Network Analysis & Systems

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UE18EC201: Network Analysis & Systems

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Network Analysis and Synthesis

Unit IV: Two-Ports

Examples (1)

- Example 1, Valkenburg, p. 328.

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- Example 2, Valkenburg, p. 330.

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- Problem 11-1, Valkenburg, p. 342.

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- Example 1, Valkenburg, p. 328.
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- Example 3, Valkenburg, p. 330.
- Problem 11-1, Valkenburg, p. 342.
- Problem 11-3, Valkenburg, p. 342.

Examples (1)

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- Example 2, Valkenburg, p. 330.
- Example 3, Valkenburg, p. 330.
- Problem 11-1, Valkenburg, p. 342.
- Problem 11-3, Valkenburg, p. 342.
- Problem 11-4, Valkenburg, p. 342.

Two-Ports (21)

Transmission Parameters:

$$\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

Two-Ports (21)

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Observe that the four parameters may be defined as follows:

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad \frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$-\frac{1}{B} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad -\frac{1}{D} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

- These are called chain parameters or *ABCD* parameters or *t* parameters or general circuit parameters (power transmission lines).

Two-Ports (22)

- The negative signs are due to a different conventions for the direction of I_2 in power transmission problems.

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$$
\begin{aligned}
V_1 &= AV_2 - BI_2 \\
I_1 &= CV_2 - DI_2
\end{aligned}
$$

- $1/A$ is the open-circuit gain.
- $1/C$ is the open-circuit transfer impedance.
- $-1/B$ is the short-circuit transfer admittance.
- $-1/D$ is the short-circuit gain.

Two-Ports (23)

Suppose that there are two two-ports: Then

$$\begin{pmatrix} V_{1a} \\ I_{1a} \end{pmatrix} = \begin{pmatrix} A_a & B_a \\ C_a & D_a \end{pmatrix} \begin{pmatrix} V_{2a} \\ -I_{2a} \end{pmatrix}$$

and

$$\begin{pmatrix} V_{1b} \\ I_{1b} \end{pmatrix} = \begin{pmatrix} A_b & B_b \\ C_b & D_b \end{pmatrix} \begin{pmatrix} V_{2b} \\ -I_{2b} \end{pmatrix}$$

If these are two are in cascade or chain, then for the composite network

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

Two-Ports (23)

When the two ports are in cascade,

$$\begin{aligned} V_1 &= V_{1a} & I_1 &= I_{1a} \\ V_2 &= V_{2b} & I_2 &= I_{2b} \\ I_{2a} &= -I_{1b} & V_{2a} &= V_{1b} \end{aligned}$$

Then

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} =$$

Two-Ports (23)

When the two ports are in cascade,

$$\begin{aligned} V_1 &= V_{1a} & I_1 &= I_{1a} \\ V_2 &= V_{2b} & I_2 &= I_{2b} \\ I_{2a} &= -I_{1b} & V_{2a} &= V_{1b} \end{aligned}$$

Then

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_a & B_a \\ C_a & D_a \end{pmatrix} \begin{pmatrix} A_b & B_b \\ C_b & D_b \end{pmatrix}$$

- This can be generalised to any number of two-ports.

Two-Ports (24)

Inverse Transmission Parameters:

$$\begin{aligned} V_2 &= A' V_1 - B' I_1 \\ I_2 &= C' V_1 - D' I_1 \end{aligned}$$

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$$-\frac{1}{B'} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad -\frac{1}{D'} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

- These parameters apply for transmission in the opposite direction.
- The properties of the inverse transmission $A'B'C'D'$ parameters are similar to the transmission $ABCD$ parameters.

Two-Ports (25)

Recall:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Compare this with

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad \frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

- Thus, $A = z_{11}/z_{21}$ and $C = 1/z_{21}$.

Two-Ports (25)

Recall:

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$-\frac{1}{B} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad -\frac{1}{D} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

- Thus, $B = \Delta/z_{21}$ and $D = z_{22}/z_{21}$, where $\Delta = z_{11}z_{22} - z_{12}z_{21}$.

Two-Ports (26)

Transmission parameters in terms of z-parameters:

$$A = \frac{z_{11}}{z_{21}}, \quad B = \frac{\Delta}{z_{21}}, \quad C = \frac{1}{z_{21}}, \quad D = \frac{z_{22}}{z_{21}}$$

- Clearly, $AD - BC = z_{12}/z_{21}$.
- For a reciprocal network,

$$AD - BC = 1$$

- Similarly for the inverse transmission parameters, reciprocity implies

$$A'D' - B'C' = 1$$

Two-Ports (27)

Hybrid, or h, parameters:

$$\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}$$

Observe that the four parameters may be defined as follows:

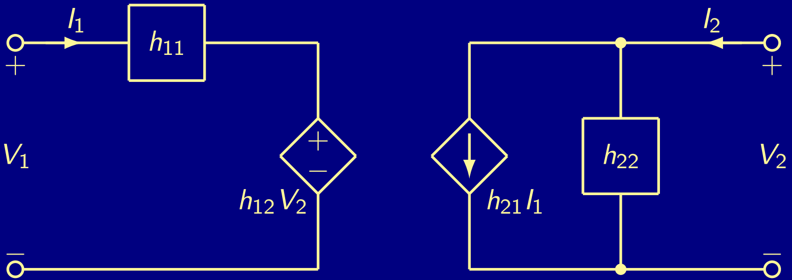
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

- h_{11} and h_{21} are respectively the s.c. input impedance and current gain.
- h_{12} and h_{22} are respectively the o.c. reverse voltage gain and output admittance.

Two-Ports (28)

- Note that the parameters are dimensionally mixed; hence, the term 'hybrid'.
- Useful for modelling transistors.



