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BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

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Professor Salivahanan is the recipient of IEEE Outstanding Branch Counsellor and Advisor Award in the Asia-Pacific region for 1966-97. He is the Senior Member of IEEE, Fellow of IETE, Fellow of Institution of Engineers (India), Life Member of ISTE and Life Member of Society for EMC Engineers.

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Tata McGraw Hill Education Private Limited
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San Juan Santiago Singapore Sydney Tokyo Toronto

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Tata McGraw-Hill

Published by Tata McGraw-Hill Publishing Company Limited,
7 West Patel Nagar, New Delhi 110 008.

Basic Electrical and Electronics Engineering

ISBN (13): 978-0-07-014612-9
ISBN (10): 0-07-014612-8

Managing Director: *Ajay Shukla*

Head—Higher Education Publishing: *Vibha Mahajan*

Manager—Sponsoring: *Shalini Jha*
Editorial Executive: *Tina Jajoriya*
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General Manager—Production: *Rajender P Ghansela*
Assistant General Manager—Production: *B L Dogra*

Typeset at Bharati Composers, D-6/159, Sector-VI, Rohini, Delhi 110 085, and printed at Print Shop Pvt Ltd., No. 4/310, Gandhi Street, Kottivakkam, Old Mahabalipuram Road, Chennai – 600096

Cover Printer: Print Shop Pvt Ltd

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Part II Electronics Engineering

PREFACE

This book has been primarily written with an objective of providing a single text covering the undergraduate syllabus on Basic Electrical and Electronics Engineering. It provides a comprehensive exposition of the principles of electrical and electronics engineering for undergraduate students of all engineering disciplines. Moreover, students pursuing diploma in EEE, ECE, CSE, IT, ICE and EIE streams as well as those appearing for competitive examinations will find this book to be a valuable reference.

The book has evolved from the lecture notes prepared for teaching the courses in engineering to undergraduate students. The authors make no claim to original research in preparing the text.

CHAPTER ORGANISATION

This book, spanning over 16 chapters, has been structured to offer comprehensive coverage of different topics prescribed by the universities. In-depth theoretical explanation along with numerous examples and practice problems facilitate sound understanding of concepts. To build a strong foundation in the subject, each chapter offers several worked-out examples, recapitulation of important formulae, review questions and practice problems. The pedagogy includes Solved Problems (105), Numerical Questions (95) and Review Questions (425).

The book is divided in two parts. Part I is related to electrical engineering and Part II is devoted to electronics engineering. The chapter-wise contents of the book are summarised below.

Part I (Electrical Engineering)

Chapter 1 on Fundamentals of Electricity and DC Circuits includes Definition, Symbol and Unit of Quantities, Multiple and Sub-Multiple Units, Computation of Resistance at Constant Temperature, Temperature Dependence of Resistance, Computation of Resistance at Different Temperatures, Computation of at Different Temperatures, Ohm's Law-Statement, Illustration and Limitation Units—Work, Power and Energy (Electrical, Thermal and Mechanical), Circuits-Identifying the Elements and the Connected Terminology, Kirchoff's Laws–Statement and Illustration, Resistances in Series and Voltage Division Technique, Resistances in Parallel and Current Division Technique, Method of Solving a Circuit by Kirchhoff's Laws, and Star to Delta and to Star Transformations. **Chapter 2** on Magnetic Circuits covers Definition of Magnetic Quantities, Magnetic Circuit, Leakage Flux, Fringing Effect and Comparison between Magnetic and Electric Circuit.

Chapter 3 on Electromagnetic induction deals with Magnetic Effect of Electric Current, Current Carrying Conductor in Magnetic Field, Law of Electromagnetic Induction, Induced Emf, Self Inductance (L), Mutual Inductance and Coupling Coefficient between Two Magnetically Coupled Circuits (K). **Chapter 4** on AC Fundamentals gives a detailed exposition of Generation of Alternative EMF Terminology, Concept of 3-Phase EMF Generation, Root mean square (RMS) or Effective value, Average value of AC, Phasor Representation of Alternating Quantities and Analysis of AC Circuit.

Chapter 5 on Single phase ac Circuits gives thorough understanding of J Operator, Complex Algebra, Representation of Alternating Quantities in Rectangular and Polar Forms, R-L Series Circuit, R-C Series Circuit and R-L-C Series Circuit. It also deals with Admittance and its Components, Simple Method of Solving Parallel AC Circuits, Resonance and Three Phase AC Circuits. **Chapter 6** on Electrical Machines covers DC Generator, DC Motor, Transformer, 3-phase Induction motor, Single Phase Induction Motors, 3-phase AC Generator or Alternator and synchronous Motors.

Chapter 7 on Measuring Instruments discusses Classification of Instruments, Basic Principles of Indicating Instruments, Moving Iron Instruments (Attraction Type and Repulsion Type) Moving Coil Instruments (Permanent Magnet Type and Dynamometer Type), Induction Type Energy Meter and Megger. **Chapter 8** on House Wiring covers Wiring Materials and Accessories, Types of Wiring, Basic Principles of Earthing and Wiring Layout for a Residential Building. The last **Chapter 9** in Part I on Transmission and Distribution of Electrical Power covers Power Generation, Transmission System, and OH and UG Systems.

Part II (Electronics Engineering)

Chapter 10 on Passive Circuit Components offers comprehensive information on Resistors, Capacitors and Inductors. **Chapter 11** on Transducers deals with Capacitive Transducer, Inductive Transducer, Linear Variable Differential Transformer (LVDT), Oscillation Transducer, Potentiometric Transducer, Electrical Strain Gauges, Resistance Thermometer, Thermistor, Thermocouple, Hall Effect, Piezoelectric Transducer and Photoelectric Transducer.

Chapter 12 on Junction Diode and its Applications gives an understanding of Semiconductor Theory, Theory of PN Junction Diode, PN Diode Applications, Zener Diode, Varactor Diode, Tunnel Diode, rectifiers, Filters, Diode Clippers, Diode Comparator, Clampers and Hall Effect. An Introduction of Bipolar Junction Transistor, Construction of BJT, Transistor Biasing, operation of NPN Transistor and PNP Transistor, Types of Transistor Amplifier Configuration, Transistor as an Amplifier, Large Signal DC, and Small Signal CE Values of Current Gain, Field Effect Transistor, Thyristor, OPTO-Electronic Devices, Display Devices, Cathode Ray Oscilloscope (CRO) and Power Conditioning Equipments are given in **Chapter 13** on Transistors and other devices.

Chapter 14 on integrated Circuits covers Advantages, limitation and Classification of ICs, Manufacturing Processes of Monolithic ICs, Linear ICs, Ideal Operational Amplifier, Operational, Ideal Voltage Transfer Curve, Open-loop and Closed-loop Op-amp Configurations, Op-amp Configurations, bandwidth with feedback, Noise, Frequency Response and Compensation, Op-Amp Applications and IC 555 Timer. **Chapter 15** on Digital Electronics covers Number System, Binary Arithmetic, 1's and 2's Complements, Binary Coded Decimal, Boolean Algebra, Logic Gates, Combinational Logic Design, Karnaugh Map Representation of Logical Functions, some Common Combinational Circuits, Sequential Circuits, A/D and D/A Converter Circuits and Logic Families. **The last Chapter 16** on Communication systems informs about Telecommunication Services, Analog and Digital Signals, Basic Principles of Modulation, Pulse Modulation Techniques, Pulse Digital Modulation-Pulse Code Modulation, Digital Modulation Techniques, data Transmission, Modem, Radio Broadcast, Television, microwave communication. Satellite Communication, Radar System , Optical Fibre Communication, ISDN and Internet.

We extend our appreciation to the Tata McGraw Hill Education team, especially Vibha Mahajan, Shalini Jha, Tina Jajoriya, Dipika Dey and Anjali Razdan for coordinating with us during the editorial, copyediting and production stages of this book. We sincerely thank Mr R Gopalakrishnan, SSN College of Engineering for the efficient word processing of some chapters.

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Part I

Electrical Engineering

FUNDAMENTALS OF ELECTRICITY AND DC CIRCUITS

1

INTRODUCTION

Can we perceive the electricity by any one of our five senses? Or can we see the electricity directly? The answer is No. We can perceive electricity through the different causes of electric current. For example, heat from a heater is due to the conversion of electrical energy to thermal energy (thermal effect). Then, what is electricity? This chapter answers this question and explains the other related concepts.

Modern Electron Theory

Several theories about electricity were developed through experiments and observations of its behaviour. But, the only theory that survived over the years to explain the nature of electricity is the Modern Electron Theory of Matter.

All matters (solid, liquid or gas) are composed of minute particles called molecules. Each molecule is made up of atoms.

The central part of an atom is called an nucleus. The nucleus contains protons and neutrons. Protons are positively charged particles while the neutrons have no charge. But, both have the same mass. The outer part of an atom is known as extra-nuclear space and contains only electrons. The mass of electrons is very small when compared to the mass of protons and neutrons. An electron is negatively charged particle having negative charge equal to the positive charge of a proton. The electrons move round the nucleus in different orbits as per the following rules:

(a) The maximum number of electrons in each shell is given by $2n^2$ where *'n'* is the principal quantum number.

(b) A sub shell can accommodate 2 $(2l + 1)$ electrons, where *'l'* is the orbital quantum number.

(c) The possible values of magnetic orbital quantum number, m_1, lies in the range $+ l$ to $- l$ including zero.

(d) The magnetic spin quantum number, m_s, can have only two values $+\frac{1}{2}$ or $-\frac{1}{2}$.

Mass number = No. of protons + No. of neutrons

Atomic number = No. of protons or No. of electrons in an atom.

From the above theory, it is clear that (i) Every matter is electrical in nature, i.e. it contains particles of electricity namely protons and electrons. (ii) Under ordinary conditions, a body does not possess any charge because the number of protons is equal to the number of electrons. Therefore, in normal condition, an atom is neutral.

Free Electrons

All the electrons in one atom move around its nucleus in different orbits. The electrons that revolve very near the nucleus, i.e. in the inner orbits, are very tightly bound to the nucleus. The binding force of electrons with the nucleus goes on decreasing if we move away from the nucleus. So, the electrons in the last orbit are very loosely bound to the nucleus. These electrons are called valence electrons. In metals, these valence electrons are so weakly attached to their nuclei that they can be removed or detached. Such electrons are called free electrons.

The free electrons move at random from one atom to another in the material. All electrons in a metal are not free electrons. One atom of a metal can provide at the most one free electron. But, one can expect a large number of free electrons in a small piece of metal, as the small piece of metal has large number of atoms.

Nature of Electricity

(i) The number of protons is equal to the number of electrons in a body under normal conditions. So, the resultant charge is zero and the boy is electrically neutral. Thus, the paper of this book does not possess any charge and is electrically neutral.

(ii) If we remove some electrons from a neutral body, there occurs a deficit of electrons in the body. Consequently, it attains a positive charge.

(iii) If we supply some electrons to a neutral body, there occurs an excess of electrons. Consequently, the body attains a negative charge.

From the above, it is clear that whether a given body exhibits electricity (i.e. charge) or not depends upon the relative number of particles of electricity, i.e. protons and neutrons and especially the electrons.

1.1 DEFINITION, SYMBOL AND UNIT OF QUANTITIES

1.1.1 Unit of Charge

The charge on an electron is so small that it is not convenient to select it as the unit of charge. Normally, coulomb is used as the unit of charge. One coulomb of charge is equal to the charge on 6.28×10^{18} electrons.

$$1 \text{ coulomb} = \text{Charge on } 6.28 \times 10^{18} \text{ electrons.}$$

Therefore, if we say that a body has a positive charge of 1 coulomb, it means that it has a deficit of 628×10^{16} electrons. The symbol for the charge is Q or q.

1.1.2 Electric Potential

When a body is charged, certain amount of work is done in charging it. This work done is stored in the form of potential energy. The charged body has the capacity to do work by moving the other charges by either attraction or repulsion. The ability of the charged body to do work is called electric potential.

The greater the capacity of a charged body to do work, the greater is its electric potential. And, the work done to charge a body to 1 coulomb is the measure of its electric potential.

Electric Potential, $V = \dfrac{\text{Work done}}{\text{Charge}} = \dfrac{W}{Q}$

The work done is measured in joules and charge in coulombs. Therefore, the unit of electric potential is Joules/Coulomb (or) volt. If $W = 1$ joule, $Q = 1$ coulomb, then $V = 1/1 = 1$ volt.

Hence, a body is said to have an electric potential of 1 volt if 1 joule of work is done to give it a charge of 1 coulomb.

When we say that a body has an electrical potential of 5 volts, it means that 5 joules of work has been done in charging the body to 1 coulomb or every coulomb of charge possesses an energy of 5 joules. The greater the joules/coulombs on a charged body, the greater is its electric potential.

1.1.3 Electric Current

The flow of free electrons in a metal is called electric current. The flow of electric current is explained with the help of the Fig. 1.1. The copper strip has a large number of free electrons. Prior to the application of voltage, the free electrons wander through the copper strip in all directions. At any section of the strip, the number of electrons that move from right to left equals the number of electrons that move from left to right. The net number of electrons crossing the section is zero. Hence, there is no flow of electric current under normal condition.

When electric pressure or voltage is applied, then the free electrons which are negatively charged will start moving towards the positive terminal round the circuit as shown in Fig. 1.1. This directed flow of electrons is called electric current.

The actual direction of current, i.e. motion of electrons, is from the negative terminal to the positive terminal of the cell through the part of the external circuit. However, prior to the modern electron theory, it was believed that electric current

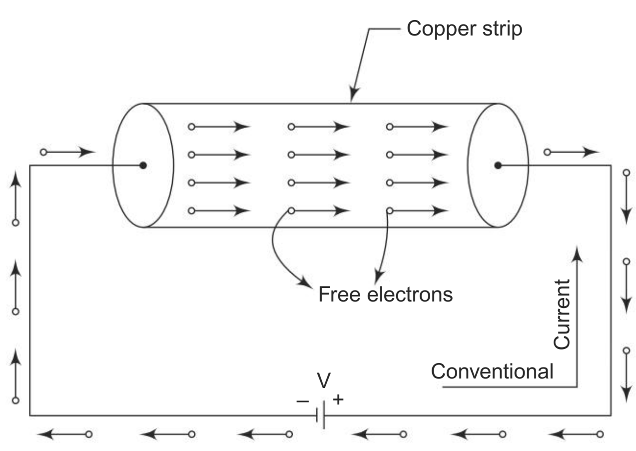


Fig. 1.1

was the movement of positive electricity from the positive to the negative terminal of the cell. This convention is so firmly established that it is still in use. This assumed direction is called conventional current.

Unit of Current The strength of electric current I is the rate of flow of electrons, i.e. charge flowing per second.

$$\text{Electric Current} = I = \frac{dq}{dt} \text{ or } \frac{Q}{t}$$

The charge Q is measured in coulomb and time t is measured in seconds. Therefore, the unit of current is coulomb/second or Ampere. If $Q = 1$ coulomb, $t = 1$ second, then $I = 1/1 = 1$ Amp.

One Ampere of current is said to flow through a wire, if at any section, one coulomb of charge flows in one second.

1.1.4 Potential Difference

The difference in the potentials of two charged bodies is called potential difference.

Consider two bodies A and B having potentials of 5 volts and 2 volts respectively as shown in Fig. 1.2 (i). Each coulomb of charge on body A has an energy of 5 joules and each coulomb of charge on body B has an energy of 2 joules. So, body A is at higher potential than body B and the potential difference is 3 volts.

If the two bodies are joined through a wire [Fig. 1.2. (ii)], the electrons will flow from body B to body A. The conventional current flow will be in the opposite direction, i.e. from body A to body B. When the two bodies attain the same potential, the flow of current stops. Potential difference is sometimes called voltage.

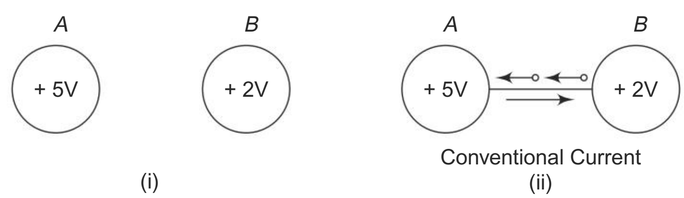


Fig. 1.2

Unit The unit of potential difference is also same as that of electric potential, i.e. volt.

The potential difference between two points is 1 volt if 1 joule of work is done in transferring 1 coulomb of charge from one point to other. (1 joule of words is done in this case, if 1 coulomb is transferred from lower potential point to higher potential point and 1 joule of work will be released as heat if 1 coulomb of charge moves from higher potential point to lower potential point.)

1.1.5 Electromotive Force (EMF) and Maintaining Potential Difference

A device that maintains potential difference between two points is said to develop electromotive force (e.m.f.). A simple example is that of a cell. Figure 1.3 shows the

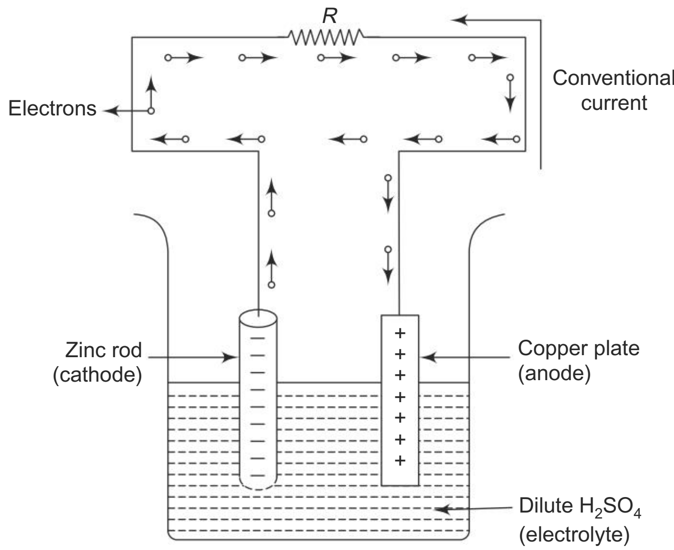


Fig. 1.3

voltaic cell. It consists of a copper plate called anode and a zinc rod called cathode immersed in dilute H_2SO_4 known as electrolyte.

The chemical reaction taking place in the cell removes electrons from the copper plate and transfers them to the zinc rod through the agent, i.e. the electrolyte dilute H_2SO_4. Consequently the copper plate attains positive charge of $+Q$ coulombs and the zinc rod attains negative charge of $-Q$ coulombs. During this process, the chemical action of the cell has done certain amount of work, say W joules. Hence, the potential difference between the two plates is W/Q volts.

If the two plates are joined by a wire (having lumped resistance R) the upper plate will attract some electrons from the zinc rod through the wire. The chemical action of the cell now transfers equal amount of electrons from copper plate to zinc rod internally (i.e. through the electrolyte) to maintain the original potential difference, i.e. W/Q volts. This process continues so long as the circuit is complete or so long as there is chemical energy. The flow of electrons through the external wire from zinc rod to the copper plate is the electric current.

Thus, the potential difference causes current to flow while an e.m.f maintains the potential difference. Both are measured in volts. Hence, e.m.f is not a force that causes a current flow. EMF is the energy supplied to the charge by an active device such as a cell or a battery.

1.1.6 Resistance

In Fig. 1.1, when the voltage is applied to the copper strip through the connecting wire, current starts to flow or the electrons start moving. While moving, the electrons collide with the atoms and molecules of the copper strip. Because of this collision or obstruction, the rate of flow of electrons or current is limited. And, we can say that there is some opposition for the flow of electrons or current.

The opposition offered by a substance to the flow of electric current is called resistance or resistance may be defined as the physical property of a substance due to which it opposes or restricts the flow of electricity (i.e. electrons) through it.

Certain substances offer very little opposition to the flow of electric current and are called conductors. For example, metals, acids, and salt solutions. Amongst pure metals, silver, copper and aluminium are very good conductors in the given order. Certain substances offer very high resistance or hindrance to the flow of electric current and are called insulators or relatively poor conductors of electricity. For example, bakelite, mica, glass, rubber, P.V.C., –dry wood, etc.

Resistance is the electric friction (which is similar to the friction mechanics) offered by the substance. It causes production of heat with the flow of electric current. Resistance of a substance is actually a distributed one. But, for practical studies, it can be lumped and denoted by R as shown in Fig. 1.4.

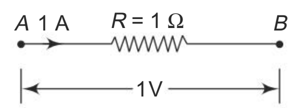


Fig. 1.4

Unit The practical unit of resistance is ohm and is represented by the symbol Ω (Omega). A wire is said to have a resistance of 1 ohm if a p.d. of 1 V across the ends causes current of 1 Amp to flow through it or a wire is said to have a resistance of 1 ohm if it releases 1 joule or develops (0.24 calorie of heat) when a current of 1 A flows through it.

1.2 MULTIPLE AND SUB-MULTIPLE UNITS

Table 1.1

Sl. No.	*Prefix*	*Its Meaning*	*Abbreviation*	*Equal*
1.	Tera	One Billion	T	10^{12}
2.	Giga	—	G	10^{9}
3.	Mega	One Million	M	10^{6}
4.	Kilo	One Thousand	k	10^{3}
5.	Centi	One Hundredth	–	10^{-2}
6.	Milli	One Thousandth	m	10^{-3}
7.	Micro	One Millionth	μ	10^{-6}
8.	Nano	—	n	10^{-9}
9.	Pico	One Billionth	p	10^{-12}

The above multiple and sub-multiple units are applicable to all units such as ohm, volt, ampere, henry, farad, etc.

1.3 COMPUTATION OF RESISTANCE AT CONSTANT TEMPERATURE

The resistance R of a conductor depends upon the following factors:

(i) Length If the length of a conductor is increased, the distance that the free electrons must travel is increased. As the distance is increased the free electrons will be obstructed by more atoms and molecules which are present in the lengthy conductor. Therefore, the resistance of the conductor is increased or in other words, the resistance R of a conductor is directly proportional to its length *(l)*

i.e. $R \propto l$

(ii) Cross Sectional Area For given free electrons to flow, if the area of cross section of the conductor is increased, the path for the flow of free electrons is increased. Hence opposition offered is decreased. And so, resistance offered by the conductor is also decreased or in other words, the resistance (R) of the conductor is inversely proportional to the area of cross section (a) of the conductor.

i.e. $R \propto l/a$

(iii) Nature at Material Different materials have different atomic structure because each material has different atomic weight and atomic number. Therefore, the resistance offered by each material will be different because of the difference in free electrons available. In other words, the resistance of the material depends upon the nature of material. For example, a steel wire offers more resistance than a copper wire of same length and same cross-sectional are.

(iv) Temperature In general, resistance of a conductor changes with temperature.

From the first three points and assuming the temperature to remain constant for the time being,

We get $R \propto l/a$

or $R = \rho\, l/a$ (1.1)

where ρ (Greek letter 'Rho') is a constant and is known as resistivity or specific resistance of the material of the conductor. Its value depends upon the nature of material.

1.3.1 Specific Resistance or Resistivity

We have, $R = \rho\, l/a$

If $l = 1$ m, $a = 1$ m^2 then $R = \rho$.

So, specific resistance of a material is the resistance offered by 1 m length of the wire of material having an area of cross-section of 1 m^2. [Fig. 1.5(i)].

or

For any two faces of a cube of the material having each side 1 m as in Fig. 1.5 (ii), $l = 1$ m, $a = 1$ m^2. So, specific resistance may be defined as the resistance between the opposite faces of a metre cube of the material [Fig. 1.5(ii)]

Unit of Resistivity

We know $R = \rho\, l/a$

or $\rho = R\, a/l$

From the above, the unit of resistivity depends upon the units of area of cross section and length. If length is measured in metres and area of cross-section (a) in m^2, then the unit of resistivity

$$= \frac{\text{Ohm} \times m^2}{m} = \text{Ohm} - m$$

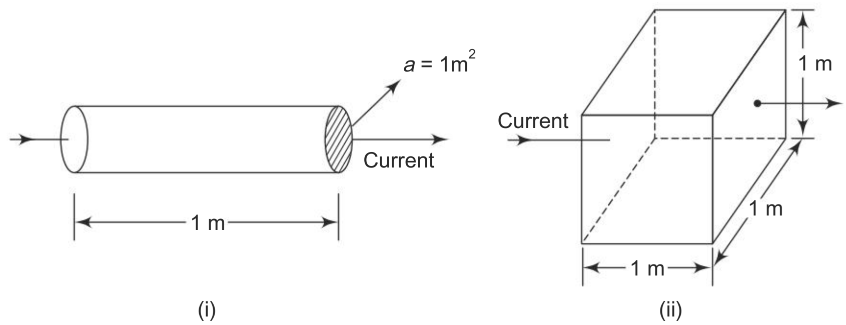


Fig. 1.5

1.3.2 Conductance

While resistance is the opposition to the flow of current, conductance is the inducement to the flow of current. Hence, conductance (G) is the reciprocal of resistance.

i.e. $G = 1/R$ (1.2)

We know $R = \rho\, l/a$

$G = (1/\rho)\,(a/l) = \sigma\, a/l$ (1.3)

where σ (Greek letter sigma) = $1/\rho$ is a constant and called conductivity or specific conductance of the material.

The unit of conductance is mho (℧), i.e. ohm spelt backward.

The unit of conductivity

$$= \frac{Gl}{a} = \text{mho}.\frac{m}{m^2} = \text{mho/m}$$

1.4 TEMPERATURE DEPENDENCE OF RESISTANCE

When the temperature of a substance increases the molecules vibrate more rapidly, impeding the movement of free electrons through the substance. On increasing the temperature, there is no further increase in free electrons in a conductor. Because of this, the resistance increases. The resistance of pure metals (e.g. copper aluminium) increases with rise in temperature and they have positive temperature co-efficient of resistance. The change in resistance is fairly regular for normal range of temperatures. So, the temperature versus resistance graph is a straight line as shown in Fig. 1.6 (for copper).

If this line is extended backward, it cuts temperature axis at – 234.5°C. It means that theoretically, the resistance of copper is zero at – 234.5°C. But, actually, the curve departs from the straight line path at very low temperatures.

A temperature rise in insulators and semiconductors creates many more free electrons than excited in the cooler state. These offset the interference to the drift movement caused by the increased molecular activity. So, resistance of such materials decreases with rise in temperature. These materials have negative temperature

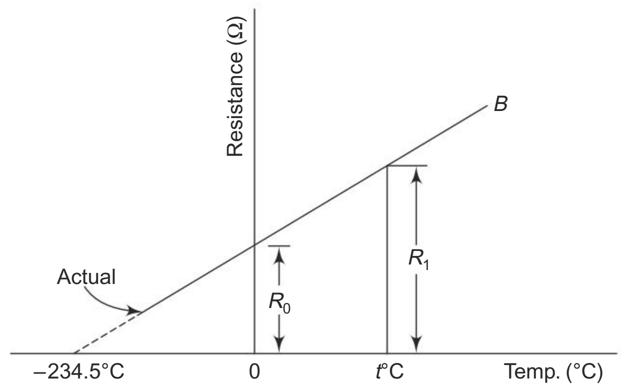


Fig. 1.6

co-efficient of resistance. Examples of such materials are, electrolytes, insulators, glass, mica, rubber, etc. and semiconductors (silicon, germanium, etc.).

In case of alloys, the resistance increases with the rise in temperature. This increase is very small and irregular. For some high resistance alloys (e.g. magnanimous, Eureka, etc.) the change in resistance is practically negligible over a wide range of temperature.

1.5 COMPUTATION OF RESISTANCE AT DIFFERENT TEMPERATURES

The change in resistance of a material with rise in temperature can be expressed by means of temperature co-efficient of resistance. Consider a conductor having resistance R'_0 at 0°C and R_t at t°C. It has been found that in the normal range of temperatures, the increase in resistance, i.e. $(R_t - R_0)$.

(i) is directly proportional to the initial resistance.

i.e. $(R_t - R_0) \propto R_0$

(ii) is directly proportional to the rise in temperature

i.e. $(R_t - R_0) \propto t$ and

(iii) also depends upon the nature of material.

Combining the first two, we get

$$R_t - R_0 \propto R_0 \, t$$

or $$R_t - R_0 = \alpha_0 R_0 \, t$$

where α_0 is a constant and is called the temperature co-efficient of resistance at 0°C. Its value depends upon the nature of material and temperature.

From Eqn. (1.4), we get

$$R_t = R_0 (1 + \alpha_0 t) \tag{1.5}$$

1.5.1 Temperature Co-efficient of Resistance

From Eqn. (1.4); we get

$$\alpha_0 = \frac{R_t - R_0}{R_0\, t}$$

α_0 = increase in resistance/ohm original resistance/rise in temperature.

From the above, we can define that temperature co-efficient of a material is the increase in resistance per ohm original resistance per °C rise in temperature. The unit of α is ohms/ohm/°C, i.e./°C.

If a conductor has resistance R_0, R_1 and R_2 at 0°C, t_1°C and t_2°C respectively, then,

$$R_1 = R_0\,(1 + \alpha_0\, t_1)$$
$$R_2 = R_0\,(1 + \alpha_0\, t_2)$$
$$\frac{R_1}{R_2} = \frac{1 + \alpha_0\, t_1}{1 + \alpha_0\, t_2} \tag{1.6}$$

The above Eqn. (1.6) is used in determining the rise in temperature of winding of electrical machines.

1.5.2 Graphical Determination of α

The value of temperature coefficient of resistance (α) can also be determined graphically from temperature versus resistance graph of the material. Figure 1.7 shows the temperature/resistance graph for a conductor which is a straight line. The resistance of the conductor is R_0 at 0°C and it becomes R_t at t°C. By definition,

$$\alpha_0 = \frac{R_t - R_0}{R_0\, t}$$

But, from the graph,

$$R_t - R_0 = CD \quad \text{and} \quad t = \text{rise in temperature} = AD.$$

$$\therefore \qquad \alpha_0 = \frac{CD}{R_0 AD}$$

But, CD/AD is the slope of the straight line AB which is the temperature/resistance graph of the material.

$$\alpha_0 = \frac{\text{Slope of temp./resistance graph}}{\text{Original resistance}} \tag{1.7}$$

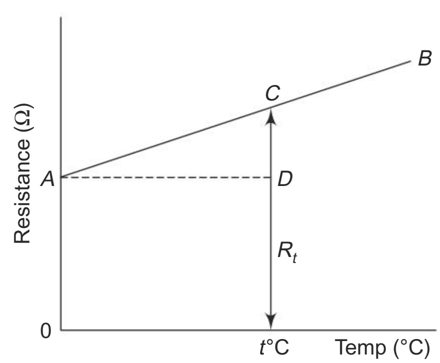


Fig. 1.7

Hence temperature coefficient of resistance at 0°C is the slope of temperature vs. resistance graph divided by the resistance at 0°C, i.e. R_0.

From the above, the following points may be noted:

(i) The value of α depends upon the temperature. At any temperature, α can be calculated by using Eq. (1.7).

$$\alpha_0 = \frac{\text{Slope of temp. vs. resistance graph}}{R_0}$$

and

$$\alpha_t = \frac{\text{Slope of temp. vs. resistance graph}}{R_t}$$

(ii) The value α_0 is maximum and it decreases as the temperature is increased. This is clear from the fact that the slope of temperature vs. resistance graph is constant and R_t has the maximum value.

1.6 COMPUTATION OF α AT DIFFERENT TEMPERATURES

Consider a conductor having resistance R_0, R_1 and R_2 at 0°C, t_1°C and t_2°C respectively. Let, α_0, α_1 and α_2 be the temperature coefficient of resistance at 0°C, t_1°*C* and t_2°C respectively. We have to establish the relationship between α_1 and α_0, α_2 and α_0 and α_1 and α_2. Figure 1.8 shows the temp. vs. resistance graph of the conductor.

We have proved in Sec. 1.6.2 that,

$$\alpha_0 = \frac{\text{Slope of graph}}{R_0}$$

Slope of graph = $\alpha_0 R_0$

Similarly

$$\alpha_1 = \frac{\text{Slope of graph}}{R_t}$$

Slope of graph = $\alpha_1 R_1$

$$\alpha_2 = \frac{\text{Slope of graph}}{R_2}$$

Slope of graph = $\alpha_2 R_2$

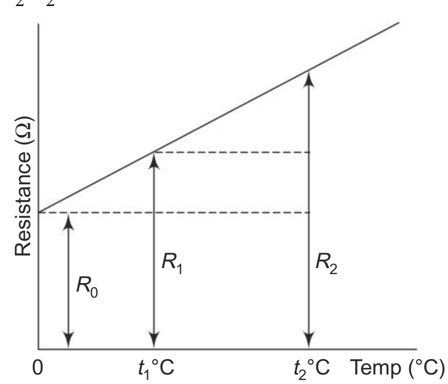


Fig. 1.8

Since the slope of temp. vs. resistance of the conductor is constant (from Fig. 1.8).

$$\alpha_0 R_0 = \alpha_1 R_1 = \alpha_2 R_2$$

or
$$\alpha_1 = \frac{\alpha_0 R_0}{R_1}$$

$$= \frac{\alpha_0 R_0}{R_0(1 + \alpha_0 t_1)} \quad [\because R_1 = R_0 (1 + \alpha_0 t_1]$$

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \tag{1.8}$$

and
$$\alpha_2 = \frac{\alpha_0 R_0}{R_2}$$

$$= \frac{\alpha_0 R_0}{R_0(1 + \alpha_0 t_2)} \quad [\because R_2 = R_0 (1 + \alpha_0 t_2)]$$

$$= \frac{\alpha_0}{1 + \alpha_0 t_2} \tag{1.9}$$

Subtracting the reciprocal of Eqn. (1.8) from the reciprocal of Eqn, (1.9), we get,

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{(1 + \alpha_0 t_2)}{\alpha_0} - \frac{(1 + \alpha_0 t_1)}{\alpha_0} = \frac{\alpha_0 (t_2 - t_1)}{\alpha_0}$$

$$= (t_2 - t_1)$$

$\therefore$
$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)} \tag{1.10}$$

Also, slope of graph $\tan\theta = \alpha_0 R_0 = \alpha_1 R_1 = \alpha_2 R_2$ and the increase in resistance as temperature rises from $t_1°C$ to t_2°C = slope of graph × rise in temperature.

$$= \tan\theta\,(t_2 - t_1)$$

$$= \alpha_1 R_1 (t_2 - t_1)$$

Resistance at t_2°C is $R_2 = R_1 + \alpha_1 R_1 (t_2 - t_1)$

or
$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)] \tag{1.11}$$

1.7 OHM'S LAW-STATEMENT, ILLUSTRATION AND LIMITATION

The relationship between the potential difference (V), the current (I) and resistance (R) in a d.c. circuit, first discovered by the scientist George Simon Ohm, is called Ohm's law.

Statement The ratio of potential difference between any two points of a conductor to the current flowing between them is constant, provided the physical conditions (e.g. temperature, etc.) do not change.

i.e.
$$V/I = \text{constant (or) } V/I = R \tag{1.12}$$

where R is the resistance between the two points of the conductor considered.

The unit of resistance (i.e. ohm) was named in honour of George Simon Ohm.

Illustration Let the P.D. between points A and B be V volts and current flowing be I amp.

Then V/I = constant = R (say)

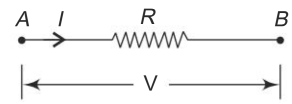


Fig. 1.9

We know that, if the voltage is doubled (2V), the current flowing will also be doubled (2I). So, the ratio V/I remains the same. (i.e. R).

Also, when voltage is measured in volts and current in amperes, then resistance will be in ohms.

Limitations

1. Ohm's Law does not apply to all non-metallic conductors. For example, for Silicon Carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity.
2. It also does not apply to non-linear devices such as Zener diode, voltage-regulator (VR) tubes and the like.
3. Ohm's law is true for metal conductors at constant temperature. If the temperature changes, the law is not applicable.

1.7.1 Special Note on Ohm's Law

1. Ohm's law can be expressed in three forms:
   i.e. $V/I = R;\quad V = IR;\quad I = V/R$
2. If Ohm's law is expressed graphically (taking voltage along Y axis and current along X axis), the graph will be a straight line passing through the origin. Obviously, the slope of the graph will give the resistance.

1.8 UNITS—WORK, POWER AND ENERGY (ELECTRICAL, THERMAL AND MECHANICAL)

1.8.1 Important Physical Quantities

(i) ***Mass*** It is the quantity of matter possessed by a body. The mass of a body is constant and is independent of place and position of the body. The SI unit of mass is kilogram (kg).

(ii) ***Force*** It is the product of mass (m in kg) and acceleration (a in m/sec^2). The unit of force is newton (N). One newton is the force required to accelerate a mass of 1 kg through an acceleration of 1 m/sec^2.

$$F = ma \text{ newtons}$$

(iii) ***Weight*** The force with which a body is attracted towards the centre of earth is called the weight of the body. If m is the mass of the body in kg and g is the acceleration due to gravity in m/sec^2, then, weight, $W = mg$ newtons. Though the value of (g) varies from place to place, its practical value is 9.81 m/sec^2.

Sometimes, weight is given in kg-wt units. One kg-wt is the weight of mass of 1 kg, i.e. 1 × 9.81 newtons.

1 kg. wt = 9.81 newtons

1.8.2 Some Cases of Mechanical Work or Energy

1. When a force of F newtons is exerted on a body through a distance 'd' meters in the direction of force, then work done = $F \times d$ N.m or Joules.
2. Suppose a force of F newtons is maintained tangentially at a radius r meters from O as in Fig. 1.10. In one revolution, the point of application of force travels through a distance of $2\pi r$ metres.

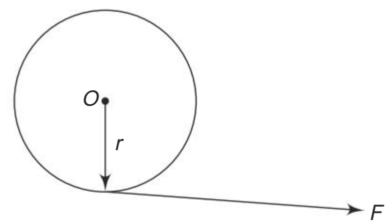


Fig. 1.10

   Work done in one revolution = Force × distance moved in one revolution

   $= F \times 2\pi r$

   $= 2\pi\, T$ Nm or Joules

   where $T = Fr$ is called torque. The SI unit of torque is clearly Nm or joules. If body makes N revolutions per minute then, work done per minute = $2\pi NT$ joules.
3. If a body of mass m kg is moving with a speed of v m/sec^2, then, kinetic energy (K.E.) possessed by the body is

   K.E. = $1/2\; mv^2$ joules
4. If a body of mass m kg is lifted vertically through a height of h mts and g is the acceleration due to gravity in m/sec^2, then,

   Potential energy of body = Work done in lifting the body
   = Force required × height
   = Weight of the body × height
   $= mg \times h$
   $= mgh$ joules

   The SI unit of mechanical work done and energy is the same and is joule.

1.8.3 Thermal Energy

The SI unit of thermal energy is calorie and one calorie is the amount of heat required to raise the temperature of 1 gram of water through 1°C which is also known as specific heat of water. If S is the specific heat of a body, then, amount of heat required to raise the temperature through θ°C is $ms\,\theta$ calories.

It has been found experimentally that 1 calorie = 4.186 joules.

So, heat required = $ms\,\theta \times 4.186$ joules.

Now, joule is also SI unit of heat.

1.8.4 Electrical Power

The rate at which work is done in an electric circuit is called electric power.

i.e. $$\text{Electric Power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to circuit, it causes current (i.e. electrons or charge) to flow through it. Clearly, certain amount of work is done in moving the electrons in the circuit.

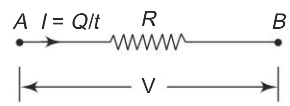


Fig. 1.11

In the Fig. 1.11,

V = p.d. across AB in volts

I = Current in Amps.

R = Resistance of AB in ohms

t = time in sec. for which current flows

The total charge that flows in t sec. is $Q = I \times t$ coulombs.

By definition, the p.d., $V = \dfrac{\text{Work done in electric circuit}}{Q}$

$$\text{Work done} = VQ = VIt$$

Electric power P = Work done/time = VIt/t

$= VI$ Joules/sec or watt

$P = VI$ watts

$P = I^2 R$ watts $\because V - IR$

$P = V^2/R$ watts $\because I = V/R$

Unit of Electric Power The basic unit of electric power is joule/sec or Watt. Note that 1 joule/sec. = 1 watt or 1 joule = 1 watt sec. Power in watts = voltage in volts × current in amps. The bigger units of electric power are kilowatts (kW) and Megawatts (MW).

$$1 \text{ kW} = 10^3 \text{ watts}$$

$$1 \text{ MW} = 10^6 \text{ watts or } 10^3 \text{ kW}$$

Other Units of Electric Power

1. Sometimes, power is measured in horse power (HP)

   1 H.P. = 735 watts.

2. If a body makes N rpm a.id the torque acting is T N.m then work done/minute = $2\pi NT$ joules (as in sub-section 1.9.2). Work done/sec = $2\pi NT/60$ joules/sec (or) watt

   i.e. Power = $2\pi NT/60$ watts

or Power = $\dfrac{2\pi NT}{60 \times 735}$ H.P.

1.8.5 Electrical Energy

The total work done in an electric circuit is called electrical energy.

i.e. Electrical energy = $P.D.$ × Total charge flow

$= VQ$ (Fig. 1.11)

$= VIt$ joules or watt.sec.

Electrical Energy $= VIt$

$= I^2 Rt$ (1.13)

$= V^2/R \times t$

Electrical energy is the product of electric power and time for which the current flows in the circuit.

The unit of electrical energy is joule or watt-sec if power is in watts and time in seconds. If power is in watts and time is in hours then unit of electrical energy is watt-hour. If power is in kilowatts and time is in hours then unit of electrical energy is kWh.

1 kWh of electrical energy is also called simply 1 unit.

Also, $$1 \text{ kWh} = 10^3 \times 60 \times 60 \text{ watt sec or joules}$$
$$36 \times 10^5 \text{ joules}$$
$$1 \text{ calorie} = 4.186 \text{ joules}$$
$$1 \text{ kcal} = 4186 \text{ joules}$$

and $$1 \text{ kWh} = \frac{36 \times 10^5}{4.186} \text{calories}$$

or $$1 \text{ kWh} = 860 \times 10^3 \text{ calories}$$

or $$1 \text{ kWh} = 860 \text{ kcal.}$$

Example 1.1 A wire of length 1 m has a resistance of 2 ohms. What is the resistance of second wire, whose specific resistance is double that of the first, if the length of wire is 3 m and the diameter is double that of first?

Solution:

For the first wire: $l_1 = 1\ m\ t,\ R_1 = 2\ \Omega\ d_1 = d(\text{say})$

$\rho_1 = \rho$ (say)

For the second wire: $l_2 = 3\ m\ t,\ d_2 = 2d$

$\rho_2 = 2\rho$

From the Eqn. (1.1),

$$R_1 = \rho_1 \frac{l_1}{a_1} = \frac{\rho \times 1}{\frac{\pi d^2}{4}}$$

i.e. $$R_1 = \frac{4\rho}{\pi d^2} \quad (1)$$

and $$R_2 = \rho_2 \frac{l_2}{a_2} = \frac{2\rho \times 3}{\frac{\pi (2d)^2}{4}}$$

i.e. $$R_2 = \frac{6\,\rho}{\pi d^2} \quad (2)$$

Dividing Eqn. (1) by (2) gives

$$\frac{R_1}{R_2} = \frac{4}{6}$$

or $$R_2 = \frac{6R_1}{4} = \frac{6 \times 2}{4}$$

i.e. $$R_2 = 3\ \Omega$$

Example 1.2 A rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (a) opposite ends and (b) opposite sides. The resistivity of copper is $1.7 \times 10^{-6}\ \Omega$ cm.

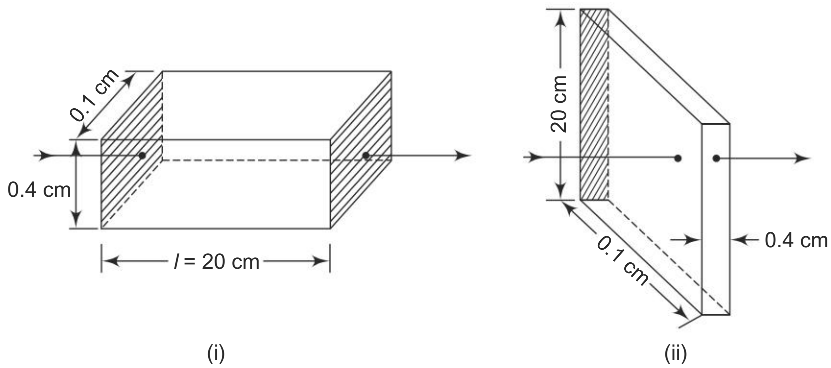


Fig. E.1.1

Solution

$l = 20$ cms; $w = 0.1$ cm; $t = 0.4$ cm; $\rho = 1.7 \times 10^{-6}\ \Omega$ cm

(*a*) Referring Fig. E.1.1 (i)

$l = 20$ cm; $a = 0.1 \times 0.4 = 0.04$ cm^2 (i.e. *wt*)

We know $R = \rho\, l/a$

$$R_1 = \frac{1.7 \times 10^{-6} \times 20}{0.04} = 0.85 + 10^{-3}\ \Omega$$

i.e. $R_1 = 0.85$ mΩ

(b) Referring to Fig. E.1.1 (ii)

$l = 0.4$ cm, $a = 0.1 \times 20 = 2$ cm^2

$$R_2 = \frac{1.7 \times 10^{-6} \times 0.4}{2} = 0.34 \times 10^{-6}\ \Omega$$

i.e. $R_2 = 0.34\ \mu\Omega$

Example 1.3 An aluminium wire 10 m long and 2 mm in diameter is connected in parallel to a copper wire 6 m long. A total current of 2 A is passed through the combination and it is found that current through the aluminium wire is 1.25 A. Calculate the diameter of copper wire. Specific resistance of copper is $1.6 \times 10^{-6}\ \Omega$ cm and that of aluminium is $2.6 \times 10^{-6}\ \Omega$ cm.

Solution

$l_a = 10$ m = 1000 cm

$d_a = 2$ mm = 0.2 cm

$\rho_a = 2.6 \times 10^{-6}\ \Omega$ cm

$l_c = 6$ m = 600 cm

$\rho_c = 1.6 \times 10^{-6}\ \Omega$ cm

$I = 2$ A

$I_a = 1.25$ A

Current through the copper wire $I_c = 2 - 1.25 = 0.75$ A

Resistance of aluminium wire $R_a = \rho_a\, l_a/a$

$$= \frac{2.6 \times 10^{-2} \times 1000}{\frac{\pi(0.2)^2}{4}}$$

$$= 0.08276\ \Omega$$

We know that current is inversely proportional to the resistance through which it flows (from Ohm's law).

Resistance of Copper wire $R_c = \dfrac{I_a}{I_c} R_a$

$$= \frac{1.25}{0.75} \times 0.08276 = 0.138\ \Omega$$

Also,

$$R_c = \rho_c \frac{l_c}{a_c} = \frac{\rho_c l_c}{\frac{\pi d_c^2}{4}}$$

$$d_c^2 = \frac{4\rho_c l_c}{R_c} = 8857 \times 10^{-6}$$

and $d_c = 0.094$ cm or 0.94 mm

Example 1.4 10 cc of copper is (i) drawn into a wire 100 m long (ii) rolled into a square sheet of 10 cm side. Find the resistance of the wire and the resistance between opposite faces of the sheet if resistivity of copper is $1.7 \times 10^{-6}\ \Omega$ cm.

Solution

$$10 \text{ cc} = 10 \text{ cm}^3 \ (1 \text{ cc} = 1 \text{ cm}^3)$$

$$l = 100 \text{ m} = 10000 \text{ cm}$$

$$\rho = 1.7 \times 10^{-6}\ \Omega \text{ cm}$$

(i) From Fig. E.1.2(i)

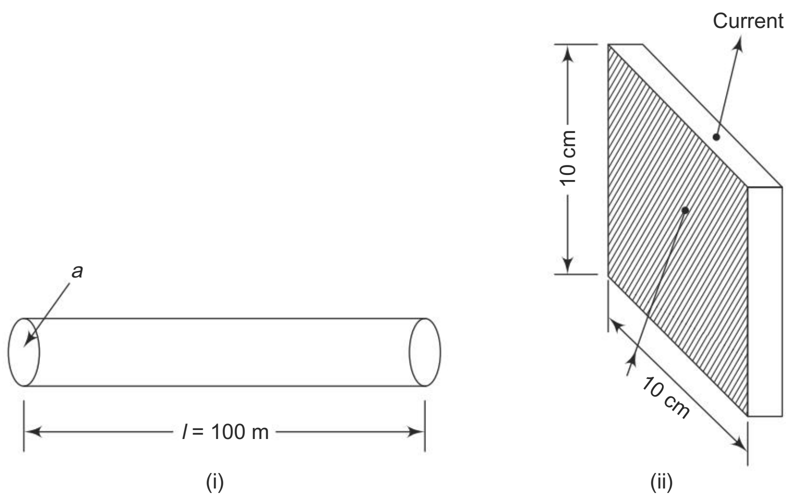


Fig. E.1.2

Area of cross-section a = Volume/length = 10/10000 $= 0.001 \text{ cm}^2$

$$R_1 = \frac{\rho l}{a} = \frac{1.7 \times 10^{-6} \times 10000}{0.001} = 17\ \Omega$$

(ii) From Fig. E.1.2(ii)

Area of cross-section, $a = 10 \times 10 = 100 \text{ cm}^2$

length (i.e. thickness) l = Volume/area = 10/100 = 0.1 cm

$$R_2 = \frac{\rho l}{a} = \frac{1.7 \times 10^{-6} \times 0.1}{100} = 1.7 \times 10^{-9}\ \Omega$$

Example 1.5 A platinum coil has a resistance of 3.146 Ω at 40°C and 3.767 Ω at 100°C. Find (i) the resistance at 0°C (ii) the temperature coefficient at 0°C and (iii) temperature coefficient at 40°C.

Solution

At $t_1 = 40\ °\text{C}$ $R_1 = 3.46\ \Omega$

At $t_2 = 100\ °\text{C}$ $R_2 = 3.767\ \Omega$

We know that $R_1 = R_0 (1 + \alpha_0 t_1)$

$R_2 = R_0 (1 + \alpha_0 t_2)$

So, $$\frac{R_1}{R_2} = \frac{1 + \alpha_0 t_1}{1 + \alpha_0 t_2}$$

$$\frac{3.146}{3.767} = \frac{1 + \alpha_0 40}{1 + \alpha_0 100}$$

on solving $$\alpha_0 = \frac{1}{264} \Big/ °\text{C}$$

$$R_0 = \frac{R_1}{1 + \alpha_0 t_1} = \frac{3.146}{1 + \frac{40}{264}}$$

i.e. $R_0 = 2.732\ \Omega$

Also, we know that $$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

So, $$\alpha_{40} = \frac{1/264}{1 + \frac{40}{264}}$$

i.e. $$\alpha_{40} = \frac{1}{304} \Big/ °\text{C}$$

Example 1.6 A copper coil has a resistance of 0.4 Ω at 12°C. Find its resistance at 52°C. Temperature coefficient of resistance of copper is given to be 0.004/°C at 0°C.

Solution

At $t_1 = 12°\text{C}$ $R_1 = 0.4\ \Omega$

$\alpha_0 = 0.004/°\text{C}$

We know that $$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} = \frac{1}{1/\alpha_0 + t_1}$$

$$\alpha_{12} = \frac{1}{\frac{1}{0.004} + 12} = \frac{1}{262} \Big/ ^\circ C$$

and $$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)]$$

$$R_2 = R_1 [1 + \alpha_{12} (t_2 - t_1)]$$

$$= 0.4 \left[1 + \frac{1}{262} (52 - 12)\right]$$

$$R_2 = 0.4611\ \Omega$$

Example 1.7 The field coil of a shunt motor has a resistance of 45 Ω at 20°C. Find the average temperature of the winding at the end of the run when the resistance is increased to 48.5 Ω. Temperature coefficient of resistance is 0.004/°C at 0°C.

Solution

At $t_1 = 20°C$ $\quad R_1 = 45\ \Omega$

At t_2 (= ?) $\quad R_2 = 48.5\ \Omega$

$$\alpha_0 = 0.004/°C$$

We know that

$$R_1 = R_0 (1 + \alpha_0 t_1)$$

$$R_2 = R_0 (1 + \alpha_0 t_2)$$

and $$\frac{R_1}{R_2} = \frac{1 + \alpha_0 t_1}{1 + \alpha_0 t_2}$$

$$\frac{45}{48.5} = \frac{1 + 20 \times 0.004}{1 + 0.004 t_2}$$

On solving $\quad t_2 = 41°C$

Example 1.8 A coil which has an initial temperature of 20°C is connected to a 180 V supply and the initial current is found to be 4 A. Sometime later, the current is found to have decreased to 3.4 A, the supply voltage being unchanged at 180 V. Determine the temperature rise of the coil. Assume the temperature coefficient of copper to be 0.0043/°C at 0°C.

Solution

$$V = 180 \text{ volts}$$

$$I_1 = 4 \text{ amp.}$$

$$t_1 = 20°C$$

$$I_2 = 3.4 \text{ A}$$

$$\alpha_0 = 0.0043/°C$$

$$t_2 = ?$$

Initial resistance of the coil, $\quad R_1 = \frac{180}{4} = 45\ \Omega$

Resistance of the coil after sometime $\quad R_2 = \frac{180}{3.4} = 52.94\ \Omega$

We know that $$\frac{R_1}{R_2} = \frac{1 + \alpha_0 t_1}{1 + \alpha_0 t_2}$$

$$\frac{45}{52.94} = \frac{1 + 0.0043 \times 20}{1 + 0.0043 t_2}$$

On solving $t_2 = 64.56°C$

Temperature rise $t = t_2 - t_1 = 64.56°C - 20°C = 44.56°C$

Example 1.9 When working normally, the temperature of the filament in a 230 V, 150 W gas filled tungsten lamp is 2750°C. Assuming a room temperature of 16°C, calculate (i) the normal current taken by the lamb (ii) the current taken at the moment of switching on. Temperature coefficient of tungsten is 0.0047/°C.

Solution

$$t_2 = 2750°C$$
$$P = 150 \text{ watts}$$
$$V = 230 \text{ volts}$$
$$t_1 = 16°C$$
$$\alpha_0 = 0.0047/°C$$

Resistance of the filament of the lamp under normal working condition,

$$R_2 = \frac{V^2}{P}$$
$$= \frac{(230)^2}{150} = 352.7\ \Omega$$

Also, we know that $R_2 = R_1\ [(1 + \alpha_1)\ (t_2 - t_1)]$

where α_1 is the temperature coefficient of resistant at 16°C (t_1)

R_1 = Resistance of the filament of the lamp at the moment of switching on

$$R_1 = \frac{R_2}{1 + \alpha_1\ (t_2 - t_1)} = \frac{352.7}{1 + 0.0437\ (2750 - 16)}$$

where $\alpha_1 = \dfrac{1}{1/\alpha_0 + t_1} = 0.00437/°C$

i.e. $R_1 = 27.23\ \Omega$

$$I_2 = \frac{V}{R_2} = \frac{230}{352.7} = 0.6521 \text{ A}$$

and $I_1 = \dfrac{V}{R_1} = \dfrac{230}{27.23} = 8.447 \text{ A}$

Example 1.10 The cost of boiling 2 kg of water in an electric kettle is 12 paise. The kettle takes 6 minutes to boil water from an ambient temperature of 20°C, calculate (i) the efficiency of the kettle and (ii) wattage or power rating of the kettle if cost of 1 kWh is 40 paise.

Solution

$$m_1 = 2 \text{ kg}$$
$$\theta_1 = 20°C$$
$$\theta_2 = 100°C \text{ (i.e. boiling point of water)}$$
$$t = 6 \text{ minutes or } \frac{6}{60}$$

i.e. $\dfrac{1}{10}$ hour

Cost of energy of 1 kWh = 40 paise (i.e. for one unit)

Cost of energy consumed = 12 paise

Heat energy required to raise the temperature of water from ambient temperature to boiling point, $H = mS\,(\theta_2 - \theta_1)$ or Output energy from the kettle where S = Specific heat of water = 1

$$H = 2 \times 1 \times (100 - 20) = 160 \text{ kcals}$$

$$H = \frac{160}{860} \text{ kWh} \quad (860 \text{ kcal} = 1 \text{ kWh})$$

$$= 0.186 \text{ kWh}$$

$$\left.\begin{array}{l}\text{Electrical energy supplied to the kettle} \\ \qquad\qquad\text{or} \\ \text{input energy to the kettle}\end{array}\right\} E = 1/40 \times \text{kWh}$$

$$= \frac{12}{40} \text{ kWh} = 0.3 \text{ kW/h}$$

Efficiency of the kettle $\eta = \dfrac{0.186}{0.3} \times 100 = 62\%$

Let P be the kW or power rating of the kettle

Input Electrical Energy = P × time taken in hours

i.e. $$0.3 = P \times \frac{6}{60}; \quad P = 3 \text{ kW}$$

Example 1.11 An electric kettle is required to raise the temperature of 2 kg of water from 20°C to 100°C in 15 minutes. Calculate the resistance of the heating element if the kettle is to be used on a 240 volts supply. Assume the efficiency of the kettle to be 80%.

Solution

m = 2 kg

θ_1 = 20°C

θ_2 = 100°C

t = 15 minutes or 1/4 hour, i.e. 0.25 hour

V = 240 volts

η = 80% or 0.8.

Heat received by water (or) Output energy from the kettle $H = mS\,(\theta_2 - \theta_1)$

$$= 2 \times 1 \times (100 - 80) = 160 \text{ kcal}$$

$$= 0.186 \text{ kWh}$$

Electrical energy supplied to the kettle, $E = \dfrac{H}{\eta} = \dfrac{0.186}{0.8}$

$$= 0.2325 \text{ kWh}$$

Power rating of the kettle, $P = \dfrac{E \text{ in kWh}}{t \text{ in hour}} = \dfrac{0.2325}{0.25}$

$$= 0.93 \text{ kW} = 930 \text{ W } (1 \text{ kW} = 1000 \text{ W})$$

$$R = \frac{V^2}{P} = \frac{(240)^2}{930} = 61.94\ \Omega$$

Example 1.12 Calculate the time taken for a 25 kW furnace having efficiency of 80% to melt 20 kg of aluminium. Take specific heat, melting point and latent heat of fusion of aluminium as 896 J/kg/°C, 657°C and 402 kJ/kg respectively.

Solution

$$m = 20 \text{ kg}$$
$$S = 896 \text{ J/kg/°C (or) } 0.896 \text{ kJ/kg°C}$$
$$L = 402 \text{ kJ/kg}$$
$$\theta_2 = 657°\text{C}$$
$$\theta_1 = 20°\text{C (Assumed)}$$
$$P = 25 \text{ kW}$$
$$\eta = 80\% \text{ or } 0.8 = \eta$$

$\left.\begin{array}{l}\text{Energy supplied to the furnace}\\\text{to melt the aluminium}\end{array}\right\}$ $E = P$ in kW × t in hours = 25 t kW/h

$\left.\begin{array}{l}\text{Heat energy required to melt the aluminium (or)}\\\text{Energy output from the furnace}\end{array}\right\}$ $H = mS(\theta_2 - \theta_1) + mL$

i.e.
$$H = 20 \times 0.896\,(657 - 20) + 20 \times 402 \text{ kJ}$$
$$= 19455 \text{ kJ}$$
$$= 19455/4.186 \text{ kcal} = 4648 \text{ kcal} = 5.4042 \text{ kW/h}$$
$$\left[\because 1 \text{ kcal} = 1/860 \text{ kWH and } 1 \text{ kJ} = \frac{1}{4.186} \text{kcal}\right]$$

and electrical energy input to the furnace $= \dfrac{H}{\eta}$

$$= \frac{5.4042}{0.8} = 6.7552 \text{ kWh}$$
$$25\,t = 6.7552 \text{ or } t = 0.27 \text{ hr or } 16.21 \text{ min}$$

Example 1.13 An electrically driven motor pump lifts 80 m^3 of water per minute through a height of 12 m. Allowing an overall efficiency of 70% for the motor and pump, calculate the input power to motor. If the pump is in operation for an average of 2 hours per day for 30 days, calculate the energy consumption in kWh and the cost of energy at the rate of 50 paise per kWh. Assume 1 m^3 of water has a mass of 1000 kg and $g = 9.81$ m/sec^2.

Solution

$$m = 80 \times 1000 = 8 \times 10^4 \text{ kg/min}$$
$$g = 9.81 \text{ m/sec}^2$$
$$T = 2 \times 30 = 60 \text{ hours}$$
$$\eta = 70\%$$
$$h = 12 \text{ m}$$

Cost of energy = 50 paise/kWh

$\left.\begin{array}{l}\text{Potential energy possessed by water per minute or}\\\text{Work done by the motor pump/minute}\end{array}\right\}$ $= mgh$

$$= 8 \times 10^4 \times 9.81 \times 12$$
$$= 94.176 \times 10^5 \text{ joules}$$

Work done by the motor/sec $= \dfrac{94.176 \times 10^5}{60}$ joules/sec or watts

i.e. Output power of motor = 156.96 kW

Input power to the motor $E = \dfrac{156.96}{0.7} = 224.229$ kW

Total energy supplied or energy consumption = $E \times T$

$= 224.229 \times 60 = 13{,}454$ kWh

Total cost of energy = Rs. 0.5×13.454 = Rs. 6,727.

Example 1.14 A 100 MW hydro-electric station is supplying full load for 10 hours a day. Calculate the volume of water which has been used. Assume effective head of station as 200 m and overall efficiency of the station as 80%.

Solution

$$P = 100 \text{ MW}$$
$$\eta = 200 \text{ m}$$
$$\eta = 80\%$$
$$t = 10 \text{ hr/day}$$

Energy supplied by the station or energy output from the station in 10 hours $\Big\} = 100 \times 10 = 1000$ MWh

and energy input to the station in 10 hours i.e., potential energy possessed by the water $\Big\} = \dfrac{100}{\eta}$

$$= \frac{1000}{0.8} - 1250 \text{ MWh}$$
$$= 1250 \times 10^6 \times 60 \times 60 \text{ W.sec. or joules}$$
$$= 45 \times 10^{11} \text{ joules}$$

let m be the mass of water in kg used in 10 hours.

Energy input to the station or potential energy supplied by the water to the station machines $\Big\} = mgh$ joules

$$mgh = 45 \times 10^{11}$$
$$m = \frac{45 \times 10^{11}}{9.81 \times 200} = 22.93578 \times 10^8 \text{ kg}$$

Mass of 1m^3 of water = 1000 kg

Volume of water used in 10 hours = 22.93578×10^5 m^3

Example 1.15 A current of 20 A flows for one hour in a resistance across which there is a voltage of 8 V. Determine the velocity in m/sec with which a mass of one tonne must move in order that the kinetic energy shall be equal in amount to the energy dissipated in the resistance.

Solution

$$I = 20 \text{ A}; V = 8 \text{ V}; t = 1 \text{ hr or } 600 \text{ sec}$$
$$m = 1 \text{ tonne, i.e. } 1000 \text{ kg}$$

Kinetic energy = Energy dissipated in the resistance

Energy dissipated in the resistance = VIt joules.

$= 8 \times 20 \times 3600$ joules $= 576 \times 10^3$ joules

Kinetic energy possessed by the body = $1/2\, mv^2 = 1/2\,(1000)\, v^2$

$= 500\, v^2$ joules

With the given data, $500\, v^2 = 576 \times 10^3$

or $v = 33.94$ m/sec

Example 1.16 A piece of resistance wire 15.6 m long and of cross-sectional area 12 mm^2, at a temperature of 0°C, passes a current of 7.9 A when connected to a d.c. supply at 240 V. Calculate: (i) resistivity of the wire, (ii) the current which will flow when the temperature rises to 55°C. The temperature coefficient of the resistance wire is 0.00029 ohm/ohm/°C.

Solution

(i) $R = V/I = 240/7.9 = 30.38\ \Omega$

But $R = \rho l/a = \rho \times 15.6/12 \times 10^{-6} = 30.38$

$\therefore$ Resistivity, $\rho = \dfrac{30.38 \times 12 \times 10^{-6}}{15.6} = 23.37 \times 10^{-6}\ \Omega\text{m} = 23.37\ \mu\Omega\text{m}.$

(ii) $R_t = R_0(1 + \alpha_0 t) = 30.38\ \Omega\ (1 + 0.00029 \times 55) = 30.86\ \Omega$

$\therefore$ Current, $I = V/R_t = 240/30.86 = 7.78\text{A}.$

Example 1.17 1 km of wire with a diameter of 11.7 mm and resistance of 0.031 Ω is drawn so that its diameter is 5 mm. What does its resistance become?

Solution

Here $R_1 = 0.031\ \Omega;\ r_1 = \dfrac{1}{2} \times 11.7\text{ mm} = 5.85 \times 10^{-3}\text{ m};$

$r_2 = \dfrac{1}{2} \times 5\text{ mm} = 2.5 \times 10^{-3}\text{ m}.$

Now $R = \rho \dfrac{l}{a} = \rho \dfrac{(v/a)}{a} = \rho \dfrac{v}{a^2}$ (where v = volume of wire)

Since volume of wire is constant and the material is same, hence,

$$R \propto 1/a^2 \text{ or } R_2/R_1 = (a_1/a_2)^2$$

or

$$R_2 = R_1\left[\frac{a_1}{a_2}\right]^2 = R_1\left[\frac{\pi r_1^2}{\pi r_2^2}\right] = R_1\ (r_1/r_2)^4$$

$$= 0.031(5.85 \times 10^{-3}/2.5 \times 10^{-3})^4\ \Omega = 0.9294\ \Omega.$$

Example 1.18 A semi-circular ring of copper has an inner radius of 8 cm, radial thickness of 4 cm, and axial thickness of 6 cm. Calculate the resistance of the ring at 50°C between its two end faces. The specific resistance of copper at 20°C = 1.724 × 10^{-6} Ωcm, and resistance temperature coefficient of copper at 0°C = 0.0043/°C.

Solution

$\rho_{20} = 1.724 \times 10^{-6}\ \Omega\text{cm} = 1.724 \times 10^{-6}\ \Omega\text{m};\ \alpha = 0.0043/°\text{C};$

l (length of the semi-circular ring between its end faces)

$$= \pi r = \pi\left[\frac{8+12}{2}\right] = \pi \times 10\text{ cm} = 0.1\ \pi\text{m};\ a = 6 \times 4 = 24\text{ cm}^2 = 2.4 \times 10^{-3}\text{ m}^2$$

$$\therefore\ R_{20} = \rho_{20} \times l/a = \frac{1.724 \times 10^{-8} \times 0.1\ \pi}{2.4 \times 10^{-3}} = 2.2567 \times 10^{-6}\ \Omega$$

$$\therefore\ R_{50} = R_{20}\left[\frac{1 + \alpha 50}{1 + \alpha 20}\right] = 2.2517 \times 10^{-6}\left[\frac{1 + 0.0043 \times 50}{1 + 0.0043 \times 20}\right]$$

$$= 2.2567 \times 10^{-6}\left[\frac{1.215}{1.086}\right]\Omega = 2.5248\ \mu\Omega.$$

Example 1.19 A copper rod, 0.5 m long and 5 mm in diameter, has a resistance of 425 $\mu\Omega$ at 15°C. Calculate the resistivity of copper at this temperature. If the same rod is drawn out into a wire having a uniform diameter of 1 mm, calculate the resistance of the wire, when its temperature is 50°C. Assume the resistivity to be unchanged and the temperature coefficient of copper to be 0.00426/°C at °C.

Solution

(i) $l_1 = 0.5$ m; $r_1 = (1/2) \times 5$ mm $= 2.5$ mm $= 2.5 \times 10^{-4}$ m;

$a_1 = \pi(2.5 \times 10^{-3})^2 = 1.9635 \times 10^{-5}$ m^2; $R_1 = 425\ \mu\Omega = 4.25 \times 10^{-4}\ \Omega$.

$$\therefore \text{ Resistivity, } \rho = \frac{R_1 a_1}{l_1} = \frac{4.25 \times 10^{-4} \times 1.9635 \times 10^{-5}}{0.5} = 1.669 \times 10^{-8}\ \Omega\text{-m}.$$

(ii) $r_2 = \frac{1}{2} \times 1$ mm $= 0.5$ mm $= 5 \times 10^{-4}$ m; $a_2 = \pi(5 \times 10^{-4})^2$

$= 7.854 \times 10^{-7}$ m^2

Now $$\frac{R_2}{R_1} = \left[\frac{a_1}{a_2}\right]^2 = \left[\frac{1.9636 \times 10^{-5}}{7.854 \times 10^{-7}}\right]^2 = 625$$

$\therefore$ $R_{15} = R_2 = 625 \times R_1 = 625 \times 4.25 \times 10^{-4}\ \Omega = 0.2656\ \Omega$

Now $$\frac{R_{50}}{R_{15}} = \frac{1 + 50\ \alpha_0}{1 + 15\ \alpha_0} = \frac{1 + 50 \times 0.00426}{1 + 15 \times 0.00426} = 1.14$$

Hence, $R_{50} = R_{15} \times 1.14 = 0.2656\ \Omega \times 1.14 = 0.3028\ \Omega$.

Example 1.20 An aluminium wire, 7.5 m long, is connected in parallel with a copper wire, 6 m long. When a current of 5 A is passed through the parallel combination, it is found that the current in aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. The resistivity of copper is 0.017 $\mu\Omega$-m and that of aluminium is 0.028 $\mu\Omega$-m.

Solution Using subscript symbols 1 and 2 respectively, we have:

$l_1 = 7.5$ m; $a_1 = \pi/4(d_i)^2 = \pi/4 \times (0.001\text{ m})^2 = 7.854 \times 10^{-5}$ m^2;

$\rho_1 = 0.028\ \mu\Omega$-m; $l_2 = 6$ m; $\rho_2 = 0.017\ \mu\Omega$-m $= 1.7 \times 10^{-8}\ \Omega$-m;

$I = 5$ A; $I_1 = 3$ A, $I_2 = 5 - 3 = 2$ A.

$$\therefore \quad R_1 = \rho_1 \frac{l_1}{a_1} = \frac{2.8 \times 10^{-8} \times 7.5}{7.854 \times 10^{-7}} = 0.2673\ \Omega.$$

$\therefore$ Voltage drop across the two ends of *Al* wire,

$$V_1 = I_1 R_1 = 3 \times 0.2673 = 0.8022\text{ V}$$

$$R_2 = \frac{\rho_2 l_2}{a_2} = \frac{1.7 \times 10^{-8} \times 6}{a_2}\ \Omega$$

$\therefore$ Voltage drop through the two ends of Cu wire,

$$V_2 = \frac{2 \times 1.7 \times 10^{-8} \times 6}{a_2} = \frac{2.04 \times 10^{-7}}{a_2}\text{ V}$$

Since the wires are connected in parallel, so $V_1 = V_2$,

i.e $0.8022 = (2.04 \times 10^{-7})/a_2$

$$\therefore \quad a_2 = \frac{2.04 \times 10^{-7}}{0.8022} = 2.54 \times 10^{-7}\ \text{m}^2$$

$$\text{or} \quad \frac{\pi d_2^2}{4} = 2.54 \times 10^{-7}\ \text{m}^2$$

$$\text{or} \quad d_2 = (2.54 \times 10^{-7} \times 4/\pi\ \text{m}^2)^{\frac{1}{2}} = 0.569 \times 10^{-3}\ \text{m}.$$

Hence, diameter of copper wire is 0.569×10^{-3} m or 0.569 mm.

Example 1.21 If α_1 and α_2 are the temperature coefficients of resistance at t_1 and t_2 respectively, then show that:

(i) $\alpha_2 = \dfrac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}$

(ii) $(t_2 - t_1) = (\alpha_1 - \alpha_2)/\alpha_1\ \alpha_2$

Solution

$$R_t = R_0(1 + \alpha_0 t)$$

If we cool conductor from t°C, then:

$$R_0 = R_t[1 + \alpha_t(-t)] = R_1[1 - \alpha_t t]$$

$$\text{or} \quad \alpha_t = \frac{R_t - R_0}{R_t \times t} = \frac{R_0(1 + \alpha_0 t) - R_0}{R_0(1 + \alpha_0 t) \times t} = \frac{\alpha_0}{(1 + \alpha_0 t_2)} \qquad \text{(i)}$$

$\therefore$ From (i) we get, $\alpha_1 = \alpha_0/(1 + \alpha_0 t_1)$, and $\alpha_2 = \alpha_0/(1 + \alpha_0 t_2)$

$$\text{or} \quad 1/\alpha_1 = (1 + \alpha_0 t_1)/\alpha_0 \qquad \text{(ii)}$$

$$\text{and} \quad 1/\alpha_2 = (1 + \alpha_0 t_2)/\alpha_0 \qquad \text{(iii)}$$

From (iii) – (ii), we get,

$$\boxed{\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2} = \frac{(1 + \alpha_0 t_2) - (1 + \alpha_0 t_1)}{\alpha_0} = (t_2 - t_1)} \qquad \text{(iv)}$$

$$\text{Now} \quad \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) = \left[\frac{[1 + \alpha_1(t_2 - t_1)]}{\alpha_1}\right]$$

$$\therefore \quad \boxed{\alpha_2 = \frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}} \qquad \text{(v)}$$

Example 1.22 A coil has a resistance of 100 ohms, when the mean temperature is 20°C; and 110 ohms, when the mean temperature is 45°C. Find its mean temperature rise, when its resistance is 124 ohms, and surrounding temperature is 15°C.

Solution

$$\frac{R_{45}}{R_{20}} = \frac{R_0(1 + 45\ \alpha_0)}{R_0(1 + 20\alpha_0)} = \frac{1 + 45\alpha_0}{1 + 20\alpha_0} = \frac{110\ \Omega}{100\ \Omega} = 1.10$$

$$\therefore \quad 1 + 45\ \alpha_0 = 1.10 + 22\ \alpha_0 \text{ or } 23\ \alpha_0 = 0.10$$

$$\alpha_0 = (1/230)/°\text{C}.$$

Let at t°C, the resistance of coil become 124 ohms.

$$R_t = 124 = R_0(1 + \alpha_0 t) = R_0[1 + t/230]$$

$$\text{Also} \quad R_{20} = 100 = R_0(1 + \alpha_0 \times 20) = R_0[1 + (20/230)]$$

Dividing (i) by (ii), we get,

$$\frac{124}{100} = 1.24 = \frac{[1 + (t/230)]}{[1 + (20/230)]} = \frac{230 + t}{250}$$

$\therefore$ $230 + t = 250 \times 1.24 = 310$

Hence, $t = 80°C$

Hence, mean temperature rise = 80 – 15°C = 65°C.

Example 1.23 A coil has a resistance of 18 Ω, when its mean temperature is 20°C; and 20 Ω, when its mean temperature rise is 50°C. Find its mean temperature rise when its resistance is 21 Ω and the surrounding temperature is 15°C.

Solution

$$\frac{R_{50}}{R_{20}} = \frac{20\ \Omega}{18\ \Omega} = \frac{R_0(1 + 50\alpha_0)}{R_0(1 + 20\alpha_0)} = \frac{1 + 50\alpha_0}{1 + 20\alpha_0}$$

$\therefore$ $$\frac{10}{9} = \frac{1 + 30\alpha_0}{1 + 20\alpha_0} \text{ or } \frac{30\alpha_0}{1 + 20\alpha_0} = \frac{1}{9}$$

or $270\alpha_0 = 1 + 20\alpha_0$ or $\alpha_0 = (1/230)/°C$

Now $$\frac{R_t}{R_{50}} = \frac{R_0(1 + \alpha_0 t)}{R_0(1 + 50\alpha_0)} = \frac{1 + \alpha_0 t}{1 + 50\alpha_0} = \frac{21}{20} = 1.05$$

$\therefore$ $1 + \alpha_0 t = 1.05 + 52.5\alpha_0$

or $$t = \frac{1}{\alpha_0}[0.05 + 52.5\alpha_0] = \frac{0.05}{\alpha_0} + 52.5$$

$$= \frac{0.05}{1/250} + 52.5 = 0.05 \times 250 + 52.5 = 65°C$$

Hence, mean temperature rise = 65 – 15°C = 50°C.

1.9 CIRCUITS-IDENTIFYING THE ELEMENTS AND THE CONNECTED TERMINOLOGY

Any arrangement of electrical energy sources (e.g. batteries, generators, etc.), resistances and other circuit elements is called an electrical network. The terms circuit and network are used synonymously in electrical literature. Figure 1.12 is an electrical circuit or network.

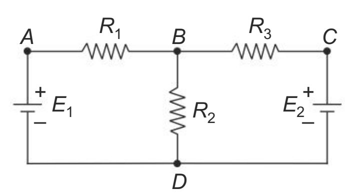


Fig. 1.12

1.9.1 Identifying the Elements

The symbols for the various circuit elements are given in Fig. 1.13.

1.9.2 Network Terminology

(i) Circuit The closed path followed by an electric current is called an electrical circuit.

The essential parts of an electric circuit are the source of power (battery, generator, etc.), the conductors to carry the current and the loads. (The device which utilises the electrical energy is called load, e.g. lamp, motor, heater, etc.).

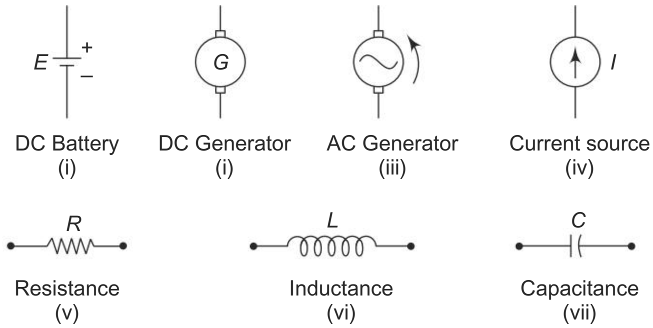


Fig. 1.13

The closed path followed by direct current (dc) is called a dc circuit.

(ii) Parameters The various elements of an electric circuit are called its parameters (e.g. resistance, capacitance, inductance, etc.) and these may be lumped or distributed.

(iii) Node Node is a junction where two or more circuit elements are connected together. Point at which two elements are connected together is generally called as minor node. (*A* and C in Fig. 1.12). Junction at which three or more elements are connected together is called as major nodes (*B* and *D* in Fig. 1.12). In circuit analysis, we will consider only the major nodes. So, hereafter, if we say node, *I* means only major node.

(iv) Branch A branch is that part of a circuit which lies between two junction points. For example, branch *BD,* branch *BAD,* branch *BCD* in Fig. 1.12.

(v) Loop A loop is any closed path of a circuit. Thus, in Fig. 1.12, *ABDA, BCDB* and *ABCDA* are the loops.

(vi) Mesh A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 1.12, loops *ABDA* and *BCDB* qualify as mesh because they cannot be further divided into other loops. But, the loop *ABCD* cannot be called as mesh because it encloses two loops *ABDA* and *BCDB*.

1.10 KIRCHOFF'S LAWS–STATEMENT AND ILLUSTRATION

Entire electric circuit analysis is based on these laws only. In the early 1 century, Gustav Kirchoff, a German scientist, gave his findings with electrical circuit in a set of two laws—a current and a voltage law—which are together known as Kirchoff's laws.

1.10.1 First Law (i.e.) Kirchoff's Current Law (KCL)

Statement The algebraic sum of currents meeting at a junction or node in an electrical circuit is zero.

Explanation An algebraic sum is one in which the sign of the quantity is taken into account. Consider five conductors, carrying current, I_1, I_2, I_3, I_4 and I_5 meeting at point O as shown in Fig. 1.14. If we assume the currents flowing towards point O as positive, then, the currents flowing away from point O will have negative sign. Now, applying Kirchoff's current law at junction O, we get

$$(+I_1) + (-I_2) + (+I_3) + (-I_4) + (+I_5) = 0$$

i.e. $$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

or $$I_1 + I_3 + I_5 = I_2 + I_4$$

i.e. sum of incoming currents = sum of outgoing currents.

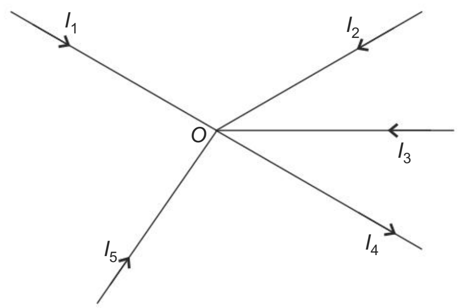


Fig. 1.14

The above law (KCL) can also be stated as:

The sum of the currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

Validity of KCL KCL is true because electric current is merely flow of electrons and they cannot accumulate at any point in the circuit.

1.10.2 Second Law-Kirchoff's Voltage Law (KVL)

Statement In any closed circuit or mesh or loop, the algebraic sum of all the voltages taken around is zero.

Validity If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in the potential at that point. This means that the sum of emfs and sum of voltage drops or rise met on the way is zero.

Algebraic emfs and Voltage Drops While applying KVL, algebraic sums are involved. So, it is necessary to assign proper signs to the emfs and voltage drops. The following sign convention may be used.

A rise in potential can be assumed positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(i) If we go from a +ve terminal of the battery or source to the –ve terminal, there is a fall in potential and so the emf should be assigned negative sign.

If we, go from the –ve terminal of the battery or source to the +ve, terminal, then, there is rise in potential and so the emf should be given positive sign. It is clear that sign of emf is independent of the direction of current through it.

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistance in the same direction as the current there is a fall in potential (current flows from higher potential point to lower potential point). So, the sign of this voltage drop is negative. If we go opposite to the direction of current flow, there is a rise in potential and hence, this voltage drop should be given positive sign. It is clear that the sign of voltage drop (i.e. *IR* drop) depends upon the direction of current flow and is independent of the polarity of the emf in the circuit under consideration.

1.10.3 Illustration of Kirchoff's Laws

Kirchoff's laws can be explained with the help of the circuit shown in Fig. 1.15.

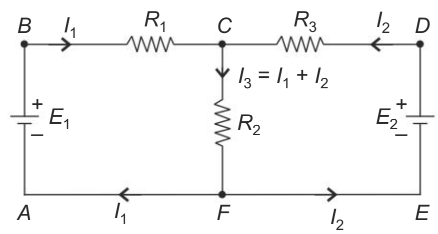


Fig. 1.15

The direction in which the currents are assumed to flow is unimportant, since if wrong direction is chosen, it will be indicated by a negative sign in the result.

(i) If we apply KCL at junction *C*, we get $I_3 = I_1 + I_2$. The strength of current in any branch can be found by KCL.

(ii) There are three loops in the circuit, namely, *ABCFA, EDCFE* and *ABCDEFA*. KVL can be applied to these loops to get the desired equations.

Loop ABCFA If we go round the loop in the order, the sign of emf E_1 is +ve because we are moving from –ve terminal of the battery to +ve terminal of the battery. The sign of $I_1 R_1$ drop is –ve as we are moving along the direction of the current flow and hence, drop in potential. The sign of I_3R_2 is also –ve, the reason is same as that of I_1R_1 drop.

So, we get $+E_1 - I_1R_1 - I_3R_2 = 0$

i.e. $$E_1 = I_1R_1 + I_3R_2$$

Loop EDCFE If we go round the loop in the order, the sign of emf E_2 is +ve, drop I_2R_3 is –ve and drop I_3R_2 is –ve

$\therefore \quad +E_2 - I_2R_3 - I_3R_2 = 0$

or

$$E_2 = I_2R_3 + I_3R_2$$

Loop ABCDEFA

On applying KVL, we get

$$+E_1 - I_1R_1 + I_2R_3 - E_2 = 0$$

or
$$E_1 - E_2 = I_1R_1 - I_2R_3$$

1.11 RESISTANCES IN SERIES AND VOLTAGE DIVISION TECHNIQUE

The circuit in which resistances are connected end to end so that there is one path for the current flow is called series circuit.

Let three resistances R_1, R_2 and R_3 be connected in series across a battery of V volts as in Fig. 1.16 (i). Obviously, the current I is same throughout the circuit. By Ohm's law, the voltages across the resistances are

$$V_1 = IR_1;\ V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3$$

also
$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$

or

or
$$\frac{V}{I} = R_1 + R_2 + R_3$$

The ratio of (V/I) is the total resistance between the points A and B and is called the total or equivalent resistance of the three resistances [Fig. 1.16(ii)].

$\therefore$
$$R_T = R_1 + R_2 + R_3 \tag{1.14}$$

also
$$\frac{1}{G_r} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} \quad \text{(in terms of conductances)}$$

Hence, when a number of resistances are connected in series the equivalent resistance is the sum of all the individual resistances.

Multiplying the Eqn. (1.14) throughout by I_2 we get, $I^2\,RT = I^2R_1 + I^2R_2 + I^2R_3$.

Therefore, the total power consumed in the series circuit is the sum of the powers consumed by the individual resistances.

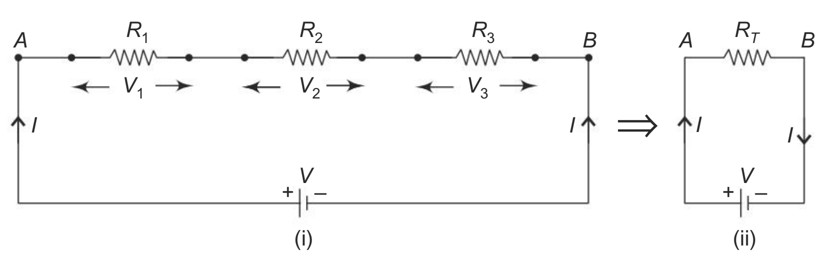


Fig. 1.16

1.11.1 Concepts of Series Circuit

(i) The current flowing in all parts of the circuit is same.

(ii) Voltage across the different elements will depend upon the resistance of elements.
(iii) Voltage drops are additive.
(iv) Resistances are additive.
(v) Powers are additive.
(vi) The applied voltage equals the sum of different voltage drops.

1.11.2 Disadvantages of Series Circuit

(i) If a break occurs at any point in the circuit, no current will flow and the entire circuit becomes useless.
(ii) If 5 numbers of lamps, each rated 230 volts are to be connected in series circuit, then, the supply voltage should be 5 × 230 = 1150 volts. But voltage available for lighting circuit in each and every house is only 230 V. Hence, series circuit is not practicable for lighting circuits.
(iii) Since electrical devices have different current ratings, they cannot be connected in series for efficient operation.

1.11.3 Voltage Division Technique

Let n resistances R_1, R_2, R_3, ... R_n be connected in series across a battery of V volts. The current flowing through all the resistances is I and is same as shown in Fig. 1.16 (iii).

The equivalent resistance $R_T = R_1 + R_2 + R_3 + R_4 + ... + R_n$

$$= \sum_{x=1}^{n} R_x$$

The current through the series circuit, $I = \dfrac{V}{R_T}$ (By Ohm's law)

Voltage across each resistance is

$$V_1 = IR_1,\ V_2 = IR_2 \ldots\ V_x = IR_x \ldots .;\ V_n = IR_n$$

i.e. $$V_1 = \frac{V}{R_T}R_1;\ V_2 = \frac{V}{R_T}R_2;\ \ldots\ V_x = \frac{V}{R}R_x$$

∴ Voltage across any resistance in the series circuit

$$= V_x \frac{R_x}{R_T} V \qquad (1.15)$$

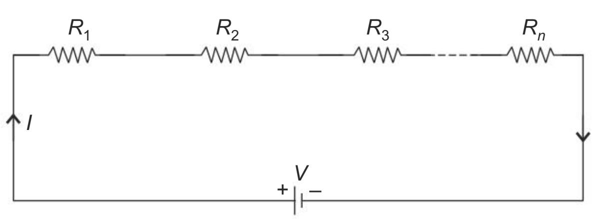


Fig. 1.16(iii)

1.12 RESISTANCES IN PARALLEL AND CURRENT DIVISION TECHNIQUE

If one end of each resistance connected to one common point and the other end of each resistance connected to another common point, there will be as many paths for current flow as the number of resistances. This is called parallel circuit.

Let three resistances R_1, R_2 and R_3 be connected in parallel across a battery of V volts as in Fig. 1.17(i). The total current I divides into three parts; I_1 flowing through R_1, I_2 through R_2 and I_3 through R_3. Obviously, the voltage across each resistance is same (i.e. V).

By Ohm's law, current through each resistance is

$$I_1 = \frac{V}{R_1};\ I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

Also,
$$I = I_1 + I_2 + I_3$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$
$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But, V/I is the total resistance between the points A and B and $I/V = \frac{1}{R_T}$ [Fig. 1.17(ii)].

$\therefore$
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (1.16)$$

Also $\quad G_T = G_1 + G_2 + G_3 \quad$ (in terms of conductances)

Hence, when a number of resistances are connected in parallel, the reciprocal of the total resistance is equal to the sum of reciprocals of individual resistances.

Multiplying the Eqn. (1.16) by V^2 throughout, we get

$$\frac{V^2}{R_T} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

i.e. the total powers consumed in a parallel circuit is equal to the sum of power consumed by the individual resistances.

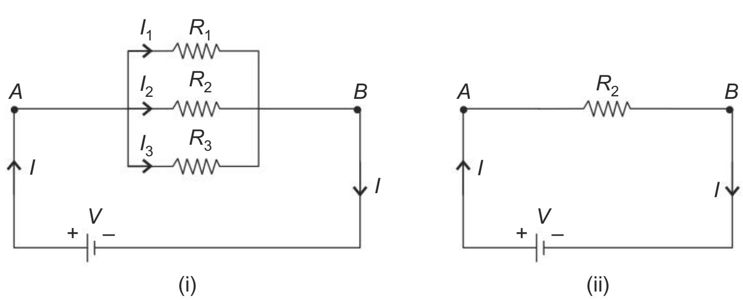


Fig. 1.17

1.12.1 Concepts of Parallel Circuit

(i) Same voltage across all elements.

(ii) All elements will have individual currents, depending upon the resistance of element.

(iii) The total resistance of a parallel circuit is always less than the smallest of the resistances.

(iv) If n resistances each of R are connected in parallel, then

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \; n \text{ terms}$$

$$= \frac{n}{R}$$

or
$$R_T = \frac{R}{n}$$

(v) Powers are additive.

(vi) Conductances are additive.

(vii) Branch currents are additive.

1.12.2 Advantages of Parallel Circuits

(i) The electrical appliances rated for the same voltage but different powers (and hence currents) can be connected in parallel without affecting each other's performance.

(ii) If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

Due to the above advantages, all electrical appliances are connected in parallel. We can switch on or off any light or appliances without affecting other lights or appliances.

1.12.3 Current Division Technique

Case (1) When Two Resistances are in Parallel Two resistances R_1 and R_2 ohms are connected in parallel across a battery of V volts. Current through R_1 is I_1 and through R_2 is I_2 (Fig. 1.18).

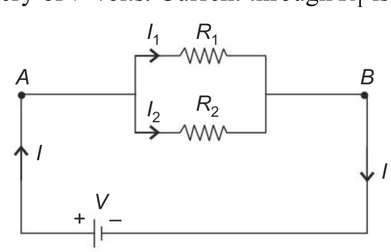


Fig. 1.18

(i) Total resistance R_T

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2}$$

or
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, the total value of two resistances connected in parallel is equal to their product divided by their sum.

(ii) Branch currents I_1, I_2

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = IR_T = I\frac{R_1R_2}{R_1 + R_2}$$

By Ohm's law, current through $R_1, I_1 = \frac{V}{R_1} = I\frac{R_2}{R_1 + R_2}$

and current through $R_2, I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$

Hence, in a parallel circuit of two resistances, the current in one resistor is the total current times the opposite resistance divided by the sum of the two resistances.

In terms of conductances,

$$I_1 = I\frac{1/G_2}{1/G_1 + 1/G_2} = I\frac{G_1}{G_1 + G_2}$$

and $$I_2 = \frac{G_2}{G_1 + G_2}I$$

Case (2) When Three Resistances are in Parallel (Refer Fig. 1.19)

$$\frac{I}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{R_2R_3 + R_3R_1 + R_1R_2}{R_1R_2R_3}$$

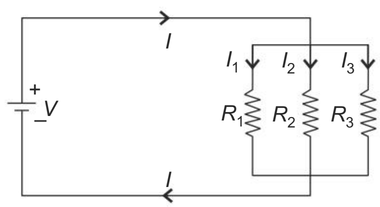


Fig. 1.19

or $$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

Also, $$V = IR_T = I\frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

By, Ohm's law,

current through R_1 is $I_1 = \frac{V}{R_1} = I\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$

current through R_2 is $I_2 = \frac{V}{R_2} = I\frac{R_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}$

and current through R_3 is $I_3 = \frac{V}{R_3} = I\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}$

In terms of conductances,

$$I_1 = I\frac{1/G_2G_3}{1/G_1G_2 + 1/G_2G_3 + 1/G_3G_1} = I\frac{G_1}{G_1 + G_2 + G_3}$$

$$I_2 = I\frac{1/G_3G_1}{1/G_1G_2 + 1/G_2G_3 + 1/G_3G_1} = I\frac{G_2}{G_1 + G_2 + G_3}$$

$$I_3 = I\frac{1/G_1G_2}{1/G_1G_2 + 1/G_2G_3 + 1/G_3G_1} = I\frac{G_3}{G_1 + G_2 + G_3}$$

Case (3) General Case If n number of different resistances are connected parallel across a battery as shown in Fig. 1.20, then, by referring to the previous cases, the current through any resistance can be found as

$$I_x = I\frac{G_x}{\sum_{x=1}^{n} G_x} \quad \text{General formula for current division}$$

1.12.4 Solving Series-Parallel Circuits

For the series parallel circuit shown in Fig. 1.21(i)

The equivalent resistance of parallel combination $= R_p = \dfrac{R_2R_3}{R_2 + R_3}$

Now, Fig. 1.21 (i) can be redrawn as shown in Fig. 1.21 (ii). The total equivalent resistance of the entire circuit,

$$R_T = R_1 + R_P$$

$\therefore$ Total current $\quad I = \dfrac{V}{R_T}$

The voltage across the parallel branch is $V_P = IR_P$

Now, by Ohm's law $I_2 = \dfrac{V_P}{R_2}$

and $\quad I_3 = \dfrac{V_P}{R_3}$

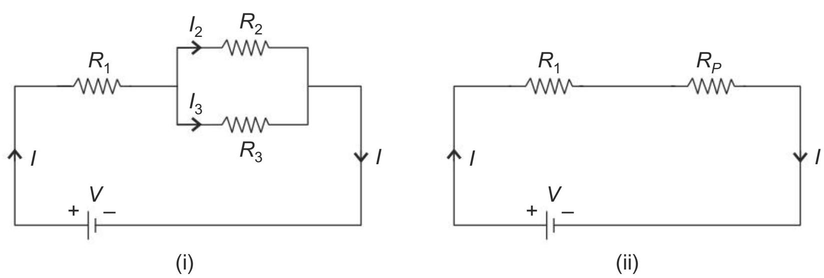


Fig. 1.21

1.12.5 Internal Resistance of Sources

(i) Voltage Sources All voltage sources (battery, generators, etc.) must have some internal resistance (r) (very small in value). This is shown as a series resistor connected external to the source in Fig. 1.22(i).

(ii) Current Sources All current sources must have some internal resistance (r) (very high in value). This is connected externally across the source, as shown in Fig. 1.22(ii).

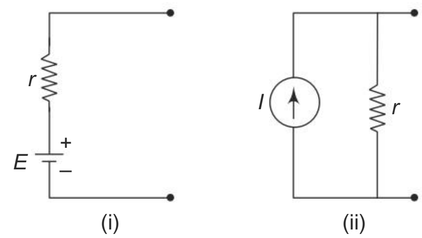


Fig. 1.22

1.13 METHOD OF SOLVING A CIRCUIT BY KIRCHOFF'S LAWS

The basic laws which govern the performance of any electrical circuit have been stated and illustrated in the earlier sections. Using these laws only can we analyse a given electrical circuit and determine the response required. The method of analysing a circuit is called 'Loop Analysis' when Kirchoff's Voltage Law is applied and it is called 'Nodal Analysis' when Kirchoff's Current Law is applied. These analyses are given in the following sections.

1.13.1 Loop Analysis

(i) Assume unknown currents in the given circuit and show their direction arbitrarily by arrows.
(ii) Choose any closed loop and find the algebraic sum of voltage drops plus the algebraic sum of emfs in that closed loop.
(iii) Equate the algebraic sum of voltage drops plus the algebraic sum of emfs to zero and obtain the equation in terms of unknown currents.
(iv) Write equations for as many closed loops as the number of unknown quantities (currents).
(v) Now obtain the unknown currents by solving the equations.
(vi) If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.

1.13.2 Nodal Analysis

(i) Identify all the independent nodes. The voltages at these nodes are unknown.
(ii) Select one of the nodes as reference node. Assign zero potential at this node.
(iii) Assign potentials at the other independent nodes.
(iv) Apply Kirchoff's current law at each node and write the governing equations.
(v) Solve for the unknown voltages.

Example 1.24 Two resistors of 4 ohms and 6 ohms are connected in parallel. If the total current is 30 A, find the current through each resistor.

Solution

$$R_1 = 4 \text{ ohm}; R_2 = 6 \text{ ohm}$$
$$I = 30 \text{ A}$$
$$I_1 = I\frac{R_2}{R_1 + R_2} = 30 \times \frac{6}{6+4} = 18 \text{ A}$$

and $I_2 = I - I_1 = 30 - 18 = 12$ A

Example 1.25 Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that total power of 100 watts may be absorbed.

Solution $R_1 = 2$ ohms, $R_2 = 3$ ohms, $R_3 = 4$ ohms and $R_4 = 5$ ohms $P = 100$ watts. Let R_T be the equivalent resistance of the parallel combinations of R_1, R_2, R_3 and R_4.

$$\therefore \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$
$$= \frac{30 + 20 + 15 + 12}{60} = \frac{77}{60}$$

and $R_T = \frac{60}{77}$ ohms.

Total power consumed $P = \frac{V^2}{R_T} = 100$ W; $V = 8.827$ volts

Example 1.26 When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel to increase the load to 16 A.

Solution

$$V = 230 \text{ V}, \quad I_1 = 12 \text{ A}, \quad I_2 = 16 \text{ A}$$

Since it is parallel circuit, it is clear that, current through the resistance

$$R = I = I_2 - I_1$$
$$= 16 - 12$$
$$= 4 \text{ A}$$

By Ohm's law, $R = V/I = 230/4$

i.e. $R = 57.5$ ohm

Example 1.27 A 50 ohm resistor is in parallel with a 100 ohm resistor. The current in 50 ohm resistor is 7.2 A. What is the value of third resistance to be added in parallel to make the line current as 12.1 A?

Solution The data given and data required are clearly marked in Fig. E.1.3. By Ohm's law and the characteristics of parallel circuit.

$$V = I_1 R_1 = 7.2 \times 50$$
$$= 360 \text{ volts}$$

Also, $V = I_2 R_2$

$$\therefore \quad I_2 = \frac{V}{R_2} = \frac{360}{100}$$
$$= 3.6 \text{ Amp.}$$

and $I_3 = I - I_1 - I_2 = 12.1 - 7.2 - 3.6$
$$= 1.3 \text{ A}$$

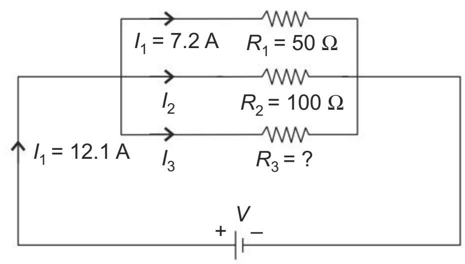


Fig. E.1.3

Once again $\quad V = I_3 R_3$

$\therefore \quad R_3 = \dfrac{V}{I_3} = \dfrac{360}{1.3}$

i.e. $\quad R_3 = 276.92$ ohms

Example 1.28 A resistor of 3.6 ohm is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms so that the total resistance of the circuits shall be 6 ohms.

Solution The data given and the data required are clearly marked in Fig. E.1.4 (i).

The equivalent resistance of the parallel combination is

$$R_P = \frac{R_1 R_2}{R_1 + R_3}$$

$$= \frac{3.6\, R_3}{3.6 + R_3}$$

Now, total circuit resistance $R_T = R_P + R_2$ from Fig. E.1.4. (ii).

i.e. $$\frac{3.6\, R_3}{3.6 + R_3} + 4.56 = 6$$

On solving $\quad R_3 = 2.4$ ohms.

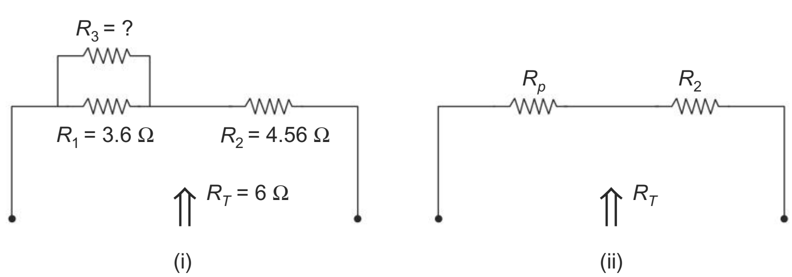


Fig. E.1.4

Example 1.29 A resistance R is connected in series with a parallel circuit comprising two resistors 12 ohms and 8 ohms respectively. The total power dissipated in the circuit is 70 watts when the applied voltage is 22 volts. Calculate the value of R.

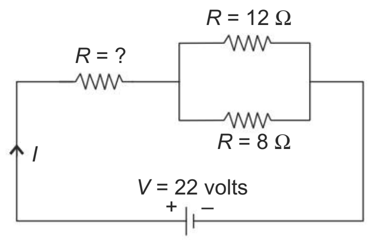


Fig. E.1.5

Solution

$$P = 70 \text{ watts}$$

Other data are marked in Fig. E.1.5.

Total current through the circuit $I = P/V$

$$= 70/22 = 3.18 \text{ Amps.}$$

Equivalent resistance of parallel combination

i.e. $$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{8 \times 12}{20}$$

$$= 4.8 \text{ ohms}$$

and voltage across the parallel combination $V_P = IR_P$

i.e. $$V_P = 4.8 \times 3.18 = 15.27 \text{ volts}$$

$\therefore$ Voltage across the resistance R is $V_R = V - V_P$

$$= 22 - 15.27$$

$$= 6.73 \text{ volts}$$

and by Ohm's law $$R = \frac{V_R}{I} = \frac{6.73}{3.18}$$

i.e. $$R = 2.12 \text{ ohms}$$

Example 1.30 Two resistors 12 ohms and 6 ohms are connected in parallel and this combination is connected in series with a 25 ohms resistance and a battery which has an internal resistance of 0.25 ohms. Determine the emf of the battery if P.D. across 6 ohms resistance is 6 volts.

Solution Data given and data required are marked in Fig. E.1.6.

As per Ohm's law $I_2 = \dfrac{V_2}{R_2} = \dfrac{6}{6} = 1 \text{ A}$

As per the characteristics of parallel circuit P.D. across

$$R_1 \text{ is } V_1 = V_2 = 6\text{V}$$

$\therefore$ $$I_1 = \frac{V_1}{R_1} = \frac{6}{12} = 0.5 \text{ A}$$

and total current $$I = I_1 + I_2 = 1.0 + 0.5$$

$$= 1.5\text{A}$$

As per the characteristics of series circuit (i.e. voltages are additive)

$$E = Ir + IR_3 + V_2 \text{ or } V_1$$

$$= 1.5 \times 0.25 + 1.5 \times 6.25 + 6 = 15.75 \text{ volts}$$

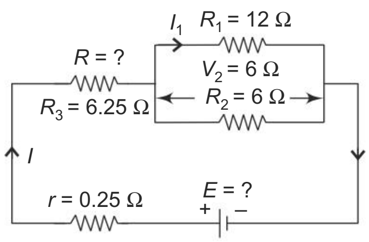


Fig. E.1.6

Example 1.31 A circuit consists of three resistors 3 ohms, 4 ohms and 6 ohms in parallel and a fourth resistor 4 ohms in series. A battery of emf 12 V and internal resistance 6 ohm is connected across the circuit. Find the total current in the circuit and terminal voltage across the battery.

Solution Data given and data required are marked in Fig. E.1.7.

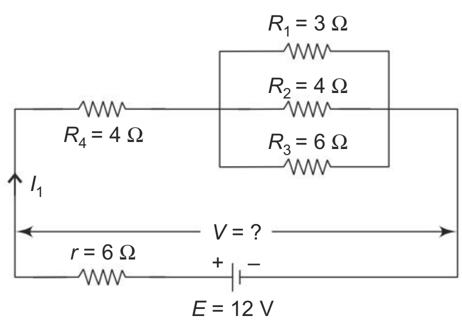


Fig. E.1.7

Equivalent resistance of the parallel combination (R_P).

i.e. $$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$1/R_P = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4+3+2}{12} = \frac{9}{12}$$

and $$R_P = \frac{12}{9} = 1.3333 \text{ ohms}$$

Total circuit resistance $R_T = R_P + R_4 + r$

$= 1.3333 + 4 + 6$

$= 11.333$ ohms

$\therefore$ Total circuit current $I = \dfrac{E}{R_T} = \dfrac{12}{11.3333}$

$= 1.059$ Amps.

And, terminal voltage of the battery, $V = E - Ir$

$= 12 - 1.059 \times 6$

$= 5.647$ volts

Example 1.32 An electrical network is arranged as shown in Fig. E.1.8. Find (i) the current in branch *AF*. (ii) the power absorbed in branch *BE* and (iii) *P.D.* across branch *CD*.

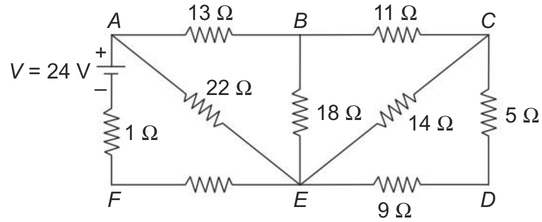


Fig. E.1.8

Solution Equivalent resistance between *E* and *C*

i.e. $$R_{EC} = \frac{(9+50)\,14}{(9+5)+14} = 7 \text{ ohms}$$

Equivalent resistance between *B* and *E*, $$R_{BE} = \frac{(11+7)+18}{(11+7)+18} = 9 \text{ ohms}$$

Equivalent resistance between *A* and *E*, $$R_{AE} = \frac{(13+9)22}{(13+9)+22} = 11 \text{ ohms}$$

Now, total equivalent circuit resistance $R_T = 11 + 1 = 12$ ohms

∴ Total current and current in branch $AF = \dfrac{24}{12} = 2$ A

Now, P.D. across $AE = V$ – P.D. in 1 ohms
$= 24 - (1 \times 2)$
$= 22$ volts

∴ Current in branch $AE = \dfrac{22}{22} = 1$ A

Current in branch $AB = 2 - 1 = 1$ A

P.D. across branch BE = P.D. across AE – Drop in branch AB
$= 22 - 13 \times 1$
$= 9$ volts

∴ Power absorbed in branch $BE = \dfrac{(9)^2}{18} = 4.5$ watts

Current in branch $BE = \dfrac{9}{18} = 0.5$A

and current in branch $BC = 1 - 0.5 = 0.5$A

At point *C*, current of 0.5 A divides into two parts one through 14 ohms and other through (5 + 9) = 14 ohms resistor.

$\therefore$ Current in branch $CDE = \dfrac{0.5}{2} = 0.25$ A

So, P.D. across $CD = 0.25 \times 5 = 1.25$ volts.

Example 1.33 Using Kirchhoff's laws, find the current in various resistors in the circuit shown in Fig. E.1.9 (i).

First, name the circuit and mark the currents with direction arbitrarily as shown in Fig. E.1.9 (ii).

Apply KVL to the loop* ABCFA*:

$$+25 - 6I_1 - 4(I_1 + I_2) + 4.5 = 0$$

i.e.
$$10I_1 - 4I_2 = 25 \quad (1)$$

Apply KVL to loop* CDEFC*:

$$-3I_2 + 45 + 4(I_1 - I_2) = 0$$

$$4I_1 - 7I_2 = -45 \quad (2)$$

On solving the simultaneous Eqns (1) and (2) we get,

$$I_1 = 6.574 \text{ A}$$
$$I_2 = 10.185 \text{ A}$$

i.e. Current in branch *FABC* is $I_1 = 6.574$ A

Current in branch *CDEF* is $I_2 = 10.185$ A

Current in branch *CF* is $(I_1 - I_2) = -3.611$ A

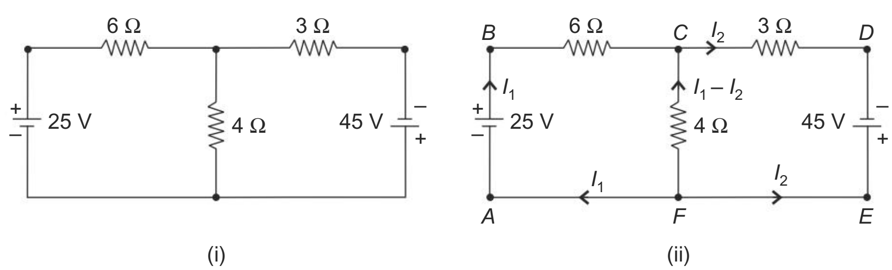


Fig. E.1.9

Negative sign indicates that the assumed current direction in branch *CF* is wrong, i.e., the actual direction of current flow is *F* to *C* (and not *C* to *F*).

Example 1.34 A Wheatstone bridge *ABCD* has the following details: *AB* = 1000 ohms: *BC* = 100 ohms; *CD* = 450 ohms; *DA* = 5000 ohms. A galvanometer of resistance 500 ohms is connected between *B* and *D*. A 4.5 volt battery of negligible resistance is connected between *A* and *C* with *A* positive. Find the magnitude and direction of current through the galvanometer.

Mark the direction of currents arbitrarily in Fig. E.1.10.

Let I be the total current delivered by the battery. This current is divided into two paths at point *A*, I_1 in branch *AB* and I_2 in branch *AD*. The current I_1 is taking two paths at junction *B*. Let I_g be the current through branch *BD* (i.e. the current

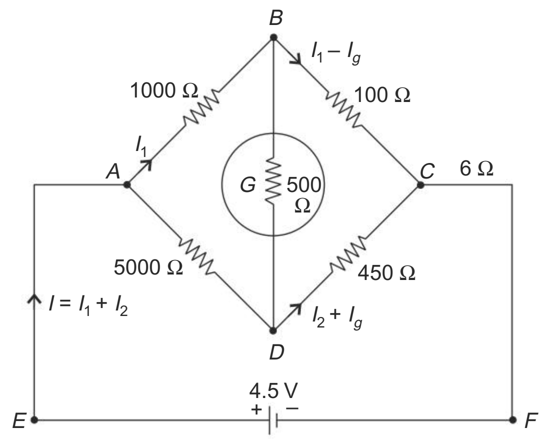


Fig. E.1.10

through galvanometer). Then, by KCL the current through the branch *BC* is $(I_1 - I_g)$. Applying KCL of node *D*, the current through branch *DC* is $(I_2 + I_g)$. Now, applying KCL at node *C*, the current through the branch *CFE* is $I_1 + I_2 = (1)$

Now apply KVL to the loop *ABDA*

$$-1000\, I_1 - 500\, I_g + 5000 I_2 = 0$$

$$2I_1 + I_g - 10 I_2 = 0 \quad (1)$$

Apply KVL to the *BCDB*

$$-100\,(I_1 - I_g) + 450\,(I_2 + I_g) + 500\, I_g = 0$$

$$-100\, I_1 + 1050\, Ig + 450\, I_2 = 0$$

or $$2I_1 - 21I_g - 9I_2 = 0 \quad (2)$$

Apply KVL to the loop *EADCFE*

$$-5000\, I_2 - 450\,(I_2 + I_g) + 4.5 = 0$$

$$5450\, I_2 + 450\, I_g = 4.5 \quad (3)$$

$$(1) - (2);\ 22\, I_g - I_2 = 0 \quad (4)$$

$$(4) \times 5450 \rightarrow 119900\, I_g - 5450\, I_2 = 0 \quad (5)$$

$$(3) + (5);\ 120350\, I_g = 4.5$$

and $$I_g = 37.391 - \mu A$$

As I_g is +ve the assumed direction is correct, i.e. *B* to *D*

Example 1.35 Calculate the current in 20 ohms resistor shown in Fig. E.1.11.

Let current I_1 is flowing in branch *FE*. I_1 divides into two paths at node *D*. Let I_2 be the current through *DA* (i.e., 20 ohm resistance), then, current through *DCBA* is $(I_1 - I_2)$. Now, if we apply KCL at node *A*, current in *AF* is I_1.

Apply KVL to loop *AFEDA*

$$+8 - 200\, I_1 - 20\, I_2 = 0$$

or $$200\, I_1 + 20\, I_2 = 8 \quad (1)$$

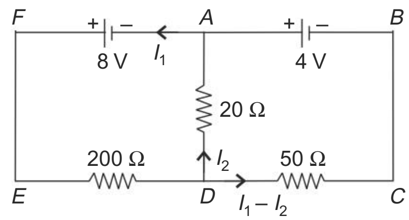


Fig. E.1.11

Apply KVL to looop *ADCDA*,

$$-4 + 50\,(I_1 - I_2) - 20I_2 = 0$$

$$50I_1 - 70I_2 - 4 \qquad (2)$$

$$(2) \times 4;\ 200I_1 - 280/_2 = 16 \qquad (3)$$

$$(3) - (1) \Rightarrow -300I_2 = 8$$

$$I_2 = -26.67 \text{ mA}$$

i.e., $I_2 = 26.67$ mA (–ve sign means that actual current direction is A to D and not D to A).

Example 1.36 The resistance of two coils is 25 ohms, when they are connected in series, and 6 ohms, when connected in parallel. Determine the individual resistance of the two coils.

Solution

Let R_1 and R_2 be the two resistance, then:

$$R_1 + R_2 = 25\,\Omega, \text{ and } \frac{R_1R_2}{(R_1 + R_2)} = 6\,\Omega = \frac{R_1R_2}{25\,\Omega}$$

or $$R_1R_2 = 150\,\Omega^2$$

$\therefore$ $$R_1 - R_2 = [(R_1 + R_2)^2 - 4R_1R_2]^{1/2} = [25^2 - 4 \times 150]^{1/2} = 5\,\Omega$$

$\therefore$ $$(R_1 + R_2) + (R_1 - R_2) = 2R_1 = 25 + 5 = 30\,\Omega$$

or $$R_1 = 15\,\Omega, \text{ and } R_2 = 10\,\Omega.$$

Example 1.37 If the total power dissipated in the network (shown in the Figure) is 16 watts, find the value of R, and the total current.

Solution

(i) $$R_{\text{effective}} = \frac{4 \times R}{4 + R} + \frac{2 \times 8}{(2 + 8)} = \frac{4R}{(4 + R)} + 1.6\,\Omega.$$

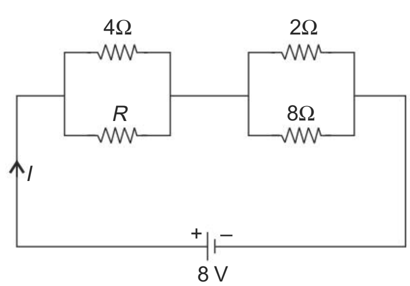


Fig. E.1.12

$\therefore$ Power, $P = 16\text{W} = V^2/R_{\text{effective}} = \dfrac{8^2}{\dfrac{4R}{(4+R)} + 1.6}$

or $\dfrac{4R}{(4+R)} + 1.6 = \dfrac{64}{16} = 4$ or $\dfrac{4R}{(4+R)} = 2.4$

or $\dfrac{R}{1} = 6\,\Omega.$

(ii) Now $R_{\text{effective}} = \dfrac{4\times 6}{4+6} + 1.6 = 4\,\Omega$

$\therefore$ Total current, $I = V/R_{\text{effective}} = 8/4 = 2A.$

Example 1.38 Determine the current flowing in the branches marked 1, 2 and 3 as shown the in Fig. E 1.13:

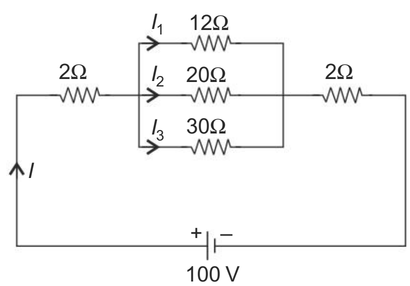


Fig. E.1.13

Solution

(i) $R_{\text{effective}} = 2 + \dfrac{1}{(1/12)+(1/20)+(1/30)} + 2 = 10\,\Omega.$

$\therefore$ Current, $I = V/R_{\text{effective}} = \dfrac{100}{100} = 10\,\text{A}.$

(ii) Let I_1, I_2 and I_3 be the respective values of current through 12 Ω, 20 Ω and 30 Ω resistances. Then

$$10\,\text{A} = I_1 + I_2 + I_3$$

$$10\,\text{A} = I_1 + \frac{I_1 \times R_1}{R_2} + \frac{I_1 \times R_1}{R_3} = I_1 + \frac{I_1 \times 12}{20} + \frac{I_1 \times 12}{30}$$

$$= I_1 + 0.6I_1 + 0.4I_1 = 2I_1$$

or $I_1 = 5\text{A}.$

$\therefore$ Current through:

12 Ω Resistance, $I_1 = 5\,\text{A}.$

12 Ω Resistance, $I_2 = 0.6 \times 5\,\text{A} = 3\,\text{A}.$

30 Ω Resistance, $I_3 = 0.4 \times 5\,\text{A} = 2\,\text{A}.$

Example 1.39 Two coils, connected in parallel across 100 V d.c. supply mains, take 10 A from mains. The power dissipated in one coil is 600 W. What is the resistance of each coil?

Solution Effective Resistance of R_1 and R_2 is $R_1 + R_2$.

$$R_{\text{effective}} = R_1R_2/(R1 + R_2) = 100\text{V}/10\text{A} = 10\,\Omega \quad \text{(i)}$$

Now Power, $P_1 = V^2/R_1 = 100^2/R_1 = 600\,\text{W}$

or $R_1 = \dfrac{10{,}000}{600} = 16.67\,\Omega.$

$\therefore$ $= \dfrac{16.67 \times R_2}{16.67 + R_2} = 10$ or $16.67\,R_2 = 166.7 + 10\,R_2$

or $R_2 = \dfrac{166.7}{6.67} = 25\,\Omega.$

Example 1.40 Calculate the supply current (I) in the following network, if 5 ohms resistor dissipates energy at the rate of 20 W.

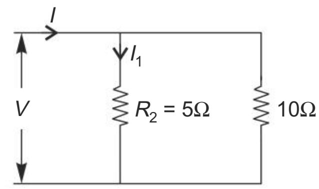


Fig. E.1.14

Solution Energy dissipated in $5\,\Omega$ resistor,

$$I_1^2 \times 5 = 20 \quad \text{or} \quad I_1 = 2\,\text{A}.$$

But $I_1 = \dfrac{10}{5+10} \times \quad I = \dfrac{2I}{3} = 2\,\text{A}$

$\therefore$ $I = 2\,\text{A} \times (3/2) = 3\,\text{A}.$

Example 1.41 Find the resistances in the network shown below, if power dissipation in R_2, and R_4 are 75 W and 30 W respectively.

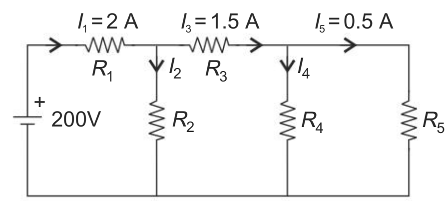


Fig. E.1.15

Solution

Current in $R_2, I_2 = 2.0 - 1.5\ \text{A} = 0.5\ \text{A},$

and current in $R_4, I_4 = 1.5 - 0.5\ \text{A} = 1.0\ \text{A}$

Now $I_2^2\,R_2 = (0.5)^2 \times R_2 = 75$ or $R_2 = 300\,\Omega$ (i)

and $I_4^2\,R_4 = I^2 \times R_4 = 30$ or $R_4 = 30\,\Omega$ (ii)

Also $R_4\,I_4 = R_5\,I_5$ or $30 \times 1 = R_5 \times 0.5$

$\therefore$ $R_5 = 60\ \Omega.$ (iii)

Also $R_1\,I_1 + R_2\,I_2 = 200$

$\therefore$ $R_1 \times 2 + 300 \times 0.5 = 200$ or $R_1 = (200 - 150)/2 = 25\,\Omega$ (iv)

Also $R_3\,I_3 + R_4\,I_4 = R_2\,I_2$

$\therefore$ $R_3 \times 1.5 + 30 \times 1 = 300 \times 0.5$ or $R_3 = (150 - 30)/1.5 = 80\,\Omega$ (v)

Example 1.42 A current of 20 A flows through two ammeters A and B joined in series. Across A, the p.d. is 0.2 V, and across B, it is 0.3 V. Find how the same current will divide between A and B, when they are joined in parallel?

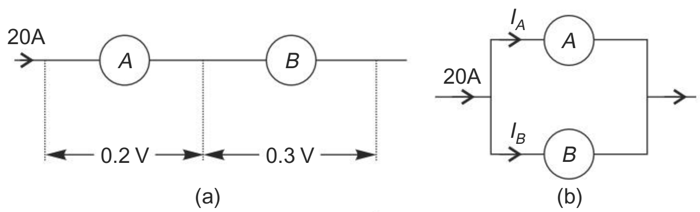


Fig. E.1.16

Solution

From Fig. E 1.16(a): $R_A = V_A/I = 0.2V/20\text{ A} = 0.01\,\Omega.$

$R_B = 0.3\text{V}/20\text{A} = 0.015\,\Omega$

From Fig. E 1.16(b): $I_A = R_B \times \text{I}/(R_A + R_B)$

$= 0.015 \times 20\text{ A}/(0.01 + 0.05) = 12\text{ A}$

$I_B = I - I_A = 20 - 12 = 8\text{ A}.$

Example 1.43 For the circuit shown in Fig. E1.17, find the resistance between *A* and *D*.

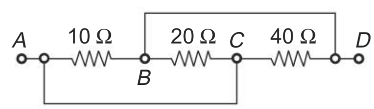


Fig. E.1.17

Solution The given circuit is:

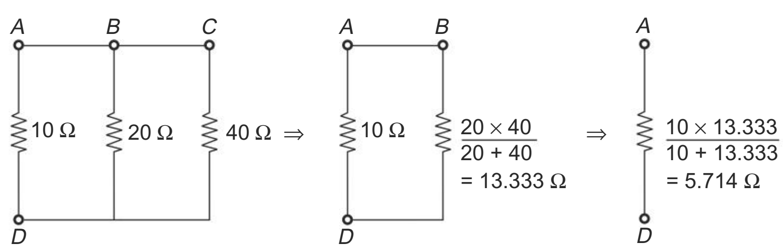


Fig. E.1.18

Hence, $R_{AD} = 5.714\,\Omega.$

Example 1.44 Two coils of resistances R_1 and R_2 are connected in parallel, and voltage of 200 V is applied to their terminals. The total current taken is 25 Amps, and the power dissipated in R_1 is 1,500 watts. Determine the magnitudes of R_1 and R_2.

Solution

Power dissipated in $R_1 = V^2/R_1 = 1{,}500$ W.

$\therefore \quad (200)^2/R_1 = 1{,}500$

or $\quad R_1 = (40{,}000/1{,}500)\,\Omega = 26.667\,\Omega.$

Now $$R_{\text{effective}} = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{V}{I} = \frac{200}{25} = 8\,\Omega$$

or $$8 = \frac{80/3 \times R_2}{80/3 + R_2} = \frac{80\,R_2}{80 + 3\,R_2}$$

which on solving gives $R_2 = 80/7 = 11.429\,\Omega$.

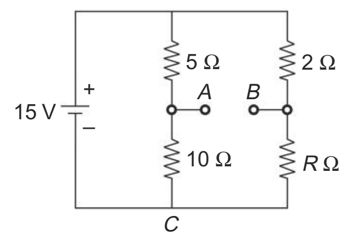


Fig. E.1.19

Example 1.45 In the circuit shown in Fig. E1.19, find the value of resistance R, when $V_{AB} = 5$ volts.

Solution

$$V_{AC} = \frac{10}{(10+5)} \times 15\text{ V} = 10\text{ V},$$

and $$V_{BC} = \frac{R}{(R+2)} \times 15\text{ V} = \frac{15R}{(R+2)}\text{V}$$

$\therefore$ $$V_{AB} = 5\text{ V} = V_{AC} - V_{BC}\left[10 - \frac{15\,R}{(R+2)}\right]\text{V} = \frac{(20-5R)}{(R+2)}\text{ V}$$

$\therefore$ $5R + 10 = 20 - 5R$ or $10R = 10$ or $R = 1\,\Omega$.

Example 1.46 Find (i) current in 15 Ω resistor, (ii) voltage across 18 Ω resistor, and (iii) power dissipated in 7 Ω resistor of the circuit given Fig. E.1.20:

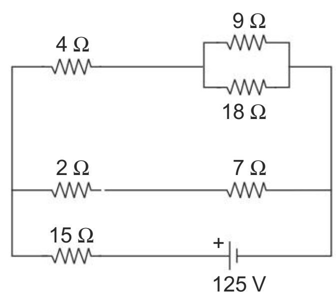


Fig. E.1.20

Solution

Resistance of Branch 1, $$R_1 = 4 + \frac{9 \times 18}{9 + 18} = 10\,\Omega$$

Resistance of Branch 2, $$R_2 = 2 + 7\,\Omega = 9\,\Omega$$

$\therefore$ $$R_{\text{effective}} = (10\,\Omega \times 9\,\Omega) + 15\,\Omega = \frac{10+9}{19} + 15 = 19.736\,\Omega$$

(i) Current in 15 Ω resistor, $I = V/R_{\text{effective}} = 125/19.736 = 6.33$ A.

(ii) Current in branch with 4 Ω resistor,

$$I_1 = \frac{I \times 9}{9 + 10} = \frac{6.33 \times 9}{19} = 3\text{A}$$

$\therefore$ Current in 18 Ω resistor $= \dfrac{I_1 \times 9}{9 + 18} = \dfrac{3 \times 9}{27} = 1$A.

Voltage across 18 Ω resistor = 18 × 1 = 18V.

(iii) Current in 7 Ω resistor, $I_2 = I - I_1 = 6.33 - 3 = 3.33$A

$\therefore$ Power dissipated in 7 Ω resistor $= I_2^2 \times 7 = (3.33)^2 \times 7 = 77.6$ W.

Example 1.47 Using KVL, determine total current drawn from the source and also current in 15 ohms resistance of the following circuit:

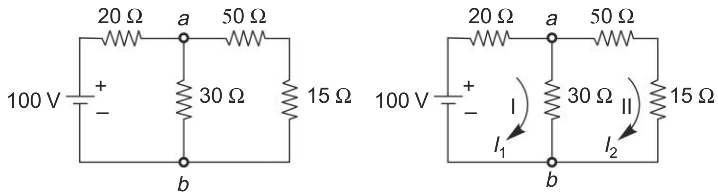


Fig. E.1.21

Solution Applying KVL to meshes I, and II, we get:

$$20I_1 + 30(I_1 - I_2) - 100 = 0$$

or $$50I_1 - 30I_2 = 100$$

or $$I_1 - 3I_2 = 10 \qquad \text{(i)}$$

$$(50 + 15)I_2 + 30(I_2 - I_1) = 0$$

$$-30I_1 + 95I_2 = 0$$

or $$I_2 = 30I_1/95 = 6I_1/19 \qquad \text{(ii)}$$

From (i) and (ii), we get:

$$5I_1 - 3 \times 6I_1/19 = 10$$

or $$95I_1 - 18I_1 = 190$$

or total current drawn, $I_1 = 190/77 = 2.4675$ A (iii)

From (ii) and (iii), we get:

$$I_2 = (6/19)I_1 = (6/19) \times (2.467) = 0.7792 \text{ A} \qquad \text{(iv)}$$

Hence, current through 15 Ω resistror,

$$I_2 = 0.7792 \text{ A}$$

Example 1.48 Find the value of R and the current flowing through it in the following network, when the current in branch OA is zero.

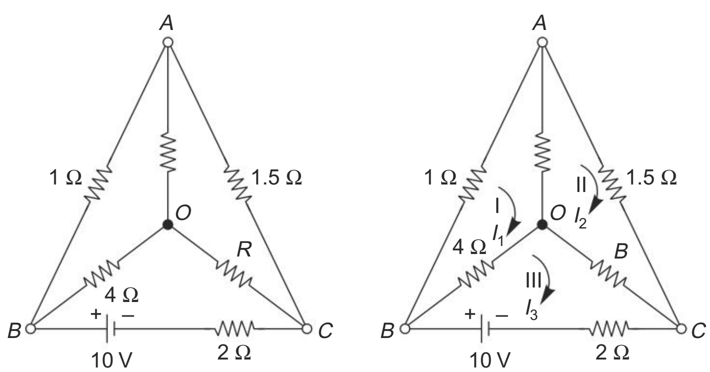


Fig. E.1.22

Solution Applying KVL to meshes I, II and III, we get,

$$1 \times I_1 + 0 + 4(I_1 - I_3) = 0$$

or $$5I_1 = 4I_3 \text{ or } I_3 = 1.25I_1 \quad \text{(i)}$$

Also current in OA branch $$I_1 - I_2 = 0$$

$\therefore$ $$I_2 = I_1 \quad \text{(ii)}$$

$$1.5I_2 + R(I_2 - I_3) + 0 = 0$$

or $$I_2(1.5 + R) = RI_3 \quad \text{(iii)}$$

or $$(1.5 + R)\, I_1 = R(1.25\, I_1) \text{ [from (i)]}$$

or $$R = 1.5/0.25 = 6\,\Omega \quad \text{(iv)}$$

$$4(I_3 - I_1) + R(I_3 - I_2) + 2\, I_3 - 10 = 0$$

or $$-4I1 - RI2 + (6 + R)I_3 = 0$$

or $$-4I_1 - 6I_1 + (6 + 6)\, 1.25\, I_1 = 10 \text{ [from (i), (ii) and (iii)]}$$

or $$-4I_1 - 6I_1 + 15I_1 = 10.$$

or $$5I_1 = 10.$$

or $$I_1 = I_2 = 10/5 = 2\text{A [from (ii)]}$$

and $$I_3 = 2 \times 1.25 = 2.5\text{A}$$

Hence, current following through R,

$$(I_3 - I_2) = 2.5 - 2.0 = 0.5\text{A}.$$

Example 1.49 A network is arranged as shown in Fig. E.1.23. Determine the value of the current in the 8 Ω resistor, using mesh equations.

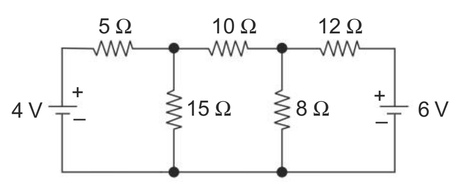


Fig. E.1.23

Solution

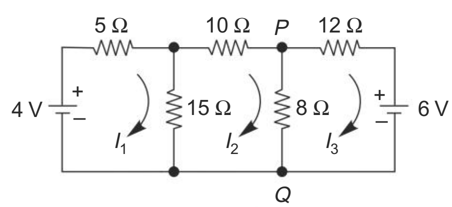


Fig. E.1.24

with reference to the Fig. E.1.24, we derive.

Loop I:

$$4 - 5I_1 - 15(I_1 - I_2) = 0 \text{ or } 20I_1 - 15I_2 = 4 \quad \text{(i)}$$

Loop II:

$$-10I_2 - 8(I_2 - I_3) - 15(I_2 - I_1) = 0 \text{ or } 15I_1 - 33I_2 + 8I_3 = 0 \quad \text{(ii)}$$

Loop III:

$$-12I_3 - 6 - 8(I_3 - I_2) = 0 \text{ or } 8I_2 - 20I_3 = 6 \quad \text{(iii)}$$

$\therefore$ 5 × (ii) + 2 × (iii) gives: $75I_1 - 149I_2 = 12$ (iv)

Solving (i) and (iv), we get: $I_2 = 0.032\text{A}$ (v)

From (i) and (v), we get: $I_1 = 0.224\text{A}$ (vi)

From (iii) and (vi), we get: $I_3 = -0.287\text{A}$ (vii)

$\therefore$ Current through the 8Ω resistor.

$$= I_2 - I_3 = 0.032\text{A} + 0.287\text{A} = 0.319\text{A} \quad \text{(from } P \text{ to } Q\text{)}$$

Example 1.50 Using mesh current analysis, find the current through the galvanometer *G* in the Wheatstone bridge shown in Fig. E.1.25.

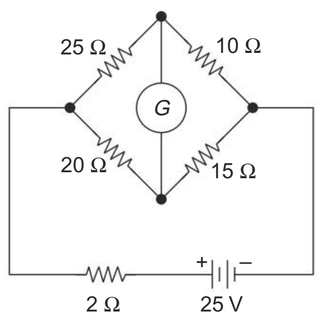


Fig. E.1.25

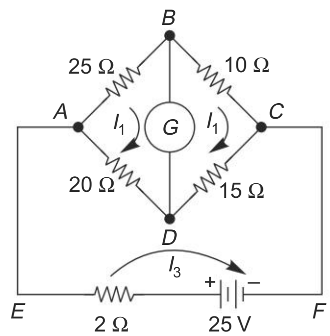


Fig. E.1.26

Solution

Refer to Fig. E.1.26. Applying KVL to loop *ABDA*, we get,

$$25I_1 + 50(I_1 - I_2) + 20(I_1 - I_3) = 0$$

or $$95I_1 - 50I_2 - 20I_3 = 0 \text{ or } 19I_1 - 10I_2 - 4I_3 = 0 \quad \text{(i)}$$

Applying KVL to loop *BCDB*, we get,

$$10I_2 + 15(I_2 - I_3) + 50(I_2 - I_1) = 0$$

or $$75I_2 - 15I_3 - 50I_1 = 0 \text{ or } 10I_1 - 15I_2 + 3I_3 = 0 \quad \text{(ii)}$$

Applying KVL to loop ADCFEA, we get,

$$20(I_3 - I_1) + 15(I_3 - I_2) + 2I_3 - 25 = 0$$

or $$37I_3 - 15I_2 - 20I_1 = 25 \text{ or } 20I_1 + 15I_2 - 37I_3 = -25 \quad \text{(iii)}$$

Now 3 × (i) – 2 × (ii) gives:

$$37I_3 - 18I_3 = 0 \text{ or } I_3 = 37I_1/18 \quad \text{(iv)}$$

Also (ii) + (iii) gives: $30I_1 - 34I_3 = -25$ (v)

From (iv) and (v) we get,

$$30\,I_1 - (34 \times 37/18)I_1 = -25$$

which on simplification gives: $I_1 = 0.62674$

Now $3 \times \text{(i)} + 4 \times \text{(ii)}$ gives: $97I_1 - 90I_2 = 0$ (vi)

or $I_2 = (97/90) \times 0.62674 = 0.67549$ (vii)

Hence, current through galvanometer,

$$= I_1 - I_2 = 0.62674 - 0.67549 \quad (\text{from } D \text{ to } B)$$

$$= 0.04875 \text{ from } D \text{ to } B \text{ or } 0.04875 \quad (A \text{ from } B \text{ to } D)$$

Example 1.51 Find the current through 30 ohms branch by mesh analysis.

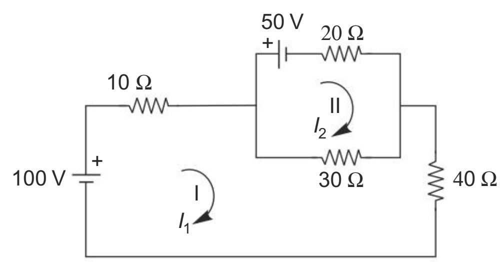


Fig. E.1.27

Solution Applying KVL to mesh I and II, we get:

$$100 - 10I_1 - 30(I_1 - I_2) - 40I_2 = 0$$

or $-80I_1 + 30I_2 = -100$ or $8I_1 - 3I_2 = 10$ (i)

$$50 - 20I_1 - 30(I_2 - I_1) = 0$$

or $30I_1 - 50I_2 = -50$

or $3I_1 - 5I_2 = -5$ (ii)

Eq. (i) × 5 – Eq. (ii) × 3 gives:

$$40I_1 - 15I_2 = 50$$

$$-9I_1 + 15I_2 = 15$$

or $31I_1 = 65$

$$I_1 = 2.0967\text{ A.} \quad \text{(iii)}$$

From Eqs. (ii) and (iii), we get:

$$6.29 - 5I_2 = -5$$

or $I_2 = 11.29/5 = 2.2587\text{ A.}$ (iv)

Hence, current in 30 Ω branch,

$$(I_2 - I_1) = 2.2587 - 2.0967 = 0.162\text{A}$$

Example 1.52 Determine the power output of each voltage source, using Kirchhoff's laws, for the network shown in Fig. E.1.28:

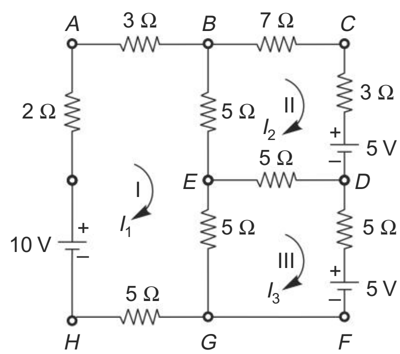


Fig. E.1.28

Solution Applying KVL to loops *ABEGHA*, *BCDEB* and *EDFGE*, we get:

$$(2 + 3 + 5)I_1 + 5(I_1 - I_2) + 5(I_1 - I_3) = 0$$

or $$20I_1 - 5I_2 - 5I_3 = 0$$

or $$4I_1 - I_2 - I_3 = 2 \qquad \text{(i)}$$

$$(7 + 3)I_2 + 5(I_2 - I_3) + 5(I_2 - I_1) = -5$$

or $$-5I_1 + 20I_2 - 5I_3 = -5$$

or $$I_1 - 4I_2 + I_3 = 1 \qquad \text{(ii)}$$

$$5(I_3 - I_2) + 5I_3 + 5(I_3 - I_1) = -5$$

or $$-5I_1 - 5I_2 + 15I_3 = -5$$

or $$I_1 + I_2 - 3I_3 = 1 \qquad \text{(iii)}$$

Solving Equation (i), (ii) and (iii), we get:

$I_1 = 0.37$A; $I_2 = -0.23$A; and $I_3 = -0.29$A.

∴ Power output from:

(i) Source of 10 V = 10 × 0.37 = 3.7 W
(ii) Source of 5 V = 5 × 0.23 = 1.15 W
(iii) Source of 5 V = 5 × 0.29 = 1.45 W

1.14 STAR TO DELTA AND TO STAR TRANSFORMATIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another in order to solve for unknown quantities.

Two circuit configurations that often account for these difficulties are the star or the Wye (*g*) and the Delta (*D*) as shown in Fig. 1.23 (i) and (ii)

(i) γ-net work
(ii) Δ-network

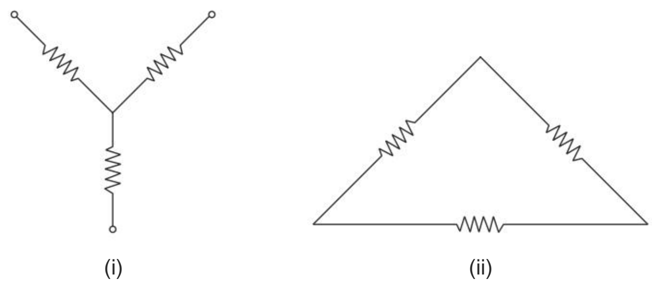


Fig. 1.23

They are also referred to as the Tee (T) network and the Pi (π) network respectively as shown in Fig. 1.24 (i) and (ii).

(i) T-network

(ii) π-network

Fig. 1.24

When we desire to convert the given network from one form to the other, we must have equations relating the resistances of the two forms. Thus the conversion gives the electrically equivalent network. This means that they draw the same amount of current when connected across a specified source.

Let us now make the two different analyses and derive the required equations.

1.14.1 Delta to Star Transformation

The given network is in delta form. We are to obtain its equivalent star form.

Consider the equivalent resistance between the terminals a and b. When the two networks are electrically equivalent, the resistances between terminals a and b as measured in star and in delta must be same. In star network as shown in Fig. 1.25. Resistance between terminals a and b is $R_{ab} = R_a + R_b$.

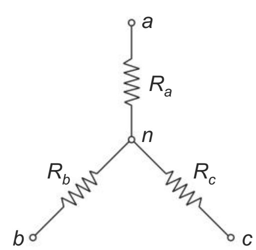


Fig. 1.25

In delta network as shown in Fig. 1.26 (i) and (ii):

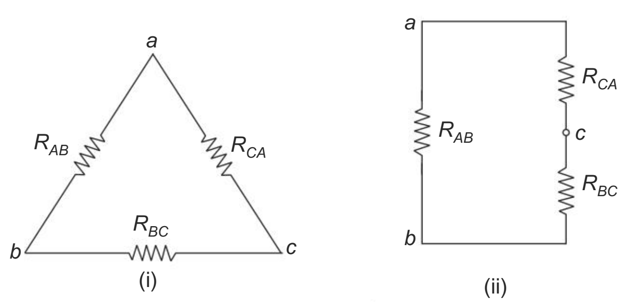


Fig. 1.26

Resistance between terminals a and b is

$$R_{ab} = \frac{R_{AB}\,(R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

Thus

$$R_a + R_b = \frac{R_{AB}\,(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \tag{1}$$

In a similar manner, equating the effective resistances between terminals b and c and a, we get the following relationships.

$$R_b + R_c = \frac{R_{BC}\,(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \tag{2}$$

$$R_c + R_a = \frac{R_{CA}\,(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \tag{3}$$

Subtracting (2) from (1),

$$R_a - R_c = \frac{R_{AB}\,R_{CA} - R_{BC}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{4}$$

Adding (3) and (4),

$$2R_a = \frac{2\,R_{CA}\,R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_a = \frac{R_{CA}\,R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \tag{1.17}$$

Similarly,

$$R_b = \frac{R_{AB}\,R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \tag{1.18}$$

$$R_c = \frac{R_{BC}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{1.19}$$

If the networks considered have conductances, the following set of Eqns 1.20 to 1.22 can be used for converting delta network into star network.

$$\frac{1}{G_a} = \frac{\dfrac{1}{G_{AB}}\,\dfrac{1}{G_{CA}}}{\dfrac{1}{G_{AB}} + \dfrac{1}{G_{BC}} + \dfrac{1}{G_{CA}}} = \frac{1}{G_{CA}\,G_{AB}}\;\frac{G_{AB}\,G_{BC}\,G_{CA}}{G_{AB}\,G_{BC} + G_{BC}\,G_{CA} + G_{CA}\,G_{AB}}$$

$$= \frac{G_{BC}}{G_{AB}\,G_{BC} + G_{BC}\,G_{CA} + G_{CA}\,G_{AB}}$$

$$-\,G_a = \frac{G_{AB}\,G_{BC} + G_{BC}\,G_{CA} + G_{CA}\,G_{AB}}{G_{BC}} \tag{1.20}$$

$$G_b = \frac{G_{AB}\,G_{BC} + G_{BC}\,G_{CA} + G_{CA}\,G_{AB}}{G_{CA}} \tag{1.21}$$

$$G_c = \frac{G_{AB}\,G_{BC} + G_{BC}\,G_{CA} + G_{CA}\,G_{AB}}{G_{AB}} \tag{1.22}$$

1.14.2 Star to Delta Transformation

The given network is in star form. We have to obtain its equivalent delta form.

Let us consider the networks consisting of conductances as shown in Fig. 1.27(i) and (ii)

In order to evaluate the required parameters, a set of three measurements are made and equations are formulated.

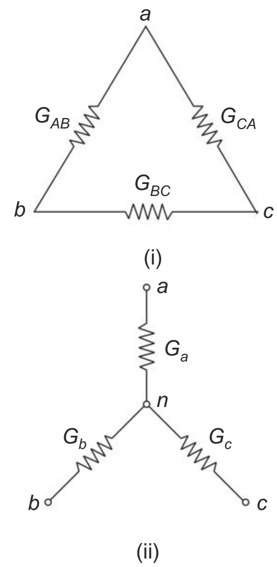


Fig. 1.27

Step 1 Terminals b and c are short circuited and the effective conductance between terminals a and b is found out.

In delta network Effective conductance between terminals a and b is $G_{ab} = G_{AB} + G_{CA}$

In star network Effective conductance between terminals a and b is

$$G_{ab} = \frac{G_a (G_b + G_c)}{G_a + G_b + G_c}$$

For the two networks to be electrically equivalent, the effective conductance as measured above must be equal.

Thus, $$G_{AB} + G_{CA} = \frac{G_a (G_b + G_+)}{G_a + G_b + G_c} \tag{1}$$

Step 2 Terminals c and a are short-circuited and the effective conductance between terminals b and c is measured.

Thus, $$G_{BC} + G_{AB} = \frac{G_b (G_c + G_a)}{G_a + G_b + G_c} \tag{2}$$

Step 3 Terminals a and b are short-circuited and the effective conductance between terminals c and a is measured.

Thus, $$G_{CA} + G_{BC} = \frac{G_c(G_a + G_b)}{G_a + G_b + G_c} \tag{3}$$

Subtracting (3) from (2) we have,

$$G_{AB} - G_{CA} = \frac{G_a G_b + G_c G_a}{G_a + G_b + G_c} \tag{4}$$

Adding, (1) and (4) we have,

$$2G_{AB} = \frac{2\, G_a G_b}{G_a + G_b + G_c}$$

or $$G_{AB} = \frac{G_a G_b}{G_a + G_b + G_c} \tag{1.23}$$

$$G_{BC} = \frac{G_b G_c}{G_a + G_b + G_c} \tag{1.24}$$

$$G_{CA} = \frac{G_b G_c}{G_a + G_b + G_c} \tag{1.25}$$

In the above set of Eqns 1.23 to 1.25 we can substitute resistances in place of conductances and thus obtain the transformation between resistive networks.

$$\frac{1}{R_{AB}} = \frac{\frac{1}{R_a}\frac{1}{R_b}}{\frac{1}{R_a}+\frac{1}{R_b}+\frac{1}{R_c}} = \frac{1}{R_a R_b}\frac{R_a R_b R_c}{R_a R_b + R_b R_c + R_c R_a}$$

$$= \frac{R_c}{R_a R_b + R_b R_c + R_c R_a}$$

$$R_{AB} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c} \tag{1.26}$$

$$R_{BC} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a} \tag{1.27}$$

$$R_{CA} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b} \tag{1.28}$$

Example 1.53 A delta network consists of equal resistances in all the three arms. Find the resistances of the arms of its equivalent star.

Solution Let R (ohms) be the resistance of the given delta network.

As the given delta network has equal resistance in all the arms, its equivalent star network will also have equal resistances in all its three arms. Let R' represent the resistance of the equivalent star network.

Then $$R' = \frac{R \,.\, R}{R+R+R} = \frac{R}{3} \text{ (ohms)}$$

Example 1.54 Determine the equivalent resistance across terminals A and B in the network shown in Fig. E.1.129.

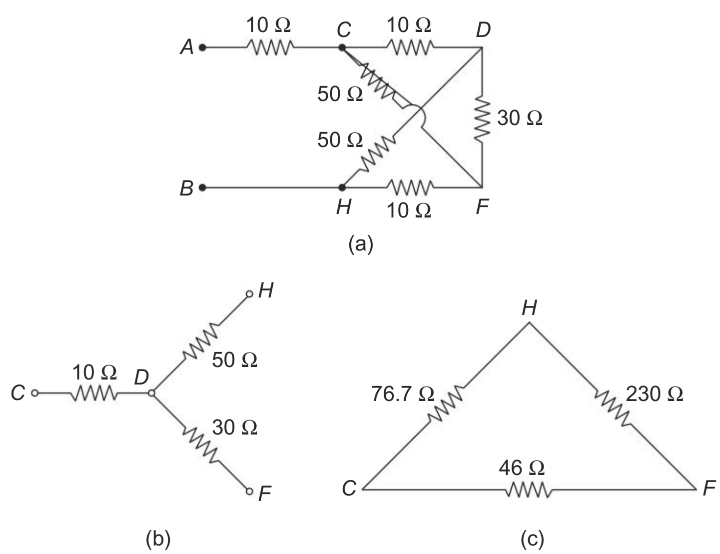


Fig. E.1.29 (a–c)

Solution Using star to delta conversion, the star point D is eliminated.

$$R_{CF} = \frac{10 \times 30 + 30 \times 50 + 50 \times 10}{50} = \frac{2300}{50} = 46\ \Omega$$

$$R_{FH} = \frac{2300}{10} = 230\ \Omega \quad R_{HC} = \frac{2300}{10} = 76.7\ \Omega$$

$$R_{CF} = \frac{50 \times 46}{50 + 46} = 23.96\ \Omega$$

$$R_{HF} = \frac{10 \times 230}{10 + 230} = 9.6\ \Omega$$

Redrawing the network, we have

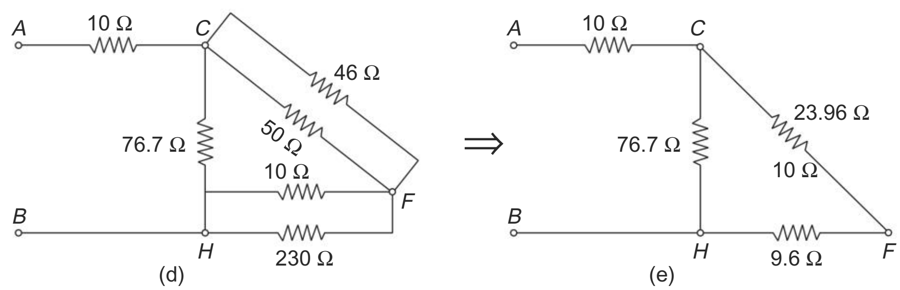


Fig. E.1.29 (d-e)

R_{CF} and R_{HF} act in series between terminals C and H. Hence they are added to eliminate node F. Thus $R_{CH} = 23.96 + 9.6 = 33.56\ \Omega$.

Between nodes C and H. 76.7 Ω and 33.56 Ω act in parallel. Thus the effective resistance is

$$R_{CH} = \frac{76.7 \times 33.56}{76.7 + 33.56} = 23.35\ \Omega$$

Equivalent resistance across between terminals A and B is 35.35 Ω

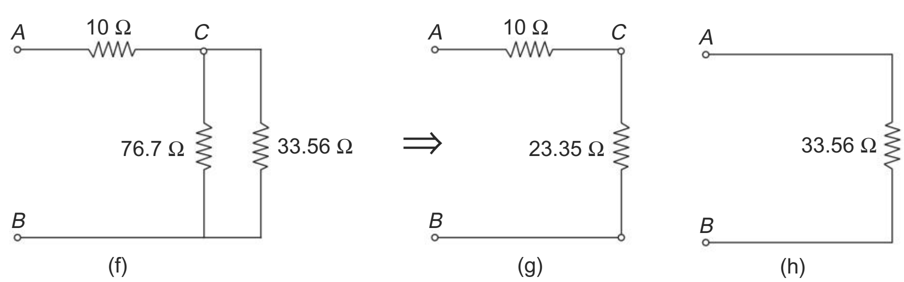


Fig. E.1.29 (f-h)

Example 1.55 A network of 9 conductors connects 6 points A, B, C, D, E, F as shown in Fig. E.1.30. Determine the resistance between A and C and the resistance between D and F.

(a) To find the resistance between A and C

The method used is carried out in the following steps:

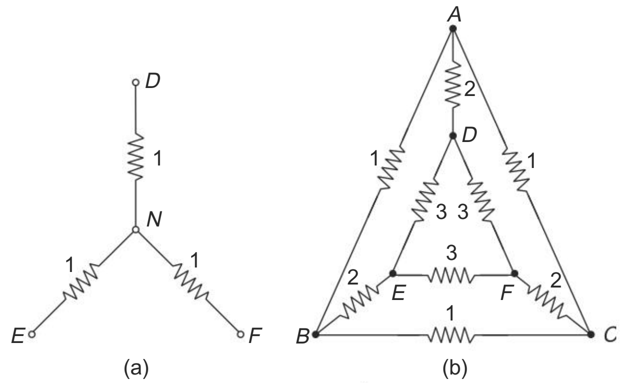


Fig. E.1.30 (a-b)

Step No. **Process done** **Network redrawn**

1. Delta *DEF* is converted into its equivalent star.
   $R_{DN} = R_{EN} R_{FI} = 1\ \Omega$
2. The series branches of the inner star network are added
   $R_{AN} = R_{AD} + R_{DN} = 3\ \Omega$
   $R_{BN} = R_{BE} + R_{EN} = 3\ \Omega$
   $R_{CN} = R_{CF} + R_{FN} = 3\ \Omega$
3. Star *ABCN* is converted into its equivalent delta
   $$R_{AB} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3} = 9\ \Omega$$
   $R_{BC} = R_{CA} = 9\ \Omega$

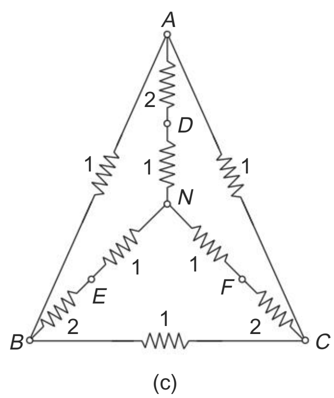


Fig. E.1.30 (c)

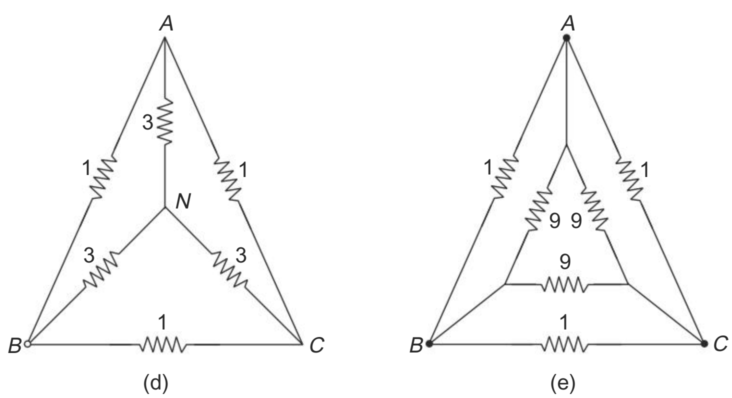


Fig. E.1.30 (d-e)

4. Parallel resistances as shown in each branch are combined into single resistance

Effective resistance = $\frac{1 \times 9}{1 \times 9}$ = 0.9 Ω

5. As viewed across terminals A and C,
Branches AB and BC act in series
Thus effective resistance = 0.9 + 0.9 = 1.8 Ω

6. 1.8 Ω and 0.9 Ω act in parallel. Effective resistance = $\frac{1.8 \times 0.9}{1.8 + 0.9}$ = 0.6 Ω

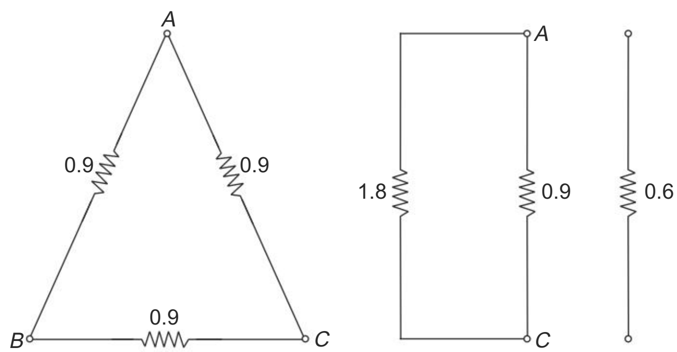


Fig. E.1.30 (f-h)

(b) Resistance between D and F: One of the possible methods is given below in terms of steps required:

Step No. **Process done** **Network redrawn**

1. Node A is eliminated using Y to Δ conversion

$$R_{BC} = \frac{1 \times 2 + 2 \times 1 + 1 \times 1}{2} = \frac{5}{2} = 2.5\ \Omega$$

$$R_{CD} = \frac{5}{I} = 5\ \Omega$$

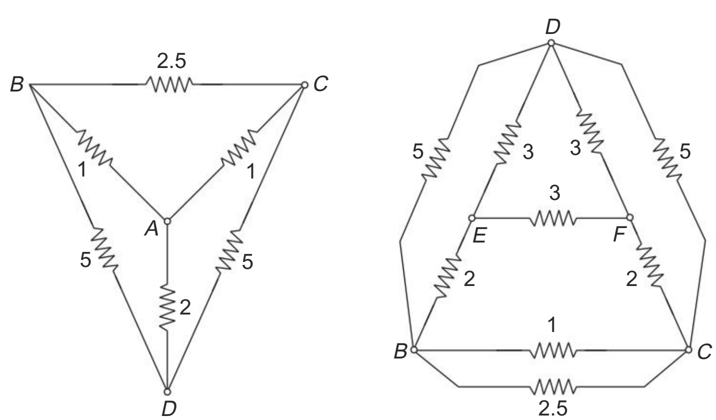


Fig. E.1.30 (i-j)

2. Node C is eliminated using Y to Δ conversion

$$R_{DB} = \frac{0.72 \times 5 + 5 \times 2 + 2 \times 0.72}{2}$$

$$= \frac{15.04}{2} = 7.52\ \Omega$$

$$R_{BF} = \frac{15.04}{5} = 3\ \Omega$$

$$R_{FD} = \frac{15.04}{0.72} = 20.9\ \Omega$$

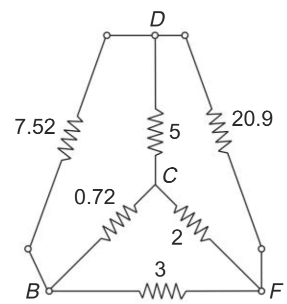


Fig. E.1.30 (k)

3. Parallel branches between nodes B and D and nodes D and F are reduced

$$\frac{7.52 \times 5}{7.52 + 5} = 3\ \Omega$$

$$\frac{20.9 \times 3}{20.9 + 3} = 2.6\ \Omega$$

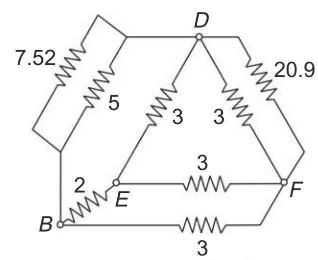


Fig. E.1.30 (l)

4. Node E is eliminated using Y to Δ conversion

$$R_{BF} = \frac{2 \times 3 + 3 \times 3 + 3 \times 3}{3} = \frac{21}{3} = 7\ \Omega$$

$$R_{FD} = \frac{21}{2} = 10.5\ \Omega$$

$$R_{DB} = \frac{21}{3} = 7\ \Omega$$

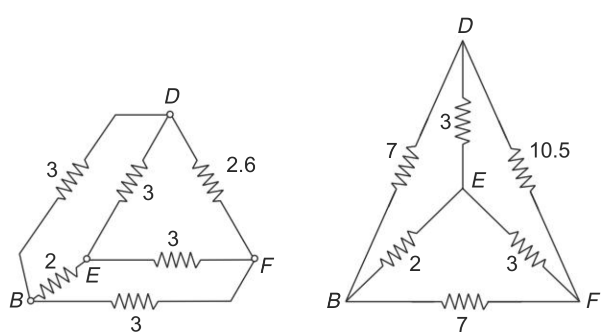


Fig. E.1.30 (m - n)

5. All the parallel branches are reduced

$$\frac{7 \times 3}{7 + 3} = 2.1\ \Omega$$

$$\frac{2.6 \times 10.5}{2.6 + 10.5} = 2.1\ \Omega$$

6. $R_{DB} + R_{BF} = 4.2\ \Omega$

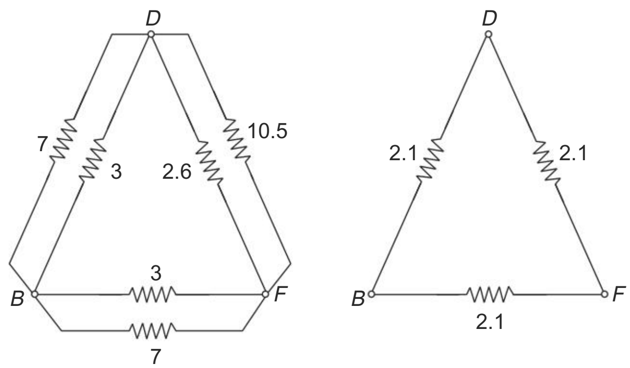


Fig. E.1.30 (o-p)

7. $$\frac{4.2 \times 2.1}{4.1 + 2.1} = 1.4\ \Omega$$

Effective resistance between terminals D and $F = 1.4\ \Omega$

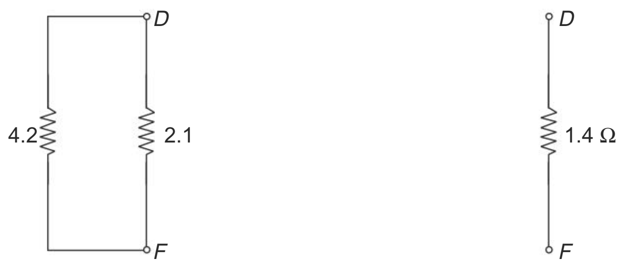


Fig. E.1.30 (q-r)

Example 1.56 Using node voltage analysis, obtain the currents flowing in all the resistors of the circuit shown in Fig. E.1.32.

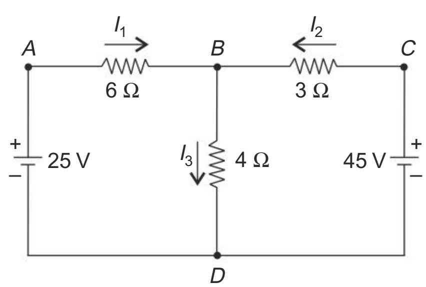


Fig. E.1.31

Solution There are four nodes A, B, C and D. Of these. node D can be taken as reference node. Voltage sources are connected between nodes A and D and nodes C and D. Thus the potentials (voltages) at nodes A and C are 25 V and 45 V respectively. Hence B is the only independent node the potential at which is unknown.

Let V_B be the voltage at node B.

Applying Kirchhoff's current law at node B, we get

$$-I_1 - I_2 + I_3 = 0$$

But

$$I_1 = \frac{V_A - V_B}{6} = \frac{25 - V_B}{6}$$

$$I_2 = \frac{V_C - V_B}{3} = \frac{45 - V_B}{3}$$

$$I_3 = \frac{V_B}{4}$$

$$\therefore \quad \frac{25 - V_B}{6} - \frac{45 - V_B}{3} + \frac{V_B}{4} = 0$$

$$\therefore \quad V_B\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{4}\right) = \frac{25}{6} + \frac{45}{3}$$

$$V_B\left(\frac{9}{12}\right) = \frac{115}{6} \qquad \therefore\ V_B = 25.56 \text{ volts}$$

$$\therefore \quad I_1 = \frac{25 - 25.6}{6} = -0.1 \text{ A} \quad \text{(Direction is from } B \text{ to } A\text{)}$$

$$I_2 = \frac{45 - 25.6}{3} = 6.47 \text{ A}$$

$$I_3 = \frac{25.6}{4} = 6.4 \text{ A}$$

Example 1.57 Compute the voltage at nodes A and B in the circuit of Fig. E.1.32 C is the reference node.

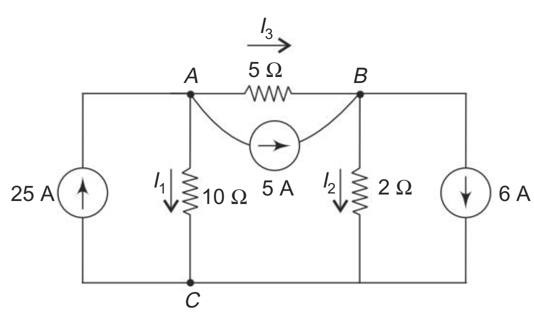


Fig. E.1.32

Solution A and B are the nodes, the voltage at which are required. Let V_A and V_B be the voltage at the nodes A and B respectively. Applying KCL at node A, we have

$$-25 + I_1 + 5 + I_3 = 0$$

or $$-25 + \frac{V_A}{10} + \frac{V_A - V_B}{5} + 5 = 0$$

or $$V_A\left(\frac{1}{10} + \frac{1}{5}\right) - \frac{V_B}{5} = 20 \quad (1)$$

Applying KCL at node B, we have

$$-I_3 - 5 + I_2 + 6 = 0$$

or $$-\frac{V_A - V_B}{5} + \frac{V_B}{2} = -1 \quad (2)$$

or $$-\frac{V_A}{5} + V_B\left(\frac{1}{5} + \frac{1}{2}\right) = -1$$

$$0.3\ V_A - 0.2\ V_B = 20 \quad (3)$$

$$-0.2\ V_A + 0.7\ V_B = -1 \quad (4)$$

(3) × 0.2, gives $0.06\ V_A - 0.04\ V_B = 4$

(4) × 0.3, gives $-0.06\ V_A + 0.21\ V_B = -0.3$

Adding, $0.17\ V_B = 3.7$

$\therefore$ $V_B = 21.7$ V

Using the value of V_B in Eqn. (3), we have

$$0.3\ V_A - 0.2 \times 21.7 = 20$$

$$-V_A = \frac{20 + 0.2 \times 21.7}{0.3} = 81.1 \text{ V}$$

Example 1.58 A bridge network $ABCD$ has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by star/delta transformation, the network resistance as viewed from the battery terminals BD.

Solution As shown in Fig. E.1.33(b), delta DAC has been reduced to its equivalent star.

$$R_D = \frac{2 \times 1}{2 + 1 + 1} = 0.5\,\Omega,\ R_A = \frac{1}{4} = 0.25\,\Omega,\ R_C = \frac{2}{4} = 0.5\,\Omega$$

Hence, the original network of Fig. E.1.33(a) is reduced to the one shown in Fig. E.1.33(d). As seen, there are two parallel paths between points N and B, one of resistance 1.25 Ω and the other of resistance 1.5 Ω. Their combined resistance is

$$= \frac{1.25 \times 1.5}{1.25 + 1.5}$$

$$= \frac{15}{22}\,\Omega$$

Total resistance of the network between points D and B is

$$= 0.5 + \frac{15}{22} = \frac{13}{11}\,\Omega$$

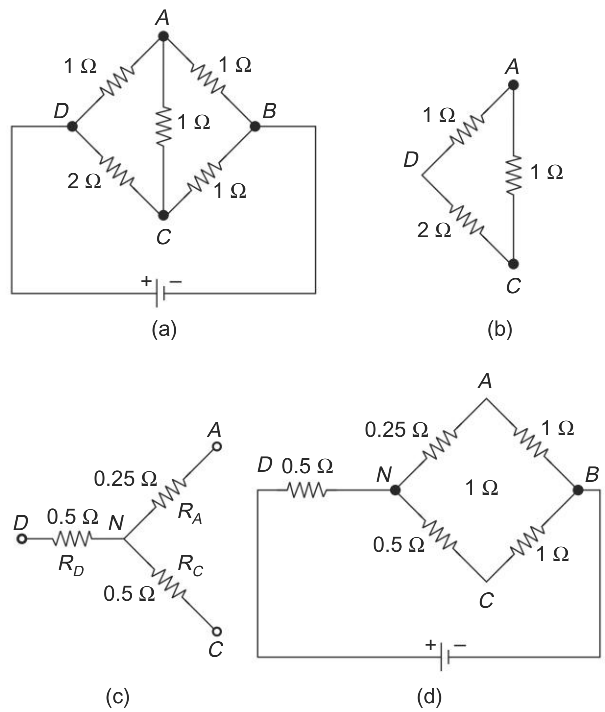


Fig. E.1.33

Example 1.59 A network of resistances is formed as follows:

$AB = 9\ \Omega$; $BC = 1\ \Omega$; $CA = 1.5\ \Omega$ forming a delta and $AD = 6\ \Omega$; $BD = 4\ \Omega$ and $CD = 3\ \Omega$ forming a star. Compute the network resistance measured between (i) A and B (ii) B and C (iii) C and A.

Solution The star of Fig. E.1.34(a) may be converted into the equivalent delta and combined in parallel with the given delta ABC. Using the rule given in Arr, the three equivalent delta resistances of the given star become as shown in Fig. E.1.34 (b).

When combined together, the final circuit is as shown in Fig. E.1.34 (c).

(i) As seen, there are two parallel paths across points A and B.

(a) One directly from A to B having a resistance of $6\,\Omega$ and

(b) The other via C having a total resistance

$$= \left[\frac{27}{20} + \frac{9}{10}\right] = 2.25\,\Omega \quad \therefore R_{AB} = \frac{6 \times 2.25}{(6 + 2.25)} = \frac{18}{11}\Omega$$

(ii)
$$R_{BC} = \frac{\frac{9}{10} \times \left[6 + \frac{27}{20}\right]}{\left[\frac{9}{10} + 6 + \frac{27}{10}\right]} = \frac{441}{550}\Omega$$

(iii)
$$R_{CA} = \frac{\frac{27}{20} \times \left[6 + \frac{9}{10}\right]}{\left[\frac{9}{10} + 6 + \frac{27}{20}\right]} = \frac{661}{550}\Omega$$

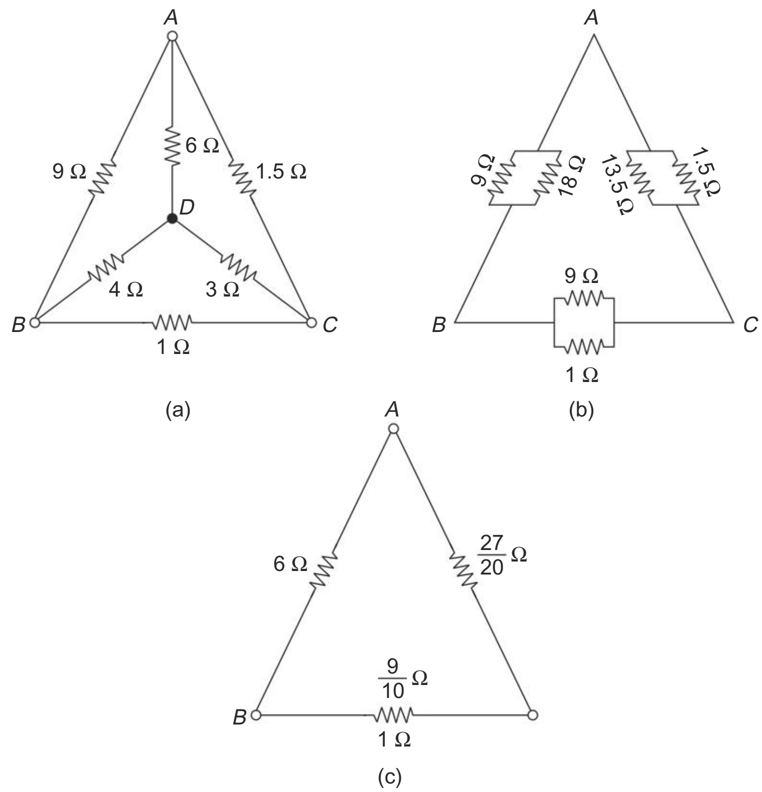


Fig. E.1.34

IMPORTANT FORMULAE

1. $I = \dfrac{Q}{t}$ or $\dfrac{dQ}{dt}$ amperes where Q and t are charge in coulombs and time in seconds respectively.
2. $V = \dfrac{\text{Word done}}{\text{Charge}} = W/Q$ volts where W is in joules and Q in Coulombs.
3. $R = \rho \dfrac{1}{a}$ ohms ρ in ohms-m or ohms-cm

   l in m or cm

   a in m^2 or cm^2
4. If a conductor has resistance of R_0, R_1 at 0°C and t°C respectively then $R_1 = R_0\,(1 + \alpha_0\, t)$ where α_0-temperature coefficient of resistance at 0°C.
5. $\dfrac{R_2}{R_1} = \dfrac{1 + \alpha_0\, t_2}{1 + \alpha_0\, t_1}$; $\alpha_0 = \dfrac{\text{Slope of temp. vs. resistance graph}}{R_0}$

   $$\alpha_t = \frac{\text{Slope of temp. vs. resistance graph}}{R_t}$$

Also, slope is constant, $R_t > R_0 \quad \therefore \alpha_0 > \alpha_t$

6. $R_2 = R_1 (1 + \alpha_1 (t_2 - t_1))$
7. $\alpha_2 = \dfrac{1}{1/\alpha_1 + (t_2 - t_1)}$
8. $1 = V/R$ ohm's law or $R = V/I$ or $V = I/R$
9. Electric power $P = VI = I^2R = \dfrac{V^2}{R}$ watts where V, I and R is volume amperes and ohms respectively.
10. Electrical energy $= VI/t = \dfrac{V^2}{R}t = I^2Rt$ joules or watt sec

    where V in volts
    I in amperes
    R in ohms
    t in secs
11. When n number of resistances are connected in series, then, equivalent total resistant R_T is
$$R_T = R_1 + R_2 + \ldots + R_n$$
12. When n number of conductance are connected in series, then, equivalent or total conductance G_T is given by
$$\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \ldots + \frac{1}{G_n}$$
13. When n number of resistances are connected in parallel then, resistance R_T is given by
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$
14. When n number of equal resistance each of R ohms are connect parallel then, total resistance R_T is given by
$$R_T = \frac{R}{n} \text{ ohms}$$
15. When n number of conductances are connected in parallel then, the total conductance G_T is given by
$$GT = G_1 + G_2 + \ldots + G_n$$
16. When two resistances R_1 and R_2 are connected in parallel, then, the equivalent resistance R_T is
$$R_T = \frac{R_1R_2}{R_1 + R_2} \quad \text{i.e. } \frac{\text{Product}}{\text{Sum}}$$
17. When two resistances R_1 and R_2 in parallel carry a total current of I, then

    Current through R_1, $I_1 = I\dfrac{R_2}{R_1 + R_2}$

    Current through R_2, $I_2 = I\dfrac{R_1}{R_1 + R_2}$

    i.e. current in one resistance = Total current $\times \dfrac{\text{Other resistance}}{\text{Total resistance}}$

18. When n number of resistances in parallel carry total current I, then current in any resistance is given by

$$I_x = \frac{G_x}{\sum_{x=1}^{n} G_x}$$

where $x = 1, 2, 3...n$

and $G_1, G_2 ... G_n$ conductances of $R_1, R_2, R_3 ... R_n$ respectively.

19. The P.D. V across the terminals of a cell is $V = E - Ir$ where

E = emf of the cell

I = load current (current drawn by the load from the cell)

r = internal resistance of the cell

20. (i) At any junction (or node) in an electrical circuit algebraic

$$\Sigma I = 0 \text{ (KCL)}$$

(ii) In any closed circuit or mesh or loop.

The algebraic ΣE + algebraic ΣIR drops = 0 (KVL)

21. When 'n' number of resistances are in parallel, with a total voltage 'V' volts applied across them, then the voltage across any resistance is given by

$$V_x = V\left(\frac{R_x}{\sum_{x=1}^{n} R_x}\right)$$

22. To convert delta resistance network into its equivalent star network:

Resistance between one node and the star point

$$= \frac{\text{Product of resistances connected to that node}}{\text{Sum of all the three resistances}}$$

23. To convert star resistance network into its equivalent delta network.

Resistance between any two nodes.

$$= \frac{\text{Sum of the product of two resistances taken at a time}}{\text{Resistance between the third node and the neutral point}}$$

24. To convert delta conductance network into its equivalent star network:

Conductance between one node and the star point.

$$= \frac{\text{Sum of the product of two conductances taken at a time}}{\text{Conductance between the other two nodes}}$$

25. To convert star conductance network into its equivalent delta network: Conductance between any two nodes.

$$= \frac{\text{Product of conductances connected to those two nodes}}{\text{Sum of all the three conductances}}$$

REVIEW QUESTIONS

1. What is the fundamental difference between emf and P.D.?
2. Why do conductors have positive temperature coefficient of resistance while insulators have negative temperature coefficient of resistance?

3. What is the importance of temperature/resistance graph of a conductor?
4. Give the characteristics of a series and parallel circuit.
5. What are the advantages of series, parallel and series parallel circuit?
6. Why are domestic appliances connected in parallel?
7. Obtain the equation for the current through any resistance, in a parallel circuit having *n* number of different resistances.
8. State and explain Kirchoff's laws.
9. How will you prove the validity of Kirchoff's laws?
10. State and explain Ohm's law. Give its limitations.
11. Obtain the equation for the voltage across any resistance in a series circuit having '*n*' number of different resistances.
12. Derive equations to relate the resistances at different temperatures.
13. Derive the formula to compute the temperature coefficient of a resistance at any temperature given its value at a particular temperature.
14. Obtain expressions for the equivalent star network resistances for a given delta network.
15. You are given a star resistive network. How will you convert it into its equivalent delta? Justify your answer.

PROBLEMS

1. Find the resistance of 1000 metres of a copper wire 25 sq. mm in cross-section. What will be the resistance of another wire of the same material, three times as long and one-half the cross-sectional area?
2. A length of wire has a resistance of 4.5 ohm. Find the resistance of another wire of the same material three times as long and twice the cross-sectional area.
3. A copper wire of diameter 1 cm has a resistance of 0.15 ohms. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire?
4. The field winding of a generator has a resistance of 12.7 ohms at 18°C and 14.3 ohms at 50°C. Find (i) temperature co-efficient at 0°C, (ii) resistance at 0°C and (iii) temperature co-efficient at 18°C.
5. The shunt winding of a motor has it resistance of 80 ohms at 15°C. Find its resistance at 50°C. Resistance temperature coefficient of copper is 0.004/°C at 0°C.
6. The resistance of the field coils of a dynamo is 173 ohms at 16°C. After working for 6 hours on full load, the resistance of the coil increases to 212 ohm. Calculate the mean temp rise of field coils. Assume temperature coefficient of resistance of copper to be 0.00426/°C at 0°C.
7. The field circuit of a 440 V shunt motor took 2.3 A when first switched on, the ambient temperature being 17°C. Later, the field current was found to remain steady at 1.9 A. Determine the temperature of the winding assuming $\alpha = 1/234.5$ per°C.
8. Two coils connected in series have resistances of 600 ohms and 300 ohms and temperature coefficients of 0.001/°C and 0.004/°C respectively at 20°C. Find the resistance of the combination at 50°C. What is the effective temperature coefficient of the combination at 20°C?
9. A current of 10 A flows through a resistor for 10 minutes and the power dissipated by the resistor is 100 watts. Find the P.D. across the resistor and the energy supplied to the circuit.

10. The electrical load in a small workshop consists of 14 lamps each rated at 240V, 60V and 3 electric fans each rated at 240V, 1 kW. What is the effective resistance of the load.
11. Three resistors 8 ohms, 6 ohms and 10 ohms are connected in series to a battery of terminal voltage 24 volts. Find the current in the circuit, P.D. across each resistor and power dissipated in each resistor.
12. A 100 watts 250 V lamp is connected in series with a 100 W, 200 V lamp across 250 supply. Calculate (i) circuit current and (ii) voltage across each lamp. Assume the lamp resistance to remain unaltered.
13. The element 500 watt electric iron is designed for use on a 200 V supply. What value of resistance is needed to be connected in series in order that the iron can be operation from 240 V supply.
14. Two coils connected in series have a resistance of 18 ohms and when connected parallel have a resistance of 4 ohms. Find the value of resistances.
15. Three resistors 4 ohms, 12 ohms and 6 ohms are connected in parallel. If the current taken is 12 A, find the current through each resistor.
16. A battery having emf of 12 V is connected across terminals *AB* of the circuit shown in Fig. P.1.1. Find (i) the current flowing in each resistance and (ii) total power absorbed by the circuit.

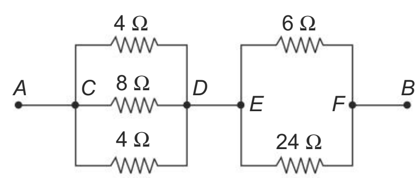


Fig. P.1.1

17. Determine the current *I* in the circuit shown in Fig. P.1.2. All resistances are in ohms.

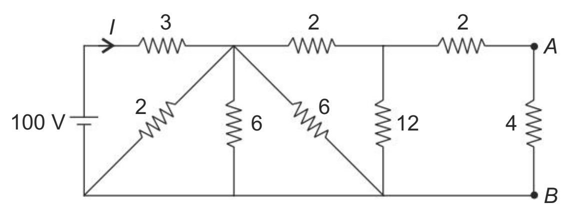


Fig. P.1.2

18. Determine the current supplied by the battery in the circuit shown in P.1.3 by using Kirchhoff's laws.

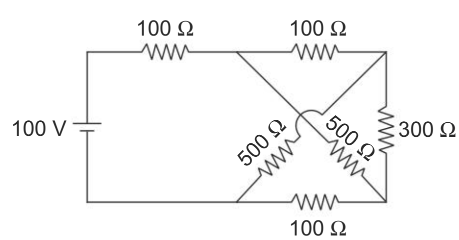


Fig. P.1.3

19. Calculate the voltage across *AB* in the network shown in Fig. P.1.4 and indicate the polarity of the voltage.

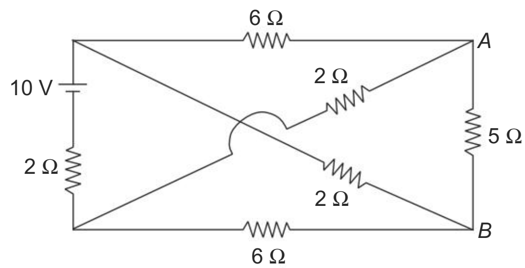


Fig. P.1.4

20. For the network shown in Fig. P.1.5, determine the current through the ammeter *A* having a resistance of 9 ohms.

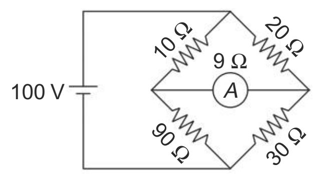


Fig. P.1.5

21. Determine the current delivered by the battery in the circuit of Fig. P.l.6. Use star-delta conversion.

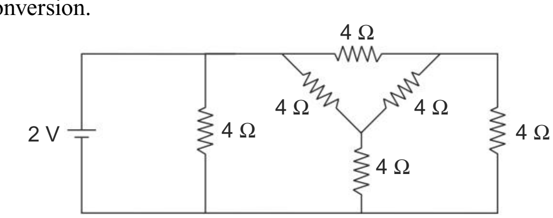


Fig. P.1.6

22. In the network of Example 1.55, determine the resistance between *A* and *D*.
23. Using star-delta conversion, determine the current supplied by the battery of 15 V in the circuit. of Fig. P.1.7. All the resistors are of 5 ohms each.
24. For the circuit of Fig. P.l.8, determine the current through and the voltage across each component using node voltage analysis. $R_1 = 1.1$ kΩ, $R_2 = 510$ Ω, $R_3 = 2.4$ kΩ.

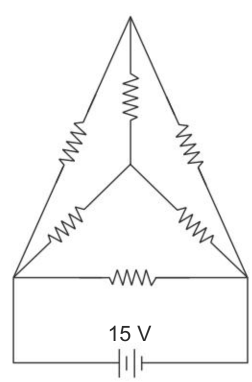


Fig. P.1.7

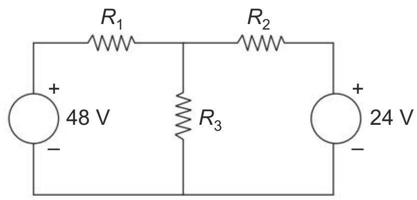


Fig. P.1.8

25. Using modal analysis, determine the current in the 50 ohms branch in the circuit of Fig. P.l.9.

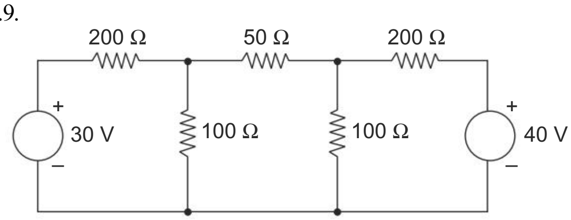


Fig. P.1.9

26. Calculate the resistance of 100 m length of wire having a uniform cross-sectional area of 0.01 mm^2 and of resistivity 50 μΩ cm. If the wire is drawn to 3 times its original length, calculate its resistance.
27. A rectangular metal strip has the dimensions x = 10 cm, y = 0.5 cm, z = 0.2 cm. Determine the ratio of+ the resistances R_x, R_y and R_z between the respective pairs of opposite faces.
28. The resistance of a copper wire is 50 ohms at 30°C. If wire is heated to a temperature of 80°C, find its resistance at this temperature. Assume temperature coefficient of resistance of copper at 0°C to be 0.00427/°C. Also find the temperature coefficient at 35°C.
29. The resistance of the field coils with copper conductors of a dynamo is 120 Ω at 25°C. The resistance of the coil increases to 140 Ω after 6 hours. Calculate the mean temperature rise of the field coil, a_0 = 0.0042/°C.
30. An armature has a resistance of 0.25 ohm at 17°C, and armature copper loss is to be maintained at 500 W. If the permissible temperature of winding is 58°C, and winding is wound with standard annealed copper wire, determine the continuous maximum rating for the armature; α_0 *for copper* = (1/234.5)/°C.
31. Find the current flowing at the instant of switching a metal filament lamp on to a 250 V supply. The working temperature of the filament and the room temperature are 2,000°C and 20°C respectively. The resistance temperature coefficient of filament material at 20°C is 0.005 per degree centigrade, and the lamp consumes 2 kwh in 20 hours.
32. Calculate the effective resistance of the following combination of resistances between points A and B.

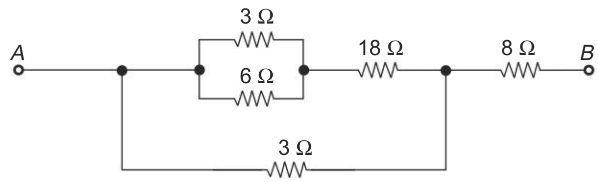


Fig. P.1.10

33. In the circuit shown, the total power dissipated is 488 W. Determine the current flowing in each resistor and the P.D. between A and B.

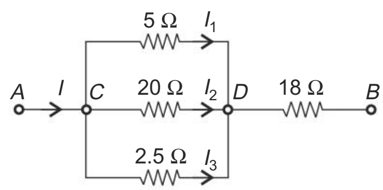


Fig. P.1.11

34. Determine the value of R, if the power dissipated in 10 ohms resistor is 40 W for the circuit shown below.

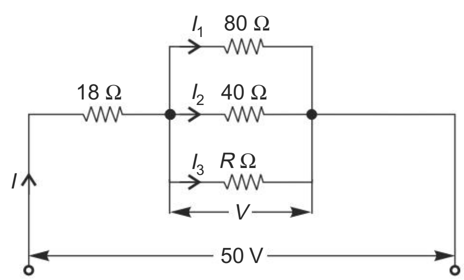


Fig. P.1.12

35. The current in the 6 Ω resistor of the network (shown in the given figure) is 2A. Find the current in all other resistors and the voltage V across the network.

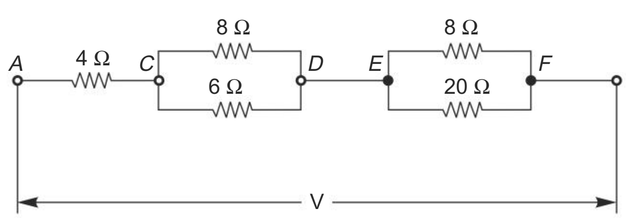


Fig. P.1.13

36. Calculate the total resistances of the network between points *A* and *B* of the circuit shown below:

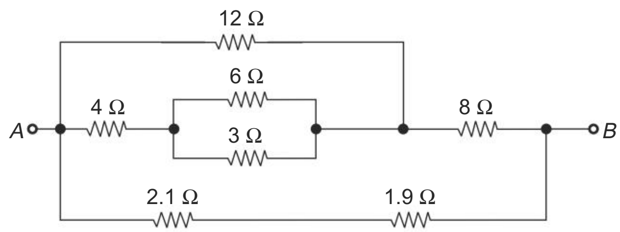


Fig. P.1.14

37. Determine the value of the equivalent resistance at the terminals *AB* of the network shown below:

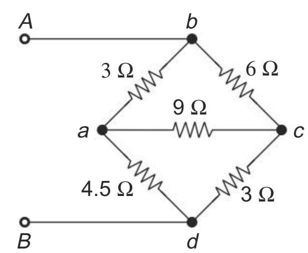


Fig. P.1.15

38. Find the resistance at the *A*-*B* terminals in the electric circuit shown below, using Δ–y *transformation*.

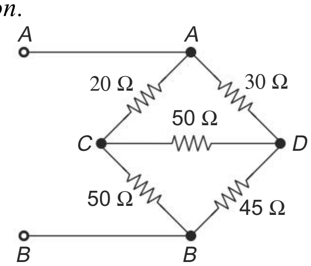


Fig. P.1.16

39. Determine the resistance between the point *X* and *Y* for the network given below:

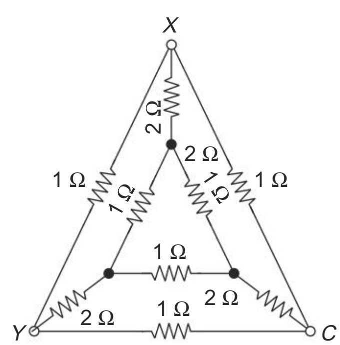


Fig. P.1.17

ANSWERS TO PROBLEMS

1. $\frac{20}{29}$ ohms; $\frac{120}{29}$ ohms
2. 6.75 ohms
3. 2.4 ohms
4. 1/236 per°C; 11.8 ohms; 1/254 per°C
5. 90.56 ohms
6. 56.5°C
7. 69.85°C
8. 954 ohms; 0.002/°C
9. 10V; 6×10^4 joules
10. 15 ohms
11. 24 ohms 1A; 8V; 6V; 10V, 8W, 6W, l0W
12. 0.244 A,152.4 V; 97.6V
13. 16 ohms
14. 12 ohms, 6 ohms
15. 6 A, 2 A and 4 A
16. 0.75 A, 0.375 A, 1.5 A, 0.375 A;.22.5 W
17. 25 A
18. 0.3 A
19. *B* is 2 V above *A*
20. 1 A
21. 4 A
22. 1.2 Ω
23. 6 A
24. 18.7 mA, 20.6 *V,* 6.6 mA, 3.5 V, 11.4 mA, 27.4 V
25. 20 mA (from *B* to *A*)
26. $R_1 = 500\,\Omega$, $R_2 = 4{,}500\,\Omega$
27. 10,000 : 25 : 4
28. 0.003715/°C, 58.36 Ω
29. 43.8°C
30. 41.47 A
31. 4.826 A
32. 10.60 Ω
33. $I = 6.5$A, $I_1 = 2$A, $I_2 = 0.5$A, $I_3 = 4$A
34. $R = 34.29\,\Omega$
35. 1.5A, 3.5A, 2.5A, 1A and $V = 46$V
36. 3 Ω
37. 4 Ω
38. 36 Ω
39. $\frac{7}{12}\,\Omega$

MAGNETIC CIRCUITS

2

INTRODUCTION

Many of the everyday electrical devices depend on proper magnetic design as much as they do on proper electrical design. For instance, in loud speakers, motors, generators and relays, the magnetic circuit is often more important than the electric circuit.

There exist in nature some materials which attract small pieces of iron. The name of this material is magnet. Each magnet has a region around it which a force field exists and even the conductors carrying electric current will be perceptibly influenced in this region. This region is called the magnetic field. This field can also be produced by a current-carrying conductor.

This chapter deals with the characteristics of such magnetic fields, simple and composite magnetic circuits and comparison between electrical and magnetic circuits.

2.1 DEFINITION OF MAGNETIC QUANTITIES

The various terms involved with magnetism are grouped as follows along with their characteristics:

2.1.1 Magnetic Flux [ϕ]

The magnetic lines of force (i.e. the amount of magnetic field quantitatively) produced by a magnet is called magnetic flux.

Its unit is Weber. 1 wb = 10^8 magnetic lines
= 10^8 Maxwells

Characteristics of Magnetic Flux Lines:

1. Magnetic flux lines do not have physical existence.
2. The direction of magnetic flux lines is defined as the direction in which an isolated north pole would move if it were placed in a magnetic field.
3. Each line of magnetic flux is a closed loop by itself.
4. Magnetic flux lines never intersect.
5. Lines of magnetic flux closer to each other and having the same directions repel at each other.
6. Lines of magnetic flux closer to each other and having opposite directions attract each other.

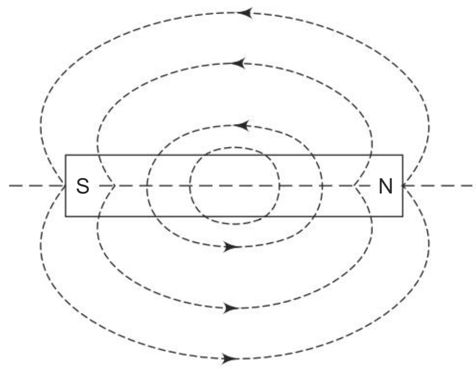


Fig. 2.1

2.1.2 Magnetic Flux Density (*B*)

Magnetic flux density is the flux per unit area at right angles to the flux.

$$B = \frac{\phi}{a} \text{ wb/m}^2$$

where ϕ(wb) is the magnetic flux and a (m^2) is the area of cross section. Its unit is wb/m^2 or Tesla.

2.1.3 Magneto Motive Force (F)

MMF is the cause for producing flux in a magnetic circuit. It is obtained as the product of the current (I amps) flowing through a coil of N turns. Its unit is Ampere.

Thus, $$F = N\,I \text{ Amps.}$$

The term, M.M.F. is normally referred to as Ampere Turns (AT).

2.1.4 Magnetic Field Intensity (or) Magnetising Force (H)

It is defined as M.M.F. per unit length of the magnetic flux path. It is a measure of the ability of a magnetised body to produce magnetic induction in other magnetic substances. Its unit is Ampere/metre.

With reference to Fig. 2.2 $$H = \frac{N\,I}{l}$$

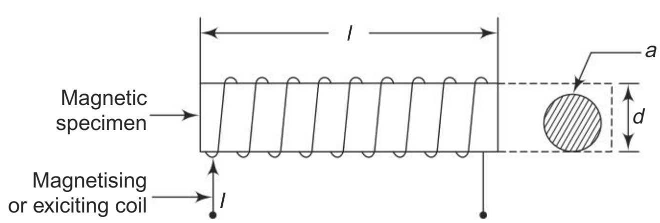


Fig. 2.2

2.1.5 Permeability (μ)

This is the property of the magnetic medium. The flux density (B) is proportional to the magnetising force which produces it.

i.e.
$$B \propto H$$
$$B = \mu H$$
$$\mu = B/H$$

where μ is the constant of proportionality and is called permeability.

2.1.6 Relative Permeability (μ_r)

The relative permeability of a medium or material is defined as the ratio of the flux density produced in that medium or material to the flux density produced in vaccum by the same magnetising force.

$$\mu_r = \frac{\text{Flux density in the medium}}{\text{Flux density in vaccum}}$$

The absolute permeability of vaccum or free space has the value of $4\pi \times 10^{-7}$ Henry/m.

The permeability for any other medium is

$$\mu = \mu_r \times \mu_0$$

The relative permeability of vaccum or free space is unity and that of air and other non-magnetic materials is very nearly equal to unity.

2.1.7 Reluctance (S)

Reluctance is the property of an magnetic circuit by which the setting up of flux is opposed. It is defined as the ratio of the magnetomotive force to the flux.

The unit of reluctance is Amp./Weber and is denoted by S.

2.1.8 Permeance (P)

It is the reciprocal of reluctance and is the readiness with which magnetic flux is developed. It is analogous to conductance in an electric circuit. Its unit is weber per Amp.

2.2 MAGNETIC CIRCUIT

Magnetic circuit is the path followed by magnetic flux. Magnetic flux follows a complete loop or circuit coming back to its starting point. In any magnet, magnetic flux leaves its north pole, passing through air, enters the magnet at its south pole and finally reaches point where they start.

It is possible to establish magnetic flux in a definite limited path by using a magnetic material of high permeability. In this manner, the magnetic flux forms a closed circuit exactly as an electric current does in an electric circuit.

Magnetic circuits can be classified into simple and composite ones. A simple magnetic circuit is made up of a single magnetic material. Thus, such a circuit reflects the magnetic properties of the material used. But, a composite magnetic circuit will have minimum of two different specimens offering different magnetic properties. Both of them may be magnetic or one may be non-magnetic (such as air). The combination of magnetic material and air is of more practical importance.

2.2.1 Analysis of Simple Magnetic Circuit

Consider a circular solenoid or a toroidal iron ring having a magnetic path of l meters, area of cross-section, a m^2 and a coil of turns carrying I amperes wound anywhere on it as in Fig. 2.3.

$$\text{M.M.F. Produced} = NI \text{ Amps.} \tag{2.1}$$

According to the definition of H, the field-strength inside the solenoid is

$$H = \frac{NI}{l} \text{ Amp./metre} \tag{2.2}$$

$$= \frac{NI}{2\pi R_m}$$

Now
$$B = \mu_0 \mu_r H \qquad (\because \quad \mu = B/H) \tag{2.3}$$

$$B = \frac{\mu_0 \mu_r NI}{l} \text{ wb/m}^2\ldots \tag{2.4}$$

Total flux produced

$$\phi = Ba = \frac{\mu_0 \mu_r a\, NI}{l} \text{ wb}$$

Also,
$$\phi = \frac{NI}{\dfrac{l}{\mu_0 \mu_r a}} = \frac{NI}{s} \text{ wb} \tag{2.5}$$

According to the definition of reluctance,

$$S = \frac{NI}{\phi}$$

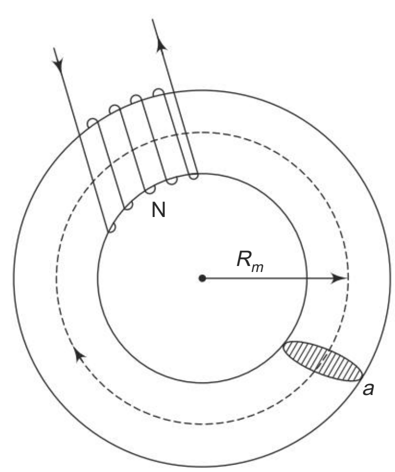


Fig. 2.3

Thus, the denominator $\frac{1}{\mu a}$ or $\frac{1}{\mu_0 \mu_r a}$ is the reluctance of the circuit and is analogous to resistance in electric circuit

$$S = \frac{1}{\mu_0 \mu_r a} \tag{2.6}$$

$$\text{Flux} = \frac{\text{mmf}}{\text{reluctance}}$$

The above equation is also known as 'Ohm's' law of magnetic circuit' because its resembles a similar expression in electric circuits,

$$\text{Current} = \frac{\text{emf}}{\text{resistance}}$$

If the amp. turns to produce a given flux (ϕ) or flux density (B) in a magnetic circuit is to be determined, the steps to be followed are as under:

$$B = \frac{\phi}{a}$$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$\text{mmf} = Hl.$$

i.e ampere turns for any part of magnetic circuit = magnetising force in that part × length of that part.

2.2.2 Analysis of Composite Magnetic Circuits

Let us consider a composite circuit shown in Fig. 2.4 consists of the three different magnetic materials of different relative permeabilities along with an air gap. It can be inferred from the figure that the flux lines find their closed path through the different magnetic materials and the air gap. Thus, a single significant magnetic flux exists. Hence, the circuit can be termed as 'Series Magnetic Circuit'.

Let l_1, l_2 and l_3 be the lengths of the various magnetic materials used in the series magnetic circuit.

a_1, a_2 and a_3 be the areas of the cross-section of the respective part of the series magnetic circuit.

l_g be the length of the air gap.

a_g be the area of cross-section at the air gap.

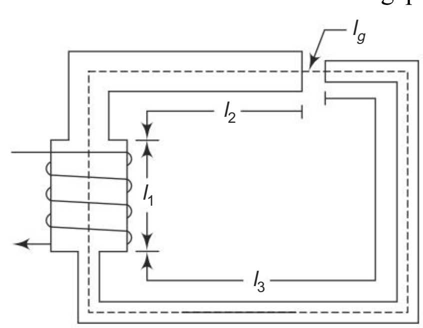


Fig. 2.4 Series composite magnetic circuit

Each part of the series circuit will offer reluctance to the magnetic flux ϕ. The amount of reluctance will depend upon the dimensions and relative permeability of that part.

Since the different parts of the circuit are in series, the total reluctance is equal to the sum of reluctances of individual parts.

$\therefore$ Total reluctance, $S = S_1 + S_2 + S_3 + S_g$

The given magnetic circuit is analogous to the electrical circuit is shown in Fig. 2.5.

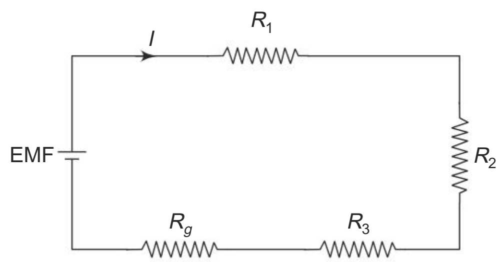


Fig. 2.5

The different resistive elements are in series.

The total resistance of the circuit is equal to the sum of various individual resistance values,

i.e.,
$$R = R_1 + R_2 + R_3 + R_g$$
$$\text{Total emf} = IR_1 + IR_2 + IR_3 + IR_g$$
$$= \text{Current} \times \text{Total resistance}$$

Substituting for the reluctance components,

$$S = \frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_2}{\mu_0 \mu_{r2} a_2} + \frac{l_3}{\mu_0 \mu_{r3} a_3} + \frac{l_g}{\mu_0 \mu_{rg} a_g}$$

$$\text{Total mmf} = \text{Flux} \times \text{Reluctance} = \phi S_1 + \phi S_2 + \phi S_3 + \phi S_g$$

$$\phi = \left[\frac{l_1}{\mu_0 \mu_{r1} a_1} + \frac{l_2}{\mu_0 \mu_{r2} a_2} + \frac{l_3}{\mu_0 \mu_{r3} a_3} + \frac{l_g}{\mu_0 \mu_r a_g}\right]$$

$$= \left[\left(\frac{\phi}{a_1 \mu_0 \mu_{r1}} \times l_1\right) + \ldots \left(\frac{\phi}{a_g \mu_0}\right)(l_g)\right] \quad (\because \quad \mu_{r_g} = 1 \text{ for air})$$

$$= \frac{B_1}{\mu_0 \mu_{r_1}} \times l_1 + \frac{B_2}{\mu_0 \mu_{r_2}} \times l_2 + \frac{B_3}{\mu_0 \mu_{r_3}} \times l_3 + \frac{B_g}{\mu_0} l_g$$

$$\text{Total mmf} = H_1 l_1 + H_2 l_2 + H_3 l_3 + M_g l_g \qquad \left[\because H = \frac{B}{\mu_0 \mu_r}\right]$$

2.2.3 Analysis of Parallel Magnetic Circuits

In a magnetic circuit, if the flux has more than one path, then it is known as a parallel magnetic circuit.

In most of the electrical machines magnetic circuits exist in parallel.

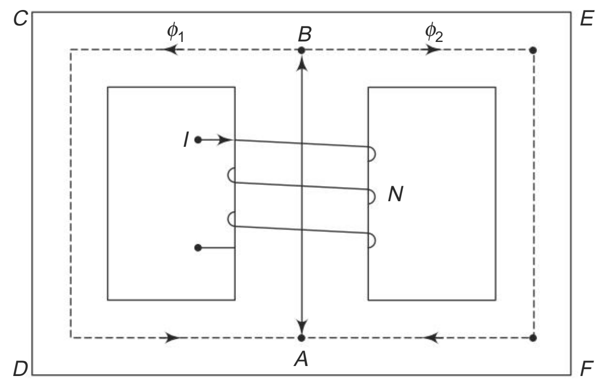


Fig. 2.6

In a parallel magnetic circuit, the total flux exists in a common section of the magnetic circuit which contains the exciting coil. Then, it divides into two or more parts, follows the different paths and recombines at the other end of the common section.

Let us consider the following parallel magnetic circuit shown in Fig. 2.6.

This magnetic circuit is analogous to the following electric circuit (Fig. 2.7).

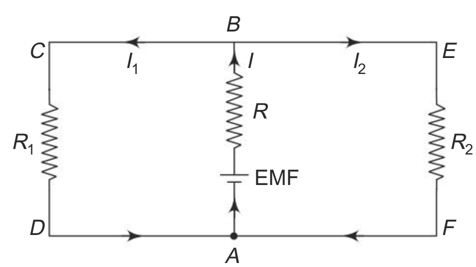


Fig. 2.7

Let ϕ = Total flux in the circuit

ϕ_1 = Flux in the path *BCDA*

ϕ_2 = Flux in the path *BEFA*

Part of circuit	*Length*	*Cross sectional area*	*Magnetising force*	*Reluctance*
BCDA	l_1	a_1	H_1	S_1
BEFA	l_2	a_2	H_2	S_2
AB	l_3	a_3	H_3	S_3

Total flux $\phi = \phi_1 + \phi_2$

mmf in the circuit $= \phi S_3 + \phi_1 S_1$

(or)

$= \phi S_3 + \phi_2 S_2$

Also, mmf $= H_3 l_3 + H_1 l_1 = H_3 l_3 + H_2 l_2$

This equation is similar to the emf equation in the electrical equivalent circuit i.e. in electrical circuit,

Let $I = I_1 + I_2$ = Total current in the circuit

R = Total Resistance of path AB

R_1 = Resistance of path $BCDA$ (Current in this path is I_1)

R_2 = Resistance of path $BEFA$ (Current in this path is I_2)

emf in the circuit $= IR = I_1 R_1$

(or)

$IR = I_2 R_2$

Total mmf required = AT for common section + AT for any one of the parallel paths.

The above analysis is based on *Kirchoff's laws for Magnetic Circuits* stated as follows:

First Law The total flux towards a node is equal to the total flux away from the node in any magnetic circuit.

Second Law In any magnetic circuit, the sum of the product of the magnetising force in each part of the magnetic circuit and the length of that part is equal to the resultant mmf.

2.3 LEAKAGE FLUX

The flux which does not follow the desired path in magnetic circuit is known as *leakage flux*.

Usually, we assume that all the flux lines take the path of the magnetic medium. But, practically, it is impossible to confine all the flux to the iron path only. It is because, to prevent the leakage of flux, there is no perfect magnetic insulator. Even in air, flux is conducted fairly well. Hence, some of the flux leaks through air as shown in Fig. 2.8 and is known as leakage flux. All the magnetic flux which completes the desired magnetic circuit is the *useful flux*.

To account for the leakage flux, the term, "leakage coefficient" is introduced.

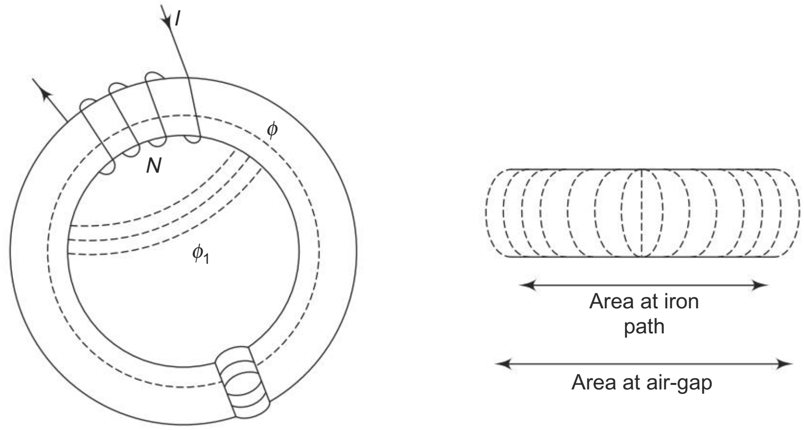
Leakage coefficient is defined as follows and it is defined by λ.

Leakage coefficient, $\lambda = \dfrac{\text{total flux}}{\text{useful flux}} = \dfrac{\phi + \phi'}{\phi}$

Usually, leakage factor is greater than unity.

2.4 FRINGING EFFECT

An air gap is often inserted in magnetic circuits out of necessity. When crossing an air gap, the magnetic lines of force have a tendency to bulge out. This is because the magnetic lines of force repel each other when they are passing through a non-magnetic material. This phenomenon is known as *fringing*. It is shown in Fig. 2.8.



(a) (b)

Fig. 2.8

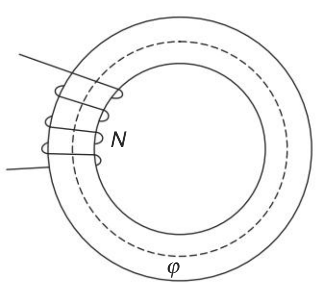
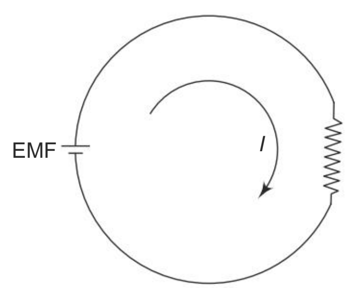
Actually, the fringing increases the effective area of the air gap. As a result, the flux density in the air gap is reduced.

2.5 COMPARISON BETWEEN MAGNETIC AND ELECTRIC CIRCUIT

The analogy between electric and magnetic circuits is depicted in Table. 2.1.

To explain the analogy, a copper ring connected to a dc source is taken as electric circuit. A toroidal ring wound with N turns of wire through which a current I flows is taken as the magnetic circuit.

Table 2.1

Magnetic Circuit	*Electric Circuit*
 **Fig. 2.9(a)**	 **Fig. 2.9(b)**
1. The closed path for magnetic flux is known as magnetic circuit.	The closed path for electric current is called an electric circuit.

2. Flux = $\dfrac{\text{mmf}}{\text{reluctance}}$	Current = $\dfrac{\text{mmf}}{\text{resistance}}$
3. mmf (Ampere)	emf (volts)
4. Reluctivity	Resistivity
5. Reluctance	Resistance
$S = \dfrac{l}{\mu_0 \mu_r a}$	$R = \dfrac{\rho l}{a}$ ohms
where $\dfrac{l}{\mu_0 \mu_r}$ = reluctivity of the magnetic circuit	where ρ = resistivity of the eletric circuit.
6. Permeance = $\dfrac{1}{\text{Reluctance}}$	Conductance = $\dfrac{1}{\text{Reluctance}}$
7. Flux density, $B = \dfrac{\phi}{a}$ wb/m^2	Current density, $J = \dfrac{I}{a}$ A/m^2
8. Magnetising force, $H = \dfrac{NI}{l}$ Amp/metre	Electric field intensity, $E = \dfrac{V}{d}$ volt/metre
9. Magnetic flux does not actually flow in a magnetic circuit	The electric current actually flows in an electric circuit
10. The reluctance of a magnetic circuit is not constant and it depends upon flux density in the material	The resistance of an electric circuit is practically constant, even though it varies very slightly with temperature
11. In a magnetic circuit, energy is required to create the flux and not to maintain it	In an electric circuit, energy is required so long as the current has to flow through it
12. There is no magnetic insulator. Even in air, the best known insulator, flux can be set up	There are many electric insulators

Example 2.1 A toroidal air cored coil with 2000 turns has a mean radius of 25 cm, diameter of each turn being 6 cm. If the current in the coil is 10 *A*, find *MMF*, (b) flux and (c) flux density.

Solution:

$N = 2000$ turns; $I = 10$ Amps; $R_m = 25$ cm

dia. of each turn, $d = 6$ cm.

(a) $MMF = NI = 2000 \times 10 = 20{,}000$ Amperes

(b) Flux = MMF/Reluctance

Reluctance = $l/a\mu$

$$l = 2\pi R_m = 2\pi \times \frac{25}{100}\text{ m} = 1.57\text{ m}$$

$$a = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (6 \times 10^{-2})^2 = 2.8 \times 10^{-3}\text{ m}^2$$

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1$$

$\therefore$ Reluctance $= \dfrac{1}{a\mu}$

$$= \frac{1.57}{2.8 \times 10^{-3} \times 4\pi \times 10^{-7} \times 1}$$

$$= 4.46 \times 10^8 \text{ At/wb}$$

$$\text{Flux} = \frac{\text{mmf}}{\text{Reluctance}} = \frac{20{,}000}{3.47 \times 10^8}$$

$$= 4.48 \times 10^{-5} \text{ wb}$$

(c) $$\text{Flux density} = \frac{\text{Flux}}{\text{Area of cross section}} = \frac{4.48 \times 10^{-5}}{2.8 \times 10^{-3}}$$

$$= 1.6 \times 10^{-2} \text{ wb/m}^2 \text{ (or) Tesla}$$

Example 2.2 The flux produced in the air gap between two electromagnetic poles is 5×10^{-2} wb. If the cross sectional area of the air gap is 0.2 m^2 find (a) flux density, (b) magnetic field intensity, (c) reluctance, and (d) permeance of the air gap. Find also the mmf dropped in the air gap given the length of the air gap to be 1.2 cm.

Solution:

$$\phi = 5 \times 10^{-2} \text{ wb}$$

$$a = 0.2 \text{ m}^2$$

$$l_g = 1.2 \text{ cm} = 0.012 \text{ m}$$

$$\mu_r = 1$$

(a) Flux density (B) $\dfrac{\phi}{a} = \dfrac{5 \times 10^{-2}}{0.2} = 0.25$ wb/sq.m

(b) Magnetic field intensity (H) $= \dfrac{B}{\mu_0 \mu_r}$

$$= \frac{0.25}{4\pi \times 10^{-7} \times 1}$$

$$= 1.9894 \times 10^5 \text{ A/m}$$

(c) Reluctance (S) of the air gap $= \dfrac{l_g}{a\mu}$

$$= \frac{0.012}{0.2 \times 4\pi \times 10^{-7} \times 1}$$

$$= 47746.48 \text{ A/wb}$$

(d) Permeance = 1/Reluctance $= \dfrac{1}{47746.48} = 2.0944 \times 10^{-5}$ wb/A

(e) mmf in the air gap $= \phi \times$ Reluctance $= 5 \times 10^{-2} \times 47747 = 2387$ A.

Example 2.3 A ring has mean diameter of 15 cm, a cross section of 1.7 cm^2 and has a radial gap of 0.5 mm in it. It is uniformly wound with 1500 turns of insulated wire and a current of 1 A produces a flux of 0.1 mwb across the gap. Calculate the relative permeability of iron on the assumption that there is no magnetic leakage.

Solution:

$$\phi = 0.1 \times 10^{-3} \text{ wb}$$

$$a = 1.7 \times 10^{-4} \text{ m}^2$$

$$l_g = 0.5 \times 10^{-3} \text{ m}$$

$$2R_m = 15 \text{ cm}$$

For air gap,
$$B = \frac{\phi}{a} = \frac{10^{-4}}{1.7 \times 10^{-4}}$$
$$= \frac{1}{1.7} = \text{Tesla}$$
$$H = \frac{B}{\mu_0} = \frac{1}{1.7 \times 4\pi \times 10^{-7}} \text{ A/m}$$
$$= 4.681 \times 10^5 \text{ A/m}$$

Ampere turns required for air gap $= H.l_g = 4.681 \times 10^5 \times 0.5 \times 10^{-3} = 234.1$

Total ampere turns provided $= 1500 \times 1 = 1500$

At available for iron path $= 1500 - 234.1 = 1265.9$

Length of iron path $= 15 \times \pi \times 10^{-2} - 0.5 \times 10^{-3} = 47.074 \times 10^{-2}$ m

$$H \text{ for iron path} = \frac{1265.9}{47.074 \times 10^{-2}} = 2689.17 \text{ A/m}$$

Now,
$$H = \frac{B}{\mu_0 \mu_r}$$
$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.588}{4\pi \times 10^{-7} \times 2689.17}$$
$$= 174$$

Relative permeability of iron = 174.

Example 2.4 A series magnetic circuit has an iron path of length 50 cm and an air gap of length 1 mm. The cross-sectional area of the iron is 6.66 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mwb in the circuit. The following points are taken from the magnetization curve for iron.

Flux density (wb/m^2)	:	1.2	1.35	1.45	1.55
Magnetizing force (A/m)	:	500	1000	2000	4000

Solution AT required for air gap:

$$l_i = 50 \text{ cm} = 0.5 \text{ m}$$
$$l_g = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$
$$\phi = 0.9 \text{ mwb} = 0.9 \times 10^{-3} \text{ wb}$$
$$a = 6.66 \text{ cm}^2 = 6.66 \times 10^{-4} \text{ m}^2$$
$$B = \frac{0.9 \times 10^{-3}}{6.66 \times 10^{-4}} = 1.35 \text{ wb/m}^2.$$
$$H_g = \frac{B}{\mu_0} = \frac{1.35}{4\pi \times 10^{-7}}$$
$$= 107.43 \times 10^4 \text{ A/m}$$
$$\text{AT required} = H_g \times l_g = 107.43 \times 10^4 \times 1 \times 10^{-3}$$
$$= 1074.3$$

AT required for iron path

$$l_i = 50 \text{ cm} = 0.5 \text{ m}$$
$$N = 400$$
$$B = \frac{\phi}{a} = \frac{0.9 \times 10^{-3}}{6.66 \times 10^{-4}}$$
$$= 1.35 \text{ wb/m}^2$$

From the given data, corresponding value of $H = 1000$ A/m.

$$\text{AT required for iron path} = H_i \times l_i$$
$$= 1000 \times 0.5$$
$$= 500$$
$$\text{Total AT required} = 1074 + 500$$
$$= 1574$$
$$\text{Exciting current required } (I) = \frac{1574}{400} = 3.94 \text{ Amps.}$$

Example 2.5 An iron rod of 1 cm radius is bent to a ring of mean diameter 30 cm and wound with 250 turns of wire. Assume the relative permeability of iron as 800. An air gap of 0.1 cm is cut across the bent ring. Calculate the current required to produce a useful of 20,000 lines if (a) leakage is neglected and (b) leakage factor is 1.1.

Solution:

$$l_g = 0.1 \text{ cm}$$
$$\frac{d}{2} = 1 \text{ cm}$$
$$2R_m = 30 \text{ cm}$$
$$N = 250 \text{ turns}$$
$$\mu_r = 800$$
$$\phi = 20{,}000 \text{ lines}$$
$$= 20{,}000 \times 10^{-8} \text{ wb}$$

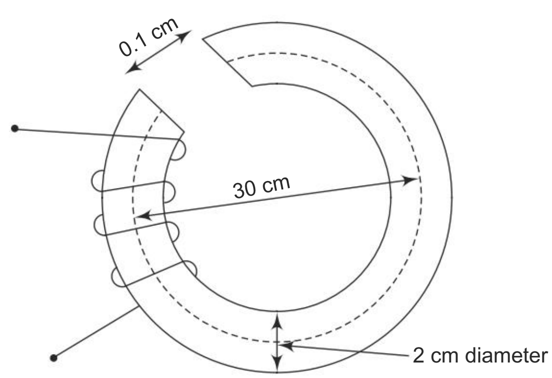


Fig. E.2.1

(a) Neglecting leakage:

Total Reluctance = Reluctance of air gap + Reluctance of iron path

$$\text{Reluctance of air gap} = \frac{l_g}{\mu_0 \mu_r a} = \frac{0.001}{(4\pi \times 10^{-7} \times 1 \times \pi \times 1 \times 10^{-4})}$$

$$= 2533029.59 \text{ A/wb}$$

l_i, Length of iron path = Total length = l_g

$$l_i = (\pi d - l_g)m = (\pi \times 0.3 - 0.001)$$

$$\text{Reluctance of iron path} = \frac{(\pi \times 0.3 - 0.001)}{4\pi \times 10^{-7} \times 800 \times \pi \times 1 \times 10^{-4}}$$

$$= 2980988.896 \text{ A/wb}$$

Total Reluctance = 5514018.49 A/wb

MMF = Flux × Reluctance

$= 20{,}000 \times 10^{-8} \times 5514018.49$ A/wb

= 1102.8 Amp. turns

$$\text{Current required} = \frac{1102.8}{\text{No. of turns}} = \frac{1120.8}{250}$$

Current required = 4.41 Amps.

(b) Including leakage:

To have a useful flux of 20,000 lines in the air gap, the *MMF* required the air gap porition is

$= \phi \times$ Reluctance of air gap

$= 20{,}000 \times 10^{-8} \times 2533039.59$

MMF for air gap = 506.606 Amp

For the iron path, the flux has to be more.

The total flux in iron path = Leakage factor × Useful flux

$= 1.1 \times 20000 \times 10^{-8}$ wb

Hence, *MMF* for iron path = $1.1 \times 20{,}000 \times 10^{-8}$ Reluctance of iron path

= 655.82 Amp.

Total *MMF* = 655.82 + 506.606 A = 1162.426 A

$$\text{Current required} = \frac{1162.426}{250} = 4.649 \text{ Amps.}$$

Current required = 4.649 Amps.

Example 2.6 The magnetic circuit shown in Fig. E.2.2 has the following dimensions: l_1 = 10 cm, $l_2 = l_3$ = 18 cm, cross sectional area of l_1 path = 6.25×10^{-4} m^2, cross-sectional area of l_2 and l_3 paths = 3×10^{-4} m^2, length of air gap = 1mm. Taking the relative permeability of the materials as 800 find the current in the 600 turn exciting coil and to establish a flux of 100×10^{-6} wb in the air gap, neglect leakage and fringing.

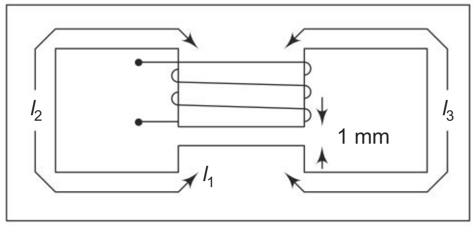


Fig. E.2.2

Solution:

$$l_1 = 10 \text{ cm}; l_2 = l_3 = 18 \text{ cm}$$
$$l_g = \text{air gap length} = 1\text{mm}$$
$$= 1 \times 10^{-3} \text{ m}^2$$
$$a_1 = 6.25 \times 10^{-4} \text{ m}^2$$
$$a_2 = Al_3 = 3 \times 10^{-4} \text{ m}^2$$
$$\mu_r = 800; N = 600$$

Air gap flux, $\phi = 100 \times 10^{-6}$ wb

For the air gap

Flux density, $B_g = \dfrac{100 \times 10^{-6}}{6.25 \times 10^{-4}} = 0.16$ Tesla

Magnetising force, $H_g = \dfrac{B}{\mu_0 \mu_r} = \dfrac{0.16}{4\pi \times 10^{-7} \times 1}$

$$= 1.27 \times 10^5 \text{ A/m}$$

$$(MMF)_a = H_g l_g$$
$$= 1.27 \times 10^5 \times 1 \times 10^{-3} = 127 \text{ A}$$

For path I_1

Flux density, $B_1 = 0.16$ Tesla

Magnetising force, $H_1 = \dfrac{B_I}{\mu_r \mu_0} = \dfrac{0.16}{800 \times 4\pi \times 10^{-7}}$

$$= 159 \text{ A/m}$$

$$(MMF)l_1 = H_1 l_1$$
$$= 159\,(10 - 0.1) \times 10^{-2} = 15.741 \text{ A}$$

Since paths l_2 and l_3 are similar, the total flux divide equally between these two paths. Since these paths are in parallel, it is necessary to consider the mmf for only one of them.

For path l_2 flux $= 50 \times 10^{-6}$ wb

Flux density $B_2 = \dfrac{50 \times 10^{-6}}{3 \times 10^{-4}} = 0.167$ Tesla

Magnetising force $H_2 = \dfrac{B_2}{\mu_r \mu_0} = \dfrac{0.167}{800 \times 4\pi \times 10^{-7}}$

$$= 166 \text{ A/m}$$

$$(MMF)l_2 = H_2 l_2$$
$$= 166 \times 18 \times 10^{-2} = 29.88 \text{ A}$$
$$\text{Total mmf} = 127 + 15.741 + 29.88$$
$$= 172.62 \text{ A}$$

Current required $\quad I = \text{Total mmf}/N$

$$= \frac{172.62}{600} = 0.287 \text{ A}$$

Example 2.7 Find the exciting current and total flux in an iron ring 10 cm in mean diameter and 10 cm^2 in cross section and wound with 150 turns of wire. The flux density is 0.1 wb/m^2 and the permeability is 800.

Solution:

$D_m = 10$ cm $\qquad N = 150 \qquad B = 0.1$ wb/m^2

$a = 10$ cm^2 $\qquad \mu r = 800$

$$\frac{\text{NI}}{\text{Reluctance}} = \text{flux} \qquad \frac{\text{NI}}{S} = \phi = Ba$$

$$S = \frac{l}{a\mu_o\mu_r}$$

$$\therefore \qquad I = \frac{BaS}{N} = \frac{0.1 \times 10^{-4} \times \pi \times 10 \times 10^{-2}}{150 \times 10 \times 10^{-4} \times 4\pi \times 10^{-7} \times 800} = 2.0833 \times 10^3 \text{ A}$$

Example 2.8 An air cored solenoid has a length of 30 cm and a diameter of 1.5 cm. Calculate its reluctance, if it has 900 turns.

Solution:

$l = 30$ cm $= 0.3$ m $\qquad d = 1.5 \times 10^{-2}$ m $\qquad N = 900$

$\mu_r = 1 \qquad I = 5$ A

$$\text{Reluctance} = \frac{1}{a\mu_o\mu_r} = \frac{0.3}{\pi \times \dfrac{(1.5 \times 10^{-2})^2}{4} \times 4\pi \times 10^{-7} \times 1}$$

$$= 1350949 \times 10^3 \text{ A/wb}$$

Example 2.9 An electromagnet, made of cast steel, has an air gap of length 1 mm and an iron path of length 30 cm. Find the magnetomotive force required to produce a flux density of 0.9 wb/m^2 in the air gap. Value of H for cast steel obtained from its magnetization curve corresponding to the given value of flux density is 800 AT/m.

Solution:

$l_g = 1 \times 10^{-3}$ m $\qquad B = 0.9$ wb/m^2

$l_i = 30$ cm $= 0.3$ m $\qquad H_i = 800$ AT/m

For iron path

$$\text{MMF required} = H_i l_i = 800 \times 0.3 = 240 \text{ AT}$$

For air gap

$$\text{Mmf required} = H_g l_g = \frac{B}{\mu_o} lg = \frac{0.9}{4\pi \times 10^{-7}} \times 10^{-3} = 716 \text{ AT}$$

$\therefore$ Total mmf required = 240 + 716 = 956 AT

Example 2.10 An iron ring of mean length 50 cm has an air gap of 1.2 mm and winding of 220 turns. If the permeability of iron is 350 when a current of 1.2 A flows through the coil, find the flux density.

Solution:

$l_i = 50 \text{ cm} = 0.5 \text{ m} \quad N = 220 \quad I = 1.2\text{A}$

$l_g = 1.2 \times 10^{-3} \text{ m} \quad \mu_r = 350$

MMF produced = $NI = 220 \times 1.2 = 264$

Let the cross-section area of the circuit = 'a' (m^2)

$$\text{Reluctance of iron path, } S_i = \frac{l_i}{a_i \mu_o u_r} = \frac{0.5}{\text{a} \times \mu_o \times 350}$$

$$\text{Reluctance of air gap, } S_g = \frac{lg}{a_g \mu_o} = \frac{1.2 \times 10^{-2}}{a \times \mu_o}$$

$$\therefore \text{ Total reluctance, } S = S_i + S_g = \frac{1}{a\mu_o}\left[\frac{0.5}{350} + 1.2 \times 10^{-3}\right] = \frac{2.68 \times 10^{-3}}{a\mu_o}$$

$$\therefore \text{ Flux in the circuit is } \phi = \frac{\text{Mmf}}{\text{Reluctance}} = \frac{264 \times a\mu_o}{2.628 \times 10^{-5}}$$

$$\therefore \text{ Flux density } \frac{\phi}{a} = \frac{264\,\mu_o}{2.628 \times 10^{-3}} = \frac{264 \times 4\pi \times 10^{-7}}{2.628 \times 10^{-3}}$$

$$= 0.126 \text{ wb/m}^2$$

IMPORTANT FORMULAE

1. Flux density $B = \dfrac{\phi}{a}$
2. Absolute permeability $(\mu) = \mu_0 \times \mu_r$
3. MMF = NI Ampere
4. Magnetic field intensity, $H = \dfrac{NI}{l}$ A/m

   Also, $H = \dfrac{B}{\mu_0 \mu_r}$ A/m
5. Reluctance, $S = \dfrac{B}{\mu_0 \mu_r a}$ A/wb
6. Reluctances in series $S_{\text{total}} = \dfrac{l_1}{\mu_0 \mu_{r1} a_1} + \dfrac{l_2}{\mu_0 \mu_{r2} a_2} + \ldots$
7. Reluctances in parallel, $\dfrac{1}{S_{\text{total}}} = \dfrac{\mu_0 \mu_{r1} a_1}{l_1} + \dfrac{\mu_0 \mu_{r2} a_2}{l_2} + \ldots$

8. Flux, $\phi = \dfrac{NI}{S}$

$$= \frac{NI}{l/\mu_0 \mu_r a}$$

REVIEW QUESTIONS

1. Define Magnetic circuit and compare electric and magnetic circuits.
2. Derive an expression for the ampere-turns required for a simple and composite magnetic circuit.
3. Define the following:
   (a) Magnetomotive force, (b) Reluctance, (c) Permeability, (d) Magnetic leakage
4. Explain the terms: m.m.f., flux and reluctance in connection with a magnetic circuit. Derive the relationship among them.
5. What is leakage coefficient? How does it affect magnetic circuits? What are its disadvantages?

PROBLEMS

1. A solenoid 1.5 m long consists of 5000 turns of wire uniformly wound over an insulated bobin having outside diameter 0.038 m. If the flux produced is 56.4×10^{-6} wb, calculate the current supplied and the flux density at the centre of the solenoid.
2. Find the ampere-turns required to produce a flux of 0.4 mwb in the air gap of a magnetic circuit which has an air gap of 0.5 mm. The iron ring has 4 cm^2 cross-section and 63 cm mean length. Take $\mu_r = 1800$ and leakage coefficient = 1.15.
3. An iron ring of 50 cm mean diameter and 10 cm^2 cross-section has 1000 turns of insulated copper wire wound uniformly on it. If it produces a flux of 2.5×10^{-3} wb when a current of 4 amperes flows in it, calculate the relative permeability (μ_r) of the iron.
4. A coil is wound uniformly with 300 turns over an iron ring having a mean circumference of 400 mm and a cross-section of 500 mm^2. If the coil has a resistance of 8 Ω and is connected across a 20 V d.c supply, calculate
   (a) The m.m.f. of the coil.
   (b) Magnetic field strength (H)
   (c) Total flux
   (d) The reluctance of the ring.
   Assume the value of $\mu_r = 900$
5. A steel ring 10 cm radius and of circular cross-section 1 cm in radius has an air gap of 1 mm length. It is wound uniformly with 500 turns of wire carrying a current of amperes. Neglect magnetic leakage. The air gap takes 60% of the total M.M.F. Find total reluctance.
6. The magnetic circuit shown in Fig. E.2.2 has the following dimensions: $l_1 = 10$ cm, $l_2 = l_3 = 18$ cm, cross-sectional area of l_1 path $= 6.25 \times 10^{-4}$ m^2, cross-sectional area of l_2 and l_3 paths $= 3 \times 10^{-4}$ m^2, length of air gap = 1 mm. Taking the relative permeability of the materials as 800, find the current in the 600 turn exciting coil to establish a flux of 100×10^{-6} wb in the air gap. Neglect leakage and fringing.

7. The magnetic circuit has square cross section of area 12.25 cm^2. An air gap of 1 mm is cut in one of its limbs. An exciting coil of 1225 turns is wound on the core. What will be the current required to flow in the exciting coil to produce an air gap flux of 5 mwb. Take the relative permeability of the core material as 2500.
8. An iron ring has a mean circumference of 50 cm. It has an air gap of 1.2 mm. It carries a uniform winding of 500 turns. Find the flux density in the core per unit current in the coil. Assume the relative permeability of iron as 700.
9. An iron ring has a cross section of 3.5 cm^2, an air gap of 1 mm and an effective iron path of 80 cm. A coil of 500 turns is uniformly wound over the ring, A current of 2 A in the coil produces an air gap flux of 0.35 mWb. Find the relative permeability of iron.
10. A magnetic circuit has an air gap of 2.5 mm. A flux density of 1.8 wb/m^2 is required in the air gap. What mmf is required for this?

ANSWERS TO PROBLEMS

1. 0.0497 wb/m^2; 11.87 A
2. 718
3. 781.25
4. 750A; 4687.5 A/m; 7.07×10^5 A/wb; 1.06 wb
5. 4.17×10^6 A/wb
6. 0.577 Amps.
7. 3.6 A
8. 0.333 wb/m^2
9. 3117
10. 3581 AT

ELECTROMAGNETIC INDUCTION

3

INTRODUCTION

The link between electricity and magnetism was discovered by Oersted in 1824. He discovered that a magnetic field exists around a current carrying conductor. That is, magnetism can be created by means of an electric current. A few years later, in 1831, Faraday, the famous English scientist discovered that a magnetic field can create an electric current in a conductor. When the magnetic flux linking a conductor changes, an emf is induced in the conductor. This phenomenon is known as *electromagnetic induction.* This important effect has brought a revolution in the engineering world. Most of the electrical devices are based on this principle. This chapter deals with the various aspects of electromagnetic induction.

3.1 MAGNETIC EFFECT OF ELECTRIC CURRENT

When an electric current flows through a conductor, magnetic field is set up all along the conductor's length. And, the magnetic lines of force are in the form of concentric circles around the conductor. The direction of lines of force depends upon the direction of current. It may be determined by any one of the following two rules:

(i) Right Hand Gripping Rule Hold the conductor in the right hand with the outstretched thumb pointing in the direction of current. Then the other fingers point in the direction of the magnetic field around the conductor.

(ii) The Right Hand Cork Screw Rule The direction of the magnetic field is the direction of rotation of a right handed corkscrew turned so as to advance along the wire in the current direction.

The following facts can be noted about the magnetic effects of electric current:

1. The greater the current through the conductor, stronger the magnetic field and vice versa.
2. The magnetic field is stronger near the conductor and it becomes weaker and weaker when we move away from the conductor.
3. The shape of the conductor determines the pattern of the magnetic field.
4. When two conductors carrying the same amount of current in opposite directions are laid side-by-side, then there is no magnetic effect produced.
5. When two conductors carrying the same current in the same direction are laid side-by-side, then, the magnetic effect produced is twice the effect caused due to one such conductor alone.

3.2 CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

Consider a current carrying conductor placed at right angles to a magnetic field as shown in Fig. 3.1.

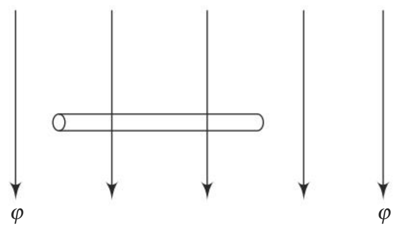


Fig. 3.1

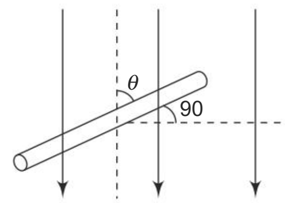


Fig. 3.2

Let I = current in Amperes through the conductor

l = effective length in metres of the conductor which is immersed in and perpendicular to the field.

B = flux density of magnetic field in wb/m^2.

By the law of interaction, the conductor experiences a force which acts in a direction perpendicular to both the field and the current.

The force (F) acting on the conductor is given by the following equation:

$$F = BIl \text{ newtons} \tag{3.1}$$

This is relation is true only when the conductor and the magnetic field are at right angles to each other. If the conductor is inclined at an angle θ to magnetic field, then the length of the conductor normal to the field, is '$l \sin \theta$' (Fig. 3.2).

Hence, the force is given by

$$F = BIl \sin \theta \text{ newton} \tag{3.2}$$

The direction of this force can be found out by applying Fleming's left hand rule.

Fleming's Left Hand Rule Stretch out the fore finger, middle finger and thumb of the left hand so that they are at right angles to one another. If the fore finger points in the direction of magnetic field (north to south) and the middle finger points towards the direction of current, then the thumb will point in the direction of motion of the conductor (Fig. 3.3).

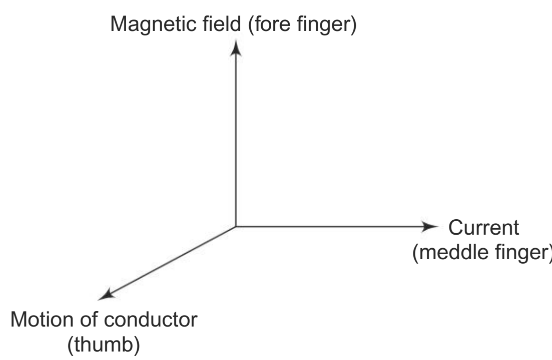


Fig. 3.3

3.3 LAW OF ELECTROMAGNETIC INDUCTION

3.3.1 Statement of Faraday's Law

Whenever the magnetic flux linking a circuit changes an emf is always induced in it. The magnitude of such an emf is proportional to the rate of change of flux linkages.

3.3.2 Lenz's Law

The law states that any induced emf will circulate a current in such a direction so as to soppose the cause producing it.

Thus, Lenz's law gives the nature of induced emfs.

Illustration Let us consider an insulated coil whose terminals are connected to a sensitive galvanometer *G* (Fig. 3.4). Initially it is placed close to a stationery bar magnet at position 1, 2. At this condition, some flux from the *N*-pole of the magnet is linked with the coil. But, there is no deflection in the galvanometer. Suppose now, the magnet is suddenly brought closer to the coil in position 3, 4. Suddenly, there is a momentary deflection in the galvanometer. This occurs only when the magnet is in motion relative to the coil. It is clear that the flux linked with the coil is increased due to this approach. Next, suppose the magnet is suddenly withdrawn away from the coil, there is a momentary deflection in an opposite direction. It also persists as long as the magnet is in motion. And the flux linked with the coil is decreased, due to the withdrawal of the magnet.

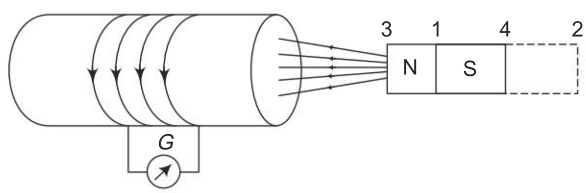


Fig. 3.4

So, the emf is induced in the coil only when there is a change of flux linkages with the coil. Stationery flux cannot induce any emf in a stationary conductor.

The same results can be found out when the bar magnet is stationery and the coil is moved suddenly away or towards the magnet.

Also, it can be noted that the amount of deflection in the galvanometer (i.e. the magnitude of induced EMF) depends on the quickness of movement of bar magnet or coil.

Hence, we can conclude that whenever the magnetic flux linking a conductor changes, an emf is always induced in it. And its magnitude depends on the rate of change of flux linkages with the coil.

3.3.3 Mathematical Explanation

Let N = No. of turns in the coil

ϕ_1 = initial flux linked the coil in Webers

ϕ_2 = final flux linked the coil in Webers

t = time taken in seconds for creating the change in flux from ϕ_1 to ϕ_2

Flux linkages mean the product of number. of turns and the flux linking the coil.

Initial flux linkages = $N\,\phi_1$

Final flux linkages = $N\,\phi_2$

Magnitude of the average induced emf, $E = \dfrac{N\,\phi_2 - N\,\phi_1}{t}$ volts

i.e. $$E = \frac{N\,(\phi_2 - \phi_1)}{t} \text{ volts} \tag{3.3}$$

Putting the above expression in its differential form, we get the emf induced at any instant as

$$e = \frac{d}{dt}(N\phi) = N\frac{d\phi}{dt} \text{ volts}$$

Incorporating Lenz's law to account for the nature of the induced emf, we have induced emf at any instant

$$e = -N\frac{d\phi}{dt} \text{ volts} \tag{3.4}$$

3.3.4 Fleming's Right Hand Rule

The direction of the induced emf can be easily found out by applying Fleming's Right Hand Rule (Fig. 3.5).

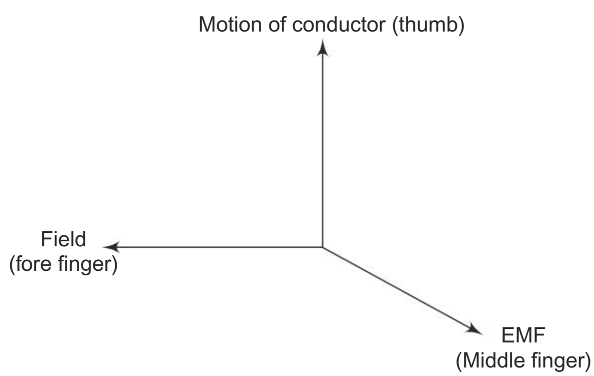


Fig. 3.5

Hold the thumb, the fore finger and the middle finger of the right hand at right angles to one another. If the thumb points to the direction of motion and the fore finger to the direction of the magnetic field, the middle finger will point the direction of induced emf.

3.4 INDUCED EMF

An emf is induced in a coil or conductor whenever there is a change in flux linkages. This change in flux linkages can be brought in the following two ways:

(i) The conductor is moved in a stationery magnetic field in such a way that there is a magnitude change in flux linkages. This kind of induced emf is known as *dynamically induced* emf (e.g generator).

(ii) The conductor is stationary and the magnetic field is moving or changing. This kind of induced emf is known as *statically induced emf* (e.g. transformer).

The following paragraphs deal at length about the two types of induced emfs.

3.4.1 Dynamically Induced emf

Consider a stationary magnetic field of flux density B wb/m^2. Let the direction of the magnetic field be as shown in Fig. 3.6. In this field, a conductor with circular cross-section is placed.

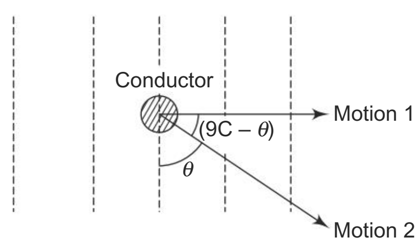


Fig. 3.6

Let 'l' be the effective length, in metres, of the conductor in the field.

Let the conductor be moved in the direction shown as 'Motion 1', i.e. at right angles to the field. In a time 'dt' seconds, the distance moved is 'dx' metres.

Area swept by the moving conductor = $l\,dx$ m^2

Magnetic flux linked by the conductor = $l\,dx\,B$ wb

If the conductor has one turn, the flux linkage

$$\psi = 1\ l\ dx\ B$$

Rate of change of flux linkages $= \dfrac{d\psi}{dt} = \dfrac{ldx\,B}{dt}$

According to Faraday's law, the emf induced, in the conductor is

$$e = \frac{Bldx}{dt} \text{ volts}$$

i.e. $$e = Blv \text{ volts} \qquad (3.5)$$

where $$v = \frac{dx}{dt} = \text{linear velocity.}$$

The direction of the induced emf is obtained by applying Fleming's right hand rule.

Let the conductor be moved with the same velocity 'v' m/sec in an inclined direction making an angle 'θ' to the direction of the field.

Then, the induced emf in the conductor is reduced by sin θ.

$\therefore$ $$e = Blv = \sin\theta \text{ volts} \quad (3.6)$$

3.4.2 Statically Induced emf

In this case, the conductor is held stationary and the magnetic field varied. It may be self induced or mutually induced.

Consider, as shown in Fig. 3.7, two coils A and B wound over a magnetic specimen. Coil A is energised using a battery of strength E volts. If switch K is initially closed, then a steady current of I amp, will flow through the coil A. It produces a flux of ϕ wb. Let us assume that this entire flux links coils A and B. When the switch is suddenly opened, the current reduces to zero. Hence, the flux linking both the coils becomes zero. As per Faraday's law, emfs are induced in both the coils A and B. Such emfs are known as *statically induced emfs*. Statically induced emfs, is also known as "transformer emf". It can be classified into two categories, namely, self induced emf and mutually induced emf.

Self Induced emf If a single coil carries a current, a flux will be set up in it. If the current changes the flux will change. The change in flux will induce an emf in the coil. This kind of emf is known as "self induced emf".

In other words, self induced emf is the emf induced in a circuit when the magnetic flux linking it changes the flux being produced by current in the *same circuit*.

The magnitude of this self-induced emf is $e = N\dfrac{d\phi}{dt}$

The direction of this induced emf would be such as to oppose any change of flux which is the very cause of its production. Hence, it is also known as the opposing or counter emf of self induction.

Mutually Induced emf It is the emf induced in one circuit due to change of flux linking it, the flux being produced by current in another circuit.

Referring to Fig. 3.7, when a change in current through coil A is created, we find that the flux linking coil B changes. Hence, an emf is induced in coil B which is mutually induced emf. The same phenomenon can be observed in coil A when coil B is energised.

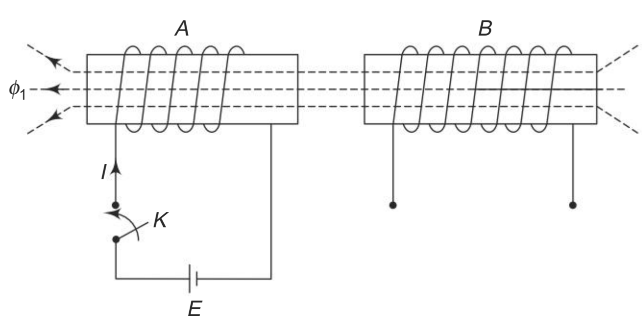


Fig. 3.7

3.5 SELF INDUCTANCE (L)

3.5.1 Definition

Self inductance of a circuit is the flux linkages per unit current in it. Its unit is Henry.

By definition, $L = \dfrac{N\phi}{I}$ Henry

3.5.2 Equation for Self-Inductance

Consider a magnetic circuit as shown in Fig. 3.8.

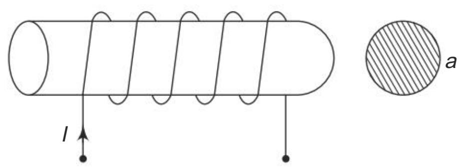


Fig. 3.8

Let N – No. of turns in the magnetising winding
I – magnetising current (Amperes)
l – length of the magnetic circuit (m).
a – cross sectional area of the magnetic circuit (m^2)
μ_r – relative permeability of the specimen.

Magnetising force, $H = \dfrac{\text{MMf}}{\text{length}} = \dfrac{NI}{l}$ Amp/metre

Magnetic flux density, $B = \mu_0\, \mu_r\, H = \mu_0\mu_r \left(\dfrac{NI}{l}\right)$ tesla

Magnetic flux, $\phi = Ba$

$$= \left(\frac{\mu_0\mu_r NI}{l}\right) a \ldots \text{wb}$$

Flux linkage of the circuit $= N\phi = \left(\dfrac{\mu_0\mu_r\, N^2 I}{l}\right) a$

Self Inductance, $L = \dfrac{N\phi}{I} = \left(\dfrac{\mu_0\mu_r N^2 I}{l}\right)\dfrac{a}{I}$

$$= \frac{N^2}{l/\mu_0\mu_r a)} = \frac{N^2}{S} = \frac{N^2}{\text{Reluctance}} \tag{3.7}$$

3.5.3 Relationship between Self-Induced emf and Self Inductance

We know that induced emf is given by

$$e = -N\frac{d\phi}{dt}$$

and $$L = \frac{N\phi}{I}$$

$$LI = N\phi$$

For a small increment of current 'di' let the incremental flux be '$d\phi$'.

$$Ldi = N d\phi$$

∴ $$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$-L\frac{di}{dt} = -N\frac{d\phi}{dt}$$

Comparing equations, we get

$$e = -L\frac{di}{dt} \text{ where } L = \frac{N^2}{S}$$

The self induced emf in a circuit is directly proportional to the rate of change of current in the same circuit.

L is the constant of the circuit called the self inductance (or) coefficient of self induction.

3.6 MUTUAL INDUCTANCE (M)

3.6.1 Definition

Mutual inductance between two circuits is defined as the flux linkages of one circuit per unit current in the other circuit. Its unit is Henry.

$$M = \frac{N_2\phi_1}{I_1}$$

or $$M = \frac{N_1\phi_2}{I_2}$$

3.6.2 Equation for Mutual Inductance

Consider two coils as shown in Fig. 3.9.

Let N_1 = No. of turns in coil 1
N_2 = No. of turns in coil 2
l = length of magnetic circuit (m)
a = cross-sectional area of magnetic circuit (m^2)
μ_r = relative permeability of the material used
I_1 = magnetising current in coil 1
ϕ_1 = magnetic flux produced by I_1.

Reluctance of the magnetic circuit, $S = \frac{1}{a\mu_0\mu_r}$

$$\phi_1 = \frac{\text{mmf}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

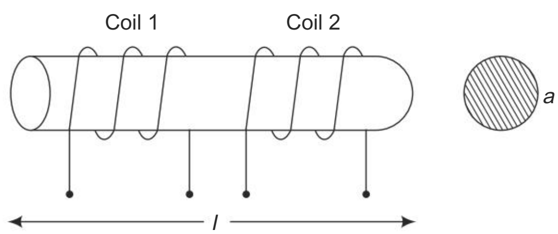


Fig. 3.9

Assume that all the flux ϕ_1 links the entire coil 2

Flux linkage of circuit 2 is

$$\psi_{21} = \frac{N_2 N_1 I_1}{S}$$

Mutual inductance, $M = \dfrac{\psi_{21}}{I_1} = \dfrac{N_1 N_2}{S}$ (3.8)

3.6.3 Relationship between M and Induced emf

Referring to Fig. 3.9, mutually induced emf in coil 2 is

$$E_{m,2} = \frac{\psi_{21}}{t} = \frac{N_2 \phi_1}{t}$$

At any instant $e_{m,2} = -N_2 \dfrac{d\phi_1}{dt}$

And, the mutual inductance between the two circuits is given as

$$M = \frac{N_2 \phi_1}{I_1}$$

$$MI_1 = N_2 \phi_1$$

At any instant, $M \dfrac{di_1}{dt} = N_2 \dfrac{d\phi_1}{dt}$

$$-M \frac{di_1}{dt} = -N_2 \frac{d\phi_1}{dt}$$

Comparing equations, we get

$$e_{m,2} = -M \frac{di_1}{dt}$$

Similarly, mutually induced emf in coil 1 due to change in current of coil 2 is,

$$e_{m,1} = -M \frac{di_2}{dt}$$

Thus, the mutually induced emf in a circuit is directly proportional to the rate of change of current in another circuit. The constant of proportionality (M) is called the mutual inductance or the coefficient of mutual induction.

3.7 COUPLING COEFFICIENT BETWEEN TWO MAGNETICALLY COUPLED CIRCUITS (K)

With reference to Fig. 3.9, the following additional notations are used:

I_2 = magnetising current in coil 2

ϕ_2 = flux caused by I_2

There will be mutual flux linkages when both the coils are energised simultaneously.

Let a fraction of flux ϕ_1, say $K_1\ \phi_1$, link coil 2

Flux linkages of coil 2 is, $\psi_{21} = N_2\ (K_1\ \phi_1)$

Mutual Inductance,

$$M = \frac{\psi_{21}}{I_1} = \frac{N_2 K_1 \phi_1}{I_1}$$

$$= \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)\left(\because \quad \phi = \frac{NI}{S}\right)$$

$$M = \frac{N_1 N_2 K_1}{S}$$

where S = Reluctance of the magnetic circuit $= \dfrac{1}{\mu_0 \mu_r a}$

Let a fraction of flux ϕ_2, say $K_2 \phi_2$, link coil 1. Flux linkages of coil 1 is $\psi_{12} = N_1(K_2\ \phi_2)$

$$M = \frac{\psi_{12}}{I_2} = \frac{N_1 K_2 \phi_2}{I_2}$$

$$= \frac{N_1 K_2}{I_2}\left(\frac{N_2 I_2}{S}\right)$$

i.e.

$$M = \frac{N_1 N_2 K_2}{S}$$

From the above equations, we get

$$M^2 = K_1 K_2\ N_1^2 N_2^2 / S^2$$

$$= K^2 \left(\frac{N_1^2}{S}\right)\left(\frac{N_2^2}{S}\right)$$

$$= K_2 L_1 L_2$$

where L_1 and L_2 are the self inductances of the coils 1 and 2 respectively

$$\therefore \qquad K^2 = \frac{M^2}{L_1 L_2};\ K = \frac{M}{\sqrt{L_1 L_2}}$$

always $K \leq 1$.

Thus, coupling coefficient, $K = \dfrac{M}{\sqrt{L_1 L_2}}$ (3.9)

Example 3.1 The field coils of 4 pole d.c generator each having 500 turns are connected in series. When the field is excited there is a magnetic flux of 0.02 wb/pole. If the field circuit is opened in 0.02 second and the residual magnetism is 0.002 wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current?

Solution: No. of poles (P) = 4

$$
\begin{aligned}
\text{No. of turns per pole } (N_1) &= 500 \\
\phi &= 0.02 \text{ wb/pole} \\
t &= 0.02 \text{ sec} \\
\text{Residual flux} &= 0.002 \text{ wb/pole} \\
\text{Total No. of turns} &= N = P \times N_1 \\
&= 4 \times 500 \\
&= 2000 \\
\text{Total initial flux} &= 4 \times 0.02 \\
&= 0.08 \text{ wb.} \\
\text{Total residiual flux} &= 4 \times 0.002 \\
&= 0.008 \text{ wb} \\
\text{Change in flux, } d\phi &= 0.08 - 0.008 \\
&= 0.072 \text{ wb} \\
\text{Time of opening the circuit, } dt &= 0.02 \text{ sec.} \\
\text{Induced emf} &= N\frac{d\phi}{dt} \\
&= 2000 \times \frac{0.072}{0.02} \\
&= 7200 \text{ volts}
\end{aligned}
$$

The direction of this emf is the same as that of the original direction of the exciting current.

Example 3.2 A coil of resistance 150 Ω is placed in a magnetic flux of 0.1 m wb. The coil has 500 turns and a galvanometer of 450 Ω resistance is connected in series with it. The coil is moved in 0.1 sec from the given field to another field of 0.3 m wb. Find the average induced emf and average current through the coil.

Solution: Resistance of the coil = 150 Ω

$$
\begin{aligned}
\phi_1 &= 0.1 \text{ m wb} \\
N &= 500 \text{ turns} \\
R_{\text{gal}} &= 450\ \Omega \\
dt &= 0.1 \text{ sec} \\
\phi_2 &= 0.3 \text{ m wb}
\end{aligned}
$$

The flux is changed from 0.1 m wb to 0.3 m wb in 0.1 sec and coil has 500 turns.

The induced emf = rate of change of flux linkages.

i.e. $$e = N\frac{d\phi}{dt} = \frac{(0.3 - 0.1) \times 10^{-3}}{0.1} \times 500 = 1\text{V}$$

Current $$I = \frac{\text{Volts}}{\text{Total resistance}} = \frac{1}{150 + 450}$$
$$= 0.0017\ A$$

Example 3.3 A conductor 10 cm long and carrying a current of 60 A lies perpendicular to a field of strength 1000 A/m. Calculate (a) the force acting on the conductor, (b) the mechanical power to move this conductor against this force with a speed of 1 m/s and (c) emf induced in the conductor.

Solution:

$$l = 10 \text{ cm} = 0.1 \text{ m}$$
$$I = 60\ A$$
$$H = 1000 \text{ A/m}$$
$$v = 1 \text{ m/s}$$

(a) $F = BIl$ Newton

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 \times 4\pi \times 10^{-4} \text{ wb/m}^2$$
$$F = 4\pi \times 10^{-4} \times 60 \times 0.1 = 7.5 \times 10^{-3} \text{ N}$$

(b) $P = F.v.$

$$= 7.5 \times 10^{-3} \times 1 = 0.0075 \text{ watt}$$

(c) $e = Blv$

$$= (4\pi \times 10^{-4}) \times (0.1) \times (1) = 4\pi \times 10^{-5} \text{ V}$$

Example 3.4 An air cored toroidal coil has 480 turns, a mean length of 30 cm and a cross-sectional area of 5 cm^2. Calculate (a) the inductance of the coil and (b) the average induced emf, if a current of 4 A is reversed in 60 millionseconds.

Solution:

$$l = 30 \text{ cm} = 0.3 \text{ m}$$
$$N = 450 \text{ turns}$$
$$a = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$
$$I = 4 \text{ Amps.}$$
$$dt = 60 \text{ m sec}$$

(a) $$L = \frac{N^2}{(1/\mu_0 \mu_r a)}$$
$$= \frac{\mu_0 \mu_r a N^2}{l}$$
$$= \frac{4\pi \times 10^{-6} \times 1 \times 5 \times 10^{-4} \times (480)^2}{0.3} \text{ Henry} \quad (\because \mu_r = 1 \text{ for air})$$
$$= 483 \times 10^{-6}\ H$$

(b) $$di = 4 - (-4) = 8 \text{ Amp}$$
$$dt = 60 \times 10^{-3} \text{ sec}$$

Average induced emf $(E) = L\dfrac{di}{dt}$

$$= 483 \times 10^{-6} \times \frac{8}{60 \times 10^{-3}}$$

$$= 0.064 \text{ V.}$$

Example 3.5 The self inductance of a coil of 500 turns is 0.25 *H*. If 60% of the flux is linked with a second coil of 10500 turns, calculate (a) the mutual inductance between the two coils and (b) emf induced in the second coil when current in the first coil changes at the rate of 100 A/sec.

Solution:

$$L_1 = 0.25\ H$$

$$N_1 = 500 \text{ turns}$$

$$N_2 = 10500 \text{ turns}$$

$$\phi_2 = \frac{60}{100}\phi_1,\ \phi_2 = 0.6\ \phi_1.$$

$$\frac{di_i}{dt} = 100 \text{ A/sec}$$

Flux/ampere in the first coil

$$= \frac{\phi_1}{I_1} = \frac{0.25}{N_1} = \frac{0.25}{500} = 5 \times 10^{-4}$$

Flux linking the second coil = $\phi_2 = 0.6\ \phi_1$

$$\frac{\phi_2}{I_1} = 0.6\ (\phi_1/I_1)$$

$$= 0.6 \times 5 \times 10^{-4}$$

$$= 3 \times 10^{-4}$$

(a) $$M = N_2\frac{\phi_2}{I_1} = 10500 \times 3 \times 10^{-4} = 3.15\ H$$

(b) $$e_m = M\frac{di_1}{dt} = 3.15 \times 100 = 315 \text{ V}$$

Example 3.6 The number of turns in a coil is 250. When a current of 2 A flows in this coil, the flux in the coil is 0.3 m wb. When this current is reduced to zero in 2 milliseconds, the voltage induced in a coil lying in the vicinity of coil is 63.75 volts. If coils, mutual inductance and number of turns in the second coil.

Solution: $N_1 = 250$; $I_1 = 2A$: $\phi_1 = 0.3$ wb

$dt = 2$ msec; $e_{m2} = 63.75$ V; $K = 0.75$

(a) Self inductance of first coil is

$$L_1 = N_1 \times \frac{\phi_1}{I_1}$$

$$= 250 \times \frac{0.3 \times 10^{-3}}{2}$$

$$= 37.5 \text{ mH.}$$

(b) Voltage in the second coil is

$$e_{m2} = M\frac{dI_1}{dt}$$

i.e. $$63.75 = M\frac{2}{2 \times 10^{-3}}$$

$$M = 63.75 \times 10^{-3} = 63.75 \text{ mH.}$$

we know that $$M = K\sqrt{L_1 L_2}$$

i.e. $$63.75 = 0.75\sqrt{3.75 \times L_2}$$

$$L_2 = 193 \text{ mH.}$$

(c) ϕ_2 the flux in the second coil

$$= k \times \phi_1$$
$$= 0.75 \times 0.3 \times 10^{-3}$$
$$= 0.255 \times 10^{-3}\ wb$$

$$e_2 = N_2\frac{d\phi_2}{dt}$$

$$63.75 = N_2\frac{0.255 \times 10^{-3}}{2 \times 10^{-3}}$$

$$N_2 = 500 \text{ turns.}$$

Example 3.7 A wire having a length of 1 m moves at right angles to its length at 50 m/sec in a uniform magnetic field of 1 wb/m^2. Determine the emf induced in the conductor when the direction of motion is

(a) at right angles to the field (b) inclined at 30° to the direction of the field.

Solution:

$l = 1$ m $\quad v = 50$ m/s $\quad B = 1$ wb/m^2

(a) $\theta = 90°$

Emf induced in the conductor $= Blv \sin\theta$

$= 1 \times 1 \times 50 \times \sin 90° = 50$ volts

(b) $\theta = 30°$

Emf induced in the conductor $= 1 \times 1 \times 50 \times \sin 390° = 25$ volts

Example 3.8 A coil of 1000 turns encloses a magnetic circuit with a cross-section area of 10 cm^2. With 4.2 A, the flux density is 1 wb/m^2 and with 9.2 A, it is 1.42 wb/m^2. Find the average inductance between these limits. Also calculate the induced emf, if the current reduces uniformly from 9.2 A to 4.2 *a* in 0.05 sec.

Solution:

$N = 1000$	For $I_1 = 4.2$ A	$B_1 = 1$ wb/m^2
$a = 10 \times 10^{-4}$ m^2	For $I_2 = 9.2$ A	$B_2 = 1.42$ wb/m^2

Inductance, $$L = N\frac{d\phi}{dI} = NA\frac{db}{dI}$$

$$= 1000 \times 10 \times 10^{-4}\left(\frac{1.42 - 1}{9.2 - 4.2}\right)$$

$$= 0.084 \text{ H}$$

Emf induced $= -L\dfrac{di}{dt} = -0.084\left[\dfrac{4.2-9.2}{0.05}\right]$

$= 8.4$ volts

Example 3.9 A solenoid has 1600 turns of wire wound over a length of 50 cm. A second coil (search coil) of 600 turns, enclosing a mean area of 18 cm^2, is placed centrally in the solenoid. Calculate (a) the mutual inductance (b) the emf induced in the search coil when the current in the solenoid changes uniformly at the rate of 300 A/sec.

Solution:

$N_1 = 1600$ $\quad N_2 = 600$

$l = 0.5$ m $\quad a = 18 \times 10^{-4}$ m^2

Flux density in the solenoid is $B = \mu o H$

$$= \frac{4\pi \times 10^{-7} \times 1600\, I}{0.5} = 1.28 \times 10^{-4}\, \pi I$$

Mutual inductance, $M = \dfrac{BaN_2}{I}$

$$= \frac{1.28 \times 10^{-3} \times \pi I \times 18 \times 10^{-4} \times 600}{1}$$

$= 4.34$ mH

Voltage induced $= M\dfrac{di}{dt} = 4.34 \times 10^{-3} \times 300 = 1302$ V.

Example 3.10 Two coils *A* and *B* lie in parallel planes. Coil *A* has 15000 turns and coil *B* has 12000 turns. 55% of flux produced by coil *A* links coil *B*. A current of 6 A in coil *A* produces 0.05 mwb, while the same current in coil *B* produces 0.08 mwb. Calculate the mutual inductance and the coupling coefficient.

Solution:

Coil A	**Coil B**
$N_A = 15000$	$N_B = 12000$
$I_A = 6$A	$I_B = 6$A
$\phi_A = 0.05 \times 10^{-3}$ wb	$\phi_B = 0.08 \times 10^{-3}$ wb

Mutual flux $= 0.55 \times 0.05 \times 10^{-3}$ wb

Self inductance of coil A, $L_A = \dfrac{0.05 \times 10^{-3} \times 15000}{6} = 0.125$ H

Self inductance of coil B, $L_B = \dfrac{0.08 \times 10^{-3} \times 12000}{6} = 0.16$ H

Mutual Inductance $= \dfrac{0.55 \times 0.05 \times 10^{-3} \times 12000}{6}$

$= 0.055$ H

Coefficient of coupling $= \dfrac{M}{\sqrt{L_A L_B}} = \dfrac{0.055}{\sqrt{0.125 \times 0.16}}$

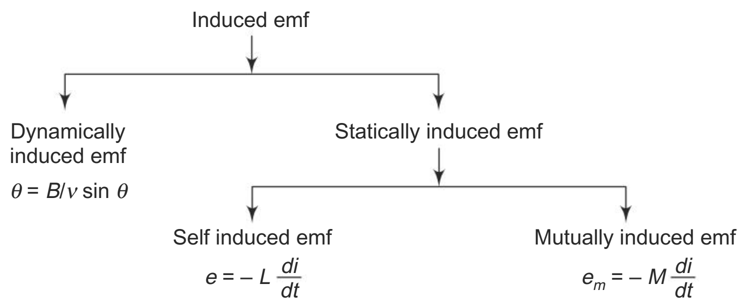
$= 0.389$

IMPORTANT FORMULAE

1. Faraday's law of electromagnetic induction gives, the induced emf

$$e = -N\frac{d\phi}{dt} \text{ volts}$$

2. The direction of the induced emf and hence current may be found either by Fleming's Right hand Rule or Lenz's law.
3. The Induced emf may be subdivided as follows:



4. Flux linkages = $N\phi$.
5. Value of the coefficient of self-inductance of a solenoid is given by

$$L = \frac{N\phi}{I} \text{ henry}$$

Also, $$L = \frac{\mu_0 \mu_r a N^2}{l} \text{ henry}$$

6. Value of coefficient of mutual inductance between two coils is given by

$$M = \frac{N_2 \phi_2}{I_1} \text{ henry}$$

$$= \frac{\mu_0 \mu_r a N_1 N_2}{l} \text{ henry}$$

7. Coefficient of coupling between two magnetically coupled coils is given by

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

REVIEW QUESTIONS

1. State and explain the laws of electromagnetic induction and the rule which determines the direction of the induced emf.
2. Write short notes
   (a) Flux linkage (b) Self-induction
   (c) Mutual induction (d) Lenz's law

3. What are the factors on which the inductance of a coil depend? Derive the necessary expression for calculating the inductance of a coil?
4. Explain the concepts of mutual and self-inductance and define the units in which these are measured.
5. Deduce from first principles an expression for the inductance of a long solenoid.
6. What is coefficient of coupling? How can it be varied? Derive an expression for the same.
7. Explain the concepts of electromagnetic force with the help of relevant rules.

PROBLEMS

1. A coil of resistance 100 ohms is placed in a magnetic field of 1 mwb. The coil has 100 turns and a galvanometer of 400 ohms resistance is connected in series with it. Find the average emf and the current if the coil is moved in 1/10th second from the given field to a field strength of 0.2 mwb.
2. A straight conductor 25 cm long carries 100 A and lies perpendicular to as uniform field of 0.5 wb/m^2. Find:
   (a) the mechanical force acting on the conductor.
   (b) the power necessary to drive the conductor against the force at an uniform speed of 1.27 m/sec.
   (c) the emf generated in the conductor.
   (d) the electrical power developed.
3. A coil of 800 turns is wound on a wooden former and a current of 5 A produces a magnetic flux of 200×10^{-6} wb. Calculate inductance of coil and induced emf when current is reversed in 0.2 second.
4. Two coupled coils have a coefficient of coupling, 0.85, $N_1 = 100$ turns and $N_2 = 800$ turns. With coils 1 open and a current of 5 A in coil 2, the flux ϕ_2 is 0.35 mwb. Find L_1, L_2 and M.
5. Two long single-layer solenoids have the same length and the same number of turns but are placed co-axially one within the other. The diameter of the inner coil is 8 cm and that of the outer coil is 10 cm. Calculate the coefficient of coupling between the coils.
6. A coil of 800 turns is wound on a wooden former and a current of 5 A produces a magnetic flux of 200×10^{-6} wb. Calculate the inductance of coil and induced emf when current is reversed in 0.2 second.
7. Two coils *A* of 50 turns and *B* of 1000 turns are wound side by side on a closed magnetic circuit made of iron. Cross section of the magnetic circuit is 80 cm^2 and its mean length is 120 cm.
   (a) Calculate the mutual inductance between the coils, if the relative permeability of iron is 2200.
   (b) Calculate the emf induced in the coil *B* when the current in coil *A* grows steadily from 0 to 10 A in 0.015 sec.
8. Two coils, connected in series, give a combined inductance 0.62 H, or 0.12 H, depending upon the relative directions of currents in them. One coil, when isolated, has a self-inductance of 0.21 H. Calculate the mutual inductance and the coupling coefficient.
9. A solenoid consists of 1500 turns of wire wound on a former of length 120 cm and diameter 4 cm. It is placed coaxially within another solenoid of the same length and

number of turns but of diameter 8 cm. Find, for this arrangement, the mutual inductance and the coupling coefficient.

10. A solenoid of length 15 cm has 600 turns of wire wound uniformly over the entire length. The wire carries an alternating current of 0.25 A at a frequency of 6000 Hz. A search coil having 60 turns enclosing a cross-sectional area of 2.5 cm^2 is placed at the middle of the solenoid. Calculate the mutual inductance between the coils and the emf induced in the search coil.

ANSWERS TO PROBLEMS

1. 0.8 V; 1.6 mA.
2. 1.25 newton; 1.587 Joules/sec
   158.75 mV; 15.87 W
3. 0.032 H; – 1.6 volt
4. 0.875×10^{-3} H; 56×10^{-3}H, 5.95×10^{-3} H.
5. $K = 0.8$.
6. 0.032 H, 1.6 V
7. 0.921H, 614V
8. 0.125 H, 0.682
9. 2.96 mH, 0.5
10. 75.3 μH, 0.709 V

AC FUNDAMENTALS

4

INTRODUCTION

We have seen so far about the analysis of dc circuits. A dc quantity is one which has a constant magnitude irrespective of time. But, an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways: (1) By its effective value, (2) by its average value and (3) by its peak value. This chapter deals with generation of alternating current, its representation and analysis of ac circuit. The analysis pertains to the basic component only.

4.1 GENERATION OF ALTERNATIVE EMF

Consider a coil of n turns placed in a magnetic field of maximum value ϕ_m Webers [see Fig. 4.1 (a)]. The coil is initially along the reference axis. In this position, the field is perpendicular to the plane of the coil.

Let the coil be rotated in the anticlockwise direction with an angular velocity of ω rad/sec.

When the coil is along the reference axis at $\omega t = 0$, it is called as zero e.m.f. position. This is because the movement of the coil at this instant $\omega t = 0$ is along the field.

Let at any instant t sec. the coil takes a position as shown in Fig. 4.1(b).

At this instant, the coil makes an angle $\theta = \omega t$ with the reference axis.

At this position, the normal component of the mangnetic flux with respect to the plane of the coil is equal to

$$\phi_m \cos\theta \qquad (\therefore \quad \theta = \omega t)$$

The normal component = $\phi_m \cos\omega t$

Flux linkages (ψ) at this instant (y) is equal to $N\phi_m \cos\omega t$. According to Faraday's law.

The emf induced in the coil at the instant under consideration.

$$e = -\frac{d\psi}{dt} = \frac{-d}{dt}(N\phi_m \cos\omega t)$$

$$= -N\phi_m\omega(-\sin\omega t)$$

$$e = (N\phi_m\omega)\sin\omega t \tag{4.1}$$

With the above expression, we can calculate the emf induced at various instants.

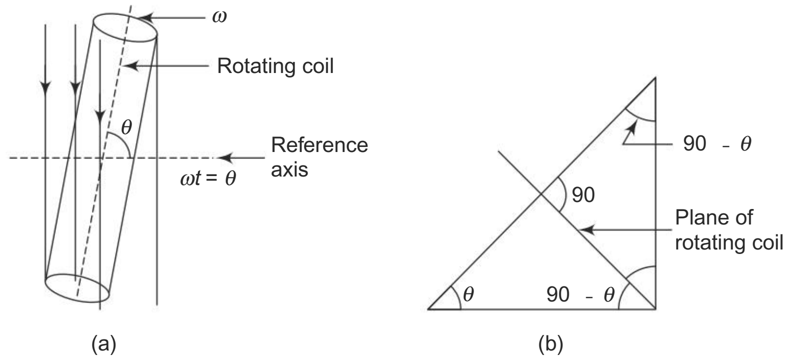


Fig. 4.1

When $\omega t = 0$ or $180°$

$$= (N\phi_m \omega) \sin 0 = (N\phi_m\, \omega) \sin 180°$$

$$e = 0$$

when $\omega t = 90°$ $e = (N\phi_m\omega) \sin 90°$

when $\omega t = 270°$ $e = N\phi_m\omega \sin (270°)$

$$e = -N\phi_m\omega$$

Let $N\phi_m\omega = E_m$ denote the maximum value of induced emf then from Eq. (4.1) we can write,

Instantaneous emf $\quad e = E_m \sin \omega t$ (Refer Fig. 4.2) $\qquad$ (4.2)

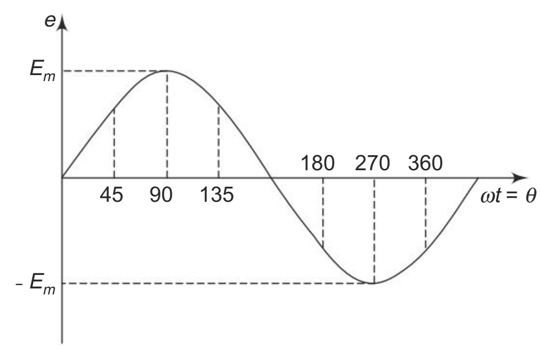


Fig. 4.2 Alternating emf wave for one complete cycle

4.2 TERMINOLOGY

1. Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Examples of waveforms are shown in Fig. 4.3.

2. Alternating Waveform This is a wave which reverses its direction at regularly recurring intervals, e.g. Fig. 4.3(a).

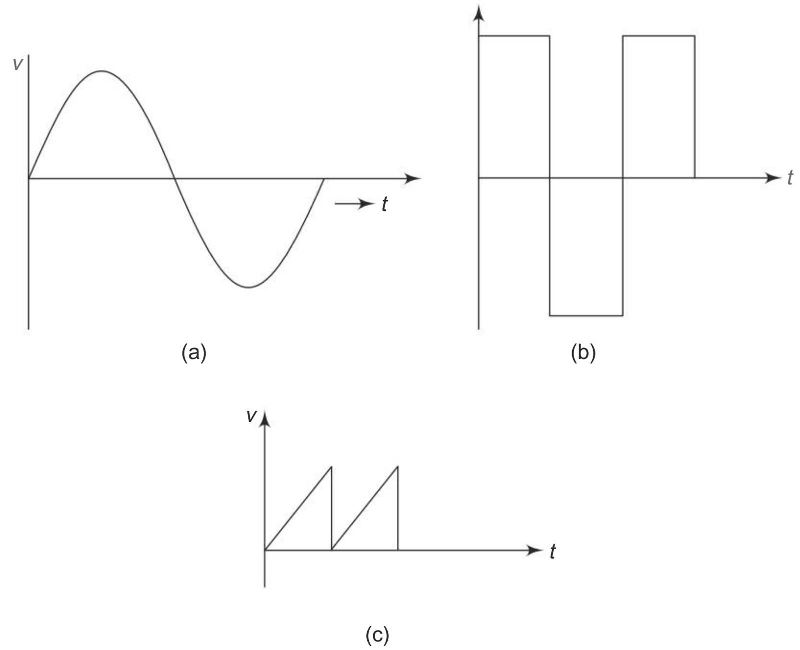


Fig. 4.3 (a) Sinusoidal waveform (b) Rectangular waveform (c) Sawtooth waveform

3. Periodic Waveform Periodic waveform is one which repeats itself after definite time intervals.

4. Sinusoidal and Non-Sinusoidal Waveform

Sinusoidal waveform It is an alternating waveform in which sine law is followed.

Non-sinusoidal waveform It is an alternating waveform in which sine law is not followed.

5. Cycle One complete set of positive and negative halves constitute a cycle.

6. Amplitude The maximum positive or negative value of an alternating quantity is called the amplitude.

7. Frequency The number of cycles per second of an alternating quantity is known as frequency. Unit for frequency is expressed as c/s or Hertz (Hz).

8. Period (T) Time period of an alternating quantity is the time taken to complete one cycle. Time period is equal to the reciprocal of frequency. Time period is expressed in secs.

9. Phase The phase at any point on a given wave is the time that has elapsed since the quantity has last passed through zero point of reference and passed positively.

10. Phase Difference The term is used to compare the phase of two waveforms or alternating quantities.

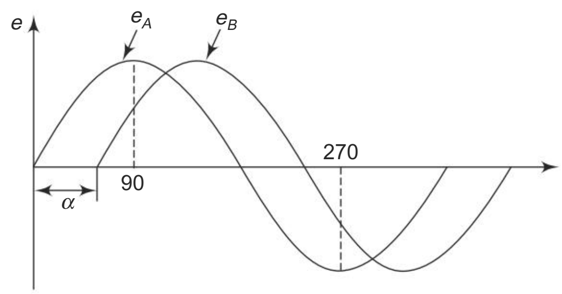


Fig. 4.4

Consider two coils AA' and BB' rotating in magnetic field of maximum value ϕ_m. Let the coils A and B be identical.

Let the two coils rotate with the same angular velocity ω. Let the coil A take the position of reference axis at $t = 0°$. The place of coil B is behind that of coil A by an angle α. Waveform representation of the emf's induced in both the coils are shown in Fig. 4.4.

The emf induced in coil A is given by $e_A = E_m \sin \omega t$ (4.3)

The emf induced in coil B is given by

$$e_B = E_m \sin (\omega t - \alpha) \tag{4.4}$$

Where α is the phase angle between e_A and e_B.

Negative sign of α denotes that e_B lags e_A.

Suppose the emf induced in coil B is taken as reference.

Then, $$e_B = E_m \sin \omega t \tag{4.5}$$

In general, a *Lagging* quantity is one which starts and progresses behind the quantity under reference.

And a *Leading* quantity is one which starts and programmes ahead of the quantity under reference.

4.3 CONCEPT OF 3-PHASE EMF GENERATION

Definition and Computation We have seen above only about single phase systems. Generally generation, transmission and distribution of electrical energy are of three phase in nature.

Three phase system is a every common poly phase system. It could be viewed as the combination of three single phase systems with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system whereas pulsating torque is produced in the case of a single phase system. Because of these advantages, the overall generation, transmission and distribution of electrical power is usually of three phase. This chapter deals with the generation of 3-phase emf.

4.3.1 Generation of Three Phase emf

Consider three similar coils RR', YY' and BB' placed in a magnetic field of maximum value φ_m Webers (Refer Fig. 4.5). The coils are displaced by an angle of 120° in space between any two. Let all the coils rotate in the anticlockwise direction at an angular velocity ω.

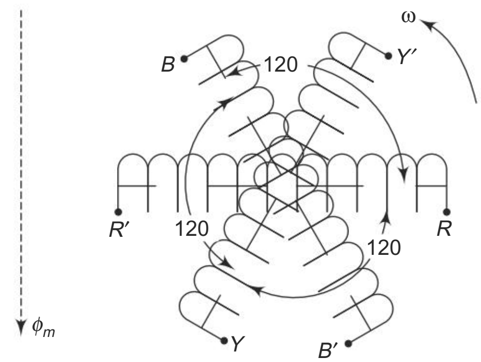


Fig. 4.5

According to Faradays law of electromagnetic induction, emfs are induced in the coils RR', YY' and BB'. The induced emf in coil YY' lags behind the induced emf in coil RR' by 120°. The induced emf in coil BB' lags behind that in coil RR' 240°.

Expressing mathematically,

$$e_R = E_m \sin \omega t \text{ (reference quantity)}$$

$$e_Y = E_m \sin(\omega t - 120°)$$

$$e_B = E_m \sin(\omega t - 240°)$$

All the three induced emfs have the same amplitude, same period and frequency. Thus, the above set of voltages are called three phase balanced system of voltages. The waveforms of the induced voltages are shown in Fig. 4.5.

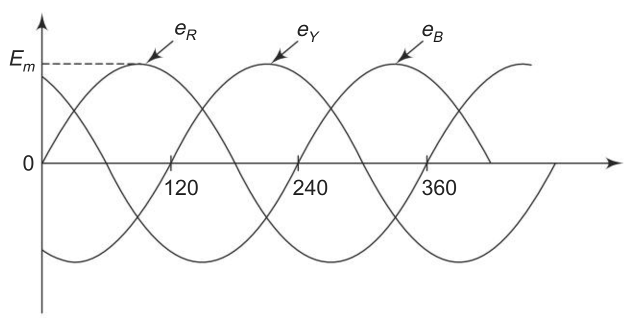


Fig. 4.6

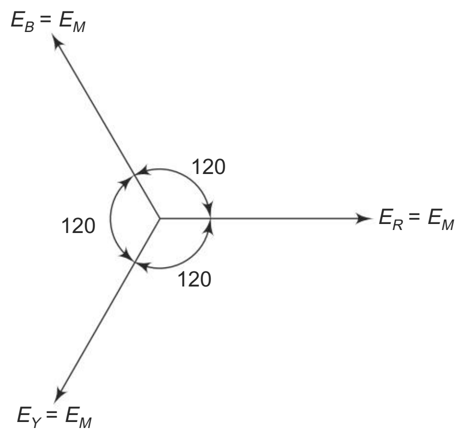


Fig. 4.7

Phasor Representation Let the emf induced in R phase, E_R be taken as reference. $\overline{E}_Y$ lags $\overline{E}_R$ by 120° and $\overline{E}_B$ lags $\overline{E}_R$ by 240°. The three phasors are represented in Fig. 4.7.

4.3.2 Phase Sequence

Phase sequence indicates the rotation of phasors in a particular direction. The order in which the different phasors reach their respective maximum value is known as phase sequence. For the system under discussion, if the phase sequence is given as RYB then the convention is R phase reaches its maximum value first, Y phase follows R and B phase follows Y in reaching the maximum value. The RYB sequence in the anticlockwise direction defines the positive phase sequence, [Refer Fig. 4.8(a)].

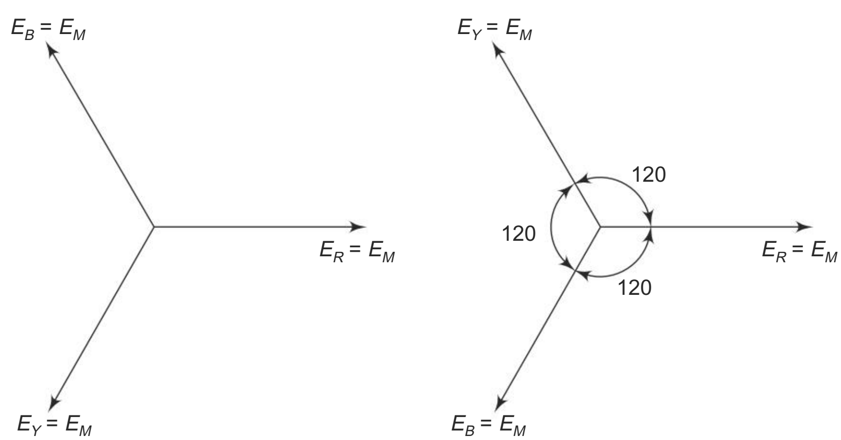


Fig. 4.8 (a) Positive phase sequence (b) Negative phase sequence

For the same system, if the phase sequence is given as *RBY*, it indicates that *R* phase reaches its maximum value first, *B* phase follows *R* and *Y* phase follows *B* in reaching the maximum values. *RBY* defines the negative phase sequence refer Fig. 4.8 (b).

4.4 ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Definition Effective or RMS value of an alternating current is deifned by that steady value of current (dc) which when flowing in a given circuit for a given time produces the same heat as would be produced by the alternating current flowing in the same circuit for the same time.

Determination of RMS Value for any Alternating Circuit Figures 4.9 and 4.10 denote an alternating current wave during its positive half cycle for both symmetrical non-sinusoidal and sinusoidal waves. Divide the time base into n number of small intervals. Erect mid-ordintates in each interval.

