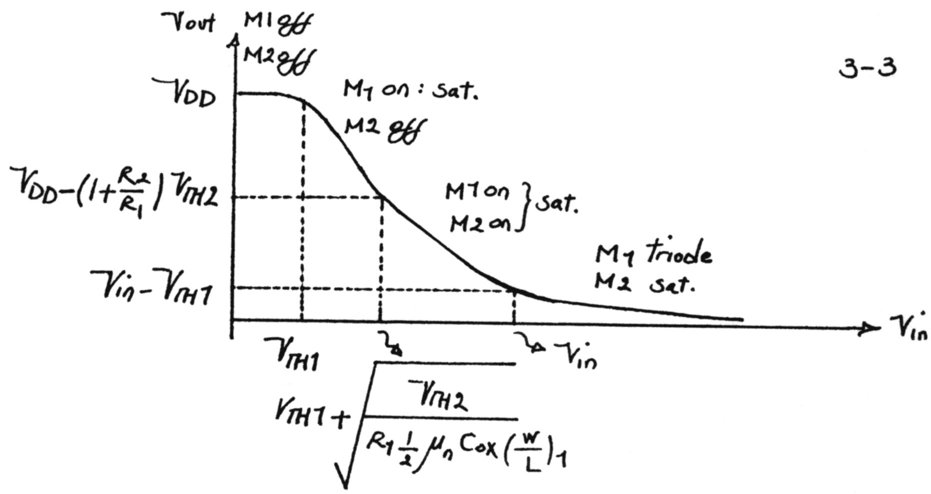


3-3

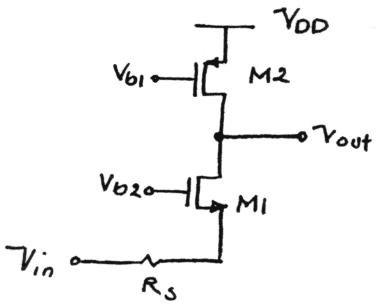


VDD
R1
R2
M2
Vout
Vin
M1

(c)

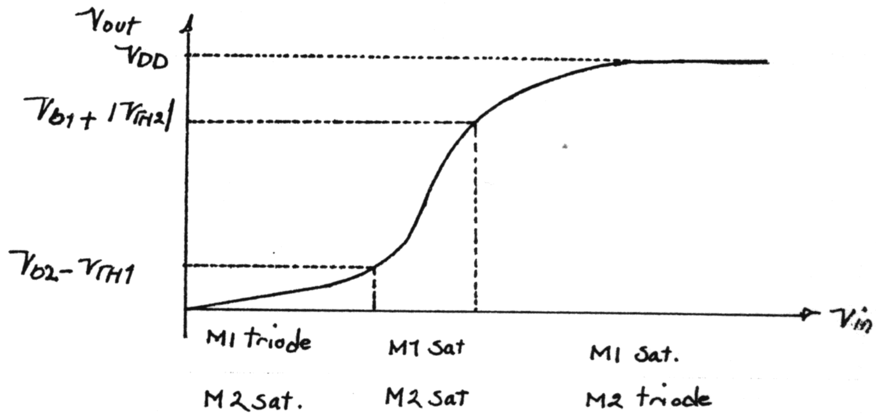


Vout
M1 off
M2 off
VDD
M1 on : sat.
M2 off
$V_{DD}-(1+\frac{R_2}{R_1})V_{TH2}$
M1 on
M2 on
sat.
M1 triode
M2 sat.
$V_{in}-V_{TH1}$
V_{TH1}
Vin
$V_{TH1}+\sqrt{\frac{V_{TH2}}{R_1\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1}}$



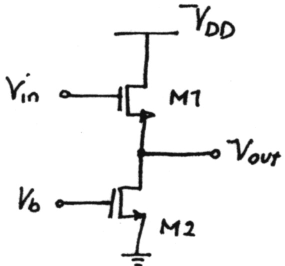
VDD
Vb1
M2
Vout
Vb2
M1
Vin
RS

(d)



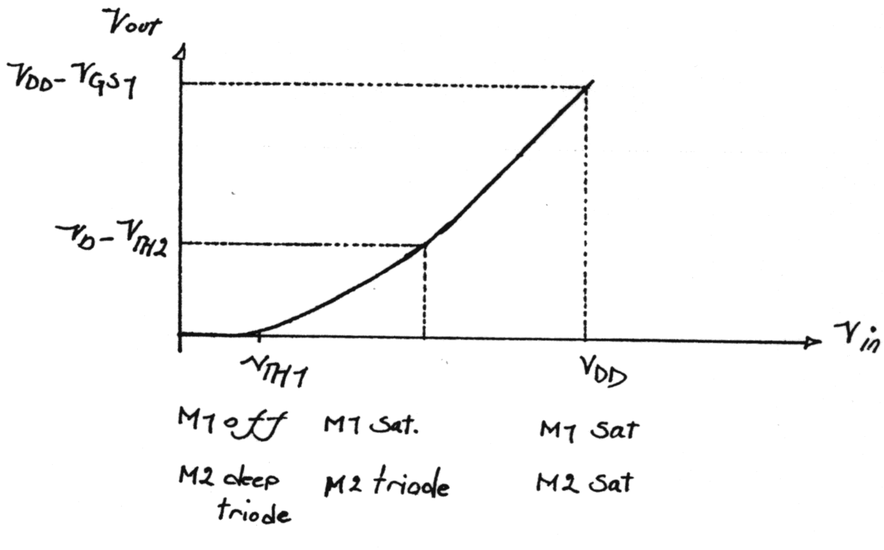
Vout
VDD
$V_{b1}+|V_{TH2}|$
$V_{b2}-V_{TH1}$
Vin
M1 triode
M2 sat.
M1 Sat
M2 Sat
M1 Sat.
M2 triode

3.2:

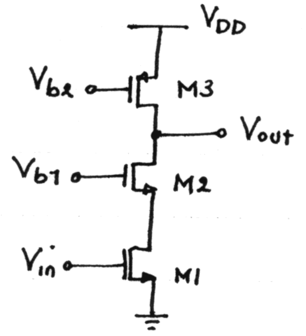


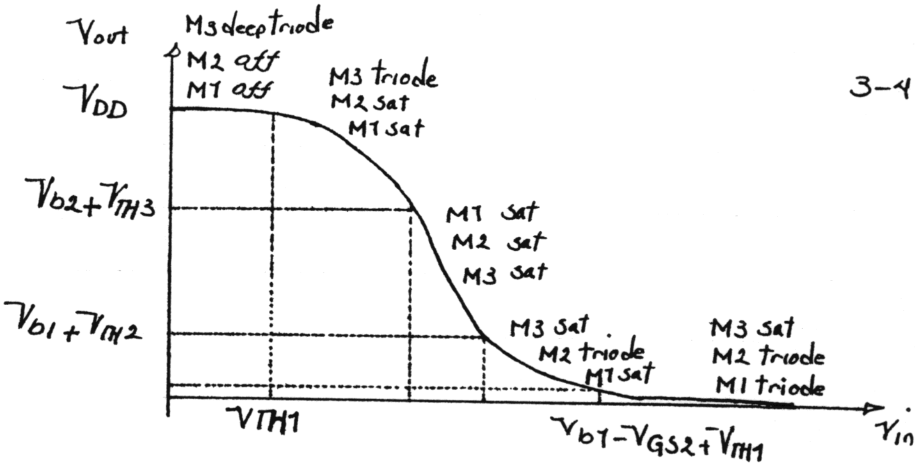
VDD
Vin
M1
Vout
Vb
M2

(a)

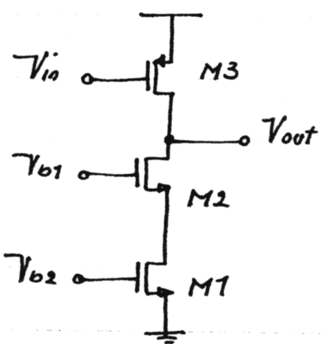


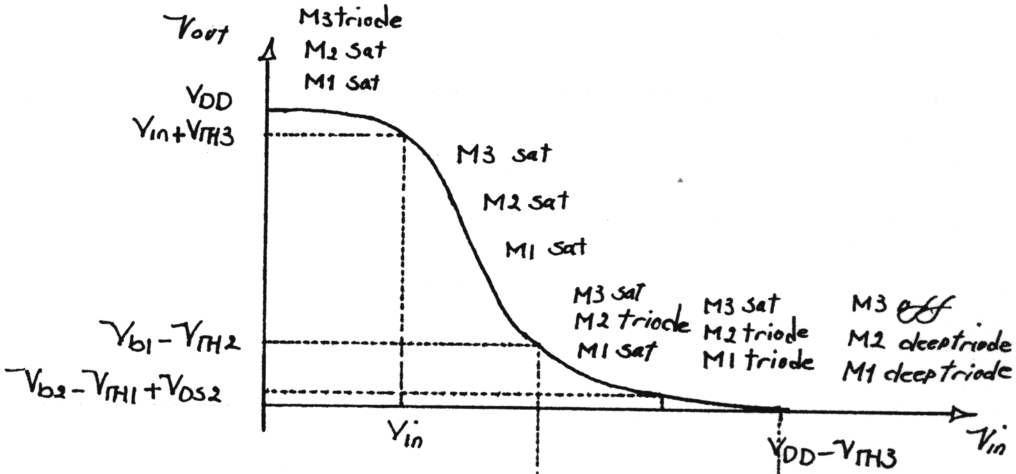
Vout
$V_{DD}-V_{GS1}$
V_b-V_{TH2}
Vin
V_{TH1}
VDD
M1 off
M2 deep triode
M1 Sat.
M2 triode
M1 Sat
M2 Sat





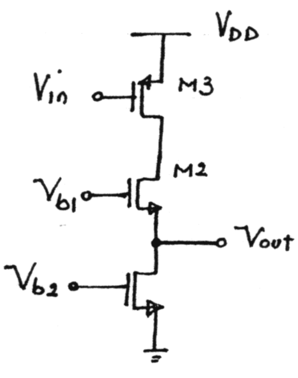
(b)

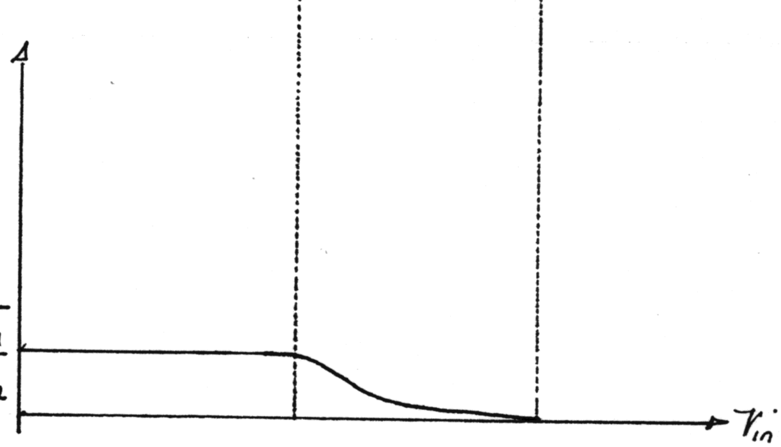




(C)

V_{DS2} is obtained from $\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b2}-V_{TH1})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{b1}-V_{b2}+V_{TH1}-V_{TH2})V_{DS2}-\frac{V_{DS2}^2}{2}\right]$



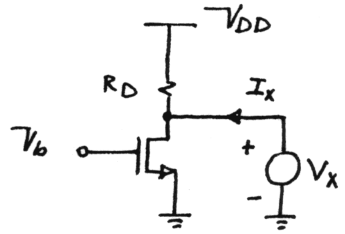


$(V_{b1}-V_{TH2})-(V_{b2}-V_{TH1})\sqrt{\frac{(\frac{W}{L})_1}{(\frac{W}{L})_2}}$

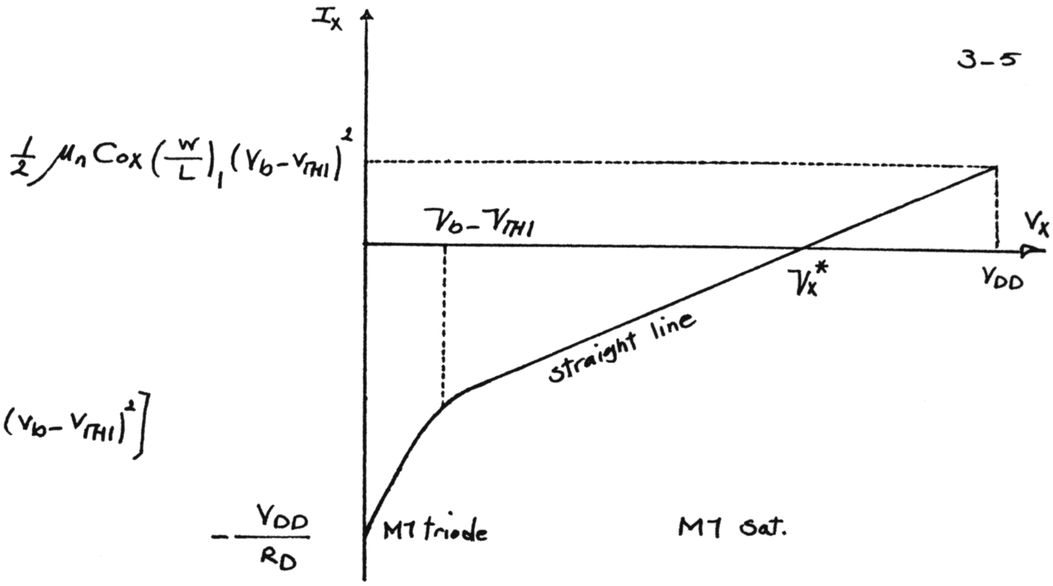
(d)

3.3.

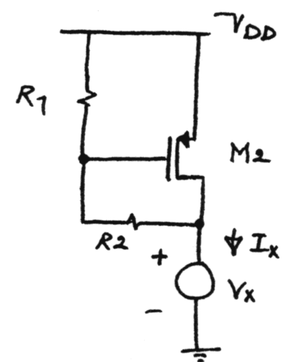
3-5



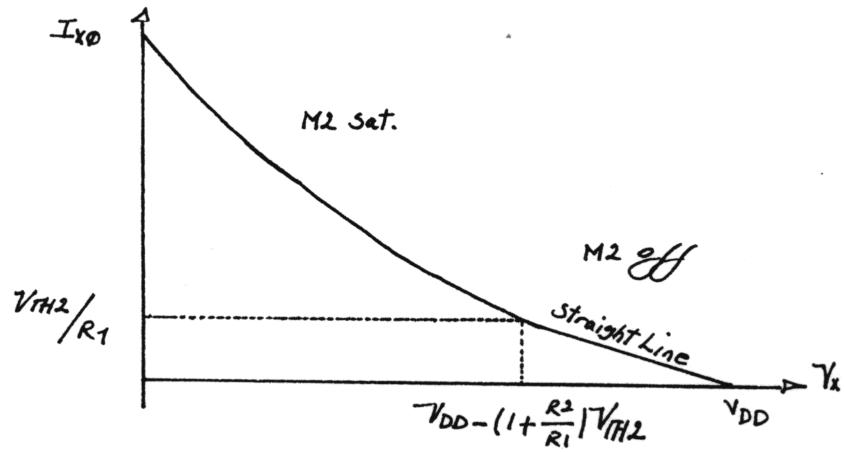
(a)



$$V_X^* = V_{DD} - R_D\left[\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2\right]$$

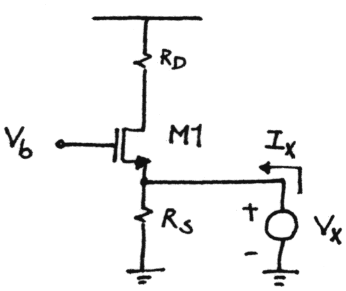


(b)

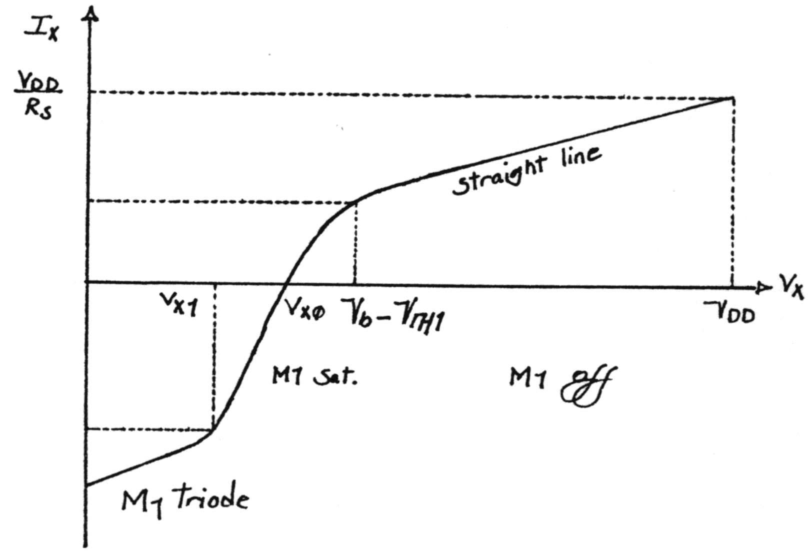


If $V_X < V_{DD} - (1+\frac{R_2}{R_1})V_{TH2}$ $\quad I_X = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[\frac{(V_{DD}-V_X)}{R_1+R_2}\cdot R_1 - V_{TH2}\right]^2 + \frac{V_{DD}-V_X}{R_1+R_2}$

If $V_X > V_{DD} - (1+\frac{R_2}{R_1})V_{TH2}$ $\quad I_X = \frac{V_{DD}-V_X}{R_1+R_2}$



(c)

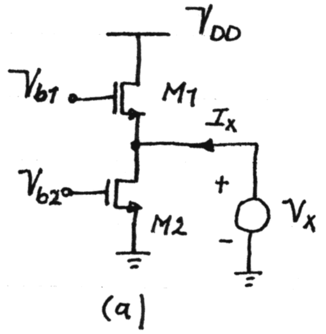


$$V_{X0} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2 \cdot R_D$$

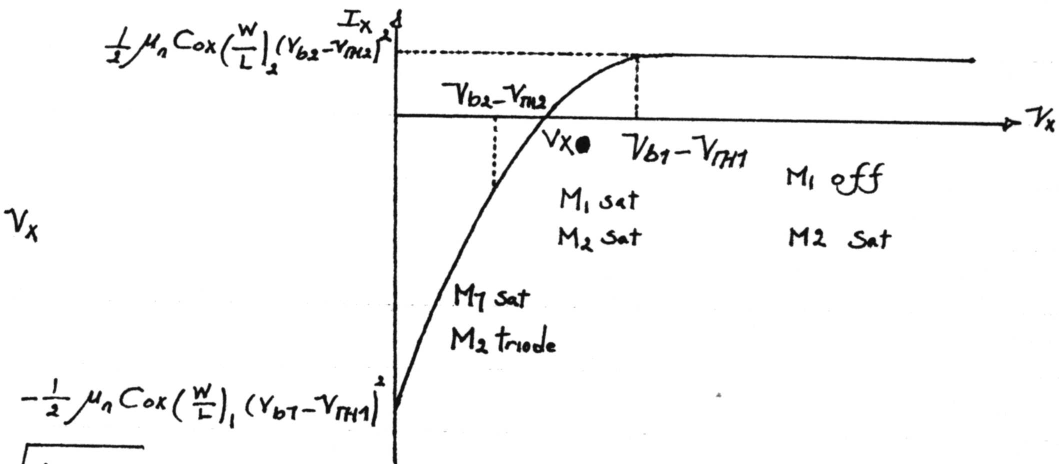
3-6

$$V_{X1} = V_b - V_{TH1} - \left(\frac{2(V_{DD} - V_b + V_{TH1})}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot R_D}\right)^{1/2}$$

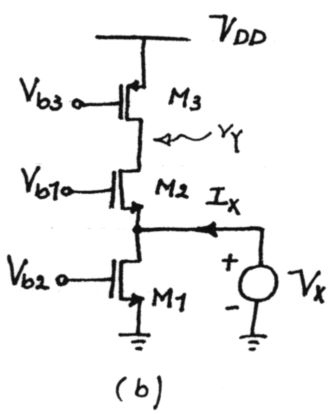
3.4.



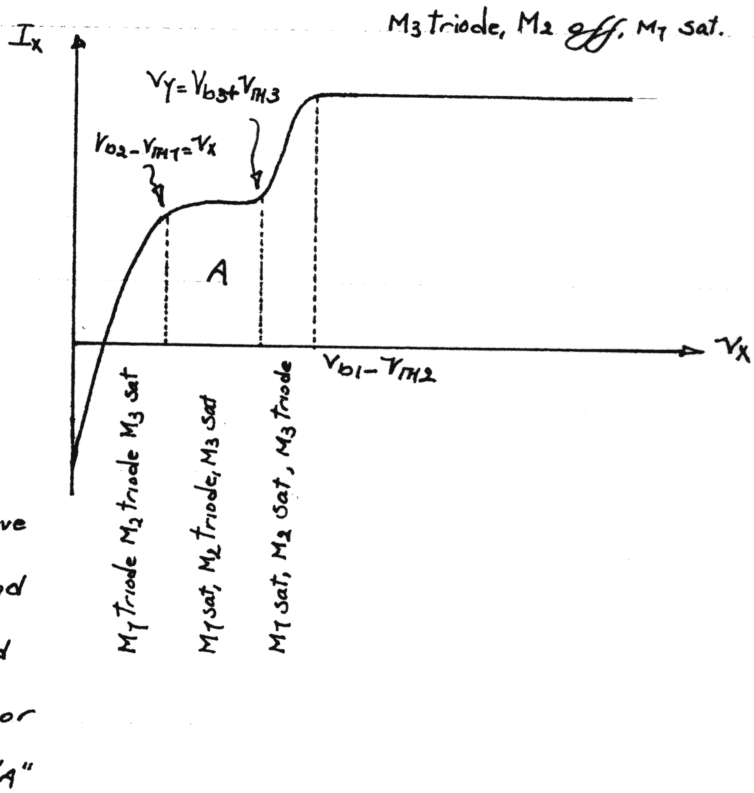
(a)



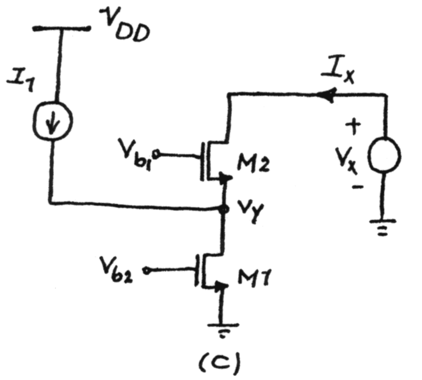
$$V_{X0} = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$



(b)

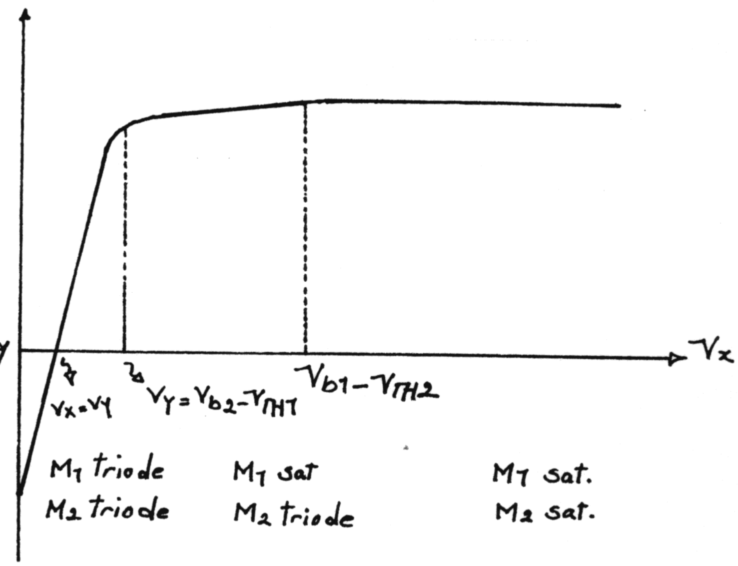


It's worth mentioning that the I_X/V_X Curve varies with the value of bias voltages and aspect ratios, therefore, some region(s), based on the aforementioned parameters, gets wider or narrower, especially the region Called "A" in the above figure.

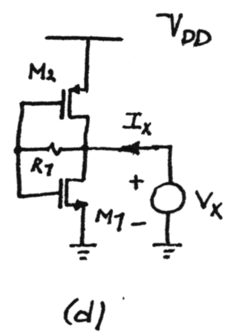


(c)

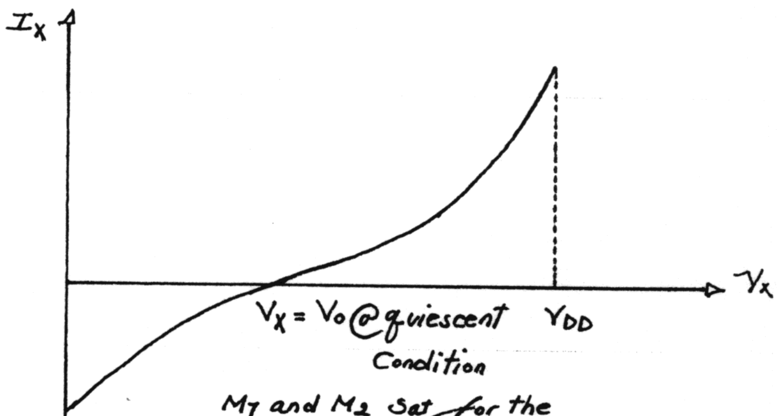
we assume $V_{b1} > V_{b2}$ and both M_1 and M_2 operate in Saturation region if $V_X = V_{DD}$



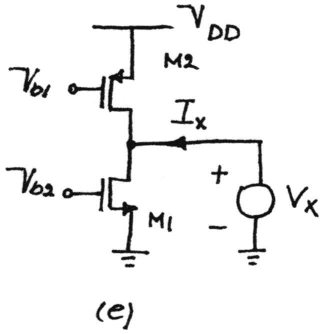
Below V_X, for which $V_X = V_Y$, drain current of M_2 flows in opposit direction, revealing the fact the drain and source terminals of M_2 are reversed. As expected, most of I_1 flow through M_2 when $V_X = 0$, because we assume that $V_{b1} > V_{b2}$.



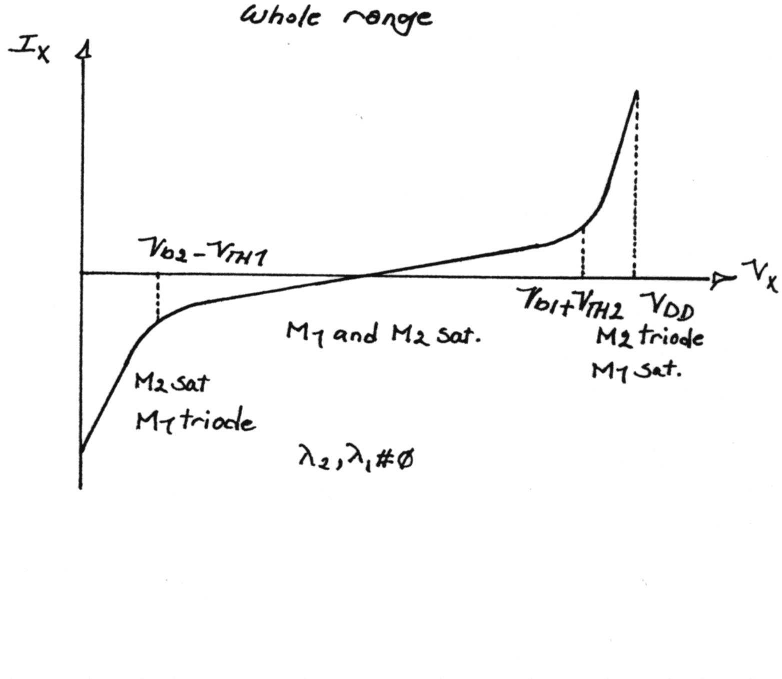
(d)



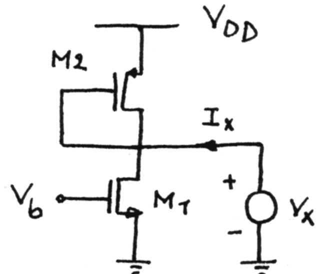
M_1 and M_2 Sat for the whole range



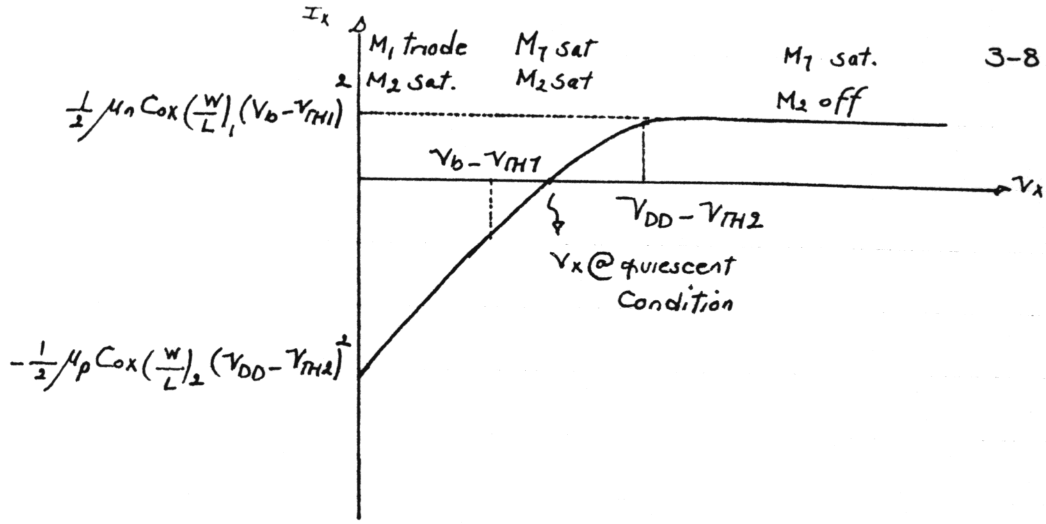
(e)



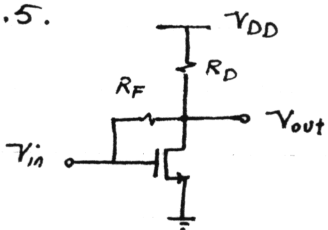
$\lambda_2, \lambda_1 \neq 0$



(f)



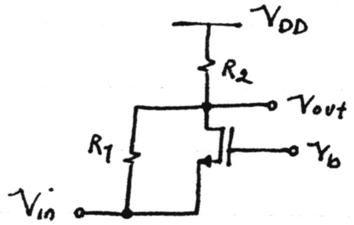
3.5.



(a)

$$\frac{V_o - V_{in}}{R_F} + g_{m1} V_{in} + \frac{V_o}{r_{o1}} + \frac{V_o}{R_D} = 0$$

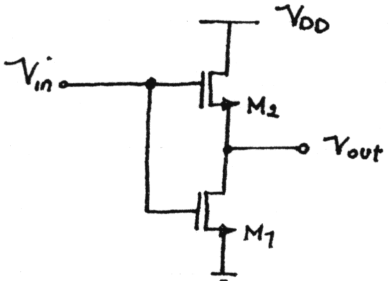
$$A_V = \frac{V_o}{V_{in}} = -\frac{g_{m1} - 1/R_F}{\frac{1}{R_F} + \frac{1}{r_{o1}} + \frac{1}{R_D}}$$



(b)

$$\frac{V_o}{R_2} + (V_o - V_{in})\left(\frac{1}{R_1} + \frac{1}{r_{o1}}\right) - g_{m1} V_{in} = 0$$

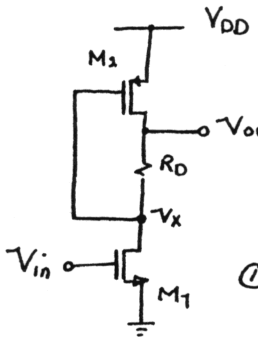
$$\frac{V_o}{V_{in}} = \frac{g_{m1} + \frac{1}{R_1} + \frac{1}{r_{o1}}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{r_{o1}}}$$



(c)

$$g_{m2}(V_{in} - V_{out}) + \frac{-V_{out}}{r_{o2}} = g_{m1} V_{in} + \frac{V_{out}}{r_{o1}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} - g_{m2}}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}$$



(d)

① $\left(g_{m1}V_{in}+\frac{V_X}{r_{o1}}\right)R_D+V_X=V_{out}$, ② $-\left(g_{m2}V_X+\frac{V_{out}}{r_{o2}}\right)=g_{m1}V_{in}+\frac{V_X}{r_{o1}}$,

② → $V_X\left(-g_{m2}-\frac{1}{r_{o1}}\right)=g_{m1}V_{in}+\frac{V_{out}}{r_{o2}}$ ② → $V_X=-\frac{g_{m1}\cdot V_{in}+V_{out}/r_{o2}}{g_{m2}+\frac{1}{r_{o1}}}$ ③

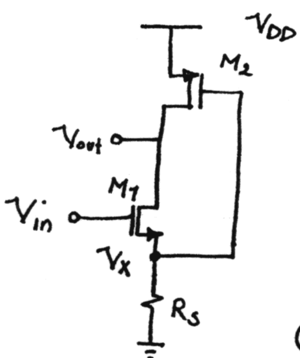
① → $g_{m1}R_DV_{in}+\left(1+\frac{R_D}{r_{o1}}\right)V_X=V_o$ ④

③, ④ → $g_mR_DV_{in}-\dfrac{\left(1+\frac{R_D}{r_{o1}}\right)\left(g_{m1}V_{in}+\frac{V_{out}}{r_{o2}}\right)}{g_{m2}+\frac{1}{r_{o1}}}=V_{out}$

$$\left[g_{m1}\cdot R_D-\frac{g_{m1}(1+R_D/r_{o1})}{g_{m2}+\frac{1}{r_{o1}}}\right]V_{in}=\left[1+\frac{\frac{1}{r_{o2}}(1+R_D/r_{o1})}{g_{m2}+\frac{1}{r_{o1}}}\right]V_{out}$$

$$\left[g_{m1}\cdot R_D\left(g_{m2}+\frac{1}{r_{o1}}\right)-g_{m1}\left(1+\frac{R_D}{r_{o1}}\right)\right]V_{in}=\left[g_{m2}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\left(1+\frac{R_D}{r_{o1}}\right)\right]V_{out}$$

$$\frac{V_{out}}{V_{in}}=\frac{g_{m1}(g_{m2}R_D-1)}{g_{m2}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\left(1+\frac{R_D}{r_{o1}}\right)}$$



(e)

$$\overset{①}{-\left(\frac{V_{out}}{r_{o2}}+g_{m2}V_X\right)}=\overset{②}{\frac{V_{out}-V_X}{r_{o1}}+g_{m1}(V_{in}-V_X)}=\overset{③}{\frac{V_X}{R_S}}$$

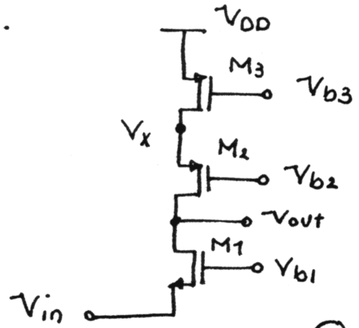
①, ③ → $V_X=-\dfrac{V_{out}}{r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)}$

②, ③ $\frac{V_{out}}{r_{o1}}+g_{m1}V_{in}=-\dfrac{V_{out}}{r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)}\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)$

$$\frac{V_{out}}{r_{o1}}\cdot r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)+g_{m1}\cdot V_{in}\cdot r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)=-V_{out}\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}(g_{m2}+1/R_S)r_{o2}}{g_{m1}+\frac{1}{R_S}+\frac{1}{r_{o1}}\left[1+r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)\right]}$$

3.6.



(a)

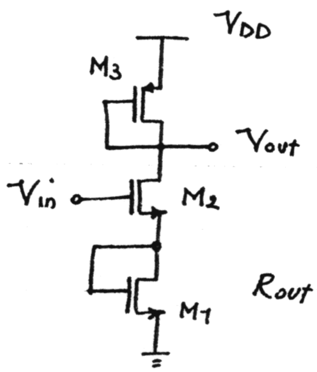
$$\underset{①}{-\frac{V_X}{r_{o3}}} = \underset{②}{g_{m2}V_X + \frac{V_X - V_{out}}{r_{o2}}} = \underset{③}{\frac{V_{out}-V_{in}}{r_{o1}} - g_{m1}V_{in}}$$

$$①,② \rightarrow V_X = \frac{V_{out}}{1 + r_{o2}\left(g_{m2} + \frac{1}{r_{o3}}\right)}$$

$$①,③ \rightarrow \frac{V_{out}-V_{in}}{r_{o1}} - g_{m1}V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2}(1+g_{m2}\cdot r_{o3})}$$

$$V_{out}\left[\frac{1}{r_{o1}} + \frac{1}{r_{o3}+r_{o2}(1+g_{m2}\cdot r_{o3})}\right] = \left(g_{m1} + \frac{1}{r_{o1}}\right)V_{in}$$

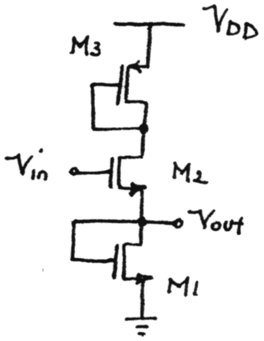
$$\frac{V_{out}}{V_{in}} = \frac{(1+g_{m1}\cdot r_{o1})\left[r_{o3}+r_{o2}(1+g_{m2}\cdot r_{o3})\right]}{r_{o1}+r_{o3}+r_{o2}(1+g_{m2}r_{o3})}$$



$$G_m = \frac{g_{m2}\cdot r_{o2}}{\left(\frac{1}{g_{m1}}\|r_{o1}\right) + \left[1+g_{m2}\left(\frac{1}{g_{m1}}\|r_{o1}\right)\right]r_{o2}}$$

$$R_{out} = \left(\frac{1}{g_{m3}}\|r_{o3}\right)\|\left\{\left[1+g_{m2}\left(\frac{1}{g_{m1}}\|r_{o1}\right)\right]r_{o2} + \left(\frac{1}{g_{m1}}\|r_{o1}\right)\right\}$$

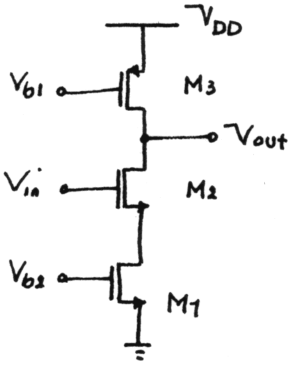
(b) $$A_V = -G_m\cdot R_{out} = -\frac{g_{m2}r_{o2}\left(\frac{1}{g_{m3}}\|r_{o3}\right)}{\left(\frac{1}{g_{m3}}\|r_{o3}\right) + \left\{\left[1+g_{m2}\left(\frac{1}{g_{m1}}\|r_{o1}\right)\right]r_{o2} + \left(\frac{1}{g_{m1}}\|r_{o1}\right)\right\}}$$



(C)

resistance seen looking up at the source of M2, $\frac{\left(\frac{1}{g_{m3}}\|r_{o3}\right)+r_{o2}}{1+g_{m2}\cdot r_{o2}}$

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{g_{m1}}\|r_{o1}\right)}{\left(\frac{1}{g_{m1}}\|r_{o1}\right) + \frac{\left(\frac{1}{g_{m3}}\|r_{o3}\right)+r_{o2}}{1+g_{m2}\cdot r_{o2}}}$$

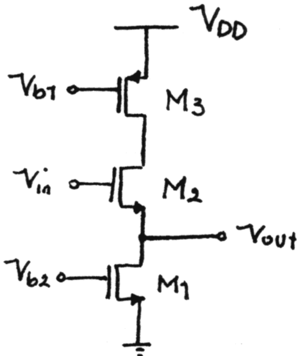


(d)

$$G_m = \frac{g_{m2} \cdot r_{o2}}{r_{o1} + \left[1 + g_{m2} \cdot r_{o1}\right] r_{o2}}$$

$$R_{out} = r_{o3} \parallel \left[(1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}\right]$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2} \cdot r_{o2} \cdot r_{o3}}{r_{o3} + (1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}}$$

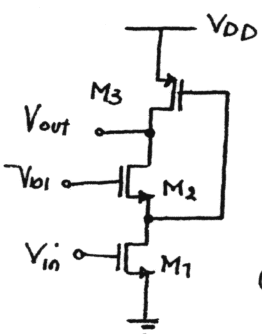


(e)

resistance seen looking up at the source of M_2

$$R_{in} = \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + \dfrac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}} = \frac{r_{o1}(1 + g_{m2} \cdot r_{o2})}{r_{o1}(1 + g_{m2} \cdot r_{o2}) + r_{o2} + r_{o3}}$$



$$\underset{①}{-\left(\frac{V_{out}}{r_{o3}} + g_{m3} V_x\right)} = \underset{②}{\left(\frac{V_{out} - V_x}{r_{o2}} - g_{m2} V_x\right)} = \underset{③}{\frac{V_x}{r_{o1}} + g_{m1} V_{in}}$$

①, ② $\rightarrow$ $$\frac{V_x}{r_{o2}} + g_{m2} V_x - g_{m3} V_x = \frac{V_{out}}{r_{o2}} + \frac{V_{out}}{r_{o3}} \rightarrow V_x = \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out}$$

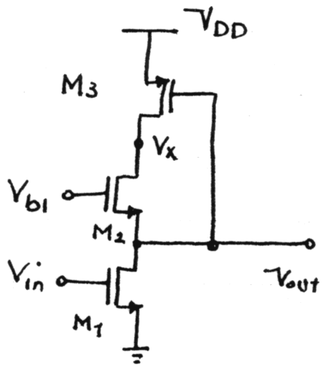
①, ③ $$-\frac{V_{out}}{r_{o3}} - g_{m3} V_x = \frac{V_x}{r_{o1}} + g_{m1} V_{in}$$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m3} + \frac{1}{r_{o1}}\right) \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out} = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{\left(g_{m3} + \frac{1}{r_{o1}}\right)\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}\right] = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{(1+g_{m3}r_{o1})(r_{o3}+r_{o2})}{r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}\right] = g_{m1}\cdot V_{in}$$ 3-12

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(1+g_{m3}\cdot r_{o1})(r_{o3}+r_{o2})}$$



(g)

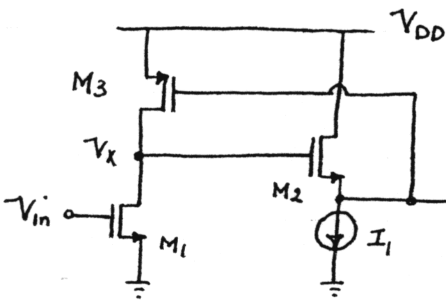
$$V_X = \frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}} \cdot V_{out}$$

$$-\frac{V_X}{r_{o3}} - g_{m3}V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1}\cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{r_{o3}}{r_{o2}}+1}V_{out} - g_{m3}V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1}V_{in}$$

$$-V_{out}\left[\frac{1+(g_{m2}-g_{m3})r_{o2}}{r_{o3}+r_{o2}} + g_{m3} + \frac{1}{r_{o1}}\right] = g_{m1}V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{o1}(r_{o2}+r_{o3})}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}\cdot r_{o1})}$$



(h)

① ②

$$-\left(\frac{V_X}{r_{o3}} + g_{m3}V_{out}\right) = g_{m1}V_{in} + \frac{V_X}{r_{o1}}$$

③

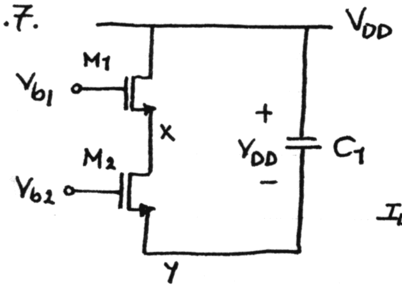
$$-\frac{V_{out}}{r_{o2}} + g_{m2}(V_X - V_{out}) = 0$$ @ output node

③ $\rightarrow$ $$V_X = \frac{\frac{1}{r_{o2}} + g_{m2}}{g_{m2}} \cdot V_{out} = \frac{1+g_{m2}r_{o2}}{g_{m2}r_{o2}}V_{out}$$

①, ② $\rightarrow$ $$-\left[\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)\frac{1+g_{m2}\cdot r_{o2}}{g_{m2}\cdot r_{o2}} + g_{m3}\right]V_{out} = g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}\cdot g_{m2}r_{o1}r_{o2}r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2}\cdot r_{o2}) + g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}}$$

3.7.



(a)

$$V_Y(t=0) = -V_{C1} + V_{DD} = -V_{DD} + V_{DD} = 0$$

$$I_{D1} = I_{D2} \rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{b1} - V_X(t=0) - V_{TH1}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

$$V_X(t=0) = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$

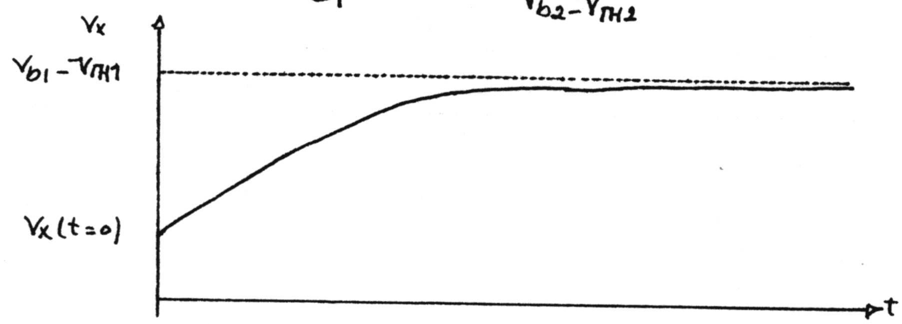
We assume that $V_X(t=0) > V_{b2} - V_{TH2}$, therefore, M_2 is always saturated.

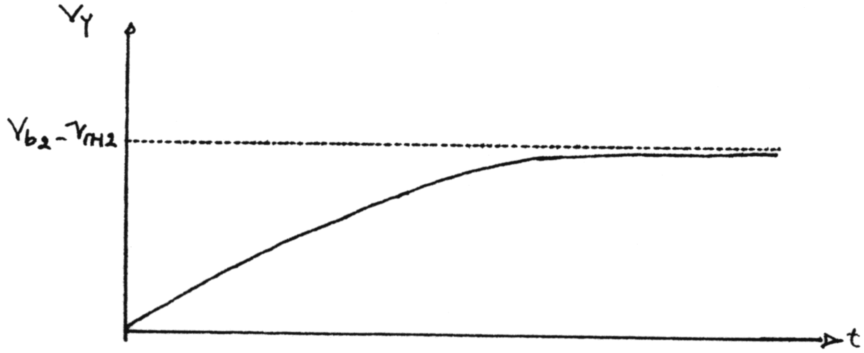
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2}_{1} = \underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_Y - V_{TH2})^2}_{2} = \underbrace{C_1 \frac{dV_Y}{dt}}_{3}$$

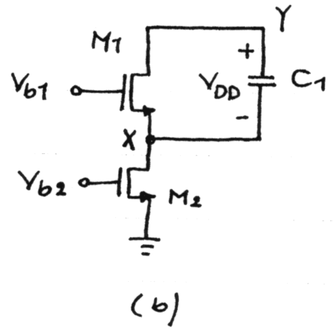
$$2, 3 \rightarrow \frac{dV_Y}{(V_{b2} - V_Y - V_{TH2})^2} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 \cdot dt$$

$$\frac{1}{V_{b2} - V_Y - V_{TH2}} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + K, \quad K = \frac{1}{V_{b2} - V_{TH2}} \quad \text{because } V_Y(t=0) = 0$$

$$V_Y = V_{b2} - V_{TH2} - \frac{1}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + \frac{1}{V_{b2} - V_{TH2}}}, \quad V_X = V_{b1} - V_{TH1} - (V_{b2} - V_Y - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} \leftarrow 2, 1$$







(b)

3-14

The drain current of M2 is zero, therefore, M2 operates in deep triode region, pulling down V_X to zero potential.

$V_X = 0$ for $0 < t < \infty$

$V_Y(t=0) = V_{DD} \rightarrow M_1$ starts in saturation.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 = -C_1\frac{dV_{C1}}{dt} = -C_1\frac{dV_Y}{dt}$$

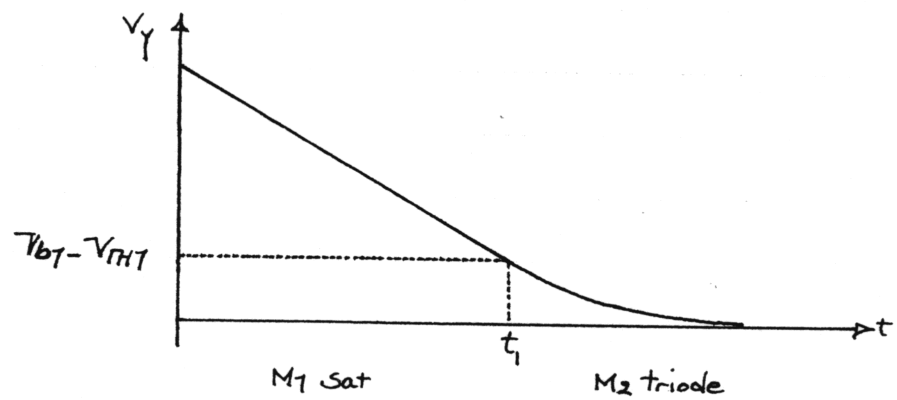
① $V_Y = V_{C1} = V_{DD} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 t$

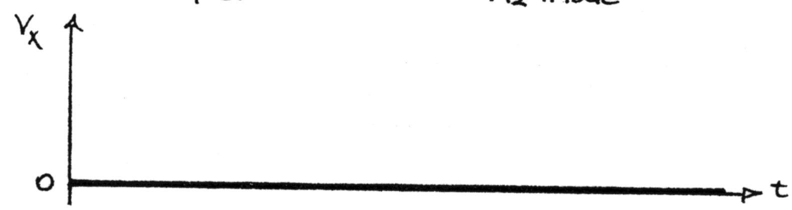
When $V_Y = V_{b1} - V_{TH1}$, ② M_1 enters triode region.

Substituting ② in ①, we calculate the time when M_1 is at the edge of triode region.

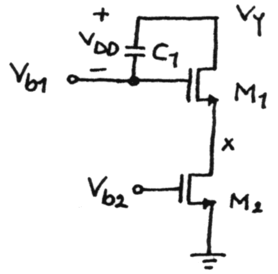
$$t_1 = \frac{V_{DD} - V_{b1} + V_{TH1}}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2}$$

for $t > t_1$: $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_{TH1}) V_Y - \frac{V_Y^2}{2}\right] = -C_1\frac{dV_Y}{dt} \rightarrow V_Y = \ldots$





3-15



(c)

$V_Y(t=0) = V_{DD} + V_{b1}$, both transistors are saturated.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2$$

$$V_X = V_{b1} - V_{TH1} - (V_{b2} - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}$$

$$C_1\frac{dV_{C1}}{dt} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 \rightarrow V_{C1} = V_{DD} - \frac{1}{2}\mu_n\frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

$$V_Y = V_{C1} + V_{b1} = V_{DD} + V_{b1} - \frac{1}{2}\mu_n\frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

@ $t = t_1$, we have $V_Y = V_{b1} - V_{TH1}$, polarity of voltage across C_1 has already changed.

$$V_{DD} + V_{b1} - \frac{1}{2}\mu_n\frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t_1 = V_{b1} - V_{TH1}$$

$$t_1 = \frac{2(V_{DD} + V_{TH1})C_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2}$$

for $t > t_1$, M_1 enters triode region. We assume that still M_2 is saturated.

$V_Y = V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} \cdot t$ where $I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$

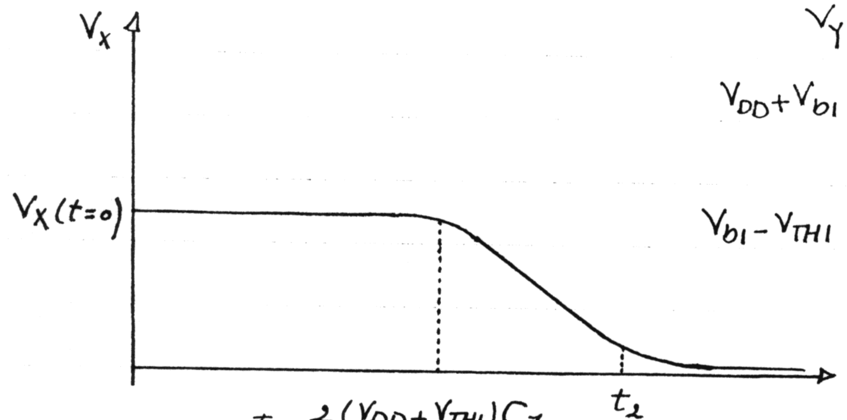
and $I_{D2} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[(V_{b1} - V_X)\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2}\cdot t - V_X\right) - \frac{\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2}\cdot t - V_X\right)^2}{2}\right]$

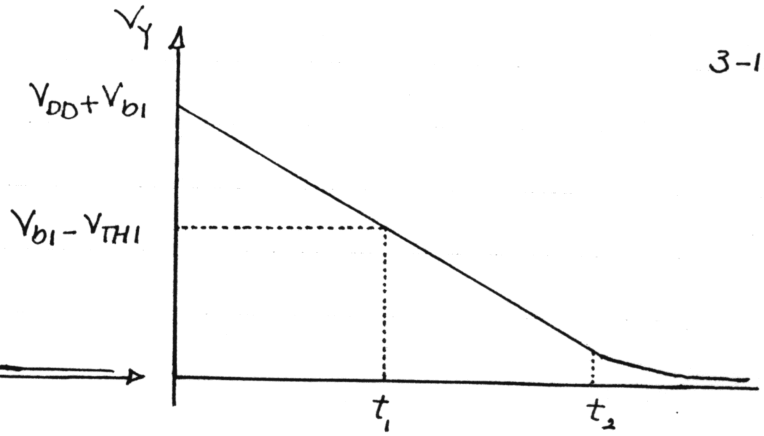
$\rightarrow$ V_X is obtained

When $V_X = V_{b2} - V_{TH2}$, M_2 enters the triode region, too.

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{b2} - V_{TH2})V_X - \frac{V_X^2}{2}\right] = \mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[(V_{b1} - V_X - V_{TH1})(V_Y - V_X) - \frac{(V_Y - V_X)^2}{2}\right] = -C_1\frac{dV_Y}{dt}$$

V_X and V_Y are obtained. This regime continues until V_X and V_Y drop to zero, and C_1 Charges up to $-V_{b1}$.



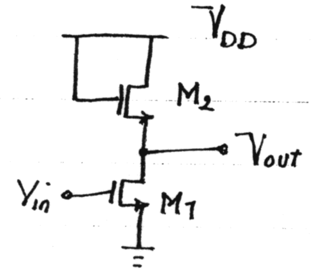


3-16

$$t_1 = \frac{2(V_{DD}+V_{TH1})C_T}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2}-V_{TH2})^2}$$

for $0<t<t_1$ M_1 Sat, M_2 Sat, for $t_1<t<t_2$ M_1 Triode, M_2 Sat.
for $t_2<t$ M_1 Triode, M_2 Triode

3.8



$$\left(\frac{W}{L}\right)_1 = \frac{50}{0.5}, \quad \left(\frac{W}{L}\right)_2 = \frac{10}{0.5}, \quad I_{D1} = I_{D2} = 0.5\,mA$$

$$\mu_n C_{ox} = 350\,\frac{Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times 3.9\ Farad/Cm}{9\times10^{-7}\,Cm} =$$

$$1.34225\times10^{-4}\ A/V^2$$

$$\mu_p C_{ox} = \frac{100\ Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times3.9\ Farad/Cm}{9\times10^{-7}\,Cm} =$$

$$3.835\times10^{-5}\ A/V^2$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_N I_1} = 20K, \quad I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2}-V_{TH2})^2(1+\lambda_N V_{DS2}),$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times20\left[3-V_o-0.7-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)\right]^2\left[1+0.1(3-V_o)\right]$$

$$2.3-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)-\sqrt{\frac{1}{2.6845\,(1.3-0.1V_o)}} = V_o \rightarrow V_o = 1.466\ V$$

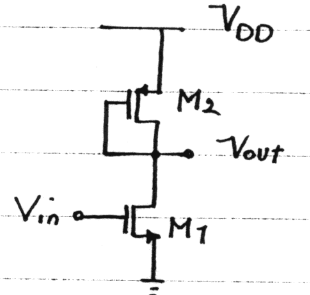
$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.66\times10^{-3}\ A/V$$

$$g_{m2} = \sqrt{2\times1.34225\times10^{-4}\times20\times0.5\times10^{-3}} = 1.63\times10^{-3}\ A/V$$

$$g_{mb2} = \frac{\gamma\cdot g_{m2}}{2\sqrt{2\phi_F+V_{SB}}} = \frac{0.45}{2\sqrt{0.9+1.466}}\times1.63\times10^{-3} = 2.3843\times10^{-4}\ A/V$$

$$R_{out} = \frac{1}{g_{m2} + g_{mb2} + r_{o2}^{-1}} \parallel r_{o1} = \frac{1}{1.63\times10^{-3} + 2.3843\times10^{-4} + (20\times10^{3})^{-1}} \parallel 20\times10^{3} \qquad 3.17$$

$R_{out} = 508\ \Omega$ $\qquad A_V = -g_{m1}\cdot R_{out} = -3.66\times10^{-3}\times 508 = -1.85$



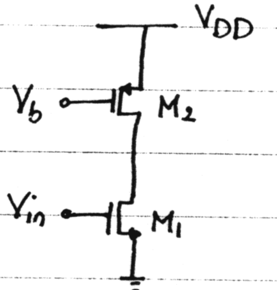
$$g_{m2} = \sqrt{2\times 3.835\times10^{-5}\times 20\times 0.5\times10^{-3}} = 8.7578\times10^{-4}$$

$$r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$$

$$R_{out} = \frac{1}{g_{m2} + r_{o2}^{-1}} \parallel r_{o1} = 974.8628\ \Omega$$

$$A_V = -g_{m1}\cdot R_{out} = -0.8537$$

3.9.



$\left(\frac{W}{L}\right)_1 = 50/0.5,\ \left(\frac{W}{L}\right)_2 = 50/2,\ I_{D1} = I_{D2} = 0.5\ mA$

$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K,\ r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 40K$

$$g_{m1} = \sqrt{2\times 1.34225\times10^{-4}\times 100\times 0.5\times10^{-3}} = 3.6636\times10^{-3}$$

$$A_V = -g_{m1}(r_{o1}\parallel r_{o2}) = -48.84$$

If we assume that M_1 is in the edge of the triode region, then, we have:

$V_{GS} - V_{TH1} = V_{DS1} = V_{out},\ I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2(1+\lambda_N V_{DS})$

$0.5\times10^{-3} = \frac{1}{2}\times 1.34225\times10^{-4}\times 100\, V_{DS}^2(1+0.1V_{DS}) \rightarrow \sqrt{\frac{1}{13.4225(1+0.1V_{DS})}} = V_{DS}$

$V_{DSmin} = V_{omin} = 0.2693$

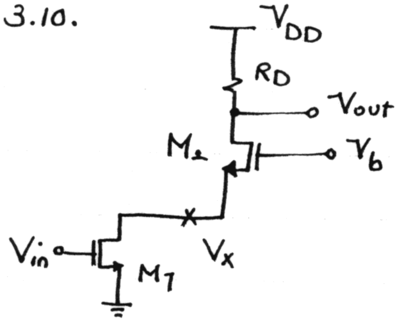
If we assume that M_2 is in the edge of the triode region, then, we have:

$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH2}|)^2(1+\lambda_p V_{SD}),\ 0.5\times10^{-3} = \frac{1}{2}\times 3.835\times10^{-5}\times 25\, V_{SD}^2(1+\lambda_p V_{SD})$

$\sqrt{\frac{1}{0.95875(1+0.05V_{SD})}} = V_{SD} \rightarrow V_{SDmin} = 0.9967\,V,\ V_{omax} = V_{DD} - V_{SDmin},$

$V_{omax} = 2V$

3.10.



$\left(\frac{W}{L}\right)_1 = 50/0.5,\ \left(\frac{W}{L}\right)_2 = 10/0.5\ \ I_{D1} = I_{D2} = 0.5\,mA$ 3–18

$R_D = 1K\Omega$

$$V_{DS,sat1} = V_{GS1} - V_{TH1} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times100}\right)^{1/2}$$

$$V_{DS,sat1} = 0.2729V$$

$$V_{X,Bias} = 0.2729 + 50\times10^{-3} = 0.3229V$$

$$V_{TH2} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = 0.7 + 0.45\left(\sqrt{0.9+0.3229} - \sqrt{0.9}\right)$$

$$V_{TH2} = 0.77073V$$

$$V_{GS2} = V_{TH2} + \left(\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2} = 0.77073 + \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times20}\right)^{1/2} = 1.38107V,$$

$$V_b = V_{GS2} + V_X$$

$$V_b = 1.38107 + 0.3229 = 1.7V,\quad g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.6636\times10^{-3}\ A/V$$

$$g_{m2} = \sqrt{2\times1.34225\times10^{-4}\times20\times0.5\times10^{-3}} = 1.6384\times10^{-3}\ A/V$$

$$g_{mb2} = \frac{0.45}{2\sqrt{0.9+0.3229}}\,1.6384\times10^{-3} = 3.3336\times10^{-4},\quad r_{o1} = r_{o2} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K$$

$$R_{out} = R_D \parallel \left\{\left[1 + (g_{m2}+g_{mb2})r_{o2}\right]r_{o1} + r_{o2}\right\} = 10^3 \parallel \left\{\left[1 + (1.6384\times10^{-3} + 3.3336\times10^{-4})20\times10^3\right]20\times10^3 + 20\times10^3\right\}$$

$$R_{out} = 998.7947\,\Omega,\quad G_m = \frac{g_{m1}\cdot r_{o1}\left[r_{o2}(g_{m2}+g_{mb2})+1\right]}{r_{o1}\cdot r_{o2}(g_{m2}+g_{mb2}) + r_{o1} + r_{o2}}$$

$$G_m = \frac{3.6636\times10^{-3}\times20\times10^3\left[20\times10^3(1.6384\times10^{-3}+3.3336\times10^{-4})+1\right]}{(20\times10^3)^2(1.6384\times10^{-3}+3.3336\times10^{-4}) + 2\times20\times10^3} = 3.5751\times10^{-3}\ A/V$$

$$A_v = -G_m R_{out} = -3.57$$

We obtain the small signal voltage gain from input to node x.

$$R_{out@X} = r_{o1} \parallel \frac{R_D + r_{o2}}{1+(g_{m2}+g_{mb2})r_{o2}} = 20\times10^3 \parallel \frac{10^3 + 20\times10^3}{1+(1.6384\times10^{-3}+3.3336\times10^{-4})20\times10^3}$$

$$R_{out@X} = 506.2$$

$$A_{VX} = -g_{m1}\cdot R_{out@X} = -1.8545$$

If $V_X = V_{X\,min} = V_{DS,sat1}$, $\Delta V_X = -50mV \rightarrow \Delta V_{in} = \frac{-50\times10^{-3}}{-1.8545} = 26.96\times10^{-3}$

$\Delta V_{out} = 26.96 \times 10^{-3} \times (-3.57) = -96.25 \times 10^{-3}$ 3-19

$V_{out\,min} = V_{DD} - R_D I_D + \Delta V_o = 3 - 1 \times 0.5 - 96.25 \times 10^{-3} = 2.4V$

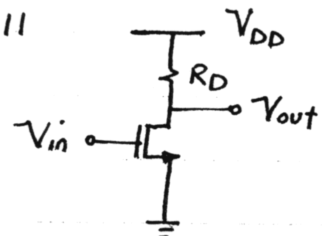
$V_{out,max} = 3V, \quad \Delta V_o = 3 - 2.5 = 0.5V, \quad \Delta V_{in} = \dfrac{0.5}{-3.57} = -0.14V$

$\Delta V_X = -1.8545\,(-0.14) = 0.2597$

$V_{X,max} = V_{X,Bias} + 0.2597 = 0.3229 + 0.2597 = 0.5826V$

If we take $V_{out,min} = V_b - V_{TH2} = 1.7 - 0.77073 = 0.92924V$, $\Delta V_o = -1.57$ which translates into a huge negative swing at x that makes the final voltage at node x negative. Therefore, M_1 limits the negative going output swing because the voltage gain from input to node x is quite large.

3.11



$(\frac{W}{L})_1 = 50/0.5, \quad R_D = 2K\Omega, \quad \lambda = \phi$

$r_{o1} = \dfrac{1}{\lambda_N I_D} = \dfrac{1}{0.1 \times 10^{-3}} = 10K$

$R_{out} = r_{o1} \parallel R_D = 10K \parallel 2K = \dfrac{5000}{3}\,\Omega$

$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 10^{-3}} = 5.1812 \times 10^{-3}$

$A_V = -g_{m1} \cdot R_{out} = -5.1812 \times 10^{-3} \times \dfrac{5000}{3} = -8.6353$

At the edge of the triode region: $V_{out} = V_{GS} - V_{TH} = V_{GS} - 0.7$, $I_{D1} = \dfrac{V_{DD} - V_{out}}{R_D} =$

$\dfrac{3 - V_{GS} + 0.7}{2 \times 10^3} = \dfrac{3.7 - V_{GS}}{2 \times 10^3}, \quad I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2$

$\dfrac{3.7 - V_{GS}}{2 \times 10^3} = \dfrac{1}{2} \times 1.34225 \times 10^{-4} \times 100\,(V_{GS} - 0.7)^2$

$13.4225\,V_{GS}^2 - 17.7915\,V_{GS} - 10.277025 = 0 \rightarrow V_{GS} = 1.137V$

I_D @ the edge of the triode $= \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100\,(1.137 - 0.7)^2 = 1.28151 \times 10^{-3}$

g_{m1} @ the edge of the triode $= \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 1.28151 \times 10^{-3}} = 5.8653 \times 10^{-3}$

$r_{o1} = \dfrac{1}{0.1 \times 1.28151\ 10^{-3}} = 7.8 \times 10^3$

A_V @ the edge of the triode $= -g_{m1}(r_{o1} \parallel R_D) = -5.8653 \times 10^{-3}\,(7.8 \times 10^3 \parallel 2 \times 10^3)$

$A_V = -9.3374$

V_o @ the edge of the triode $= V_{DD} - R_D \times I_D = 3 - 2 \times 1.2815 \times 10^{-3} = 0.4369\,V$ 3-20

$V_{DS} = V_{DS,sat1} - 50\times10^{-3} = 0.4369 - 50\times10^{-3} = 0.3869\,V$

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.3869}{2\times10^{3}} = 1.3065\times10^{-3}$$

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{GS} - V_{TH1}) V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$1.3065\times10^{-3} = 1.34225\times10^{-4} \times 100 \left[(V_{GS} - 0.7)\, 0.3869 - \frac{(0.3869)^2}{2}\right] \Rightarrow V_{GS} = 1.145$$

$$g_{m1} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{DS}$$

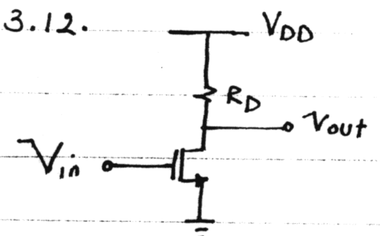
g_{m1} @ the point where 50mV into the triode $= 1.34225\times10^{-4} \times 100 \times 0.3869 = 5.1942\times10^{-3}$

$$R_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1} - V_{DS}) \rightarrow R_o = \frac{1}{1.34225\times10^{-4}\times100(1.145-0.7-0.38)}$$

$R_o = 1.2835\times10^{3}\ \Omega$

A_V @ 50 mV into the triode region $= -5.1942\times10^{-3}(1.2835\times10^{3} \,\|\, 2\times10^{3}) = -4$

3.12.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K, \quad \lambda = 0$

$$I_{D1} @ V_{out} = 1V = \frac{V_{DD} - V_o}{R_D} = \frac{3-1}{2\times10^{3}} = 10^{-3}\,A$$

$$V_{in} = V_{TH1} + \left(\frac{2 I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = 0.7 + \left(\frac{2\times10^{-3}}{1.34225\times10^{-4}\times100}\right)^{1/2} \rightarrow V_{in} @ V_{out}=1V = 1.086\,V$$

$$I_{D1} @ V_{out} = 2.5V = \frac{3-2.5}{2\times10^{3}} = 2.5\times10^{-4},\ V_{in} @ V_{out}=2.5V = 0.7 + \left(\frac{2\times2.5\times10^{-4}}{1.34225\times10^{-4}\times100}\right)^{1/2} = 0.893\,V$$

$$g_{m1} @ V_{out}=1V = \sqrt{2\times1.34225\times10^{-4}\times100\times10^{-3}} = 5.1812\times10^{-3}$$

$$g_{m1} @ V_{out}=2.5V = \sqrt{2\times1.34225\times10^{-4}\times100\times2.5\times10^{-4}} = 2.59\times10^{-3}$$

$r_{o1}@V_{out}=7V = \dfrac{1}{0.1\times10^{-3}} = 10K$, $R_{out} = r_{o1} \| R_D = 10000 \| 2000 = \dfrac{5000}{3}\,\Omega$ 3-21

$A_V@V_{out}=7V = -g_{m1}\cdot R_{out} = -5.1812\times10^{-3} \times \dfrac{5000}{3} = -8.6353$

$r_{o1}@V_{out}=2.5V = \dfrac{1}{0.1\times2.5\times10^{-4}} = 40K$, $R_{out} = r_{o1} \| R_D = 40000 \| 2000 = 1.9\times10^{3}$

$A_V@V_{out}=2.5V = -g_{m1}\cdot R_{out} = -2.59\times10^{-3} \times 1.9\times10^{3} = -4.9221$

3.13. $\left(\frac{W}{L}\right) = 50/0.5$ $|I_D| = 0.5\,mA$

& $100/1$

For NMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \dfrac{1}{\lambda_N I_D} = \dfrac{1}{0.1\times0.5\times10^{-3}} = 20K$

$g_m = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.6636\times10^{-3}$

$g_m r_o = 73.27$

For PMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \dfrac{1}{\lambda_P I_D} = \dfrac{1}{0.2\times0.5\times10^{-3}} = 10K$

$g_m = \sqrt{2\times3.835\times10^{-5}\times100\times0.5\times10^{-3}} = 1.9583\times10^{-3}$

$g_m r_o = 19.5831$

For NMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \dfrac{1}{\frac{0.1}{2}\times0.5\times10^{-3}} = 40K$

$g_m = 3.6636\times10^{-3}$, $g_m r_o = 146.5469$

For PMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \dfrac{1}{\frac{0.2}{2}\times0.5\times10^{-3}} = 20K$

$g_m = 1.9583\times10^{-3}$, $g_m r_o = 39.1663$

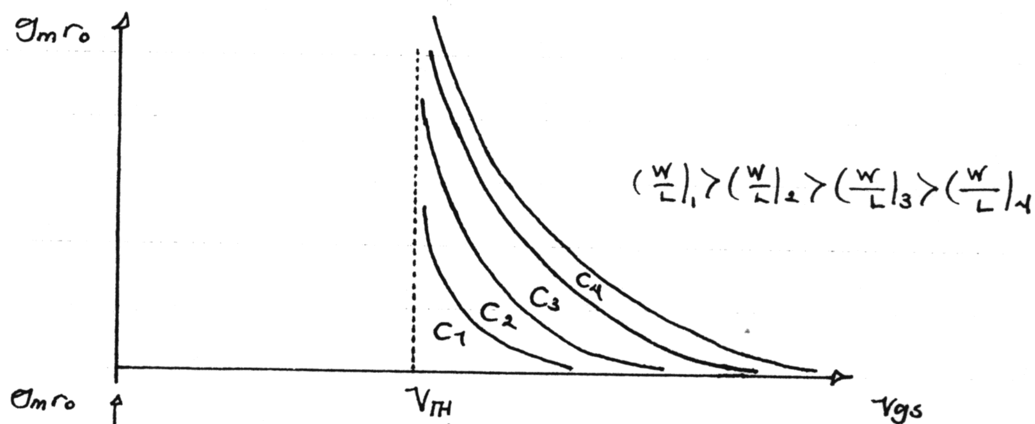
3.14. $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$ (1)

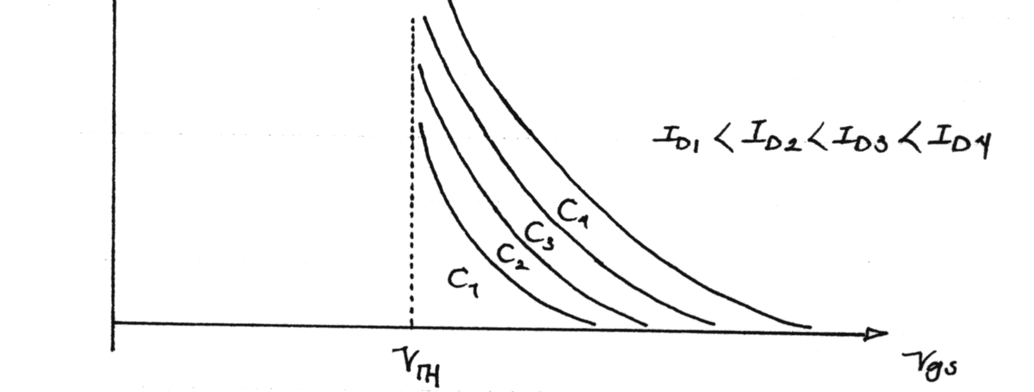
$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})(1+\lambda V_{DS})$ (2)

Substituting $(1+\lambda V_{DS})$ from ① in ②, we have. 3-22

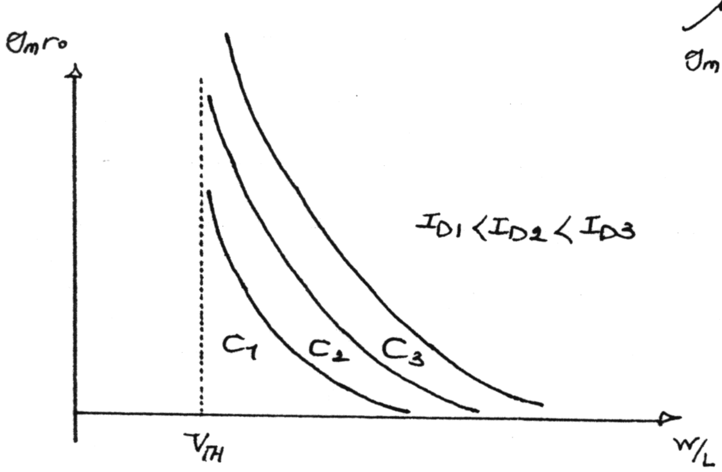
$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2} = \frac{2I_D}{V_{GS}-V_{TH}}$$

$$g_m r_o = \frac{2I_D}{V_{GS}-V_{TH}}\,\frac{1+\lambda V_{DS}}{\lambda I_D} = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})} = \frac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$$

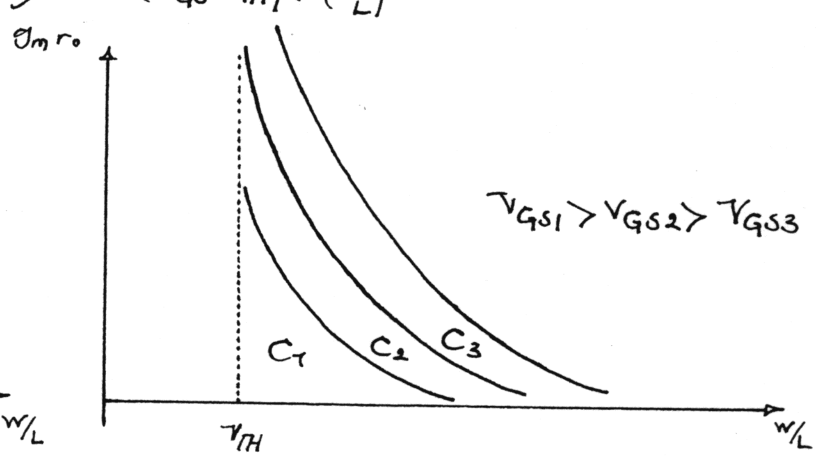




3.15. From 3.14. we have: $g_m r_o = \dfrac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$



(a)



(b)

3.16. $\frac{W}{L} = 50/0.6$ $\quad V_G = +1.2V$ $\quad V_S = 0$ $\quad 0 < V_D < 3$ $\quad V_{bulk} = 0$

$V_{Dsat} = V_{GS} - V_{TH} = 1.2 - 0.7 = 0.5V$, for a saturated device $g_m r_o = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})}$

@ the edge of the triode region $g_m r_o = \frac{2(1+0.5\times 0.1)}{0.1(1.2-0.7)} = 42$

We cannot neglect the channel-length modulation in the triode region, because it would lead to a discontinuity at the transition point between the saturation and the triode region.

@ triode region

$$g_m = \frac{\partial I_D}{\partial V_{gs}} = \mu_n C_{ox}\left(\frac{W}{L}\right) V_{DS}(1+\lambda V_{DS})$$

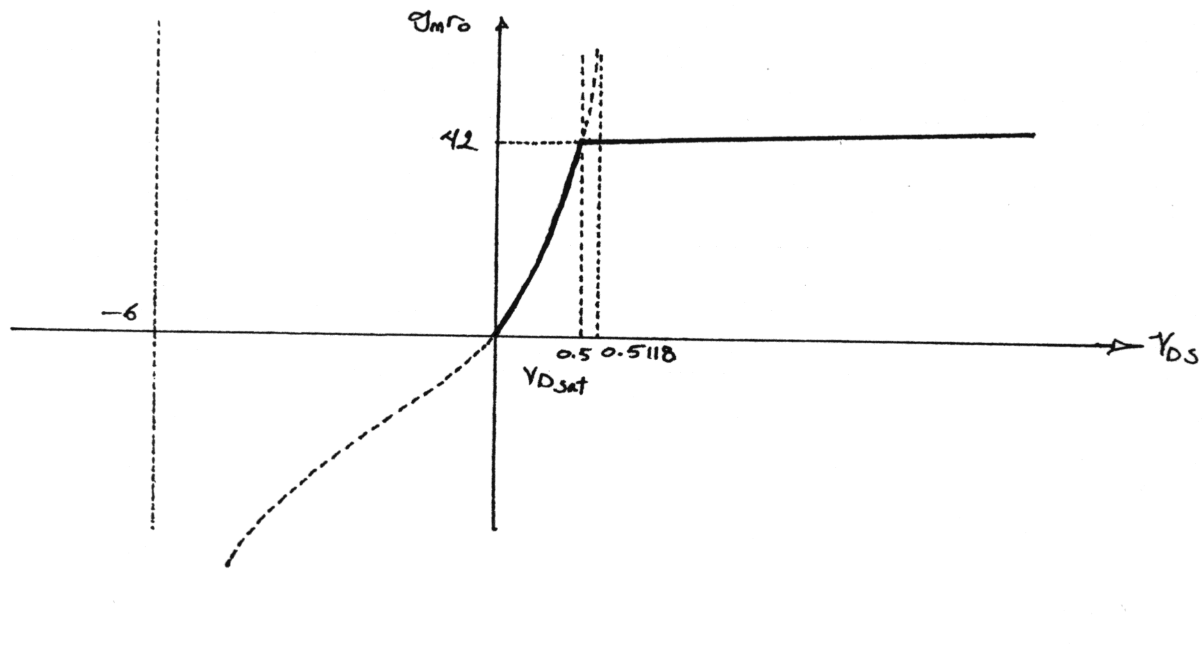
$$g_o = \frac{\partial I_D}{\partial V_{ds}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left\{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]\right\}$$

in the triode region $g_m r_o = \dfrac{(1+\lambda V_{DS})V_{DS}}{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]}$

In Saturation $\quad g_m r_o = \dfrac{2(1+0.1V_{DS})}{0.1(1.2-0.7)} = 40 + 4V_{DS} \qquad V_{DS} > 0.5V$

In triode $\quad g_m r_o = \dfrac{(1+0.1V_{DS})V_{DS}}{(0.5-V_{DS})(1+0.1V_{DS}) + 0.1\times 0.5 V_{DS}(1-V_{DS})}$

$$g_m r_o = \frac{0.1V_{DS}^2 + V_{DS}}{-0.15V_{DS}^2 - 0.9V_{DS} + 0.5}$$



$V_{bulk} = -1V$, $V_{SB} = +1V$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1} - \sqrt{0.9}\right) = 0.8933\ V$$

In Saturation $\quad g_m r_o = \dfrac{2(1+0.1V_{DS})}{0.1(1.2-0.8933)} = 65.2262 + 6.5226\,V_{DS}$

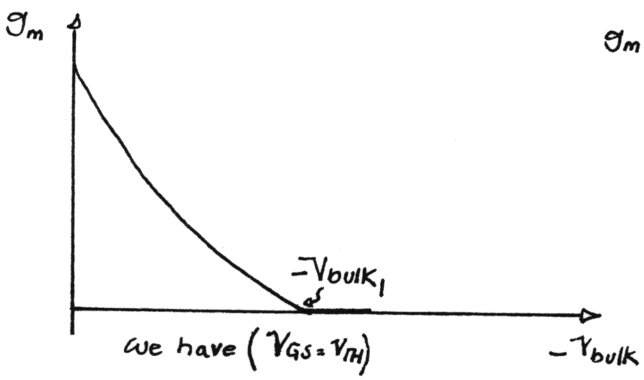
$V_{DSsat} = V_{GS} - V_{TH} = 1.2 - 0.8933 = 0.3066\ V$, @the edge of the Triode $g_m r_o = 67.2262$

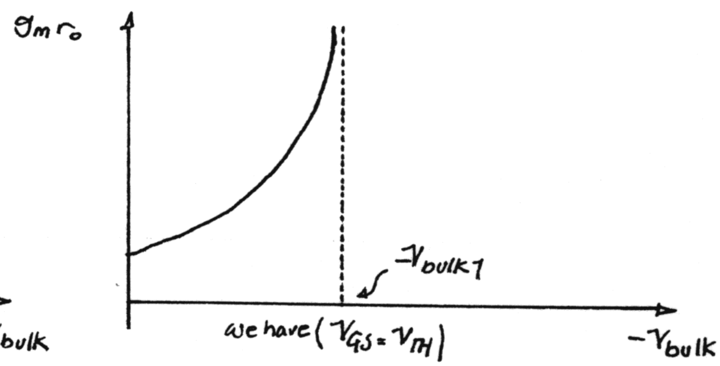
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{(1.2-0.8933-V_{DS})(1+0.1V_{DS}) + 0.1\left[(1.2-0.8933)V_{DS} - 0.5V_{DS}^2\right]}$$

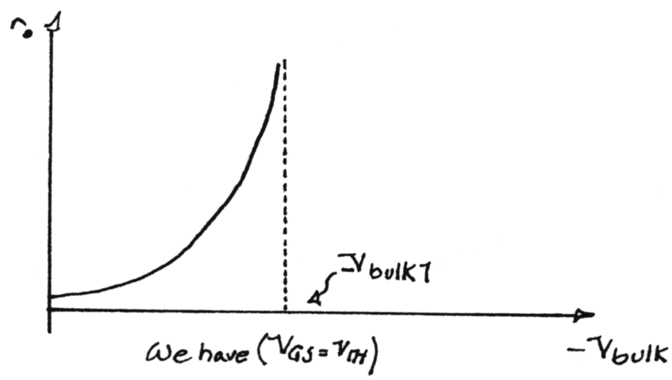
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{-0.15V_{DS}^2 - 0.9386V_{DS} + 0.3066}$$

3.17. $\quad g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS} - V_{TH0} - \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)\right](1+\lambda V_{DS})$

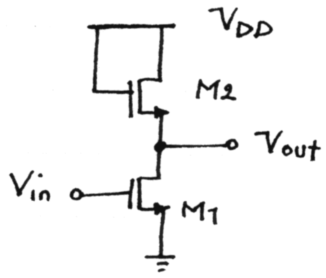
$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2\lambda}\ ,\quad g_m r_o = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})}$$







3.18.



$(\frac{W}{L})_1 = 50/0.5 \quad (\frac{W}{L})_2 = 10/0.5 \,, \quad \lambda = \gamma = 0$

M_1 at the edge of the triode region, $\rightarrow V_{out} = V_{in} - V_{TH1}$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_2 (V_{DD} - V_{in} + V_{TH1} - V_{TH2})^2$$

$$(\frac{W}{L})_1^{1/2}(V_{in} - V_{TH1}) = (\frac{W}{L})_2^{1/2}(V_{DD} - V_{in}) \rightarrow (V_{in} - V_{TH1}) = \sqrt{\frac{(\frac{W}{L})_2}{(\frac{W}{L})_1}}\,(V_{DD} - V_{in})$$

$$V_{in} = \left(\sqrt{\frac{(\frac{W}{L})_2}{(\frac{W}{L})_1}}\,V_{DD} + V_{TH1}\right) \Big/ \left(1 + \sqrt{\frac{(\frac{W}{L})_2}{(\frac{W}{L})_1}}\right) = \left[(\frac{10}{50})^{1/2} \times 3 + 0.7\right] \Big/ \left[1 + (\frac{10}{50})^{1/2}\right] = 1.41V$$

$$A_V = -\sqrt{\frac{(\frac{W}{L})_1}{(\frac{W}{L})_2}} = -\sqrt{\frac{50}{10}} = -2.236\,.$$

At the edge of the triode region $V_{out} = 1.41 - 0.7 = 0.71\,V$

50 mV into the triode region $V_{out} = 0.71 - 50\times10^{-3} = 0.66V$

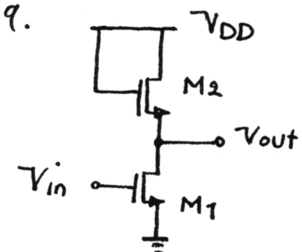
$$\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}(\frac{W}{L})_1 \left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right]$$

$$V_{in} = \frac{(\frac{W}{L})_2}{(\frac{W}{L})_1}\,\frac{(V_{DD} - V_{out} - V_{TH2})^2}{V_{out}} + \frac{V_{out}}{2} + V_{TH1} = \frac{10}{50}\,\frac{(3 - 0.66 - 0.7)^2}{0.66} + \frac{0.66}{2} + 0.7$$

$V_{in} = 1.8437\,,\quad I_D = \mu_n C_{ox}(\frac{W}{L})_1\left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right],\quad \frac{\partial I_D}{\partial V_{in}} = \mu_n C_{ox}(\frac{W}{L})_1 \cdot V_{out}$

$$A_V = -\frac{\mu_n C_{ox}(\frac{W}{L})_1 \cdot V_{out}}{\mu_n C_{ox}(\frac{W}{L})_2 (V_{DD} - V_{out} - V_{TH2})} = -\frac{\frac{50}{0.5} \times 0.66}{\frac{10}{0.5} \times (3 - 0.66 - 0.7)} = -2.015$$

3.19.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad \lambda = 0$

$V_{out} = V_{in} - V_{TH1}, \quad V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)$

$I_{D1} = I_{D2} \Rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} + V_{TH1} - V_{TH2,0} - 0.45\left(\sqrt{0.9 + V_{out}} - \sqrt{0.9}\right)\right)^2$

$\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \left(\frac{W}{L}\right)_2 \left[V_{DD} - V_{in} - 0.45\left(\sqrt{0.9 + V_{in} - 0.7} - \sqrt{0.9}\right)\right]^2$

$V_{in} = \sqrt{\frac{1}{5}}\left[3 - V_{in} - 0.45\left(\sqrt{0.2 + V_{in}} - \sqrt{0.9}\right)\right] + 0.7 \rightarrow$ After enough iterations $\rightarrow$

$V_{in} = 1.3685, \quad V_{out} = 0.6685, \quad \eta_2 = \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}} = \frac{0.45}{2(0.9 + 0.6685)^{1/2}} = 0.1796$

$A_V = -\frac{g_{m1}}{g_{m2}(1+\eta_2)} = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}}\frac{1}{1+\eta_2} = -\sqrt{\frac{50}{10}}\frac{1}{1+0.1796} = -1.8955$

$V_{out} = 0.6685 - 50\times 10^{-3} = 0.6185$

$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right]$

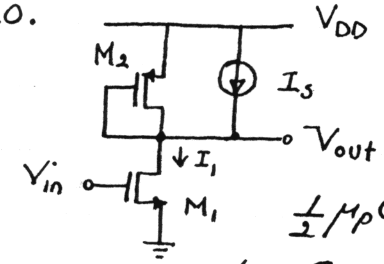
$V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 0.6185} - \sqrt{0.9}\right) = 0.8276$

$24.1453 = \frac{50}{0.5}\left[(V_{in} - 0.7)\,0.6185 - \frac{0.6185^2}{2}\right] \rightarrow V_{in} = 1.3996$

$\eta_2 = \frac{0.45}{2(0.9 + 0.6185)^{1/2}} = 0.1825 \qquad A_V = -\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})(1+\eta_2)} =$

$\frac{\left(\frac{W}{L}\right)_1 V_{out}}{\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})(1+\eta_2)} = \frac{50\times 0.6185}{10(3 - 0.6185 - 0.8276)(1 + 0.1825)} = -1.6829$

3.20.



