$$\varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

εL = change in length $\Delta L = \dfrac{PL}{AE}$

The following examples illustrate the application of these simple formulae.

Example 2.1 Application of Hooke's law

A circular bar 20 mm in diameter and 200 mm long is subjected to a force of 20 kN. Find the stress, strain, and elongation in the bar if the value of $E = 80$ GPa.

Solution Area of the bar $= \pi \times 20^2/4 = 314.16$ mm^2

Stress $\sigma = P/A = 20 \times 10^3/314.16 = 63.66$ N/mm^2

$E = 80$ GPa $= 80{,}000$ MPa $= 80{,}000$ N/mm^2

Strain $= \sigma/E = 63.66/80{,}000 = 7.9 \times 10^{-4}$

Example 2.2 Young's modulus of elasticity

The ratio of Young's moduli of elasticity of two materials is 2.35. Find the ratio of the stresses and elongations in two bars of these materials if they are of the same length and same area and subjected to the same force P.

Solution Stress $\sigma = P/A$. Load P and area remaining the same, ratio of stresses = 1.0

Strain in one bar $\varepsilon_1 = \sigma/E_1$; strain in the other bar $\varepsilon_2 = \sigma/E_2$

Length being the same, the ratio of elongations will be the same as ratio of strains

Ratio of elongations $= \varepsilon_1/\varepsilon_2 = (\sigma/E_1)/(\sigma/E_2) = E_2/E_1 = 1/2.35 = 0.42$

Example 2.3 Young's modulus of elasticity

A bar of cross-sectional area 314 mm^2 elongates by 0.8 mm over a length of 600 mm when subjected to a tensile force of 12000 N. Find the Young's modulus of elasticity of the material of the bar.

Solution Stress $\sigma = 12{,}000/314 = 38.22$ N/mm^2

Strain $\varepsilon = \sigma/E = 38.22/E$. Elongation $= \varepsilon L = 38.22\ L/E$

Elongation = 0.8 mm

Therefore, $38.22 \times 600/E = 0.8$ mm

$E = 38.22 \times 600/0.8 = 28665$ N/mm^2 $= 28.67$ GPa

Example 2.4 Young's modulus of elasticity of a circular pipe

A circular pipe of internal diameter 30 mm and thickness 4 mm is subjected to a force 30 kN and the elongation was measured as 1 mm. If the length of the pipe is 2 m, find the value of Young's modulus of elasticity and the stress in the pipe.

Solution Internal diameter of the pipe = 30 mm; thickness = 4 mm; external diameter = 38 mm; area of the pipe = $\pi(38^2 - 30^2)/4 = 427.26$ mm^2

Stress in the pipe material $\sigma = P/A = 30{,}000/427.26 = 70.2$ N/mm^2

Elongation = 1 mm; Length of pipe = 2 m = 2000 mm

Strain $\varepsilon = 1/2000 = 5 \times 10^{-4}$

E = stress/strain $= 70.2/(5 \times 10^{-4}) = 140{,}400$ N/mm^2 $= 140.4$ GPa

Example 2.5 Stress in uniform bar

A uniform steel rod, 6 mm ϕ and 0.5 m long, is subjected to a tensile force of 3 kN. Find the stress in the bar and its elongation. $E = 200$ GPa.

Solution Tensile force = 3 kN = 3000 N

$$\text{Area of cross section} = \frac{\pi \times 6^2}{4} = 9\pi \text{ mm}^2$$

$$\text{Stress} = \frac{3000}{9\pi} = 106 \text{ N/mm}^2$$

Change in lengh,

$$\Delta L = \frac{PL}{AE} = \frac{3000 \times (0.5 \times 1000)}{9\pi \times 200 \times 10^9/10^6}$$

(Note that 1 GPa = 10^9 Pa = 10^3 MPa = 10^3 N/mm^2.)

$$\Delta L = 0.265 \text{ mm}$$

Example 2.6 Determining stress and E

The length of an aluminium rod 10 mm ϕ and 400 mm long increases to 400.15 mm when subjected to a tensile force of 2 kN. Find the stress in the bar and the value of E for aluminium.

Solution Area of bar $= \dfrac{\pi \times 10^2}{4} = 25p$ mm^2

$$\text{Stress} = \frac{2 \times 1000}{25\pi} = 25.46 \text{ N/mm}^2$$

Change in length,

$$\Delta L = 400.15 - 400 = 0.15 \text{ mm}$$

$$\Delta L = \frac{PL}{AE}, \quad E = \frac{PL}{A\Delta L}$$

$$E \text{ for aluminium} = \frac{2000 \times 400}{25\pi \times 0.15} = 67{,}906 \text{ N/mm}^2 = 67.9 \text{ GPa}$$

Example 2.7 Uniform bar subjected to loads along length

A bar of uniform cross section 20 mm ϕ is subjected to loads as shown in Fig. 2.5(a). Find the total elongation of the bar and the maximum stress in the bar. $E = 200$ GPa. (All lengths are in mm.)

Solution The total elongation will be the sum of the elongations of the individual sections *AB, BC,* and *CD*. The free body diagrams of each of these sections are shown in Fig. 2.5(b).

We have to determine the force P such that the induced stresses in both bronze and steel portions are less than their respective allowable stresses. Allowable stress is the maximum stress that can be permitted to be induced in a member subjected to external load. We shall first assume that maximum stress induced in the bronze bar is 90 Mpa and determine the maginitude of force P.

$$P = A_B \times \sigma_B = 300 \times 90 = 27 \times 10^3 \ N = 27 \ KN$$

Each portion of the compound bar is subjected to the force equal to the applied force. Therefore we can find the induced stress in the steel portion based on the calculated value of force P.

$$\sigma_{S_i} = \frac{P}{A_S} = \frac{27 \times 10^3}{200} = 135 \ Mpa$$

The induced stress 135 Mpa in steel is greater than the corresponding allowable stress $\sigma_S = 130 \ Mpa$. Therefore we will now find the force P based on the allowable stress of steel.

$$P = \sigma_S \times A_S = 130 \times 200 = 26 \times 10^3 \ N = 26 \ KN$$

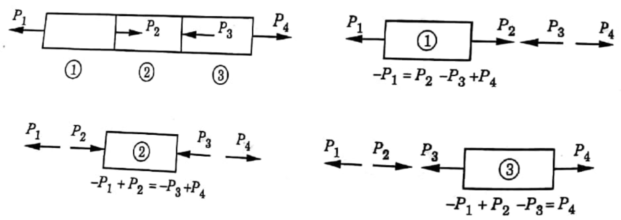
Induced stress in Bronze portion

$$\sigma_{B_i} = \frac{P}{A_B} = \frac{26 \times 10^3}{300} = 86.7 \ Mpa < \sigma_B$$

$P = 26 \ KN$ is the maximum force that can be applied on the bar, since the induced stresses in both portions are within the limits.

1.1.14 Principle of Superposition

Most of machine members are subjected to combination of various loads. The combined effect of load in the form of stress and strain may be found by using the principle of superposition, which can be stated as **Total effect of several loads applied on a body is the sum of effects of individual loads applied separately.**



$$dL = dL_1 + dL_2 + dL_3$$

Fig 1.1.16

To find the force acting in a portion, calculate the algebraic sum of forces individually on its each side as shown in Fig. 1.1.16. The forces acting on either sides of an individual portion should be equal in magnitude and opposite in direction.

As an example consider a stepped bar subjected to various forces as shown in fig. 1.1.16. Based on the superposition principle and equilibrium requirement, forces acting on each portion of the body are found and the corresponding stresses are calculated. Stress induced in each portion is calculated as the ratio of corresponding load and cross-sectional area. Net deformation dL in the stepped bar is sum of individual deformations in each portion. This principle can be applied only when

- The effects such as stress and strain are directly proportional to the loads which produce them.
- The strains produced are small.

SOLVED PROBLEMS

EXAMPLE 1.1.12 A brass bar having cross-sectional area 300 mm^2 is subjected to axial forces as shown in Fig. E 1.1.12. Find the total elongation of the bar. Take E as 84 Gpa.

Soln :

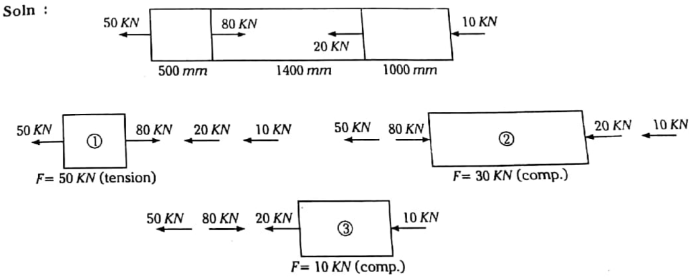


Fig. E. 1.1.12

Portion (1)

$$\delta L_1 = \frac{PL}{AE} = \frac{50 \times 10^3 \times 500}{300 \times 84 \times 10^3} = 0.99 \ mm$$

Portion (2)

$$\delta L_2 = \frac{PL}{AE} = \frac{-30 \times 10^3 \times 1400}{300 \times 84 \times 10^3} = -1.67 \ mm$$

Portion (3)

$$\delta L_3 = \frac{PL}{AE} = \frac{-10 \times 10^3 \times 1000}{300 \times 84 \times 10^3} = -0.397\ mm$$

Net deformation

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$= 0.99 - 1.67 - 0.397 = -1.08\ mm$$

EXAMPLE 1.1.13 Different portions of a stepped bar are subjected to the forces as shown in fig E 1.1.3. Determine i) stress induced in each portion and ii) Net deformation in the bar. Take E = 200 Gpa.

Soln :

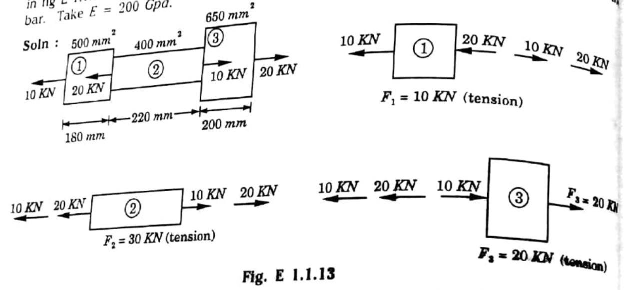


Fig. E 1.1.13

Stress σ and deformation dL induced in each portion are found based on the load acting on the corresponding portion.

PORTION (1)

$$\sigma_1 = \frac{F_1}{A_1} = \frac{10 \times 10^3}{500} = 20\ Mpa$$

$$d\,L_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{\left(10 \times 10^3\right) \times 180}{500 \times 2 \times 10^5} = 0.018\ mm$$

PORTION (2)

$$\sigma_2 = \frac{F_2}{A_2} = \frac{30 \times 10^3}{400} = 75\ Mpa$$

$$d\,L_2 = \frac{F_2 L_2}{A_2 E_2} = \frac{\left(30 \times 10^3\right) \times 220}{400 \times 2 \times 10^5} = 0.0825\ mm$$

PORTION (3)

$$\sigma_3 = \frac{F_3}{A_3} = \frac{20 \times 10^3}{650} = 30.8\ Mpa$$

$$d\,L_2 = \frac{F_3 L_3}{A_3 E_3} = \frac{\left(20 \times 10^3\right) \times 200}{650 \times 2 \times 10^5} = 0.031\ mm$$

$$dL = dL_1 + dL_2 + dL_3 = 0.018 + 0.0825 + 0.031 = 0.132\ mm \quad \text{Ans.}$$

EXAMPLE 1.1.14 A stepped bar with three different, portions has a fixed support at one of its ends. The stepped bar is subjected to forces as shown in fig. E 1.1.4. Determine the stresses and deformations induced in each portion. Also find the net deformation induced in the stepped bar. Take E = 200 Gpa.

Soln :

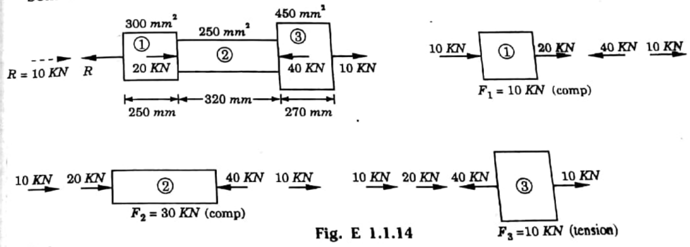


Fig. E 1.1.14

Soln :

The fixed end exerts reaction R whose direction is assumed as shown by solid arrow in fig. E 1.1.14. Considering equilibrium of the bar we get

$$\Sigma F_x = 0 \qquad -R + 20 - 40 + 10 = 0$$

$$R = -10\ KN$$

Assumed direction of reaction R is wrong, since the calculated value has negative sign. Therefore reverse its direction as shown by dotted arrow in fig. E 1.1.14.

PORTION (1)

$$\sigma_1 = \frac{F_1}{A_1} = \frac{10 \times 10^3}{300} = 33.3\ Mpa\ (\text{comp})$$

$$d\,L_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{-10 \times 10^3 \times 250}{300 \times 2 \times 10^5} = -0.042\ mm$$

$$P_1 + P_2 = P \qquad (19)$$

Compatibility condition shows, $\Delta_1 = \Delta_2$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2} \qquad (20)$$

From the above two equations the two unknowns P_1 and P_2 can be found uniquely.

Example 2.22 A compound bar of length 500 mm consists of a strip of aluminium 50 mm wide × 20 mm thick and a strip of steel 50 mm wide × 15 mm thick rigidly joined at ends. If the bar is subjected to a load of 50 kN, find the stresses developed in each material and the extension of the bar. Take elastic modulus of aluminium and steel as 1×10^5 N/mm² and 2×10^5 N/mm² respectively.

Solution

$L = 500$ mm

$P = 50$ kN

$A_a = 20 \times 50 = 1000$ mm²

$A_s = 15 \times 50 = 750$ mm²

$E_a = 1 \times 10^5$ N/mm²

$E_s = 2 \times 10^5$ N/mm²

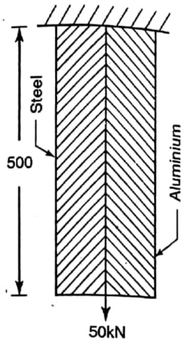


Fig. 2.36

The compound bar is shown in Fig. 2.36.

Let P_a be the force shared by aluminium strip and P_s the load shared by steel strip. From static equilibrium condition,

$$P_a + P_s = 50 \times 10^3 \qquad (1)$$

From compatibility condition, $\Delta_a = \Delta_s$.

$$\frac{P_a L_a}{A_a E_a} = \frac{P_s L_s}{A_s E_s} \qquad (2)$$

Since $L_a = L_s$

$$\frac{P_a}{1000 \times 1 \times 10^5} = \frac{P_s}{750 \times 2 \times 10^5}$$

$$P_s = 1.5\, P_a \qquad (3)$$

From equation (1) and (3) we get,

$$2.5 P_a = 50 \times 10^3$$

or $P_a = 20 \times 10^3$ N

Substituting it in equation (1), we get,

$$P_s = 30 \times 10^3$$

∴ Stress in aluminium strip $= \dfrac{P_a}{A_a}$

$$= \frac{20 \times 10^3}{1000} = 20 \text{ N/mm}^2$$

and stress in steel strip $= \dfrac{30 \times 10^3}{750} = 40$ N/mm²

$$\Delta_a = \Delta_s = \frac{P_s L_s}{A_s E_s} = \frac{30 \times 10^3 \times 500}{750 \times 2 \times 10^5} = 0.1 \text{ mm} \qquad \textbf{(Ans)}$$

Example 2.23 A compound bar consists of a circular rod of steel of diameter 20 mm rigidly fitted into a copper tube of internal diameter 20 mm and thickness 5 mm as shown in Fig. 2.37. If the bar is subjected to a load of 100 kN, find the stresses developed in the two materials.

Take $E_s = 2 \times 10^5$ N/mm²

and $E_c = 1.2 \times 10^5$ N/mm²

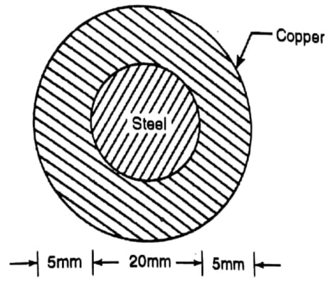


Fig. 2.37

Solution

Now, $A_s = \dfrac{\pi}{4} \times 20^2 = 100\pi$ mm²

and external diameter of copper tube $= 20 + 2 \times 5 = 30$ mm

∴ Area of copper tube $A_c = \dfrac{\pi}{4} \times (30^2 - 20^2) = 125\pi$ mm²

From static equilibrium condition,

$$P_s + P_c = 100 \qquad (1)$$

where P_s is load shared by steel in kN

and P_c is the load shared by copper in kN.

From compatibility condition, $\Delta_s = \Delta_c$

$$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$$

$$\frac{P_s}{100\pi \times 2 \times 10^5} = \frac{P_c}{125\pi \times 1.2 \times 10^5}$$

or $P_s = 1.3333\, P_c$

∴ From (1), $P_c = \dfrac{100}{2.3333} = 42.857$ kN

$P_s = 1.3333\, P_c = 57.143$ kN

$$\text{Stress in steel} = \frac{57.143 \times 10^3}{100\pi} = 181.89 \text{ N/mm}^2$$

$$\text{Stress in copper} = \frac{42.857 \times 10^3}{125\pi} = 109.134 \text{ N/mm}^2$$

$$D^2 = d^2 e^{wx/\sigma} = 30^2 \cdot e^{7.5\times 9.81\times 10^{-6}\times 3000/30}$$

or $$= 900 \times 1.0148 = 913.34$$

or $$D = 30.22 \text{ mm}$$

LO 11

Analyse statically indeterminate systems

1.11 STATICALLY INDETERMINATE SYSTEMS

When a system comprises two or more members of different materials, the forces in various members cannot be determined by the principle of statics alone. Such systems are known as *statically indeterminate systems*. In such systems, additional equations are required to supplement the equations of statics to determine the unknown forces. Usually, these equations are obtained from deformation conditions of the system and are known as *compatibility equations*. A compound bar is a case of an indeterminate system and is discussed below:

Compound Bar

A bar consisting of two or more bars of different materials in parallel is known as a *composite* or *compound bar*. In such a bar, the sharing of load by each can be found by applying equilibrium and the compatibility equations.