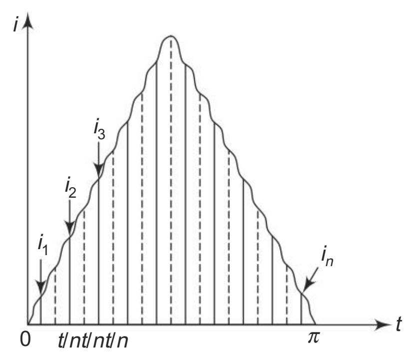


Fig. 4.9 Non sinusoidal wave

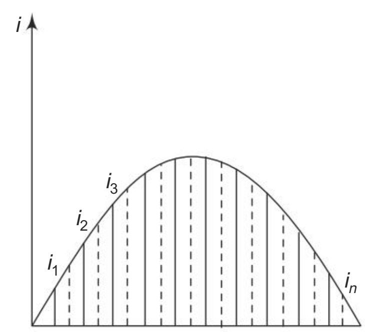


Fig. 4.10 Sinusoidal wave

Duration of each interval is equal to t/n secs. Let i_1, i_2 and i_3, ... i_n be the mid ordinates and these represent the current values in each of the small intervals.

Let this current be applied to a circuit of resistance R ohms.

Heat produced in the first interval = $0.24 \times 10^{-3}\, i_1^2\, R\, t/n$ ... kcal.

Heat produced in the second interval = $(0.24 \times 10^{-3})\, i_2^2\, R\, t/n$ ... kcal.

Heat produced in the last interval = $(0.24 \times 10^{-3}) \times i_n^2\, R\, t/n$ ... kcal

Total heat produced = Heat produced in the first interval +

Heat produced in the second interval + ... +

Heat produced in the last interval

i.e Total heat produced = Sum of the heat produced in n intervals

$$= 0.24 \times 10^{-3}\, Rt \left[\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}\right] \tag{4.7}$$

Let I amp be the steady current flowing through the same resistance R ohms for the same time t secs. Then, heat produced

$$= 0.24 \times 10^{-3} \times I^2 Rt \text{ kcal} \tag{4.7a}$$

By the definition of RMS value,

$$0.24 \times 10^{-3} I^2 Rt = 0.24 \times 10^{-3} Rt \frac{i_1^2 + i_2^2 + i_n^2}{n}$$

$$I_2 = \frac{i_1^2 + i_2^2 + i_3^2 + i_n^2}{n}$$

$$I = \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}\right)^{1/2}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \ldots .+ i_n^2}{n}} \tag{4.8}$$

RMS value of the alternating current, I_{RMS} = Square root of the mean values of the sum of squares of the current components.

Analytical Method to Obtain the RMS Value for Sinusoidal Currents

Let the alternating current be represented by

$$i = I_m \sin \omega t$$

$$= I_m \sin \theta \, (\theta = \omega t)$$

$$i^2 = I_m^2 \sin^2 \theta$$

Mean square of $\quad \text{AC} = \int_0^{2x} \frac{I_m^2 \sin^2 \theta}{2\pi} d\theta$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{I - \cos 2\theta}{2} d\theta$$

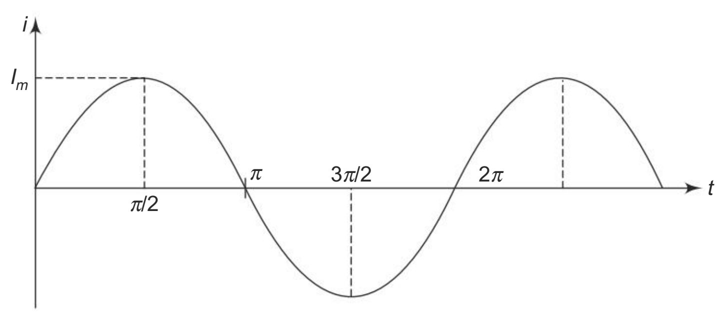


Fig. 4.11

$$= \frac{I_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2\pi}\,\frac{2\pi}{2} = \frac{I_m^2}{2}$$

RMS value of the alternating sinusoidal current is

$$I = \sqrt{\frac{I_m^2}{2}} = \frac{l_m}{\sqrt{2}} = 0.707\ l_m$$

$$I_{\text{RMS}} = 0.707\ I_m \tag{4.9}$$

Similarly, for an alternating voltage

$$V_{\text{RMS}} = V = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \ldots\ V_n^2}{n}} \tag{4.10}$$

For a sinusoidal voltage $= V = \dfrac{V_m}{\sqrt{2}} = 0.707\ V_m$

The RMS value of a wave can also be obtained by the formula

$$\text{RMS value} \qquad \sqrt{\frac{\text{Area under the square curve for one cycle}}{\text{Period}}} \tag{4.11}$$

4.5 AVERAGE VALUE OF AC

Definition The average value of an ac is given by that steady current which transfers across a circuit the same charge as would be transferred by the ac across the same circuit in the same time.

Determination of Average Value

With reference to Figs. 4.9. and 4.10, the average value,

$$I_{\text{av}} = \frac{i_1 + i_2 + i_3 + \ldots\ i_n}{n} \tag{4.12}$$

Average value can be easily obtained by first finding the average value for a small interval of time and then integrating over the curve, i.e.

$$I_{\text{av}} = \frac{1}{T}\int_0^T idt \tag{4.13}$$

This is nothing but the ratio of the area under the curve over one complete cycle to the base.

Average Value for Symmetrical and Unsymmetrical Waves

In the case of a symmetrical alternating current or voltage wave there exists two exactly similar half cycles whether sinusoidal or non-sinusoidal. In this case, the average value over a complete cycle is zero. Hence, for symmetrical waves the average value is taken for only one half cycle.

In the case of unsymmetrical waves, the average values more always be taken over the whole cycle.

Analytical Method to Obtain the Average Value for Sinusoidal Current

Let $$i = I_m \sin \theta$$

Since this is a symmetrical wave it has two equal half cycles namely positive and negative halves.

Considering one half cycle for this symmetrical wave the average value is obtained by

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i d\theta = \frac{1}{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{I_m}{\pi}(-\cos \theta)_0^x$$

$$= \frac{I_m}{\pi}(1 + 1) = \frac{I_m}{\pi} \times 2$$

$$I_{av} = \frac{2\,I_m}{\pi}$$

$$I_{av} = 0.637\, I_m \qquad (4.14)$$

where I_m is the maximum value of current.

For a sinusoidal voltage wave,

$$V_{av} = 0.637\, V_m.$$

Form Factor and Peak Factor The relation between average, RMS and maximum values can be expressed by two factors namely form factor and peak factor. Form factor (K_f): Form factor is defined as the ratio of RMS value to the average value.

i.e. $$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} \qquad (4.15)$$

Peak factor or Cost factor) (K_p): Peak factor is defined as the ratio of peak value to the R.M.S. value. (4.16)

i.e. $$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}}$$

For a sinusoidal wave,

$$\text{Form factor } (K_f) = \frac{0.707\, I_m}{0.637\, I_m} = 1.11 \qquad (4.17)$$

$$\text{Peak factor } (K_f) = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414. \qquad (4.18)$$

Example 4.1 Find the average and RMS value of the following waveforms (Fig. E.4.1). Find also the form factor and peak factor

Solution: (a)

(i) Unsymmetrical saw tooth waveform is given

$$\text{Average value} = \frac{\text{Area under one complete cycle}}{\text{Period}}$$

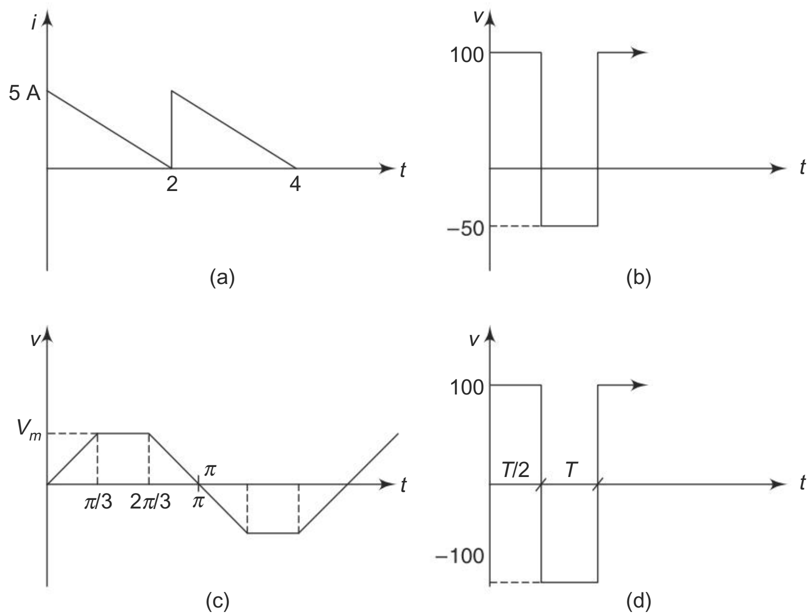


Fig. E.4.1

$$\text{Area under one complete cycle} = \frac{1}{2} \times 2 \times 5 = 5$$

$$\text{Period or base} = 2 \text{ secs}$$

$$\text{Average value} = 5/2 \text{ Amps}$$

$$I = 2.5 \text{ Amps}$$

(ii) ***RMS* value** (Refer Fig. E.4.2)

$$\text{RMS Value} = \sqrt{\frac{\text{Area under the squared curve for one cycle}}{\text{Period}}}$$

$$\text{Area under the squared curve} = 1/3 \times 5^2 \times 2 = 50/3$$

$$\text{Base} = 2$$

$$\therefore \qquad \text{RMS value} = \frac{50}{6} = 8.333$$

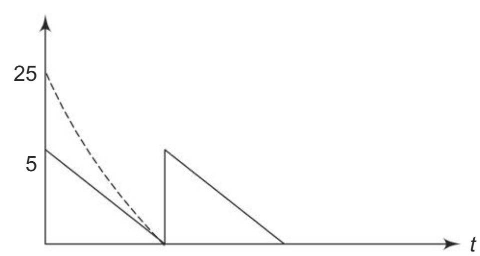


Fig. E.4.2

$$\text{RMS value} = 2.887 \text{ Amps}$$

(iii) **Form factor**

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{2.887}{2.5}$$

$$= 1.155.$$

(iv) **Peak factor**

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}} \quad K_p = \frac{5}{2.887} = 1.732.$$

Solution (b) Unsymmetrical rectangular wave is given.

(i) $$\text{Average value} = \frac{\text{Area under one complete cycle}}{\text{Period}}$$

Area under one cycle = Area of the rectangle from 0 to $T/2$ + area of the rectangle from $T/2$ to T

$$= 100 \times T/2 + (-50 \times T/2)$$

$$= T/2\,(100 - 50) = 25 \times T$$

$$\text{Area under one cycle} = 25\text{ T}$$

$$\text{Average value} = \frac{25\,T}{T} = 25 \text{ volts}$$

(ii) **RMS value** Refer Fig. E.4.3

$$RMS \text{ value} = \sqrt{\frac{\text{Area under squared curve}}{\text{Period}}}$$

$$\text{Area under one squared curve} = 100^2 \times T/2 + (-50)^2 \times T/2$$

$$= 100^2 \times T/2 + 2500\,T/2$$

$$= (10{,}000 + 2{,}500)\,T/2$$

$$= 12500\,T/2$$

$$\text{Area under one squared curve} = 6{,}250\,T$$

$$\text{RMS value} = \sqrt{\frac{6250T}{T}} = \sqrt{6250}$$

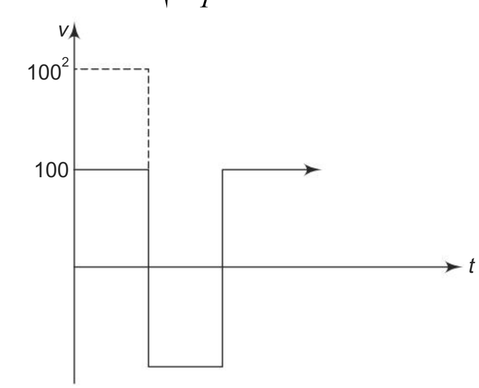


Fig. E.4.3

$$\text{RMS value} = 79.057 \text{ volt}$$

(iii) **Form factor**

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{79.057}{25}$$

$$\text{Form factor} = 3.16$$

$$\text{Peak factor} = \frac{100}{79.057} = 1.26$$

Solution (c) Symmetrical trapezoidal waveform is given

(i) $$\text{Average value} = \frac{\text{Area under one half cycle}}{\text{Period (Base)}}$$

Area under one half cycle = Area of the triangle from 0 to $\pi/3$ + Area of the rectangle for the period $\pi/3$ to $2\pi/3$ + Area of the triangle from $2\pi/3$ to π.

$$= \frac{1}{2} \times V_m \times \frac{\pi}{3} + V_m \times \frac{\pi}{3} + \frac{1}{2} \times V_m \times \frac{\pi}{3}$$

$$= V_m \frac{\pi}{3}\left(\frac{1}{2} + 1 + \frac{1}{2}\right)$$

$$\text{Average value} = \frac{(2/3)\, V_m\, \pi}{\pi} = \frac{2}{3} V_m$$

(ii) **RMS value** Refer Fig. E.4.4

$$\text{RMS value} = \sqrt{\frac{\text{Area under the squared curve}}{\text{Period}}}$$

Area under the squared curve = 1 + 2 + 3

$$= \frac{1}{3} V_m^2 \times \frac{\pi}{3} + V_m^2 \times \frac{\pi}{3} + \frac{1}{3} V_m^2 \frac{\pi}{3}$$

$$= V_m^2 \times \frac{\pi}{3}\left(\frac{1}{3} + 1 + \frac{1}{3}\right) = \frac{5}{9} V_m^2\, \pi$$

$$\text{RMS value} = \sqrt{\frac{5}{9\pi} V_m^2\, \pi}$$

$$\text{RMS value} = 0.745\, V_m \text{ volts}$$

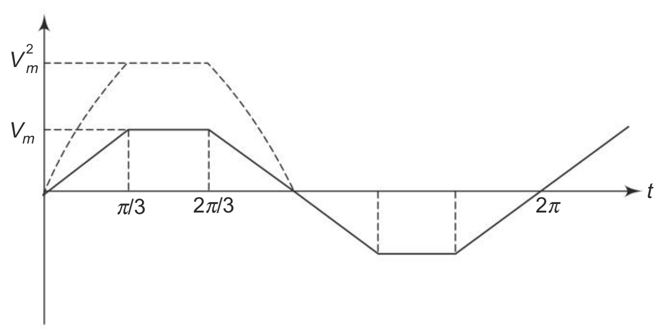


Fig. E.4.4

(iii) **Form factor**

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.745\ V_m}{2/3V_m} = 1.11$$

(iv) $$\text{Peak factor} = \frac{\text{Peak Value}}{\text{RMS Value}} = \frac{V_m}{0.745\ V_m} = 1.342$$

Solution (d) Symmetrical rectangular waveform is given

(i) **Average value**

$$\text{Average value} = \frac{\text{Area under one half cycle}}{\text{Period}}$$

$$= \frac{100 \times T/2}{T/2} = 100\ \text{V}$$

(ii) **RMS value** Refer to Fig. E.4.5

$$\text{RMS value} = \sqrt{\frac{\text{Area under squared curve}}{\text{Period}}}$$

$$= \sqrt{\frac{100^2 \times T/2}{T/2}} = 100\ \text{volts}$$

(iii) **Form factor**

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{100}{100} = 1$$

(iv) **Peak factor**

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}} = \frac{100}{100} = 1$$

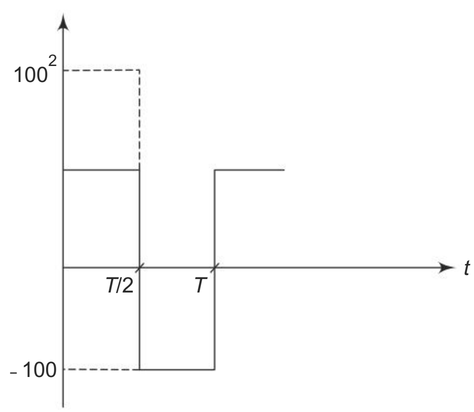


Fig. E.4.5

Example 4.2

Calculate (i) form factor and (ii) peak factor of (a) a half rectified sine wave and (b) full wave rectified sine wave.

A Half Wave Rectified Sine Wave

(i) $$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

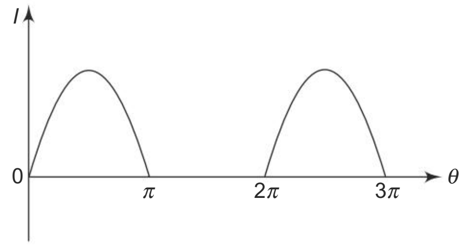


Fig. E.4.6

$$i = I_m \sin\theta \quad \text{for } 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

$$\text{Mean square value} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\theta + \int_{\pi}^{2\pi} \theta d\theta\right]$$
$$= \frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta d\theta$$
$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos^2\theta}{2} d\theta = \frac{I_m^2}{4\pi}\int_0^{\pi} (1-\cos 2\theta)d\theta$$
$$= \frac{I_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$
$$= \frac{I_m^2}{4\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 - \frac{\sin 2\times 0}{2}\right)$$
$$= \frac{I_m^2}{4\pi}\times\pi = \frac{I_m^2}{4}$$
$$\text{RMS value} \sqrt{\frac{I^2m}{4}} = \frac{Im}{2}$$

Average value Half-wave rectified wave is an unsymmetrical wave.

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$
$$\text{Area under one complete cycle} = \int_0^{\pi} I_m \sin\theta\, d\theta + \int_x^{2\pi} 0\, d\theta$$
$$= I_m(-\cos\theta)_0^{\pi}$$
$$= I_m(-\cos\pi + \cos 0) = I_m(1+1) = 2I_m$$
$$\text{Average value} = \frac{2I_m}{2\pi} = I_m/\pi$$
$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{I_{m/2}}{I_{m/\pi}} = \frac{I_m\times\pi}{2\times I_m} = \frac{\pi}{2} = 1.57$$
$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}} = \frac{I_m}{I_{m/2}} = 2$$

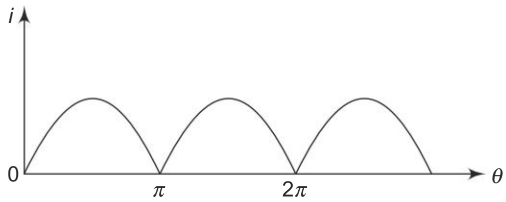


Fig. E.4.7

(b) Full-wave rectified sine wave

$$\text{Mean square value} = \frac{\text{Area under one squared curve}}{\text{Period}}$$

$$\text{Mean square value} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta = \frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left(\pi - \frac{\sin 2r}{2} - 0 + \frac{\sin 0}{2}\right)$$

$$= \frac{I_m^2}{2\pi} \times \pi = \frac{I_m^2}{2}$$

$$\text{RMS value} = \frac{I_m}{\sqrt{2}}$$

Average Value

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta \qquad (\because \text{ the given wave is symmetrical})$$

$$= \frac{I_m}{\pi}(-\cos\theta)_0^{\pi} = \frac{I_m}{\pi}(1+1) = \frac{2I_m}{\pi}$$

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

$$= \frac{I_m/\sqrt{2}}{\dfrac{2I_m}{\pi}} = \frac{I_m \times \pi}{\sqrt{2} \times 2I_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2}$$

Example 4.3 Calculate the root mean square value, form and peak factors of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, – 5, – 10 V, etc. What would be the root mean square value of a sine wave having the same peak value?

Solution: RMS value of the given waveform

$$V = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \ldots + V_{10}^2}{10}}$$

$$= \sqrt{\frac{5^2 + 10^2 + 20^2 + 50^2 + 60^2 + 50^2 + 20^2 + 10^2 + 5^2 + 0^2}{10}}$$

$$= \sqrt{\frac{9650}{10}} = 31.06 \text{ volts}$$

Average Value

$$V_{av} = \frac{1}{10}(5 + 10 + 20 + 50 + 60 + 50 + 20 + 10 + 5 + 0) = 23 \text{ volts}$$

$$\text{Form factor } (K_f) = \frac{V}{V_{av}} = \frac{31.06}{23} = 1.35$$

$$\text{Peak factor } (K_p) = \frac{V_m}{V} = \frac{60}{31.06} = 1.931$$

RMS value of a sinusoidal wave with the same peak value

$$\frac{V_m}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.43 \text{ volts}$$

Example 4.4 An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Also find the time taken to reach 96 A for the first time.

Solution: $f = 60$ Hz; $I_m = 120$ A, $i = 96$ A

$$i = I_m \sin \omega t = 120 \sin (2\pi \times 60) \quad t = 120 \sin (377)\, t$$

$$t = \frac{\sin^{-1}(96/120)}{377} = 0.00246 \text{ sec} \quad t = 2.46 \text{ milli sec.}$$

Example 4.5 A 50 Hz current has a peak amplitude of 100 A. Find the rate of change of current in amperes per second at time t where (a) $t = 0.0025$ sec, (b) $t = 0.005$ and (c) $t = 0.01$ sec after $i = 0$ and is increasing.

Solution: $f = 50$ Hz; $I_m = 100$ A

$$\text{Rate of change of current} = \frac{di}{dt}$$

$$\text{Instantaneous value of current} = I_m \sin \omega t$$

$$i = 100 \sin \omega t$$

$$\frac{di}{dt} = 100\, \omega \cos \omega t$$

where ω = angular frequency = $2\pi f = 2\pi \times 50 = 314$ rad/sec

$$\frac{di}{dt} = 100 \times 314 \cos (314t)$$

$$\frac{di}{dt} = 31{,}400 \cos (314t) \text{A/sec.}$$

(a) $t = 0.0025$ sec

$$\frac{di}{dt} = 31400 \cos (314 \times 0.0025)$$

$= 31400 \cos (45°)$ $(\because 314 \times 0.0025 = 0.785 \text{ rad} = 45°)$.

$= 22203$ A/sec

(b) At $t = 0.005$ sec.

$$\frac{di}{dt} = 31400 \cos (314 \times 0.005)$$

$= 31400 \cos (90°)$ $(\because 314 \times 0.005 = 1.5 \text{ rad} = 90°)$.

$= 0$

(c) At $t = 0.01$ sec after $i = 0$ and is increasing

$$\frac{di}{dt} = 31400 \cos (314 \times 0.01)$$

$= 31400 \cos 180°$ $(\because 314 \times 0.01 = 3.14 \text{ rad} = 180°)$

$= -31400$ A/sec.

Example 4.6 A coil having 200 turns and the area of cross section 250 sq. cm is rotated about its axis perpendicular to that of uniform magnetic field strength 0.5 tesla at a speed of 1200 r.p.m. Determine

(a) Equation for instantaneous induced emf.
(b) Maximum value of induced emf.
(c) Instantaneous values of induced emf in the following cases.
  (i) The plane of the coil is parallel to the field.
  (ii) The plane of the coil is perpendicular to the field
  (iii) The plane of the coil is inclined at angle of 45° to the field.

Solution:

$$N = 200; \quad a = 250 \text{ cm}^2; \quad B_m = 0.5 \text{ tesla}$$

Speed = 1200 r.p.m.

(a) Equation for the instantaneous induced emf is

$$e = E_m \sin \omega t$$

$$\phi_m = B_m \times a = 0.5 \times 250 \times 10^{-4} \text{ wb} = 125 \times 10^{-4} \text{ wb}$$

$$\omega = 2\pi f$$

$$\text{Frequency} = \text{Revolutions per second} = \frac{1200}{60} = 20 \text{ rps}$$

$$\omega = 2 \times \pi \times 20 = 125.67 \text{ rad/sec}$$

Equation for the instantaneous emf

$$e = E_m \sin \omega t$$

$$= 200 \times 125.67 \times 125 \times 10^{-4} \sin (125.67t)$$

$$e = 314 \sin (125.67t)$$

Maximum induced emf $E_m = N\omega\phi_m$

$E_m = 314$ volts

(c) (i) $\theta = \omega t = 90°$, $e = 314 \sin 0$

$e = 314$ volts

(ii) $\theta = \omega t = 0°$ $e = E_m \sin \omega t = 314\ (0°)$

$e = 0$ volt

(iii) $\theta = \omega t = 45°$

$$e = 314 \sin 45$$
$$e = 222 \text{ volts}$$

Example 4.7 Find the root mean value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with peak value of 10 A.

Solution: $I = 10$ A, $I_m = 10$ A

Instantaneous quantity of the sinusoidal current $= I_m \sin \omega t$
$= 10 \sin \omega t$

or $i = 10 \sin \theta \quad (\because \quad \theta = \omega t)$

and,

The resultant current in the wire = DC component + AC component
$= 10 + 10 \sin \theta$ (Refer Fig. E.4.8)

At any instant, the expression for the value of current is given by

$$i = 10 + 10 \sin \theta$$

i.e. $i = 10\,(1 + \sin \theta)$

$$i^2 = 100\,(1 + \sin \theta)^2$$

$$\text{Mean square of the AC} = \int_0^{2\pi} \frac{100(1 + \sin \theta)^2}{2\pi}\, d\theta$$

$$= \frac{100}{2\pi} \int_0^{2\pi} (1 + \sin \theta)^2\, d\theta$$

$$= \frac{100}{2\pi} \int_0^{2\pi} (1 + 2 \sin \theta + \sin 2\theta)\, d\theta$$

$$= \frac{100}{2\pi} \int_0^{2\pi} \left[1 + 2 \sin \theta + \left(\frac{1 - \cos 2\theta}{2}\right)\right] d\theta$$

$$= \frac{100}{2\pi} \left(\theta - 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)_0^{2\pi}$$

$$= \frac{100}{2\pi} \left(2\pi - 2 \cos 2\pi + \frac{2\pi}{2} - \frac{\sin 4\pi}{4}\right) - \left(0 - 2 \cos 0 + 0 - \frac{\sin 0}{4}\right)$$

$$= \frac{100}{2\pi} \left(2\pi - 2 + \frac{2\pi}{2}\right) - (-2)$$

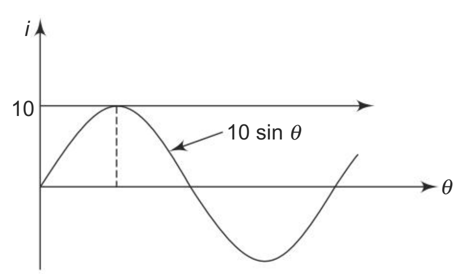


Fig. E.4.8

$$= \frac{100}{2\pi} \cdot 3\pi = 150 \text{ Amps}$$

$$\text{RMS value} = \sqrt{\text{Mean square value}} = \sqrt{150} = 12.25 \text{ A.}$$

Example 4.8 Find the relative heating effects of two current waves of equal peak value one sinusoidal and the other rectangular in waveform.

Solution: Refer Fig. E.4.9

Maximum value of rectangular current wave = Max. value of sinusoidal current wave

Let I_m be the maximum value. Heating effect due to current wave = I^2RT where I is the RMS value.

RMS value of the rectangular wave (I) $= \sqrt{\frac{I_m^2 \times \pi}{\pi}} = I_m$

At any instant, the expression for the sinusoidal current wave $i = I_m \sin\theta$

$$\text{RMS value} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} \, d\theta} = \sqrt{\frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right)} = \sqrt{\frac{I_m^2}{2}}$$

RMS value of sinusoidal current $= \frac{I_m}{\sqrt{2}}$.

Heating effect due to the sinusoidal current wave = I^2RT

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 RT = \frac{I_m^2}{2} RT$$

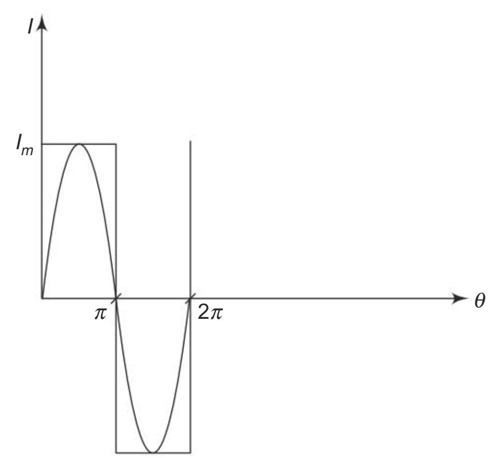


Fig. E.4.9

$$\text{Relative heating effects} = \frac{I_m^2}{2} RT : I_m^2 RT$$

$$= \frac{1}{2} : 1 = 1 : 2$$

Example 4.9 Calculate (i) the maximum value (ii) RMS value of the following quantities (a) 40 sin ωt (b) $(A + B)\sin(\omega t - \pi/2)$, (c) $F = 10 \sin \omega t - 17.3 \cos \omega t$.

Solution: (a) Maximum value = 40

$$\text{R.M.S. value} = \frac{40}{\sqrt{2}} = 28.28 \text{ units}$$

(b) $(A + B)\sin(\omega t - \pi/2)$

$$\text{Maximum value} = A + B$$

$$\text{R.M.S. value} = \frac{A + B}{\sqrt{2}}$$

(c) $F = 10 \sin \omega t + 17.3 \sin(\omega t - 90°)$

$$\text{Maximum value} = 10^2 + 17.3^2 = 19.98 \text{ units.}$$

$$\text{RMS value} = \frac{19.98}{\sqrt{2}} = 14.13 \text{ units.}$$

Example 4.10 An alternating current varying sinusoidally with a frequency of 50 c/s has a RMS value of 20 A. Write down the equation for the instantaneous value and find this value at (a) 0.0025 sec. (b) 0.0125 sec after passing through zero and increasing positively, (c) At what time, measured from zero, will the value of the instantaneous current be 14.14 A.

Solution: $f = 50$ c/s $I = 20$ A

$$I_m = I \times \sqrt{2}$$

(a) $t = 0.0025$ sec.

Equation for the instantaneous value, $i = I_m \sin \omega t$

$$i = 20\sqrt{2} \sin 2\pi f t$$

$$= 20\sqrt{2} \sin 2\pi \times 50 \times 0.0025 = 20 \text{ A}$$

(b) $t = 0.0125$ sec.

$$i = I_m \sin \omega t = 20\sqrt{2} \sin 2\pi \times 50 \times 0.0125 = -20 \text{ A.}$$

$$i = I_m \sin \omega t = 14.14 \text{ A;}$$

$$20\sqrt{2} \sin \omega t = 14.14\ 20\sqrt{2} \sin(2\pi \times 50)t = 14.14$$

$$t = 1.6675 \times 10^{-3} \text{ sec.}$$

Example 4.11 Determine the root mean square value of the resultant current in a wire which carries simultaneously a direct current of 5A and a sinusoidal alternating current of peak value 10A

Solution:

$$I_1 = 5\text{A (d.c)} \quad I_2 = \frac{10}{\sqrt{2}} \text{ (a.c.)}$$

$$\text{RMS value of the resultant current} = I = \sqrt{I_1^2 + I_2^2}$$

$$= \sqrt{10^2 + \left(\frac{10}{\sqrt{2}}\right)^2}$$

$$= 12.25 \text{ A}$$

Example 4.12 An alternating sinusoidal current at 50 Hz has an amplitude of 141.4 A. Find the current and the rate of change of current at instants 0.0025 sec, 0.05 sec and 0.01 sec after the instant at which the current is zero and is increasing positively.

Solution:

$$i = 141.4 \sin \omega t$$

$$\omega = 2\pi f = 2\pi \times 50 = 100\,\pi$$

$$\therefore \quad i = 141.4 \sin 100\,\pi t$$

$$\frac{di}{dt} = 141.4 \times 100\,\pi \cos 100\,\pi t = 14140\,\pi \cos 100\,\pi t$$

(a) At $t = 0.0025$ sec

$$i = 141.4 \sin\ 100\,\pi \times 0.0025 = 100 \text{ A}$$

$$\frac{di}{dt} = 14140\,\pi \cos 100\,\pi \times 0.0025 = 31411 \text{ A/sec}$$

(b) At $t = 0.005$ sec

$$i = 141.4 \sin 100\,\pi \times 0.005 = 141.4 \text{ A}$$

$$\frac{di}{dt} = 14140 \cos 100\,\pi \times 0.005 = 0$$

(c) At $t = 0.01$ sec

$$i = 141.4 \sin 100\,\pi \times 0.01 = 0$$

$$\frac{di}{dt} = 14140 \cos 100\,\pi \times 0.01 = -\,14140 \text{ A/sec}$$

Example 4.13 A moving coil ammeter, a hot wire ammeter and a 60 Ω resistor are in series with a rectifier across a 120 V, 50 Hz sine wave voltage. The resistance of the rectifier is 50 Ω in one direction and 500 Ω in the other direction. Calculate the currents measured by the ammeters, total power and the power dissipated in the rectifier.

Solution: $R = 60\ \Omega$ $\quad V = 120$ V

Rectifier resistance: $R_f = 50\ \Omega$ $\quad f = 50$ Hz

$R_r = 500\ \Omega$

Peak value of applied voltage = $120\sqrt{2} = 169.7$ V

Peak value of current in the forward direction $= \dfrac{169.7}{60 + 50} = 1.54$ A

Peak value of current in the reverse direction $= \dfrac{169.7}{60 + 500} = 0.30$ A

Moving coil ammeter will read the average value.

∴ Reading of moving coil ammeter (over a period of 0 to 2π)

$$= \frac{2 \times 1.54 - 2 \times 0.3}{2\pi} = 0.4 \text{ A}$$

Hot wire ammeter will read the rms value of current.

∴ The mean square of current over a period 0 to 2π

$$= \frac{1}{2\pi}\left[\frac{\pi}{2}\{(1.54)^2 + (0.3)^2\}\right] = 0.615 \text{ A}$$

$\therefore$ RMS value of current measured by the hot wire ammeter = $\sqrt{0.615}$ = 0.784 A

RMS value of current in the forward direction = $\frac{1.54}{\sqrt{2}}$ = 1.09 A

RMS value of current in the reverse direction = $\frac{0.3}{\sqrt{2}}$ = 0.21 A

$\therefore$ Total average power dissipated = $\frac{R_1 I_f^2 + R_2 I_r^2}{2}$

$$= \frac{(600 + 50)\,(1.09)^2 + (60 + 500)\,(0.21)^2}{2}$$

$$= \frac{130.69 + 24.69}{2} = 77.69 \text{ W}$$

Power dissipated in the rectifier = $\frac{R_f I_f^2 + R_r I_r^2}{2} = \frac{50 \times 1.09^2 + 500 \times 0.21^2}{2}$ = 40.7 W

4.6 PHASOR REPRESENTATION OF ALTERNATING QUANTITIES

As shown in Fig. 4.12 (a) consider a phasor $OP = I_m$, where I_m is the maximum value of alternating current. Let this phase rotate in the anticlockwise direction at a speed of ω rad/sec. The waveform of alternating sinusoidal current is shown in Fig. 4.12 (b).

Thus, any alternating sinusoidal quantity can be represented by a rotating phasor, provided the following conditions are satisfied.

(a) The rotating phasor should be equal to the peak value of the quantity.

(b) The rotating phasor should initially start at zero and then move its positive direction, i.e. anticlockwise direction.

(c) The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Generally RMS values are used for phasor representation.

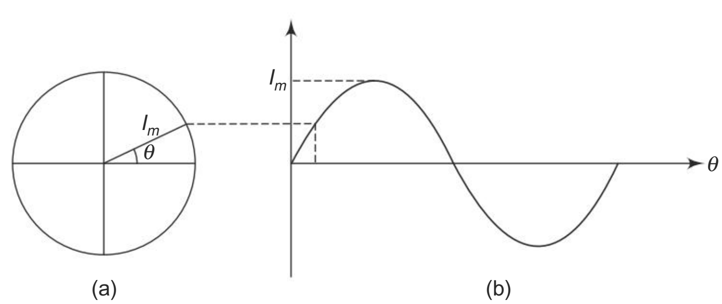


Fig. 4.12

4.7 ANALYSIS OF AC CIRCUIT

The response of electric circuits to alternating current can be studied by passing an alternating current through the basic circuit elements resistor (R), inductor (L) and capacitor (C).

4.7.1 Pure Resistive Circuit

Consider the circuit in Fig. 4.13 in which a resistor of value R ohms is connected across an alternating voltage source.

Let the sinusoidal voltage applied across the resistance be

$$v = V_m \sin \omega t \tag{4.19}$$

The resulting current has an instantaneous value, i

By Ohm's law,

$$v = iR$$

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \tag{4.20}$$

where $I_m = \dfrac{V_m}{R}$ represents the peak value of the circuit current.

Comparing the voltage and the current Eqns. (4.19) and (4.20), we find that the applied voltage and the resisting current are in phase with each other.

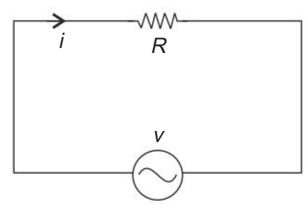


Fig. 4.13

Phasor Representation In a pure resistive circuit, there is no phase difference between the voltage applied and the resulting current, i.e the phase angle $\phi = 0$.

If the voltage is taken as the reference phasor, the phasor representation for voltage and current in a pure resistive circuit is given in Fig. (4.14).

Fig. 4.14

Waveform Representation The waveform for applied voltage and resulting current are shown in Fig. 4.15. Since the current and

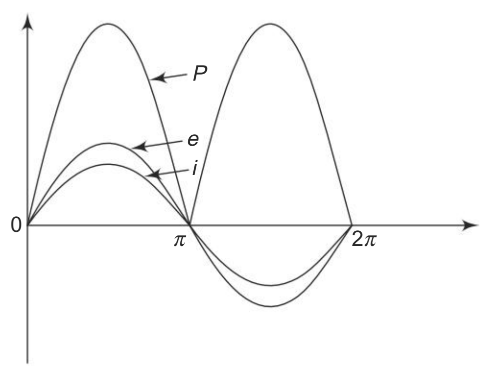


Fig. 4.15

voltage are in phase, the waveform reach their maximum and minimum values at the same instant.

Impedance (Z) It is the ratio of the voltage to current in an a.c. circuit.

$$Z = \frac{v}{i} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m}{V_m/R} = R \tag{4.21}$$

Average Power The instantaneous power (p) is given by

$$p = vi = V_m \sin \omega t \, I_m \sin \omega t \tag{4.22}$$

$$p = V_m I_m \sin^2 \theta \; (\because \omega t = \theta))$$

Average power for one cycle, P

$$= \frac{V_m I_m}{\pi} \int_0^{\pi} \sin^2 \theta \, d\theta$$

$$= \frac{V_m I_m}{\pi} \int_0^{\pi} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{1}{\pi} \frac{V_m I_m}{\pi} \left[\left(\theta - \frac{\sin 2\theta}{2} \right) \right]_0^{\pi}$$

$$= \frac{1}{\pi} \frac{V_m I_m}{\pi} \left(2\pi - 0 - \sin \frac{4\pi}{2} + \sin \frac{0}{2} \right)$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

$$\text{Average power} = VI \text{ watts} \tag{4.23}$$

Power Factor It is the cosine of the phase angle between voltage and current

$$\cos \phi = \cos 0 = 1 \text{ (unity)} \tag{4.24}$$

4.7.2 Pure Inductive Circuit

Consider the circuit of Fig. (4.16). In this circuit, an alternating voltage is applied across a pure inductor of self inductance L Henry.

Let the applied alternating voltage be

$$v = V_m \sin \omega t \tag{4.25}$$

We know that the self induced emf always opposes the applied voltage.

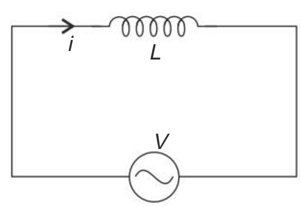


Fig. 4.16

$$v = L \frac{di}{dt} \tag{4.26}$$

$$\therefore \quad i = \frac{1}{L} \int \cdot v \, dt = \frac{1}{L} \int V_m \sin \omega t \, dt = \frac{V_m}{L} \left(\frac{-\cos \omega t}{\omega} \right)$$

$$= -\frac{V_m}{\omega L} \cos \omega t = I_m \sin (\omega t - \pi/2) \tag{4.27}$$

Comparing Eqns. (4.25) and (4.27) we can say that the current through an inductor lags the applied voltage by 90°.

Waveform Representation The current waveform is lagging behind the voltage waveforms by 90°. The waveforms are shown in Fig. 4.17.

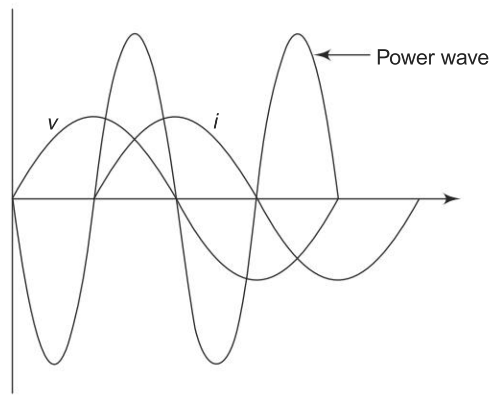


Fig. 4.17

Phasor Representation Taking the voltage phasor as reference, the current phasor is shown to lag the voltage by 90° (Fig. 4.18).

Impedance (Z)

$$Z = \frac{\text{Maximum value of } V}{\text{Maximum value of } I}$$

$$= \frac{V_m}{I_m} = \frac{V_m}{(V_m/\omega L)} = \omega L \tag{4.28}$$

$Z = \omega L$ is called inductive reactance and is denoted by X_L.

$$X_, = \omega L = 2\pi f L \tag{4.29}$$

Power Instantaneous power $P = vi$

$$= V_m \sin \theta I_m \sin (\theta - \pi/2) \tag{4.30}$$

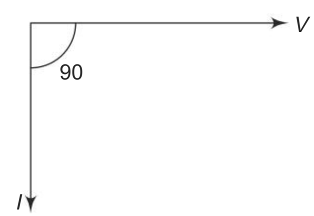


Fig. 4.18

Average Power

$$P = \frac{-1}{\pi} \int_0^{\pi} V_m I_m \sin \theta \cos \theta \, d\theta$$

$$= \frac{-1}{\pi} \int_0^{\pi} \frac{V_m I_m}{2} \sin 2\theta \, d\theta$$

$$= + \frac{V_m I_m}{2\pi} \left(\frac{\cos 2\theta}{2} \right)_0^{\pi}$$

$$= \frac{V_m I_m}{4\pi} (\cos 2\pi - \cos 0) = 0 \tag{4.31}$$

Thus, a pure inductor does not consume any real power.

Power Factor In a pure inductor the phase angle between the current and the voltage phasors is 90°.

i.e. $\phi = 90°; \cos \theta = \cos 90° = 0$

Thus the power factor of a pure inductive circuit is zero lagging.

4.7.3 Pure Capacitive Circuit

Consider the circuit of Fig. 4.19 in which a capacitor of value C Farad is connected across an alternating voltage source.

Let the sinusoidal voltage applied across the capacitance be

$$v = V_m \sin \omega t \tag{4.32}$$

The characteristic equation of a capacitor is

$$V = \frac{1}{C}\int i dt$$

$$i = C\frac{dV}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega c\, V_m \cos \omega t$$

$$i = I_m \cos \omega t \text{ where, } I_m = \omega C \cdot V_m$$

$$i = I_m \sin(\omega t + 90) \tag{4.33}$$

Comparing Eqns (4.32) and (4.33) we find that there is a plane difference of 90° between the voltage and the current in a pure capacitor.

The current in a pure capacitor leads the applied voltage by an angle of 90°.

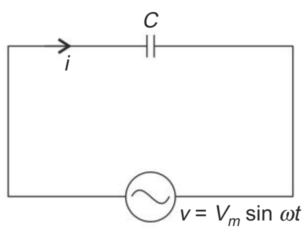


Fig. 4.19

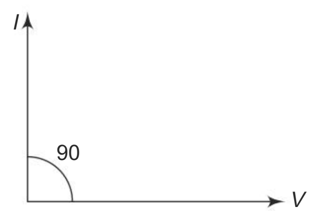


Fig. 4.20

Phasor Representation In the phasor representation, voltage phasor is taken as the reference. The current phasor leads an angle of 90° (Fig. 4.20).

Waveform Representation The current waveform is ahead of the voltage waveform by an angle of 90°. The waveforms are shown in Fig. 4.21.

Impedance (Z)

$$Z = \frac{\text{Maximum value of voltage}}{\text{Maximum value of current}}$$

$$= \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C} = x_c$$

where, x_c is the capacitive reactance

$$x_c = \frac{1}{\omega c} = \frac{1}{2\pi f c} \tag{4.34}$$

Power Instantaneous power is $p = vi$

$$p = V_m \sin \theta\, I_m \cos \theta$$

$$p = V_m I_m \sin \theta \cos \theta \tag{4.35}$$

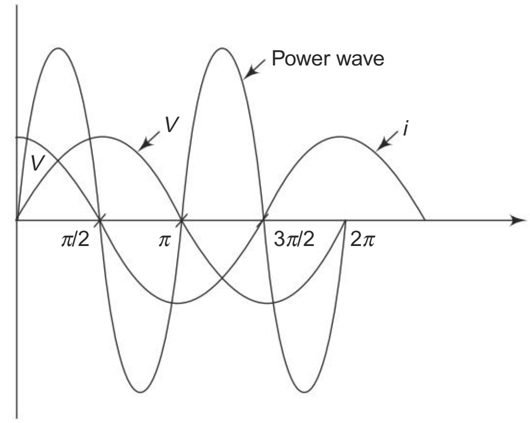


Fig. 4.21

$$\text{Average power} = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos d\theta$$

$$= \frac{V_m I_m}{2\pi}\int_0^{\pi} \sin 2\,\theta\, d\theta = 0$$

Thus, the pure capacitor does not consume any real power.

Power Factor (cos ϕ) In the pure capacitive circuit, the phase angle between the voltage and current is 90° lead.

$$\text{p.f.} = \cos 90 = \text{zero (lead)} \tag{4.36}$$

Example 4.14 A voltage 100 sin ωt is applied to a 10-ohm resistor. Find the current, the instantaneous and the average powers.

Solution:

$$e = 100 \sin \omega t$$

$$R = 10 \text{ ohms}$$

$$i = e/R = 100/10 \sin \omega t = 10 \sin \omega t$$

$$p = ei \quad \therefore\ I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

$$= 100 \sin \omega t \times 10 \sin \omega t = 1000 \sin^2 \omega t$$

$$\text{Average power, } P = EI = \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}} = \frac{1000}{2} = 500 \text{ W}$$

Example 4.15 A voltage $e = 340 \sin 314t$ is applied to a circuit and the resulting current, $I = 42.5 \sin 314t$. Identify and hence find the values of the component. Find the value of power consumed.

Solution:

$$e = 340 \sin 314t$$

$$i = 42.5 \sin 314\, t$$

From the above voltage and current equations, we find that they are in phase with each other. Hence, the basic component connected in the circuit must be resistor.

$$R = e/i = \frac{340 \sin 314t}{42.5 \sin 314t} = 8 \text{ ohms}$$

$$P = EI = \frac{340}{\sqrt{2}} \times \frac{42.5}{\sqrt{2}} = 7225 \text{ watts}$$

Example 4.16 A pure inductance $L = 0.2$ H has an applied voltage of $e = 100 \sin 314t$. Find the instantaneous current, instantaneous and average powers, inductive reactance and the R.M.S. current.

Solution:

$$e = 100 \sin 314t$$

$$L = 0.2 \text{ H}$$

$$i = \frac{1}{L}\int e dt = \frac{1}{0.2}\int 100 \sin 314t$$

$$= -1.592 \cos 314t$$

Instantaneous power $= (100 \sin 314t)(-1.592 \cos 314t) = -79.6 \sin 628\, t$

Average power, $P = 0$

Inductive reactance, $X_L = \omega L = 314 \times 0.2 = 62.8$ ohms

R.M.S. current $I = \dfrac{E}{X_L}$

$$= \frac{100/\sqrt{2}}{62.8} = 1.126 \text{ Amps.}$$

Example 4.17 A coil of wire may be considered as a pure inductance of 0.225 connected to a 120 volt 50 c/s source. Calculate (a) inductive reactance, (b) current, (c) the maximum power delivered to the inductor, (d) the average power, and (e) write the equations of the voltage and current.

Solution:

(a) $L = 0.225$ H $\quad e = 120$ V, $\quad f = 50$ c/s

Inductive reactance $X_L = 2\pi f L = 2\pi \times 50 \times 0.225 = 70.65$ ohms.

(b) Instantaneous current $i = \dfrac{1}{0.225}\displaystyle\int \sqrt{2} \times 120 \sin 314t\, dt$

$$= \frac{\sqrt{2} \times 120}{0.225}\left(\frac{-\cos 314t}{314}\right)$$

$$= \frac{-120 \times \sqrt{2}}{0.225 \times 314} \cos 314\, t = -2.4 \cos 314\, tA.$$

$\therefore$ $I = \dfrac{2.4}{\sqrt{2}} = 1.699$ A.

(c) Maximum power delivered in the inductor $= \dfrac{E_m I_m}{2}$

$$= \frac{120 \times \sqrt{2} \times 2.4}{2}$$

$$= 203.68 \text{ watts.}$$

(d) Average power $= 0$

(e) The equation for voltage $= E_m \sin \omega t$

$= 169.68 \sin 314\, t$

The equation for current, $i = -2.4 \cos 314\, t$

Example 4.18 A pure inductance, $L = 0.01$ H takes a current, $10 \cos 1500\, t$. Calculate (a) the inductive reactance, (b) the equation for the voltage across it, and (c) at what frequency will the inductive reactance be equal to 40 ohms.

Solution:

$L = 0.01$ H, $i = 10 \cos 1500\, t$, $= 1500$ rad/sec

(a) Inductive reactance $X_L = 2\pi f L = 1500 \times 0.01 = 1.5$ ohms.

(b) The equation for the voltage across is

$$e = L\frac{di}{dt}$$

$$= 0.01 \frac{d}{dt}(10 \cos 1500t)$$

$$= -150 \sin 1500t$$

$$= 150 \cos (1500t + 90°)$$

(c) $X_L = 40$ ohms

$2\pi f L = 40$

$2\pi f \times 0.01 = 40$

∴ Frequency $f = \dfrac{40}{2\pi \times 0.01}$

$= 636.619 \text{ Hz} \cong 637$ Hz.

Example 4.19 A 135 μF capacitor has a 150 volts 50 cycle supply. Calculate (a) capacitive reactance, (b) equation for the current, (c) Instantaneous power, (d) average power, (e) RMS current, (f) maximum power delivered to the capacitor.

Solution:

$C = 135\ \mu$F, $E = 150$ V, $f = 50$ c/s

(a) Capacitive reactance $X_c = 1/2\, \pi f c$

$$= \frac{1}{2 \times \pi \times 50 \times 135 \times 10^{-6}} = 23.578 \text{ ohms.}$$

(b) Equation for the current

$$i = \frac{150 \times \sqrt{2}}{28.578} \sin (2\pi f t + \pi/2)$$

$$= 8.99 \sin (314\, t + \pi/2) \text{ A}$$

(c) Instantaneous power, $p = EI \sin 2\, \omega t$

$$= 150 \times \frac{8.99}{\sqrt{2}} \sin 2\, \omega t$$

$p = 953/68 \sin 2\, \omega t$ watts

(d) The average power $P = 0$

(e) The R.M.S. current

$$I = \frac{I_m}{\sqrt{2}} = \frac{8.99}{\sqrt{2}} = 6.35 \text{ Amps.}$$

(f) The maximum power delivered $= 150 \times \sqrt{2} \times 6.35 \times \sqrt{2}$

$= 1905$ watts.

Example 4.20 A potential difference of $100 + 100\sqrt{2}\sin 314t$ is applied to a circuit having a resistance of 5Ω in series with a reactance of 12Ω. Find the following: Impedance, power expended and power factor of the circuit.

Solution:

$$v = 100 + 100\sqrt{2}\sin 314t \text{ Volts}$$
$$\omega = 314$$
$$R = 5\ \Omega$$
$$X = 12\ \Omega$$

Impedance, $Z = R + jX = 5 + j12 = 13\angle 67.4^\circ\ \Omega$

$$I_{dc} = \frac{100}{5} = 20\text{A}$$

RMS current of ac component, $I_{ac} = \dfrac{100\sqrt{2}}{\sqrt{2}} \times \dfrac{1}{3} = 7.69$ A

∴ Total power $= RI_{dc}^2 + RI_{ac}^2$

$$= 5\,(20^2 + 7.69^2) = 2296 \text{ W}$$

Supplied voltage (RMS) $= \sqrt{100^2 + 100^2} = 141.4$ V

Current (RMS) $= \sqrt{20^2 + 7.69^2} = 21.4$ A

∴ Circuit impedance $= \dfrac{141.4}{21.4} = 6.6\ \Omega.$

Power factor $= \dfrac{\text{Power expended}}{VI} = \dfrac{2296}{141.4 \times 21.4} = 0.759$

Example 4.21 Two single phase alternators supply 300 A and 350 A respectively to a common load at a phase difference of 30°. Find the resultant current and its phase relationship to its components.

Solution:

$$\bar{I}_1 = 300\angle 0^\circ \text{ A}$$
$$\bar{I}_2 = 350\angle 30^\circ \text{ A}$$

Resultant current supplied is

$$\bar{I}_1 = \bar{I}_1 + \bar{I}_2 = 300\angle 0^\circ + 350\angle 30^\circ$$
$$= 300 + j0 + 303 + j175$$
$$= 603 + j175 = 627.9\angle 16^\circ \text{ A}$$

Thus, the resultant current is 627.9 A and it leads 300 A by 16°.

Example 4.22 A voltage $v = 230\sin 100\,\pi t$ is applied to a coil having resistance 100 Ω and inductance 319 mH. Find the expression for the current and the power taken by the coil.

Solution:

$$v = 230\sin 100\,\pi t$$
$$R = 100\ \Omega$$
$$L = 319 \text{ mH}$$
$$X_L = \omega L = 100\,\pi \times 319 \times 10^{-3} = 100\ \Omega$$

Impedance, $= R + jX_L = 100 + j100 = 141.4\angle 45°$

$\therefore$ Expression for the current is

$$i = \frac{\text{Im}\sin(\omega t - \phi)}{Z} = \frac{230}{141.4}\sin(100\,\pi t - 45°)$$

$$= 1.626\sin(100\,\pi t - 45°)$$

Power taken by the coil $= (\text{RMS current})^2 \times R$

$$= \left(\frac{1.626}{\sqrt{2}}\right)^2 \times 100 = 132.2\text{ W}$$

Example 4.23 Two sources of emfs having values $e_1 = 230\sin\omega t$ volts and $e_2 = 230\sin\left(\omega t\ \frac{\pi}{6}\right)$ volts respectively are in series. What is the resultant voltage? Express it as a vector with reference to e_1. Calculate the RMS current and the power supplied to a circuit of impedance (8 + j6) Ω.

Solution:

$$E_1 = 230\angle 0° + 230 + j0$$

$$E_1 = 230\angle 30° + 200 + j115$$

$\therefore$ Resultant voltage (peak value), $E = E_1 + E_2 = 230 + jo + 200 + j115$

$$= 430 + j115 = 445\angle 15°$$

$\therefore$ Resultant voltage (RMS value) $= \frac{445}{\sqrt{2}}\angle 15° = 315\angle 15°$

$$\text{Current} = \frac{315}{Z} = \frac{315\angle 15°}{8 + j6} = \frac{315\angle 15°}{10\angle 36.86°} = 31.5\angle -21.86°$$

$\therefore$ Power supplied $= 315 \times 31.5 \times \cos(-21.86°)$

$$= 9209\text{ W}$$

Example 4.24 A sinusoidal voltage has two components $e_1 = 230\sin 100\,\pi t$ and $e_2 = 35\sin 700\,\pi t$. It is applied across a capacitor 20 μF. Calculate the effective value of current due to each component of voltage.

Solution:

$$e_1 = 230\sin 100\,\pi t \qquad C = 20 \times 10^{-6}\text{ F}$$

$$e_2 = 230\sin 700\,\pi t$$

Current due to component e_1 is

$$I_1 = \frac{I_1m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}\,X_C} = \frac{V_m\,\omega t}{\sqrt{2}}$$

$$= \frac{230 \times 100\,\pi \times 20 \times 10^{-6}}{\sqrt{2}} = 1.02\text{ A}$$

Current due to component e_2 is

$$I_2 = \frac{I_2m}{\sqrt{2}} = \frac{35 \times 700\pi \times 20 \times 10^{-6}}{\sqrt{2}} = 1.09\text{ A}$$

Example 4.25 A voltage $v = 230 \sin 314t + 60 \sin 942t$ is applied to a circuit having a resistor, an inductor and a capacitor connected in parallel. The values are: 10Ω, 10Ω with 0.03H and 100 μF. Determine the RMS value of the current in each branch, the total current, the power input and the power factor.

Solution:

$$v = 230 \sin 314t + 60 \sin 942t$$

(fundamental) (third harmonic)

R = 10Ω, inductor resistance = 10Ω, L = 0.03H

$C = 100 \times 10^{-6}$ F

Branch with Resistor (R)

Fundamental current $I_1m = \dfrac{230}{10} = 23$ A $\quad \therefore \quad I_1 = \dfrac{23}{\sqrt{2}} = 16.26$ A

Third harmonic component $I_3m = \dfrac{60}{10} = 6$ A $\quad \therefore \quad I_3 = \dfrac{6}{\sqrt{2}} = 4.24$ A

$\therefore$ RMS current $= \sqrt{16.26^2 + 4.24^2} = 16.8$ A

Power, $P_R = (16.8)^2 \times 10 = 2822$ W

Branch with inductor (L)

Impedance to fundamental component $= 10 + j314 \times 0.03$

$$= 10 + j\,9.42\ \Omega$$

$$= 13.74 \angle 43.29°\ \Omega$$

$\therefore$ Fundamental current, $I_1m = \dfrac{230}{13.74} = 16.74$ A

$\therefore$ $I_1 = \dfrac{16.74}{\sqrt{2}} = 11.84$ A (rms)

Third harmonic component of current, $I_{3m} = \dfrac{60}{13.74} = 4.367$ A

$\therefore$ $I_1 = \dfrac{4.367}{\sqrt{2}} = 3.09$ A

$\therefore$ Total RMS current in the inductor is

$$I_L = \sqrt{11.84^2 + 3.09^2} = 12.24 \text{ A}$$

Power, $P_L = I_L^2 \times R_L = (12.24)^2 \times 10 = 1498$ W

Branch with Capacitor

Reactance to fundamental component $= \dfrac{1}{314 \times 100 \times 10^{-6}} = 31.85\ \Omega$

$\therefore$ $I_{1m} = \dfrac{230}{31.85} = 7.22$ A

$I_1 = \dfrac{7.22}{\sqrt{2}} = 5.1$ A

Reactance to third harmonic component $= \dfrac{1}{942 \times 100 \times 10^{-6}} = 10.61\ \Omega$

$\therefore$ $I_{3m} = \dfrac{60}{10.61} = 5.66$A

$$I_3 = \frac{5.66}{\sqrt{2}} = 4.0 \text{ A}$$

Total RMS current in the capacitor is

$$I_c = \sqrt{5.1^2 + 4.0^2} = 6.48 \text{ A}$$

Power in the capacitor = 0

Total power dissipated = $P_R + P_L + P_C$

$$= 2822 + 1498 + 0 = 4320 \text{ W}$$

Calculation of the Total Current

Fundamental component = $\bar{I}_{R1} + \bar{I}_{L1} + \bar{I}_{C1}$

$$= 16.26\angle 0° + 11.84\angle 43.29° + 5.1\angle 90°$$

$$= 16.26 + j\,0 + 8.62 - j\,8.12 + 0 + j\,5.1$$

$$= 24.88 - j\,3.02 = 25.06\angle 6.9° \text{ A}$$

Third harmonic component = $\bar{I}_{R3} + \bar{I}_{L3} + \bar{I}_{C3}$

$$= 4.24\angle 0° + 3.09\angle -43.29° + 4\angle 90°$$

$$= 4.24 + j0 + 2.25 - j2.12 + 0 + j4$$

$$= 6.49 + j1.88 = 6.76\angle 16.16° \text{ A}$$

Total current (RMS value) = $\sqrt{25.06^2 + 6.76^2} = 25.96\text{A}$

Voltage applied (RMS value) = $\sqrt{\left(\frac{230}{\sqrt{2}}\right)^2 + \left(\frac{60}{\sqrt{2}}\right)^2} = 168 \text{ V}$

$$\text{Power factor} = \frac{\text{Power}}{V_1} = \frac{4320}{168 \times 25.96} = 0.99$$

IMPORTANT FORMULAE

1. Instantaneous emf (e) = emf E_m sin ωt
2. Root mean square value

   $= \sqrt{\text{Mean of the sum of the squares of the component}}$
3. (a) For an unsymmetrical wave

   $$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

   (b) For a symmetrical wave

   $$\text{Average value} = \frac{\text{Area under the curve for one half cycle}}{\text{Period for half cycle.}}$$
4. Form Factor = $\frac{\text{RMS value}}{\text{Average value}}$

5. Peak factor = $\dfrac{\text{Peak value}}{\text{RMS value}}$
6. Table showing the impedance and their phase angle

Table 4.1

Circuit element	*Impedance Z*	*Phase angle* ϕ	*Power factor cos* ϕ
Resistance (R)	R	0°	1
Inductance (L)	X_L	– 90°	zero lagging
Capacitance (C)	X_c	90°	zero leading

REVIEW QUESTIONS

1. Derive an expression for the alternating sinusoidal emf.
2. Define RMS value and obtain the same for a sinusoidal voltage wave.
3. Define average value and obtain the same for a half wave rectified voltage wave.
4. (i) Define (a) frequency, (b) phase, (c) form factor, and (d) peak factor
   (ii) Explain the term phase difference.
5. Derive expressions for voltage, current and power in a resistor supplied with an alternating sinusoidal voltage.
6. Derive expressions for voltage, current and power in an inductor supplied with sinusoidal voltage.
7. Derive expressions for voltage, current and power in a capacitor with a sinusoidal voltage.

PROBLEMS

1. Find the form factor and the peak factor for the waveform given in Fig. P.4.1

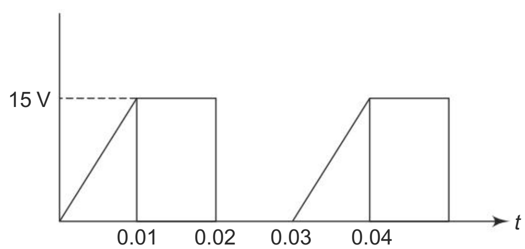


Fig. P.4.1

2. Find effective and average values for the following waveforms of Fig. P.4.2
3. A voltage given by $e = 240 \sin 377\, t$ is applied to a circuit and the circuit current found to be $i = 40 \sin 377\, t$. Identify the circuit element and determine its value. A calculate the power consumed, if any.
4. A voltage $e = 200 \sin \omega t$ when applied to a resistant is found to give a power 100 watts. Find the value of resistance and the equation for current.

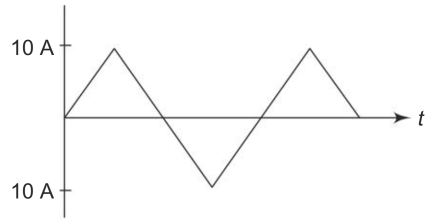


Fig. P.4.2

5. A pure inductance of 254.63 mH is connected to 120V, 50 Hz source. Calculate inductive reactance, the current, the maximum power delivered and the average power. Write the equation for the voltage and the current.
6. An alternating current is defined by the equation $i = 230 \sin\left(314t + \frac{\pi}{4}\right)$. Determine the following: peak value, average value, RMS value, frequency and time period.
7. Three alternating sinusoidal voltages, acting in series, are impressed on a circuit. They are: $v_1 = 115 \sin 314t$, $v_2 = 120 \sin (314t - 30°)$, $v_3 = 230 \sin (314t - 90°)$. Calculate for the resultant voltage (a) expression for the instantaneous value (b) frequency (c) RMS value.
8. An alternating voltage is defined by $v = 220 \sin 377t$. It is applied to a circuit having a resistance of 22 Ω. Determine (a) the RMS value (b) the frequency (c) the power loss.
9. An emf is given by $v = 415 \sin 346t$. When it is applied to a circuit, the resulting current is found to be $i = 20.75 \sin (346t - 60°)$. Determine (a) RMS values of voltage and current (b) frequency (c) phase difference between voltage and current (d) impedance (e) power factor.

ANSWERS TO PROBLEMS

1. 1.334, 1.499
2. 5.77, 5
3. 6 ohms, 4800 W
4. 200 ohms, sin ωt
5. 80 ohms, 1.5 A,\ 360 W, 0, V = 169 sin314t, $i = 2.1 \sin (314t - \pi/2)$.
6. 230A, 146.5A, 162.6A, 50Hz, 0.02 sec
7. 363.4 (sin 314t –52.6°), 50Hz, 257 V
8. 155.6V, 60Hz, 1100 W
9. 293.4V, 14.67A, 55Hz, 60°(lag), 20Ω, 0.5(lag)

SINGLE PHASE AC CIRCUITS

5

INTRODUCTION

In the last chapter we had seen what happens when an ac voltage is applied independently to the following circuit elements.

(i) Resistance
(ii) Inductance
(iii) Capacitance

We can connect all these three elements in series and/or parallel. Such circuits can be solved by the application of Kirchhoff's Laws but the difference is that here we take into account the magnitude and the phase of voltages and currents. That is the phasor sum or difference is computed. In this chapter we shall examine the performance of different circuits having combinations of *R*, *L* and *C*.

5.1 J OPERATOR

Alternating voltage is a phasor quantity having both magnitude and phase. When such a voltage is applied to a circuit, the resulting current is also a complex quantity. The property of the circuit obtained as the ratio of voltage to current is again a complex quantity. Thus the analysis of ac circuits involves complex quantities which requires a knowledge of the properties of complex variables. For this a complex operator j is used. The symbol j is assigned a value of $\sqrt{-1}$. Any quantity multiplied by j means that the quantity is rotated through 90° in the counter-clockwise direction.

When the operator j is operated on vector *OA,* we get the new vector *OB,* which is displaced by 90° in counter-clockwise direction from *OA* (Fig. 5.1).

$$\overline{OB} = j\,\overline{OA}$$

Similarly,

Operation of j twice on $\overline{OA}$ gives $\overline{OC} = j^2\,\overline{OA}$

Operation of j thrice on $\overline{OA}$ gives $\overline{OD} = j^3\,\overline{OA}$

Operation of j four times on $\overline{OA}$ brings $\overline{OA} = j^4\,\overline{OA}$ back to its original position after rotating it through 360°.

Hence, it is seen that successive applications of the operator j to the vector O produce successive 90° steps of rotation of the vector in the

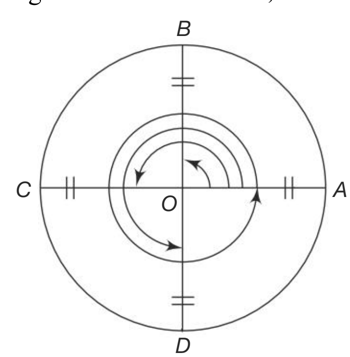


Fig. 5.1

counter clockwise direction. It can be noted that the magnitude of the vector is not altered in any way.

Properties of j Operator

$$j = 90° \text{ counter-clockwise rotation} = \sqrt{-1}$$
$$j^2 = 180° \text{ counter-clockwise rotation} = (\sqrt{-1})^2 = -1$$
$$j^3 = 270° \text{ counter-clockwise rotation} = (\sqrt{-1})^3 = -(\sqrt{-1}) = -j$$
$$j^4 = 360° \text{ counter-clockwise rotation} = (\sqrt{-1})^4 = +1$$

It is important to note that

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

5.2 COMPLEX ALGEBRA

A vector can be specified in terms of its x- and y-components.

For example, $\overline{OP_1}$ (Fig. 5.2) can be completely described by writing that its horizontal component is a_1 and vertical component is b_1. But instead of stating this in words, it can be expressed symbolically as $\overline{P}_1 = a_1 + j\,b_1$, where j is an operator indicating that (i) component b_1 is perpendicular to component a_1 (ii) the two terms are not to be treated like terms in any algebraic expression. The vector written in this way is said to be in "complex form". In mathematics, a_1 is known as real component and b_1 as imaginary component. But in electrical engineering, these are known as in phase (or active) and quadrature (or reactive) components respectively.

The numerical value of vector P_1 is $\sqrt{a_1^2 + b_1^2}$.

Its angle with x-axis is given by $\theta = \tan^{-1}\left(\dfrac{b_1}{a_1}\right)$.

Similarly other vectors $\overline{P}_2$, $\overline{P}_3$ and $\overline{P}_4$ can be expressed in complex form as follows:

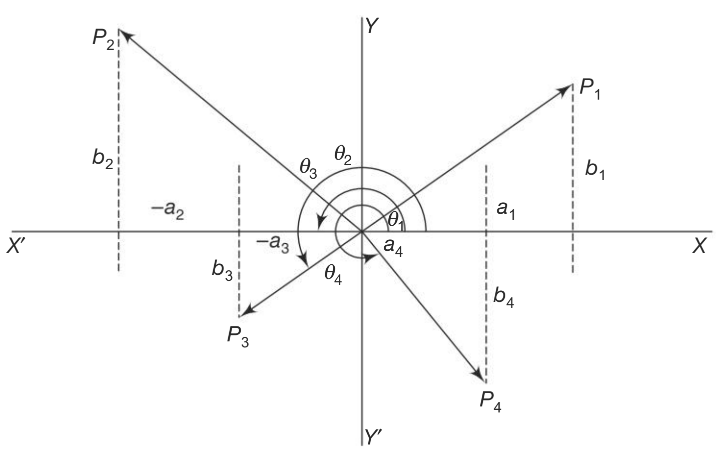


Fig. 5.2

$$\overline{P}_2 = -a_2 + jb_2$$
$$\overline{P}_3 = -a_3 - jb_3$$
$$\overline{P}_4 = a_4 - jb_4$$

Complex Conjugate The conjugate of a complex number differs only in the algebraic sign of imaginary component. Therefore $a + jb$ and $a - jb$ are complex conjugates.

5.2.1 Addition and Subtraction of Complex Quantities

Let us consider two vector quantities $\overline{P}_1 = a_1 + jb_1$ and $\overline{P}_2 = a_2 + jb_2$. It is required to find their sum and difference.

Addition Let the resultant vector be $\overline{P}$

The magnitude and angle of the above vectors given in Table 5.1.

Table 5.1

Vector	*Magnitude*	*Included angle*	*Angle with reference to positive x axis*
$\overline{P}_2 = -a_2 + jb_2$	$\sqrt{a_2^2 + b_2^2}$	$\tan^{-1}\left(\frac{b_2}{a_2}\right)$	$\theta_2 = 180° - \tan^{-1}\left(\frac{b_2}{a_2}\right)$
$\overline{P}_3 = -a_3 - jb_3$	$\sqrt{a_3^2 + b_3^2}$	$\tan^{-1}\left(\frac{b_3}{a_3}\right)$	$\theta_3 = 180° + \tan^{-1}\left(\frac{b_3}{a_3}\right)$
$\overline{P}_4 = -a_4 - jb_4$	$\sqrt{a_4^2 + b_4^2}$	$\tan^{-1}\left(\frac{b_4}{a_4}\right)$	$\theta_4 = -\tan^{-1}\left(\frac{b_4}{a_4}\right)$ or $360° - \tan{-1}\left(\frac{b_4}{a_4}\right)$

$$\overline{P} = \overline{P}_1 + \overline{P}_2$$
$$= a_1 + jb_1 + a_2 + jb_2$$
$$= (a_1 + a_2) + j(b_1 + b_2)$$

The magnitude of vector P

$$= \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$$

Angle of vector P with x-axis

$$= \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$$

Subtraction

$$\overline{Q} = \overline{P}_1 - \overline{P}_2 = a_1 + jb_1 - (a_2 + jb_2) = a_1 + jb_1 - a_2 - jb_2$$
$$= (a_1 - a_2) + j(b_1 - b_2)$$

Magnitude of vector $Q = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Angle of vector Q with x-axis $= \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right)$

5.2.2 Multiplication and Division of Complex Quantities

Let us find the multiplication and division of two vectors $\overline{P}_1$ and $\overline{P}_2$

Multiplication

$$\overline{R} = \overline{P}_1 \times \overline{P}_2 = (a_1 + jb_1)\,(a_2 + jb_2) = a_1a_2 + jb_1a_2 + jb_2a_1 + j^2b_1b_2$$
$$= (a_1a_2 - b_1b_2) + j\,(a_1b_2 + b_1a_2) \quad \therefore j^2 = -1$$

Magnitude of $\overline{R} = \sqrt{(a_1a_2 - b_1b_2)^2 + (a_1b_2 + b_1a_2)^2}$

Its angle with respect to x-axis $= \tan^{-1}\left(\dfrac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$

Division

$$\overline{S} = \frac{\overline{P}_1}{P_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)}{(a_2 + jb_2)} \times \frac{(a_2 - jb_2)}{(a_2 - jb_2)}$$
$$= \frac{a_1a_2 + jb_1a_2 - ja_1b_2 - j^2b_1b_2}{a_2^2 + b_2^2}$$
$$= \frac{a_1a_2 + b_1b_2 + jb_1a_2 - ja_1b_2}{a_2^2 + b_2^2}$$
$$\overline{S} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + j\frac{(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$$

Magnitude and phase angle can be found in the usual way.

5.3 REPRESENTATION OF ALTERNATING QUANTITIES IN RECTANGULAR AND POLAR FORMS

The phasors such as alternating quantities can be represented in 1. Rectangular and 2. Polar form.

(1) Rectangular Form

From Fig. 5.3 in $\Delta\, OAR$

$$\cos\theta = \frac{OA}{OR} \Rightarrow OA = R \times \cos\theta$$
$$\sin\theta = \frac{RA}{OR} \Rightarrow RA = R \times \sin\theta$$

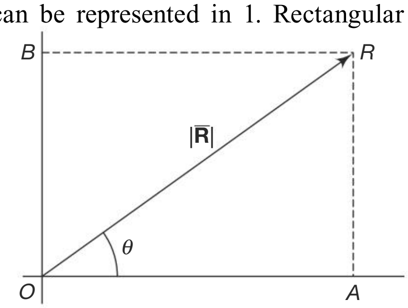


Fig. 5.3

It is seen that the x-component of $\overline{R}$ is $R \cos\theta$ and y-component of $\overline{R}$ is R s

Thus the phasor R can be represented as $\overline{R} = R\cos\theta + jR\sin\theta$.

This is equivalent to the form $\overline{R} = a + jb$ and it is called rectangular for. In general, the rectangular form is $\overline{R} = a \pm jb$.

(ii) Polar Form In Fig. 5.3 $/\overline{R}/$ represents the magnitude of vector. θ represents its inclination counter-clockwise direction) with x-axis.

Hence, the expression for $\overline{R}$ can be written in the simplified form $|\overline{R}|$ where $|\overline{R}| = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1}(b/a)$. For angles in clockwise direction, expression becomes $|\overline{R}|$ $\angle -\theta$.

In general, the expression is written as $/R/ \angle \pm \theta$. This is called polar form. This is simply a short hand or symbolic style of writing $Re^{\pm j\theta}$. Thus, we easily transform polar form to rectangular form and vice versa.

Multiplication of two complex quantities is $A \angle\theta \times B \angle\phi = AB \angle\theta$

Division of two complex quantities is $A \angle\theta \div B \angle\phi = \dfrac{A}{B} \angle\theta - \phi$.

Example 5.1 Express the following complex numbers in polar form

(i) $7 - j5$ (ii) $-9 + j6$ (iii) $-8 - j8$ (iv) $6 + j6$

Solution By using $R \to P$ conversion in the calculation

(i) $7 - j5 = 8.602 \angle -35.54°$

(ii) $-9 + j6 = 10.82 \angle 146.3°$

(iii) $-8 - j8 = 11.31 \angle -135 = 11.31 \angle 225$

(iv) $6 + j6 = 8.49 \angle 45°$

Example 5.2 Do the following operations and write the result in polar form.

(i) $10 \angle 60° + 8 \angle -45°$

(ii) $(5 + j4) \times (-4 - j6)$

(iii) $(-2 - j5) \div (5 + j7)$

Solution

(i) $10 \angle 60° + 8 \angle -45°$

For addition and subtraction, rectangular form is more convenient. So, first let us convert the complex numbers into rectangular form.

$$10 \angle 60° = 5 + j8.66$$
$$8 \angle -45° = 5.66 - j5.66$$
$$10 \angle 60° + 8 \angle -45 = 5 + j8.66 + 5.66 - j5.66$$
$$= 10.66 + j3$$
$$10.66 + j3 = 11.07 \angle 15.72°$$

$\therefore$ In polar form.

$$10 \angle 60° + 8 \angle -45° = 11.07 \angle 15.72°$$

(ii) $(5 + j4) \times (-4 - j6)$

For multiplication and division polar form is more convenient. So, first let us convert the complex numbers into polar form.

$$5 + j4 = 6.40 \angle 38.66°$$
$$-4 - j6 = 7.211 \angle -123.69°$$
$$(5 + j4) \times (-4 - j6) = 6.4 \times 7.211 \angle 38.66° - \angle 123.69°$$
$$= 46.15 \angle -85.03°$$

(iii) $(-2 - j5) + (5 + j7)$; $-2 - j5 = 5.39 \angle -111.8°$

$$5 + j7 = 8.6 \angle 54.46°$$
$$(-2 - j5) + (5 + j7) = \frac{5.39 \angle -111.8°}{8.6 \angle 54.46°}$$
$$= 0.627 \angle -166.26°$$

5.4 R–L SERIES CIRCUIT

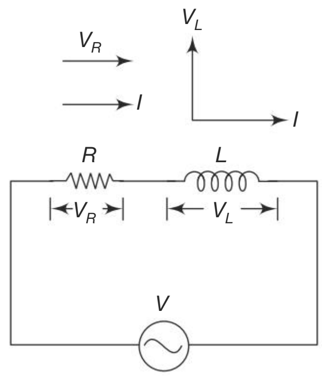


Fig. 5.4

Let us consider a circuit in which a pure resistance R ohms and a purely inductive coil of inductance L henries are in series (Fig. 5.4).

Let $v = V_m \sin \omega t$ be the applied voltage

i = Circuit current at any instant

I = Effective value of circuit current

V_R = Potential difference across resistor

V_L = Potential difference across inductor

f = Frequency of applied voltage

The same current I flows through R and L. Hence I is taken as reference vector.

Voltage across $R = \overline{V}_R = I\,R$ in phase with I.

Voltage across $L = \overline{V}_L = IX_L$ leading I by 90°

At any instant, applied voltage

$$V = V_R + V_L \quad \text{(refer Fig. 5.5)} \tag{5.1}$$

Applied voltage $$V = IR + jIX_L = \overline{I}\,(R + jX_L) \tag{5.2}$$

$$V/I = R + jX_L = \overline{Z} = \text{impedance of circuit (Fig. 5.6).}$$

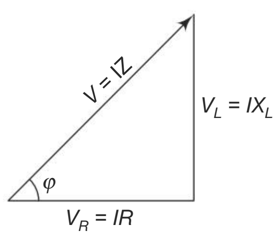


Fig. 5.5

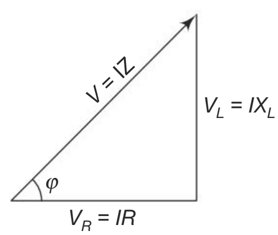


Fig. 5.6

Magnitude of $$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{\left((IR)^2 + IX_L\right)^2}$$

$$= \sqrt{I^2\left(R^2 + X_L^2\right)}$$

$$= I\sqrt{R^2 + X_L^2} \tag{5.3}$$

Current $$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$\frac{V}{I} = \sqrt{R^2 + X_L^2} = \text{Magnitude of impedance of the circuit } Z$$

$$|\overline{Z}| = \sqrt{R^2 + X_L^2} \tag{5.4}$$

$$\text{Impedance}^2 = \text{Resistance}^2 + \text{Reactance}^2 \tag{5.5}$$

From the voltage triangle of Fig. 5.5, if common parameter I is removed, the same becomes an impedance triangle, shown in Fig. 5.6. From the $\Delta\, ABC$,

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}} \tag{5.6}$$

$$\therefore \qquad \phi = \tan^{-1}\frac{X_L}{R} \tag{5.7}$$

ϕ is called the phase angle and it is the angle between V and I. Its value lies between 0 and 90° and $Z = R + jX_L$. The real part R is resistance and the imaginary part X_L reactance.

$$Z = R + jX_L = Z\angle\phi$$

Power Factor As defined in Section 4.8, p.f. = cos ϕ.

From the $\Delta\, ABC$, shown in Fig. 5.6,

$$\cos\phi = \frac{R}{Z} \tag{5.8}$$

Referring to Fig. 5.5, the current I lags the total voltage V. So, power factor of an R–L circuit is lagging. Also.

$$\cos\phi = \cos\left[\tan^{-1}\left(\frac{X_L}{R}\right)\right] \tag{5.9}$$

Power Calculation The principal current I can be resolved into two components.

(i) A component I_a in phase with voltage. This is called the active or real or wattful component.

$$I_a = I\cos\phi \tag{5.10}$$

(ii) A component I_r at right angles to V

This component is called the reactive or quadrature or wattiess component.

$$I_r = I\sin\phi \tag{5.11}$$

$$I = \sqrt{I_a^2 + I_r^2} \tag{5.12}$$

Actual Power (P) There is a real power consumption in any circuit when a current component is phase with voltage. It is measured in watts.

Active or real power $\quad P = VI_a$

$$P = VI\cos\phi = VI \times \frac{R}{Z} \quad \text{(Fig. 5.6)}$$

$$= \frac{V}{Z} \times I \times R = I^2R \tag{5.13}$$

Thus, it can be inferred that the actual power consumption is dependent on the power factor of the circuit.

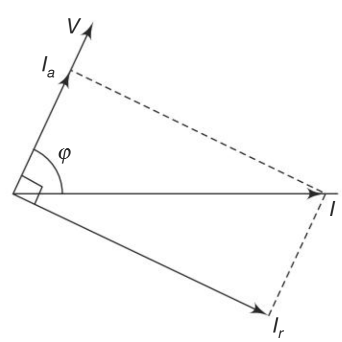


Fig. 5.7

Reactive or Quadrature Power (Q) There is a reactive power consumption in any circuit when a current component is in quadrature with voltage. It is measured in Volt Ampere Reactive.

Reactive Power = $V \times$ quadrature component of current

$$Q = VI \sin \phi \tag{5.14}$$

$$= VI \frac{X_L}{Z} = I^2 X_L$$

The unit of reactive power is VAR. (Reactive volt. amp).

Complex or Apparent Power (S) It is calculated as the product of voltage and current. It is measured in volt ampere.

Complex or Apparent power $S = V \times I = I^2 Z$

$$= \frac{V^2}{Z}$$

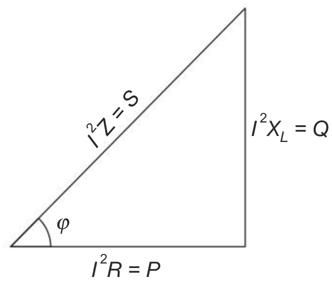


Fig. 5.8 Power triangle

All the above power components are shown in Fig. 5.8.

From Fig. 5.8

$$\overline{S} = P + jQ \tag{5.16}$$

Magnitude of $$S = \sqrt{P^2 + Q^2} \tag{5.17}$$

Waveform This is shown in Fig. 5.9. The current I lags behind the applied voltage V by an angle ϕ. Hence if the applied voltage is given as

$$v = V_m \sin \omega t \tag{5.18}$$

then the current equation is

$$i = I_m \sin (\omega t - \phi) \tag{5.19}$$

where $$I_m = \frac{V_m}{Z}$$

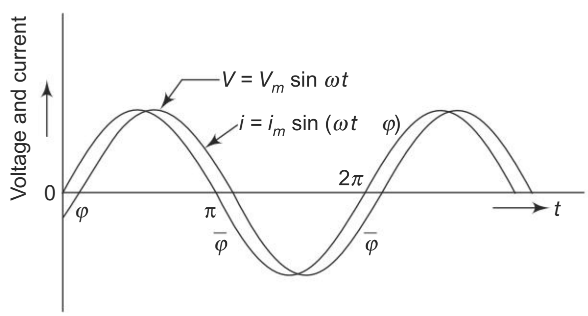


Fig. 5.9

Example 5.3 In a series circuit containing pure resistance and pure inductance, the current and voltage expressed as $i\,(t) = 5 \sin (314\, t + 2\,\pi/3)$ and $V\,(t) = 20 \sin (314\, t + 5\,\pi/6)$.

(i) What is the impedance of the circuit?

(ii) What are the values of resistance, inductance and power factor?

(iii) What is the average power drawn by the circuit?

Solution

$$i(t) = 5 \sin (314t + 2\ \pi/3)$$
$$v(t) = 20 \sin (314t + 5\ \pi/6)$$

$$\text{Phase angle of current} = \frac{2\pi}{3} \text{ radians} = 2 \times \frac{180^\circ}{3} = 120^\circ$$

$$\text{Phase angle of voltage} = \frac{5\pi}{6} \text{ radians} = 5 \times \frac{180^\circ}{6} = 150^\circ$$

∴ Current lags the voltage by 150 −120° = 30°.

Lagging p.f. means that it is an R–L circuit.

$$\therefore \quad \text{p. f.} = \cos 30^\circ = 0.866 \text{ (lagging)}$$

$$\text{Now impedance} \quad Z = \frac{V_m}{I_m} = \frac{20}{5} = 4 \text{ ohms}$$

$$\cos \phi = \frac{R}{Z} \Rightarrow R = Z \cos \phi = 4 \times \cos 30^\circ$$

Value of resistance = 3.46 Ω

$$Z = \sqrt{R^2 + X_L^2}$$

$$\therefore \quad X_L = \sqrt{Z^2 - R^2} = \sqrt{4^2 - 3.46^2} = 2\ \Omega$$

$$X_L = \omega L = 314 \times L$$

$$\left[\because v(t) = 15 \sin \left(314t + \frac{5\pi}{6}\right). \text{ It is of the form } V_m \sin \omega t\right]$$

$$L = \frac{2}{314} = 6.37 \times 10^{-3}\ H$$

$$\text{Average power} = VI \cos \phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866$$

$$\left[\because \text{R.M.S. value} = \frac{\text{Max. value}}{\sqrt{2}}\right]$$

$$= 43.3 \text{ W}$$

Example 5.4 An inductive coil takes 10 A and dissipates 1000 W when connected to a supply at 250 V, 25 Hz. Calculate the impedance, the effective resistance, the reactance, the inductance and the power factor.

Solution

$$I = 10 \text{ A}$$

$P = 1000\ \Omega$ (unless specified, i.e. given power is actual power)

$$V = 250 \text{ V};\ f = 25 \text{ Hz.}$$

Inductive coil will also have certain resistance. So it is equivalent to an R–L circuit.

$$\text{Power} = VI \cos \phi = I^2 R = 1000$$
$$(10)^2 \times R = 1000$$
$$R = \frac{1000}{100} = 10\ \Omega$$

$$\text{Impedance} \qquad Z = \frac{V}{I} = \frac{250}{10} = 25\ \Omega.$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{25^2 - 10^2} = 22.91\ \Omega$$

$$X_L = \omega L = 2\ \pi f L$$

$$22.91 = 2\pi \times 25 \times L$$

$$L = 0.146\ H$$

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{10}{25} = 0.4 \text{ lagging}$$

Otherwise,

$$P = VI\cos\phi$$

$$1000 = 250 \times 10 \times \cos\phi$$

$$\cos\phi = \frac{1000}{2500} = 0.4 \text{ lagging}$$

Example 5.5 A current of 5 A flows through a non-inductive resistance in series with a when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V across the coil 200 V, calculate (i) the impedance, resistance and reactance the coil, (ii) the power absorbed by the coil and (iii) the total power. Draw phasor diagram.

Solution Referring Fig. E.5.1

$$V = 250\text{ V}, f = 50\text{ Hz}, V_R = 125\text{ V}, I = 5\text{ A}$$

Voltage across coil = 200 V

Resistance $R = \dfrac{V_R}{I} = \dfrac{25}{5} = 25\ \Omega$

The impedance of the coil $Z_1 = \dfrac{200}{5} = 40$ ohms

$$Z_1 = \sqrt{R_1^2 + X_L^2} = 40 \text{ ohms}$$

$$R_1^2 + X_L^2 = 1600 \text{ ohms}$$

The total impedance of the circuit

$$= Z = \frac{V}{I} = \frac{250}{5} = 50 \text{ ohms}$$

$$= |\overline{Z}| = |R + R_1 + jX_L| = \sqrt{(R + R_1)^2 + (X_L)^2} = 50 \text{ ohms}$$

$$(R + R_1)^2 + X_L^2 = 2500 \tag{2}$$

(2) – (1) gives,

$$(R + R_1)^2 + X_L^2 - R_1^2 - X_L^2 = 2500 - 1600$$

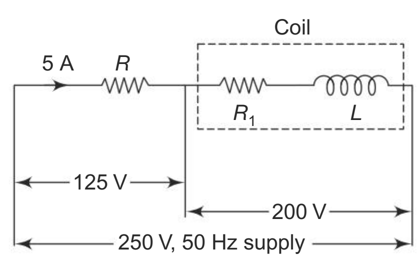


Fig. E.5.1

$$R^2 + R_1^2 + 2RR_1 - R_1^2 = 900$$

$$R^2 + 2RR_1 = 900$$

$$25^2 + (2 \times 25 \times R_1) = 900$$

$$50\, R_1 = 900 - 625 = 275$$

$$R_1 = 5.5 \text{ ohms}$$

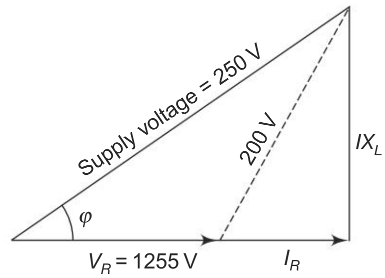


Fig. E.5.2

Substituting this value in Eqn. (1)

$$(5.5)^2 + X_L^2 = 1600\ \Omega$$

$$X_L^2 = 1600 - (5.5)^2 = 1569.75$$

Reactance of coil, $X_L = 39.62$ ohms

Power absorbed by the coil $= I^2R_1 = 5^2 \times 5.5 = 137.5$ W

$$\text{Total power} = I^2\,(R + R_1) = 5^2\,(25 + 5.5) = 762.5 \text{ W}$$

Phasor Diagram As shown in Fig. E.5.2

Example 5.6 When a resistor and an inductor in series are connected to 240 V supply, a current of 3 A flows lagging 37° behind the supply voltage. The voltage across the inductor is 171 V. Find the resistance of resistor and the resistance and reactance of the inductor.

Solution Referring Fig. E.5.3

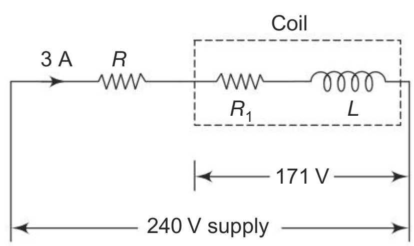


Fig. E.5.3

$$V = 240 \text{ V};\ I = 3 \text{ A};\ \phi = 37°;\ \text{p. f. is lagging}$$

Voltage across inductor = 171 V

Impedance of coil $= \dfrac{171}{3} = 57$ ohms; impedance of coil $= \sqrt{R_1^2 + X_L^2} = 57$ ohms

$$R_1^2 + X_L^2 = 3249$$

The total impedance of the circuit $= \dfrac{240}{3} = 80$ ohms

$$\text{Total impedance} = |R + (R_1 + jX_L)|$$

$$= \sqrt{(R + R_1)^2 + X_L^2} = 80 \text{ ohms}$$

$$(R + R_1)^2 + X_L^2 = 6400$$

p.f. angle $\phi = 37°$; $\therefore$ p. f. $= \cos\phi = 0.799$

$$\cos\phi = \frac{\text{Total resistance}}{\text{Total impedance}} = 0.799 = \frac{R + R_1}{80}\ R + R_1 = 63.92 \text{ ohms}$$

Substituting this value in Eqn. (2)

$$63.92^2 + X_L^2 = 6400; \quad X_L^2 = 2314.23$$

Reactance of inductor $X_L = 48.11$ ohms

Substituting the value of X_L in Eqn (1)

$$R_1^2 + (48.11)^2 = 3249$$

$$R_1^2 = 3249 - (48.11)^2 = 934.43$$

Resistance of inductor $R_1 = 30.57\ \Omega$

$R + R_1 = 63.92\ \Omega$; $R + 30.57 = 63.92$

Resistance of resistor $R = 33.35\ \Omega$.

Example 5.7 When a voltage of 100 V at 50 Hz is applied to a choking coil *A*, the current taken is 8 *A* and the power is 120 W. When applied to a coil *B*, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series?

Solution

Supply voltage = 100 V at 50 Hz

with coil *A*, $I = 8$ A, $P = 120$ W; with coil *B*, 1 = 10 A. $P = 500$ W

For coil A

Refer Fig. E.5.4

The impedance $Z_1 = \dfrac{\text{Voltage}}{\text{Current}} = \dfrac{100}{8} = 12.5\ \Omega$

$$\text{Power} = I_1^2 R_1 = 120 \text{ W}; \quad 120 = 8^2 \times R_1; \quad R_1 = \frac{120}{64} = 1.875\ \Omega$$

Reactance $X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{12.5^2 - 1.875^2} = 12.36\ \Omega$

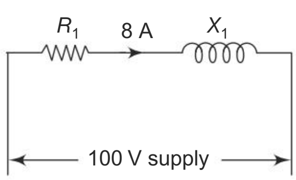


Fig. E.5.4

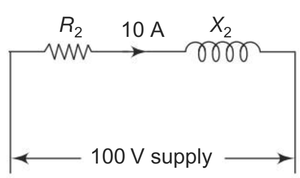


Fig. E.5.5

For coil B

Refer Fig. E.5.5

The impedance $Z_2 = \dfrac{100}{10} = 10$ ohms; Power $= 500 = I_1^2 R_2 = 10^2 R_2$

$$R_2 = \frac{500}{100} = 5\ \Omega$$

Reactance $X_2 = \sqrt{Z_2^2 - R_2^2} = \sqrt{10^2 - 5^2} = 8.66\ \Omega$

When coils A and B are in series

Refer to Fig. E.5.6

Total resistance $R = R_1 + R_2$

$R = 1.875 + 5 = 6.875\ \Omega$

Total reactance $X = X_1 + X_2 = 12.36 + 8.66 = 21.02\ \Omega$

Total impedance $Z = \sqrt{R^2 + X^2} = \sqrt{6.875^2 + 21.02^2} = 22.1\ \Omega$

$$\text{Current drawn} = \frac{V}{Z} = \frac{100}{21.1} = 4.52\text{ A}$$

$$\text{Power taken} = I^2 R = 4.52^2 \times 6.875 = 140.46\text{ W.}$$

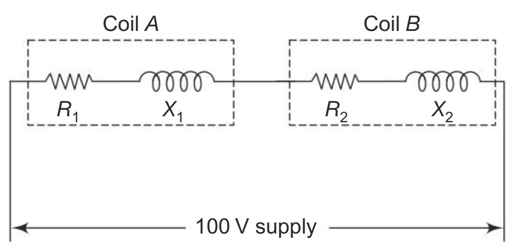


Fig. E.5.6

5.5 R-C SERIES CIRCUIT

Let us consider the circuit shown in Fig. 5.10 in which a pure resistance R ohm and a pure capacitance of C Farad are in series.

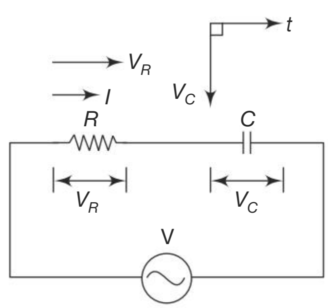


Fig. 5.10

Let $v = V_m \sin \omega t$ be the applied voltage

i = Circuit current at any instant

I = Effective value of circuit current

V_R = Potential difference across resistor

V_C = Potential difference across capacitor

f = Frequency of applied voltage

The same current I flows through R and C

Voltage across $R = \overline{V}_R = IR$ in phase with I

Voltage across $C = \overline{V}_C = IX_C$ lagging I by 90°.

At any instant (Fig. 5.11)

Applied voltage $\overline{V} = \overline{V}_R + \overline{V}_C$ (5.20)

Applied Voltage $\overline{V} = \overline{I}R - j\,\overline{I}X_C$

$$\overline{V} = \overline{I}\,(R - jX_C) \tag{5.21}$$

$$\frac{\overline{V}}{\overline{I}} = R - jX_C = \overline{Z} = \text{impedance of circuit} \tag{5.22}$$

$$\text{Magnitude of } V = \sqrt{V_R^2 + (-V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2} = \sqrt{I^2R^2 + I^2X_C^2} = I\sqrt{R^2 + X_C^2} \tag{5.23}$$

$$\text{Current } I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$\frac{V}{I} = \sqrt{R^2 + X_C^2} = \text{Magnitude of impedance of the circuit } Z$$

$$|\overline{Z}| = \sqrt{R^2 + X_C^2}; \quad |\overline{Z}|^2 = R^2 + X_C^2 \tag{5.24}$$

From the voltage triangle of Fig. 5.11 if the common parameter I is removed the same becomes an impedance triangle shown in Fig. 5.12.

From the $\Delta\, ABC$ tan $\phi = \dfrac{X_C}{R} = \dfrac{1/\omega C}{R} = \dfrac{1}{\omega CR}$ (5.25)

$$\phi = \tan^{-1}\frac{1}{\omega CR} \tag{5.26}$$

ϕ is called phase angle and it is the angle between V and I. Its value lies between 0 and −90°; $\overline{Z} = R - jX_C = Z\angle -\phi$. The real part R is resistance and the imaginary part $-X_C$ is reactance.

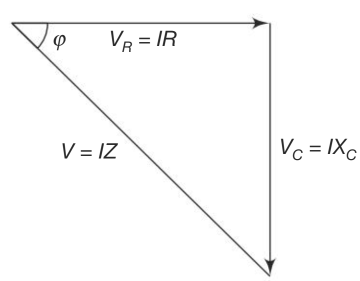


Fig. 5.11

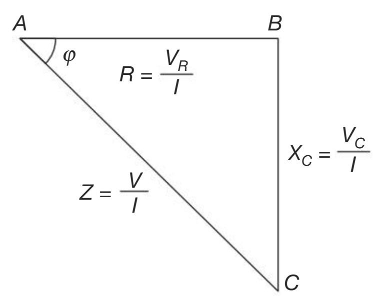


Fig. 5.12

Power Factor p.f. = cos ϕ where ϕ is the angle between voltage and current. From the ΔABC, shown in Fig. 5.12,

$$\cos\phi = \frac{R}{Z} \tag{5.27}$$

Referring to Fig. 5.11, the current I leads the applied voltage V. So the power factor of an R–C circuit is leading.

$$\cos \phi = \cos\left[\left(\tan^{-1}\frac{X_C}{R}\right)\right] \tag{5.28}$$

Power Calculation As in the case of R–L series circuit, here also we can define actual power reactive power and apparent power.

Actual or real power measured in watts is

$$P = VI_a; \quad P = VI\cos\phi$$

Reactive or quadrature power measured in volt ampere Reactive is

$$Q = VI\sin\phi$$

complex or apparent power measured in volt Ampere is $S = VI$. All the above power components are shown in Fig. 5.13.

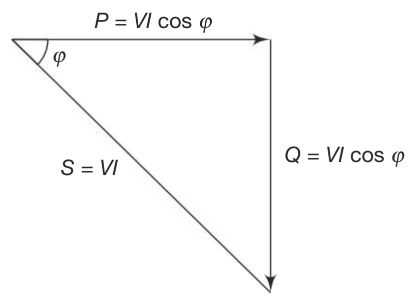


Fig. 5.13

Here also, $\bar{S} = P + jQ$

Magnitude of $S = \sqrt{P^2 + Q^2}$

Waveform Referring to Fig. 5.14 the current I leads the applied voltage V by an angle ϕ. Hence if the applied voltage is given as

$$v = V_m \sin \omega t$$

Then the current equation is $i = I_m \sin(\omega t + \phi)$ (5.29)

where $I_m = \dfrac{V_m}{Z}$

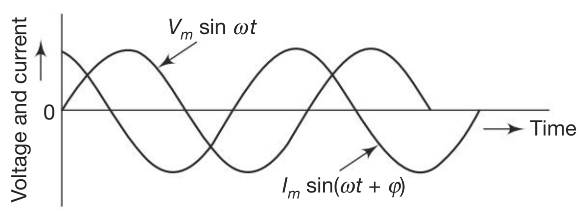


Fig. 5.14

Example 5.8 A resistance of 100 ohm is connected in series with a 50 μF capacitor to a supply at 200 V, 50 Hz. Find the (i) impedance, current, power factor and phase angle and (ii) The voltage across resistor and across capacitor. Draw the phasor diagram.

Solution

$$R = 100\ \Omega \qquad C = 50 \times 10^{-6}\ \text{F}$$

$$V = 200\ \text{V}, \qquad f = 50\ \text{Hz}$$

Impedance $\bar{Z} = R - jX_C$

$$X_C = \frac{1}{\omega_C} = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.66\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 63.66^2}$$

$$= 118.54\ \Omega$$

$$\text{Current } I = \frac{V}{Z} = \frac{200}{118.54} = 1.69\ \text{A}$$

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{100}{118.54} = 0.844 \text{ leading}$$

$$\text{Phase angle} = \phi = \cos^{-1} 0.844$$

$$= 32.48°$$

$$\text{Voltage across resistor} = IR = 1.69 \times 100$$

$$V_R = 169\ \text{V}$$

$$\text{Voltage across capacitor} = IX_C$$

$$V_C = 1.69 \times 63.66 = 107.59\ \text{V}$$

Phasor Diagram

It is shown in Fig. E.5.7 a

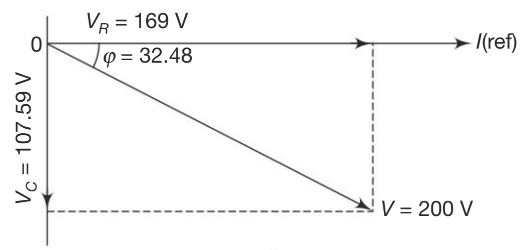


Fig. E.5.7 a

Example 5.9 In a circuit, the applied voltage of 150 V lags, the current of 8 A by 40°

(i) Is p.f. lagging or leading?
(ii) What is the value of p.f.?
(iii) Is the circuit inductive or capacitive?
(iv) What is the value of active and reactive power?

Solution

$$V = 150\ \text{V}, \quad I = 8\ \text{A}, \quad \phi = 40°$$

(i) The applied voltage lags behind the current. That means the current leads the voltage.

$\therefore$ p.f. is leading

(ii) p.f. = $\cos\phi = \cos 40 = 0.766$ (leading)

(iii) p.f. leading means capacitive circuit

(iv) Active power = $VI\cos\phi = 150 \times 8 \times 0.766$
$= 919.2$ W

Reactive power = $VI\sin\phi = 150 \times 8 \times 0.643$
$= 771.6$ VAR

Example 5.10 Find the circuit constants of a two element series circuit which consumes 700 W with 0.707 leading p.f. The applied voltage is $V = 141.4 \sin 314\, t$.

Solution

$$v = 141.4 \sin 314t$$

$$P = 700 \text{ W}, \quad \text{p.f.} = 0.707 \text{ leading}$$

Leading p.f. means $R-C$ circuit

Max. value of supply voltage = 141.4 V

R.M.S. value of supply voltage $= \dfrac{141.4}{\sqrt{2}} = 99.98$ V

$$\cos\phi = 0.707 \text{ leading; Power} = VI\cos\phi$$

$$700 = 99.98 \times I \times 0.707; \quad I = 9.9 \text{ A}$$

Impedance $\quad Z = \dfrac{V}{I} = \dfrac{99.98}{9.9} = 10.09$ ohms

$$\cos\phi = \frac{R}{Z} \Rightarrow R = Z\cos\phi$$

$$R = 10.09 \times 0.707 = 7.13\ \Omega$$

$$\sin\phi = \frac{X_C}{Z} \Rightarrow X_C = Z\sin\phi$$

$$\phi = \cos^{-1} 0.707 = 45°;\ \sin\phi = 0.707$$

$$X_C = 10.09 \times 0.707 = 7.13 \text{ ohms}$$

$$\frac{1}{\omega C} = 7.13; \quad \frac{1}{314 \times C} = 7.13 \Rightarrow C = \frac{1}{314 \times 7.13}$$

$$C = 4.466 \times 10^{-4} \text{ F}$$

$$= 446.6 \times 10^{-6} \text{ F}$$

$$C = 446.6\ \mu\text{F}$$

Example 5.11 A series $R-C$ circuit takes a power of 7000 watts when connected to 200 V 50 Hz supply. The voltage across the resistor is 130 volts. Calculate (i) the resistance current, p.f., capacitance and impedance. (ii) write the equation for the voltage and current.

Solution

$$V = 200 \text{ V, 50 Hz. } P = 7000 \text{ W, } V_R = 130 \text{ V}$$

$$\text{Power} = I^2R = \frac{V_R^2}{R} = 7000 \text{ W}$$

$$\frac{130^2}{R} = 7000$$

$\therefore$ Resistance $R = 2.414$ ohms

$$\text{Current } I = \frac{V_R}{R} = \frac{130}{2.414} = 53.85 \text{ A}$$

$$\text{Total impedance} = Z = \frac{V}{I} = \frac{200}{53.85}$$

Impedance $\quad Z = 3.71$ ohms

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{3.71^2 - 2.414^2}$$

$$X_C = 2.82 \text{ ohms } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\frac{1}{2\pi \times 50 \times C} = 2.82$$

$$C = \frac{1}{2\pi \times 50 \times 2.82} = 1.1287 \times 10^{-3}$$

$$= 1128.7 \times 10^{-6} \text{ F}$$

Capacitance $C = 1128.7\ \mu\text{F}$

$$\cos\phi = \frac{R}{Z} = \frac{2.414}{3.71} = 0.65$$

$$\text{p.f.} = 0.65 \text{ leading}$$

$$\phi = \cos^{-1} 0.65 = 49.4°$$

Equations

R.M.S. value of voltage = 200 V

$$\text{Max. value} = 200 \times \sqrt{2}$$

$$= 282.84 \text{ volts}$$

If the voltage is taken as reference,

$$v = 282.84 \sin 2\pi f t = 282.84 \sin (2\pi \times 50)\, t$$

$$= 282.84 \sin 314.16\, t$$

Current leads the voltage by 49.40°

∴ Equation for current is

$$i = 53.85 \times \sqrt{2} \sin (314.16\, t + 49.4°)$$

$$i = 76.155 \sin (314.16\, t + 49.4°)$$

Example 5.12 A resistance of 10 ohms and a capacitor of 50 μF are connected in series across a 200 V, 50 Hz supply. Find the following: impedance, current, power factor, power.

Solution

$$\overline{V} = 20\boxed{0°} \text{ volts.} \quad R = 100\ \Omega$$

$$f = 50 \text{ Hz} \qquad C = 50\ \mu\text{F}$$

$$X_c = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

Impedance, $Z = R - jX_c = 100 - j\,63.66 = 118.54 - 32.5°\ \Omega$

Current, $I = \dfrac{V}{Z} = \dfrac{200\ 0°}{118.54\ -32.5°} = 1.69\ 32.5°\ \text{A}$

Power factor, $= \cos 32.5° = 0.843$ (lead)

Power, $P = VI\cos\phi = 200 \times 1.69 \times 0.843 = 285$ W

Example 5.13 An RLC series circuit consists of capacitor of value 0.05 μF. It resonates at 500 Hz. Calculate the value of inductance L.

Solution:

$$C = 0.05\ \mu\text{F}$$

$$F = 500 \text{ Hz}$$

At resonance, $X_L = X_c$. But $X_c = \dfrac{1}{2\pi \times 500 \times 0.05 \times 10^{-6}} = 6366.2\ \Omega$

$\therefore$ Inductance, $\quad L = \dfrac{X_L}{2\pi f} = \dfrac{6366.2}{2\pi \times 500} = 2.03$ H

Example 5.14 An inductive circuit has a resistance of 50 Ω and an inductance of 0.5 H. It is connected across a voltage of 200 V, 50 Hz supply. Calculate the current and the power supplied. What alteration is required, if the same amount of current has to be drawn at unity power factor?

Solution

$R = 50\ \Omega, \qquad L = 0.5$ H

$F = 50$ Hz, $\qquad V = 200$ V

Impedance, $\quad Z = R + jX_L = 50 + j2\pi \times 50 \times 0.5$

$= 50 + j157.08\ \Omega = 164.85\ 72.34°\ \Omega$

Current supplied, $\quad I = \dfrac{V}{Z} = \dfrac{200\ \text{O}}{164.85\ 72.34°} = 1.21\ -72.34°$A

Power supplied, $\quad P = VI \cos \phi$

$= 200 \times 1.21 \times \cos(-72.34°)$

$= 73.42$ W

When the circuit has to draw the current at unity power factor, then the reactance of the circuit must be zero. Hence, a capacitance whose reactance equals the inductive reactance should be connected in the circuit.

Thus, $\quad X_C = X_L = 2\pi\ 50 \times 0.5 = 157.08$

$\therefore$ Capacitance, $\quad C = \dfrac{1}{2\pi + 50 + 157.08} = 0.2\mu$F

When the same magnitude of current has to be drawn, additional resistance is to be added in series.

$\therefore \qquad 50 + r = \dfrac{200}{1.21} = 165.3\ \Omega \qquad \therefore\ r = 165.3 - 50 = 115.3\ \Omega$

Hence a resistance of 115.3 Ω and a capacitance of 0.2 μF should be connected in series with the existing resistance and inductance.

Example 5.15 A coil of resistance 50 Ω and inductance 9 H is connected in series with a capacitor across a variable frequency source. If the maximum current is 1 A at 75 Hz, find the capacitance.

Solution $\quad R = 50\ \Omega \qquad L = 9$ H

$I_o = 1$A $\qquad f = 75$ Hz

i.e. at resonance.

Hence $\quad X_c = X_L = 2\pi \times 75 \times 9 = 4241.15\ \Omega$

$\therefore \quad C = \dfrac{1}{2\pi \times 75 \times 4241}.15 = 0.5\ \mu$F.

5.6 R-L-C SERIES CIRCUIT

Consider a circuit as shown in Fig. 5.15 having resistance R ohms, inductance L henrys and capacitance C farads all connected in series.

Let the applied voltage be $v = V_m \sin \omega t$

i = Circuit current at any instant

I = Effective value of circuit current
V_P = Potential difference across resistor
V_L = Potential difference across inductance
V_C = Potential difference across capacitance
f = Frequency of applied voltage

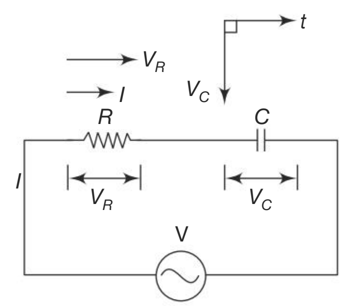


Fig. 5.15

The same current I flows through R, L and C. Hence I is taken as reference vector.

Voltage across $R = \overline{V}_R = IR$ in phase with current

Voltage across $L = \overline{V}_L = IX_L \angle 90°$ $\because$ voltage leads current by 90°

$= jIX_L$

Voltage across $C = \overline{V}_C = IX_C \angle -90°$ $\because$ voltage lags current by 90°

$= -jIX_C$

Applied voltage $\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_C$ (5.30)

$= IR + \bar{J}IX_L - \bar{J}IX_C$

$= I(R + jX_L - jX_C)$ (5.31)

$$\frac{\overline{V}}{I} = \text{impedance of the circuit}$$

$$\overline{Z} = R + jX_L - jX_C$$

$$\overline{Z} = R + j(X_L - X_C) \text{ ohms} \tag{5.32}$$

$$= |\overline{Z}| \angle \phi$$

$$|\overline{Z}| = \sqrt{R^2 + (X_L - X_C)^2} \tag{5.33}$$

$X_L - X_C = X$ is called net reactance. If $X_L > X_C$ the circuit will behave like R–L circuit. If $X_C > X_L$, the circuit will behave like R–C circuit.

Magnitude of $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ (5.34)

Magnitude of current

$$I = \frac{V}{|\overline{Z}|}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{5.35}$$

Phasor Diagram Case (i) If $X_L > X_C$

X will be inductive in nature $X_L > X_C$

$\therefore$ The circuit will behave like an R–L circuit (Refer Fig. 5.16).

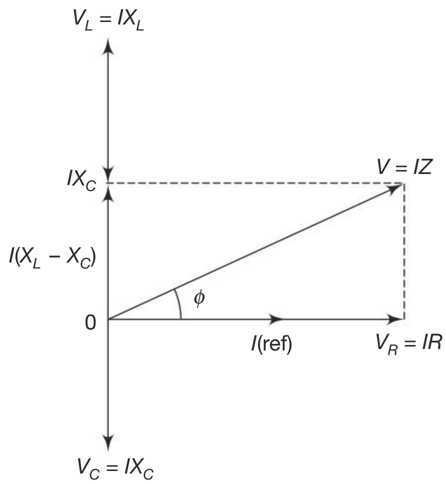


Fig. 5.16

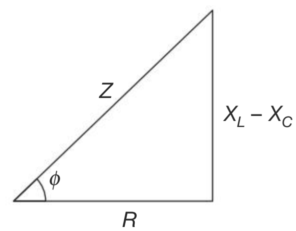


Fig. 5.17

From Fig. 5.16, we can draw impedance triangle (Fig. 5.17)

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{(\omega L - 1/\omega C)}{R} \tag{5.36}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) \tag{5.37}$$

$$\text{p.f.} = \cos\phi = \frac{R}{Z}$$

From Fig. 5.16 we can see that current lags applied voltage by an angle ϕ

$\therefore$ p.f. is lagging.

Case (ii) If $X_C > X_L$

X will be capacitive in nature if $X_C > X_L$

$\therefore$ The circuit will behave like a R-C circuit (Fig. 5.18).

From Fig. 5.18, we can draw the impedance diagram (Fig. 5.19)

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$= \frac{(1/\omega c) - \omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{(1/\omega c) - \omega L}{R}\right)$$

$$\text{p.f.} = \cos\phi = \frac{R}{Z}$$

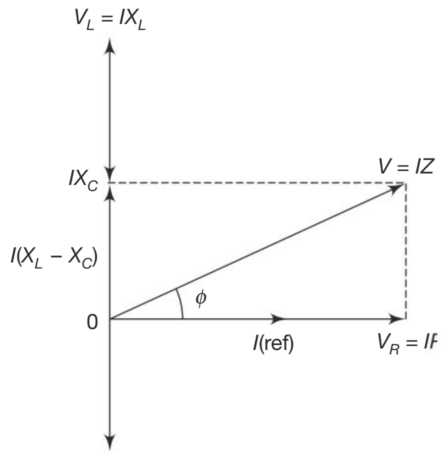


Fig. 5.18

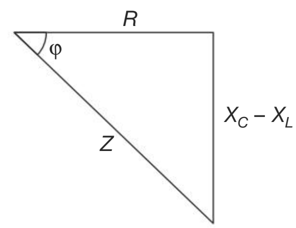


Fig. 5.19

From Fig. 5.18, we can see that current leads applied voltage by an angle ϕ
$\therefore$ p.f. is lead.

Power Calculation As in the case of R–L circuit, here also

Actual or Real power $P = VI \cos \phi$ watts

Reactive or Quadrature power $Q = VI \sin \phi$ VAR

Complex or apparent power $S = VI$ volt Amp

$$\text{Magnitude of } \overline{S} = \sqrt{P^2 + Q^2}$$

$$\text{p.f.} = \cos \phi = \frac{\text{Real power}}{\text{Apparent power}}$$

Waveform Referring to Figs. 5.20 and 5.21 if the applied voltage is given as

$$v = V_m \sin \omega t$$

then the current equation is

$$i = I_m \sin (\omega t \pm \phi) \tag{5.39}$$

+ for case (ii) i.e. $X_C > X_L$
− for case (i) i.e. $X_L > X_C$

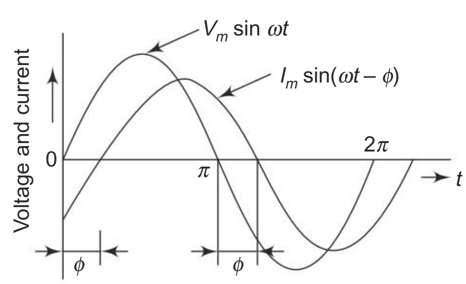


Fig. 5.20

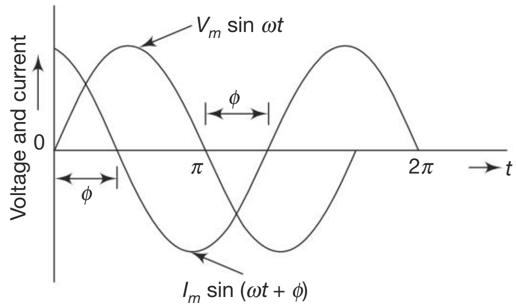


Fig. 5.21

Example 5.16 A coil of resistance 10 ohms and inductance 0.1 H is connected in series with a 150 μF capacitor across 200 V, 50 Hz supply. Calculate (i) inductive reactance, capacitive reactance, impedance, current and power factor and (ii) the voltage across the coil and capacitor respectively.

Solution Referring Fig. E.5.8

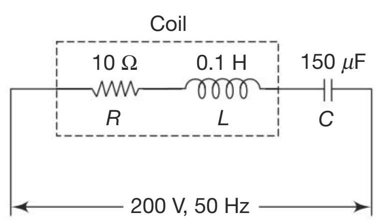


Fig. E.5.8 (b)

$$R = 10 \text{ ohms}; L = 0.1\text{H}, C = 150\ \mu\text{F}$$

$$V = 200 \text{ V}, f = 50 \text{ Hz}$$

Inductive reactance $X_L = \omega L$

$$= 2\pi f L = 2\pi \times 50 \times 0.1$$

$$X_L = 31.42\ \Omega$$

Capacitive reactance $= X_C = 1/\omega_c = \dfrac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$

$$X_C = 21.22\ \Omega.$$

Impedance $Z = R + j X_L - j X_C$

$$|\overline{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{10^2 + (31.42 - 21.22)^2}$$

$$= 14.28\ \Omega$$

Current $I = \dfrac{V}{Z} = \dfrac{200}{14.28} = 14\text{A}$

$$\text{p.f} = \cos\phi = \frac{R}{Z} = \frac{10}{14.28} = 0.7$$

As $X_L > X_C$, $XL - X_C$ is inductive

$\therefore$ p.f is lagging; p.f = 0.7 lagging

Voltage across coil = $I \times$ impedance of coil

Impedance of coil = $\sqrt{R^2 + X_L^2} = \sqrt{10^2 + 31.42^2} = 32.97$ ohms

Voltage across coil = $14 \times 32.97 = 461.62$ volts

Voltage across capacitor = IX_C

$= 14 \times 21.22 = 297.08$ volts

Example 5.17 In the circuit of Fig. E.5.9, at a frequency $f = 500$ Hz, the current lags the voltage by 50°. Find R and voltage across each circuit element. Draw the phasor diagram.

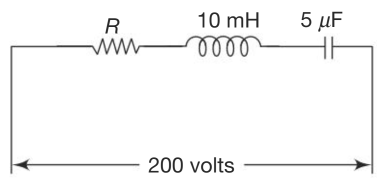


Fig. E.5.9

Solution

$L = 10$ mH, $C = 5\ \mu$F $f = 500$ Hz

$\phi = 50°$ lagging

Inductive reactance $X_L = \omega L = 2\pi f L$

$= 2\pi \times 500 \times 10 \times 10^{-3}$

$X_L = 31.42\ \Omega$

Capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 500 \times 5 \times 10^{-6}}$

$= 63.66\ \Omega$

$X_C > X_L \quad \therefore X_C - X_L$ is capacitive

$\therefore$ p.f is leading.

From impedance Δ

$$\tan \phi = \frac{X_L - X_C}{R}; \tan = \tan 50° = \frac{31.42 \sim 63.66}{R}$$

$$R = 27.09\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{27.09^2 + (31.42 - 63.66)^2} = 42.11\ \Omega$$

$$\text{Current } I = \frac{V}{Z} = \frac{200}{42.11} = 4.75 \text{ A}$$

Voltage across resistance $V_R = IR = 4.75 \times 27.09 = 128.68$ V

Voltage across inductance $V_L = IX L$

$= 4.75 \times 31.42 = 149.25$ V

Voltage across capacitor $V_C = IXC$

$= 4.75 \times 63.66 = 302.39$ V

Phasor Diagram

This is shown in Fig. E.5.10

Example 5.18 A 230 V, 50 Hz voltage is applied to a coil of $L = 5\ H$ and $R = 2\ \Omega$ in series with a capacitance C. What value must C have in order that the p.f. across the oil shall be 250 V.

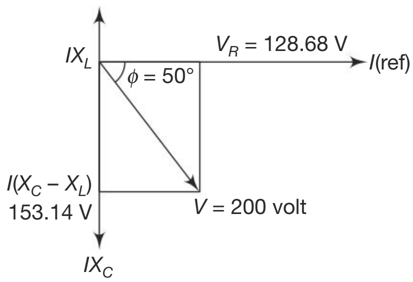


Fig. E.5.10

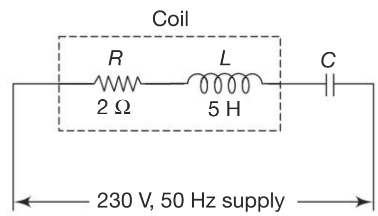


Fig. E.5.11

Solution Referring Fig. E.5.11

$$V = 230 \text{ V } f = 50 \text{ Hz} \quad R = 2 \text{ ohms} \quad L = 5 \text{ H.}$$

$$\text{Voltage across coil} = 250 \text{ V}$$

$$\text{The impedance of the coil} = Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{2^2 + (2\pi \times 50 \times 5)^2}$$

$$Z_1 = 1570.79 \ \Omega$$

For the voltage across the coil to be 250 V

$$\text{Current must be} = \frac{250}{Z_1}$$

$$I = \frac{250}{1570.79} = 0.159 \text{ A}$$

$$\text{Now the total impedance of the circuit} = \frac{V}{I} = \frac{230}{0.159}$$

$$Z = 1446.54 \text{ ohms}$$

$$|\bar{Z}| = \sqrt{R^2 + (X_L - X_C)^2}; \ 1446.54 = \sqrt{2^2 + (1570.79 - X_C)^2}$$

$$2.092 \times 10^6 = 4 + (1570.79 - X_C)^2$$

$$= 4 + 2.47 \times 10^6 + X_C^2 - 3141.58 \, X_C$$

$$X_C^2 - 3141.58 \, X_C + 378004 = 0$$

$$X_C = \frac{3141.58{:}1 \pm \sqrt{3141.58^2 - (4 \times 378004)}}{2}$$

$$= 3016.26 \ \Omega, \ 125.34 \ \Omega$$

X_C cannot be greater than Z.

$\therefore \ X_C \ngtr 1446.54 \ \Omega$

$\therefore$

$$X_C = 125.34 \ \Omega; \ \frac{1}{2\pi \times 50 \times C} = 125.34$$

$$C = \frac{1}{2\pi \times 50 \times 125.34} = 2.539 \times 10^{-5} \text{ F}$$

$$= 25.39 \times 10^{-6} \text{ F} = 25.39 \ \mu\text{F}$$

Example 5.19 A resistance R, an inductance $L = 0.5$ H and a capacitance C are connected in series. When a voltage $v = 350 \cos (3000t - 20°)$ volts is applied to this series combination, the current flowing is $15 \cos (3000\,t - 60°)$ amperes. Find R and C.

Solution

$$L = 0.5 \text{ H},\ v = 350 \cos (3000\,t - 20°)$$

$$i = 15 \cos (3000\,t - 60°)$$

Phase difference between applied voltage and current. is $\phi = -60° + 20° = -40°$ with current lagging.

$$\therefore \qquad X_L > X_C \text{ (p.f. is lagging)}$$

$$\omega = 2\pi f = 3000$$

$$\text{Total Impedance } Z = \frac{V_m}{I_m} = \frac{350}{15} = 23.33\ \Omega$$

$$\phi = 40°$$

$$\tan\phi = \frac{X_L - X_C}{R};\ \tan(40°) = \frac{X_L - X_C}{R}$$

$$XL - X_C = 0.839\,R$$

$$X = 0.839\,R \quad (\because X = X_L - X_C)$$

$$\text{Total impedance } Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{R^2 + (0.839\,R)^2} = \sqrt{1.704\,R^2}$$

$$= 1.305\,R$$

$$R = \frac{Z}{1.305} = \frac{23.33}{1.305};\ R = 17.88\ \Omega$$

$$X = 0.839\,R$$

$$X_L - X_C = 0.839 \times 17.88 = 15\ \Omega$$

$$X_L = \omega L = 3000 \times 0.5 = 1500\ \Omega$$

$$1500 - X_C = 15$$

$$X_C = 1485\ \Omega$$

$$X_C = \frac{1}{\omega_C}$$

$$1485 = \frac{1}{3000\,\text{C}};\ C = 0.224\ \mu\text{F}$$

5.7 ADMITTANCE AND ITS COMPONENTS

Admittance of a circuit is defined as reciprocal of impedance. In general. the impedances will be complex numbers of the form

$$\overline{Z} = R + jX \tag{5.40}$$

The real part R is called resistance (in ohms) and the imaginary part X is called reactance (in ohms). Similarly admittance also can be expressed in terms of real part and imaginary part.

$$\frac{1}{\overline{Z}} = \frac{1}{R - jX} = \frac{R - jX}{(R + jX)\,(R - jX)}$$

[Multiplying both numerator and denominator by the conjugate of denominator $(R - jX)$]

$$\frac{1}{\overline{Z}} = \frac{R + jX}{R^2 + X^2};\ \overline{Y} = \frac{R}{R^2 + X^2} - j = \frac{X}{R^2 + X^2} \tag{5.41}$$

$\frac{1}{\overline{Z}}$ is called admittance and it is represented as $\overline{Y}$. Unit is siemen ℧.

The real part $\frac{R}{R^2 + X^2}$ is called conductance (in ℧) and the imaginary part $\frac{X}{R^2 + X^2}$ is called susceptance (in ℧).

In general, admittance

$$\overline{Y} = G \pm JB \tag{5.42}$$

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{X}{R^2 + X^2}$$

If

$$\overline{Z} = R + jX_L,\ \overline{Y} = \frac{1}{Z} = G - jB_L \tag{5.43}$$

If

$$\overline{Z} = R - jX_C,\ \overline{Y} = G + jB_C \tag{5.44}$$

Let us consider a circuit shown in Fig. 5.22 having three impedances Z_1, Z_2, Z_3 in parallel.

Using Kirchhoff's current law, $\overline{I} = \overline{I}_1 + \overline{I}_2 + \overline{I}_3$

$$\frac{\overline{V}}{\overline{Z}_{eq}} = \frac{\overline{V}}{\overline{Z}_1} + \frac{\overline{V}}{\overline{Z}_2} + \frac{\overline{V}}{\overline{Z}_3};\ \frac{1}{\overline{Z}_{eq}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3}$$

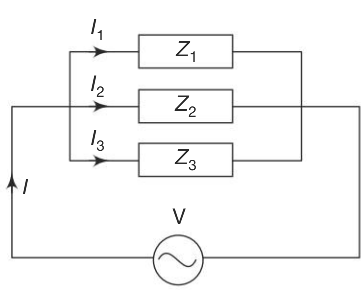


Fig. 5.22

$$\therefore \qquad \overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3 \tag{5.45}$$

When two or more impedances are in parallel, the equivalent admittance sum of the individual admittances.

If $$\overline{Y}_1 = G_1 + jB_1, \overline{Y}_2 = G_2 + jB_2, \overline{Y}_3 = G_3 + jB_3$$

$$\overline{Y}_{eq} = G_{eq} + jB_{eq}; \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3 = G_1 + jB_1 + G_2 + jB_2 + G_3 + jB$$

$$\overline{Y}_{eq} = (G_1 + G_2 + G_3) + j(B_1 + B_2 + B_3) \tag{5.46}$$

$$G_{eq} + jB_{eq} = (G_1 + G_2 + G_3) + j(B_1 + B_2 + B_3)$$

$$\therefore \qquad G_{eq} = G_1 + G_2 + G_3 \tag{5.47}$$

$$B_{eq} = B_1 + B_2 + B_3 \tag{5.48}$$

$\therefore$ The sum of individual conductance is equal to the equivalent conductance. The sum of individual susceptances is equal to the equivalent susceptance.

For solving series circuits, we use impedances and for parallel circuits, admittances. Consider the impedance triangle shown in Fig. 5.6 of an *R-L* series circuit. Its admittance triangle is shown in Fig. 5.23.

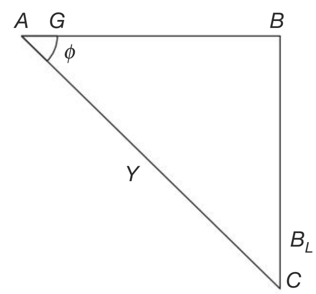


Fig. 5.23

From Fig. 5.23

$$\cos\phi = \frac{G}{Y} \Rightarrow \text{conductance } G = Y\cos\phi \tag{5.49}$$

$$\sin\phi = \frac{B_L}{Y} \Rightarrow \text{susceptance } B_L = Y\sin\phi \tag{5.50}$$

The inductive susceptance is regarded as negative.

The capacitive susceptance is regarded as positive.

$$\text{p.f.} = \cos\phi = \frac{G}{Y}$$

5.8 SIMPLE METHOD OF SOLVING PARALLEL AC CIRCUITS

In parallel circuits, the current divides itself into as many parts as the number of branches. The voltage across each branch will be same. In ac we must remember that the total current is the phasor sum of the branch currents. The parallel circuit can be analysed as follows.

Let us consider a parallel circuit having two branches as shown in Fig. 5.24.

Branch 1

Impedance of branch 1 = $\overline{Z}_1 = R_1 + jX_L = Z_1 \angle\phi_1$

$$\text{Admittance of branch 1} = \overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{\overline{Z}_1 \angle\phi_1} = Y_1 \angle -\phi_1 \tag{5.51}$$

For *R–L* Circuit, p.f. is lagging.

$\therefore$ Phase angle is $-\phi_1$

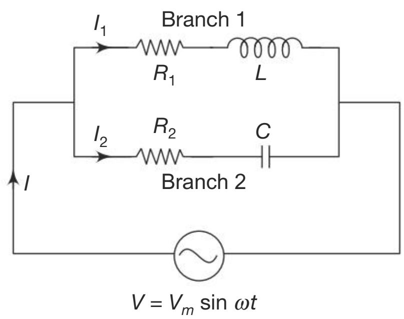


Fig. 5.24

Current in that branch is $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1} = \bar{V}\bar{Y}_1 = I_1 \angle -\phi_1$

where $$I_1 = \frac{V}{Z_1} = VY_1$$

Branch 2

Impedance of branch 2 = $\bar{Z}_2 = R_i - jX_C = Z_2 \angle + \phi_2$

Admittance of branch 2 = $Y_2 = \dfrac{1}{\bar{Z}_2} = \dfrac{1}{Z_2 \angle \phi_2} = Y_2 \angle + \phi_2$ (5.52)

Current in that branch is $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2} = \bar{V}\bar{Y}_2 = I_2 \angle + \phi_2$

Now, for calculating the total current and p.f. we can use phasor diagram.

In parallel circuits, voltages across all the branches are same. So voltage is taken as reference phasor (Fig. 5.25).

The total current drawn from the supply is

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = I \angle \pm \phi \tag{5.53}$$

Power consumed by branch $I = P_1 = VI_1 \cos \phi_1$ (5.54)

where $\cos \phi_1$ is the p.f. of branch 1

Power consumed by branch 2 = $P_2 = VI_2 \cos \phi_2$ (5.55)

where $\cos \phi_2$ is the p.f. of branch 2

Total power $$P = P_1 + P_2 \tag{5.56}$$

This total power is otherwise equal to $VI \cos \phi$

i.e. $$P = VI \cos \phi \tag{5.57}$$

$\cos \phi$ = power factor of the entire circuit

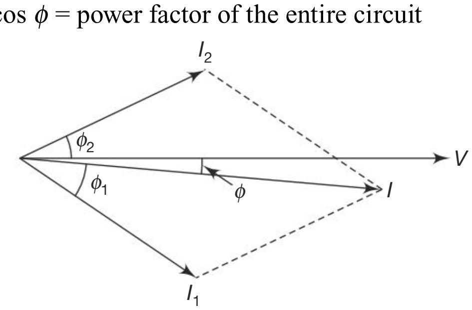


Fig. 5.25

Example 5.20 Calculate the admittance *Y*, the conductance *G* and the susceptance *B* of a circuit consisting of 10 Ω in series with an inductor of 0.1 H when the frequency is 50 Hz.

Solution $R = 10\ \Omega, L = 0.1\ \text{H}, f = 50\ \text{Hz}$

$$X_L = \omega L = 2\pi L = 2\pi \times 50 \times 0.1 = 31.42 \text{ ohms}$$

$$\text{Impedance } \overline{Z} = R + jX_L$$

$$\overline{Z} = 10 + j31.42 \text{ ohms}$$

$$\text{Admittance } \overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{10 + j\,31.42}$$

$$\overline{Y} = \frac{(10 - j\,31.42)}{(10 + j\,31.42)\,(10 - j\,31.42)}$$

[∵ Multiplying numerator and denominator by $(10 - j\,31.42)$]

$$\overline{Y} = \frac{10 - j\,31.42}{10^2 + 31.42^2} = \frac{10 - j\,31.42}{1087.22} = 9.2 \times 10^{-3} - j0.029 \text{ mho}$$

Conductance $G = 0.0092$ mho; Susceptance $B = 0.029$ mho

$$|\overline{Y}| = \sqrt{0.0092^2 + 0.029^2} = 0.0304 \text{ mho}$$

Example 5.21 Calculate the admittance $G + jB$ if the impedance is $10 + j5$ ohms.

Solution

$$\overline{Z} = 10 + j5 \text{ ohms}$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{10 + j5} = \frac{10 - j5}{(10 + j5)(10 - j5)}$$

$$= \frac{10 - j5}{10^2 + 5^2} = 0.08 - j0.04 \text{ mho}$$

$$G + jB = 0.08 - j\,0.04 \text{ mho}$$

Example 5.22 An impedance of $(7 + j5)$ ohms is connected in parallel with another circuit having an impedance of $(10 - j8)$ ohm. The supply voltage is 230 V, 50 Hz. Calculate (i) the admittance, the conductance and susceptance of the combined circuit I (ii) the total current taken from mains and its p.f.

Solution Referring to Fig. E.5.12.

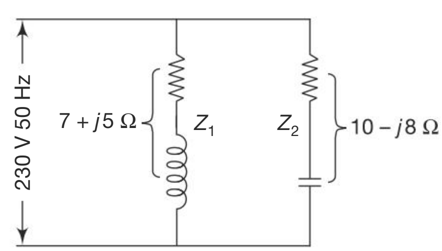


Fig. E.5.12

$\bar{Z}_1 = 7 + j_5$ ohms; $\bar{Z}_2 = 10 - j8$ ohms Supply = 230 V, 50 Hz

$$\text{Admittance of branch 1} = \frac{1}{\bar{Z}_1} = \frac{1}{7+j5} = \frac{7-j5}{(7+j5)(7-j5)} = \frac{7-j5}{7^2+5^2} = \frac{7-j5}{74}$$

$$\bar{Y}_1 = 0.095 - j\,0.068 \text{ mho}$$

$$\text{Admittance of branch 2} = \frac{1}{\bar{Z}_2} = \frac{1}{10-j8}$$

$$\bar{Y}_2 = \frac{10+j8}{(10-j8)(10+j8)} \frac{10+j8}{10^2+8^2} = \frac{10+\text{j}8}{164} = 0.061 + \text{j}0.049 \text{ mho}$$

Admittance of the combined circuit $\bar{Y} = \bar{Y}_1 + \bar{Y}_2$

$$\bar{Y} = 0.095 - j0.068 + 0.061 + j0.049$$

$$\bar{Y} = 0.156 - j0.019 \text{ mho}$$

Conductance $G = 0.156$ mho Susceptance $B = 0.019$ mho (inductive)

Let supply voltage = $230\angle 0°$ Current $I = \bar{V}\bar{Y}$

$$= 230\angle 0 \times (0.156 - j\,0.019)$$

$$= 230\angle 0 \times 0.157\angle -6.94°$$

$$= 36.11\angle -6.94° \text{A}$$

$$\text{p.f.} = \cos\phi = \cos(-6.94°)$$

$$= 0.992 \text{ lagging} \qquad \because \phi \text{ is } -\text{ve}$$

Example 5.23 When a 240 V, 50 Hz supply is applied to a resistor of 15 ohms is parallel with an inductor, the total current is 22.1 A. What value must the frequency have for the total current to be 34 A?

Solution Referring to Fig. E.5.13,

Supply = 240 V, 50 Hz = 15 ohms $I = 22.1$ A

$$\text{Conductance of the circuit } G = \frac{1}{R} = \frac{1}{15} = 0.067 \text{ mho}$$

$$\text{Susceptance of the circuit } B = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{(2\pi \times 50)L}$$

$$= \frac{0.00318}{L}$$

Admittance of the circuit = $G - jB$

$$= 0.067 - j\,0.00318/L$$

$$I = 22.1 \text{ A}, V = 240 \text{ volts}$$

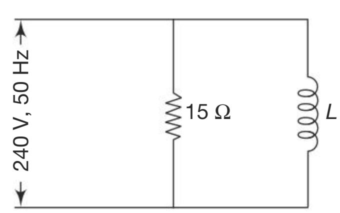


Fig. E.5.13

$$\text{Total admittance } Y = \frac{1}{V} = \frac{22.1}{240} = 0.092 \text{ mho}$$

$$|\overline{Y}| = \sqrt{0.067^2 + \frac{0.00318^2}{L^2}} = 0.092\ \Omega$$

$$0.067^2 + \left(\frac{0.00318}{L}\right)^2 = 0.0085$$

$$0.0045 + \frac{1.011 \times 10^{-5}}{L^2} = 0.0085$$

$$\frac{1.011 \times 10^{-5}}{L^2} = 0.0085 - 0.0045 = 0.004$$

$$L^2 = \frac{1.011 \times 10^{-5}}{0.004} = 2.53 \times 10^{-3};\ L = 0.05 \text{ H}$$

When the current is 34 A,

$$\text{Total admittance } Y = \frac{1}{V} = \frac{34}{240} = 0.142 \text{ mho}$$

$$\text{Conductance } G = \frac{1}{R} = 0.067 \text{ mho}$$

$$B = \frac{1}{2\pi f L} = \frac{1}{2\pi \times 0.05 \times f} = \frac{3.183}{f}$$

$$\text{Admittance} = \sqrt{0.067^2 + \left(\frac{3.183}{f}\right)^2} = 0.142\ \mho$$

$$0.067^2 + \left(\frac{3.183}{f}\right)^2 = 0.142^2 = 0.02$$

$$\frac{10.13}{f^2} = 0.02 - 0.00449 = 0.016;\ f^2 = \frac{10.13}{0.016} = 633.125$$

$$f = 25.2 \text{ Hz}$$

Example 5.24 A coil of resistance 15 ohms and inductance 0.05 H is connected in parallel with non-inductive resistor of 20 ohms. Find (i) the current in each branch and the total current supplied and (ii) the phase angle of the combination when a voltage of 200 V at 50 Hz is applied. Draw the phasor diagram.

Solution Refer Fig. E.5.14

$R_1 = 15$ ohms, $L = 0.05$ H, $R_2 = 20$ ohms, Supply = 200 V, 50 Hz

(i) Admittance of branch = 1 = $\overline{Y}_1 = \dfrac{1}{R_1 + jXL}$

$$Y_1 = \frac{1}{15 + j\,(2\pi \times 50 \times 0.05)} = \frac{1}{15 + j\,15.71} = \frac{15 + j15.71}{471.8}$$

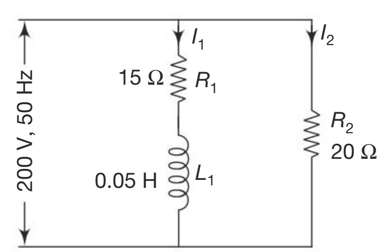


Fig. E.5.14

$= 0.032 - j0.033$ mho

Current in branch $1 = \bar{I}_1 = \bar{V}\bar{Y}_1 = 200 \times (0.032 - j0.033)$

$= 200 \times 0.0459 \angle -45.88°$

$\bar{I}_1 = 9.18 \angle -45.88°$A

Admittance of branch $2 = \bar{Y}_2 = \dfrac{1}{20} = 0.05$ mho

Current in branch $2 = \bar{V}\bar{Y}_2 = 200 \times 0.05$

$\bar{I}_2 = 10 \angle 0°$ A

Total current supplied $\bar{I} = \bar{I}_1 + \bar{I}_2$

$\bar{I} = 9.18 \angle -45.88° + 10 \angle 0°$

$= 6.39 - j6.59 + 10 = 16.39 - j6.59$

Total current $I = 17.67 \angle -21.9°$ A

(ii) Phase angle of the combination = – 21.9°

Phasor diagram is shown in Fig. E.5.15.

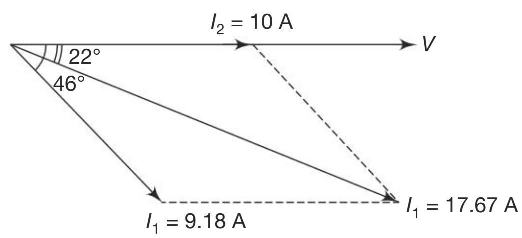


Fig. E.5.15

Example 5.25 A coil of inductance 6 mH and resistance 40 ohms is connected across a supply which has a sinusoidal EMF of 100 V and a frequency of 800 Hz. Also across the supply is a circuit consisting of perfect capacitor 4 μF in series with resistance of 50 ohms. Find (i) the total current taken from the supply and (ii) the phase angle between the currents in the coil and the capacitor. Draw the phasor diagram.

Solution Referring to Fig. E.5.16

$$R_1 = 40\ \Omega \quad L = 6\text{ mH}, \quad R_2 = 50\ \Omega \quad C = 4\ \mu\text{F}$$

Supply = 100 V, 800 Hz

$$X_L = \omega L = 2\pi f L = 2\pi \times 800 \times 6 \times 10^{-3} = 30.16 \text{ ohms}$$

$$X_C = \frac{1}{\omega c} = \frac{1}{2\pi \times 800 \times 4 \times 10^{-6}} = 49.74 \text{ ohms}$$

Admittance of branch $1 = \bar{Y}_1 = \dfrac{1}{R_1 + jX_L}$

$$\bar{Y}_1 = \frac{1}{40 + j30.16} = \frac{40 - j30.16}{2509.63};\ \bar{Y}_1 = 0.016 - j0.012 \text{ mho}$$

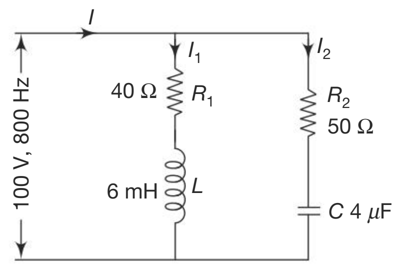


Fig. E.5.16

Admittance of branch 2 $= \overline{Y}_2 = \dfrac{1}{R_2 - jX_C}$

$$\overline{Y}_2 = \frac{1}{50 - j49.74} = \frac{50 + j49.74}{50^2 - j49.74^2} = \frac{50 + 49.74}{4974.07}$$

$$= 0.01 + j0.0099 \text{ mho}$$

$$\text{Current } \overline{I}_1 = \overline{V}\,\overline{Y}_1 = 100\,(0.016 - j0.012)$$

$$= 1.6 - j1.2\text{A} = 2 \angle -36.87° \text{ A}$$

$$\text{Current } \overline{I}_2 = \overline{V}\,\overline{Y}_2 = 100\,(0.01 + j0.0099) = 1 + j0.99 \text{ A} = 1.41 \angle 44.71° \text{ A}$$

$$\text{Total current } \overline{I} = \overline{I}_1 + \overline{I}_2$$

$$= 1.6 - j1.2 + 1 + j0.99$$

$$= 2.6 - j0.21 = 2.61 \angle -4.62° \text{ A}$$

Phase angle between current in the coil and capacitor

$= \text{phase angle between } I_1 \text{ and } I_2 = 44.71 + 36.87$

$= 81.58°$

The phasor diagram is shown in Fig. E.5.17.

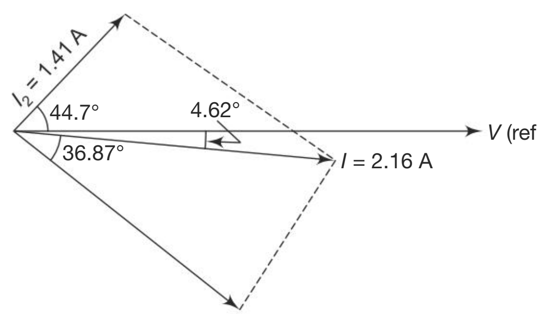


Fig. E.5.17

Example 5.26 Two circuits, the impedances of which are given by $\overline{Z}_1 = 10 + j15$ ohms $\overline{Z}_2 = 6 - j8$ ohms are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch?

Solution Referring to Fig. E.5.18

$$\overline{Z}_1 = 10 + j15 \text{ ohms}; \overline{Z}_2 = 6 - j8 \text{ ohms}, I = 15 \text{ A}$$

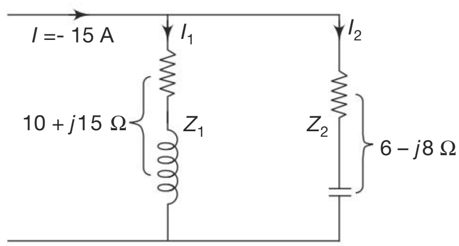


Fig. E.5.18

Current through branch 1

$$\bar{I}_1 = \frac{6 - j8}{10 + j15 + (6 - j8)} \times 15 \; (\because \text{ using current division}$$

$$= \frac{6 - j8}{16 + j7} \times 15 = \frac{15 \times 10 \angle -53.13^\circ}{17.46 \angle 23.63^\circ}$$

$$\bar{I}_1 = 8.59 \angle -76.76^\circ \text{ A}$$

Current through branch $2 = \bar{I}_2 = \dfrac{10 + j15}{10 + j15 + (6 - j8)} \times 15$

$$= \frac{10 + j15}{16 + j7} \times 15$$

$$\bar{I}_2 = \frac{18.03 \angle 56.31^\circ}{17.46 \angle 23.63^\circ} \times 15 = 15.49 \angle 32.68^\circ$$

Power consumed by branch $1 = I_1^2 R_1 = 8.59^2 \times 10 = 737.88$ W

Power consumed by branch $2 = I_2^2 R_2 = 15.49^2 \times 6$

$= 1439.6$ W

Example 5.27 Two coils A and B are connected in parallel and a voltage of 200 V at 50 Hz applied to their common terminals. The coils have resistances of 10 ohms and 5 ohms and inductances of 0.023 H and 0.035 H respectively. Find (i) the current in each coil and total current, and (ii) p.f. of the combination.

Solution Referring to Fig. E.5.19.

Supply = 200 V, 50 Hz,

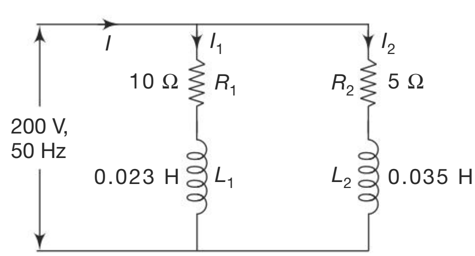


Fig. E.5.19

$$R_1 = 10 \text{ ohms}, L_1 = 0.023 \text{ H}, R_2 = 5 \text{ ohms}, L_2 = 0.035 \text{ H}$$

$$X_{L_1} = \omega L_1 = 2\pi f L_1$$

$$= 2\pi \times 50 \times 0.023 = 7.23 \text{ ohms}$$

$$X_{L_2} = \omega L_2 = 2\pi f L_2 = 2\pi \times 50 \times 0.035 = 10.99 \text{ ohms}$$

Admittance of branch 1 $= \overline{Y}_1 = \dfrac{1}{10 + j7.23}$

$$= \frac{10 + j7.23}{152.27} = 0.066 - j0.047 \ \mho$$

Admittance of branch 2 $= Y_2 = \dfrac{1}{5 + j10.99} \mho$

$$Y_2 = 5 - \frac{j10.99}{145.78} = 0.034 - 0.075 \ \mho$$

Total admittance $= \overline{Y} = \overline{Y}_1 + \overline{Y}_2$

$$\overline{Y} = 0.066 - j0.047 + 0.034 - j0.075$$

$$= 0.1 - j0.122 \ \mho$$

Current through branch 1 $= \overline{I}_1 = \overline{V}\,\overline{Y}_2$

$$\overline{I}_1 = 200 \times (0.066 - j0.047)$$

$$= 200 \times 0.081 \angle -35.46^\circ = 16.2 \angle -35.46^\circ \text{ A}$$

Current through branch 2 $= \overline{I}_2 = \overline{V}\,\overline{Y}_2$

$$= 200 \times (0.034 - j\,0.075) = 200 \times 0.082 \angle -65.61^\circ$$

$$= 16.4 \angle -65.61^\circ \text{ A}$$

Total current $\overline{I} = \overline{I}_1 + \overline{I}_2$

$$= 16.2 \angle -35.46^\circ + 16.4 \angle -65.61^\circ = 13.19 - j9.39 + 6.77 - j14.94$$

$$= 19.96 - j24.33 = 31.47 \angle -50.64^\circ \text{ A}$$

p.f of combination = cos (–50.64) = 0.634 lagging

Phasor diagram is shown in Fig. E.5.20.

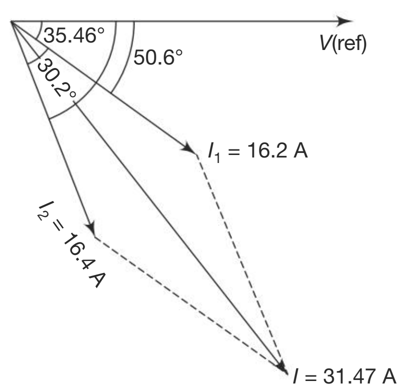


Fig. E.5.20

Example 5.28 Find the magnitude and phase of currents in the two branches and the total current in the circuit shown in Fig. E. 5.21.

Solution

$$X_1 = 2\pi \times 50 \times 0.015 = 4.71\ \Omega$$
$$X_2 = 2\pi \times 50 \times 0.00 -25.13\ \Omega$$

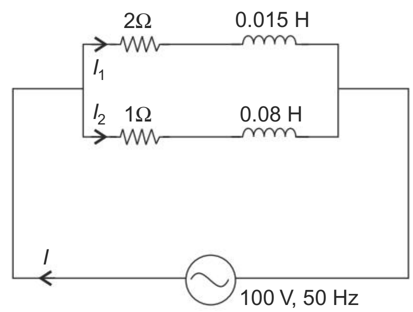


Fig. E.5.21

Impedance $\bar{Z}_1 = R_1 + jx_1 = 2 + j\,4.71 = 5.11\ 67°\ \Omega$

Impedance $\bar{Z}_2 = R_2 + jx_2 = 1 + j\,25.13 = 25.15\ 87.7°\ \Omega$

Applied voltage, $\bar{V} = 100\quad 0°$

Branch current, $\bar{I}_1 = \dfrac{100\ 0}{5.11\ 67°} = 19.57\ \ 67° = 7.65 - j\,18$ A

Branch current, $\bar{I}_2 = \dfrac{100\ 0}{25.15\quad 87.7°} = 3.98\ 87.7° = 0.16 - j\,3.97$ A

$$\text{Total current} = \bar{I} = \bar{I}_1 + \bar{I}_2 = 7.16 - j\,21.97 = 23.1 - 72°\ \text{A}$$

Example 5.29 In a parallel RC circuit, $R = 8\ \Omega$, $X_C = 12\ \Omega$. What is the admittance?

Solution

$$R = 8\,\Omega \qquad\qquad X_C = -j12\,\Omega.$$

$$\text{Admittance} = \frac{1}{R} + \frac{1}{X_c} = \frac{1}{8} + \frac{1}{-j12} = 0.125 + \text{j}0.083\ ℧$$

Example 5.30 In a parallel *RL* circuit, $R = 3\Omega$, $X_L = 4\Omega$. What is the admittance?

Solution $R = 3\Omega \qquad\qquad X_L = j4\Omega$

$$\text{Admittance} = \frac{1}{3} + \frac{1}{j4} = 0.33 - j0.25\ ℧$$

5.9 RESONANCE

Introduction Inductive reactance $X_L = \omega L$; capacitive reactance $X_C = \dfrac{1}{\omega C}$. The inductive reactance increases as the frequency is increased, but the capacitive reactance decreases with higher frequencies. Thus inductive reactance and capacities reactance have opposite properties. So, for any LC combination, there must be one frequency at which X_L equals X_C. This case of equal and opposite reactances is called resonance. The frequency at which the two reactances are equal is to resonant frequency. Now let us study this resonance condition for both series and parallel circuits.

5.9.1 Resonance in Series A.C. Circuits

Let us consider the series R–L–C circuit.

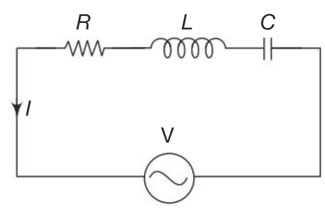


Fig. 5.26

The applied voltage $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

$$= \bar{I}R + j\bar{I}X_L - j\bar{I}X_C$$

$$\bar{V} = \bar{I}\,(R + jX_L - jX_C) \tag{5.58}$$

$$\frac{\bar{V}}{\bar{I}} = R + j(X_L - X_C) \tag{5.59}$$

Total impedance $\bar{Z} = R + j\,(X_L - X_C)$.

The impedance curve is shown in Fig. 5.27.

At a certain frequency, $X_L = X_C$. When $X_L = X_C$ the circuit is said to be in resonance.

At resonant condition.

Total impedance Z is minimum and is equal to R

∴ The circuit will be purely resistive circuit.

∴ p.f. = cos ϕ = 1

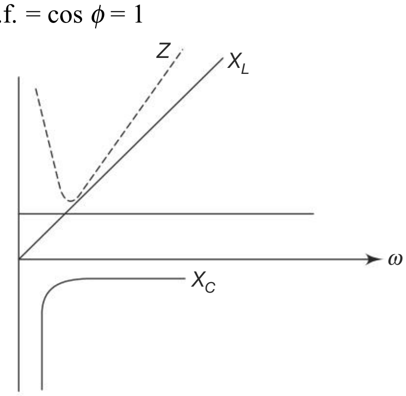


Fig. 5.27

$$\text{current } I = \frac{V}{Z} = \text{maximum } I_0 \quad (\because Z \text{ is minimum}) \tag{5.60}$$

$$V_L = IX_L;\ V_C = IX_C$$

Since $X_L = X_C$ the two voltages are equal in magnitude and opposite in phase. Hence they cancel out each other. The phasor diagram is shown in Fig. 5.28 (a) and 5.28 (b).

$\therefore$ V and I are in phase and hence p.f. is unity.

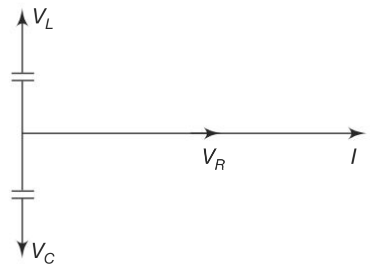


Fig. 5.28 (a)

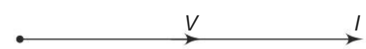


Fig. 5.28 (b)

Resonant Frequency The frequency at which resonance occurs is called resonant frequency.

$$X_L = X_C \tag{5.61}$$

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2\, LC = 1$$

$$\omega_r^2 = \frac{1}{\sqrt{LC}}$$

$$w_r = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{5.62}$$

when $f < f_r$, X_C will be greater than X_L. $\therefore$ p.f. is leading

when $f > f_r$, X_L will be greater than X_C. $\therefore$ p.f. is lagging.

Q Factor It is defined as the voltage magnification at resonance.

At resonance, current I_0 is maximum and is equal to $\frac{V}{R}$. Voltage across inductance

$$V_L = I_0 X_L \tag{5.63}$$

Voltage across capacitance $V_C = I_0 X_C$

Supply voltage $V = I_0 Z$

$$= I_0 R \tag{5.64}$$

$\therefore$ Voltage magnification $= \frac{V_L}{V}$ or $\frac{V_C}{V}$

$$Q = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega L}{R} \tag{5.65}$$

$$Q \text{ factor} = \frac{V_C}{V} = \frac{I_0 X_C}{I_0 R} = \frac{1}{\omega_r CR} \tag{5.66}$$

$$Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$$

$$= \frac{\frac{1}{\sqrt{LC}} \times L}{R} \qquad \because f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{\sqrt{L/C}}{R} = \frac{\sqrt{L}}{R\sqrt{C}} = 1/R\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Bandwidth Bandwidth of a circuit is given by the band of frequencies which lies between two points on either side of the resonant frequency where current falls to $1/\sqrt{2}$ of its maximum value. This band of frequencies also provide resonance effects.

There are two frequencies (f_1 and f_2) on either side of f_r at which current is equal to $\frac{I_0}{\sqrt{2}}$. Power at these two frequencies = I^2R.

$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$= \frac{I_0^2}{2} \times R = \frac{1}{2} \times I_0^2 R$$

$$= \frac{1}{2} \times \text{power at resonance}$$

$\therefore$ frequencies f_1 and f_2 are called half power frequencies.

$$\text{Bandwidth} = \Delta f = f_2 - f_1 \tag{5.67}$$

$$\text{Otherwise B.W.} = \frac{\text{Resonant frequency}}{Q \text{ factor}} = \frac{f_r}{Q} \tag{5.68}$$

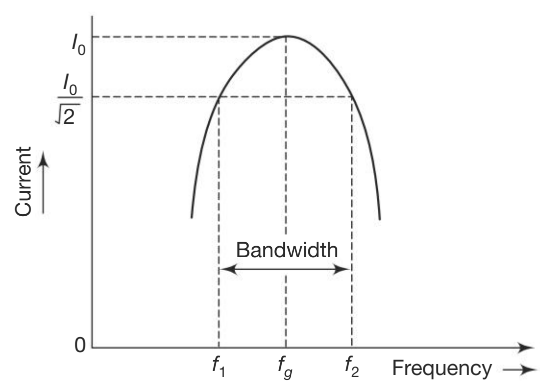


Fig. 5.29

Resonance Curve Resonance curve is shown in Fig. 5.29.

5.9.2 Resonance in Parallel AC Circuits

Let us consider a parallel circuit consisting of two branches as shown in Fig. 5.30.

The impedance of branch 1 = $R_1 + jX_L$

The impedance of branch 2 = $R_2 - jX_C$

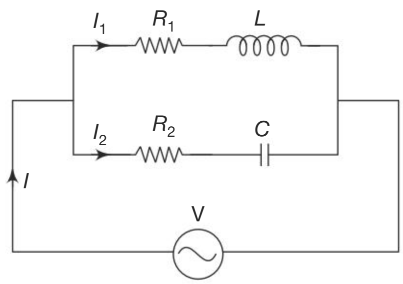


Fig. 5.30

The admittance of the circuit

$$\overline{Y}_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} = \overline{Y}_1 + \overline{Y}_2$$

$$= \frac{1}{R_1 + j\,X_L} + \frac{1}{R_2 + j\,X_C}$$

$$= \frac{(R_1 - j\,X_L)}{(R_1 + j\,X_L)\,(R_1 - j\,XL)} + \frac{R_2 + j\,X_C}{(R_2 + j\,X_C)\,(R_2 - j\,X_C)}$$

$$= \frac{R_1 - j\,X_L}{R_1^2 + X_L^2} + \frac{R_2 + j\,X_C}{R_2^2 + X_C^2}$$

$$= \frac{R}{R_1^2 + X_L^2} + \frac{R_2 - j}{R_2^2 + X_C^2}\left(\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right)$$

At resonance, the circuit is purely resistive.

$\therefore$ There should be no imaginary term in Y_{eq}.

$$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega_r L}{R_1^2 + X_L^2} = \frac{1}{\omega_r C\,(R_2^2 + X_C^2)}$$

$$\frac{LC}{R_1^2 \omega_r^2\, L^2} = \frac{C^2}{\omega_r^2\, R_2^2 + 1}$$

$$LC\,(1 + \omega_r^2 C^2 R_2^2) = C^2\,(R_1^2 + \omega_r^2 L^2)$$

$$LC + \omega_r^2\, LC^3\, R_2^2 = R_1^2\, C^2 + \omega_r^2\, L^2 C^2$$

$$\omega_r^2\, LC^3\, R_2^2 - \omega_r^2\, L^2 C^2 = R_1^2\, C^2 - LC$$

$$\omega_r^2\, LC^2\,(CR_2^2 - L) = R_1^2\, C^2 - LC$$

$$\omega_r^2\, LC\,(CR_2^2 - L) = R_1^2\, C - L$$

$$\omega_r^2 = \frac{CR_1^2 - L}{LC\,(CR_2^2 - L)}$$

$$2\pi f_r = \omega_r = \frac{1}{\sqrt{LC}}\sqrt{\left(\frac{CR_1^2 - L}{CR_2^2 - L}\right)}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}} \qquad (5.69)$$

Let $R_2 = 0$ which is a valid assumption

then $$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{-L}}$$

$$= \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L - CR_1^2}{L}} \qquad (5.70)$$

If R_1 is also negligible

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L}{L}}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ and same as for series resonance (5.71)

If R_1 is not neglected

$$Y_{eq} = \frac{R_1}{R_1^2 + X_L^2} = \frac{R_1}{R_1^2 + \omega_r^2\, L^2}$$

$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{L - CR_1^2}{L - CR_2^2}}$$

$$= \frac{1}{\sqrt{LC}}\sqrt{\frac{L - CR_1^2}{L}} \qquad \because\ R_2 \cong 0$$

$$Y_{eq} = \frac{R_1}{R_1^2 + \dfrac{L}{C}\dfrac{(L - CR_1^2)}{L}} = \frac{R_1}{R_1^2 + \dfrac{L}{C} - \dfrac{CR_1^2}{C}} = \frac{R_1}{\dfrac{L}{C}} = \frac{CR_1}{L}$$

$\therefore$ Impedance at resonance condition

$$= Z = \frac{1}{Y_{eq}} = \frac{L}{CR_1} \qquad (5.72)$$

$\dfrac{L}{CR_1}$ is known as equivalent or dynamic impedance at resonance.

At resonance condition, the imaginary part of admittance is zero.

$\therefore$ Admittance is having minimum value. Impedance is having maximum value.

Current at resonance

$$I_0 = \frac{V}{Z} = \frac{V}{L/CR_1} = \frac{VCR_1}{L} \tag{5.73}$$

Since the impedance is maximum, current will be minimum at resonant condition.

Q-factor Q factor is defined as current magnification at resonant condition.

$$Q = \frac{|I_L|}{|I|} = \frac{|I_C|}{|I|} \tag{5.74}$$

$$\text{Total current } I = \frac{V}{Z} = \frac{VCR_1}{L}$$

$$\text{Current } I_L = \frac{V}{X_L} = \frac{V}{\omega_r L} \quad [\because \; R_1 \text{ is practically very small}]$$

$$Q = \frac{V/\omega_r L}{\frac{VCR_1}{L}} = \frac{L}{\omega_r CR_1 L} = \frac{L}{\omega_r CR_1} \tag{5.75}$$

$$\text{Current } I_C = \frac{V}{X_C} = \omega_r CV$$

$$Q \text{ factor} = \frac{|I_C|}{|I|} = \frac{\omega_r CV}{\frac{VCR_1}{L}} = \frac{\omega_r L}{R_1}$$

$$Q = \frac{\omega_r L}{R_1} = \frac{L}{\omega_r CR_1} \tag{5.76}$$

Bandwidth As in the case of series circuit here also there are two frequencies f_1 and f_2 on either side of f_r. The current at these frequencies will be $I_0 \times \sqrt{2}$.

These two frequencies are called half power frequencies. Bandwidth = $\Delta f = f_2 - f_1$

$$= \frac{\text{Resonant frequency}}{Q \text{ factor}} = \frac{f_r}{Q} \tag{5.77}$$

Resonance Curve Resonance curve is shown in Fig. 5.31.

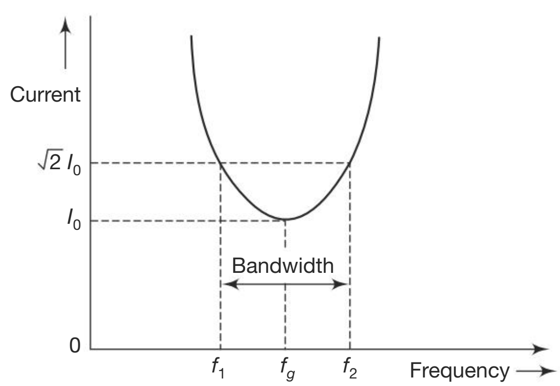


Fig. 5.31

Example 5.31 A series RLC circuit with $R = 10$ ohms. $L = 10$ mH and $C = 1\ \mu$F has an applied voltage of 200 V at resonant frequency. Calculate the resonant frequency, the current in the circuit and the voltages across the elements at resonance. Find also the quality factor and bandwidth.

Solution $R = 10$ ohms; $L = 10$ mH, $C = 1\ \mu$F.

$$\text{Supply voltage} = 200 \text{ V}$$

$$\text{Resonant frequency } f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10\times10^{-3}\times1\times10^{-6}}} = 1591.55 \text{ Hz}$$

$$\text{At resonance, current } I_0 = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

$$\text{Voltage across resistance } V_R = I_0R = 20\times10 = 200 \text{ V}$$

$$\text{Voltage across inductance } V_L = I_0X_L = I\times 2\pi f_rL$$

$$= 20\times2\pi\times1591.55\times10\times10^{-3}$$

$$= 2000 \text{ volts}$$

$$\text{Voltage across capacitance } V_C = I\,X_C = I\times\frac{1}{2\pi f_r C}$$

$$V_C = 20\times\frac{1}{2\pi\times1591.55\times1\times10^{-6}}$$

$$V_C = 2000 \text{ volts}$$

$$\text{Quality factor } Q = \frac{\omega_rL}{R}$$

$$= \frac{2\pi\times1591.55\times10\times10^{-3}}{10}$$

$$Q = 10$$

$$\text{Bandwidth} = \frac{f_r}{Q}$$

$$= \frac{1591.55}{10} = 159.155$$

$$\text{B.W.} = 159.16 \text{ Hz}$$

Example 5.32 A series RLC circuit is connected to a 200 V, 50 Hz supply. When L is varied, the maximum current obtained is 0.4 amperes and the voltage across the capacitor then is 330 volts. Find the circuit constants.

Solution Supply $= 220$ V, 50 Hz

Maximum current $= 0.4$ Amp

Voltage across capacitor $V_C = 330$ V

Maximum current corresponds to resonant condition.

$\therefore$ At resonant condition, current $I_0 = 0.4$ A

Voltage across capacitance $V_C = 330$ volts.

$$\text{Capacitive reactance } X_c = \frac{V_C}{I_0}$$

$$= \frac{330}{0.4} = 825 \text{ ohms}$$

$$C = \frac{1}{2\pi f x_c} = \frac{1}{2\pi \times 50 \times 825}$$

$$= 3.85 \times 10^{-6} \text{ F}$$

$$C = 3.85\ \mu\text{F}$$

$$\text{At resonance condition } X_C = X_L$$

$$X_L = 825 \text{ ohms}$$

$$2\pi f L = 825$$

$$L = \frac{825}{2\pi \times 50} = 2.63 \text{ H}$$

Total impedance of the circuit $Z = R$ at resonant frequency

$$\therefore \quad \text{Resistance } R = \frac{V}{I_{\max}} = \frac{200}{0.4} = 550 \text{ ohms}$$

Example 5.33 Find the resonant frequency for the circuit as shown in Fig. E.5.22.

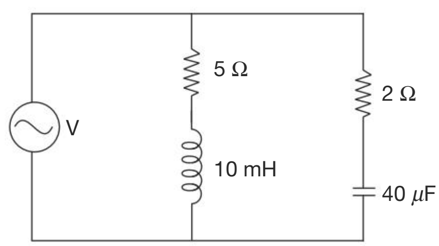


Fig. E.5.22

Solution

$$R_1 = 5 \text{ ohms}, L = 10 \text{ mH},$$

$$R_2 = 2 \text{ ohms}, C = 40\ \mu\text{F}$$

$$\text{Resonant frequency} \quad f_r = \frac{1}{2\pi \times \sqrt{LC}} \times \sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

$$f_r = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \times 40 \times 10^{-6}}} \sqrt{\frac{(40 \times 10^{-6} \times 5^2) - 10 \times 10^{-3}}{(40 \times 10^{-6} \times 2^2) - 10 \times 10^{-3}}}$$

Example 5.34 A coil of 20 ohms resistance has an inductance of 0.2 H and is connected in parallel with a 100 μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistance. Find also the value of that resistance.

Solution The circuit is shown in Fig. E.5.23.

The circuit will act as a non-inductive resistance at resonant condition only.

$$\text{Resonant frequency } f_r = \frac{1}{2\pi\sqrt{LC}} \times \sqrt{\frac{L - CR_1^2}{L}}$$

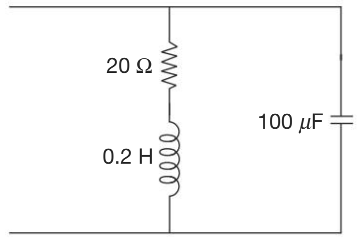


Fig. E.5.23

$$f_r = \frac{1}{2\pi\sqrt{0.2 \times 100 \times 10^{-6}}}\sqrt{\frac{0.2 - (100 \times 10^{-6} \times 20^2)}{0.2}}$$

$$f_r = 31.83 \text{ Hz}$$

Value of resistance = value of impedance at resonant condition

$$= \frac{L}{CR_1}$$

$$= \frac{0.2}{100 \times 10^{-6} \times 20}$$

$$= 100\ \Omega$$

Example 5.35 A series RLC circuit with $Q = 250$ is resonant at 1.5M Hz. Find the frequencies at half power points and bandwidth.

Solution:

$$Q = 250 \qquad f_r = 1.5 \times 10^6 \text{ Hz}$$

Bandwidth, $\Delta_f = \dfrac{f_r}{Q} = \dfrac{1.5 \times 10^6}{250} = 6000 \text{ Hz}$

∴ Half power frequencies $= f_r + \Delta f, \quad f_r - \Delta f,$

$= 1506000, 1494000.$

Example 5.36 Find the resonant frequency in the ideal parallel LC circuit with $L = 40$ mH and $C = 0.01\ \mu$F.

Solution:

$$L = 40 \times 10^{-3}\ H \qquad C = 0.01 \times 10^{-6} \text{ F}$$

Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{40 \times 10^{-3}} \times 0.01 \times 10^6}$$

$= 7957.7$

$\cong 7958$ Hz.

Example 5.37 A120 V source whose frequency is variable, is impressed across a series RLC circuit with $R = 50\ \Omega$, $L = 0.5$ H and $C = 50\ \mu$F. Determine: the following, resonant frequency of the circuit, current at resonance, voltage developed across L and C at resonance, Q factor, bandwidth, and new value of capacitance required to change the resonance frequency to 300 rad/sec.

Solution

$V = 120$ volts, $R = 50\ \Omega$, $L = 0.5$H

$C = 50 \times 10^{-6}$ F

Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.5 \times 50 \times 10^{-6}}} = 31.8$ Hz.

Current at resonance, $I_o = \dfrac{V}{R} = \dfrac{120}{50} = 2.4$ A

Voltage developed across inductor $= jI_o \times L$

$= j2.4 \times 2\pi \times 31.8 \times 0.5$

$= j240$ V

Voltage developed across capacitor$= -j240$ V

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{50}\sqrt{0.\frac{5}{50 \times 10^{-6}}} = 2.$$

$$\text{Band width } = \frac{f_r}{Q} = 31.\frac{8}{2} = 15.9$$

Given resonance is to occur at 300 rad/sec.

i.e, $2\pi f_r = 300.$

Let C_r be the new value of capacitance required.

then, $2\pi f_r = \dfrac{1}{\sqrt{LC_n}}.$

$\therefore$ $C_n = \dfrac{1}{L.(2\pi fr)^2} = \dfrac{1}{0.5 \times 300^2} = 22\mu$F.

Example 5.38 An inductor has a Q factor of 45 at a frequency of 600 kHz. Calculate its Q factor at a frequency of 1000 kHz assuming that its resistance is 50 percent greater at this frequency than at 600 kHz.

Solution: $Q = 45$ $f_1 = 600$ kHz. Resistance $= R$

$f_2 = 1000$ kHz. Resistance $= 1.5\ R$

$\therefore$ Q factor at the new frequency

$$= \frac{W_2 L}{1.5\,R}$$

But $L = \dfrac{QR}{\omega_1} = \dfrac{45 \times R}{2\pi \times 600 \times 10^3}$

$\therefore$ New Q factor $= \dfrac{2\pi \times 1000 \times 10^3 \times 45 \times R}{1.5R \times 2\pi \times 600 \times 10^3}$

$= 50$

Example 5.39 A series RLC circuit with $R = 4\,\Omega$, $L = 100\ \mu$H, $C = 250$ pF, is connected to a constant voltage generator of variable frequency. Determine the resonant frequency, Q factor and the half power frequencies.

Solution

$R = 4\,\Omega$ $L = 100 \times 10^{-6}$ H C$= 250 - 10^{-12}$ F

Resonant frequency, $f_r = 2\pi\sqrt{LC}$

$$= \frac{1}{2\pi\sqrt{100 \times 10^{-6} \times 250 \times 10^{-12}}}$$

$$= 1 \text{ MHz} = 1000000 \text{ Hz}$$

$$Q \text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{4}\sqrt{\frac{100 \times 10^{-6}}{250 \times 10^{-12}}} = 158$$

$$\text{Band width} = \frac{f_r}{Q} = \frac{10^6}{15^8} = 6330 \text{ Hz}$$

$\therefore$ Half power frequencies $= f_r + \Delta f, \quad f_r - \Delta f$

$$= 1006330, 993670$$

Example 5.40 A series RLC circuit with $R = 10\,\Omega$, $L = 1$ mH, $C = 1000$ pF, is connected across a sinusoidal source of 20 V with variable frequency. Calculate the following: resonant frequency, Q factor and the half power frequencies.

Solution

$R = 10\ \Omega \quad L = 1 \times 10^{-3}$ H $\quad C = 1000 \times 10^{-12}$ F $\quad V = 20$ volts.

Resonant frequency, $\quad f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 1000 \times 10^{-12}}} = 0.159 \times 10^6 \text{ Hz.}$$

$$Q \text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{1 \times 10^{-3}}{1000 \times 10^{-12}}} = 100$$

Bandwidth, $\quad \Delta f = \dfrac{fr}{Q} = \dfrac{0.159 \times 10^6}{100} = 0.159 \times 10^4 = 1590$ Hz

$\therefore$ Half power frequencies $= f_r + \Delta f, \quad f_r - \Delta f$

$$= 160590, 157410.$$

5.10 THREE PHASE AC CIRCUITS

We have seen in Section 4.3.2, how 3-phase balanced emf can be generated. We have also mentioned about phase sequence. In this section we analyse the two possible types of connections and the method of measuring power in a 3-phase circuit.

Three Phase Connections There are two possible connections in 3-phase system. One is star (or wye) connection and the other is delta (or mesh) connection. Each type of connection is governed by characteristic equations relating the currents and the voltages. Phasor diagrams plays a vital role in this analysis.

5.10.1 Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called the starpoint or the neutral point. The free ends of the three phase coils will be operating at specific potentials with respect to the potential at the star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system (Fig. 5.32) and three phase three wire system (Fig. 5.33).

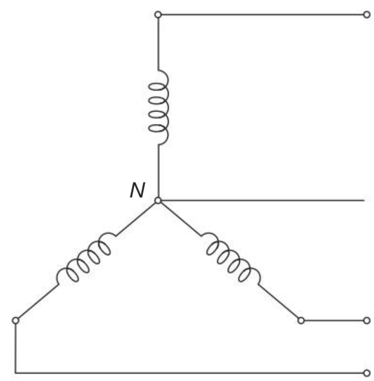


Fig. 5.32

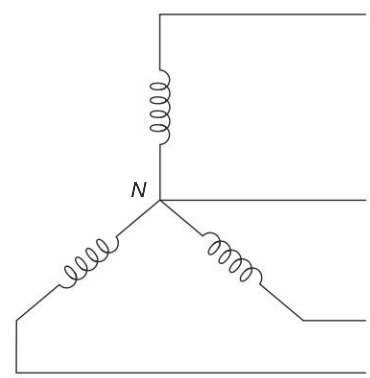


Fig. 5.33

Analysis Let us analyse the relationship between currents and relationship between voltages. We also arrive at the power equations.

Notations Defined

E_R, E_Y, E_B	:	Phase voltages of R, Y and B phases
I_R, I_Y, I_B	:	Phase currents
V_{RY}, V_{YB}, V_{BR}	:	Line voltages
I_{L1}, I_{L2}, I_{L3}	:	Line currents

In a balanced system,

$$E_R = E_Y = E_B = E_P \qquad V_{RY} = V_{YB} = V_{BR} = V_L$$
$$I_R = I_Y = I_B = I_P \qquad I_{L1} = I_{L2} = I_{L3} = I_L$$

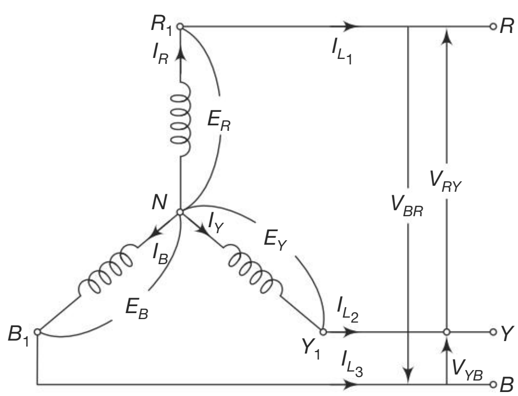


Fig. 5.34

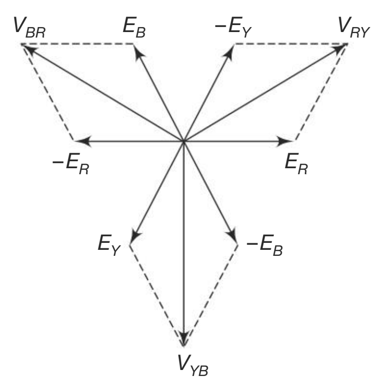


Fig. 5.35

Current Relationship Applying Kirchhoff's current law at nodes R_1, Y_1, B_1 we get $I_R = I_{L1}$; $I_Y = I_{L2}$; $I_B = I_{L3}$.

This means that in a balanced star connected system, phase current equals the line current

$$I_P = I_L.$$

Voltage Relationship Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R, V_{RY} and E_Y. We have

$$\overline{E}_R - \overline{E}_Y = \overline{V}_{RY}$$

Using law of parallelogram,

$$|\overline{V}_{RY}| = V_{RY} = \sqrt{E_R^2 + E_Y^2 + 2\,E_R\,E_Y \cos 60°}$$

$$= \sqrt{E_p^2 + E_P^2 + 2\,E_P\,E_P\,(\%0)} = E_P\sqrt{3} \tag{5.78}$$

Similarly,

$$\overline{E}_Y - \overline{E}_B = \overline{V}_{YB} \quad \text{and} \quad \overline{E}_B - \overline{E}_R = \overline{V}_{BR}$$

$\therefore$

$$\overline{V}_{YB} = E_P\sqrt{3} \text{ and } \quad V_{BR} = E_P\sqrt{3}$$

Thus,

$$V_L = \sqrt{3}\,E_P \tag{5.79}$$

Line voltage = $\sqrt{3}$ phase voltage

Power Relationship Let $\cos\phi$ be the power factor of the system.

Power consumed in one phase = $E_P\,I_P \cos\phi$

Power consumed in three phases = $3E_P\,I_P \cos\phi$

$$= 3\,\frac{V_L}{\sqrt{3}}\,I_L \cos\phi$$

$$= \sqrt{3}\,V_L\,I_L \cos\phi \text{ watts} \tag{5.80}$$

Reactive power in one phase = $E_P\,I_P \sin\phi$

Total reactive power = $3E_P\,I_P \sin\phi$

$$= \sqrt{3}\,V_L\,I_L \sin\phi \text{ VAR} \tag{5.81}$$

Apparent power per phase = $3\,E_P\,I_P$

Total apparent power = $3E_P\,I_P = \sqrt{3}\,V_L\,I_L$ volt amp (5.82)

5.10.2 Delta Connection

Here the dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Let us now analyse the above connection.

The system is a balanced one. Hence the currents and the voltages will be balanced. Notations used in the star connection are used here.

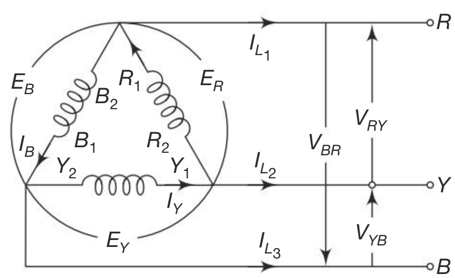


Fig. 5.36

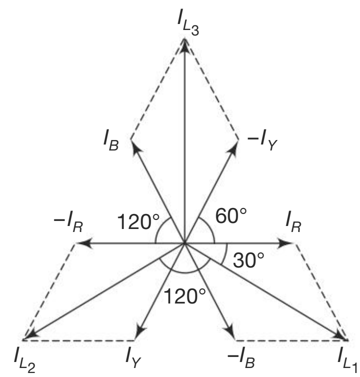


Fig. 5.37

$E_R = E_Y = E_B = E_P$, phase voltage
$I_R = I_Y = I_B = I_P$, phase voltage
$V_{RY} = V_{YB} = V_{BR} = V_L$, line voltage
$I_{L1} = I_{L2} = I_{L3} = I_L$, line voltage

Voltage Relationship Applying Kirchhoff's voltage law to the loop consisting of E_R and V_{RY}, we have $E_R = V_{RY}$.

Similarly, $E_Y = V_{YB}$ and $E_B = V_{BR}$,

Thus $E_P = V_L$.

Phase Voltage = Line Voltage

Current Relationship Applying Kirchhoff's current law at the junction of R_1 and B_2, we have $\bar{I}_R - \bar{I}_B = \bar{I}_L 1$.

Referring to the phasor diagram and applying the law of parallelogram, we have

$$I_L I = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60°}$$

$$= \sqrt{I_P^2 I_P^2 + 2 I_P I_P (\%0)}$$

$$= I_P\sqrt{3} \tag{5.83}$$

Similarly, we have

$$\bar{I}_Y - \bar{I}_R = \bar{I}_L 2 \quad \text{and} \quad \bar{I}_B - \bar{I}_Y = \bar{I}_L 3$$

Hence, $I_L 2 = I_P\sqrt{3}$ and $I_L^3 = I_P\sqrt{3}$

Thus, line current = $\sqrt{3}$ phase current

$$I_L = \sqrt{3}\, I_P \tag{5.84}$$

Power Relationship Let $\cos\phi$ be the power factor of the system.

Power per phase = $E_P I_P \cos\phi$

Total power for all the three phases = $3 E_P I_P \cos\phi$

$$= 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3}\, V_L I_L \cos\phi \text{ watts} \tag{5.85}$$

$$\text{Reactive power in one phase} = E_P I_P \sin\phi$$

$$\text{Total reactive power} = 3 E_P I_P \sin\phi$$

$$= \sqrt{3}\, V_L I_L \sin\phi \text{ VAR} \tag{5.86}$$

$$\text{Apparent power per phase} = E_P I_P$$

$$\text{Total apparent power} = 3 E_P I_P = \sqrt{3}\, V_L I_L \text{ volt amp} \tag{5.87}$$

5.10.3 Measurement of Power in 3-Phase Circuits

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phases are not of same value. As a result, the currents flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The four conditions are represented in Figs. 5.38 to 5.41.

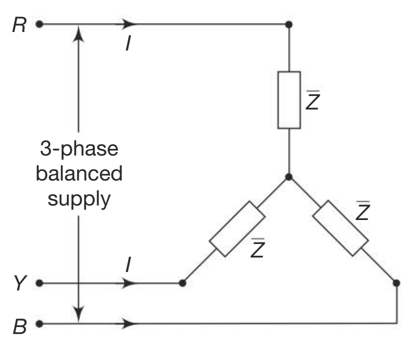


Fig. 5.38

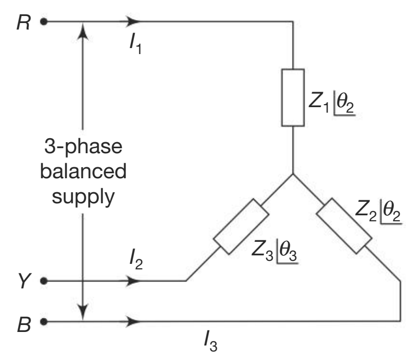


Fig. 5.39

How do we measure the power consumed in three phase circuits? This is done by using wattmeters. It is enough to use two wattmeters to measure the power consumed in three phase circuits-balanced or unbalanced. And if the circuit is balanced we can also find the circuit power factor from the readings of the two wattmeters.

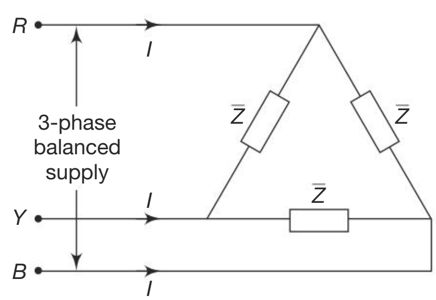


Fig. 5.40

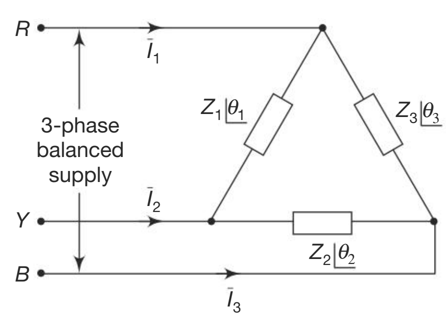


Fig. 5.41

Before analysing the measurement of power using wattmeters, let us mention here how the wattmeter measures power. The wattmeter, basically an indicating instrument, consists of a current coil and a pressure (or voltage) coil. The current coil is connected in series with the load and the pressure coil, across the load. Thus, the deflecting torque produced is proportional to the power (in watts) being measured. The pointer gives a proportionate deflection on a scale calibrated in terms of watts.

Case (a) Star Connected Load In this section we analyse the measurement of power when the load is star connected. The following assumptions are made:

(i) The 3-phase supply to which the load in connected, is balanced.
(ii) The phase sequence is R, Y, B
(iii) The load is balanced
(iv) The load is R-L in nature.

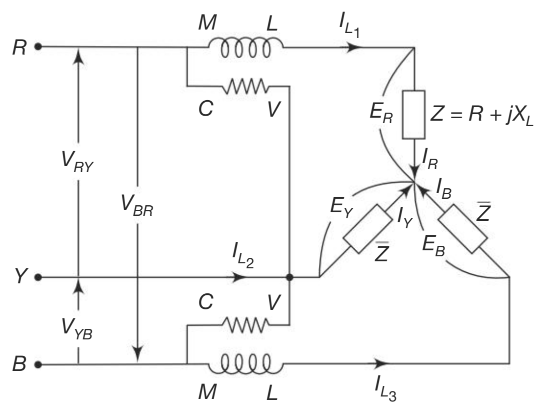


Fig. 5.42

The usual notations are used (as in section 5.10.1)

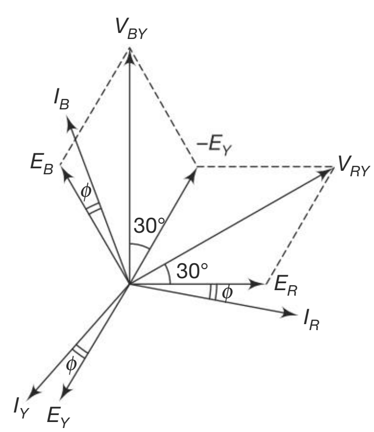


Fig. 5.43

For wattmeter 1

$$\text{Current measured} = \bar{I}_L 1 = \bar{I}_R$$
$$\text{Voltage measured} = \bar{V}_{RY}$$
$$\text{Phase angle between them} = 30° + \phi$$
$$\text{Power measured} = P_1 = V_{RY} I_R \cos (30° + \phi)$$
$$= V_L I_L \cos (30° + \phi) \quad (5.88)$$

For wattmeter 2

$$\text{Current measured} = \bar{I}_{L3} = \bar{I}_B$$
$$\text{Voltage measured} = \bar{V}_{BY}$$
$$\text{Phase angle between them} = 30° - \phi$$
$$\text{Power measured} = P_2 = V_{RY} I_R \cos (30° - \phi)$$
$$= V_L I_L \cos (30° - \phi) \quad (5.89)$$

Now,
$$P_1 + P_2 = V_L I_L \cos (30° + \phi) + V_L I_L \cos (30° - \phi) = V_L I_L$$
$$[\cos 30° \cos \phi - \sin 30° \sin \phi + \cos 30° \cos \phi - \sin 30° \sin \phi]$$
$$= V_L I_L 2 \frac{\sqrt{3}}{2} \cos \phi$$
$$= \sqrt{3} V_L I_L \cos \phi = \text{Total power in a 3-phase circuit} \quad (5.90)$$

Thus, two wattmeters connected appropriately in a 3-phase circuit can measure the total power consumed in the circuit.

To Obtain Expression for Power Factor

$$P_2 - P_1 = V_L I_L \cos (30° - \phi) - V_L I_L \cos (30° + \phi)$$
$$= V_L I_L 2 \sin 30° \sin \phi = V_L I_L \sin \phi \quad (1)$$
$$\frac{P_2 - P_1}{P_2 + P_1} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} = \frac{\tan \phi}{\sqrt{3}} \quad (2)$$
$$\therefore \quad \tan \phi = \sqrt{3} \left[\frac{P_2 - P_1}{P_2 + P_1}\right] \quad (5.91)$$
$$\therefore \text{ Power factor, } \cos \phi = \cos \left\{\tan^{-1} \sqrt{3}\left[\frac{P_2 - P_1}{P_2 + P_1}\right]\right\} \quad (5.92)$$

Case (b) Delta Connected Load In this section we analyse the measurement of power when the load is delta connected. The following assumptions are made:

(i) The 3-phase supply to which the load is connected is balanced.
(ii) The phase sequence is *R*, *Y*, *B*.
(iii) The load is balanced.
(iv) The load is *R*–*L* is nature.

With the usual notations as in Section 5.10.1, the expression for the power measured by the wattmeters are found as follows.

For Wattmeter 1

$$\text{Voltage measured} = \bar{V}_{RY} = \bar{E}_R$$
$$\text{Current measured} = \bar{I}_{L1} = \bar{I}_R - \bar{I}_B$$
$$\text{Phase angle between them} = 30° + \phi$$
$$\therefore \quad \text{Power measured} = P_1 = \bar{V}_{RY} I_{L1} \cos (30° + \phi)$$
$$= V_L I_L \cos (30° + \phi) \quad (5.93)$$

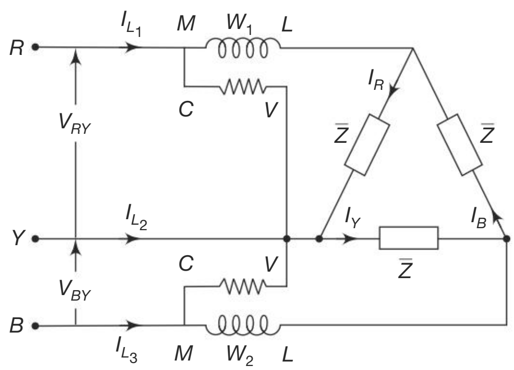


Fig. 5.44

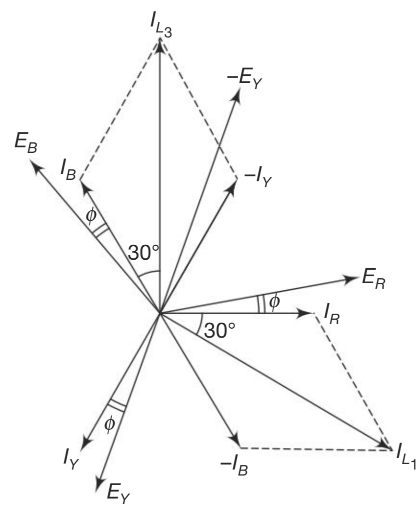


Fig. 5.45

For Wattmeter 2

Voltage measured = $\overline{V}_{BY} = -\overline{E}_Y$

Current measured = $\bar{I}_{L3} = \bar{I}_B - \bar{I}_Y$

Phase angle between them = $30° - \phi$

$\therefore$ Power measured = $P_2 = E_B I_{L3} \cos(30° - \phi)$

$$= V_L I_L \cos(30° - \phi) \tag{5.94}$$

Hence $P_1 + P_2 = V_L I_L [\cos(30° + \phi) + \cos(30° - \phi)]$

$$= V_L I_L \, 2 \cos 30° \cos \phi$$

$$= \sqrt{3}\, V_L I_L \cos \phi \tag{5.95}$$

= Total power consumed in the circuit.

$$P_2 - P_1 = V_L I_L [\cos(30° - \phi) - \cos(30° + \phi)]$$

$$= V_L I_L 2 \sin 30° \sin \phi$$

$$= V_L I_L \sin \phi \tag{5.96}$$

$$\tan \phi = \sqrt{3}\left(\frac{P_2 - P_1}{P_2 + P_1}\right) \tag{5.97}$$

$$\text{p.f.} = \cos\left\{\tan^{-1} \sqrt{3}\left[\frac{P_2 - P_1}{P_2 + P_1}\right]\right\} \tag{5.98}$$

It can be noted from the above two analyses that the readings of the individual wattmeters remain the same for the quantum of load irrespective of the nature of connection. In fact it will be interesting for the reader to prove that two wattmeters are sufficient to measure the three phase power whatever be the load condition (**Hint:** work out in terms of instantaneous values).

Some Interesting Cases to Note

Case (1) Let $\phi = 0$. This means that power factor of the load is unity.

$$P_1 - P_2 = V_L I_L \cos 30°$$

Both the wattmeters give equal reading.

Case(2) Let $\cos \phi = 0.5$

This means that $\phi = 60°$

$$\therefore \quad P_1 = V_L I_L \cos (30 + 60°) = 0$$

Thus wattmeter 1 will give a zero reading.

Case (3) Let cos $\phi < 0.5$

This means that $\phi > 60°$

$$\therefore \quad P_1 = V_L I_L \cos (30 + \phi)$$

$$= V_L I_L \cos (\phi) \text{ when } \phi > 90°$$

$\therefore$ P_1 will be negative.

Hence, when the power factor of the load is less than 0.5 lagging, wattmeter one will read a negative power.

Case (4) Let ϕ be leading.

This means that the load is $R-C$ combination.

Then $$\tan \phi = \sqrt{3}\left(\frac{P_2 - P_1}{P_1 + P_2}\right)$$

Example 5.41 Two wattmeters are used to measure the power in a three phase load. Determine the power and the power factor if the wattmeters read (i) 1000 W each, both positive and (ii) 1000 W each, of opposite sign.

Solution

Case (i)

$$P_1 = 1000 \text{ W}$$

$$P_2 = 1000 \text{ W}$$

$$\text{Total power} = P_1 + P_2 = 2000 \text{ W}$$

$$\text{Power factor} = \cos \phi$$

$$= \cos (\tan^{-1} \sqrt{3}\ (0))$$

$$= 1$$

Case (ii) $P_1 = 1000$ W

$P_2 = -1000$ W

Total power $= P_1 + P_2 = 0$ watts

Power factor $= \cos\phi = \cos = $ Zero

Example 5.42 A balanced star connected load of $(3 + j4)\ \Omega$ impedance is connected to 400 V, three phase supply. What is the real power consumed by the load?

Solution

$V_L = 400$ Impedance/phase $= Z = 3 + j4 = 5 + \angle 53°\ \Omega$

In a star connected system, phase voltage = (Line Voltage)

$$E_P = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$\therefore$ Current in each phase $= \dfrac{E_P}{Z} = \dfrac{231}{5\angle 53°}$ A

$= 46.02 - \angle 53°$ A

$\therefore$ Line current = 46.02A

$\therefore$ Total power consumed in the load $= \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 46.02 \times \cos(-53°) = 19188$ W

Example 5.43 A 3-phase balanced wye connected load has 400 line to line voltage and 10 A line current. Determine the line to neutral voltage and phase current.

Solution $V_L = 400$ V. $I_L = 10$ A

The system is star connected.

$\therefore$ Phase current = Line current = 10 A

$$\text{Phase Voltage} = \frac{\text{Line Voltage}}{\sqrt{(3)}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Example 5.44 A circuit has two impedances connected in parallel. The values are $Z_1 = (6 - j8)\ \Omega$ and $Z_2 = (16 + j12)\ \Omega$. The current in branch with $(6 - j8)\Omega$ is $(12 + j16)$ Amp. Calculate the current in the other branch, total current, voltage applied to the circuit and power consumed in the circuit.

Solution

$Z_1 = (6 - j8)\ \Omega$ $\qquad I_1 = (12 - j16)$ A

$Z_2 = (16 + j12)\ \Omega$

Applied voltage, $V = I_1 Z_1 = (12 + j16) \quad (6 - j8)$

$= 200 + jo = 200\ 0°$ V

$\therefore$ Current in the other branch is $I_2 = \dfrac{200\ 0°}{16 + j12} = (8 - j6)$ A

Total current is $I = I_1 + I_2$

$= 12 + j16 + B - j6 = 20 + j10 = 22.36\ 26.5°$ A

Power consumed in the circuit is

$$P = VI\cos\phi = 200 \times 22.36 \times \cos 26.5° = 4000 \text{ W}$$

Three Phase Circuits

Example 5.45 Determine the three-phase power supplied to a delta connected circuit with an impedance of 4 + *j*6 ohms in each phase, when a 3-phase, 415 V, 50 Hz is applied across it.

Solution

$$\bar{V} = 415\ 0° \text{ Volts} = \text{Line voltage}$$

Impedance in each phase $= Z = 4 + j6 = 7.21 \quad 56.3°\ \Omega$

Current in each phase, $\bar{I}p = \dfrac{\text{Voltage across each phase}}{\text{Impedance in each phase}}$

$$= \frac{415\ 0}{7.21\ 56.3°} = 57.55 - 56.3° \text{ A}$$

Line current, $I_L = \sqrt{3}\ I_p = 99.68$ A

∴ Power supplied to the circuit is $P = \sqrt{3}\ V_L\ I_L \cos\phi$

$$= \sqrt{3} \times 415 \times 99.68 \times \cos 56.3° = 39755\ \Omega$$

Example 5.46 When three balanced impedances are connected in delta across a 3-phase, 400-V, 50 Hz supply, the line current drawn is 20 A at a lagging power factor of 0.3. Determine the value of impedance connected in each phase.

Solution

Line Voltage $V_L = 400$ V $\qquad f = 50$ Hz

Line Current, $I_L = 20$A $\qquad \cos\phi = 0.3$ (lagging)

Phase current, $I_p = \dfrac{I_L}{\sqrt{3}} = 11.55$ A

∴ Impedance in each phase, $Z = \dfrac{V_p}{I_p} = \dfrac{400}{11}.55 = 34.63\ \Omega$

Phase angle, $\phi - cos^{-1}\ (0.3) - 72.54°$

∴ $\bar{Z} = 34.63\ 72.54° = 10.39 + j33\ \Omega$

Example 5.47 The readings on two wattmeters connected to measure power are 6.0 kW, and 1.0 kW; the latter reading being obtained after reversal of the current in the current coil. Calculate the power and the power factor of the load.

Solution

$$P_1 = 6.0 \text{ kW}$$

$$P_2 = -1.0 \text{ kW}$$

Total power, $P = P_1 + P_2 = 5$ kW

Power factor, $\cos\phi = \cos\left[\dfrac{\tan^{-1} \times \left(\sqrt{3}(P_2 - P_1)\right)}{P_2 + P_1}\right]$

$$= \cos\left[\frac{\tan^{-1} \times (\sqrt{3}(7))}{5}\right] = 0.38$$

Example 5.48 A balanced star connected load of (3 − *j*4) Ω impedance is connected to 400-V, 50 Hz, 3-phase supply. What is the power consumed in the load? At what power factor, is that power consumed?

Solution:

$$Z = 3 - j4\ \Omega = 5 - 53.1°$$

Line voltage, $V_L = 400$ V

Phase voltage, $V_p = \dfrac{VL}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}$

$\therefore$ Phase current, $I_p = \dfrac{V_p}{Z} = \dfrac{400}{\sqrt{3}} \times \dfrac{1}{5} = 46.19$ A

Line current $I_L = I_p = 46.19$ A

Power factor $= \cos = 53.1° = 0.6$ (lead)

Power consumed $= \sqrt{3}\ V_L\ I_L \cos\phi$

$= \sqrt{3} \times 400 \times 46.19 \times 0.6 = 19188$ W

Example 5.49 A 3-phase balanced wye connected load has 400 V line-to-line voltage and 10 A line current. Determine the line to neutral voltage and phase current.

Solution

Balanced, *Y* load.

$V_L = 400$ volts

$I_L = 10$ A

$\therefore$ Line to neutral voltage is $V_p = \dfrac{V_L}{\sqrt{3}} = 231$ V

Phase current $I_p = I_L = 10$ A

Example 5.50 The power into a balanced 3-phase inductive load measured by two wattmeters are 2000 W and 1000 W. The line voltages are 400 V, 50 Hz. Determine the power, kVA and power factor of the load.

Solution:

$P_1 = 2000$ W $\qquad P_2 = 1000$ W

$V_L = 400$ Volts $\qquad f = 50$ Hz

Power, $P = P_1 + P_2 = 3000$ W

Power factor, $\cos\phi = \cos\left[\tan^{-1}\sqrt{3}\left(\dfrac{P_2 - P_1}{P_2 + P_1}\right)\right]$

$$= \cos\left[\tan^{-1}\sqrt{3}\left(\frac{1000 - 2000}{1000 + 2000}\right)\right] = 0.866 \text{ (lag)}$$

$\therefore$ $\text{kVA} = \dfrac{P}{\cos\phi} = \dfrac{3}{0} \cdot 866 = 3.46$

IMPORTANT POINTS/FORMULAE

1. The operator $j = 1\ \angle 90°$

   $j^2 = 1\ \angle 180°$
2. $a + jb$ is rectangular form of complex quantities.
3. $A\ \angle\phi$ is polar form of complex quantities.
4. For addition and subtraction of complex quantities, rectangular form is more convenient.
5. For multiplication and division of complex quantities, polar form is more convenient.

6. Summary of results of series circuit

	Type of impedance	*Value of impedance*	*Phase angle for current*	*Power factor*
1	Resistance and inductance	$\sqrt{R^2 + (\omega L)^2}$	$0 < \phi < 90°$ lag	1 > p.f. > 0 lag
2	Resistance and capacitance	$\sqrt{R^2 + (1/\omega L)^2}$	$0 < \phi < 90°$ lead	1 > p.f. > 0 lead
3	*R-L-C*	$\sqrt{R^2 + (\omega L - 1/\omega C)^2}$	between 0° and 90° lag or lead	between 0 and 1 lagging if $X_L > X_C$ and leading if $X_C > X_L$

7. The impedance of a circuit is $Z = R \pm jX$ ohms. The real part is resistance and the imaginary part is reactance.
8. The active (or) real power $P = VI \cos \phi = I^2R$ watts; Reactive or quadrature power $Q = VI \sin \phi$ in VAR. Complex. or Apparent power $S = VI$ in volt amp.
9. Admittance is reciprocal of impedance $\overline{Y} = \dfrac{1}{Z}$.
10. Admittance $\overline{Y} = G \pm jB$ mho. The real part G is conductance and the imaginary part B is susceptance. For inductive circuit, susceptance is negative.
11. When two or more impedances are in parallel, the equivalent admittance $\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 \pm \overline{Y}_3 + \ldots$
    Similarly $G_{eq} = G_1 + G_2 + G_3 + \ldots$
    $B_{eq} = B_1 + B_2 + B_3 + \ldots$
12. The conductance $G = Y \cos \phi$
    susceptance $B = Y \sin \phi$
13. An *R-L-C* circuit is said to be in electrical resonance when $X_L = X_C$. Under resonance conditions, net reactance $X = X_L \sim X_C = 0$. Resonant frequency $f_r \dfrac{1}{2\pi\sqrt{LC}}$ Hz.
14. A parallel circuit is said to be in resonance when the admittance is purely conductive. The resonant frequency of a parallel circuit consisting of an *R-L* branch and pure capacitive branch is given by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

15. Comparison of series and parallel resonant circuits.

	Item	*Series Circuit (R-L-C series)*	*Parallel Circuit (RL in parallel with C)*
(i)	Impedance at resonance	Minimum	Maximum
(ii)	Current at resonance	Maximum $= \dfrac{V}{R}$	Minimum $= \dfrac{V}{(L/CR)}$
(iii)	Effective impedance	R	L/CR
(iv)	Power factor at resonance	Unity	Unity
(v)	Resonant frequency	$\dfrac{1}{2\pi\sqrt{LC}}$	$\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

(vi) It magnifies	voltage	current
(vii) Magnification is	$\frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$	$\frac{\omega_r L}{R} = \frac{1}{\omega_r RC}$

REVIEW QUESTIONS

1. Explain what is meant by impedance of a circuit and determine its value for a circuit having resistance R, inductance L and capacitance C. Also derive an expression for power factor.
2. Derive expressions for impedance, p.f., current of a RL series circuit supplied with ac voltage source. Draw also the phasor diagram.
3. A circuit is having resistance R in series with a capacitor C. An ac voltage is applied to this circuit. Derive equations for voltage, current, impedance and p.f. Draw the phasor diagram.
4. What is an impedance triangle? Draw the impedance triangle for R-L, R-C, and R-L-C series circuits.
5. Define: admittance, conductance and susceptance. Derive expressions for all these for a series RC circuit.
6. Show that the resonant frequency of a series R-L-C circuit is $f_r = \frac{1}{2\pi\sqrt{LC}}$. Also derive the expressions for Q factor.
7. Derive the expression for resonant frequency of a parallel circuit having R_1 and L in one branch and R_2 and C in the second branch.
8. An inductive coil of resistance R and inductance L is connected in parallel with a capacitor C. Derive the expressions for resonant frequency and Q factor.
9. What do you mean by a balanced 3-phase system?
10. Define: Unbalanced three phase system?
11. What is phase sequence?
12. Analyse the current relationship and the voltage relationship in a 3-phase star connected system. Also derive the equation for power is such a system.
13. Obtain the relationship between currents and the relationship between voltages in a delta connected system. Hence derive the equation for power in such a system.
14. Can you obtain the power factor of a balanced 3 phase inductive load in terms of the two wattmeter readings connected to measure power? Prove.
15. Prove that the resultant voltage at any instant is zero in a balanced three phase system.

PROBLEMS

1. Convert the following numbers into polar form
   (i) $8 - j10$ (ii) $7 + j8$ (iii) $-15 + j90$ (iv) $-6 - j12$
2. Convert the following into rectangular form
   (i) $7\angle 180°$ (ii) $17\angle 45°$ (iii) $20\angle 60°$ (iv) $8\angle 140°$

3. If $A = 1 + j7$, $B = 3 + j3$, $C = -7 - j9$ $D = 5 - j12$ find
   (i) $A + B$ and $C + D$ and express the results in polar form
   (ii) $A - B$ and $C - D$ and express the results in polar form
   (iii) $A \times B$, $C \times D$, A/B and express the results in rectangular form
4. A non-inductive load takes 10 A at 100 V. Calculate the inductance of a reactor to be connected in series in order that the same current be supplied from 220 V, 50 Hz mains. What is the phase angle between supply voltage and current? Neglect the resistance of the reactor.
5. A 200 V, 50 Hz supply is connected to a 20 ohms resistor in series with a coil. The reading of a voltmeter across the resistor is 120 V and across the coil is 144 V. Calculate the power and reactive volt. amperes in the coil and the power factor of the circuit. Also determine the resistance and the inductance of the coil.
6. A non-inductive 10 ohms resistor is in series with a coil of resistance 1.3 ohms and inductance 0.018 H. If a voltage of maximum value 100 V at a frequency of 100 Hz is applied to this circuit, what will be the voltage across resistor?
7. Find an expression for the current and the calculate the power when a voltage represented by $v = 283 \sin 100\, \pi t$ is applied to a coil having $R = 50$ ohms and $L = 0.159$ H.
8. A coil having resistance R ohms and inductance L henries is connected across a variable-frequency alternating current supply of 110 V. An ammeter in the circuit showed 15.6 when the frequency was 80 Hz and 19.7 A when the frequency was 40 Hz. Find the values of R and L.
9. A voltage of 125 V at 50 Hz is applied to a series combination of non-inductive resistor and a capacitor of 50 μF. The current is 1.25 A. Find (i) the value of resistor (ii) power drawn by the network, and (iii) p.f. of the network. Draw the phasor digram.
10. A capacitor and resistor are connected in series to and a.c supply of 50 V and 50 Hz. The current is 2 A and the power dissipated in the circuit is 80 W. Calculate the resistance and capacitance value.
11. A resistance $R = 8$ ohms. and an unknown capacitor C are in series. $V_R = 34 \sin (2000t + 45°)$. If the current leads the applied voltage by 60°, find the value of C.
12. A reactor having negligible resistance and an inductance of 0.07 H is connected in series with a resistor of 20 ohms resistance across 200 V, 50 Hz supply. Find (i) the current flowing in the circuit and p.f, (ii) the voltage across resistor and reactor, and (iii) the maximum value of energy stored in the coil.
13. A coil of power factor 0.8 is in series with a 100 μF capacitor and then connected to a 50 c/s supply. The potential difference across the coil is found to be equal to that across capacitor by measurement. Find the resistance and inductance of the coil.
14. A pure inductor, a non-inductive resistor and a capacitor are connected in series. The supply emf is 85 V at 50 Hz, the p.d. across the inductor is 40 V

and the p.d. across the resistor and capacitor together is 85 V. The current is 5 A. Calculate the values of all components and power factor of circuit.

15. A series circuit having a resistance of 10 Ω, an inductance of 0.025 H and a variable capacitance is connected to 100 V; 25 Hz supply. Calculate the capacitance when the value of current is 8 A. At this value of capacitance, also calculate (i) the circuit impedance, (ii) the circuit power factor, and (iii) the power consumed.
16. A coil of resistance 10 ohms and an inductance of 1 H and a capacitance of 15.83 μF are connected in series across 100 V supply. The current drawn is found to be 10 A. Determine the frequency of supply.
17. When an inductive coil is connected across a 250 V, 50 Hz supply, the current is found to be 10 A and the power absorbed s 1.25 kW. Calculate the impedance, the resistance and the inductance of the coil.

    A capacitor which has a reactance twice that of the coil, is now connected in series with the coil across the same supply. Calculate the p.d. across the capacitor.
18. Calculate the admittance $G + jB$ if the impedance is $6 + j8$ ohms.
19. The two impedances $\overline{Z}_1 = 10 - j15$ ohms and $\overline{Z}_2 = 4 + j8$ ohms are connected in parallel. The supply voltage is 100 V, 25 Hz. Calculate (i) the admittance, conductance and susceptance of the combined circuit, and (ii) the total current drawn and its p.f.
20. Two impedances $14 + j5$ ohms and $18 + j10$ ohms are connected in parallel across a 200 V, 50 Hz supply. Determine (i) the admittance of each branch and of the entire circuit, (ii) the total current, power and power factor, and (iii) the capacitance which when connected in parallel with the original circuit will make the resultant power factor unity.
21. A circuit *A* of resistance 8 ohms and inductive reactance 6 ohms is in parallel with a circuit *B* of resistance 3 ohms and inductive reactance 4 ohms. Find (i) the conductance, susceptance and admittance of combined circuits, and (ii) complex expressions for the current in each circuit and for the total current.
22. A circuit is made of two branches in parallel, one having a resistance of 10 ohms, in series with an inductive reactance of 20 ohms, the other having a resistance of 15 ohms in series with a capacitive reactance of 15 ohms. The supply voltage is 200 V. Find the total current, power and power factor.
23. A coil having a resistance of 45 ohms and an inductance of 0.4 H is connected in parallel with a capacitor having capacitance of 20 μF across a 230 V, 50 Hz system. Calculate (i) current taken from the supply and p.f. of the combination, and (ii) the total energy absorbed in 3 hours.
24. One branch *A* of a parallel circuit consists of a coil, the resistance and inductance of which are 30 ohms and 0.1 H respectively. The other branch *B* consists of 100 μF capacitor in series with 20 ohms resistor. If the combination is connected to 240 V, 50 Hz mains calculate the line current and power. Draw the phasor diagram.

25. A voltage 200 $\angle$ 25° volts is applied to a circuit composed of two parallel branches. If the branch currents are 10 $\angle$ 40° A and 20 $\angle$–30° A respectively determine the kilo voltamperes, kilovars and kilowatts in each branch and in the whole circuit. What is the p.f. of the combined load?
26. A coil having resistance of 10 Ω and an inductance of 0.4 H is connected in series with a condenser of capacitance 40 μF. A voltage of 2000 volts at variable frequency is applied to the combination. At what frequency, will the current be maximum? Calculate this current and the voltage drops across the coil and condenser for this frequency. Find the voltage magnification at resonance.
27. A series *RLC* circuit has R = 5 ohms, L = 0.01 H, C = 10 μF, calculate the resonant frequency, Q factor and bandwidth.
28. A coil of resistance 3 ohms and inductance 0.01 H is connected in series with a capacitor C across 240 volts main. What is the value of C in order that maximum current may flow at a frequency of (i) 50 Hz (ii) 60 Hz? Find in each case the circuit current and the voltage across C.
29. Find the value of R which will make the circuit of Fig. P.5.1 resonant at 200 Hz.
30. What resistance in series with the capacitor of Problem 29 will make the circuit resonant at 1600 Hz.
31. Calculate the phase angle for a balanced three phase circuit in which two wattmeters connected to measure three phase power read 1000 and 800 watts, respectively. Also calculate the phase angle if the meters read 1000 W and – 800 W respectively.
32. The power flowing into a balanced 3-phase inductive load measured by two wattmeters are 2000 W and 1000 W. The line voltage are 400 V, 50 Hz. Determine the power, KVA and power factor of the load.
33. A 3 phase, 400 V supply is given to a balanced star connected load of impedance 8 + j6 ohms in each branch. Find the line current, power factor and total power.
34. Each phase of a 3-phase a.c. generator produces a voltage of 3810 volts and can carry a maximum current of 218 amperes. Find the line voltage, maximum line current and total KVA capacity of the generator if it is (a) star connected (b) delta connected.
35. A balanced three-phase load takes 5 kW and 20 kvar. Find the readings of two wattmeters connected to measure the total power.
36. In the circuit shown in the Fig. P.5.2, calculate current through each impedance, the power supplied to the circuit and the power factor of the circuit.
37. Two impedances $Z_1 = (15 + j10)\Omega$ and $Z_2 = (8 - j6)\ \Omega$ connected in parallel draw a total current of 20 A. Calculate the current and the power taken by each impedance
38. An alternating current of frequency 120 Hz flows through a series circuit consisting of non inductive resister 12 Ω, a coil of resistance 3 Ω and inductive reactance 15 Ω. Calculate the current, the voltage across the resistor and the voltage across the coil at the instant when the applied voltage 115 V is maximum.

39. A part of a specific circuit has a coil of resistance 50 Ω and inductance 1 H. The resonant frequency is 60 Hz. If the supply voltage is 230 V, 50 Hz, the find (a) the current, (b) power factor (c) the voltage across the coil.
40. Three inductors of equal values are star-connected. They take 6 KW at a power factor of 0.82 when connected to a 415 V, 50 Hz 3-ϕ, 3-wire supply. Find the load resistance and the inductance in each phase.

ANSWERS TO PROBLEMS

1. (i) $12.01 \angle -51.34°$ (ii) $10.63 \angle 48.81°$
   (iii) $91.24 \angle 9.46°$ (iv) $13.42 \angle -116.56°$
2. (i) -7 (ii) $12.02 + j\,12.02$
   (iii) $10 + j\,17.32$ (iv) $-6.13 + j\,5.14$
3. (i) $10.77 \angle 68.2°$, $21.09 \angle -95.5°$ (ii) $4.47 \angle 116.6°$, $12.37 \angle 165.0°$
   (iii) $-18 - j24$, $-143 + j\,39$, $1.33 + j\,1$
4. 0.062 H; 63°
5. 121 W; 855 VAR; 0.7; 3.37 Ω, 0.076 H.
6. 44.2 V
7. $4 \sin\left(100\pi t - \dfrac{\pi}{4}\right)$; 400 W
8. 4.95 ohms.; 0.01 H
9. 77.12 Ω; 120.5 W; 0.77 (lead)
10. 20 ohms; 212 μF
11. 36 μF
12. 6.73 *A*; 0.673; 148; 134.6; 3.17 joules
13. 25.5 ohms, 60.8 mH
14. 16.52 ohms, 0.025 H, 795.8 μF; 0.97 lagging
15. 556 μF, 12.5 ohms, 0.8 leading, 640 W
16. 40 Hz
17. 25 ohms, 12.5 ohms; 68.7 mH, 433 V
18. $0.06 - j\,0.08$ mho
19. (i) 0.097 mho, 0.081 mho, − 0.054 mho (ii) 9.7 A, 0.83 lagging
20. (i) $0.0634 - j\,0.0226$ mho; $0.0424 - j\,0.0236$ mho $0.1058 - j\,0.0462$ ℧
    (ii) 23.1 A, 4.232 kW; 0.915 (iii) 147 μF
21 (i) 0.2 mho, 0.22 mho, 0.297 mho
   (ii) $\bar{I}_1 = \bar{V}(0.08 - j\,0.06)$ $\bar{I}_2 = \bar{V}(0.12 - j\,0.16)$ $\bar{I}_3 = \bar{V}(.02 - j0.22)$
22. 10.69 A, 2116.62 W, 0.99 lagging
23. (i) 0.61, 0.957 lag (ii) 0.402 kWh
24. 7.38 *A*, 1740 W
25. 2 kVA, 0.518 kVAR, 1.93 kW; 4kVA, 3.28 kVAR, 2.29kW; 5.0 kV A, 2.76 kV AR, 4.22kW; 0.84 lagging
26. 39.8 Hz, 20 A, 2010 V, 2000 V, 10
27. 503, 6.3, 80

28. (i) 1013 μF (ii) 704 μF (iii) 80A, 251 V, 302 V
29. 12.97 ohms
30. 28.06 ohms
31. 100.9°; 86.3°
32. 3000 W, 3464 kVA, 0.866
33. 23.1A, 0.8, 12803 W
34. 6600 V, 218 A, 2492 kVA; 3810 V, 378 V, 2492 kVA
35. –3.27, 8.27 kW
36. I_A = 16.24 * (* – 32.16 ↑ o) A, I ↓ B – 13.62 * (* – 51.16 ↑ o) A, I ↓ C – 5.56 * (* 20.65 ↑ o) A, 3162 W, 0.846 (lag)
37. $I_1 = 8.57 \angle{-46.8°}$, P_1 = 904.6W $I_2 = 15.42 \angle{23.8°}$, P_2 = 217.5W
38. 4.42A, 53.04V, 67.58V
39. 1.57A, 0.342(lead), 499V
40. 19.3 Ω, 43 mH

ELECTRICAL MACHINES

6

INTRODUCTION

The broad based term, "Electrical Energy Systems Engineering" accounts for generation, transmission, distribution and utilization of electrical energy. At every stage, different machineries are used each one of them serving specific purpose. For example alternators used to generate alternating current (generation), transformers are used for stepping up and stepping down the voltage levels (transmission and distribution), dc motors, synchronous motors, stepper motors and induction motors all used in very many industrial and domestic applications. In this chapter, the principle of operation of various machines will be presented. The explanation are accompanied by the required sketches.

6.1 DC GENERATOR

6.1.1 Principle

The generator is a dynamic machine in which mechanical energy is converted into electrical energy. It operates on the principle based on the Faraday's Law of electromagnetic induction. The emf generated is to be classified as dynamically induced emf. The basic requirements for the dynamically induced emf to exist are the following:

(i) A steady magnetic field
(ii) A conductor capable of carrying current
(iii) The conductor to move in the magnetic field

The working principle of a dc generator is illustrated in Fig. 6.1. It shows a steady magnetic field produced by the pole pieces of a magnet N and S. A single turn coil *ABCD* is placed in the field produced between the pole pieces. The coil is rotated by means of a prime mover. Thus, as per Faraday's law, an emf is induced in the coil. Such an emf is basically alternating. This bidirectional induced emf is made unidirectional using the commutator. Figure 6.2, illustrates the use of commutator.

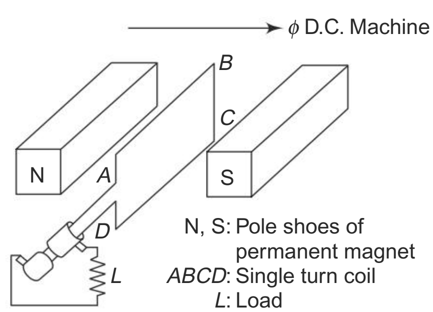


Fig. 6.1

6.1.2 Construction

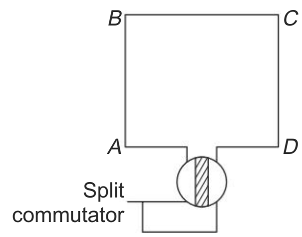


Fig. 6.2

For the satisfactory operation of a dc generator, it should consists of a stator and a rotor.

The stator accommodates the yoke, the main field system and the brushes. The rotor has the armature and the commutator as its main parts. Figure 6.3 shows these parts. Each of these parts is described as follows:

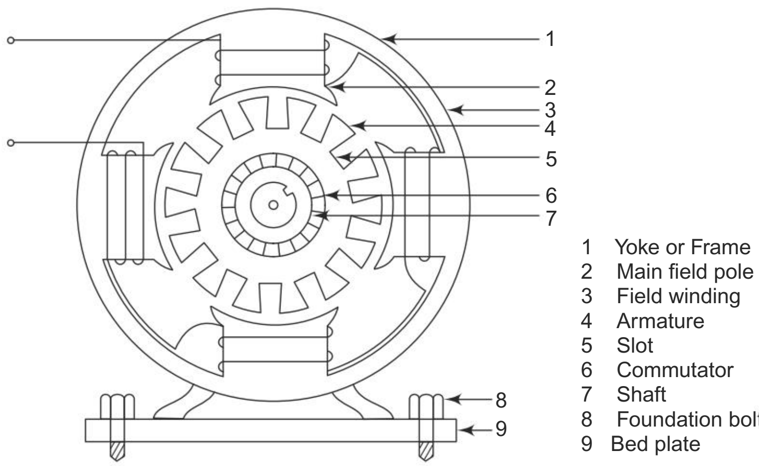


Fig. 6.3

Yoke or Frame It is the outermost solid metal part of the machine. It forms part of magnetic circuit and protects all the inner parts from mechanical damage.

Field System This consists of main field poles and field winding. The field poles are made of laminations of a sutiable magnetic material. Such a magnetic material has very high relative permeability and very low hysteresis loss. The pole face is in the form of horse shoe so that a uniform flux distribution is obtained in the air gap between the poles and the rotating part. The field winding is placed over the each pole and all these are connected in series. Again the field winding is so arranged on the different poles that when a direct current is passed through this winding, the poles get magnetized to N and S polarities alternately. Thus, the field system is responsible for producing the required working flux in the air gap.

Brushes A set of brushes made of carbon or graphic are fixed such that they are always in gentle touch with the revolving armature. The generator is connected to external circuits by means of these brushes. Thus, the brushes are used to tap the generated electrical energy off the rotating part of the generator.

Armature The armature of a dc generator is in the form of laminated slotted drum. Slots are provided over the entire periphery of the armature and these slots are

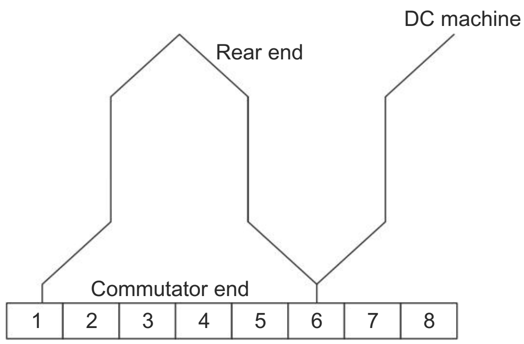


Fig. 6.4

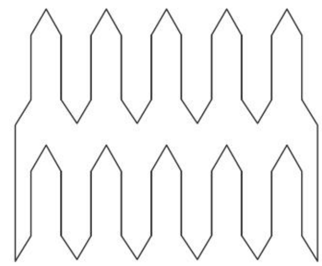


Fig. 6.5

cut axially, i.e. they are parallel to the shaft. In these slots are embedded armature conductors. All the conductors are series connected to form a single armature winding. The conductors are bent at the rear end of the armature and are connected to the commutator segments at the front end otherwise called the connected end. This is shown in Fig. 6.4. The armature winding may have two or more parallel paths depending on the type of winding used (Fig. 6.5). It is in those armature conductors mechanical energy is converted into electrical energy.

There are two ways an armature is wound.

(1) Wave winding: The number of parallel paths formed between the armature terminals is two irrespective of the number of poles.

(2) Lap winding: Here the number of parallel paths is to irrespective of the number of poles in the machine. (The further details of armature winding is beyond the scope of this book).

Commutator The commutator is similar in shape to armature. But, it has less diameter than that of the armature. Required number of segments are provided over the complete periphery of the commutator. There is an electrical insulation between every pair of segments. A minimum of two conductors are connected to each segment. But, at the same time the two conductors making a single coil are connected to different commutator segments. The brushes are so placed that they are always keeping to such with the revolving commutator segments.

6.1.3 EMF Equation or Equation for the emf Generated

Let P – Number of poles in the generator

ϕ – flux per pole in webers

z – total number of armature conductors

A – Number of parallel paths formed by the armature winding between the armature terminals

$A = 2$, for wave wound armature winding

$A = P$, for lap wound armature winding

N – speed of rotation of armature in RPM

E_g – emf induced across the armature terminals or emf induced in any one parallel path of the armature winding.

According to Faraday's Law of electromagnetic induction, average emf induced in one conductor = $\dfrac{d\phi}{dt}$ [no. of turns = 1]

Here $d\phi$ – flux cut by the conductor in one revolution = $P\phi$ (wb)

and dt – time taken by the conductor for one revolution = $60/N$ (sec.)

$\therefore$ Average emf generated in one conductor = $\dfrac{P\phi N}{60}\,\dfrac{\text{wb}}{\text{sec}}$ or volts

No. of conductors connected in series in one parallel path = Z/A

$\therefore$ EMF generated/path or generated EMF, $E_g = \dfrac{P\phi N\,Z}{60A}$

Note: The above is the emf is generated in the armature on open circuit. This means that no load is connected to the generator.

6.1.4 Types of DC Generator

Depending upon how the field windings are excited, dc generators can be classified as separately excited and self excited generators. Separately excited generators receive the required electric power for their field system from a separate dc source. Self excited generators receive the required power for their having system from the electrical power developed in their armature. To avoid having a separate dc source, self excited generators are preferred. Depending upon the field winding connection with the armature winding and their location is the generator circuit, self excited generators can be classified as

(i) Shunt generators
(ii) Series generators
(iii) Compound generators

6.1.5 Electrical Equivalent Circuits, Current and Voltage Equations of DC Generators

Let R_a, R_{sh} and R_{se} – resistance of armature, shunt field and series field windings respectively.

I_a, I_{sh} and I_{se} – current through the armature, shunt field and series field windings respectively

I_L – Load current

V, V_{sh} – voltage across the load or terminal voltage of the generator and voltage across the shunt field winding respectively.

V_b, V_{ar} – voltage drop due to the brush contact resistance, voltage drop due to armature reaction respectively.

E_g – emf generated on no-load condition

Terminal Markings

Z, ZZ – shunt field winding terminals

Y, YY – series field winding terminals

A, AA – Armature terminals

Separately Excited DC Generator Figure 6.6. shows the electrical equivalent circuit of a separately excited dc generator. The field winding is connected to a separate dc source. The load is connected across the armature terminals.

Current equation is, $I_a = I_L$ (1)

voltage equation is $E_g = V + I_a R_a + V_b + V_{ar}$ (2)

$V_b = 2 \times$ drop due to each brush contact

If V_b and V_{ar} negligible, then

$$E_g = V + I_a + R_a \quad (6.1)$$

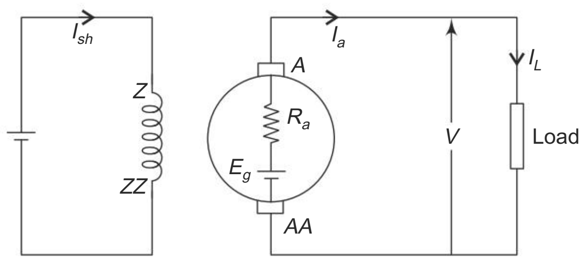


Fig. 6.6

DC Shunt Generator Figure 6.7 shows the electrical equivalent of a self excited dc shunt generator or simply dc shunt generator. The shunt field winding is connected across the armature terminals.

Current equation is, $I_a = I_L + I_{sh}$; But $I_{sh} = V/R_{sh}$ (1)

Voltage equation is, $E_g = V + I_a R_a + V_b + V_{ar}$ (2)

or $E_g = V + I_a R_a$; (if V_e and V_{ar} are negligible) (6.2)

Series Generator Figure 6.8 shows the electrical equivalent circuit of a dc series generator. The series field is connected in series with the armature and the load is connected across this series combination.

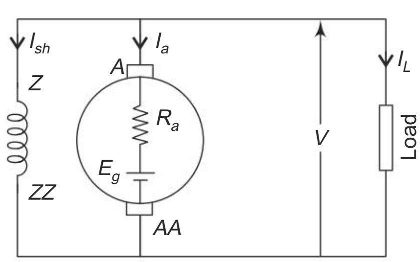


Fig. 6.7

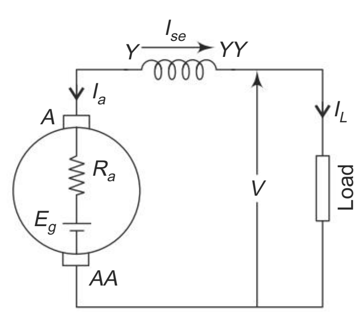


Fig. 6.8

Current equation is, $I_a = I_L = I_{se}$ (1)

Voltage equation is, $E_g = V + I_a (R_a + R_{se}) + V_b + V_{ar}$ (2)

or $E_g = V + I_a (R_a + R_{se})$; (if V_a and V_{ar} are negligible) (6.3)

The sum $(R_a + R_{se})$ can be called as machine resistance or internal resistance of the machine.

Long Shunt Compound Generator The electrical equivalent circuit of a long shunt compound generator is shown in Fig. 6.9. The shunt field winding is connected across the series combination of armature and series field windings and across the load. This is characterised by the fact that series field current equals the armature current.

Current equation is, $I_L = I_{se} = I_a + I_{sh}$; Here $I_{sh} = V/R_{sh}$ (1)

Voltage equation is, $E_g = V + I_a (R_a + R_{se}) + V_b + V_{ar}$ (2)

or $E_g = V + I_a (R_a + R_{se})$; (if V_a and V_{ar} are negligible) (6.4)

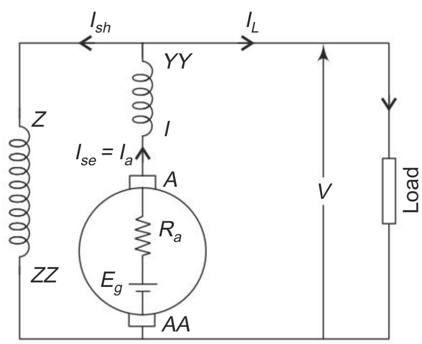


Fig. 6.9

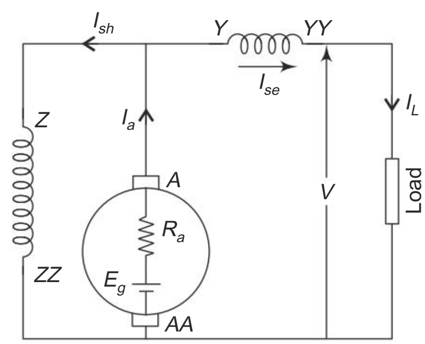


Fig. 6.10

Short Shunt Compound Generator Figure 6.10 shows the electrical equivalent circuit of a short shunt compound generator. Shunt field winding is connected across the series combination of load and series field winding and across the armature terminal. This is characterised by the fact that the series field current equals the load current.

Current equation is, $I_L = I_{se}$ (1)

$I_a = I_L + I_{sh}$ (2)

Here $I_{sh} = V_{sh}/R_{sh}$ and $V_{sh} = V + I_L R_{se}$ (3)

Voltage equation is $E_g = V + I_a R_a + I_L R_{se} + V_b + V_{ar}$ (or)

or $E_g = V + I_a R_a + I_L R_{se}$; (if V_a and V_{ar} are negligible). (6.5)

In compound generators if the fluxes due to both the main field (shunt field) and series field windings aid each other, then it is known as cumulative compounding. If the fluxes oppose each other, it is called differential compounding.

Armature Reaction The interaction of armature mmf with main field mmf is called armature reaction.

When the armature circuit is closed through load, current starts flowing through the armature winding. This armature current sets up a magnetic field. This field opposes the main field as per Lenz's law. This interaction between armature and main field causes two effects.

(i) Demagnetising effect—because of this net flux in the generator is reduced and so the emf generated and hence the terminal voltage are reduced.

(ii) Cross magnetising effect—because of this there is sparking between brushes and commutator segment resulting in poor communication.

6.1.6 DC Generators Characteristics

DC Shunt Generator

Open circuit characteristics (*OCC*) or *No-load characteristics* or *magnetisation characteristics*.

By removing the load connected in the circuit of Fig. 6.6 and Fig. 6.7, we obtain the no-load condition. When the generator is run at its rated speed with no-load, the terminal voltage appearing across the armature can be taken as the induced voltage. Thus, it is possible to obtain this open circuit voltage as a function of the field current. By varying the field current using the regulating rheostat in the field circuit, we can obtain the variation in the no-load voltage and plot the same. The curve thus obtained is known as open circuit characteristics, the typical shape of which is shown in Fig. 6.11.

We can observe that the major portion of this characteristics (2-3) is linear. There are two non-linear portions namely 1-2 and 3-4. Beyond the operating point 4, we say that saturation has occurred and so there will not be any appreciate increase in the generated voltage even for large increase in field current.

Because of the residual flux in the magnetic poles there is a small amount of induced voltage even when the field current is zero. As $E_g \alpha \phi$ or I_f, the initial portion

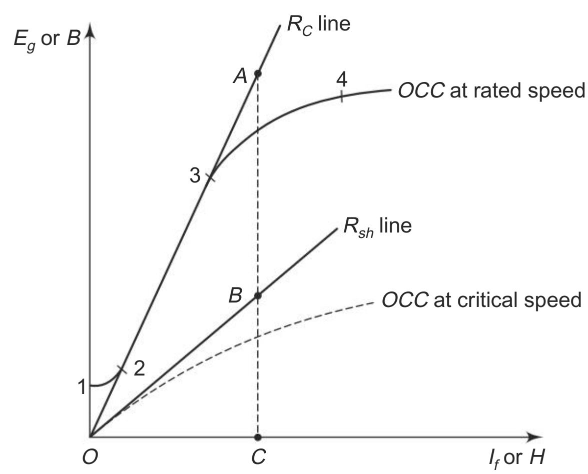


Fig. 6.11

of OCC is a straight line. After a certain value of field current, the magnetic poles are saturated.

So the OCC is almost flat in shape, i.e. the increase in voltage is very small even if I_f goes on increasing.

The non-build up of voltage of dc shunt generators under open circuited condition depends on the following factors.

(a) residual flux (b) reverse connection of shunt field coil (c) shunt field circuit resistance and (d) speed of armature.

(i) If there is no residual flux, no emf is induced. As a result there is no further increase in field flux and the induced emf is zero. In this case the field winding alone can be connected to a separate dc source. So that it can retain some magnetisation (i.e. residual flux).

(ii) The residual flux in the machine and the flux set up by the shunt field winding must aid each other so that the net air gap flux increases which in turn develops voltage. Otherwise, the residual flux may get wiped off and hence induced emf drops to zero. So the field winding terminals should be properly connected.

(iii) The field circuit resistance must be equal to or less than the circuit field resistance R_e. Otherwise the generator will fail to build up voltage.

(iv) The speed of generator must be equal to or greater than the critical speed N_C, of the generator. Otherwise the generator will fail to build up voltage.

Critical field resistance (R_e) It is the maximum value of resistance in the field circuit with which the generator will just build up voltage. Beyond this value of resistance the machine will fail to build up voltage.

Critical speed (N_e) It is the minimum speed at which the generators will just build up voltage. Below this critical speed it will fail to build up voltage.

(or)

It is that speed at which the shunt field resistance (R_{sh}) of the generator will be its critical field resistance.

Both the critical speed N_e and the critical field resistance R_e of a generator can be obtained from its *OCC* at rated speed discussion as follows.

Draw the *OCC* at rated speed N_R. Draw the tangent to the initial portion of the *OCC*. The slope of this tangent gives $R_C = \frac{AC}{OC}$. Now, measure the value R_{sh} of the generator and draw the line representing the field circuit resistance. Draw any ordinate which cuts the R_e line, R_{sh} line and X-axis at A, B and C. The critical speed is calculated as

$$N_C = \frac{BC}{AC} \times \text{Rated speed } (N_R)$$

Load characteristics of dc shunt generator With the circuits given in Fig. 6.6 and Fig. 6.7 load characteristics can be obtained. The generators should be made to run at its rated speed. The generator field current should be adjusted so that rated voltage is built up at no-load. Now by varying the load, the terminal voltage V as a function of load current I_L can be obtained as shown in Fig. 6.12.

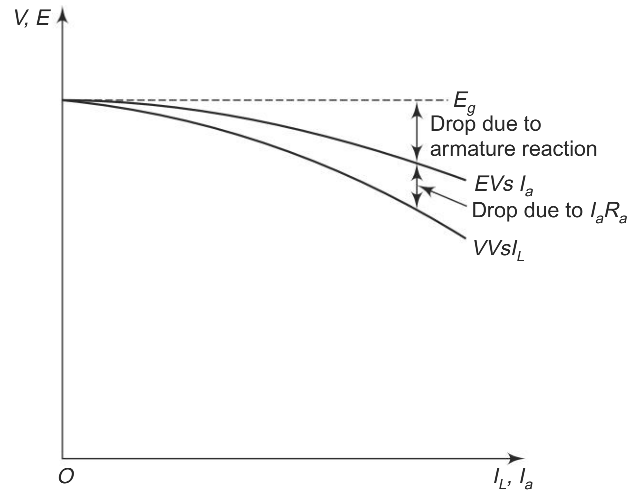


Fig. 6.12

The terminal voltage of the generator decreases on loading. This is because of (i) voltage drop due to armature resistance, R_a (ii) voltage drop due to armature reaction and (iii) the cumulative effect of (i) and (ii) which results in further reduction in flux and hence the generated emf. So the generated emf and the terminal voltage go on decreasing.

There are two kinds of load characteristics

(a) **External characteristics** Variation of load terminal voltage with respect to the load current [i.e. V_{vs} I_L]. These two quantities are external quantities and they can be measured directly. Hence the name external characteristics.

(b) **Internal characteristics** Variation of induced voltage in loaded condition as a function of the armature current [i.e. E_{vs} I_a]. These two quantities are internal quantities for the generation and E cannot be measured directly, but it can be estimated. Hence the name internal characteristics.

Voltage regulation Voltage regulation of a dc generator is defined as the change in terminal voltage when the load is reduced from rated value to zero, expressed as a percentage of the rated load voltage.

$$\text{Voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}}; \tag{6.6}$$

V_{NL} – No load terminal voltage
V_{FL} – Rated terminal voltage

A well-designed generator will have low voltage regulation. The regulation of dc generator on full load is about 5 to 10%.

Characteristics of dc Series Generator The *OCC* of series generator can be obtained with the help of the circuit shown in Fig. 6.13. The shape and explanation of *OCC* is similar and same as that for *DC* shunt generator.

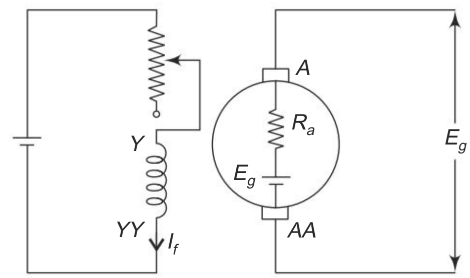


Fig. 6.13

The load characteristics of the dc series generator can be obtained with the help of the circuit shown in Fig. 6.8. The *OCC* and load characteristics are shown in Fig. 6.14.

The load characteristics of the dc series generator is using rising characteristics whereas that of the dc shunt generator is drooping characteristics. The difference between curves I and II represents the voltage drop due to internal resistance of the generator, i.e. $R_a(R_a + R_{se})$ drop. The difference between curves II and III represents the drop due to armature reaction. If the load goes on increasing, then the terminal voltage may become zero because of heavy armature reaction.

The slope of tangent drawn to the initial portion of the external characteristic represents the critical resistance of the generator. It is the maximum value of resistance in the load circuit including R_a and R_{se} with which the machine can just build up voltage. Beyond this value of resistance, the generator fails to build up voltage.

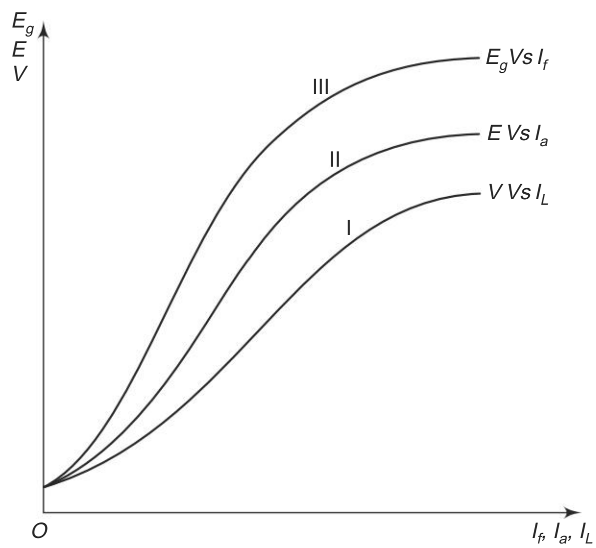


Fig. 6.14

Characteristics DC compound generator The terminal voltage of the dc shunt generator reduces on loading. This drop in voltage may be objectionable when the generator is feeding lighting load, as illumination reduces when the terminal voltage is reducing. To reduce this effect or offset this effect a few series field turns may be included so that the fluxes due to both shunt and series fields aid each other. So net flux increases when the load increases and hence the induced emf and terminal voltage increase. The characteristics of dc compound generator for various degree of compounding is shown in Fig. 6.15.

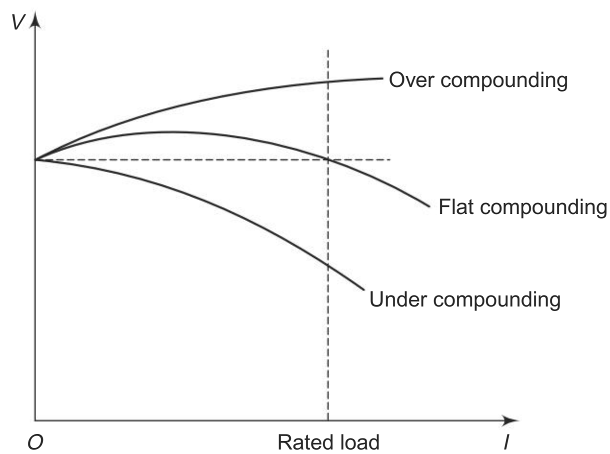


Fig. 6.15

If the series field ampere-turns are such that they induce the same voltage at rated load as at no-load, the generator is said to be "flat compounded". If the series field amp-turns are such that rated load voltage is greater than the no-load voltage, the generator is said to be 'over compounded'. If the series field amp-turns are such that rated load voltage is less than the no-load voltage, the generator is said to be 'under compounded'.

6.1.7 Applications of DC Generators

DC Shunt Generator

1. Separately excited generators are preferred where the characteristics of dc shunt generators is not upto the expected level.
2. They can be used to excite the field magnets of ac generators.
3. A s the drop in voltage is very small, these generators can be used for supplying loads needing constant voltage.
4. They are used as source for battery charging purpose.
5. These generators are used for electroplating and electrolytic purpose.

DC Series Generators

1. They are used for series arc lighting.
2. They are used for series incandescent lighting.

3. They are used as booster, for the purpose of compensating the drop in voltage in the lines on loading.
4. Used for regenerative braking of dc locomotives.

Compound Generators

1. By means of compound generators it is possible to give constant voltage at the line by proper compounding.
2. Differentially compounded generator may be used for welding purposes.
3. They are used to supply power to railway circuits, incandescent lamps, elevator motors, etc.

6.2 DC MOTOR

6.2.1 Principle

Whenever a current carrying conductor is kept in a stationery magnetic field an electromagnetic force is produced. This force is exerted on the conductor and hence the conductor is moved away from the field. This is the principle used in d.c. motors.

6.2.2 Construction

The construction of dc motor is exactly similar to dc generators. The salient parts of a dc motor are yoke or frame, main field system, brushes, armatures and commutator.

6.2.3 Working

In a dc motor, both the armature and the field windings are connected to a dc supply. Thus, we have current carrying armature conductors placed in a stationery magnetic field. Due to the electromagnetic torque on the armature conductors, the armature starts revolving. Thus, electrical energy is converted into mechanical energy in the armature. When the armature is in motion, we have revolving conductors in a stationery magnetic field. As per Faraday's Law of electromagnetic induction, an emf is induced in the armature conductors. As per Lenz's law, this induced emf opposes the voltage applied to the armature. Hence, it is called the counter or back emf. There also occurs a potential drop in the armature circuit due to its resistance. Thus, the applied voltage has to overcome the back emf in addition to supplying the armature circuit drop and producing the necessary torque for the continuous rotation of the armature.

Figure 6.16 describes the circuit symbols for the armature, the shunt field and the series field of a dc machine irrespective of whether the dc machine is generating or motoring. Figure 6.17 gives the electrical circuit of a d.c. shunt motor where

E_b = back EMF
I_a = current flowing in the armature circuit
R_a = resistance of armature circuit
V = applied voltage

Thus, the characteristics equation of a dc motor is $V = E_b + I_aR_a$, (6.7)
where I_aR_a represents the potential drop in the armature circuit.

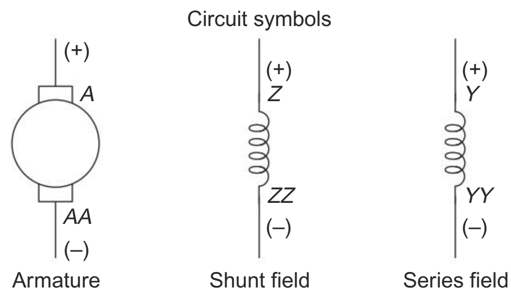


Fig. 6.16

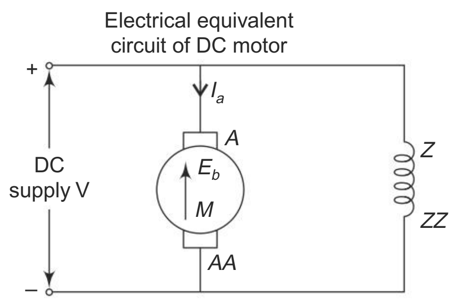


Fig. 6.17

6.2.4 Types of DC Motors and Electrical Equivalent

DC Shunt Motor The electrical equivalent circuit is given in Fig. 6.17 and the voltage equation also given there.

DC Series Motor Figure 6.18 shows the electrical equivalent of a dc series motor.

The voltage equation is

$$V = E_b + I_a(R_a + R_{se}) \tag{6.8}$$

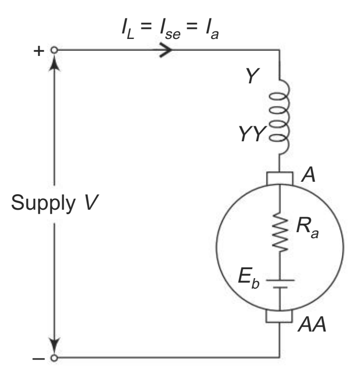


Fig. 6.18

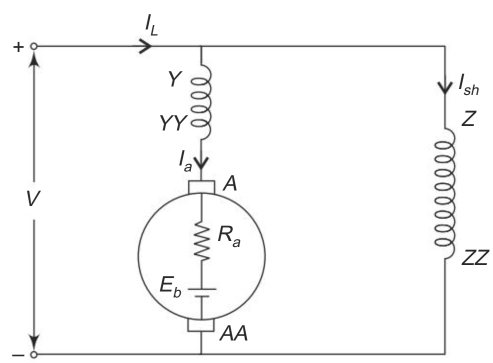


Fig. 6.19

DC Compound Motor

(a) Long shunt compound motor The electrical equivalent of long shunt compound motor is shown in Fig. 6.19.

The current equation is $I_L = I_a + I_{sh}$

Voltage equation is $V = E_b + I_a(R_a + R_{se})$ (6.9)

(b) Short shunt compound motor Figure 6.20 shows the electrical equivalent of short shunt compound motor.

Current equation is, $I_{se} = I_L$ (1)

$I_L = I_a + I_{sh}$ (2)

$I_{sh} = V_{sh}/R_{sh}$ (3)

where $V_{sh} = V - I_L R_{se}$

Voltage equation is $V = E_b + I_L R_{se} + I_a R_a$ (6.10)

Here the effect of V_e and V_{ar} is neglected. Depending upon the series field winding connection, with respect to the shunt field windings connection, we can get differential and cumulative compounding actions.

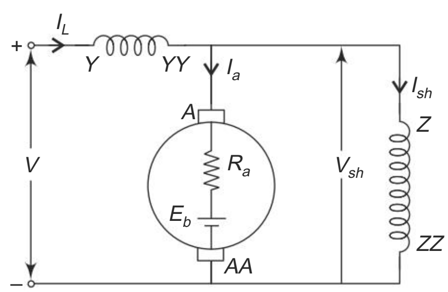


Fig. 6.20

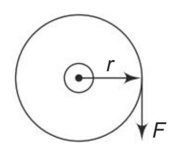


Fig. 6.21

6.2.5 Torque and Speed Equations

Let a circumferential force of F Newtons act tangentially in a pulley of radius r meters, as shown in Fig. 6.21. The pulley is running at a speed of N rpm.

Work done by this force in one revolution = $F \times 2\pi r$ Joules (1)

Work done by this force/sec = $F \times 2\pi r \times N/60$ Joules/sec (2)

i.e. Power developed = $T\omega$ watts (3)

where T(-torque) = $F \times r$ N-m (4)

$$\omega \text{ (angular speed)} = \frac{2\pi N}{60} \text{ rad/sec.}$$

Armature Torque T_a of Motor Let T_a (N-m) be the torque developed in the armature of motor running at N rpm.

Power developed in the armature $= T_a \dfrac{2\pi N}{60}$ rad/sec (1)

Also, electrical equivalent of mechanical power developed in the armature $= E_b I_a$

$$\therefore \quad T_a \frac{2\pi N}{60} = E_b I_a \text{ or } T_a \omega = E_b I_a \quad (2)$$

$$T_a = \frac{E_b I_a}{\omega} \quad (3)$$

Substituting $\quad E_b = \dfrac{P\phi ZN}{60A}$ and $\omega = \dfrac{2\pi N}{60}$

$$T_a = \frac{P\phi Z}{2\pi A} I_a; \text{ (N-m)} \quad (6.12)$$

or $T_a \,\alpha\, \phi\, I_a$; as P, Z, A are constants for a given motor.

The above equation is known as torque equation.

Torque available on the motor shaft $T_{sh} = T_a - T_s$

T_s-torque lost due to iron and mechanical losses or stray losses

$$T_s = \frac{\text{Stray losses in watts}}{\omega} \text{ ... N-m}$$

Speed Equation

$$E_b = \frac{P\phi ZN}{60A} \quad (6.13)$$

or $\quad E_b \,\alpha\, \phi\, N$; as P, Z, A are fixed (1)

or $\quad N \alpha \dfrac{E_b}{\phi}$ (2)

or $\quad N \alpha \dfrac{V - I_a R_a}{\phi}$ (3)

Speed Regulation It is defined as the change in speed when the load on the motor is reduced from rated value to zero, expressed as a percentage of rated load speed, N

$$\% \text{ speed regulation} = \frac{N \cdot L \text{ speed} - F.L \text{ speed}}{F \cdot L \text{ speed}} \times 100 \quad (6.14)$$

$$= \frac{dN}{N} \times 100$$

6.2.6 Characteristics of DC Motors

The characteristics of dc show the relationship between the following quantities.

(i) Torque and armature current (T vs I_a). It is also known as the electrical characteristic.

(ii) Speed and armature current (N vs I_a).

(iii) Speed and armature torque (N vs T_a). It is also known as mechanical characteristics.

The above characteristics can be obtained for all types of motors with the help torque and speed equations.

Characteristics of dc Shunt Motor

(i) T_a vs I_a Characteristics The flux, ϕ reduces slightly on heavy loads only. Thus, flux can be considered to remain constant.

$$T_a \; \alpha \; I_a$$

So, T_a vs I_a characteristics is a straight line characteristics as shown in Fig. 6.23.

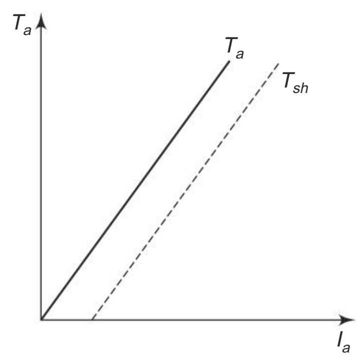


Fig. 6.22

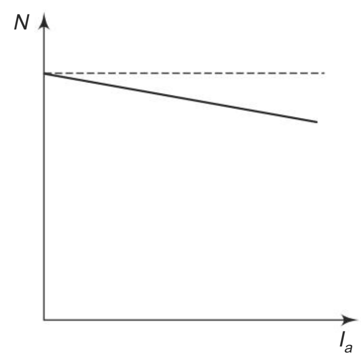


Fig. 6.23

(ii) N vs I_a Characteristic

$$N \alpha \frac{Eb}{\phi} \tag{1}$$

For shunt motor ϕ is almost constant on load $\quad \therefore N \; \alpha \; E_b$

$$N \; \alpha \; V - I_a R_a \tag{2}$$

As load increases, I_a increases, but the drop $I_e R_a$ is very small as R_e is very small. So there will be a small change in speed from no-load to full load as shown in Fig. 6.23. The drop in speed from no-load to full-load is 5 to 10% of no-load speed.

Shunt motor is a constant speed motor

(iii) N vs T_a Characteristics This can be deduced from the above two characteristics and is shown in Fig. 6.24.

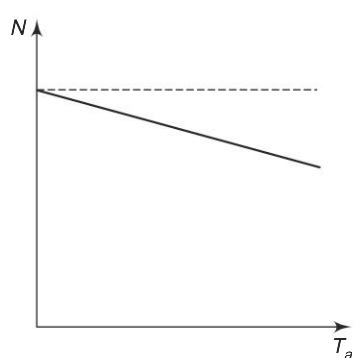


Fig. 6.24

Characteristics of dc Series Motors

(i) T_a vs I_a Characteristics

$$T_a \; \alpha \; f I_a; \tag{1}$$

In series motor $\phi \; \alpha \; I_{Se}$ and $I_{Se} = I_a$ (2)

At light loads, $T_a \; \alpha \; I_a^2$ (prior to saturation of field poles)

At heavy loads, $T_a \; \alpha \; I_a$ (after saturation of magnetic poles, ϕ is constant for any value of I_a)

So, T_a vs I_a characteristics is a parabola prior to saturation, and it is a straight line after saturation, as shown in Fig. 6.25. On heavy loads, the series motor exerts higher starting torque as $T_a \; \alpha \; I_a^2$ prior to saturation.

(ii) N vs I_a Characteristic

$$N \alpha \frac{E_b}{\phi}; \tag{1}$$

$$E_b = V - I_a (R_a + R_{se}) \text{ and } \phi \; \alpha \; I_{se} \, (I_{se} = I_a) \tag{2}$$

Change in E_b is very small with changes in load. So it can be regarded as constant on all loading

$$\therefore \qquad N \alpha \, 1/\phi \quad \text{or} \quad N \alpha \, 1/I_a \tag{3}$$

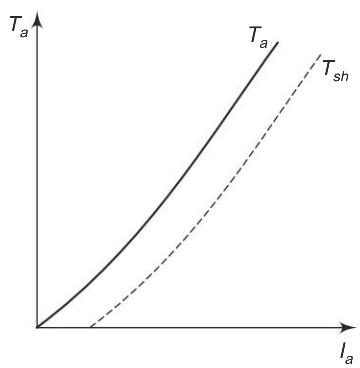


Fig. 6.25

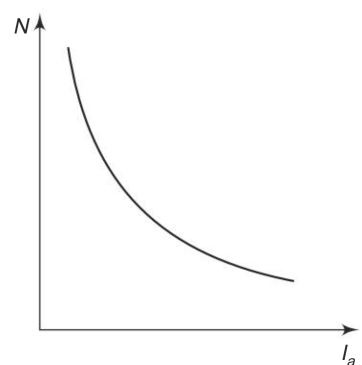


Fig. 6.26

The speed-armature current characteristic is a regular parabola as shown in Fig. 6.26. On light load or no-load, the armature current I_a is very small, and so the motor will run at dangerously high speeds. So, series motor should always be started with some load on it. Series motor is a variable speed motor.

(iii) N vs T_a Characteristics From the above two characteristics, this characteristics can be deduced, as shown in Fig. 6.27.

Characteristics of Compound Motors

Cumative Compound Motor The fluxes produced by the two field windings, aid each other. On light loads, the torque developed is similar to that in a shunt

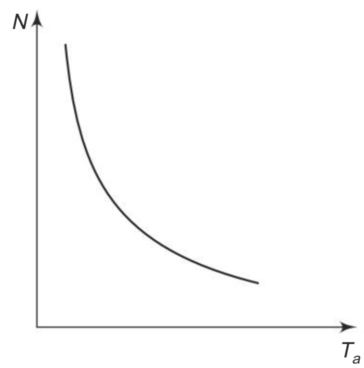


Fig. 6.27

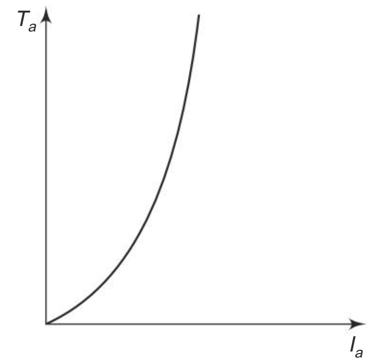


Fig. 6.28

motor. On heavy loads, the net flux increases and torques rises rapidly as shown in Fig. 6.28.

Differential Compound Motor On no-load and light loads, the effect of series field flux is small. So, torque produced is as that in a shunt motor. On heavy loads the effect of series field is more and reduces the net flux. Hence torque developed reduces and may become zero. On further loading the series field dominates the main field, torque is developed in the opposite direction and motor will run in the opposite direction. The characteristics is shown in Fig. 6.29.

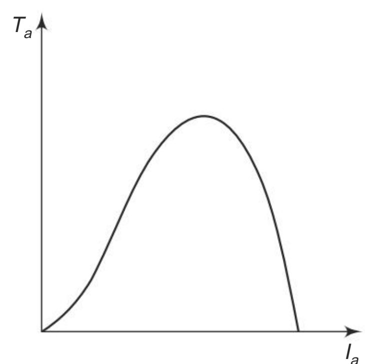


Fig. 6.29

6.2.7 Losses in DC Machines

The various losses that occur dc machines can be grouped under three heads which are copper losses, iron losses and mechanical losses.

(i) Copper Losses Power loss due to flow of current through the various winding resistances of the dc machine is known as copper loss.

(a) Armature and Series Field Copper Losses, i.e. $I_a^2 R_a$ and $I_{se}^2 R_{se}$ The armature and series field currents I_a and I_{se} vary with load current. So, these losses are variable losses.

(b) Shunt Field Copper Losses, i.e. $I_{sh}^2 R_{sh}$ or VI_{sh} As the supply voltage and voltage across the shunt field winding do not vary much, the shunt field current and hence the shunt field copper is almost constant.

(ii) Magnetic losses or Iron Losses or Core Losses When the armature rotates it experience magnetic reversal which results in hysteresis loss in the armature core of the machine. When the armature rotates, there is induced emf in the armature core which results in eddy current flow in the armature core and hence the eddy current losses. To reduce the hysteresis loss, material having low hysteresis coefficient is preferred. To reduce the eddy current loss the armature core is laminated.

Hysteresis loss $W_h = \eta B_m^{1.6} f V$ .... watts (6.15)

Eddy current loss $W_e = kB_m^2 f^2 t^2 V^2$ .... watts (6.16)

where, η – hysteresis coefficient of armature core material

B_m – maximum value of flux density in the core

f – frequency of magnetic reversals $= \dfrac{PN}{120}$

V – Volume of armature core material

t – thickness of each armature core lamination

k – eddy current constant for the armature core material

If B_m and V are fixed,

$$W_h \propto f$$

and $$W_e \propto f^2$$

The shunt motor will run at almost constant speed as load varies.

$\therefore$ f is also constant.

So, the cores losses for dc shunt machine are almost constant.

(iii) Mechanical Losses Power loss due to mechanical friction between the moving parts of the machine and the power loss due to friction between moving parts of the machine and air (air friction or windage loss) are combinedly known as mechanical losses. These two losses depend on the speed of machine. All the machines will normally be operated at a specific speed. In such cases and in cases where the speed varies very little with load, mechanical losses remain unchanged with load.

The magnetic and mechanical losses are combinedly called as rotational or stray losses, (W_s). The sum of stray losses and shunt field copper losses is known as constant losses or standing losses (W_k).

Efficiency of Motor

Electrical Power input $P_i = VI_L$	Copper losses W_{cu}	Electrical equivalent of power developed with armature $P_m = E_b I_a$ (T_a)	Stray losses W_s (T_s)	Mechanical power output available at the shaft P_o (T_{sh})

$$\% \text{ Efficiency of motor} = \frac{P_o}{P_i} \times 100$$

Efficiency of Generator

Mechanical Power input or Power output of the Driving engine	Stray losses W_s (T_s)	Electrical power developed in the armature $E_g I_a$	Copper Losses W_{cu}	Electrical power output available across the load $P_o = VI_L$

$$P_i = \frac{2\pi \times NT_{sh}}{60}$$

T_{sh}-shaft
torque of engine

$$\% \text{ efficiency of generator} = \frac{P_o}{P_i} \times 100$$

6.2.8 Speed Control of DC Motors

The speed equation of dc motors is, $N \alpha \frac{E_b}{\phi}$ or $N \alpha \frac{V - I_a R_a}{\phi}$. From this equation it is clear that speed of a motor depends on three factors. (i) Flux per pole (ii) Armature circuit resistance and (iii) Supply voltage. Based on these factors the speed of a dc motor can be controlled by the following methods.

(i) Field control method
(ii) Armature rheostatic control method
(iii) Variable supply voltage control method

Field Control Methods

DC Shunt Motor

$$N \alpha \frac{E_b}{\phi} \text{ (or) } N \alpha \frac{V - I_a R_a}{\phi}$$

As R_a is small the armature drop is very small and so $N \alpha \frac{V}{\phi}$.

As supply voltage V is constant and $\phi \alpha I_f$, we can get, $N \alpha \frac{I}{I_f}$.

By varying the field current I_f, the speed of dc shunt motor can be controlled. The necessary circuit diagram is shown in Fig. 6.30. With rated voltage and field current the motor will run at rated speed. So, by means of field control the speed above rated speed can be controlled and obtained. Higher speed can be obtained with low field current. With low field current and heavy armature reaction, the commutator will become poor.

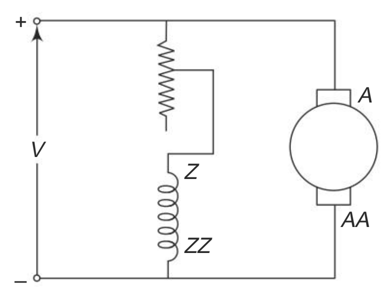


Fig. 6.30

Advantages

(i) Speed above rated speed can be controlled.
(ii) As field current is low, the loss in the external rheostat will be less.

Disadvantages

(i) Speed below rated speed cannot be obtained.
(ii) Weakening the field results in poor commutation.

DC Series Motor The speed control of dc series motor by field control technique can be obtained by the following methods.

(i) **Using field diverter** A variable resistance is connected across the series field as shown in Fig. 6.31. By varying this resistance, the current through the field is reduced and the speed increased.

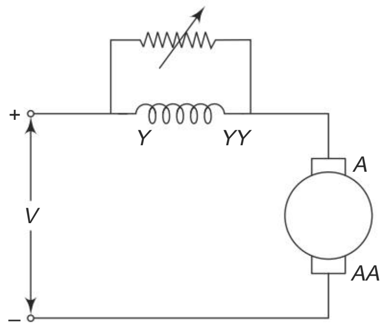


Fig. 6.31

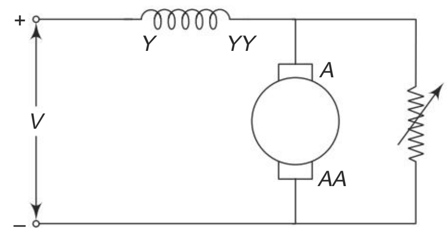


Fig. 6.32

(ii) **Using armature diverter** A variable rheostat is connected across the armature terminals of the motor as shown in Fig. 6.32. For given load torque, when the resistance value is changed the armature current is reduced. To meet the load torque the flux is increased by drawing more current from the source. So the speed of the motor is reduced. The speed can be varied by varying the diverter resistance.

(iii) **Field tapping** The series field is tapped for different number of turns as shown in Fig. 6.33 with full field turns the speed of motor is minimum. When the number of field turns is reduced, the ampere turns and hence the flux is reduced. So the speed of the motor is increased. This type of speed control is often used in a electric traction motor.

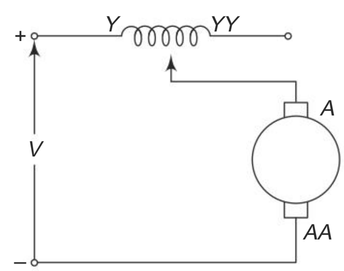


Fig. 6.33

(iv) **By Paralleling field coils** For a 4-pole dc motor there are 4 fields coils. The four coils can be connected as shown in Fig. 6.35. For given load current, the current through the field coil and the flux per pole is 100%, 50% ands 25% when the coils are connected as in Fig. 6.34. (i), (ii) and (iii) respectively. The speed is increased in steps. For getting intermediate speeds, a diverter resistance can be used.

Armature Rheostat Control Methods

DC shunt motor A controller resistance is connected in series with the armature as shown in Fig. 6.35. If the control resistance value is increased, the voltage across the armature is reduced and the speed is reduced as $N \alpha \dfrac{V - I_a(R_a + R)}{\phi}$. With fixed R and load changing the speed of the motor is reduced which is shown in Fig. 6.36.

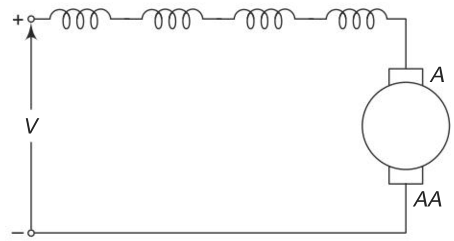


Fig. 6.34 (i)

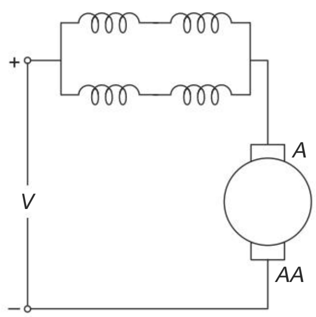


Fig. 6.34 (iii)

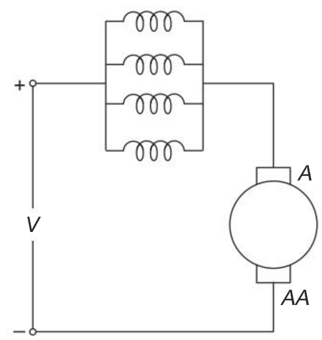


Fig. 6.34 (ii)

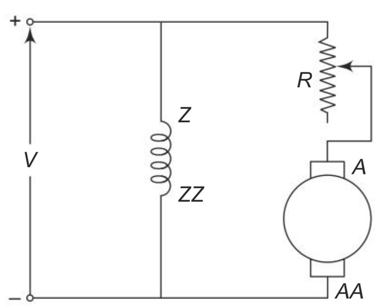


Fig. 6.35

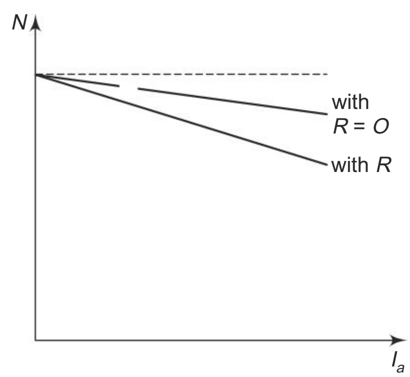


Fig. 6.36

DC Series Motor A controller resistance R, is connected in series with the motor as shown in Fig. 6.37. As R is zero the motor runs at maximum speed for a given load. When R is included the back emf is reduced as $E_b = V - I_a(R_a + R_{ge} + R)$ and the speed is reduced. If R is fixed and the load is varied, the speed reduces as E_b is reduced. This characteristic is shown in Fig. 6.38.

Voltage Control Method: (Ward Leonard System) The speed of shunt motor M_1 is to be controlled. The field of M_1 can be connected to a dc source

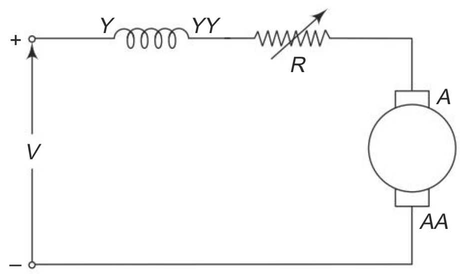


Fig. 6.37

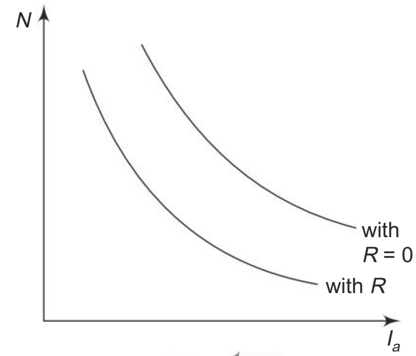


Fig. 6.38

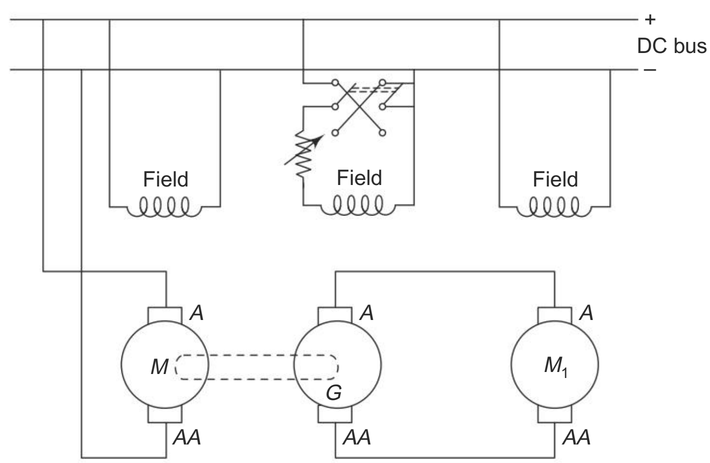


Fig. 6.39

separately. If the armature is supplied with variable voltage, the speed of M_1 is controlled. To achieve this a motor-generator set is used as shown in Fig. 6.39. The fields of this motor and generator are also connected to the DC source. The motor is run at a specified speed by proper control of its field current. The generator voltage can be varied varying its field current. So, the speed of M_1 is changed. The direction of motor M_1 can be changed by changing the direction of field current of generator by a proper change over switch. This method of speed control becomes costlier as its needs a motor-generator set.

Note: The speed of dc compound motor can be controlled by using the methods and techniques discussed for dc shunt and series motor with necessary changes.

6.2.9 Starting of D.C. Motors

If the motor is directly switched on to the supply, the current drawn by the motor will be many times its full load current. This is because at the moment of switching on, there is no back emf developed by the motor. So, the current drawn by the motor is controlled by its internal resistance which is very small in value. This current is $I_a = \dfrac{V}{R_a}$ which is sometimes around 25 to 30 times of full load current of motor. This starting current will affect the power system supply quality. Specifically, there will be a heavy drop in system voltage momentarily. So, the starting current has to be limited. This can be achieved by connecting a resistance in the armature circuit. Once the motor picks up speed the back emf is developed and so the external resistance can be gradually cut out from the circuit. This is shown in Fig. 6.40. This resistance can be called as starting resistance.

To start the dc motor a device called starter can be used. The main function of the starter is to control the starting current drawn by the motor. The main component of a starter is resistance R_s. In addition to this they have protective devices like no-voltage release coil and overload release coil. There are 2-point starters meant for starting dc series motors 3-point and 4-point starters which are used for starting dc shunt and compound motors. The construction and working of a 3-point starter alone is discussed here.

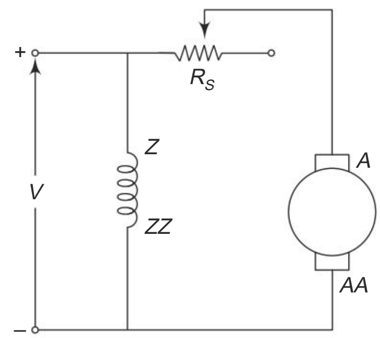


Fig. 6.40

DC 3-point Starter The components and the internal wiring of a 3-point starter is shown in Fig. 6.41.

When the starter handle is moved from OFF position to stud (1) of R_s, the armature circuit of motor is closed through the starting resistance R_s and the overload release coil [OLR coil], thereby the starting current is controlled. Field circuit of motor is also closed through the voltage coil [NVC]. The motor starts running and back emf develops. Now the starter handle can be moved gradually from one stud to another and finally to ON.

The starter handle should be moved against the restraining force offered by the spring mounted in the starter handle. Once the starter handle comes to ON position, the NVC holds the starter handle firmly, i.e. the electromagnetic attracts the soft iron piece attached to the starter handle.

Whenever supply fails, the NVC gets de-energised and the electromagnetic N is demagnetized. Because of spring "force, the starter" handle comes to OFF position. When the motor is over loaded, electromagnetic force offered by the OLR coil is sufficient to attract the iron strip *P*. The contacts *CC* are bridged which results in short circuiting of NVC. So in this case also the electromagnet. "*N*" is demagnetized and the starter handle comes to OFF position. In this way the motor is protected against failure of supply and against overloading.

The main drawback here is, under normal running condition of motor, the resistance R_s included in the field circuit, reduces the field current unnecessarily. This results in less torque production and poor commutation. This problem is removed by

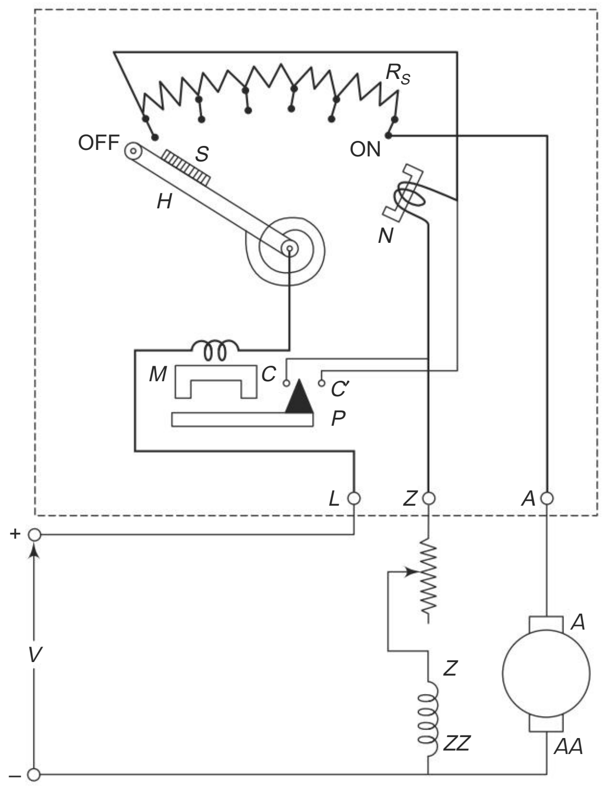


Fig. 6.41

attaching a metal plate in the form of an arc just below the starting resistance. There is no electrical contact between this metal arc and the starting resistance. It is so positioned that when the handle touches stud 1 it also touches the metal arc through which the field gets the full voltage applied.

This three-point starter is not suitable in cases where the motor speed is controlled by weakening its field. This would result in de-energising of the no voltage release coil and so the handle will automatically come back to the OFF position. This disadvantage is overcome in the 4-point starter. In that starter there is one more parallel circuit comprising of the brass strip, no voltage release coil and a series protective resistance. The field winding constietutes another parallel circuit having its own regulator.

6.3 TRANSFORMERS

6.3.1 Principle

The transformer works on the principle of electromagnetic induction. In this case, the conductors are stationary and the magnetic flux is varying with respect to time. Thus, the induced emf comes under the classification of statically induced emf.

The transformer is a static piece of apparatus used to transfer electrical energy from one circuit to another. The two circuits are magnetically coupled. One of the circuits is energised by connecting it to a supply at specific voltage magnitude, frequency and waveform. Then, we have a mutually induced voltage available across the second circuit at the same frequency and waveform but with a change in voltage magnitude if desired. These aspects are indicated in Fig. 6.42.

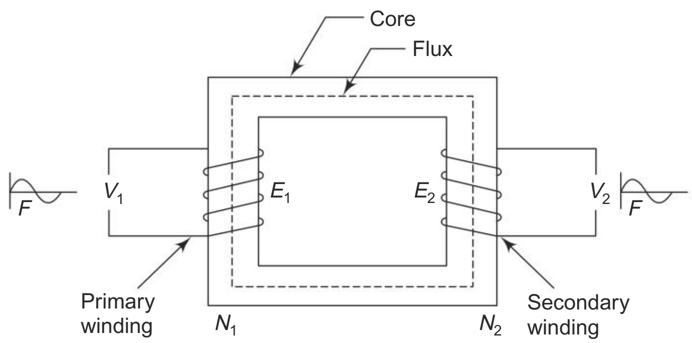


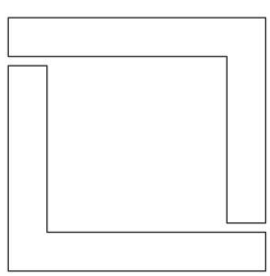
Fig. 6.42

6.3.2 Construction

The following are the essential requirements of a transformer:

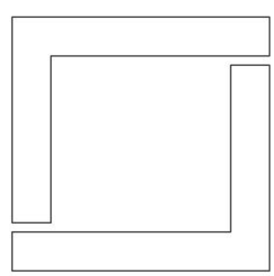
(a) A good magnetic core
(b) Two windings
(c) A time varying magnetic flux

The transformer core is generally laminated and is made out of a good magnetic material such as transformer steel or silicon steel. Such a material has high relative permeability and low hysteresis loss. One type of transformer, called the core type is constructed using L-shaped laminations or stampings. Figures 6.43–6.45 indicate the stacking arrangement of the stampings and the finished core.



First set of stampings

Fig. 6.43



Second set of stampings

Fig. 6.44

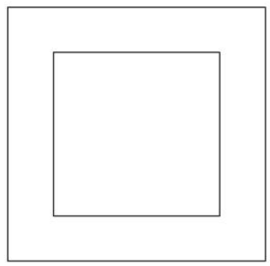


Fig. 6.45

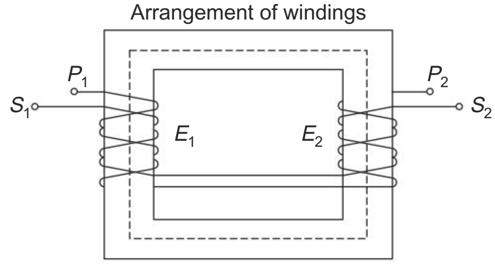


Fig. 6.46

There are two windings in a transformer. They are called primary and secondary windings. Generally, the windings are made of copper. The arrangement of windings on the transformer core is shown in Fig. 6.46.

6.3.3 Working

Let us say that a transformer has N_1 turns in its primary winding and N_2 turns in its secondary winding. The primary winding is connected to a sinusoidal voltage of magnitude V_1 at a frequency 'f' hertz. A working flux of ϕ webers is set up in the magnetic core. This working flux is alternating and sinusoidal as the applied voltage is alternating and sinusoidal. When this flux link the primary and the secondary winding, emfs are induced in them. The emf induced in the primary is the self induced emf and that induced in the secondary is the mutually induced emf. Let the induced voltages in the primary and the secondary be E_1 and E_2 volts respectively. These voltages will have sinusoidal waveform and the same frequency as that of the applied voltage. The currents which flow in the closed primary and the secondary circuits are respectively I_1 and I_2.

In any transformer, $K = \dfrac{N_2}{N_1}$, defines the transformation ratio.

Three categories of transformer action are possible:

$E_2 < E_1$ (i.e. $V_2 < V_1$) ... step down transformer

$E_2 = E_1$ (i.e. $V_2 = V_1$) ... 1:1 or equal ratio transformer

$E_2 > E_1$ (i.e. $V_2 > V_1$) ... Step up transformer

The induced emfs are proportional to the number of turns. In any transformer, the primary ampere turns equals the secondary ampere turns.

i.e. $$N_1 I_1 = N_2 I_2$$

Thus, we have $$\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

Whenever any load is put on the transformer (connected to secondary winding) the primary of the transmission draws the required amount of current in order to keep the working flux constant. Thus, the transformer works with a perfect static balance.

6.4 THREE PHASE INDUCTION MOTOR

6.4.1 Principle

When a three phase balanced voltage is applied to a three phase balanced winding, a rotating magnetic field is produced. This field has a constant magnitude and rotates in space with a constant speed. If a stationary conductor is placed in this field, an emf will be induced in it. By creating a closed path for the induced current to flow, an electromagnetic torque can be exerted on the conductor. Thus, the conductor is put in rotation.

6.4.2 Construction

The important parts of a three phase induction motor are schematically represented in Fig. 6.47. Broadly classified, they are stator and rotor. Each of these is described below.