Two-Ports (29)

Inverse Hybrid, or g, Parameters:

$$
\begin{aligned}
I_1 &= g_{11} V_1 + g_{12} I_2 \\
V_2 &= g_{21} V_1 + g_{22} I_2
\end{aligned}
$$

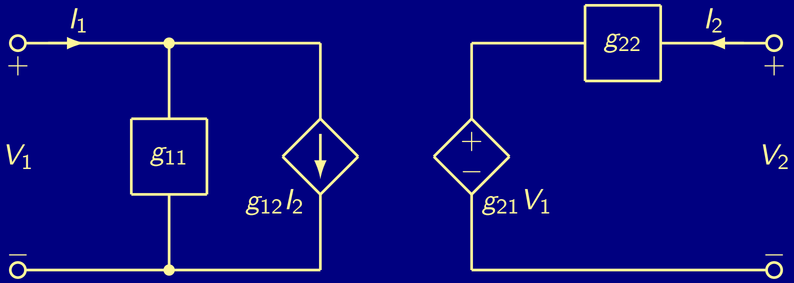
Observe that the four parameters may be defined as follows:

$$
g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}
$$

$$
g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}
$$

- g_{11} and g_{21} are respectively the o.c. input admittance and voltage gain.
- g_{12} and g_{22} are respectively the s.c. reverse current gain and output impedance.

Two-Ports (30)



Examples (2)

- Example 4, Valkenburg, p. 335.

Examples (2)

- Example 4, Valkenburg, p. 335.
- Problem 11-10, Valkenburg, p. 343.

Examples (2)

- Example 4, Valkenburg, p. 335.
- Problem 11-10, Valkenburg, p. 343.
- Problem 11-11, Valkenburg, p. 343.

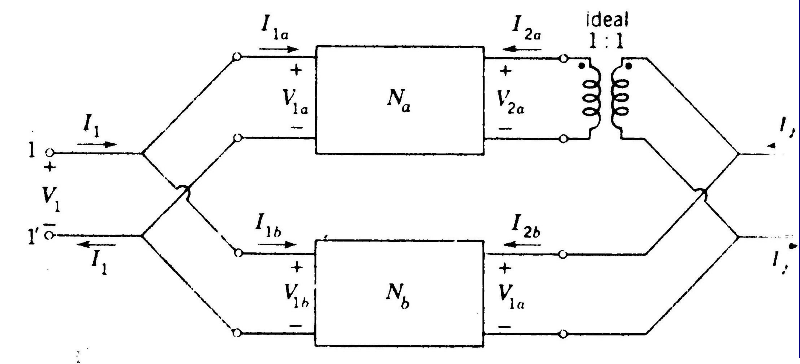
Examples (2)

- Example 4, Valkenburg, p. 335.
- Problem 11-10, Valkenburg, p. 343.
- Problem 11-11, Valkenburg, p. 343.

Reading Assignment: Section 11-6: Relationships Between Parameter Sets.

Two-Ports (31)

Parallel Connection of Two-Ports:



Two-Ports (32)

- Assumption: The parallel connection will not alter the nature of the networks.
- For example, T-networks shorts out the lowest resistor, and hence the network is altered.
- This is circumvented by the 1:1 turns ratio ideal transformer.
- Or, when the two networks have a common ground. In this case, the ideal transformer is not required.

Two-Ports (33)

- Short-circuit admittance functions are useful in characterising parallel two-ports.
- For Network A,

$$\begin{aligned} I_{1a} &= y_{11a}V_{1a} + y_{12a}V_{2a} \\ I_{2a} &= y_{21a}V_{1a} + y_{22a}V_{2a} \end{aligned}$$

- For Network B,

$$\begin{aligned} I_{1b} &= y_{11b}V_{1b} + y_{12b}V_{2b} \\ I_{2b} &= y_{21b}V_{1b} + y_{22b}V_{2b} \end{aligned}$$

Two-Ports (34)

- Assuming parallel connection can be made,

$$V_1 = V_{1a} = V_{1b}, \quad V_2 = V_{2a} = V_{2b}$$
$$I_1 = I_{1a} + I_{1b}, \quad I_2 = I_{2a} + I_{2b}$$

Two-Ports (34)

- Assuming parallel connection can be made,

$$V_1 = V_{1a} = V_{1b}, \quad V_2 = V_{2a} = V_{2b}$$
$$I_1 = I_{1a} + I_{1b}, \quad I_2 = I_{2a} + I_{2b}$$

- Therefore, from KCL,

$$\begin{aligned} I_1 &= (y_{11a} + y_{11b})V_1 + (y_{12a} + y_{12b})V_2 \\ I_2 &= (y_{21a} + y_{21b})V_1 + (y_{22a} + y_{22b})V_2 \end{aligned}$$

Two-Ports (34)

- Assuming parallel connection can be made,

$$V_1 = V_{1a} = V_{1b}, \quad V_2 = V_{2a} = V_{2b}$$
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- Therefore, from KCL,

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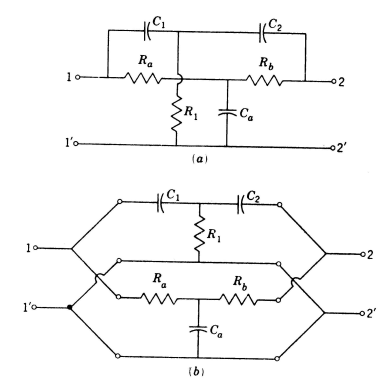
- This can be extended to any number of networks.

Two-Ports (35)

Twin T-networks with a common ground:



Two-Ports (36)

A bridged-T-network and its equivalent as a parallel two-port: