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SECTION ⇒ DS

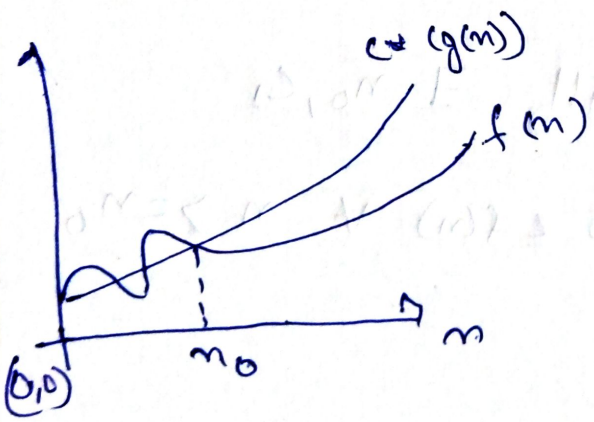
Q.1 ⇒ Asymptotic Notation: There are language to express the required time & space by an algorithm to solve a given problem.

(1) Big-O Notation: It is notation for the worst case analysis of an algorithm. (Upper bound)

According to it for a two func $f(n)$ & $g(n)$

$f(n) = O(g(n))$, if and only if there exist n_0 & c such that.

$0 \leq f(n) \leq c * g(n)$ for all $n \geq n_0$



Ans ⇒ $n+n^2 = O(n^2)$

here $f(n) = n+n^2$, $g(n) = n^2$.

$n+n^2 \leq n^2+n^2$ ($\because n < n^2$, $n^2 = n^2$)

$n+n^2 \leq 2n^2$ (here $c=2$) for $n_0=1$

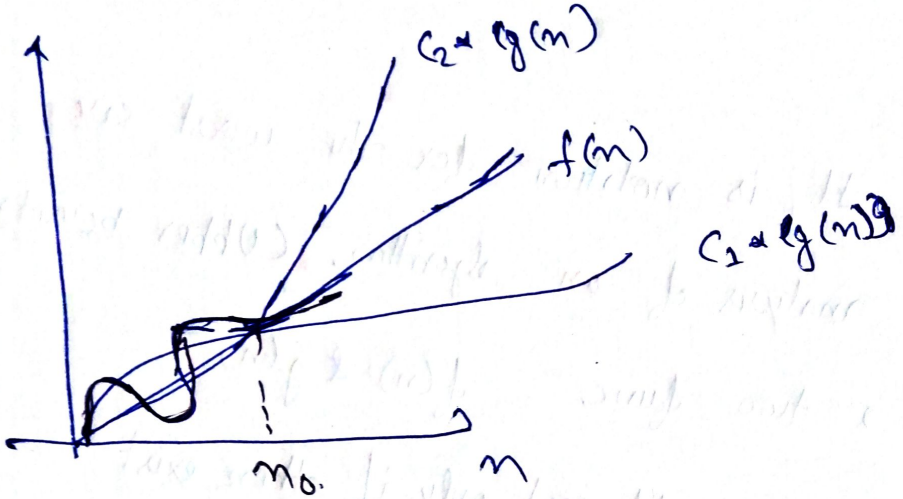
so $f(n) = O(g(n))$

or $n+n^2 = O(n^2)$

⊙ Big theta (Θ): For avg case time complexity (tightly bound)

for any two function $f(n)$ & $g(n)$

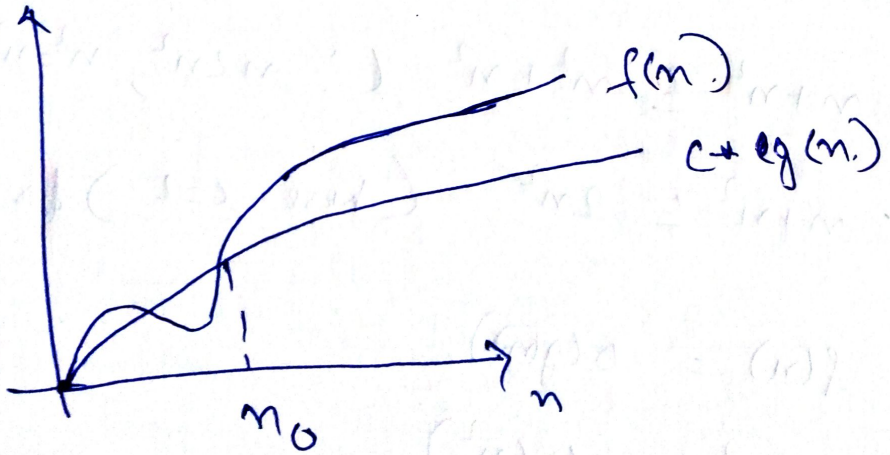
$f(n) = \Theta(g(n))$ if and only if there exists n_0, c_1, c_2 such that $0 \leq c_1 * g(n) \leq f(n) \leq c_2 * g(n)$ [for $n \geq n_0$]



⊙ Big Omega (Ω): for best case complexity (lower bound)

$f(n) = \Omega(g(n))$ iff $\exists\ n_0, c_1$

$\exists\ 0 \leq c_1 * g(n) \leq f(n)$ $\forall\ n \geq n_0$



Q.2 T.C. of for (i=1 to n) { i=i*2 }

Series → 1, 2, 4, 8, 16 ---- n (G.P.)

$a = 1$, $r = 2$

$t_k = ar^{k-1} \Rightarrow n = a*2^{k-1}$

$\Rightarrow n = 2^{k-1}$

$\Rightarrow 2^k = 2n$

$\Rightarrow k = 2\log_2 n$

so T.C. → $O(\log_2 n)$

Q.3 → $T(n) = \{ 3T(n-1) \}$ if $n>0$, otherwise 1 }

$T(n) = 3T(n-1)$ ---(i)

let $n = n-1$, $T(n-1) = 3T(n-2)$

$T(n) = 3^2 T(n-2)$

or $T(n) = 3^3 T(n-3)$

or $T(n) = 3^n T(n-n)$

$T(n) = 3^n T(0) = 3^n$

so T.C. → $O(3^n)$

Q.4) $T(n) = \begin{cases} 2T(n-1) - 1 & \text{if } n > 0, \\ 1 & \text{otherwise} \end{cases}$

$T(n) = 2T(n-1) - 1$

let $n = n-1$, $T(n-1) = 2T(n-2) - 1$

so $T(n) = 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2 - 1$

let $n = n-2$, $T(n-2) = 2T(n-3) - 1$

so $T(n) = 2^2(2T(n-3) - 1) - 2 - 1$

$= 2^3 T(n-3) - 2^2 - 2 - 1$

or

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \cdots - 2^1 - 2^0$

$T(0) = 1$, let $n-k = 0$ so $k = n$

$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - \cdots - 2^1 - 2^0$

$= 2^n - 2^{n-1} - 2^{n-2} - \cdots - 2^1 - 2^0$

$= 2^n - (2^{n-1} + 2^{n-2} + \cdots + 2^1 + 2^0)$ } G.P

$T(n) = 2^n - \frac{1(2^n - 1)}{2 - 1} = 2^n - 2^n + 1$

so T.C. ⇒ $O(1)$

Q.5 ⇒

```
int i=1, s=1;
while (s<=n){
        i++; s= s+i;
        printf("#");
}
```