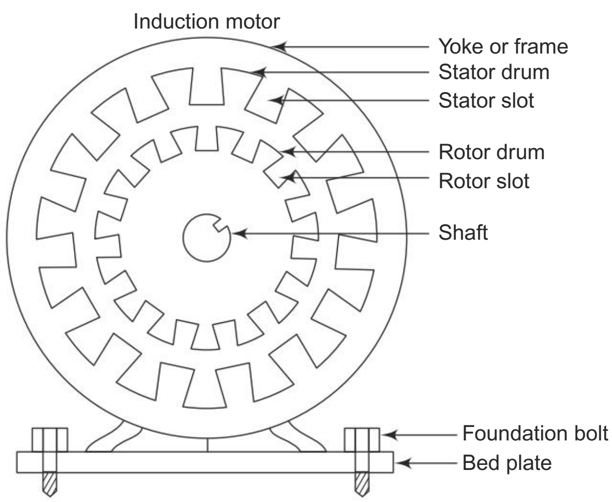


Fig. 6.47

Stator This is the stationary part of the motor. It consists of an outer solid circular metal part called the yoke or frame and a laminated cylindrical drum called the stator drum. This drum ash number of slots provided over the entire periphery of it. Required numbers of stator conductors are embedded in the slots. These conductors are electrically connected in series and are arranged to form a balanced three phase winding. The stator is wound to give a specific number of poles. The stator winding may be star or delta connected.

Rotor This is the rotating part of the induction motor. It is also in the form so slotted cylindrical structure. The air gap between stator and rotor is as minimum as mechanically possible. There are two types of rotors—squirrel cage rotor and slip-ring or wound rotor.

Figure 6.48 shows the construction of a squirrel cage rotor. in this type, each rotor slot accommodates a rod or bar made of a good conducting material. These rotor rods are short circuited at both ends by means of end rings made of the same metal as that of the rotor conductors. Thus, the rotor circuit forms a closed path for any current to flow through.

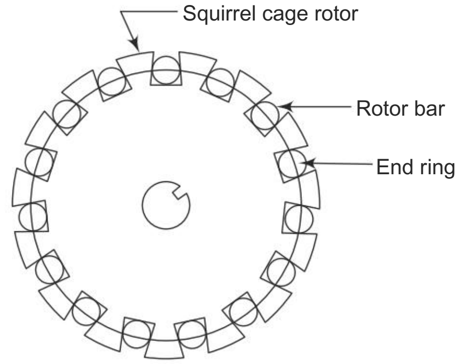


Fig. 6.48

Figure 6.49 shows the rotor winding of a slip ring or wound rotor. In this case, large number of conductors are embedded in the rotor slots. These conductors are electrically connected to form a balanced three phase winding. Again, the rotor is wound to give the same number of poles as the stator. Three similar ends of the rotor winding are joined together thus making a common point. The other three ends are connected to three slip rings which are mounted to the shaft. These slip rings are used to create a short circuit among the three phase windings of the rotor.

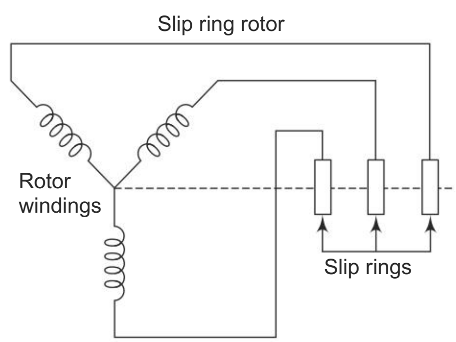


Fig. 6.49

6.4.3 Working

A three phase balanced voltage is applied across the three phase balanced stator winding. A rotating magnetic field is produced. This magnetic field completes its path through the stator, the air gap and the rotor. In this process, the rotor conductors, which are still stationary, are linked by the time varying stator magnetic field. Therefore, an emf is induced in the rotor conductors. When the rotor circuit forms a closed path, a rotor current is circulated. Thus, the current carrying rotor conductors are placed in the rotating magnetic field. Hence, as per the law of interaction, an electromagnetic force is exerted on the rotor conductors. Thus, the rotor starts revolving.

According to Lenz's law, the nature of the rotor induced current is to oppose the cause producing it. Here the cause is the rotating magnetic field. Hence, the rotor rotates in the same direction as that of the rotating magnetic field.

In practice, the rotor speed never equals the speed of the rotating magnetic field (called the synchronous speed). The difference in the two speeds is called slip. The current drawn by the stator is automatically adjusted whenever the motor is loaded.

6.4.4 Starting of 3-phase Induction Motors

If a 3-phase induction motors is directly switched on to the supply, they draw starting current equal to 5 to 8 times of full load current and develop starting torque equal to only 1 to 1.5 times of load torque. This amount of starting current is objectionable because it affects the system voltage momentarily. The starting torque developed may not be sufficient to accelerate the loads which are at rest. To limit the starting current and to improve the starting torque, 3-phase induction motors are started by means of suitable starters. However small capacity (upto 5 HP) motors can be started by using direct on line starters or directly switched on to the supply.

3-phase Squirrel Cage Induction Motor The following are the different methods by which 3-phase squirrel cage induction motors can be started.

(i) Primary resistor or reactor starters
(ii) Auto-transformer starter
(iii) Star-Delta ($Y - \Delta$) starters

In all the above three methods reduced voltage is applied to the terminals of starter winding. Once the motor picks up speed, then normal voltage is applied to the motor. The working of $Y - \Delta$ starter is explained here.

Star–Delta ($Y - \Delta$) Starter This starter is used for induction motor, which is normally working on delta connected stator winding. This starter is having a TPDTS and TPSTS. The necessary wiring of $Y - \Delta$ starter with the starting winding of induction motor is shown in Fig. 6.50. The supply is given to the motor by closing the TPSTS. The starter handle i.e TPDTS is put on "start" position. Now the starter winding is connected to the supply in star connection. So, the starting current is reduced by 1/3 times the starting if the motor is directly switched on. This is well explained in Fig. 6.51. The starting torque developed is about 1.5 to 2 times the full load torque. Once the motor runs on normal speed, the TPDTS can be put on "RUN" position, now the motor winding is connected in delta.

Rotor Resistance Starter for 3-phase Slip Ring Induction Motor The connection of rotor resistance starter with rotor windings of a 3-phase slip ring induction motor is shown in Fig. 6.52.

$$\therefore \quad \text{Starting Current} = \frac{1}{3} \times \text{Current when the motor is delta connected}$$

$$= \frac{1}{3} \times \text{Running current}$$

The normal voltage is applied to the starter windings, keeping the starter resistance at maximum or "start" position. The motor current and hence the supply current is controlled by this resistance. With starting resistance in the rotor circuit, the rotor power factor is increased. This increase the starting torque developed is

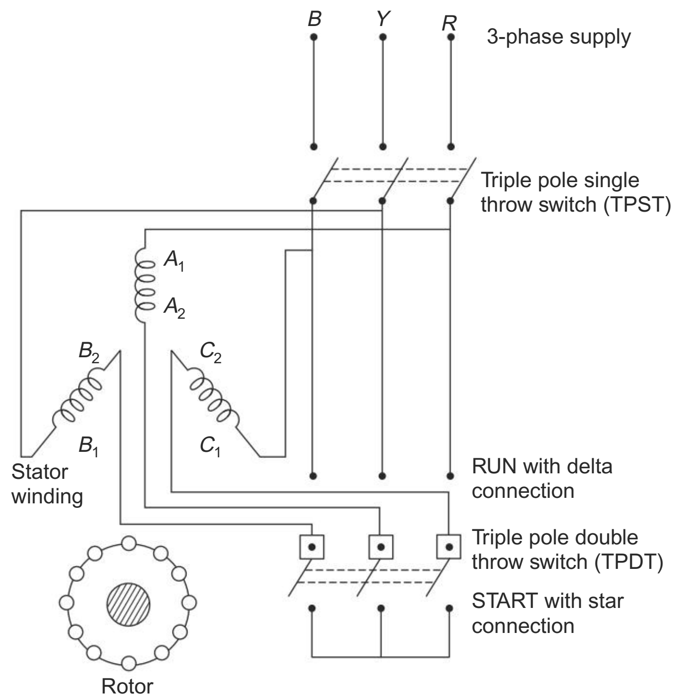


Fig. 6.50

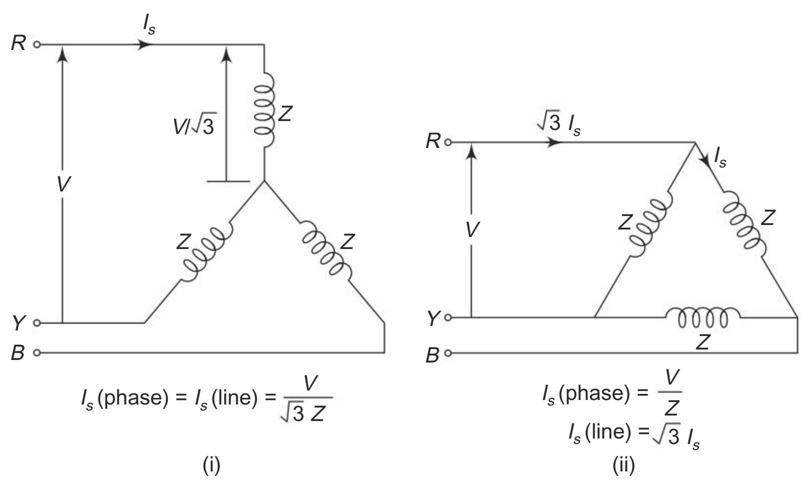


Fig. 6.51

propotional to the rotor power factor. When motor speed reaches the normal value, the starter resistance can be gradually cut so that rotor winding is short circuited through the slip rings.

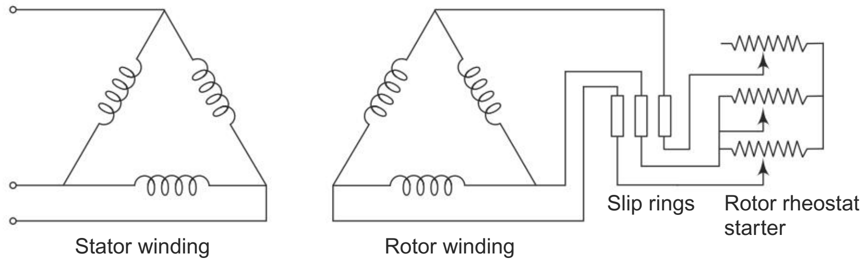


Fig. 6.52

6.5 SINGLE PHASE INDUCTION MOTORS

Single phase induction motors are widely used in domestic, industrial and machine tool applications. The capacity of this motor varies from fractional horse power to 5 HP. Fractional horse power motors are used in variety of applications.

6.5.1 Construction and Non-Self Starting of 1-phase Induction Motor

The construction of 1-phase is similar to 3-phase induction motors. The starter has a single winding. The rotor is squirrel cage type as in 3-phase induction motor. When starter winding is energised with single phase supply an alternating flux is set up, but it is not a revolving field as in 3-phase induction motor. This alternating flux when acting on stationery motor cannot produce rotation. Only a revolving starting. However if the rotor is given an initial start by hand or by other means, then motor may start and run.

To make the single phase induction motors self starting, the following type of 1-phase induction motors were developed.

(a) Split-phase induction motors
  (i) Resistance-start motor
  (ii) Capacitor-start motor
  (iii) Permanent split-capacitor motor
  (iv) Two-value capacitor motor
(b) Shaded-pole induction motor
(c) Reluctance-start induction motor
(d) Repulsion-start induction motors

Only two of the above types of 1-phase are discussed in the following sections.

6.5.2 Split Phase Resistance Start 1-Phase Induction Motor

In addition to the main winding of the motor, an auxillary winding is also placed in the starter. Both these windings are uniformly distributed in the stator slots. The two windings are displaced by 90° (electrical) in space. The main winding is called running winding and the auxillary winding is called the starting winding. The arrangement of these two windings in stator is shown in Fig. 6.53. The main winding is

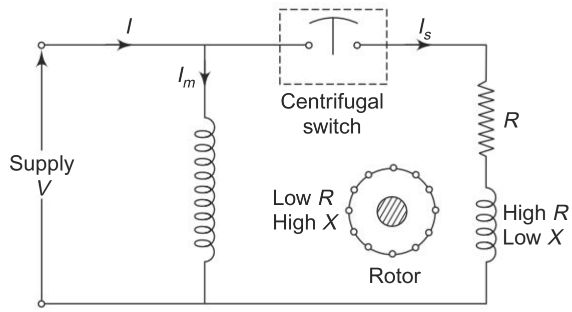


Fig. 6.53

highly inductive and the auxillary winding should be highly resistive. For this purpose, the auxiliary winding is provided with less number of turns. To make it highly resistive external resistance can be used in the auxiliary winding circuit. A centrifugal switch placed in the auxiliary winding circuit. The switch is in closed condition when the motor is at rest.

The 1-phase supply is given to the two windings, which are I parallel across the supply. The currents through the main winding and starting winding are displaced by an angle α as shown in Fig. 6.54. This angle α should be kept nearer to 90° by proper design of starting winding. The starting torque developed is proportional to sin α. When the motor attain 75% of rated speed, the centrifugal switch is opened and hence starting winding is disconnected from the supply. The starting torque developed is 150 to 200% full load torque of the motor.

This type of motor is used in oil burns, machine tools, grinders, dish washes, washing machines, airblowers and air compressors.

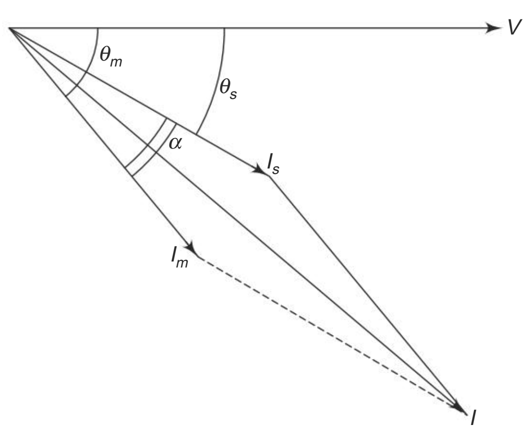


Fig. 6.54

6.5.3 Split Phase Capacitor Start Induction Run Single Phase Induction Motor

The single phase supply given to the stator winding, can be split into two phases by connecting a suitable capacitor in series with the starting winding, as shown in Fig. 6.55 (i). The value of capacitance should be such that the phase angle α between I_s and I_m should be nearer to 90°. Then the fluxes due to this current is also displaced by the same angle α. This causes revolving field. The starting torque developed depends on sin α. The starting torque developed is about 300 to 450% of full load torque. This is more as compared to the previous type motor as α is more here. The value of α in the previous type motor is around 40° only. When the motor attains 75% of rated speed, the centrifugal switch opens and the starting winding is isolated from the circuit.

This motor is used where high starting torque is needed under loaded condition. They are used for pumps, refrigerations units, air-conditioners, large size washing machines, etc.

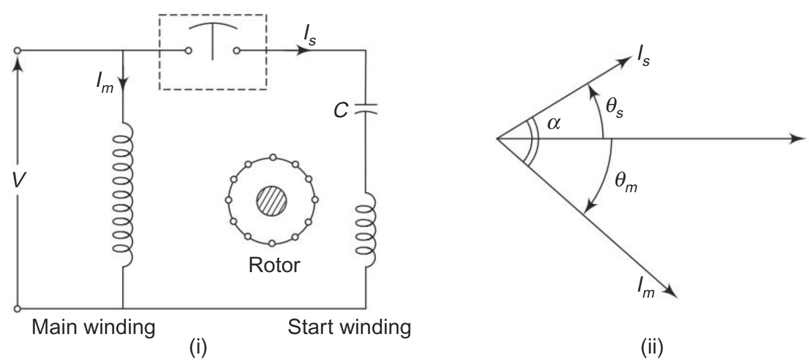


Fig. 6.55

6.6 3-PHASE AC GENERATOR OR ALTERNATOR

An alternator works on the principle of electromagnetic induction. If a conductor is placed in a moving magnetic field an emf is induced in the stationery conductor as per Faraday's first law of electromagnetic induction.

6.6.1 Construction

The two important parts of AC generators are stator and rotor. The construction of ac generator showing all main parts is given in Fig. 6.56.

Stator The stator consists of a cast iron or mild steel frame, which supports the armature core. This frame acts as an enclosure and provides a closed path for the magnetic flux. The armature core is made of laminated sheets. The material for armature core may be special magnetic iron or steel alloys. The inner periphery of

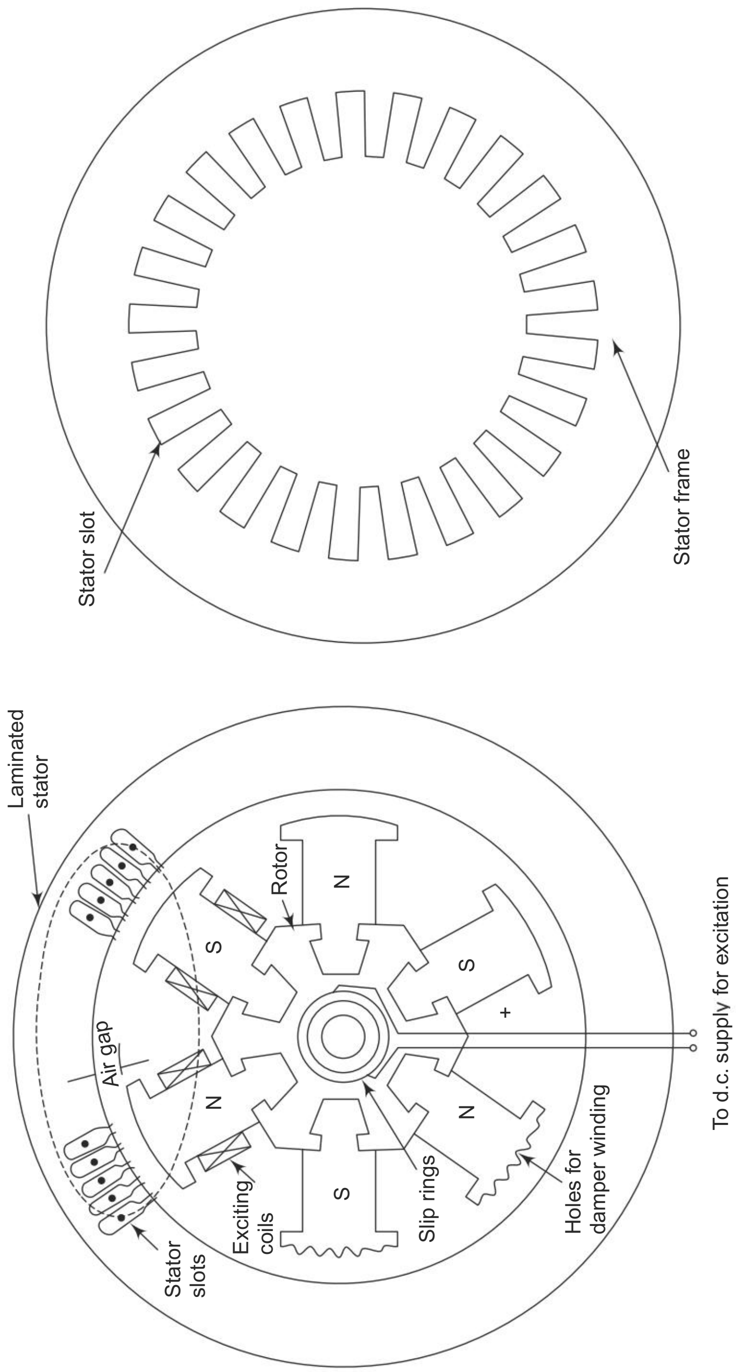

Stator slot
Stator frame
Laminated stator
Rotor
N
S
S
Air gap
N
S
+
N
Exciting coils
Slip rings
Holes for damper winding
Stator slots
To d.c. supply for excitation

Fig. 6.56

the armature core is slotted, in order to accommodate the armature winding. The armature winding may be single layer or double layer. The 3-phase armature winding should be a balanced one. The number of turns and size of wire should be same for all the 3-phase winding and the 3-phase winding are displaced in space by 120° (electrical) between them. The three ends of three phase winding are connected together to make the starprint, and the other three terminals of the three phase windings are brought out of the generator.

Rotor The rotor is like a flywheel having alternate N and S poles on its outer periphery. These electromagnets are magnetised by means of low dc voltage of 125 or 250 V. As rotor and hence the field magnets are rotating, this dc excitation voltage is given through slip rings which are fixed on the frame.

There are two types of rotors used in ac generators.

(i) *Salient or projecting pole type* It is used for engine driven generators, which are run at low and medium speed. The rotor has even number of projecting poles, whose cores are boiled to a heavy magnetic wheel of cast iron. The axial length of rotor is short and the diameter is large.

(ii) *Smooth cylindrical type* It is used for steam turbine driven generators or turbo alternators which are run at high speed. The rotor is made up of cast iron and cylindrical in shape. The outer periphery of the rotor is slotted to receive the field windings. The field windings are wound, such that N and S poles occur alternately. The number of field poles may be two or four. The axial length of rotor is large and its diameter is less.

6.6.2 Working of Alternator

The field magnets are magnetised by applying 125 volts or 250 volts through slip rings. The field windings are connected such that, alternate N and S poles are provided. The rotor and hence the field magnets are driven by the prime movers (steam driven turbine or engine driven). As the rotor rotates, the armature conductors are cut by the magnetic flux. Hence emf is induced in the armature conductors. As the magnetic poles are alternately N and S pole, the emf acts in one direction and then in the other direction. Hence an alternating emf induced in the stator conductors. The frequency of induced emf is depends on the number of N and S poles moving past an armature conductor in one second. The frequency of induced emf is given by,

$$f = \frac{P\,N}{120}; \text{ P} \rightarrow \text{No. of magnetic poles} \qquad (6.17)$$

$$N \rightarrow \text{Speed of rotor in rpm.}$$

The direction of induced emf can be obtained by Fleming's Right hand rule.

6.7 SYNCHRONOUS MOTOR

Synchronous motor runs only at synchronous speed, $N_s = \dfrac{120\,f}{P}$ on no-load and loaded conditions. It maintains speed while running. The only way to change the speed is by varying the supply frequency. It is not a self starting motor. This motor has to run upto its synchronous speed by some method, and then it can be synchronised.

6.7.1 Construction

The construction of synchronous motor is same as that of 3-phase ac generator. A synchronous machine may be run as synchronous motor and as synchronous generator.

6.7.2 Working Principle of Operation

When a 3-phase balanced winding is supplied with a balanced 3-phase supply, a rotating magnetic field of constant magnitude and synchronous speed is produced. Let the motor stator have 2-poles marked as N_s and S_s as shown in Fig. 6.57. The stator poles are rotating at synchronous speed. Let us assume that it moves in clockwise direction. The rotor is positioned as shown in Fig. 6.58 initially with reference to the position *AB* marked. As like poles repel, the rotor will tend to move in anticlockwise direction. At the end of first half cycle of supply, the rotor poles occupy the position as shown in Fig. 6.58 with respect to the reference plane *AB*. Now, because of attractive force between the unlike poles, the rotor will tend to move in the opposite, i.e. clockwise direction. So, with rapid movement of the stator poles, the rotor is subjected to a torque which is also rapidly reversing. But because of the inertia of the rotor, it will not respond to such a rapid reversing torque. So, the rotor remains stationary, which shows that it is not a starting motor.

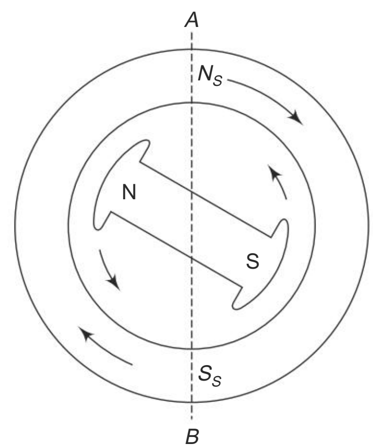


Fig. 6.57

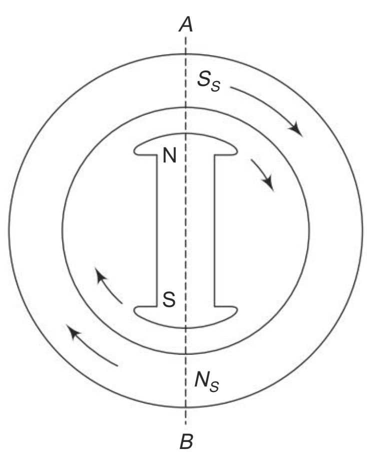


Fig. 6.58

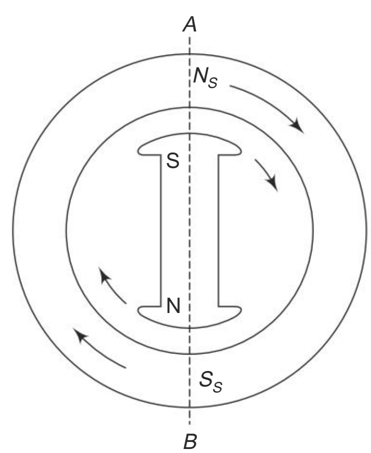


Fig. 6.59

Now consider the position of the rotor as shown in Fig. 6.59. The stator and rotor poles attract each other. Let the rotor also be rotating in clockwise direction at synchronous speed as that of stator field. So, the stator and rotor poles attract each other, at any instant, which results in unidirectional torque production, in clockwise direction in this case.

6.7.3 Sychronous Motor Starting Method

The rotor with unexcited magnetic poles may be made to run at synchronous speed by a dc motor which is mechanically coupled to the rotor of synchronous motor. The 3-phase ac supply is given to the stator winding of 3-phase generator. The dc supply to magnetic poles is switched on. The rotor magnetic poles and the stator magnetic poles are magnetically locked and will run at synchronous speed. Now dc motor can be switched off. The synchronous motor will drive the dc motor as generator.

The methods for starting synchronous motor are:

(i) A dc motor coupled to the synchronous motor shaft: (discussed above)
(ii) Using the damper windings as a squirrel cage induction motor.
(iii) Using the field excited generator as a dc motor [similar to method (i)].

6.7.4 Synchronous Motor-application

(i) It is used in power house and in major substations to improve the power factor.
(ii) Used in textile mills, rubber mills, mining and other big industries for power factor improvement purpose and to maintain the facility of system voltage.
(iii) It is used to drive continuously operating and constant speed equipments like fans, blowers, centrifugal pumps, air comparators, etc.

REVIEW QUESTIONS

1. Explain with the help of a diagram, how a dc generator works. Why is commutator used in it?
2. Explain the production of back emf in a dc motor. Hence, describe the working of the same.
3. Draw a neat diagram showing parts of a dc machine. Explain briefly the function of each.
4. Explain the principle of operation of a transformer. What are its applications?
5. Describe the working of a three phase induction motor. What are its applications?
6. Derive the equation for the emf generators in a dc generator.
7. How do you classify generators?
8. Draw the circuit diagram and give the characteristic equations for (i) separately excited dc generator (ii) dc shunt generators (iii) dc series generator and (iv) long-shunt and short-shunt dc compound generators.
9. What do you mean by the term "compounding" with reference to dc generators?
10. What is armature reaction?
11. What is open characteristics of a dc generator?
12. Draw the O.C.C of a shunt generators and draw the line to represent the critical field resistance.

13. What are external and internal characteristics of dc generators? What significant information are conveyed from them?
14. Define the terms: Critical field resistance, critical speed, voltage regulations with reference to dc generators.
15. Draw the equivalent circuit and give the performance equations of different types of dc motors.
16. Derive equations for torque and speed in dc motor.
17. What are the important characteristics plotted for dc motors?
18. Analyse the mechanical characteristics of dc shunt, series and compound motors.
19. Discuss the various losses in a dc motor.
20. How do you compute the efficiency of a dc motor and that of a dc generator?
21. List the basic factors on which the speed of a dc motor depend?
22. Make a brief analysis of the field control method for controlling the speed.
23. Analyse the armature control method of controlling the speed of dc motors.
24. Explain the Ward Leonard Method of speed control?
25. What is the need for Starter in a dc motor?
26. Draw a neat diagram of a dc 3-point starter and explain its working.
27. Is a starter necessary for starting a 3-phase induction motor? why?
28. Describe the working of a star-delta starter.
29. What is the principle of a rotor resistance starter?
30. Why is not a single phase induction motor self starting?
31. What are the methods adopted to make the single phase induction motors self start?
32. Describe the construction and working of a phase resistance start 1-phase induction motor.
33. Discuss the working of a split phase capacitor start induction run 1-phase motor.
34. What is an alternator?
35. Give the constructional details of an alternator.
36. Explain the working principle of an alternator.
37. Explain the principle of operation of a synchronous motor?
38. What methods can be employed to start a synchronous motor?
39. List the applications of synchronous motor.
40. List the applications of 1-phase induction motor.

MEASURING INSTRUMENTS

7

INTRODUCTION

A measuring instrument is a piece of apparatus used to measure a quantity such as voltage, current, power, energy, resistance, etc. It may indicate, by a deflection, the quantity under measurement or give consumption of electricity and electrical energy during a specified period or produce a continuous record of the variations in a quantity. Many principles are utilized in the operation of such instruments. This chapter deals with certain deflecting and an integrating type of measuring instruments. All of them make use of the electromagnetic or magnetic effect for their working.

7.1 CLASSIFICATION OF INSTRUMENTS

Electrical measuring instruments are classified as follows:

I.	Depending on the quantity measured	e.g.	Voltmeter, Ammeter, Wattmeter, Energymeter, Ohmmeter.
II.	Depending on the different principles used for their working	e.g.	Moving Iron type, Moving coil type, Dynamometer type, Induction type.
III.	Depending on how the quantity is measured?	e.g.	Deflecting type, Integrating type, Recording type.

7.2 BASIC PRINCIPLES OF INDICATING INSTRUMENTS

These instruments give a deflection proportional to the quantity being measured. For their satisfactory operation, the following torques shall act upon the moving system of the instruments.

7.2.1 Deflecting Torque

This torque acts on the moving system of the instrument to give the required deflection. It exists as long as the instrument is connected to the supply. It is produced by any one of the effects such as magnetic, electromagnetic, induction, chemical effects. The deflecting torque shall ensure a deflection proportional to the magnitude of the quantity being measured. The magnitude of the deflecting torque produced is actually dependent on the quantity to be measured.

7.2.2 Opposing Torque

This torque always opposes the deflecting torque. The moving system attains a steady deflected position when the opposing torque equals the deflecting torque. The components of the opposing torque are inertia torque, control torque and damping torque. These are explained in the following sub-sections.

(a) Inertia Torque This is due to the inertia of the moving system. The deflecting has to overcome this and make the moving system move from its rest position.

(b) Control Torque This torque is always present in the instrument whether it is connected to the supply or not. The control torque increases with the deflection of the moving system. It opposes the deflecting torque. The moving system is brought to a steady deflected position when the control torque is balanced by the deflecting torque. The control torque is also essential to bring back the moving system to its *initial* or *rest* or *zero* position once the instrument is disconnected from the supply. The control torque can be produced using spring or gravity as explained below:

(i) Spring control (Refer Fig. 7.1). Two helical springs of rectangular cross section are connected to the spindle of the moving system. With the movement of the pointer, the springs bet twisted in the opposite direction. Thus, the required amount of control is effected on the moving system. Also, once the instrument is disconnected from the supply, the pointer (moving system) is brought back to its initial position due to the twisted spring.

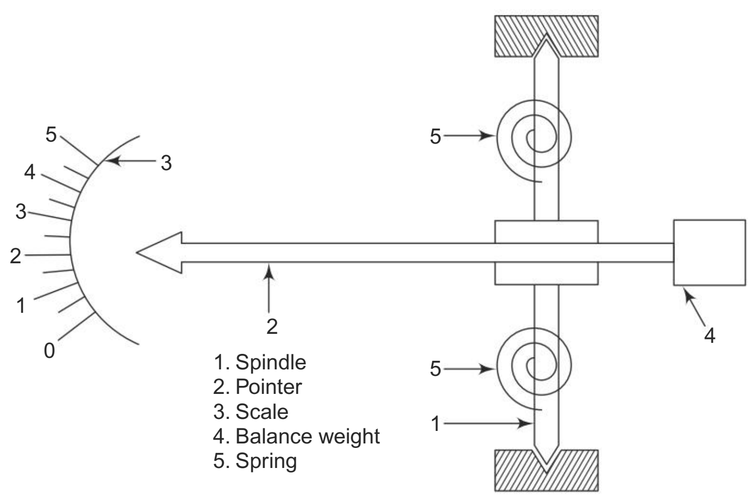


Fig. 7.1

In spring controlled instruments, the scale will be linear if the deflecting torque is proportional to the quantity being measured.

(ii) Gravity control (Refer Fig. 7.2). In this method, adjustable small weights are added to some part of the moving system. When the pointer deflects, this weight

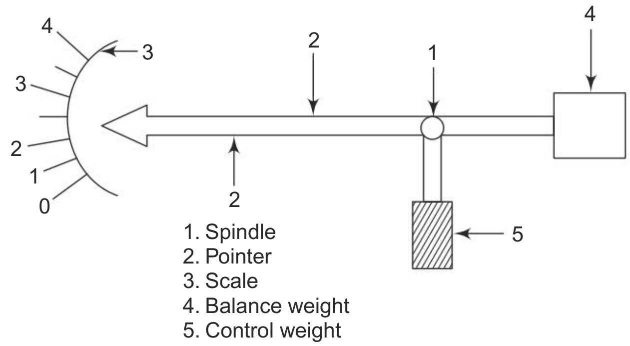


Fig. 7.2

also takes a deflected position. The gravitational force acting on the moving weight produces the required control torque. The instruments making use of gravity control arrangement will have a non-uniform scale. The scale will be cramped in the initial range and more uniform towards the end. Use of gravity control imposes the restriction that the instrument shall be used in a vertical position only.

(c) Damping Torque This torque is produced only when the instrument is in operation. This ensures that the moving system takes just the required time to reach its final deflected position. Then, the instrument is said to be 'dead-beat'. If sufficient damping torque is not produced, the pointer make underdamped oscillations before reaching the steady deflection. If the damping torque is more than the required value, the pointer becomes sluggish and it takes longer than the required time to reach the final deflection (Refer Fig. 7.3).

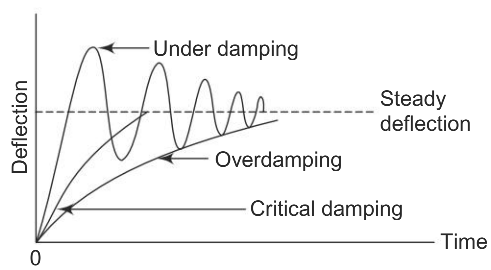


Fig. 7.3

The required damping torque can be produced by the following methods:

(i) Air friction damping Two methods of producing damping torque using air are shown in Figs. 7.4 and 7.5. Figure 7.4 describes the arrangement where a piston attached to the spindle of the moving system is positioned to move inside an air chamber. There is a very small clearance between the piston and the chamber. When the spindle moves be to the deflecting torque, the piston moves inside the air chamber. The suction and the compression actions on the air inside the air chamber

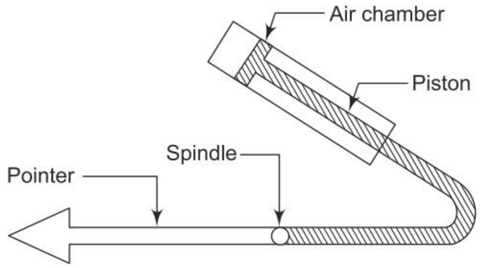


Fig. 7.4

produces the necessary damping torque. Generally, the air chamber and the piston are made of aluminium.

Another arrangement is shown in Fig. 7.5. There is a sector shaped box containing air. In this box move a pair of vanes attached to the spindle of the instrument. The movement of the vanes in the air produces the required damping torque. Both the box and the vanes are made of aluminium.

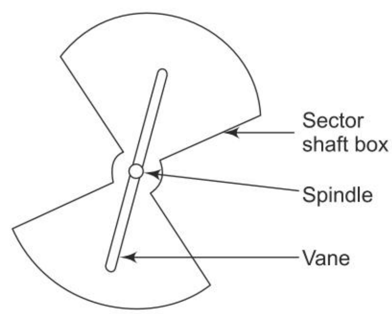


Fig. 7.5

(ii) Eddy current damping (Refer Fig. 7.6) A thin disc of a conducting but non-magnetic material (like copper or aluminium) is mounted on the spindle. The spindle carries the moving system and the pointer. A permanent magnet is used to produce the required magnetic flux. The position of the disc is such that while in motion, it cuts the magnetic flux. Hence, eddy currents are produced in the disc. These currents flow in such a direction that the motion of the disc is opposed. Thus, the required damping torque is produced. This is method is more effective than the other method.

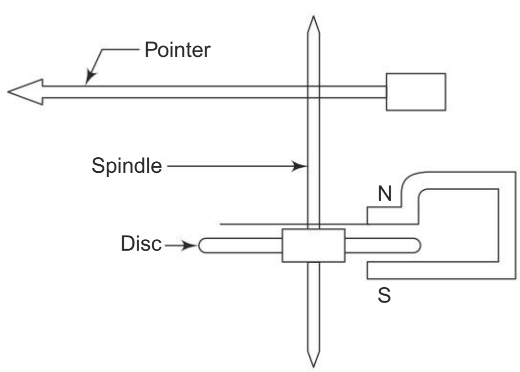


Fig. 7.6

7.3 MOVING IRON INSTRUMENTS–ATTRACTION TYPE

Moving Iron Instruments are used mainly to measure voltage or current. These are two types of moving iron instruments namely attraction type and repulsion type.

Principle It is well known that a soft iron piece gets magnetised when it is brought into a magnetic field produced by a permanent magnet. The same phenomenon happens when the soft iron piece is brought near either of the ends of a coil carrying current. The iron piece is attracted towards that portion where the magnetic flux density is more. This movement of the soft iron piece is used to measure the current or voltage which produces the magnetic field.

Construction (Refer Fig. 7.7). The instrument consists of a working coil. It carries the current to be measured or a current proportional to the voltage to be measured. A soft iron disc is attached to the spindle. To the spindle, a pointer is also attached. The pointer is made to move over a calibrated scale. The moving iron (soft iron disc) is pivoted such that it is attracted towards the centre of the coil where the magnetic field is maximum.

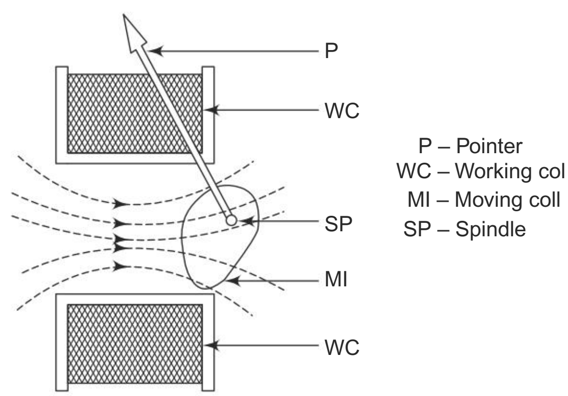


Fig. 7.7

Working The working coil carries a current which produces a magnetic field. The moving disc is attracted towards the centre of the coil where the flux density is maximum. The spindle is, therefore, moved. Thus, the pointer, attached to the spindle gives a proportional deflection.

Deflecting Torque Produced by the current or the voltage to be measured. It is proportional to the square of the current or voltage. Hence, the instrument can be used to measure d.c. or a.c scale is non-uniform.

Control torque: Spring or gravity
Damping: Air friction damping

7.4 MOVING IRON INSTRUMENT–REPULSION TYPE

Principle Two iron pieces kept with close proximity in a magnetic field get magnetized to the same polarity. Hence, a repulsive force is produced. If one of the two pieces is made movable, the repulsive force will act on it and move it on to one side. This movement is used to measure the current or voltage which produces the magnetic field.

Construction (Refer Fig. 7.8). The instrument consists of a working coil which carries a current proportional to voltage or the current to be measured. There are two iron pieces-fixed and moving. The moving iron is connected to the spindle to which is attached a pointer. It is made to move over a calibrated scale.

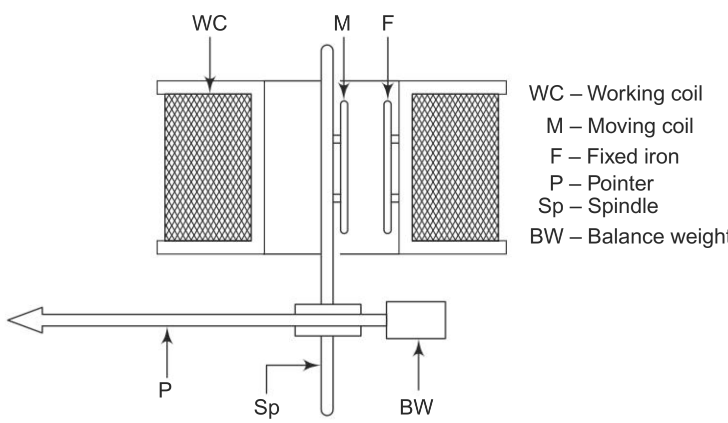


Fig. 7.8

Working When the operating coil carries current, a magnetic field is produced. This field magnetises similarly both the soft iron pieces. Thus, a repulsive force is produced which acts on the moving iron and pushes it away from its rest position. Thus, the spindle moves and hence the pointer gives a proportionate deflection. Whatever be the direction of current in the coil, the two irons are always similarly magnetised.

Deflecting Torque Produced by the current or the voltage to be measured it is proportional to the square of the current or voltage. Hence, the instrument can be used for dc and ac.

Control torque: Spring or Gravity
Damping: Pneumatic (i.e air damping)

7.5 MOVING COIL INSTRUMENTS–PERMANENT MAGNET TYPE

Principle A current carrying coil is placed in a magnetic field, a force is exerted. It tends to act on the coil and moves it away from the field. This movement of the coil is used to measure current or voltage.

Construction (Refer Figs. 7.9 and 7.10). N and S refer to the pole pieces of a permanent magnet. A soft iron core in the form of a cylinder is placed in the space between the poles (C). In the permanent magnetic field is placed a rectangular coil of many turns (MC) wound on a former (AF). The former is made of aluminium or copper. To the moving coil is attached the spindle (S_P). Two helical springs (S_g) are connected to the spindle to give the necessary control torque. A pointer (p) attached to the spindle is made to move over a calibrated scale.

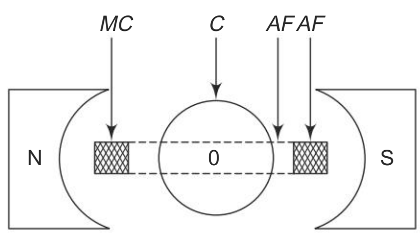


Fig. 7.9

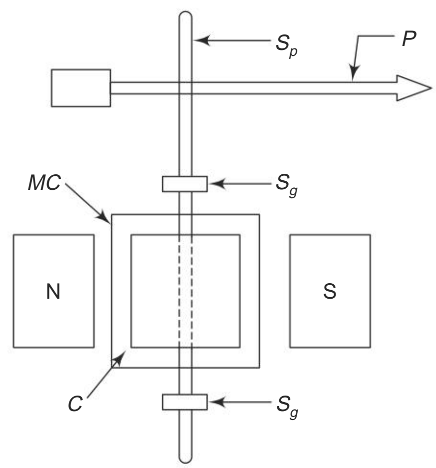


Fig. 7.10

Working A magnetic field of sufficient density is produced by the permanent magnet. The moving coil carries the current or a current proportional to the voltage to be measured. Hence, an electromagnetic force is produced which tends to act on the moving coil and moves it away from the field. This movement makes the spindle move and so the pointer gives a proportionate deflection.

Deflecting torque	... It is directly proportional to the current or the voltage to be measured. So, the instrument can be used to measure direct current and dc voltage.
Control torque	... Spring control
Damping torque	... Eddy current damping. When the moving coil made of aluminium former is moved due to the force exerted on it, it cuts the magnetic flux lines produced by the permanent magnet. Hence, eddy currents are induced in the former.

As per Lenz's law, these eddy currents produce the required damping torque opposing the motion of the moving coil.

7.6 MOVING COIL INSTRUMENTS–DYNAMOMETER TYPE

Principle Working principle of this type of instrument is same as that of permanent magnet moving coil type. But, the difference is that there is no permanent magnet in this instrument. Both the operating fields are produced by the current and/or the voltage to be measured.

Construction (Refer Fig. 7.11). The fixed coil (*FC*) is made in two sections. In the space between these two sections, a moving coil (*MC*) is placed. The moving coil is attached to the spindle to which is attached a pointer. The pointer is allowed to move over a calibrated scale. Two helical springs are attached to the spindle to give the required control torque. A piston attached to the spindle is arranged to move inside an air chamber.

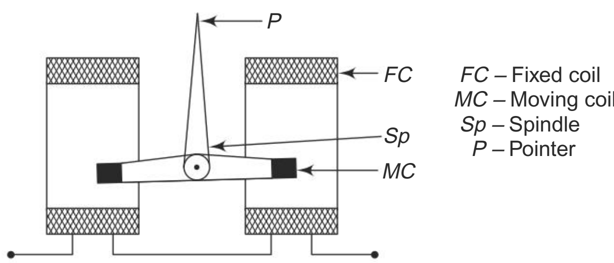


Fig. 7.11

Working The fixed coil and the moving coil carry currents. Thus, two magnetic field are produced. Hence, an electromagnetic force tends to act on the moving coil and makes it move. This makes the pointer give a proportionate deflection.

Deflecting Torque

(a) *As voltmeter* The two coils are electrically in series. They carry a current proportional to the voltage to be measured. The deflecting torque is proportional to $(\text{voltage})^2$. Hence, the instrument can be used for measuring dc and ac voltages.

(b) *As ammeter* The two coils are electrically in series. They carry the current to be measured. The deflecting torque is proportional to $(\text{current})^2$. Hence, the instrument can be used for measuring dc and ac.

(c) *As wattmeter* Fixed coils carry the system current. Moving coil carrys a current proportional to the system voltage. The design is such that the deflecting torque is proportional to VI cos ϕ, i.e power to be measured.

Control torque: Spring Control

Damping torque: Air damping

7.7 INDUCTION TYPE ENERGY METER

Energy meter is an integrating meter. It gives the quantity of electrical energy consumed over a specified period.

Principle When a conducting metal part is placed in an alternating magnetic field, eddy currents are induced in the metal part. The magnetic flux produced by these eddy currents are made to interact with another magnetic field. Thus, the required operating torque is produced. The instrument can work on *alternating current* only.

Construction The salient parts of an induction type energy meter are schematically shown in Fig. 7.12.

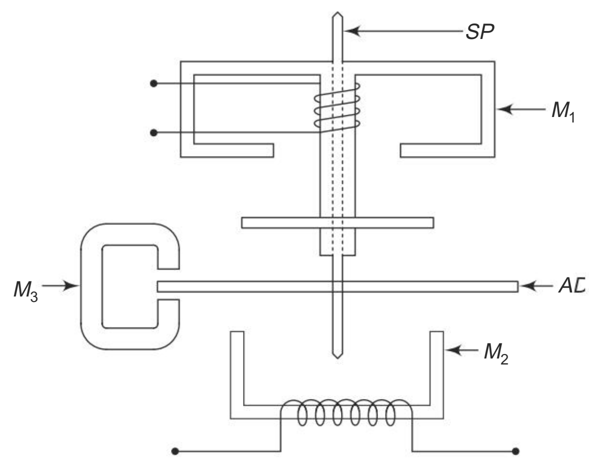


Fig. 7.12

The instrument consists of the following parts:

M_1 — Shunt magnet. The coil over this carries a current proportional to the system voltage.

M_2 — Series magnet. The coil over this carries a current proportional to the system current.

AD — Aluminium disc connected to the spindle (SP).

M_3 — Brake magnet. This is a permanent magnet. It is so arranged that the aluminium disc is in the gap between the pole pieces of it. As a result of this, when in rotation, the aluminium disc cut the permanent magnetic flux.

Working

Rotating torque: The current in the shunt magnet produces a flux ϕ_{sh} which in turn produces are eddy current i_{sh} in the disc. Similarly, the current in the series magnet set up of a flux ϕ_{se} which produced an eddy current i_{se} in the disc. Due to the interaction between the sets of fluxes and the eddy currents, a torque is exerted on the disc and so the disc is put in rotation. Such a torque is continuously exerted and so the disc continues to rotate as long as the instrument is connected to the supply. That torque is called the rotating torque. It is proportional to the power consumed and the rotation of the disc accounts for the time. Thus, the energy is recorded.

Braking torque This is similar to damping torque in deflecting instruments. When in rotation, the aluminium disc cuts the magnetic flux produced by the brake magnet. Hence, the induced currents interact with the permanent magnetic flux and produce the braking torque. As per Lenz's law, the braking torque opposes the rotating torque. The aluminium disc attains a steady speed when the braking torque balances the rotating torque.

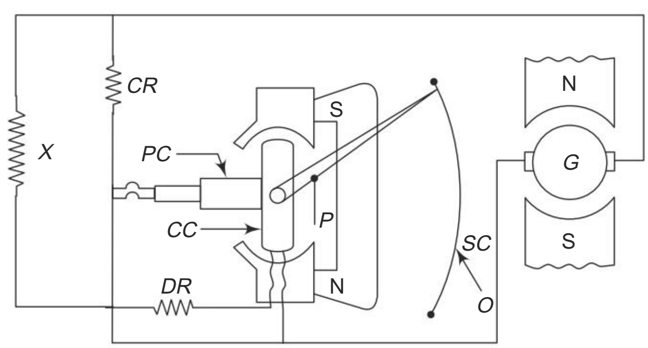
Registering mechanism The instrument has a suitable registering mechanism by which the consumption of energy is recorded correctly.

7.8 MEGGER

Megger is the most portable insulation tester. It is used to measure very high resistances of the order of megaohms.

Principle The instrument works on the principle of ratiometer/ohmmeter. The required deflecting torque is produced by both the system voltage and the current. Due to interaction between the magnetic fields produced by the voltage and the current, the deflecting torque is produced. The required coils are so positioned that the deflecting torque is proportional to the ratio, *VII*.

Construction (Refer Fig. 7.13). It consists of (i) A small hand driven d.c. generator (ii) A moving element which has 2 coils, a deflecting coil (or current coil) and a controlling coil (or potential coil) (iii) Calibrated scale in mega ohms (iv) Pointer and (v) Permanent magnet.



G –Generator
PM –Permanent magnet
SC –Scale(in mag ohms)
P –Pointer
PC –Pressure coil
CC – Current coil
CR – Control circuit resistance
DR – Deflecting circuit resistance
X – Unknown resistance

Fig. 7.13

The two coils are rigidly mounted at right angles to each other. They are connected to the small hand driven generator. The coils, move in the air gap of a permanent magnet. To protect the coils under short circuit, a limiting resistor is connected in series with the coils.

Operation Resistance to be measured is connected across the test terminals, i.e. connected in series with the deflecting coil and across the generator. When currents are supplied to the coils, then they have torques in opposite directions.

If the resistance to be measured is high, no current will flow through the deflecting coil. The controlling coil will, therefore, set itself perpendicular to the magnetic axis and hence, sets the pointer at infinity.

If the resistance to be measured is small, a high current flows through the deflecting coil and the resulting torques sets the pointer to zero.

For intermediate values of resistances, depending upon the torque production, the pointer is set at a point between zero and infinity.

The hand driven generator is of permanent magnet type and it is designed to generate from 500 to 2500 volts.

REVIEW QUESTIONS

1. What are the basic requirements of an indicating instrument? Briefly discuss them.
2. Why is damping torque necessary in indicating instruments? What are the methods of producing the same? Explain with necessary sketches.
3. Justify the necessity of controlling torque in indicating instruments. Discuss the various methods of producing the same. Compare them.
4. What is the working principle of a moving iron voltmeter? Explain its working.
5. What do you understand by attraction type and repulsion type instruments? What are the important difference between moving coil and moving iron instruments?
6. Describe the working principle of a moving coil ammeter.
7. Explain the working of a dynamometer type instrument. What are the specific requirements of the same when it is used as a wattmeter?
8. Name the instruments used for measuring the electrical power consumed during a specific period. Discuss how the same functions.
9. What is a megger? Explain its working principle.

HOUSE WIRING

8

INTRODUCTION

House wiring is to deal with the distribution system within the domestic premises. The wiring requirements may vary among the different consumers. House wiring is generally done for consumption of electrical energy at 230 V, 1-phase or at 400 V, 3-phase. In the latter case, the total load in the house is expected to be divided among the three phases. An earth wire is also run connecting all the power plugs from where large quantity of electrical energy is tapped by using electrical appliances like heater, electric iron, hot plate, etc. This chapter deals with the wiring materials and accessories, different systems of wiring and earthing methods.

8.1 WIRING MATERIALS AND ACCESSORIES

8.1.1 Switches

A switch is used to make or break the electric circuit. In must make the contact firmly. Under some abnormal conditions it must retain its rigidity and keep its alignment between switch blades and contacts correct to a fraction of cm. Different types of switches are as follows:

(a) Surface Switch or Tumbler Switch These switches are mounted on the mounting block directly fixed over the surface of the wall. Such types of switches project out of the surface of the wall. These switches can be classified into single-way and two-way switches.

Single-way switch: —|_

Two-way switch: —\— used for wiring circuits which are to be controlled from two points independently.

(b) Flush Switch The flush switch is fixed in-fixed with the wall and it does not project out. These switches can also be classified into single-way and two-way switches.

(c) Pull Switches or Ceiling Switches The pull switches are fixed on the ceiling and all the live parts are out of reach of the operator. The switch has a strong mechanical action and is usually operated with a single pull on the chord for on or off.

(d) Rotary Snap Switches The rotary switch consists of an insulated handle to which are fixed the blades. These blades move in steps by the movement of the handle and make contact with the terminals to which are connected the wires in the electric circuits. The movement of the handle is controlled by a cam or a spring. As the handle is moved by a quarter turn the blade is released and moves over quickly (with the help of spring) to make or break the circuit. These switches are available in single- or two-way patterns.

(e) Push Button Switch This type of switch consists of one blade only. The blade is given a rocking action by press buttons and its movement is controlled by a cam and a spring. Thus, the blade opens or closes with quick motion.

(f) Iron-clad Water Tight Switches Such switches are of cast iron and have robust construction. A cook gasket is fitted between the case and the cover which makes it watertight.

8.1.2 Lamp Holders

A lamp holder is used to hold the lamp for lighting purposes. The different types are as follows:

Pendent holder
Batten holder — for incandescent bulbs
Screw lamp holders — for bulbs rated 200W and above
Fluorescent lamp holders — for fluorescent tubes
Starter holders — for tube light starters

8.1.3 Lamp Holder Adopter

It is used for tapping temporary power for small portable electric appliances from lamp holders. Such a practice is not advised.

8.1.4 Ceiling Roses

These are used to provide a tapping to the lampholder through the flexible wire or a connection to a fluorescent tube or a ceiling fan.

8.1.5 Mounting Blocks

These are nothing but wooden round blocks. They are used in conjunction with ceiling roses, batten lamp holders, surface switches, ceiling switches, etc.

8.1.6 Socket Outlets

The socket outlets have all insulated base with moulded or socket base having three terminal sleeves. The cover is again moulded with corresponding three holes. The two thin terminal sleeves are meant for making connection to the load circuit wires and the third terminal sleeve, larger in cross-section, is used for an earth connection.

8.1.7 Plugs

These are used for tapping power from socket outlets.

8.1.8 Main Switch

This is used at the consumer's premises so that he may have self-control of the entire distribution circuit. The different classifications are double poled and triple poled.

8.1.9 Distribution Fuse Boards

In industries or in very big buildings, where a number of circuits are to be wired, distribution fuse boards are used. They are usually iron clad and are designed with a large space for wiring and splitting the circuits. The fuse bank in the distribution board can easily be removed.

8.1.10 Accessories Used

Screw driver, side cutting plier, long nose plier, slip plier, pocket knife, hammer, woodsaw, hacksaw, chiseld, scratch awl, hand drill, auger bit, rawlplug tool, centre punch, blow lamp, wire gauge.

8.2 TYPES OF WIRING

The type of wiring to be adopted is dependent on various factors, like durability, safety, appearance, cost and consumer's budget. Different types of wiring are explained below.

8.2.1 Cleat Wiring

In this system, the V.I.R. conductors are supported in porcelain cleats (vulcanised India rubber wire in porcelain cleats).

8.2.2 Wooden Casing Capping

The system of wiring is most commonly adopted for residential buildings. It consists of rectangular wooden blocks, called casting, made from first class seasoned teak wood or any other wood free from any defect. It has usually two grooves into which the wires are laid. The casing at the top is covered by means of capping which is a rectangular strip of wood of the same width as that of casing and is screwed to it. Two or three wires of the same polarity may be run in one groove. But wires of opposite polarity need not be run in one groove.

8.2.3 T.R.S. Wiring

It is Tough Rubber sheathed wiring. The T.R.S. cables are available in single, twin or three cores with circular or oval shape. The cable is quite flexible and has an insulation which resists rough usage, moisture, climate variations, acids and alkalies but is slightly affected by lubricating oils. So T.R.S. cables may be run on the surface of the walls or buried in plaster.

8.2.4 Conduit Wiring

In this system of wiring, the V.I.R conductors are run in metallic tubes called conduits. It is the best system of wiring which provides mechanical protection, safety against fire and shock if bonding and earthing are well done. This is most desirable for workshops and public buildings.

8.2.5 Wiring Circuits

(a) Staircase Wiring Refer Fig. 8.1.

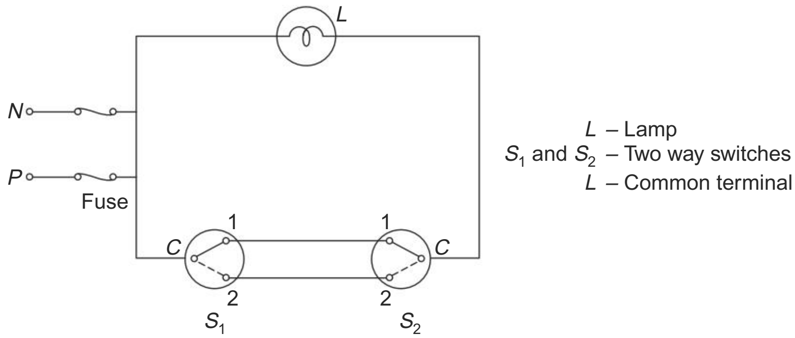


Fig. 8.1

In this wiring a single lamp is controlled from two places. For this purpose two numbers of two-way switches are used. Application of this type of wiring is in staircase.

On analysis of the given wiring circuit, we can make the inferences as given in Table. 8.1.

Table 8.1

Position of switch	*Position of switch*	*Condition of lamp*
S_1	S_2	
1	1	ON
1	2	OFF
2	1	OFF
2	2	ON

(b) Fluorescent Tube Circuit with a glow starting switch is shown in Fig. 8.2.

Description Fluorescent tube (T) has filaments on either ends. They are coated with electron emitting material. The inside of the tube has a phosphorous coating which is used to convert ultraviolet radiation into visible light and to give the required colour sensation. A ballast (B) is used to give a transient high voltage so as to initiate the electron movement. It is an iron cored coil having high inductance. G is a glow starter. A capacitance (C is used for improving the power factor of the circuit. Another capacitor (C_1) is used to suppress radio interference.

Working With the switch "S" closed, the circuit gets closed. The current flows through the ballast and the starter. The glow switch suddenly breaks thereby breaking the circuit. Due to the high inductive property of the ballast, a transient high voltage is available across the filaments. Hence electrons are emitted and travel through the tube. Such a continuous flow of electrons produces the sensation of light to human eyes.

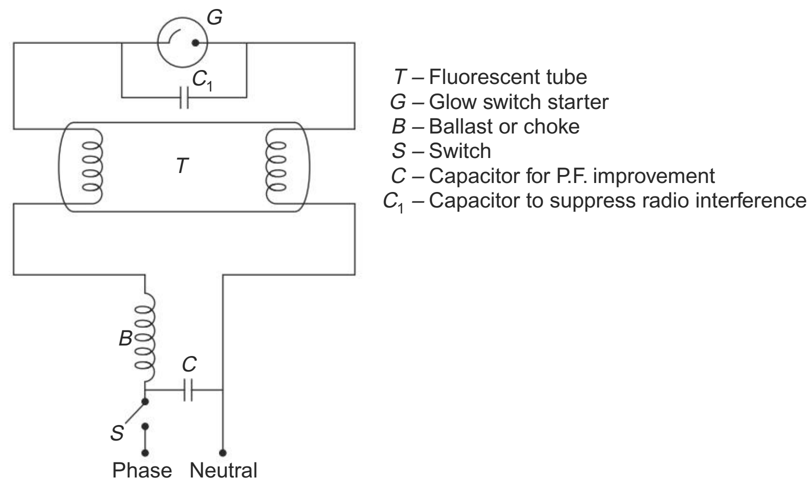


Fig. 8.2

8.3 BASIC PRINCIPLES OF EARTHING

8.3.1 Earthing and its Necessity

Earthing means generally connected to the mass of the earth. It shall be in such a manner as to ensure at all times an immediate and safe discharge of electric current due to leakages, faults, etc.

All metallic parts of every electrical installation such as conduit, metallic sheathing, armouring of cables, metallic panels, frames, iron clad switches, instrument frames, household appliances, motors, starting gears, transformers, regulators etc, shall be earthed using one continuous bus (barewire). If one earth bus for the entire installation is found impracticable, more than one earthing system shall be introduced. Then, the equipment and appliances shall be divided into sub-groups and connected to the different earth buses.

The earthing conductors, when taken out doors to the earthing point, shall be encased in pipe securely supported and continued up to a point not less than 0.3 more below ground level. No joints are permitted in an earth bus. Whenever there is a lightning conductor system installed in a building, its earthing shall not be bonded to the earthing of the electrical installation.

Before electric supply lines or apparatus are energized, all earthing system shall be tested for electrical resistance to ensure efficient earthing. It shall not be more than two ohms including the ohmic value of earth electrode.

8.3.2 Earthing through a G.I. Pipe

In this method a G.I. pipe used as an earth electrode. The size of the pipe depends upon the current to be carried and type of soil in which the earth electrode is buried.

For ordinary soils the length of the G.I. pipe used as an earth elctrode is 2 m long and 38 mm in diameter or 1.37 m long and 51 mm in diameter. For dry and

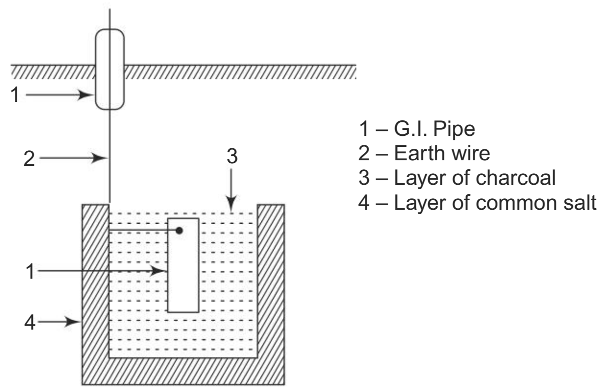


Fig. 8.3

rocky soils the length may be increased to about 2.75 metres and 1.85 metres respectively. The pipe is placed vertically, burying to a depth not less than 2 metres in as moist a place as possible, preferably in close proximity of water tap, water pipe or water drain and at least 0.6 metre away from all building foundations, etc as shown in Fig. 8.3. The pipe shall be completely covered by 80 mm of Charcoal with the layer of common salt 30 mm all around it. The charcoal and salt decreases the earth resistance.

8.3.3 Earthing through a Plate

A G.I. or copper plate is used as an earth electrode. If a G.I. plate is used it shall be of dimensions 0.3 m × 0.3 m and 6.35 mm thick and if a copper plate is used it shall be of dimensions 0.3 m × 0.3 m and 3.2 mm thick. The plate is buried to a depth of not less than 2 m in as moist a place as possible preferably in close proximity of water tap, water pipe or water drain and at least 0.6 m away from all building foundations, etc. The plate shall be completely covered by 80 mm of charcoal with a layer of common salt of 30 mm all around it, keeping the faces of the vertical as shown in Fig. 8.4.

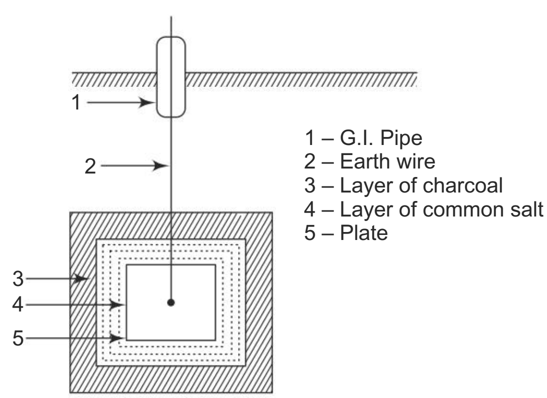


Fig. 8.4

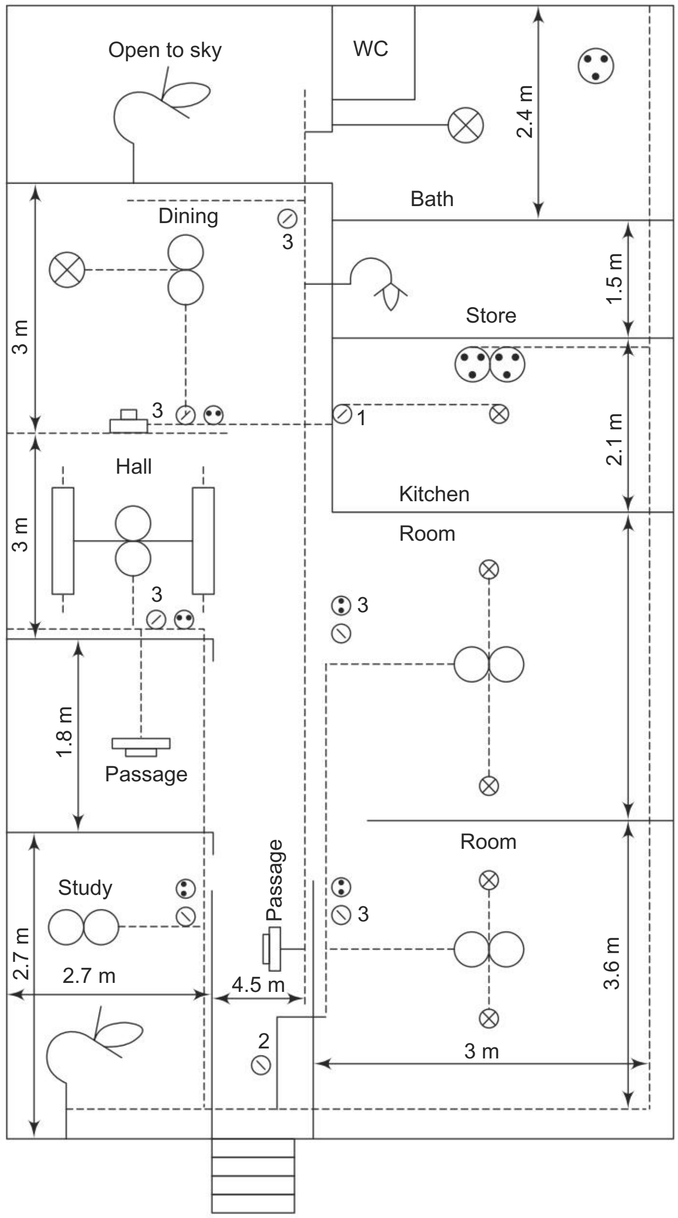


Fig. 8.5

Notes:

1. It is advisable to use only copper electrodes for earthing *a*, *d*, *c* installation to avoid corrosion due to electrolytic action.
2. Arrangements must be made to keep the electrode wet during summers to keep the earth resistance low.

3. The practice of using common salt in earthing arrangement is not recommended now because it corrodes the G.I. pipe used for earthing. There is a likelihood of the pipe or earth wire getting cut and hence the earthing effect ceases.

8.4 WIRING LAYOUT FOR A RESIDENTIAL BUILDING

The wiring layout for a certain residential building is shown in Fig. 8.5. The supply is taken from overhead transmission lines through service mains. It goes to the energy metre, the main switch and then to the distribution board. Required number of lamp points, fan points, plug points, and power plug points are provided. The neutral wire runs throughout the installation. The mains is an ironclad double pole with a fuse in the live line and a limit in the neutral.

REVIEW QUESTIONS

1. Why is the neutral of the supply earthed?
2. Discuss the different types of wiring bringing out the merits and demerits of each.
3. Draw a neat wiring diagram for staircase lighting and explain its working.
4. Draw the wiring diagram of fluorescent lamp. Describe its working.
5. Write a short note on wiring for residential building.
6. Give an account of the various wiring materials used.
7. What is the necessity of earthing in domestic wiring? Discuss in brief the various methods of earthing employed.

TRANSMISSION AND DISTRIBUTION OF ELECTRICAL POWER

9

INTRODUCTION

Electrical power is generated in power stations which are mostly located in remote places. This is because the generating stations are installed in the areas, where the resources for power generation are available. The bulk power generated is to be transmitted to different load centers. The transmission of electrical power is done at high voltages. The power received at the load centers is to be distributed to the consumers at normal voltage. There is a complex network of conductors between the power station and the consumers. This network is divided into major parts called transmission and distribution systems. Generation, transmission and distribution systems combinedly known as electric supply system or electric power system.

9.1 POWER GENERATION

Electrical power is generated normally at 11 kV or at 6.6 kV by number of generators in parallel. Generation at still higher voltage will impose some technical problems. Power generated at 11 kV is stepped upto 110 kV or 230 kV or 400 kV or still higher voltage with the help of power transformers. A typical power supply system is shown in Fig. 9.1.

9.2 TRANSMISSION SYSTEM

The powers generated at the generating stations is transmitted at higher voltage to the main load centers. This transmission system is known as primary transmission system. The voltage level here may be 1500 kV, 700 kV, 400 kV or 230 kV.

There are advantages when power is transmitted at high voltages— (i) The volume of conductor material required is low. (ii) For a given amount of power transmitted, the current through the line is reduced, when the transmission system voltage is high. This reduces the line losses and hence efficiency of transmission is increased. (iii) As the current is reduced the voltage in the line is reduced and hence the line regulation is improved.

The power from the main load centers is transmitted to different sub-load centers at voltages 33 kV, 66 kV or even at 110 kV. This part of transmission is known as secondary transmission system.

The primary transmission system employs overhead lines in general. The secondary transmission of power is either by means of OH lines or underground cable. In rural areas, OH lines are used and in city areas, underground cable system is used.

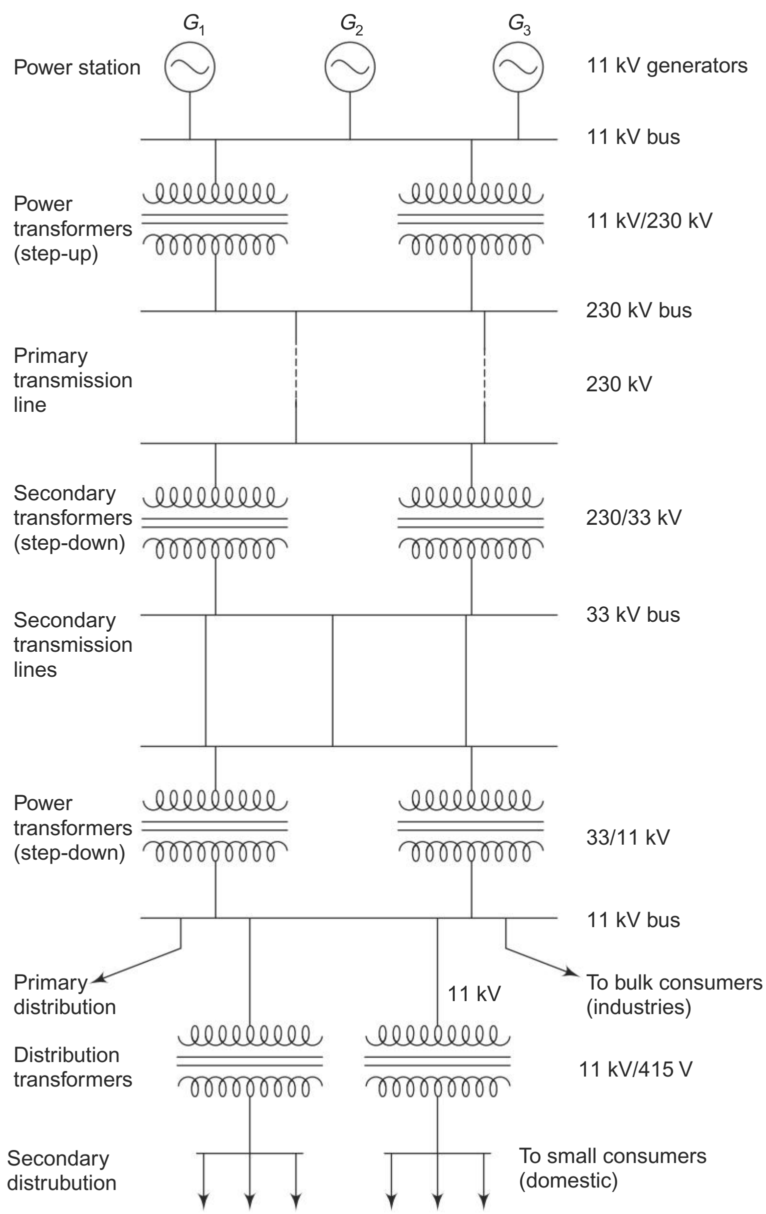


Fig. 9.1

9.2.1 Transmission System

When power is transmitted by dc there are a lot of advantages when compared to a.c. transmission systems.

(i) The amount of insulation required is less in dc transmission as compared to that for ac transmission.

(ii) The size of towers, cross arms required are small for dc transmission system.
(iii) It improves the stability of the system.
(iv) Power factor of dc transmission system is unity.
(v) There is no changing current.
(vi) There is skin effect. Skin effect increases the effective resistance of the line conductors.

The main disadvantages of HVDC transmission are— (i) There is no equipment like a transformer, available for step-up or step-down the voltage. So this has to be done on the ac side only. (ii) Power cannot be generated at higher voltages because of commutation problem (iii) There are limitations with HVDC switching devices and circuit measures.

The HVDC transmission system is more economical when the distance of transmission is more than 600 km for OH line.

The components used in a substation for dc transmission purpose are more and are shown in Fig. 9.2. The capital cost of dc transmission system is higher when compared to that of ac transmission system, when the length of the transmission line system is less than 600 km, and less when the length of the transmission line system is greater than 600 km.

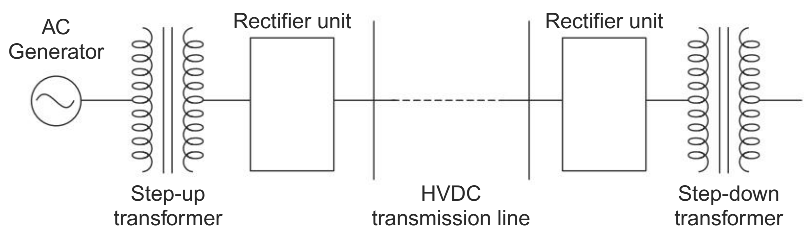


Fig. 9.2

9.2.2 Distribution System

The power is received at the sub-load centre at voltages of 33 kV, 66 kV or 110 kV. The voltage level is stepped down to 11 kV at the sub-load centers. The power is distributed through the 11 kV lines, which runs along the main roads. This distribution system is referred to as primary distribution system.

Number of 11 kV/415 volts transformers are connected enroute 11 kV lines. Now the power is distributed through distribution lines, which run along all the streets of town/city/villages. This distribution line is called a distributor. The power is distributed with the help of 3-phase, 4-wire lines/cables. Voltage levels available are 1-phase or 230 V and 3-phase 415 volts. The lines feeding the power from the distribution transformer to the distributors are called feeders.

From the distributors, power is supplied to the consumers by means of lines called service lines. The feeder distribution and service lines combinedly, form the secondary distribution system which is shown in Fig. 9.3. The distributors may be radial or ring. The secondary distribution of power is by means of ac only. This is because all the loads are designed to work on ac only, which are most efficient and less in cost.

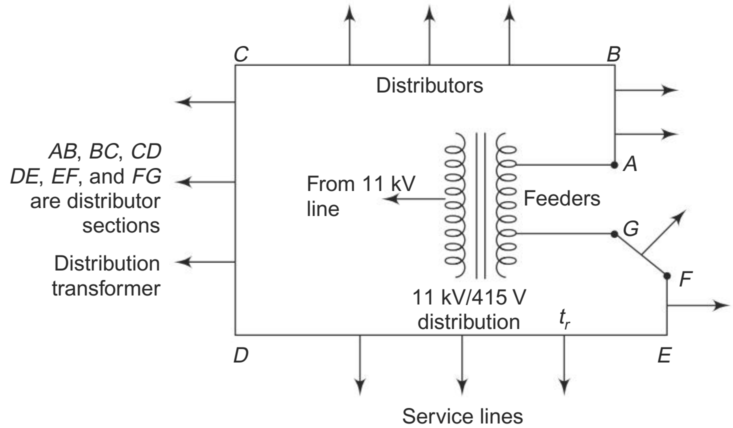


Fig. 9.3

9.2.3 Main Components of Transmission and Distribution Systems

(i) Conductors, which are used to carry the electrical power from one place to another place.
(ii) Supports, which may be RCC poles, MS, tubular poles or towers. These are used to keep the conductors at a proper height from the ground.
(iii) Cross arms, which are attached to the poles or towers and are used to support the conductors.
(iv) Insulators, which are used to isolate the conductors at higher potential from the ground.
(v) Miscellaneous items like lighting arresters, ground wires, stay wires, etc.

9.3 COMPARISON OF OVERHEAD (OH) AND UNDERGROUND (UG) SYSTEMS

In city area, secondary distribution of power is done by UG cable system. Sometimes the primary distribution is by means of UG cable system. The OH line and UG cable systems are compared based on the following factors.

(i) Public Safety: UG cable is preferred, as there is less chance of any hazard to the public.
(ii) Initial cost: the initial investment on UG cable system is about 10 times the rate on OH line system. So, OH line system is preferred.
(iii) Flexibility: OH line system is more flexible than the UG cable system. OH line system can be modified easily.
(iv) Faults: The chances of occurrence of fault in an UG cable system is less when compared to the OH line system, as it is exposed to the atmosphere.
(v) Appearance: The general appearance of UG cable system is better.

(vi) Fault location and Repair: Fault location and rectification of fault is easier in OH line system.

(vii) Useful life: The useful life period of an OH line system is only 50% that of UG cable system.

(viii) Maintenance cost: The maintenance cost of UG cable system is low, as the chance of fault is less in an UG cable system.

(ix) Interference with communication circuits. It is less with an UG cable system and more with an OH line system.

From the above comparison, it is clear that systems have advantages and limitations. However in city areas we bother only about safety and not about the economics involved. So in this condition, UG cable system is preferred.

REVIEW QUESTIONS

1. Draw the one line diagram of a typical power supply system. Mark all the components.
2. What are the advantages of dc transmission systems over ac transmission systems?
3. Mention the disadvantages of a dc transmission system.
4. Draw the diagram showing the components of a distribution system.
5. Bring out a table of comparison between lines and underground cable systems.

PART II

Electronics Engineering

PASSIVE CIRCUIT COMPONENTS

10

INTRODUCTION

The most commonly used passive circuit components in electronic and electrical circuits are resistors, capacitors and inductors.

10.1 RESISTORS

Physical materials resist the flow of electrical current to some extent. Certain materials such as copper offer very low resistance to current flow, and hence they are called as *conductors*. Other materials such as ceramic which offer extremely high resistance to current flow are called as *insulators*. In electric and electronic circuits, there is a need for materials with specific values of resistance in the range between that of a conductor and an insulator. These materials are called resistors and their values of resistance are expressed in ohms (Ω).

The resistance R of a given material is proportional to its length L and inversely proportional to its area of cross section A. Thus R varies as L/A and $R = \rho(L/A)$, where ρ is the constant of the material known as its *specific resistance* or *resistivity.*

The various types of resistors are given in Table 10.1.

Table 10.1 Various Types of Resistors

Fixed resistors	*Variable resistors*
Carbon composition	Potentiometer
Carbon film	Rheostat
Metal film	Trimmer
Wire wound	

10.1.1 Fixed Resistors

Carbon Composition This is the most widely used fixed resistor in discrete circuits. The construction of the carbon composition resistor is shown in Fig. 10.1. The carbon resistors are made of finely divided carbon mixed with a powdered insulating material such as resin or clay in the proportions needed for the desired resistance value. These are then placed in a casing (moulded plastic) with lead wires of tinned copper. Resistances of this types are available in the range from few ohms to hundred megaohms and typical power ratings of 1/8 to 2 W.

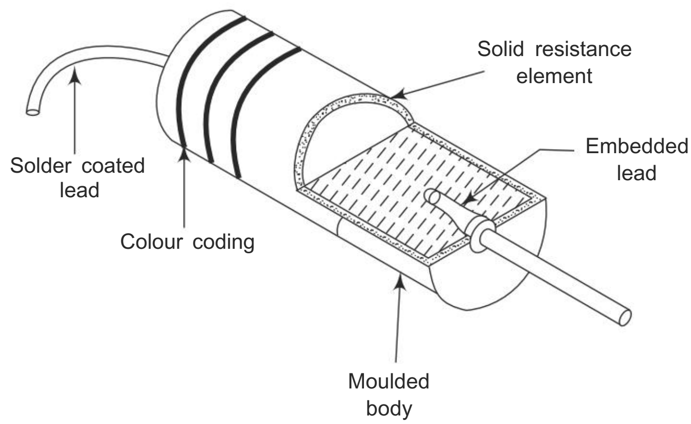


Fig. 10.1 Cutaway View of a Carbon Composition Resistor

Carbon Film This is yet another type of carbon resistor. Its basic structure is shown in Fig. 10.2. It is manufactured by depositing a carbon film on a ceramic substrate. In this process only approximate values of resistance are obtained by either trimming the layer thickness or by cutting helical grooves of suitable pitch along its length. During this process, the value of the resistance is monitored constantly. Cutting of grooves is stopped as soon as the desired value of resistance is obtained. Contact caps are fitted at both ends, and then the lead wires made of tinned copper are welded to these end caps. This type of resistors are available commercially in the range of 10 Ω to 10 MΩ with a power rating of up to 2 W. Carbon film resistors are less noisy than carbon composition resistors and they are of low cost.

Metal Film Resistor Construction of this type of resistor is similar to the carbon film resistors, the only difference being the material used for constructing the film. In metal film resistors, the film usually consists of an alloy of tin and antimony metals. Metal film resistors are available as thin and thick film type components.

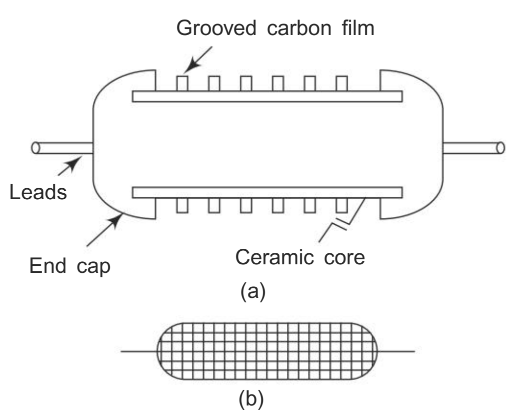


Fig. 10.2 Carbon Film Resistor (a) Construction (b) Carbon Film Resistor

(a) Thin film resistor The resistance element in this type is a film having a thickness of the order of one-millionth of an inch. Typically, the thin film is deposited on a ceramic substrate under a high vacuum and this technique is called vacuum deposition. Metals used for deposition include Nickel and Chromium. This type of resistors are available in the range of 10 Ω to 1 MΩ with a power rating of up to 5 W.

(b) Thick film resistor The resistance element in this type is a film having a thickness greater than one millionth of an inch. Four different types of thick film resistors are available, viz. tin-oxide, metal-glaze, cermet and bulk film resistors.

(i) Tin oxide type In this type of resistors, tin oxide in vapour form is usually deposited on a ceramic substrate under high temperature. The vapour reacting with the substrate, which is heated, results in a tightly formed resistance film. This type of resistor is available in the range of a few ohms to 2.5 MΩ with a power rating of up to 2 W.

(ii) Metal-glaze type In this type, a powdered glass and fine metal particle (palladium and silver) mixture is deposited on a ceramic substrate. This combination is then heated to a high temperature, typically 800°C. This results in a fusion of metal particles to the substrate. This type of resistors are manufactured in the range of a few ohms to 1.5 MΩ with a power rating of up to 5 W.

(iii) Cermet type A cermet film resistor is made by screening a mixture of precious metals and binder materials on a ceramic substrate. The word 'cermet' is derived from ceramic and metal. As in the case of metal-glaze resistor, the combination is then heated to a high temperature. It is available in the range of 10 Ω to 10 MΩ with a maximum power rating of 3 W.

(iv) Bulk film type In this type, the metal film is etched on a glass substrate. As metal film and glass have unequal coefficients of expansion, the metal film is compressed slightly by the glass substrate. The compressed film has a negative temperature coefficient which cancels out the inherent positive temperature coefficient of the film. As a result, the bulk film resistor has a temperature coefficient close to zero. This type of resistor is available in the range of 30 Ω to 600 kΩ with a maximum power rating of 1 W.

Wire Wound Resistor It has a wide range of applications. It is used as an ultra precision resistor in instrumentation and a power resistor in industrial applications. Wire wound resistors are manufactured in two types: (i) Power style wire wound resistor, and (ii) Precision style wire wound resistor.

(i) Power style type It is made by winding a single layer length of special alloy wire, in the form of a coil around an insulating core. The ends of the winding are attached to metal pieces inserted in the core. Tinned copper wire leads are attached to metal pieces. The unit is then covered with a coating, such as vitreous enamel (an inorganic glass like moisture) or silicone. This coating protects the winding against moisture and breakage. The resistance wire lead must have carefully controlled resistance per unit length of wire and low temperature coefficient, and be able to operate at high temperatures. Alloys used include nickel-chromium-aluminium and nickel-chromium-iron (Nichrome). The cylindrical core is ceramic, steatite or a vitreous

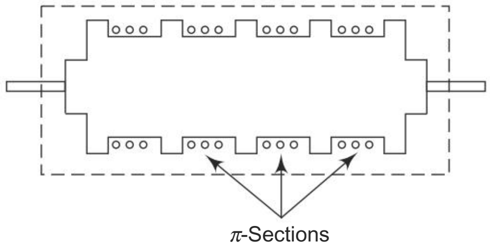


Fig. 10.3 Precision Wire Wound Resistor

material. This type of resistor is available in the range from less than an ohm to greater than a megaohm with a power rating as high as 1500 W. To increase its power rating, the resistor is sometimes placed in a metal housing such as aluminium.

(ii) Precision style wire wound resistor They are wound on ceramic (steatite) tubes or sometimes epoxy moulded tubes as shown in Fig. 10.3. The wire is wound in alternate directions in the adjacent π-sections to minimise inductive effect at high frequencies. This type of winding is referred to as 'bifilar'. Wires of low temperature coefficient alloys (e.g., Manganin) are employed. The leads are usually anchored to the tubes firmly and joined to the wire ends by brazing. The unsealed type of resistors are impregnated with varnish while the sealed types are given a dip of epoxy resin or encased in ceramic or glass tubes.

Yet another type of construction in which the inductance is virtually reduced to zero is shown in Fig. 10.4. Here, a thick film serpentine pattern is deposited on a ceramic core. Similar in behaviour to biflilar winding opposite magnetic fields produced by current flowing in adjacent resistance paths cancel each other, thus reducing the inductance to zero. Precision style wire wound resistors are available in the range of a fraction of an ohm to 10 MΩ with a power rating of the order of 2 W.

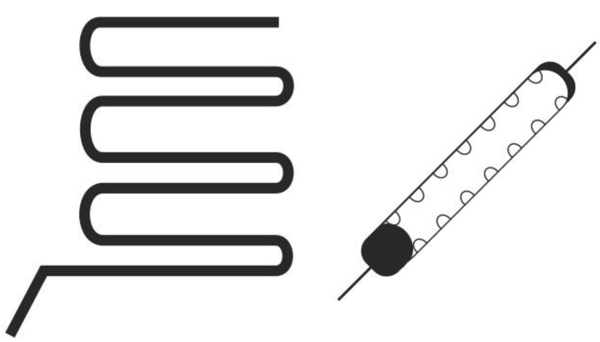


Fig. 10.4 A Serpentine Resistor Pattern Used to Reduce Inductance in Precision Wire Wound Resistor

10.1.2 Variable Resistors

These are the resistors whose resistance can be changed between zero and a certain maximum value. They are used in electronic circuits to adjust the value of voltages and currents. For example, they are used as volume control in radio and brightness control in television. Variable resistors can be broadly classified into three types, viz. Potentiometer, Rheostat, Trimmer.

Potentiometer It is a variable resistor either of carbon or wire wound type. It is smaller in size compared to a rheostat and is usually referred to as "POT."

(i) Carbon potentiometer: The construction of carbon potentiometer is shown in Fig. 10.5. These are manufactured either in film or moulded track types. Both consist of an annular ring of carbon resistance, formed on a plastic base, over which a movable contact can slide. There is a slip ring (a continuous metal ring) which is also contacted by the movable contact. Three terminals are provided, two of them connected to the ends of the carbon track and one to the slip ring. A shaft runs through a bush in the center of the base to which the movable contact is attached. The assembly is enclosed by a case of sheetmetal over which the resistance value and taper are also engraved. The taper refers to variation in resistance along the track which may be a logarithmic or linear variation.

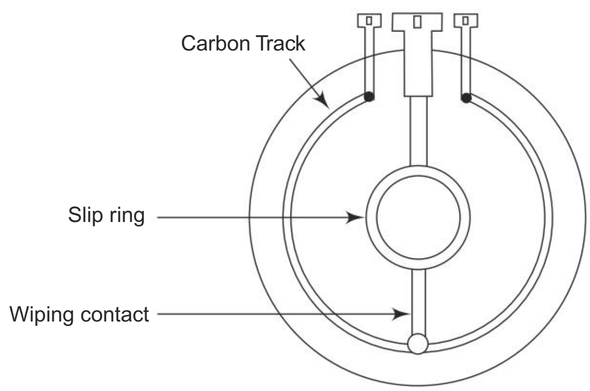


Fig. 10.5 Carbon Potentiometer

The track may be either the film type or a moulded type. In the film type, a mixture of carbon, graphite and resin in the form of a paste is sprayed on to a plastic (phenolic) sheet, with suitable marks in the form of rings. If a nonlinear taper is required, the ring width may be varied circumferentially or the spray may be made from different composition mixtures along the track length.

In the moulded track type, the resistance material and the base plate are moulded together with the slip ring, the terminals and the bushing all being inserted in the moulding operation. The integral moulding of base, track and terminals gets rid of solders, rivets, welds and provides good humidity resistance and fewer mechanical joints.

(ii) Wire wound potentiometers They are of two types: (a) Singleturn, and (b) Multiturn.

(a) *Singleturn Wire Wound Potentiometer* As shown in Fig. 10.6 (a), on a flat strip of insulating card made of flexible plastics or anodised aluminium (aluminium oxide formed on aluminium), the resistance wire, usually copper alloy wire for low resistance pots and the nickel chromium for high resistance pots, is wound and then the strip is bent round a cylindrical surface. Contact by means of a slider metal or beryllium copper, which is spring loaded, is made on the inside periphery or the outer edge. Contact from the slider is made through a slip ring or by a coiled spring. The winding is usually of two or three linear resistance sections to approximate ideal taper (such as log). Singleturn pots are available in the range from 50 Ω to 5 MΩ and in power ratings of 2 to 3 W. It is commonly used as a gain control element in an amplifier and as brightness and contrast controls in TV receivers.

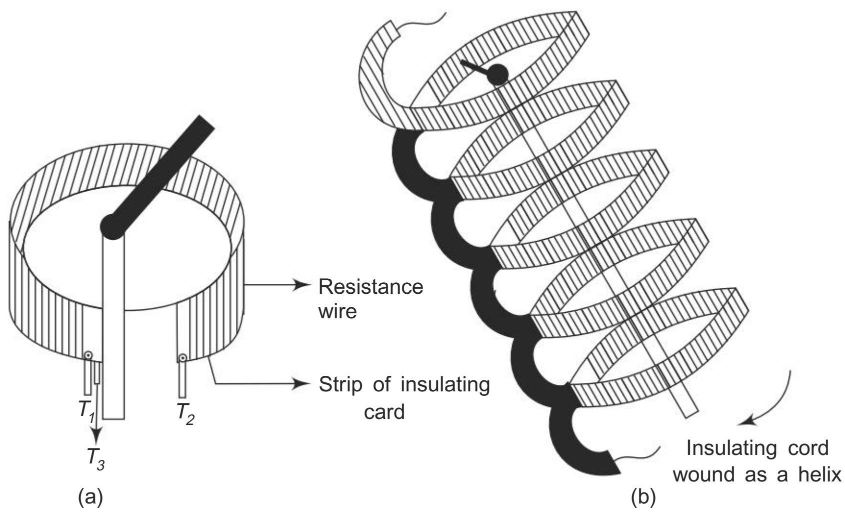


Fig. 10.6 (a) Single Turn Wire Wound Potentiometer, and (b) Multiturn (Helical) Potentiometer

(b) *Multiturn Wire Wound Potentiometer* Multiturn or helical pots are used in applications that require the precise setting of a resistance value. An example is the setting of coefficients in an analog computer. Commonly, they have up to 10 turns. As shown in Fig. 10.6 (b) the resistance element is wound on a long strip and then formed into a helix and held in a plane using silicone varnish. The contact is of precision metal and has multiple "fingers". Usually, the groove between the helical turns of the element may be used to guide the contact. Because of its construction, the wire wound pot has appreciable stray inductance and capacitance which may be a problem at high frequency operation. They are available in the range from 50 Ω to 250 kΩ with a power rating of up to 5 W.

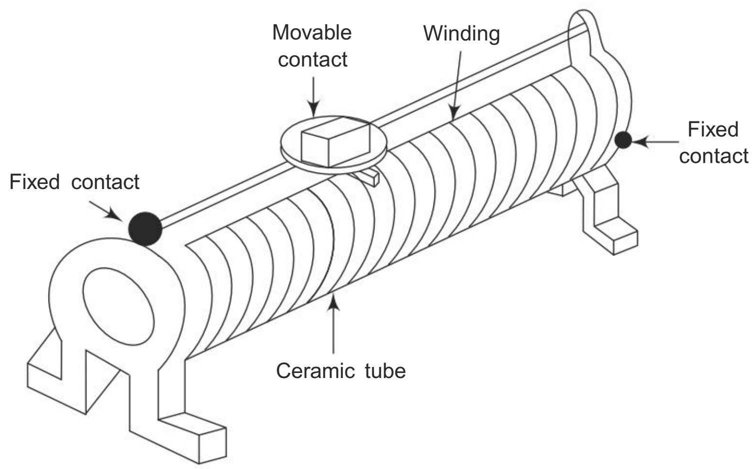


Fig. 10.7 Rheostat

Rheostat A wire wound pot that can dissipate more than 5 W is referred to as a rheostat. As shown in Fig. 10.7, the resistance wire is wound on an open tube of ceramic which is covered with vitreous enamel, except for the track of the movable contact. The rheostat is capable of withstanding temperatures up to 300 °C. It is used to control motor speed, X-ray tube voltages, Welding current, Ovens and in many other high power applications.

Trimmer A trimmer or trimming potentiometer is used where the resistance must be adjustable but not continuously variable. It finds a very important place in calibration and balancing of electronic equipments. These are screw activated and the resistance can be adjusted by a screw driver. Figure 10.8 shows the commonly used rotary trimmer. Here, the resistance track is made of carbon and cermet. The carbon track along with the movable slide is housed in a steel casing. Typically, their resistance range is from a few ohms to 5 MΩ and the power rating is 1 W.

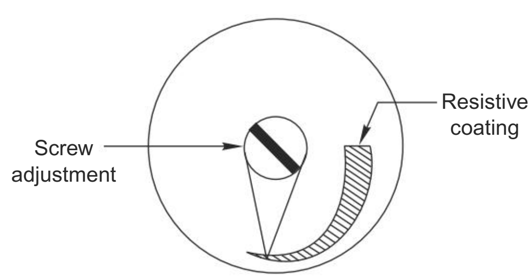


Fig. 10.8 Rotary Trimmer

10.1.3 Tolerance

In the manufacture of resistors where thousands of resistors are made in a day, it is not possible to adjust every ordinary resistor to an exact value. The term tolerance denotes the acceptable deviation in the resistance value of a resistor. The usual specified tolerances are 5%, 10% and 20% for ordinary resistors, while precision resistors have a tolerance close to 0.1%. For example, the resistance marked as 1 kΩ with a 10% tolerance can have any value between 900 Ω and 1100 Ω.

Table 10.2 gives the tolerance values for the various types of resistors.

Table 10.2 Tolerances for various resistors

Type of resistor	*Tolerance*
Fixed	
(i) Carbon composition	
(ii) Carbon film	5% or above
(iii) Metal film	
(a) Thin film	< 0.5%
(b) Thick film	
* Tin oxide	< 1%
* Metal-glaze	< 1%
* Cermet type	≥ 1%
* Bulk film	≥ 0.005%
(iv) Wire wound	
* Power style	5% to 20%
* Precision style	< 0.5%
Variable	
(i) Potentimeter	
(a) Carbon type	10% to 20%
(b) Wire wound	
* Singleturn	10% to 20%
* Multiturn	3%
(ii) Trimmer	10%

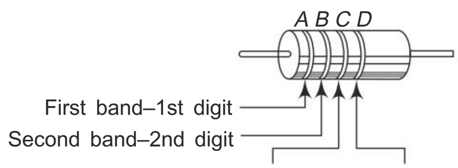
10.1.4 Colour Coding of Resistors

Resistors are coded to indicate the resistance value and tolerance. As shown in Fig. 10.9, there are four colour bands (*A*, *B*, *C* and *D*), one by the side of the another starting from the left end. The first two bands (*A* and *B*) denote the first and second digits of the resistance value and the third band (*C*) indicates how many zeroes follow the first two digits. Tolerance is given by the fourth band (*D*).

Figure 10.9 shows the colour code and tolerance values for the various colours. For example, a resistor with the following colour bands sequence

Yellow – Violet – Orange – Gold

denotes a 47000 Ω with 5% tolerance resistor.



Colour	Digit	Multiplier	Tolerance
Black	0	1	—
Brown	1	10	1%
Red	2	100	2%
Orange	3	1000	—
Yellow	4	10000	—
Green	5	100000	—
Blue	6	1000000	—
Violet	7	10000000	—
Gray	8	—	—
White	9	—	—
Gold	—	0.1	5%
Silver	—	—	10%
No colour	—	—	20%

Fig. 10.9 Standard Colour Code for Composition and Some Axial Type Resistors

10.2 CAPACITORS

Next to resistors, capacitors are the most widely used passive elements in circuits. Capacitors are the devices which can store electric charge. They are used in tuned circuits, timing circuits, filters, amplifier circuits, oscillator circuits and relay circuits. They are also used for power factor correction and for starting single phase motors.

The reactance of a capacitor of capacitance C at frequency f is given by

$$X_C = \frac{1}{2\pi f C} \Omega$$

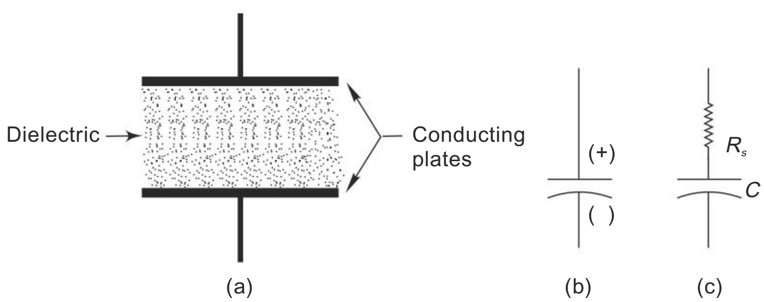


Fig. 10.10 Capacitor (a) Basic Structure (b) Symbol and (c) Series Equivalent Circuit

The capacitive reactance (X_C) varies inversely with the frequency of the applied a.c. voltage. Therefore, the capacitor allows higher frequency currents more easily than lower frequency currents. For d.c. voltage or current. For d.c. voltages, i.e. $f = 0$, $X_C = \infty$. Hence, a capacitor blocks (cannot conduct) the d.c. voltage or current.

A capacitor essentially consists of two conducting plates separated by a dielectric material as shown in Fig. 10.10. The capacitance of a parallel-plate capacitor is given by

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

where A = Area of each plate in m^2.

d = distance between parallel plates in m

ε_0 = dielectric constant (permittivity) of the free space $= \dfrac{10^{-9}}{36\pi}$ F/m $= 8.854 \times 10^{-12}$ F/m

ε_r = relative dielectric constant (permittivity).

For large capacitance, the area A and the value of the relative permittivity of the dielectric ε_r must be large, while d must be very small. Larger capacitances can be obtained by using high permittivity dielectrics of smaller thickness.

When a voltage V is applied across the capacitor plates, the electrons accumulate on the side of the capacitor connected to the positive terminal of the voltage source. The plate connected to the positive terminal of the voltage source loses electrons. This accumulation of electrons produces a negative charge on one side of the capacitor, while the opposite side has a positive charge. Thus, with the application of voltage, the electrons are simply redistributed from one side of the capacitor to the other. This redistribution process of the electrons is called the charging of the capacitor. The charging continues until the potential difference across the capacitor is equal to the applied voltage. The more the applied voltage, the stronger is the electric field and more charge is stored in the dielectric. The amount of charge q stored in the capacitor is, therefore, proportional to the applied voltage. Also, a large capacitance can store more charge. These relations can be mathematically expressed as

$$q = CV \text{ coulombs}$$

where q = charge stored in the dielectric

V = voltage across the capacitor

C = capacitance of the capacitor.

From the above equation, when one coulomb is stored in the dielectric with a potential difference of one volt, the capacitance is one farad.

A charged capacitor has energy stored in the electric field existing between its plates. The expression for stored energy is

$$W = \frac{1}{2} CV^2$$

In a capacitor, the dielectric strength is the ability of a dielectric to withstand a potential difference without breakdown. This voltage rating is very important because breakdown of the insulator provides a conducting path through the dielectric. Since the breakdown voltage increases with greater thickness, capacitors of higher voltage ratings have more distance between the plates. The increased distance between the plates reduces the capacitance. The typical values of dielectric constant and dielectric strength of various dielectric materials are shown in Table 10.3.

Table 10.3 Dielectric properties

Sl.No.	*Material*	*Dielectric constant*	*Dielectric strength volts/mil (breakdown)*
1.	Air or vacuum	1	20
2.	Plastic film	2–3.5	1000–5000
3.	Oil	2–5	500–1000
4.	Paper	2.5–6	500–1000
5.	Mica	3–8	600–1500
6.	Glass	4–7	<500
7.	Electrolytic	7–11	2000
8.	Ceramics	80–12000	100–300

Capacitors are generally classified according to the dielectric used. Most commonly used dielectrics are air, electrolyte, ceramic, plastic, mica and paper. Only electrolytic capacitors have polarity. The capacitance of a fixed capacitor is never constant except under certain fixed conditions. It changes with temperature, frequency and age, and the capacitance value marked on the capacitor strictly applies only at room temperature and at low frequencies. Some of the fixed capacitors are discussed in the following sections.

10.2.1 Fixed Capacitors

Electrolytic Capacitors Electrolytic capacitors normally have the smallest volume and cost for a given capacitance. These types of capacitors are mainly used in applications where a large value of capacitance in a small volume is required, such as filter in a power supply. Electrolytic capacitors are very widely used in filters, time constant circuits, bypass, coupling/decoupling, smoothing and power electronic applications. They are manufactured in many shapes and configurations from as low as 0.1 μF upto 1.0 F, for voltages upto 500 V dc. They are generally polarised and are used on dc only. They are available in non-polarised form and are used in motor-start applications. Based on the type of metal, used for etching anode and or cathode, they are classified as (i) Aluminium type and (ii) Tantalum type.

In the case of polarised electrolytic capacitor, a plus sign or minus sign is printed on the package near one of the two leads. The lead near the plus sign must be connected to a higher dc potential point when the capacitor is put in a circuit. If it is connected reversely, then the capacitor may be short circuited and permanently damaged. For non-polarised electrolytic capacitor, there is no such restriction when it is connected in a circuit.

Polarised Electrolytic Capacitor The basic structure of polarised electrolytic foil capacitor is shown in Fig. 10.11(a). Certain metals like aluminium, tantalum, vanadium and bismuth are used to form anode and cathode foils. Then a very thin layer of aluminium or tantalum oxide is electrochemically formed on the anode foil.

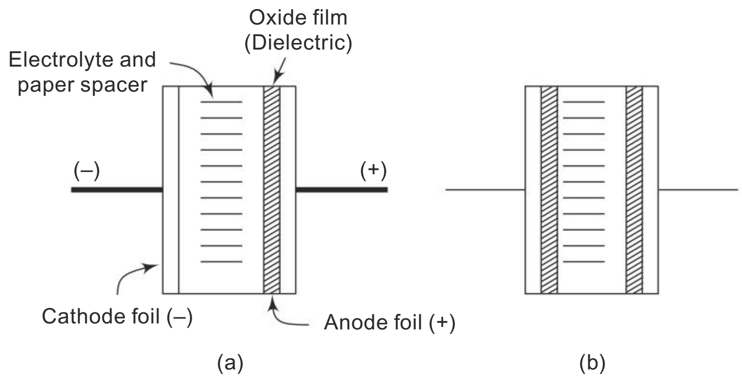


Fig. 10.11 An Elementary Electrolytic Capacitor (a) Polarised and (b) Non-polarised

The oxide layer becomes the dielectric for the capacitor. Such a very thin film of dielectric results in a large capacitance value. The cathode and oxide coated anode is separated by a paper spacer which is soaked in an electrolytic solution. This spacer is required to prevent short circuiting between the anode and cathode foils. For high voltage rating capacitors, the spacer is thicker than for low voltage rating capacitors. The most common shape of electrolytic capacitor is the roller type, inside which the paper spacer is sandwiched between the anode and cathode foils.

Non-polarised Electrolytic Capacitor The basic structure of a non-polarised electrolytic capacitor is shown in Fig. 10.11(b). This type has two oxide coated anodes. For an equal size polarised capacitor, the non-polarised type has one-half the capacitance for the same voltage rating. They are generally used in applications such as ac motor starting, crossover networks and large pulse signals.

(a) Aluminium type This type of electrolytic capacitors are available in polarised or non-polarised form. The construction of this type of capacitors is similar to the construction of polarised or non-polarised capacitors, discussed in the previous section. This type of capacitors have high dc leakage and low insulation resistance. They are low in cost and have a high volumetric efficiency. The disadvantages are that their shelf-life is limited and their capacitance deteriorates with time and use.

(b) Tantalum type: Tantalum electrolytic capacitors have long shelf-life, stable operating characteristics, increased operating temperature range and greater volumetric efficiency. The disadvantages of tantalum type in comparison to the aluminium type are its greater cost and lower voltage rating.

Tantalum electrolytic capacitors can be divided into three types, namely, (i) Foil, (ii) Wet anode and (iii) Solid anode.

(i) *Foil type* The tantalum foil type is similar in construction to aluminium foil electrolytic type.

(ii) *Wet anode type* As shown in Fig. 10.12, the wet anode type capacitor is made by moulding a mixture of tantalum powder and binder into the shape of a pellet. Under high temperature and in a vacuum, the pellet mixture is

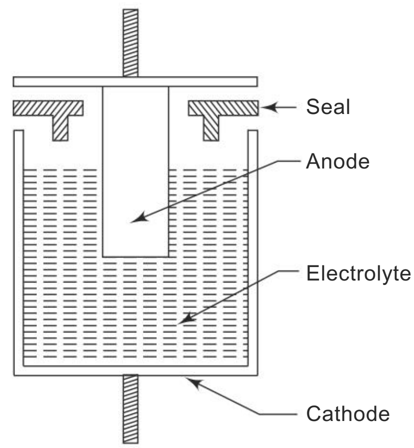


Fig. 10.12 Wet Anode Type Tantalum Capacitor

welded together (sintered) and due to this sintering, the binder and impurities are driven off. The result is a porus pellet on which a layer of tantalum oxide is electrochemically formed. The volumetric efficiency of wet anode type is about three times that of the foil type.

(iii) *Solid anode type* As shown in Fig. 10.13, this type of capacitor is made by sintering an anode pellet on which a layer of tantalum oxide is formed. The pellet is then formed with a manganese dioxide layer, which serves as a solid dielectric. The cathode connection is made by coating this pellet by carbon and silver paint. The solid-anode construction is most widely used. It has the longest life and leakage current of the three types of tantalum capacitors.

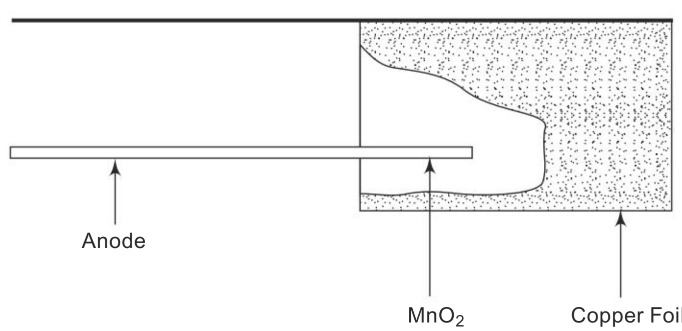


Fig. 10.13 Solid-anode Type Tantalum Capacitor

Ceramic Capacitors These capacitors use ceramic dielectric with thin metal films as electrodes bonded to the ceramic. There are three basic types of a ceramic dielectrics: (i) low permittivity, (ii) medium permittivity, and (iii) high permittivity types.

Low permittivity ceramic capacitors can be made to exhibit zero temperature coefficient and they find wide use in temperature compensation networks. The permittivity usually ranges from 6 to 400. It operates at voltages as high as 6 kV and temperatures up to 125 °C. Its capacitance value is generally limited to 0.001 μF.

In the medium permittivity ceramic capacitors, the permittivity is in the range from 500 to about 4000 and is relatively stable over a wide temperature range of −55 to + 125 °C with a maximum capacitance change of ±15%.

In the high permittivity ceramic capacitors, the permittivity ranges from 5000 to 30000. They have capacitance values up to 2.2 μF and maximum working voltage of 100 V. However, these capacitors change their values appreciably with temperature, dc voltage and frequency.

The medium and high permittivity ceramic capacitors are used for bypass and decoupling applications or frequency discrimination where *Q*-factor and stability are not of major importance.

The ceramic capacitors are available as disc, tubular, monolithic and barrier type capacitors.

Disc capacitor In the disc type capacitor, silver is fixed onto both sides of ceramic to form conductor plates. The sheets are then baked and cut to different sizes of 0.5 mm. The terminal leads are attached by pressure contact or soldering. The disc capacitors have high capacitance per unit volume. Round disc capacitors with axial leads are used for high voltage operation. The discs are lacquered or encapsulated in plastic or phenolic moulding. They are available up to a value of 0.01 μF. They have voltage rating of upto 750 V dc (350 V ac). Some disc type capacitors are shown in Fig. 10.14.

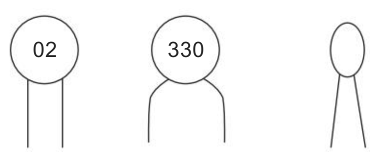


Fig. 10.14 Disc Type Ceramic Capacitors

Tubular capacitor The construction of tubular capacitor is shown in Fig. 10.15. The required materials are ground and mixed thoroughly. The mixture is compressed and fixed with suitable fluxes and moulded into tubes. Leads are formed and soldered. The outer surface may be protected by lacquer in the uninsulated types. The tubular capacitors are available in the range of 5 pF to 1000 pF for voltage ranges up to 5 kV, and up to 10000 pF in the lower voltage ranges.

Monolithic capacitor The monolithic capacitors are formed by interleaving layers of ceramic and platinum electrodes. The first stage in the manufacturing process is the preparation of a slurry or slip in either aqueous or organic solution. Organic binders and plasticizers are added to the slurry to give strength and flexibility to the film. This is then cast on a carrier substrate in thicknesses of 0.025 mm to 0.75 mm and cut into strips. The next step is to apply screen electrodes of

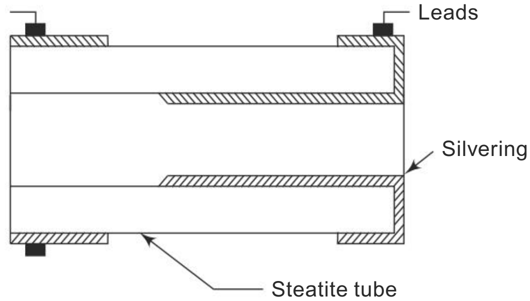


Fig. 10.15 Tubular Ceramic Capacitor

platinum or palladium to the film. The electrode configuration is designed in such a way that each layer of film when blanked will carry an electrode. The deposition of metallic layer extends to alternate faces and accurate location of electrode pattern is required. After the required number of layers have been formed for a specific value of capacitance, the strips are subjected to consolidation under pressure. Then external electrodes are applied in order to pick up the edges of the internal electrodes and form a parallel connection. This entire structure is fired to form a monolithic block. Sintering is done at high temperature (1300°C) to reduce the volume of the block by 40%. Some monolithic type ceramic capacitors are shown in Fig. 10.16.

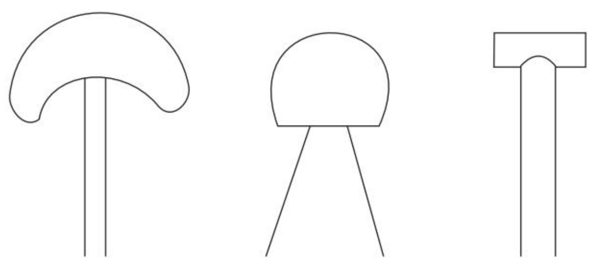


Fig. 10.16 Monolithic Type Ceramic Capacitors

Barrier layer capacitors Barium titanate is used as the dielectric medium in these capacitors. If such a ceramic disc is reduced at 1300°C, it becomes highly non-conducting medium. If reoxidised, a conducting layer of ceramic forms upon the surface. This barrier of conducting to non-conducting layer possesses high values of dielectric constants. Suitable additives will improve the dielectric constant and the loss angle. Silver electrodes are fired on to this barrier layer and the entire assembly is reoxidised in hot air once again. This is encapsulated to prevent the entrance of moisture. The construction of a barrier type capacitor is shown in Fig. 10.17.

These capacitors have much lower insulation resistance and are used only for low voltage applications. They have temperature limitation and can operate from −40°C to 80°C. These are used in very high frequency circuit applications.

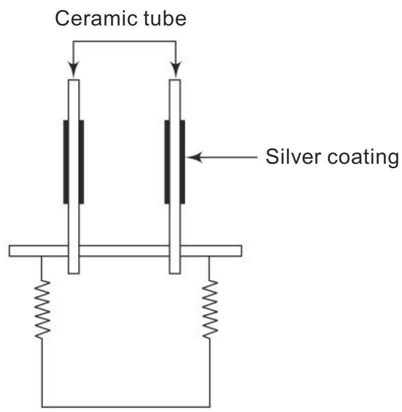


Fig. 10.17 Barrier Type Capacitor

Uses of ceramic capacitors The ceramic capacitors are widely used as bypass capacitors and in decoupling and biasing applications. The major drawback is that the capacitance value varies with the temperature. Ceramic capacitors of the insulated type are used in TV receivers. Tubular capacitors are used to isolate the antennas in receivers. Capacitors with little temperature–capacitance variations are used to compensate for impedance-temperature changes in circuits.

Plastic Capacitors These capacitors use a plastic film as a dielectric. The plastics used include polystyrene, polycarbonate, Teflon and polyester (mylar). Plastic capacitors are available in typical ranges of 500 pF to 10 μF. When sealed properly, these plastics exhibit good mechanical strength, resistance to heat and chemical inertness. The chief characteristics of these capacitors are their very high insulation resistance, high reliability, small physical size and low dielectric absorption.

Polystyrene capacitors Polystyrene is a hydrocarbon material and has a lower permittivity than polyester and polycarbonate. These capacitors are made by rolling the polystyrene film with aluminium foils. These are compactly wound and the leads are soldered at the ends. The entire assembly is carefully heat treated so that the plastic softens (softening temperature, 90°C) and makes good contact with the metal foil. The winding requires particular attention because it may accumulate electrostatic charges and attracts dust. Polystyrene is not suitable for metallising because of lack of available oxygen for the self-healing action and low tensile strength.

They exhibit a low dissipation factor, small capacitance change with temperature and very good stability. They tend to be large in size, and their maximum operating temperature is 85°C. Polystyrene capacitors are normally used in coupling, resonant and measuring circuits. As these capacitors have low dissipation factor, they can be used for ac applications.

Polyester capacitors Polyester is a tough polymer with high tensile strength, free from pinholes and with good insulating properties over a reasonable wide temperature range. Its combination of good insulation properties over a wide temperature range and high mechanical strength are the outstanding characteristics. It also has a low moisture absorption. The high mechanical strength (up to approximately 210 °C) makes polyester ideally suited for metallising by vacuum evaporation. It consists of sufficient oxygen to produce good self-healing. Its maximum operating temperature is 125°C.

Due to the good all-round mechanical and electrical properties and the high dielectric constant of polyester film, these capacitors are the most widely used film or paper capacitors. Polyester capacitors are normally used for coupling/decoupling applications at low and medium frequencies but find many other applications in general, and in power electronics as well as in certain ac applications. Polyester is ideal for dc applications because of its high dielectric strength and resistivity.

Polycarbonate capacitors Polycarbonate is a polyester of carbonic acid bisphenols. It combines good physical properties with a lower dissipation factor than polyester. As it does not have as much available oxygen, its self-healing properties are not quite as good as for polyester. It has a higher insulation resistance. It can operate at temperatures as high as 125 °C. Polycarbonate allows the manufacture of close-tolerance (typically ± 1%), stable, small physical size, long life capacitors and is mainly used for dc applications, though it can also be used for ac applications.

A comparison chart of some of the main characteristics of the three types of plastic dielectric capacitors is given in Table 10.4.

Table 10.4 Characteristics of different types of plastic capacitors

	Polystyrene	*Polyester*	*Polycarbonate*
Permittivity (at 1 kHz, 20 °C)	2.5	3.3	2.9
Dissipation factor (at 1 kHz,20 °C)	0.0002	0.005	0.001
Dielectric strength (V/µm) (at 20 °C)	200	304	184
Insulation resistance (Ω – F) (at 20 °C)	1×10^6	3×10^4	6×10^4
Dielectric absorption (%) (at 20 °C)	0.01	0.2	0.05
Maximum operating temperature (°C)	85	125	125
Film molecular structure	Non-polar	Polar	Polar
Metallisation	Not available	Available	Available

Mica Capacitors The dielectric used in mica capacitors is muscovite or white mica, ruby or rose coloured mica and amber mica. It has a dielectric constant between 3 and 8 and a dielectric strength of 600–1500 V/mil. Its breakdown voltage widely varies from 500 V to 20 kV. It has low losses and the power factor varies depending on the dryness from 0.01 to 2% and can respond to very high frequencies. It can withstand temperatures up to 400 °C. It has better temperature coefficient and frequency characteristic. It is highly stable and its capacitance variation is within 1%. The drawbacks are that it absorbs moisture and it is big in size. It is good for cheap and rugged capacitors. Mica capacitors are used for small capacitance values of 50 to 500 pF. They are used as coupling capacitors at high frequency, in radio frequency transmitters and also in measuring circuits, bypassing circuits and r.f. resonant circuits. However ceramic and glass capacitors have replaced them in most applications.

There are two types of mica capacitors. They are (i) Stacked mica capacitors and (ii) Silvered mica capacitors.

(i) Stacked mica capacitor Perfect mica sheets are selected and stacked between tin foil sections as shown in Fig. 10.18 to provide the required capacitance. Stacking should be done under controlled pressure, as excess pressure may increase the capacitance value. Alternate metal foils are connected together and brought out to opposite side to provide capacitor leads. These capacitors are used over a wide temperature range (−55 to + 150°C) and have a high insulation resistance. Their capacitance values range from 1.0 pF to 0.1 μF.

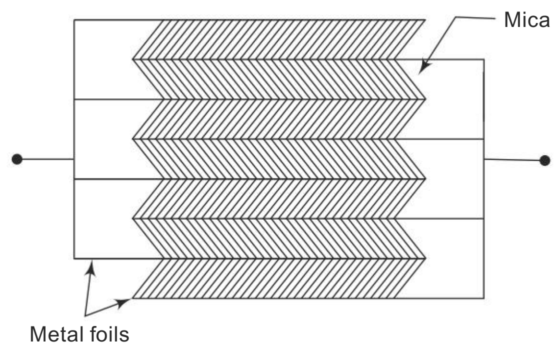


Fig. 10.18 Stacked Mica Capacitor

(ii) Silvered mica capacitor As shown in Fig. 10.19, the silvered mica capacitor is made of stacked silver coated mica sheets alternatively and sealed in a wax modulated case. The coating of mica is done at 500°C with silver oxide solution. Compared to the stacked mica capacitor, it is smaller in size and has greater mechanical stability and uniform characteristics.

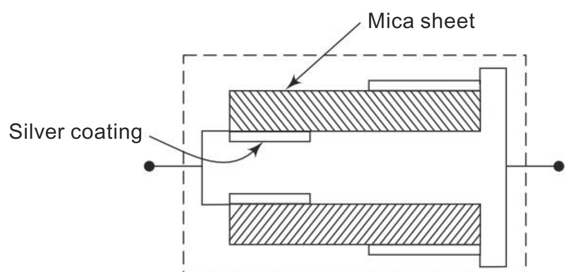


Fig. 10.19 Silvered Mica Capacitor

Paper Capacitors The dielectric in paper capacitors is Kraft paper impregnated with wax or resin. It is generally packaged as a rolled sandwich. Paper capacitors are available in the range of 500 pF to 50 μF, can withstand high voltages, can operate in ambient temperatures as high as 125°C, and are inexpensive. The two types of paper capacitors are: (i) Impregnated paper capacitor, and (ii) Metallised paper capacitor.

(i) Impregnated paper capacitor It consists of two thin sheets of paper between metal foils compactly rolled and impregnated with an impregnant like mineral oil, mineral wax, petroleum jelly, castor oil, polyisobutylene or chlorinated diphenyl. Two types of construction, called the 'tab' and 'extended foil' are common. In the tab type, a metal tab is inserted after rolling and the lead wires are soldered to the tabs. In the extended foil type, sheets of foil are allowed to extend over the margin of the paper, one foil on one side and another foil at the other side. The extended foil type is inexpensive and suitable for higher voltage operation, but has a disadvantage of occupying more space.

(ii) Metallised paper capacitor Instead of a separate foil, a layer of metal can be sprayed on to the paper. The layer is very thin, about 50 microns. A single sheet of metallised paper can be used up to 200 volts. In some capacitors, a layer of metallised paper along with a plastic film of Teflon and Mylar are used. This rolled capacitor is impregnated with an impregnant and enclosed in a metal case. This type has low insulation resistance and power factor, and it occupies small space. They are used for power factor correction and motor starting. They are also used as bypass and filter capacitors in audio frequency applications.

10.2.2 Variable Capacitors

Capacitor shown in Fig. 10.20. It is composed of two set of plates, usually made of aluminium. One set, known as rotor, is mounted on a shaft and meshes with a set of fixed metal plates, known as stator. The two set of plates are insulated and are never permitted to touch each other.

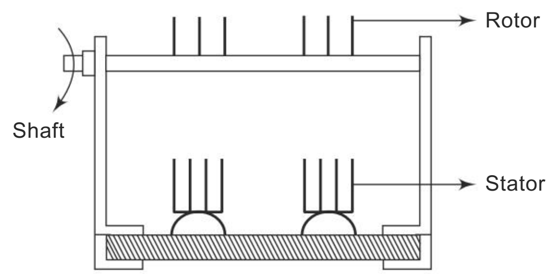


Fig. 10.20 Variable Tuning Capacitor

When the shaft is rotated, the effective area that exists between the rotor and stator plates varies, thereby varying the capacitance. The shape of the plates determines the manner in which capacitance varies with shaft rotation. The variation will be linear if the moving plates are semicircular.

Capacitance values for air variable capacitors range from a few picofarads upto 500 pF with a maximum voltage rating of 9 kV. For higher operating voltages upto 60 kV, a vacuum variable capacitor is used.

10.2.3 Dissipation Factor

An ideal capacitor when discharged gives up all the electrical energy that was supplied to it in charging. But practical capacitors dissipate some of the energy delivered to them, i.e. they suffer from losses due to (i) dielectric losses—particularly at higher frequencies; (ii) insulation or leakage resistance; (iii) resistance of the leads and plates. The insulation resistance and dielectric strength are lowered if the capacitor is operated at high temperature, and high voltage and humidity. The leakage losses increase with frequency, applied voltage and temperature. The voltage rating of capacitors has to be reduced if operated at higher temperatures and under humidity conditions.

The power factor of a capacitor expresses the ratio of the power wasted per cycle to the power stored per cycle in a capacitor. The power factor (PF) is the ratio of the equivalent series resistance to the total impedance of the capacitor, i.e.

$$\text{PF} = \frac{R_s}{Z} = \frac{R_s}{\sqrt{R_s^2 + X_c^2}}$$

The dissipation factor (DF) of a capacitor is the ratio of the equivalent series resistance to the capacitive reactance, i.e. DF = R_s/X_c. For low losses (i.e., $R_s << X_c$), the power factor is equal to dissipation factor. The equivalent series resistance is due to capacitor plates with leads and insulation or leakage resistance. For an ideal capacitor $R_s = 0$ and hence PF = 0. Due to the losses, a practical capacitor has a non-zero power factor. For a practical capacitor the power factor angle Φ is not 90° but slightly differs, given by $90° - \theta$. This angle θ is called the loss angle of a capacitor and tan θ is usually specified as the *dissipation factor* (*DF*). Dissipation factor is another parameter used to describe the quality of a capacitor. The reciprocal of the dissipation factor can be considered as the Q of the capacitor and is the ratio of the capacitor reactance (X_c) to the equivalent series resistance (R_s).

$$Q = \frac{1}{DF} = \frac{X_c}{R_s} = \frac{1}{\omega C R_s} = \tan \Phi = \tan (90° - \theta) = \cot \theta = \frac{1}{\tan \theta}$$

Thus, higher Q means better quality of the capacitor.

The characteristics and applications of various types of capacitors are summarised in Table 10.5.

10.3 INDUCTORS

Inductor is the third passive component used in electronic circuits. It stores energy in the form of magnetic field and delivers it as and when required.

Whenever current passes through a conductor, lines of magnetic flux are generated around it. This magnetic flux opposes any change in current due to the induced emf. This opposition to the change in current is known as inductance and the component producing inductance is known as inductor. The unit of inductance is Henry (symbol H). The induced emf is actually given by

$$e = -L\frac{di}{dt}$$

Table 10.5 Characteristics and applications of capacitors

Type	*Approximate Capacitance range and tolerance*	*Power factor*	*Maximum working voltage (in volts)*	*Operating temperature*	*Typical applications*
Aluminium electrolytic	(i) 2 μF – 100 μF – 15% (ii) 50 μF – 1000 μF – 15%	0.02 – 0.2	600 50 – 200	85°C – 125°C 85°C – 125°C	Rectifier filters and smoothing Decoupling and bypassing in a.f.
Tantalum electrolytic	1 μF – 2000 μF – 25%	0.02 – 0.2	6 – 150	85°C – 125°C	Coupling, decoupling, bypassing in amplifiers
Silvered ceramic	2 pF – 0.001 μF 1 – 20%	0.001	350	85°C – 150°C	r.f. amplifiers, r.f. bypass, decoupling and resonant circuit
Ceramic with high dieletric constant	50 pF – 0.01 μF 20%	0.001	350	85°C – 125°C	r.f. amplifiers, r.f. bypass, decoupling and resonant circuit
Polyesterene	100 pF – 0.5 μF – 1/2 – 5%	0.002	125 – 500	65°C – 85°C	Resonant, coupling, measuring and ac applications
Polyester	0.005 – 2 μF – 10 – 20%	0.005	500	65°C – 125°C	Coupling, decoupling, smoothing and dc applications
Stacked mica	50 pF – 0.01 μF – 2 – 20%	0.001 – 0.005	2000	85°C – 120°C	r.f. coupling, bypassing circuits
Silvered mica	10 pF – 0.1 μF – 1/2 – 20%	0.001 – 0.005	1000	85°C – 150°C	r.f. resonant and measuring circuits
Tubular rolled paper	0.005 μF – 10 – 20%	0.004 – 0.01	1000	85°C – 125°C	a.f. coupling and decoupling, bypass, filter, motor-start
Metallised paper	0.005 μF – 25%	0.005 – 0.015	400	85°C – 125°C	a.f. coupling and decoupling, bypass, filter, motor-start
Air/polystyrene variable capacitance	15 pF – 500 pF	1000.0	1000	85°C – 125°C	Tuning circuits in receivers and transmitters
Mica trimmers and padders	4 pF – 70 pF 100pF – 600pF	—	500	80°C	Tracking and alignment of receivers
Air trimmers	8 pF – 100 pF	—	500	80°C	Tracking and alignment of receivers

where e = induced emf in volts at any instant
L = inductance in Henry
di/dt = rate of change of current.

The negative sign in the above equation indicates that the induced emf opposes the cause for the change in current.

An inductor is usually a coil of copper wire wound around a core made up of a ferromagnetic material. The value of the inductor depends upon the following factors: (i) number of turns, (ii) permeability of the core material, and (iii) size of core.

Inductors can be divided into two categories (i) fixed inductors and (ii) variable inductors.

10.3.1 Fixed Inductors

Fixed type inductors can be further divided into three categories depending on the type of core used. They are (i) air inductors, (ii) iron core inductors, and (iii) ferrite core inductors.

These three types of inductors are shown in Fig. 10.21.

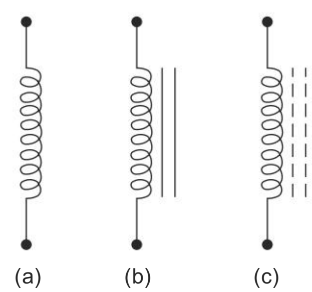


Fig. 10.21 (a) Air Core Inductor (b) Iron Core Inductor and (c) Ferrite Core Inductor

Air Core Inductors In radio frequency applications where very low values of inductance (from a fraction of a μH to a few μH) is required, air core inductors are generally used. Air core inductors consist of a few turns of wire wound on a hollow former.

Iron Core Inductors They have a coil containing a number of turns of copper wire wound on a hallow former and the core material passes through the former in such a way that it forms a closed magnetic path for the magnetic flux. The former is made up of paper or plastic material. The core is generally made up of silicon steel (a ferromagnetic material having high permeability) in the form of thin laminated sheets. Laminated sheets are used instead of solid mass to reduce the hysteresis and eddy current losses. Iron core transformers are used in low frequency applications such as filter circuits in power supplies, chokes in fluorescent tubes or as a reactive element in ac circuits. The values of inductors are generally in the order of a few Henries.

Ferrite Core Inductors Iron core inductors are not suitable for high frequency applications (because of enormous increase in hysteresis and eddy current losses). This difficulty is overcome by the use of ferrite materials as the core. A ferrite is basically an insulator having very high permeability.

Ferrites are made up of non-metallic compounds consisting mainly of ferric oxide in combination with one or two bivalent metal oxides. They are hard, dense ceramics and because of their high resistivity they can be used in the form of solid cores. The values of such inductors range a few microhenries to a few millihenries.

The typical applications of ferrite core inductors are in (i) r.f. chokes for supply decoupling purposes, (ii) switching regulated type dc power supplies, and (iii) various types of filters used in communication equipment.

10.3.2 Variable Inductors

In certain applications such as tuned circuits, it is required to vary the inductance from a minimum value to a maximum value. Ferrite core variable inductors are generally used for this purpose. In such inductors the hollow former on which the coil is wound has screw threads in the inner hollow portion. Similar matching threads are provided on the ferrite core which can be screwed in or out of the former. Because of the change of the position of the ferrite core the value of the inductance changes. It is maximum when the ferrite core is fully in. The variable ferrite core inductor is shown in Fig. 10.22.

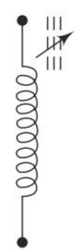


Fig. 10.22 Variable Inductor

Fig. 10.23 Equivalent Circuit of an Inductor

10.3.3 Q of an Inductor

Though there is no power loss in an ideal inductor, losses do occur in a practical inductor. These losses are of two types: (i) I^2R loss (ohmic loss in the copper wire), and (ii) hysteresis and eddy current losses in the core.

An equivalent circuit of a practical inductor is shown in Fig. 10.23 in which R_s is the effective resistance of the inductor which accounts for the total losses.

$$\text{The quantity } Q = \frac{\omega L_s}{R_s}$$

which is the ratio of the inductive reactance (ωL_s) to the effective resistance is known as the Q of an inductor. This is also called as the quality factor or figure of merit of an inductor.

REVIEW QUESTIONS

1. What is meant by resistance? Describe the various types of resistors.
2. What are the uses of resistors in electronic circuits?
3. Write short notes on wire wound resistors.
4. Describe different types of potentiometers.
5. Explain the tapering of potentiometers.
6. Why is colour coding used in resistors? Briefly explain its significance.
7. What is meant by tolerance in resistors?
8. Write short notes on: (a) Film resistors (b) Rheostat, and (c) Trimmer.
9. Explain the action of capacitor for dc and ac voltages.
10. Define the dielectric strength referred to a capacitor.
11. List the various capacitors used in electronic circuits and state uses of each type.
12. Compare the constructional features and characteristics of (a) Paper, (b) Mica, (c) Ceramic, and (d) Plastic capacitors.
13. Desctibe briefly polarised and non-polarised electrolytic capacitors.
14. Compare the constructional features and characteristics of aluminum and tantalum electrolytic capacitors.
15. What are the specifications of a capacitor? State the factors affecting the capacitance of a capacitor.
16. State and explain the various losses of a capacitor
17. Explain what is meant by dissipation factor of a capacitor.
18. Write short notes on variable capacitors.
19. Why is colour coding used in capacitors? Briefly explain its significance
20. What is meant by tolearance in capacitors?
21. What are the factors that influence the value of inductance in an inductor?
22. Give a detailed account of fixed inductors.
23. Write down the applications of various types of inductors.
24. What are the losses that occur in an inductor?
25. Explain the quality factor of an inductor.
26. Classify the inductors and explain briefly.

TRANSDUCERS

11

INTRODUCTION

A transducer is a device which converts the energy from one form to another form. This energy may be electrical, mechanical, chemical, optical or thermal.

Transducers may be classified according to their application, method of energy conversion, nature of the output signal, and so on. All these classifications usually result in overlapping areas. A sharp distinction among the types of transducers is difficult.

The transducer that gives electrical energy as output is known as *electrical transducer.* The output electrical signal may be voltage, current, or frequency and production of these signals is based upon resistive, capacitive, inductive effects etc. For measuring non-electrical quantities, a detector is used which usually converts the physical quantity into a displacement, that activates the electrical transducer.

The *displacement transducers,* such as capacitive, oscillation, potentiometric, photoelectric (phototube) and piezoelectric, use the principle of converting a mechanical force into displacement and then into electrical parameters. Here, the mechanical elements used for converting this applied force into displacement are called force-summing devices.

The transducers may also be classified as (i) Active and (ii) Passive transducers. *Active transducers,* also known as self generating type, develop their own voltage or current as the output signal. The energy required for production of this output signal is obtained from the physical phenomenon being measured.

Passive transducers, also known as externally powered transducers, derive the power required for energy conversion from an external power source. However, they may also absorb some energy from the physical phenomenon under study.

A few examples of active and passive transducers are given in Table 11.1.

The opto-electronic transducer such as photoconductive cell, photovoltaic cell, solar cell, phototube and photomultiplier tube use the principle of converting light energy into electrical energy.

Some of the basic requirements of a transducer are given below.

(i) Linearity The input-output characteristics of the transducer should be linear.

(ii) Ruggedness The transducer should withstand overloads, with measures for overload protection.

(iii) Repeatability The transducer should produce identical output signals when the same input signal is applied at different times under the same environmental conditions.

(iv) *High stability and reliability* The output from the transducer should not be affected by temperature, vibration and other environmental variations and there should be minimum error in measurements.

(v) *Good dynamic response* In industrial, aerospace and biological applications, the input to the transducer will not be static but dynamic in nature, i.e. the input will vary with time. The transducer should respond to the changes in input as quickly as possible.

(vi) *Convenient instrumentation* The transducer should produce a sufficiently high analog output signal with high signal-to-noise ratio, so that the output can be measured either directly or after suitable amplification.

(vii) *Good mechanical characteristics* The transducer, under working conditions, will be subjected to various mechanical strains. Such external forces should not introduce any deformity and affect the performance of the transducer.

Of the many effects that are used in transducers, the principal effects used are variation of resistance, inductance, capacitance, piezoelectric effect and thermal effects which are described in the following sections.

Table 11.1 Active and passive transducers

Active transducers	*Passive transducers*
Thermocouple	Resistance
Piezoelectric transducer	Potentiometric device
Photovoltaic (Photojunction) cell	Resistance strain gauge
Moving coil generator	Resistance thermometer
Photoelectric (Photoemission) cell	Thermistor
	Photoconductive cell
	Inductance
	Linear Variable Differential Transformer (LVDT)
	Capacitance
	Voltage and current
	Devices using Hall effect
	Photoemissive cell
	Photomultiplier tube

11.1 CAPACITIVE TRANSDUCER

The capacitance of a parallel-plate capacitor is given by

$$C = \varepsilon_0 \, \varepsilon_r \frac{A}{d}$$

where A = area of each plate in m^2

d = distance between parallel plates in m

ε_0 = dielectric constant (permittivity) of free space in F/m

ε_r = relative dielectric constant (permittivity)

The capacitance is directly proportional to the area of the plate (A) and inversely proportional to the distance between the parallel plates (d). Obviously, any variation in A or d causes a corresponding variation in the capacitance. This principle of variation in d is used in the capacitive transducer, shown in Fig. 11.1.

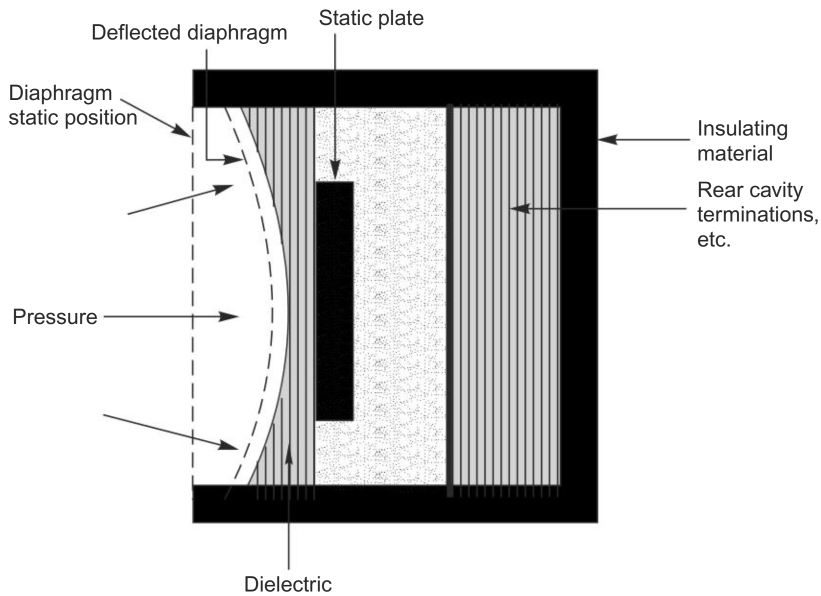


Fig. 11.1 Capacitive Transducer

When a force is applied to a diaphragm which acts as one plate of a capacitor, the distance between the diaphragm and the static plate is changed. The resulting change in capacitance can be measured with an a.c. bridge or an oscillator circuit in which the change in frequency can be measured by an electronic counter and it is a measure of the magnitude of the applied force. In capacitor microphone, the same principle is used in which sound pressure varies the capacitance between the fixed plate and a movable diaphragm.

The capacitive transducer can measure static and dynamic changes. The drawback of this transducer is its sensitivity to temperature variations.

11.2 INDUCTIVE TRANSDUCER

When a force is applied to the ferromagnetic armature, the air gap, as shown in Fig. 11.2, is changed thereby varying the reluctance of the magnetic circuit. Thus the applied force is measured by the change of inductance in a single coil.

The inductive transducer enables static and dynamic measurements. Its drawback is that it has limited frequency response.

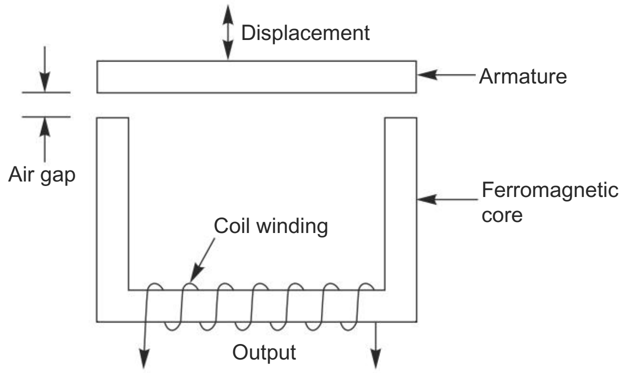


Fig. 11.2 Inductive Transducer

11.3 LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

The most widely used inductance transducer is the Linear Variable Differential Transformer (LVDT) and is shown in Fig. 11.3(a). It consists of a primary coil and two exactly similar secondary coils with a rod shaped magnetic core positioned centrally inside the coil. An alternating current is fed into the primary and voltages V_{o1} and V_{o2} are induced in the secondary coils. As these coils are connected in series opposition, the output voltage $V_o = V_{o1} - V_{o2}$. If the core is placed ideally in the central position (null position or reference position), $V_{o1} = V_{o2}$ and hence the output voltage $V_o = 0$. In practice due to incomplete balance, a residual voltage usually remains with the core in this position. As shown in Fig. 11.3, when the core is displaced from the null position, the induced voltage in the secondary towards which the core

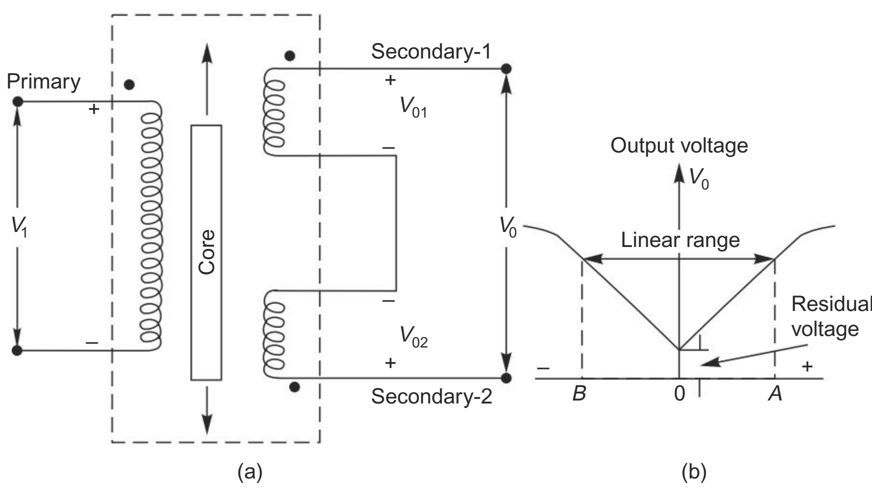


Fig. 11.3 Linear Variable Differential Transformer [LVDT]

has moved increases while that in the other secondary decreases. This results in a differential voltage output from the transformer.

The output voltage produced by the displacement of the core is linear over a considerable range [Fig. 11.3(b)] but flattens out at both ends, and the voltage phase changes by 180° as the core moves through the center position.

LVDT provides continuous resolution and shows low hysteresis and hence, repeatability is excellent under all conditions. As there are no sliding contacts, there is less friction and less noise.

It is sensitive to vibrations and temperature. The receiving instrument must be selected to operate on ac signals or a demodulator network must be used if a dc output is required.

11.4 OSCILLATION TRANSDUCER

Figure 11.4 shows the basic circuit of an oscillation transducer. The force-summing device is used to change the distance between the parallel plates of the capacitor, thereby changing the value of capacitance (similarly, the inductance value can also be changed) in the stable LC oscillator. The change in oscillator frequency caused by the externally applied force can be measured by an electronic counter.

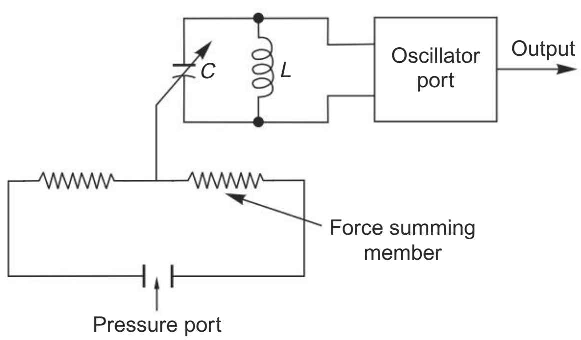


Fig. 11.4 Oscillation Transducer

This transducer measures both the static and dynamic phenomena and is used in telemetry systems. Its limited range, poor thermal stability and low accuracy restrict its use to low accuracy applications.

11.5 POTENTIOMETRIC TRANSDUCER

The basic circuit of a potentiometric transducer is shown in Fig. 11.5. A potentiometric transducer consists of a resistance element that is contacted by a movable slider. A force-summing member is used to move the slider thereby changing the resistance and correspondingly, the output voltage changes. The same principle can be used to vary the resistance in a bridge circuit.

This transducer has high electric efficiency and provides a sufficient output to permit control operations without further amplification.

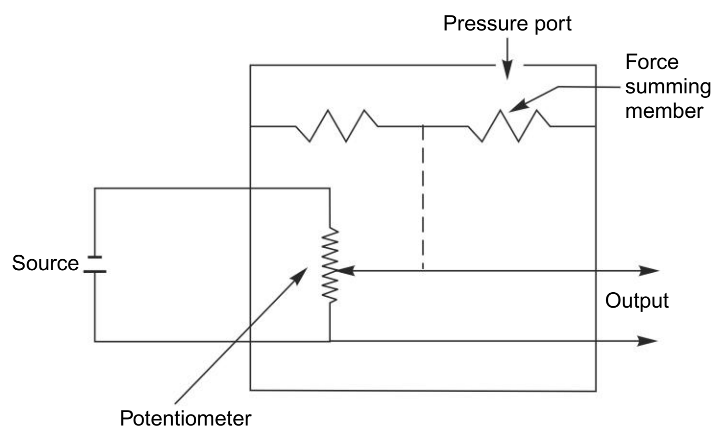


Fig. 11.5 Potentiometric Transducer

11.6 ELECTRICAL STRAIN GAUGES

If a metal conductor is stretched or compressed, its resistance changes because of dimensional changes (length and cross sectional area) and resistivity change. If a wire is under tension and increases its length from l to $l + \Delta l$, i.e. the strain $S = \Delta l/l$, then its resistance increases from R to $R + \Delta R$.

The sensitivity of a strain gauge is described in terms of a characteristic called the gauge factor G, defined as the unit change in resistance per unit change in length, i.e.

$$G = \frac{\Delta R/R}{\Delta l/l} = \frac{\Delta R/R}{S}$$

11.6.1 Unbonded Strain Gauge

The schematic diagram of a typical displacement transducer wherein the measuring forces are transmitted to the platform containing the unbonded wire structure by means of a force rod is shown in Fig. 11.6. The resistance wires have equal lengths.

When an external force is applied to the strain gauge, the armature moves in the direction indicated. Elements A and D increase in length, whereas elements B and C decrease in length. The change in resistance of the four wires is proportional to their change in length and this change can be measured with a Wheatstone bridge as shown in Fig. 11.6 (c).

Thus, the external force causes variation in resistance of the wires, unbalancing the bridge and causing an output voltage V_o proportional to the pressure. The bridge is balanced if

$$\frac{R_A}{R_C} = \frac{R_B}{R_D}$$

11.6.2 Bonded Wire Strain Gauge

A bonded wire strain gauge consists of a grid of fine resistance wire of diameter of about 25 μm. The wire is cemented to a base. The base may be a thin sheet of paper

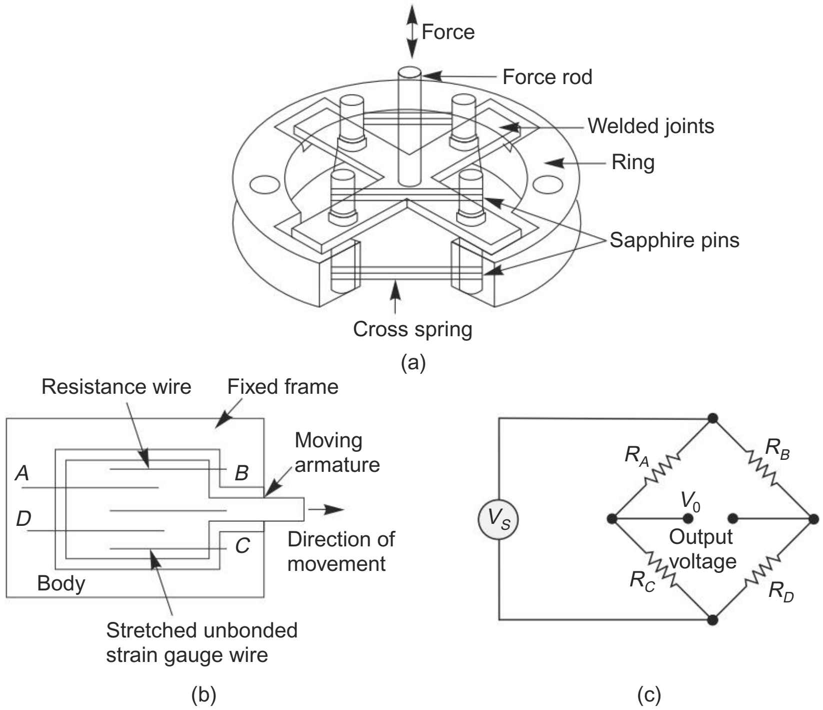


Fig. 11.6 Unbonded Strain Gauge

or a very thin Bakelite sheet. The wire is covered with a thin sheet of material so that it is not damaged mechanically. The base is bonded to the structure under study with an adhesive material. It acts as a bonding material. It permits a good transfer of strain from base to wires.

The commonly used types of bonded strain gauges are shown in Fig. 11.7.

11.7 RESISTANCE THERMOMETER

The resistance of most electrical conductors varies with temperature according to the relation

$$R = R_0 (1 + \alpha T + \beta T^2 + ...)$$

where R_0 = resistance at temperature T_0 (usually 0°C),

R = resistance at T,

α, β = constants, and T is the rise in temperature above T_o

Over a small temperature range, depending on the material, the above equation reduces to

$$R = R_0 (1 + \alpha T)$$

where α is the temperature coefficient of resistance.

Important properties of materials used for resistance thermometers are (i) high temperature coefficient of resistance, (ii) stable properties so that the resistance

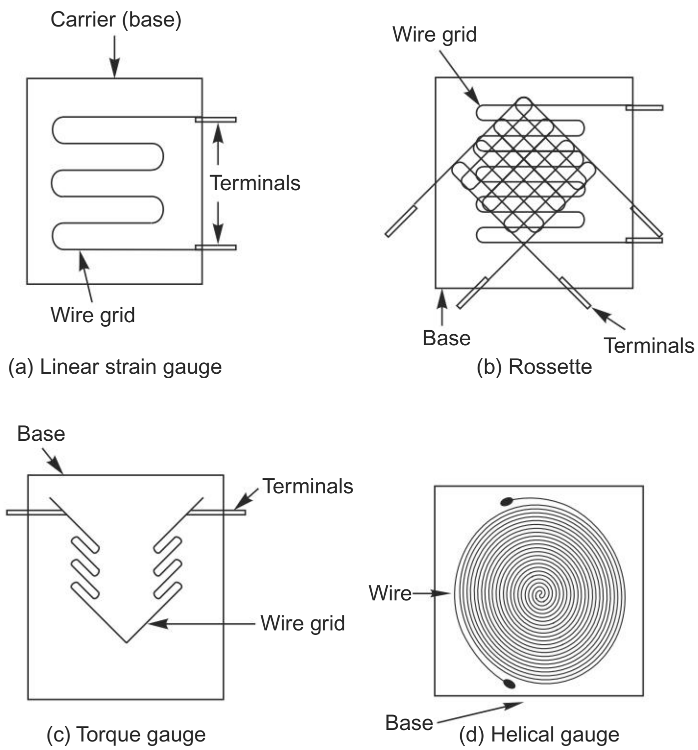


Fig. 11.7 Bonded Strain Gauges

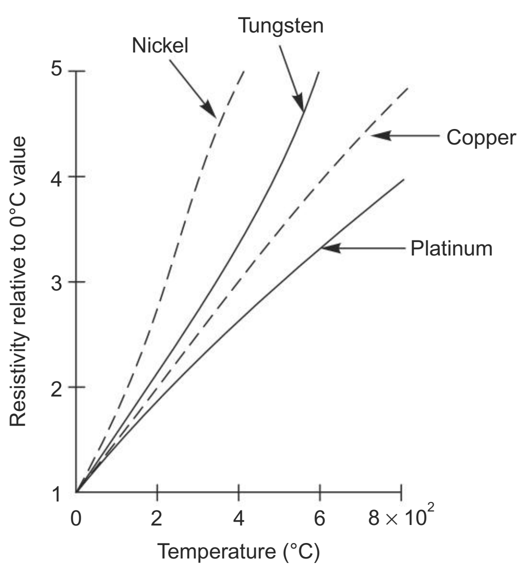


Fig. 11.8 Variation of resistivity with temperature of materials used for resistance thermometers

characteristic does not drift with repeated heating and cooling or mechanical strain, and (iii) a high resistivity to permit the construction of small sensors.

The variation of resistivity with temperature of some of the materials used for resistance thermometers is shown in Fig. 11.8. From the figure it can be seen that tungsten has a suitable temperature coefficient of resistance but is brittle and difficult to form. Copper has a low resistivity and is generally confined to applications where the sensor size is not restricted. Both platinum and nickel are widely used because they are relatively easy to obtain in pure state.

Platinum has an advantage over nickel in that its temperature coefficient of resistance is linear over a larger temperature range. The resistance–temperature relationship for platinum resistance elements is determined from the Callendar equations

$$T = \frac{100\,(R_T - R_0)}{R_{100} - R_0} + d\left(\frac{T}{100} - 1\right)\frac{T}{100}$$

where T is the temperature and R_T is the resistance at temperature T, R_0 is the resistance at 0°C, R_{100} is the resistance at 100°C and d is the Callendar constant (approximately 1.5).

The construction of industrial platinum resistance thermometer is shown in Fig. 11.9.

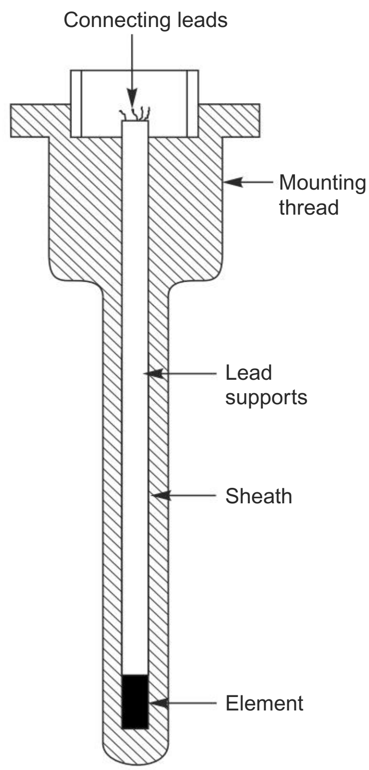


Fig. 11.9 Industrial Platinum Resistance Thermometer

11.8 THERMISTOR

Thermistor or Thermal resistor is a two-terminal semiconductor device whose resistance is temperature sensitive. The value of such resistors decreases with increase in temperature. Materials employed in the manufacture of the thermistors include oxides of cobalt, nickel, copper, iron, uranium and manganese.

The thermistor has very high temperature coefficient of resistance, of the order of 3 to 5% per°C, making it an ideal temperature transducer. The temperature coefficient of resistance is normally negative. The resistance at any temperature T, is given approximately by

$$R_T = R_0 \exp \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

where R_T = thermistor resistance at temperature T (K),
R_o = thermistor resistance at temperature T_o (K), and
β = a constant determined by calibration.

At high temperatures, this equation reduces to

$$R_T = R_0 \exp\left(\frac{\beta}{T}\right)$$

The resistance–temperature characteristic is shown in Fig. 11.10. The curve is non-linear and the drop in resistance from 5000 to 10 Ω occurs for an increase in temperature from 20 to 100 °C. The temperature of the device can be changed internally or externally. An increase in current through the device will raise its temperature carrying a drop in its terminal resistance. Any externally applied heat source will result in an increase in its body temperature and drop in resistance. This type of action (internal or external) lends itself well to control mechanisms.

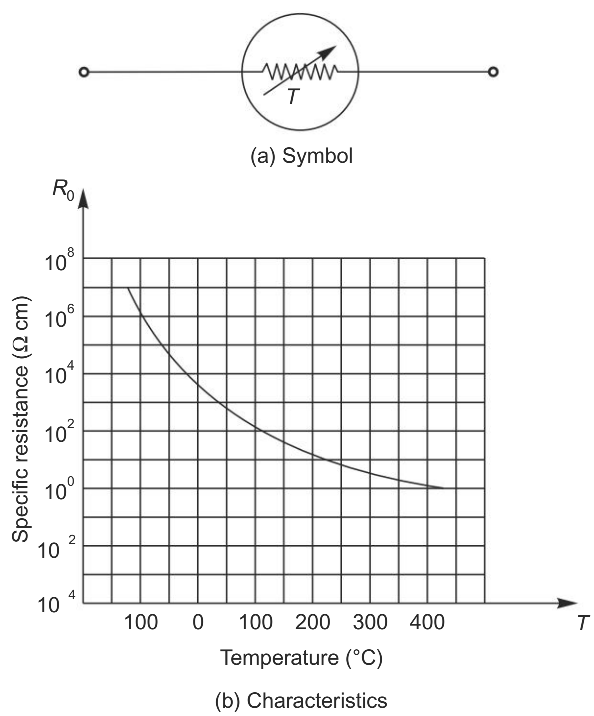


Fig. 11.10 Symbol and Resistance-temperature Characteristics of Thermistor

Three useful parameters for characterizing the thermistor are the time constant, dissipation constant, and resistance ratio. The time constant is the time for a thermistor to change its resistance by 63% of its initial value, for zero-power dissipation. Typical values of time constant range from 1 to 50 s.

The dissipation factor is the power necessary to increase the temperature of thermistor by 1 °C. Typical values of dissipation factor range from 1 to 10 mW/°C.

Resistance ratio is the ratio of the resistance at 25 °C to that at 125 °C. Its range is approximately 3–60.

Thermistors are used to measure temperature, flow, pressure, liquid level, voltage or power level, vacuum, composition of gases and thermal conductivity and also in compensation network.

11.9 THERMOCOUPLE

A thermocouple is a junction between two dissimilar metals or semiconductors that generates a small voltage, typically in the millivolt range, with coefficient of about 50 μV/°C.

Various thermocouple materials and methods of construction are used depending on the temperature, environment and required sensitivity. A typical thermocouple circuit for temperature measurement is shown in Fig. 11.11. It consists of two junctions, reference and sensing maintained at different temperatures. Each junction is made by welding two dissimilar metals together. The reference junction is maintained at a fixed temperature, usually 0°C and the output voltage depends upon the temperature of the sensing junction. As only a relatively small output of the order of 50 μV/°C is obtained, it is necessary to amplify the output for calibration and measurement.

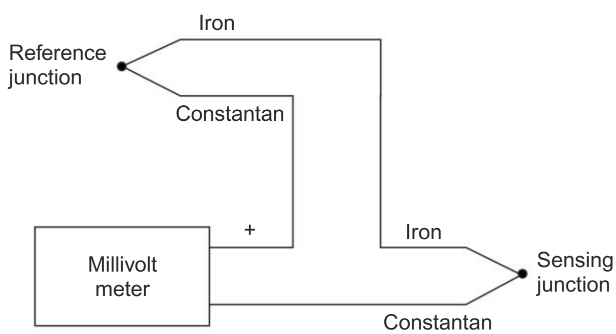


Fig. 11.11 A typical Thermocouple Circuit

The iron-constantan thermocouple is used for measuring temperatures up to 760°C and Chromel–Alumel thermocouple is used for temperature measurement up to 1370°C.

11.10 HALL EFFECT

When a transverse magnetic field B is applied to a metal or a semiconductor carrying a current I, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as the *Hall effect*.

The Hall effect is used to find whether a semiconductor is N or P type and to determine the carrier concentration. The mobility μ can also be calculated with simultaneous measurement of the conductivity σ.

The schematic arrangement of the semiconductor, the magnetic field and the current flow pertaining to the Hall effect are shown in Fig. 11.12.

An electron of charge e travelling in a magnetic field B with drift velocity V experiences a force given by

$$F = eE = BeV$$

In an N-type semiconductor, the current is carried by electrons and these electrons will be forced downwards toward side 1 which becomes negatively charged with respect to side 2. Therefore the Hall voltage V_H appearing between surfaces 1 and 2 is given by

$$V_H = \frac{R_H}{w} BI$$

where w is the width of the conductor in the direction of the magnetic field and the hall coefficient $R_H = 1/\rho$ where ρ is the charge density. If terminal 2 becomes charged positively with respect to terminal 1, the semiconductor must be N-type and $\rho = ne$, where n is the electron concentration. On the other hand, if the polarity of V_H is positive at terminal 1 with respect to terminal 2, the semiconductor must be P-type and $\rho = pe$, where p is the hole concentration. The conductivity σ and the mobility μ are related by the equation $\sigma = \rho\mu$ or $\mu = \sigma R_H$.

The advantage of Hall effect transducers is that they are non-contact devices with high resolution and small size. Some of the other applications are in measurement of velocity, rpm, sorting, limit sensing, and non-contact current and magnetic field measurements.

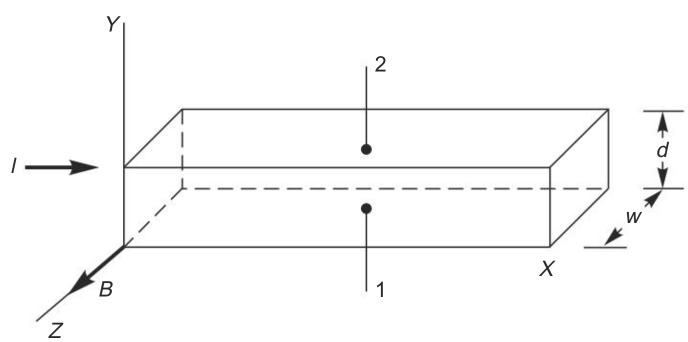


Fig. 11.12 Schematic Arrangement to Observe the Hall Effect

11.11 PIEZOELECTRIC TRANSDUCER

If the dimensions of asymmetrical crystalline materials, such as quartz, rochelle salt and barium titanite, are changed by the application of a mechanical force, the crystal produces an emf. This property is used in piezoelectric transducers.

The basic circuit of a piezoelectric transducer is shown in Fig. 11.13. Here, a crystal is placed between a solid base and the force-summing member. An externally applied force exerts pressure on the top of the crystal and it produces an emf across the crystal which is proportional to the magnitude of the applied pressure.

As this transducer has a very good high frequency response, it is used in high frequency accelerometers. As it needs no external power source, it is called as self-generating transducer. The main drawbacks are that it cannot measure static conditions and the output voltage is affected by temperature variations of the crystal.

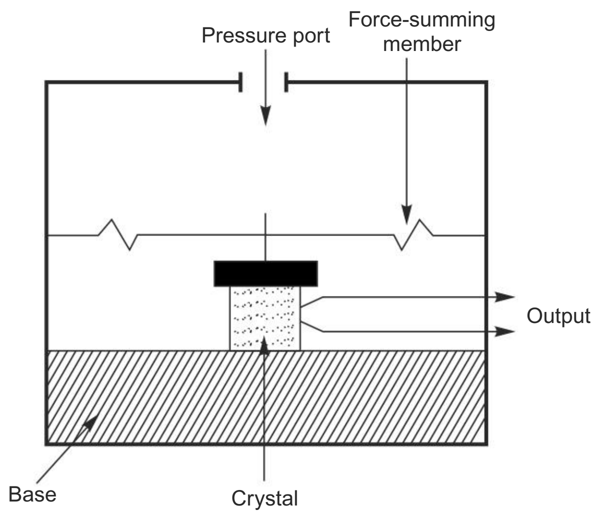


Fig. 11.13 Piezoelectric Transducer

11.12 PHOTOELECTRIC TRANSDUCER

This is an opto-electronic or optical transducer shown in Fig. 11.14. It uses a phototube and a light source separated by a small window whose aperture is controlled by the force-summing device. The quantity of incident light on the photosensitive cathode is varied according to the externally applied force thereby changing the anode current.

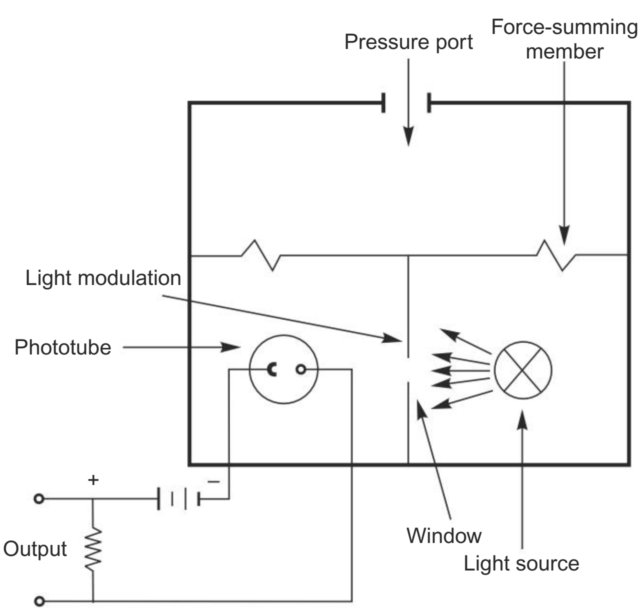


Fig. 11.14 Photoelectric Transducer

This device measures both static and dynamic phenomena and it has high efficiency. It does not respond to high frequency light variation.

REVIEW QUESTIONS

1. What is a transducer? Briefly describe any one of the displacement transducers.
2. What are active and passive transducers? Why are they called so?
3. What are the basic requirements of a transducer?
4. Explain the transduction principle used in the capacitor microphone.
5. Discuss the working principle of an inductive transducer.
6. Explain in detail the working of a Linear Variable Differential Transformer.
7. Describe the construction of oscillation transducers. How will you measure pressure with such a type of transducer?
8. Explain the Hall effect and its applications.
9. What is meant by gauge factor of a strain gauge?
10. Discuss with suitable diagrams the salient features of unbonded and bonded strain gauges.
11. Explain the principle of operation of a resistance thermometer.
12. What is a thermocouple?
13. How is a thermocouple used for temperature measurement?
14. Write short notes on
    (a) Thermistor
    (b) Capacitive transducer
    (c) Photoelectric transducer
    (d) Piezoelectric transducer

JUNCTION DIODE AND ITS APPLICATIONS

12

INTRODUCTION

Electronics is the science and technology of passage of charged particles in a vaccum or in a gas or in a semiconductor. Basically it is a study of electron devices and their utilization. An electron device is that in which electrons flow through a vaccum or gas or semiconductor. In the beginning of 20th century electronics began to take technological shape and it has enjoyed an explosive development in the last five decades.

Electronics has a wide range of applications such as rectifications, amplifications, power generation, industrial control, photo-electricity and communications, etc. The electronic industry turns out a variety of items in the range of Consumer electronics, Control and industrial electronics, Communication and broadcasting equipments, Bio-medical euipments, Calculators, Computers, Microprocessors, Aerospace and defence equipments and components.

Hence, the study of electronics is very much essential and important. This Chapter describes the junction diode and its applications in detail.

12.1 SEMICONDUCTOR THEORY

Depending on their conductivity, materials can be classified into three types as conductors, semiconductors and insulators. Conductor is a good conductor of electricity. Insulator is a poor conductor of electricity. Semiconductor has its conductivity lying between these two extremes.

Materials can be classified into these three types depending on the number of valence electrons in the atom. Electrons in the outermost orbit of an atom are valence electrons. In a good conductor, the number of valence electrons will be 1 or 2, (e.g. copper).

In an insulator, the outermost orbit will be completely filled (e.g. xenon).

In a semiconductor, the outermost orbit will be partially filled. For example, the number of valence electrons is 4 in Germanium (Ge) and Silicon (Si).

12.1.1 Energy Band Structure

The range of energies that an electrons may possess in an atom is known as the energy band. The three important energy bands are explained below.

(i) Valence Band The range of energy possessed by valence electrons is known as valence band.

(ii) Conduction Band In good conductors, the valence electrons are loosely attached to the nucleus, so that even on applications of small electric field, some of the valence electrons may get detached from the nucleus to become free electrons. These free electrons which are responsible for the conduction of current in good conductors are called conduction electrons and the range of energies possessed by these electrons is known as conduction band.

(iii) Forbidden Band The energy band in between the conduction band and the valence band is called forbidden band.

Classifications of Materials According to Energy Bands

Conductor In good conductors, as shown in Fig. 12.1, the valence and conduction bands overlap each other. Due to this overlapping, even the application of small electric field causes the free electrons to constitute electric current.

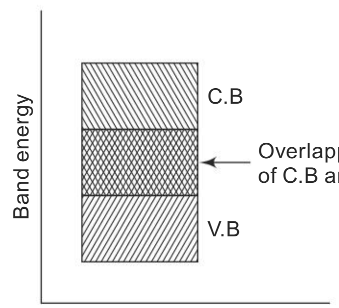


Fig. 12.1 Conductor

Insulator In insulator, as shown in Fig. 12.2, the forbidden energy gap E_g is very large and hence almost all the electrons are in the valence band and the conduction band is almost empty. Hence, no electrons are available for conduction.

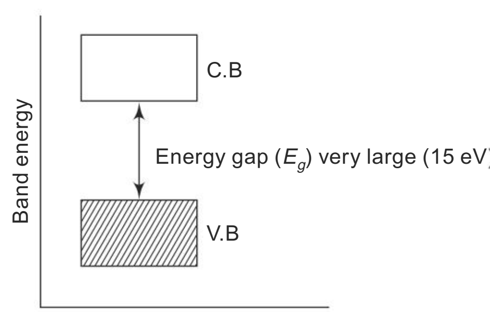


Fig. 12.2 Insulator

Semiconductor Semiconductors are those substances whose electrical conductivity lies in between conductors and insulators. In terms of energy band shown in Fig. 12.3, the valence band is almost filled (partially filled) and conduction band is almost empty.

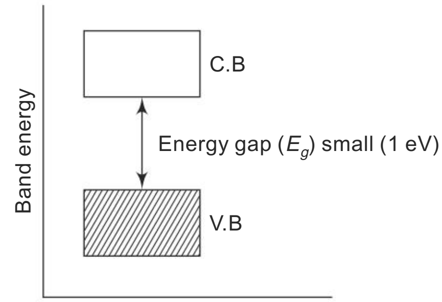


Fig. 12.3 Semiconductor

A comparatively smaller electric field (smaller than required for insulator) is required to push the electrons from the valence band to conduction band. At low temperatures, the valence band is completely filled and the conduction band is completely empty. Therefore a semiconductor virtually behaves as an insulator at low temperature. However even at room temperature some electrons crossover to the conduction band giving conductivity to the semiconductor. As temperature increases, the number of electrons crossing over to the conduction band increases and hence electrical conductivity increases. Hence a semiconductor has negative temperature coefficient of resistance.

Classifications of Semiconductors

Intrinsic Semiconductor A pure semiconductor is called intrinsic semiconductor. As already explained in the first chapter, even at the room temperature, some of the valence electrons may acquire sufficient energy to enter the conduction band to form free electrons. Under the influence of electric field, these electrons constitute electric current. A missing electron in the valence band leaves a vacant space there, which is known as a hole, as shown in Fig. 12.4. Holes also contribute to electric current.

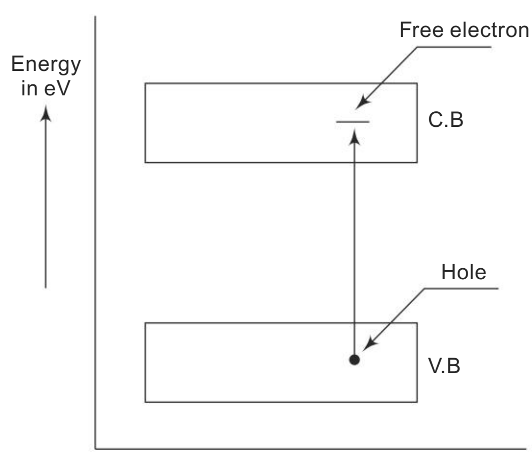


Fig. 12.4 Creation of Electron-Hole Pair in a Semiconductor

In an intrinsic semiconductor, even at room temperature, electron-hole pairs are created. When electric field is applied across an intrinsic semiconductor, the current conduction takes place by two processes, namely, free electrons and holes. Under the influence of electric field, total current through the semiconductor is the sum of currents due to free electrons and holes.

Though the total current inside the semiconductor is due to free electrons and holes, the current in the external wire is fully by electrons. In Fig. 12.5, holes being positively charged move towards the negative terminal of the battery. As the holes reach the negative terminal of the battery, electrons enter the semiconductor near the terminal (X) and combine with the holes. At the same time the loosely held electrons near the positive terminal (Y) are attracted away from their atoms into the positive terminal. This creates new holes near the positive terminal which again drift towards the negative terminal.

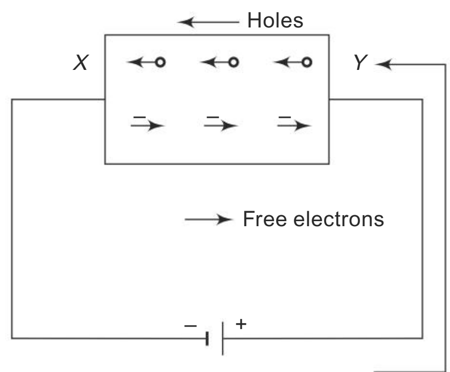


Fig. 12.5 Current Conduction in Semiconductor

Extrinsic Semiconductor Due to the poor conduction at room temperature, the intrinsic semiconductor, as such, is not useful in the electronic devices. Hence the current conduction capability of the intrinsic semiconductor should be increased. This can be achieved by adding a small amount of impurity to the intrinsic semiconductor, so that it becomes impurity semiconductor or extrinsic semiconductor. This process of adding impurity is known as doping.

The amount of impurity added is extremely small, say 1 to 2 atoms of impurity for 10^6 intrinsic atoms.

N-type Semiconductor A small amount of pentavalent impurities such as arsenic, antimony or phosphorus is added to the pure semiconductor (germanium or silicon crystal) to get N-type semiconductor.

Germanium atom has four valence electrons and antimony has five valence electrons. As shown in Fig. 12.6, each antimony atom forms a covalent bond with surrounding four germanium atoms. Thus four valence electrons of antimony atom form covalent bond with four valence electrons of individual germanium atom and fifth valence electron is left free which is loosely bound to the antimony atom.

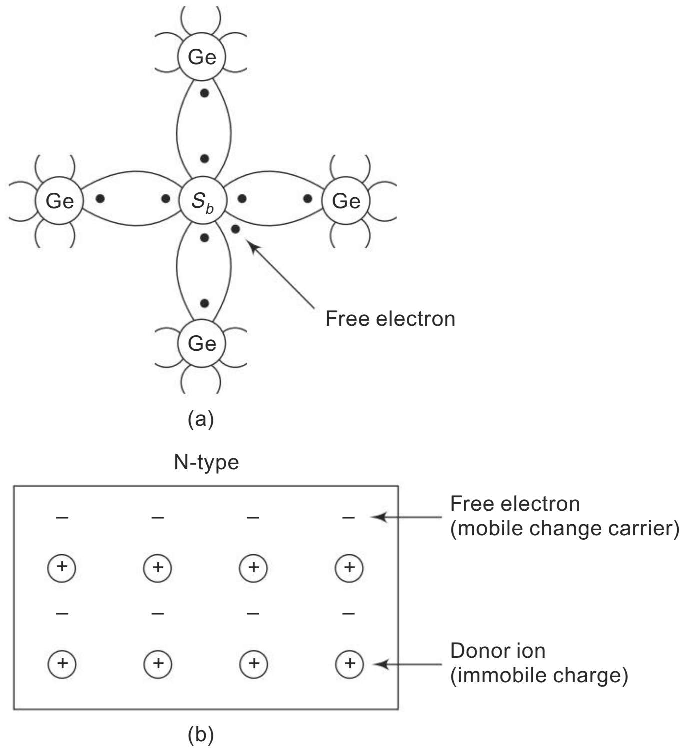
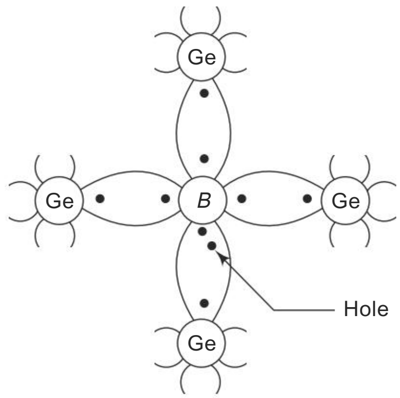


Fig. 12.6 N-type Semiconductor: (a) Formation of Covalent Bonds, and (b) Charged Carriers

This loosely bound electron can be easily excited from the valence band to the conduction band by the application of electric field or increasing the thermal energy. Thus every antimony atom contributes one conduction electron without creating a hole. Such pentavalent impurities are called donor impurities because it donates one electron for conduction. On giving an electron for conduction, the donor atom becomes positively charged ion because it loses one electron. But it cannot take part in conduction because it is firmly fixed in the crystal lattice.

Thus, the addition of pentavalent impurity (antimony) increases the number of electrons in the conduction band thereby increasing the conductivity of N-type semiconductor. As a result of doping, the number of free electrons far exceeds the number of holes in an N-type semiconductor. So electrons are called majority carriers and holes are called minority carriers.

P-type Semiconductor A small amount of trivalent impurities such as aluminium or boron is added to the pure semiconductor to get the *P*-type semiconductor. Germanium (Ge) atom has four valence electrons and boron has three valence electrons as shown in Fig. 12.7. Three valence electrons in boron form covalent bond with four surrounding atoms of Ge leaving one bond incomplete which gives rise to



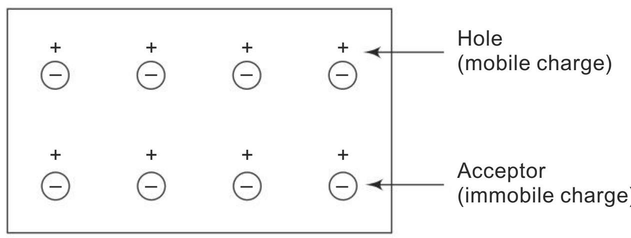


Fig. 12.7 P-type Semiconductor: (a) Formation of Covalent Bonds, and (b) Charged Carriers

a hole. Thus trivalent impurity (boron) when added to the intrinsic semiconductor (germanium) introduces a large number of holes in the valence band. These positively charged holes increase the conductivity of P-type semiconductor. Trivalent impurities such as boron is called acceptor impurity because it accepts free electrons in the place of holes. As each boron atom donates a hole for conduction, it becomes a negatively charged ion. As the number of holes is very much greater than the number of free electrons in a P-type material, holes are termed as majority carriers and electrons as minority carriers.

12.2 THEORY OF PN JUNCTION DIODE

In a piece of semiconductor material, if one half is doped by P-type impurity and the other half is doped by N-type impurity, a PN junction is formed. The plane dividing the two halves or zones is called PN junction. As shown in Fig. 12.8, the N-type material has high concentration of free electrons while P-type material has high concentration of holes. Therefore at the junction there is a tendency for the free electrons to diffuse over to the P-side and holes to the N-side. This process is called *diffusion.* As the free electrons move across the junction from N-type to P-type, the donor ions become positively charged. Hence a positive charge is built

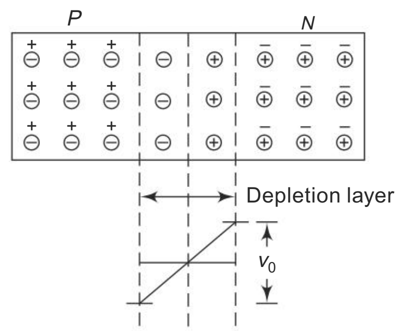


Fig. 12.8 Formation of PN Junction

on the N-side of the junction. The free electrons that cross the junction uncover the negative acceptor ions by filling in the holes. Therefore a net negative charge is established on the P-side of the junction. This net negative charge on the P-side prevents further diffusion of electrons into the P-side. Similarly, the net positive charge on the N-side repels the holes crossing from P-side to N-side. Thus a barrier is set up near the junction which prevents further movement of charge carriers, i.e. electrons and holes. This is called potential barrier or junction barrier V_0. V_0 is 0.3 V for germanium and 0.72 V for silicon.

The electrostatic field across the junction caused by the positively charged N-type region tends to drive the holes away from the junction and negatively charged P-type region tends to drive the electrons away from the junction. Thus the junction region is depleted to mobile charge carriers. Hence it is called depletion layer.

12.2.1 Under Forward Bias Condition

When positive terminal of the battery is connected to the P-type and negative terminal to the N-type of the PN junction diode, the bias applied is known as forward bias.

Operation As shown in Fig. 12.9, the applied potential with external battery acts in opposition to the internal potential barrier. Under the forward bias condition, the applied positive potential repels the holes in P-type region so that the holes move

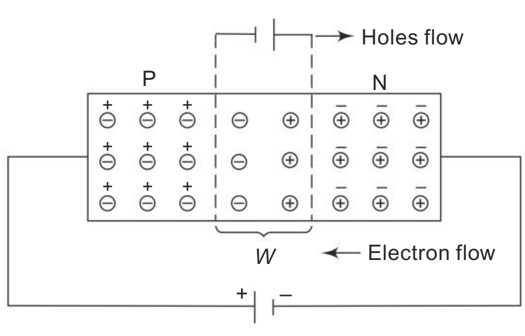


Fig. 12.9 PN Junction Under Forward Bias

towards the junction and the applied negative potential repels the electrons in the N-type region and the electrons move towards the junction. Eventually when the applied potential is more than the internal barrier potential, the depletion region and internal potential barrier disappear.

V–I Characteristics of a Diode Under Forward Bias Under forward bias condition, the *V–I* characteristics of a PN junction diode are shown in Fig. 12.10. As the forward voltage (V_F) is increased, for $V_F < V_0$, the forward current I_F is almost zero (region *OA*), because the potential barrier prevents the holes from P-region and electrons from N-region to flow across the depletion region in the opposite direction.

For $V_F > V_0$, the potential barrier at the junction completely disappears and hence, the holes cross the junction from P-type to N-type and the electrons cross the junction in the opposite direction, resulting in relatively large current flow in the external circuit.

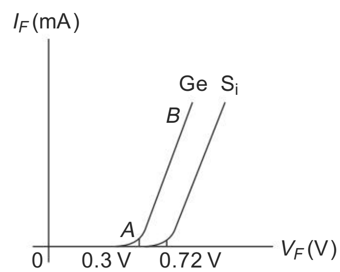


Fig. 12.10 *V-I* Characteristics of a Diode Under Forward Bias Condition

A feature worth noting in the forward characteristics shown in Fig. 12.10 is the cut in or threshold voltage (V_r) below which the current is very small. It is 0.3 V and 0.72 V for germanium and silicon respectively. At the cut in voltage, the potential barrier is overcome and the current through the junction starts to increase rapidly.

12.2.2 Under Reverse Bias Condition

When the negative terminal of the battery is connected to the P-type and positive terminal of the battery is connected to the N-type of the PN junction, the bias applied is known as reverse bias.

Operation Under applied reverse bias as shown in Fig. 12.11, holes which form the majority carriers of the P-side move towards the negative terminal of the battery and electrons which form the majority carrier of the N-side are attracted towards the positive terminal of the battery. Hence the width of the depletion region which is depleted of mobile charge carriers increases. Thus the electric field produced by applied reverse bias, is in the same direction as the electric field of the potential barrier. Hence, the resultant potential barrier is increased, which prevents the flow of majority carriers in both directions. Therefore, theoretically no current should flow in the external circuit. But in practice, a very small current of the order of a few microamperes flows under reverse bias as shown in Fig. 12.12. Electrons forming covalent

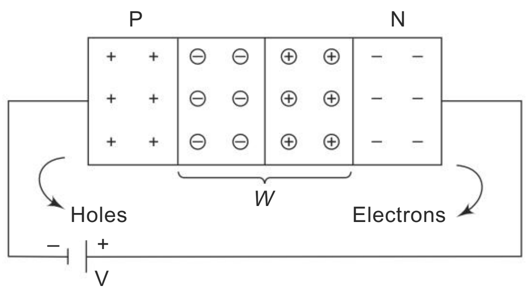


Fig. 12.11 PN Junction Under Reverse Bias

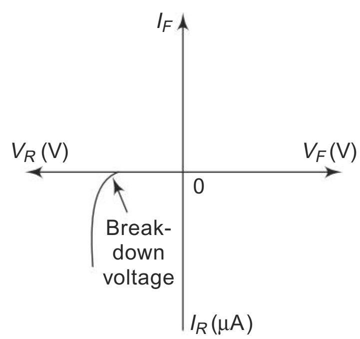


Fig. 12.12 *V–I* Characteristics Under Reverse Bias

bonds of the semiconductor atoms in the P- and N-type regions may absorb sufficient energy from heat and light to cause breaking of some covalent bonds. Hence electron–hole pairs are continually produced in both the regions. Under the reverse bias condition, the thermally generated holes in the P-region are attracted towards the negative terminal of the battery and the electrons in the N-region are attracted towards the positive terminal of the battery. Consequently, the minority carriers, electrons in the P-region and holes in the N-region, wander over to the junction and flow towards their majority carrier side giving rise to a small reverse current. This current is known as *reverse saturation current*. The magnitude of reverse saturation current mainly depends upon junction temperature because the major source of minority carriers is thermally broken covalent bonds.

For large applied reverse bias, the free electrons from the N-type moving towards the positive terminal of the battery acquire sufficient energy to move with high velocity to dislodge valence electrons from semiconductor atoms in the crystal. These newly liberated electrons, in turn, acquire sufficient energy to dislodge other parent electrons. Thus, a large number of free electrons are formed which is commonly called as an avalanche of free electrons. This leads to the breakdown of the junction leading to very large reverse current. The reverse voltage at which the junction breakdown occurs is known as *breakdown voltage*.

The PN junction diode will perform satisfactorily only if it is operated within certain limiting values. They are:

(i) Maximum forward current It is the highest instantaneous current under forward bias condition that can flow through the junction.

(ii) Peak inverse voltage It is the maximum reverse voltage that can be applied to the PN junction. If the voltage across the junction exceeds PIV, under reverse bias condition, the junction gets damaged.

(iii) Maximum power rating It is the maximum power that can be dissipated at the junction without damaging the junction. Power dissipation is the product of voltage across the junction and current through the junction.

12.3 PN DIODE APPLICATIONS

An ideal PN junction diode is a two terminal polarity sensitive device that has zero resistance (diode conducts) when it is forward biased and infinite resistance (diode does not conduct) when reverse biased. Due to this characteristic the diode finds a number of applications as follows.

(i) rectifiers in dc power supplies
(ii) switch in digital logic circuits used in computers
(iii) clamping network used as dc restorer in TV receivers and voltage multipliers
(iv) clipping circuits used as wave shaping circuits used in computers, radars, radio and TV receivers
(v) demodulation (detector) circuits.

The same PN junction with different doping concentration finds special applications as follows:

(i) detectors (APD, PIN photo diode) in optical communication circuits
(ii) Zener diodes in voltage regulators
(iii) varactor diodes in tuning sections of radio and TV receivers
(iv) light emitting diodes in digital displays
(v) LASER diodes in optical communications
(vi) Tunnel diodes as a relaxation oscillator at microwave frequencies.

12.4 ZENER DIODE

When the reverse voltage reaches breakdown voltage in normal PN junction diode, the current through the junction and the power dissipated at the junction will be high. Such an operation is destructive and the diode gets damaged. Whereas diodes can be designed with adequate power dissipation capabilities to operate in the breakdown region. One such a diode is known as Zener diode. Zener diode is heavily doped than the ordinary diode.

From the *V–I* characteristics of the Zener diode, shown in Fig. 12.13, it is found that the operation of Zener diode is same as that of ordinary PN diode under forward-biased condition. Whereas under reverse-baised condition, breakdown of the junction occurs. The breakdown voltage depends upon the amount of doping. If the diode is heavily doped, depletion layer will be thin and, consequently, breakdown occurs at lower reverse voltage and further, the breakdown voltage is sharp. Whereas a lightly doped diode has a higher breakdown voltage. Thus breakdown voltage can be selected with the amount of doping.

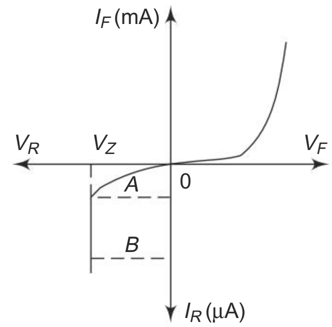


Fig. 12.13 *V–I* Characteristics of a Zener Diode

The sharp increasing current under breakdown conditions are due to the following two mechanisms.

(1) Avalanche breakdown
(2) Zener breakdown.

12.4.1 Avalanche Breakdown

As the applied reverse bias increases, the field across the junction increases correspondingly. Thermally generated carriers while traversing the junction acquire a large amount of kinetic energy from this field. As a result the velocity of these carriers increases. These electrons disrupt covalent bonds by colliding with immobile ions and create new electron-hole pairs. These new carriers again acquire sufficient energy from the field and collide with other immobile ions thereby generating further electron–hole pairs. This process is cumulative in nature and results in generation of avalanche of charge carriers within a short time. This mechanism of carrier generation is known as Avalanche multiplication. This process results in flow of large amount of current at the same value of reverse bias.

12.4.2 Zener Breakdown

When the P and N regions are heavily doped, direct rupture of covalent bonds takes place because of the strong electric fields, at the junction of PN diode. The new electron-hole pairs so created increase the reverse current in a reverse biased PN diode. The increase in current takes place at a constant value of reverse bias typically below 6 V for heavily doped diodes. As a result of heavy doping of P and N regions, the depletion region width becomes very small and for an applied voltage of 6 V or less, the field across the depletion region becomes very high, of the order of 10^7 V/m, making conditions suitable for Zener breakdown. For lightly doped diodes, Zener breakdown voltage becomes high and breakdown is then predominantly by Avalanche multiplication. Though Zener breakdown occurs for lower breakdown voltage and Avalanche breakdown occurs for higher breakdown voltage, such diodes are normally called Zener diodes.

12.4.3 Applications

From the Zener characteristics shown in Fig. 12.13, under the reverse bias condition, the voltage across the diode remains almost constant although the current through the diode increases as shown in region *AB*. Thus, the voltage across the Zener diode serves as a reference voltage. Hence, the diode can be used as a voltage regulator.

In Fig. 12.14 it is required to provide constant voltage across load resistance R_L, whereas the input voltage may be varying over a range. As shown, Zener diode is reverse biased and as long as the input voltage does not fall below V_Z (Zener breakdown voltage), the voltage across the diode will be constant and hence the load voltage will also be constant.

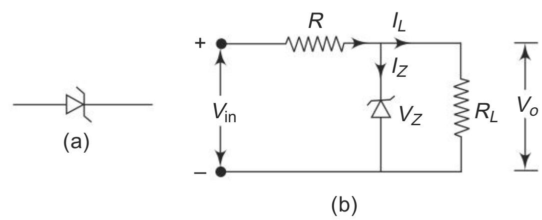


Fig. 12.14 Zener Diode. (a) Circuit Symbol and (b) as a Voltage Regulator

12.5 VARACTOR DIODE

The varactor, also called a varicap, tuning or voltage variable capacitor diode, is also a junction diode with a small impurity dose at its junction, which has the useful property that its junction or transition capacitance is easily varied electronically.

When any diode is reverse biased, a depletion region is formed, as seen in Fig. 12.15(a). The larger the reverse bias applied across the diode, the width of the depletion layer *W* becomes wider. Conversely, by decreasing the reverse bias voltage, the depletion region width *W* becomes narrower. This depletion region is devoid of majority carriers and acts like an insulator preventing conduction between the N and P regions of the diode, just like a dielectric, which separates the two plates of a capacitor. The varactor diode with its symbol is shown in Fig. 12.15 (b).

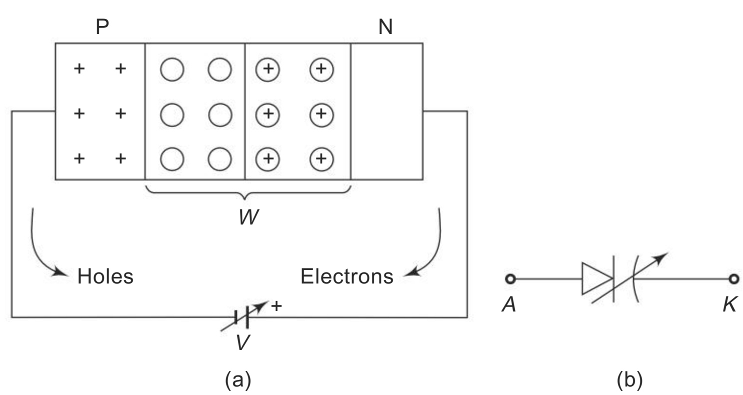


Fig. 12.15 (a) Depletion Region in a Reverse Biased (b) Circuit Symbol of PN Junction Varactor Diode

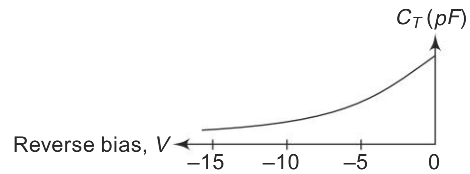


Fig. 12.16 Characteristics of Varactor Diode

As the capacitance is inversely proportional to the distance between the plates ($C_T \propto l/W$), the transition capacitance C_T varies inversely with the reverse voltage as shown in Fig. 12.16. Consequently, an increase in reverse bias voltage will result in an increase in the depletion region width and a subsequent decrease in transition capacitance C_T. At zero volt, the varactor depletion region W is small and the capacitance is large at approximately 600 pF. When the reverse bias voltage across the varactor is 15 V, the capacitance is 30 pF.

The varactor diodes are used in FM radio and TV receivers, AFC circuits, self adjusting bridge circuits and adjustable bandpass filters. With improvement in the type of materials used and construction, varactor diodes find application in tuning of LC resonant circuit in microwave frequency multipliers and in very low noise microwave parametric amplifiers.

12.6 TUNNEL DIODE

The Tunnel or Esaki diode is a thin-junction diode which exhibits negative resistance under low forward bias conditions.

An ordinary PN junction diode has an impurity concentration of about 1 part in 10^8. With this amount of doping the width of the depletion layer is of the order of 5 microns. This potential barrier restrains the flow of carriers from the majority carrier side to the minority carrier side. If the concentration of impurity atoms is greatly increased to the level of 1 part in 10^3, the device characteristics are completely changed. The width of the junction barrier varies inversely as the square root of the impurity concentration and therefore, is reduced from 5 microns to less than 100 Å (10^{-8} m). This thickness is only about 1/50th of the wavelength of visible light. For such thin potential energy barriers, the electrons will penetrate through the junction rather than surmounting them. This quantum mechanical behaviour is referred to as tunneling and hence, these high-impurity-density PN junction devices are called tunnel diodes.

The V–I characteristic for a typical germanium tunnel diode is shown in Fig. 12.17. It is seen that at first forward current rises sharply as applied voltage is increased, where it would have risen slowly for an ordinary PN junction diode (which is shown as dashed line for comparison). Also, reverse current is much larger for comparable back bias than in other diodes due to the thinness of the junction. The interesting portion of the characteristic starts at the point A on the curve, i.e. the peak voltage. As the forward bias is increased beyond this point, the forward current drops and continues to drop until point B is reached. This is the valley voltage. At B,

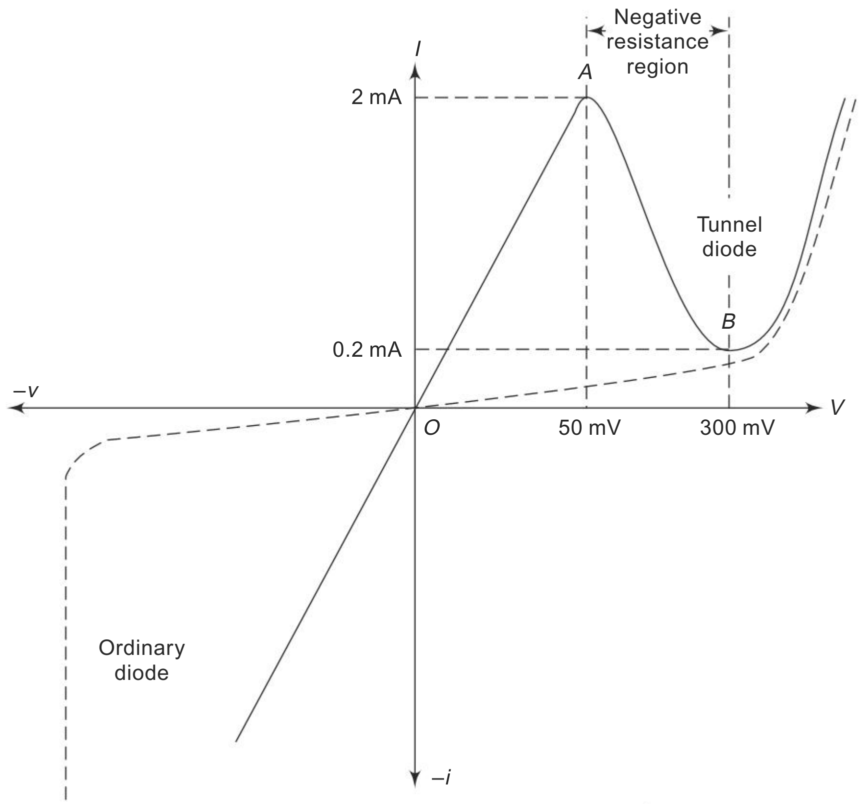


Fig. 12.17 *V–I* Characteristic of Tunnel Diode

the current starts to increase once again and does so very rapidly as bias is increased further. Beyond this point, characteristic resembles that of an ordinary diode. Apart from the peak voltage and valley voltage, the other two parameters normally used to specify the diode behaviour are the peak current and the peak-to-valley current ratio, which are 2 mA and 10 respectively, as shown.

The *V–I* characteristic of the tunnel diode illustrates that it exhibits dynamic resistance between *A* and *B*. Figure 12.18 shows energy level diagrams of the tunnel diode for three interesting bias levels. The shaded areas show the energy states occupied by electrons in the valence band, whereas the cross hatched regions represent energy states in the conduction band occupied by the electrons. The levels to which the energy states are occupied by electrons on either side of the junctions are shown by dotted lines. When the bias is zero, these lines are at the same height. Unless energy is imparted to the electrons from some external source, the energy possessed by the electrons on the N-side of the junction is insufficient to permit them to climb over the junction barrier to reach the P-side. However, quantum mechanics show that there is a finite probability for the electrons to tunnel through the junction to reach the other side, provided there are allowed empty energy states in the P-side of the junction at the same energy level. Hence, the forward current is zero.

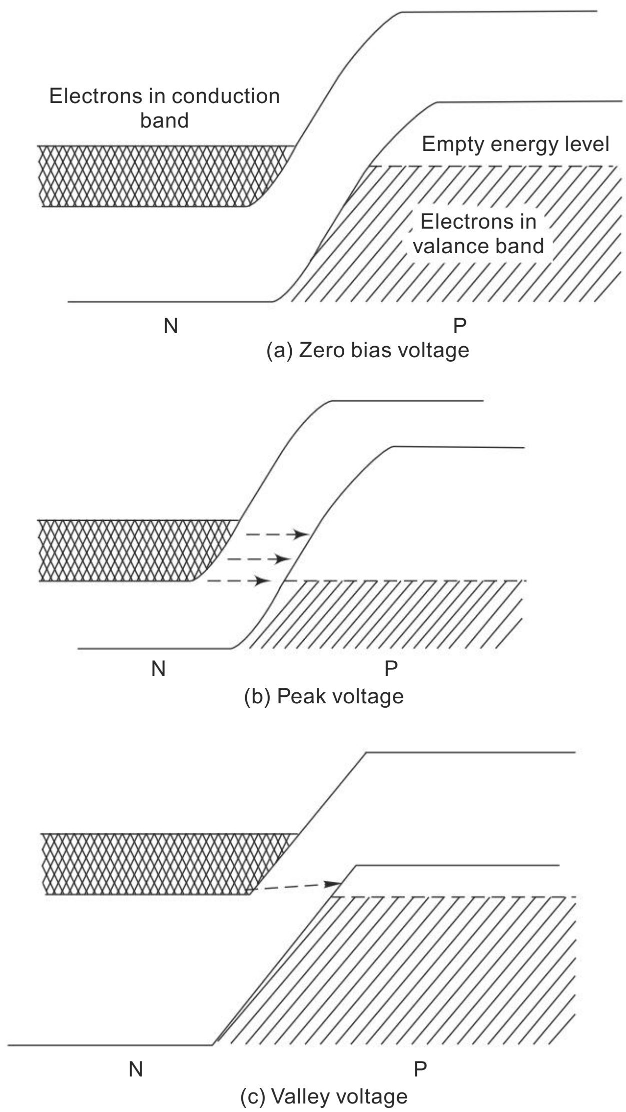


Fig. 12.18 Energy Level Diagrams of Tunnel Diode

When a small forward bias is applied to the junction, the energy level of the P-side is lower as compared with the N-side. As shown in Fig. 12.18(b), electrons in the conduction band of the N-side see empty energy level on the P-side. Hence, tunneling from N-side to P-side takes place. Tunneling in other directions is not possible because the valence band electrons on the P-side are now opposite to the forbidden energy gap on the N-side. The energy band diagram shown in Fig. 12.18(b), is for the peak of the diode characteristic.

When the forward bias is raised beyond this point, tunneling will decrease as shown in Fig. 12.18(c). The energy of the P-side is now depressed further, with the result that fewer conduction band electrons on the N-side are opposite to the unoccupied P-side energy levels. As the bias is raised, forward current drops. This corresponds to the negative resistance region of the diode characteristic. As forward bias is raised still further, tunneling stops altogether and it behaves as a normal PN junction diode.

Equivalent Circuit

The equivalent circuit of the tunnel diode when biased in the negative resistance region is as shown in Fig. 12.19(a) and Fig. 12.19(b) shown the symbol of tunnel diode. In the circuit, R_s is the series resistance and L_s is the series inductance which may be ignored except at highest frequencies. The resulting diode equivalent circuit is thus reduced to parallel combination of the junction capacitance C_j and the negative resistance $-R_n$. Typical values of the circuit components are $R_s = 6\ \Omega$, $L_s = 0.1$ nH, $C_j = 0.6$ pF and $R_n = 75\Omega$.

Applications

1. Tunnel diode is used as an ultra-high speed switch with switching speed of the order of ns or ps
2. As logic memory storage device
3. As microwave oscillator
4. In relaxation oscillator circuit
5. As an amplifier.

Advantages

1. Low noise
2. Ease of operation
3. High speed
4. Low power

Disadvantages

1. Voltage range over which it can be operated is 1 V or less
2. Being a two terminal device, there is no isolation between the input and output circuit.

12.7 RECTIFIERS

Rectifier is defined as an electronic device used for converting ac voltage into unidirectional voltage. A rectifier utilizes unidirectional conduction device like a vacuum diode or PN junction diode. Rectifiers are classified depending upon the period of conduction as Half-wave rectifier and Full-wave rectifier.

12.7.1 Half-wave Rectifier

It converts an ac voltage into a pulsating dc voltage using only one half of the applied ac voltage. The rectifying diode conducts during one half of the ac cycle only. Figure 12.19 shows the basic circuit and waveforms of a half wave rectifier.

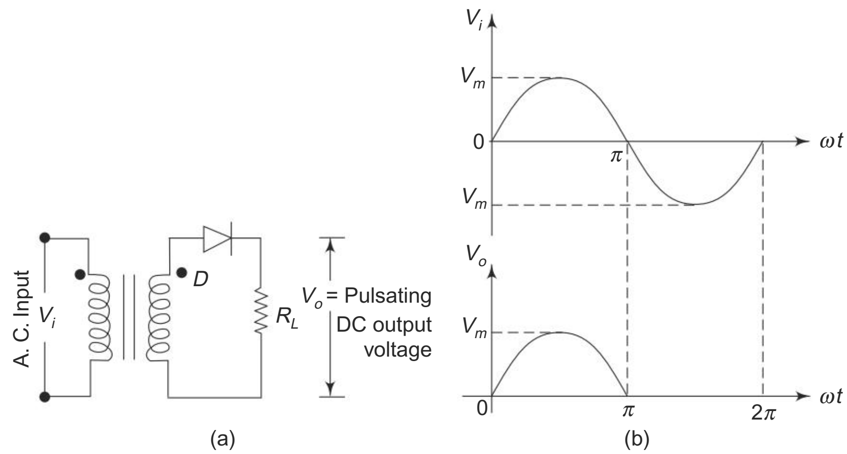


Fig. 12.19 (a) Basic circuit of a Half-wave Rectifier and (b) Input and Output Waveforms of Half-wave Rectifier

Let V_i be the voltage to the primary of the transformer and given by the equation

$$V_i = V_m \sin \omega t;\ V_m >> V_r$$

where V_r is the cut-in voltage of the diode. During the positive half-cycle of the input signal, the anode of the diode becomes more positive with respect to the cathode and hence, diode D conducts. For an ideal diode, the forward voltage drop is zero. So the whole input voltage will appear across the load resistance, R_L.

During negative half-cycle of the input signal, the anode of the diode becomes negative with respect to the cathode and hence, diode D does not conduct. For an ideal diode, the impedance offered by the diode is infinity. So the whole input voltage appears across diode D. Hence, the voltage drop across R_L is zero.

***Ripple factor* (Γ)** The ratio of rms value of ac component to the dc component in the output is known as ripple factor (Γ)

$$\Gamma = \frac{\text{rms value of ac component}}{\text{dc value of component}} = \frac{V_{r,\,\text{rms}}}{V_{\text{dc}}}$$

where $$V_{r,\,\text{rms}} = \sqrt{V_{\text{rms}}^2 - V_{\text{dc}}^2}$$

$$\Gamma = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1}$$

V_{av} is the average or the dc content of the voltage across the load and is given by

$$V_{\text{av}} = V_{\text{dc}} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0.d(\omega t)\right]$$

$$= \frac{V_m}{2\pi}[-\cos \omega t]_0^{\pi} = \frac{V_m}{\pi}$$

Therefore, $$I_{\text{dc}} = \frac{V_{\text{dc}}}{R_L} = \frac{V_m}{\pi R_L} = \frac{I_m}{\pi}$$

If the values of diode forward resistance (r_f) and the transformer secondary winding resistance (r_s) are also taken into account, then

$$V_{dc} = \frac{V_m}{\pi} - I_{dc}(r_s + r_f)$$

$$I_{dc} = \frac{V_{dc}}{(r_s + r_f) + R_L} = \frac{V_m}{\pi(r_s + r_f + R_L)}$$

RMS voltage at the load resistance can be calculated as

$$V_{rms} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{1}{4\pi}\int_0^{\pi}(1 - \cos 2\,\omega t)\, d(\omega t)\right]^{\frac{1}{2}} = \frac{V_m}{2}$$

Therefore $$\Gamma = \sqrt{\left[\frac{V_m/2}{V_m/\pi}\right]^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

From this expression it is clear that the amount of ac present in the output is 121% of the dc voltage. So the half-wave rectifier is not practically useful in converting ac into dc.

Efficiency (η) The ratio of dc output power to ac input power is known as rectifier efficiency (η).

$$\eta = \frac{\text{dc output power}}{\text{ac input power}} = \frac{P_{dc}}{P_{ac}}$$

$$= \frac{\dfrac{(V_{dc})^2}{R_L}}{\dfrac{(V_{rms})^2}{R_L}} = \frac{\left(\dfrac{V_m}{\pi}\right)^2}{\left(\dfrac{V_m}{2}\right)^2} = \frac{4}{\pi^2} = 0.406 = 40.6\%$$

The maximum efficiency of a half-wave rectifier is 40.6%.

Peak inverse voltage (PIV) It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half cycle. For half-wave rectifier, PIV is V_m.

Transformer utilisation factor (TUF) In the design of any power supply, the rating of the transformer should be determined. This can be done with a knowledge of the dc power delivered to the load and the type of rectifying circuit used.

$$\text{TUF} = \frac{\text{dc power delivered to the load}}{\text{ac rating of the transformer secondary}}$$

$$= \frac{P_{dc}}{P_{ac}\text{ rated}}$$

In the half-wave rectifying circuit, the rated voltage of the transformer secondary is $V_m/\sqrt{2}$, but the actual rms current flowing through the winding is ony $\frac{I_m}{2}$, not $I_m/\sqrt{2}$.

$$\text{TUF} = \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{V_m}{\sqrt{2}} \times \frac{I_m}{2}} = \frac{\frac{V_m^2}{\pi^2}\frac{1}{R_L}}{\frac{V_m}{\sqrt{2}}\frac{V_m}{2R_L}} = \frac{2\sqrt{2}}{\pi^2} = 0.287$$

The TUF for a halfwave rectifier is 0.287.

Form factor

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}}$$

$$= \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} = 1.57$$

Peak factor

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}}$$

$$= \frac{V_m}{V_m/2} = 2$$

Example 12.1 A half-wave rectifier, having a resistive load of 1000 Ω rectifies an alternating voltage of 325 V peak value and the diode has a forward resistance of 100 Ω. Calculate (a) peak, average and rms value of current (b) dc power output (c) ac input power, and (d) efficiency of the rectifier.

(a) Peak value of current, $I_m = \frac{V_m}{r_f + R_L} = \frac{325}{100 + 1000} = 295.45 \text{ mA}$

Average current, $I_{dc} = \frac{I_m}{\pi} = \frac{295.45}{\pi} \text{ mA} = 94.046 \text{ mA}$

RMS value of current, $I_{rms} = \frac{I_m}{2} = \frac{295.45}{2} = 147.725 \text{ mA}$

(b) DC power output, $P_{dc} = I_{dc}^2 \times R_L$

$$= (94.046 \times 10^{-3})^2 \times 1000 = 8.845 \text{ W}$$

(c) AC input power, $P_{ac} = (I_{rms})^2 \times (r_f + R_L)$

$$= (147.725 \times 10^{-3})^2 (1100) = 24 \text{ W}$$

(d) Efficiency of rectification, $\eta = \frac{P_{dc}}{P_{ac}} = \frac{8.845}{24} = 36.85\%$

Example 12.2 A half-wave rectifier is used to supply 24 V dc to a resistive load of 500 Ω and the diode has a forward resistance of 50 Ω. Calculate the maximum value of the ac voltage required at the input.

Average value of load current, $I_{dc} = \frac{V_{dc}}{R_L} = \frac{24}{500} = 48 \text{ mA}$

Maximum vlaue of load current, $I_m = \pi \times I_{dc} = \pi \times 48 \text{ mA} = 150.8 \text{ mA}$

Therefore, maximum ac voltage required at the input,

$$V_m = I_m \times (r_f + R_L)$$

$$= 150.8 \times 10^{-3} \times 550 = 82.94 \text{ V}$$

Example 12.3 An ac supply of 230 V is applied to a half-wave rectifier circuit through transformer of turns ratio 5 : 1. Assume the diode is an ideal one. The load resistance is 300 Ω. Find (a) dc output voltage (b) PIV (c) maximum, and (d) average values of power delivered to the load.

(a) The transformer secondary voltage $= \dfrac{230}{5} = 46$ V

Maximum value of secondary voltage, $V_m = \sqrt{2} \times 46 = 65$ V

Therefore, dc output voltage, $V_{dc} = \dfrac{V_m}{\pi} = \dfrac{65}{\pi} = 20.7\text{V}$

(b) PIV of a diode $= V_m = 65$ V

(c) Maximum value of load current, $I_m = \dfrac{V_m}{R_L} = \dfrac{65}{300} = 0.217$ A

Therefore, maximum value of power delivered to the load,

$$P_m = I_m^2 \times R_L = (0.217)^2 \times 300 = 14.1 \text{ W}$$

(d) The average value of load current, $I_{dc} = \dfrac{V_{dc}}{R_L} = \dfrac{20.7}{300} = 0.069$ A

Therefore, average value of power delivered to the load,

$$P_{dc} = I_{dc}^2 \times R_L = (0.069)^2 \times 300 = 1.43 \text{ W}$$

12.7.2 Full-wave Rectifier

It converts an ac voltage into a pulsating dc voltage using both half cycles of the applied ac voltage. It uses two diodes of which one conducts during one half-cycle while the other diode conducts during the other half-cycle of the applied ac voltage. Figure 12.20 shows the basic circuit and waveforms of full-wave rectifier.

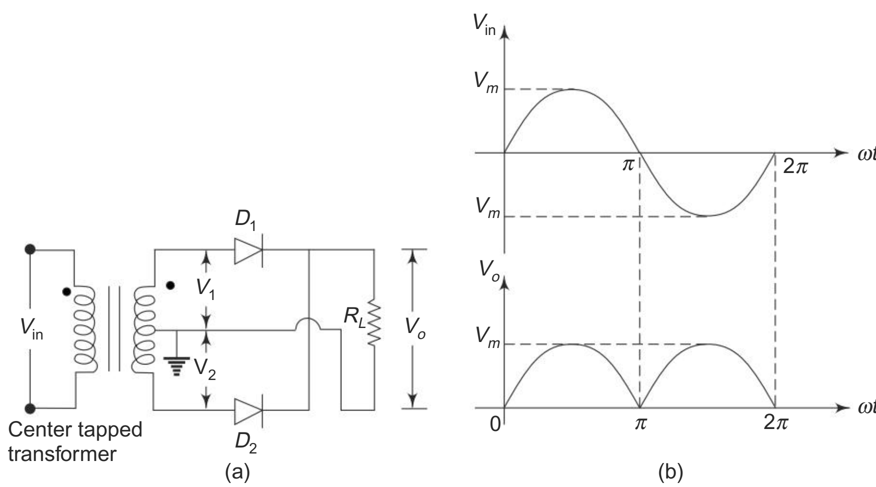


Fig. 12.20 Full Wave Rectifier

During positive half of the input signal, anode of diode D_1 becomes positive and at the same time the anode of diode D_2 becomes negative. Hence D_1 conducts and D_2 does not conduct. The load current flows through D_1 and the voltage drop across R_L will be equal to the input voltage.

During the negative half-cycle of the input, the anode of D_1 becomes negative and the anode of D_2 becomes positive. Hence, D_1 does not conduct and D_2 conducts. The load current flows through D_2 and the voltage drop across R_L will be equal to the input voltage.

***Ripple factor* (Γ)** $\quad \Gamma = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}$

The average voltage or dc voltage available across the load resistance is

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$

$$= \frac{V_m}{\pi}[-\cos \omega t]_0^{\pi} = \frac{2V_m}{\pi}$$

$$I_{dc} = \frac{V_{dc}}{R_L} = \frac{2V_m}{\pi R_L} = \frac{2I_m}{\pi} \text{ and } I_{rms} = \frac{I_m}{\sqrt{2}}$$

If the diode forward resistance (r_f) and the transformer secondary winding resistance (r_s) are included in the analysis, then

$$V_{dc} = \frac{2V_m}{\pi} - I_{dc}(r_s + r_f)$$

$$I_{dc} = \frac{V_{dc}}{(r_s + r_f) + R_L} = \frac{2V_m}{\pi(r_s + r_f + R_L)}$$

RMS value of the voltage at the load resistance is

$$V_{rms} = \sqrt{\left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)\right]} = \frac{V_m}{\sqrt{2}}$$

Therefore, $\quad \Gamma = \sqrt{\left(\frac{V_m/\sqrt{2}}{2V_m/\pi}\right)^2 - 1} = \sqrt{\frac{\pi^2}{8} - 1} = 0.482$

***Efficiency* (η)** The ratio of dc output power to ac input power is known as rectifier efficiency (η).

$$\eta = \frac{\text{dc output power}}{\text{ac input power}} = \frac{P_{dc}}{P_{ac}}$$

$$= \frac{(V_{dc})^2/R_L}{(V_{rms})^2/RL} = \frac{\left[\frac{2V_m}{\pi}\right]^2}{\left[\frac{V_m}{\sqrt{2}}\right]^2} = \frac{8}{\pi^2} = 0.812 = 81.2\%$$

The maximum efficiency of a full-wave rectifier is 81.2%.

Transformer utilisation factor (TUF) The average TUF in a full-wave rectifying circuit is determined by considering the primary and secondary winding separately and it gives a value of 0.693.

Form factor

$$\text{Form factor} = \frac{\text{rms value of the output voltage}}{\text{average value of the output voltage}}$$

$$= \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

Peak factor

$$\text{Peak factor} = \frac{\text{peak value of the output voltage}}{\text{rms value of the output voltage}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

Peak inverse voltage for full-wave rectifier is $2V_m$ because the entire secondary voltage appears across the non-conducting diode.

Example 12.4 A 230 V, 60 Hz voltage is applied to the primary of a 5 : 1 step-down, center-tap transformer used in a full wave rectifier having a load of 900 Ω. If the diode resistance and secondary coil resistance together has a resistance of 100 Ω, determine (a) dc voltage across the load, (b) dc current flowing through the load, (c) dc power delivered to the load, (d) PIV across each diode, and (e) ripple voltage and its frequency.

The voltage across the two ends of secondary $= \dfrac{230}{5} = 46$ V

Voltage from center tapping to one end, $V_{\text{rms}} = \dfrac{46}{2} = 23$ V

(a) dc voltage across the load, $V_{\text{dc}} = \dfrac{2V_m}{\pi} = \dfrac{2 \times 23 \times \sqrt{2}}{\pi} = 20.7$ V

(b) dc current flowing through the load, $I_{\text{dc}} = \dfrac{V_{\text{dc}}}{(r_s + r_f + R_L)} = \dfrac{20.7}{1000} = 20.7$ mA

(c) dc power delivered to the load,

$$P_{\text{dc}} = (I_{\text{dc}})^2 \times R_L = (20.7 \times 10^{-3})^2 \times 900 = 0.386 \text{ W}$$

(d) PIV across each diode = 2 $V_m = 2 \times 23 \times \sqrt{2} = 65$ V

(e) Ripple voltage, $V_{r,\,\text{rms}} = \sqrt{(V_{\text{rms}})^2 - (V_{\text{dc}})^2}$

$$= \sqrt{(23)^2 - (20.7)^2} = 10.05 \text{ V}$$

Frequency of ripple voltage = 2 × 60 = 120 Hz

Example 12.5 A full-wave rectifier has a center-tap transformer of 100–0–100 V and each one of the diodes is rated at $I_{\max} = 400$ mA and $I_{\text{av}} = 150$ mA. Neglecting the voltage drop across the diodes, determine (a) the value of load resistor that gives the largest dc power output, (b) dc load voltage and current, and (c) PIV of each diode.

(a) We know that the maximum value of current flowing through the diode for normal operation should not exceed 80% of its rated current.

Therefore, $I_{\max} = 0.8 \times 400 = 320$ mA

The maximum value of the secondary voltage

$$V_m = \sqrt{2} \times 100 = 141.4 \text{ V}$$

Therefore, the value of load resistor that gives the largest dc power output

$$R_L = \frac{V_m}{I_{\max}} = \frac{141.4}{320 \times 10^{-3}} = 442 \ \Omega$$

(b) DC (load) voltage, $V_{dc} = \dfrac{2V_m}{\pi} = \dfrac{2 \times 141.4}{\pi} = 90 \text{ V}$

DC load current, $I_{dc} = \dfrac{V_{dc}}{R_L} = \dfrac{90}{442} = 0.204 \text{ A}$

(c) PIV of each diode $= 2\ V_m = 2 \times 141.4 = 282.8 \text{ V}$

Example 12.6 A full-wave rectifier delivers 50 W to a load of 200 Ω. If the ripple factor is 1%, calculate the ac ripple voltage across the load.

DC power delivered to the load

$$P_{dc} = \frac{V_{dc}^2}{R_L}$$

Therefore, $V_{dc} = \sqrt{P_{dc} \times R_L} = \sqrt{50 \times 200} = 100 \text{ V}$

The ripple facor, $\Gamma = \dfrac{V_{ac}}{V_{dc}}$

i.e $0.01 = \dfrac{V_{ac}}{100}$

Therefore, the ac ripple voltage across the load, $V_{ac} = 1$ V

12.7.3 Bridge Rectifier

The need for a center tapped transformer in a full-wave rectifier is eliminated in the bridge rectifier. As shown in Fig. 12.21 the bridge rectifier has four diodes connected to form a bridge. The ac input voltage is applied to diagonally opposite ends of the bridge. The load resistance is connected between the other two ends of the bridge.

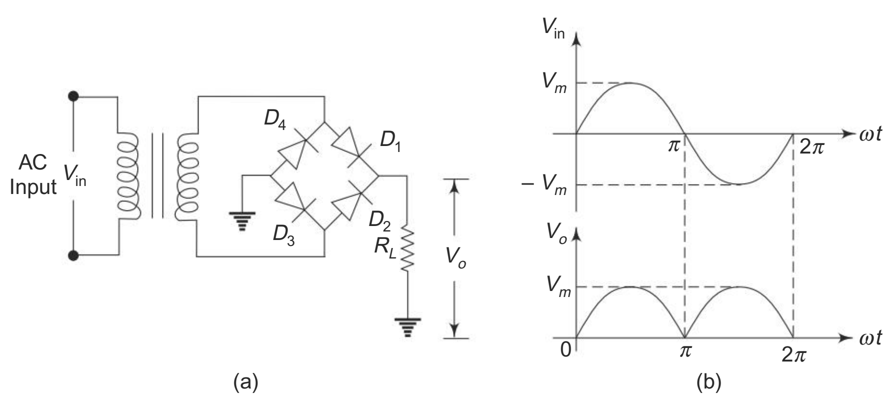


Fig. 12.21 Bridge Rectifier

For the positive half-cycle of the input ac voltage, diodes D_1 and D_3 conduct, whereas diodes D_2 and D_4 do not conduct. The conducting diodes will be in series through the load resistance R_L. So the load current flows through R_L.

During the negative half-cycle of the input ac voltage, diodes D_2 and D_4 conduct, whereas diodes D_1 and D_3 do not conduct. The conducting diode D_2 and D_4 will be in series through the load R_L and the current flows through R_L in the same direction as in the previous half-cycle. Thus a bidirectional wave is converted into a unidirectional one.

The average values of output voltage and load current for bridge rectifier are the same as for a center-tapped full wave rectifier. Hence

$$V_{dc} = \frac{2V_m}{\pi} \quad \text{and} \quad I_{dc} = \frac{V_{dc}}{R_L} = \frac{2V_m}{\pi R_L} = \frac{2I_m}{\pi}$$

If the values of the transformer secondary winding resistance (r_s) and diode forward resistance (r_f) are considered in the analysis, then

$$V_{dc} = \frac{2V_m}{\pi} - I_{dc}(r_s + r_f)$$

$$I_{dc} = \frac{2V_m}{\pi R_L} = \frac{2V_m}{\pi(r_s + r_f + R_L)}$$

The maximum efficiency of a bridge rectifier is 81.2% and the ripple factor is 0.48. The PIV is V_m.

Advantages of the bridge rectifier In the bridge rectifier the ripple factor and efficiency of the rectification are the same as for the full-wave rectifier. The PIV across either of the non-conducting diodes is equal to the peak value of the transformer secondary voltage, V_m. The bulky center tapped transformer is not required. Transformer utilisation factor is considerably high. Since the current flowing in the transformer secondary is purely alternating, the TUF increases to 0.812, which is the main reason for the popularity of a bridge rectifier. The bridge rectifiers are used in applications allowing floating output terminals, i.e. no output terminal is grounded.

The bridge rectifier has only one disadvantage that it requires four diodes as compared to two diodes for center-tapped full wave rectifier. But the diodes are readily available at cheaper rate in the market. Apart from this, the PIV rating required for the diodes in a bridge rectifier is only half of that for a center tapped full-wave rectifier. This is a great advantage, which offsets the disadvantage of using extra two diodes in a bridge rectifier.

Example 12.7 A 230 V, 50 Hz voltage is applied to the primary of a 4 : 1 step-down transformer used in a bridge rectifier having a load resistance of 600 Ω. Assuming the diodes to be ideal, determine (a) dc output voltage, (b) dc power delivered to the load, (c) PIV, and (d) output frequency.

(a) The rms value of the transformer secondary voltage,

$$V_{rms} = \frac{230}{4} = 57.5 \text{ V}$$

The maximum value of the secondary voltage

$$V_m = \sqrt{2} \times 57.5 = 81.3 \text{ V}$$

Therefore, dc output voltage, $V_{dc} = \dfrac{2V_m}{\pi} = \dfrac{2 \times 81.3}{\pi} = 52 \text{ V}$

(b) DC power delivered to the load,

$$P_{dc} = \frac{V_{dc}^2}{R_L} = \frac{52^2}{1000} = 2.704 \text{ W}$$

(c) PIV across each diode = V_m = 81.3 V

(d) Output frequency = 2 × 50= 100 Hz

Comparison of Rectifiers

The comparison of rectifiers is given in Table 12.1.

Table 12.1 A Comparison of Rectifiers

Particulars	*Type of rectifier*		
	Half-wave	*Full-wave*	*Bridge*
No. of diodes	1	2	4
Maximum efficiency	40.6%	81.2%	81.2%
V_{dc} (no load)	V_m/π	$2V_m/\pi$	$2V_m/\pi$
Average current/diode	I_{dc}	$I_{dc}/2$	$I_{dc}/2$
Ripple factor	1.21	0.48	0.48
Peak inverse voltage	V_m	$2V_m$	V_m
Output frequency	f	2f	2f
Transformer utilisation factor	0.287	0.693	0.812
Form factor	1.57	1.11	1.11
Peakfctor	2	$\sqrt{2}$	$\sqrt{2}$

12.8 FILTERS

The output of a rectifier contains dc component as well as ac component. Filters are used to minimise the undesirable ac, i.e. ripple leaving only the dc component to appear at the output.

The ripple in the rectified wave being very high, the factor being 48% in the full-wave rectifier; majority of the applications which cannot tolerate this, will need an output which has been further processed.

Figure 12.22 shows the concept of a filter, where the full wave rectified output voltage is applied at its input. The output of a filter is not exactly a constant dc level. But it also contains a small amount of ac component. Some important filters are:

(i) Inductor filter
(ii) Capacitor filter
(iii) LC or L-section filter, and
(iv) CLC or π-type filter

12.8.1 Inductor Filter

Figure 12.23 shows the inductor filter. When the output of the rectifier passes through an inductor, it blocks the ac component and allows only the dc component to reach the load.

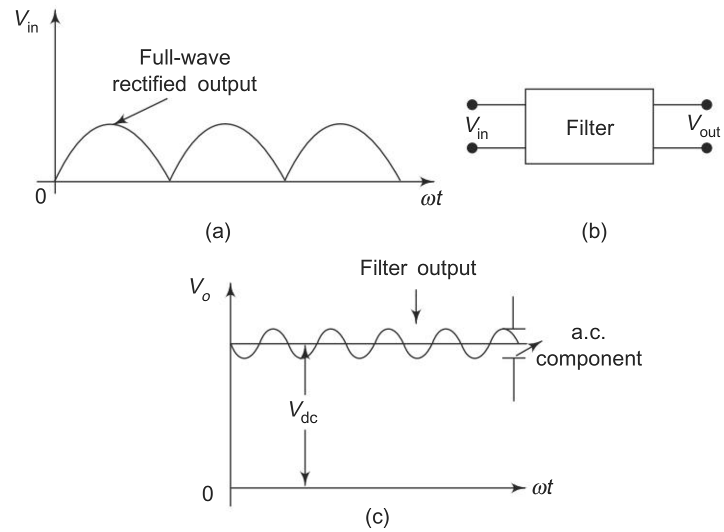


Fig. 12.22 Concept of a Filter

The ripple factor of the Inductor filter is given by

$$\Gamma = \frac{R_L}{3\sqrt{2}\,\omega L}$$

It shows that the ripple factor will decrease when L is increased and R_L is decreased. Clearly, the inductor filter is more effective only when the load current is high (small R_L). The larger value of the inductor can reduce the ripple and at the same time the output dc voltage will be lowered as the inductor has a higher dc resistance.

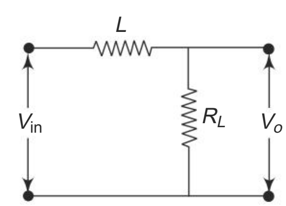


Fig. 12.23 Inductor Filter

The operation of the inductor filter depends on its well known fundamental property to oppose any change of current passing through it.

To analyse this filter for a full-wave, the Fourier series can be written as

$$V_o = \frac{2V_m}{\pi} - \frac{4V_m}{\pi}\left[\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \frac{1}{35}\cos 6\,\omega t + \ldots\right]$$

The dc component is $\frac{2V_m}{\pi}$.

Assuming the third and higher terms contribute little output, the output voltage is

$$V_o = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t$$

The diode, choke and transformer resistances can be neglected since they are very small as compared with R_L. Therefore, the dc component of current $I_m = \frac{V_m}{R_L}$. The impedance of series combination of L and R_L at 2ω is

$$Z = \sqrt{R_L^2 + (2\omega L)^2} = \sqrt{R_L^2 + 4\omega^2 L^2}$$

Therefore, for the ac component,

$$I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}}$$

Therefore, the resulting current i is given by,

$$i = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi} \cdot \frac{\cos(2\omega t - \varphi)}{\sqrt{R_L^2 + 4\omega^2 L^2}}$$

where $\varphi = \tan^{-1}\left(\frac{2\omega L}{R_L}\right)$

The ripple factor, which can be defined as the ratio of the rms value of the ripple to the dc value of the wave is

$$\Gamma = \frac{\dfrac{4V_m}{3\pi\sqrt{2}\sqrt{R_L^2 + 4\omega^2 L^2}}}{\dfrac{2V_m}{\pi R_L}} = \frac{2}{3\sqrt{2}} \times \frac{1}{\sqrt{1 + \dfrac{4\omega^2 L^2}{R_L^2}}}$$

If $\frac{4\omega^2 L^2}{R_L^2} >> 1$, then a simplified expression for ripple factor is

$$\Gamma = \frac{R_L}{3\sqrt{2}\ \omega L}$$

In case, the load resistance is infinity, i.e. the output is an open circuit, then the ripple factor is

$$\Gamma = \frac{2}{3\sqrt{2}} = 0.471$$

This is slightly less than the value of 0.482. The difference being attributable to the omission of higher harmonics as mentioned. It is clear that the inductor filter should only be used where R_L is consistently small.

Example 12.8 Calculate the value of inductance to use in the inductor filter connected to a full-wave rectifier operating at 60 Hz to provide a dc output with 4% ripple for a 100 Ω load.

We know that the ripple factor for inductor filter is $\Gamma = \frac{R_L}{3\sqrt{2}\ \omega L}$

Therefore, $$0.04 = \frac{100}{3\sqrt{2}(2\pi \times 60 \times \mathrm{L})} = \frac{0.0625}{L}$$

$$L = \frac{0.0625}{0.04} = 1.5625 \text{ H}$$

12.8.2 Capacitor Filter

An inexpensive filter for light loads is found in the capacitor filter which is connected directly across the load, as shown in Fig. 12.24 (a). The property of a capacitor is that it allows ac component and blocks the dc component. The operation of a capacitor

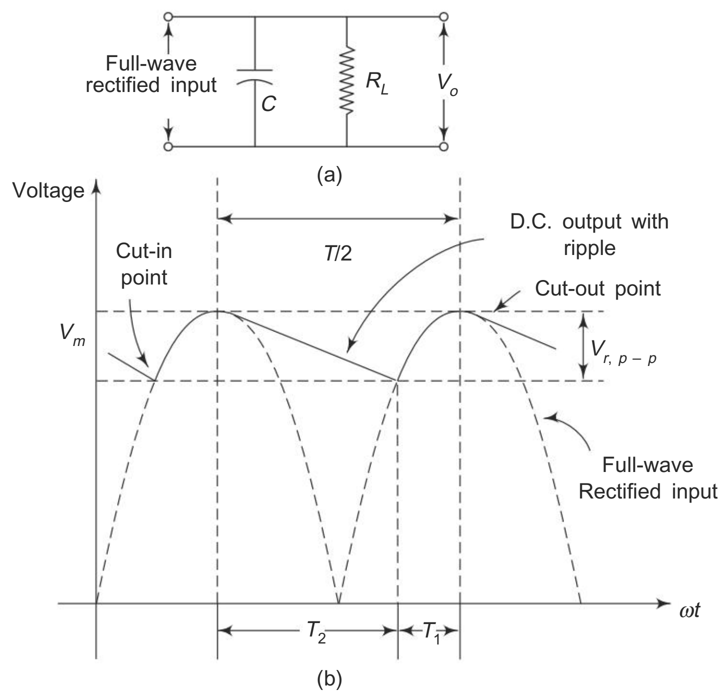


Fig. 12.24 (a) Capacitor Filter, (b) Ripple Voltage Triangular Waveform

filter is to short the ripple to ground but leave the dc to appear at the output when it is connected across a pulsating dc voltage.

During the positive half-cycle, the capacitor charges up to the peak value of the transformer secondary voltage, V_m, and will try to maintain this value as the full-wave input drops to zero. The capacitor will discharge through R_L slowly until the transformer secondary voltage again increases to a value greater than the capacitor voltage. The diode conducts for a period which depends on the capacitor voltage (equal to the load voltage). The diode will conduct when the transformer secondary voltage becomes more than the 'cut-in' voltage of the diode. The diode stops conducting when the transformer voltage becomes less than the diode voltage. This is called cut-out voltage.

Referring to Fig. 12.24 (b) with slight approximation, the ripple voltage waveform can be assumed as triangular. From the cut-in point to the cut-out point, whatever charge the capacitor acquires is equal to the charge the capacitor has lost during the period of non-conduction, i.e. from cut-out point to the next cut-in point.

The charge it has acquired $= V_{r,\,p-p} \times C$

The charge it has lost $= I_{dc} \times T_2$

Therefore, $V_{r,\,p-p} \times C = I_{dc} \times T_2$

If the value of the capacitor is fairly large, or the value of the load resistance is very large, then it can be assumed that the time T_2 is equal to half the periodic time of the waveform.

i.e. $$T_2 = \frac{T}{2} = \frac{1}{2f}, \text{ then } V_{r,\,p-p} = \frac{I_{dc}}{2fC}$$

With the assumptions made above, the ripple waveform will be triangular in nature and the rms value of the ripple is given by

$$V_{r,\,\text{rms}} = \frac{V_{r,\,p-p}}{2\sqrt{3}}$$

Therefore from the above equation, we have

$$V_{r,\,\text{rms}} = \frac{I_{dc}}{4\sqrt{3}}fC$$

$$= \frac{V_{dc}}{4\sqrt{3}\,fCR_L}, \text{ since } I_{dc} = \frac{V_{dc}}{R_L}$$

Therefore, ripple factor $\Gamma = \dfrac{V_{r,\,\text{rms}}}{V_{dc}} = \dfrac{1}{4\sqrt{3}\,fCR_L}$

The ripple may be decreased by increasing C or R_L (or both) with a resulting increase in dc output voltage.

If $f = 50$ Hz, C in μF and R_L in Ω, $\Gamma = \dfrac{2890}{CR_L}$.

Example 12.9 Calculate the value of capacitance to use in a capacitor filter connected to a full-wave rectifier operating at a standard aircraft power frequency of 400 Hz, if the ripple factor is 10% for a load of 500 Ω.

We know that the ripple factor for capacitor filter is

$$\Gamma = \frac{1}{4\sqrt{3}\,fCR_L}$$

Therefore, $$0.01 = \frac{1}{4\sqrt{3} \times 400 \times C \times 500} = \frac{0.722 \times 10^{-6}}{C}$$

$$C = \frac{0.722 \times 10^{-6}}{0.01} = 72.2\ \mu\text{F}$$

12.8.3 LC Filter

We know that the ripple factor is directly proportional to the load resistance R_L in the inductor filter and inversely proportional to R_L in the capacitor filter. Therefore, if these two filters are combined as LC filter or L-section filter as shown in Fig. 12.25, the ripple factor will be independent of R_L.

If the value of the inductance is increased, it will increase the time of conduction. At some critical value of inductance, one diode, either D_1 or D_2 in full-wave rectifier, will always be conducting.

From Fourier series, the output voltage can be expressed as

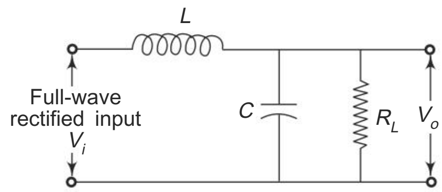


Fig. 12.25 LC Filter

$$V_o = \frac{2V_m}{\pi} - \frac{4\,V_m}{3\pi}\cos 2\,\omega t$$

The dc output voltge, $V_{dc} = \dfrac{2V_m}{\pi}$

Therefore, $I_{rms} = \dfrac{4V_m}{3\pi\sqrt{2}} \cdot \dfrac{1}{X_L} = \dfrac{\sqrt{2}}{3} \cdot \dfrac{V_{dc}}{X_L}$

This current flowing through X_c creates the ripple voltage in the output.

Therefore, $V_{r,\,rms} = I_{rms} \cdot X_C = \dfrac{\sqrt{2}}{3} \cdot V_{dc} \cdot \dfrac{X_C}{X_L}$

The ripple factor, $\Gamma = \dfrac{V_{r,rms}}{V_{dc}} = \dfrac{\sqrt{2}}{3} \cdot \dfrac{X_C}{X_L}$

$$= \frac{\sqrt{2}}{3} \cdot \frac{1}{4\omega^2 CL}, \text{ since } X_C = \frac{1}{2\omega C} \text{ and } X_L = 2\omega L$$

If f = 50 Hz, C is in μF and L is in Henry, ripple actor $\Gamma = \dfrac{1.194}{LC}$.

Example 12.10 Design a filter for full wave circuit with LC filter to provide an output voltage of 10 V with a load current of 200 mA and the ripple is limited to 2%.

The effective load resistance $R_L = \dfrac{10}{200 \times 10^{-3}} = 50\ \Omega$

We know that the ripple factor, $\Gamma = \dfrac{1.194}{\text{LC}}$

i.e. $0.02 = \dfrac{1.194}{\text{LC}}$

i.e. $LC = \dfrac{1.194}{0.02} = 59.7$

Critical value of $L = \dfrac{R_L}{3\omega} = \dfrac{50}{3 \times 2\pi f} = 53$ mH

Taking L = 60 mH (about 20% higher), C will be about 1000 μF.

12.8.4 Multiple LC Filters

Better filtering can be achieved using two or more L-section filters as shown in Fig. 12.26.

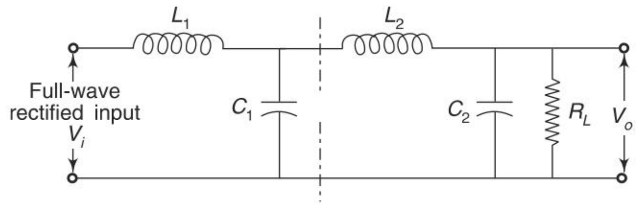


Fig. 12.26 Multiple LC Filter

The ripple factor, $\Gamma = \frac{\sqrt{2}}{3} \cdot \frac{X_{C2}}{X_{L2}} \cdot \frac{X_{C1}}{X_{L1}}$

12.8.5 CLC or π-section Filter

Figure 12.27 shows the CLC or π-type filter which basically consists of a capacitor filter followed by an LC section.

This filter offers a fairly smooth output, and is characterised by a highly peaked diode currents and poor regulation. Proceeding the analysis in the same ways as that for the single L-section filter, we obain

$$\Gamma = \sqrt{2} \cdot \frac{X_{C1}}{R_L} \cdot \frac{X_{C2}}{X_L}$$

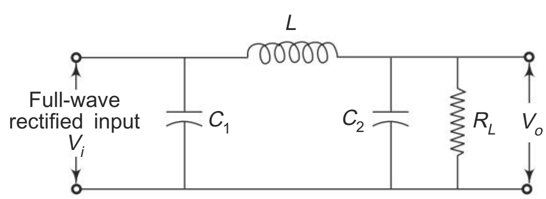


Fig. 12.27 CLC or π-type Filter

The term 'R_L' in the above equation should be noted.

If $f = 50$ Hz, C in μF, L in H and R_L in Ω then $\Gamma = \frac{5700}{LC_1C_2R_L}$

Example 12.11 Design a CLC or π-section filter for $V_{dc} = 10$ V, $I_L = 200$ mA and $\Gamma = 2\%$.

$$R_L = \frac{10}{200 \times 10^{-3}} = 50\ \Omega$$

$$0.02 = \frac{5700}{LC_1C_2 \times 50} = \frac{114}{LC_1C_2}$$

If we assume $L = 10$ H and $C_1 = C_2 = C$, we have

$$0.02 = \frac{5700}{LC_1\,C_2 \times 50} = \frac{114}{LC^2} = \frac{11.4}{C^2}$$

$$C^2 = 570;\ \text{therfore},\ C = \sqrt{570} \approx 24\ \mu\text{F}$$

12.8.6 R-C Filters

Consider the CLC filter with the inductor L replaced by a resistor R. This type of filter called R-C filter is shown in Fig. 12.28. The expression for the ripple factor can be obtained by replacing X_L by R giving.

$$\Gamma = \sqrt{2}\,\frac{X_{C1}}{R_L}\cdot\frac{X_{C2}}{R}$$

Therefore, if resistor R is chosen equal to the reactance of the inductor which it replaces, the ripple remains unchanged.

The resistance R will increase the voltage drop and hence, the regulation will be poor. This type of filters are often used for economic reasons, as well as the space and weight requirement of the iron-cored choke for the LC filter. Such R–C filters are often used only for low current power supplies.

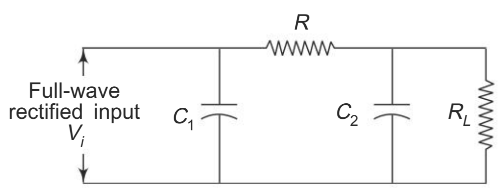


Fig. 12.28 R-C Filter

12.9 DIODE CLIPPERS

The circuit with which the waveform is shaped by removing (or clipping) a portion of the input signal without distorting the remaining part of the alternating waveform is called a *clipper*. Clipping circuits are also referred to as voltage (or current) limiters, amplitude selectors, or slicers. These circuits find extensive use in radars, digital computers, radio and television receivers etc.

The clipping circuits employ the components like diode, resistor and dc battery. The diodes used are assumed to be ideal for the discussion in this chapter. But in practice, the resistor (R) is used to limit the current flowing through the diode when it is forward biased. In order to get the flat output waveform at the clipping level, the value of R is chosen in such a way that it satisfies the conditions of $R_r > R > R_f$ and $R = \sqrt{R_f R_r}$, where R_f is the forward resistance and R_r is the reverse resistance of the diode.

There are four general categories of clippers, viz. (i) positive clipper (ii) negative clipper (iii) biased clipper and (iv) combination clipper.

1. Positive clipper In the series positive clipper as shown in Fig. 12.29 (a), when the input voltage is positive, the diode does not conduct and acts as an open

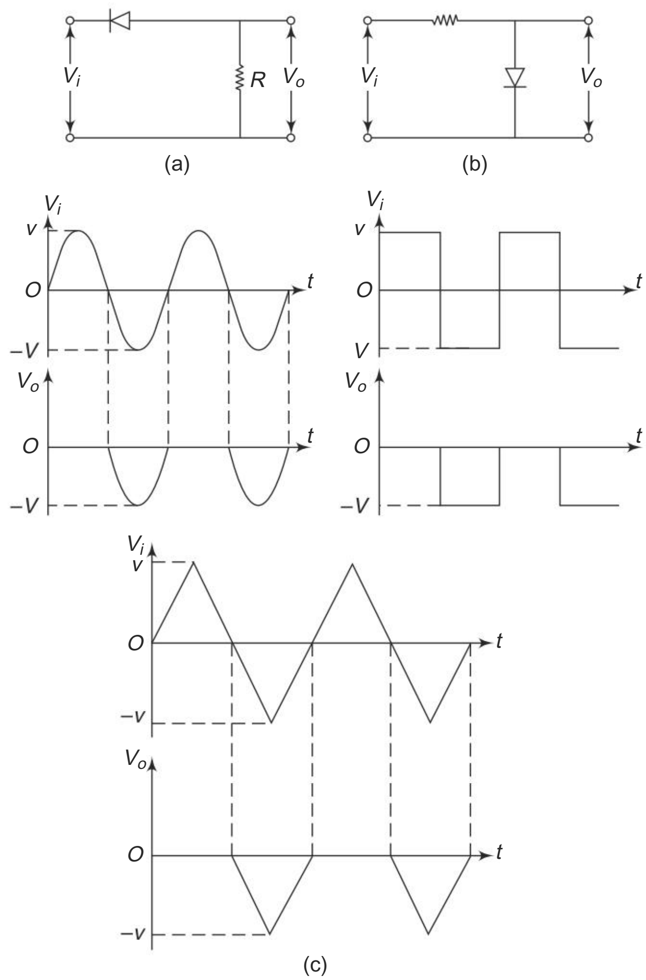


Fig. 12.29 Positive clipper (a) Series, (b) Shunt, and (c) Input and output signal waveforms

circuit and hence the positive half cycle does not appear at the output, i.e. the positive half cycle is clipped off. When the input signal is negative, the diode conduct and acts as a closed switch (short circuit), the negative half cycle appears at the output as shown in Fig. 12.29 (c).

In the shunt positive clipper as shown in Fig. 12.29 (b), when the input voltage is positive, diode conducts and acts as short-circuit and hence there is zero signal at the output, i.e. the positive half cycle is clipped off. When the input signal is negative, the diode does not conduct and acts as an open switch, the negative half cycle appears at the output as shown in Fig. 12.29 (c).

It is evident from the above discussion that positive clippers act as half wave rectifier. Thus the positive clipper has clipped the positive half cycle completely and allowed to pass the negative half cycle of the input signal.

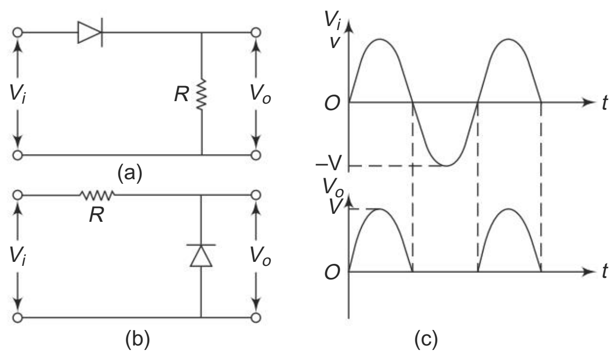


Fig. 12.30 Negative clipper (a) Series, (b) Shunt, and (c) Input and output signal waveforms

2. Negative clipper In the negative clipping circuit, the diode is connected in a direction opposite to that of a positive clipper. In the series negative clipper as shown in Fig. 12.30 (a), during the positive half cycle of the input signal, the diode conducts and acts as a short-circuit and hence, the positive half cycle of the input signal will appear at the output as shown in Fig. 12.30 (c). During the negative half cycle of the input signal, the diode does not conduct and acts as an open circuit. The negative half cycle will not appear at the output, i.e. the negative half cycle is clipped off, as shown in Fig. 12.30 (c).

It is evident from the above discussion that the negative clippers of both series and shunt types work as half wave rectifier. Thus, the negative clipper has clipped the negative half cycle completely and allowed to pass the positive half cycle of the input signal.

3. Biased clipper In some applications, it is required to remove a small portion of positive or negative half cycle of the signal voltage and hence the biased clipper is used. The name *bias* is designated because the adjustment of the clipping level is achieved by adding a biasing voltage in series with the diode or resistor.

(a) *Biased positive clipper* Figure 12.31 shows the circuits of shunt and series type positive clipper along with the input and output voltage waveforms. In the biased series positive clipper as shown in Fig. 12.31 (a), the diode does not conduct as long as the input voltage is greater than $+V_R$ and hence, the output remains at $+V_R$. When the input voltage becomes less than $+V_R$, the diode conduct and acts as a short circuit. Hence, all the input signal having less than $+V_R$ as well as negative half cycle of the input wave will appear at the output, as shown in Fig. 12.31 (c).

In the biased shunt positive clipper as shown in Fig. 12.31 (b), the diode conducts as long as the input voltage is greater than $+V_R$ and the output remains at $+V_R$ until the input voltage becomes less than $+V_R$. When the input voltage is less than $+V_R$, the diode does not conduct and acts as an open switch. Hence all the input signal having less than $+ V_R$ as well as negative half cycle of the input wave will appear at the output, shown in Fig. 12.31 (c).

The clipping level can be shifted up or down by varying the bias voltage V_R.

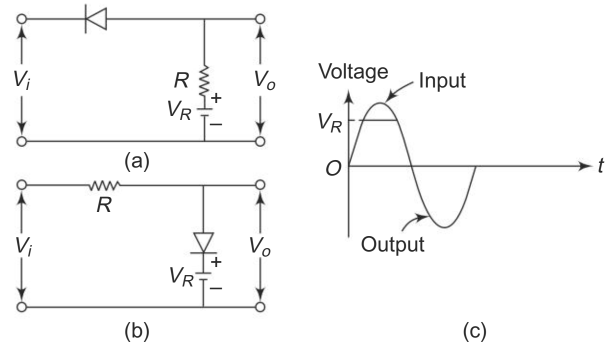


Fig. 12.31 Biased Clipper (a) Series, (b) Shunt, and (c) Input and output signal waveforms

(b) *Biased positive clipper with reverse polarity of the battery V_R* Figure 12.32 shows the biased series and shunt clippers with reverse polarity of V_R along with the input and output voltage waveforms. Here, the entire signal above $-V_R$ is clipped off.

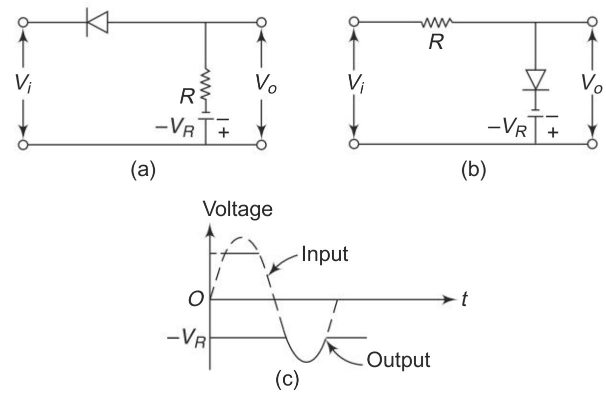


Fig. 12.32 Biased positive clipper with reverse polarity or V_R. (a) series, (b) shunt, and (c) Input and output signal waveforms

(c) *Biased negative clipper* In the biased series negative clipper shown in Fig. 12.33 (a), when the input voltage $V_i \leq -V_R$ the diode does not conduct and clipping takes place. In the biased shunt clipper shown in Fig. 12.33 (b), when the input voltage $V_i \leq -V_R$ the diode conducts and clipping takes place. The clipping level can be shifted up and down by varying the bias voltage ($-V_R$).

(d) *Biased negative clipper with reverse polarity of the battery V_R* Figure 12.34 shows the biased series and shunt clippers with reverse polarity of V_R along with the input and output waveforms, as shown in Fig. 12.33 (c). Here, the entire signal below $+V_R$ is clipped off.

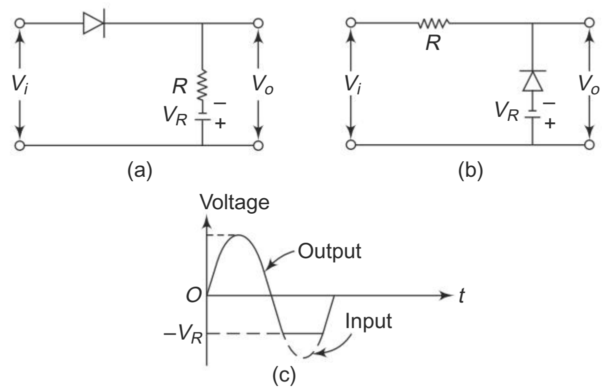


Fig. 12.33 Biased negative clipper (a) Series, (b) Shunt, and (c) Input and output signal waveforms

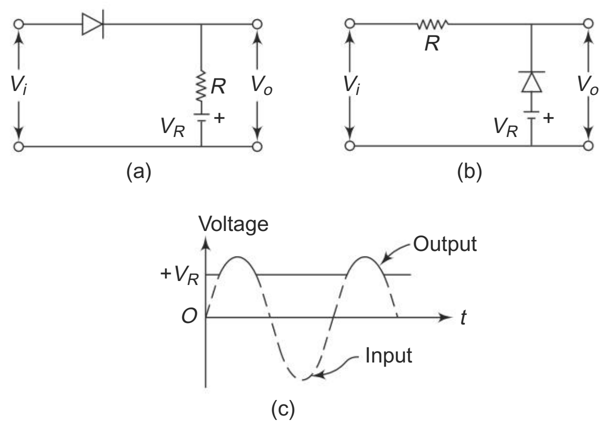


Fig. 12.34 Biased negative clipper with reverse polarity of V_R (a) Series, (b) Shunt, and (c) Input and output signal waveforms

4. Combination clipper This is the combination of a biased positive clipper and a biased negative clipper. Figure 12.35 shows the combination clipper along with the input and output voltage waveforms. When the input signal voltage $V_i \geq +V_{R1}$, diode D_1 conducts and acts as a closed switch, while diode D_2 is reverse biased and D_2 acts as an open switch. Hence, the output voltage cannot exceed the voltage level of $+V_{R1}$ during the positive half cycle.

Similarly, when the input signal voltage $V_i \leq -V_{R2}$, diode D_2 conducts and acts as a closed switch, while diode D_1 is reverse biased and D_1 acts as an open switch. Hence the output voltage V_o cannot go below the voltage level of $-V_{R2}$ during the negative half cycle.

It is evident that, the clipping levels may be changed by varying the values of V_{R1} and V_{R2}. If $V_{R1} = V_{R2}$, the circuit will clip both the positive and negative half cycles at

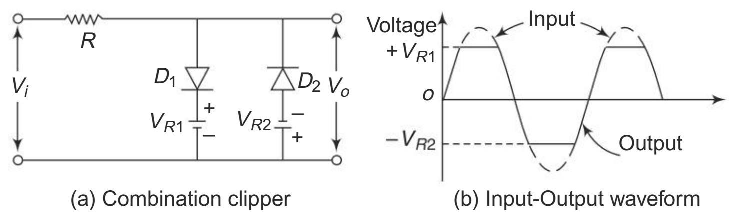


Fig. 12.35 (a) Combination clipping circuit and (b) Input and output waveforms

the same voltage levels and hence, such a combination clipper is called symmetrical clipper.

5. *Two level slicer* The circuit of a two level slicer shown in Fig. 12.36 (a) is similar to the combination clipper but with the diode connections reversed. The circuit has a slice cut from positive half cycle of the input signal as shown in Fig. 12.36 (b).

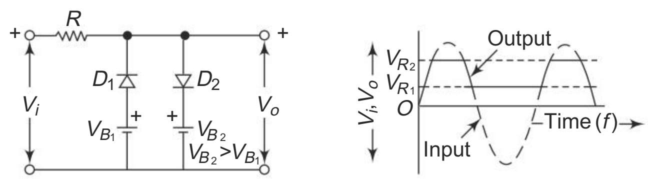


Fig. 12.36 (a) Two level slicer circuit and (b) Its input and output waveforms

Example 12.12 Determine the output waveform for the network of Fig. 12.37 for the input sinusoidal and square waveforms. Assume the diode is ideal.

Here, the addition of bias voltage +3 V shifts the entire input signal by +3 V. Then the circuit clips off the negative half cycle lying below the zero axis. The resultant output waveforms are shown in Fig. 12.37.

Example 12.13 Determine V_o for the network of Fig. 12.38 for the waveform given. Assume the diode is ideal.

When the input voltage $V_i \leq 3$ V the diode is forward biased, resulting in a short-circuited diode. Then the circuit clips off the wave which is below +3 V. The resultant output waveform is shown in Fig. 12.38.

Example 12.14 Determine V_o for the network of Fig. 12.39 for the waveform given. Assume the diodes are ideal.

Solution: When the input voltage $V_i \geq 5$V, the diode D_1 conducts when $V_i \leq -3$V, diode D_2 conducts. Hence, the positive half cycle of the input voltage is clipped at +5V and the negative half cycle is clipped at –3V, as shown in Fig. 12.39.

Example 12.15 Determine V_o for the network of Fig. 12.40 for the waveform given. Assume the diode is ideal.

Solution: When the input voltage $V_i \leq 3$ V, D_1 conducts. When $V_i \geq 6$ V, diode D_2 conducts. Hence, both D_1 and D_2 will not conduct and act as open switches and

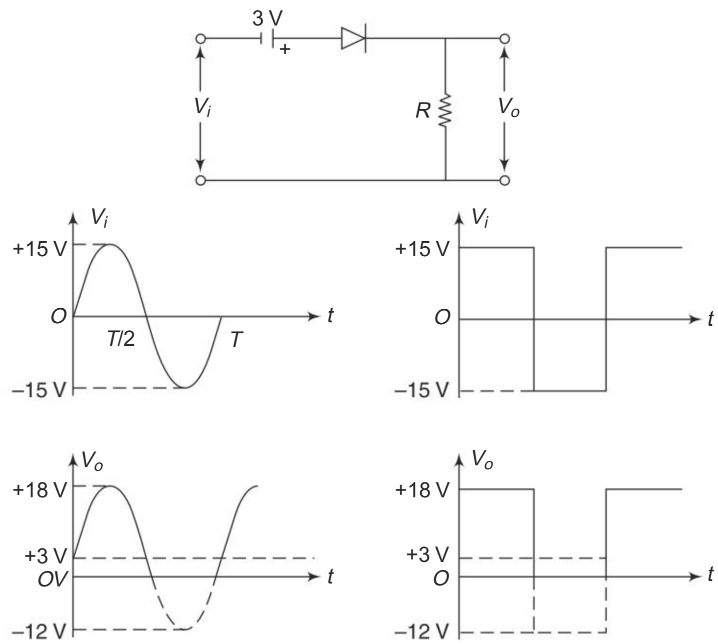


Fig. 12.37 Clipper circuit

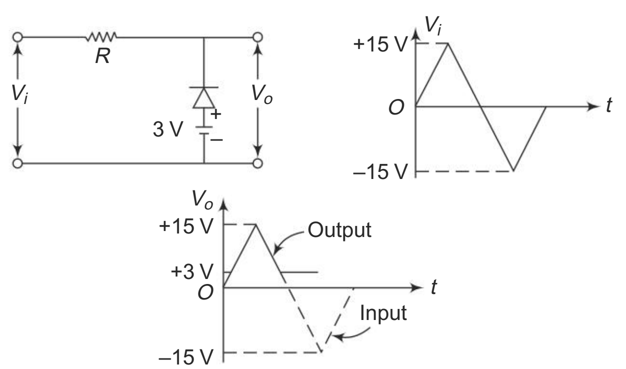


Fig. 12.38

allow the signal to pass during 3 V ≥ V_i ≤ 6 V. The resultant waveform is shown in Fig. 12.40.

12.10 DIODE COMPARATOR

The nonlinear circuit which was used to perform the operation of clipping may also be used to perform the operation of comparison and this circuit shown in Fig. 12.41

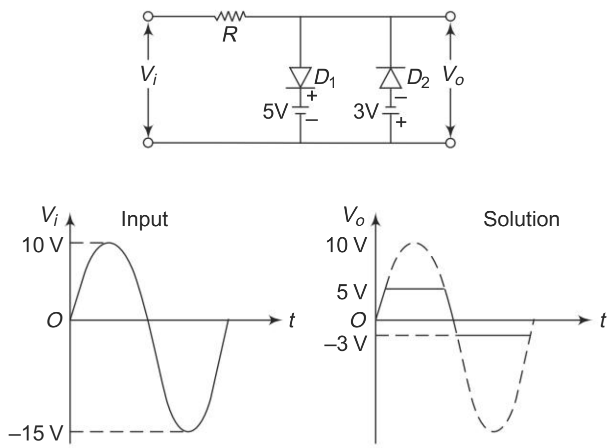


Fig. 12.39

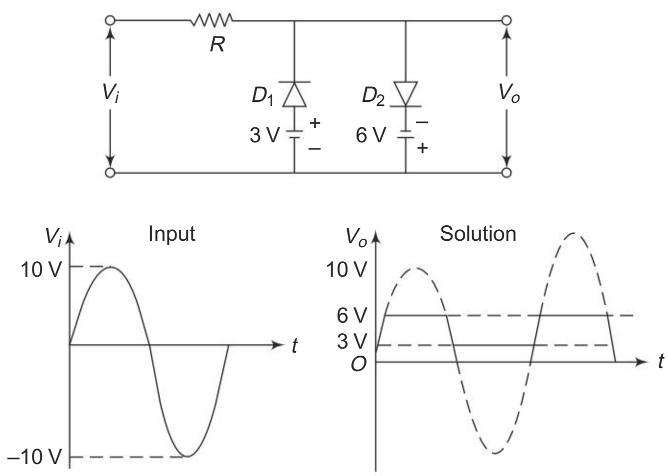


Fig. 12.40

(a) is called the comparator. The comparator circuit compares an input signal $V_i(t)$ with a reference voltage V_R. The comparator output is independent of the signal until it attains the reference level. When the signal and reference level become equal, there will be a sharp pulse at the comparator output.

The input signal is considered as a ramp. At $t = t_1$, $V_i(t) = V_R + V_\gamma$, and $V_0 = V_R$ until $t = t_1$. Beyond $t = t_1$, the output rises with the input signal. At some later time $t = t_2$, the device responds in the range ΔV_0 for a precise input voltage ΔV_t corresponding to Δt.

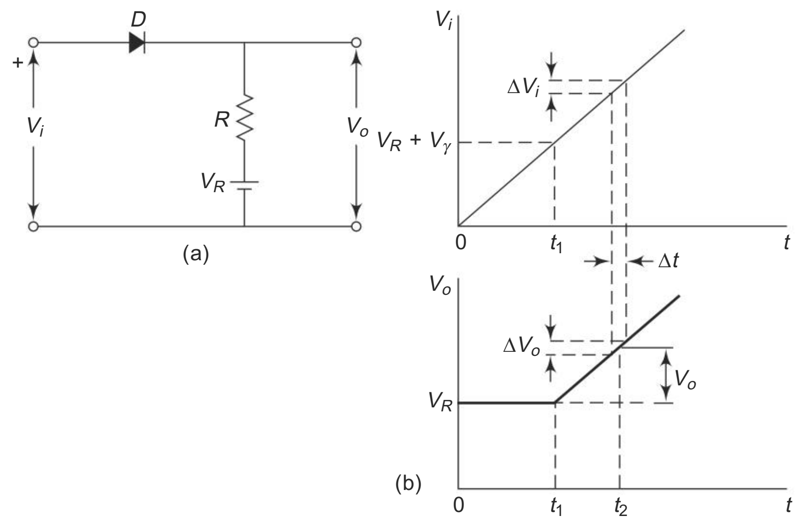


Fig. 12.41 (a) A diode comparator (b) the output waveform corresponding to a ramp input signal, V_i

When $V_R = 0$, the output will respond everytime the input passes through zero. This arrangement is called a zero-crossing detector. The most important systems using comparators are

(i) Square waves from a sine wave
(ii) Timing markers generator from a sine wave.
(iii) Phase meter
(iv) Amplitude—distribution analyzer
(v) Pulse time modulation
(vi) Pulse, square wave and triangular wave generators, and
(vii) Analog to digital converter

12.11 CLAMPERS

Clamping network shifts (clamps) a signal to a different dc level, i.e. it introduces a dc level to an ac signal. Hence, the clamping network is also known as dc restorer. These circuits find application in television receivers to restore the dc reference signal to the video signal.

The clamping network has the various circuit components like a diode, a capacitor and a resistor. The time constant for the circuit $\tau = RC$ must be large so that the voltage across the capacitor does not discharge significantly when the diode is not conducting.

Consider the clamper circuit shown in Fig. 12.42. The diodes used are assumed to be ideal. A square wave with maximum amplitude of V is given as the input to the network. During the positive half cycle, the diode conducts, i.e. it acts like a short circuit. The capacitor charges to V volts. During this interval, the output which is taken

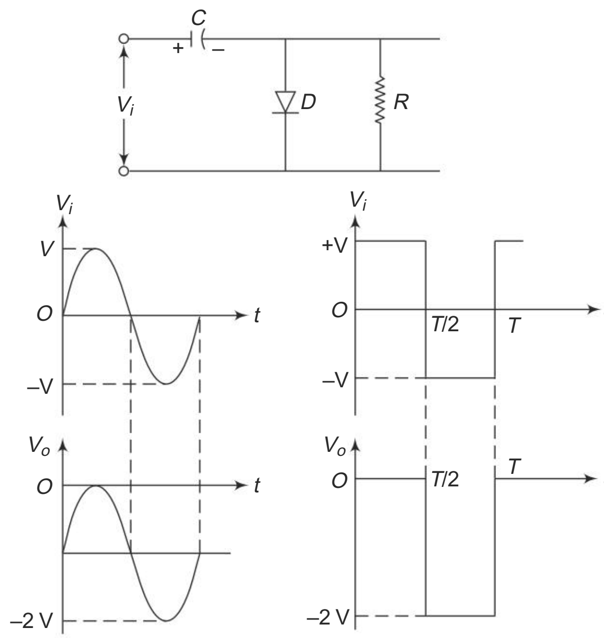


Fig. 12.42 Clamper circuit

across the short circuit will be V_o = 0V. During the negative half cycle, the diode is open. The output voltage can be found out by applying Kirchhoff's law.

$$-V - V - V_0 = 0$$

Therefore, $V_o = -2$ V

The analysis of the clamper circuit can be done as follows.

Determine the portion of the input signal that forward biases the diode. When the diode is in short circuit condition, the capacitor charges upto a level determined by the voltage across the capacitor in its equivalent open circuit state. During the open circuit condition of the diode, it is assumed that the capacitor will hold on to all its charge and therefore voltage. In the clamper networks, the total swing of the output is equal to the total swing of the input signal.

Example 12.16 Determine V_o for the network shown in Fig. 12.43.

Solution: The frequency of the given input signal is 2000 Hz. Hence, the period of the signal is 0.5 ms. During the negative half of the input signal, the diode is forward biased and it acts like a short circuit and the capacitor charges to 20 V. This can be found out by applying Kirchhoff's law in the input side.

$$15 + V_c - 5 = 0$$

and $C_c = 20$ V

The voltage across the resistor will be equal to the dc voltage 5 V.

During the positive half of the input signal, the diode is reverse biased and it acts like an open circuit. Hence, the 5V battery has no effect on V_o. Applying Kirchhoff's voltage law around the outside loop, we get + 15 + 20 – V_o = 0.

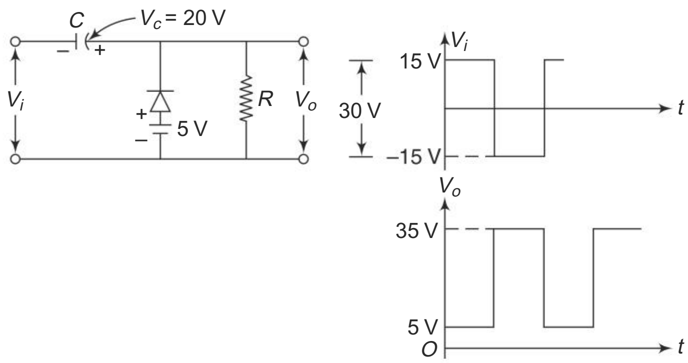


Fig. 12.43

Therefore, V_o = 35 V.

The resulting output appears in Fig. 12.43. Here, the output swing of 30 V is equal to the input swing of 30 V.

Example 12.17 Determine V_o for the clamping circuit shown in Fig. 12.44 for the given sinusoidal input signal.

During the negative half of the input signal, the diode conducts, and acts like a short circuit. Now, the output voltage, V_o = 0 V. The capacitor is charged to 10 V with polarities shown in Fig. 12.44 and it behaves like a battery.

During the positive half of the input signal, the diode does not conduct, and acts like an open circuit. Hence, the output voltage, V_o = 10 V + 10 V = 20 V. This gives positively clamped voltage and the resultant output waveform is shown below.

Example 12.18 Determine V_o for the clamping circuit shown in Fig. 12.45 for the given sinusoidal input signal.

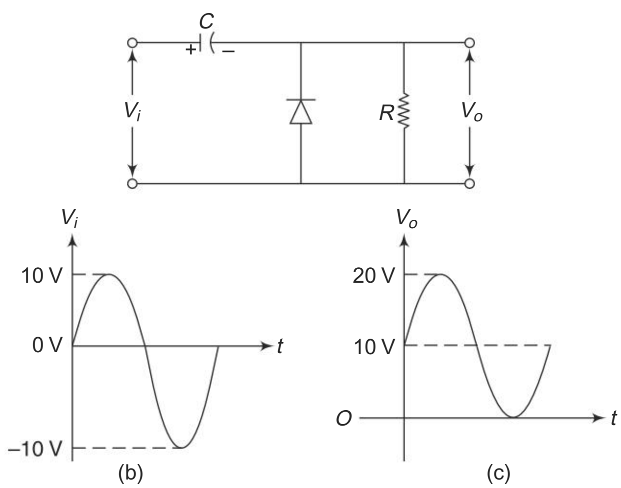


Fig. 12.44

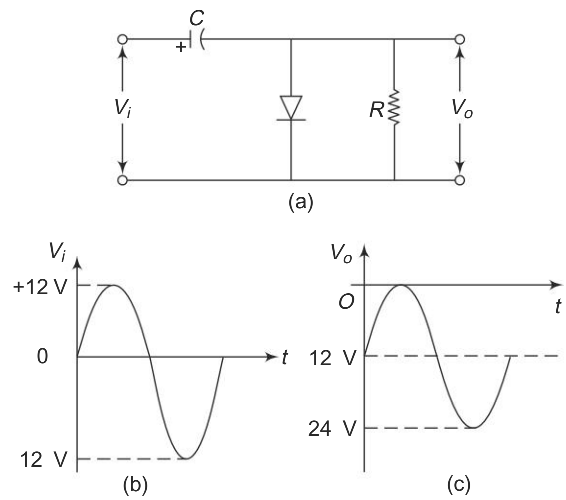


Fig. 12.45

During the positive half of the input signal, the diode conducts and acts like a short circuit. Now, the output voltage, $V_o = 0$ V. The capacitor is charged to 12V with polarities shown in Fig. 12.45 and it behaves like a battery.

During the negative half of the input signal, the diode does not conduct and acts like an open circuit. Hence, the output voltage, $V_o = -12\text{ V} - 12\text{ V} = -24$ V. This gives negatively clamped voltage and the resultant output waveform is shown in the Fig. 12.45 (c).

12.12 HALL EFFECT

When a transverse magnetic field B is applied to a specimen (thin strip of metal or semiconductor) carrying current I, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as the *Hall effect*.

A Hall effect measurement experimentally confirms the validity of the concept that it is possible for two independent types of charge carriers, electrons and holes, to exist in a semiconductor.

The schematic arrangement of the semiconductor, the magnetic field and the current flow pertaining to the Hall effect are shown in Fig. 12.46. Under the equilibrium condition, the electric field intensity, E, due to the Hall effect must exert a force on the carrier of charge, q, which just balances the magnetic force, i.e.

$$qE = Bqv_d$$

where v_d is the drift velocity. Also, the electric field intensity due to Hall effect is

$$E = \frac{V_H}{d}$$

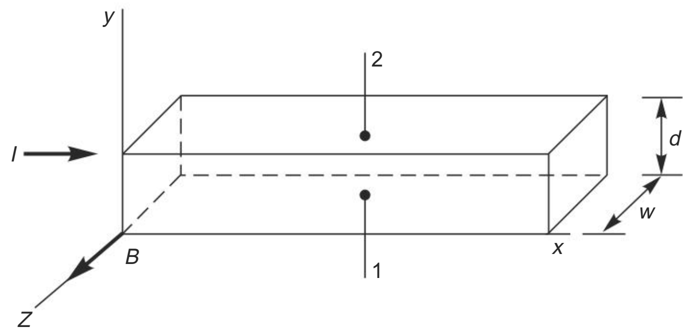


Fig. 12.46 Schematic Arrangement to Observe the Hall Effect

where d is the distance between surfaces 1 and 2, and V_H is the Hall voltage appearing between surfaces 1 and 2. In an N-type semiconductor, the current is carried by electrons and these electrons will be forced downward towards side 1 which becomes negatively charged with respect to side 2.

The current density (J) is related to charge density (ρ) by

$$J = \rho v_d$$

Further, the current density (J) is related to current(I) by

$$J = \frac{I}{\text{Area}} = \frac{I}{wd}$$

where w is the width of the specimen in the direction of magnetic field (B).

Combining the above relations, we get

$$V_H = Ed = B\, v_d d = \frac{BJd}{\rho} = \frac{BI}{\rho w}$$

The Hall coefficient, R_H, is defied by

$$R_H = \frac{1}{\rho}$$

so that $V_H = \frac{R_H}{w} BI$. A measurement of the Hall coefficient R_H determines not only the sign of the charge carriers but also their concentration. The Hall coefficient for a P-type semiconductor is positive, whereas it is a negative for an N-type semiconductor. This is true because the Hall voltage in a P-type semiconductor is of opposite polarity to that in an N-type semiconductor.

The advantage of hall effect transducers is that they are non-contact devices with high resolution and small size.

Applications The Hall effect is used to find whether a semiconductor is N- or P-type and to determine the carrier concentration. If terminal 2 becomes charged positively with respect to terminal 1, the semiconductor must be N-type and $\rho = nq$, where n is the electron concentration. On the other hand, if the polarity of V_H is positive at terminal 1 with respect to terminal 2, the semiconductor must be P-type and $\rho = pq$, where p is the hole concentration.

The mobility (μ) can also be calculated with simultaneous measurement of the conductivity (σ). The conductivity and the mobility are related by the equation $\sigma = \rho\mu$ or $\mu = \sigma R_H$.

Therefore, the conductivity for N-type semiconductor is $\sigma = nq\mu_n$ and for P-type semiconductor, $\sigma = pq\mu_p$, where μ_n is the electron mobility and μ_p is the hole mobility.

Thus, if the conductivity of a semiconductor is also measured along with R_H, then mobility can be determined from the following relations.

For N-type semiconductor, $\mu_n = \dfrac{\sigma}{nq} = \sigma R_H$

and for P-type semiconductor, $\mu_p = \dfrac{a}{pq} = \sigma R_H$

Since V_H is proportional to B for a given current I, Hall effect can be used to measure the a.c. power and the strength of magnetic field and sense the angular position of static magnetic fields in a magnetic field meter. It is also used in an instrument called Hall effect multiplier which gives the output proportional to the product of two input signals. If I is made proportional to one of the inputs and B is made proportional to the second signal, then from the equation, $V_H = \dfrac{BI}{\rho w}$, V_H will be proportional to the product of two inputs. Hall devices for such applications are made from a thin wafer or film of indium antimonide (InSb) or indium arsenide. As the material has a very high electron mobility, it has high Hall coefficient and high sensitivity.

An electrical current can be controlled by a magnetic field because the magnetic field changes the resistances of some elements with which it comes in contact. In the magnetic bubble memory, while read-out, the Hall effect element is passed over the bubble. Hence, a change in current of the circuit will create, say, a *one*. If there is no bubble, there will be a *zero* and there will be no current change in the output circuit. The read-in device would have an opposite effect, wherein the Hall device creates a magnetic field when supplied with a pulse of current. This, in turn, creates a little domain and then a magnetic bubble is created.

Some of the other applications are in measurement of velocity, rpm, sorting, limit sensing and non-contact current meaurements.

Example 12.19 An N-type semiconductor has a Hall coefficient of 200 cm^3/C and its conductivity is 10 s/m. Find its electron mobility.

Given: $R_H = 200$ cm^3/C and $\sigma = 10$ s/m

Therefore, the electron mobility, $\mu_n = \sigma R_H = 10 \times 200 = 200$ cm^2/V-s

Example 12.20 The conductivity of an N-type semiconductor is 10 s/m and its electron mobility is 50×10^{-4} m^2/V-s. Determine the electron concentration.

Given: $\sigma = 10$ s/m and $\mu_n = 50 \times 10^{-4}$ m^2/V-s

We know that the electron mobility, $\mu_n = \dfrac{\sigma}{nq}$

Therefore, the electron concentation,

$$n = \frac{\sigma}{\mu q} = \frac{10}{50 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

$$= 12.5 \times 10^{21}\ \text{m}^{-3}$$

Example 12.21 A current of 20 A is passed through a thin metal strip, which is subjected to a magnetic flux density of 1.2 Wb/m^2. The magnetic field is directed perpendicular to the current. The thickness of the strip in the direction of the magnetic field is 0.5 mm. The Hall voltage is 60 V. Find the electron density.

Given: $I = 20$ A, $B = 1.2$ Wb/m^2, $V_H = 60$ V and $w = 0.5$ mm

We know that the number of conduction electrons, i.e. electron with density,

$$n = \frac{BI}{V_H q_w} = \frac{1.2 \times 20}{60 \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-3}} = 5 \times 10^{21}\ \text{m}^3$$

REVIEW QUESTIONS

1. What is electronics?
2. What is semiconductor?
3. How does the energy band structure of a semiconductor differ from that of a conductor and an insulator?
4. What is meant by doping in a semiconductor?
5. What is the need for adding impurities in a semiconductor?
6. Explain "majority and minority carriers" in a semiconductor?
7. What is a PN junction? How is it formed?
8. Describe the action of PN junction diode under forward bias and reverse bias.
9. Explain how undirectional current flow is possible through a PN junction diode.
10. Explain *V–I* characteristics of a PN junction diode.
11. Explain the following terms in a PN junction diode.
    (a) maximum forward current
    (b) Peak inverse voltage and
    (c) maximum power rating
12. Draw the energy band diagram of a PN junction and explain working of a diode.
13. Give the equation for the current in a semiconductor diode. With the help of above equation, explain in detail the V-I characteristics of a semiconductor diode.
14. Explain the terms: (i) static resistance (ii) dynamic resistance (iii) junction resistance and (iv) reverse resistance of a diode.
15. Define the term transition capacitance C_T of a PN diode.
16. Explain the term diffusion capacitance C_D of a forward biased diode.
17. Explain the effect of temperature of a diode.
18. Mention the applications of PN junction diode.
19. Explain Avalanche breakdown and zener breakdown.
20. Draw the *V–I* characteristic of zener diode and explain its operation.
21. Show that the zener diode can be used as a voltage regulator.
22. Show that a reverse biased PN junction can be used as a variable capacitor.
23. Explain the principle behind the varactor diode and list out its applications.

24. What is "tunneling?"
25. From the energy band diagram explain the V–I characteristic of a tunnel diode.
26. Draw the equivalent circuit of a tunnel diode and explain it.
27. List out the applications of tunnel diode and mention its advantages and disadvantages.
28. What is a rectifier?
29. Show that a PN diode works as rectifier.
30. Define the following terms.
    (a) ripple factor, (b) peak inverse voltage, (c) efficiency, (d) transformer utilisation factor, (e) form factor, and (f) peak factor.
31. Draw the circuit diagram of an half wave rectifier, and explain its operation.
32. Derive expressions for rectification efficiency, ripple factor, transformer utilisation factor, form factor and peak factor of an half wave rectifier with resistive load.
33. A half wave rectifier has a load of 3.5 kΩ. If the diode resistance and secondary coil reesistance together have a resistance of 800 Ω and the input voltage has a signal voltage of peak value 240 V, calculate
    (i) Peak, average and rms value of current flowing
    (ii) dc power output
    (iii) ac power input
    (iv) Efficiency of the rectifier

    [*Ans.* (i) 58.81 mA, 17.78 mA and 27.9 mA (ii) 1.1 W (iii) 3.35 W (iv) 32.9%]
34. Explain the action of a full-wave rectifier and give waveforms of input and output voltages.
35. Derive expressions of dc or average value of voltage and rms value of voltage of a full wave rectifier with resistive load.
36. Derive an expression for a ripple factor in a full-wave rectifier with resistive load.
37. Determine the value of ripple factor in the full-wave rectifier operating at 50 Hz with a 100 μF capacitor filter and 100 Ω load. [*Ans.* 29%)
38. Show that a full-wave rectifier is twice as efficient as a half-wave rectifier.
39. Describe the action of a full-wave bridge rectifier.
40. What are the advantages of a bridge rectifier?
41. Compare half wave, full-wave and bridge rectifiers.
42. What is the need for filters in power supplies?
43. Explain the various types of filters used in power supplies.
44. Obtain the ripple factor of a full-wave rectifier with shunt capacitor filter.
45. Derive an expression for the ripple factor in a full-wave rectifier using inductor filter.
46. Compare the performance of inductive, L-section and π-section filters.
47. An L–C filter is to be used to provide a dc output with 1% ripple from a full-wave rectifier operating at 50 Hz. Assuming $L/C = 0.01$, determine the required values of L and C. [*Ans.* 1.093 H, 109.27 μF]
48. In a full-wave rectifier using an L-C filter, it is known that $L = 10$ H, $C = 100$ μF and $R_L = 500$ Ω. Calculate I_{dc}, V_{dc}, I_{ac}, V_{ac}, if $V_m = 30$ V and $f = 50$ Hz.

    [*Ans.* 38.2 mA, 19.1 V, 1.43 mA, 22.7 mV]
49. What is clipper? With the help of circuit diagram and waveforms describe the operation of positive and negative clipper.

50. Describe the operation of biased clipper and combination clipper.
51. Sketch the circuit of a double ended clipper using ideal PN diode which limit the output between ±8 V.
52. What is a diode comparator? Explain its operation with the help of necessary circuit diagrams and waveforms.
53. What is a clamper? Discuss with the help of circuit diagram and waveforms, the operation of a clamping circuit.
54. Differentiate between positive clamper and negative clamper.
55. Sketch V_0 for each clipping network as shown in Fig. 12.47 for the input shown. Assume the diodes are ideal.

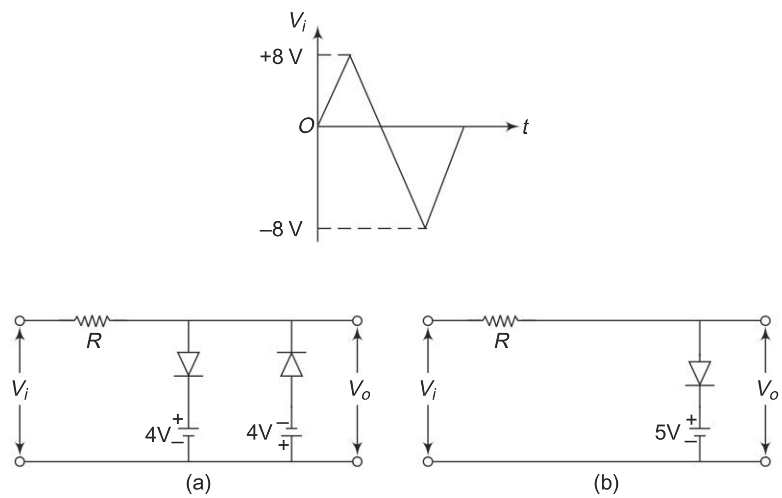


Fig. 12.47

56. Sketch V_o for each clipping network as shown in Fig. 12.48 for the input shown. Assume the diodes are ideal.
57. Sketch V_o for each clamping network as shown in Fig. 12.49 for the input shown. Assume the diodes are ideal.
58. The input waveform as shown in Fig. 12.50 is given to (a) positive clamper and (b) negative clamper. Determine the output waveforms.
59. With the help of circuit diagram and waveforms describe the operation of a two level slicer.
60. Explain Hall effect. How can Hall effect be used to determine some of the properties of a semiconductor?
61. Describe the applications of Hall effect?
62. A sample of N-type semiconductor has a Hall coefficient of 150 cm^3/coulomb. If its resistivity is 0.15 Ω-cm, estimate the electron mobility in the sample.

[*Ans.* 1000 cm^2/V-s]

63. The conductivity of a pure silicon bar is 5×10^{-4}/Ω-m. The magnetic flux density is 0.1 Wb/m^2 and the thickness of the bar in the direction of the magnetic field is 3 mm.

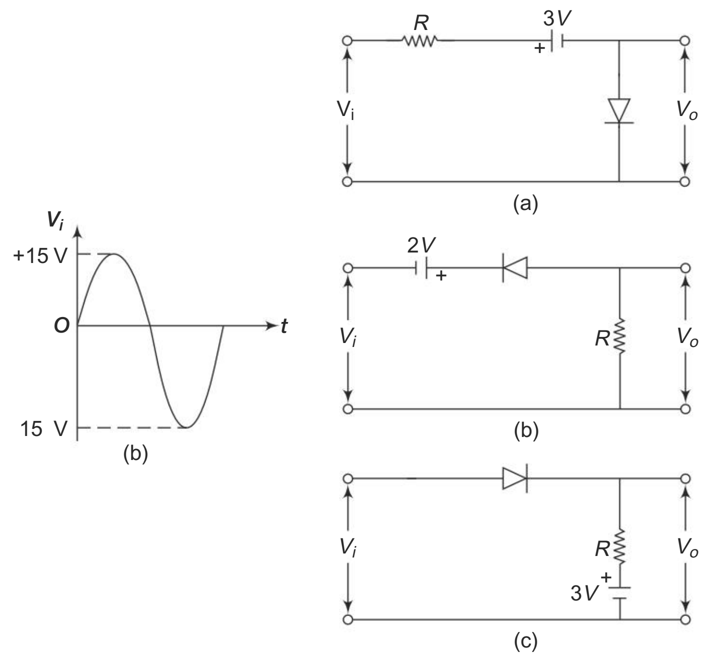


Fig. 12.48

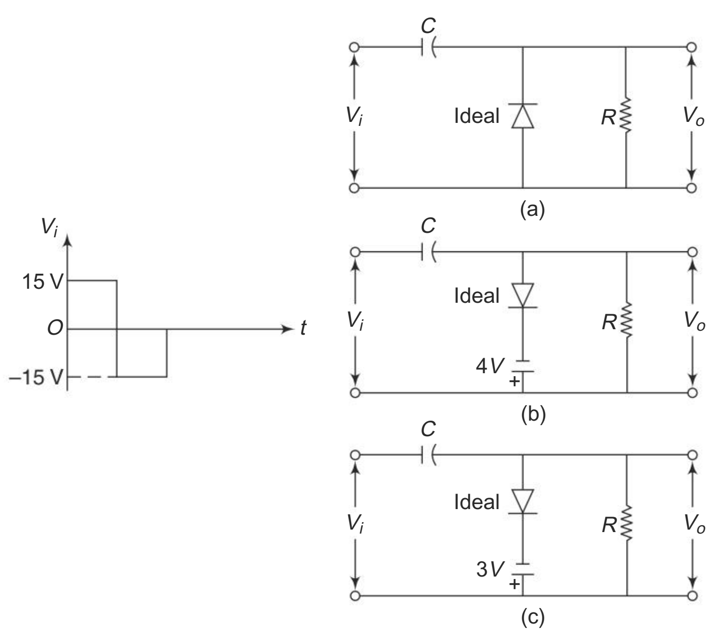


Fig. 12.49

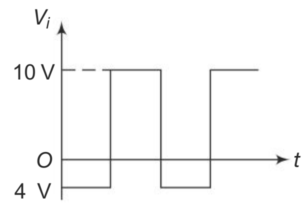


Fig. 12.50

The measured values of Hall voltage and current are 50 mV and 10 µA, respectively. Find the hole mobility. [*Ans.* 0.075 m^2/V-s]

64. Describe with the help of a relevant diagram, the construction of an LED and explain its working.
65. List the applications of an LED.
66. Explain the principle and working of photodiode.
67. Write the equation for the volt-ampere characteristics of a photodiode. Define each term in the equation.
68. Explain the volt-ampere characteristics of a semiconductor photodiode.
69. List the applications of a photodiode.
70. Describe with neat diagrams the operation of a phototransistor and state its applications.
71. With output characteristics, explain how phototransistor responds to the incident light.
72. Describe the principle of operation of an LCD.
73. What are the advantages and disadvantages of LCD?
74. What are the relative advantages of LCD over LED?

TRANSISTORS AND OTHER DEVICES

13

13.1 INTRODUCTION OF BIPOLAR JUNCTION TRANSISTOR

A Bipolar Junction Transistor (BJT) is a three terminal semiconductor device in which the operation depends on the interaction of both majority and minority carriers and hence the name Bipolar. The BJT is analogous to a vacuum triode and is comparatively smaller in size. It is used in amplifier and oscillator circuits, and as a switch in digital circuits. It has wide applications in computers, satellites and other modern communication systems.

13.2 CONSTRUCTION OF BJT

The BJT consists of a silicon (or germanium) crystal in which a thin layer of N-type Silicon is sandwiched between two layers of P-type silicon. This transistor is referred to as PNP. Alternatively, in a NPN transistor, a layer of P-type material is sandwiched between two layers of N-type material. The two types of the BJT are represented in Fig. 13.1.

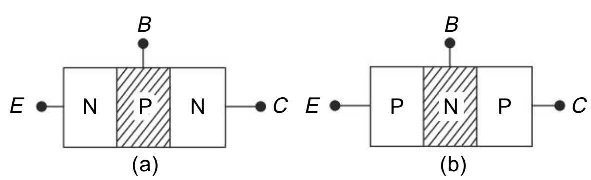


Fig. 13.1 Transistor (a) NPN and (b) PNP

The symbolic representation of the two types of the BJT is shown in Fig. 13.2. The three portions of the transistor are Emitter, Base and Collector, shown as *E*, *B* and *C*, respectively. The arrow on the emitter specifies the direction of current flow when the *EB* junction is forward biased.

Emitter is heavily doped so that it can inject a large number of charge carriers into the base. Base is lightly doped and very thin. It passes most of the injected charge carriers from the emitter into the collector. Collector is moderately doped.

13.3 TRANSISTOR BIASING

As shown in Fig. 13.3, usually the emitter-base junction is forward biased and collector-base junction is reverse biased. Due to the forward bias on the emitter-base

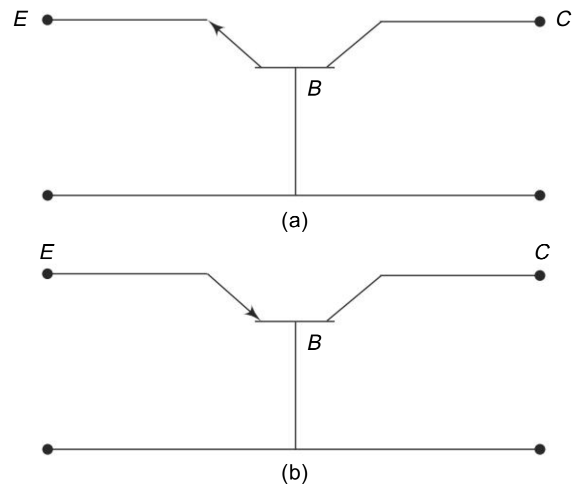


Fig. 13.2 Circuit symbol (a) NPN transistor and (b) PNP transistor

junction an emitter current flows through the base into the collector. Though, the collector-base junction is reverse biased, almost the entire emitter current flows through the collector circuit.

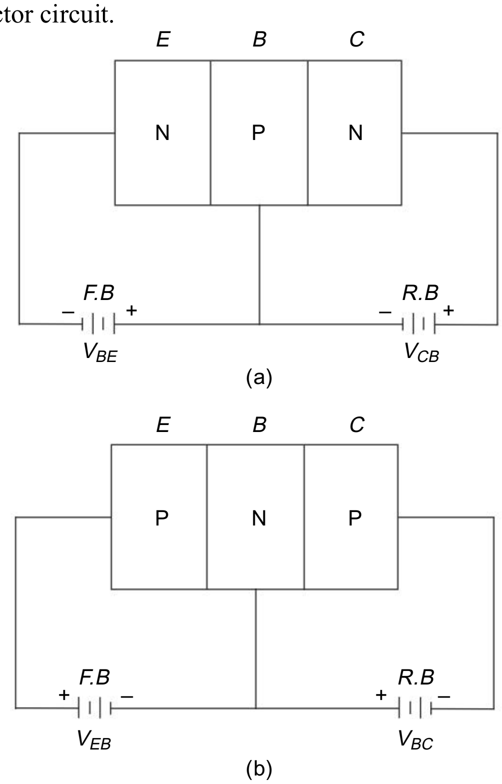


Fig. 13.3 Transistor biasing (a) NPN transistor and (b) PNP transistor

13.4 OPERATION OF NPN TRANSISTOR

As shown in Fig. 13.4, the forward bias applied to the emitter base junction of an NPN transistor causes a lot of electrons from the emitter region to crossover to the base region. As the base is lightly doped with P-type impurity, the number of holes in the base region is very small and hence the number of electrons that combine with holes in the P-type base region is also very small. Hence a few electrons combine with holes to constitute a base current I_B. The remaining electrons (more than 95%) crossover into the collector region to constitute a collector current I_C. Thus the base and collector current summed up gives the emitter current, i.e. $I_E = -(I_C + I_B)$.

In the external circuit of the NPN bipolar junction transistor, the magnitudes of the emitter current I_E, the base current I_B and the collector current I_C are related by $I_E = I_C + I_B$.

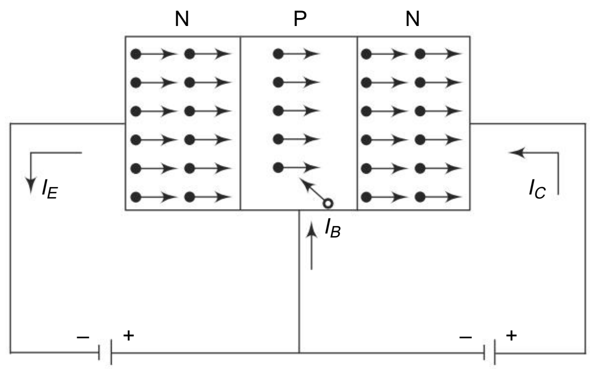


Fig. 13.4 Current in NPN transistor

13.5 OPERATION OF PNP TRANSISTOR

As shown in Fig. 13.5, the forward bias applied to the emitter-base junction of a PNP transistor causes a lot of holes from the emitter region to crossover to the base region as the base is lightly doped with N-types impurity. The number of electrons in the base region is very small and hence the number of holes combined with electrons in the N-type base region is also very small. Hence a few holes combined with electrons to constitute a base current I_B. The remaining holes (more than 95%) crossover into the collector region to constitute a collector current I_C. Thus the collector and base current when summed up gives the emitter current, i.e. $I_E = -(I_C + I_B)$.

In the external circuit of the PNP bipolar junction transistor, the magnitudes of the emitter current I_E, the base current I_B and the collector current I_C are related by

$$I_E = I_C + I_B \tag{13.1}$$

This equation gives the fundamental relationship between the currents in a bipolar transistor circuit. Also, this fundamental equation shows that there are current amplification factors α and β in common base transistor configuration and common emitter transistor configuration respectively for the static (d.c.) currents, and for small changes in the currents.

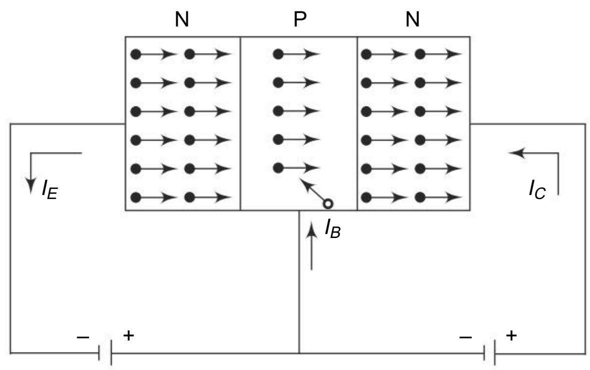


Fig. 13.5 Current in PNP Transistor

Large-signal current gain (α) The large signal current gain of a common base transistor is defined as the ratio of the negative of the collector-current increment to the emitter-current change from cutoff ($I_E = 0$) to I_E, i.e.

$$\alpha = -\left(\frac{I_C - I_{CBO}}{I_E - 0}\right) \tag{13.2}$$

where I_{CBO} (or I_{CO}) is the reverse saturation current flowing through the reverse biased collector-base junction, i.e. the collector to base leakage current with emitter open. As the magnitude of I_{CBO} is negligible when compared to I_E, the above expression can be written as

$$\alpha = \frac{I_C}{I_E} \tag{13.3}$$

Since I_C and I_E are flowing in opposite directions, α is always positive. Typical value of α ranges from 0.90 to 0.995. Also, α is not a constant but varies with emitter current I_E, collector voltage V_{CB} and temperature.

General Transistor Equation In the active region of the transistor, the emitter is forward biased and the collector is reverse biased. The generalised expression for collector current I_C for collector junction voltage V_C and emitter current I_E is given by

$$I_C = -\alpha I_E + I_{CBO}(1 - e^{V_C/V_T}) \tag{13.4}$$

If V_C is negative and $|V_c|$ is very large compared with V_T, then the above equation reduces to

$$I_C = -\alpha I_E + I_{CBO} \tag{13.5}$$

If V_C, i.e. V_{CB}, is few volts, I_C is independent of V_C. Hence the collector current I_C is determined only by the fraction α of the current I_E flowing in the emitter.

Relation among I_C, I_B ***and*** I_{CBO} From Eqn. (13.5), We have

$$I_C = -\alpha I_E + I_{CBO}$$

Since I_C and I_E are flowing in opposite directions,

$$I_E = -(I_C + I_B)$$

Therefore, $$I_C = -\alpha[-(I_C + I_B)] + I_{CBO}$$

$$I_C - \alpha I_C = \alpha I_B + I_{CBO}$$

$$I_C(1-\alpha) = \alpha I_B + I_{CBO}$$

$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{I_{CBO}}{1-\alpha}$$

Since
$$\beta = \frac{\alpha}{1-\alpha} \tag{13.6}$$

the above expression becomes

$$I_C = (1+\beta)\, I_{CBO} + \beta\, I_B \tag{13.7}$$

Relation among I_C, I_B and I_{CEO} In the common-emitter (CE) transistor circuit, I_B is the input current and I_C is the output current. If the base circuit is open, i.e, $I_B = 0$, then a small collector current flows from the collector to emitter. This is denoted as I_{CEO}, the collector-emitter current with base open. This current I_{CEO} is also called the collector to emitter leakage current.

In this CE configuration of the transistor, the emitter-base junction is forward-biased and collector-base junction is reverse-biased and hence the collector current I_C is the sum of the part of the emitter current I_E that reaches the collector, and the collector-emitter leakage current I_{CEO}. Therefore, the part of I_E, which reaches collector is equal to $(I_C - I_{CEO})$.

Hence, the *large-signal current gain* (β) is defined as,

$$\beta = \frac{(I_C - I_{CEO})}{I_B} \tag{13.8}$$

From the equation, we have

$$I_C = \beta I_B + I_{CEO} \tag{13.9}$$

Relation between I_{CBO} and I_{CEO} Comparing Eqs (13.7) and (13.9), we get the relationship between the leakage currents of transistor common-base (CB) and common-emitter (CE) configurations as

$$I_{CEO} = (1+\beta)\, I_{CBO} \tag{13.10}$$

From this equation, it is evident that the collector-emitter leakage current (I_{CEO}) in CE configuration is $(1+\beta)$ times larger than that in CB configuration. As I_{CBO} is temperature-dependent, I_{CEO} varies by large amount when temperature of the junctions changes.

Expression for Emitter Current

The magnitude of emitter-current is

$$I_E = I_C + I_B$$

Substituting Eqn. (13.7) in the above equation, we get

$$I_E = (1+\beta)\, I_{CBO} + (1+\beta)\, I_B \tag{13.11}$$

Substituting Eqn. (13.6) into Eqn. (13.11), we have

$$I_E = \frac{1}{1-\alpha} I_{CBO} + \frac{1}{1-\alpha} I_B \tag{13.12}$$

DC current gain (β_{dc} or h_{FE}) The dc current gain is defined as the ratio of the collector current I_C to the base current I_B. That is,

$$\beta_{\text{dc}} = h_{FE} = \frac{I_C}{I_B} \tag{13.13}$$

As I_C is large compared with I_{CEO}, the large signal current gain (β) and the d.c. current gain (h_{FE}) are approximately equal.

13.6 TYPES OF TRANSISTOR AMPLIFIER CONFIGURATION

When a transistor is to be connected in a circuit, one terminal is used as an input terminal, the other terminal is used as an output terminal and the third terminal is common to the input and output. Depending upon the input, output and common terminal, a transistor amplifier can be connected in three configurations. They are: (i) Common base (CB) configuration, (ii) Common emitter (CE) configuration, and (iii) Common collector (CC) configuration.

(i) CE configuration This is also called grounded base configuration. In this configuration, emitter is the input terminal, collector is the output terminal and base is the common terminal.

(ii) CB configuration This is also called grounded emitter configuration. In this configuration, base is the input terminal, collector is the output terminal and emitter is the common terminal.

(iii) CC configuration This is also called grounded collector configuration. In this configuration, base is the input terminal, emitter is the output terminal and collector is the common terminal.

The supply voltage connections for normal operation of an NPN transistor in the three configurations are shown in Fig. 13.6.

13.6.1 CB Configuration

The circuit diagram for determining the static characteristics curves of an NPN transistor in the common base configuration is shown in Fig. 13.7.

Input characteristics To determine the input characteristics, the collector-base voltage V_{CB} is kept constant at zero volt and the emitter current I_E is increased from zero in suitable equal steps by increasing V_{EB}. This is repeated for higher fixed values of V_{CB}. A curve is drawn between emitter current I_E and emitter-base voltage V_{EB} at constant collector-base voltage V_{CB}. The input characteristics thus obtained are shown in Fig. 13.8.

When V_{CB} is equal to zero and the emitter-base junction is forward biased as shown in the characteristics, the junction behaves as a forward biased diode so that emitter current I_E increases rapidly with small increase in emitter-base voltage V_{EB}. When V_{CB} is increased keeping V_{EB} constant, the width of the base region will decrease. This effect results in an increase of I_E. Therefore, the curves shift towards the left as V_{CB} is increased.

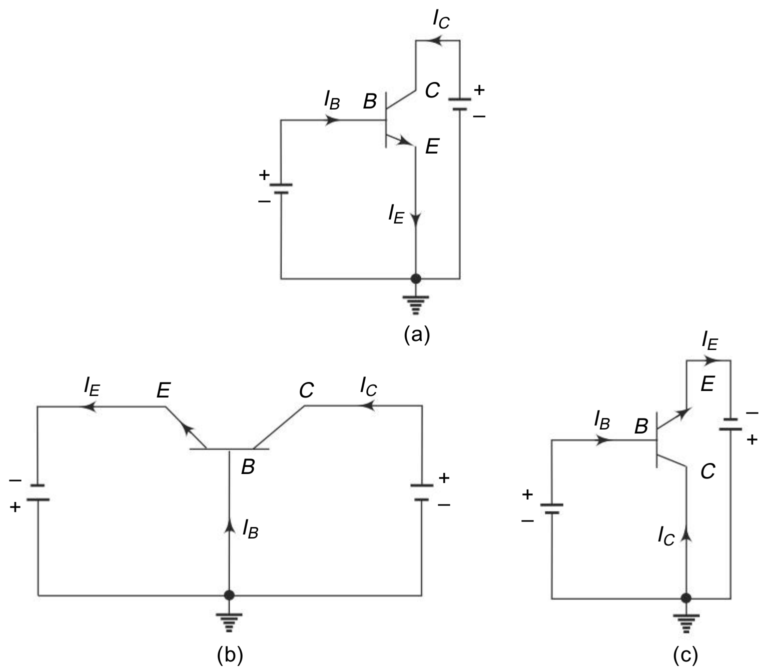


Fig. 13.6 Transistor configuration: (a) common base (b) common emitter and (c) common collector

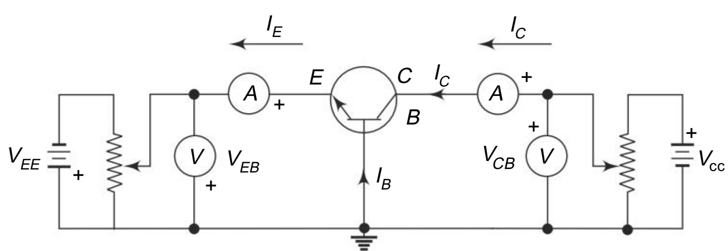


Fig. 13.7 Circuit to determine CB static characteristics

Output characteristics To determine the output characteristics, the emitter current I_E is kept constant at a suitable value by adjusting the emitter-base voltage V_{EB}. Then V_{CB} is increased in suitable equal steps and the collector current I_C is noted for each value of I_E. This is repeated for different fixed values of I_E. Now the curves of I_C versus V_{CB} are plotted for constant values of I_E and the output characteristics thus obtained is shown in Fig. 13.9.

From the characteristics, it is seen that for a constant value of I_E, I_C is independent of V_{CB} and the curves are parallel to the axis of V_{CB}. Further, I_C flows even when V_{CB} is equal to zero. As the emitter-base junction is forward biased, the majority carriers,

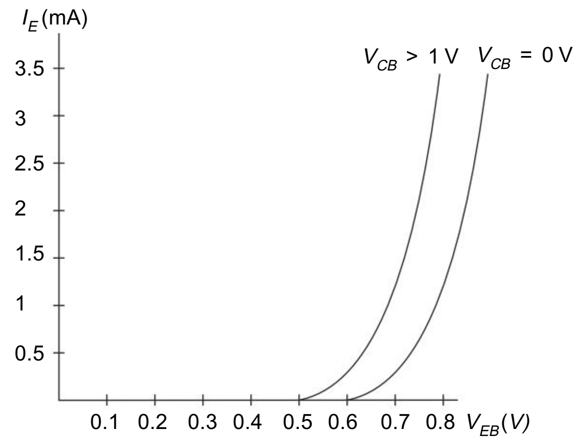


Fig. 13.8 CB Input characteristics

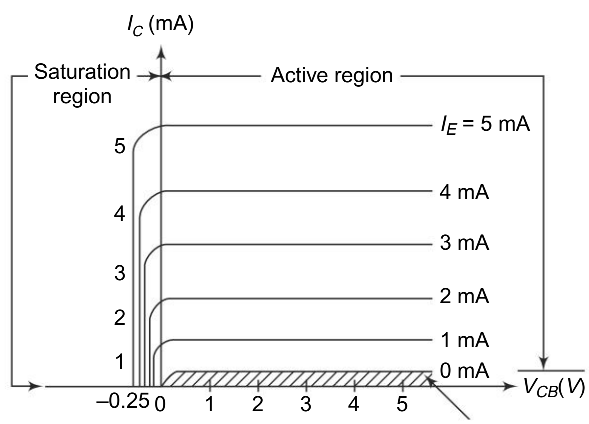


Fig. 13.9 CB Output Characteristics

i.e. electrons, from the emitter are injected into the base region. Due to the action of the internal potential barrier at the reverse biased collector-base junction, they flow to the collector region and give rise to I_C even when V_{CB} is equal to zero.

Early effect or base-width modulation As the collector voltage V_{CC} is made to increase the reverse bias, the space charge width between collector and base tends to increase, with the result that the effective width of the base decreases. This dependency of base-width on collector-to-emitter voltage is known as the *Early effect*. This decrease in effective base-width has three consequences:

(i) There is less chance for recombination within the base region. Hence, α increases with increasing $|V_{CB}|$.

(ii) The charge gradient is increased within the base, and consequently, the current of minority carriers injected across the emitter junction increases.

(iii) For extremely large voltages, the effective base-width may be reduced to zero, causing voltage breakdown in the transistor. This phenomenon is called the *punch through*.

For higher values of V_{CB}, due to Early effect, the value of α increases. For example, α changes, say from 0.98 to 0.985. Hence, there is a very small positive slope in the CB output characteristics and hence the output resistance is not zero.

Transistor parameters The slope of the CB characteristics will give the following four transistor parameters. Since these parameters have different dimensions, they are commonly known as common base *hybrid parameters* or *h-parameters*.

(i) Input impedance (h_{ib}) It is defined as the ratio of the change in (input) emitter voltage to the change in (input) emitter current with the (output) collector voltage V_{CB} kept constant. Therefore,

$$h_{ib} = \frac{\Delta V_{EB}}{\Delta I_E}, \ V_{CB} \text{ constant} \tag{13.14}$$

It is the slope of CB input characteristics I_E versus V_{EB} as shown in Fig. 13.8. The typical value of h_{ib} ranges from 20 Ω to 50 Ω.

(ii) Output admittance (h_{ob}) It is defined as the ratio of change in the (output) collector current to the corresponding change in the (output) collector voltage with the (input) emitter current I_E kept constant. Therefore,

$$h_{ob} = \frac{\Delta_{IC}}{\Delta V_{CB}}, \ I_E \text{ constant} \tag{13.15}$$

It is the slope of CB output characteristics I_C versus V_{CB} as shown in Fig. 13.9. The typical value of this parameter is of the order of 0.1 to 10 μ mhos.

(iii) Forward current gain (h_{fb}) It is defined as a ratio of the change in the (output) collector current to the corresponding change in the (input) emitter current keeping the (output) collector voltage V_{CB} constant. Hence,

$$h_{fb} = \frac{\Delta I_C}{\Delta I_E}, \ V_{CB} \text{ constant} \tag{13.16}$$

It is the slope of I_C versus I_E curve. Its typical value varies from 0.9 to 1.0.

(iv) Reverse voltage gain (h_{rb}) It is defined as the ratio of the change in the (input) emitter voltage and the corresponding change in (output) collector voltage with constant (input) emitter current, I_E. Hence,

$$h_{rb} = \frac{\Delta V_{EB}}{\Delta V_{CB}}, \ I_E \text{ constant} \tag{13.17}$$

It is the slope of V_{EB} versus V_{CB} curve. Its typical value is of the order of 10^{-5} to 10^{-4}.

13.6.2 CE Configuration

Input characteristics To determine the input characteristics, the collector to emitter voltage is kept constant at zero volt and base current is increased from zero in equal steps by increasing V_{BE} in the circuit shown in Fig. 13.10.

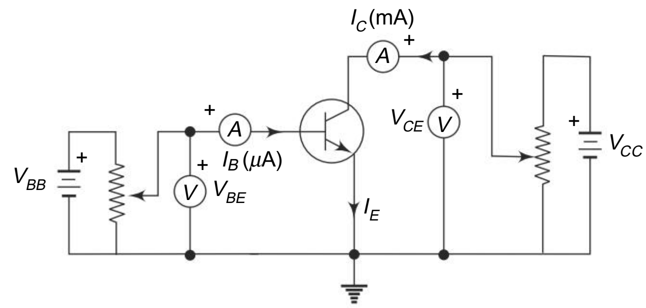


Fig. 13.10 Circuit to determine CE static characterisics

The value of V_{BE} is noted for each setting of I_B. This procedure is repeated for higher fixed values of V_{CE}, and the curves of I_B V_s V_{BE} are drawn. The input characteristics thus obtained are shown in Fig. 13.11.

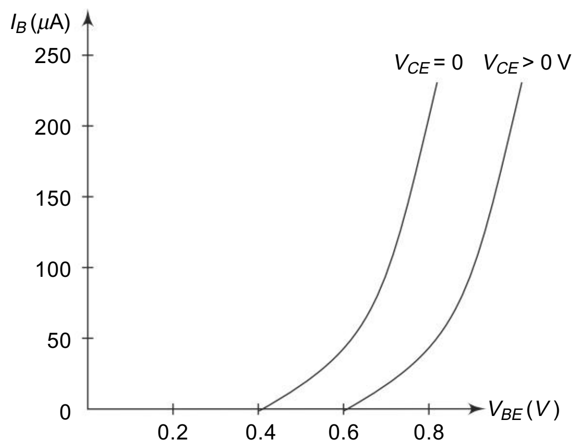


Fig. 13.11 CE input characteristics

When V_{CE} = 0, the emitter-base junction is forward biased and the junction behaves as a forward biased diode. Hence the input characteristic for V_{CE} = 0 is similar to that of a forward-biased diode. When V_{CE} is increased, the width of the depletion region at the reverse biased collector-base junction will increase. Hence the effective width of the base will decrease. This effect causes a decrease in the base current I_B. Hence, to get the same value of I_b as that for V_{CE} = 0, V_{BE} should be increased. Therefore, the curve shifts to the right as V_{CE} increases.

Output characteristics To determine the output characteristics, the base current I_B is kept constant at a suitable value by adjusting base-emitter voltage, V_{BE}. The magnitude of collector-emitter voltage V_{CE} is increased in suitable equal steps from zero and the collector current I_C is noted for each setting of V_{CE}. Now the curves of I_C versus V_{CE} are plotted for different constant values of I_B. The output characteristics thus obtained are shown in Fig. 13.12.

From Eqs (13.6) and (13.7), we have

$$\beta = \frac{\alpha}{1-\alpha} \qquad \text{and} \qquad I_C = (I + \beta)\, I_{CBO} + \beta I_B$$

For larger values of V_{CE}, due to Early effect, a very small change in α is reflected in a very large change in β. For example, when $\alpha = 0.98$, $\beta = \dfrac{0.98}{1 - 0.98} = 49$. If α increases to 0.985, then $\beta = \dfrac{0.985}{1 - 0.985} = 66$. Here, a slight increase in α by about 0.5% results in an increases in β by about 34%. Hence, the output characteristics of CE configuration show a larger slope when compared with CB configuration.

The output characteristics have three regions, namely, saturation region, cutoff region and active region. The region of curves to the left of the line OA is called the *saturation region* (hatched), and the line OA is called the saturation line. In this region, both junctions are forward biased and an increase in the base current does not cause a corresponding large change in I_C. The ratio of $V_{CE(\text{sat})}$ to I_C in this region is called saturation resistance.

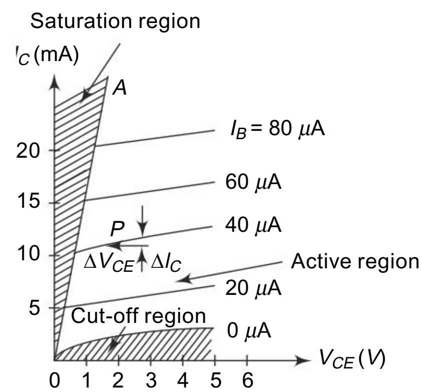


Fig. 13.12 CE output characteristics

The region below the curve for $I_B = 0$ is called the *cut-off region* (hatched). In this region, both junctions are reverse biased. When the operating point for the transistor enters the cut-off region, the transistor is OFF. Hence, the collector current becomes almost zero and the collector voltage almost equals V_{CC}, the collector supply voltage. The transistor is virtually an open circuit between collector and emitter.

The central region where the curves are uniform in spacing and slope is called the *active region* (unhatched). In this region, emitter-base junction is forward biased and the collector-base junction is reverse biased. If the transistor is to be used as a linear amplifier, it should be operated in the active region.

If the base current is subsequently driven large and positive, the transistor switches into the saturation region via the active region, which is traversed at a rate that is dependent on factors such as gain and frequency response. In this ON condition, large collector current flows and collector voltage falls to a very low value, called $V_{CE\text{sat}}$, typically around 0.2 V for a silicon transistor. The transistor is virtually a short circuit in this state.

High speed switching circuits are designed in such a way that transistors are not allowed to saturate, thus reducing switching times between ON and OFF times.

Transistor parameters The slope of the CE characteristics will give the following four transistor parameters. Since these parameters have different dimensions, they are commonly known as common emitter *hybrid parameters* or *h-parameters.*

(i) Input impedance (h_{ie}) It is defined as the ratio of the change in (input) base voltage to the change in (input) base current with the (output) collector voltage V_{CE} kept constant. Therefore,

$$h_{ie} = \frac{\Delta V_{BE}}{\Delta I_B},\ V_{CE} \text{ constant} \tag{13.18}$$

It is the slope of CE input characteristics I_B versus V_{BE} as shown in Fig. 13.11. The typical value of hie ranges from 500 to 2000 Ω.

(ii) Output admittance (h_{oe}) It is defined as the ratio of change in the (output) collector current to the corresponding change in the (output) collector voltage with the (input) base current I_B kept constant. Therefore,

$$h_{oe} = \frac{\Delta I_C}{\Delta V_{CE}},\ I_B \text{ constant} \tag{13.19}$$

It is the slope of CE output characteristic I_C versus V_{CE} as shown in Fig. 13.12. The typical value of this parameter is of the order of 0.1 to 10 μ mhos.

(iii) Forward current gain (h_{fe}) It is defined as a ratio of the change in the (output) collector current to the corresponding change in the (input) base current keeping the (output) collector voltage V_{CE} constant. Hence,

$$h_{fe} = \frac{\Delta I_C}{\Delta I_B},\ V_{CE} \text{ constant} \tag{13.20}$$

It is the slope of I_C versus I_B curve. Its typical value varies from 20 to 200.

(iv) Reverse voltage gain (h_{re}) It is defined as the ratio of the change in the (input) base voltage and the corresponding change in (output) collector voltage with constant (input) base current, I_B. Hence,

$$h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}},\ I_B \text{ constant} \tag{13.21}$$

It is the slope of V_{BE} versus V_{CE} curve. Its typical value is of the order of 10^{-5} to 10^{-4}.

13.6.3 CC Configuration

The circuit diagram for determining the static characteristics of an NPN transistor in the common collector configuration is shown in Fig. 13.13.

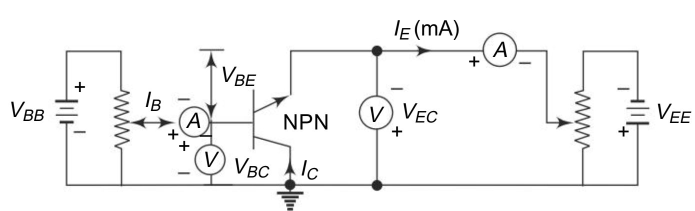


Fig. 13.13 Circuit to determine CC static characteristics

Input characteristics To determine the input characteristics, V_{EC} is kept at a suitable fixed value. The base-collector voltage V_{BC} is increased in equal steps and the corresponding increase in I_B is noted. This is repeated for different fixed values of V_{EC}. Plots of V_{BC} versus I_B for different values of V_{EC} shown in Fig. 13.14 are the input characteristics.

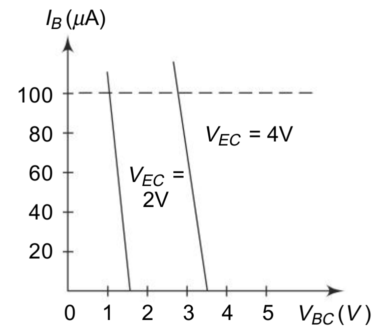


Fig. 13.14 CC input characteristics

Output characteristics The output characteristics shown in Fig. 13.15 are the same as those of the common emitter configuration.

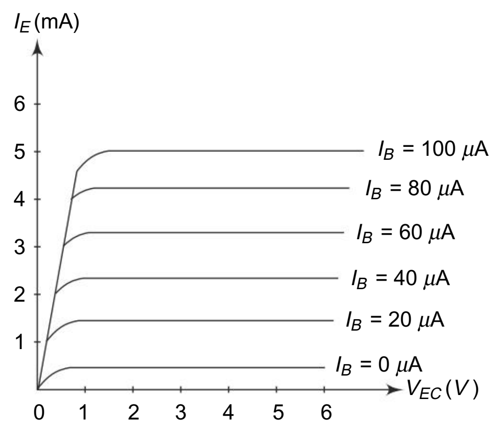


Fig. 13.15 CC output characteristics

13.6.4 Comparison

Table 13.1 A comparison of CB, CE and CC configurations

Property	*CB*	*CE*	*CC*
Input resistance	Low (about 100 Ω)	Moderate (about 750 Ω)	High (about 750 kΩ)
Output resistance	High (about 450 kΩ)	Moderate (about 45 kΩ)	Low (about 25 Ω)
Current gain	1	High	High
Voltage gain	About 150	About 500	Less than 1
Phase shift between input & output voltages	0 or 360°	180°	0 or 360°
Applications	for high frequency circuits	for audio frequency circuits	for impedance matching

13.6.5 Current Amplification Factor

In a transistor amplifier with a.c. input signal, the ratio of change in output current to the change in input current is known as the current amplification factor.

In the CB configuration the current amplification factor, $\alpha = \dfrac{\Delta I_C}{\Delta I_E}$ (13.22)

In the CE configuration the current amplification factor, $\beta = \dfrac{\Delta I_C}{\Delta I_B}$ (13.23)

In the CC configuration the current amplification factor, $\gamma = \dfrac{\Delta I_E}{\Delta I_B}$ (13.24)

Relationship between α and β We know that $\Delta I_E = \Delta I_C + \Delta I_B$

By definition, $\Delta I_C = \alpha \Delta I_E$

Therefore, $\Delta I_E = \alpha \Delta I_E + \Delta I_B$

i.e $\Delta I_B = \Delta I_E (1 - \alpha)$

Dividing both sides by ΔI_C, we get

$$\frac{\Delta I_B}{\Delta I_C} = \frac{\Delta I_E}{\Delta I_C}(1 - \alpha)$$

Therefore,

$$\frac{1}{\beta} = \frac{1}{\alpha}(1 - \alpha)$$

$$\beta = \frac{\alpha}{(1 - \alpha)}$$

Rearranging, we also get $\alpha = \dfrac{\beta}{(1 + \beta)}$ or $\dfrac{1}{\alpha} - \dfrac{1}{\beta} = 1$ (13.25)

From this relationship, it is clear that as a approaches unity, β-approaches infinity.

The CE configuration is used for almost all transistor applications because of its high current gain, β.

Relation among α, β and γ In the CC transistor amplifier circuit, I_B is the input current and I_E is the output current.

From Eq. (13.22), $\gamma = \dfrac{\Delta I_E}{\Delta I_B}$

Substituting $\Delta I_B = \Delta I_E - \Delta I_C$, we get

$$\gamma = \frac{\Delta I_E}{\Delta I_E - \Delta I_C}$$

Dividing the numerator and denominator on RHS by ΔI_E, we get

$$\gamma = \frac{\dfrac{\Delta I_E}{\Delta I_E}}{\dfrac{\Delta I_E}{\Delta I_E} - \dfrac{\Delta I_C}{\Delta I_E}} = \frac{1}{1 - \alpha}$$

Therefore, $$\gamma = \frac{1}{1-\alpha}(\beta + 1) \tag{13.26}$$

13.7 TRANSISTOR AS AN AMPLIFIER

A load resistor R_L is connected in series with the collector supply voltage V_{CC} of CB transistor configuration as shown in Fig. 13.16.

A small change in the input voltage between emitter and base, say ΔV_i, causes a relatively larger change in emitter current, say ΔI_E. A fraction of this change in current is collected and passed through R_L and is denoted by symbol α'. Therefore the corresponding change in voltage across the load resistor R_L due to this current is $\Delta V_0 = \alpha' R_L \Delta I_E$.

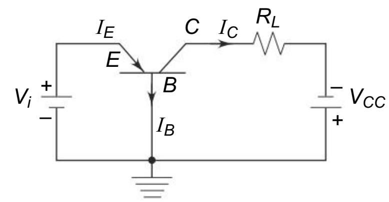


Fig. 13.16 Common base transistor configuration

Here, the voltage amplification $A_v = \dfrac{\Delta V_0}{\Delta V_1}$ is greater than unity and thus the transistor acts as an amplifier.

13.8 LARGE SIGNAL DC, AND SMALL SIGNAL CE VALUES OF CURRENT GAIN

We know from the characteristics of CE configuration, the current amplification factor is $\beta \equiv \dfrac{\alpha}{1-\alpha}$.

and $$I_C = (1 + \beta)\, I_{CBO} + \beta I_B$$

The above equation can be expressed as

$$I_C - I_{CBO} = \beta I_{CBO} = \beta I_{CBO} + \beta I_B$$

Therefore, $$\beta = \frac{I_C - I_{CBO}}{I_B - (-I_{CBO})}$$

The output characteristics of CE configuration show that in the *cut-off* region, the values $I_E = 0$, $I_C = I_{CBO}$ and $I_B = -I_{CBO}$. Therefore, the above equation gives the ratio of the collector-current increment to the base-current change from cut-off to I_B, and hence β *is called the large-signal current gain of common gain of common-emitter transistor.* The d.c. current gain of the transistor is given by

$$\beta_{\text{d.c.}} = h_{FE} = \frac{I_C}{I_B}$$

Based on this h_{FE} value, we can determine whether the transistor is in saturation or not. For any transistor, in general, I_B is large compared to I_{CBO}. Under this condition, the value of $h_{FE} = \beta$.

The small-signal CE forward short-circuit gain β' is defined as the ratio of a collector-current increment ΔI_C for a small base-current change ΔI_B at a fixed collector-to-emitter voltage V_{CE}.

i.e. $$\beta' \equiv \left.\frac{\partial I_C}{\partial I_B}\right|_{V_{CE}}$$

If β is independent of currents, then $\beta' = \beta = h_{FE}$. However, β is a function of current, then $\beta' = \beta + (I_{CBO} + I_B)\dfrac{\partial \beta}{\partial I_B}$. By using $\beta' = h_{fe}$ and $\beta = h_{FE}$. Therefore, the above equations becomes

$$h_{fe} = \frac{h_{FE}}{1 - (I_{CBO} + I_B)\dfrac{\partial h_{FE}}{\partial I_C}}$$

In Fig. 13.17, the h_{FE} versus I_C shows a maximum and hence $h_{fe} > h_{FE}$ for smaller currents, and $h_{fe} < h_{FE}$ for larger currents. Therefore, the above equation is valid only for the active region.

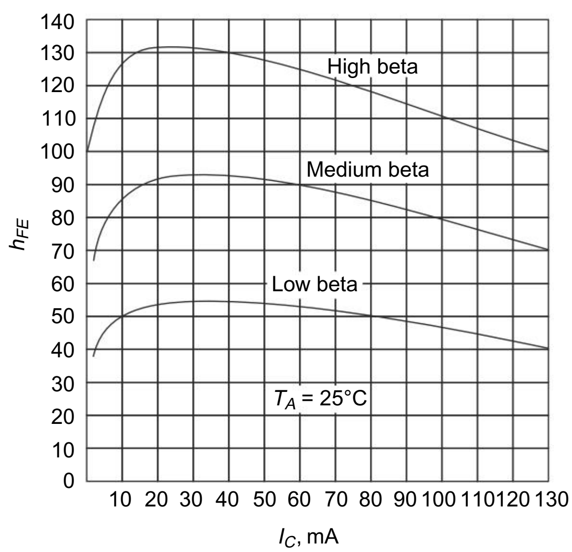


Fig. 13.17 Characteristics curves of d.c. current gain h_{FE} (at V_{CE} = – 0.25 V) versus collector current for low, medium and high beta values.

Example 13.1 In a common-base transistor circuit, the emitter current I_E is 10 mA and the collector current I_C is 9.8 mA. Find the value of the base current I_B.

Solution:

$$I_E = 10 \text{ mA and } I_C = 9.8 \text{ mA}$$

We know that emitter current is

$$I_E = I_B + I_C$$

i.e., $$10 = I_B + 9.8$$

Therefore, $$I_B = 0.2 \text{ mA}$$

Example 13.2 In a common-base connection, the emitter current I_E is 6.28 mA and the collector current I_C is 6.20 mA. Determine the common-base d.c. current gain.

Solution:

Given: $I_E = 6.28$ mA and $I_C = 6.20$ mA

We know that common-base dc current gain,

$$\alpha = \frac{I_C}{I_E} = \frac{6.20 \times 10^{-3}}{6.28 \times 10^{-3}} = 0.987$$

Example 13.3 The common-base dc current gain of a transistor is 0.967. If the emitter current is 10 mA, what is the value of base current?

Solution:

Given $\alpha = 0.967$ and $I_E = 10$ mA.

The common-base dc current gain (α) is

$$\alpha = 0.967 = \frac{I_C}{I_E} = \frac{I_C}{10}$$

Therefore, $I_C = 0.967 \times 10 = 9.67$ mA

The emitter current $I_E = I_B + I_C$

i.e., $10 = I_B + 9.67$

Therefore, $I_B = 0.33$ mA

Example 13.4 The transistor has $I_E = 10$ mA and $\alpha = 0.98$. Determine the values of I_C and I_B.

Solution:

Given: $I_E = 10$ mA and $\alpha = 0.98$

the common-base dc current gain, $\alpha = \dfrac{I_C}{I_E}$

i.e., $0.98 = \dfrac{I_C}{10}$

Therefore $I_C = 0.98 \times 10 = 9.8$ mA

The emitter current $I_E = I_B + I_C$

i.e., $10 = I_B + 9.8$

Therefore, $I_B = 0.2$ mA

Example 13.5 If a transistor has a α of 0.97, find the value of β. If $\beta = 200$, find the value of α.

Solution:

If $\alpha = 0.97, \beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.97}{1-0.97} = 32.33$

If $\beta = 200, \alpha = \dfrac{\beta}{\beta+1} = \dfrac{200}{200+1} = 0.995.$

Example 13.6 A transistor has $\beta = 100$. If the collector current is 40 mA, find the value of emitter current.

Solution:

Given: $\beta = 100$ and $I_C = 40$ mA

$$\beta = 100 = \frac{I_C}{I_B} = \frac{40}{I_B}$$

Therefore, $I_B = \dfrac{40}{100} = 0.4$ mA and

$$I_E = I_B + I_C = (0.4 + 40) \times 10^{-3} = 40.4 \text{ mA}$$

Example 13.7 A transistor has $\beta = 150$. Find the collector and base currents, if $I_E = 10$ mA.

Solution:

Given: $\beta = 150$ and $I_E = 10$ mA

The common-base current gain, $\alpha = \dfrac{\beta}{\beta + 1} = \dfrac{150}{150 + 1} = 0.993$

Also, $\alpha = \dfrac{I_C}{I_E}$

i.e., $0.993 = \dfrac{I_C}{10}$

Therefore, $I_C = 0.993 \times 10 = 9.93$ mA

The emitter current $I_E = I_B + I_C$

i.e., $10 \times 10^{-3} = I_B + 9.93 \times 10^{-3}$

Therefore, $I_B = (10 - 9.93) \times 10^{-3} = 0.07$ mA

Example 13.8 Determine the values of I_B and I_E for the transistor circuit if $I_C = 80$ mA and $\beta = 170$.

Solution:

Given: $\beta = 170$ and $I_C = 80$ mA

We know that (β), $\beta = 170 = \dfrac{I_C}{I_B} = \dfrac{80 \times 10^{-3}}{I_B}$

Therefore, $I_B = \dfrac{80 \times 10^{-3}}{170} = 0.47$ mA

and $I_E = I_B + I_C = (0.47 + 80)$ mA $= 80.47$ mA

Example 13.9 Determine the values of I_C and I_E for the transistor circuit of $\beta = 200$ and $I_B = 0.125$ mA.

Solution:

Given: $I_B = 0.125$ mA and $\beta = 200$

Therefore, $\beta = 200 = \dfrac{I_C}{I_B} = \dfrac{I_C}{0.123 \times 10^{-3}}$

Therefore, $I_C = 200 \times 0.125 \times 10^{-3} = 25$ mA

and $I_E = I_B + I_C = (0.125 + 25) \times 10^{-3} = 25.125$ mA

Example 13.10 Determine the values of I_C and I_B for the transistor circuit if $I_E = 12$ mA and $\beta = 100$.

Solution:

Given: $I_E = 12$ mA and $\beta = 100$

We know that base current,

$$I_B = \frac{I_E}{1+\beta} = \frac{12 \times 10^{-3}}{1+100} = 0.1188 \text{ mA}$$

and collector current, $I_C = I_E - I_B = (12 - 0.1188) \times 10^{-3} = 11.8812$ mA

Example 13.11 A transistor has $I_B = 100\ \mu$A and $I_C = 2$ mA. Find (a) β of the transistor; (b) α of the transistor, (c) emitter current I_E, (d) if I_B changes by + 25 μA and I_C changes by + 0.6 mA, find the new value of β.

Solution:

Given: $I_B = 100\ \mu\text{A} = 100 \times 10^{-6}$ A and $I_C = 2 \text{ mA} = 2 \times 10^{-3}$ A.

(a) To find β of the transistor

$$\beta = \frac{I_C}{I_B} = \frac{2 \times 10^{-3}}{100 \times 10^{-6}} = 20$$

(b) To find α of the transistor

$$\alpha = \frac{\beta}{\beta + 1} = \frac{20}{1+20} = 0.952$$

(c) To find emitter current, I_E

$$I_E = I_B + I_C = 100 \times 10^{-6} + 2 \times 10^{-3} \text{ A}$$

$$= (0.01 + 2) \times 10^{-3} = 2.01 \times 10^{-3} \text{ A} = 2.01 \text{ mA}$$

(d) To find new value of β when $\Delta I_B = 25\ \mu$A and $\Delta I_C = 0.6\ \mu$A

Therefore, $I_B = (100 + 25) \text{ mA} = 125\ \mu$A

$I_C = (2 + 0.6) \text{ mA} = 2.6$ mA

New value of β of the transistor,

$$\beta = \frac{I_C}{I_B} = \frac{2.6 \times 10^{-3}}{125 \times 10^{-6}} = 20.8$$

Example 13.12 For a transistor circuit having $\alpha = 0.98$, $I_{CBO} = I_{CO} = 5\ \mu$A and $I_B = 100$ mA, find I_C and I_E.

Solution:

Given $\alpha = 0.98$, $I_{CBO} = I_{CO} = 5\ \mu$A and $I_B = 100\ \mu$A

The collector current is

$$I_C = \frac{\alpha . I_B}{1-\alpha} + \frac{I_{CO}}{1-\alpha} = \frac{0.98 \times 100 \times 10^{-6}}{1 - 0.98} = 5.15 \text{ mA.}$$

The emitter current is

$$I_E = I_B + I_C = 100 \times 10^{-6} + 5.15 \times 10^{-3} = 5.25 \text{ mA}$$

Example 13.13 A germanium transistor used in a complementary symmetry amplifier has $I_{CBO} = 10\ \mu$A at 27°C and $h_{FE} = 50$. (a) Find I_C when $I_B = 0.25$ mA and (b) assuming h_{FE} does not increase with temperature, find the value of new collector current, if the transistor's temperature rises to 50 °C.

Solution:

Given: $I_{CBO} = 10\ \mu A$ and $h_{FE}\ (= \beta) = 50$

(a) **To find the value of collector current when I_B** = 0.25 mA

$$I_C = \beta I_B + (1 + \beta)\, I_{CBO}$$
$$= 50 \times (0.25 \times 10^{-3}) + (1 + 50) \times (10 \times 10^{-6})$$
$$A = 13.01 \text{ mA}$$

(b) **To find the value of new collector current if temperature rises to 50ºC**

We know that I_{CBO} doubles for every 10°C rise in temperature. Therefore,

$$I'_{CBO}\ (\beta = 50) = I_{CBO} \times 2\frac{(T^2 - T^1)}{10} = 10 \times 2\frac{(50 - 27)}{10}\ \mu A$$
$$= 10 \times 2^{2.3}\ \mu A = 49.2\ \mu A$$

Therefore, the collector current at 50°C is

$$I_C = \beta \cdot I_B + (1 + \beta) I'_{CBO}$$
$$= 50 \times (0.25 \times 10^{-3}) + (1 + 50) \times 49.2 \times 10^{-6} \text{ A} = 15.01 \text{ mA}$$

Example 13.14 When the emitter current of a transistor is changed by 1 mA, there is a change in collector current by 0.99 mA. Find the current gain of the transistor.

Solution: The current gain of the transistor is

$$\alpha = \frac{\Delta I_C}{\Delta I_E} = \frac{0.99 \times 10^{-6}}{1 \times 10^{-3}} = 0.99$$

Example 13.15 The dc current gain of a transistor in CE mode is 100. Determine its d.c. current gain in CB mode.

Solution:

The d.c. current gain of the transistor in CB mode is $\alpha_{dc} = \dfrac{\beta_{dc}}{1 + \beta_{dc}}$.

$$= \frac{100}{1 + 100} = 0.99$$

Example 13.16 When I_E of a transistor is changed by 1 mA, its I_C changes by 0.995 mA. Find its common base current gain α, and common-emitter current gain β.

Solution: Common-base current gain is

$$\alpha = \frac{\Delta I_C}{\Delta I_E} = \frac{0.995 \times 10^{-3}}{1 \times 10^{-3}} = 0.995$$

Common-emitter current gain is

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.995}{1 - 0.995} = 199$$

Example 13.17 The current gain of a transistor in CE mode is 49. Calculate its common-base current gain. Find the base current when the emitter current is 3 mA.

Solution:

Given $\beta = 49$

We know that $\alpha = \dfrac{\beta}{1 + \beta}$

Therefore, the common base current gain is

$$\alpha = \frac{49}{1+49} = 0.98$$

We also know that $\alpha = \frac{I_C}{I_E}$

Therefore, $I_C = \alpha I_E = 0.98 \times 3 \times 10^{-3} = 2.94$ mA

Example 13.18 Determine I_C, I_E and α for a transistor circuit having $I_B = 15$ mA and $\beta = 150$.

Solution: The collector current,

$$I_C = \beta I_B = 150 \times 15 \times 10^{-6} = 2.25 \text{ mA}$$

The emitter current,

$$I_E = I_C + I_B = 2.25 \times 10^{-3} + 15 \times 10^{-6} = 2.265 \text{ mA}$$

Common-base current gain,

$$\alpha = \frac{\beta}{1+\beta} = \frac{150}{151} = 0.9934$$

Example 13.19 Determine the base, collector and emitter currents and V_{CE} for a CE circuit shown in Fig. 13.18. For V_{CC} = 10V, V_{BB} = 4V, R_B = 200 kΩ, R_C = 2 kΩ, V_{BE} (on) = 0. 7 V, β = 200.

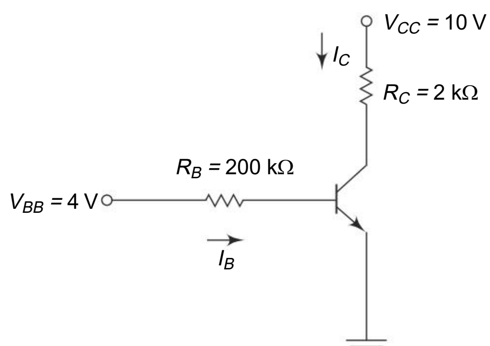


Fig. 13.18

Solution: Referring to Fig. 13.18 the base current is

$$I_B = \frac{V_{BB} - V_{BE}\,(\text{on})}{R_B} = \frac{4 - 0.7}{200 \times 10^3} = 16.5\ \mu\text{A}$$

The collector current is

$$I_C = \mu I_B = 200 \times 16.5 \times 10^{-6} = 3.3 \text{ mA}$$

The emitter current is

$$I_E = I_C + I_B = 3.3 \times 10^{-3} + 16.5 \times 10^{-6} = 3.3165 \text{ mA}$$

Therefore, $V_{CE} = V_{CC} - I_C R_C = 10 - 3.3 \times 10^{-3} \times 2 \times 10^3 = 3.4$ V

Example 13.20 Calculate the values of I_C and I_E for a transistor with $\alpha_{\text{d.c.}}$ = 0.99 and I_{CBO} = 5 μA. I_B is measured as 20 μA.

Solution:

Given: $\alpha_{\text{d.c.}} = 0.99$, $I_{CBO} = 5\ \mu$A and $I_B = 20\ \mu$A

$$I_C = \frac{\alpha_{\text{d.c.}}}{1-\alpha_{\text{d.c}}} + \frac{I_{CBO}}{1-\alpha_{\text{d.c.}}} = \frac{0.99 \times 20 \times 10^{-6}}{1-0.99} + \frac{5 \times 10^{-6}}{1-0.99} = 2.48 \text{ mA.}$$

Therefore, $I_E = I_B + I_C = 20 \times 10^{-6} + 2.48 \times 10^{-3} = 2.5$ mA

Example 13.21 The reverse leakage current of the transistor when connected in CB configuration is 0.2 μA and it is 18 μA when the same transistor is connected in CE configuration. Calculate $\alpha_{\text{d.c.}}$ and $\beta_{\text{d.c.}}$ of the transistor.

Solution:

The leakage current $I_{CBO} = 0.2\ \mu$A

$$I_{CEO} = 18\ \mu\text{A}$$

Assume that $I_B = 30\ \mu$A

$$I_E = I_B + I_C$$

$$I_C = I_E - I_B = \beta I_B + (1+\beta) I_{CBO}$$

We know that $I_{CEO} = \dfrac{I_{CBO}}{1-\alpha} = (1+\beta) I_{CBO}$

$$\beta = \frac{I_{CEO}}{I_{CBO}} - 1 = \frac{18}{0.2} - 1 = 89$$

$$I_C = \beta I_B + (1+\beta)\, I_{CBO}$$

$$= 89\,(30 \times 10^{-6}) + (1+89)(0.2 \times 10^{-6})$$

$$= 2.67 \text{ A}$$

$$\alpha_{\text{d.c.}} = 1 - \frac{I_{CBO}}{I_{CEO}} = 1 - \frac{0.2 \times 10^{-6}}{18 \times 10^{-6}} = 0.988$$

$$\beta_{\text{d.c.}} = \frac{I_C - I_{CBO}}{I_B - I_{CEO}}$$

$$= \frac{2.67 - 0.2 \times 10^{-6}}{30 \times 10^{-3} - 18 \times 10^{-6}} = 89$$

Example 13.22 If $\alpha_{\text{d.c.}} = 0.99$ and $I_{CEO} = 50\ \mu$A, find emitter current.

Solution:

Given: $\alpha_{\text{d.c.}} = 0.99$ and $I_{CBO} = 50\ \mu$A.

Assume that $I_B = 1$ mA

$$I_C = \frac{\alpha_{\text{d.c.}} I_B}{1-\alpha_{\text{d.c.}}} + \frac{I_{CBO}}{1-\alpha_{\text{d.c.}}} = \frac{0.99(1 \times 10^{-3})}{1-0.99} + \frac{50 \times 10^{-6}}{1-0.99}$$

$$= \frac{0.99 \times 10^{-3}}{0.01} + \frac{50 \times 10^{-6}}{0.01} = 99 \text{ mA} + 5 \text{ mA} = 104 \text{ mA.}$$

$$I_E = I_C + I_B = 104 \text{ mA} + 1 \text{ mA} = 105 \text{ mA.}$$

13.9 FIELD EFFECT TRANSISTOR

FET is a device in which the flow of current through the conducting region is controlled by an electric field. Hence the name Field Effect Transistor (FET). As current conduction is only by majority carries, FET is said to be a unipolar device.

Based on the construction, the FET can be classified into two types as Junction FET (JFET) and Metal Oxide Semiconductor FET (MOSFET).

Depending upon the majority carriers, JFET has been classified into two types named as (1) N-channel JFET with electrons as the majority carriers and (2) P-channel JFET with holes as the majority carriers.

13.9.1 Construction of N-Channel JFET

It consists of an N-type bar which is made of silicon. Ohmic contacts, (terminals) made at the two ends of the bar, are called Source and Drain.

Source (S) This terminal is connected to the negative pole of the battery. Electrons which are the majority carriers in the N-type bar enter the bar through this terminal.

Drain (D) This terminal is connected to the positive pole of the battery. The majority carriers leave the bar through this terminal.

Gate (G) Heavily doped P-Type silicon is diffused on both sides of the N-type silicon bar by which PN junctions are formed. These layers are joined together and the called Gate *G*.

Channel The region BC of the N-type bar in the depletion region is called the channel. Majority carriers move from the source to drain when a potential difference V_{DS} is applied between the source and drain.

13.9.2 Operation of N-channel JFET

When $V_{GS} = 0$ and $V_{DS} = 0$ When no voltage is applied between drain and source, and gate and source, the thickness of the depletion regions around the PN junction is uniform as shown in Fig. 13.19.

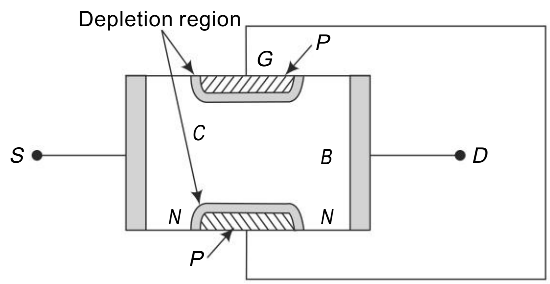


Fig. 13.19 JFET construction

When V_{DS} = 0 and V_{GS} is decreased from zero In this case PN junctions are reverse biased and hence the thickness of the depletion region increases. As V_{GS} is decreased from zero, the reverse bias voltage across the PN junction is increased and hence the thickness of the depletion region in the channel increases until the two depletion regions make contact with each other. In this condition, the channel is said to be cutoff. The value of V_{GS} which is required to cutoff the channel is called the cutoff voltage $V_{C.}$

When V_{GS} = 0 and V_{PS} is increased from zero Drain is positive with respect to the source with $V_{GS} = 0$. Now the majority carriers (electrons) flow through the N-channel from source to drain. Therefore the conventional current I_D flows from drain to source. The magnitude of the current will depend upon the following factors:

1. The number of the majority carriers (electrons) available in the channel, i.e. the conductivity of the channel.
2. The length l of the channel.
3. The cross sectional area A of the channel at B.
4. The magnitude of the appled voltage V_{DS}. Thus the channel acts as a resistor of resistance R given by

$$R = \frac{\rho l}{A} \tag{1}$$

$$I_D = \frac{V_{DS}}{R} = \frac{A\,V_{DS}}{\rho l} \tag{2}$$

where ρ is the resistivity of the channel. Because the resistance of the channel and the applied voltage V_{DS}, there is a gradual increase of positive potential along the channel from source to drain. Thus the reverse voltage across the PN junctions increases and hence the thickness of the depletion regions also increases. Therefore the channel is wedge shaped, as shown in Fig. 13.20.

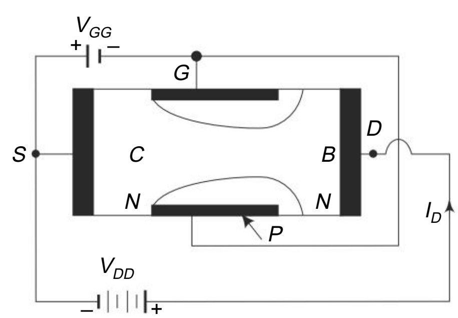


Fig. 13.20 JEFT Under Applied Bias

As V_{DS} is increased, the cross-sectional area of the channel will be reduced. At a certain value V_P of V_{DS}, the cross-sectional area at B becomes minimum. At this voltage, the channel is said to be pinched off and the drain voltage V_P is called the pinch-off voltage.

As a result of the decreasing cross-section of the channel with the increase of $V_{DS,}$ the following results are obtained.

(i) As V_{DS} is increased from zero, I_D increases along *OP*, and the rate of increase of I_D with V_{DS} decreases as shown bin Fig. 13.21.

(ii) When $V_{DS} = V_P$, I_D becomes maximum. When V_{DS} is increased beyond V_P, the length of the pinch-off region increases. Hence there is no further increase of $I_{D.}$

(iii) At a certain voltage corresponding to the point *B*, I_D suddenly increases. This effect is due to the avalanche multiplication of electrons caused by breaking of covalent bonds of silicon atoms in the depletion region between the gate and the drain. The drain voltage at which the breakdown occurs is denoted by BV_{DGO}. The variation of I_D with V_{DS} when $V_{GS} = 0$ is shown in Fig. 13.21 by the curve *OPBC*.

When V_{GS} is negative and V_{DS} is increased When the gate is maintained at a negative voltage less than the negative cutoff voltage, the reverse voltage across the junction is further increased. Hence for a negative value of V_{GS}, the curve of I_D versus V_{DS} is similar to that for $V_{GS} = 0$, but the values of V_P and BV_{DGO} are lower, as shown in Fig. 13.21.

From the curves, it is seen that above the pinch-off voltage, at a constant value of V_{DS}, I_D increases with an increase of V_{GS}. Hence a JFET is suitable for use as a voltage amplifier, similar to a transistor amplifier.

It can be seen from the curve that for the voltage $V_{DS} = V_P$, the drain current is not reduced to zero. If the drain current is to be reduced to zero, the ohmic voltage drop along the channel should also be reduced to zero. Further, the reverse biasing to the gate-source PN junction essential for pinching off the channel would also be absent.

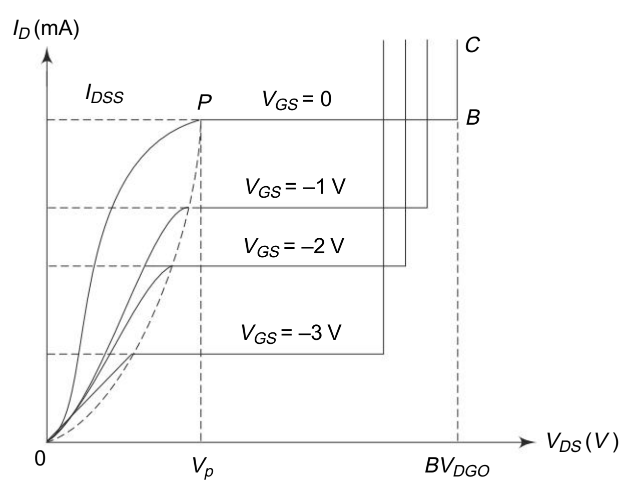


Fig. 13.21 Drain Characteristics

The drain current I_D is controlled by the electric field that extends into the channel due to reverse biased voltage applied to the gate; hence this device has been given the name "Field Effect Transistor."

In a bar of P-type semiconductor, the gate is formed due to N-type semiconductor. The working of the P-channel JFET will be similar to that of the N-channel JFET with proper alterations in the biasing circuits; in this case holes will be the current carriers instead of electrons. The circuit symbols for N-channel and P-channel JFET is are shown in Fig. 13.22. It should be noted that the direction of the arrow points in the direction of conventional current which would flow into the gate if the PN junction was forward biased.

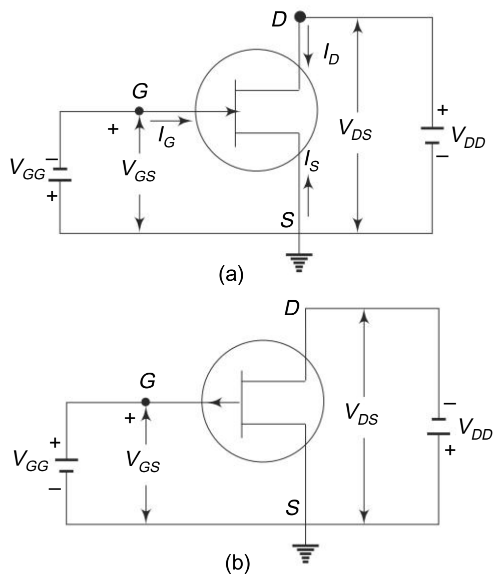


Fig. 13.22 Circuit Symbols for N and P-Channel JFET

13.9.3 Comparison of JFET and BJT

1. FET operation depends only on the flow of majority carriers—holes for P-channel FETs and electrons for N-channel FETs. Therefore they are called Unipolar devices. Bipolar transitor (BJT) operation depends on both minority and majority current carriers.
2. FETs are less noisy than BJTs.
3. FETs exhibit a much higher input impedance (> 100 MΩ) than BJTs.
4. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
5. FET is normally less sensitive to temperature.
6. FET amplifiers have less voltage gain and produce more signal distortion except for small signal operation.

13.9.4 Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

MOSFET is the common term of the Insulated Gate Field Effect Transistor (IGFET). There are two forms of MOSFET: (i) Enhancement MOSFET and (ii) Depletion MOSFET.

Principle By applying a transverse electric field across an insulator, deposited on the semiconducting material, the thickness and hence the resistance of a conducting channel of a semiconducting material can be controlled.

Enhancement MOSFET

Construction The construction of an N-channel Enhancement MOSFET is shown in Fig. 13.23. Two highly doped N^+ regions are diffused in a lightly doped substrate of P-type silicon substrate. One N^+ region is called the source *S* and the other one is called the drain *D*. They are separated by 1 mil (10^{-3} inch). A thin insulating layer of SiO_2 is grown over the surface of the structure and holes are cut into the oxide layer, allowing contact with source and drain. Then a thin layer of metal aluminum is formed over the layer of SiO_2. This metal layer covers the entire channel region and it forms the gate *G*.

The metal area of the gate, in conjunction with the insulating oxide layer of SiO_2 and the semiconductor channel forms a parallel plate capacitor. This device is called the insulated gate FET because of the insulating layer of SiO_2. This layer gives an extremely high input resistance for the MOSFET.

Operation If the substrate is grounded and a positive voltage is applied at the gate, the positive charge on *G* induces an equal negative charge on the substrate side between the source and drain regions. Thus an electric field is produced between the source and drain regions. The direction of the electric field is perpendicular to the plates of the capacitor through the oxide. The negative charge of electrons which are minority carriers in the P-type substrate forms an inversion layer. As the positive voltage on the gate increases, the induced negative charge in the semiconductor

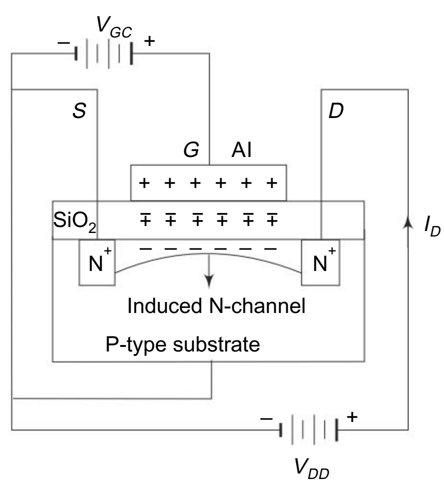


Fig. 13.23 N-Channel Enhancement MOSFET

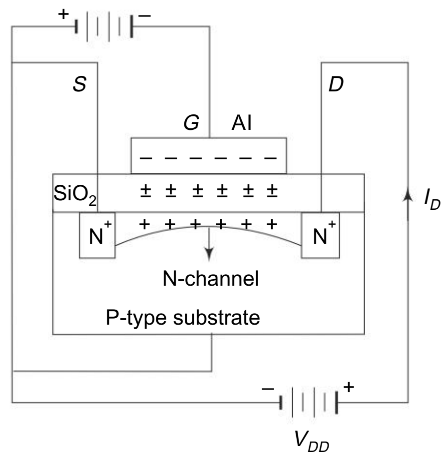


Fig. 13.24 N-Channel Depletion MOSFET

increases. Hence the conductivity increases and current flows from source to drain through the induced channel. Thus the drain current is enhanced by the positive gate voltage as shown in Fig. 13.25.

Depletion MOSFET The construction of an N-channel depletion MOSFET is shown in Fig. 13.24 where an N-channel is diffused between the source and drain to the basic structure of MOSFET.

With $V_{GS} = 0$ and the drain D at a positive potential with respect to the source, the electrons (majority carriers) flow through the N-channel from S to D. Therefore, the conventional current I_D flows through the channel D to S. If the gate voltage is made negative, positive charge consisting of holes is induced in the channel through SiO_2 of the gate-channel capacitor. The introduction of the positive charge causes depletion of mobile electrons in the channel. Thus a depletion region is produced in the channel. The shape of the depletion region depends on V_{GS} and V_{DS}. Hence the channel will be wedge shaped as shown in Fig. 13.25. When V_{DS}, is increased, I_D

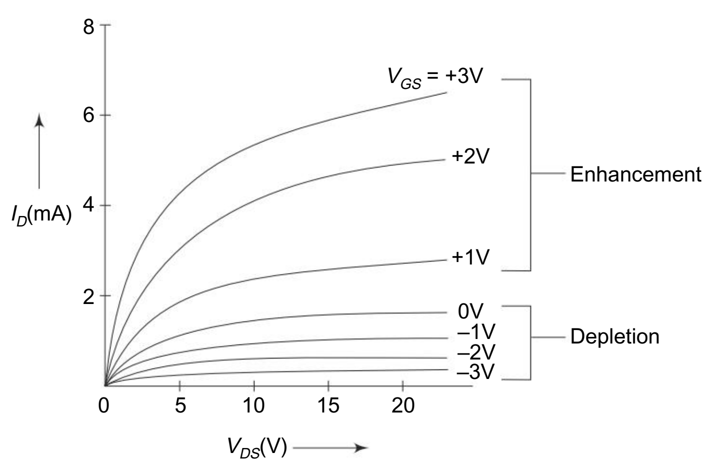


Fig. 13.25 Volt-Ampere Characteristics of MOSFET

increases and it becomes practically constant at a certain value of V_{DS}, called the pinch-off voltage. The drain current I_D almost gets saturated beyond the pinch-off voltage.

Since the current in an FET is due to majority carriers (electrons for an N-type material), the induced positive charges make the channel less conductive, and I_D drops as V_{GS} is made negative.

The depletion MOSFET may also be operated in an enhancement mode. It is only necessary to apply a positive gate voltage so that negative charges are induced into the N-type channel. Hence the conductivity of the channel increases and I_D increases. The volt-ampere characteristics are indicated in Fig. 13.25.

13.10 THYRISTOR

Thyristor, in general, is a semiconductor device having three or more junctions. Such a device acts as a switch without any bias and can be fabricated to have voltage ratings of several hundred volts and current ratings from a few amperes to almost thousand amperes. The family of thyristors consists of PNPN diode (Shockley diode), SCR, TRIAC, DIAC, UJT etc.

13.10.1 PNPN diode (Shockley diode)

As showing in Fig. 13.26, it is a four-layer PNPN silicon device with two terminals. When an external voltage is applied to the device in such a way that anode is positive with respect to cathode, junctions J_1 and J_3 are forward biased and J_2 is reverse biased. Then the applied voltage appears across the reverse bias junction J_2. Now the current flowing through the device is only reverse saturation current.

However, as this applied voltage is increased, the current increases slowly until the so called firing or breakover voltage (V_{BO}) is reached. Once firing takes place, the current increases abruptly and the voltage drop across the device decreases sharply. At this point, the diode switches over from 'OFF' to 'ON' state. Once the device is fired into conduction, a minimum amount of current known as *holding current*, I_H,

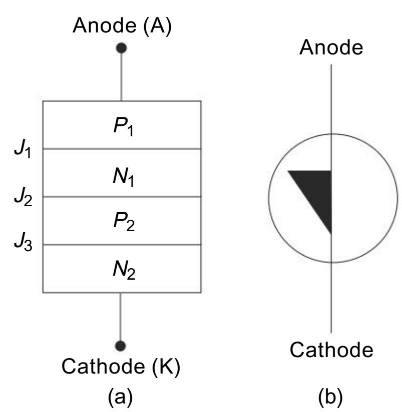


Fig. 13.26 PN-PN Diode: (a) Basic Structure and (b) Circuit Symbol

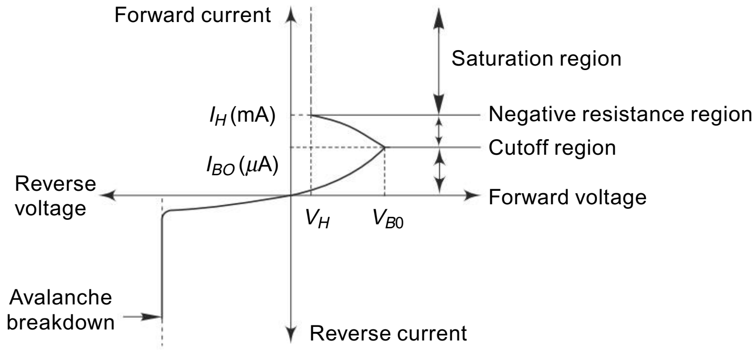


Fig. 13.27 Characteristics Curve of PNPN Diode

is required to flow to keep the device in ON state. To turn the device OFF from ON state, the current has to be reduced below I_H by reducing the applied voltage close to zero, i.e. below *holding voltage*, V_H. Thus the diode acts as a switch during forward bias condition. The characteristic curve of a PNPN diode is shown in Fig. 13.27.

13.10.2 SCR (Silicon Controlled Rectifier)

The basic structure and circuit symbol of SCR is shown in Fig. 13.28. It is a four-layer, three-terminal device in which the end P-layer acts as anode, the end N-layer acts as cathode and P-layer nearer to cathode acts as gate.

Characteristics of SCR The characteristics of SCR are shown in Fig. 13.29. SCR acts as a switch when it is forward biased. When the gate current $I_G = 0$, operation of SCR is similar to PNPN diode. When $I_G < 0$, the amount of reverse bias

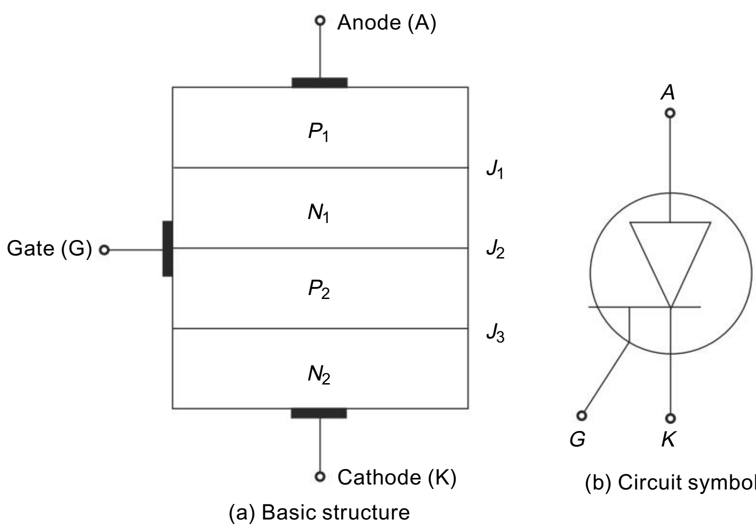


Fig. 13.28 Basic Structure and Circuit Symbol of SCR

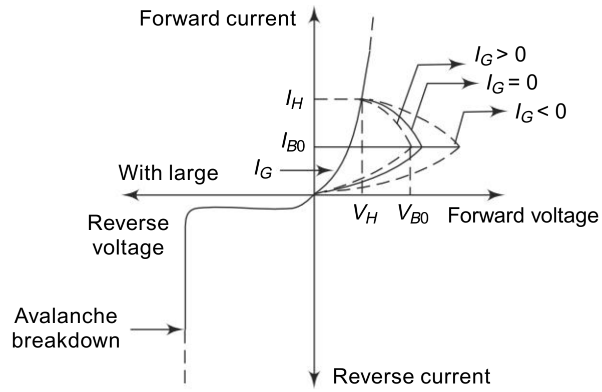


Fig. 13.29 Characteristics of SCR

applied to J_2 is increased. So the breakover voltage V_{BO} is increased. When $I_G > 0$, the amount of reverse bias applied to J_2 is decreased thereby decreasing the break-over voltage. With very large positive gate current, breakover may occur at a very low voltage such that the characteristic of SCR is similar to that of ordinary PN diode. As the voltage at which SCR is switched 'ON' can be controlled by varying the gate current I_G, it is commonly called as controlled switch. Once SCR is turned ON, the gate loses control, i.e. the gate cannot be used to switch the device OFF. One way to turn the device OFF is that the anode current is lowered below I_H by reducing the supply voltage below V_H, keeping the gate open.

SCR is used in relay control, motor control, phase control, heater control, battery chargers, inverters, regulated power supplies and as static switches.

Two Transistor Version of SCR As shown in Fig. 13.30, SCR can be split into two parts and displaced mechanically from one another but connected electrically.

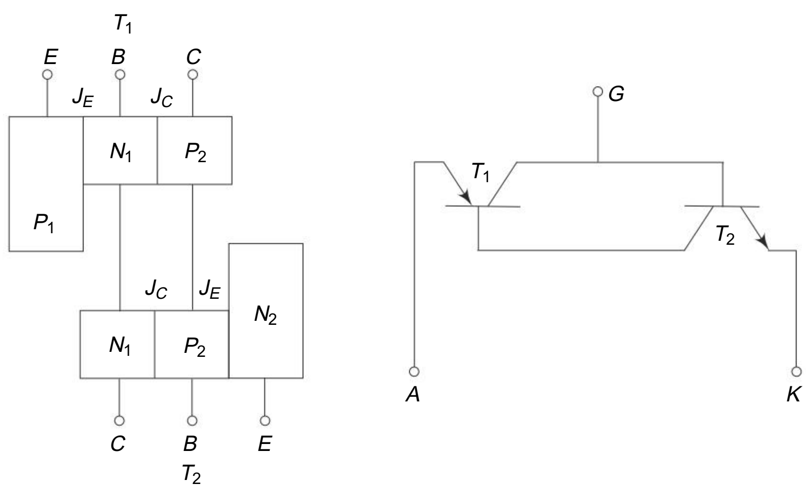


Fig. 13.30 Two Transistor Version of SCR

Thus the device may be considered to be constituted by two transistors T_1 (PNP) and T_2 (NPN) connected back to back. When gate is kept open, i.e. $I_G = 0$, SCR acts as a PNPN diode.

13.10.3 TRIAC (Triode ac switch)

TRIAC is a three terminal semiconductor switching device which can control alternating current in a load. The basis structure and equivalent circuit of a TRIAC are shown in Fig. 13.31. TRIAC is equivalent to two SCRs connected in parallel but in the reverse direction as shown in Fig. 13.32. So TRIAC will act as a switch for both directions. The characteristics of TRIAC are shown in Fig. 13.33.

Triac is used for light control, motor speed control and as static switch to turn ac power ON and OFF.

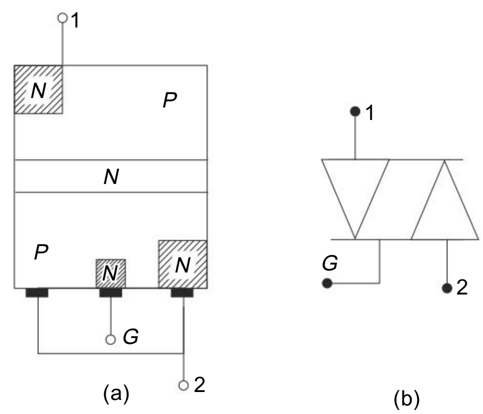


Fig. 13.31 TRIAC: (a) Basic Structure and (b) Circuit Symbol

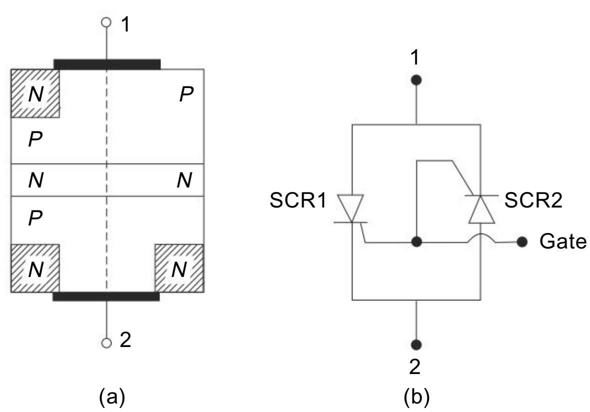


Fig. 13.32 Two-SCR Version of TRIAC: (a) Basic Structure and (b) Equivalent Circuit

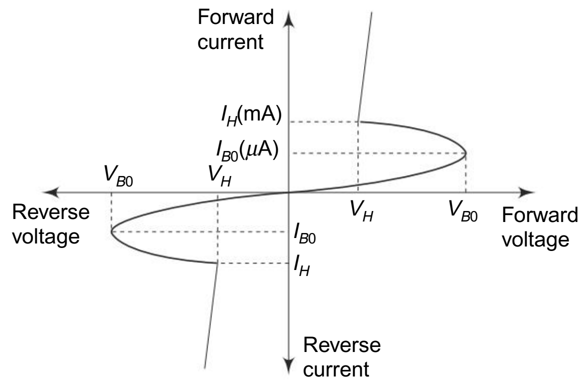


Fig. 13.33 Characteristics of TRIAC

TRIAC is used for light control, motor speed control and as static switch to turn ac power ON and OFF.

13.10.4 DIAC (Diode as switch)

As shown in Fig. 13.34, DIAC is a three layer, two terminal device. It acts as a bidirectional diode. It does not have any gate terminal. From the characteristics of DIAC shown in 13.35, it acts as a switch in both directions.

DIAC is used as triggering device in TRIAC phase control circuits used for light dimming, motor speed control and heater control.

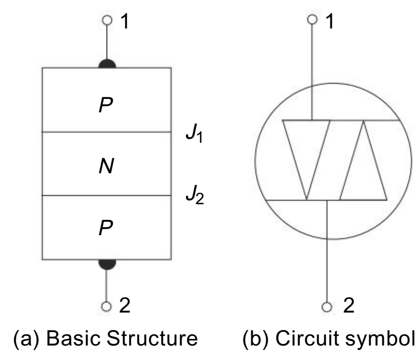


Fig. 13.34 DIAC

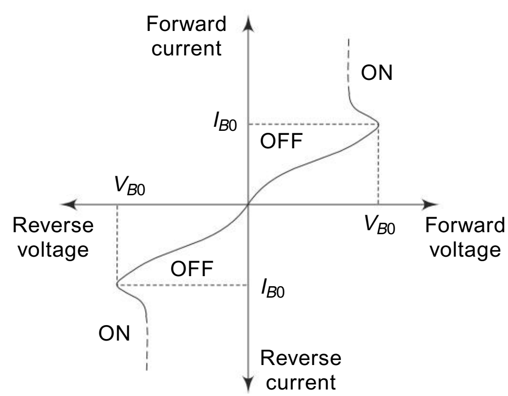


Fig. 13.35 Characteristics of DIAC

13.10.5 UJT (Unijunction Transistor)

UJT is a three terminal semiconductor device. As it has only one PN junction and three leads, it is commonly called as Unijunction Transistor.

The basic structure of UJT is shown in Fig. 13.36 (a). It consists of a lightly-doped silicon bar with a heavily-doped P-type material alloyed to its one side closer to B_2 for producing single PN junction. The circuit symbol of UJT is shown in Fig. 13.36 (b). Here the emitter leg is drawn at an angle to the vertical and the arrow indicates the direction of the conventional current.

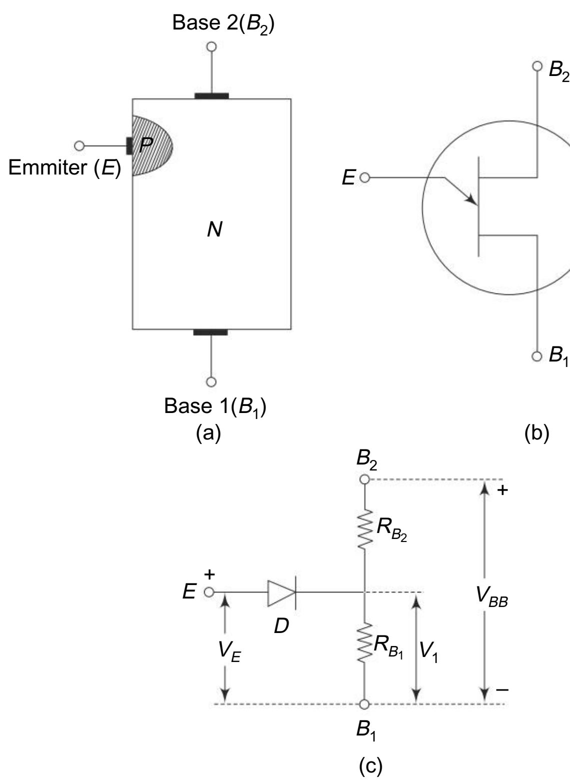


Fig. 13.36 UJT: (a) Basic Structure (b) Circuit Symbol and (c) Equivalent Circuit

Characteristics of UJT Referring to Fig. 13.36 (c), the interbase resistance between B_2 and B_1 of the silicon bar is $R_{BB} = R_{B1} + R_{B2}$. With emitter terminal open, if voltage V_{BB} is applied between the two bases, a voltage gradient is established along the N-type bar. The voltage drop across R_{B1} is given by $V_1 = \eta\ V_{BB}$ where the intrinsic stand-off ratio $\eta = R_{B1}\ (R_{B1} + R_{B2})$. This voltage V_1 reverse biases the PN junctions and emitter current is cut off. But a small leakage current flows from B_2 to emitter due to minority carriers. If a positive voltage V_E is applied to the emitter, the PN junction will remain reverse biased so long as V_E is less than V_1. If V_E exceeds V_1 by the cutin voltage V_r, the diode becomes forward biased. Under this condition, holes are injected into N-type bar. These holes are repelled by the terminal B_2 and are attracted by the terminal B_1. Accumulation of holes in E to B_1 region reduces the resistance in this section and hence emitter current I_E is increased and is limited by V_E. The device is now in the 'ON' state.

If a negative voltage is applied to the emitter, PN junction remains reverse biased and the emitter current is cut off. The device is now in the 'OFF state.

As shown in Fig. 13.37, up to the peak point P, the diode is reverse biased. At P, the diode starts conducting and holes are injected into the N-layer. Hence resistance decreases thereby decreasing V_E for the increase I_E. So there is a negative resistance region from peak point P to valley point V. After the valley point, the device is driven into saturation and behaves like a conventional forward biased PN junction diode.

A unique charateristics of UJT is, when it is triggered, the emitter current increases regeneratively until it is limited by emitter power supply. Due to this negative resistance property, UJT can be employed in a variety of applications, viz. sawtooh wave generator, pulse generator, switching, timing and phase control circuits.

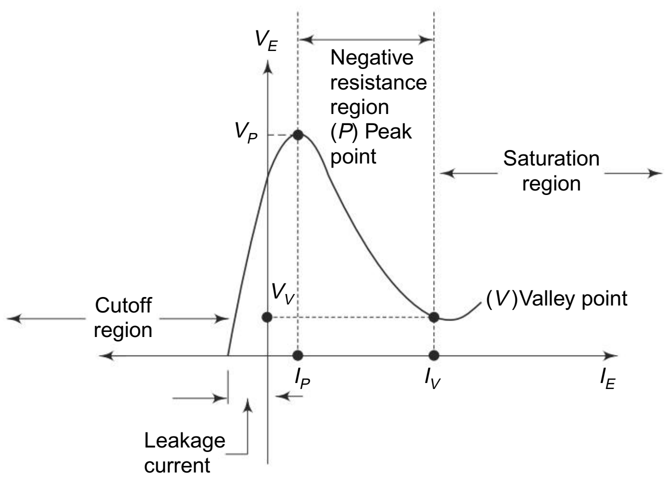


Fig. 13.37 Characteristics of UJT

13.11 OPTO-ELECTRONIC DEVICES

When radiation is incident on a semiconductor, some absorption of light by the material takes place, and its conductivity increases. This effect is called photoconductive effective which is described below.

Energy content of a photon is $E = h_f$, where h is the Planck's constant (6.626×10^{-34} J – s) and f is the frequency of the incident light. If frequency f is very low so that $E < E_g$ where E_g is the forbidden band energy between valence band and conduction band, the energy is inadequate in transferring an electron from valence band to conduction band and hence light passes through the material with very little absorption. However, if $E \geq E_g$, electrons in the valence band absorb the incident photons and get shifted to the conduction band. Thus electron-hole pairs are generated by the incident light in addition to those created thermally. The increased current results in increased conductivity. Hence such a material is called photoconductor or photoresistor.

Therefore, for photoconduction to take place in an intrinsic semiconductor, the photon must possess energy atleast equal to the forbidden energy gap E_g. Thus the minimum frequency f_c to cause photoconduction is given by

$$f_c = \frac{E_g}{h}$$

13.11.1 Photoconductive Cell

Photoconductive cell or Light Dependent Resistor (LDR) is made of a thin layer of semiconductor material such as cadmium sulfide or lead sulfide. The semiconductor layer is enclosed in a sealed housing. A glass window in the housing permits light to fall on the active material of the cell.

Photoconductive cell (PC) exhibits the peculiar property that its resistance decreases in the presence of light and increases in the absence of light. The cell simply acts as a conductor whose resistance changes when illuminated.

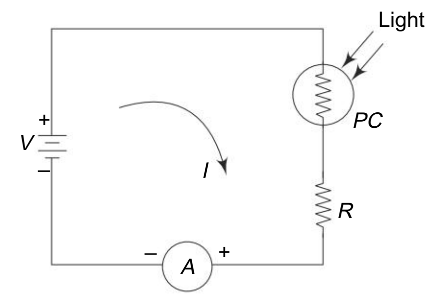


Fig. 13.38 Photoconductive Cell Connected in a Simple Circuit

A simple circuit for a photoconductive cell is shown in Fig. 13.38. Here the resistance of the photoconductive cell, in series with R, limits the amount of current l in the circuit. The ammeter A is used to measure the current l. When no light falls on the cell, its resistance is very high and the current l is low. Hence the voltage drop V_0 across R is relatively low. When the cell is illuminated, its resistance becomes very low. Hence the current l increase and the voltage V_0 increases. Thus the simple circuit arrangement with slight modification can be used in control circuits to control the current.

13.11.2 Photovoltaic Cell

Photovoltaic cell, a light-sensitive semiconductor device, produces a voltage when illuminated which may be used directly to supply small amount of electric power. The voltage increases as the intensity of light falling on the semiconductor junction of this cell increases. The photovoltaic cell consists of a piece of semiconductor material such as silicon, germanium or selenium which is bonded to a metal plate, as shown in Fig. 13.39 (a). The current symbol for photovoltaic cells are used in the low power devices

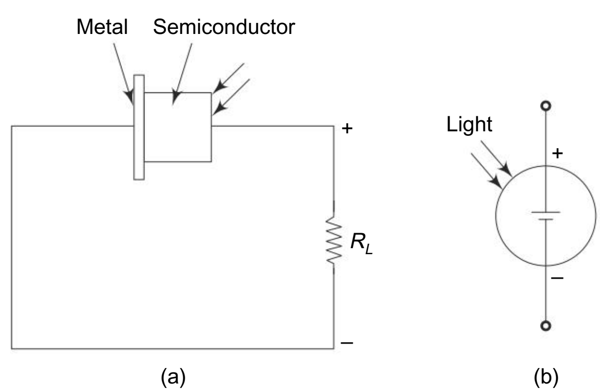


Fig. 13.39 (a) Construction and (b) Circuit Symbol for Photovoltaic Cell

such as light meters. Nowadays, with an improvement in the efficiency of these cells, more power has been produced, as in solar cells which are photovoltaic devices.

13.11.3 Solar Cell

When sun light is incident on a photovoltaic cell, it is converted into electric energy. Such an energy converter is called Solar cell or Solar battery and is used in satellites to provide the electrical power. This cell consists of a single semiconductor crystal which has been doped with both *P* and *N* type materials. When light falls on the PN junction, a voltage appears across the junction. About 0.6 V is developed by the solar cell in bright sunlight. The amount of power the cell can deliver depends on the extent of its active surface. An average cell will produce about 30 mW per sq. in. of surface, opeatring in a load of 4 Ω. To increase the power output, banks of cells are used in series an parallel combinations.

13.11.4 Phototube

The phototube is a radiant energy device that controls its electron emission when exposed to incident light. Figure 13.40 shows the circuit diagram of a phototube. The anode and the photosensitive cathode are placed in a high vacuum glass envelope. When sufficient voltage is applied between the photocathode and the anode, the collector current is directly proportional to the amount of incident light.

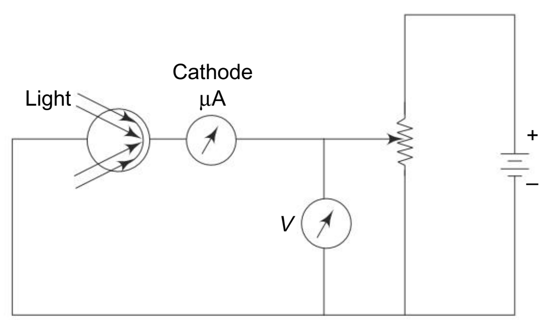


Fig. 13.40 Circuit Diagram of a Phototube

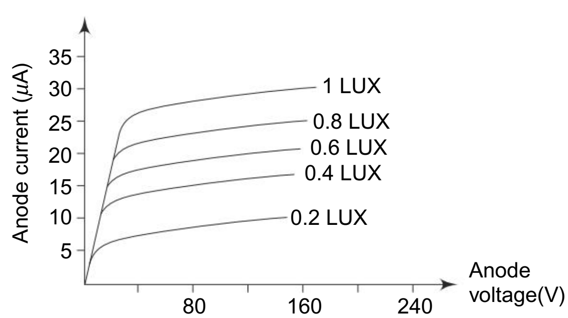


Fig. 13.41 Characteristics Curves of a Phototube

Typical voltage-current characteristics of a high vacuum phototube are shown in Fig. 13.41. The current through the tube is extremely small, usually in the range of a few microamperes. Hence the phototube is connected to an amplifier to provide an useful output.

13.11.5 Photomultiplier

Emitted currents from photoelectric surfaces are very small, especially with low light levels. These currents can be directly amplified in a device called *electron multiplier* or *photomultiplier.*

The photomultiplier tube consists of an evacuated glass envelope containing a photocathode, an anode and several additional electrons, called *dynodes*, each at a higher voltage than the previous dynode.

Photomultiplier or Multiplier phototube uses secondary emission to provide current multiplication in excess of a factor of 10^6. Figure 13.42 shows the arrangements of a photomultiplier. It consists of six dynodes which are maintained at increasing potentials in sequence from photocathode C to the anode A. When light falls on the cathode, electrons are emitted and directed at a high velocity toward dynode d_1 which is a higher potential. They bombard the treated surface of dynode d_1 which has a secondary emission coefficient, δ, above unity and dynode d_1 liberates secondary electrons for every primary electron striking it. These secondary electrons are focused to a second dynode d_2 which is at a relatively higher potential above that of d_1. This surface may also have a secondary emission coefficient, δ, so that δ^2 electrons leave due to every electron originally leaving the photoelectric cathode C. This process of secondary emission is repeated 'n' times on 'n' electrodes or dynodes as they are called, giving on overall current gain of δ^n. In practice, δ may be in the range of 5 to 10, and as many as 9 cascaded dynodes are used for current gains as high as 10^9. The dynodes are shaped to form curved electric fields which will focus

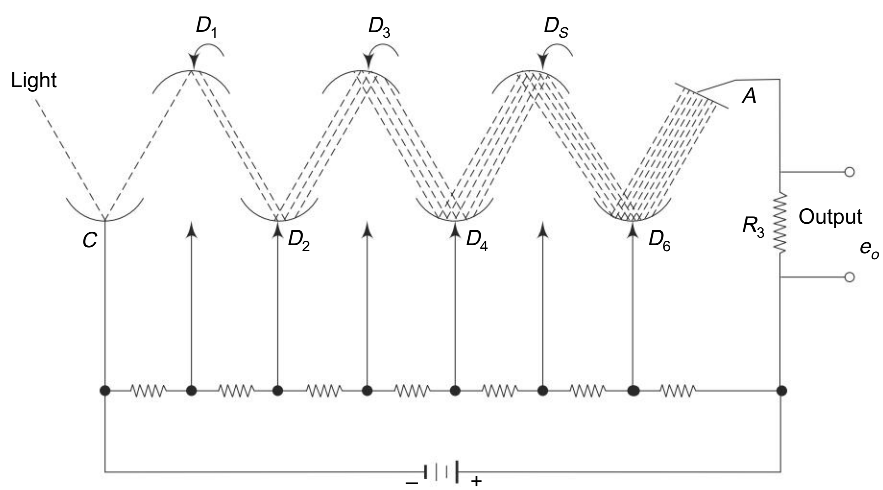


Fig. 13.42 Photomultiplier Arrangement

the electrons on to each succeeding dynode. If the electrons are deflected from their normal path between stages due to magnetic fields and miss a dynode, the gain falls. To minimise this effect, μ-metal magnetic shields are often placed around the photomultiplier tube. Thus, the original emission of electrons from the photocathode are multiplied many times and are finally collected by the anode.

The characteristics of a photomultiplier depend upon the voltage per dynode and upon the potential between the last dynode and collector. Performance in terms of a last dynode-to-collector voltage is shown in Fig. 13.43.

Typical anode current ratings range from a minimum of 100 μA to a maximum of 1 mA. Luminous sensitivities range from 1 A per lumen or less, to over 2000 A per lumen. The extreme luminous sensitivity possible with these devices is 100 A per lumen, i.e., only 10^{-5} lumen is needed to produce 1 mA of output current.

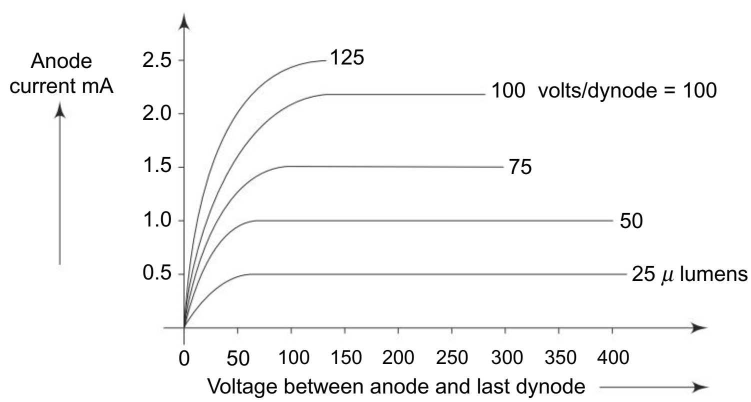


Fig. 13.43 Characteristics of a Photomultiplier

Applications Since they have fast response, low noise, ultra high sensitivity and small dark current, they find wide applications in space explorations, laser communications, scintillators and radiation directors of X-rays, gamma rays and energetic particles found in unclear physics.

13.11.6 Photodiode

Silicon photodiode is a light sensitive device, also called *photodetector*, which converts light signals into electrical signals. The construction and symbol of a photodiode are shown in Fig. 13.44. A lens permits light to fall on the junction. When light falls on the reverse biased PN photodiode junction, hole-electron pair are created. The movement of these hole-electron pairs in a properly connected circuit results in current flow. The current is proportional to the intensity of light and is also affected by the frequency of the light falling on the junction of the photodiode.

The characteristics of a photodiode are shown in Fig. 13.45. The reverse current increases in direct proportion to the level of illumination. Even when no light is applied, there is a minimum reverse leakage current, called *dark current*, flowing through the device. Photodiodes are used as light detectors, demodulators and

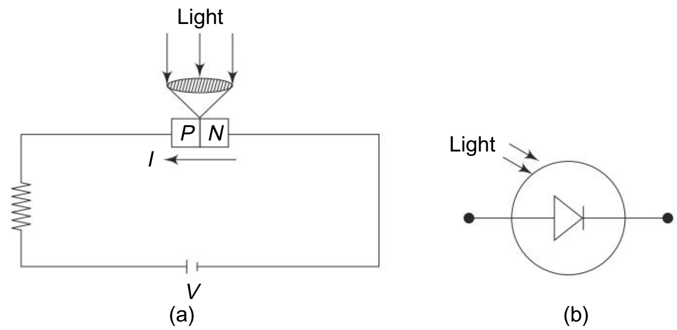


Fig. 13.44 Photodiode

encoders; they are also used in optical communication systems and switching circuits.

13.11.7 Phototransistor

The current produced by a photodiode is very low which cannot be directly used in control applications. Therefore this current should be amplified before applying to control circuits. The phototransistor is a light detector which combines a photodiode and a transistor amplifier. When the phototransistor is illuminated, it permits a greater flow of current.

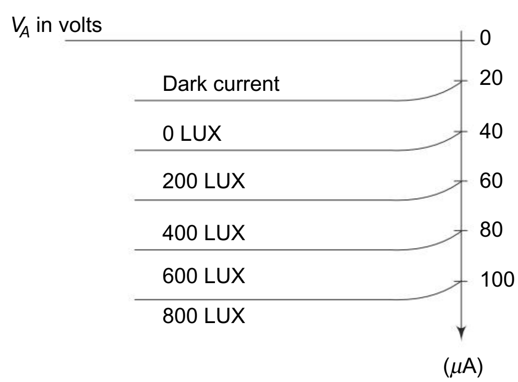


Fig. 13.45 Characteristics of Photodiode

Figure 13.46 shows the circuit of an NPN phototransistor. A lens focusses the light on the light on the base-collector junction. Although the phototransistor has three sections, only two leads, the emitter and the collector leads, are generally used. In this device, base current is supplied by the current created by the light falling on the base-collector photodiode junction.

Current in a phototransistor is dependent mainly on the intensity of light entering the lens and is less affected by the voltage applied to the external circuit. Figure 13.49 shows a graph of collector current I_C as a function of collector-emitter voltage V_{CE} and as a function of illumination H.

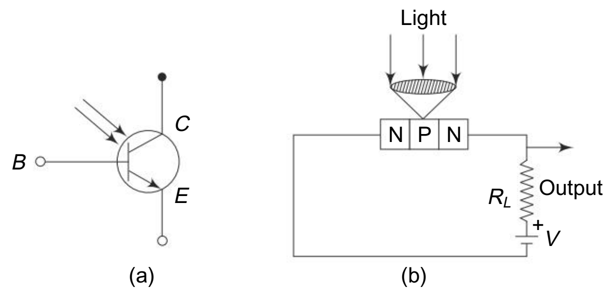


Fig. 13.46 (a) NPN Phototransistor: (a) Symbol and (b) Biasing Arrangement

13.12 DISPLAY DEVICES

13.12.1 Light Emitting Diode (LED)

The Light Emitting Diode (LED) is a PN junction device which emits light when forward biased, by a phenomenon called electroluminescence. In all semiconductor PN junctions, some of the energy will be radiated as heat and some in the form of photons. In silicon and germanium, greater percentage of energy is given out in the form of heat and the emitted light is insignificant. In other materials such as gallium phosphide (GaP) or gallium arsenide phosphide (Ga AsP), the number of photons of light energy emitted is sufficient to create a visible light source. Here, the charge carrier recombination takes place when electrons from the N-side cross the junction and recombine with the holes on the P-side.

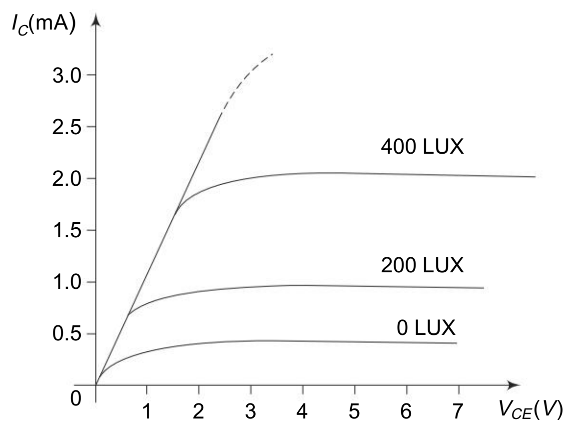


Fig. 13.47 Characteristics of Phototransistor

LED under forward bias and its symbol are shown in Fig. 13.48 (a) and (b), respectively. When an LED is forward biased, the electrons and holes move towards the junction and recombination takes place. As a result of recombination, the electrons lying in the conduction bands of N-region fall into the holes lying in the valence band

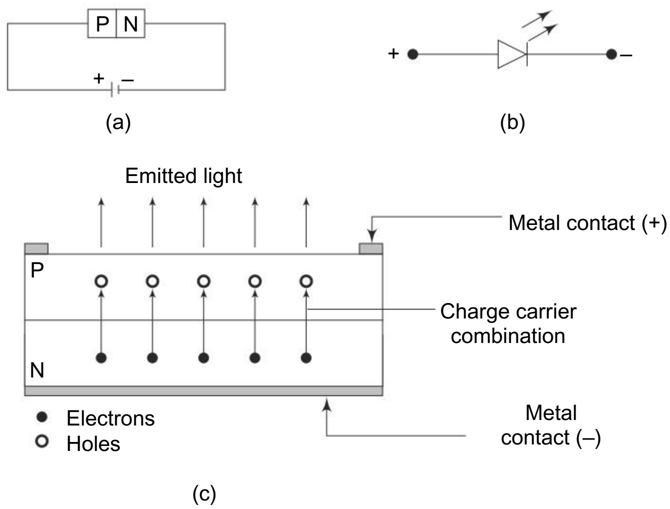


Fig. 13.48 LED (a) LED Under Forward Bias, (b) Symbol, and (c) Recombinations and Emission of Light

of a P-region. The difference of energy between the conduction band and the valence band is radiated in the form of light energy. Each recombination causes radiation of light energy. Light is generated by recombination of electrons and holes whereby their excess energy is transferred to an emitted photon. The brightness of the emitted light is directly proportional to the forward bias current.

Figure 13.48 (c) shows the basic structure of an LED showing recombinations and emission of light. Here, an N-type layer is grown on a substrate and a P-type is deposited on it by diffusion. Since carrier recombination takes place in the P-layer, it is kept uppermost. The metal anode connections are made at the outer edges of the P-layer so as to allow more central surface area for the light to escape. LEDs are manufactured with domed lenses in order to reduce the reabsorption problem. A metal (gold) film is applied to the bottom of the substrate for reflecting as much light as possible to the surface of the device and also to provide cathode connection. LEDs are always encased to protect their delicate wires.

The efficiency of generation of light increases with the injected current and with a decrease in temperature. The light is concentrated near the junction as the carriers are available within a diffusion length of the junction.

LEDs radiate different colours such as red, green, yellow, orange and white. Some of the LEDs emit infrared (invisible) light also. The wavelength of emitted light depends on the energy gap of the material. Hence, the colour of the emitted light depends on the type of material used is given as follows.

Gallium arsenide (GaAs)–infrared radiation (invisible)

Gallium phosphide (GaP)–red or green

Gallium arsenide phosphide (GaAsP)–red or yellow

In order to protect LEDs, resistance of 1 kΩ or 1.5 kΩ must be connected in series with the LED. LEDs emit no light when reverse biased. LEDs operate at voltage levels from 1.5 to 3.3 V, with the current of some tens of milliamperes. The power requirement is typically from 10 to 150 mW with a life time of 1,00,000 + hours. LEDs can be switched ON and OFF at a very fast speed of 1 ns.

They are used in burglar alarm systems, picture phones, multimeters, calculators, digital meters, microprocessors, digital computers, electronic telephone exchange, intercoms, electronic panels, digital watches, solid state video displays and optical communication systems. Also, there are two-lead LED lamps which contain two LEDs, so that a reversal in biasing will change the colour from green to red, or vice-versa.

When the emitted light is coherent, i.e., essentially monochromatic, then such a diode is referred to as an Injection Laser Diode (ILD). The LED and ILD are the two main types used as optical sources. ILD has a shorter rise time than LED, which makes the ILD more suitable for wide-bandwidth and high-data-rate applications. In addition, more optical power can be coupled into a fiber with an ILD, which is important for long distance transmission. A disadvantage of the ILD is the strong temperature dependence of the output characteristic curve.

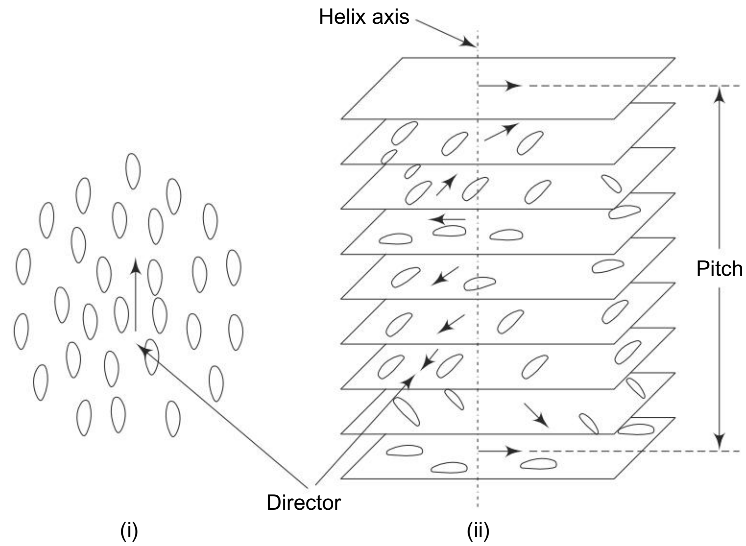
13.12.2 Infrared Emitters

The infrared emitting diodes are PN junction gallium arsenide devices which emit a beam of light when forward biased. When the junction is energised, electrons from the N-region will recombine with the excess holes of the P-material in a specially formed recombination region sandwiched between the P-and N-type materials. This recombination, which tends to restore the equilibrium carrier densities, can result in the emission of photons from the junction. The radiant energy from the device is infrared with a typical peak at 0.9 μm, which ideally matches the response of silicon photodiode and phototransistors.

These infrared emitting diodes are used in shaft encoders, data-transmission systems, intrusion alarms, card and paper tape readers, and high density mounting applications. The shaft encoder can produce 150 μW of radiant energy at 1.2 V and 50 mA.

13.12.3 Liquid Crystal Display (LCD)

Liquid Crystal Displays (LCDs) are used for display of numeric and alphanumeric character in dot matrix and segmental displays. The two liquid crystal materials which are commonly used in display technology are nematic and cholesteric whose schematic arrangements of molecules is shown in Fig. 13.49 (a). The most popular liquid crystal structure is the Nematic Liquid Crystal (NLC). In this type, all the molecules align themselves approximately parallel to a unique axis (director), while retaining the complete translational freedom. The liquid is normally transparent, but if subjected to a strong electric field disruption of the well ordered crystal structure takes place causing the liquid to polarise and turn opaque. The removal of the applied electric field allows the crystal structure to regain its original form and the material becomes transparent.



(a)

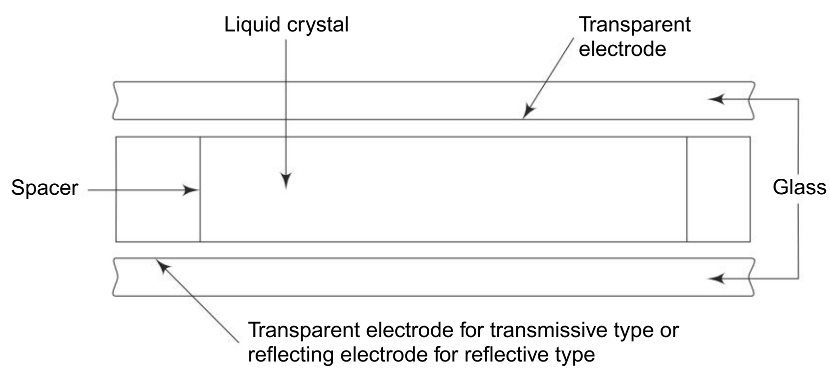


Fig. 13.49 (a) Schematic Arrangement of Molecules in Liquid Crystal, (i) Nematic, (ii) Cholesteric and (b) Construction of a Dynamic Scattering LCD

Based on the construction, LCDs are classified into two types. They are (i) Dynamic scattering type, and (ii) Field effect type.

Dynamic scattering type The construction of a dynamic scattering liquid crystal cell is shown in Fig. 13.49 (b). The display consists of two glass plates, each coated with tin oxide (SnO_2) on the inside with transparent electrodes separated by a liquid crystal layer, 5 to 50 μm thick. The oxide coating on the front sheet is etched to produce a single or multi-segment pattern of characters, with each segment properly insulated from each other. A weak electric field applied to a liquid crystal tends to align molecules in the direction of the field. As soon as the voltage exceeds a certain threshold value, the domain structure collapses and the appearance is changed. As the voltage grows further, the flow becomes turbulent and the substance turns optically inhomogeneous. In this disordered state, the liquid crystal scatters light.

Thus, when the liquid is not activated, it is transparent. When the liquid is activated, the molecular turbulence causes light to be scattered in all directions and the cell appears to be bright. This phenomenon is called dynamic scattering.

Field Effect Type The construction of a field effect LCD display in similar to that of the dynamic scattering type, with the exception that two thin polarising optical filters are placed at the inside of each glass sheet. The LCD material is of twisted nematic type which twists the light (change in direction of polarisation) passing through the cell when the latter is not energised. This allows light to pass through the optical filters and the cell appears bright. When the cell is energised, no twisting of light takes place and the cell appears dull.

Liquid crystal cells are of two types: (i) Transmittive type, and (ii) Reflective type. In the transmittive type cell, both glass sheets are transparent so that light from a rear source is scattered in the forward direction when the cell is activated.

The reflective type cell has a reflecting surface on one side of the glass sheet. The incident light on the front surface of the cell is dynamically scattered by an activated cell. Both types of cells appear quite bright when activated even under ambient light conditions.

Liquid crystals consume small amount of energy. In a seven segment display the current drawn is about 25 μA for dynamic scattering cells and 300 μA for field effects cells. LCDs required a.c. voltage supply. A typical voltage supply to dynamic scattering LCDs is 30 V peak-to-peak with 50 Hz. LCDs are normally used for seven-segment displays.

Advantages of LCD

(i) The voltages required are small.

(ii) They have a low power consumption. A seven segment displays requires about 140 W (20 W per segment), whereas LEDs require about 40 mW per numeral.

(iii) They are economical.

Disadvantages of LCD

(i) LCDs are very slow devices. The turn ON and OFF times are quite large. The turn ON time is typically of the order of a few ms, while the turn OFF is 10 ms.

(ii) When used on d.c., their life span is quite small. Therefore, they are used with a.c, supplies having a frequency less than 50 Hz.

(iii) They occupy a large area.

Comparison between LED and LCD Table 13.2 gives the comparison between LED and LCD.

Table 13.2 Comparison between LED and LCD

LED	*LCD*
Consumes more power—requires 10–250 mW power per digit	Essentially acts as a capacitor and consumes very less power—requires 10–200 μW power per digit
Because the high power requirement, it requires external interface circuitry when driven from ICs	Can be driven directly from IC chips
Good brightness level	Moderate brightness level
Operable within the temperature range –40 to 85°C	Temperature range limited to –20 to 60° C
Life time is around 100,000 hours	Life time is limited to 50,000 hours due to chemical degradation
Emits light in red, orange, yellow, green and white	Invisible in darkness—requires external illumination
Operating voltage range is 1.5 to 5 V d.c.	Operating voltage range is 3 to 20 V a.c.
Response time is 50 to 500 ns	Has a slow decay time—response times is 50 to 200 ms
Viewing angle 150°	Viewing angle 100°

13.12.4 Alphanumeric Displays

Display devices provide a visual of numbers, letters, and various signs in response to electrical input, and serve as constituents of an electronic display system. Display devices can be classified as passive displays and active displays.

(i) *Passive displays* Light controllers—they are modulators of light in which the light pattern gets modified on application of electric field, e.g. LCD.

(ii) *Active displays* Light emitters—they are generators of light, e.g. LED.

The optical devices described so far were capable of operating in an OFF/ON mode. LEDs are used as low consumption indicator lamps. Also, both LEDs and LCDs are potentially more useful as elements in alphanumeric display panels. There are two possible arrangements of optical displays, viz. seven segment and dot matrix, the choice being based on the display size, definition and allowed circuit complexity.

One way of producing an alphanumeric display is to make a *seven-segment* monolithic device, as shown in Fig. 13.50 (a), which can display all numerals and nine letters. Each segment contains LED/LCD which can be turned ON or OFF to form the desired digit. Each segment of the array has to be switched in response to a logic signal. For example, Fig. 13.50 (a) shows the response to a logic signal corresponding to 2, in which segments *a*, *b*, *d*, *e* and *g* have been switched ON and *c* and *f* remain OFF. Similarly, when all segments are ON, the digit formed is 8. If only the centre segment, *g*, is OFF the digit will be zero. Common anode and Common cathode seven segment LED displays are shown in Fig. 13.50 (b). Common anode type LED

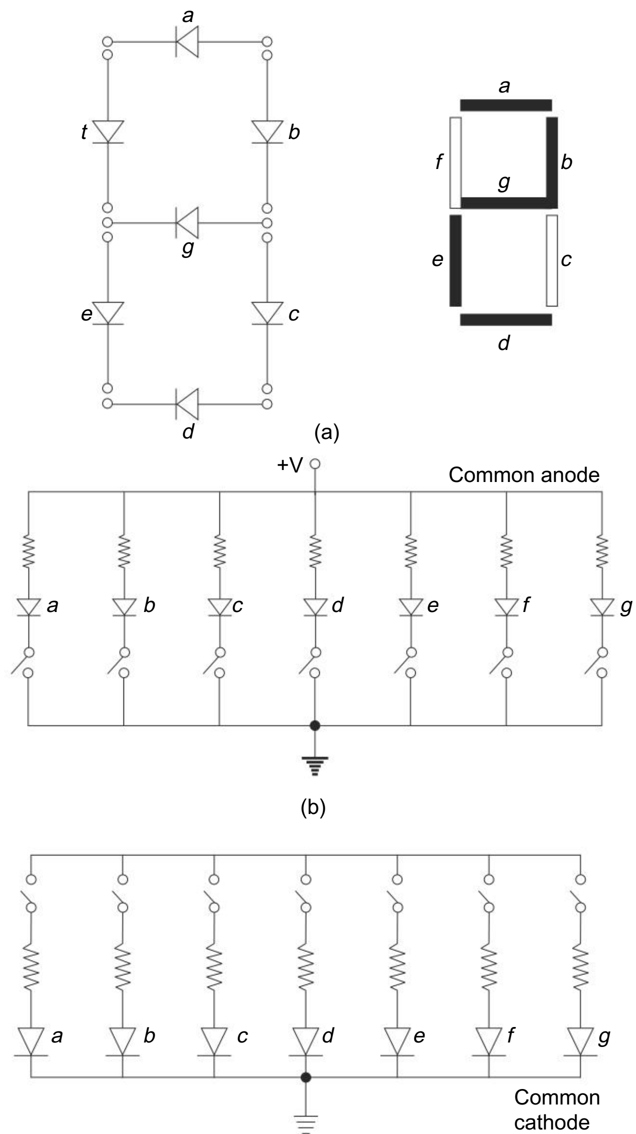


Fig. 13.50 Optoisolators (a) Photodiode (b) Photo-Darlington (c) Common Cathode Configurations

displays require an active LOW configuration, whereas an active HIGH circuitry is necessary for the common type LED display.

The seven segment displays are used in digital clocks, calculators, microwave ovens, digital multimeters, microprocessor trainer kits, stereo tuners, etc.

Another method of producing an alphanumeric displays is to make a *dot matrix* of LEDs/LCDs in a monolithic structure. Commonly used dot matrices for this display

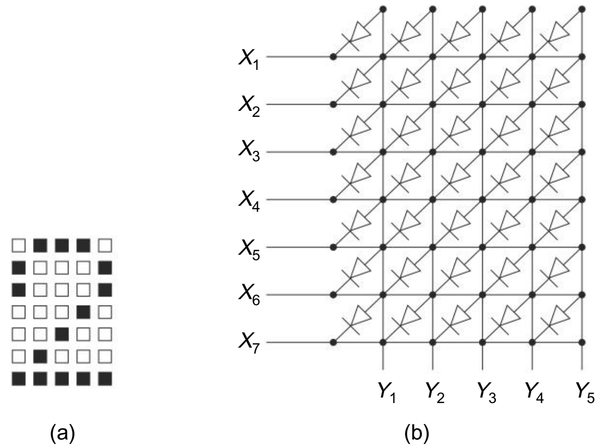


Fig. 13.51 (a) 5 × 7 Dot Matrix and (b) Wiring Pattern for 5 × 7 Dot Matrix

are 5 × 7, × 5 × 8 and 7 × 9, which can display 64 different characters including the alphabets, numerals and various symbols, by driving the appropriate horizontal and vertical inputs in a predetermined sequence using character generator ROMs. Here rows and columns are activated one by one for spans of time in a sequence. Due to persistence of human eye, in illusion of continuous display is obtained. A 5 × 7 dot matrix assembly using LEDs and the corresponding wiring pattern is shown in Fig. 13.51.

LED displays are available in many different sizes and shapes. The light emitting region is available in lengths from 0.25 to 2.5 cm.

13.12.5 Optocoupler

An optocoupler is a solid-state component in which the light emitter, the light path and the light detector are all enclosed within the component and cannot be changed externally. As the optocoupler provides electrical isolation between circuits, it is also called *optoisolator*. An optoisolator allows signal transfer without coupling wires, capacitors or transformers. It can couple digital (ON/OFF) or analog (variable) signals.

The schematic representation for an optocoupler appears in Fig. 13.52. The optoisolator, also referred to as an optoelectronic coupler, generally consists of an infrared LED and a photodetector such as PIN photodiode for fast switching, phototransistor Darlington pair, or photo-SCR combined in a single package. Optoisolators transduce input voltage to proportional light intensity by using LEDs. The light is transduced back to output voltage using light sensitive devices. GaAs LEDs are used to provide spectral matching with the silicon sensors.

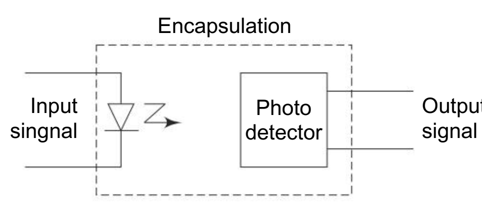


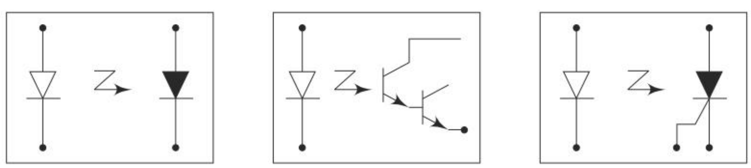
Fig. 13.52 Schematic Representation of an Optocoupler

The wavelength response of each device is made to be as identical as possible to permit the highest measure of coupling possible. There is a transparent insulating cap between each set of elements embedded in the structure (not visible) to permit the passage of light. They are designed with very small response times in such a way that they can be used to transmit data in the MHz range.

The rigid structure of this package permits one-way transfer of the electrical signal from the LED to the photodetector, without any electrical connection between the input and output circuitry. The extent of isolation between input and output depends on the kind of materials in the light path and on the distance between the light emitter and the light detector. A significant advantage of the optoisolator is its high isolation resistance, of the order of 10^{11} Ω with isolation voltages upto 2500 V between the input and output signals, and this feature allows it to be used as an interface between high voltage and low voltage systems. Application for this device includes the interfacing of different types of logic circuits and their use in level-and-position-sensing circuits.

In the optoisolator, the power dissipation of LED and phototransistor are almost equal and I_{CEO} is measured in nano-amperes. The relative output current is almost constant when the case temperature varies from 25 to 75°C. The V_{CE} voltage affects the resulting collector current only very slightly. The switching time of an optoisolator decreases with increased current, while for many devices it is exactly the reverse. It is only 2 μs for a collector current of 6 mA and a load resistance of 100 Ω.

The schematic diagrams for a photodiode, photo-Darlington pair and photo SCR optoisolator are illustrated in Fig. 13.53.



(a) (b) (c)

Fig. 13.53 Optoisolators (a) Photodiode (b) Photo-Darlington and (c) Photo-SCR

13.13 CATHODE RAY OSCILLOSCOPE (CRO)

The CRO is a versatile electronic testing and measuring instrument that allows the amplitude of the signal which may be voltage, current, power etc. to be displayed primarily as a function of time. It is used for voltage, frequency and phase angle measurement and also for examining the waveforms, from d.c. or very low frequency to very high frequencies.

Figure 13.54 shows the basic block diagram of a CRO. It comprises the main sections of (i) Horizontal and vertical voltage amplifiers, (ii) Power supply circuits, and (iii) Cathode Ray Tube (CRT).

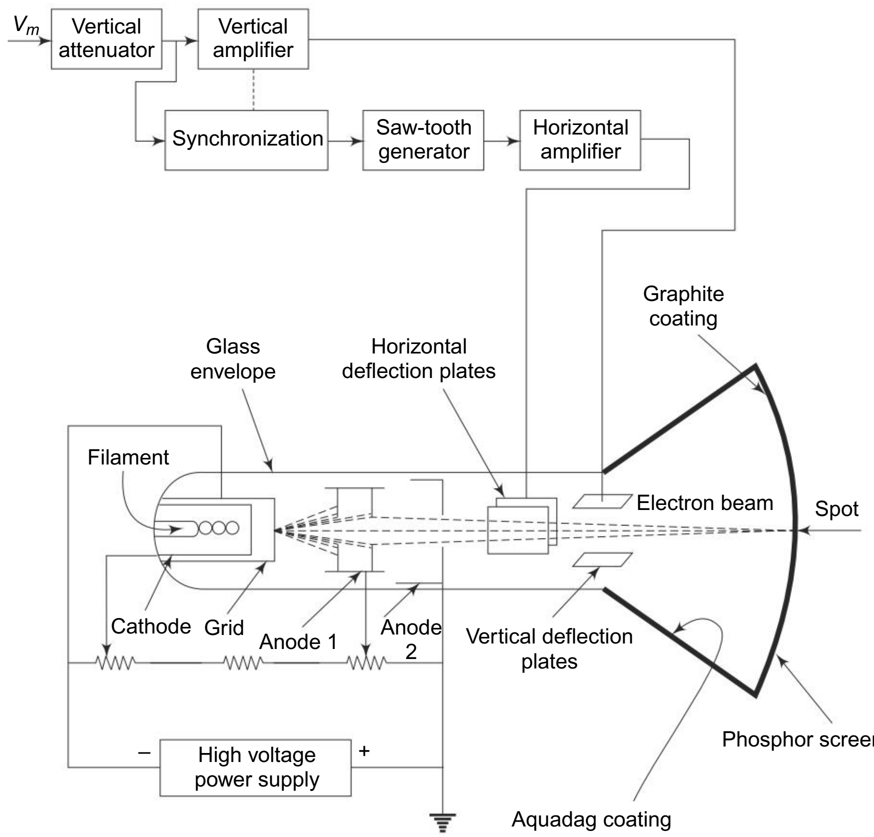


Fig. 13.54 Schematic diagram of a CRO

13.13.1 Vertical and Horizontal Voltage Amplifiers

These amplifiers are connected between the input terminals and the deflection plates. The function of the amplifiers is to increase the deflection sensitivity for weak input voltages.

The input signal is fed through a calibrated attenuator and a wide band high gain vertical amplifier to the vertical deflection plates of the CRT. The horizontal amplifier which is connected to the horizontal plates of the CRT is fed from an internally generated time base, usually a sawtooth waveform generator, alternatively the horizontal amplifier can also be fed from an externally connected X input.

The horizontal sweep (sawtooth) signal is triggered by a portion of the input signal applied to the vertical plates. A finite amount of time (in the range of sec.) is elapsed before the sawtooh waveform is applied to the horizontal plates. Hence, to observe the starting edge of the input signal fully, it should be delayed by the same amount of time in the delay line.

13.13.2 Power Supply Circuits

The power supply unit provides high voltages required by the CRT to generate and to accelerate the electron beam in addition to supplying the required operating voltage for the other circuits of the oscilloscope. The CRT requires high voltages, of the order of a few thousand volts, for acceleration and a low voltage for the heater of the electron gun which emits electrons. The CRO has various control switches on the panel. The intensity of the spot and focusing can be adjusted by the respective control knobs.

13.13.3 Cathode Ray Tube (CRT)

The CRT is the heart of the oscilloscope. It is a vacuum tube of special geometrical shape and converts an electrical signal into visual one. A heated cathode emits electrons which are accelerated to high velocity and are brought to focus on a fluorescent screen. When the electron beam strikes the screen of the CRT, a spot light is produced. The electron beam on its journey, is deflected in response to the electrical signal under study. As a result, the waveform of the electrical signal is displayed. As shown in Fig. 13.54, the CRT has various parts which are described below.

Glass envelope and screen It houses the electron gun, vertical and horizontal plates, and a screen on the conical front end. The inner walls of the CRT between neck and screen are usually coated with a conducting material (graphite) called acquadag. This conductive coating is electrically connected to the accelerating anode so that electrons which accidentally strike the wall are returned to the anode. It prevents the wall of the tube from charging to a high negative potential.

The screen is coated with a suitable fluorescent material depending on the required colour of the spot. Some of the substances which give characteristic fluorescent colours are

Zinc orthosilicate: green (used in CRT for general purpose)
Calcium tungstate: blue (used in CRT for fast photography)
Zinc sulfide or
Zinc cadmium sulphate: white (used in television receiver tubes).

Electron gun It produces a focused beam of electrons. It consists of an indirectly heated cathode, a control grid, a focusing anode and an accelerating anode. The control grid is at a negative potential with respect to cathode, whereas the two anodes are maintained at a high positive potential with respect to cathode. These two anodes act as an electrostatic lens to converge the electron beam at a point on the screen. The cathode consists of a nickel cylinder coated with an oxide coating that provides plenty of electrons. The control grid encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. The focusing anode focuses the electron beam to a sharp point by controlling the positive potential on it. The positive potential (about 10,000 V) on the accelerating anode is much higher than that on the focusing anode so that this anode accelerates the narrow beam to high velocity. Therefore, the electron gun generates a narrow, accelerated beam of electrons which produces a spot of light when it strikes the screen.

Deflection plates The electron beam comes under the influence of vertical and horizontal deflection plates before it strikes the screen.

When no voltage is applied to the vertical deflection plates, the electron beam produces a spot of light at the center of the screen. If the upper plate is positive with respect to the lower plate, the electron beam is deflected upwards and strikes the screen above its center. If the upper plate is negative with respect to the lower plate, the electron beam is deflected downwards and strikes the screen below its center. Thus the electron beam is made to move up and down vertically by controlling the voltage on the vertical plates thereby producing spots of light on the screen.

When a sinusoidal voltage is applied to the vertical deflection plates, the upper plate is positive during the positive half cycle and negative during the negative half cycle thereby producing a continuous vertical line on the screen.

The electron beam is made to move horizontally from side to side at a uniform rate by applying a sawtooth wave which varies linearly with time across the horizontal deflection plates.

Thus, the spot of light can be moved all over the surface of the screen by the simultaneous action of both vertical and horizontal deflection plates. In order to get the exact pattern of the signal on the screen, the signal voltage is given to the vertical deflection plates and sawtooth wave to the horizontal deflection plates.

Types of CRTs Conventionally, CRTs form the basis of cathode ray oscilloscopes (CROs), TVs and console/monitors. They are useful in displaying numeric, alphanumeric and graphic displays with high resolution. These are of two types:

(i) Electrostatic (used in CROs)
(ii) Electromagnetic (used in TVs)

There are also storage CRTs using digital storage, mesh storage, phosphor storage and transfer storage. Flat CRTs are also available.

13.13.4 Applications of CRO

The modulation index of Amplitude Modulation (AM) waves can be measured using a CRO. The voltage-current characteristics of PN junction diode and transistor, and characteristics of a transformer core can be displayed on CRO. Some more applications are discussed below.

1. Measurement of voltage If the signal is applied to the vertical deflection plates only, a vertical line appears on the screen. The height of the line is proportional to peak voltage of the applied signal.

The amplitude of the signal can be measured by applying the signal to the vertical plates and the sweep is applied to the horizontal plates using internal sweep circuitry. The vertical scale on the CRT screen is marked in centimeters. Each centimeter is further subdivided into 5 parts so that each represents 0.2 cm. If the peak amplitude of the waveform is 1.7 cm and the scale selected by the dial setting is 1 V/cm, then the amplitude of the signal is 1.4 cm × cm × 1 V/cm = 1.4 V.

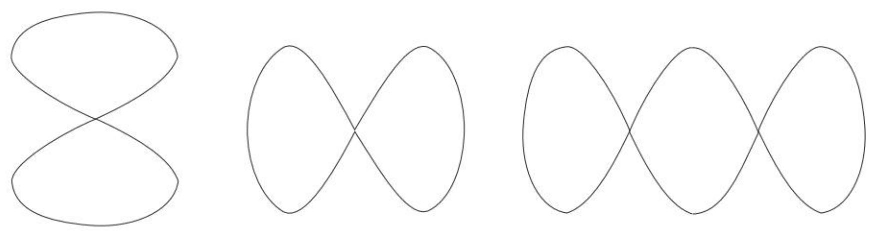
2. Measurement of current When a current is to be measured, it is passed through a known resistance and the voltage it is measured.

3. Measurement of frequency

(a) *Using signal waveform*: The signal for which the frequency (f) is to be measured is given to the vertical input. The number of divisions occupied by one complete cycle of the waveform is measured. The number of divisions multiplied by the time base setting in sec. is equal to the time period (T) of one cycle. The frequency (f) of the waveform is inverse of the time period T, i.e. $f = 1/T$.

(b) *Using Lissajous figure*: If sinusoidal voltages are applied to both vertical and horizontal inputs of CRO, some interesting figures are displayed, which are known as *Lissajous figures*. Two sine waves of the same frequency produce a Lissajous figure which may be a straight line, an ellipse or a circle, depending on the phase and amplitude of the two signals.

Two sine waves of equal amplitudes but different frequencies will produce a figure from which the relationship between the two frequencies can be understood. For example, Fig 13.55 (a) shows that the vertical input signal has twice the frequency of the horizontal input signal. Similarly, Fig. 13.55 (b) indicates that horizontal input signal has twice the frequency of the vertical one. Figure 13.55 (c) shows three loops indicating that vertical input signal has thrice the frequency of the horizontal one.



(a) $2f_v = f_H$ (b) $f_v = 2f_H$ (c) $f_v = 3f_H$

Fig. 13.55 Frequency measurement (Lissajous figures) (a) $2f_V = f_H$ (b) $f_V = 2f_H$ and (c) $f_V = 3f_H$

A known frequency (f_H) is applied to horizontal input and unknown frequency (f_y) to the vertical input. Then a Lissajous pattern with loops is obtained. The unknown frequency (f_v) can be measured by the following relationship.

$$\frac{f_V}{f_H} = \frac{\text{No. of loops cut by horizontal line}}{\text{No. of loops cut by vertical line}}$$

4. Measurement of phase difference The phase difference between two sinusoidal signals of same frequency can be calculated from the amplitudes A and B of the Lissajous pattern (an ellipse) shown in Fig. 13.56. The phase difference (deg), $\theta = \sin^{-1}(A/B)$.

Lissajous figures are formed when two sine waves are applied simultaneously to the vertical and horizontal deflecting plates of a CRO. In general, the shape of the Lissajous figures depends on amplitude, phase difference and ratio of frequency of the two waves. Two sine waves of the same frequency and amplitude may produce straight line an ellipse or a circle, depending on their phase difference as shown in Fig. 13.57.

5. Test of distortion on amplifier CRO is useful to measure the distortion using Lissajous figure. Figure 13.58 shows the connection for testing the frequency distortions of an amplifier network. The audio oscillator is adjusted to a known

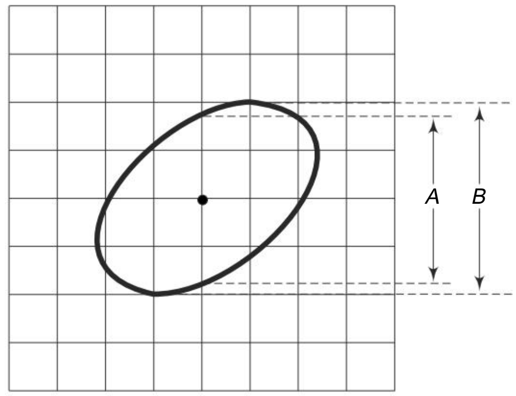


Fig. 13.56 Phase difference measurement

frequency and is connected to the deflecting plates $x - x'$. The input signal obtained at the output of the amplifier is connected to the deflecting plates $y - y'$. If the amplifier produces higher harmonics of the input frequency due to the nonlinearities of the active device used, the CRO screen shows the hoops in Lissajous figure which indicates the presence of distortion. A single display indicates the presence of distortion. A straight line display indicates the absence of distortion in the amplifier

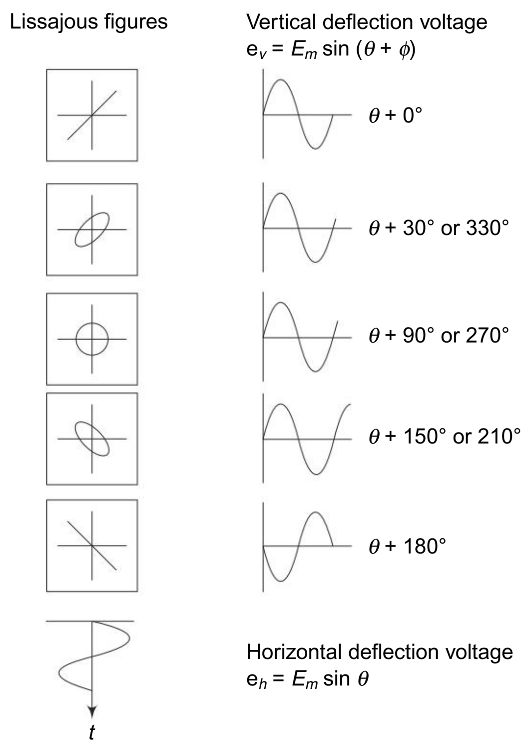


Fig. 13.57 Lissajous figure depending on the phase difference of the two waves

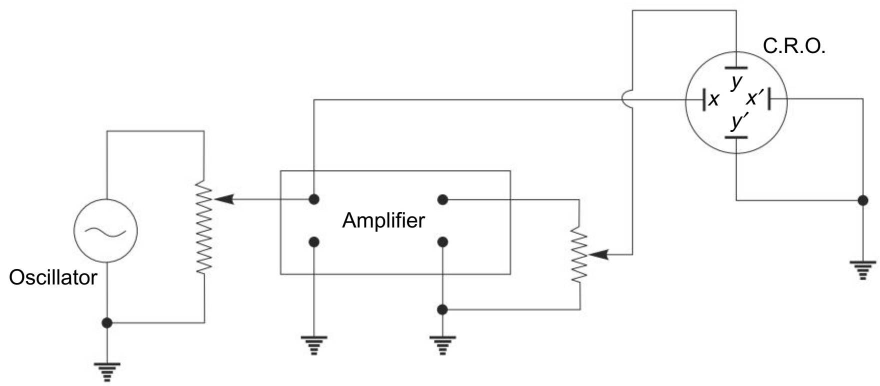


Fig. 13.58 Distortion test on amplifier

13.14 POWER CONDITIONING EQUIPMENTS

All electronic circuits need d.c. power supply either from battery or power pack units. It may not be economical and convenient to depend upon battery power supply. Hence, many electronic equipment contain circuits which convert the a.c. supply voltage into d.c. voltage at the required level. The unit containing these circuits is called the *Linear Mode Power Supply* (LPS). In the absence of a.c. main supply, the d.c. supply from battery can be converted into required a.c. voltage which may be used by computer and other electronic systems for their operation. Also, in certain applications, d.c. to d.c. conversion is required. Such a power supply unit that converts d.c. into a.c. or d.c. is called *Switched Mode Power Supply* (SMPS).