$\left(\frac{W}{L}\right)_1 = 20/0.5,\ I_1 = 1\,mA,\ I_S = 0.75\,mA,\ \lambda = 0$ 3.27

M_1 at the edge of the triode region $V_{out} = V_{in} - V_{TH1}$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

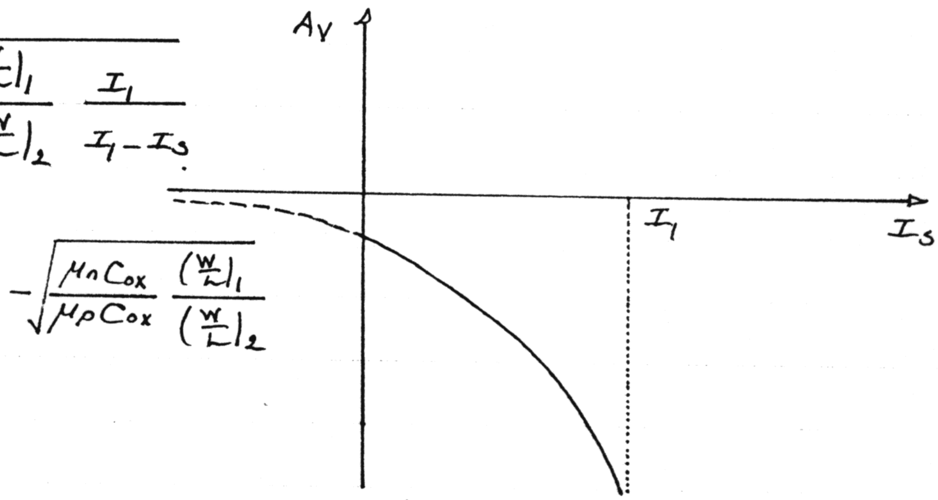
$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

$$(V_{in} - V_{TH1})^2 = \frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1} \rightarrow V_{in} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + V_{TH1} = 0.7 + \sqrt{\frac{2\times 10^{-3}}{1.34225\times 10^{-4}\times 20/0.5}} = 1.31$$

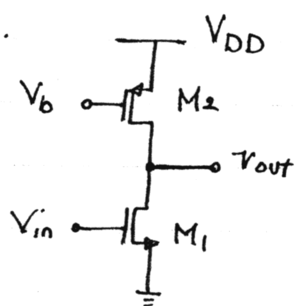
$$\frac{1}{2}\times 3.835\times 10^{-5}\left(\frac{W}{L}\right)_2 (3 - 1.31 + 0.7 - 0.8)^2 + 0.75\times 10^{-3} = 10^{-3},\quad \left(\frac{W}{L}\right)_2 = 5.159$$

$$A_V = -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}} \times \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \times \frac{I_1}{I_2}} = -\sqrt{\frac{1.34225\times 10^{-4}}{3.835\times 10^{-5}} \times \frac{20/0.5}{5.159} \times \frac{10^{-3}}{2.5\times 10^{-4}}} = -10.418$$

3.21. $$A_V = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}}\ \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}\ \frac{I_1}{I_1 - I_S}}$$



3.22.



output voltage Swing = 2.2

$I_{D1} = I_{D2} = 1\,mA$

$A_V = 100$

$$V_{OUT,min} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2},\quad V_{out,max} = V_{DD} - \left(\frac{2I_{D2}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2}$$

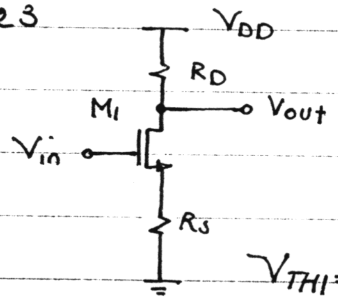
$$V_{DD} - \left(\frac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2} - \left(\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = 2.2, \quad r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.1\times10^{-3}} = 10K$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.2\times10^{-3}} = 5K, \quad r_{o1} \| r_{o2} = \frac{10^4}{3}, \quad g_{m1}(r_{o1}\|r_{o2}) = 100 \rightarrow g_{m1} = \frac{100\times3}{10^4} = 0.03$$

$$2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \times10^{-3} = 9\times10^{-4} \rightarrow \left(\frac{W}{L}\right)_1 = \frac{9\times10^{-4}}{2\times1.34225\times10^{-4}\times10^{-3}} = 3352.5796$$

$$3 - \left(\frac{2\times10^{-3}}{1.34225\times10^{-4}\times3352.5796}\right)^{1/2} - 2.2 = \left(\frac{2\times10^{-3}}{3.835\times10^{-5}\left(\frac{W}{L}\right)_2}\right)^{1/2} \rightarrow \left(\frac{W}{L}\right)_2 = 96.97$$

3.23



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K \quad R_S = 200\Omega$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K, \quad V_S = R_S I_D = 200\times0.5\times10^{-3} = 0.1$$

$$V_{TH1} = V_{TH1,0} + \gamma\left(\sqrt{2|\phi_f| + V_{SB}} - \sqrt{2|\phi_f|}\right) = 0.7 + 0.45\left(\sqrt{0.9+0.1} - \sqrt{0.9}\right)$$

$$V_{TH1} = 0.723, \quad V_{out} = V_{DD} - R_D \cdot I_D = 3 - 2\times10^{3}\times0.5\times10^{-3} = 2$$

$$V_{DS} = 2 - 0.1 = 1.9$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})(1+\lambda V_{DS}) = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (1+\lambda V_{DS}) I_D}$$

$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{50}{0.5}(1+0.1\times0.9)\times0.5\times10^{-3}} = 3.8249\times10^{-3}$$

$$\eta_1 = \frac{0.45}{2(0.1+0.9)^{1/2}} = 0.225 \quad G_m = \frac{g_{m1}\cdot r_{o1}}{R_S + \left[1 + (1+\eta_1)g_{m1}\cdot R_S\right]r_{o1}} =$$

$$G_m = \frac{3.8249\times10^{-3}\times20\times10^{3}}{200 + \left[1+(1+0.225)\,3.8249\times10^{-3}\times200\right]20\times10^{3}} = 1.9644\times10^{-3}$$

$$R_{out} = \left[1 + (g_m + g_{mb})r_o\right]R_S + r_o$$

Seen looking down at the drain of M1

$R_{out} = \left[1 + (1+0.225)\, 3.8249\times10^{-3}\right] 200 + 20\times10^{3} = 20.2\times10^{3}$ 3-29

$R_{out,total} = R_{out} \,//\, R_D = 1819.8274$, $A_V = -G_m \cdot R_{out,total} = -1.96\times10^{-3}\times 1819.8$

$A_V = -3.57$

$V_{out} = V_{in} - V_{TH1}$ @ the edge of the triode region

$V_{in} = V_{GS1} + R_S I_D$

$V_{DD} - R_D I_D = V_{out}$, $V_{DD} - R_D I_D = V_{GS1} + R_S I_D - V_{TH1}$, $V_{DD} - (R_S + R_D) I_D = V_{GS1} - V_{TH1}$

$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{DD} - (R_S + R_D) I_D\right]^2 =$

$\frac{1}{2}\times 1.34225\times10^{-4}\times\frac{50}{0.5}\left[3 - (2000+200) I_D\right]^2$

$I_D = 6.71125\times10^{-3}(3 - 2200 I_D)^2 \rightarrow 32482.45 I_D^2 - 89.5885 I_D + 60.40125\times10^{-3} = 0$

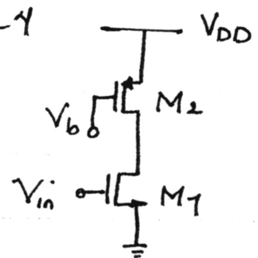
$I_{D1} = 1.5844\times10^{-3}$ (not acceptable), $I_{D2} = 1.17355\times10^{-3}$ (acceptable!)

$V_{in} = V_{DD} - R_D I_D + V_{TH1} = 3 - 2000\times1.17355\times10^{-3} + 0.7 = 1.35288\,V$

$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{50}{0.5}\times1.17355\times10^{-3}} = 5.6128\times10^{-3}$

$G_m = \frac{g_{m1}}{1 + g_{m1}\cdot R_S} = 2.6443\times10^{-3}$ $A_V = -G_m R_D = -2.6443\times10^{-3}\times2000 = -5.2887$

3.24



$A_V = -5$, $\left(\frac{W}{L}\right)_1 = 20/0.5$, $I_{D1} = 0.5\,mA$, $V_b = 0$

$g_{m1} = 2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (1 + \lambda V_{DS}) I_D$

$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{20}{0.5}(1 + 0.1 V_{DS})\times0.5\times10^{-3}}$

$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K$, $r_{o2} = \frac{1}{\lambda_P I} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$

The key point here is that the channel length modulation effect in M_1 cannot be neglected because its drain-source voltage is quite large. We take this effect into account with a few iterations.

First we let $V_{DS1}=0$, then we have, $g_{m1}=2.31711\times10^{-3}$ (as $A_v=-5$) 3-30

$R_{out,total}=2157.86\,\Omega$

$$r_{o2}=\frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{SG}-|V_{TH2}|-V_{SD}\right)}=2418.8356$$

$$0.5\times10^{-3}=\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[\left(V_{SG}-|V_{TH2}|\right)V_{SD}-\frac{V_{SD}^2}{2}\right]$$, by dividing these two relations together.

$$1.2094=\frac{(3-0.8)V_{SD}-0.5V_{SD}^2}{3-0.8-V_{SD}}=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.8188V_{SD}+5.3214=0$$

$V_{SD}=0.8987$, now second iteration starts, with the aid of the value we obtain for V_{SD} (or V_{DS}) from the first iteration, we have:

$g_{m1}=2.5489\times10^{-3}$, $R_{out}=1961.6020\,\Omega$

$$r_{o2}=2174.9182,\quad 1.087459=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5749V_{SD}+4.7848=0$$

$V_{SD}=0.8336\,V$

Third iteration starts now:

By substituting the value of V_{SD} from the second iteration in the relation for g_{m1}, we get:

$g_{m1}=2.5558\times10^{-3}$, $R_{out}=1956.3119$, $r_{o2}=2168.4169\,\Omega$

$$1.0842=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5684V_{SD}+4.77051=0$$

$V_{SD}=0.8315$

By doing the forth iteration:

$g_{m1}=2.5560\times10^{-3}$

$R_{out}=1956.1662$, $r_{o2}=2168.2379$, $1.0841189=\dfrac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}}$

$V_{SD}^2-6.5682V_{SD}+4.77012=0$

$$\boxed{V_{SD}=0.8315}$$

3.31

$$I = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{SG} - |V_{TH2}|) V_{SD} - \frac{V_{SD}^2}{2} \right], \left(\frac{W}{L}\right)_2 = \frac{0.5 \times 10^{-3}}{3.835 \times 10^{-5} \left[(3-0.8)0.8315 - \frac{0.8315^2}{2} \right]}$$

$$\left(\frac{W}{L}\right)_2 = 8.7878$$

If M_1 is at the edge of the triode region: $V_{out} = V_{in} - V_{TH1} = V_{in} - 0.7$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = I_{D2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{DD} - |V_{TH2}|)(V_{DD} - V_o) - \frac{(V_{DD} - V_o)^2}{2} \right] \times \left[1 + 0.2(V_{DD} - V_o)\right]$$

$$V_{out} = \sqrt{\frac{2 \times 3.835 \times 10^{-5}}{1.34225 \times 10^{-4}} \frac{8.7878}{40} \left[2.2(3 - V_o) - \frac{(3 - V_o)^2}{2} \right] (1.6 - 0.2 V_o)}$$

$V_o = 0.6663$, $V_{in} = 1.3663$, $g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out} =$
$1.34225 \times 10^{-4} \times \frac{20}{0.5} \times 0.6663 = 3.5773 \times 10^{-3}$

However, M_2 is no longer in triode region because $V_o = 0.66 < V_b + |V_{TH2}| = 0.8$
Therefore, we should recalculate V_o with the assumption that M_2 is saturated

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o)\right]$$

$536.9 V_o^2 + 32.623 V_o - 260.9845 = 0$, $V_{out} = 0.6674$, $V_{in} = 1.3674$

$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH}) = 3.5837 \times 10^{-3}$, $I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = 1.196 \times 10^{-3}$

$$r_{out} = r_{o1} \,||\, r_{o2} = \frac{1}{(\lambda_p + \lambda_N) I} = 2786.962\,\Omega$$

$$A_v = -g_{m1} \cdot r_{out} = -9.9877$$

$V_{out} = 0.8$, $\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o)\right]$

$$1.34225 \times 10^{-4} \times 40 \times (V_{GS} - 0.7)^2 = 3.835 \times 10^{-5} \times 8.7878 (3 - 0.8)^2 \left[1 + 0.2(3 - 0.8)\right]$$

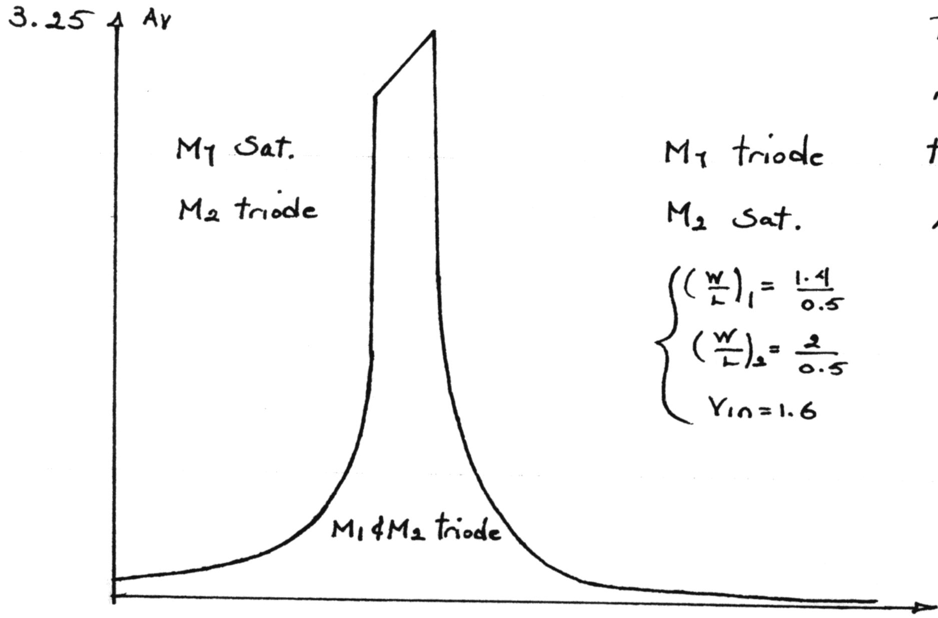
$V_{in} = 1.3614$

$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = 3.5512 \times 10^{-3}$$

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = 1.1744 \times 10^{-3}$$

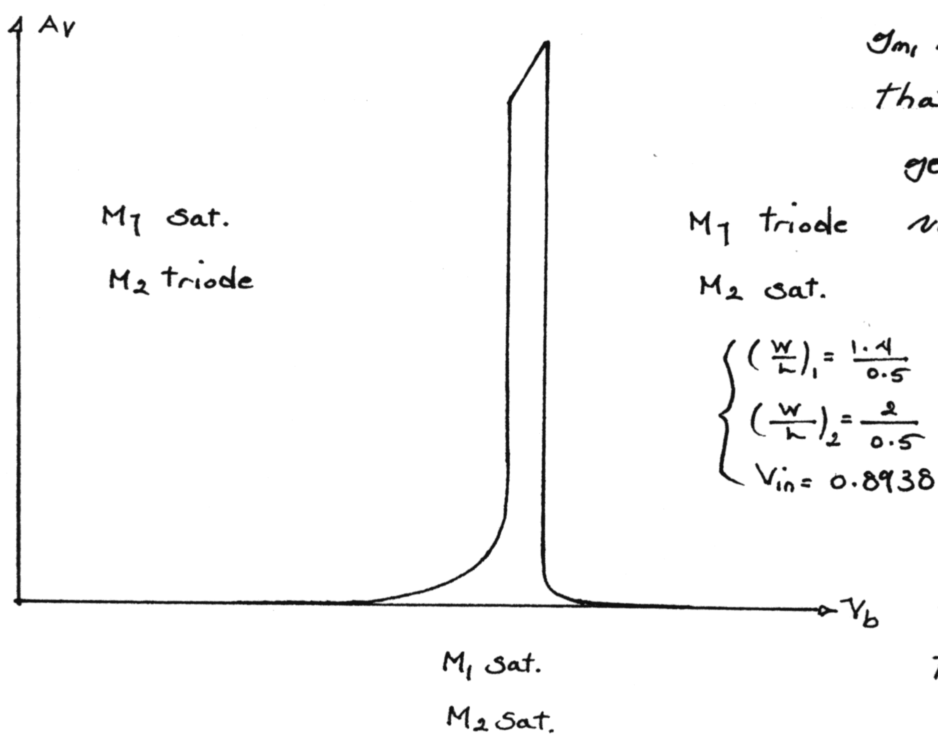
$$r_{out} = \frac{1}{(\lambda_P + \lambda_N) I} = 2838.2553$$

$$A_V = -g_{m1} \cdot r_{out} = -10.08$$

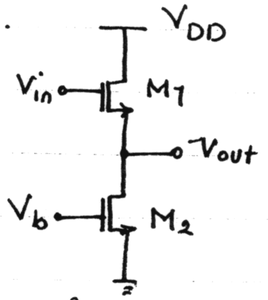


For M_1 to enter the triode region before M_2 is saturated, the overdrive voltage of M_1 must be increased.

Comparing the two curves, we observe that at $V_b = 0$ small signal voltage gain in (a) is higher than that V_b in (b). That is because g_{m1} in (a) is higher than that in (b). However, generally, small signal voltage gain in (a) is less than that in (b), because when v_b sweeps all the way from 0 to V_{DD}, nowhere are both devices simultaneously in the saturation region.



3.26. 3.39



$V_{in} - V_{out} = 1V, \; I_{D1} = I_{D2} = 0.5\,mA, \; V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_1 (1-0.7)^2$

$\left(\frac{W}{L}\right)_1 = 82.77 \qquad V_{GS2} = 0.5+1 = 1.5$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_2 (1.5-0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$

$\gamma = 0.45\,V^{-1}, \; V_{in} = 2.5V, \; V_{in} - V_{out} = 1, \; I_{D1} = I_{D2} = 0.5\,mA, \; V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, \; V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5V$

$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_0} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1.5} - \sqrt{0.9}\right) = 0.97022$

$I_{D1} = I_{D2} = 0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4} S_1 (1-0.97)^2 = \frac{1}{2}\times1.34225\times10^{-4} S_2 (1.5-0.7)^2$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

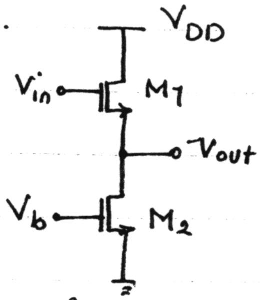
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9+0.8} - \sqrt{0.9}\right) = 0.8598$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times8278\,(V_{in} - 0.8 - 0.8598)^2$

$V_{in} = 1.6897$

3.26.



$V_{in} - V_{out} = 1V$, $I_{D1} = I_{D2} = 0.5\ mA$, $V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_1 (1-0.7)^2$$

$\left(\frac{W}{L}\right)_1 = 82.77$ $\qquad V_{GS2} = 0.5 + 1 = 1.5$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_2 (1.5-0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$$

$\gamma = 0.45\ V^{-1}$, $V_{in} = 2.5V$, $V_{in} - V_{out} = 1$, $I_{D1} = I_{D2} = 0.5\ mA$, $V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5$, $V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5V$

$$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_f| + V_0} - \sqrt{2|\phi_f|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1.5} - \sqrt{0.9}\right) = 0.97022$$

$$I_{D1} = I_{D2} = 0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4} S_1 (1-0.97)^2 = \frac{1}{2}\times1.34225\times10^{-4} S_2 (1.5-0.7)^2$$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

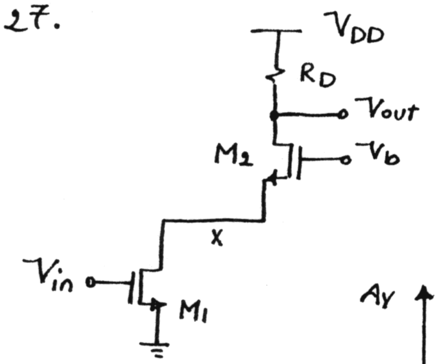
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

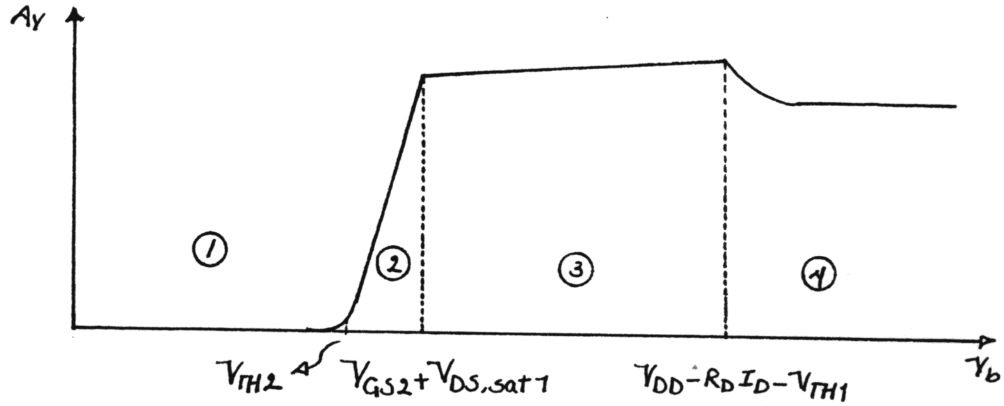
$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9+0.8} - \sqrt{0.9}\right) = 0.8598$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times8278\,(V_{in} - 0.8 - 0.8598)^2$$

$V_{in} = 1.6897$

3.27.

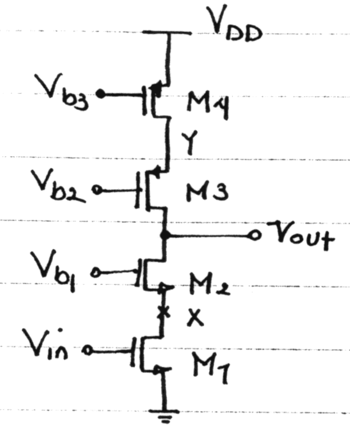




1. In this region V_b is less than V_{TH2}, so M_1 and M_2 are off. It is worth mentioning that M_2 is saturated-off and M_1 is off in triode region.
2. V_b is increasing above V_{TH2}, as a result, a current establishes in circuit. M_1 operates in triode region and M_2 does in saturation. The higher V_b, the higher the drain-source voltage of M_1, increasing the output impedance of M_1 which, in turn, causes the small signal voltage gain of the circuit increases.
3. Both devices are in saturation region and the maximum gain is attainable in this region. The slight increase in A_V is because of increasing the transconductance of M_1 with increasing V_X (or V_b).
4. M_2 enters the triode region, as a result, the total output impedance decreases down to the limit of $r_{o1} \| R_D$. Consequently, the small signal voltage gain experiences a similar change.

3.28

3.35



output swing $= 1.9\,V$

$I_{bias} = 0.5\,mA$

$\gamma = 0 \qquad \left(\frac{W}{L}\right)_{1-4} = \left(\frac{W}{L}\right)$

$V_{b1} - V_{TH1} < V_{out} < V_{b2} + V_{TH3}$

$V_{b2} + V_{TH3} - (V_{b1} - V_{TH1}) = 1.9$

$V_{b2} + 0.8 - V_{b1} + 0.7 = 1.9\ ,\ V_{b2} - V_{b1} = 0.4$

$$0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S (V_{b1} - V_X - V_{TH2})^2 =$$
$$\frac{1}{2}\mu_p C_{ox} S (V_Y - V_{b2} - |V_{TH3}|)^2 = \frac{1}{2}\mu_p C_{ox} S (V_{DD} - V_{b3} - |V_{TH4}|)^2$$

$$V_{DD} - V_{SDmin,4} - V_{SDmin,3} - V_{DSmin,1} - V_{DSmin,2} = 1.9$$

$$1.1 = \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2}$$

$$1.1 = 2\sqrt{2I_D}\left(\frac{1}{\sqrt{\mu C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)\frac{1}{\sqrt{S}} \rightarrow S = \frac{8I_D\left(\frac{1}{\sqrt{\mu_p C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)^2}{1.1^2}$$

$$S = \frac{8\times0.5\times10^{-3}\left(\frac{1}{\sqrt{1.34225\times10^{-4}}} + \frac{1}{\sqrt{3.835\times10^{-5}}}\right)^2}{1.1^2} = 202.98 \rightarrow S = 203$$

$$V_{DSmin,1} = \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} = \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times203}\right)^{1/2} = 0.1915$$

$$V_{SDmin,4} = \left(\frac{2\times0.5\times10^{-3}}{3.835\times10^{-5}\times203}\right)^{1/2} = 0.3584$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times203\,(V_{b1} - V_X - 0.7)^2$$

$V_{b1} - V_X = 0.8915$

$$0.5\times10^{-3} = \frac{1}{2}\times3.835\times10^{-5}\times203\,(V_Y - V_{b2} - 0.8)^2$$

$V_Y - V_{b2} = 1.1584$

$V_{b2} - V_{b1} = 0.4$

If $V_X = 0.1915 \rightarrow V_{b1} = 1.083$, $V_{b2} = 1.483$, $V_Y = 2.6414$

$V_{SD4} = V_{DD} - V_Y = 0.3586$, as a result, M_7 and M_2 are at the edge of the triode region.

$$g_{m1} = \sqrt{2\mu_n C_{ox} S (1 + \lambda V_{DS}) I_D} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 203 \times 0.5 \times 10^{-3}}$$

$$g_{m1} = g_{m2} = 5.2199 \times 10^{-3}$$

$$r_{o1} = r_{o2} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K \qquad r_{o3} = r_{o4} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$$

$$G_m = \frac{g_{m1} \cdot r_{o1} \cdot (1 + g_{m2} \cdot r_{o2})}{r_{o1} \cdot r_{o2}\, g_{m2} + r_{o1} + r_{o2}} = \frac{5.2199 \times 10^{-3} \times 20 \times 10^{3} (20 \times 10^{3} \times 5.2199 \times 10^{-3} + 1)}{(20 \times 10^{3})^2 \times 5.2199 \times 10^{-3} + 2 \times 20 \times 10^{3}}$$

$G_m = 5.17 \times 10^{-3}$, neglecting the body effect.

$$R_{out} = \left[(1 + g_{m2} r_{o2}) r_{o1} + r_{o2} \right] \| \left[(1 + g_{m3} r_{o3}) r_{o1} + r_{o3} \right]$$

$$R_{out} = \left[(1 + 5.2199 \times 10^{-3} \times 20 \times 10^{3}) 20 \times 10^{3} + 20 \times 10^{3} \right] \| \left[(1 + 2.79 \times 10^{-3} \times 10 \times 10^{3}) 10 \times 10^{3} + 10 \times 10^{3} \right]$$

$R_{out} = 262.1766 \times 10^{3}$, $A_V = -G_m R_{out} = -5.17 \times 10^{-3} \times 262.1766 \times 10^{3}$

$A_V = -1355.45$ $\qquad g_{m3} = g_{m4} = \sqrt{2 \times 3.835 \times 10^{-5} \times 203 \times 0.5 \times 10^{-3}} = 2.79 \times 10^{-3}$

Chapter 4: Differential Amplifiers

4.1

(a) $A_V \cong -\dfrac{g_{mN}}{g_{mP}} = -\sqrt{\dfrac{\mu_n (W/L)_N}{\mu_P (W/L)_P}}$ (4.52)

$A_V = -\sqrt{\dfrac{350}{100} \times \dfrac{50/0.5}{50/1}} = -\sqrt{7} = \underline{-2.65}$

(b) $A_V = -g_{mN}(r_{ON} \| r_{OP})$ (4.53)

$I_D = \dfrac{I_{SS}}{2} = 0.5^{mA}$ $\mu_n C_{ox} = 350 \times \dfrac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 0.134 \ ^{mA}/_{V^2}$

$g_{m_N} = \sqrt{2 I_D \mu_n C_{ox} \dfrac{W}{L}} = \sqrt{2 \times 0.5^m \times 0.134^m \times 100} = 3.66 \ m\Omega^{-1}$

$L_N = 0.5^{\mu} \Rightarrow \lambda_n = 0.1 \rightarrow r_{ON} = \dfrac{1}{\lambda_n I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$L_P = 1^{\mu}$; $\lambda_P = 0.2$ for $L = 0.5^{\mu}$; $\lambda \propto \dfrac{1}{L} \rightarrow \lambda_P = 0.1$

$r_{OP} = \dfrac{1}{\lambda_P I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$A_V = -g_{mN}(r_{ON} \| r_{OP}) = -3.66^m (20^K \| 20^K) = \underline{-36.6}$

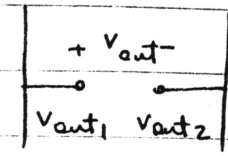
$(V_{in,cm})_{min} = 0.4 + V_{GS1}$ for both circuits

$V_{GS1} = V_{TH} + \sqrt{\dfrac{2 I_D}{\mu_n C_{ox} (W/L)_N}} = 0.7 + \sqrt{\dfrac{2 \times 0.5^m}{0.134^m \times 100}} = 0.7 + 0.27 = 0.97^V$

$\rightarrow (V_{in,CM})_{min} = 0.4 + 0.97 = 1.37^V$

max output voltage swing:

(a) $(V_{out1,2})_{max} = V_{DD} - |V_{TH,P}| = 3 - 0.8 = 2.2^V$



There are two constraints for $(V_{out1,2})_{min}$:

1) M_1 enters triode: $(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n}$

$= 0.4 + 0.97 - 0.7 = 0.67^V$

2) all of I_{SS} goes through M_3:

$$(V_{out1,2})_{min} = V_{DD} - \left|V_{GS3}\big|_{I_D = I_{SS}}\right| = V_{DD} - |V_{TH,P}| + \sqrt{\frac{2\,I_{SS}}{\mu_p C_{ox} (\frac{W}{L})_3}}$$

$$= 3 - 0.8 - \sqrt{\frac{2 \times 1^m}{38.3^{\mu} \times 50}} = 3 - 0.8 - 1.02 = 1.18^V$$

$$\mu_p C_{ox} = 100 \times \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 38.3\ ^{\mu A}/_{V^2} \qquad \Rightarrow (V_{out1,2})_{min} = 1.18^V$$

Max swing of $V_{out1,2} = 2.2 - 1.18 = 1.02^V$

Max swing of $V_{out} = 2 \times 1.02 = 2.04^V$

(b) $(V_{out1,2})_{max} = V_{DD} - |V_{GS3} - V_{TH,P}| = 3 - 0.72 = 2.28^V$

$(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n} = 0.67^V$

Max swing of $V_{out} = 2(2.28 - 0.67) = 3.22^V$

4.2 $I_{SS} = 1^{mA}$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

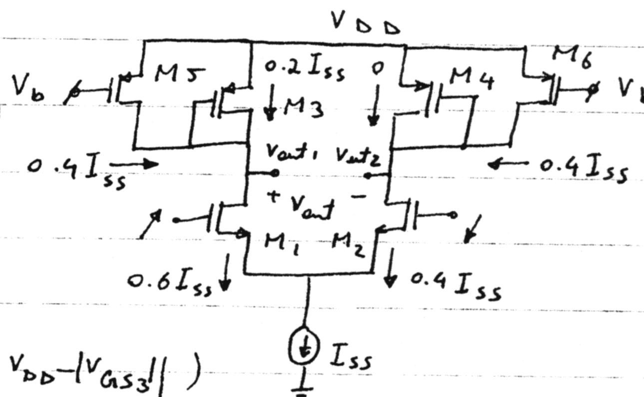
$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2\,\frac{I_{SS}}{2}}} = -4.18$$

(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4^{mA}$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^m}{38.3^{\mu} \times 100}} = 1.74$$

(c)



$$(V_{out1,2})_{max} = \min(V_b + |V_{TH,P}|, V_{DD} - |V_{TH,P}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2^V$$

$$(V_{out1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D = 0.6 I_{SS}} - V_{TH,n},\ V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D = 0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6\, I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299^V$$

$$|V_{GS3}|\Big|_{I_D = 0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2\, I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323^V = 1.12^V$$

$$(V_{out1,2})_{min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88^V$$

Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^V$

4.2 $I_{SS} = 1\ mA$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

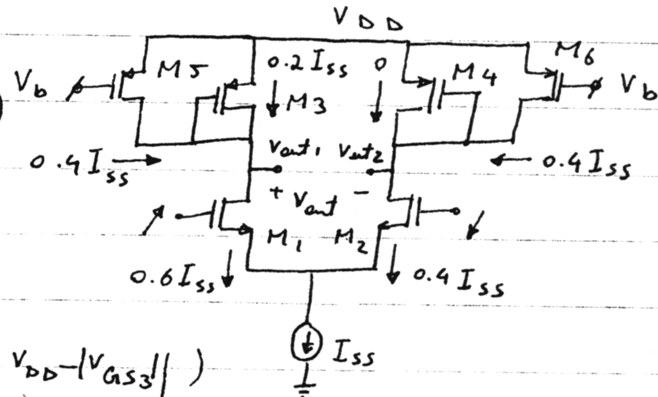
$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2 \frac{I_{SS}}{2}}} = -4.18$$

(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4\ mA$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^m}{38.3^\mu \times 100}} = 1.74$$

(c)



$$(V_{out1,2})_{max} = \min(V_b + |V_{TH,P}|, V_{DD} - |V_{TH,P}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2^V$$

$$(V_{out1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D = 0.6 I_{SS}} - V_{TH,n}, V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D = 0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299^V$$

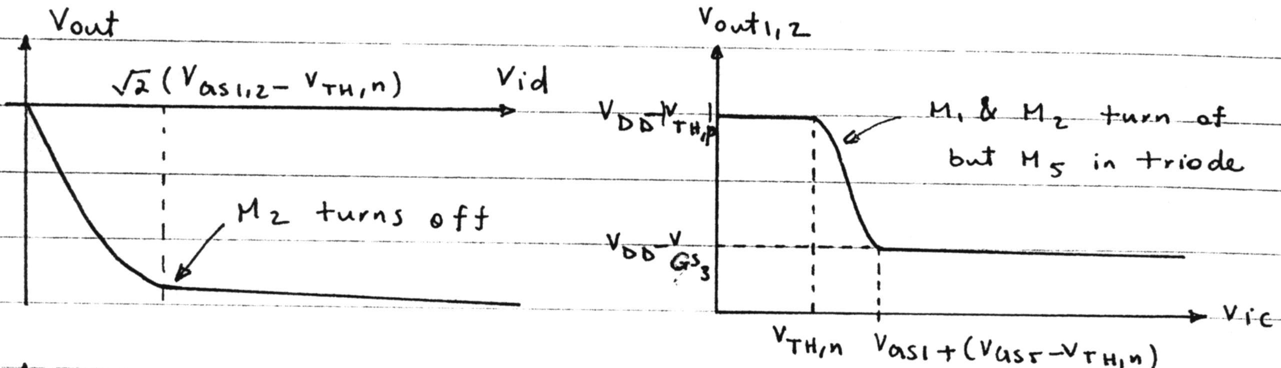
$$|V_{GS3}|\Big|_{I_D = 0.2 I_{SS}} = |V_{THP}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323^V = 1.12^V$$

$$(V_{out1,2})_{min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88^V$$

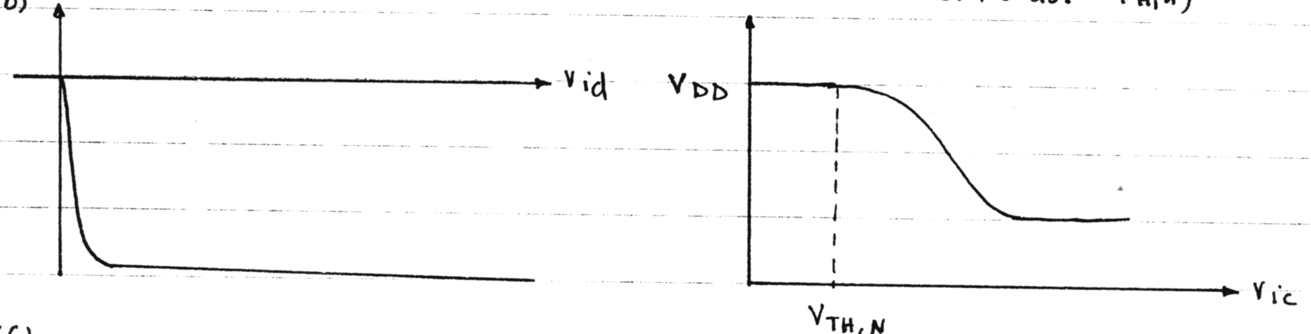
Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^V$

4.3

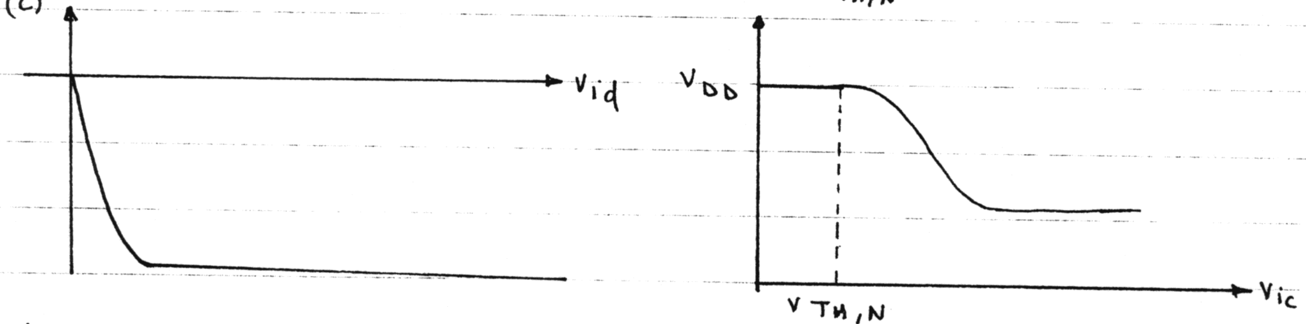
(a)


V_{out}
$\sqrt{2}\,(V_{GS1,2} - V_{TH,n})$
V_{id}
M_2 turns off
$V_{out1,2}$
$V_{DD} - V_{TH,p}$
M_1 & M_2 turn of but M_5 in triode
$V_{DD} - V_{GS_3}$
V_{ic}
$V_{TH,n}$
$V_{GS1} + (V_{GS5} - V_{TH,n})$

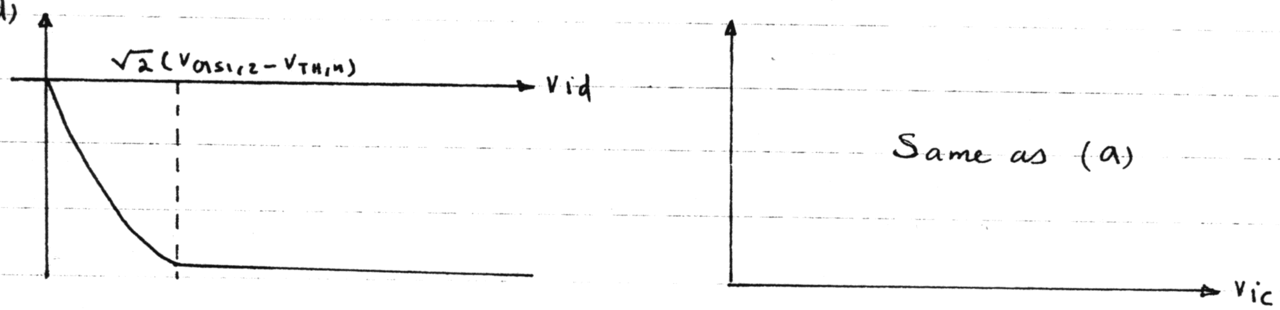
(b)


V_{id}
V_{DD}
V_{ic}
$V_{TH,N}$

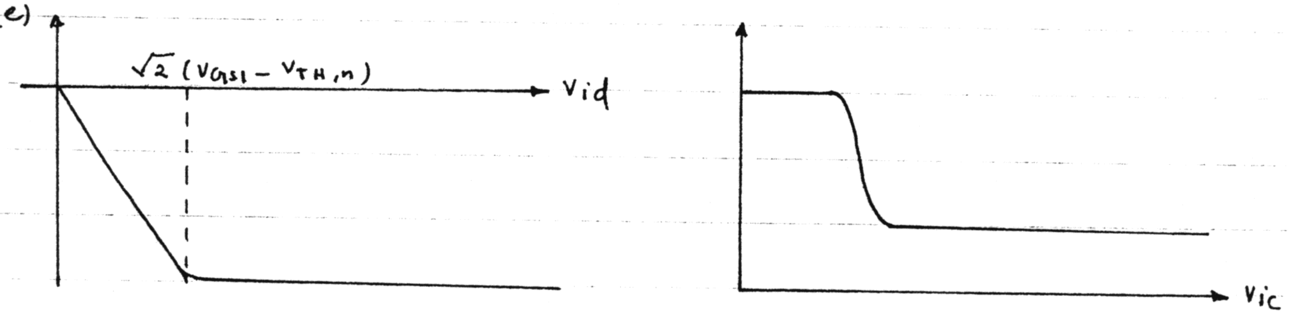
(c)


V_{id}
V_{DD}
V_{ic}
$V_{TH,N}$

(d)

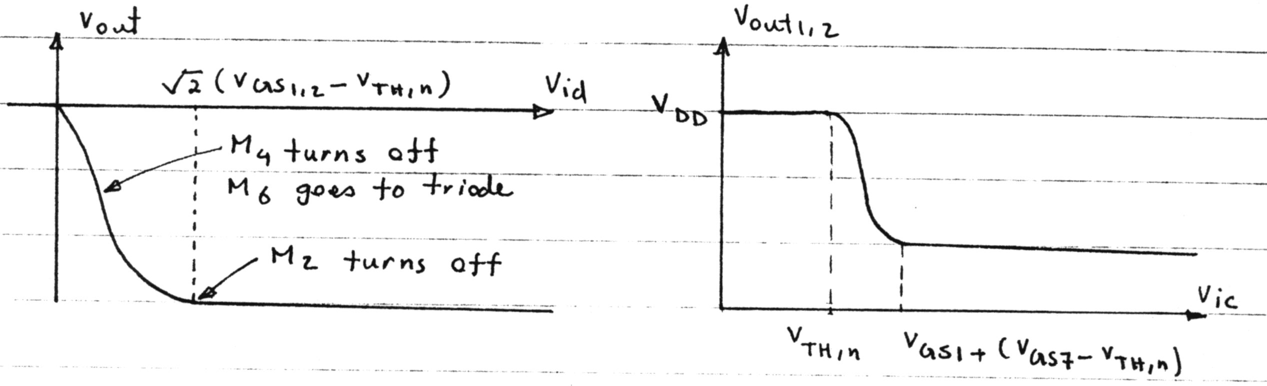

$\sqrt{2}\,(V_{GS1,2} - V_{TH,n})$
V_{id}
Same as (a)
V_{ic}

(e)

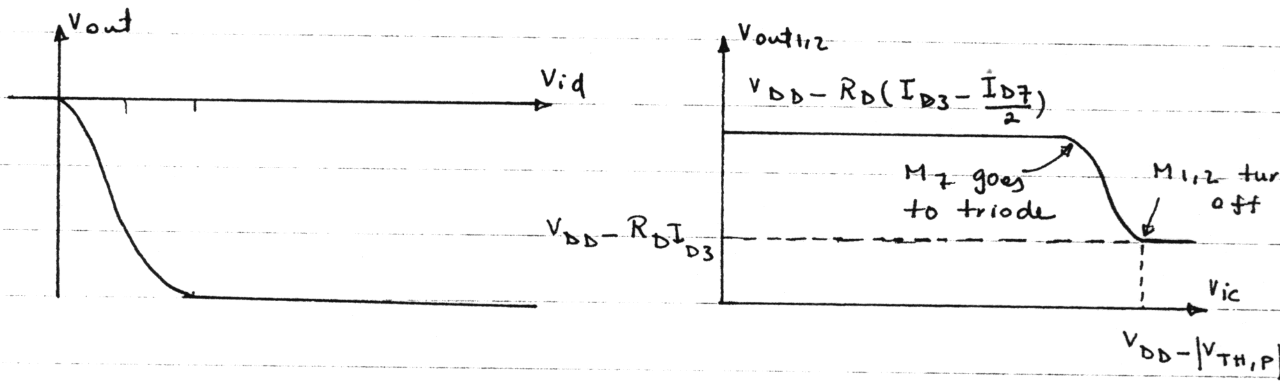

$\sqrt{2}\,(V_{GS1} - V_{TH,n})$
V_{id}
V_{ic}

4.4

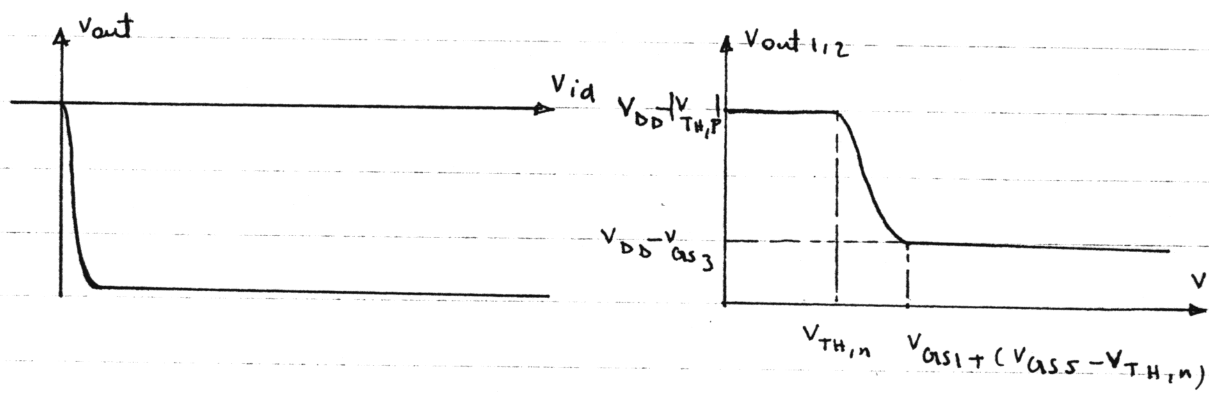
(a)


V_{out}
$\sqrt{2}\,(V_{GS1,2} - V_{TH,n})$
V_{id}
M_4 turns off
M_6 goes to triode
M_2 turns off
$V_{out1,2}$
V_{DD}
V_{ic}
$V_{TH,n}$
$V_{GS1} + (V_{GS7} - V_{TH,n})$

(b)


V_{out}
V_{id}
$V_{out1,2}$
$V_{DD} - R_D(I_{D3} - \frac{I_{D7}}{2})$
M_7 goes to triode
$M_{1,2}$ turns off
$V_{DD} - R_D I_{D3}$
V_{ic}
$V_{DD} - |V_{TH,P}|$

(c)


V_{out}
V_{id}
$V_{out1,2}$
$V_{DD} - |V_{TH,P}|$
$V_{DD} - V_{GS3}$
V_{ic}
$V_{TH,n}$
$V_{GS1} + (V_{GS5} - V_{TH,n})$

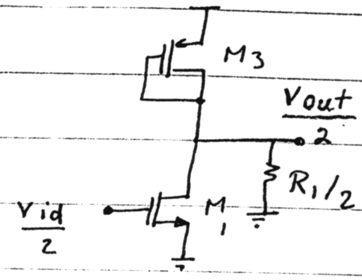
4.5

Fig. 4.35 Using half circuit we have:

(a) we define $V_{id} = V_{in1} - V_{in2}$

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1}\left(\frac{1}{g_{m3}} \,\|\, \frac{R_1}{2} \,\|\, r_{o1} \,\|\, r_{o3}\right)$$

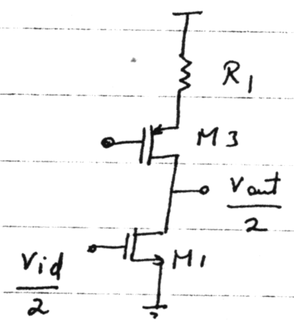
$$\approx -g_{m1}\left(\frac{1}{g_{m3}} \,\|\, \frac{R_1}{2}\right) = -\frac{g_{m1}R_1}{2+g_{m3}R_1}$$



(b)

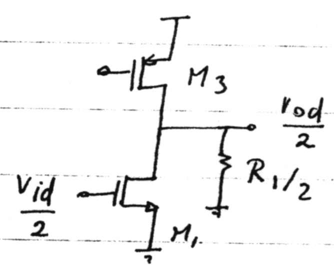
$$A_v = -g_{m1}\left[r_{o1} \,\|\, (R_1 g_{m3} r_{o3} + R_1 + r_{o3})\right]$$

$$\approx -g_{m1}\left(r_{o1} \,\|\, R_1 g_{m3} r_{o3}\right)$$



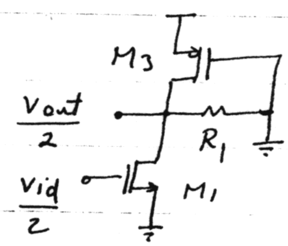
(c)

$$A_v = -g_{m1}\left(r_{o1} \,\|\, r_{o3} \,\|\, \frac{R_1}{2}\right)$$

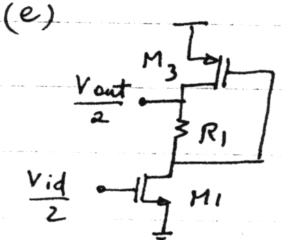


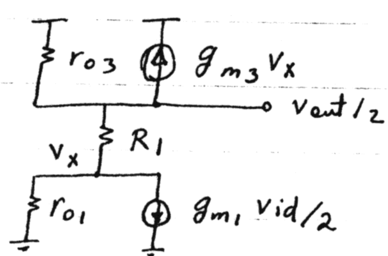
(d)

$$A_v = -g_{m1}\left(r_{o1} \,\|\, r_{o3} \,\|\, R_1\right)$$



(e)





KCL: $$\frac{V_x}{r_{o1}} + \frac{V_x - V_{out}/2}{R_1} + g_{m1}\frac{V_{id}}{2} = 0$$

$$\frac{V_{out}/2}{r_{o3}} + \frac{V_{out}/2 - V_x}{R_1} + g_{m3}V_x = 0 \Rightarrow V_x = \frac{(r_{o3}+R_1)V_{out}}{2r_{o3}(1-R_1 g_{m3})}$$

$$\left(\frac{R_1 + r_{o1}}{R_1 r_{o1}} \times \frac{r_{o3}+R_1}{r_{o3}(1-R_1 g_{m3})} - \frac{1}{R_1}\right) V_{out} + g_{m1}V_{id} = 0$$

$$\frac{(R_1+r_{o1})(R_1+r_{o3}) - r_{o1}r_{o3}(1-R_1 g_{m3})}{R_1 r_{o1} r_{o3}(1-R_1 g_{m3})} V_{out} + g_{m1}V_{id} = 0$$

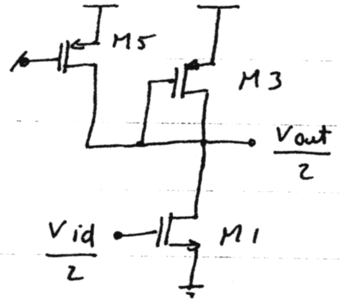
$$\frac{R_1 + r_{o1} + r_{o2} + r_{o1}r_{o3}g_{m3}}{r_{o1}r_{o3}(1-R_1 g_{m3})} V_{out} + g_{m1}V_{id} = 0$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{id}} = -\frac{g_{m1}r_{o1}r_{o3}(1-R_1 g_{m3})}{R_1 + r_{o1} + r_{o3} + r_{o1}r_{o3}g_{m3}} \approx -\frac{g_{m1}}{g_{m3}}(1-R_1 g_{m3})$$

$$R_1 g_{m3} < 1$$

Fig. 4.36

(a)

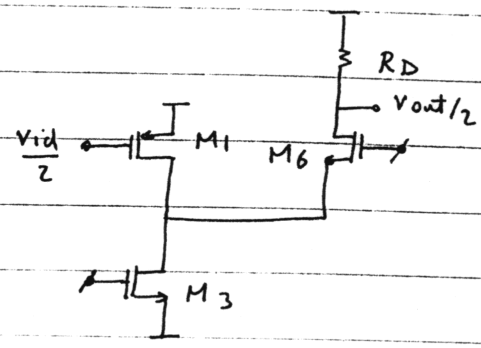


$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right)$$

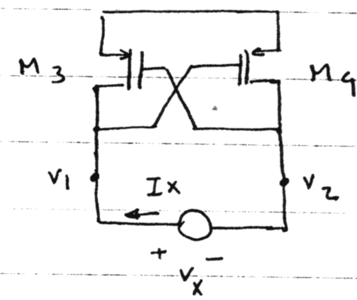
$$\approx -\frac{g_{m1}}{g_{m3}}$$

(b)

$$A_v = -g_{m1}\left(R_D \,\|\, [r_{o6}\, g_{m6}(r_{o1} \| r_{o3})]\right)$$



(c)



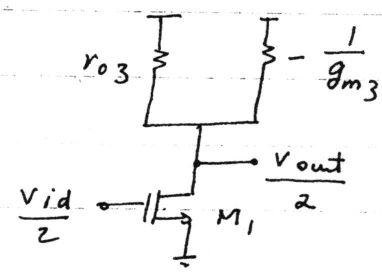
if we neglect r_{o3} & r_{o4} at the moment, we have:

$$I_x = g_{m3} V_2 \qquad g_{m3} = g_{m4} = g_{m3,4}$$

$$I_x = -g_{m4} V_1$$

$$2I_x = -g_{m3,4}(V_1 - V_2) = -g_{m3,4} V_x$$

$$\Rightarrow \frac{V_x}{I_x} = -\frac{2}{g_{m3,4}}$$



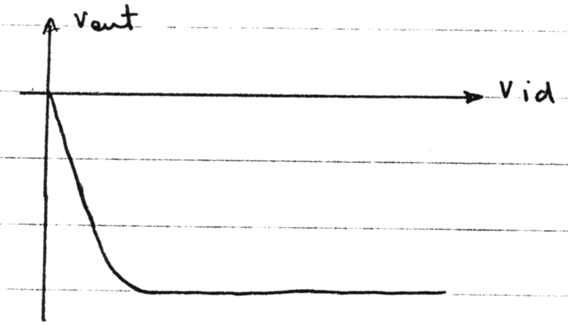
$$A_v = -g_{m1}\left(r_{o1} \| r_{o3} \| \frac{-1}{g_{m3}}\right)$$

$$A_v = -\frac{g_{m1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m3}} \qquad \left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} > g_{m3}\right)$$

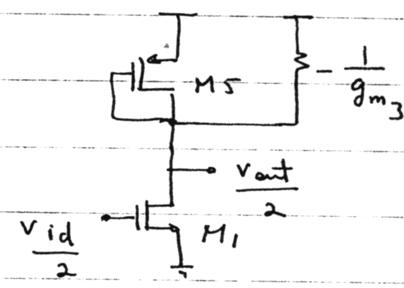
if $g_{m3} \geq \frac{1}{r_{o1}} + \frac{1}{r_{o3}}$ then the circuit is not stable and small signal model is not valid.