Consider the case of a solid bar enclosed in a hollow tube as shown in Fig. 1.27. Let the subscripts 1 and 2 denote the solid bar and the hollow tube respectively.

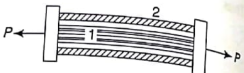


Fig. 1.27

Equilibrium Equation As the total load must be equal to the load taken by individual members,

$$P = P_1 + P_2 \tag{i}$$

Compatibility Equation The deformation of the bar must be equal to the tube.

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2} \quad \text{or} \quad P_1 = \frac{P_2 A_1 E_1}{A_2 E_2} \tag{ii}$$

Inserting (ii) in (i),

$$P = \frac{P_2 A_1 E_1}{A_2 E_2} + P_2 = \frac{P_2 A_1 E_1 + P_2 A_2 E_2}{A_2 E_2} = \frac{P_2 (A_1 E_1 + A_2 E_2)}{A_2 E_2}$$

or $$P_2 = \frac{P \cdot A_2 E_2}{A_1 E_1 + A_2 E_2} \tag{1.12}$$

Similarly, $$P_1 = \frac{P \cdot A_1 E_1}{A_1 E_1 + A_2 E_2} \tag{1.13}$$

Example 1.15 Three equally spaced rods in the same vertical plane support a rigid bar *AB*. Two outer rods are of brass, each 600 mm long and of 25 mm in diameter. The central rod is of steel that is 800 mm long and 30 mm in diameter. Determine the forces in the rods due to an applied load of 120 kN through the midpoint of the bar. The bar remains horizontal after the application of load. Take $E_s/E_b = 2$.

Solution

Given A rigid bar system as shown in Fig. 1.28.

$$E_s/E_b = 2.$$

To find Forces in brass and steel rods

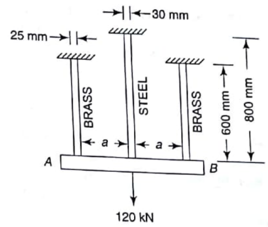


Fig. 1.28

Applying compatibility equation

As the bar remains horizontal after the application of load, the elongation of each of the brass bars and of the steel bar are the same, $\Delta_b = \Delta_s$

or $$\frac{P_b L_b}{A_b E_b} = \frac{P_s L_s}{A_s E_s}$$

or $$P_b = \frac{L_s}{L_b} \cdot \frac{E_b}{E_s} \left(\frac{d_b}{d_s}\right)^2 P_s = \frac{800}{600} \cdot \frac{1}{2} \left(\frac{25}{30}\right)^2 P_s$$

or $$P_b = 0.463 P_s$$

Applying equilibrium equation

$$2P_b + P_s = P$$

or $$2 \times 0.463\, P_s + P_s = 120$$

or $$1.926\, P_s = 120$$

or $$P_s = 62.3 \text{ kN and } P_b = 28.84 \text{ kN}$$

Example 1.16 Three equidistant vertical rods, each of 20 mm diameter, support a load of 25 kN in the same plane as shown in Fig. 1.29. Initially, all the rods are adjusted to share the load equally. Neglecting any chance of buckling, and taking E_s = 205 GPa and E_c = 100 GPa, determine the final stresses when a further load of 20 kN is added.

Solution

Given Three equidistant vertical rods supporting a load of 25 kN as shown in Fig. 1.29.

$L_s = 3.6$ m $E_s = 205$ GPa

$L_c = 2.8$ m $E_c = 100$ GPa

$d = 20$ mm

Initial load = 25 kN

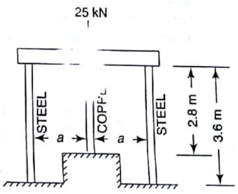


Fig. 1.29

To find Final stresses when a further load of 20 kN is added

$$A = (\pi/4)\, 20^2 = 100\, \pi \text{ mm}^2$$

Finding of initial stress in each rod

$$\sigma_i = \frac{25\,000}{100\pi \times 3} = 26.53 \text{ MPa}$$

Finding of additional stresses in each rod

On adding a further load of 20 kN, let the increase of stress in the steel rod be σ_s and in the copper rod be σ_c.

Example 2.22 **Stress in a composite bar of steel and cast iron**

A steel rod, 60 mm ϕ and 1 m long, is encased by a cast iron (CI) sleeve 8 mm thick and of internal diameter 60 mm. The assembly is subjected to a load of 40 kN. Find the stresses in the two materials and the elongation of the assembly. E for steel = 200 GPa and E for cast iron = 100 GPa.

