9.1 (p325)

$\alpha = 1000\ \text{s}^{-1}$ and $\omega_o = 800$ rad/s, with $R = 100\ \Omega$.

(a) $\alpha = \dfrac{1}{2RC}$ so $C = \dfrac{1}{2R\alpha} = \underline{5\ \mu\text{F}}$

(b) $\omega_o = \dfrac{1}{\sqrt{LC}}$ so $L = \dfrac{1}{C\omega_o^2} = \underline{312.5\ \text{mH}}$

(c) $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = \underline{-400\ \text{s}^{-1}}$

(d) $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = \underline{-1600\ s^{-1}}$

9.2 (p329)

(a) $i_L(0^-) = 3 \times \dfrac{24}{24+48} = \underline{1\ \text{A}}$

(b) $v_C(0^-) = 48 i_L(0^-) = \underline{48\ \text{V}}$

(c) Since the capacitor voltage cannot change in zero time, $v_C(0^+) = v_C(0^-) = 48\ \text{V}$. Thus,
$i_R(0^+) = \dfrac{48}{24} = \underline{2\ \text{A}}$

(d) $i_L(0^+) = i_L(0^-)$. With the source off at $t = 0^+$, $i_C(0^+) + i_L(0^+) + i_R(0^+) = 0$ so
$i_C(0^+) = -1 - 2 = \underline{-3\ \text{A}}$

(e) $\alpha = \dfrac{1}{2RC} = 5\ \text{s}^{-1}$ (48–Ω ohm resistor shorted) and $\omega_o = \dfrac{1}{\sqrt{LC}} = 4.899$ rad/s

$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ [1]

where $s_1 = -4\text{s}^{-1}$ and $s_2 = -6\text{s}^{-1}$

$$i_C(t) = C\frac{dv_c}{dt} = C\left[A_1 s_1 e^{s_1 t} + A_2\, s_2\, e^{s_2 t}\right]$$
$$= \frac{1}{240}\left[-4\, A_1\, e^{-4t} - 6\, A_2\, e^{-6t}\right]$$

$$i_C(0^+) = -3 = \frac{1}{240}(-4A_1 - 6A_2) \qquad [2]$$

also, from Eq. [1]

$$v_C(0^+) = 48 = A_1 + A_2 \qquad [3]$$

Solving Eqns. [2] and [3], we find that $A_1 = -216$ V and $A_2 = 264$ V

so $v_C(t) = -216e^{-4t} + 264e^{-6t}$ V and hence $v_C(0.2) = \underline{-17.54 \text{ V}}$

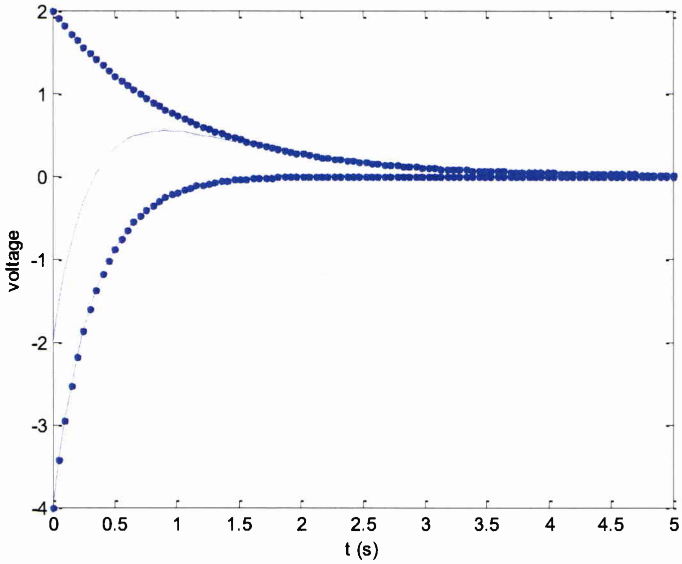
9.3 (p331)

We have a parallel RLC circuit with
$\alpha = \frac{1}{2RC} = 41.67\times10^9 \text{ s}^{-1}$ and $\omega_0 = \frac{1}{\sqrt{LC}} = 20\times10^9$ rad/s.
Since $\alpha > \omega_0$, the circuit is overdamped, so we compute
$s_1 = -5.113\times10^9 \text{ s}^{-1}$ and $s_2 = -78.23\times10^9 \text{ s}^{-1}$.

Thus, we expect a resistor current $i_R(t) = Ae^{s_1 t} + Be^{s_2 t} = Ae^{-5.113\times10^9 t} + Be^{-78.23\times10^9 t}$ A.
Noting that $v_C = 3i_R$ and $v_C(0) = 0$, we can write 3A + 3B = 0. [1]
Define ic as flowing out of the top node. Then,
noting that $i_C(0^+) = -i_R(0^+) - i_L(0+) = 0/3 + 6 = 6$ and
that $i_C(0^+) = C\left.\frac{dv}{dt}\right|_{t=0^+} = 4\times10^{-12}\left[3As_1e^{s_1 t} + 3Bs_2e^{s_2 t}\right]_{t=0^+} = 4\times10^{-12}\left[3As_1 + 3Bs_2\right] = -6$ [2]
we can solve these two equations for A = 6.838 and B = –A.
Therefore, $\underline{i_R(t) = 6.838\left(e^{-78.23\times10^9 t} - e^{-5.113\times10^9 t}\right) \text{ amperes.}}$

9.4 (p333)

(a) We begin by sketching the two terms independently, then adding graphically, as shown below (the total is drawn in a solid line).



(b) The maximum value of this function is 544.3 mV (see part (c), or estimate from graph). Solving using MATLAB for the time at which this value drops to 5.443 mV,

```
>>solve('2*exp(-ts) - 4*exp(-3*ts) = 5.443e-3')
```

from which the only sensible answer is 5.906 s.

(c) Using MATLAB, we find the time at which the maximum occurs by setting the derivative of the function equal to zero:

```
>> solve('-2*exp(-t) + 12*exp(-3*t) = 0','t')

ans =

   0.8959

>> 2*exp(-1/2*log(6))-4*exp(-3*1/2*log(6))

ans =

   0.5443
```

So the maximum value of 544.3 mV occurs at t = 895.9 ms.

9.5 (p338)

(a) At $t = 0$, resistor R_2 is shorted.

$$\omega_o = \frac{1}{\sqrt{LC}} = 500 \text{ rad/s.}$$

$$\alpha = \frac{1}{2RC} = 500\text{ s}^{-1},\ \text{so}\ R_1 = \underline{1000\ \Omega}$$

(b) The voltage across the capacitor cannot change in zero time, so $v(0^+) = v(0^-)$.

Looking at the circuit for $t = 0^-$, $v(0^-) = 0.5\dfrac{(1000)}{1000 + R_2}R_2 = 100$

Solving, $R_2 = \underline{250\ \Omega}$

(c) $v(t) = e^{-500t}(At + B)$. Define i_C, i_{R_1}, i_L flowing downwards.

$$v(0^+) = v(0^-) = 100 = B,\ \text{so}\ v(t) = e^{-500t}(At + 100)$$

$$i_L(0^-) = 0.5\frac{1000}{1000 + 250} = 0.4\text{ amperes} = i_L(0^+)$$

$$i_{R_1}(0^+) = \frac{100}{1000} = 0.1\text{ amperes so } i_C(0^+) = -0.4 - 0.1 = -0.5\text{ amperes}$$

$$i_C(t) = C\frac{dv}{dt} = 10^{-6}\left[(-500)e^{-500t}(At + 100) + Ae^{-500t}\right]$$

$$i_C(0^+) = -0.5 = 10^{-6}\left[(-500)(100) + A\right]$$

so $A = -450\ 000\ \text{V}\cdot\text{s}^{-1}$

Thus, $v(t) = e^{-500t}(-4.5\times10^5 t + 100)$

and $v(0.1) = \underline{-\ 212.3\ \text{V}}$

9.6 (p344)

$$\alpha = \frac{1}{2RC} = 100\text{s}^{-1},\ \omega_o = \frac{1}{\sqrt{LC}} = 224\text{ rad/s, so } \omega_d = 200\text{ rad/s}$$

For $t < 0$, $v = 3\left(\dfrac{100}{100 + 50}\right) = 2\text{ V} = v(0^-) = v(0^+)$

For the inductor, defining i_L as flowing downward, $i_L(0^-) = \dfrac{5}{500} = 0.01\text{ A} = i_L(0^+)$

(a) For $t > 0$, $i_C = C\dfrac{dv}{dt}$

where $v(t) = e^{-100t}(A\cos 200t + B\sin 200t)$

$v(0^+) = 2 = \text{A}$

Define i_C and i_R flowing downward. Then $i_C + i_L + i_R = 0$

At $t = 0^+$, $i_C(0^+) = -i_L(0^+) - i_R(0^+)$

$$= -0.01 - \frac{v(0^+)}{500}$$

$$= -0.01 - \frac{2}{500} = -0.014 \text{ A}$$

$$i_C(t) = C\frac{dv}{dt}$$

$$= 10\times10^{-6}\left[-100e^{-100t}(2\cos 200t + B\sin 200t) + e^{-100t}(-400\sin 200t + 200B\cos 200t)\right]$$

$$i_C(0^+) = -0.014 = 10\times10^{-6}\left[-100(2) + (200B)\right]$$

Solving, $B = -6$

Thus, $\left.\dfrac{dv}{dt}\right|_{t=0^+} = \dfrac{-0.014}{10\times10^{-6}} = \underline{-1400\ \dfrac{\text{V}}{\text{s}}}$

(b) $v(t) = e^{-100t}(2\cos 200t - 6\sin 200t)$ so $v(1 \text{ ms}) = \underline{0.695 \text{ V}}$

(c) These are several ways to solve this, including graphically. Using the iterative routine on a calculator, we find that $\underline{t = 1.609 \text{ ms}}$ is the first zero for $t > 0$. $\left(t = \dfrac{1}{200}\tan^{-1}\left(\dfrac{1}{3}\right) \text{ exactly}\right)$

9.7 (p349)

(a) $\alpha = \dfrac{R}{2L} = \underline{100 \text{ s}^{-1}}$

(b) $\omega_o = \dfrac{1}{\sqrt{LC}} = \underline{223.6 \text{ rad/s}}$ $\therefore$ underdamped with $\omega_d = 200$ rad/s

(c) $i(0^-) = 1$ A

since $i(0^+) = i(0^-)$, $i(0^+) = \underline{1 \text{ A}}$

(d) • define v_C, v_L with '+' on top.

$v_C(0^+) = v_C(0^-) = 100\times1 = 100$ V

$v_L(0^+) = v_C(0^+) - 100i(0^+) = 100 - 100 = 0.$ Since $v_L = L\dfrac{di}{dt}$, $\left.\dfrac{di}{dt}\right|_{t=0^+} = \underline{0}$

(e) $i(t) = e^{-100t}(A\cos 200t + B\sin 200t)$
$i(0) = 1 = A$

$$\frac{di}{dt} = e^{-100t}(-200A\sin 200t + 200B\cos 200t) - 100e^{-100t}(A\cos 200t + B\sin 200t)$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = (200B) - 100A = 0 \quad \text{so} \quad B = \frac{100}{200} = 0.5$$

Thus, $i(t) = e^{-100t}(\cos 200t + 0.5\sin 200t)$ and i (12 ms) = – 120.4 mA

9.8 (p351)

We're given a hint that "plug and chug" might not be a good route to try here, so instead let's simply note that as we have a series circuit, the inductor current (the quantity of interest) flows through each element. Further, $i_L = i_C = -3v_C$ (the last term coming from the dependent source).

Armed with the knowledge that $i_C = C\frac{dv_C}{dt}$, we can then write $C\frac{dv_C}{dt} = -3v_C$. Rewriting, $0.01\frac{dv_C}{dt} + 3v_C = 0$ or $\frac{dv_C}{dt} + 300v_C = 0$, which as a solution of the form $v_C = v_C(0^+)e^{-300t} = 10e^{-300t}$ V, $t > 0$. Since we can obtain $i_C(t)$ by differentiation and multiplying by C, $i_L = i_C = -30e^{-300t}$ A, $t > 0$.

9.9 (p355)

(a) $i_L(0^+) = i_L(0^-) = 10$ A $\qquad (i_s = 10$ A, $t < 0)$

(b) $v_C(0^+) = v_C(0^-) = 20\ (10) = 200$ V

(c) Since the inductor current cannot change in zero time, $v_R(0^+) = 20(10) = 200$ V

(d) $i_L(\infty) = -20$ A due to $i_s \to -20$ A

(e) $\alpha = \frac{R}{2L} = 10\ 000\,\text{s}^{-1}$ and $\omega_o = 1/\sqrt{LC} = 10\ 000$ rad/s
$\therefore$ circuit is critically damped. So,

$$i_L(t) = e^{-10^4 t}(A_1 t + A_2) - 20$$

Applying initial conditions,

$$i_L(0^+) = 10 = A_2 - 20 \qquad [1]$$

and so $A_2 = 30$

$$\frac{di_L}{dt} = -10^4 e^{-10^4 t}(A_1 t + A_2) + A_1 e^{-10^4 t}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -10^4 A_2 + A_1 = \frac{1}{L}\left[v_C(0^+) - v_R(0^+)\right]$$

or $\quad A_1 - 30\times10^4 = 10^3(200 - 200)$

so $\quad A_1 = 30\times10^4$

Thus, $i_L(t) = e^{-10^4 t}(30\times10^4 t + 30) - 20$

$= \underline{2.073\text{ A}}$

9.10 (p359)

(a) $i_L(0^+) = i_L(0^-) = \dfrac{10}{50} = \underline{0.2\text{ A}}$ $(v_S = 10\text{ V}, t < 0)$

(b) $v_C(0^+) = v_C(0^-) = 50\times\dfrac{10}{50} = \underline{10\text{ V}}$

(c) $i_L(\infty) = \dfrac{30}{50} = \underline{0.6\text{ A}}$ $(v_S \to 30\text{ V})$

(d) $\alpha = \dfrac{1}{2RC} = 10\text{s}^{-1}$ and $\omega_o = \dfrac{1}{\sqrt{LC}} = 8\text{ rad/s}$

Thus, the circuit is overdamped and

$$i_L(t) = Ae^{s_1 t} + Be^{s_2 t} + 0.6$$

where $s_1 = -4\text{s}^{-1}$ and $s_2 = -16\text{ s}^{-1}$

$$i_L(0) = 0.2 = A + B + 0.6 \qquad [1]$$

$$\frac{di_L}{dt} = -4Ae^{-4t} - 16Be^{-16t}$$

so $\left.\frac{di_L}{dt}\right|_{t=0^+} = -4A - 16B = \frac{1}{L}\left[-v_C(0)+30\right]$ or $-4A - 16B = \frac{20}{15.625}$ [2]

Solving Eqns. [1] and [2] simultaneously, A = – 0.4267 and B = 0.02667

so that $i_L(t) = -426.7e^{-4t} + 26.67e^{-16t} + 600$ mA

Thus, $i_L(0.1)$ = 319.4 mA

9.11 (p361)

$$i_L(0^-) = \frac{12}{1+5} = 2 \text{ A} \qquad = i_L(0^+)$$

$$v(0^-) = 12\frac{5}{1+5} = 10 \text{ V} \qquad = v(0^+)$$

- define i flowing out of "+" reference of $v(t)$.

$$v = L\frac{di}{dt} \text{ and } i = -C\frac{dv}{dt} \qquad \text{Thus, } v = -LC\frac{d^2v}{dt^2} \text{ or } \frac{d^2v}{dt^2} = -25v$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{-i_L(0^+)}{C} = \frac{-2}{0.005} = -400 \text{ V}$$

so an initial voltage of +400 V is required where – 6 V was needed previously. At the $v(t)$ node, an initial voltage of +10 V is required where 0 V was previously needed. Previously a gain of – 9 was obtained using R_1 = 10 kΩ and R_f = 90 kΩ. Now we require a gain of – 25, so replace R_f with 250 kΩ.