4.6

(a)



(b)



Similar to what we had in the previous problem (Fig 4.36 (c)), M_3 gives a negative resistance at the output.

$$A_V = -g_{m1}\left(\frac{1}{g_{m5}} \,\Big|\Big|\, \frac{-1}{g_{m3}}\right) \qquad (\lambda = \infty)$$

$$A_V = -\frac{g_{m1}}{g_{m5} - g_{m3}} \qquad (g_{m3} \text{ must be less than } g_{m5})$$

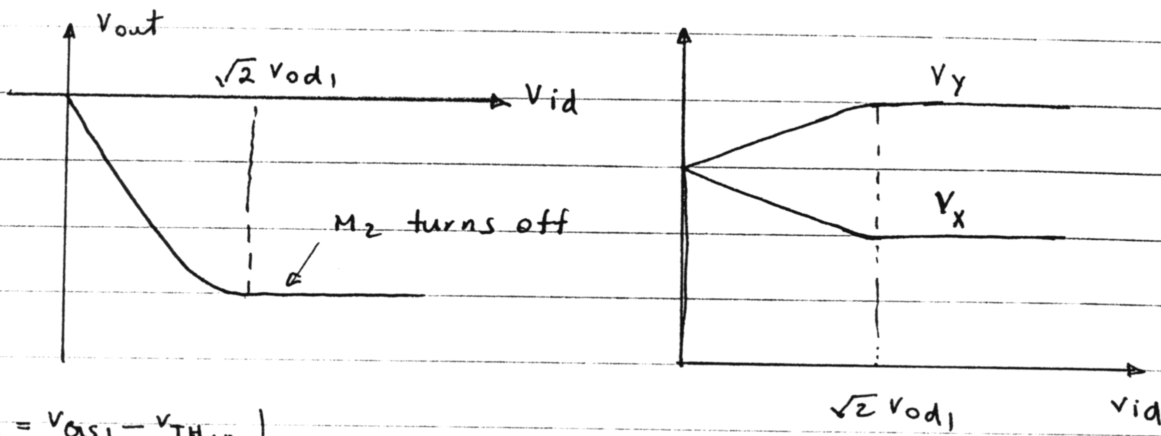
$$g_m = \mu_p C_{ox} \frac{W}{L}(V_{GS} - V_T) \qquad V_{GS\,3,4} = V_{GS\,5,6}$$

$$\Rightarrow \quad \frac{g_{m3,4}}{g_{m5,6}} = \frac{(W/L)_{3,4}}{(W/L)_{5,6}} = 0.8$$

$$\Rightarrow \quad A_V = -\frac{g_{m1}}{g_{m5} - 0.8\,g_{m5}} = -\frac{5\,g_{m1}}{g_{m5}}$$

4.7

(a)

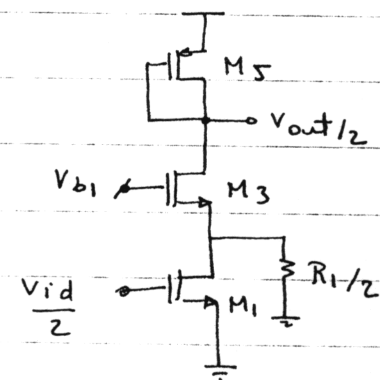


$$V_{od1} = V_{GS1} - V_{TH,n} \Big|_{I_D = \frac{I_{SS}}{2}}$$

(b)

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1}\left(\frac{R_1}{2} \Big\| \frac{1}{g_{m3}}\right) g_{m3}\, g_{m5}^{-1}$$

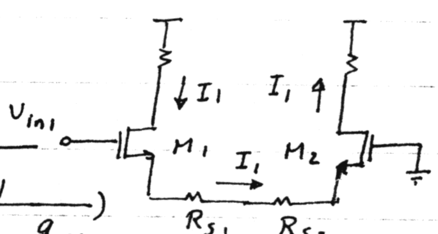
$$= -\frac{g_{m1}}{g_{m5}\left(1 + \frac{2}{R_1 g_{m3}}\right)}$$



4.8

By using superposition, we have:

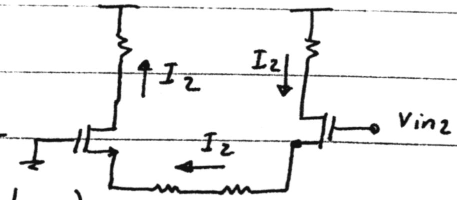
$$G_{m1} = \frac{I_1}{V_{in1}}\Bigg|_{V_{in2}=0} = \frac{g_{m1}}{1 + (g_{m1} + g_{mb1})\underbrace{\left(R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}\right)}_{R_{eq1}}}$$



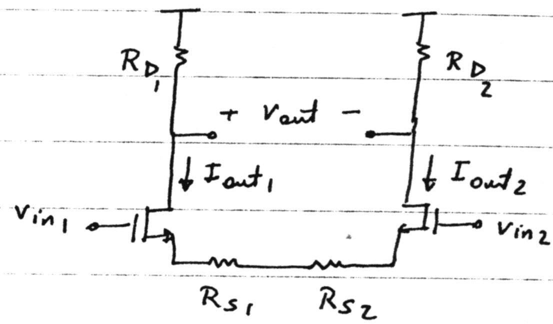
$$R_{eq1} = R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}$$

(Transconductance of CS)

Similarly:



$$G_{m2} = \left.\frac{I_2}{V_{in2}}\right|_{V_{in1}=0} = \frac{g_{m2}}{1 + (g_{m2} + g_{mb2})\left(R_{S1} + R_{S2} + \frac{1}{g_{m1} + g_{mb1}}\right)}$$



$$I_{out1} = I_1 - I_2$$

$$I_{out2} = I_2 - I_1$$

$$V_{out} = V_{out1} - V_{out2} = -R_{D1} I_{out1} + R_{D2} I_{out2}$$

$$V_{out} = -R_{D1}(I_1 - I_2) - R_{D2}(I_1 - I_2)$$

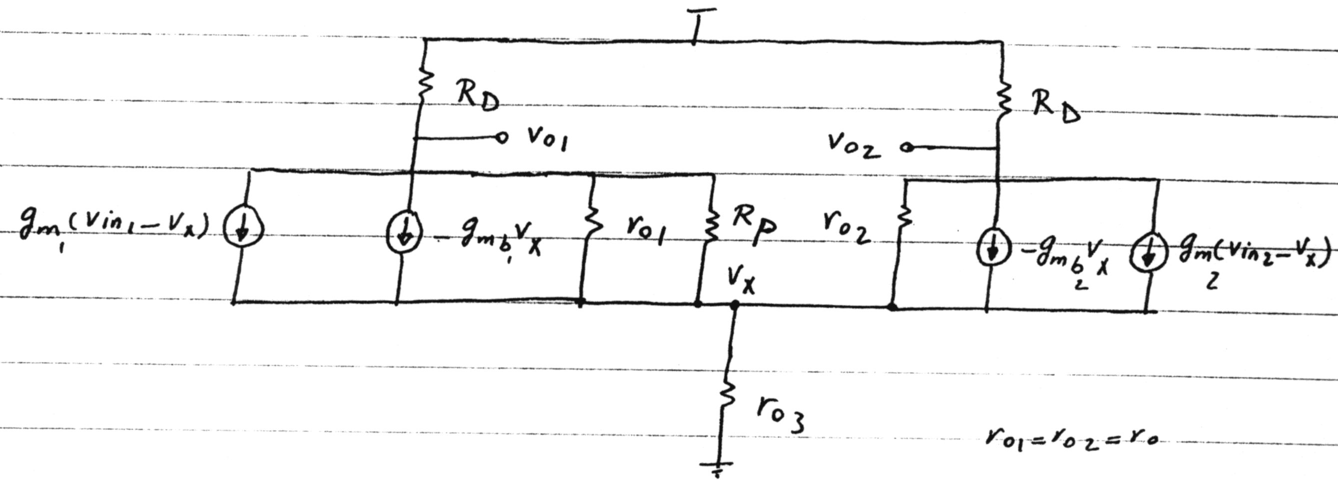
$$V_{out} = -(R_{D1} + R_{D2})(I_1 - I_2)$$

$$V_{out} = -(R_{D1} + R_{D2})(G_{m1} V_{in1} - G_{m2} V_{in2})$$

or equivalantly:

$$V_{out} = -(R_{D1} + R_{D2})\left[(G_{m1} + G_{m2})\frac{V_{in1} - V_{in2}}{2} + (G_{m1} - G_{m2})\frac{V_{in1} + V_{in2}}{2}\right]$$

4.9



$r_{o1}=r_{o2}=r_o$

$g_{m_1}=g_{m_2}=g_m$

$g_{mb_1}=g_{mb_2}=g_{mb}$

KCL:

$$\begin{cases} ① \quad \dfrac{V_{o1}}{R_D}+\dfrac{V_{o1}-V_x}{r_o}+g_m(V_{in_1}-V_x)-g_{mb}V_x+\dfrac{V_{o1}-V_x}{R_P}=0 \\ ② \quad \dfrac{V_{o2}}{R_D}+\dfrac{V_{o2}-V_x}{r_o}+g_m(V_{in_2}-V_x)-g_{mb}V_x=0 \\ \dfrac{V_{o1}}{R_D}+\dfrac{V_{o2}}{R_D}+\dfrac{V_x}{r_{o3}}=0 \Rightarrow V_x=-r_{o3}\dfrac{V_{o1}+V_{o2}}{R_D} \end{cases}$$

we define:

$$\begin{cases} V_{od}=V_{o1}-V_{o2} \\ V_{oc}=\dfrac{V_{o1}+V_{o2}}{2} \end{cases} \Rightarrow \begin{cases} V_{o1}=V_{oc}+\dfrac{V_{od}}{2} \\ V_{o2}=V_{oc}-\dfrac{V_{od}}{2} \end{cases}$$

Now by substituting V_{o1}, V_{o2} and V_x in ① and ② we have:

$$\begin{cases} (V_{oc}+\dfrac{V_{od}}{2})(\dfrac{1}{R_D}+\dfrac{1}{R_P}+V_o)+g_mV_{in_1}-(\dfrac{1}{R_P}+g_m+g_{mb}+\dfrac{1}{r_o})(-\dfrac{2r_{o3}}{R_D}V_{oc})=0 \\ (V_{oc}-\dfrac{V_{od}}{2})(\dfrac{1}{R_D}+\dfrac{1}{r_o})+g_mV_{in_2}-(g_m+g_{mb}+\dfrac{1}{r_o})(-\dfrac{2r_{o3}V_{oc}}{R_D})=0 \end{cases}$$

or:

$$\left[\frac{1}{R_D}+\frac{1}{R_P}+\frac{1}{r_o}+(\frac{1}{R_D}+\frac{1}{r_o}+g_m+g_{mb})(\frac{2r_{o3}}{R_D})\right]V_{oc}$$

$$+\frac{1}{2}(\frac{1}{R_D}+\frac{1}{R_P}+\frac{1}{r_o})V_{od}+g_mV_{in_1}=0 \qquad ③$$

$$\left[\frac{1}{R_D}+\frac{1}{r_o}+\left(g_m+g_{mb}+\frac{1}{r_o}\right)\left(\frac{2r_{o3}}{R_D}\right)\right]V_{oc}-\frac{1}{2}\left(\frac{1}{R_D}+\frac{1}{r_o}\right)V_{od}$$

$$+\ g_m V_{in2}=0 \qquad (4)$$

From equation (3) and (4) V_{od} and V_{oc} can be solved in terms of V_{in1} and V_{in2}.

Now if $\lambda=\gamma=0$, we have:

(5)
$$\begin{cases}\dfrac{V_{o1}}{R_D}+g_m(V_{in1}-V_X)+\dfrac{V_{o1}-V_X}{R_P}=0\\[2ex] \dfrac{V_{o2}}{R_D}+g_m(V_{in2}-V_X)=0 \Rightarrow V_X=V_{in2}+\dfrac{V_{o2}}{g_mR_D}\\[2ex] V_{o1}+V_{o2}=0\ ,\quad V_{out}=V_{o1}-V_{o2}\Rightarrow V_{o1}=\dfrac{V_{out}}{2}\quad V_{o2}=-\dfrac{V_{out}}{2}\end{cases}$$

(5) $\Rightarrow$
$$\frac{V_{out}}{2R_D}+g_mV_{in1}-g_mV_{in2}+\frac{V_{out}}{2R_D}+\frac{V_{out}}{2R_P}-\frac{V_{in2}}{R_P}+\frac{V_{out}}{2R_Pg_mR_D}=0$$

$$V_{out}\left(\frac{1}{R_D}+\frac{1}{2R_P}+\frac{1}{2g_mR_PR_D}\right)=-g_m(V_{in1}-V_{in2})+\frac{V_{in2}}{R_P}$$

$$V_{in2}=\frac{V_{in1}+V_{in2}}{2}-\frac{V_{in1}-V_{in2}}{2}$$

$$\Rightarrow V_{out}\left(\frac{1}{R_D}+\frac{1}{2R_P}+\frac{1}{2g_mR_DR_P}\right)=-\left(g_m+\frac{1}{2R_P}\right)(V_{in1}-V_{in2})+\frac{(V_{in1}+V_{in2})}{2R_P}$$

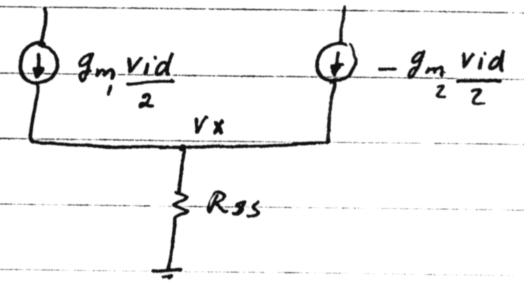
$$V_{out}=\left[-\left(g_m+\frac{1}{2R_P}\right)(V_{in1}-V_{in2})+\frac{V_{in1}+V_{in2}}{2R_P}\right]\left(R_D\,\|\,2R_P\,\|\,2g_mR_DR_P\right)$$

$$CMRR=\frac{g_m+\frac{1}{2R_P}}{\frac{1}{R_P}}=\frac{2R_Pg_m+1}{2}$$

$$A_{dm-dm}=-\left(g_m+\frac{1}{2R_P}\right)\left(R_D\|2R_P\|2g_mR_DR_P\right)\qquad A_{cm-dm}=-\frac{R_D\|2R_P\|2g_mR_DR_P}{R_P}$$

4.10

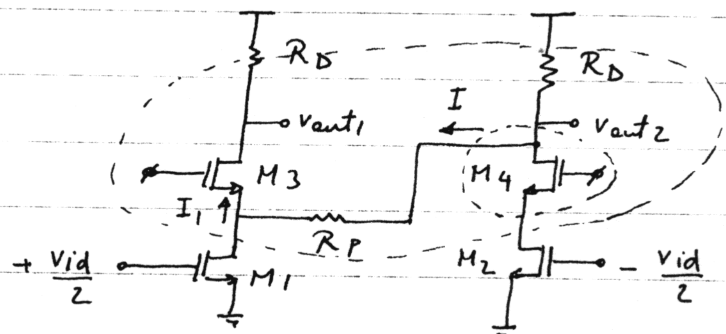
$\lambda = 0$, so for a differential input, symmetry in the input is enough ~~to~~ for the tail node to be ground. in other words:



$$g_{m1}\frac{V_{id}}{2} - g_{m2}\frac{V_{id}}{2} = R_{SS}V_x$$

$$g_{m1} = g_{m2} \Rightarrow V_X = 0$$

$(V_{GS1} = V_{GS2} \Rightarrow g_{m1} = g_{m2}$
but $g_{m3} \neq g_{m4}$)



For the cutsets shown:

$$\frac{V_{out1}}{R_D} + \frac{V_{out2}}{R_D} + \frac{V_{id}}{2}g_{m1} - \frac{V_{id}}{2}g_{m2} = 0 \quad \Rightarrow \quad V_{out1} + V_{out2} = 0 \quad (1)$$

KCL: $$\frac{V_{out2}}{R_D} + I - \frac{V_{id}}{2}g_{m2} = 0 \quad \overset{g_{m1}=g_{m2}}{\Longrightarrow} \quad I = \frac{V_{id}\,g_{m1}}{2} - \frac{V_{out2}}{R_D} \quad (2)$$

KVL: $V_{out2} = IR_P + \dfrac{I_1}{g_{m3}}$, but $I_1 = \dfrac{V_{out1}}{R_D}$

$$\Rightarrow V_{out2} = IR_P + \frac{V_{out1}}{R_D\,g_{m3}} \quad (3)$$

(2), (3) $\Rightarrow$ $$V_{out2} = \left(\frac{V_{id}\,g_{m1}}{2} - \frac{V_{out2}}{R_D}\right)R_P + \frac{V_{out1}}{R_D\,g_{m3}}$$

$V_{out1} = -V_{out2}$ $\Rightarrow$ $$-V_{out1}\left(1 + \frac{R_P}{R_D} + \frac{1}{R_D\,g_{m3}}\right) = \frac{g_{m1}R_P}{2}V_{id}$$

$$A_{vd} = \frac{V_{out1} - V_{out2}}{V_{id}} = 2\frac{V_{out1}}{V_{id}} = -\frac{g_{m1} R_P}{1 + \frac{R_P}{R_D} + \frac{1}{R_D g_{m3}}}$$

$$= -\frac{g_{m1} R_D}{1 + \frac{R_D}{R_P} + \frac{1}{R_P g_{m3}}} = -\frac{g_{m1} R_D}{1 + \frac{1}{R_P}\left(R_D + \frac{1}{g_{m3}}\right)}$$

Since $\lambda = 0 \Rightarrow r_{o5} = \infty \Rightarrow A_{cm} = 0 \Rightarrow CMRR = \infty$

4.11

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH,P})V_{DS} - V_{DS}^2\right] \cong \mu_p C_{ox}\frac{W}{L}(V_{GS}-V_{TH,P})V_{DS}$$

$$R_{on} = \frac{|V_{DS}|}{I_D} = \frac{1}{\mu_p C_{ox}|V_{GS_3}-V_{TH,P}|}$$

if $R_{on} = 2^{K\Omega} \Rightarrow |V_{GS_3} - V_{TH,P}| = \frac{1}{2^K \times 38.3^{\mu} \times 100} = 0.131^V$

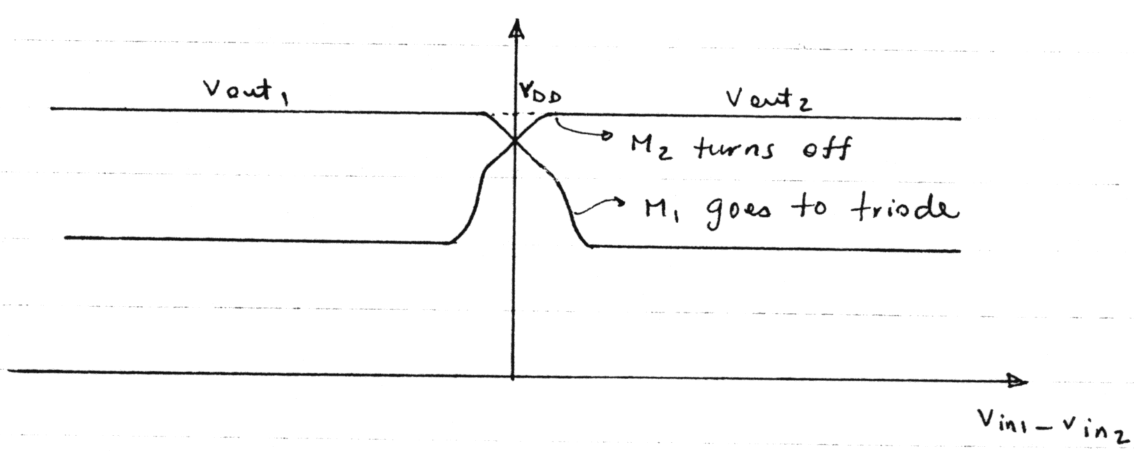
$I_{SS} = 20^{\mu A} \Rightarrow V_{GS1} = V_{TH,N} + \sqrt{\frac{2 I_D}{\mu_n C_{ox}\frac{W}{L}}} = 0.7 + \sqrt{\frac{2\times 10^{\mu}}{0.134^m \times 100}} = 0.739^V$

$V_{DD} = |V_{GS3}| - V_{GS1} + V_{in,cm}$

$\Rightarrow V_{in,cm} = 3 - (0.131 + 0.8) + 0.739 = 2.81^V$

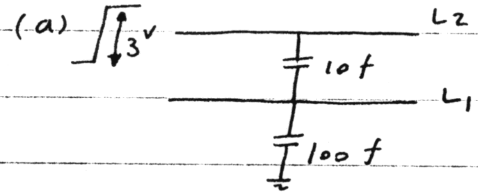
$|V_{DS3}| = R\frac{I_{SS}}{2} = 2^K \times 10^{\mu} = 20^{mV} \Rightarrow |V_{DS3}| < |V_{GS3} - V_{TH,P}| \Rightarrow M_3$ & M_4 are in triode

$V_{D1} - V_{G1} = (3 - 20^m) - 2.81 = 0.17^V > -V_{TH,n} \Rightarrow M_1$ & M_2 are in sat.



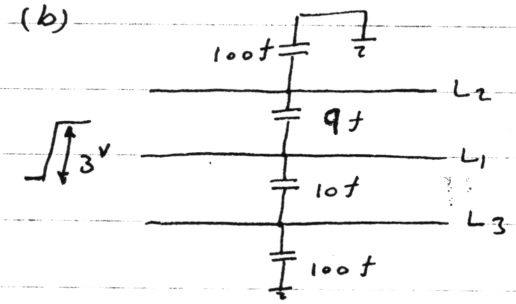
4.12

(a)



$$\Delta V_{L_1} = \frac{10}{10+100} 3^V = \frac{3}{11} = 0.273^V$$

(b)



$$\Delta V_2 = \frac{9}{9+100} \times 3^V = 0.248^V$$

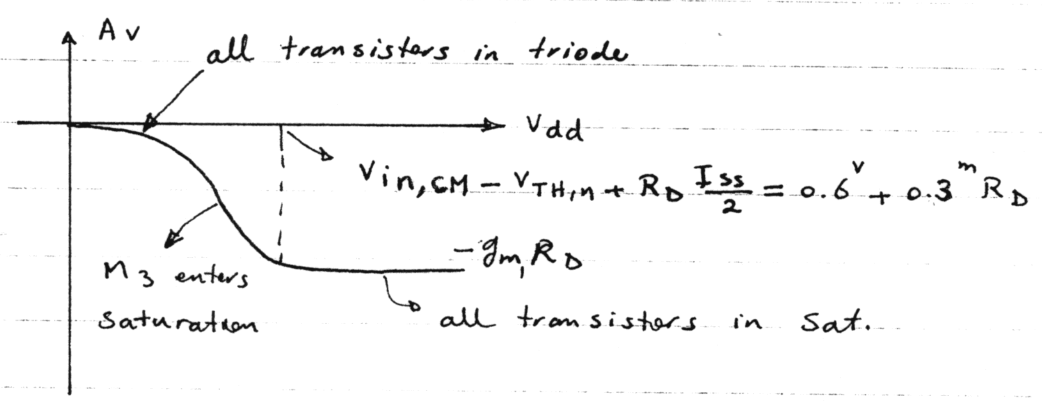
$$\Delta V_3 = \frac{10}{10+100} \times 3^V = 0.273^V$$

$$\Rightarrow \Delta V_{23} = 0.273 - 0.248 = 25^{mV}$$

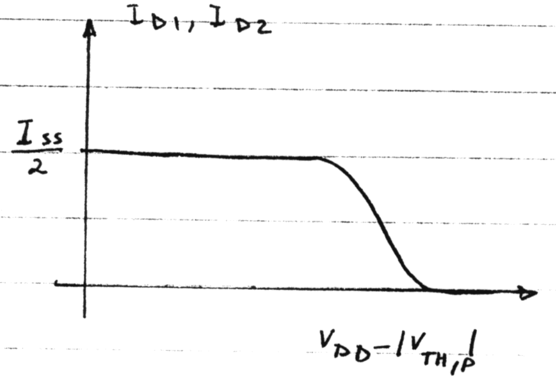
4.13

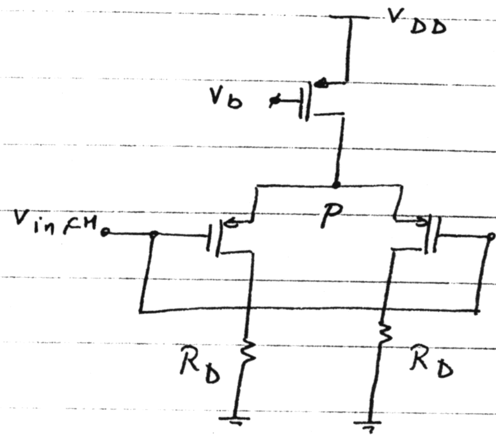
Fig. 4.8 (a) $\quad I_{SS} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TH,n})^2$

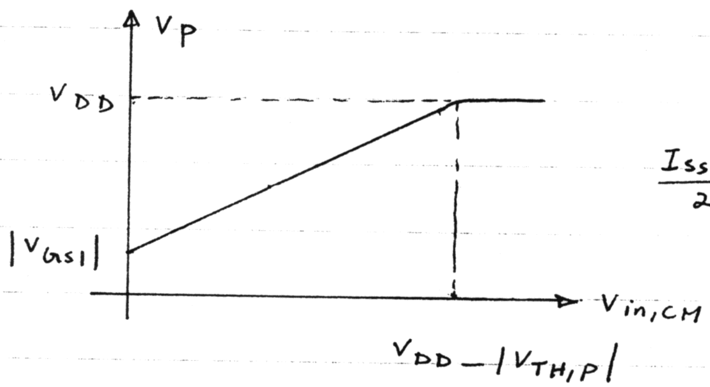
$$V_{GS3} = V_b = 1^V \Rightarrow I_{SS} = 0.5 \times 0.134^m \times 100 (1-0.7)^2 = 0.603^{mA}$$

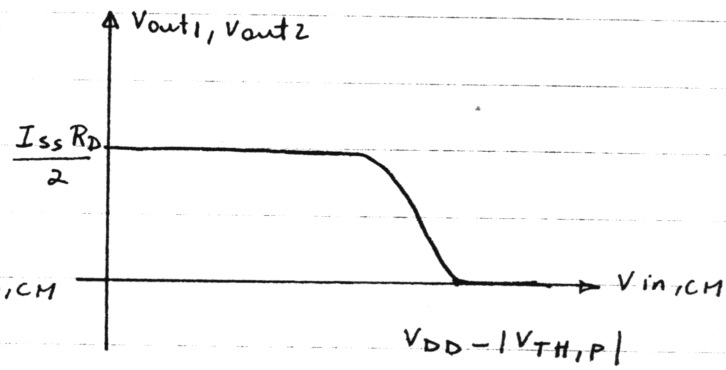


4.14









$$I_{D1,2} = \frac{V_{out1,2}}{R_D}$$

4.15 (a) $(V_{out1,2})_{max} = V_{DD} = 3^V$

$$(V_{out1,2})_{min} = V_{in,CM} - V_{TH,N} = 1.2 - 0.7 = 0.5^V$$

Max swing at $V_{out} = 2(3 - 0.5) = 5^V$

(b) $A_v = -g_m R_D \qquad g_m = \sqrt{2 \mu_n C_{ox} (\frac{W}{L}) I_D} = \sqrt{2 \times 0.134^m \times 100 \times \frac{0.5^m}{2}} = 2.59^m$

to get max swing: $R_D \frac{I_{SS}}{2} = \frac{(V_{out1,2})_{max} - (V_{out1,2})_{min}}{2} = 1.25$

$\Rightarrow R_D = 5^{k\Omega} \qquad \Rightarrow \qquad A_v = -2.59^m \times 5^k = -13$

4.16

(a) $V_{GS} - V_{TH} = \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\dfrac{2\times 0.5^m}{0.134^m \times 100}} = 0.273^V$

(b) $\Delta I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\dfrac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$ (4.9)

$$\Delta I_D = \frac{1}{2}\times 0.134^m \times \frac{50}{0.5}\times 50^m \sqrt{\frac{4\times 1^m}{0.134^m \times \frac{50}{0.5}} - (50^m)^2}$$

$\Delta I_D = 182\ \mu A \Rightarrow \begin{cases} I_{D1} = 0.5^m + \dfrac{0.182^m}{2} = 0.591^m \\ I_{D2} = 0.5^m - \dfrac{0.182^m}{2} = 0.409^m \end{cases}$

(c)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}^2}} \quad (4.10)$$

$\Delta V_{in} = 50^{mV} \Rightarrow G_m = 3.61\ m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \Rightarrow G_{m_0} \doteq 3.66\ m\Omega^{-1}$.

$$G_m = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}^2}}$$

if we define $A = \dfrac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$, $B = \left(\dfrac{G_m}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}\right)^2$

$\Rightarrow B = \dfrac{(A - 2\Delta V_{in}^2)^2}{A - \Delta V_{in}^2}$ $\quad 4\Delta V_{in}^4 - (4A - B)\Delta V_{in}^2 + A^2 - AB = 0$

$$\Delta V_{in}^2 = \frac{4A - B \pm \sqrt{(4A-B)^2 - 16(A^2 - AB)}}{8}$$

taking the smaller value: $\Delta V_{in}^2 = \dfrac{4A - B - \sqrt{8AB + B^2}}{8}$

So for different value of G_m, B can be calculated and then ΔV_{in} is found.

$G_{m_{10\%}} = 0.9 \times 3.66^m = 3.294\ m\Omega^{-1} \Rightarrow |\Delta V_{in}| = 139^{mV}$ 10% drop

$G_{m_{90\%}} = 0.1 \times 3.66^m = 0.366\ m\Omega^{-1} \Rightarrow |\Delta V_{in}| = 372^{mV}$ 90% drop

4.17

(a) $V_{od} = V_{GS} - V_{TH} = 0.386^V$

(b) $\Delta I_D = 0.129^{mA} \rightarrow I_{D1} = 0.565^{mA}$

$I_{D2} = 0.435^{mA}$

(c) $G_m = 2.57\ m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \Rightarrow G_{m_0} = 2.59\ m\Omega^{-1}$

$G_{m_{10\%}} = 0.9 \times G_{m_0} = 0.9 \times 2.59^m \Rightarrow \Delta V_{in} = 197^{mV}$

$G_{m_{90\%}} = 0.1 \times G_{m_0} = 0.1 \times 2.59^m \Rightarrow \Delta V_{in} = 526^{mV}$

For a given current, by reducing $\frac{W}{L}$, overdrive voltage increases while G_m decreases. In this case for a fixed ΔV_{in}, I_D and G_m change less so the circuit has a wider linear range.

4.18

(a) $V_{od} = 0.386^{V}$

(b) $\Delta I_D = 0.258^{mA}$ $\left\{ \begin{array}{l} I_{D1} = 1.13^{mA} \\ I_{D2} = 0.87^{mA} \end{array} \right.$

(c) $G_m = 5.14 \; m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \rightarrow G_{m0} = 5.18 \; m\Omega^{-1}$

$G_{m_{10\%}} = 0.9 \times 5.18^{m} \Rightarrow \Delta V_{in} = 197^{mV}$

$G_{m_{90\%}} = 0.1 \times 5.18^{m} \Rightarrow \Delta V_{in} = 526^{mV}$

In this case V_{od} and G_m have increased but the linearity range of G_m is same as (p. 4.17)

4.19 $I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH,n})^2$

$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{2W}{L} (V_{in2} - V_{TH,n})^2$

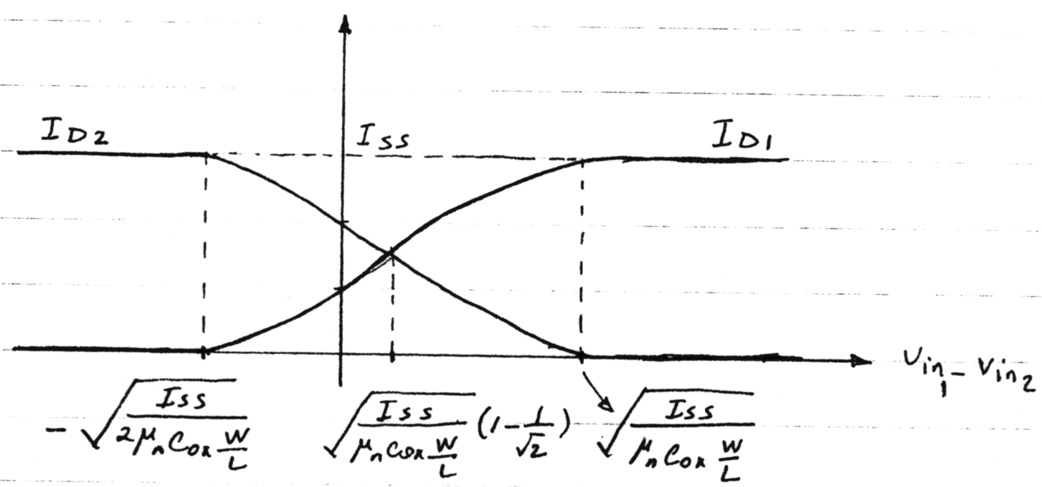
$I_{D1} + I_{D2} = I_{SS}$

if $I_{D1} = I_{D2}$ then:

$$I_{D1} = \frac{I_{SS}}{2} \Rightarrow V_{in1} = V_{TH,n} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_{D2} = \frac{I_{SS}}{2} \Rightarrow V_{in2} = V_{TH,n} + \sqrt{\frac{I_{SS}}{2\mu_n C_{ox} \frac{W}{L}}}$$

$$V_{in1} - V_{in2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \left(1 - \frac{1}{\sqrt{2}}\right)$$



if $V_{in1} = V_{in2} \Rightarrow I_{D1} = \frac{I_{SS}}{3}$, $I_{D2} = \frac{2 I_{SS}}{3}$

4.20

$$A_{dm-dm} = -g_m R_D$$

$$A_{cm-dm} = \frac{g_m}{1 + 2 g_m R_{SS}} R_D - \frac{g_m}{1 + 2 g_m R_{SS}} (R_D + \Delta R_D)$$

$$A_{cm-dm} = -\frac{g_m}{1+2g_m R_{SS}}\Delta R_D$$

$$SNR = \frac{(A_{dm-dm}\cdot V_{in,dM})^2}{(A_{cm-dm}\cdot V_{in,cm})^2} = \left(\frac{g_m R_D \times 10^m}{\frac{g_m}{1+2g_m R_{SS}}\Delta R_D \times 100^m}\right)^2$$

$$SNR = \frac{(1+2g_m R_{SS})^2}{\left(\frac{\Delta R}{R}\right)^2}\times\left(\frac{1}{10}\right)^2$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 3.66^{m\Omega^{-1}} \qquad (I_D = 0.5^m)$$

$$L_{SS} = 0.5\ \mu m \Rightarrow \lambda = 0.1\ V^{-1} \Rightarrow R_{SS} = \frac{1}{\lambda I_D} = \frac{1}{0.1\times 1^m} = 10^{K\Omega}$$

$$\Rightarrow SNR = \left(\frac{1+2\times 3.66^m\times 10^K}{0.05}\right)^2\left(\frac{1}{10}\right)^2 = 22000$$

or $SNR = 10\log 22000 = 43.4\ dB$

$$CMRR = \left|\frac{A_{dm-dm}}{A_{cm-dm}}\right| = \frac{g_m R_D}{\frac{g_m\Delta R_D}{1+2g_m R_{SS}}} = \frac{1+2g_m R_{SS}}{\Delta R_D/R_D}$$

$CMRR = 1484$ or $CMRR = 20\log 1484 = 63.4\ dB$

4.21 $$A_{cm-dm} = -\frac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1} \qquad g_{m1}+g_{m2} = 2g_m$$

$$SNR = \left(\frac{A_{dm-dm}\times V_{in-dm}}{A_{cm-dm}\times V_{in-cm}}\right)^2 = \left(\frac{g_m R_D}{\frac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1}}\right)^2\left(\frac{10^m}{100^m}\right)^2$$

$$SNR = \left(\frac{2 g_m R_{SS} + 1}{\frac{\Delta g_m}{g_m}} \right)^2 \times \left(\frac{1}{10}\right)^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \Rightarrow \Delta g_m = -\mu_n C_{ox} \frac{W}{L} \Delta V_{TH}$$

$$|\Delta g_m| = 0.134^m \times 100 \times 1^{mV} = 13.4 \ \mu\Omega^{-1}$$

$$\Rightarrow SNR = \frac{(2 \times 3.66^m \times 10^K + 1)^2}{\left(\frac{13.4\mu}{3.66^m}\right)^2} \times \left(\frac{1}{10}\right)^2 = 4.1 \times 10^6$$

or $SNR = 10 \log 4.1 \times 10^6 = 66.1 \ dB$

$$CMRR = \left| \frac{A_{dm-dm}}{A_{cm-dm}} \right| = \frac{1 + 2 g_m R_{SS}}{\Delta g_m / g_m} = 20300$$

or $CMRR = 20 \log 20300 = 86.1 \ dB$

4.22 (a)

$$\left(\frac{W}{L}\right)_{SS} = \frac{50}{0.5}, \quad I_{SS} = 0.5^{mA}$$

$$V_{od_{SS}} = V_{GS3} - V_{TH} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{SS}}} = 0.273^V$$

$$I_{D1} = \frac{I_{SS}}{2} = 0.25^{mA} \Rightarrow V_{od_1} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = 0.193^V$$

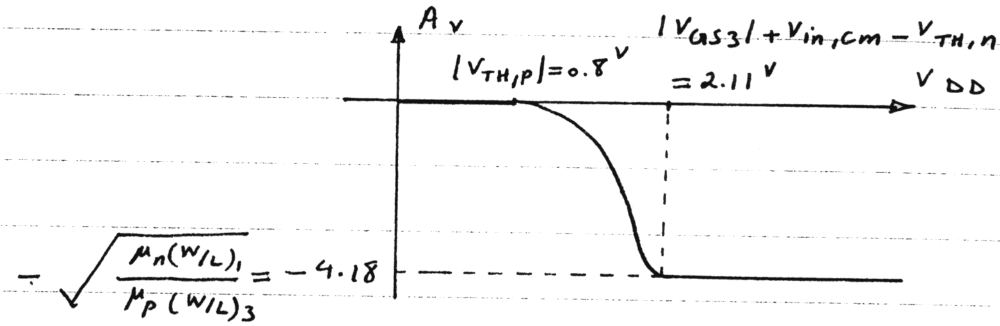
$$(V_{in,cm})_{min} = V_{GS1} + V_{od_{SS}} = 0.7 + 0.193 + 0.273 = 1.17^V$$

$$(V_{in,cm})_{max} = V_{DD} - |V_{GS3}| + V_{TH,N}$$

$$|V_{GS3}| = |V_{TH,p}| + \sqrt{\frac{2 I_{D3}}{\mu_p C_{ox} (\frac{W}{L})_3}} = 1.61^V$$

$$(V_{in,cm})_{max} = 3 - 1.61 + 0.7 = 2.09^V$$

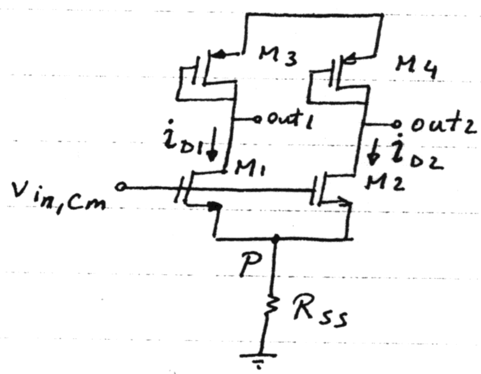
(b) $V_{in,cm} = 1.2^V$



4.23

This mismatch in V_{TH} of M_1 and M_2 makes I_{D1} and I_D unequal. Therefore $g_{m1} \neq g_{m2}$, $g_{m3} \neq g_{m4}$. Using the equivalant circuit below to calculate A_{cm-dm}, we have:

$$i_{D1} = g_{m1} (V_{in,cm} - V_P)$$



$$i_{D2} = g_{m2} (V_{in,cm} - V_P)$$

$$V_{out1} = -\frac{i_{D1}}{g_{m3}} = -\frac{g_{m1}}{g_{m3}} (V_{in,cm} - V_P)$$

$$V_{out2} = -\frac{i_{D2}}{g_{m4}} = -\frac{g_{m2}}{g_{m4}} (V_{in,cm} - V_P)$$

$$\frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2 I_{D1} \mu_n C_{ox} (\frac{W}{L})_{1,2}}}{\sqrt{2 I_{D3} \mu_p C_{ox} (\frac{W}{L})_{3,4}}} = \sqrt{\frac{\mu_n (\frac{W}{L})_{1,2}}{\mu_p (\frac{W}{L})_{3,4}}}, \quad \text{Similarly} \quad \frac{g_{m2}}{g_{m4}} = \sqrt{\frac{\mu_n (\frac{W}{L})_{1,2}}{\mu_p (\frac{W}{L})_{3,4}}}$$

$$\Rightarrow V_{out1} = V_{out2} \Rightarrow A_{cm-dm} = 0, \quad CMRR = \infty$$