Solution If the loads shared by steel and cast iron are P_s and P_c, then $P_s + P_c = 40$ kN (Fig. 2.18).

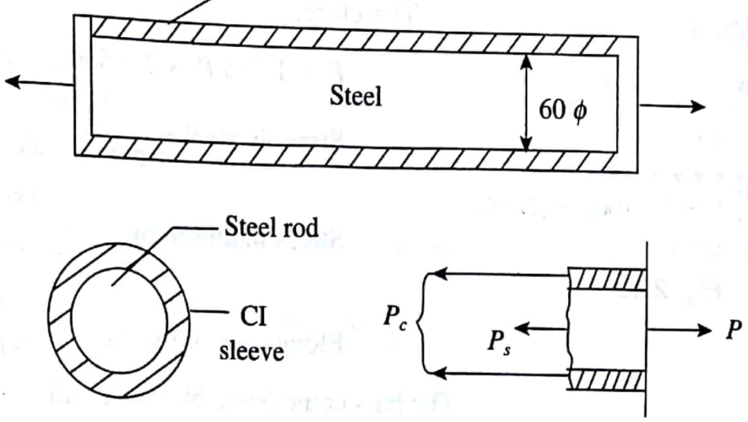


Fig. 2.18

$$\text{Area of cross section of steel rod} = \frac{\pi \times 60^2}{4} = 2827.4 \text{ mm}^2$$

$$\text{Area of cross section of cast iron} = \frac{\pi}{4}(76^2 - 60^2) = 1709 \text{ mm}^2$$

Since the strains in the two materials are equal,

$$\frac{P_s}{A_s E_s} = \frac{P_c}{A_c E_c}, \quad \frac{P_s}{2827.4 \times 200{,}000} = \frac{P_c}{1709 \times 100{,}000}, \quad P_s = 3.31 P_c$$

$$3.31 P_c + P_c = 40{,}000, \quad P_c = 9281 \text{ N}$$

$$P_s = 30{,}719 \text{ N}$$

$$\text{Stress in steel} = \frac{30{,}719}{2827.4} = 10.86 \text{ N/mm}^2$$

$$\text{Stress in cast iron} = \frac{9281}{1709} = 5.43 \text{ N/mm}^2$$

$$\text{Elongation of the assembly} = \frac{10.86 \times 1000}{200{,}000} = 0.0543 \text{ mm}$$

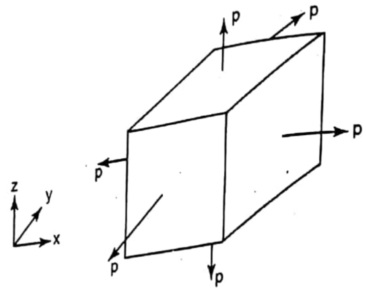


Fig. 2.62

Hence,
$$e_x = \frac{p}{E} - \mu\frac{p}{E} - \mu\frac{p}{E}$$
$$= \frac{p}{E}(1-2\mu)$$

Similarly
$$e_y = \frac{p}{E}(1-2\mu)$$
$$e_z = \frac{p}{E}(1-2\mu) \qquad (1)$$

Volumetric strain $e_v = e_x + e_y + e_z$
$$= \frac{3p}{E}(1-2\mu)$$

From definition, bulk modulus K is given by
$$K = \frac{p}{e_v} = \frac{p}{\dfrac{3p\,(1-2\mu)}{E}}$$

or
$$E = 3K(1-2\mu) \qquad (2)$$

Relationship between EGK:

We know
$$E = 2G(1+\mu) \qquad (a)$$
and
$$E = 3K(1-2\mu) \qquad (b)$$

By eliminating μ between the above two equation we can get the relationship between E, G, K, free from the term μ.

From equation (a)
$$\mu = \frac{E}{2G} - 1$$

Substituting it in equation (b), we get
$$E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]$$
$$= 3K\left(1 - \frac{E}{G} + 2\right) = 3K\left(3 - \frac{E}{G}\right)$$

$$= 9K - \frac{3KE}{G}$$
$$\therefore \quad E\left(1 + \frac{3K}{G}\right) = 9K$$
or
$$E\left(\frac{G+3K}{G}\right) = 9K \qquad (c)$$
or
$$E = \frac{9KG}{G+3K} \qquad (2.26a)$$

Equation (c) may be expressed as
$$\frac{9}{E} = \frac{G+3K}{KG}$$
i.e.
$$\boxed{\frac{9}{E} = \frac{3}{G} + \frac{1}{K}} \qquad (2.26b)$$

Example 2.37 A bar of 20 mm diameter is tested in tension. It is observed that when a load of 37.7 kN is applied, the extension measured over a guage length of 200 mm is 0.12 mm and contraction in diameter is 0.0036 mm. Find Poisson's ratio and elastic constants E, G, K.