Series ⇒ 1, 3, 6, 10, 15, 21, 28 ----- n

1st iteration ⇒ s = s+1

2nd iteration ⇒ s = s+1+2

till ⇒ $1+2+3+\dots+k <= n$

$$\frac{k*(k+1)}{2} <= n$$

or $O(k^2) <= n$

or $k = O(\sqrt{n})$

so T.C. = $O(\sqrt{n})$

Q6 ⇒

```
for (i=1 ; i*i <= n ; i++)
    count ++
```

let loop run till k i=k

$k^2 <= n$

$k <= \sqrt{n}$

no T.C ⇒ $O(\sqrt{n})$

Q7 ⇒

```
for (i=n/2 ; i<=n ; i++)                    O(n)
    for (j=1 ; j<=n ; j=j*2)                O(log n)
        for (k=1 ; k<=n ; k=k*2)            O(log n)
```

no T.C ⇒ $O(n \log^2 n)$

Q8 ⇒

```
function (int n) {
    if (n==1) return;
    for (i=1 to n) {
        for (j=1 to n) {
            print (n+n);
        }
    }
    function (n-3);
}
```

Recurrence Relation ⇒ $T(n) = T(n-3) + n^2$

or $T(n) = T(n-6) + 2*n^2$

$T(n) = T(n-9) + 3n^2$

or $T(n) = T(n-3k) + kn^2$

$T(1) = 0$, $n-3k = 1$ $\Rightarrow k = \frac{n-1}{3}$

so $T(n) = T(1) + \frac{(n-1)}{3} n^2$

so T.C. ⇒ $O(n^3)$

Q.9 ⇒

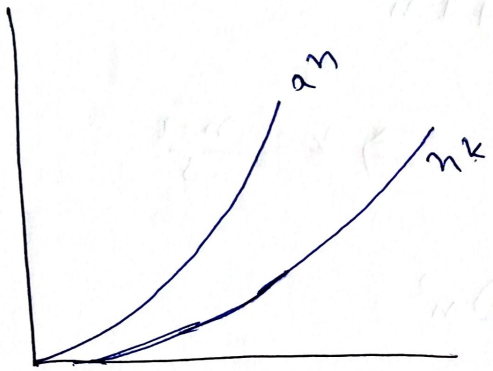
```
for (i=1 to n){
    for (j=1 ; j<=n ; j=j+i)
        printf ("*");
}
```

i	j	times
1	1→n	n
2	1→n	n/2
3	1→n	n/3
⋮	⋮	⋮
n	1→n	n/n

T.C. = ~~$O(n^2)$~~ $O(n \log n)$

Q.10⇒ Find asymptotic relation btw n^k & a^n, $k \geq 1$ & $a > 1$ are constants. Find c & n_0 for which relations holds.

Sol⇒



$n^k = O(a^n)$

$n^k \leq a^n$, c ∀ $c > 0$ & $n \geq n_0$

let $n = n_0$

$n_0^k \leq c \cdot a^{n_0}$

[so let $k = a = 3$]

[$n_0^3 \leq c \cdot 3^{n_0}$ so $c \geq 1$ & $n_0 \geq 1$]

Q.11 ⇒

```
void fun (int n){
    int i=0, j=1;
    while (i<n){
        i = i+j;
        j++;
    }}
```

Series ⇒ 0, 1, 3, 6, 10, 15 --

let at last iteration :

$$n = 0 + 1 + 2 + 3 + 4 + 5 + \cdots + k$$

$$n = \frac{k(k+1)}{2}$$

$$n = \frac{k^2+1}{2}$$

$$n \cong k^2$$

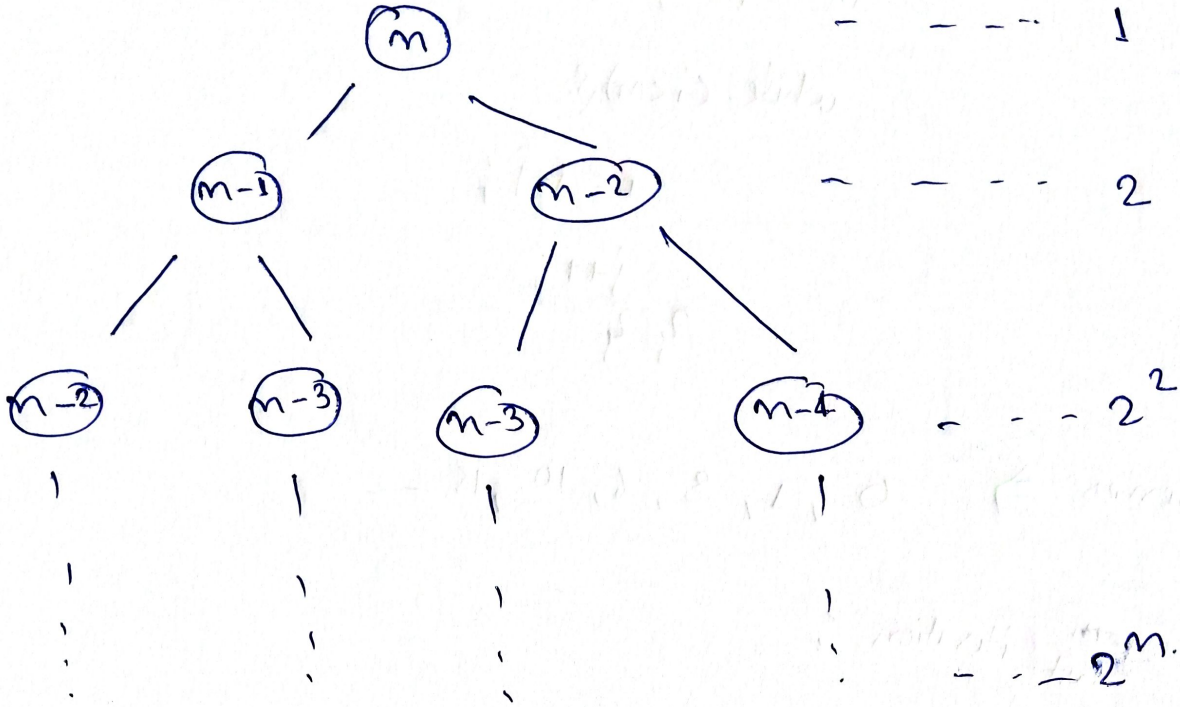
$$k \cong \sqrt{n}$$

so T.C. ⇒ $O(\sqrt{n})$.

Q.12 ⇒ Recurrence relation for fibonacci series..

$$T(n) = T(n-1) + T(n-2) + 1$$

using Recurrence tree method:



$$T.C. = 1 + 2 + 4 + \cdots + 2^n = 1\frac{(2^{n+1}-1)}{2-1} = 2^{n+1}-1$$

so $T.C. = O(2^n)$

Space Complexity: Space complexity of fibonacci series using recursion is proportional to height of recurrence tree.

so $S.C. \Rightarrow O(n)$

Q.13 ⇒ Write code for complexity.

(i) $n \log n$

```
for (i to n)
{
    for (j=1, j<=n, j*=2)
        O(1) statements
}
```

(ii) n^3

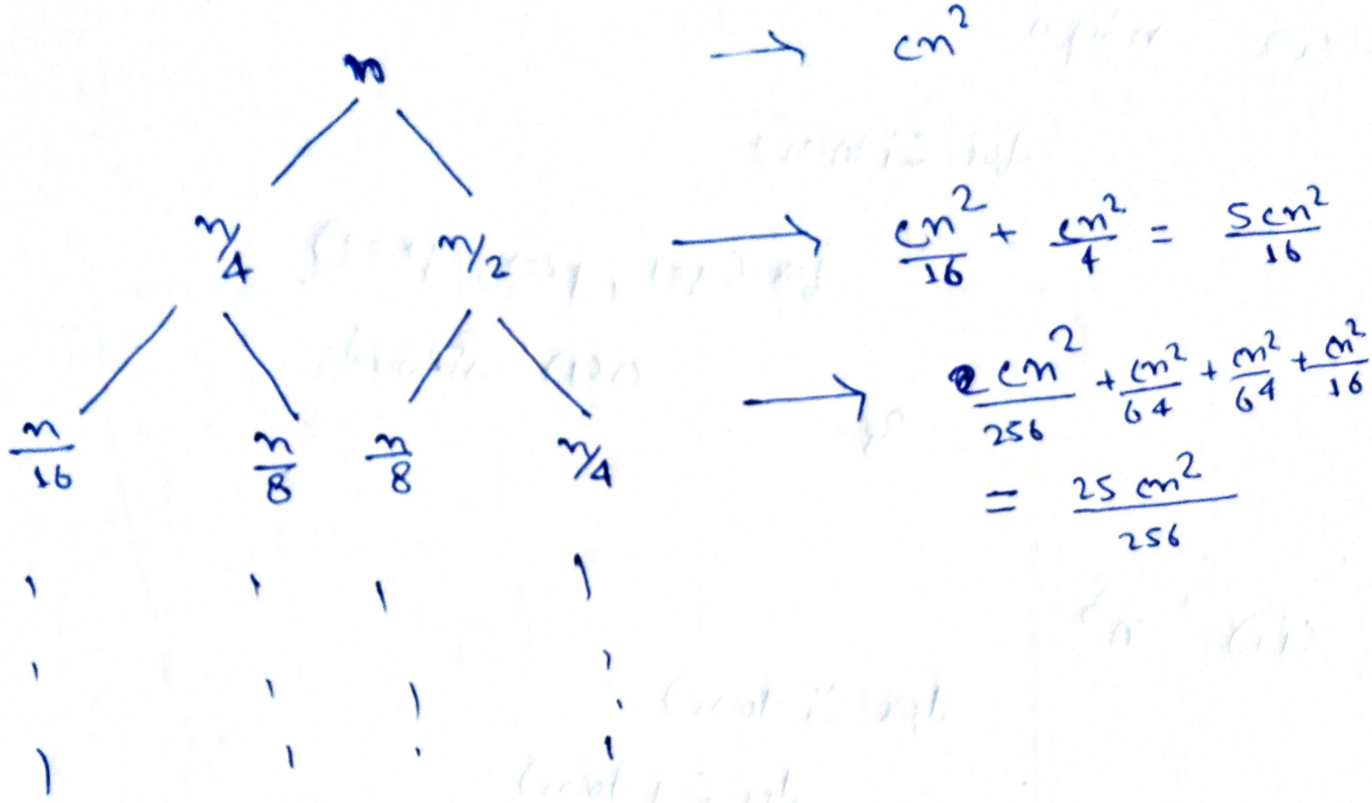
```
for (i to n)
    for (j to n)
        for (k to n)
            O(1) statements
```

(iii) $\log(\log n)$

~~for (int i=0; i<n;~~

```
int i=n;
while (i>0)
{ ---
  ---
  i = √i ;
}
```

Q.14) $T(n) = T(n/4) + T(n/2) + cn^2$



$$\text{No } T(n) = c(n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \cdots)$$

here $r = \frac{5}{16}$ no $S_n = \frac{1}{1-r}$

$$T(n) = cn^2\left(1 + \frac{5}{16} + \frac{25}{256} + \cdots\right)$$

$$= cn^2\left(\frac{1}{1-\frac{5}{16}}\right) = cn^2 \times \frac{16}{11}$$

$$\text{No T.C} \Rightarrow \Theta(n^2)$$

Q.15 ⇒

```
int fun (int n)
{
    for (i to n)
        for (j=1 ; j<n ; j+=i) {
            O(1) task
        }
}
```

i	j	times
1	1→n	n-1
2	1→n	(n-1)/2
3	1→n	(n-1)/3
⋮	⋮	⋮
n	1→n	n-1/n
		n log n

[T.C. ⇒ O(n log n)]

Q.16 ⇒

```
for (i=2 ; i<=n ; i=pow(i,k))
{
    O(1)
}
```

Sol ⇒ Series = $2, 2^k, 2^{2k}, 2^{3k}, \ldots,$

let last term be 2^{x*k}

$n = 2^{k^x}$

$\log n = x \log 2^k$

Q.16⇒ for (int i = 2; i<=n; i = pow(i,k))

{ O(1);

}

$i = 2, 2^k, 2^{k^2}, 2^{k^3} \cdots, 2^{k^x}$

$n = 2^{k^x}$

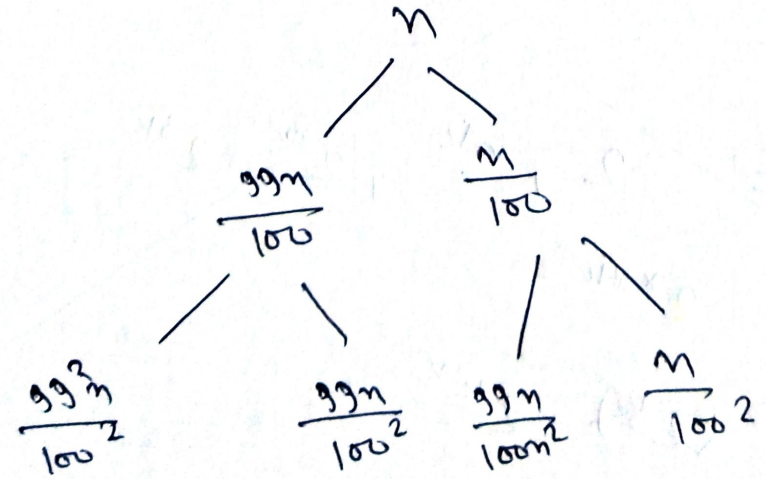
$\log n = k^x \log 2$

$\frac{\log \log n}{\log 2} = x \log k$

$x = \frac{\log \log n}{\log 2 + \log k}$

so T.C. ⇒ $O(\log \log n)$

Q.17⇒ ~~$T(n) = T(n-1) + n$~~ $T(n) = T(\frac{99n}{100}) + T(\frac{n}{100})$



If we take longer branch i.e. $\frac{99n}{100}$

T.C. $\Rightarrow \log_{\frac{100}{99}} n \simeq \log n$

$n = \left(\frac{99}{100}\right)^k \qquad k = \log_{\frac{100}{99}} n$

$T(n) = n \left(\frac{\log \frac{100}{99}}{100}\right)^n = O(n \log_{99} n)$

Q18 ⇒ Increasing of growth.

(a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

(b) $1 < \log\log n < \sqrt{\log(n)} < \log n < \log 2n < 2\log n < n < 2n < 4n < n^2 \log n < n^2 < \log(n!) < 2^{2n} < n!$

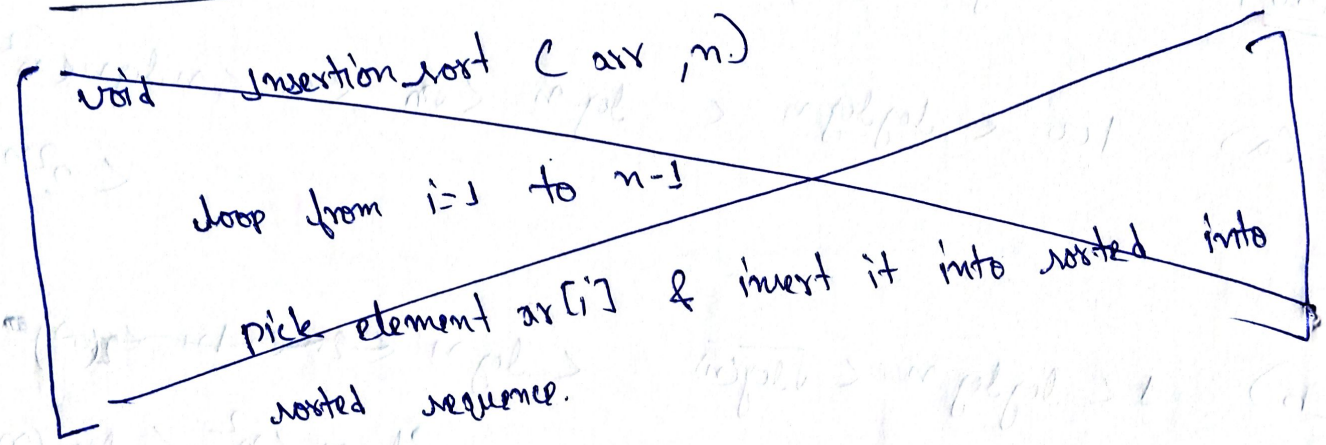
(c) $96 < \log_8 n < \log_2 n < 5n < n\log_8(n) < n\log_2 n < 8n^2 < 7n^3 < \log(n!) < n$

$\log n! < 8^{12n} < n!$

Q.19 ⇒ Linear Search:

```
for (i=0 to k-1)
{
    if (ar[i] = key)
    {
        return i;
    }
    return -1;
}
```

Q.20 ⇒ Iterative Insertion Sort:




```
void insertion_sort (int arr[], int n)
{
    int i, temp, j;
    for i<-1 to n
    {
        temp <- arr[i];
        j <- i-1;
        while (j>=0 AND arr[j] > temp)
        {
            arr[j+1] <- arr[j];
```

```
            j = j-1;
        }
        arr[j+1] = temp;
    }
}
```

Recursive Insertion sort :-

```
void recursive_insertion_sort (int arr[], int n)
{
    if (n <= 1)
        return
    recursive_insertion_sort (arr, n-1)
    val = arr[n-1]
    pos = n-2
    while (pos >= 0 && arr[pos] > val){
        arr[pos+1] = arr[pos]
        pos = pos-1
    }
    arr[pos+1] = val
}
```

It is called online sorting because it provided one sorted element at a time & ~~sequence of sorted as consider~~ produces a partial solution without considering future elements.

Q.21=>	Algorithm	Time complexity: Best case	Average Case	Worst Case
(1)	Bubble sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
(2)	Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
(3)	Merge sort	$O(n\log n)$	$O(n\log n)$	$O(n\log n)$
(4)	Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
(5)	Quick sort	$O(n\log n)$	$O(n\log n)$	$O(n^2)$
(6)	Heap sort	$O(n\log n)$	$O(n\log n)$	$O(n\log n)$

Q.22=> Algorithm	Inplace	Stable	Online Sorting
Bubble Sort	~	~	✗
Selection Sort	~	✗	✗
Merge Sort	✗	~	✗
Insertion Sort	~	~	~
Quick Sort	✗	✗	✗
Heap Sort	~	✗	✗

Q.23 ⇒ Recursive Binary Search:

```
int b_search ( int arr[], int l, int r, int x).
{
    if (l > r)
        return -1;
    int m = (l+r)/2
    if (arr[m] = x)
        return m;
    else if (arr[m] < x)
        return b_search (arr, m+1, r, x);
    else
        b_search (arr, l, m-1, x
}
```

Iterative Binary Search:

```
int binarysearch (int arr[], int n, int x)
{
    l=0, r=n-1;
    while ( l<r)
    {   m = (l+r)/2
        if (arr[m]=x)        return m;
        else if ( arr[m]<x)  l = m+1;
        else                 r = m-1;
}
```

return -1;

}

Time & Space Complexity of Iterative Binary search ⇒ O(log n) & O(1)

Time & Space Complexity of Recursive Binary search ⇒ O(log n), O(log n)

Q24⇒ Recurrence Relation for Binary Search ⇒

$$T(n) = T(n/2) + 1$$