4.24

P 4.20 $\quad CMRR = \dfrac{1 + 2g_{m1}R_{SS}}{\Delta R_D / R_D}$

$$R_{D1} = \frac{1}{g_{m3}} , \quad R_{D2} = \frac{1}{g_{m4}}$$

$$\frac{\Delta R_D}{R_D} = \frac{R_{D1} - R_{D2}}{R_{D1}} = 1 - \frac{R_{D2}}{R_{D1}} = 1 - \frac{g_{m3}}{g_{m4}} = 1 - \frac{\sqrt{2\mu_p C_{ox} (\frac{W}{L})_3 I_D}}{\sqrt{2\mu_p C_{ox} (\frac{W}{L})_4 I_D}}$$

$$\frac{\Delta R_D}{R_D} = 1 - \sqrt{\frac{10}{11}} = 0.0465$$

$\Rightarrow \quad CMRR = 2248 \quad$ or $\quad CMRR = 20 \log 2247 = 67 \text{ dB}$

4.25

(a) $\quad A_V = -g_{m1} (r_{o1} \| r_{o3})$

$g_{m1} = 3.66^{m\Omega^{-1}} \qquad r_{o1} = \dfrac{1}{\lambda I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$r_{o3} = \dfrac{1}{\lambda I_D} = \dfrac{1}{0.2 \times 0.5^m} = 10^{K\Omega} \quad \Rightarrow \quad A_V = -3.66^m (10^k \| 20^k) = -24.4$

(b) $\quad (V_{out1,2})_{min} = 1.5 - V_{TH,n} = 1.5 - 0.7 = 0.8^V$

$$(V_{out1,2})_{max} = V_{DD} - (|V_{GS3}| - |V_{TH,P}|)$$

$$= V_{DD} - \sqrt{\frac{2 I_D}{\mu_p C_{ox} (\frac{W}{L})_3}} = 3 - \sqrt{\frac{2 \times 0.5^m}{38.3^{\mu} \times 100}} = 2.49^V$$

Max swing of $V_{out} = 2(2.49 - 0.8) = 3.38^V$

4.26

P 4.20 : $CMRR = \dfrac{1 + 2g_{m1} R_{SS}}{\Delta R_D / R_D}$

$$R_D = \frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5} \approx \frac{1}{g_{m3}}$$

$$I_{D3} + I_{D5} = I_{D4} + I_{D6} = \frac{I_{SS}}{2} \Rightarrow \Delta I_{D3} = -\Delta I_{D5} \qquad (1)$$

$$I_{D5} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH,P})^2 \Rightarrow \frac{\partial I_{D5}}{\partial V_{TH,P}} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH,P})$$

$$\Delta I_{D5} \approx \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH,P}|) \Delta V_{TH,P} \qquad (2)$$

$$g_{m3} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}} \qquad \frac{\partial g_{m3}}{\partial I_{D3}} = \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}}$$

$$\Rightarrow \Delta g_{m3} \approx \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \Delta I_{D3} \qquad (3)$$

$$R_D = \frac{1}{g_{m3}} \qquad \frac{\partial R_D}{\partial g_{m3}} = -\frac{1}{g_{m3}^2} \Rightarrow \frac{\Delta R_D}{R_D} \approx -\frac{\Delta g_{m3}}{g_{m3}}$$

$$(1),(2),(3) \Rightarrow \frac{\Delta R_D}{R_D} = -\frac{\Delta g_{m3}}{g_{m3}} = -\sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \times \frac{\Delta I_{D3}}{\sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}}} = -\frac{\Delta I_{D3}}{2 I_{D3}}$$

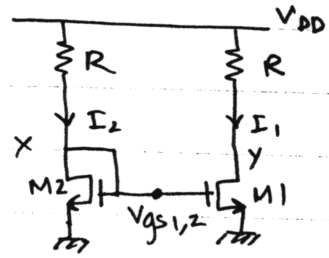
$$= \frac{\Delta I_{D5}}{2 I_{D3}} = \frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH,P}|) \Delta V_{TH,P}}{2 I_{D3}} = \frac{I_{D5}}{I_{D3}} \cdot \frac{\Delta V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|}$$

or $\dfrac{\Delta R}{R_D} = \dfrac{I_{D5}}{I_{D3}} \cdot \dfrac{V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|} \times \dfrac{\Delta V_{TH,P}}{V_{TH,P}}$

but $\dfrac{I_{D5}}{I_{D3}} = 4$

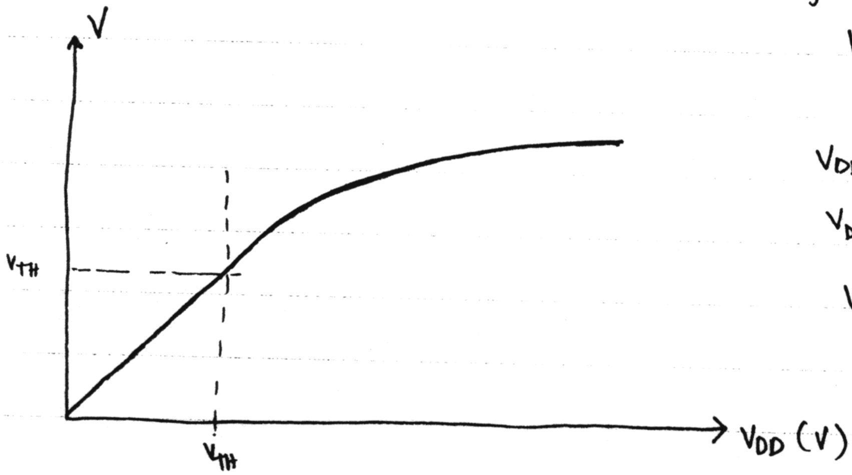
$$\Rightarrow \frac{\Delta R_D}{R_D} = 4 \cdot \frac{V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|} \cdot \frac{\Delta V_{TH,P}}{V_{TH,P}}$$

5.1 (a) M1 and M2 are off when $V_{DD} < V_{TH1,2}$.
At this time, $V_{x,y} = V_{DD}$ because $I_{1,2} = 0$.
Once $V_{DD} \geq V_{TH1,2}$, M1 and M2 turn on.
Since M1 and M2 are symmetric and the V_{gs} for both are the same $I_1 = I_2$ and $V_x = V_y = V_{gs2,1}$. V_{gs} is a quadratic solution as a function off V_{DD} and follows the below relationship.



$$V_{gs2} = V_{DD} - I_2 R$$
$$= V_{DD} - \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs2} - V_{th})^2 \cdot R$$

$$K = \frac{1}{2}\mu C_{ox}\frac{W}{L}$$



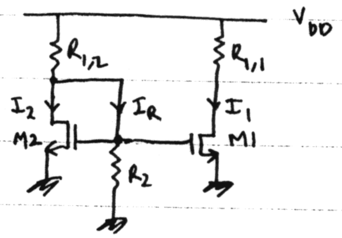
$V_{DD} < V_{TH}$: $V_X = V_Y = V_{DD}$.

$V_{DD} \geq V_{TH}$:

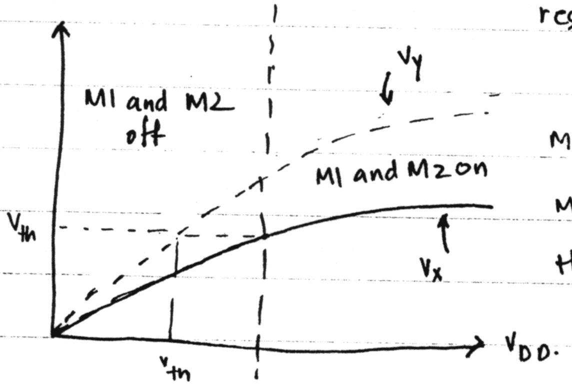
$$V_{gs} = \frac{2V_{th} - \frac{1}{KR} + \sqrt{\left(2V_{th} - \frac{1}{KR}\right)^2 - 4\left(V_{th}^2 - \frac{V_{DD}}{KR}\right)}}{2}$$

(b) We have the same solution as 5.1(a) because if M1, M2 are symmetric and $V_{gs1} = V_{gs2}$ then $V_x = V_y$ and $I_1 = I_2$, In this case, no current ever flows through R_2.

5.1 (c)



When $V_{DD} < V_{Th1,2}$ no current flows through M1 and M2. the only current that flows is I_R through $R_{1,2}$ and R_2. V_X is set by the resistor divider $V_X = V_{DD} \cdot \frac{R_2}{R_{1,2}+R_2}$ and $V_Y = V_{DD}$

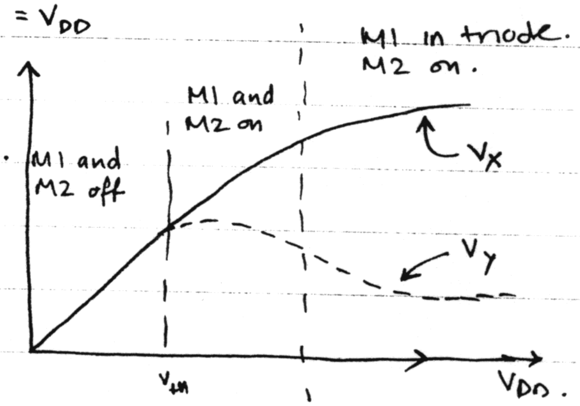


M1 and M2 turn on only when $V_X \geq V_{th1,2}$. Once M1 and M2 are on, I_1 and I_2 are equal because they have the same V_{gs}. But $V_Y > V_X$ because the current through $R_{1,2}$ consist of I_2 plus I_R which is greater than the current through $R_{1,1}$.

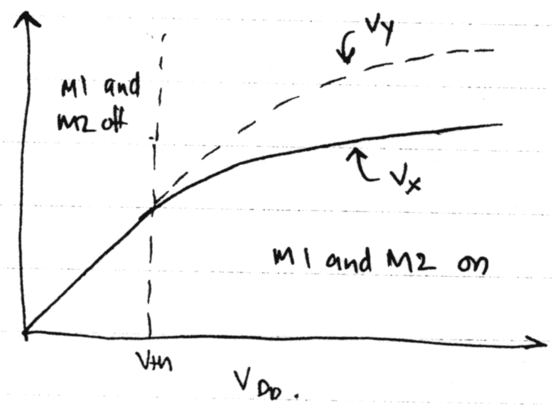
(d) When $V_{DD} < V_{th1,2}$, $I_1 = I_2 = 0$ and $V_X = V_Y = V_{DD}$

Once $V_{DD} \geq V_{th1,2}$, M1 and M2 turn on.

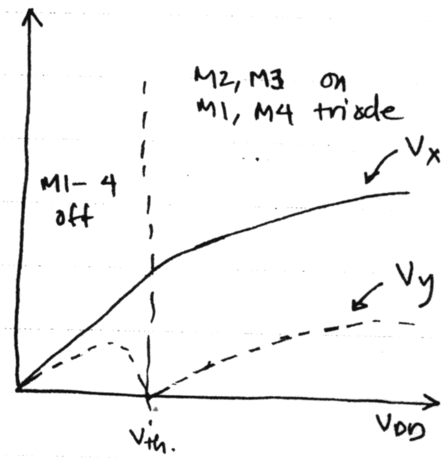
the R_2 at the source of M2 causes $V_{gs2} < V_{gs1}$. Thus $I_2 < I_1$ and $V_X > V_Y$. At some point, V_{gs1} becomes so large that M1 goes into triode as seen in the graph.



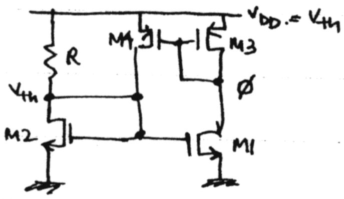
(e) Here when $V_{DD} > V_{th1,2}$, $V_{gs2} > V_{gs1}$ and $I_1 < I_2$. Thus $V_X < V_Y$, but because M2 is diode connected, M2 never goes into triode.



5.2 (a) When $V_{DD} < V_{th1,2}$, all transistors are off. $V_X = V_{DD}$ and V_Y is floating between GND and V_{DD} since it is isolated from either node.



Once $V_{DD} = V_{th1,2}$ all transistors turn on and the voltages at nodes are as follows: $V_X = V_{DD} = V_{th2}$ and $V_Y = V_{DD} - V_{th3} \cong \emptyset$ if we assume $V_{th2} = V_{th3}$. M1 and M4



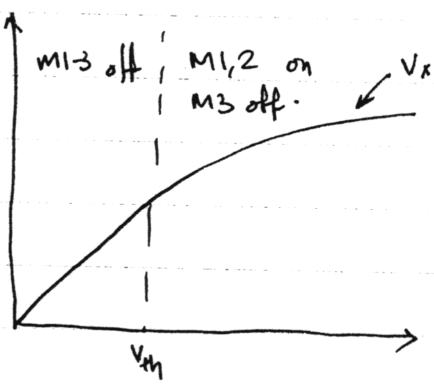
are in triode and stay in triode always as $V_{DD} > V_{th2}$.

As $V_{DD} > V_{th2}$, $V_X = V_{gs2}$ and $V_Y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$

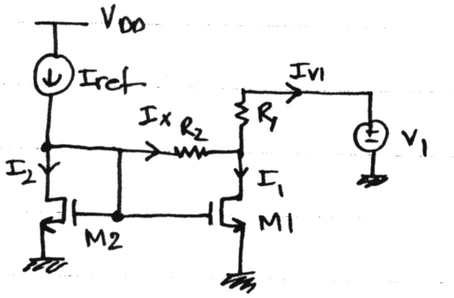
$V_X > V_Y$.

(b) when $V_{DD} < V_{th1,2}$ $I_1, I_2 = \emptyset$ and $V_X = V_Y = V_{DD}$

when $V_{DD} \geq V_{th1,2}$ $I_1 = I_2$ because $V_{gs1} = V_{gs2}$ and therefore $V_X = V_Y$. Since $V_X \cong V_Y$, $V_{gs3} = 0$ and M3 is always off.



5.3 (a)



for V_1: $0 \le V_1 \le V_{DD}$ M1 and M2 are always on: $I_1 = I_2$

V_y and V_x are related as a function of R_1, R_2 and V_1.

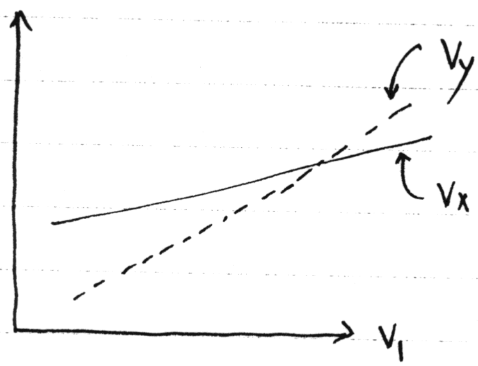
(1) $2I_1 = I_{ref} - I_{V1} = I_{ref} - \frac{V_y - V_1}{R_1}$

(2) $I_1 = \frac{V_x - V_y}{R_2} - \frac{V_y - V_1}{R_1}$

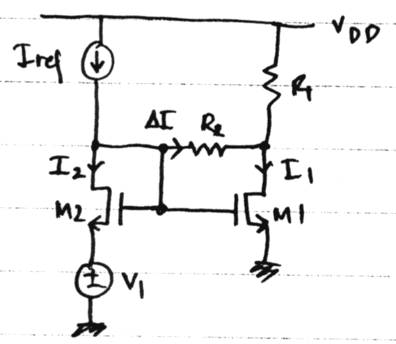
Solving these 2 equations for V_y, we get

$$V_y = \frac{R_2 V_1 + 2R_2 V_x - I_{Ref}}{R_1 + R_2}$$

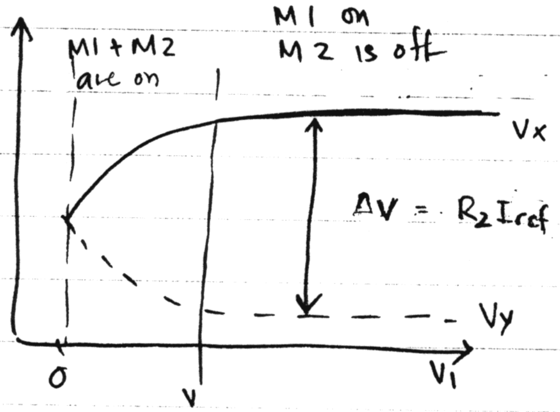
as V_1 increase, $I_{1,2}$ increase and therefore V_x and V_y increase. the slope of V_y is greater because it is a linear combination of $V_x + V_1$, but V_y starts off less than V_x because of the constant subtraction of I_{ref}/R_1+R_2.

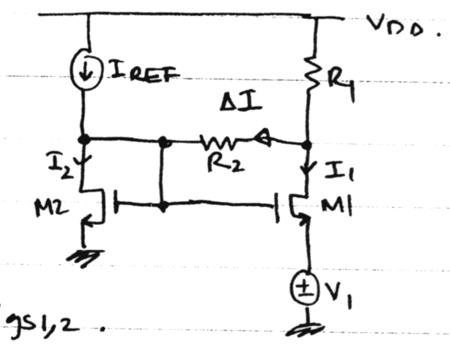


5.3 (b) When $V_1 = 0$, $I_1 = I_2 \doteq I_{ref}$ and $V_X \cong V_Y$, $\Delta I = 0$



as V_1 increases, I_2 gradually decreases and part of Iref flows throw R_2. V_X increases and $V_Y = V_X - R_2 \Delta I$, decreases. Finally when V_1 is large enough suchthat M2 turns off $V_Y = V_X - R_2 I_{ref}$ and both V_X and V_Y are set at a constant voltage



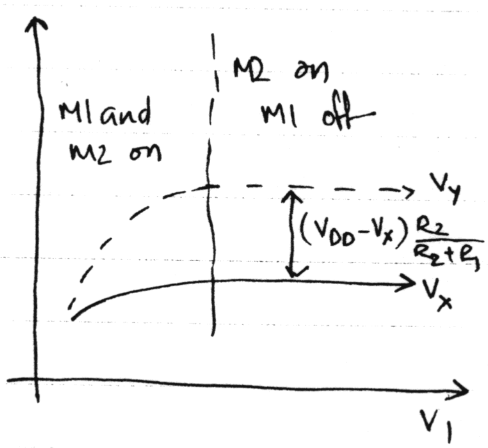


(C) When $V_1 = 0$, $I_1 = I_2$, $V_X \sim V_Y$. there maybe small variations if $V_{DD} - R_1 I_1 \neq V_{gs1,2}$.

as V_1 increases, I_1 decreases and the extra current flows through M2. I_2 increases.

$V_Y = V_X + \Delta I \, R_2$.

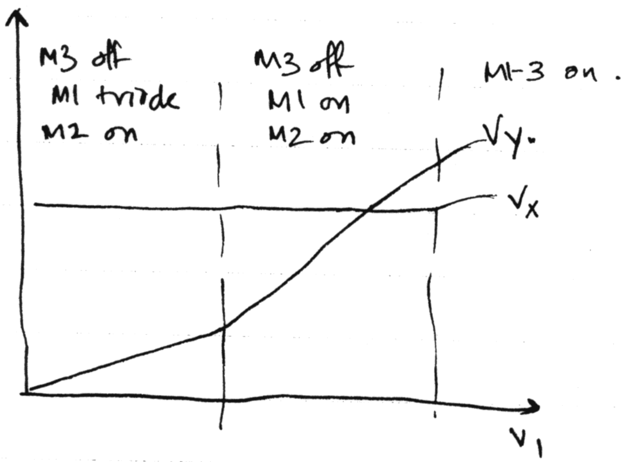
Once V_1 gets large enough, M1 shuts off. $V_Y = \dfrac{(V_{DD} - V_X) R_2}{R_1 + R_1} + V_X$



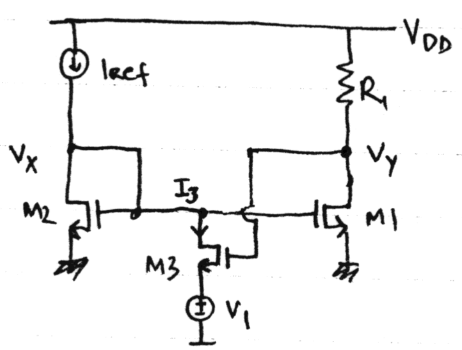
5.4 (a) V_x is constant until V_1 gets high enough that $V_Y - V_X$ is greater than V_{th3}.

Initially M1 is in triode with $V_Y = V_1 \frac{1}{1+g_{m1}R_2}$ until M1 is in saturation.

When M1 in sat., $V_Y = V_1 - I_{REF} R_1$

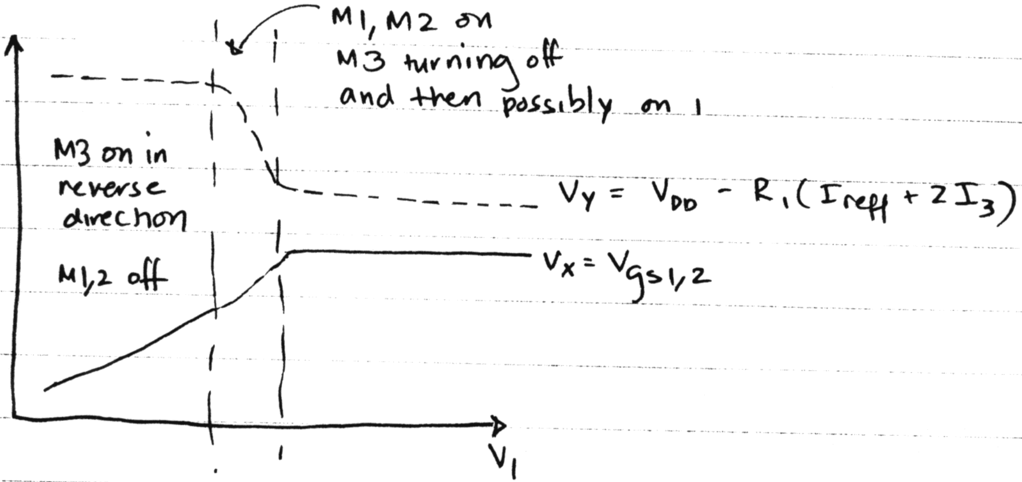


(b). When $V_1 = 0$, M1 and M2 are off and M3 is on, but in a flipped position, Source and drain switch as show below.
$V_Y = V_{DD}$ and $V_X = V_1 + V_{DS3}$ where $V_{DS3} = I_{REF}/g_{m3}$
As V_1 increase, V_X increases in the same amount until $V_X = V_{th1,2}$.
Now that M1 and M2 turn on, V_Y drops down due to $I_1 R_1$ until M3 turns off. Once M3 is off, $V_X = V_{gs1,2}$ and $V_Y = V_{DD} - R_1 I_{Ref}$.

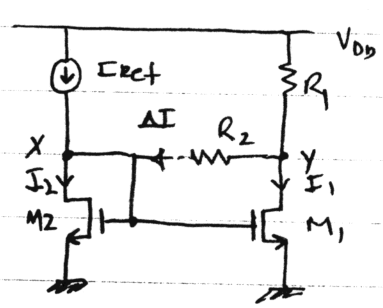


If at this point, $V_{th3} < V_Y - V_X$, M3 will turn on to increase the current through M2 and hence M1 so $V_Y = V_{DD} - R_1(I_{ref} + 2I_3)$

5.7



5.5



When $I_{ref} = 0$, current I_1 and I_2 are supplied by V_{DD} through R_1

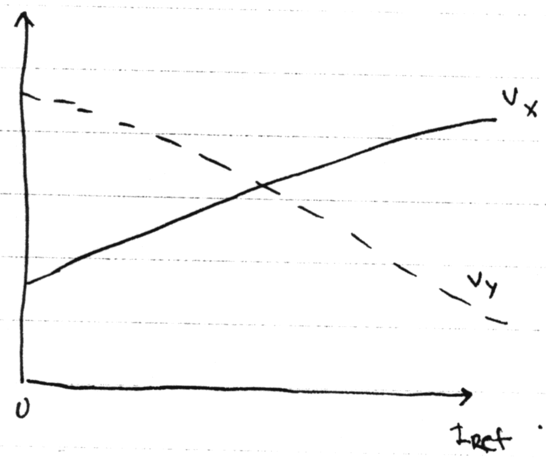
The initial points can be solved with

$$I_{1,2} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs1,2} - V_{th})^2 \qquad (1)$$

$$V_{gs2,1} = V_{DD} - (2R_1 + R_2)I_{1,2} \qquad (2)$$

where $V_x = V_{gs2,1}$

$$V_y = V_x + \frac{V_{DD} - V_x}{2R_1 + R_2} R_2 .$$



As I_{ref} increases, I_2 increases and hence $V_{gs2} = V_{gs1} = V_x$ increases. Following KCL, we can find V_y as a function of V_x, I_{ref}

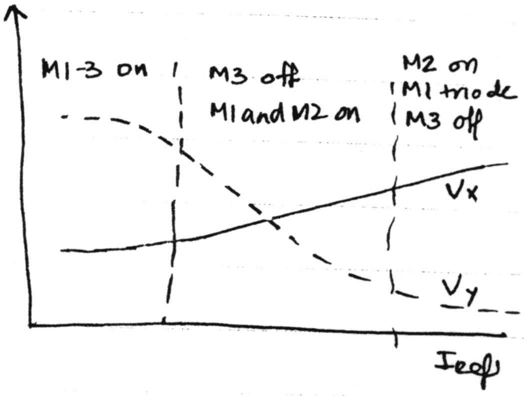
$$V_y = V_{DD}\frac{R_2}{2R_1 + R_2} - \frac{I_{ref} R_1 R_2}{2R_1 + R_2} + \frac{2R_1 V_x}{2R_1 + R_2}$$

5.5(b) Initially when $I_{ref} = 0$, this ckt is on with the follow condition $I_1 = I_2 = I_3$ since all transistors are assumed equal and all on.

$I_{1\text{-}3} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs1\text{-}3} - V_{th})^2$ (1)

$2V_{gs1\text{-}3} = V_{DD} - R_1 I_{1\text{-}3}$ (2)

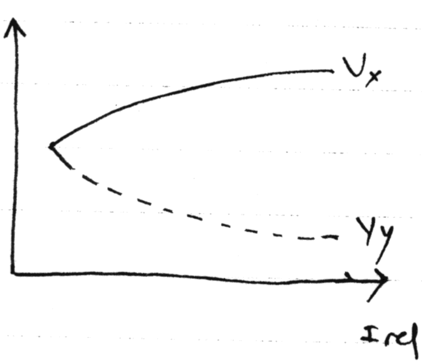
where $V_X = V_{gs1\text{-}3}$ and $V_Y = 2V_{gs1\text{-}3}$.



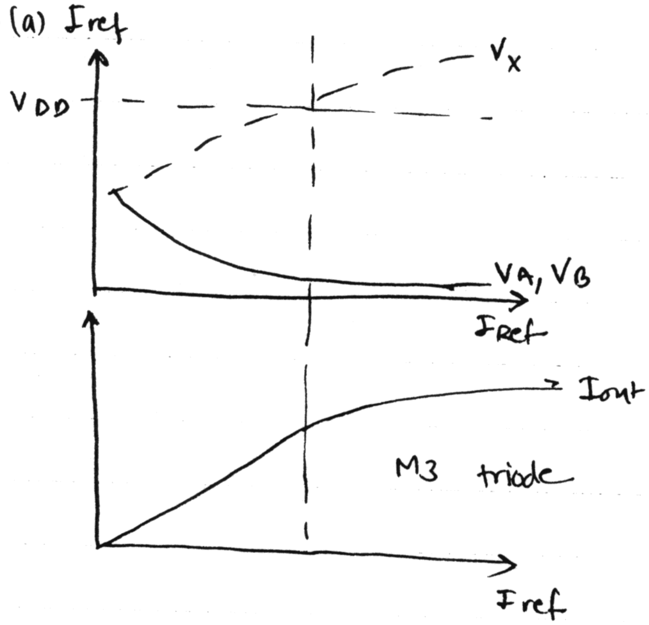
Once Iref increase, $V_{gs1,2}$ goes up and V_y drops down, decreasing V_{gs3} until M3 turns off. Then M1 and M2 act as a typical current mirror for Iref. Then when Iref get so large, the $I_{REF}R_1$ drop increases to a point when M1 goes into triode and can't sustain the Iref current.

(c) At Iref = 0, all transistors are in saturation mode with $V_x = V_y$. Once Iref turns on, M1 goes into triode and then M4 goes into triode also

$V_x \approx V_{gs2}$ and $V_y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$



5.6 (a)



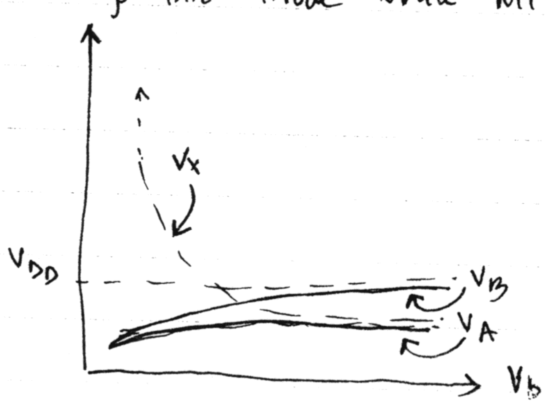
Iout follows Iref for all Iref until $V_X > V_{DD}$. Even if M1 and M2 go into triode, they still generate similar currents since V_A and V_B match.
As Iref increase, $V_{A,B}$ decrease since $V_{gs\,4,3}$ increases and V_X increases since $V_{gs1,2}$ also increase.

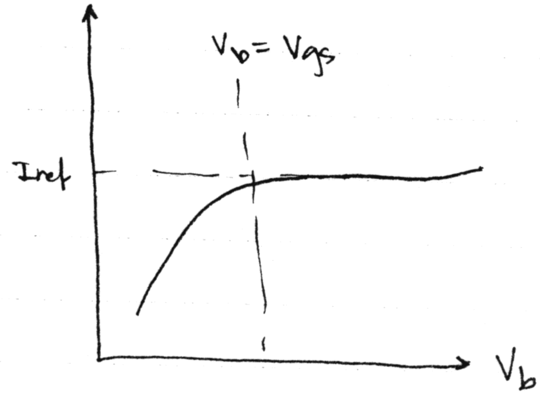
Once $V_X > V_{DD}$, M_3 goes into triod and reduces Iout w.r.t. Iref.

(b) If V_b is less than $1\,V_{gs}$ for I_{REF} current, V_X goes up to a very large voltage to allow for I_{REF} to flow through M_2 and M_4 if we assume channel resistance.

$$I = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

Once $V_b = 1V_{gs}$, V_A and V_B are ØV and Iout = Iref.
As V_b increases, V_A and V_B increase to turn M1 and M2 on. V_X is then equal to $1V_{gs}$. As V_B increase further, M3 and M4 go into triode while M1 and M2 are still in sat.





5.7 (a) $\gamma = 0$

$K_o = \frac{1}{2} \mu C_{ox}$

$I_{REF} = K_o \frac{W_o}{L_o'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{DS1})$

$I_{out} = K_o \frac{W_o}{L_o'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{DS2})$

where $V_{DS1} = V_{gs1}$ and $V_{DS2} = 2V_{gs1} - V_{gs4} - V_{gs3}$

if we assume $I_{out} \sim I_{REF}$, $V_{gs3} \approx V_{gs1}$,

so $V_{DS2} = V_{gs1} - V_{gs4}$

$V_{gs1} = V_{th} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}}$ $\quad V_{gs4} = V_{th} + \sqrt{\frac{I_1}{K_o} \frac{L_4'}{W_4}}$ $\quad L' = L - 2L_D$

$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = \frac{1 + \lambda \left(\sqrt{\frac{I_{REF} L_o'}{K_o W_o}} - \sqrt{\frac{I_1 L_4'}{K_o W_4}} \right)}{1 + \lambda \left(V_{th} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} \right)}$$

(b) $\gamma \neq 0$

$V_{th} = V_{tho} + \gamma \left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right)$ $\quad \phi_f \simeq .45\,V$ (work func.)

$V_{SB} \equiv$ Source - substrate volt

Find V_{gs1}, V_{gs0}, V_{gs4} and V_{gs3}

$V_{gs1} = V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}}$

$V_{gs0} = V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + V_{GS1}} - \sqrt{2\phi_f} \right)$

$V_{gs3} = V_{tho} + \sqrt{\frac{I_{out}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + V_{DS2}} - \sqrt{2\phi_f} \right)$

if we assume $I_{out} \sim I_{REF}$ and $V_{DS2} \sim \sqrt{\frac{I_{REF}}{K_o} \frac{L_o}{W_o}} - \sqrt{\frac{I_1 L_4}{K_o W_4}}$

we can estimate V_{gs3}

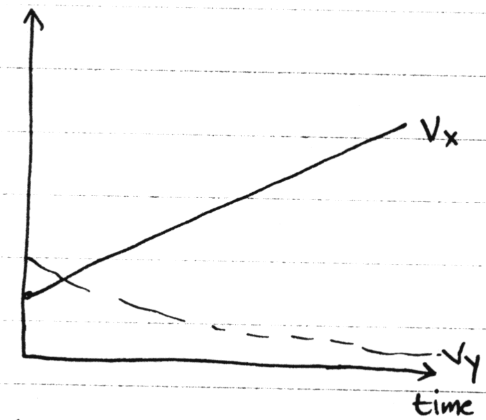
$$V_{gs3} \cong V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + \sqrt{\frac{I_{REF} L_o'}{K_o W_o}} - \sqrt{\frac{I_1 L_4'}{K_o W_4}}} - \sqrt{2\phi_f} \right)$$

5.7 $V_{gs4} = V_{th0} + \sqrt{\frac{I_1}{k_0}\frac{L_4'}{W_4}} + \gamma\left(\sqrt{2\phi_f + V_{gs3} + V_{DS2}} - \sqrt{2\phi_f}\right)$

Now we can plug everything in to the final solution

$$\frac{I_{out}}{I_{ref}} \approx \frac{1 + \lambda(V_{gs1} + V_{gs0} - V_{gs4} - V_{gs3})}{1 + \lambda(V_{gs1})}$$

5.8 (a)

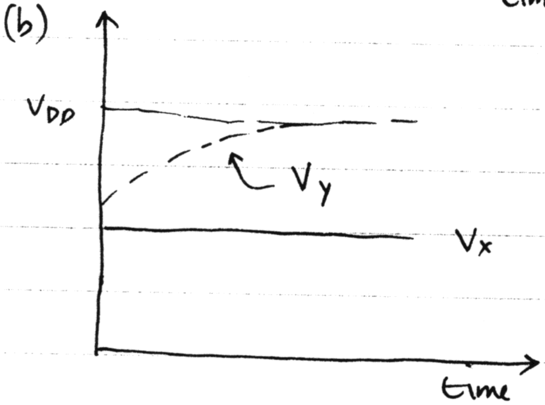


C_1 is continuously charged w/ I_{REF} so V_c and V_x increase with time $V_c = \int_0^t \frac{I_{REF}}{C_1}\,dt$

$V_x = V_c + V_{gs2}$

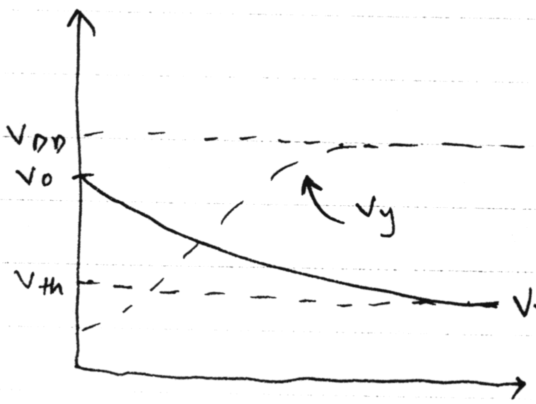
$V_{gs} = V_x$ and M_1 goes into triode

(b)



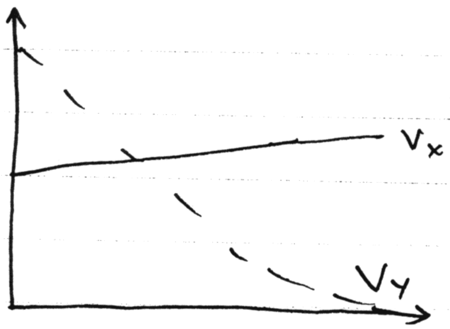
M2 is on with fixed $V_x = V_{gs2}$
C_1 is charged with current I_1 until M1 turns off.
$V_y = V_{DD} - I_1 R_1$ where I_1 goes from I_{ref} to $\emptyset$ A.

(c)



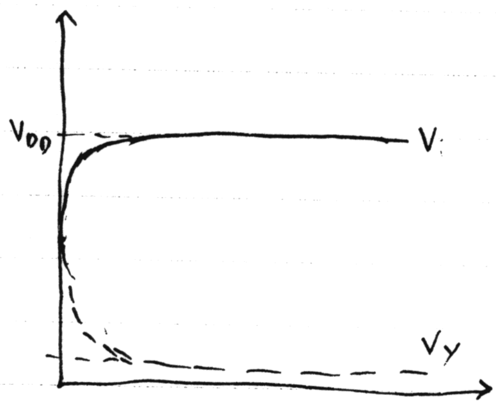
With C_1 initially charged w/ V_0, M2 is on and discharges C_1 until $V_x = V_{th}$ and $I_2 = 0$.
$V_y = V_{DD} - R_1 I_2$ and as I_2 goes to $\emptyset$, V_y goes to V_{DD}.

5.8 (d)



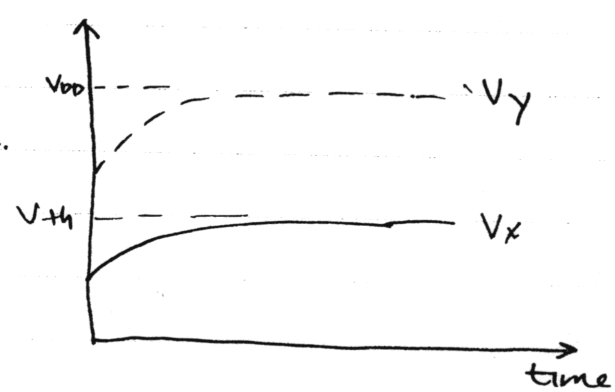
M2 and M1 are initially both on. M1 discharges all the charge in C_1 such that $V_y \to 0$ and M1 turns off. Since current through M1 reduces, current through M2 increases and V_x increases slightly

(e)

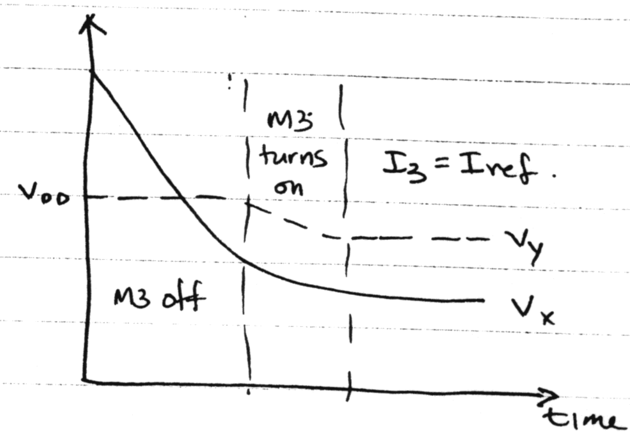


Since M2 can sustain no current, V_x goes up to V_{DD}. this pushes M1 into triode and Drops V_y to very small voltage. The Voltage accross C_1 drops to V_{th} to sustain $I_2 = 0$.

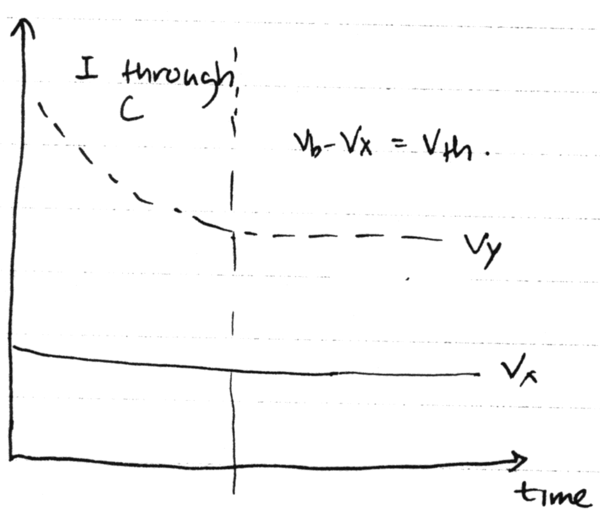
5.9 (a) At $t=0$, when C_1 is charged to 0V, $V_x = V_B - V_{gs3}$ where the $I_3 = I_{ref}$ and $V_y = V_{DD} - R_1 I_{ref}$. As the current flows into C_1, the cap charges up and shuts off M1. V_x is charged up to $V_b - V_{th}$ to sustain $I_{M1} = 0$ and $V_y = V_{DD}$



5.9 (b) At $t=0$, V_x is so high that M3 is off. Current for M1 and M2 is generated by Iref and as I_1 flows through M1, C_1 discharges enough to allow current flow throw M3. Once $I_3 = I_{REF}$, C_1 no longer discharges and has a constant set voltage.

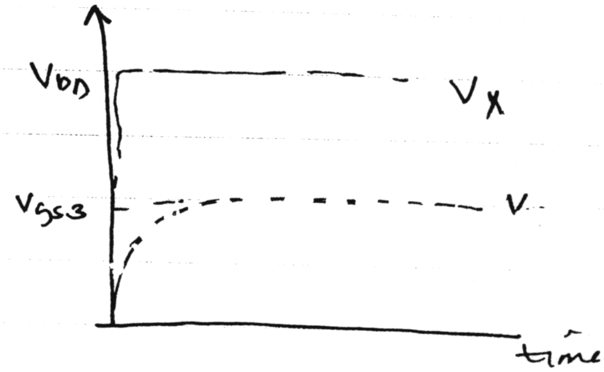


(c) At $t=0$, all transistors are on. $V_Y = V_{gs2} + V_{DD}$ and V_X is such that $I_3 = k_0' \frac{W_3}{L_3}(V_b - V_x - V_{th})^2 (1+\lambda V_{DS3})$. as C_1 discharges, V_{DS3} decreases and V_{gs3} increases, lowering V_x. At the point where $V_b - V_x = V_{th}$ and $I_3 = I_{\_} = 0$, V_Y stops decresing.



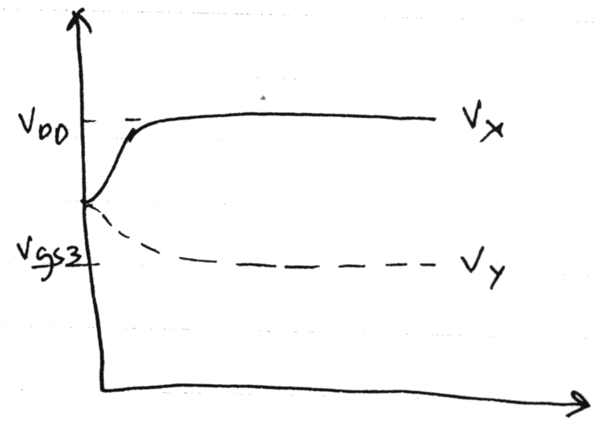
5.10 (a) at $t=0$, $V_{gs2} = 3V$ and forces current through M2. Since the source of M2 is attached to the gate of M1, no current can flow and $V_X = V_{DD}$

C_1 charges up such that $I_3 = I_1$ and $V_y = V_{DD} - V_{gs3}$

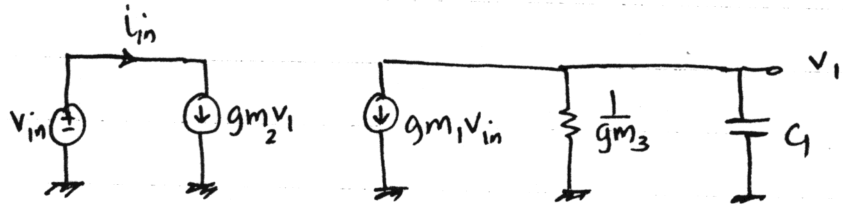


(b) Similar to 5.10(a), but since current can flow through C_1 to charge capacitor, V_X doesn't instantaneously reach V_{DD}, but slowly charges to V_{DD}.

$V_y = V_{DD} - V_{gs3}$ also.



5.11 small signal model.



$$Z_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{in}}{g_{m_2} v_1} \qquad v_1 = -g_{m_1}\left(\frac{1}{C_1 s + g_{m_3}}\right) V_{in}$$

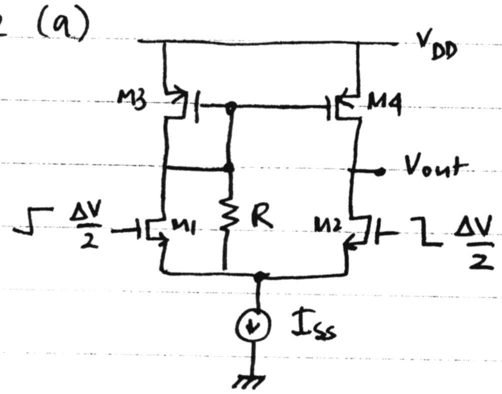
$$= -\frac{C_1 s + g_{m_3}}{g_{m_2} g_{m_1}}$$

if all transistors are equal $g_{m_1} = g_{m_2} = g_{m_3}$.

$$Z_{in} = -\frac{C_1 s}{g_m^2} - \frac{1}{g_m}$$

negative $C = -\frac{C_1}{g_m^2}$ and negative $R = -\frac{1}{g_m}$.

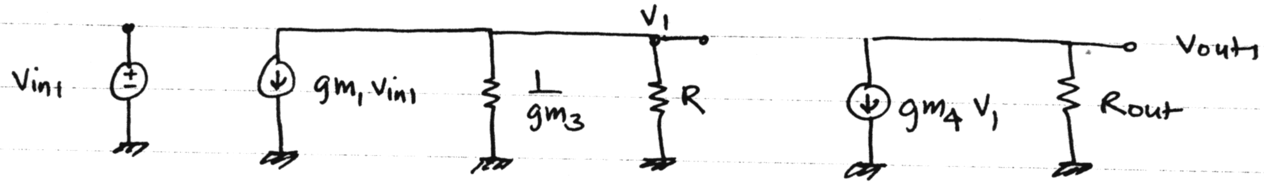
5.12 (a)



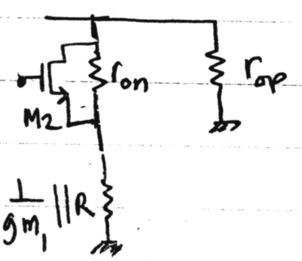
Solve for gain by super position

$V_{in} = \Delta V = \frac{\Delta V}{2} - \left(-\frac{\Delta V}{2}\right) = V_{in1} + V_{in2}$

1. for $+\frac{\Delta V}{2}$, ground M2 gate and find gain.



$R_{out_1} \Rightarrow$



$$R_{out} = r_{op} \parallel \left(\frac{1}{g_{m1}} + r_{on} + \frac{g_{m2}}{g_{m1}} r_{on}\right)$$

$$\simeq r_{op} \parallel 2r_{on} \simeq \frac{2}{3} r_o = \frac{2}{3}\frac{1}{\lambda I_o} = \frac{4}{3}\frac{1}{\lambda I_{ss}}$$

$$\frac{V_{out1}}{V_{in1}} = +g_{m1}\left(\frac{1}{g_{m3}} \parallel R_o\right) g_{m4} R_{out}$$

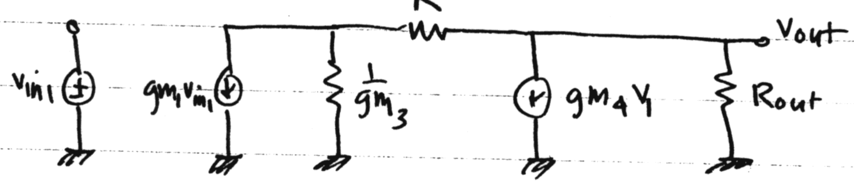
2. for $-\frac{\Delta V}{2}$

$V_{out} = -g_{m2} V_{in2} R_{out}$.

assume $g_{m1} = g_{m2}$ and $g_{m3} = g_{m4}$

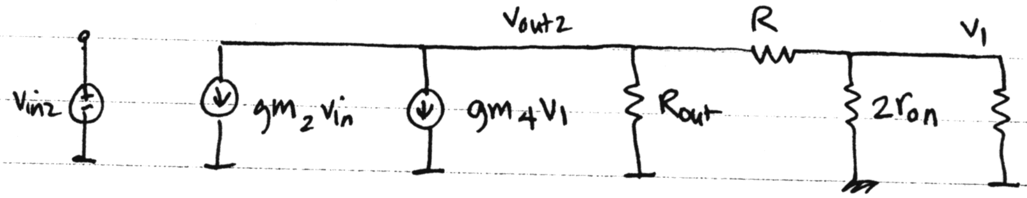
$$\text{Gain} = \frac{V_{out1} + V_{out2}}{\Delta V} = \frac{g_{m1}}{2} R_{out}\left[1 + \frac{R g_{m3}}{1 + R g_{m3}}\right]$$

(b) for $v_{in_1} = +\frac{\Delta V}{2}$



$$\frac{V_{out_1}}{V_{in1}} = g_{m_1} R_{out} \left[\frac{R g_{m_3} - 1}{R g_{m_3} + 2 R_{out} + 1} \right]$$

for $v_{in2} = -\frac{\Delta V}{2}$



$$V_1 = V_{out_2} \cdot \frac{2r_{on} \| \frac{1}{g_{m_3}}}{R + 2r_{on} \| \frac{1}{g_{m_3}}} = V_{out_2} \cdot \frac{2r_{on}}{R + 2r_{on} + 2g_{m_3} r_{on} R}$$

$$V_{out2} = \left(g_{m_2} v_{in2} - g_{m_4} V_{out2} \frac{2r_{on}}{R + 2r_{on} + 2g_{m_3} r_{on} R} \right) \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m_3}} \right) \right]$$

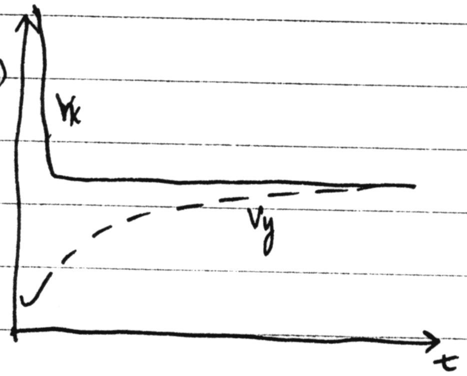
$$\frac{V_{out2}}{V_{in2}} = \frac{-g_{m_2} \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m_3}} \right) \right]}{1 + \frac{g_{m_4} 2r_{on}}{R + r_{on} + 2g_{m_3} r_{on} R} \cdot \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m_3}} \right) \right]}$$

$$\text{Gain} = \frac{V_{out_1} + V_{out2}}{\Delta V}$$

5.13 $V_{min} = V_p + V_{DSat1,2}$ $\quad V_p = V_{CM1,2} - V_{gs1,2}$

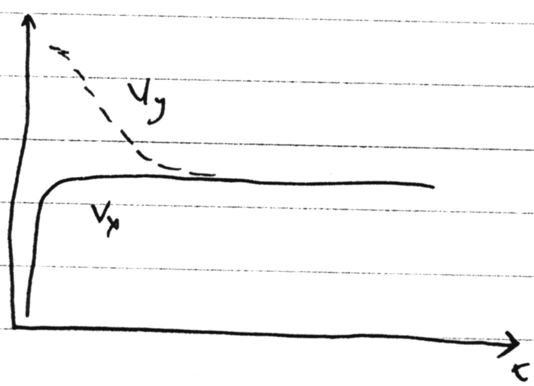
$= V_{CM1,2} - V_{gs1,2} + V_{DS1,2}$

5.14 (a)



M3 and M4 are initially off. M1 is on and M2 is in triode. the current through M1 initially comes from the C_1 charge until Vx drops in voltage and M3 and M4 turn on. M2 is still in triode until the current from M4 charges up Vy to the same voltage as Vx.

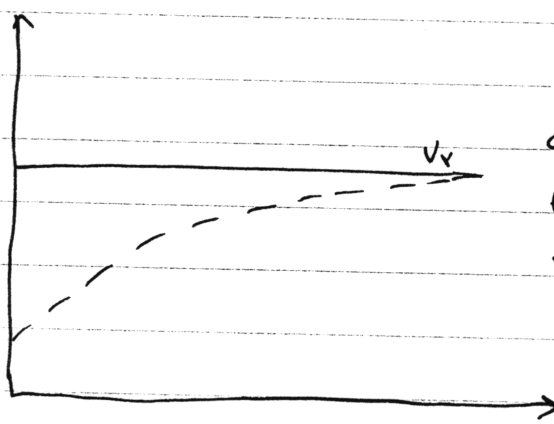
(b)



Vx starts at $1.5V - V_{gs1,2}$ where M1 is in triode and M3 is on strong. M4 and M2 can't sustain that high current w/ tail current $= I_{SS}$ so Vy goes up enough to put M4 in triode and reduce current. Current through M3 charge C_1 and Vy reduced its voltage w/ discharge in parasitics

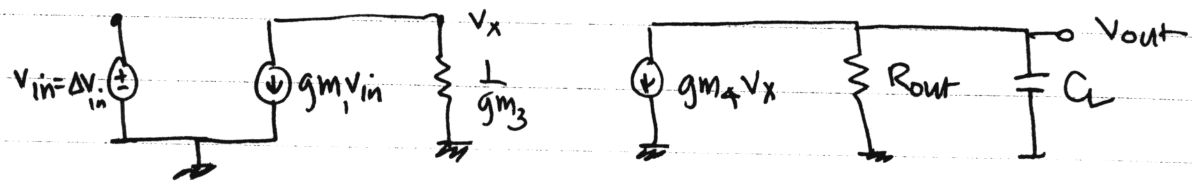
M3

(c)



Initial short between source & drain of M2 puts M2 in triode w/ minimal current flow. Current from M4 is used to charge up C_1. As C_1 charges up, some current starts flowing through M2 until V_{DS2} is high enough that M2 is in sat and all current is diverted to M2.

5.15 initial value ⇒ $V_{in} = V_1$ $V_{out} = V_x$

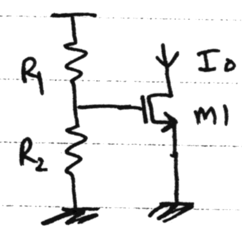


$$\Delta V_{out} = g_{m_1} \Delta V_{in} \frac{1}{g_{m_3}} g_{m_4} R_{out} \quad \text{(final)}$$

final value ⇒ $V_{in} = V_1 - \Delta V_{in}$ $V_{out} = V_x - \Delta V_{out}$

$\tau_c = R_{out} C_L$

5.16



$\frac{W}{L} = \frac{50\mu}{15\mu}$; $\lambda = 0$; $I = .5\,mA$; $K_p = \mu C_{ox} = 137\times 10^{-4}$

$L_D = 90\,nm$

a) $R2/R1$

$$V_{gs1} = V_{DD} \frac{R_2}{R_1 + R_2} = \sqrt{\frac{2I}{K_p \frac{W}{L'}}} + V_{th} \qquad L' = L - 2L_D$$

Let $RX = R_2/R_1$

$$R_x = \frac{\sqrt{2I/K_p \frac{W}{L'}} + V_{th}}{V_{DD} - \left(\sqrt{2I/K_p \frac{W}{L'}} + V_{th}\right)} = 0.4386$$

b) $$I_o = \frac{1}{2} \mu C_{ox} \frac{W}{L'} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)^2$$

$$\frac{\left(\frac{\partial I_o}{\partial V_{DD}}\right)}{I_o} = \frac{\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right) \frac{R_x}{1+R_x}}{\frac{1}{2} \mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)^2}$$

$$= \frac{2}{V_{DD} - V_{th}\left(1 + \frac{1}{R_x}\right)} = 2.84$$

5.16 (c) $\frac{\partial I_o}{\partial V_{th}} = -\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)$

$\Delta I_o \simeq -\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right) \Delta V_{th} = -233 \mu A$

$\Delta I_o = I_o(V_{th} = .75) - I_o(V_{th} = .7) = -205 \mu A$

(d) $\frac{\partial I_o}{\partial T} = -\frac{3}{2} \left(\frac{T}{T_0} \right)^{-3/2} \cdot \frac{1}{T} \cdot I_o$ * $\quad T = T_o + \Delta T$

$\Delta I_o \simeq -\frac{3}{2} \left(\frac{T}{T_o} \right)^{-3/2} \frac{1}{T} \cdot I_o \Delta T = -103 \mu A$ *

$\Delta I_o = I_o(T = 370K) - I_o(T = 300K) = -135 \mu A$ *

(e) $\Delta I_{worst\text{-}case} = I_{worst case} - I_o$

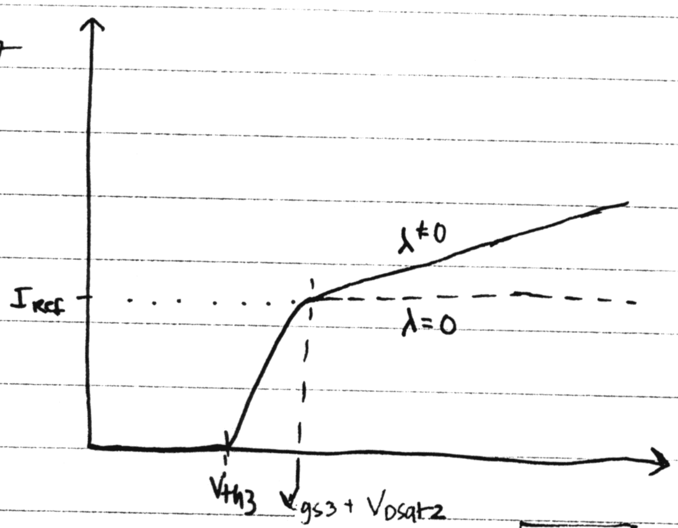
$I_{worst case} = \frac{1}{2} \mu_o \left(\frac{T_o + \Delta T}{T_o} \right)^{-3/2} \left((V_{dd} - \Delta V_{dd}) \frac{R_x}{1+R_x} - (V_{th} + \Delta V_{th}) \right)^2$

$= 43 \mu A$

$\Delta I_{worst case} = -457 \mu A$

* Note as temperature changes, so does V_{th}. In this calculation, we do not include the temperature effects on threshold voltage.

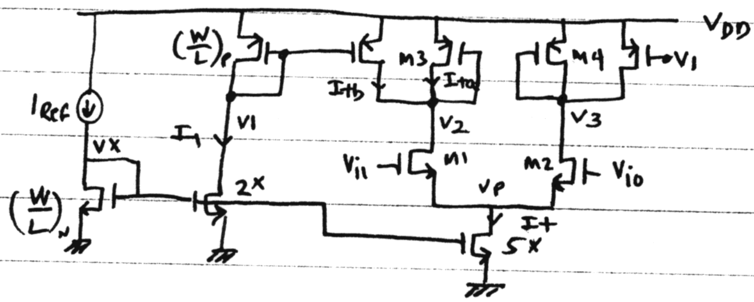
5.17



$V_{gs3} = V_{th} + \sqrt{\frac{2 I_{ref}}{K_p W/L}}$

$I_{out} = 0$ until $V_{DD} = V_{th3,4}$ when M3 and M4 are off, M2 in triode and M1 on. Once $V_{DD} > V_{th3,4}$, there is current flow but very little until M2 is in saturation at $V_{DD} = V_{gs3} + V_{Dsat2}$, $I_{out} \doteq I_{ref}$. at $V_{DD} = V_{gs1} + V_{gB}$, $I_{out} = I_{ref}$ exactly and grows as V_{DS2} increases

5.18



$(W/L)_N = \frac{10}{.5}$ $\quad (W/L)_{1,2}$ arbitrary

$(W/L)_P = \frac{10}{.5}$

$I_{REF} = 100\mu A$

$K_p = 38 \frac{\mu A}{V^2}$ $\quad K_n = 139 \frac{\mu A}{V^2}$

(a) $\lambda = 0$

$I = \frac{1}{2} K_p \frac{W}{L - 2L_D} (V_{gs} - V_t)^2$

$V_1 = V_{DD} - V_{gs(P)} = V_{DD} - V_{tp} - \sqrt{\frac{2 \cdot 2 \cdot I_{Ref}}{K_p \frac{W}{L-2L_D}}} = 1.619\ V$

$V_2 = V_3 = V_{DD} - V_{gs(3)} = V_{DD} - V_{tp} - \sqrt{\frac{I_{ref}}{K_p \frac{W}{L-2L_D}}} = 1.909\ V$

$V_p = V_{cm} - V_{gs(1)} = 1.3 - V_{tn} - \sqrt{\frac{2.5 \cdot I_{ref}}{K_n\ W}} = .3747\ V \quad (\gamma = 0)$

V_p for $\gamma = .45$ is found iteratively by finding V_{th} iteratively also.

$V_{tn}(V_p = .3747) = V_{TO} + \gamma\sqrt{2\phi + V_{SB}} - \gamma\sqrt{2\phi}$ $\quad \gamma = .45,\ \phi = .45$

$= .78$

$V_p(V_{tn} = .78) = .29$ until $V_{tn}(\text{final}) = .767$

$V_p(\text{final}) = .307$

5.18 (b) $\lambda = .2$

$$V_{gs}(0) = V_{tn} + \sqrt{\frac{2 I_{ref}}{K_n \cdot \frac{W}{L-2L_D}}} = 0.9253$$

for I_t, initially assume $V_p = .307$ from part (a)

iterate with

$$\left\{ \begin{array}{l} I_t = I_{Ref} \cdot 5 \cdot \dfrac{1+\lambda V_p}{1+\lambda V_{gs}(0)} \\ \text{and} \\ V_p = V_{cm\,1,2} - V_{gs(1,2)} \quad \text{with} \quad V_{tn\,1,2} = V_{to} + \gamma\sqrt{2\phi + V_p} - \gamma\sqrt{2\phi} \end{array} \right.$$

$I_t = 448\ \mu A$ and $V_p = 0.317$

for V_1, iterate also

$$\left\{ \begin{array}{l} I_1 = I_{Ref} \cdot 2 \cdot \dfrac{1+\lambda V_1}{1+\lambda V_{gs}(0)} \\ V_1 = V_{DD} - |V_{gsp}| = V_{DD} - V_{tp} - \sqrt{\dfrac{2 \cdot I_1}{K_p \frac{W}{L-2L_D}}} \end{array} \right.$$

final $I_1 = 222\ \mu A$, $V_1 = 1.59\ V$

for V_2, V_3, iterate also.

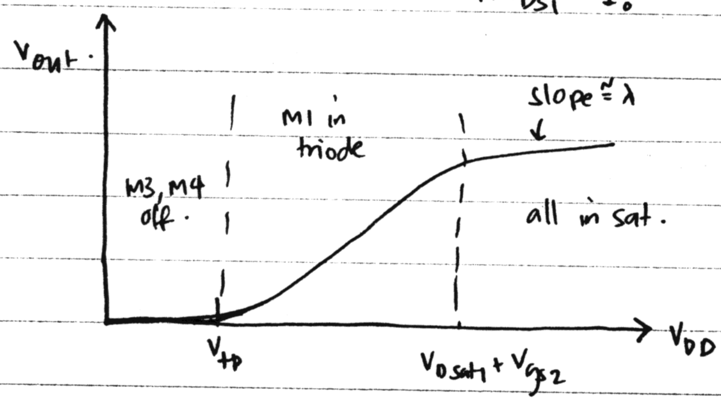
$$\left\{ \begin{array}{l} I_{ta} = I_t/2 - I_{tb} \\ V_{2,3} = V_{DD} - |V_{gs3,6}| = V_{DD} - V_{tp} - \sqrt{\dfrac{2 \cdot I_{ta}}{K_p \frac{W}{L-2L_D}}} \\ I_{tb} = I_1 \cdot \dfrac{1+\lambda |V_{gs3,6}|}{1+\lambda |V_{gsp}|} \end{array} \right.$$

$I_{ta} = 17.3\ \mu A$; $I_{tb} = 207\ \mu A$; $V_{2,3} = 2.029\ V$

5.19 $V_{DD} < V_{tp}$: M2 and M3 off , $I_3 = 0$, $V_{out} = 0$

$V_{tp} \le V_{DD} < V_{Dsat1} + V_{gs2}$: M1 in triode, linearly approaching saturation. M2 and M3 are on with Vout increasing linearly.

$V_{DD} > V_{Dsat1} + V_{gs2}$: All transistors are on and as V_{DD} increases, V_{DS1} increases so current increases as $\lambda V_{DS1} \cdot I_o$



5.20 $\gamma = 0$ $\left(\frac{W}{L}\right)_{1-3} = \frac{40}{.5}$ $I_{ref} = .3mA$. $K_n = 138 \frac{\mu A}{V^2}$ $L_D = 80\,nm$

(a) $V_b = 2V_{gs1}$ $V_{gs1} = V_{tn} + \sqrt{\frac{2I_{ref}}{K_n \frac{W}{L-2L_D}}} = .892V$

$V_b = 1.78\,V$

(b) $I_{out} = I_{ref} \cdot \frac{(1 + \lambda(V_{gs1} + \Delta V_b))}{1 + \lambda V_{gs1}} = 295\,\mu A$ $\Delta V_b = -.100V$

$\Delta I_{out} = \frac{I_{ref} \cdot \lambda \Delta V_b}{1 + \lambda V_{gs1}}$

5.20 (c) V_p increases by 1V

V_Y increases by ΔX

solve for 2 unknowns and 2 equations

$$\begin{cases} I_{out} = I_{ref}\left(\dfrac{1+\lambda(V_{gs1}-\Delta X)}{1+\lambda V_{gs1}}\right) & (M2) \quad \text{eq 1.} \\ I_{out} = \frac{1}{2}K_n \dfrac{W}{L-2L_D}(V_{gs1}-\Delta X)^2\left(\dfrac{1+\lambda(V_{gs1}+1V-\Delta X)}{1+\lambda V_{gs1}}\right) & (M3) \quad \text{eq 2} \end{cases}$$

$\Delta X = 13\,mV$

$V_y = V_{gs1} + \Delta X = .905\,V$

5.21 (a) $V_X = V_{gs1} = V_{tn} + V_{Dsat1} = .863$

$V_{Dsat1} = \sqrt{\dfrac{2I_{ref}}{K_n \frac{W}{L-2L_D}}} = .163 \qquad V_{tn} = .7V$

$V_b \geq V_{Dsat1} + V_{gs2}$

$V_{gs2} = V_{tn0} + \gamma\sqrt{2\phi + V_{Dsat1}} - \gamma\sqrt{2\phi} + \sqrt{\dfrac{2I_{REF}}{K_n\frac{W}{L-2L_D}}}$

$= .7 + .037 + .163 = .900v$

as V_b increases, M2 and M4 go into triode and $V_A \sim V_X$ and $V_B \sim V_{M4,drain}$. As lond as $V_{M4,drain}$ does not drop Below V_{Dsat1}, I_{out} will reasonably follow I_{ref}.

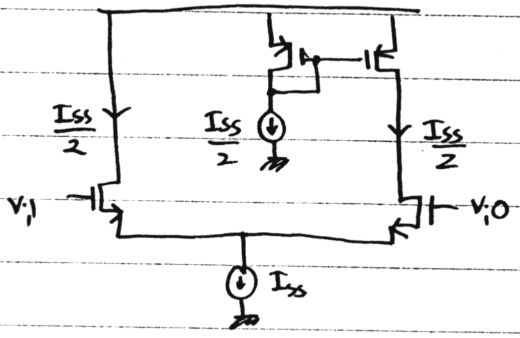
b) $V_{M4,drain} \uparrow 1V$, $V_B \uparrow \Delta X$

use eq 1.and 2. from 5.20 to solve for change in current.

$I_{out} = 301.1\,\mu A \qquad \Delta X = 21\,mV$

5.22 Assume $\lambda = 0$ for bias purpose and $\lambda = .2$ for small signal analysis.

(a) DC Bias



$$g_{m1,2} = \frac{I_{ss}}{V_{Dsat\,1,2}} = 3.18\ mS$$

$$V_{dsat1,2} = \sqrt{\frac{I_{ss}}{K_n \frac{W}{L-2L_D}}} = .157$$

$$r_{on} = r_{op} = \frac{1}{\lambda \frac{I_{ss}}{2}} = 20K$$

small signal: $\text{Gain} = \frac{v_{out}}{V_{i1} - V_{i0}} = \frac{1}{2} g_{m1,2} R_{out} = 21.2$

$$R_{out} = r_{op} \parallel \left(2r_{on} + \frac{1}{g_{m1}}\right) \approx \frac{2}{3} r_o = 13.33\ K$$

(b) Maximum output voltage swing.

$$V_{out}(min) = V_{cm} - V_{gs1,2}\left(I = \frac{I_{ss}}{2}\right) + V_{dsat\,1,2}$$
$$= V_{cm} - V_{tn1,2}$$
$$= 1.3 - .778 = .522\,v$$

$V_{out}(max) = V_{dd}$ (M4 in triode)

$$V_{out}(max) - V_{out}(min) = V_{DD} - V_{cm} + V_{tn1,2}$$
$$= 3.V - 1.3V + .778 = 2.48V$$

$$V_{tn} = V_{to} + \gamma\sqrt{2\phi + .36} - \gamma\sqrt{2\phi} = .778$$

5.23. Assume $I_3 = I_4$ though $V_{th3} \neq V_{th4}$

(a) $I_3 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{gs3} - V_{th3})^2 (1 + \lambda |V_{gs3}|)$

$I_4 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{g3} - V_{th4})^2 (1 + \lambda (|V_{gs3}| - \Delta x))$

$K' = \frac{1}{2} K_p \frac{W}{L-2L_D}$

$K' (V_{Dsat3})^2 (1 + \lambda |V_{gs3}|) = K' (|V_{dsat3}| - 1mV)^2 (1 + \lambda |V_{gs3}| + \lambda \Delta x)$

$$\Delta x \cong \frac{1mV \cdot (1 + \lambda |V_{gs3}|)}{\lambda |V_{dsat3}|}$$

$V_F = V_{gs3} - \Delta x$.

(b) $CMRR \triangleq \left| \frac{A_{dm}}{A_{cm}} \right|$

$A_{dm} = g_{m1,2} (r_{o3,4} \| r_{o1,2})$

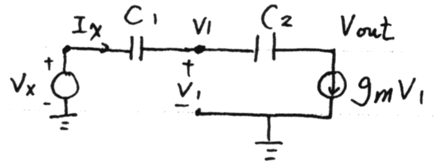
$\quad\;\; \simeq g_{m1,2} (\frac{1}{2} r_o)$

$$A_{cm} \cong \frac{-1}{1 + 2 g_{m1,2} r_o} \cdot \frac{g_{m1,2}}{g_{m3,4}}$$

$CMRR = (1 + 2 g_{m1,2} r_o) \, g_{m3,4} \, (r_{o1,2} \| r_{o3,4})$

Chapter 6

6.1 (a)



g_m : transconductance of M_1

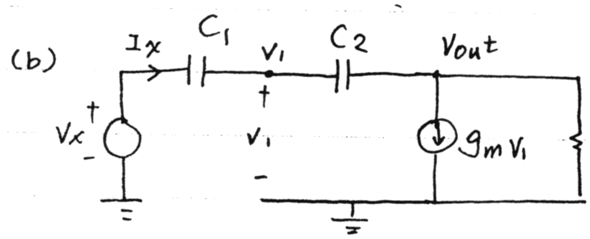
$$I_X = sC_1(V_X - V_1) = sC_2(V_1 - V_{out}) = g_m V_1$$

$$\therefore \quad sC_1 V_X = (g_m + sC_1)V_1 \quad \Rightarrow V_1 = \left[\frac{sC_1}{g_m + sC_1}\right] V_X$$

$$\Rightarrow I_X = g_m V_1 = \left[g_m sC_1 / g_m + sC_1 \right] V_X$$

$$\Rightarrow Z_{in} = \frac{V_X}{I_X} = \frac{g_m + sC_1}{g_m s C_1} \quad ※$$

(b)



$g_m = g_{m1} + g_{m2}$

$r_o = r_{o1} \parallel r_{o2}$

g_{m1}, g_{m2} : transconductance for M_1, M_2

r_{o1}, r_{o2} : output resistance for M_1, M_2

$$\therefore I_X = \overbrace{sC_1(V_X - V_1) = sC_2(V_1 - V_{out})}^{②} = \underbrace{sC_2(V_1 - V_{out}) = g_m V_1 + \frac{V_{out}}{r_o}}_{①}$$

from ① :

$$\frac{V_{out}}{V_1} = \frac{sC_2 - g_m}{sC_2 + \frac{1}{r_o}}$$

from ② :

$$(sC_1 + sC_2)V_1 = sC_1 V_X + sC_2 V_{out}$$

$$= sC_1 V_X + \frac{s^2C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} V_1$$

$$\Rightarrow \left[sC_1 + sC_2 - \frac{s^2C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_X$$

$$\Rightarrow \left[\frac{s^2C_1C_2 + \frac{sC_1}{r_o} + \frac{sC_2}{r_o} + g_m s C_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_x$$

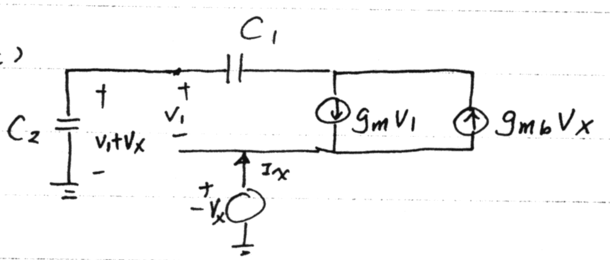
$$\therefore V_1 = \left[\frac{sC_1C_2 + \frac{C_1}{r_o}}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$I_x = sC_1(V_x - V_1) = sC_1 \cdot \left[\frac{\frac{C_2}{r_o} + g_m C_2}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2}{sC_1C_2\left(\frac{1}{r_o} + g_m\right)}$$

#

(c)



$$sC_2(V_1 + V_x) + g_m V_1 = g_{mb} V_x$$

$$\Rightarrow (sC_2 + g_m)V_1 = (g_{mb} - sC_2)V_x$$

$$\frac{V_x}{V_1} = \frac{sC_2 + g_m}{g_{mb} - sC_2}$$

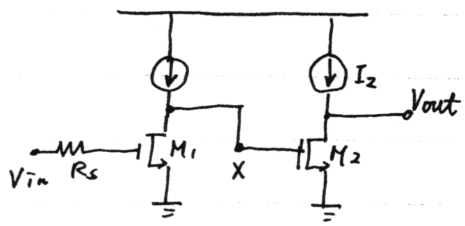
$$I_x = -g_m V_1 + g_{mb} V_x$$

$$= \left[-g_m \cdot \frac{g_{mb} - sC_2}{sC_2 + g_m} + g_{mb} \right] V_x = \left[\frac{(g_m + g_{mb})sC_2}{sC_2 + g_m} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_2 + g_m}{sC_2(g_m + g_{mb})}$$

#

6.2 (a)



There are three poles associated with this circuit.

The first pole @ Vout

$$\omega_{p,out} = \frac{1}{r_{o2} \cdot (C_{gd2} + C_{DB2})}$$

The pole @ the input

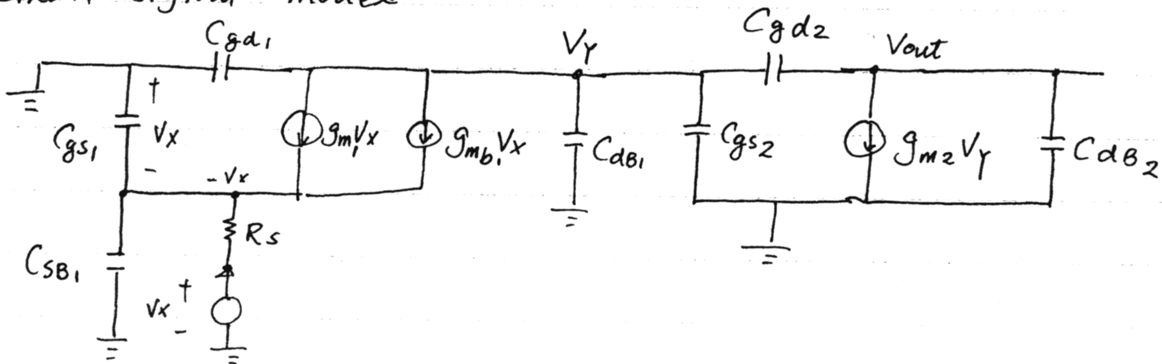
$$\omega_{p,in} = \frac{1}{R_S \cdot [(1+g_{m1} r_{o1}) C_{gd1} + C_{gs1}]}$$

The pole @ node X

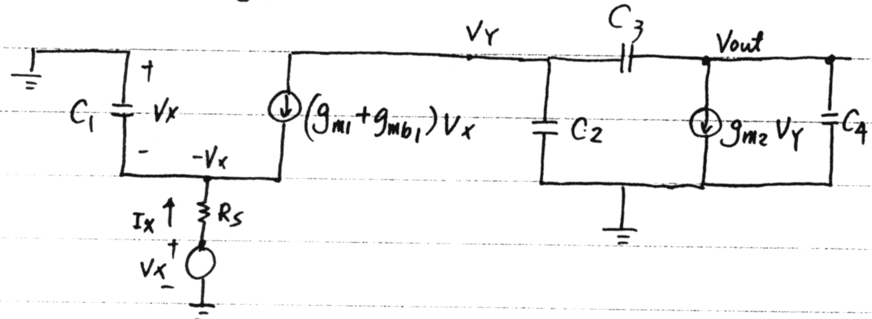
$$\omega_{p,X} = \frac{1}{r_{o1} \cdot [(C_{gd1} + C_{DB1} + C_{gs2}) + (1+g_{m2} r_{o2}) \cdot C_{gd2}]}$$

Please note that the above approximation is based on Miller effect. In order to get more accuracy approximation, transfer function has to be derived.

(b) Small signal model



Redraw small signal model



where,

$C_1 = C_{gs1} + C_{SB1}$

$C_2 = C_{gs2} + C_{dB1} + C_{gd1}$

$C_3 = C_{gd2}$

$C_4 = C_{dB2}$

KCL @ V_{out} : $sC_3(V_Y - V_{out}) = g_{m2}V_Y + sC_4 V_{out}$

$$\Rightarrow \frac{V_{out}}{V_Y} = \frac{-g_{m2} + sC_3}{s(C_3+C_4)}$$

KCL @ V_Y : $(g_{m1}+g_{mb1})V_X + sC_2 V_Y + sC_3(V_Y - V_{out}) = 0$

$$(g_{m1}+g_{mb1})V_X = -V_Y\left(sC_2 + \frac{s^2C_3C_4 + sC_3 \cdot g_{m2}}{s(C_3+C_4)}\right)$$

$$\frac{V_Y}{V_X} = -\frac{g_{m1}+g_{mb1}}{[s(C_2C_3+C_2C_4+C_3C_4)+C_3g_{m2}]/(C_3+C_4)}$$

KCL @ V_X : $\frac{V_{in}+V_X}{R_S} + sC_1V_X + (g_{m1}+g_{mb1})V_X = 0$

$$\frac{V_X}{V_{in}} = -\frac{1}{sC_1R_S + (1+(g_{m1}+g_{mb1})\cdot R_S)}$$

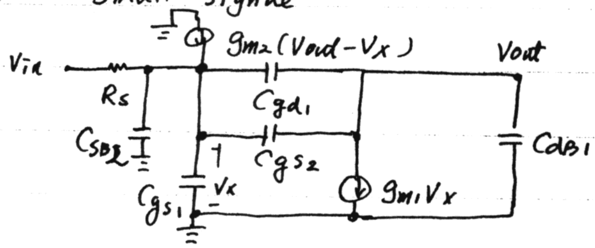
Thus. there are three poles

$\omega_{p0} = 0$

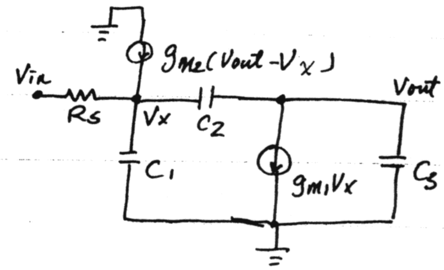
$$\omega_{p1} = \frac{-C_3 g_{m2}}{C_2C_3+C_2C_4+C_3C_4} \quad ※$$

$$\omega_{p2} = \frac{-(1+(g_{m1}+g_{mb1})\cdot R_S)}{C_1 R_S} \quad ※$$

6.3 (a) Small signal



$\Rightarrow$



$C_1 = C_{gs1} + C_{SB2}$

$C_2 = C_{gd1} + C_{gs2}$

$C_3 = C_{dB1}$

KCL @ Vout :

$$sC_2(V_x - V_{out}) = g_{m1}V_x + sC_3 V_{out}$$

$$\therefore \frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2+C_3)} \quad \text{— (1)}$$

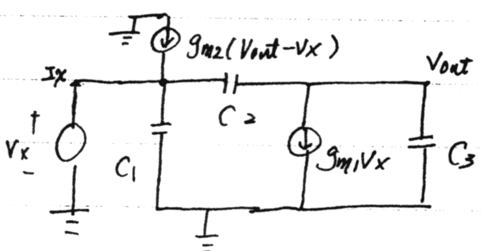
KCL @ Vx :

$$\frac{V_{in}-V_x}{R_s} + g_{m2}(V_{out}-V_x) = sC_1V_x + sC_2(V_x - V_{out})$$

$$\Rightarrow \frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m2} + sC_1 + sC_2\right) - (g_{m2}+sC_2)\cdot\left[\frac{sC_2 - g_{m1}}{s(C_2+C_3)}\right]V_x$$

$$= V_x \cdot \frac{s^2(C_1C_2+C_2C_3+C_1C_3) + s\left(\frac{1}{R_s}(C_2+C_3) + g_{m1}C_2 + g_{m2}C_3\right) + g_{m1}g_{m2}}{s(C_2+C_3)}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x}\cdot\frac{V_x}{V_{in}} = \frac{\frac{1}{R_s}(sC_2 - g_{m1})}{s^2(C_1C_2+C_2C_3+C_1C_3) + s\left[\frac{1}{R_s}(C_2+C_3) + g_{m1}C_2 + g_{m2}C_3\right] + g_{m1}g_{m2}} \quad ☆$$



$$I_x = sC_1V_x + sC_2(V_x - V_{out}) + g_{m2}(V_x - V_{out})$$

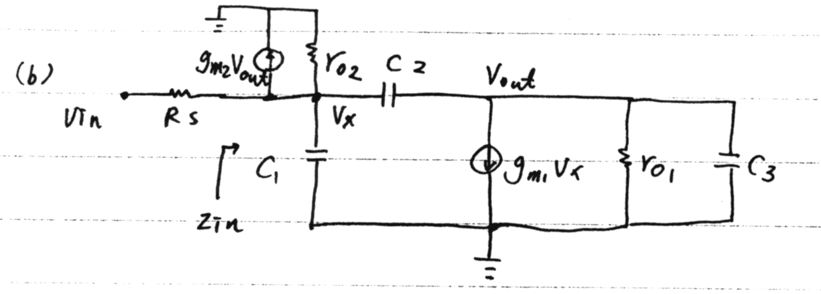
from (1): $$V_x - V_{out} = \left(\frac{sC_3 + g_{m1}}{s(C_2+C_3)}\right)V_x$$

$$\therefore I_x = \left[sC_1 + \frac{s^2C_2C_3 + g_{m1}sC_2 + g_{m2}sC_3 + g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$= \left[\frac{s^2(C_1C_2+C_2C_3+C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2+C_3)}{s^2(C_1C_2+C_2C_3+C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}} \quad ☆$$

(b)



$C_1 = C_{gs1} + C_{dB2}$

$C_2 = C_{gd1} + C_{gd2}$

$C_3 = C_{dB1} + C_{gs2}$

KCL @ V_{out} : $sC_2(V_x - V_{out}) = g_{m1}V_x + V_{out}\left(\frac{1}{r_{o1}} + sC_3\right)$

$$\frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2+C_3) + \frac{1}{r_{o1}}}$$

KCL @ V_x : $\frac{V_{in}-V_x}{R_s} = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$

$$\frac{V_{in}}{R_s} = \left(sC_1 + sC_2 + \frac{1}{r_{o2}} + \frac{1}{R_s}\right)V_x - \frac{(-g_{m2}+sC_2)(sC_2-g_{m1})}{s(C_2+C_3)+\frac{1}{r_{o1}}}\cdot V_x$$

$$= \frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_s}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_s}\right)}{s(C_2+C_3)+\frac{1}{r_{o1}}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_s}(sC_2 - g_{m1})}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_s}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_s}\right)}$$ ✱

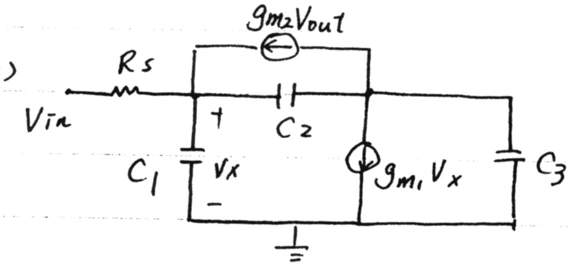
For Z_{in}

$$I_x = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$$

$$= \left[\frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}{s(C_2+C_3)+\frac{1}{r_{o1}}}\right]V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2+C_3)+\frac{1}{r_{o1}}}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 - g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}$$

(c)



$C_1 = C_{gs1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1}$

$C_3 = C_{dB1} + C_{SB2} + C_{gs2}$

KCL@ Vout : $sC_2(V_X - V_{out}) = g_{m1}V_X + sC_3V_{out} + g_{m2}V_{out}$

$$\Rightarrow \frac{V_{out}}{V_X} = \frac{sC_2 - g_{m1}}{s(C_2+C_3) + g_{m2}}$$

KCL@Vx : $\frac{V_{in}-V_X}{R_S} + g_{m2}V_{out} = sC_1V_X + sC_2(V_X - V_{out})$

$$\frac{V_{in}}{R_S} = \left(\frac{1}{R_S} + sC_1 + sC_2\right)V_X - (g_{m2}+sC_2)V_{out} = \left(\frac{1}{R_S} + sC_1 + sC_2\right)V_X - \frac{(g_{m2}+sC_2)(sC_2-g_{m1})}{s(C_2+C_3)+g_{m2}}$$

$$\frac{V_i}{V} = \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_S}(C_2+C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left(\frac{g_{m2}}{R_S} + g_{m1}g_{m2}\right)}{s(C_2+C_3) + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_X} \cdot \frac{V_X}{V_{in}}$$

$$= \frac{\frac{1}{R_S}[sC_2 - g_{m1}]}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_S}(C_2+C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left(\frac{g_{m2}}{R_S} + g_{m1}g_{m2}\right)}$$

#

For Z_{in}

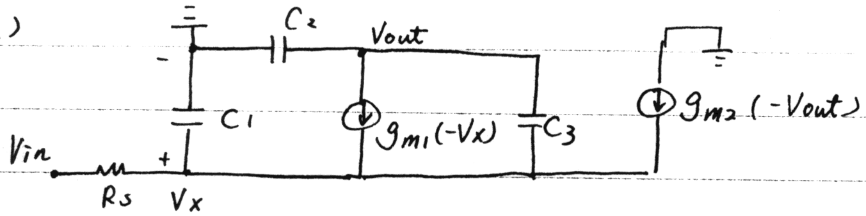
$I_X = sC_1V_X - g_{m2}V_{out} + sC_2(V_X - V_{out})$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}{s(C_2+C_3) + g_{m2}}\right]V_X$$

$$Z_{in} = \frac{s(C_2+C_3) + g_{m2}}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}$$

#

(d)



$C_1 = C_{gs1} + C_{SB1} + C_{DB2}$
$C_2 = C_{gd1} + C_{gs2} + C_{dB1}$
$C_3 = C_{gd2}$

KCL @ Vout : $-sC_2 V_{out} = -g_{m1}V_X + sC_3(V_{out} - V_X)$

$$\frac{V_{out}}{V_X} = \frac{sC_3 + g_{m1}}{s(C_2 + C_3)}$$

KCL @ V_X : $\dfrac{V_{in} - V_X}{R_S} = sC_1 V_X + g_{m1}V_X + sC_3(V_X - V_{out}) + g_{m2}V_{out}$

$$\frac{V_{in}}{R_S} = \left[\frac{1}{R_S} + s(C_1 + C_3) + g_{m1}\right]V_X + \frac{(sC_3 + g_{m1})(g_{m2} - sC_3)}{s(C_2 + C_3)}V_X$$

$$= V_X\left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_S} + \frac{C_3}{R_S} + g_{m1}C_2 + g_{m2}C_3\right] + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{sC_3 + g_{m1}}{s^2R_S(C_1C_2 + C_2C_3 + C_1C_3) + s[C_2 + C_3 + R_S(g_{m1}C_2 + g_{m2}C_3)] + g_{m1}g_{m2}R_S}$$ #

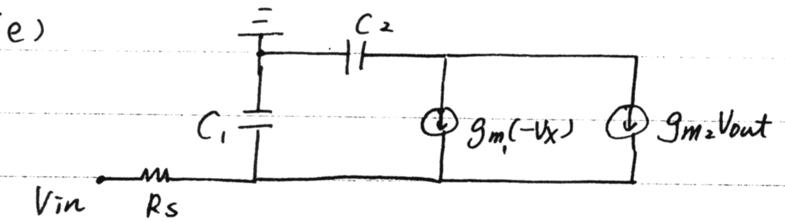
For Z_{in}

$$I_X = sC_1V_X + g_{m1}V_X + sC_3(V_X - V_{out}) - g_{m2}V_{out}$$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]V_X$$

$$\therefore Z_{in} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}$$ #

(e)



$C_1 = C_{gs1} + C_{SB1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1} + C_{SB2} + C_{gs2} + C_{dB1}$

KCL @ Vout $\Rightarrow -sC_2 V_{out} = -g_{m1} V_x + g_{m2} V_{out}$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1}}{sC_2 + g_{m2}}$$

KCL @ Vx $\Rightarrow \dfrac{V_{in} - V_x}{R_s} = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$

$$\Rightarrow \frac{V_{in}}{R_s} = \left[\frac{1}{R_s} + sC_1 + g_{m1}\right] V_x - \frac{g_{m1} g_{m2}}{(sC_2 + g_{m2})} V_x$$

$$= \frac{s^2 C_1 C_2 + s\left[\left(\frac{1}{R_s} + g_{m1}\right) C_2 + g_{m2} C_1\right] + \frac{g_{m2}}{R_s}}{sC_2 + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{s^2 R_s C_1 C_2 + s[(1 + g_{m1} R_s) C_2 + g_{m2} R_s C_1] + g_{m2}}$$ ✲

For Zin

$I_x = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$

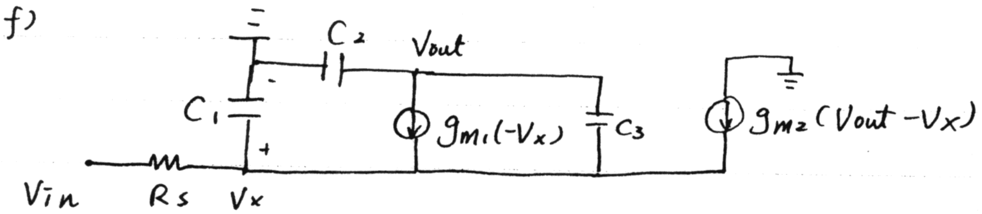
$$= \left[\frac{s^2 C_1 C_2 + s[g_{m1} C_2 + g_{m2} C_1]}{sC_2 + g_{m2}}\right]$$

$$\therefore Z_{in} = \frac{sC_2 + g_{m2}}{s^2 C_1 C_2 + s(g_{m1} C_2 + g_{m2} C_1)}$$ ✲

6.10

Pg 9+

(f)



$C_1 = C_{SB1} + C_{SB2} + C_{gs1}$

$C_2 = C_{gd1} + C_{gd2} + C_{dB1}$

$C_3 = C_{gs2}$

KCL @ Vout : $sC_2(-V_{out}) = g_{m1}(-V_x) + sC_3(V_{out} - V_x)$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1} + sC_3}{s(C_2 + C_3)}$$

KCL @ V_x: $\frac{V_{in} - V_x}{R_s} + g_{m1}(-V_x) + g_{m2}(V_{out} - V_x) = sC_1 V_x + sC_3(V_x - V_{out})$

$$\frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m1} + g_{m2} + sC_1 + sC_3\right) - \frac{(g_{m2} + sC_3)(g_{m1} + sC_3)}{s(C_2 + C_3)} \cdot V_x$$

$$= \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2}}{s(C_2 + C_3)}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} + sC_3}{s^2R_s(C_1C_2 + C_2C_3 + C_1C_3) + s\left[C_2 + C_3 + R_s(g_{m1}C_2 + g_{m2}C_2)\right] - g_{m1}g_{m2}R_s}$$

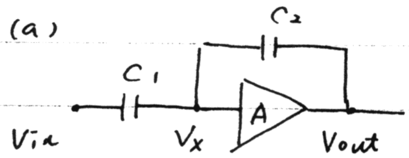
For Zin

$I_x = g_{m1}V_x + g_{m2}(V_x - V_{out}) + sC_1V_x + sC_3(V_x - V_{out})$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2}}{s(C_2 + C_3)}\right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_2) - g_{m1}g_{m2}}$$

6.4 (a)



$A = -\infty$

(i) At low frequency, V_X is like virtual ground

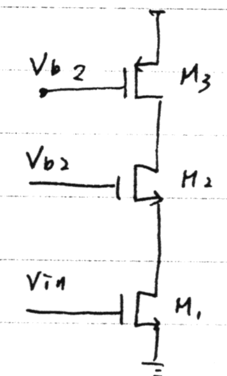
$sC_1 V_{in} = -sC_2 V_{out}$

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}$$

(ii) At high frequency, C_1, C_2 is like a short circuit

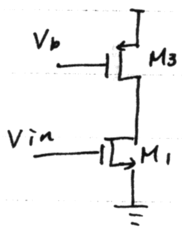
$$\frac{V_{out}}{V_{in}} = 1$$

(b) At low frequency, the equivalent circuit is shown as



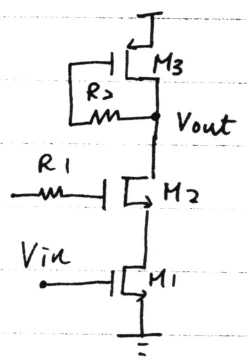
$A_v \cong -g_{m1} r_{o3} \to \infty$, if $\lambda = 0$

(ii) At high frequency, the equivalent circuit



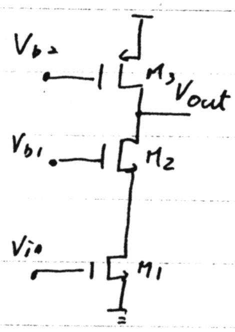
$A_v = -g_{m1}(r_{o1} // r_{o3}) \rightarrow \infty$ if $\lambda = 0$

(c) (i) At low frequency, the equivalent circuit



R_1, R_2 can be ignored

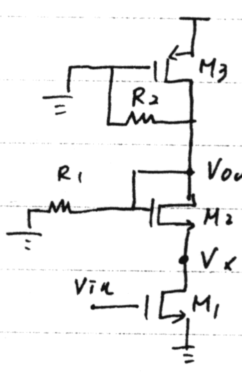
$\Rightarrow$



The impedance @ Vout $= \frac{1}{g_{m3}}$

$A_v \cong -g_{m1} \cdot 1/g_{m3} = -\frac{g_{m1}}{g_{m3}}$

(ii) At high frequency.



$\frac{V_x}{V_{in}} = -g_{m1} \cdot \underbrace{\frac{1}{g_{m2}}}$

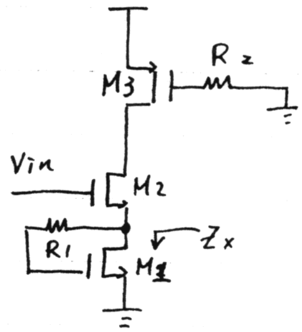
the impedance looking into V_x

The inpedance @ Vout $= R_1 // R_2$

$\therefore A_v = \left(-g_{m1} \cdot \frac{1}{g_{m2}}\right) \cdot g_{m2} \cdot (R_1 // R_2) = -g_{m1}(R_1 // R_2)$

#

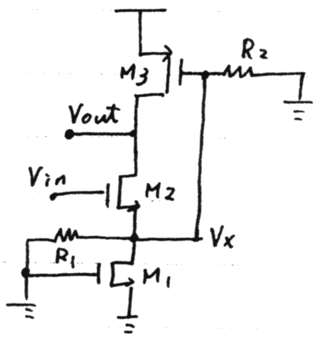
(d) (i) At low frequency, the equivalent circuit is



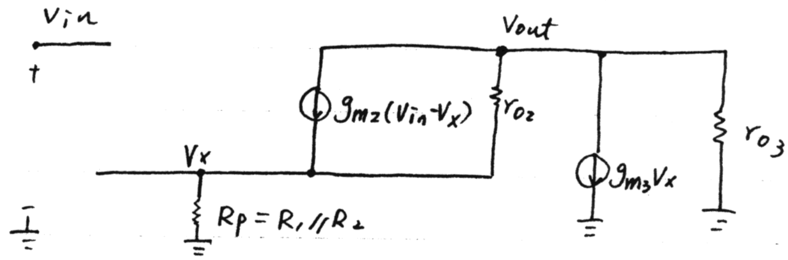
$$\frac{V_{out}}{V_{in}} = \frac{g_{m2}\left(r_{o3} \parallel (1+g_{m2}r_{o2})Z_x\right]}{1+g_{m2}Z_x} \cong \frac{g_{m2}(r_{o3}\parallel r_{o2})}{1+\frac{g_{m2}}{g_{m1}}} \rightarrow \infty \quad \text{if } \lambda = 0$$

$$Z_x = \frac{1}{g_m}$$

(ii) At high frequency



Small-signal model



KCL @ Vx, Vout : $\frac{V_x}{R_p} = g_{m2}(V_{in} - V_x) + \frac{V_{out}-V_x}{r_{o2}} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right)$

$$\frac{V_x}{R_p} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right) \Rightarrow \frac{V_{out}}{V_x} = -r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)$$

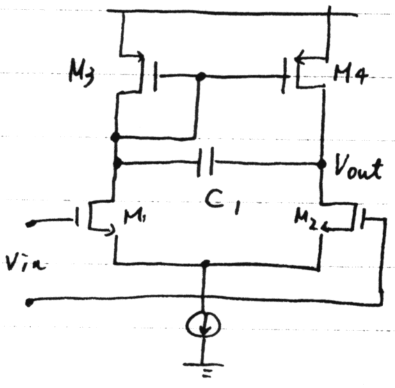
$$g_{m2}(V_{in} - V_x) + \frac{V_{out}-V_x}{r_{o2}} = \frac{V_x}{R_p} \Rightarrow g_{m2}V_{in} = \left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right)V_x + \frac{V_{out}}{r_{o2}}$$

$$\Rightarrow g_{m2}V_{in} = \left[-\left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right)\frac{1}{r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)} + \frac{1}{r_{o2}}\right]V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_{m2}r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)}{\left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right) - \frac{r_{o3}}{r_{o2}}\left(g_{m3}+\frac{1}{R_p}\right)} \rightarrow \infty \quad \text{if } \lambda = 0$$

#

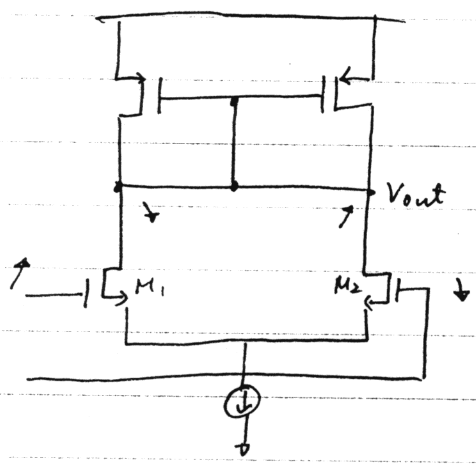
6.5 (a) (i) At low frequency



C_1 is like an open circuit @ very low frequency

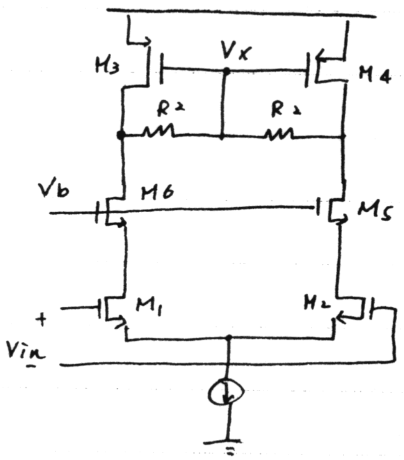
$$\Rightarrow \frac{V_{out}}{V_{in}} = -g_{m1}(r_{o2} // r_{o4}) \to \infty \text{ if } \lambda = 0$$

(ii) At very high frequency, C_1 is like a short circuit



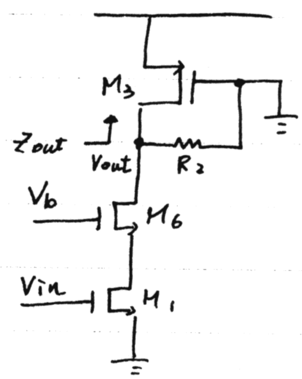
Gain $= 0$

(b) (i) At low frequency, the equivalent circuit is



half circuit →

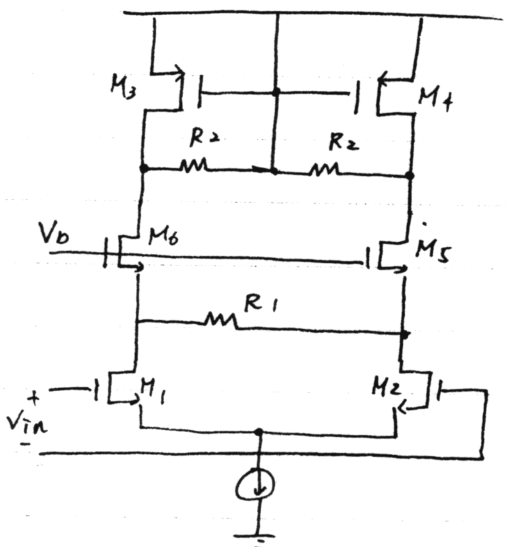
Vx is virtual ground



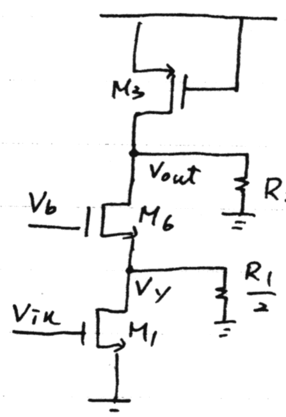
$Z_{out} \cong r_{o3} // R_2 \cong R_2$

$A_V = -g_{m1} \cdot (R_2 // r_{o3}) \cong -g_{m1} R_2$ ✡

(ii) At high frequency



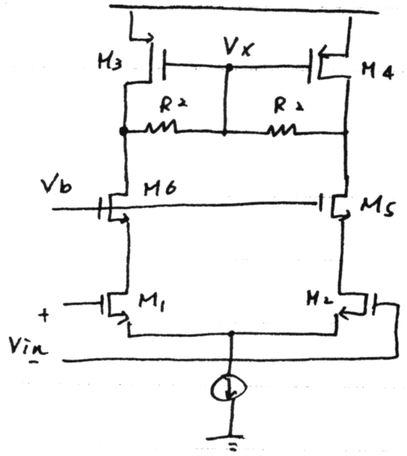
half circuit →



$\frac{V_y}{V_{in}} = -g_{m1} \left(\frac{1}{g_{m6}} // \frac{R_1}{2} \right)$, $\frac{V_{out}}{V_y} \cong + g_{m6} \cdot R_2$

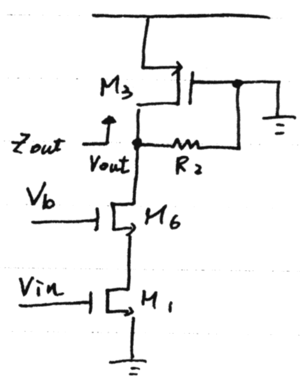
$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$$ ✡

(b) (i) At low frequency, the equivalent circuit is



half circuit →

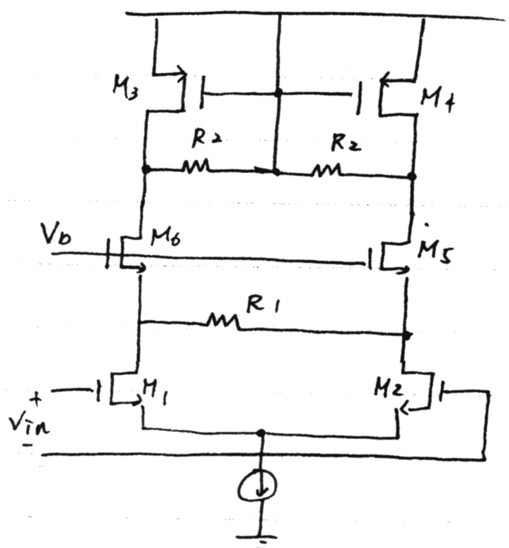
Vx is virtual ground



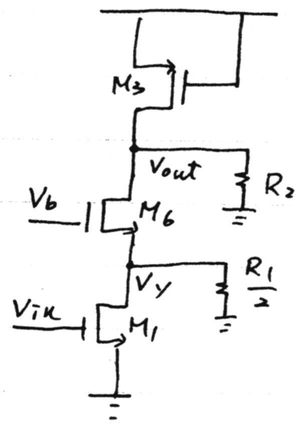
$Z_{out} \cong r_{o3} // R_2 \cong R_2$

$A_v = -g_{m1} \cdot (R_2 // r_{o3}) \cong -g_{m1} R_2$ ✻

(ii) At high frequency



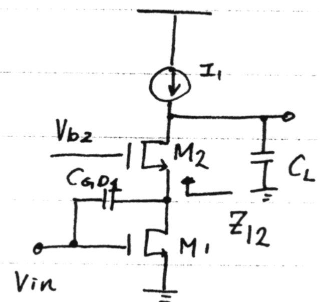
half circuit →



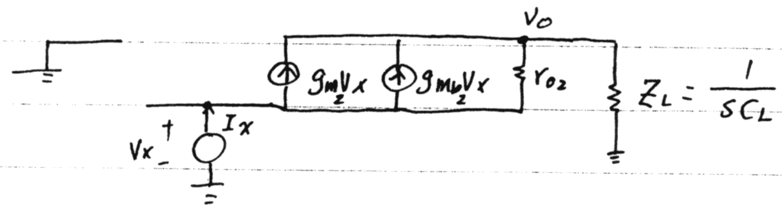
$\frac{V_y}{V_{in}} = -g_{m1} \left(\frac{1}{g_{m6}} // \frac{R_1}{2} \right)$, $\frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$

$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$ ✻

6.6



The impedance Z_{12} can be derived from the following small signal model



KCL @ V_o : $\frac{V_o}{Z_L} + \frac{V_o - V_x}{r_{o2}} = (g_{m2} + g_{mb2})V_x \Rightarrow \left(\frac{1}{Z_L} + \frac{1}{r_{o2}}\right)V_o = \left(g_{m2} + g_{mb2} + \frac{1}{r_{o2}}\right)V_x$

$$\Rightarrow V_o = \left(\frac{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right)V_x$$

$$\Rightarrow I_x = \frac{V_o}{Z_L} = \left[\frac{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right]V_x \Rightarrow \frac{V_x}{I_x} = Z_{12} = \frac{1 + \frac{Z_L}{r_{o2}}}{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}$$

$$\Rightarrow Z_{12} = \frac{r_{o2} + Z_L}{1 + (g_{m2} + g_{mb2})r_{o2}}$$

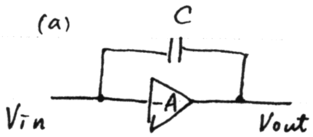
The miller multiplication for $C_{GD1} = 1 + g_{m1} Z_{12}$

$$= 1 + \frac{g_{m1}(r_{o2} + Z_L)}{1 + (g_{m2} + g_{mb2})r_{o2}} \quad \text{— (1)}$$

If C_L is relatively large $\Rightarrow \left|\frac{1}{sC_L}\right| << r_{o2}$

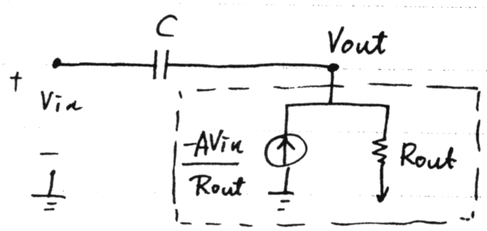
eq (1) can be approximated as $\cong 1 + \frac{g_{m1} r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} \cong 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}$

6.7 (a)



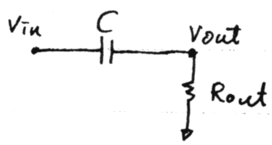
Assume the amplifier output resistance Rout

The small signal model is as follows



← The Norton equivalent circuit for amplifier

As we can see, the above circuit forms a high pass network



Thus, when there is a step ΔV at the input, output will follow Input, a step ΔV, first. Then, it will settle down to $-AVin$ as the steady state

(b) KCL @ Vout :

$$-\frac{AV_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in})$$

$$\Rightarrow \left(sC - \frac{A}{R_{out}}\right)V_{in} = \left(\frac{1}{R_{out}} + sC\right)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sCR_{out}}$$ ✱

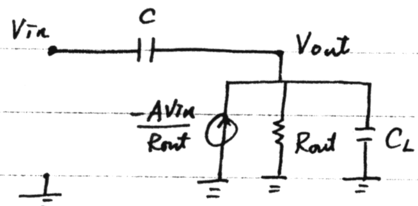
for the step response, $x(t) = u(t)$, $t \geq 0 \rightarrow X(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s} \cdot \frac{V_{out}}{V_{in}}(s) = \frac{sCR_{out} - A}{s(1 + sCR_{out})} = \frac{-A}{s} + \frac{(A+1)\cdot R_{out}C}{1 + sCR_{out}}$$

$$\Rightarrow y(t) = -A\,u(t) + (A+1)\,e^{-\frac{t}{R_{out}C}},\ t \geq 0$$

For a ΔV input step, output $= -A\cdot\Delta V + (A+1)\cdot\Delta V\cdot e^{-\frac{t}{R_{out}C}}$ ✱

6.8 (a) Small-signal circuit model



when input has ΔV jump, Vout will follow and the output jump $= \left(\frac{C}{C_L + C}\right) \Delta V$ ✱

(b) The transfer function $H(s)$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \quad : \quad \text{KCL @ } V_{out}$$

$$\Rightarrow \quad -\frac{A V_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in}) + sC_L V_{out}$$

$$\Rightarrow \quad V_{in}\left(sC - \frac{A}{R_{out}}\right) = V_{out}\left(\frac{1}{R_{out}} + sC + sC_L\right)$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sR_{out}(C + C_L)} \quad ✱$$

step response

$$Y(s) = \frac{1}{s} \cdot \frac{sCR_{out} - A}{[1 + sR_{out}(C + C_L)]} = \frac{-A}{s} + \frac{[(A+1)C + AC_L]R_{out}}{1 + sR_{out}(C + C_L)} = \frac{-A}{s} + \frac{\frac{(A+1)C + AC_L}{C + C_L}}{s + \frac{1}{R_{out}(C + C_L)}}$$

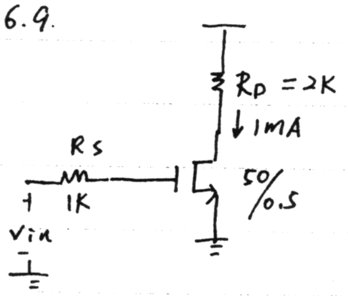
$$y(t) = -A\,u(t) + \frac{(A+1)C + AC_L}{C + C_L} \cdot e^{-\frac{t}{R_{out}(C + C_L)}} u(t)$$

For a step ΔV @ the input

$$\text{output} = -A\,\Delta V + \left[\frac{(A+1)C + A \cdot C_L}{C + C_L}\right] \cdot \Delta V \cdot e^{-\frac{t}{R_{out}(C + C_L)}}$$

✱

6.9.



$\lambda = 0.1$

$C_{ox} = \dfrac{\epsilon_{SiO_2}}{t_{ox}} = \dfrac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} = 3.835 \times 10^{-7}$

$\mu_n = 350$

$I_D = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\dfrac{50}{0.5 - 2 \times 0.08}\right)(V_{gs} - V_T)^2 (1 + 0.1 \times V_{ds})$

$= 67.113 \times 10^{-6} \times \dfrac{50}{0.34} \times 1.1 \times (Vin - 0.7)^2$

$\Rightarrow Vin = 1.0035$

$g_m = {}^{2I_D}/_{(V_{gs} - V_t)} = 6.59 \times 10^{-3}$

$C_{gs} = \frac{2}{3} C_{ox} W L + C_{ov} \cdot W$

$= \frac{2}{3} \times 3.835 \times 10^{-7} \times 50 \times (0.5 - 0.08 \times 2) \times 10^{-8} + 3.835 \times 10^{-7} \times 0.08 \times 10^{-4} \times 50 \times 10^{-4}$

$= 53.7 \times 10^{-15}$

$C_{gd} = C_{gdo} \cdot W = 0.4 \times 10^{-11} \times 50 \times 10^{-6} = 2 \times 10^{-16}$

$C_{dB} = \dfrac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{(1 + \frac{1}{0.9})^{0.6}} + \dfrac{0.35 \times 10^{-11} \times 103 \times 10^{-6}}{(1 + \frac{1}{0.9})^{0.2}} = 2.714 \times 10^{-14}$

According to eq (20)

$zero = \dfrac{g_m}{C_{gd}} = \dfrac{6.59 \times 10^{-3}}{2 \times 10^{-6}} = 3.3 \times 10^{13}$ rad/sec ✲

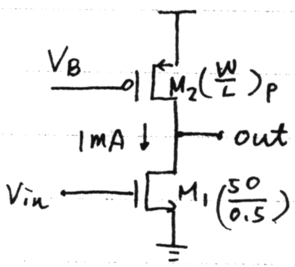
pole is the root of $R_S R_D (C_{gs} C_{gd} + C_{gs} C_{dB} + C_{gd} C_{dB}) s^2$
$+ [R_S (1 + g_m R_D) C_{gd} + R_S C_{gs} + R_D (C_{gd} + C_{dB})] s + 1$ ... from eq (6.20)

$\Rightarrow 2.95 \times 10^{-21} s^2 + 1.112 \times 10^{-10} s + 1 = 0$

$\omega_{p1} = -14.82 \times 10^9$ rad/sec ✲

ω_{p2} -22.88×10^9 rad/sec ✲

6.10



(a) The maximum output level = 2.6 V

$\rightarrow$ V_B can be as low as $2.6 - |V_{THP}| = 2.6 - 0.8 = 1.8V$

Let's choose output DC bias @ 1.5V, such that M_1, M_2 are both in saturation region

Thus, $I_{D_1} = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2\times0.08}\right)(V_{in} - 0.7)^2(1 + 0.1\times1.5)$

$\Rightarrow V_{in} = 0.997 \cong 1V$

Also, $I_{D_2} = 10^{-3} = \frac{1}{2} \times 100 \times 3.835 \times 10^{-7} \times \left(\frac{W_P}{0.5 - 2\times0.09}\right)(3 - 1.8 - 0.8)^2(1 + 0.2\times1.5)$

$W_P \cong 80.5\mu m = 81\mu m$

Therefore, we can choose $\underline{(\frac{W}{L})_P = (\frac{81\mu m}{0.5\mu m})}$, with gate bias 1.8V so that M_1, M_2 are both in saturation region and $V_{out} \cong 1.5V$

$V_{out,low} = V_{in} - |V_{THN}| = 0.3V$

Thus, the maximum output peak-to-peak swing $= 2.6 - 0.3 = \underline{1.3V}$ ✱

(b) This problem is similar to problem 6.9 except

$R_D \rightarrow (r_{o1} // r_{o2})$

$C_{DB} = C_{DB1} + C_{DB2} + C_{gd2}$

$\therefore g_{m1} = \frac{2I_D}{V_{gs} - V_t} = 2\times10^{-3} / 0.297 = 6.73\times10^{-3}$

$r_{op} = 1 \Big/ \left(\frac{\lambda_P I_D}{1 + \lambda_P V_{DS}}\right) \cong 6.5K$

$r_{ON} = 1 \Big/ \left(\frac{\lambda_N I_D}{1 + \lambda_N V_{DS}}\right) = 11.5K$

$\therefore R_D = r_{op} // r_{ON} = 4.1K$

$R_S = 1K$

$C_{gs1} = \frac{2}{3} C_{ox} W_1 L_1 + C_{ox} W \cdot \Delta L = 58.8\times10^{-15} F$

$C_{gd1} = 2\times10^{-16}$

$$C_{dB_1} = \frac{0.56\times10^{-3}\times75\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.6}} + \frac{0.35\times10^{-11}\times103\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.2}} = 23.38\times10^{-15}\,F$$

$$C_{dB2} = \frac{0.94\times10^{-3}\times121.5\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.5}} + \frac{0.32\times10^{-11}\times165\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.3}} = 70.33\times10^{-15}\,F$$

$$C_{gd2} = 81\times0.3\times10^{-11}\times10^{-6} = 0.243\times10^{-15}\,F$$

$$\omega_z = -\frac{g_{m_1}}{C_{gd_1}} = -\frac{6.59\times10^{-3}}{2\times10^{-16}} = -3.3\times10^{13}\ \text{rad/sec}$$

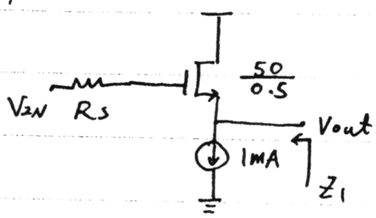
$\omega_{p_1}, \omega_{p_2}$ is the root of the equation

$$R_S R_D (C_{gs_1}C_{gd_1} + C_{gs_1}C_{dB} + C_{gd_1}C_{dB}) + [R_S(1+g_{m_1}R_D)C_{gd_1} + R_S C_{gs_1} + R_D(C_{gd_1}+C_{dB})] + 1$$

$\Rightarrow$ $\omega_{p_1} = -2.2\times10^{9}$ rad/sec

$\omega_{p_2} = -17.36\times10^{9}$ rad/sec

6.11



Assume $\gamma = 0$

$$g_m = 2I/(V_{gs}-V_t)$$

$$= \frac{2\times10^{-3}}{0.3} = 6.67\times10^{-3}$$

From eq (6.49) $Z_1 = \dfrac{R_s C_{gs}\cdot s + 1}{g_m + C_{gs} s}$

Since $\frac{1}{g_m} < R_s$

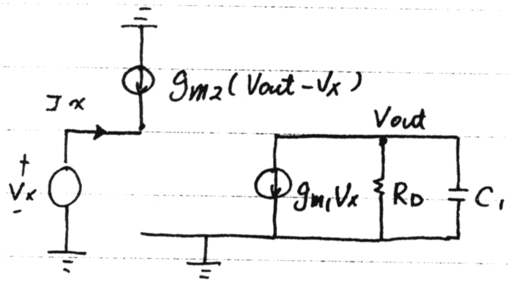
, thus Z_1 is inductive and the equivalent inductance is

$$= \frac{C_{gs}}{g_m}\left(R_s - \frac{1}{g_m}\right)$$

$C_{gs} = 58\times10^{-15}$ F

$\therefore L = 8.56\times10^{-8}$ H

6.12 (a)



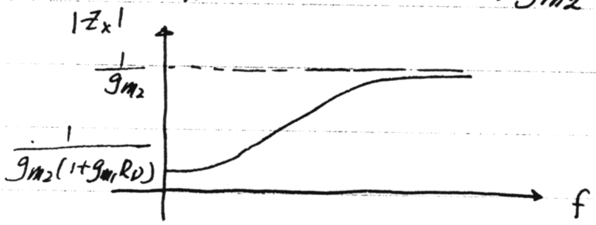
$$V_{out} = -g_{m1}V_X\left(R_D // \frac{1}{sC_1}\right)$$

$$I_X = -g_{m2}(V_{out}-V_x) = -g_{m2}V_{out} + g_{m2}V_X = \left[g_{m2}g_{m1}\left(R_D//\frac{1}{sC_1}\right)+g_{m2}\right]V_x$$

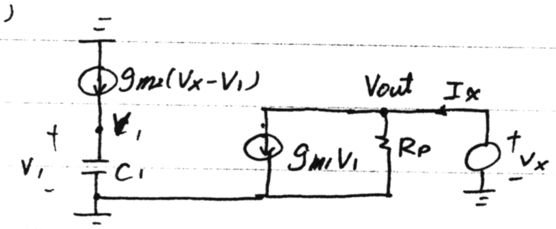
$$Z_X = \frac{V_X}{I_X} = \frac{1}{g_{m2}g_{m1}\frac{R_D/sC_1}{R_D+1/sC_1}+g_{m2}} = \frac{1}{g_{m2}\left[\left(\frac{g_{m1}R_D}{1+sR_DC_1}\right)+1\right]}$$

Thus, $Z_X(s\to 0) = 1/g_{m2}(1+g_{m1}R_D)$

$Z_X(s\to\infty) = 1/g_{m2}$



(b)



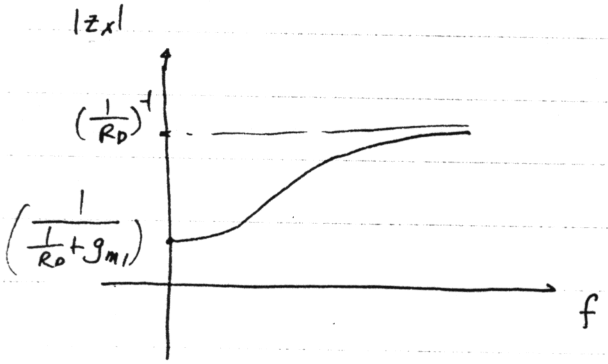
KCL @ V_1 : $g_{m2}(V_X-V_1) = sC_1V_1$

$$\Rightarrow V_1 = \left(\frac{g_{m2}}{sC_1+g_{m2}}\right)V_X$$

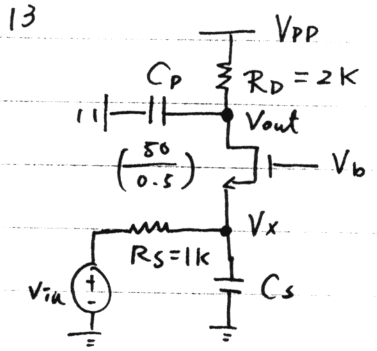
KCL @ V_{out} : $I_X = \frac{V_X}{R_D} + g_{m1}V_1$

$$= \frac{V_X}{R_D} + \frac{g_{m1}g_{m2}}{sC_1+g_{m2}}V_X$$

$$\Rightarrow Z_X = \frac{1}{\frac{1}{R_D}+\frac{g_{m1}g_{m2}}{sC_1+g_{m2}}}$$



6.13



from eq (6.53)

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_s} \cdot \frac{1}{\left(1 + \frac{C_s}{g_m + g_{mb} + R_s^{-1}} \cdot s\right)(1 + R_D C_D s)}$$

Assume V_b is chosen appropriately such that $V_x \sim 0$ (no body effect)

$g_m \sim 6.59 \times 10^{-3}$

$g_{mb} = \left[\gamma / 2\sqrt{2\phi_f} \right] g_m = 1.563 \times 10^{-3}$

$C_S = C_{SB} + C_{gs} = 42.4 \times 10^{-15} + 58.8 \times 10^{-15}$ } from problem 9

$C_D = C_{DB} = 27.14 \times 10^{-15}$ F

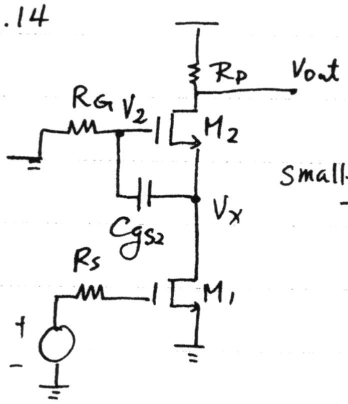
$$A_v\,(\text{low frequency}) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_s} = 1.44$$

$$\omega_{p_1} = -\frac{g_m + g_{mb} + R_s^{-1}}{C_s} = -\frac{6.59\times10^{-3} + 1.563\times10^{-3} + 10^{-3}}{42.4\times10^{-15} + 58.8\times10^{-15}} = -9.044\times10^{10} \text{ rad/s}$$

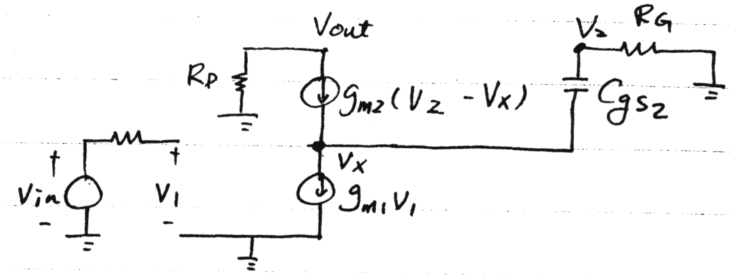
$$\omega_{p_2} = -\frac{1}{R_D C_D} = -\frac{1}{2\times10^{3} \times 2.714\times10^{-14}} = -1.84\times10^{10} \text{ rad/s}$$

Compared with the pole locations in problem 9, the poles for Common-gate configuration are much larger because there is no Miller-effect for C_{gd} in this case

6.14



small-signal model



KCL @ V_2 : $\frac{V_2}{R_G} = sC_{gs2}(V_x - V_2)$

$$\Rightarrow \frac{V_2}{V_X} = \frac{sC_{gs2}}{\frac{1}{R_G} + sC_{gs2}} = \frac{sR_G C_{gs2}}{1 + sR_G C_{gs2}} \quad \text{— ①}$$

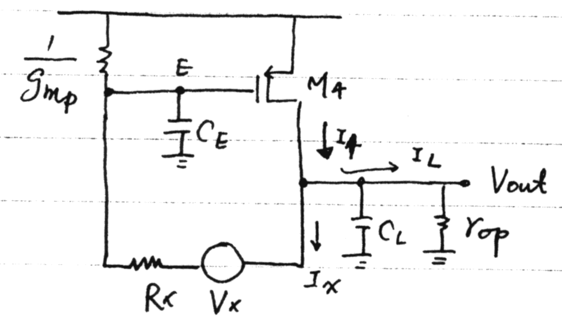
KCL @ Vout : $V_{out} = -g_{m2}(V_2 - V_X)R_D = \left[\frac{g_{m2}R_D}{1 + sR_G C_{gs2}}\right] V_X \quad \text{— ②}$

KCL @ V_X : $g_{m1}V_1 = g_{m1}V_{in} = (g_{m2} + sC_{gs2})(V_2 - V_X)$

$$= \frac{-(g_{m2} + sC_{gs2})}{1 + sR_G C_{gs2}} V_X \quad \text{— ③}$$

from ②, ③, $\frac{V_{out}}{V_{in}} = \frac{-g_{m1}g_{m2}R_D}{g_{m2} + sC_{gs2}}$ ※

6.15



For zero frequency, $I_L = 0$ & $I_4 = I_x$

$I_4 = -g_{mp} V_E$

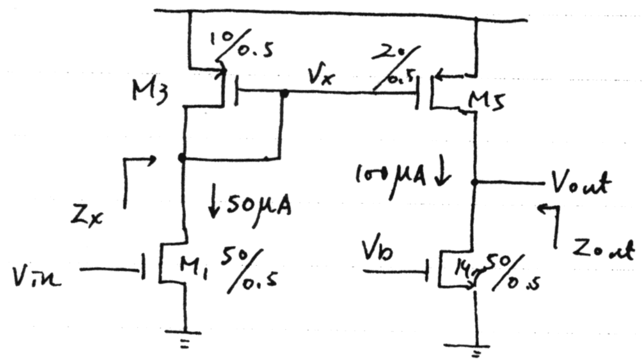
$I_x = V_E (g_{mp} + sC_E)$

$\therefore \quad I_4 = I_x \Rightarrow \quad -g_{mp} = g_{mp} + sC_E$

$$S_z = \frac{-2g_{mp}}{C_E}$$

☆

6.18 Half circuit can be drawn as follows



Since $R_s = 0$, According to (6.20) & (6.76)

$$\frac{V_{out}}{V_{in}}(s) = -\frac{(C_{gd1}\cdot s - g_{m1})\cdot \frac{1}{g_{m3}}}{\frac{1}{g_{m3}}(C_{gd1}+C_X)s+1}\cdot g_{m5}\cdot(r_{o5}//r_{o7})\cdot\frac{1}{1+(r_{o5}//r_{o7})\cdot C_L\cdot s}$$

where $C_L = C_{DB7} + C_{gd7} + C_{DB5}$

$C_X = C_{gs3} + C_{DB1} + C_{gs5} + C_{DB3} + C_{gd5}(1+g_{m5}(r_{o5}//r_{o7}))$

$Z_{out} = (r_{o5}//r_{o7})$

$Z_X = \frac{1}{g_{m3}}$

First of all, let's calculate V_x operating point

$$I_{d3} = 50\times10^{-6} = \frac{1}{2}\times100\times3.835\times10^{-7}\times\frac{10}{0.5-0.09\times2}(3-V_X-0.8)^2(1+0.2(3-V_X))$$

$$V_X \doteqdot 1.94V$$

For V_{in} operating point

$$I_{d1} = 50\times10^{-6} = \frac{1}{2}\times350\times3.835\times10^{-7}\times\frac{50}{0.5-0.08\times2}(V_{gs1}-0.7)^2(1+0.1\cdot1.94)$$

$$V_{gs1} \triangleq 0.765V$$

$$\Rightarrow g_{m1} = \frac{2I_{d1}}{(V_{gs1}-V_t)} \cong 1.54\times10^{-3}$$

$$g_{m3} = \frac{2I_{d1}}{(3-1.94-0.8)} \cong 3.73\times10^{-4} \Rightarrow g_{m5} = 2\cdot g_{m3} = 7.46\times10^{-4}$$

$$g_{ds5} = 2\times 50\times 10^{-6}\cdot \lambda \Big/ (1+\lambda\cdot 1.06) = 10^{-4}\cdot 0.2 \Big/ 1.212 \cong 1.649\times 10^{-5}$$

$$g_{ds7} = 10^{-4}\times 0.1 \Big/ (1+0.196) \cong 8.36\times 10^{-6}$$

$$r_{o5} // r_{o7} = 40290$$

$$C_L = C_{DB5} + C_{DB7} + C_{gd7} = \left[\frac{0.94\times 10^{-3}\times 30\times 10^{-12}}{\left(1+\frac{1.06}{0.9}\right)^{0.5}} + \frac{0.32\times 10^{-11}\times 43\times 10^{-6}}{\left(1+\frac{1.06}{0.9}\right)^{0.3}}\right]$$

$$+ \left[\frac{0.56\times 10^{-3}\times 75\times 10^{-12}}{\left(1+\frac{1.94}{0.9}\right)^{0.6}} + \frac{0.35\times 10^{-11}\times 103\times 10^{-6}}{\left(1+\frac{1.94}{0.9}\right)^{0.2}}\right] + 50\times 0.4\times 10^{-17}$$

$$= 19.22\times 10^{-15} + 21.36\times 10^{-15} + 2\times 10^{-16} = 40.78\times 10^{-15}$$

$$C_{gs3} = \frac{2}{3}\times 3.835\times 10^{-7}\times 10\times 0.32 + 3.835\times 10^{-7}\times 10\times 0.09 = 11.633\times 10^{-15}$$

$$C_{dB1} \cong C_{dB7} \cong 21.36\times 10^{-15}$$

$$C_{gs5} = 2\cdot C_{gs3} = 23.266\times 10^{-15}$$

$$C_{dB3} = \frac{1}{2}\cdot C_{DB5} = 9.61\times 10^{-15}$$

$$C_{gd5} = 6.3\times 10^{-11}\times 20\times 10^{-6} = 6\times 10^{-17}$$

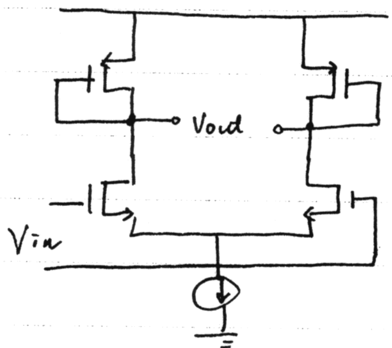
$$C_x = \left[11.633 + 21.36 + 23.27 + 9.61 + 0.06\left(1 + g_{m5}\cdot(r_{o5}//r_{o7})\right)\right]\times 10^{-15} = 67.732\times 10^{-15}$$

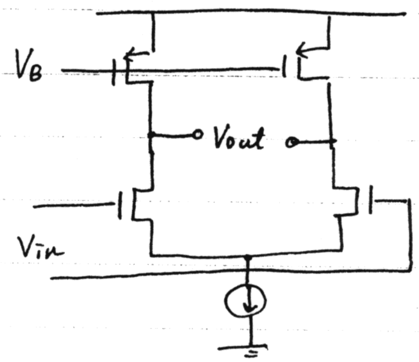
$$\therefore\ \omega_z = \frac{g_{m1}}{C_{gd1}} = 7.7\times 10^{12}\ \text{rad/sec}$$

$$\omega_{p1} = -\frac{1}{C_L\cdot(r_{o5}//r_{o7})} = -\frac{1}{40290\times 40.78\times 10^{-15}} = 6.08\times 10^{8}\ \text{rad/sec}$$

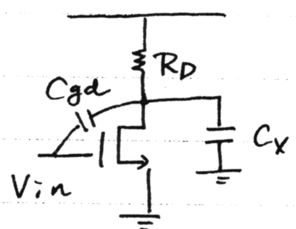
$$\omega_{p2} = -\frac{g_{m3}}{(C_{gd1}+C_x)} = \frac{-3.73\times 10^{-4}}{2\times 10^{-16} + 67.732\times 10^{-15}} = 5.5\times 10^{9}\ \text{rad/sec}$$

6.17 (a)





Both of these two differential pair can be simplified as common-source amplifier with different load resistance and capacitance



Since $R_S = 0$, equation (6.20) can be simplified as

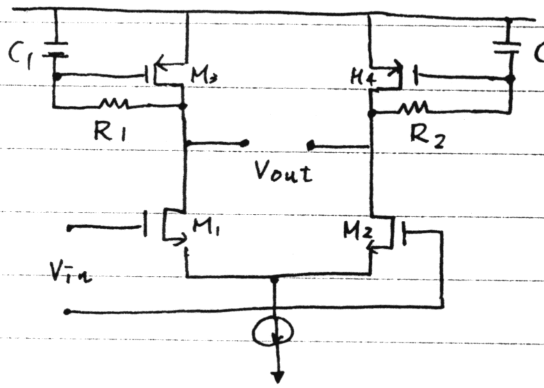
$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{gd}s - g_m)R_D}{s[R_D(C_{gd}+C_X)]+1}$$

where $\begin{cases} R_D = \frac{1}{g_{mp}} \text{ for diode connected load} \\ C_X = C_{dBN} + C_{dBp} + C_{gsp} \end{cases}$

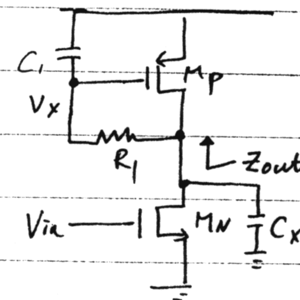
$\begin{cases} R_D \cong (r_{ON} // r_{op}) \text{ for current mirror load} \\ C_X = C_{dBN} + C_{dBp} + C_{gdp} \end{cases}$

Although there is right-half-plane zero, however this zero is much larger than the dominant pole.
Therefore, the maximum phase shift it can achieve is $\sim 90°$ before the gain is down to unity.

(b)



half-circuit →



(i) At low frequency. C_1 is open circuit,

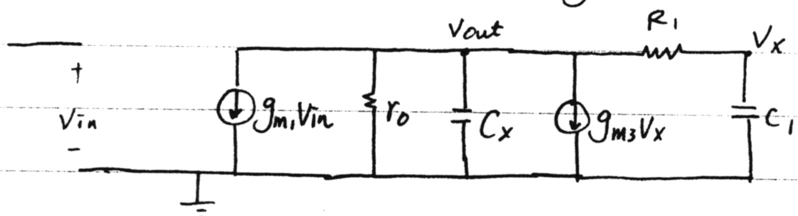
M_p is like a diode-connected device $\rightarrow Z_{out} \sim \frac{1}{g_{mp}}$

(ii) At high frequency, C_1 is short circuit.

M_p is like a current source device $\rightarrow Z_{out} \sim (r_{oN} // r_{op})$

Since $(r_{oN} // r_{op}) >> \frac{1}{g_{mp}}$, Z_{out} exhibits an inductive behavior

For transfer function, small-signal model



, $r_o = (r_{o1} // r_{o3})$

KCL @ V_X : $\frac{V_{out} - V_X}{R_1} = sC_1 V_X \Rightarrow V_{out} = (1 + sR_1C_1)V_X$

$$\Rightarrow V_X = \frac{V_{out}}{1 + sR_1C_1}$$

KCL @ Vout : $-g_{m1}V_{in} = V_{out}\left(\frac{1}{r_o} + sC_X\right) + \frac{1}{R_1}(V_{out} - V_X) + g_{m3}V_X$

$$= V_{out}\left(\frac{1}{r_o} + sC_X + \frac{1}{R_1}\right) + \left(g_{m3} - \frac{1}{R_1}\right) \cdot \frac{V_{out}}{1 + sR_1C_1}$$

$$-g_{m1}V_{in} = V_{out}\left(\frac{\frac{1}{r_o}+\frac{1}{R_1}+sC_x+(\frac{R_1}{r_o}+1)sC_1+s^2R_1C_1C_x+g_{m3}-\frac{1}{R_1}}{1+sR_1C_1}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}(1+sR_1C_1)}{s^2R_1C_1C_x+s(C_1+\frac{R_1C_1}{r_o}+C_x)+(g_{m3}+\frac{1}{r_o})}$$

From the above transfer function,

$$\omega_z = -\frac{1}{R_1C_1}$$

the sum of two poles $= -\frac{1}{R_1C_1C_x}(C_1+\frac{R_1C_1}{r_o}+C_x) = -\frac{1}{R_1C_1}(1+\frac{C_1}{C_x}(1+\frac{R_1}{r_o}))$

usually $C_1 > C_x$, C_1 at least $= C_{gs3}$

Thus, the sum of two poles $> -\frac{2}{R_1C_1}$, which means that at least one of the poles are larger than zero $\Rightarrow$ It's quite impossible to produce $135°$ phase shift

Thus, this circuit still can't produce $135°$ phase shift.

However, it's more likely for it to generate $90°$ phase shift @ unity-gain frequency.

8.1

$$V_{in} = I_{in}(Z_{in} + G_{22}) + G_{21} V_{out}$$

$$\frac{V_{out} - A_o I_{in} Z_{in}}{Z_{out}} = -G_{11} V_{out} - G_{12} I_{in}$$

$$\Rightarrow \quad I_{in}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) = V_{out}\left(\frac{1}{Z_{out}} + G_{11}\right)$$

$$V_{in} = V_{out}\left[G_{21} + \frac{(Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}{\frac{A_o Z_{in}}{Z_{out}} - G_{12}}\right]$$

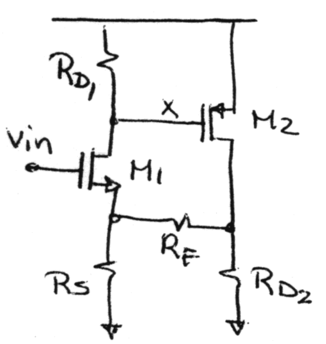
$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_o Z_{in} - G_{12} Z_{out}}{Z_{out}}}{G_{21}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) + (Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}$$

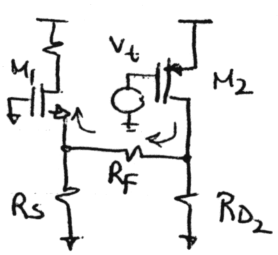
$$A_{v\,open\,loop} = \frac{1}{Z_{out}}(A_o Z_{in} - G_{12} Z_{out}) \frac{1}{Z_{in} + G_{22}} \cdot \frac{1}{\frac{1}{Z_{out}} + G_{11}}$$

$$= \frac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} \cdot \frac{1}{Z_{in} + G_{22}} (A_o Z_{in} - G_{12} Z_{out})$$

if $G_{12} \ll A_o Z_{in}/Z_{out}$ then the second term can be neglected

8.2





The current through R_S:

$$-\frac{g_{m2} \cdot V_t R_{D2}}{R_{D2} + R_F + (R_S \| \frac{1}{g_{m2}})}$$

The current through M_1:

$$-\frac{g_{m2} V_t R_{D2}}{R_{D2} + R_F + (R_S \| \frac{1}{g_{m2}})} \cdot \frac{R_S}{R_S + \frac{1}{g_{m2}}}$$

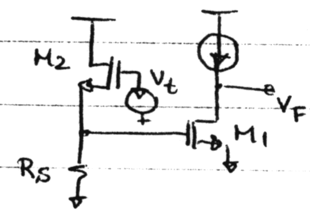
This current is multiplied by R_{D1} to produce V_F

loop gain: $\dfrac{g_{m2} R_{D2} R_S R_{D1}}{(R_{D2}+R_F)(R_S+\frac{1}{g_{m2}}) + R_S \cdot \frac{1}{g_{m2}}}$

This result is accurate, whereas $G_{21} Av_{open}$ is approximate because it neglects the signal propagating thru the feedback network from the input to the output.

8.3

voltage-current



loop gain: $\dfrac{R_S}{R_S + \frac{1}{g_{m2}}} \cdot g_{m1} r_{o1}$

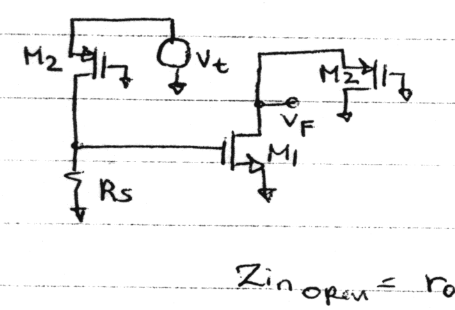
$\dfrac{V_{in}}{R_S} \times (R_S \| \frac{1}{g_{m2}}) g_{m1} r_{o1} = V$

$\Rightarrow \left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = g_{m1} r_{o1} (R_S \| \frac{1}{g_{m2}})$ $\qquad Zin_{open} = \dfrac{1}{g_{m2}}$

$Zout_{open} = r_{o1}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{closed} \underset{r_{o1}\to\infty}{=} \dfrac{1}{g_{m2}} \Rightarrow Av_{closed} = \dfrac{1}{g_{m2} R_S}$

$Zin_{closed} \underset{r_{o1}\to\infty}{=} 0$ $\qquad Zout_{closed} = \dfrac{R_S + \frac{1}{g_{m2}}}{g_{m1} R_S} = \dfrac{1}{g_{m1}} + \dfrac{1}{g_{m1} g_{m2} R_S}$

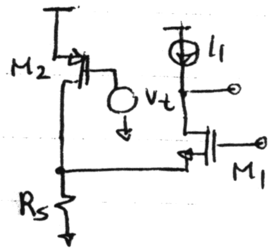


$\dfrac{V_F}{V_t} = g_{m2} R_S \dfrac{g_{m1}}{g_{m2}} = g_{m1} R_S$

$Zin_{open} = r_{o2}$ $\qquad Zout_{open} = \dfrac{1}{g_{m2}}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = R_S \cdot \dfrac{g_{m1}}{g_{m2}}$

$Av_{closed} = \dfrac{g_{m1}/g_{m2}}{1 + g_{m1} R_S}$ $\qquad Zin_{closed} = \infty$ $\qquad Zout_{closed} = \dfrac{1}{g_{m2}(1+g_{m1} R_S)}$



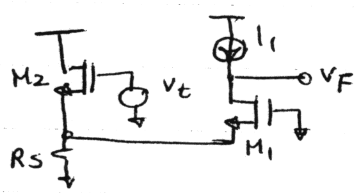
$$R_S\, g_{m2} V_t \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}} = V_F \Rightarrow \text{loop gain: } \frac{R_S\, g_{m2}\, r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$$R_{out_{open}} = g_{m1} r_{o1} R_S + r_{o1} + R_S \approx g_{m1} r_{o1} R_S$$

$$R_{in_{open}} = \frac{1}{g_{m1}}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{open} = \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$r_{o1} \to \infty$ $\quad A_v = \dfrac{1}{R_S\, g_{m2}}$ $\quad R_{in} = 0$ $\quad R_{out} = \dfrac{g_{m1}\left(R_S + \frac{1}{g_{m1}}\right)}{g_{m2}}$



$$\text{loop gain: } V_t \cdot \frac{R_S \,\|\, \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + R_S \,\|\, \frac{1}{g_{m1}}} \cdot g_{m1} r_{o1} = V_F$$

$$R_{in_{open}} = \frac{1}{g_{m1} + g_{m2}} \qquad R_{out_{open}} = g_{m1} r_{o1} \left(R_S \,\|\, \frac{1}{g_{m2}}\right)$$

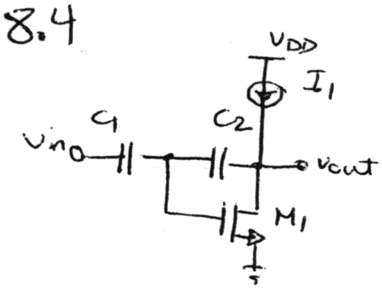
$$A_{v_{open}} \Rightarrow \frac{V_{in}}{R_S}\left(R_S \,\|\, \frac{1}{g_{m2}}\right) \times g_{m1} r_{o1} = V_{out} \Rightarrow A_{v_{open}} = \frac{g_{m1} r_{o1} \left(R_S \,\|\, \frac{1}{g_{m2}}\right)}{R_S}$$

$r_o \to \infty$

$$A_{v_{closed}} = \frac{\left(R_S \,\|\, \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \,\|\, \frac{1}{g_{m1}}\right)}{R_S \left(R_S \,\|\, \frac{1}{g_{m1}}\right)}$$

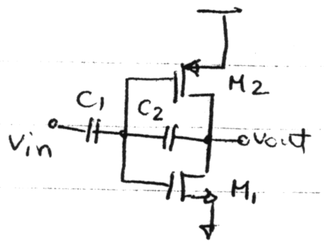
$$R_{in_{closed}} = 0 \qquad R_{out_{closed}} = \frac{\left(R_S \,\|\, \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \,\|\, \frac{1}{g_{m1}}\right)}{R_S \,\|\, \frac{1}{g_{m1}}}$$

8.4



$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_m}$$

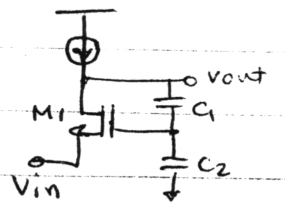
$$R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1 + C_2} g_{m1} r_o} = \frac{C_1 + C_2}{g_{m1} C_2}$$



using the results in part (a)

$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out} = \frac{C_1 + C_2}{(g_{m1} + g_{m2}) C_2}$$



$$\text{loop gain} = g_{m1} r_o \frac{C_1}{C_1 + C_2}$$

$$R_{in\,closed} = \frac{1}{g_{m1}} + \frac{1}{C_2 s}$$

$$R_{out\,closed} = \frac{r_o}{1 + g_{m1} r_o \frac{C_1}{C_1 + C_2}} = \frac{C_1 + C_2}{g_{m1} C_1}$$

8.5

$$-\frac{1}{\left(1+\frac{1}{g_{m1}r_{o1}}\right)\frac{C_2}{C_1}+\frac{1}{g_{m1}r_{o1}}} = -0.95\,\frac{C_1}{C_2} \xRightarrow{g_{m1}r_{o1}=50} \frac{C_1}{C_2}=1.63$$

Open loop output impedance: r_o

loop gain: $\frac{C_2}{C_1+C_2}\,g_m r_o$

closed loop $R_{out} = \frac{r_o}{1+\frac{C_2}{C_1+C_2}g_m r_o} = 0.49\,r_o$

8.6

$$R_{in\,closed} = \frac{1}{g_{m1}}\cdot\frac{1}{1+g_{m2}R_D\frac{C_1}{C_1+C_2}} \qquad I_1 = I_2 \Rightarrow g_{m2}=\sqrt{2}\,g_{m1}$$

$$= \frac{1}{g_{m1}}\cdot\frac{1}{1+1000\sqrt{2}\,g_{m1}} = 50 \Rightarrow g_{m1} = 3.42\ m\mho$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \Rightarrow I_D = \frac{(3.42\times 10^{-3})^2}{2\times 1.342\times 10^{-4}\times 100} = 435\,\mu A$$

8.7

$$\frac{V_x}{I_x} = \frac{R_D}{1+\frac{g_{m2}R_S(g_{m1}+g_{mb1})R_D}{(g_{m1}+g_{mb1})R_S+1}\cdot\frac{C_1}{C_1+C_2}} \overset{R_D\to\infty}{=} \frac{(g_{m1}+g_{mb1})R_S+1}{g_{m2}R_S(g_{m1}+g_{mb1})}\cdot\frac{C_1+C_2}{C_1}$$

if $(g_{m1}+g_{mb1})R_S \gg 1 \Rightarrow \frac{V_x}{I_x} = \frac{1}{g_{m2}}\cdot\frac{C_1+C_2}{C_1}$

8.8

If f_{-3dB} of each stage is ω_o

$$\left|\frac{1}{(1+\frac{s}{\omega_o})^n}\right| = \frac{1}{\sqrt{2}} \Rightarrow \left(1+\left(\frac{\omega}{\omega_o}\right)^2\right)^n = 2$$

if we indicate the Gain x f_{-3dB} as K = const

$$\Rightarrow \left(\frac{K}{\omega_o}\right)^n = 500$$

$$\Rightarrow \frac{\ln 2}{\ln\left(1+\left(\frac{\omega}{\omega_o}\right)^2\right)} = \frac{\ln 500}{\ln\left(\frac{K}{\omega_o}\right)} \Rightarrow 1+\left(\frac{\omega}{\omega_o}\right)^2 = \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}}$$

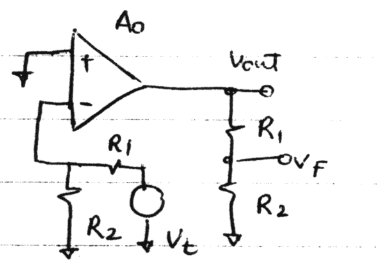
$$\Rightarrow \omega = \omega_o \sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1}$$

$$\frac{d\omega}{d\omega_o} = 0 \Rightarrow \sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1} + \frac{\omega_o}{2} \cdot \frac{1}{\sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1}} \cdot \left(-\frac{\ln 2}{\ln 500} \cdot \frac{1}{\omega_o} \cdot \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}}\right.$$

$$\Rightarrow \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1 - \frac{1}{2}\frac{\ln 2}{\ln 500}\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} = 0 \Rightarrow \frac{K}{\omega_o} = 1.67$$

$\Rightarrow$ Gain per stage = 1.67   Stage BW = 598 MHz

8.9



$$Av_{open} = A_o \frac{R_2}{R_1+R_2+R_o} \qquad R_{out_{open}} = R_o \| (R_1+R_2)$$

Loop gain=

$$\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)$$

$$Av_{closed} = \frac{A_o \frac{R_2}{R_o+R_1+R_2}}{1+\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)}$$

$$R_{out_{closed}} = \frac{R_o \| (R_1+R_2)}{1+\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)}$$

8.10

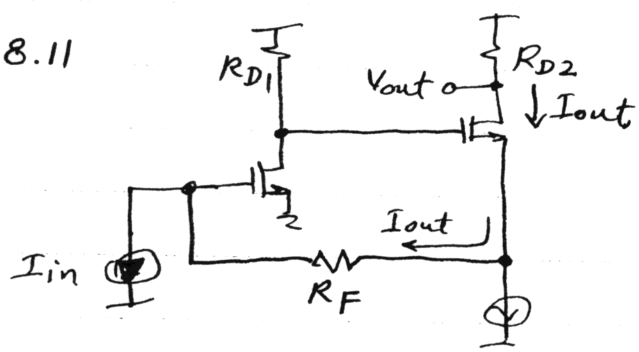
$$\frac{1+\frac{C_2}{C_1}}{\left(1+\frac{C_2}{C_1}\right)\frac{1}{g_{m1}(r_{o2}\|r_{o4})}+1} = 0.95\left(1+\frac{C_2}{C_1}\right)$$

↑ 5% gain error

$$g_{m1}(r_{o2}\|r_{o4}) \approx 24.4$$

$$\Rightarrow \quad 1+\frac{C_2}{C_1} \leq 1.28$$

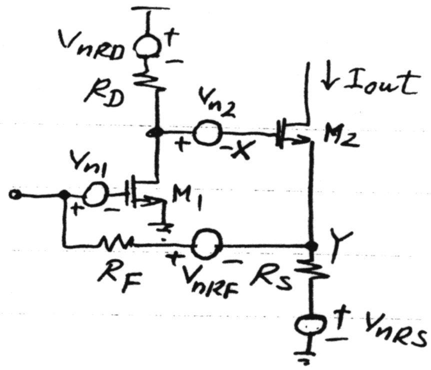
8.11



I_{out} fully flows through R_F $\therefore \Rightarrow$ $I_{out} = I_{in}$ and $V_{out} = I_{out} \cdot R_{D2}$.

Thus, the transimpedance is equal to R_{D2}.

(continued on next page)



8.11 (cn'td)

$$\underbrace{-(I_{out}R_S + V_{nRS}}_{V_Y} + V_{nRF} + V_{n1})g_{m1}R_D + V_{nRD} + V_{n2} = V_X$$

$$(V_X - V_Y)g_{m2} = I_{out}$$

$$\Rightarrow g_{m2}\left[(I_{out}R_S + V_{nRS} + V_{nRF} + V_{n1})(-g_{m1}R_D) + V_{nRD} + V_{n2} - (I_{out}R_S + V_{nRS})\right] = I_{out}$$

$$\Rightarrow I_{out}\left[1 + g_{m2}R_S(g_{m1}R_D + 1)\right] = g_{m2}\left[(-g_{m1}R_D - 1)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}\right]$$

$$\Rightarrow I_{out} = \frac{g_{m2}\left[-(1+g_{m1}R_D)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}\right]}{1 + g_{m2}R_S(1+g_{m1}R_D)}$$

If we apply a current of I_{in} to the input, the resulting output current is obtained as:

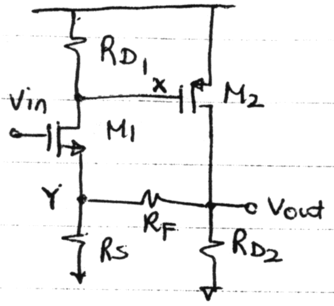
$$\left\{\underbrace{\left[(I_{in}+I_{out})R_S + I_{in}R_F\right](-g_{m1}R_D)}_{V_X} - \underbrace{(I_{in}+I_{out})R_S}_{V_Y}\right\}g_{m2} = I_{out}$$

$$\frac{I_{out}}{I_{in}} = \frac{\left[-g_{m1}R_D(R_S+R_F) - R_S\right]g_{m2}}{1 + g_{m2}R_S(1+g_{m1}R_D)}$$

Dividing the output noise current by the gain yields the input-referred noise current:

$$I_{n,in} = \frac{-(1+g_{m1}R_D)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}}{-g_{m1}R_D(R_S+R_F) - R_S}$$

8.12



$I_{D_2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D_1} = 761\,\mu A$

$$\begin{cases} \dfrac{V_Y}{2K} + \dfrac{V_Y - V_{out}}{2K} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \dfrac{V_{out}}{2K} + \dfrac{V_{out} - V_Y}{2K} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$$

$I_{D_1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342\times10^{-4} \times 100\,(V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$$\begin{cases} g_{m_1} = \dfrac{2\times0.761m}{1.037-0.7} = 4.52\,m\mho \\ g_{m_2} = \dfrac{2\times1\,mA}{1.522-0.8} = 2.77\,m\mho \end{cases}$$

$A_{V_{open}} = \dfrac{-2K}{1K + \dfrac{1}{4.52\,m\mho}}\left[-2.77\,m\mho\,[2K \,\|\, 4K]\right] = 6.04$

$A_{V_{closed}} = \dfrac{6.048}{1 + 3.024} = 1.503$

8.13

problem 8.9

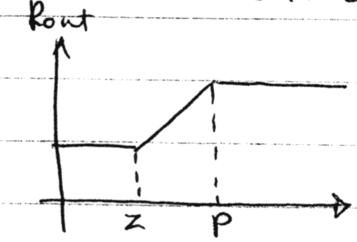
$$R_{out} = \frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + \dfrac{R_2^2}{R_1+R_2} \cdot \dfrac{A_o}{1+\dfrac{s}{\omega_o}}}$$

Zero: ω_o

pole: $\omega_o + \dfrac{A_o R_2^2}{(R_1+R_2)(R_o+R_1+R_2)}\,\omega_o$

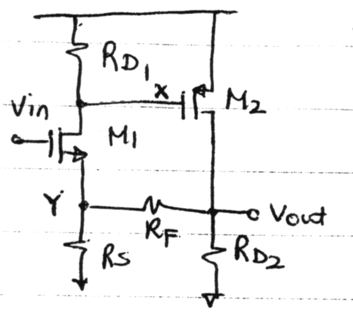
DC value: $\dfrac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + A_o R_2^2/R_1+R_2}$

final value: $R_o \| (R_1+R_2)$



The output impedance is less reduced, as the loop gain gets smaller.

8.12



$I_{D_2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}(\frac{W}{L})_2(V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D_1} = 761\,\mu A$

$\begin{cases} \frac{V_Y}{2K} + \frac{V_Y - V_{out}}{2K} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \frac{V_{out}}{2K} + \frac{V_{out} - V_Y}{2K} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$

$I_{D_1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342 \times 10^{-4} \times 100\,(V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$\begin{cases} g_{m_1} = \frac{2 \times 0.761m}{1.037 - 0.7} = 4.52\,m℧ \\ g_{m_2} = \frac{2 \times 1\,mA}{1.522 - 0.8} = 2.77\,m℧ \end{cases}$

$A_{v_{open}} = \frac{-2^K}{1^K + \frac{1}{4.52\,m℧}}\left[-2.77^{m℧}\left[2^K \| 4^K\right]\right] = 6.048$

$A_{v_{closed}} = \frac{6.048}{1 + 3.024} = 1.503$

8.13

problem 8.9

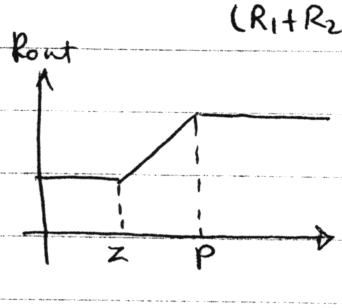
$$R_{out} = \frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + \frac{R_2^{\,2}}{R_1+R_2} \cdot \frac{A_o}{1 + \frac{s}{\omega_o}}}$$

Zero: ω_o

pole: $\omega_o + \frac{A_o R_2^2}{(R_1+R_2)(R_o+R_1+R_2)}\,\omega_o$

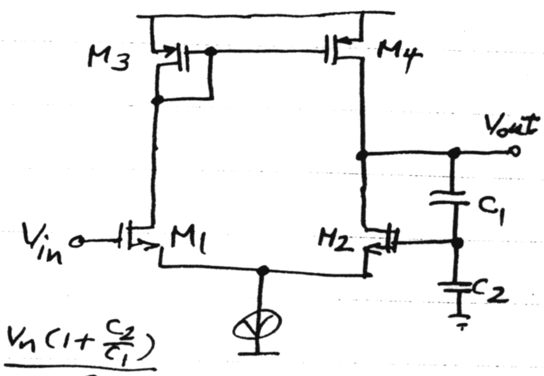
DC value: $\frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + A_o R_2^2/R_1+R_2}$

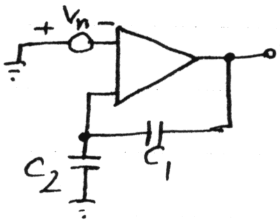
final value: $R_o \| (R_1+R_2)$



The output impedance is less reduced, as the loop gain gets smaller.

8.14 The input-referred noise voltage of the circuit is the same as that of the open-loop circuit:





$V_{out} = V_n\left(1+\frac{C_2}{C_1}\right)$

$\Rightarrow$ input-referred noise $= \dfrac{V_n\left(1+\frac{C_2}{C_1}\right)}{1+\frac{C_2}{C_1}}$

$= V_n$

The noise produced by $M_1 - M_4$ referred to the input is:

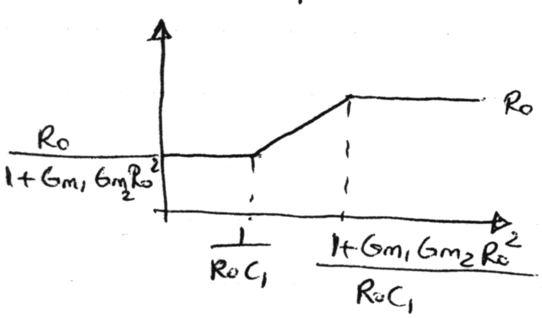
$$\overline{V_n^2} = 4kT\left(\frac{2}{3g_{m1,2}} + 2\,\frac{2\,g_{m3,4}}{3\,g_{m1,2}^2}\right) + 2\,\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\,\frac{K_P}{(WL)_{3,4}C_{ox}f} \times \frac{g_{m3,4}^2}{g_{m1,2}^2}$$

8.15

a) $Z_{in\,open} = R_o$

$$Z_{in\,closed} = \frac{R_o}{1 + G_{m1}G_{m2}R_o\,\frac{R_o}{1+R_oC_1s}} = \frac{R_o(1+R_oC_1s)}{R_oC_1s + 1 + G_{m1}G_{m2}R_o^2}$$

Zero: $\dfrac{1}{R_oC_1}$ Pole: $\dfrac{1+G_{m1}G_{m2}R_o^2}{R_oC_1}$ DC: $\dfrac{R_o}{1+G_{m1}G_{m2}R_o^2}$ final: R_o



b) Heavy feedback at lower frequency. As frequency increases, feedback weakens since the output impedance of the feedforward amplifier reduces

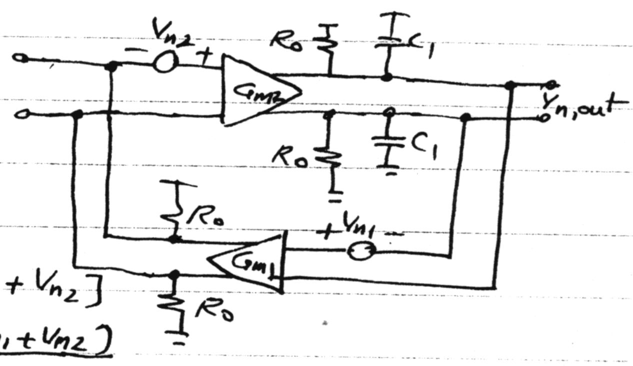
8.15 (c) For input-referred noise voltage, we short the input, hence $\overline{V_{n,out}^2} = 4kT\times 2\left(\frac{2}{3}g_m + \frac{1}{R_0}\right)\left(\left|R_0 \| \frac{1}{C_1 s}\right|\right)$.

Dividing this by the voltage gain, $g_m^2\left(\left|R_0 \| \frac{1}{C_1 s}\right|\right)^2$, we have

$$\overline{V_{n,in}^2} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_0}\right).$$

For the input noise current, we leave the input open.

Here, V_{n1} and V_{n2} represent the input noise of each differential pair (including the noise of resistors).



$$-V_{n,out} = G_{m2}\left(R_0 \| \frac{1}{C_1 s}\right)\left[(V_{n,out} + V_{n1})G_{m1}R_0 + V_{n2}\right]$$

$$\Rightarrow V_{n,out} = -\frac{G_{m2}\left(R_0 \| \frac{1}{C_1 s}\right)(G_{m1}R_0V_{n1} + V_{n2})}{1 + G_{m2}\left(R_0 \| \frac{1}{C_1 s}\right)G_{m1}R_0}$$

If we apply current between the two input terminals with value I_{in}, the output voltage is obtained as:

$$-V_{out} = (V_{out}G_{m1} + I_{in})R_0 \cdot G_{m2}\left(R_0 \| \frac{1}{C_1 s}\right)$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = -\frac{G_{m2}R_0\left(R_0 \| \frac{1}{C_1 s}\right)}{1 + G_{m1}G_{m2}R_0\left(R_0 \| \frac{1}{C_1 s}\right)}$$

Dividning $V_{n,out}$ by this gain gives the input-referred noise current:

$$I_{n,in} = \frac{G_{m1}R_0V_{n1} + V_{n2}}{R_0} \Rightarrow \overline{I_{n,in}^2} = G_{m1}^2\overline{V_{n1}^2} + \frac{\overline{V_{n2}^2}}{R_0^2}$$

8.16

a) Due to symmetry of the π network, no current flows through R_F.

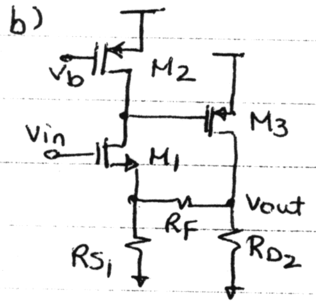
$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - R_{S1} \cdot I_{D1} - V_{Tn})^2 = 0.5\text{ mA}$

$(V_{in} - 2.2)^2 = \frac{2 \times 0.5 \times 10^{-3}}{1.342 \times 10^{-4} \times 100} \Rightarrow V_{in} = 2.473\text{ V}$

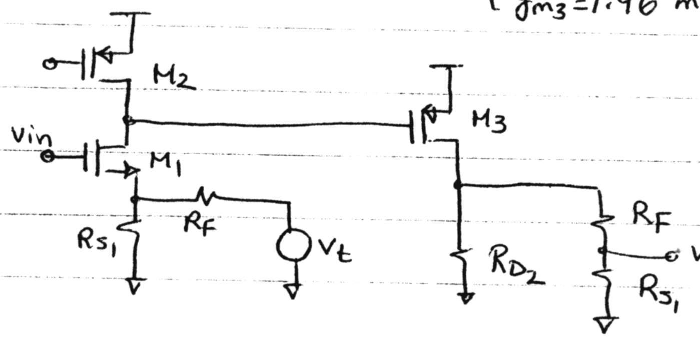
$g_{m1} = 3.66\text{ m}\mho$, $g_{m2} = 1.96\text{ m}\mho$, $g_{m3} = 1.96\text{ m}\mho$

$r_{o1} = 20K$, $r_{o2} = 10K$, $r_{o3} = 10K$

b)



$\Rightarrow$



$$Av_{open} = \frac{r_{o2}}{R_{S1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right] = 18.42$$

$$Rout_{open} = r_{o3} \| R_{D2} \| (R_F + R_{S1}) = 1.667\text{ K}\Omega$$

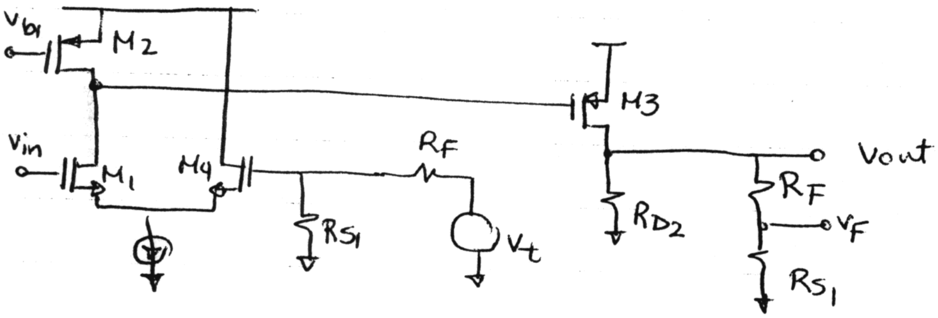
Loop gain: $\frac{V_t}{R_F} \times (R_F \| R_{S1}) \times \frac{r_{o2}}{R_{S1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[r_{o3} \| R_{D2} \| (R_F + R_{S1})\right] \times \frac{R_{S1}}{R_F + R_{S1}}$

$= V_F \Rightarrow$ loop gain $= 4.605$

$$A_v = \frac{18.42}{1 + 4.605} = 3.286$$

$$Rout = \frac{1.667^k}{1 + 4.605} = 297\ \Omega$$

8.17



$r_{o1} = r_{o4} = 20K$

$r_{o3} = 10K \quad r_{o2} = 10^K$

$g_{m1} = g_{m4} = 3.66\ m℧$

$g_{m2} = g_{m3} = 1.96\ m℧$

$$Av_{open} = \frac{1}{2} g_{m1} (r_{o1} \| r_{o2})\ g_{m3} (r_{o3} \| R_{D2} \| (R_F + R_{S1})) = 39.85$$

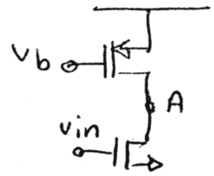
$$R_{out} = r_{o3} \| R_{D2} \| (R_F + R_{S1}) = 1.667\ k$$

$$V_t \quad \frac{R_{S1}}{R_{S1}+R_F} \times \frac{g_{m1}}{2} \times (r_{o1} \| r_{o2})\ g_{m3} (r_{o3} \| R_{D2} \| (R_F + R_{S1}))\ \frac{R_{S1}}{R_{S1}+R_F} = V_F$$

loop gain = 9.96

$$A_v = \frac{39.85}{1+9.96} = 3.63 \qquad R_{out} = \frac{1.667^K}{1+9.96} = 153\Omega$$

Smaller output impedance compared to 8.16.



8.18

a) $I_{D_n} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS_n} - V_{THn})^2 \Rightarrow V_{GS_n} = 0.973V$

$V_{GSp} = 1.311 \Rightarrow 3 - V_b = 1.311 \Rightarrow V_b = 1.69$

$V_{in} = R_1 \times I + V_{GS_n}$

M_3: saturation $\Rightarrow -V_A + V_b > |V_{THp}| \Rightarrow V_A < 1.689 - 0.8 = 0.889$

M_4: saturation $\Rightarrow -V_{out} + V_b > V_{THP} \Rightarrow V_{out} < 0.889 \Rightarrow R_1 I < 0.889 \Rightarrow R_1 \leq 177$

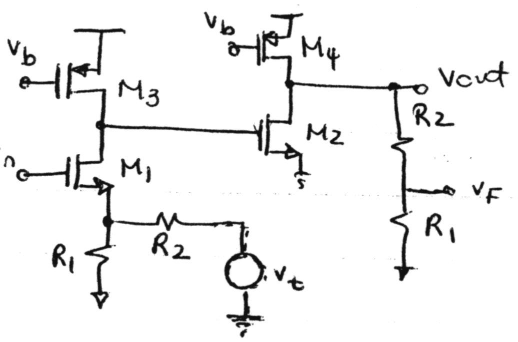
M_1: saturation $\Rightarrow V_A > R_1 I + (V_{GS_1} - V_{THn}) \Rightarrow 0.273 + R \times 0.5m < 0.889$

$\Rightarrow R_1 < 1232$

M_2: saturation: $V_{out} = R_1 I > V_A - V_{tn} \Rightarrow R_1 > \frac{0.889 - 0.7}{0.5\times 10^{-3}} = 378\,\Omega$

$378 \leq R_1 \leq 1232 \Rightarrow 1.162 \leq V_{in} \leq 1.589$

b)



$R_1 = 805\,\Omega$ $\quad r_{o1} = r_{o2} = 20K$

$g_{m1} = 3.66\,m℧$ $\quad r_{o3} = r_{o4} = 10K$

$g_{m2} = 3.66\,m℧$

open loop gain $= \frac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}(r_{o4} \| (R_1 + R_2) \| r_{o2}) = 97.6$

Output impedance $= r_{o4} \| (R_1 + R_2) \| r_{o2} = 2422\,\Omega$

loop gain: $\frac{V_t}{R_2} \times (R_1 \| R_2) \times \frac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}(r_{o4} \| (R_1 + R_2) \| r_{o2}) \frac{R_1}{R_1 + R_2} = V_F$

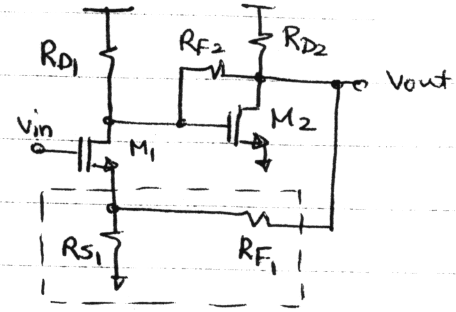
$\Rightarrow$ loop gain $= \frac{1}{3000} \times 635 \times 97.6 \times \frac{805}{3805} = 4.37$

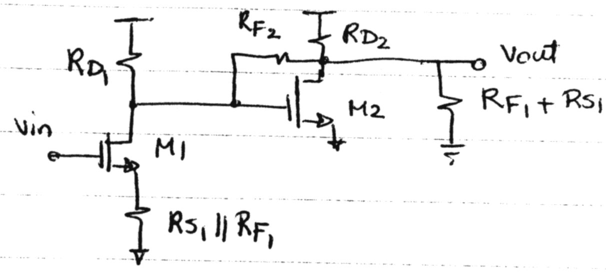
$A_v = \frac{97.6}{1 + 4.37} = 18.17$

$R_{out} = \frac{2422}{1 + 4.37} = 451\,\Omega$

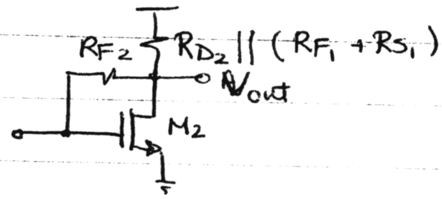
8.19

voltage - voltage





next we consider



$$R_{in_2} = \frac{R_{F2}}{1 + g_{m2}\left[R_{D2} \parallel (R_{F_1} + R_{S_1})\right]} = 261\,\Omega$$

$$R_{out_2} = \frac{R_{D2} \parallel (R_{F_1} + R_{S_1})}{1 + g_{m2}\left[R_{D2} \parallel (R_{F_1} + R_{S_1})\right]} = 174\,\Omega$$

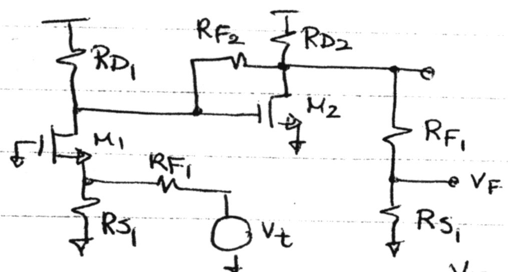
$$A_{V2} = \frac{R_D}{R_D + R_{F_2}}(-g_{m_2}R_{F_2} + 1) = -3.6$$

$$R_D = R_{D2} \parallel (R_{F_1} + R_{S_1}) = 1333$$

$$\text{Open loop gain} = -\frac{R_{D_1} \parallel R_{in2}}{R_{S_1} \parallel R_{F_1} + \frac{1}{g_{m_1}}} \cdot A_{V_2} = \frac{231}{1000 + 200} \times 3.6 = 0.69$$

Open loop output impedance = $R_{out_2} = 174\,\Omega$

loop gain:



$$\frac{V_t}{R_{F_1}} \times (R_{S_1} \parallel R_{F_1}) \times \frac{R_{D_1}}{R_{S_1} \parallel R_{F_1} + \frac{1}{g_{m_1}}} \times A_{V_2} \times \frac{R_{S_1}}{R_{S_1} + R_{F_1}} = V_F$$

$$\frac{V_F}{V_t} = \frac{1}{2000} \times 1000 \times \frac{1000}{1200} \times 3.6 \times \frac{1}{2} = 0.75$$

$$A_{V_{closed}} = \frac{0.69}{1 + 0.75} = 0.394 \qquad R_{out_{closed}} = \frac{174}{1 + 0.75} = 99.5\,\Omega$$

8.20 $\quad I_{D_1} = I_{D_2} \Rightarrow V_{in} = 1.2538 \quad \Rightarrow I_D = 2.316\ mA$

$g_{m_1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{THn}) = 1.342 \times 10^{-4} \times 100 \times (1.2538 - 0.7) = 7.432\ m℧$

$g_{m_2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{THp}) = 3.835 \times 10^{-5} \times 100 (3 - 1.2538 - 0.8) = 3.628\ m℧$

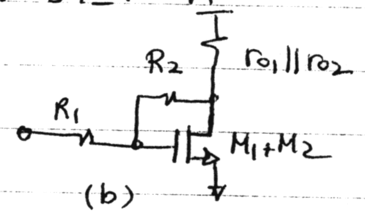
$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1 \times 2.316 \times 10^{-3}} = 4.317\ k$

$r_{o2} = \frac{1}{\lambda_2 I_2} = \frac{1}{0.2 \times 2.316 \times 10^{-3}} = 2.159\ k$

(a)

a) $\quad A_v = -(g_{m_1} + g_{m_2})(r_{o1} \| r_{o2}) = -(7.432 + 3.628)1.439 = 15.91$

$R_{out} = r_{o1} \| r_{o2} = 1439\ \Omega$



b) $\quad A_v = \frac{1}{R_1} \cdot \frac{-(R_1 \| R_2)(g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2})}{1 + (g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1 + R_2}}$

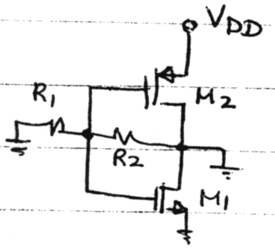
(eq. 8.70)

$= \frac{1}{1} \times \frac{0.909(7.432 + 3.628)1.258}{1 + (7.432 + 3.628)1.258 \times \frac{1}{11}} = 5.58$

$R_{out} = \frac{R_2 \| r_{o1} \| r_{o2}}{1 + (g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1 + R_2}}$

$R_{out} = 1.258^k / 2.26 = 556\ \Omega$

(b) We figure out sensitivity for (b), (a) is a special case where $R_1 = 0 \quad R_2 = \infty$



$G_m = g_{m2}$

R_{out} = same as before

$A_v = \frac{g_{m2}(R_2 \| r_{o1} \| r_{o2})}{1 + (g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1 + R_2}} = \frac{3.628 \times 1.258}{1 + 13.913 \times \frac{1.258}{11}} = 1.76$

if $R_1 = 0$ and $R_2 = \infty$

$A_v = g_{m2}(r_{o1} \| r_{o2}) = 3.628 \times 1.439 = 5.217$

8-21

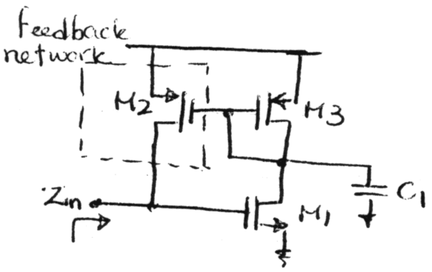
a) $\overline{v_{n\,out}^2} = 4KT\frac{2}{3}(g_{m1}+g_{m2})(r_{o1}||r_{o2})^2$

$\overline{v_{n\,in}^2} = \frac{4KT\frac{2}{3}}{g_{m1}+g_{m2}} =$

b) $\overline{v_{n\,in}^2} = 4KT R_1 + \frac{\left[\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m2})\right]R_o^2}{A_v^2} = 4KT R_1 + \frac{\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m}}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})}$

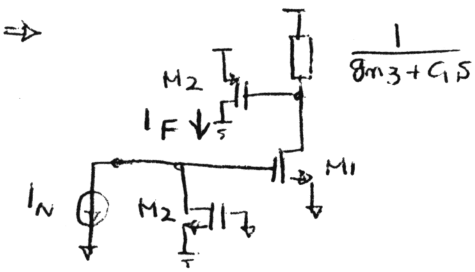
$= 4KT R_1 + \frac{4KT\frac{2}{3}}{g_{m1}+g_{m2}} + \frac{4KT/R_2}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})^2}$

8.22



Voltage - Current

$\Rightarrow$



$$\left|\frac{I_F}{I_N}\right| = r_{o2} \cdot g_{m1} \cdot \frac{1}{g_{m3} + C_1 s} \cdot g_{m2}$$

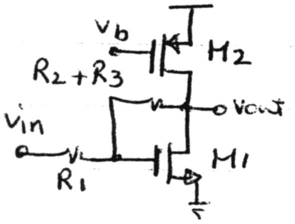
$$Z_{in_{open}} = r_{o2}$$

$$Z_{in_{closed}} = \frac{r_{o2}}{1 - r_{o2}\, g_{m1} g_{m2} \frac{1}{g_{m3}+C_1 s}} \overset{r_{o2}\to\infty}{=} -\frac{g_{m3}+C_1 s}{g_{m1} g_{m2}}$$

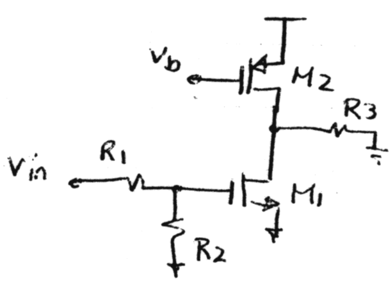
$$= -\frac{g_{m3}}{g_{m1} g_{m2}} - \underbrace{\frac{C_1}{g_{m1} g_{m2}}} \cdot s$$

8.23

Very low freq.



Very high freq.



eq. (8.70)

$$A_v = \frac{1}{R_1} \cdot \frac{-(R_1 \| (R_2+R_3))\, g_{m1} (R_2+R_3)}{1 + g_{m1}(R_2+R_3) \frac{R_1}{R_1+R_2+R_3}}$$

$$= \frac{-1}{2^k} \frac{(2^k \| 4^k) \frac{1}{200} \times 4^k}{1 + \frac{1}{200} \times 4^k \times \frac{1}{3}}$$

$$= -1.739$$

$$A_v = \frac{R_2}{R_1+R_2}(-g_{m1} R_3)$$

$$= \frac{1}{2}\left(-\frac{1}{200} \times 2000\right) = -5$$