Solution

Now,
$$P = 37.7\text{ kN} = 37700\text{ N}$$
$$\text{Area } A = \frac{\pi}{4} \times 20^2 = 100\pi\text{ mm}^2$$
$$\text{Gauge length } L = 200\text{mm}$$
$$\Delta = 0.12\text{ mm}$$
$$\Delta d = 0.0036\text{ mm}$$
$$\text{Linear strain} = \frac{\Delta}{L} = \frac{0.12}{200} = 0.0006$$
$$\text{Lateral strain} = \frac{\Delta d}{d} = \frac{0.0036}{20} = 0.00018$$
$$\therefore \text{ Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.00018}{0.0006}$$
$$\mu = 0.3 \qquad \textbf{(Ans)}$$

Now,
$$\Delta = \frac{PL}{AE}$$
$$0.12 = \frac{37700 \times 200}{100\pi \times E}$$
or
$$E = 200004.71\text{ N/mm}^2 \qquad \textbf{(Ans)}$$

Using the relation
$$E = 2G(1+\mu)$$

We get
$$G = \frac{E}{2(1+\mu)} = \frac{200004.71}{2(1+0.3)} = 76924.89\text{ N/mm}^2 \qquad \textbf{(Ans)}$$

From the relation, $E = 3K(1-2\mu)$, we get

$$K = \frac{E}{3(1-2\mu)} = \frac{200004.71}{3(1-2\times 0.3)} = 166670.59\ \text{N/mm}^2 \qquad \textbf{(Ans)}$$

... diameter and 500 mm long is subjected to a

$$\varepsilon_v = \frac{dv}{v} = \frac{\sigma}{K} \qquad (1.3.12)$$

Substituting the value of ε_v from eqn 1.3.12 into eqn 1.3.11 we get

$$\frac{\sigma}{K} = 3\frac{\sigma}{E}\left(1-\frac{2}{m}\right)$$

$$\boxed{E = 3K\left(1-\frac{2}{m}\right)}$$

1.3.9 Relation Between E, G and K

The relation between Young's modulus and modulus of rigidity and the relation between Young's modulus and bulk modulus are given by

$$E = 2G\left(1+\frac{1}{m}\right) \qquad (1.3.13)$$

$$E = 3K\left(1-\frac{2}{m}\right) \qquad (1.3.14)$$

Substituting the value of E from eqn 1.3.13 into eqn 1.3.14 we get

$$2G\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)$$

$$\frac{1}{m}(2G+6K) = 3K-2G$$

$$\frac{1}{m} = \frac{3K-2G}{2G+6K} \qquad (1.3.15)$$

Substitute the value of $1/m$ into eqn 1.3.13 we get

$$E = 2G\left(1+\frac{3K-2G}{2G+6K}\right) = 2G\left(\frac{2G+6K+3K-2G}{2G+6K}\right)$$

$$\boxed{E = \frac{9GK}{3K+G}}$$

SOLVED PROBLEMS

EXAMPLE 1.3.1 A material has Young's modulus E = 200 *Gpa* and Poisson's ratio ν = 0.3. Determine, modulus of rigidity G and bulk modulus K.

Soln : Data : E = 200 *Gpa*, ν = 0.3

Modulus of rigidity

$$E = 2G\left(1+\frac{1}{m}\right)$$

$$2 \times 10^5 = 2 \times G\,(1+0.3)$$

$$G = 0.8 \times 10^5\ N/mm^2 \text{ or } 80\ Gpa$$ Ans.

Bulk Modulus

$$E = 3K\left(1-\frac{2}{m}\right)$$

$$2\times10^5 = 3\times K(1-2\times0.3)$$

$$K = 1.67\times10^5\ N/mm^2 \text{ or } 167\ Gpa$$ Ans.

EXAMPLE 1.3.2 A uniform round bar of 24.5 *mm* diameter, when subjected to an axial tensile force 56.6 *KN* undergoes a change in diameter by 0.0042 *mm*. Taking modulus of rigidity as 80 *Gpa*, determine Poisson's ratio and Young's modulus.

Soln : Data : d = 24.5 *mm*, P = 56.6 *KN*, δd = 0.0042 *mm*, G = 80 *Gpa*

Tensile stress

$$\sigma = \frac{P}{A} = \frac{56.6\times10^3}{\frac{\pi}{4}\times24.5^2} = 120\ Mpa$$

$$\text{Lateral strain } \varepsilon_L = \frac{\text{Change diameter}}{\text{Original diameter}}$$

$$\varepsilon_L = \frac{0.0042}{24.5} = 1.72\times10^{-4}$$

Lateral strain

$$\varepsilon_L = \frac{\sigma}{mE}$$

$$mE = \frac{\sigma}{\varepsilon_L} = \frac{120}{1.72 \times 10^{-4}} = 6.976 \times 10^5 \quad (1)$$

$$E = 2G\left(1 + \frac{1}{m}\right)$$

Multiplying both sides by m

$$mE = 2G(1 + m)$$

Substituting the value of mE from eqn(1)

$$6.976 \times 10^5 = 2 \times 0.8 \times 10^5 (1 + m)$$

$$\frac{1}{m} = 0.3 \quad \text{Ans.}$$

From eqn (1)

$$mE = 6.976 \times 10^5$$

$$\frac{1}{0.3} \times E = 6.976 \times 10^5$$

$$E = 2.1 \times 10^5 \, N/mm^2 \; (210 \, Gpa) \quad \text{Ans.}$$

EXAMPLE 1.3.3 When a 12 mm diameter specimen is subjected to a tensile force 20 KN, a deformation of 0.3 mm is observed over a gauge length of 150 mm. Reduction in diameter is 0.0079. Determine i) Young's modulus, ii) Poisson's ratio iii) Modulus of rigidity and iv) Bulk modulus.

Soln :

Data : $d = 12 \, mm$, $P = 20 \, KN$, $L = 150 \, mm$, $dL = 0.3 \, mm$, $\delta d = 0.0079$

$$\text{Cross-sectional area } A = \frac{\pi}{4} \times 12^2 = 113 \, mm^2$$

$$\text{Stress } \sigma = \frac{P}{A} = \frac{20 \times 10^3}{113} \doteq 177 \, N/mm^2$$

$$\text{Deformation } dL = \frac{\sigma L}{E} \rightarrow 0.3 = \frac{177 \times 150}{E}$$

$$E = 0.89 \times 10^5 \, N/mm^2 \quad \text{Ans.}$$

$$\text{Longitudinal strain } \varepsilon_{long} = \frac{dL}{L} = \frac{0.3}{150} = 0.002$$

$$\text{Lateral strain } \varepsilon_{lat} = \frac{\text{Change in diameter}}{\text{original diameter}} = \frac{0.0079}{12} = 0.00066$$

$$\text{Poisson's ratio } \frac{1}{m} = \frac{\varepsilon_{lat}}{\varepsilon_{long}} = \frac{0.00066}{0.002} = 0.33 \quad \text{Ans.}$$

Modulus of rididity

$$G = \frac{E}{2\left(1 + \frac{1}{m}\right)} = \frac{0.89 \times 10^5}{2(1 + 0.33)} = 0.33 \times 10^5 \, N/mm^2 \quad \text{Ans.}$$

Bulk Modulus

$$K = \frac{E}{3\left(1 - \frac{2}{m}\right)} = \frac{0.89 \times 10^5}{3(1 - 2 \times 0.33)} = 0.87 \times 10^5 \, N/mm^2 \quad \text{Ans.}$$

EXAMPLE 1.3.4 The diameter of a specimen is found to reduce by 0.004 mm, when it is subjected to a tensile force of 19 KN. Initial diameter of the specimen is 20 mm. Taking modulus of rigidity for the material of specimen as $0.4 \times 10^5 \, N/mm^2$, determine the values of Young's modulus and Poisson's ratio.

Soln :

Data : $d = 20 \, mm$, $\delta d = 0.004 \, mm$, $P = 19 \, KN$, $G = 0.4 \times 10^5 \, N/mm^2$

$$\text{Lateral strain } \varepsilon_{lat} = \frac{\delta d}{d} = \frac{0.004}{20} = 2 \times 10^{-4}$$

$$\text{Stress } \sigma = \frac{P}{A} = \frac{19 \times 10^3}{\frac{\pi}{4}(20)^2} = 60.5 \, N/mm^2$$

$$\text{Longitudinal strain } \varepsilon_{long} = \frac{\sigma}{E} = \frac{60.5}{E}$$

$$\text{Poisson's ratio } \frac{1}{m} = \frac{\varepsilon_{lat}}{\varepsilon_{long}} = \frac{2 \times 10^{-4}}{\frac{60.5}{E}} = \frac{2 \times 10^{-4} E}{60.5} \quad (i)$$

$$\text{We have } G = \frac{E}{2(1 + 1/m)}$$

Substitute the value of $1/m$ from eqn (i)

$$0.4 \times 10^5 = \frac{E}{2\left(1 + \frac{2 \times 10^{-4} E}{60.5}\right)}$$