1. Linear power supply (LPS): a.c./d.c. power supply-Converter
2. Switched mode power supply (SMPS): (i) d.c./a.c. power supply-Inverter (ii) d.c./a.c. power-Inverter

An a.c./d.c. power supply converts a.c. mains (230 V, 50 Hz) into required d.c. voltages and is found in all mains operable system.

DC/DC power supplies or d.c./d.c. converters are used in portable systems. DC/AC power supplies or inverters are used in portable mains operable systems and as a supplement to a.c. mains in non-portable mains operable system, where a disruption in the power supply can affect the job being done by the system. An inverter is a form of UPS (Uninterrupted Power Supply) or SPS (Standby Power Supply) and is very popular in computer systems.

Based on the regular concept, power supplies are classified as either *linear* or *switched mode power supply*. The main difference between LPS and SMPS is seen from their block diagrams given in Figs. 13.59 (a) and (b).

The power conditioning devices are SMPS UPS, Constant Voltage Transformer (CVT), Servo Stabilizer and Isolation Transformer. They must produce at least 40% greater than the Power Consumption of the Computer and Peripherals connected to it.

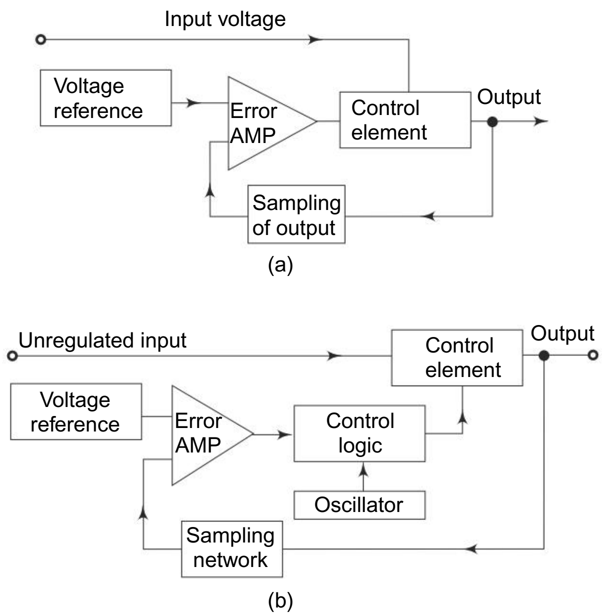


Fig. 13.59 (a) Principle of a Normal Linear Mode Power Supply, (b) Block Diagram of a Switched Mode Power Supply

13.14.1 Linear Mode Power Supply

The basic building blocks of the linear power supply are shown in Fig. 13.60. A transformer supplies a.c. voltage at the required level. This bidirectional a.c. voltage is converted into a unidirectional pulsating d.c. using a rectifier. The unwanted ripple contents of this pulsating d.c. are removed by a filter to get pure d.c. voltage. The output of the filter is fed to a regulator which gives a steady d.c. output independent of load variations and input supply fluctuations.

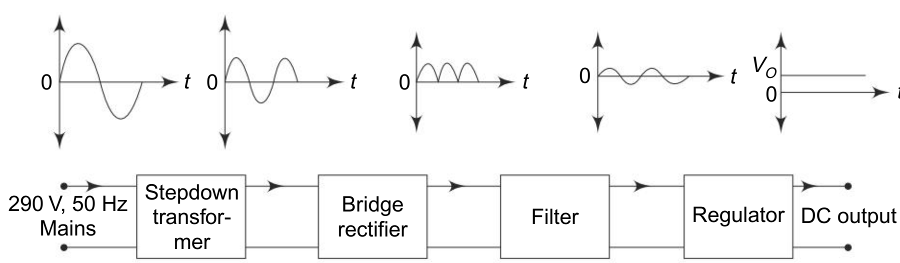


Fig. 13.60 Basic Building Block of Linear Mode Power Supply

13.14.2 Switched Mode Power Supply (SMPS)

D.C. to d.c. converters and d.c. to a.c. converters belong to the category of Switched Mode Power Supplies (SMPS). The SMPS operating from mains, without using an input transformer at line frequency 50 Hz is called "off-line switching supply". In off-line switching supply, the a.c. mains is directly rectified and filtered and the d.c. voltage so obtained is then used as an input to a switching type d.c. to d.c. converter.

The various types of a voltage regulators used in LPS, fall in the category of dissipative regulator, as it has a voltage control element (transistor or zener diode) which dissipates the power equal to the voltage difference between an unregulated input voltage and a fixed output voltage multiplied by the current flowing through it. The switching regulators solve the above problem. The switching regulator acts as a continuously variable power converter and hence, its efficiency is negligibly affected by the voltage difference. Therefore, the switching regulator is also known as 'non-dissipative regulator'.

In a switching power supply, the active device that provides regulation is always operated in a switched mode, i.e. it is operated either in cut-off or in saturation. The input d.c. is chopped at a high frequency (15 to 50 kHz) using an active device (bipolar transistor, power MOSFET or SCR) and the converter transformer. Here, the size of the ferrite core reduces inversely with frequency. The lower limit is defined at about 15 kHz by the requirement for silent operation and the upper limit of 50 kHz is to limit losses in the choke and in the active switching elements. The transformed chopped waveform is rectified and filtered. A sample of the output voltage is used as the feedback signal for the drive circuit for the switching transistor to achieve regulation.

Figure 13.59 (b) shows the concept of a switching regulator in simple form. The added elements are control logic and oscillator. The oscillator allows the control element to be switched ON and OFF. The control element usually consists of a transistor switch, an inductor and a diode. For each switch ON, energy is pumped into the magnetic field associated with the inductor which is a transformer winding in practice. This energy is then released to the load at the desired voltage level. By varying the duty cycle or frequency of switching, one can vary the stored energy in each cycle and thus control the output voltage. As a switch can only be ON or OFF, it either allows energy to pass or stop, but does not dissipate energy itself. Since only the energy required to maintain the output voltage at a load current is drawn, there is no dissipation and hence, a higher efficiency is obtained. Energy is pumped in discrete lumps, but the output voltage is kept steady by capacitor storage.

The major feature of SMPS is the elimination of physically massive power transformers and other power line magnetic. The net result is a smaller, lighter package and reduced manufacturing cost, resulting primarily from the elimination of the 50 Hz components.

D.C. to D.C. Converter The block diagram of d.c. to d.c. converter (SMPS) is shown in Fig. 13.61 (a). Here, the primary power received from a.c. main is rectified and filtered as high voltage d.c. It is then switched at a high rate of speed approximately 15 kHz to 50 kHz and fed to the primary side of a step-down transformer. The step-down transformer is only a fraction of the size of a comparable 50 Hz unit thus relieving the size and weight problems. The output at the secondary side of the transformer is rectified and filtered. Then it is sent to the output of the power supply. A sample of this output is sent back to the switch to control the output voltage.

As the load increases, output voltage tends to fall. Most switching power supplies regulate their output using a method called *Pulse-Width Modulation* (*PWM*). The power switch which feeds are primary side of the step-down transformer is driven by a pulse-width modulated oscillator. When the duty cycle is at 50%, then the maximum amount of energy will be passed through the step-down transformer. As the duty cycle is decreased, less energy will be passed through the transformer.

The width or ON time of the oscillator is controlled by the voltage feedback from the secondary rectifier output, and forms a closed loop regulator. As shown in Fig. 13.61 (b), the pulse width given to the power switch is inversely proportional to the output voltage.When the output voltage drops, the switch is ON for longer time, resulting in more energy delivered to the transformer and a higher output voltage. As the output voltage raises, the ON time becomes shorter until the loop stabilises.

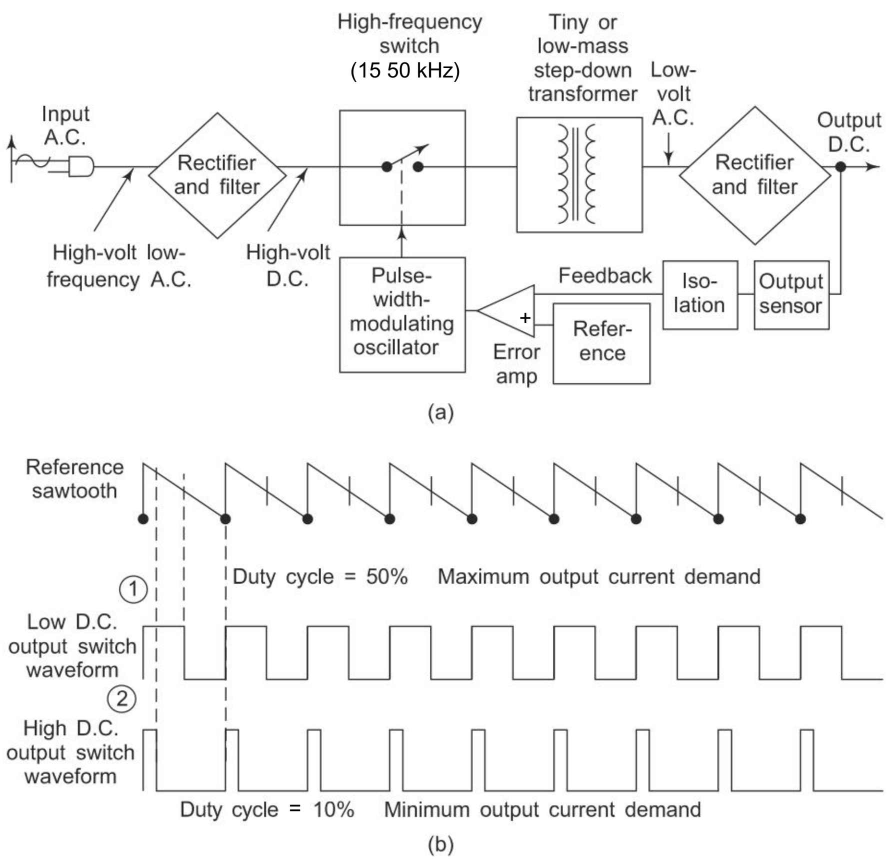


Fig. 13.61 (a) Block Diagram of Switching Power Supply, (b) Switching Power Supply Waveforms

13.14.3 D.C. to A.C. Inverter/Uninterrupted Power Supply (UPS)

The block diagram of *Standby Power Supply* (*SPS*), otherwise called as *off-line UPS* is shown in Fig. 13.62. It is a system that uses a special circuit that senses the a.c. line current. If the sensor detects a loss of power on the line, the system quickly switches over to a standby power system.

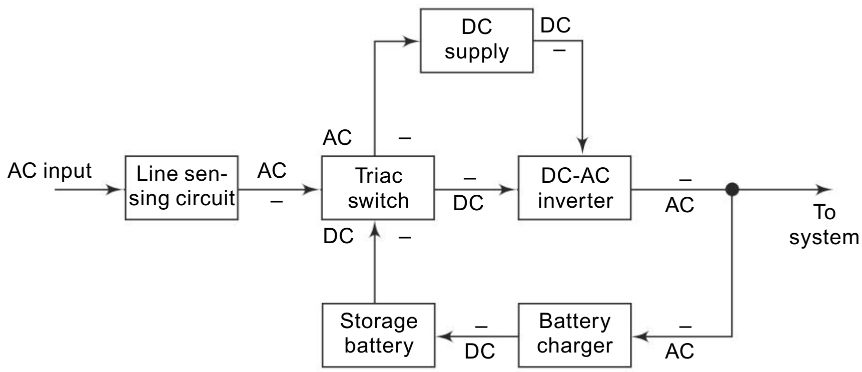


Fig. 13.62 Block Diagram of Standby Power Supply

The SPS transfers the load to the inverter which draws its power from the attached batteries. The switching process requires a small but measurable amount of time. First, the failure of the electrical supply must be sensed. Even the fastest electronic voltage sensors take a finite time to detect a power failure. Even after a power failure is detected, there is another slight pause before the computer receives its fresh supply of electricity. If the switch is not fast enough, the system shuts down and reboots which defeats the purpose of the use of the backup power supply.

The name *Uninterrupted Power Supply* is self-explanatory. Its output is never interrupted because it does not need to switch its output from line power to battery. Rather, its battery is constantly and continuously connected to the output of the system through its inverter. It is always supplying power from the battery to the computer. While a.c. power is available to the UPS, it keeps the UPS battery charged through the rectifier circuits. When the power fails, the charging of the battery is stopped, but the system gets continuous supply from the battery.

It is independent of all the variations of the electrical lines. It is the computer's own generating station keeping it safe from the polluting effects of lightning and load transients. Dips and surges can never reach the computer. The computer gets a smooth, constant electrical supply. The duration for which the UPS powers a system depends not on the rating of the UPS in volt-amperes, but that of the rating of the batteries powering it in ampere-hours. Normally, the UPS comes with a bypass switch. It helps to directly connect the system to the incoming a.c. supply if there is any problem with the UPS.

Advantages of SMPS

1. Efficiency is high because of less heat dissipation.
2. As the transformer size is very small, it will have a compact unit.

3. Protection against excessive output voltage by quick acting guard circuits.
4. Reduced harmonic feedback into the supply main.
5. Isolation from main supply without the need of large mains transformer.
6. Generation of law and medium voltage supplies are easy.
7. Switching supplies can change and unregulated input of 24 V into a regulated output of 1000 V d.c.
8. Though RF interference can be a problem in SMPS unless properly shielded, SMPS in TV sets is in synchronisation with the line frequency (15.625 kHz) and thus switching effects are not visible on the screen.
9. SMPS are also used in personal computers, video projectors and measuring instruments.

13.14.4 Constant Voltage Transformer

Introduction A transformer that maintains an approximately constant voltage ratio over the range from zero to rated output is called Constant Voltage Transformer (CVT). The CVT is also a constant current device.

The CVT finds its applications in heating or lighting apparatus, electronic equipment where all the voltages are changed to DC and UPS for computer, telex, FAX, VCR, Electromedical instruments, etc.

Construction and Working of a CVT In an ordinary transformer, the primary and secondary coils are closely coupled. Any change in primary voltage is directly transferred to the secondary, in the ratio of number of turns.

In a CVT the primary and secondary coils are wound in separate sections of the transformer as shown in Fig. 13.63. In between the coils a shunt path is provided for the flux to flow but an air gap is provided in this shunt path. A capacitor is connected in parallel with the secondary

When a voltage is applied in the primary starting from zero and slowly increased, initially all the flux generated by the primary voltage will pass through the lower half of the transformer core because the air gap in the shunt path will prevent it from taking this path. As a result, the rise in secondary voltage is proportional to the primary. But as the voltage in the secondary coil rises, its inductance value also

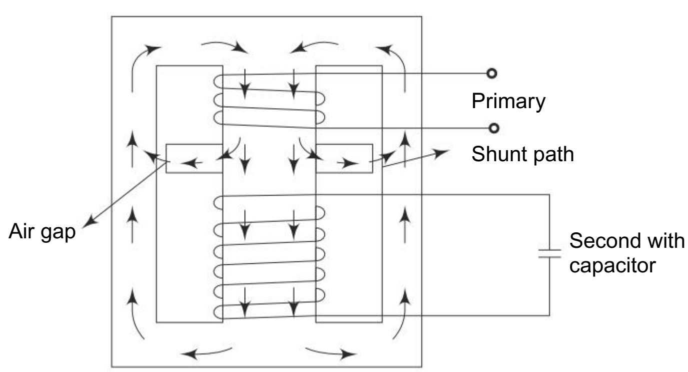


Fig. 13.63 Constant Voltage Transformer

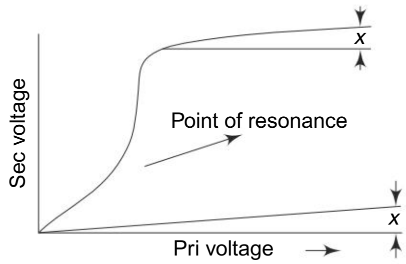


Fig. 13.64 Transfer Characteristics of CVT

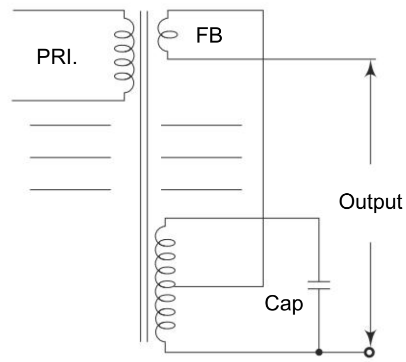


Fig. 13.65 Feedback (FB) Coil Arrangement in CVT

increases. At a certain point, the impedance of the coil will become equal to the impedance of the capacitor, i.e. $X_L = X_C$ or $2\pi f L = \dfrac{1}{2\pi f C}$.

This is the condition of resonance, and at this point a high current will flow in the LC circuit. This high current will result in sudden rise in voltage across the secondary and the core in this section of the transformer will saturate as shown in Fig. 13.64. Once the core gets saturated, it prevents the entry of further flux coming from the primary side. Therefore, any increase in flux due to the increase in primary voltage has to take an alternate shunt path. Hence, very little increase in secondary voltage takes place. This little increase can also be nullified by a Feedback [FB] winding connected as shown in Fig. 13.65. The output winding can be separated from the capacitor winding if low voltage is required.

Types of CVT There are two types of CVT based on the wave shape, viz (i) Sinusoidal type and (ii) Non-Sinusoidal type.

The wave shape in a CVT gets distorted due to the saturation in the core. The distortion is 15 to 17 percent in a non-sinusoidal CVT and the distortion is 3 to 5 percent in the sinusoidal type. The sinusoidal type consists of an extra coil, acting as a choke, filters out the harmonics in the sinusoidal CVT. In non-sinusoidal CVT, the ripple factor after conversion of AC to DC is much lower when compared with the sinusoidal type. To measure the voltage in the non-sinusoidal type CVT, either a moving iron type voltmeter or a trure RMS multimeter is suitable.

Advantages

(i) Since CVT is a constant current device, one can overload it or short circuit it for hours together without fearing a transformer burnout. Hence, a fuse is never required.

(ii) It has longer life because it has no electronic and moving parts in the circuitry.

(iii) It suppresses the surges coming from the main.

(iv) It has got good accuracy in the output regulation.

(v) It has fast response, i.e. 1/50th of a second.

(vi) Even if any fault occurs, the output will not go beyond the prescribed limit and hence it does not require any high voltage cut-off device.

Disadvantages

(i) The output voltage depends on the input frequency.

(ii) It cannot give a high starting current to motors due to its constant current characteristics.

(iii) Some CVTs make a humming sound which may be irritating in a silent laboratory or a computer room.

(iv) There is flux leakage in CVT and should be kept away from magnetic devices like floppies, cassettes, etc. This flux can also cause ripples on a computer on TV screen if kept too close.

13.14.5 Servo Stabilizer

Commonly switchable stabilizers are used to boost or buck the incoming voltage to give an output voltage within a desired range. These stabilizers have large transformers with a number of taps or windings, outputs set at different voltage levels. Relays on the board select and switch to the taps that will supply the voltage, i.e. nearly appropriate to the line voltage required.

In a servo stabilizer, as a shown in Fig. 13.66, the correction voltage whether it is to buck or boost is given continuously without any switch over or off-line. The auto transformer is a toroidally wound transformer and has two windings, one each in different directions. There is a moving contact which runs over the winding. One winding boosts and the other bucks. The moving contact is rotated by a servo motor.

There is an error amplifier which outputs an error voltage when there is a difference in the d.c. reference voltage input and the d.c. voltage rectified (from the output line) input. The error voltage drives the motor in a direction to add or subtract the correction voltage with the output voltage from the transformer T_2. As the error amplifier continuously monitors the output voltage, we get appropriately constant voltage output. The output voltage required can be set by adjusting the d.c. reference input to the error amplifier. Servo stabilizer cannot offer either spikes/surge protection or isolation. The drawback of the servo stabilizer is that it introduces some line noises. If a servo stabilizer is used, it must have power on relay, either manual or automatic.

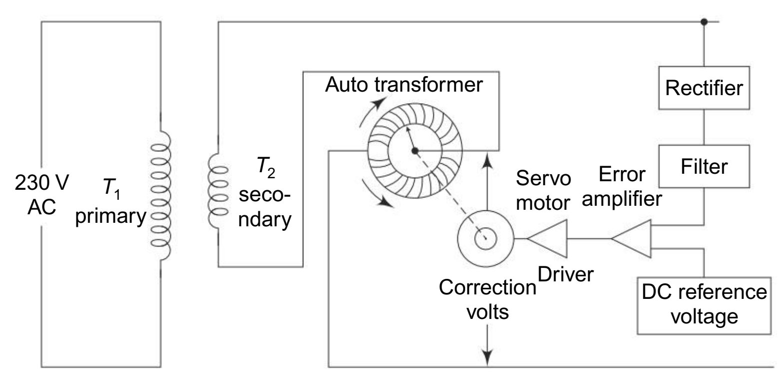


Fig. 13.66 Servo Stabilizer

13.14.6 Isolation Transformer

Power line problems can be broadly classified into over voltage, under voltage, and noise. The deadiest power line problem is the over voltage. It is the lightning like high voltage spikes that sneak into the computer and damage its silicon components. The over voltage problems are of two types, namely (i) Spikes and (ii) Surges. Spikes are high voltage transients that last for short durations of a few microseconds. Surges are high voltage transients that last for longer durations and will stretch for many milliseconds. The metal oxide varister is commonly used as a over voltage protection device. This device can accept voltages as high as 6000 volts and divert any power above a certain level to ground. The excess as 6000 volts and divert any power above a certain level to ground. The excess energy does not disappear but turns into heat possibly destroying the varisters. It is connected between the system and the power line to prevent over voltages from reaching the system. Other than metal oxide varisters, semiconductors, ionized spark-gaps and ferro-resonant transformers are also used as surge protectors.

One way of suppressing spikes and surges in the mains supply is to isolate them magnetically using isolation transformer. Isolation transformer shown in Fig. 13.67 has primary and secondary winding with winding ratio 1:1. As primary and secondary are electrically isolated, the isolation transformer effectively suppresses surges, spikes and line noises. It cannot offer any protection against under voltage or over voltage conditions or blackout/power failure conditions.

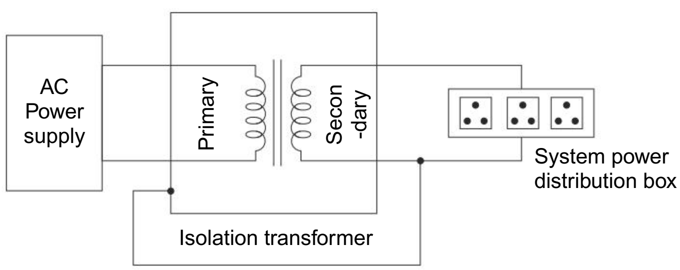
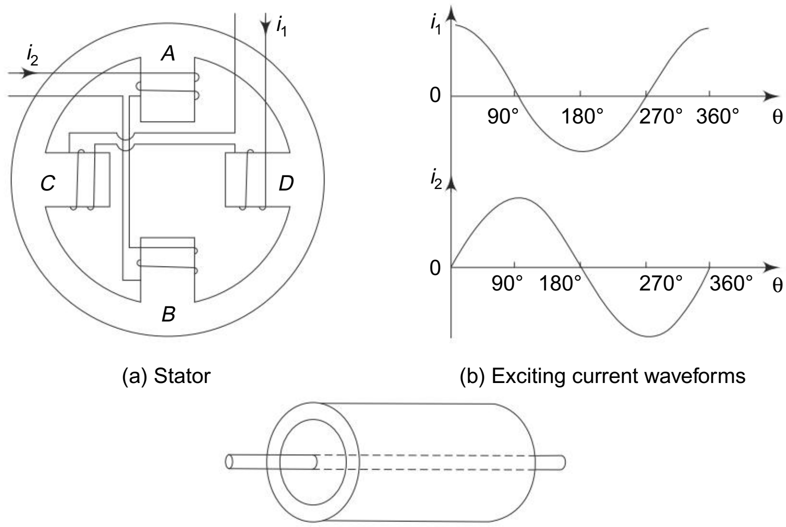


Fig. 13.67 Isolation Transformer

13.14.7 Servomotors

The servomotors are used to convert an electrical signal (control voltage) applied to them into an angular displacement of the shaft. They are normally coupled to the controlled device by a gear train or some mechanical linkage. The servomotors are broadly classified as D.C. servomotor and A.C. servomotor depending on the supply used. Their ratings vary from a fractional kilowatt to few kilowatts.

A.C. Servomotor The A.C. servomotor is a two-phase induction motor with some special design features. The simple constructional feature of A.C. servomotor is shown in Fig. 13.68. It comprises of a stator winding and a rotor, which may take one of several, forms namely (i) Squirrel cage, (ii) Drag cup and (iii) Solid Iron.



(a) Stator

(b) Exciting current waveforms

(c) Rotor

Fig. 13.68 Simple Construction of Two Phase ac Servo Motor

The stator consists of two pole-pairs *A–B*, and *C–D* mounted on the inner periphery of the stator, such that their axes are at an angle of 90° in space. The rotor bars are placed on the slots and short-circuited at both ends by end rings. The diameter of the rotor is kept small in order to reduce inertia and to obtain good accelerating characteristics.

Each pole-pair carries a winding. The exciting current in the windings should have a phase displacement of 90° as shown in Fig. 13.68 (b). The voltages applied to these windings are not balanced. Under normal operating conditions, a fixed voltage from a constant voltage source is applied to one phase, which is called the reference phase. The other phase called the control phase is energized by a voltage of variable magnitude and polarity, which is at 90° out of phase with respect to the fixed phase. The control phase is usually applied from a servo amplifier. The direction of rotation reverses if the phase sequence is reversed.

When two-phase supply is given to the stator windings, a rotating magnetic field is established in the two-phase stator. The rotating flux intersects the conductor in the rotor and induces voltage, which in turn produces current to flow in these conductors. The currents are determined by the magnitude of the voltage and the stator impedance. When the rotor is stationary, the induced voltage in the rotor coil will be of the same frequency as the supply voltage. The current due to the induced voltage produces starting torque. This torque begins to accelerate the rotor until reaches it operating speed, which is determined by friction, and load torque.

For higher values of rotor resistance, the torque-speed characteristics are linear. For servo application, the motor characteristics should be linear with negative slope. Therefore, ordinary two-phase induction motor with low rotor resistance is not suitable for servo applications.

Advantages of A.C. Servomotors

1. Low cost and low maintenance, since there is no commutator and brushes.
2. High efficiency.

Applications of A.C. Servomotors

1. Used in X-Y recorders.
2. Used in disk drives, tape drives printers, etc.

D.C. Servomotor D.C. servomotors are used for large power applications such as in machine tools and Robotics. D.C. servomotors are D.C. motors driven by current from D.C. electronic amplifier. The basic characteristics of such servomotors are that the torque developed is proportional to the applied control voltage. The D.C. servomotors can be classified as (a) Series and shunt excited motor, and (b) Separate excited motor.

(a) Series and shunt excited motor The characteristics of the series excited and shunt excited motors are highly non-linear. Such motors are seldom used in servo systems. However, in systems where linearity is not so important but where high torque is required, a modified version of the series motor called the split field motor is used.

(b) Separate excited motor Most of the D.C. motors in servo applications are of the separately excited type. The output of the D.C. servo amplifier can be connected either to the field terminals or to the armature terminals of the motor. When the field is energized by the amplifier signal, the mothor is said to be field controlled. If the armature is energized by the amplifier, the motor is *armature controlled*.

Field Controlled D.C. Servomotor Figure 13.69 shows a field controlled D.C. servomotor. The armature is driven by a constant current source. The torque developed and hence the speed of the motor, will be directly proportional to the field flux, generated by the field current flowing through the field winding. When the error signal is zero, there will be no field current and hence no torque is developed. The direction of rotation of the motor would depend on the polarity of the field and hence on the nature of the error signal. If the field polarity is reserved, the motor will develop torque in the opposite direction.

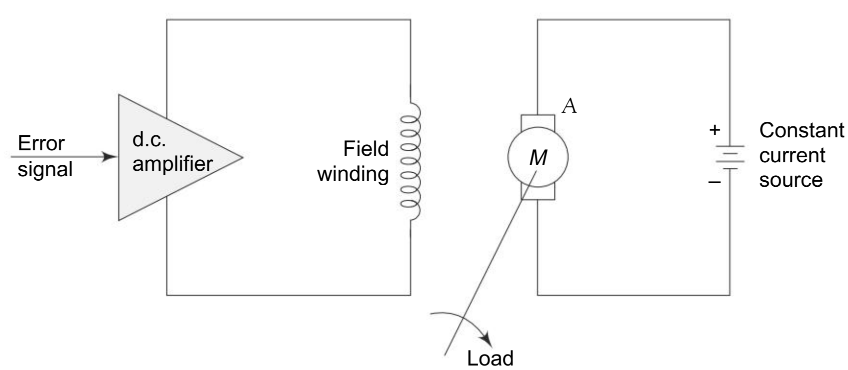


Fig. 13.69 Field Controlled Type D.C. Servomotor

Advantages of D.C. Servomotors

In D.C. servomotors,

(i) easier speed control from zero to full speed in both directions is possible
(ii) linearity characteristics are achieved easily
(iii) high torque to inertia ratio gives quick response to control signals

13.14.8 Stepper Motors

The stepper motor has gained importance in recent years because of the ease with which it can be interfaced with digital circuits. The stepper motor completes a full rotation by sequencing through a series of discrete rotational steps (stepwise rotation). Each step position is an equilibrium position, so that without further excitation the rotor position stays at the latest step. Thus continuous rotation is achieved by a train of input pulses, each of which causes an advance of one step. The number of steps per revolution and the rate at which the applied pulses determine the rotational rate. The two most widely used types of stepper motors are (i) Variable reluctance motors, and (ii) Permanent magnet motor.

A variable reluctance stepper motor consists of a single or several stacks of stators and rotors. The stators have a common frame and rotors have a common shaft as shown in Fig. 13.70. Both stators and rotors have toothed structure as shown in Fig. 13.71. The stator and rotor teeth are of same size and therefore aligned as shown in Fig. 13.72. The stators are pulse excited, while rotors are unexcited. When the stator is excited in a particular stator and rotor set, the rotor is pulled towards the nearest minimum reluctance position where stator and rotor teeth are aligned. The static torque acting on the rotor is a function of the angular misalignment θ. There

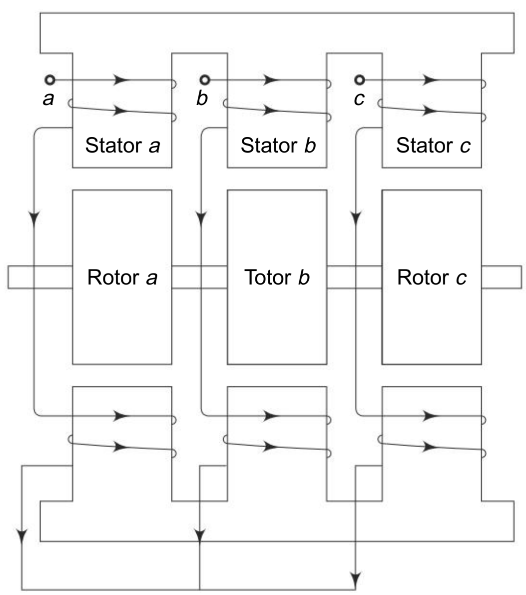


Fig. 13.70 Longitudinal Cross-sectional View of 3-stack Variable Reluctance Motor

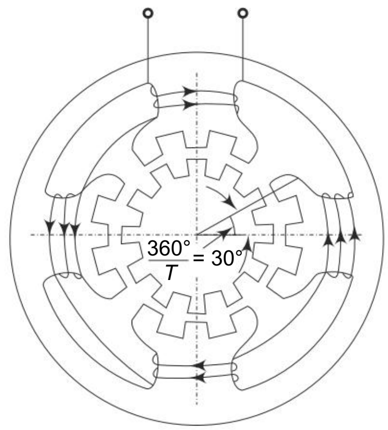


Fig. 13.71 Cross Sectional View of Stator and Rotor (12 teeth) of a Variable Reluctance Stepper Motor

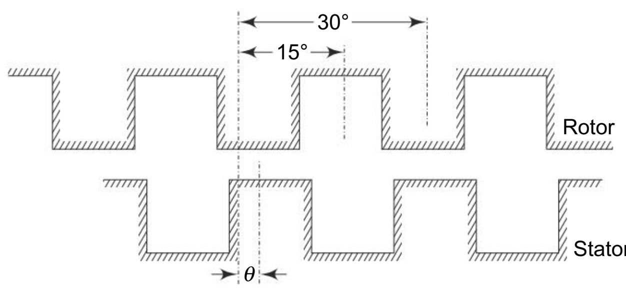


Fig. 13.72 View of Teeth of a Pair of Stator-rotor

are two positions of zero torque (*i*) $\theta = 0$, in which rotor and stator aligned and (*ii*) $\theta = \dfrac{180°}{T}$ (T = number of rotor teeth), in which rotor teeth are aligned with stator slots.

Applications of Stepper Motors

The stepper motors are used in

(i) driving the paper feed mechanism in line printers and printing terminals.

(ii) floppy disk drives to provide precise positioning of the magnetic heads of the disks.

(iii) plotters to drive X and Y coordinate pens.

(iv) many supporting roles in the manufacture of packaged foodstuffs, commercial end products and even in the production of science fiction movies.

REVIEW QUESTIONS

1. What is electronics?
2. What is semiconductor?
3. How does the energy band structure of a semiconductor differ from that of a conductor and an insulator?
4. What is meant by doping in a semiconductor?
5. What is the need for adding impurities in a semiconductor?
6. Explain "majority and minority carriers" in a semiconductor.
7. What is a PN junction? How is it formed?
8. Describe the action of PN junction diode under forward bias and reverse bias.
9. Explain how unidirectional current flow is possible through a PN junction diode.
10. Explain V–I characteristics of a PN junction diode.
11. Explain the following terms in a PN junction diode.
    (a) maximum forward current
    (b) peak inverse voltage and
    (c) maximum power rating
12. Mention the application of PN junction diode.
13. Explain the mechanism of avalanche breakdown and zener breakdown.
14. Bring out the important differences between zener breakdown and avalanche breakdown of a PN junction
15. With help of V–I characteristics show how a zener diode is used as a voltage regulator.
16. What is a bipolar junction transistor? How are its terminals named?
17. Explain the operation of NPN and PNP transistors.
18. What are the different configurations of BJT?
19. Explain how you will obtain the static characteristics of junction transistor in CE configuration.
20. Explain the input and output characteristics of a transistor in CB configuration.
21. Explain the laboratory setup for obtaining the common collector characteristics of a transistor.
22. What is the relation between I_B, I_E and I_C in C_B configuration?
23. Define α, β, γ of a transistor. Show how α and β are related to each other.
24. Compare the performance of a transistor in different configurations.
25. List the applications of BJT.
26. Why is a field effect transistor called so?
27. Explain the construction of N-channel JFET.
28. With the help of neat sketches and characteristic curves, explain the operation of the junction FET.
29. What is pinch-off voltage?
30. Compare JFET with BJT.
31. With the help of suitable diagrams explain the working of different types of MOSFET.
32. How does the constructional feature of a MOSFET differ from that of a JFET.
33. What is a thyristor? Mention some of them.
34. Describe the operation of Shockley diode.

35. With the help of V–I characteristics describe the working principle of an SCR.
36. Draw the two-transistor model of SCR and explain its breakdown operation.
37. What is a Bipolar junction transistor? How are its terminals named?
38. Explain the operation of NPN and PNP transistor.
39. What are the different configurations of BJT?
40. Explain the input and output characteristics of a transistor in CB configuration.
41. Explain the early effect and its consequences.
42. Derive the relationship α and β.
43. Why does the CE configuration provide large current amplification while the CB configuration does not?
44. Draw the circuit diagram of an NPN junction transistor CE configuration and describe the static input and output characteristics. Also, define active, saturation and cutoff regions, and saturation resistance of a CE transistor.
45. How will you determine h-parameters from the characteristics of CE configuration?
46. Determine the h-parameters from the characteristics of CB configuration.
47. What is the relation between I_B I_E and I_C in CB configuration?
48. Explain the laboratory setup for obtaining the CC characteristics.
49. Compare the performance of a transistor in different configurations.
50. Define α, β and γ of a transistor. Show how are they related to each other?
51. Explain how a transistor is used as an amplifier.
52. From the characteristics of CE configuration, explain the large signal, d.c.. and small signal CE values of current gain.
53. Calculate the vales of I_C and I_E for a transistor with αd.c. = 0.99 and I_{CBO} = 5 μA. I_B is measured as 20 μA. [**Ans.** I_C = 2.48 mA, I_E = 2.5 mA]
54. If α d.c. = 0.99 and I_B, find emitter current. [**Ans.** I_C = 104 mA, I_E = 105 mA]
55. If I_C is 100 times larger than I_B, find the value of β_{dc} [**Ans.** 100]
56. Find the value of α_{dc}, if β_{dc} is equal to 100. [**Ans.** 0.99]
57. Find the voltage gain of a transistor amplifier if its output is 5 V r.m.s and the input is 100 mV r.m.s [**Ans.** 50]
58. Find the value of α d.c., when I_C = 8.2 mA and I_E = 8.7 mA. [**Ans.** 0.943]
59. If α_{dc} is 0.96 and I_E = 9.35 mA, determine I_a [**Ans.** 8.98 mA]
60. Why is a field effect transistor called so?
61. Explain the construction of N-channel JFET.
62. With the help of a near sketches and characteristic curves, explain the operation of the junction FET.
63. What is pinch-off voltage?
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65. With the help of suitable diagrams explain the working of different types of MOSFET.
66. How does the constructional feature of a MOSFET differ types of MOSFET.
67. What is a thyristor? Mention some of them.
68. Describe the operation of Shockley diode.
69. With the help of V–I characteristics describe the working principle of an SCR.
70. Draw the two-transistor model of SCR and explain its breakdown operation.
71. What is a TRIAC? Sketch its characteristics and describe its operation.
72. DIAC is a bidirectional device Explain.

73. What is the advantage of Traic over SCR?
74. Draw the equivalent circuit of UJT and explain its operation with the help of emitter characteristics.
75. What are intrinsic stand-off ratio, peak point and valley point of a UJT.
76. Mention some of the applications of UJT.
77. What is the effect of light on semiconductors?
78. State photo-emissive effect.
79. Describe the principle involved in the photoconductive cell and photovoltaic cell.
80. What is photoconductive effect? Describe the constructional features of the photoconductive cell.
81. Distinguish between photovoltaic cell and solar cell.
82. What is LDR? Explain the use of LDR in the field of light measurements.
83. What is photovoltaic effect?
84. Give a brief description of the following optical transducers.
    (a) Vacuum phototube
    (b) Multiplier phototube
    (c) Photovoltaic cell and
    (d) Photoconductive devices
85. Draw and explain the basic circuit of a phototube amplifier.
86. Explain the principle and working of photodiode.
87. Describe with neat diagrams the theory of phototransistors.
88. With the help of output characteristics, explain how a phototransistor responds to the incident light.
89. Describe with the help of a relevant diagram, the construction of an LED and explain is working.
90. State the application of an LED.
91. Compare the working principle of LED and solar cell.
92. Describe the principle of operation of an LCD.
93. What are the advantages and disadvantages of LCD?
94. What are the relative advantages of LCD over LED?
95. In what respect is an LED different from an ordinary PN junction diode? State applications of LEDs.
96. Draw and explain a seven-segment LED display.
97. Draw and explain a dot matrix (35 elements) LED display.
98. Describe the operation of optocoupler.
99. What are digital instruments? Compare analog and digital instruments.
100. Describe the working of a CRO with the help of block diagram.
101. Explain how frequency and phase can be measured using a CRO.
102. Describe the applications of CRO.
103. Describe the distortion test on amplifiers using a CRO.
104. Explain the operation of switched mode player supply in detail with a block diagram.
105. What are the two methods of conversion techniques used in SMPS/Uninterrupted power supply?
106. What is an UPS? How does it operate?
107. List out the advantages of SMPS?
108. Explain the working principles of Constant voltage transformer.

109. What are the advantages of Constant voltage transformer?
110. What are the advantages and disadvantages of CVT?
111. Explain briefly the function of a servo stabilizer with a necessary diagram.
112. Describe the function of Isolation transformer.
113. Where do you use the isolation transformer?
114. What are the uses of servomotor?
115. Compare an A.C. and D.C. servomotor?
116. What are the uses of servomotor?
117. Describe the working principle of any one type of A.C. servomotor.
118. Describe the working principle of any one type of D.C. servomotor.
119. What is stepper motor?
120. What are the advantages of stepper motors?
121. Describe the working principle of any one type of stepper motor.

INTEGRATED CIRCUITS 14

INTRODUCTION

Integrated circuits (ICs) are miniaturized solid state devices and components. Most of the ICs are silicon chips with devices such as transistors, diodes, capacitors, resistors and operational amplifiers (op-amps). An IC can contain many thousands of devices. This chapter deals with the fabrication of monolithic ICs, and basic concepts and applications of op-amps and IC 555 Timer. An op-amp is a high gain, direct coupled amplifier. The voltage gain can be controlled by providing a feedback externally. IC technology is successful in offering low cost, little power consumption, high performance and versatile op-amps.

14.1 ADVANTAGES AND LIMITATIONS OF ICs

The following are the advantages of ICs over discrete components.

(i) Small size (around 20,000 components/square inch)
(ii) Improved performance (more complex circuits may be fabricated)
(iii) Low cost
(iv) High reliability and ruggedness
(v) Low power consumption
(vi) Less vulnerability to parameter variation
(vii) Easy troubleshooting
(viii) Simple design of systems
(ix) Standard packaging
(x) Increased operating speed (due to the absence of parasitic capacitance effect)
(xi) Less weight and portable
(xii) Battery operated systems due to low power supply requirement

The limitations of of ICs are as follows:

(i) As IC is small in size, it is unable to dissipate large amount of power. Increase in current may produce enough heat which may destroy the device.
(ii) At present, coils, inductors and transformers cannot be produced in IC form.

14.2 CLASSIFICATION OF ICs

IC technology has been advancing rapidly, increasing the complexity and functionality of the circuits fabricated. This necessitates the need for categorizing ICs based on their complexity density levels as given in the following table.

Type of IC	*No. of Gates*	*No. of Transistors*
Small Scale Integration (SSI)	3–30 per chip approx.	100
Medium Scale Integration (MSI)	30–300 per chip approx.	100–1,000
Large Scale Integration (LSI)	300–3,000 per chip approx.	1,000–20,000
Very Large Scale Integration (VLSI)	More than 3,000 per chip	20,000–10,00,000
Ultra Large Scale Integration (ULSI)	-	10^6–10^7
Giant-Scale Integration (GSI)	-	More than 10^7

The area for SSI chip is 1 sq.mm (1600 sq.mil) and for LSI chip it is 1 sq.cm (1,60,000 sq.mil).

The ICs can be classified as shown in Fig. 14.1. On the basis of fabrication process used, ICs can be classified as *monolithic* circuits and *hybrid* circuits. The word *monolithic* means *single stone* and as the name implies, the entire circuit is fabricated on a single chip of semiconductor. In monolithic integrated circuits, the components like transistors, diodes, resistors and capacitors are formed simultaneously by a diffusion process. Then the process of metalization is used in interconnecting these components to form the required circuit. The dielectric or PN junction is used to provide electrical isolation in monolithic ICs. The monolithic circuit technology is ideal for applications requiring identical characteristics of components in very large quantities. Therefore, they cost very less and provide higher order of reliability.

A hybrid circuit contains individual component parts attached to a ceramic substrate. The components are interconnected by the use of either metallization patterns or bonding wires. The hybrid circuits improve the circuit performance,

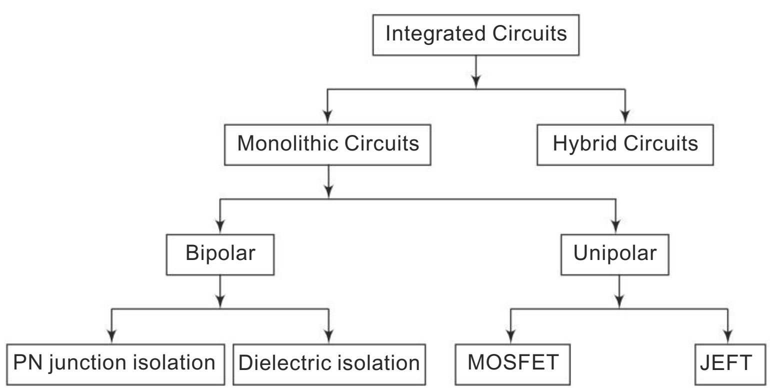


Fig. 14.1 Classification of ICs

since passive component values can be trimmed to precision at higher values. This technology is more suitable to customdesigned circuits of small volume fabrications. Hybrid ICs are categorized as thin film and thick film, based on the method used to form the resistors, capacitors, and related interconnections on the substrate.

Based on the active devices used, ICs can be classified as bipolar (using BJT) and unipolar (FET). Depending on the isolation technique employed to separate the individual components in the ICs, the bipolar ICs may further be classified as (i) PN junction isolation and (ii) dielectric isolation. On the basis of the type of field effect generation in the FET devices, the unipolar ICs may further be classified as JFET and MOSFET.

The integrated circuits can be divided into two major categories namely, linear or analog and digital. The ICs can more relevantly be classified also as (i) Analog ICs, (ii) Digital ICs and (iii) Mixed signal ICs based on the type and combinations of signals they process. The classification is made based on the treatment of the signal. An analog *singal* is the one that is defined over a continuous range of time, while, the digital signal is the one that is defined only at discrete points of time, by discrete values of amplitude. The linear ICs such as op-amps, voltage regulators, voltage comparators and timers are related to all the design phases of electronics in which signals are represented by continuous or analog quantities. The digital ICs such as logic gates, flip-flops, counters, digital clock chips, calculator chips, memory chips and microprocessors deal with discrete quantitites. In a digital IC, the information is represented by binary digits and involves logic and memory. The mixed signal ICs involve both the analog and digital signal processing.

14.3 MANUFACTURING PROCESSES OF MONOLITHIC ICs

The components normally fabricated in an IC are transistors, diodes, resistors and capacitors. Fabrication of a transistor is the most complicated of all and the fabrication of all the other components can be done in conjunction with the transistor fabrication processes. The various steps involved in the fabrication of monolithic IC by the epitaxial diffusions process are:

1. Silicon Wafer preparation
2. Epitaxial growth
3. Oxidation
4. Photolithography
5. Isolation diffusion
6. Base and Emitter diffusion
7. Pre-ohmic Etch
8. Metallisation
9. Circuit Probing
10. Scribing and Separating into Chips
11. Mounting and Packaging
12. Encapsulation

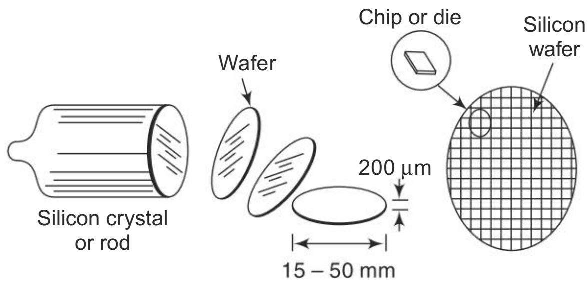


Fig. 14.2(a) Silicon Wafer Preparation

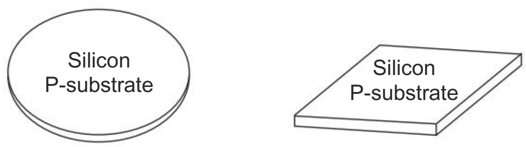


Fig. 14.2(b) Enlarged Views of Circular and Rectangular P-type Silicon Substrates

Silicon Wafer Preparation The starting material in the fabrication of an IC is a slice of single crystal silicon. Impurities of P-type (or N-type) can be added to the melt to give the required resistivity to the final silicon ingots (bars). A P-type silicon bar is cut into thin silicon called wafers as shown in Fig. 14.2(a). These wafers are polished to mirror finish to serve as the base or substrate for hundreds of ICs. The enlarged views of circular and rectangular P-type silicon substrates is shown in Fig. 14.2(b).

Epitaxial Growth An N-type silicon layer of about 15 micrometer thickness is grown on the P-type substrate by placing the wafer in a furnace at 1200°C and introducing a gas containing phosphorous (donor impurity). The resulting structure is shown in Fig. 14.3. All active and passive components are formed on the thin N-type epitaxial layer grown over the P-type substrate. This layer becomes the collector region for the transistor or cathode for a diode or an element for a capacitor.

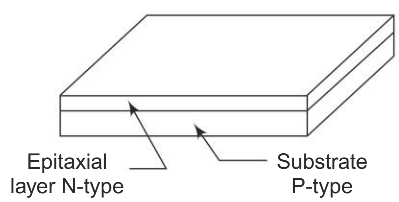


Fig. 14.3 Epitaxial Growth

Oxidation As shown in Fig. 14.4, a thin layer of silicon dioxide (SiO_2) is grown over the N-type layer by exposing the silicon wafer to oxygen atmosphere at about 1000°C.

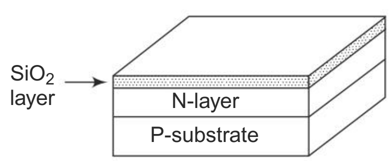


Fig. 14.4 Oxidation

Photolithography The prime use of photolithography in IC manufacture is to selectively etch (remove) the SiO_2 layer.

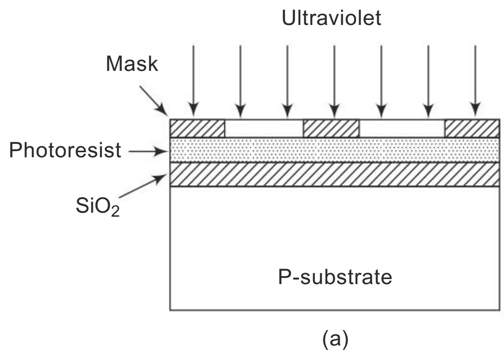


Fig. 14.5(a) Masking and Exposure to Ultraviolet Radiation

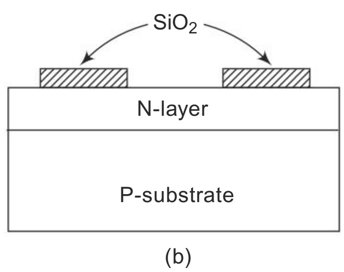


Fig. 14.5(b) Selective Openings in the SiO_2 Layer

As shown in Fig. 14.5(a) the surface of the oxide is first covered with a thin uniform layer of photosensitive emulsion (Photoresist). The mask, a black and white negative of the required pattern, is placed over the structure. When exposed to ultraviolet light, the photoresist under the transparent region of the mask becomes polymerized. The mask is then removed and the wafer is treated chemically that removes the unexposed portions of the photoresist film. The polymerised region is cured so that it becomes resistant to corrosion. The chip is dipped in an etching solution of hydrofluoric acid which removes the oxide layer not protected by the polymerised photoresist, thereby creating openings in the SiO_2 layer as shown in Fig. 14.5(b) through which P-type or N-type impurities, the polymerised photoresist is removed with sulphuric acid and by a mechanical abrasion process.

Isolation Diffusion The wafer is next subjected to P-type diffusion process which results in insolated islands of N-type layer as shown in Fig. 14.6, in which the transistor or some other component is fabricated. The N-type island forms the collector region of an NPN transistor. The heavily doped P-type regions marked P^+ result in improved isolation

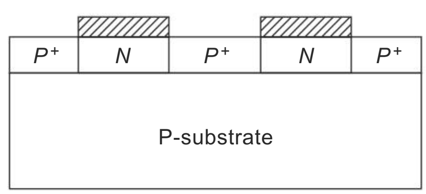


Fig. 14.6 Isolation Diffusion

between the active and passive components that will be formed in the various N-type islands of the epitaxial layer.

Base and Emitter Diffusion By once again using the photolithography and isolation diffusion processes, the P-type base region of the transistor is now diffused in the N-type island (collector region) as shown in Fig. 14.7(a). Similarly, the N-type emitter region of the transistor is diffused into the P-type base region as shown in Fig. 14.7(b). However, this is not required to fabricate a resistor where the resistivity of the P-type base region itself will serve the purpose. In this way one NPN transistor and one resistor are fabricated simultaneously.

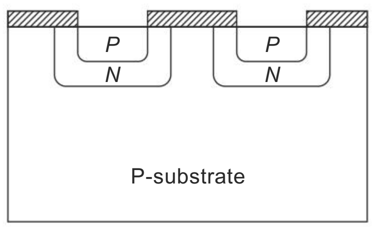


Fig. 14.7(a) Base Diffusion

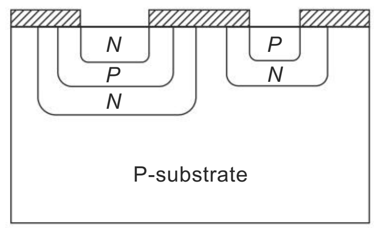


Fig. 14.7(b) Emitter Diffusion

Pre-ohmic Etch In order to get a good metal ohmic (non-rectifying) contact with the various diffused regions, N^+ regions are diffused into the structure as shown in Fig. 14.8. This is done by the oxidisation, photolithography and isolation diffusion processes.

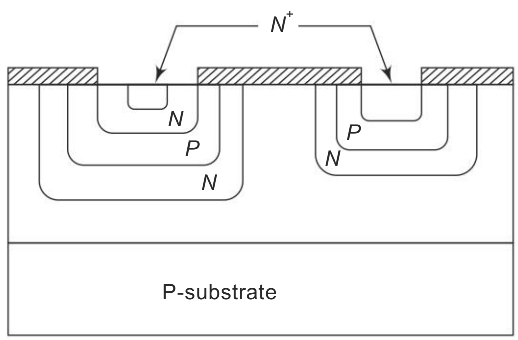


Fig. 14.8 Pre-ohmic Etch

Metallisation Metallisation is done for making interconnection between the various components fabricated in an IC and producing bonding pads around the circumference of the IC chip for later connection of wires. Metallisation is carried out by evaporating aluminium over the entire surface and then selectively etching away the aluminium to leave behind the desired interconnection and bonding pads as shown in Fig. 14.9.

Circuit Probing The performance of each integrated circuit fabricated on a wafer, is checked electrically by placing probes on the bonding pads. Faulty

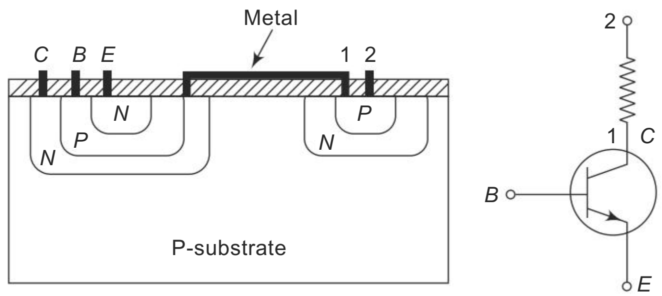


Fig. 14.9 Metallisation

chips are marked and discarded after the wafer is scribed and broken down into individual chips.

Scribing and Separating into Chips The wafer in which hundreds of ICs are fabricated is broken into individual chips by scribing with a diamond-tipped tool.

Mounting and Packaging The individual chip cannot be directly handled because it is very small and brittle. Hence it is soldered to a gold plated header through which leads have already been connected. The standard packages available are top-hat (TO) package, flat package and dual-in-line plastic package shown in Fig. 14.10 (a), (b) and (c) respectively.

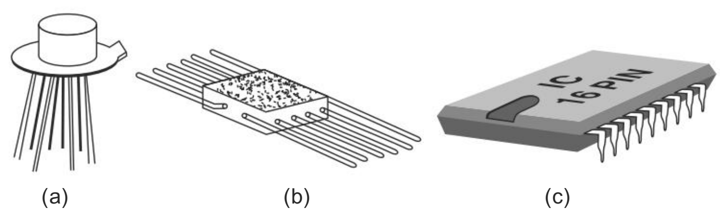


Fig. 14.10 (a) Top-hat (TO) Package, (b) Flat Package and (c) Dual-in-line Plastic Package

Encapsulation Encapsulation of an IC is essential to protect it against mechanical and chemical damage when it is in use. This is done by placing a cap over the circuit and sealing in an inert atmosphere.

14.3.1 Monolithic Diodes

The diodes used in integrated circuits are made by using the transistor structure. The three most popular are shown in Fig. 14.11. They are obtained from the transistor structure by using the emitter-base diode, with the collector short-circuited to the base; the emitter-base diode, with the collector open; and the collector-base diode, with the emitter open-circuited. The choice of the diode depends on the performance and application desired. Collector-base diodes have the higher collector-base voltage breaking rating, and they are suitable for common-cathode

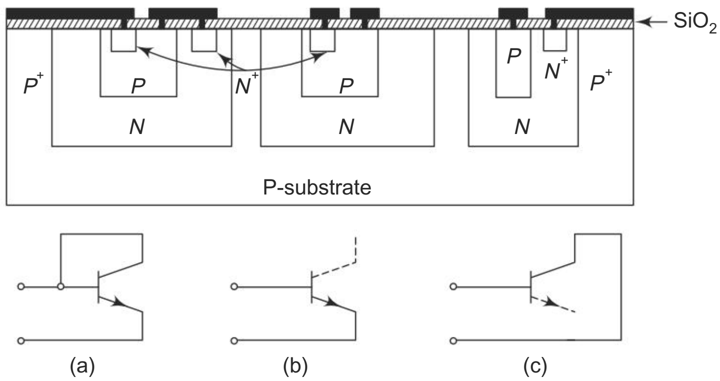


Fig. 14.11 Cross-section of various diode structures (a) Emitter-Base diode with collector shorted to base (b) emitter-base diode with collector open and (c) collector-base diode (no emitter diffusion)

diode arrays diffused within a single isolation island. The emitter-base diffusion is very popular for the fabrication of diodes.

14.3.2 Monolithic Resistors

A resistor in a monolithic IC is obtained by utilising the bulk resistivity of the diffused area. The N-type emitter diffusion and P-type base diffusion are commonly used to realise a monolithic resistor. Figure 14.12 shows the cross-sectional view of monolithic resistors.

In monolithic resistors, the resistance value is given by the formula,

$$R = R_s\, Xl/w$$

where R = Resistance offered (in ohms)

R_s = Sheet resistance (in ohms/square*)

l = Length of the diffused area

w = Width of the diffused area

[*1 square = 1 mil X 1 mil where 1 mil = 25 μm]

The sheet resistance of the base and emitter diffusion is 200 ohm/square and 2.2 ohm/square respectively. For higher values of resistances, the diffused region can be formed in zig-zag fashion, having a greater effective length.

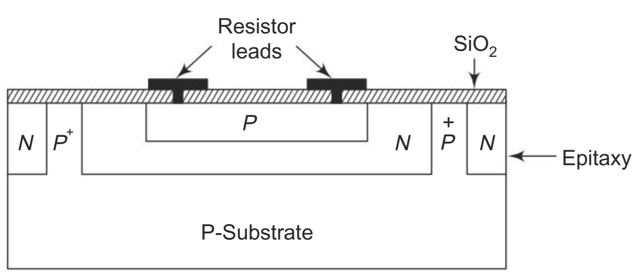


Fig. 14.12 Monolithic Resistor

14.3.3 Monolithic Capacitors

Monolithic capacitors are not frequently used in integrated circuits since they are limited in range and performance. The junction capacitor, shown in Fig. 14.13, is a reverse biased PN junction formed by the collector-base or emitter-base diffusion of the transistor. The capacitance is proportional to the area of the junction and inversely proportional to the depletion layer thickness.

The capacitance C is directly proportional to the area of cross section, A, and inversely proportional to the thickness of the depletion layer, T.

The capacitance value can be around 1.2 nF/mm^2.

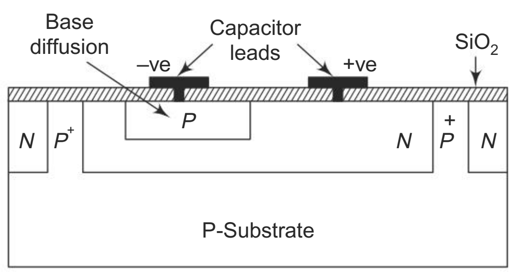


Fig. 14.13 Monolithic Capacitor

14.4 LINEAR ICs

14.4.1 Operational Amplifier

The operational amplifier (OP-AMP) is a basic block that is used in a broad range of electronic circuit applications. The op-amp performs the mathematical operations such as addition, subsraction, multiplication and division.

14.5 IDEAL OPERATIONAL AMPLIFIER

The "ideal" operational amplifier is a differential input, single-ended output device. The characteristics of an ideal op-amp are as follows:

1. Infinite input resistance, R_i = infinity
2. Zero output resistance, $R_o = 0$
3. Infinite voltage gain, A_V = infinity
4. Infinite bandwidth, BW = infinity
5. Infinite common-mode rejection ratio
6. Infinite slew-rate
7. Zero offset, i.e. when $V_1 = V_2$, $V_o = 0$
8. Characteristics do not drift with temperature.

The advantage of an ideal op-amp is that it can be used to perform a large number of mathematical operations, or generate a number of circuit functions by applying passive feedback around the amplifier. If the input and output impedance levels are respectively high and low with respect to the feedback impedances (connected externally), and if the voltage gain is sufficiently high, then the resulting amplifier performance becomes solely determined by the external feedback components.

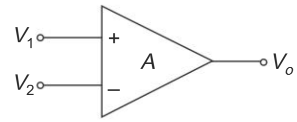


Fig. 14.14 Schematic Symbol of an Op-amp.

Figure 14.14 shows the schematic symbol of an op-amp. It has two inputs and one output. The upper input is called the noninverting input and is marked with a plus sign to indicate that V_o is in phase with V_1. The lower input is known as the inverting input; it is marked with a minus sign to indicate that V_o is 180° out of phase with V_2. If the overall voltage gain of the op-amp is A, then

$$V_o = A(V_1 - V_2) \tag{14.1}$$

Thus the output voltage is equal to the voltage gain times the difference of the two input voltages.

14.6 OPERATIONAL AMPLIFIER STAGES

The first stage of an op-amp is almost a differential amplifier and the last stage is usually a class B push–pull emitter follower. Figure 14.15 (a) shows the block diagram of a typical op-amp and Fig. 14.15 (b) shows a simplified schematic diagram for a typical op-amp. This circuit is equivalent to the 741 and many later-generation op-amps.

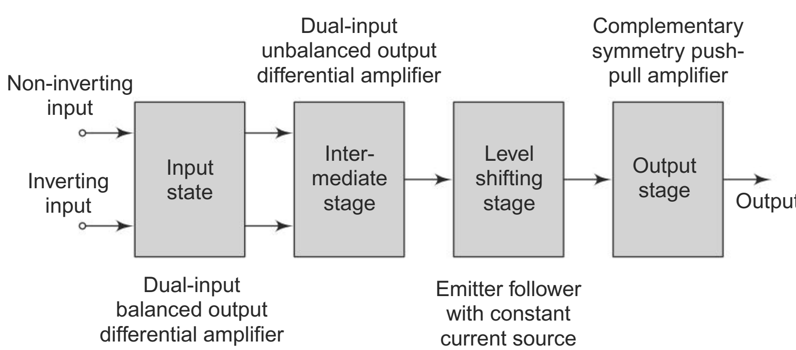


Fig. 14.15 (a) Block Diagram of a Typical Op-amp

14.6.1 Input Stage

The input stage is a dual-input, balanced-output differential amplifier. This stage provides most of the voltage gain of the amplifier and also establishes the input resistance of the op-amp. In Fig. 14.15(b), Q_{13} and Q_{14} form a current mirror circuit. Therefore, Q_{14} sources tail current to the input differential amplifier stage (Q_1 and Q_2). The differential amplifier drives a current mirror consisting of Q_3 and Q_4. An input signal V_{in} produces an amplified current out of this mirror that goes into the base of Q_5. The input stage should have the following characteristics:

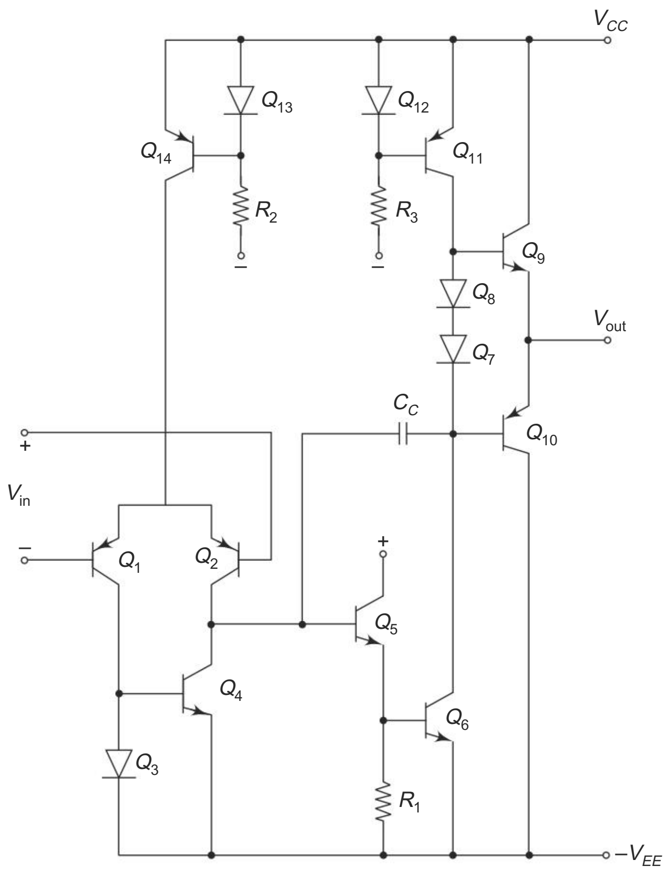


Fig. 14.15(b) Simplified Schematic for 741 and Similar Op-amps

(i) High input resistance (typ. 10 M ohm)
(ii) Low input bias current (typ. 0.5 μA)
(iii) Small input offset voltage (typ. 10 mV)
(iv) Small input offset current (typ. 0.2 mA)
(v) High CMRR (typ. 70 dB)
(vi) High open-loop voltage gain (typ. 10^4)

14.6.2 Intermediate Stage

In most of the amplifiers, an intermediate stage (dual input, unbalanced output differential amplifier) is provided which increases the overall gain of the op-amp. Because of direct coupling between the first two stages, the d.c. level at the output of the intermediate stage is well above the ground potential. This requires a level translator as the succeeding stage in order to bring the d.c. level back to the ground potential.

14.6.3 Level Shifter Stage

The level shifter (translator) circuit is used after the intermediate stage to shift the d.c. level at the output of the intermediate stage downward to zero volts with respect to ground. Usually, the third stage is an emitter follower (Q_5) using constant current source. It steps up the input impedance of the stage consisting of Q_6 by a factor of BETA. Note that Q_6 is the driver for the output stage. Incidentally, the plus sign on the Q_5 collector means it is connected to the V_{CC} supply, similarly, the minus signs at the bottom of R_2 and R_3 mean these are connected to the V_{EE} supply.

14.6.4 Output Stage

The last stage is a complementary symmetry push–pull amplifier (Q_9 and Q_{10}). Q_{11} is part of a current mirror that sources current through the compensating diodes (Q_7 and Q_8). Q_{12} is the input half of the mirror, and biasing resistor R_3 sets up the desired mirror current. C_C, called as a compensating capacitor (typically 30 pF), has a pronounced effect on the frequency response. It is needed to prevent oscillations and unwanted signals produced within the amplifier. The output stage should have the following desirable properties:

(i) Large output voltage swing capability.
(ii) Large output current swing capability.
(iii) Low output resistance.
(iv) Short circuit protection.

An emitter follower as the output stage can provide low output resistance and class B and class AB stage can provide large amount of output power.

14.7 OPERATIONAL AMPLIFIER PARAMETERS

The electrical parameters of the operational amplifier are defined in the following paragraphs.

Input Offset Voltage The input voltage is the error voltage needed to null or zero the quiescent output voltage of an op-amp. That is, the voltage that must be applied between the two input terminals of an op-amp to null the output. For instance, the 741 has a worst-case input offset voltage of 5 mV, i.e., when using 741 ICs, we have to apply a difference input of up to 5 mV to null the quiescent output voltage. The smaller the value of input offset voltage, the better the input terminals are matched.

Input Offset Current Op-amp fabrication uses monolithic integrated circuit construction, which can produce very well-matched devices for input stages, etc. by using identical geometry for a pair of devices diffused at the same time on the same chip. Nevertheless, there is always some mismatch, which gives rise to the input offset current. The input offset current is the difference between the two input currents. The 741 has an input offset current of 20 nA. When working with 741 ICs, we may find 20 nA more current in one base than the other. These unequal base currents produce a false difference voltage that gets amplified to produce a false output voltage. The smaller the input offset current, the better is the op-amp's performance.

Input Bias Current Input bias current is the average of the currents that flow into the inverting and non-inverting input terminals of an op-amp. Input bias current affects all applications of op-amps. For the op-amp to function, it is necessary to supply a small current, from picoamperes for op-amps with FET inputs to microamperes for junction transistor type inputs. The smaller the input bias current, the smaller the possible unbalance. The 741 has an input bias current of 80 nA.

Common-mode Rejection Ratio The common-mode rejection ratio (CMRR) serves as a figure of merit of an op-amp and is defined as the ratio of the differential voltage gain A_d to the common-mode voltage gain A_{cm}; that is,

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| \tag{14.2}$$

The differential voltage gain, A_d, is the gain of the op-amp when two different voltages are applied at the two inputs and is same as the large-signal voltage gain, which is specified on the data sheet. The common-mode voltage gain, A_{cm}, is the gain of the op-amp when the two terminals of the op-amp are applied with the same voltage from the same source. Generally, the common-mode voltage gain is very small and the differential voltage gain is very large; therefore, the CMRR is very large. Ideally, an op-amp has zero common-mode voltage gain and hence, the CMRR is infinite. CMRR, being a very large value, is normally expressed in decibels. For 741 ICs, CMRR is 90 dB typically. The higher the value of CMRR, the better is the matching between two input terminals and the smaller is the output common-mode voltage. A high CMRR means that the op-amp has a better ability to reject common-mode signals like electrical noise.

Example 14.1 For a given op-amp, CMRR = 10^5 and differential gain A_d = 10^5. Determine the common-mode gain A_{cm} of the op-amp.

Solution:

$$\text{CMRR} = \frac{A_d}{A_{cm}} = 10^5$$

Therefore, the common-mode gain, $A_{cm} = \dfrac{A_d}{\text{CMRR}}$

$$= \frac{10^5}{10^5} = 1$$

Slew Rate Slew rate is an important parameter because it limits the bandwidth for large signals. Slew rate limiting affects all amplifiers where capacitance on internal nodes, or as part of the external load, has to be charged and discharged as voltage levels change. It is defined as the maximum rate of change in output voltage per unit of time and is expressed in volts per microseconds, i.e.,

$$\text{SR} = \left.\frac{dV_o}{dt}\right|_{\max} v/\mu\text{s} \tag{14.3}$$

Slew rate indicates how fast the output of an op-amp can change in response to changes in the input frequency. For example, the op-amp 741 has a slew rate of 0.5 V/μs and its implication can be explained by considering Fig. 14.16. If we overdrive a 741 with a large step input (Fig. 14.16 a), the output slews as shown in Fig. 14.16 b. It takes 20 microseconds (SR = 0.5V/μs, i.e. for 0.5 volt change, 741 takes 1 microseconds; therefore for 10 volts change, it takes 10/0.5 = 20 microseconds) for the output voltage to change from 0 to 10 V. It is impossible for the 741 to change faster than this.

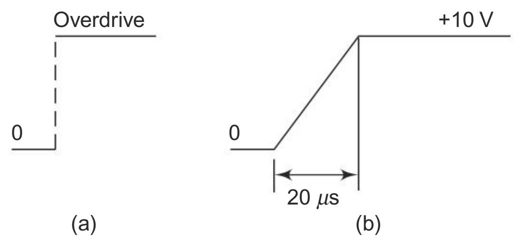


Fig. 14.16 Overloading an Op-amp Produces Slew Rate Limiting

Slew rate limiting can occur even with a sinusoidal signal. Figure 14.17 a shows a sinusoidal output with a peak of 10 V. The initial slope of the sine wave where it passes through zero is important. As long as this initial slope is less than the slew rate of the op-amp, there is no problem. But when the frequency increases, the initial slope of the sine wave may be greater than the slew rate, and this results in the distortion of the output waveform as shown in Fig. 14.17 b. The higher the frequency is, the smaller is the swing and the output waveform will be more triangular.

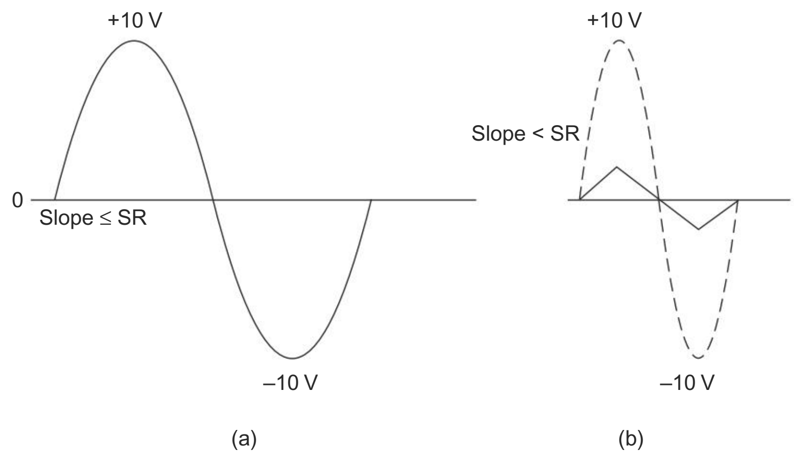


Fig. 14.17 Slew Rate Distortion of Sine Wave

Generally, the slew rate is specified for unity gain and hence, let us assume that a high frequency, large amplitude sinusoidal input is given to an op-amp voltage follower. The equation for the input sinusoidal signal is thus,

$$V_s = V_m \sin \omega t \tag{14.4}$$

With no slew rate limitations, the output voltage of the voltage follower is thus,

$$V_o = V_m \sin \omega t \tag{14.5}$$

The rate of change of the output is,

$$\frac{dVo}{dt} = \omega V_m \cos \omega t \tag{14.6}$$

The maximum rate of change of the output occurs when $\cos \omega t = 1$, i.e.,

$$\frac{dV_o}{dt}(\max) = \omega V_m \tag{14.7}$$

Thus, the slew rate is,

$$SR = \frac{dV_o}{dt}(\max) = 2\pi f V_m \text{ V/s}$$
$$= 2\pi f V_m 10^{-6} \text{ V}/\mu\text{s} \tag{14.8}$$

Equation (14.8) suggests that for faithful reproduction of the sinusoid, ωV_m must be less than or equal to the slew rate. For distortionless output, the slew rate determines the maximum frequency of operation, $f_{\max}$, for a desired output swing. The frequency $f_{\max}$ is called *full power bandwidth* and is defined as the maximum frequency at which an undistorted sinusodial output can be obtained with peak voltage V_m. For most general purpose op-amps full power bandwidth ranges from 5 to 50 kHz for a peak output swing of 10V.

Example 14.2 The output voltage of a certain op-amp circuit changes by 20 V in 4μs. What is its slew rate?

Solution: The slew rate, $SR = \frac{dV_o}{dt} = \frac{20\text{ V}}{4\,\mu\text{s}} = 5\text{ V}/\mu\text{s}$

Example 14.3 The 741C is used as an inverting amplifier with a gain of 50. The sinusodial input signal has a variable frequency and maximum amplitude of 20 mV peak. What is the maximum frequency of the input at which the output will be undistorted? Assume that the amplifier is initially nulled.

Solution: The 741C has a typical slew rate of 0.5 V/μs. Using Eq. (14.8), the slew rate is,

$$SR = \frac{2\pi f v_m}{10^6} = 0.5 \text{ V}/\mu\text{s}$$

The maximum output voltage, $V_m = A\, V_{id}$

$$= 50 \times 20 \text{ mV} = 1 \text{ V (peak)}$$

The maximum frequency of the input for which undistorted output is obtained is given by,

$$f_{\max} = \frac{SR}{2\pi V_m} \times 10^6$$
$$= \frac{0.5}{2\pi \times 1} \times 10^6 = 79.6 \text{ kHz}$$

Example 14.4 An inverting amplifier using the 741C must have a flat response up to 40 kHz. The gain of the amplifier is 10. What maximum peak-to-peak input signal can be applied without distoring the output?

Solution: The 741C has a typical slew rate of 0.5 V/μs. Using Eq. (14.8), the slew rate is,

$$SR = \frac{2\pi f V_m}{10^6} = 0.5 \text{ V}/\mu\text{s}$$

The maximum output voltage, $V_m = \frac{SR \times 10^6}{2\pi f}$

$$= \frac{0.5 \times 10^6}{2\pi \times 40 \times 10^3} = 1.99 \text{ V peak}$$

$$= 3.98 \text{ V peak-to-peak}$$

The maximum peak-to-peak input voltage for undistorted output is,

$$V_{id} = \frac{V_m}{A} = \frac{3.98}{10} = 0.398 \text{ V peak-to-peak.}$$

Gain–bandwidth Product The voltage gain of the op-amp decreases at high frequencies. This is due to the parasitic junction capacitance and minority-carrier charge storage in devices making up the circuit. This aspect of op-amp is characterised by the gain-bandwidth product, which is the bandwidth of the op-amp when the voltage gain is unity. Equivalent terms for gain-bandwidth product are closed-loop bandwidth, unity gain bandwidth and small-signal bandwidth. For general purpose op-amps, the gain-bandwidth product is in the 1 to 20 MHz range.

Input Resistance The op-amp having bipolar input stage has an input resistance in the range of 100 kΩ to 1 MΩ. Usually, the voltage gain is large enough that this input resistance has little effect on circuit performance in closed-loop feedback configurations. FET input stages have very high input resistance.

Output Resistance The output resistance of general purpose op-amps is in the order of 40 to 100 Ω. This resistance does not significantly affect the closed-loop performance of the op-amp.

14.8 EQUIVALENT CIRCUIT OF OP-AMP

The equivalent circuit of an op-amp is shown in Fig. 14.18. The equivalent circuit is useful in analyzing the operating principles of op-amps and in observing the effects of feedback. This circuit includes the values of open-loop gain A, input resistance R_i and output resistance R_o. The voltage source AV_{id} is an equivalent Thevenin voltage source, and R_o is the Thevenin equivalent resistance looking back into the output terminal of an op-amp.

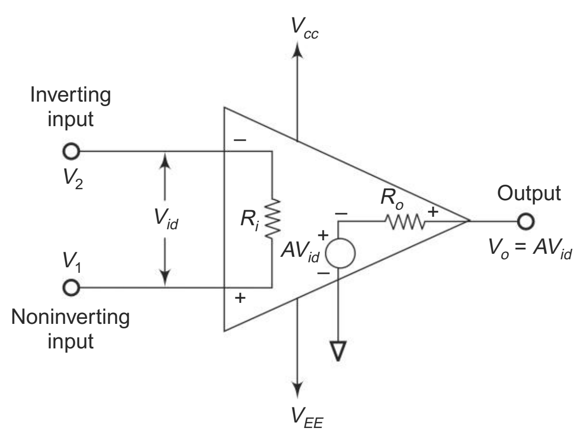


Fig. 14.18 Equivalent Circuit of an Op-amp

For the circuit shown in Fig. 14.18, the output voltage is

$$V_o = AV_{id} = A(V_1 - V_2) \tag{14.9}$$

where A = large-signal voltage gain

V_{id} = difference input voltage

V_1 = voltage at the noninverting input terminal

V_2 = voltage at the inverting input terminal.

The output voltage is directly proportional to the algebraic difference between the two input voltages. In other words, the op-amp amplifies the difference between the two input voltages; it does not amplify the input voltages themselves. The polarity of the output voltage depends on the polarity of the difference voltage.

14.9 IDEAL VOLTAGE TRANSFER CURVE

The equation for the output voltage of an op-amp, as given by Eq. (14.9), is used in analysing the characteristics of an op-amp. In Fig. 14.19, the output voltage of an ideal op-amp is plotted as a function of input differential voltage, keeping the large signal gain constant.

We notice that for very small values of the input differential voltage the output voltage increases linearly, however, the output voltage cannot exceed the saturation voltages. The saturation voltages are a function of the supply voltages. The output voltage is directly proportional to the input difference voltage only until it reaches the saturation voltages and thereafter output voltage remains constant. The curve plotted in Fig. 14.19 is called an ideal voltage transfer curve because output offset voltage is assumed to be zero. The curve, if drawn to scale, would be almost vertical because of the very large values of the large-signal gain, A.

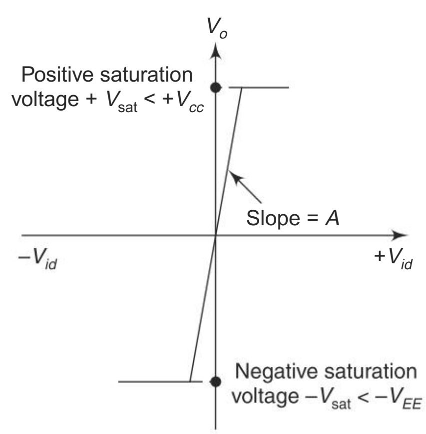


Fig. 14.19 Ideal Voltage Transfer Curve

14.10 OPEN-LOOP OP-AMP CONFIGURATIONS

The term open-loop indicates that no feedback, in any form, is given to the input from the output. When connected in open-loop, the op-amp functions as a very high gain amplifier. There are three open-loop configurations of op-amp, namely,

1. Differential amplifier
2. Inverting amplifier
3. Noninverting amplifier.

The above configurations are classified based on the number of inputs used and the terminal to which the input is applied. The op-amp is a versatile device and it amplifies both a.c. and d.c. input signals. Thus, the input signals can be either a.c. or d.c. voltages.

14.10.1 Open-Loop Differential Amplifier

In this configuration, the inputs are applied to both the inverting and the noninverting input terminals of the op-amp and the device amplifies the difference between the input voltages. Figure 14.20 shows the open-loop differential amplifier.

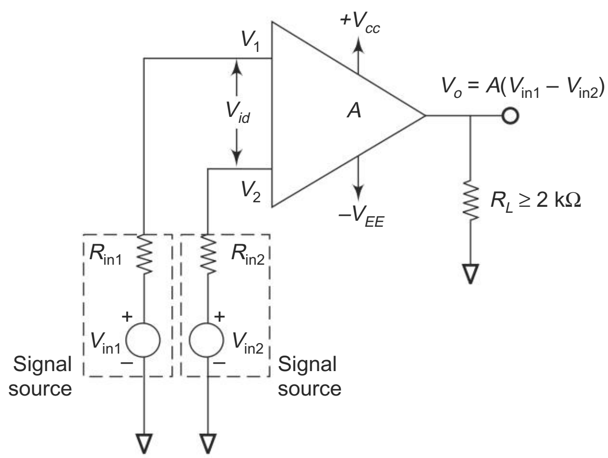


Fig. 14.20 Open-loop Differential Amplifier

The input voltages are represented by V_{in1} and V_{in2}. The source resistance R_{in1} and R_{in2} are negligibly small in comparison with the very high input resistance of the op-amp, and thus the voltage drop across these source resistances are assumed to be zero. The output voltage V_o is given by

$$V_o = A(V_{in1} - V_{in2}) \tag{14.10}$$

where A is the large-signal voltage gain. Thus, the output voltage is equal to the voltage gain, A, times the difference between the two input voltages. This is why this configuration is called a *differential amplifier.* In open-loop configurations, the large-signal voltage gain A is also called open-loop gain.

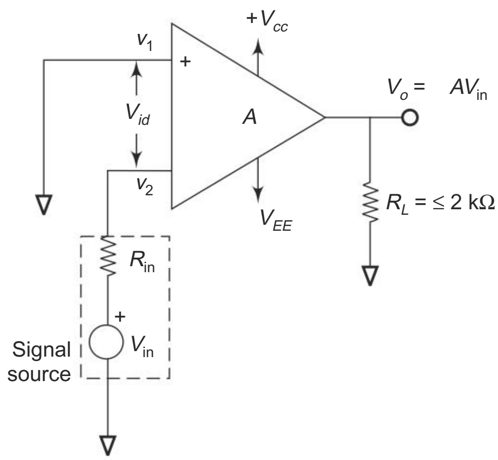


Fig. 14.21 Open-loop Inverting Amplifier

14.10.2 Inverting Amplifier

In this configuration the input signal is applied to the inverting input terminal of the op-amp and the noninverting input terminal is connected to the ground. Figure 14.21 shows the open-loop inverting amplifier.

The output voltage is 180° out of phase with respect to input and hence, the output voltage V_o is given by

$$V_o = -AV_{\text{in}} \tag{14.11}$$

Thus, in an inverting amplifier the input signal is amplified by the open-loop gain A and is also phase shifted by 180°.

14.10.3 Noninverting Amplifier

Figure 14.22 shows the open-loop noninverting amplifier. The input signal is applied to the noninverting input terminal of the op-amp and the inverting input terminal is connected to the ground.

The input signal is amplified by the open-loop gain A and the output is in phase with the input signal.

$$V_o = AV_{\text{in}} \tag{14.12}$$

In all the above open-loop configurations, only very small values of input voltages can be applied. Even for voltage levels slightly greater than zero the output is driven into saturation, which is obvious from the ideal transfer characteristics of op-amp shown in Fig. 14.19. Thus, when operated in the open-loop configuration, the output of the op-amp is either negative or positive saturation or switches between positive and negative saturation levels. This prevents the open-loop configurations of op-amps from being used in linear applications.

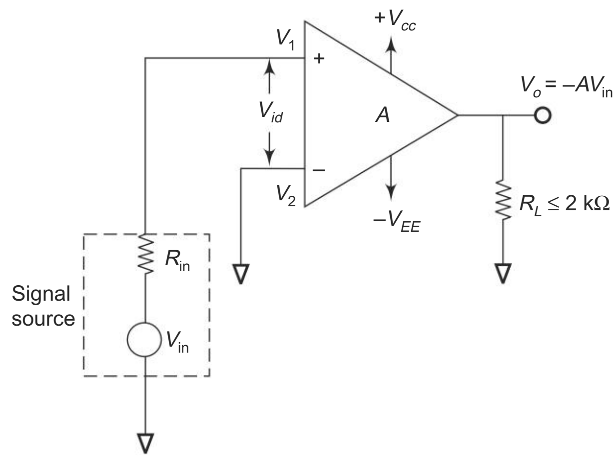


Fig. 14.22 Open-loop Noninverting Amplifier

14.10.4 Limitations of Open-Loop OP-Amp Configurations

In open-loop configurations, clipping of the output waveform occurs when the output voltage exceeds the saturation levels. This is because of very high open-loop gain of the op-amps and due to this fact only the smaller signals, of the order of microvolts or less, having very low frequency can be amplified accurately without distortion. However, signals of this magnitude are very susceptible to noise and almost impossible to obtain in the laboratory.

The open-loop gain of the op-amp is not a constant and varies with changing temperature and power supply. Also, the bandwidth of most open-loop op-amps is negligibly small. This makes the open-loop op-amps unsuitable for a.c. applications. The open-loop bandwidth of the 741 is approximately 5 Hz, but in almost all a.c. applications the bandwidth requirement is much larger than this. For the reasons stated, the open-loop op-amp is generally not used in linear applications. However, the open-loop op-amp configurations are also useful in certain nonlinear applications like square-wave generation, a stable multivibrators, etc.

14.11 CLOSED-LOOP OP-AMP CONFIGURATIONS

The op-amp can be effectively utilized in linear applications by providing feedback from the output to the input either directly or via another network. If the signal fed back is out of phase by 180° with respect to the input, then the feedback is referred to as negative feedback or degenerative feedback. Conversely, if the signal fed back is in phase with that at the input, then the feedback is referred to as positive feedback or regenerative feedback.

An op-amp that used feedback is called as a closed-loop amplifier. The most commonly used closed-loop configurations are (i) Inverting amplifier (voltage shunt feedback) and (ii) Noninverting amplifier (voltage series feedback).

14.11.1 Inverting Amplifier

The inverting amplifier is shown in Fig. 14.23 (a), and in Fig. 14.23 (b), the same inverting amplifier is redrawn in a different way so as to illustrate how voltage shunt feedback is provided. The input signal drives the inverting input of the op-amp through resistor R_1.

The op-amp has an open-loop gain of A, so that the output signal is much larger than the error voltage. Because of the phase inversion, the output signal is 180° out of phase with the input signal. This means the feedback signal opposes the input signal and the feedback is negative or degenerative.

Virtual Ground The open-loop gain of an op-amp is extremely high, typically 200,000 for a 741. If the output voltage is 10 V, the input differential voltage, V_{id}, is only

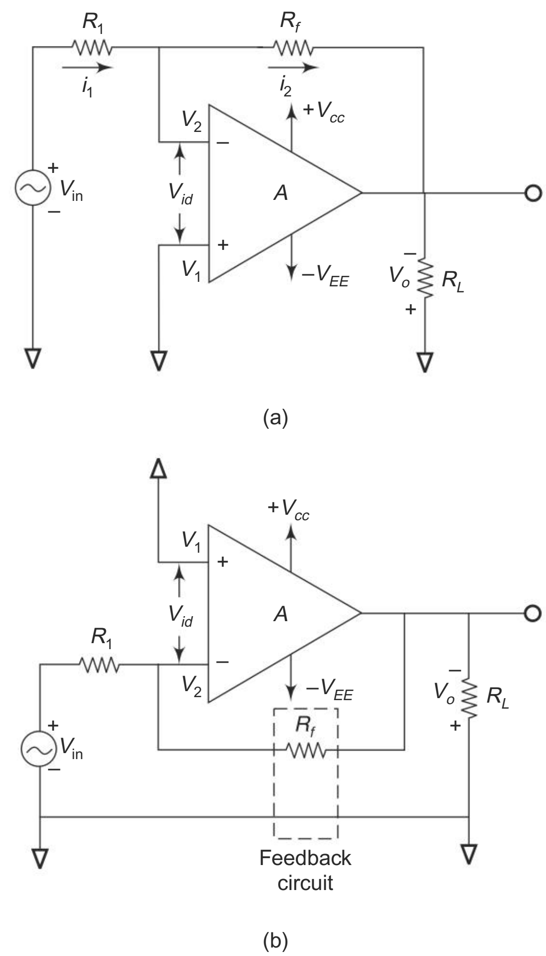


Fig. 14.23 Closed-loop Inverting Amplifier

$$V_{id} = \frac{V_o}{A} = \frac{10}{200{,}000} = 0.05 \text{ mV}$$

Furthermore, the open-loop input impedance of a 741 is around 2 MΩ. Therefore, for an input differential voltage of 0.05 mV, the input current is only

$$i_{\text{in}} = \frac{V_{id}}{R_{\text{in}}} = \frac{0.05 \text{ mV}}{2 \text{ M}\Omega} = 0.025 \text{ nA}.$$

Since the input current is so small compared to all other signal currents, this can be approximated to zero. For any input voltage applied at the inverting input, the input differential voltage, V_{id}, is negligibly small and the input current is ideally zero. Hence, the inverting input appears to be a ground point for all input voltages. Hence, the inverting input of Fig. 14.23 (a) acts as a virtual ground. The term virtual ground signifies a point whose voltage with respect to ground is zero, and yet no current can flow into the point.

The expression for the closed-loop voltage gain of an inverting amplifier can be obtained from Fig. 14.23 (a). Because the inverting input is a virtual ground, all of the input voltage appears across R_1. This sets up a current through R_1 that equals.

$$i_1 = \frac{V_{\text{in}}}{R_1} \tag{14.13}$$

All of this current must flow through R_f, because the virtual ground accepts negligible current. The left end of R_f is ideally grounded, and so the output voltage appears wholly across it. Therefore,

$$V_o = -i_2 R_f = -\,i_1 R_f = -\frac{R_f}{R_1} V_{\text{in}} \tag{14.14}$$

The closed-loop voltage gain, A_f, is given by

$$A_f = \frac{V_o}{V_{\text{in}}} = \frac{R_f}{R_1} \tag{14.15}$$

The above equation shows that the gain of the inverting amplifier is set by selecting a ratio of feedback resistance R_f to the input resistance R_1. The ratio R_f/R_1 can be set to any value, even to less than 1. Because of this property of the gain equation, the inverting amplifier configuration with feedback is more popular and lends itself to a majority of applications.

Example 14.5 For the inverting amplifier of Fig. 14.23, R_f = 10 kΩ and R_1 = 1 kΩ. Calculate the closed-loop voltage gain A_f.

Solution: The closed-loop voltage gain $A_f = -\frac{R_f}{R_1}$

$$= -\frac{10 \text{ k}\Omega}{1 \text{ k}\Omega} = -\,10$$

14.11.2 Noninverting Amplifier

The noninverting amplifier with negative feedback is shown in Fig. 14.24. The input signal drives the noninverting input of the op-amp. The op-amp provides an internal gain, A. The external resistors R_1 and R_f form the feedback voltage divider circuit. Since the returning feedback voltage drives the inverting input, it opposes the input voltage and hence, the feedback is negative or degenerative.

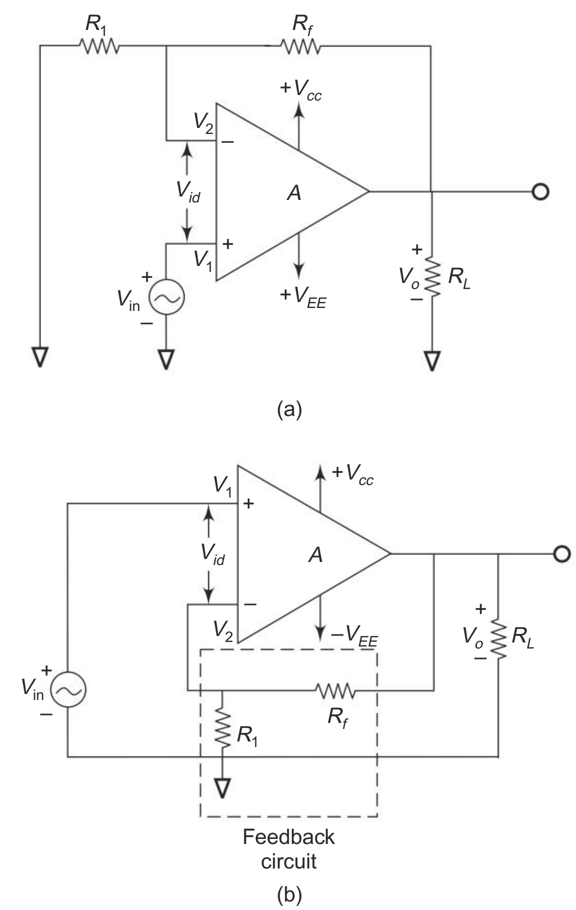


Fig. 14.24 Closed-loop Noninverting Amplifier

The feedback factor of the feedback voltage divider network is

$$\beta = \frac{R_1}{R_1 + R_f} \tag{14.16}$$

Therefore, the approximate closed-loop gain is

$$A_f \approx \frac{1}{\beta} = \frac{R_1 + R_f}{R_1} \tag{14.17}$$

$$= 1 + \frac{R_f}{R_1} \tag{14.18}$$

From the above equation, it is noted that the closed-loop gain is always greater than 1 and depends on the ratio of the feedback resistors. If precision resistors are used in the feedback network, a precise value of closed-loop gain can be achieved. The closed-loop gain does not drift with temperature changes or op-amp replacements.

Example 14.6 For the non-inverting amplifier of Fig. 14.24, R_1 = 1 kΩ and R_f = 10 kΩ. Calculate the closed-loop voltage gain of the amplifier and the feedback factor β.

Solution: The closed-loop voltage gain, $A_f = 1 + \dfrac{R_f}{R_1}$

$$= 1 + \frac{10\ \text{k}\Omega}{1\ \text{k}\Omega} = 11$$

The feedback factor, $\beta = \dfrac{R_1}{R_1 + R_f}$

$$= \frac{1\ \text{k}\Omega}{1\ \text{kW} + 10\ \text{k}\Omega} = 0.091$$

14.12 BANDWIDTH WITH FEEDBACK

The bandwidth of an amplifier is defined as the range of frequencies for which the gain remains constant. The gain-bandwidth product of an op-amp is always a constant. The gain of an op-amp and its bandwidth are inversely proportional to one another. The bandwidth of an op-amp can be increased by providing feedback signal to its input. Consider the following statement. The product of closed-loop gain and closed-loop bandwidth is same as the product of open-loop gain and open-loop bandwidth. That is,

$$A_f BW_{cl} = A BW_{ol} \tag{14.19}$$

The op-amp 741 has an open-loop gain of 200,000 and a bandwidth of about 5 Hz. Therefore, the product of its open-loop gain and bandwidth is

$$ABW_{ol} = 200{,}000 \times 5\ \text{Hz} = 1\ \text{MHz}.$$

The open-loop gain-bandwidth product of a 741 is 1 MHz. The left-hand side of Eq. (14.19) is the product of closed-loop gain and closed-loop bandwidth. No matter what the values of R_1 and R_F, the product of closed-loop gain and closed-loop bandwidth must equal the open-loop gain-bandwidth product, i.e. for 741 the closed-loop gain-bandwidth product must also be 1 MHz.

The open-loop response is shown in Fig. 14.25. The open-loop gain has a maximum value of 200,000. When the operating frequency increases to 5 Hz, the open-loop gain is down to 0.707 of its maximum value. The gain keeps dropping off with increasing frequency. After the upper cut-off frequency, f_{OL}, the gain drops by 20 dB/decade. The unity-gain frequency is the frequency where the open-loop gain has decreased to unity. In Fig. 14.25, f_{unity} equals 1 MHz.

The curve in Fig. 14.25 represents the closed-loop response of an op-amp. It can be seen from this figure that the open-loop gain decreases continuously until it approaches the value of closed-loop gain. Then, the closed-loop gain starts to decrease and at f_{CL}, the closed-loop gain is down to 0.707 of its maximum value. Thereafter, both the curves superimpose and decrease to unity at f_{unity}.

If the feedback resistors are changed, the closed-loop gain will change to a new value and so will the closed-loop cut-off frequency. But, because the gain-bandwidth product is constant, the closed-loop curve superimposes the open-loop curve beyond cut-off frequency.

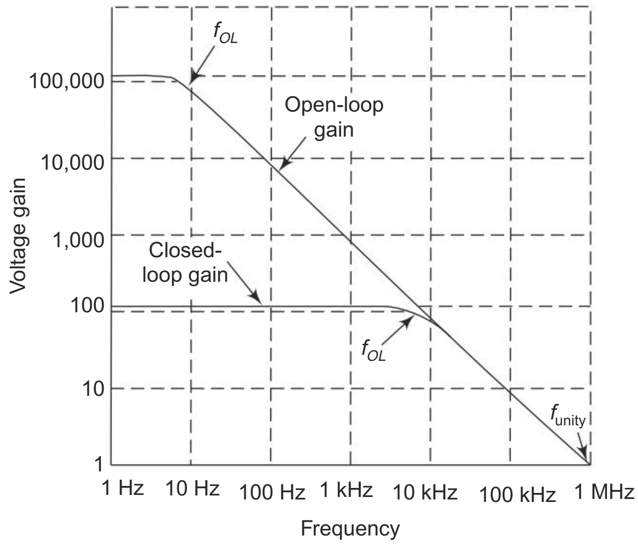


Fig. 14.25 Closed-loop and Open-loop Responses

14.13 NOISE

Noise is a major source of interference with the desired signal in electronic systems. Any unwanted signal associated with the desired signal is noise. Noise is random in nature and difficult to predict or analyze. In any electronic system, noise can come from many external sources as well as be self-induced, a result of the circuitry itself. Examples of external noise sources are switching or rotating machinery, ignition systems, and various control circuits. Natural phenomena such as lightning may also be an external noise source.

The internal noise may be caused by the a.c. random voltages and currents generated within the conductors and semiconductors of one circuit as a result of the switching of another circuit. The rate of change of current and voltage per unit of time, the speed of operation of the circuit, and the type of coupling between two circuits are some of the factors that determine the amount of noise induced in a given circuit.

Different types of noise phenomena are associated with op-amps: Schottky noise, thermal noise, and $1/f$ noise are the most important. The thermal noise increases with an increase in temperature. Like thermal noise, the amount of Schottky noise is greater with wider bandwidths and larger resistances; on the other hand, $1/f$ noise increases with a decrease in frequency f. Wider bandwidths of operation for a given value of source resistance generate more noise; also, larger source resistances for a given bandwidth increase the noise level appreciably.

To reduce the effect of electrical noise on ICs, several schemes have been commonly used. Physical shielding of the ICs and associated wiring helps to prevent external electromagnetic radiation from inducing noise into the internal circuity. Special buffering and filtering circuits can be used between the electronic circuits and signal leads. To provide a path for any radio frequency (RF), all linear IC power supply terminals should generally be bypassed to ground. The breadboard layout should be such that the bypass capacitors are as near the IC terminals as possible. Internal noise generation can be reduced by keeping input and output lead lengths as short as practically possible. One common tie point should be used near the IC for all grounds. In a high electrical noise environment, an IC with a high degree of noise immunity will minimize the amount of special care needed for proper circuit operation.

14.14 FREQUENCY RESPONSE AND COMPENSATION

In Sec. 14.12, we saw that the gain of an op-amp is frequency dependent. The gain A decreases as the operating frequency increases. This variation in gain as a function of frequency imposes a limitation on the performance and applications of the op-amps. In this section, the factors responsible for variations in open-loop gain as a function of frequency are investigated.

The gain of the op-amp is a complex number which is a function of frequency. At a given frequency, the gain will have a specific magnitude as well as a phase angle. This means that the variation in operating frequency will cause the variation in gain magnitude and its phase angle. The manner in which the gain of the op-amp responds to different frequencies is called the frequency response.

Generally, for an amplifier, as the operating frequency increases, the gain of the amplifier decreases, and the phase shift between the output and input signals increases. In the case of an op-amp, the change in gain and phase shift as a function of frequency is attributed to the internally integrated capacitors, as well as the stray capacitors. These capacitors are due to the physical characteristics of semiconductor devices and the internal construction of the op-amp.

The magnitude plot shows the way in which the gain of the op-amp changes with variation in frequency. The later-generation op-amps, such as the 741, 351 and 771, have phase shifts less than 90° even at cross-over frequencies. The cross-over frequency, also referred to as the unity gain bandwidth (UGB), is the maximum usable frequency of a given op-amp. The UGB, the range of frequencies upto to f_{unity} in Fig. 14.25, is 1 MHz for 741 op-amp.

The rate of change of gain as well as the phase shift can be changed using specific components with the op-amp. The most commonly used components are resistors and capacitors. The network formed by such components and used for modifying the rate of change of gain and the phase shift is called a compensating network. The phase lag and phase lead networks are the most commonly used compensating networks in op-amps. These two networks are indicative of their functions since phase lag contributes a negative phase angle and phase lead a positive phase angle. Thus, the main purpose of the compensating networks is to modify the performance of an op-amp circuit over the desired frequency range by controlling its gain and phase shift.

14.14.1 Internal Compensation

In internally compensated op-amps, the compensating network is designed into the circuit to control the gain and the phase shift of the op-amp. The op-amp 741C is an internally compensated op-amp. The open-loop frequency response curve for internally compensated op-amps is same as that shown in Fig. 14.25. For internally compensated op-amps like 741, the gain remains essentially constant from 0 Hz to the upper cut-off frequency f_{OL} and thereafter rolls off at a constant rate of 20 dB/decade. Thus, the open-loop bandwidth is the frequency band extending from 0 Hz to f_{OL}, or simply f_{OL}.

In the 741 op-amp, a 30 pF capacitor is the internal compensating component (C_C in Fig. 14.25 (b)), which helps to control the open-loop gain to allow it to roll off at a rate of 20 dB/decade. In fact, even if the 741 is configured as a closed-loop amplifier, inverting or noninverting, using only resistive components, the gain will always roll off at a rate of 20 dB/decade, regardless of the value of its closed-loop gain. The internally compensated op-amps are sometimes simply called compensated op-amps and generally have very small open-loop bandwidths.

14.14.2 External Compensation

In externally compensated op-amps, the external compensating components are added at designated terminals in noncompensated op-amps. For proper operation, the manufacturer recommends appropriate compensating components for the uncompensated op-amps. The op-amp 709C is a noncompensated (externally compensated) op-amp. The open-loop frequency response curves of these op-amps, as well as the connection diagram of the 709C for the external compensating components, are shown in Fig. 14.26.

Three compensating components, a resistor and two capacitors, are required for 709C. The nature of the frequency response curve depends on the values of the compensating components used. The roll-off rate with various compensating components that are specified along the gain versus frequency curves is about 20 dB/decade. Note that the open-loop bandwidth of a 709C decreases from the outermost compensated curve to the innermost. That is, if C_1 = 10 pF, R_1 = 0 ohm, and C_2 = 3 pF, the bandwidth is approximately 5 kHz, while if C_1 = 5000 pF, R_1 = 1.5 kΩ, and C_2 = 200 pF, the bandwidth is 100 Hz. The frequency compensation circuit for μA 709 is shown in Fig. 14.26 (b). The uncompensated op-amps offer relatively broader open-loop bandwidths.

14.15 OP-AMP APPLICATIONS

The operational amplifier, or op-amp, was originally developed in response to the requirements of analog computer designers. An op-amp is a high gain, direct coupled amplifier. The voltage gain can be controlled by the externally connected feedback components. The op-amp can be used in amplifier and signal processing applications involving d.c. to several MHz of frequency ranges. The operational amplifier circuits can be designed with various types of active devices. However, IC technology is remarkably successful in offering low-cost, high-performance, versatile and

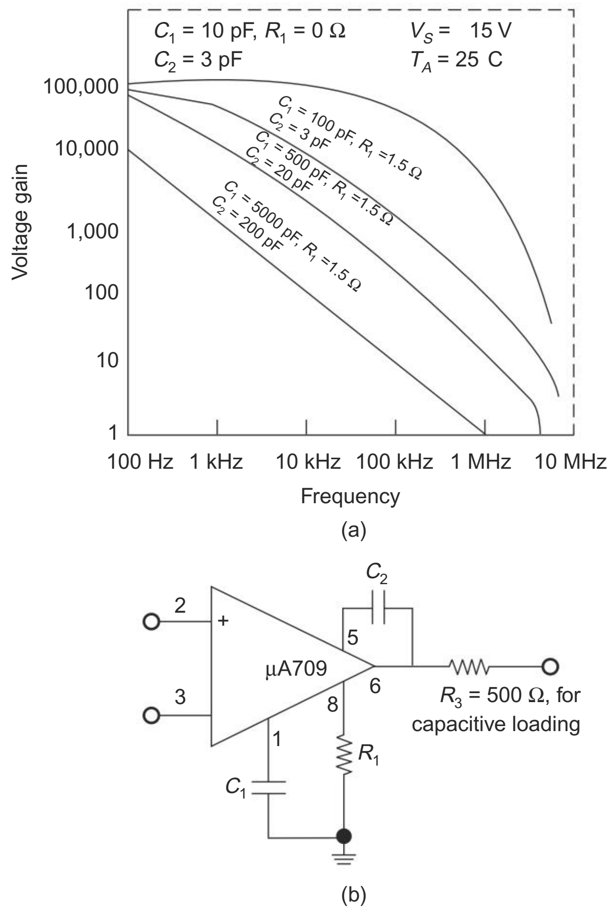


Fig. 14.26 Frequency Response of Externally Compensated Op-amp μA 709

economic op-amps in a monolithic form, and the op-amp became a widely accepted building block of modern signal processing and conditioning circuits.

Since the IC op-amps are inexpensive, versatile and easy to use, they are used for negative feedback amplifier applications, and they also find applications in wave-shaping, filtering and solving mathematical operations. Some common applications are discussed below.

14.15.1 Sign Changer (Phase Inverter)

Figure 14.27 shows the basic inverting amplifier configuration using an op-amp with input impedance Z_1 and feedback impedance Z_f. If the impedances Z_1 and Z_f are equal in magnitude and phase, then the closed-loop voltage gain is –1, and the input signal will undergo a 180° phase shift at the output. Hence, such a circuit is also called *phase inverter*. If two such amplifiers are connected in cascade, then the output from the second stage is the same as the input signal without any change of sign. Hence, the outputs from the two stages are equal in magnitude but opposite in phase and such a system is an excellent paraphase amplifier.

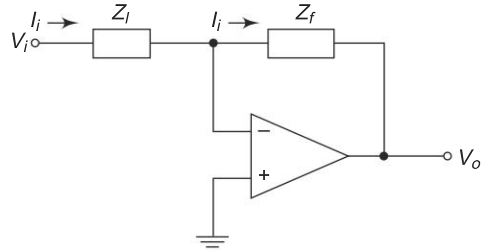


Fig. 14.27 Inverting Op-amp with Voltage Shunt Feedback

14.15.2 Scale Changer (Inverting Amplifier)

Referring to Fig. 14.27, if the ratio $Z_f = Z_1 = k$, a real constant, then the closed-loop gain is $-k$, and the input voltage is multiplied by a factor $-k$ and the scaled output is available at the output. Usually, in such applications, Z_f and Z_1 are selected as precision resistors for obtaining precise and scaled value of input voltage.

14.15.3 Voltage Follower

If $R_1 = \infty$ and $R_f = 0$ in the noninverting amplifier configuration discussed in Chapter 3, then the amplifier acts as an unity-gain amplifier or voltage follower as shown in Fig. 14.28. That is,

$$A_v = 1 + \frac{R_f}{R_1} \text{ or } \frac{R_f}{R_1} = A_v - 1$$

Since $\frac{R_f}{R_1} = 0$ we have $A_v = 1$.

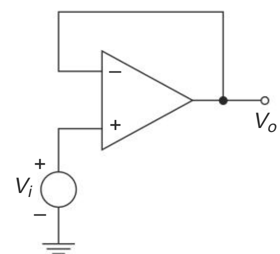


Fig. 14.28 Voltage Follower

The circuit consists of an op-amp and a wire connecting the output to the input, i.e. the output voltage is equal to the input voltage, both in magnitude and phase. In other words, $V_o = V_i$.

Since the output voltage of this circuit follows the input voltage, the circuit is called *voltage follower.* It offers very high input impedance of the order of MΩ and very low output impedance. Therefore, this circuit draws negligible current from the source. Thus, the voltage follower can be used as a buffer between a high impedance source and a low impedance load for *impedance matching* applications.

14.15.4 Adder or Summing Amplifier

The adder, also called summing amplifier, is shown in Fig. 14.29. The output of this arrangement is the linear addition of a number of input signals. Since a virtual ground exists at the inverting input of op-amp at the node *a*,

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_n}{R_n}$$

and

$$V_o = -R_f I = -\left[V_1 \frac{R_f}{R_1} + V_2 \frac{R_f}{R_2} + \ldots + \frac{V_n R_f}{R_n} \right] \tag{14.20}$$

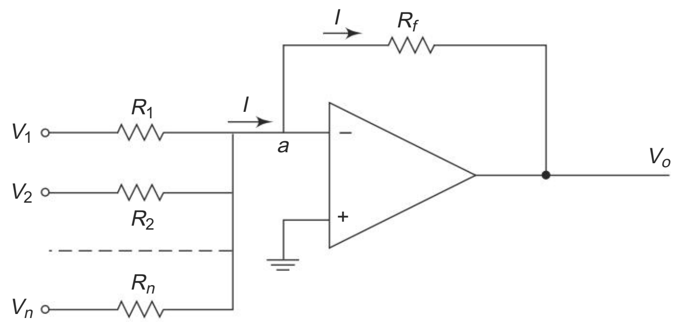


Fig. 14.29 Summing Amplifier

If $\qquad R_1 = R_2 = ... = R_n = R$, then

$$V_o = -\frac{R_f}{R}(V_1 + V_2 + ... + V_n) \tag{14.21}$$

Therefore, the output is proportional to the sum of the individual inputs. In other words, the output signal is the sum of all the inputs multiplied by their associated gains. It can then be expressed as

$$V_o = V_1 A_{V1} + V_2 A_{V2} + V_3 A_{V3} + ... \; V_n A_{Vn}$$

where A_{v1}, A_{v2} ... A_{vn} are the individual gains of the inputs. The summing amplifier may have equal gain for each of the inputs, and then it is referred to as an *equal-weighted configuration*.

The advantage of this method of summation of signals is that a very large number of inputs can be added together, thus requiring only one additional resistor for each additional input with individual gain controls.

Theoretically, individual gain controls may be produced by making the input resistors variable, or by making the feedback resistor R_f variable with the use of potentiometer. However, for ac signals, the summation is not so straight forward, since ac signals of different frequency and phase relationships do not add correctly. Then, an RMS calculation can be made for finding the effective value.

A *level-shifter* circuit can be realized by use of a two-input summing circuit, in which, one input can be the ac signal, and the second input can be the d.c. value by whose value the ac signal is to be shifted. The d.c. value acts as the *offset* for the ac signal.

Example 14.7 The summing amplifier shown in Fig. 14.29 has the following inputs, $R_f = R_1 = R_2 = R_3 = R = 1\ \text{k}\Omega$, $V_1 = +2V$, $V_2 = +3V$, $V_2 = +4V$ and the supply voltages are ± 15V. Determine the output voltage. Assume that the op-amp is initially nulled.

Solution:

The output voltage is given by

$$V_o = -\frac{R_f}{R}(V_1 + V_2 + ... + V_n) = \frac{1 \times 10^3}{1 \times 10^3}(2 + 3 + 4) = -9\text{V}$$

14.15.5 Subtractor

The basic op-amp can be used as a *subtractor* as shown in Fig. 14.30. To analyze the operation of the circuit, assume that all resistors are equal in value of R, i.e., $R_1 = R_2 = R_3 = R_f = R$. The output voltage can be determined by using the super-position principle. If $V_1 = 0$, i.e. V_1 is grounded, then the output voltage V_{o1} will be due to the input voltage V_2 alone. Hence the circuit shown in Fig. 14.30 becomes a noninverting amplifier of unity gain with input voltage $V_2/2$ at the noninverting input terminal and the output voltage is given by

$$V_{o2} = [V_2/2](1 + R/R) = V_2$$

Similarly, if $V_2 = 0$, then the output voltage V_{o1} will be due to V_1 alone. Hence, the circuit becomes an inverting amplifier of unity gain and the output voltage is given by

$$V_{o1} = -V_1$$

Now, considering that both the inputs are applied, the output voltage V_o is

$$V_o = V_{o2} + V_{o1} = V_2 - V_1 \tag{14.22}$$

Thus, the output voltage is proportional to the difference between the two input voltages. Hence, it acts as a *difference amplifier* with unity gain.

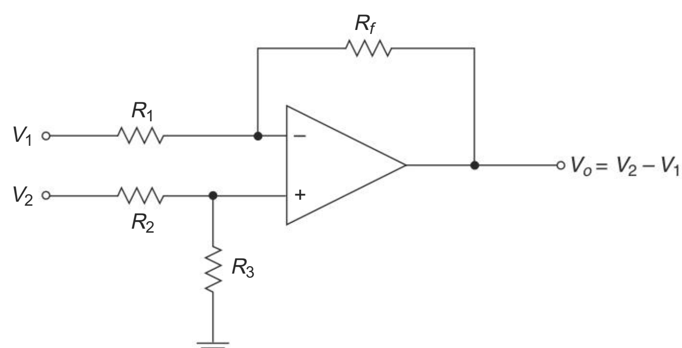


Fig. 14.30 Subtractor

14.15.6 Integrator

A circuit in which the output voltage waveform is the time integral of the input voltage waveform is called *integrator* or *integrating amplifier*. Integrator produces a summing action over a required time interval and the circuit is based on the general parallel-inverting voltage feedback model.

Ideal Integrator In order to achieve integration, the basic inverting amplifier configuration shown in Fig. 14.27 can be used with the feedback element Z_f replaced by a capacitor C_f as shown in Fig. 14.31.

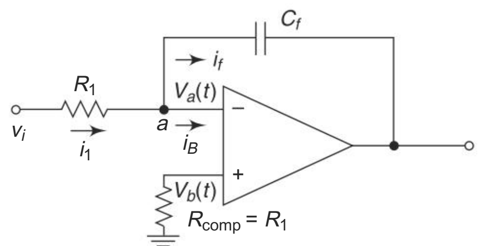


Fig. 14.31 Integrator Circuit

The expression for the output voltage $v_o(t)$ can be obtained by writing Kirchoff's current equation at node a as given by

$$i_I = i_B + i_f$$

Since i_B is negligibly small,

$$i_I = i_f$$

The current through the capacitor $i_C(t) = C\dfrac{dv_c(t)}{dt}$

Therefore, $$\frac{v_i(t) - v_a(t)}{R_1} = C_f\frac{d}{dt}(v_a(t) - v_o(t))$$

However, $v_b(t) = v_a(t) = 0$ because the gain of the op-amp A_v is very large Therefore,

$$\frac{v_i(t)}{R_1} = C_f\frac{d}{dt}(-v_o(t))$$

Integrating both sides with respect to time, we get the output voltage as defined by

$$\int_0^t \frac{v_i(t)}{R_1} = \int_0^t C_f\frac{d}{dt} = (-v_o(t))dt = -C_f v_o(t) + v_o(0)$$

Therefore,

$$v_o(t) = \frac{-1}{R_1C_f}\int_0^t v_i(t)dt + v_o(0) \tag{14.23}$$

where $v_o(0)$ is the initial output voltage. Equation (14.23) indicates that the output voltage is directly proportional to the negative integral of the input voltage and inversely proportional to the time constant $R_1 C_f$.

In frequency domain, the above equation becomes

$$V_o(s) = -\frac{1}{sR_1C_f}V_i(s) \tag{14.24}$$

Letting $s = j\omega$ in steady state, we get

$$V_o(j\omega) = -\frac{1}{j\omega R_1C_f}V_i(j\omega) \tag{14.25}$$

Hence, the magnitude of the transfer function of the integrator is

$$|A| = \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = \left| \frac{j}{\omega(R_1 C_f)} \right| = \frac{1}{\omega R_1 C_f} \tag{14.26}$$

At $\omega = 0$, the gain of the integrator is infinite. Also the capacitor acts as an open circuit and hence there is no negative feedback. Thus, the op-amp operates in open loop and hence the gain becomes infinite (or the op-amp saturates). In practice, the output will never become infinite. As the frequency increases, the gain of the integrator decreases.

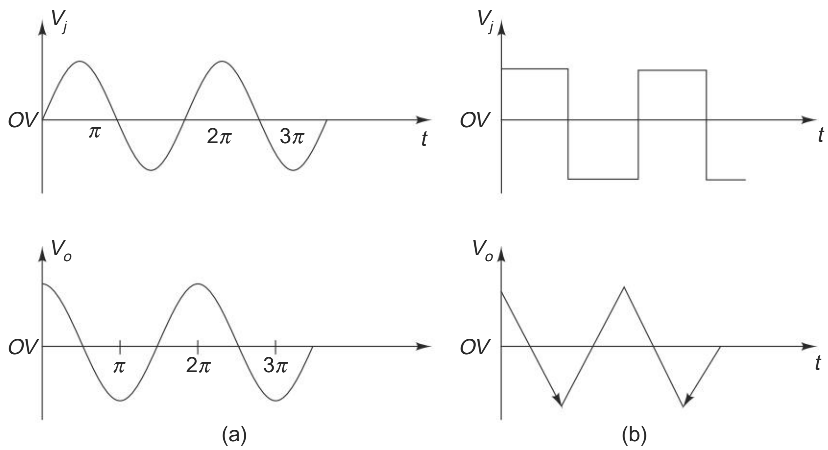


Fig. 14.32 (a) Sine-wave input and its integrated cosine output and (b) Square wave input and its triangular output

14.15.7 Differentiator

The differentiator can perform the mathematical operation of differentiation, i.e. the output voltage is the differentiation of the input voltage. This operation is very useful to find the rate at which a signal varies with time.

The ideal differentiator may be constructed from a basic inverting amplifier shown in Fig. 14.27, if the input resistor R_1 is replaced by a capacitor C_1. The ideal differentiator circuit is shown in Fig. 14.33`.

The expression for the output voltage can be obtained from Kirchoff's Current Law written at node a as follows:

$$i_C = I_B + i_f$$

Since $I_B \approx 0$,

$$i_C = i_f$$

$$C_1 \frac{d}{dt}(v_i - v_a) = \frac{v_a - v_o}{R_f}$$

But $\quad v_a = v_b \approx 0\,\text{V}$, because A is very large.

Therefore, $$C_1 \frac{dv_i}{dt} = -\frac{v_o}{R_f}$$

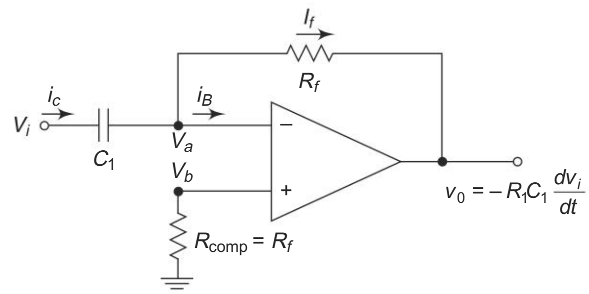


Fig. 14.33 Differentiator

or $$v_o = -R_f C_1 \frac{dv_i}{dt} \tag{14.27}$$

Thus the output v_o is equal to the $R_f C_1$ times the negative instantaneous rate of change of the input voltage v_i with time. A differentiator performs the reverse of the integrator's function. The upper cut-off frequency is given by

$$f_a = \frac{1}{2\pi R_f C_1}$$

Comparison between an Integrator and a Differentiator The process of integration involves the accumulation of signal over time, and hence sudden changes in the signal are suppressed. Therefore, an effective smoothing of the signal is achieved, and hence, integration can be viewed as *low-pass filtering.*

The process of differentiation involves the identification of sudden changes in the input signal. Constant and slowly changing signals are suppressed by a differentiator. Therefore, the differentiator can be viewed as a form of *high-pass filtering.*

14.15.8 Precision Rectifier

The signal processing applications with very low voltage, current and power levels require rectifier circuits. The ordinary diodes cannot rectify votages below the *cut-in* voltage of the diode. A circuit which can act as an *ideal diode* or *precision signal-processing rectifier circuit* for rectifying volatges which are below the level of *cut-in voltage* of the diode can be designed by placing the diode in the feedback loop of an op-amp.

Precision Diodes Figure 14.34(a) shows the precision diode. It is a single diode that can function as a non-inverting precision half-wave rectifier circuit. If v_i in the circuit of Fig. 14.34(b) is positive, the op-amp output V_{OA} also becomes positive. Then, the closed loop condition is achieved for the op-amp and the output voltage $v_o = v_i$. When $v_i < 0$, the voltage V_{OA} becomes negative, and the diode is reverse biased. The loop is then broken and the output $v_o = 0$.

Thus, the circuit acts like a voltage follower for input voltage level $v_i > \frac{0.7}{10^4} = 70$ μV, considering a open-loop gain of 10^4. The output voltage v_o follows the input voltage during the positive half cycle for the input voltage higher than 70 μV. When v_i is negative or less than 70 μV, the output of op-amp V_{OA} becomes negative, and the diode becomes reverse biased. The loop is then broken and thus $v_o = 0$. No current is delivered to the load R_L.

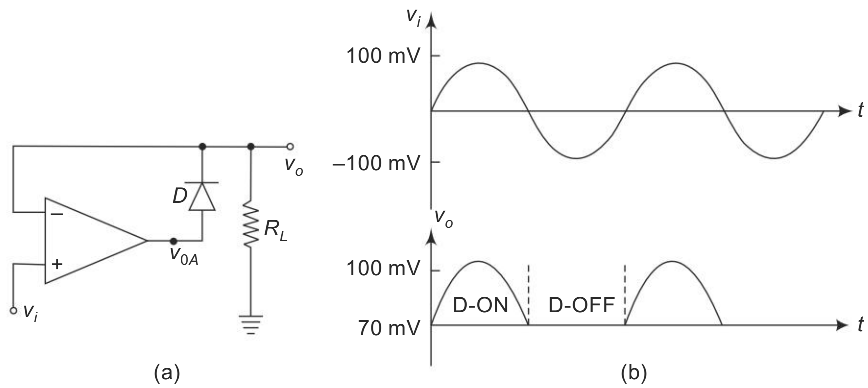


Fig. 14.34 (a) Precision diode and (b) Input and output waveforms

The precision diodes are used in Half-wave rectifier, Full-wave rectifier, Peak value detector, clipper and clamper circuits.

14.15.9 Active Filters

A filter is a frequency selective circuit that allows only a certain band of the desired frequency components of an input signal to pass through and attenuates the signals of undesired frequency components.

The filters are of two types namely (i) analog filters and (ii) digital filters. The analog filters are further classified as passive filters and active filters. The passive filters utilize only resistors, inductors and capacitors. An active network is a circuit obtained by interconnecting passive elements (resistors and capacitors) and active elements (transistors, tunnel diodes and operational amplifiers). An active filter uses an op-amp in order to minimize the effect of loading on the frequency characteristics of the filter.

The filters are widely used in communication, signal processing and sophisticated electronic instruments. The applications of filters also include the suppression of power-line hum, rejection of very low or high-frequency interference and noise, bandwidth limiting and specialized spectral shaping.

A filter can be realized in any one of the following four basic response types:

(i) Low-pass filter (LPF)
(ii) High-pass filter (HPF)
(iii) Bandpass filter (BPF)
(iv) Band-reject filter (BRF), Bandstop or Band-elimination filter (BEF).

Low-pass filter (LPF) A low-pass filter allows only low frequency signals upto a certain break point f_H to pass through, while suppressing high frequency components.

High-pass filter (HPF) A high-pass filter allows only frequencies above a certain break point to pass through and attenuates the low frequency components.

Bandpass filter (BPF) The bandpass filter is the combination of high and low-pass filters, and this allows a specified range of frequencies to pass through.

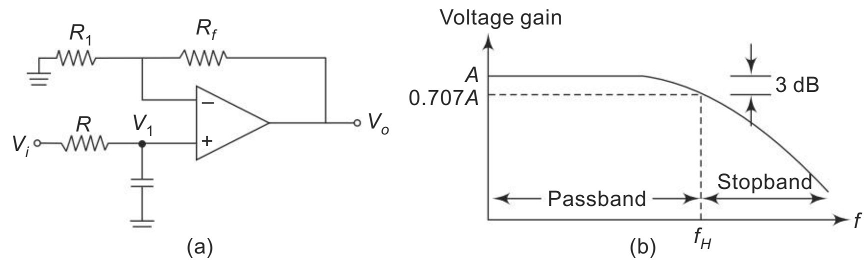


Fig. 14.35 (a) First order low-pass filter and (b) Its frequency response

First order active low-pass filter Figure 14.35 (a) is an active low-pass filter with single RC network connected to the noninverting terminal of op-amp. The input resistor R_i and feedback resistor R_f are used to determine the gain of the filter in the passband. At low frequencies the capacitor appears open, and the circuit acts like a noninverting amplifier with a voltage gain of $\left(1 + \frac{R_f}{R_i}\right)$. As the frequency increases, the capacitive reactance decreases, causing the voltage gain to drop off as shown in Fig. 14.35(b).

The gain of the low-pass filter is given by

$$\frac{V_o}{V_i} = \frac{A}{1 + j\left(\frac{f}{f_H}\right)}$$

where $\frac{V_o}{V_i}$ is the gain of the low-pass filter which is a function of frequency,

$A = 1 + \left(\frac{R_f}{R_i}\right)$ is the passband gain of the filter,

f = the frequency of the input signal and

$f_H = \frac{1}{2\pi RC}$ is the high cutoff frequency of the filter.

The frequency response of the filter can be determined by using the magnitude of the gain of the low-pass filter, which is expresses as

$$\left|\frac{V_0}{V_i}\right| = \frac{A}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

Second order active low-pass filter The circuit of second order low-pass filter is shown in Fig. 14.36. Here, equal value of components is used for simplicity.

The noramlised expression for the second order low-pass filter is given by

$$H(j\omega) = \frac{A}{s^2 + \sqrt{2}s + 1}$$

where normalized frequency $s = j\left(\frac{\omega}{\omega_H}\right)$.

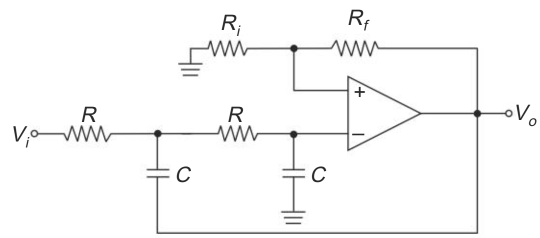


Fig. 14.36 Second order active low-pass Filter

First order active high-pass filter The active high pass filter with single RC network connected to non-inverting terminal op-amp is shown in Fig. 14.37(a). The input resistor R_i and feedback resistor R_f are used to determine the gain of the filter in the passband. At low frequencies the capacitor appears open, and the voltage gain approaches zero. At high frequencies the capacitor appears shorted, and the circuit becomes a non-inverting amplifier with a voltage gain of $\left(1+\frac{R_f}{R_i}\right)$.

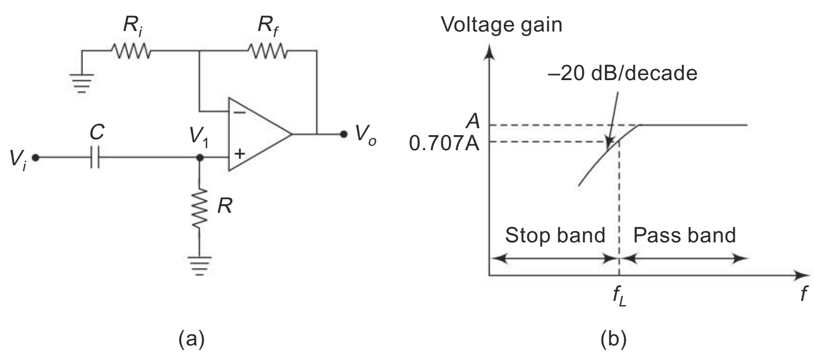


Fig. 14.37 (a) First order active high-pass filter and (b) Frequency response

The output voltage V_o of the first order active high-pass filter is

$$V_0 = \left(1+\frac{R_f}{R_i}\right)\frac{j2\pi fRC}{1+j2\pi fRC}V_i$$

Therefore, the gain of the filter becomes

$$\frac{V_o}{V_i} = A\left(\frac{j\left(\frac{f}{f_L}\right)}{1+j\left(\frac{f}{f_L}\right)}\right)$$

where passband gain of the filter is $A = 1+\left(\frac{R_f}{R_i}\right)$, f is the frequency of the input signal and the lower cut-off frequency of the filter is $f_L = \frac{1}{2\pi RC}$. The frequency response of the filter is obtained from the magnitude of the filter.

That is, $$|H(jf)| = \left|\frac{V_o}{V_i}\right| = \frac{A\left(\dfrac{f}{f_L}\right)}{\sqrt{1+\left(\dfrac{f}{f_L}\right)^2}} = \frac{A}{\sqrt{1+\left(\dfrac{f_L}{f}\right)^2}}.$$

At very low frequencies, i.e., $f < f_L$, the gain is $\cong A$. At the frequency $f = f_L$, the gain falls to 0.707 time the maximum gain A. The range of frequency above f_L is called the pass band. For the frequency $f < f_L$, the gain decreases at a constant rate of –20 dB/decade. The frequency range below the cut-off frequency is called stop band. The frequency response of the first order active high pass filter is shown in Fig. 14.37(b).

Note: The second order high pass filter is obtained from the low-pass filter by applying the transformation.

$$\left.\frac{s}{\omega_o}\right|_{\text{low – pass}} = \left.\frac{\omega_o}{s}\right|_{\text{high – pass}}$$

Hence, the resistors R and capacitors C are interchanged in a low-pass active filter to get a high-pass active filter.

Band-pass filter A band-pass filter can be constructed simply by cascading a low-pass filter whose cut-off frequency is f_H and a high-pass filter whose cut-off frequency is f_L, provided $f_H > f_L$.

Band-reject filter A band-reject filter is obtained by parallel connecting a high-pass filter whose cut-off frequency is f_L and a low-pass filter whose cut-off frequency is f_H, provided $f_H < f_L$.

14.15.10 Comparators

An op-amp comparator compares an input voltage signal with a known voltage, called the *reference voltage*. In its simplest form, the comparator consists of an op-amp operated in open-loop, when fed with two analog inputs, it produces one of the two saturation voltages, $\pm V_{sat}$ (= V_{CC}) at the output of the op-amp. Figure 14.37 (a) shows an op-amp configured for use as a non-inverting comparator. A fixed reference voltage v_{ref} is applied to (–) and a time-varying signal v_i is applied to (+) input. When the non-inverting input v_i is less than the reference voltage V_{ref}, i.e. $v_i < V_{ref}$, the output voltage v_o is at $-V_{sat} \cong -V_{EE}$. On the other hand, when v_i is greater than V_{ref} i.e. $v_i > V_{ref}$, the output voltage v_o is at $+V_{sat} \cong +V_{CC}$. Thus, the output v_o changes from one saturation level to another depending on the volatge difference between v_i and V_{ref}. The diodes D_1 and D_2 are connected to protect the op-amp from excessive input voltages of v_i as shown in Fig. 14.38(a). The transfer characteristics of comparator is shown in Fig. 14.38(b).

14.15.11 Square-wave Generator

Figure 14.39 (a) shows the circuit of a square-wave generator with the output of an op-amp fed back to the (+) input terminal. The resistors R_1 and R_2 form a voltage divider network, and a fraction $\beta = \dfrac{R_2}{R_1 + R_2}$ of the output is fed back to the input. The output can take values of $+\beta V_{sat}$ or $-\beta V_{sat}$. The voltage $\pm \beta V_{sat}$ acts as V_{ref} at

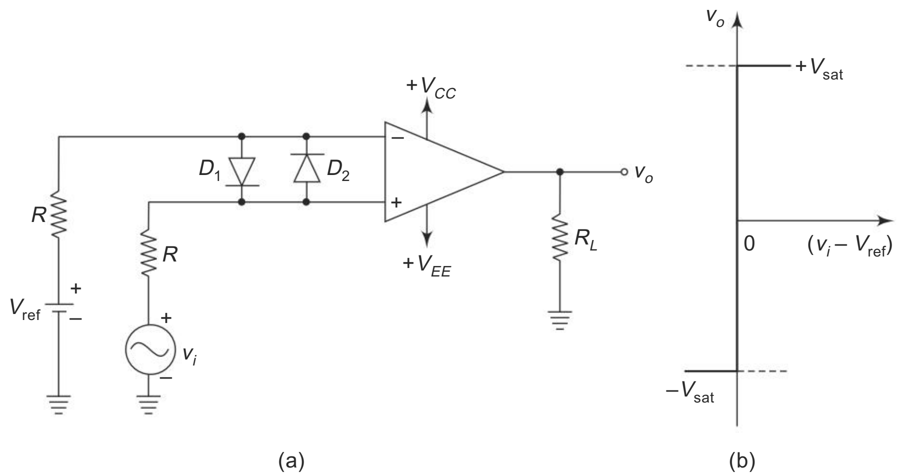


Fig. 14.38 (a) Non-inverting Comparator and (b) Transfer Characteristics

the (+) input terminal. The output is also connected to the (–) input terminal through an integrating low-pass RC network. When the voltage v_c across capacitor C just exceeds V_{ref}, switching takes place resulting in a square-wave output. That is,

(i) when $v_o = +V_{sat}$, C charges from $-\beta V_{sat}$ to $+\beta V_{sat}$ and switches v_o to $-V_{sat}$ and

(ii) when $v_o = -V_{sat}$, C charges from $+\beta V_{sat}$ to $-\beta V_{sat}$ and switches v_o to $+V_{sat}$.

As shown in Fig. 14.38(b), the total time period is given by

$$T = 2RC \ln \frac{1+\beta}{1-\beta} = 2\,RC \ln\left(\frac{R_1 + 2R_2}{R_1}\right).$$

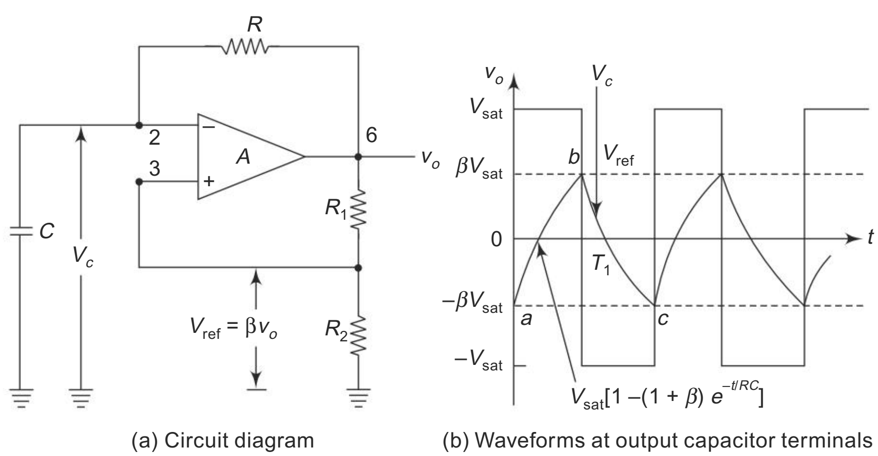


Fig. 14.39 Square-wave generator using an op-amp (a) Circuit diagram and (b) Waveforms at output and capacitor terminals

Hence, the frequency of oscillation is $f_o = \frac{1}{T} = \frac{1}{2RC \ln\left(\frac{1+\beta}{1-\beta}\right)}$

Considering $R_1 = R_2$, we have $\beta = \frac{R}{2R} = 0.5$, $T = 2RC \ln 3$ and $f_o = \frac{1}{2RC \ln 3}$

14.16 IC 555 TIMER

In most of the industrial and scientific applications, the timer circuit plays an important role. The monolithic integrated circuit 555 can be used for accurate time ranges of microseconds to hours. Its performance is independent of supply voltage and temperature variations. This device has a large number of applications in digital electronics such as square-wave generator, linear saw-tooth generator, pulse generator, etc. It can be used as an accurate time delay generator. When it is used as an oscillator, the frequency and duty cycle can be controlled accurately.

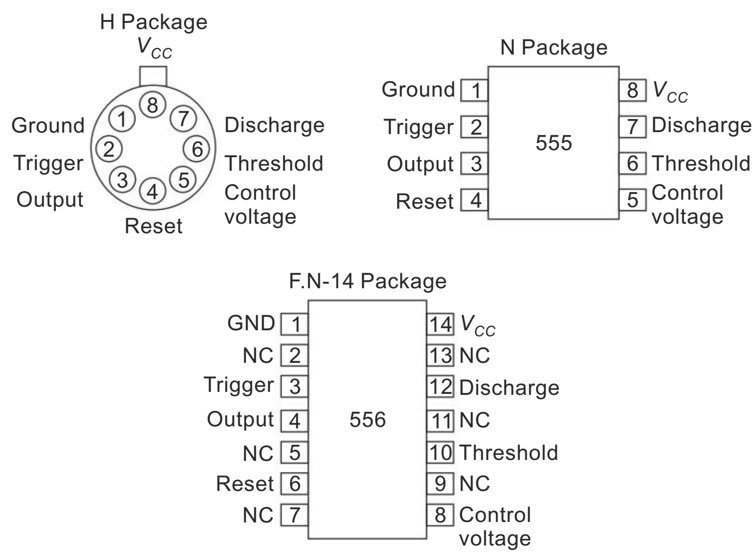


Fig. 14.40 Pin configurations of IC 555 timer

The 555 intergrated circuit was first introduced by Signetics Corporation as Type SE555/NE555. It is available in 8-pin circular style TO-99 Can, 8-pin mini-DIP and 14-pin DIP as shown in Fig. 14.40. The 555 IC is widely popular and various manufacturers provide the IC. The IC 556 contains two 555 timers in a 14-pin package, and Exar's XR-2240 contains a 555 timer with a programmable binary counter in a single 16-pin package.

The 555 timer can be operated with a dc supply voltage ranging from +5V to + 18 V. This feature makes the IC compatible to TTL/CMOS logic circuits and op-amp based circuits. The IC 555 timer is very versatile and its applications include oscillator, pulse generator, square and ramp wave generators, *one-shot* multivator, safety

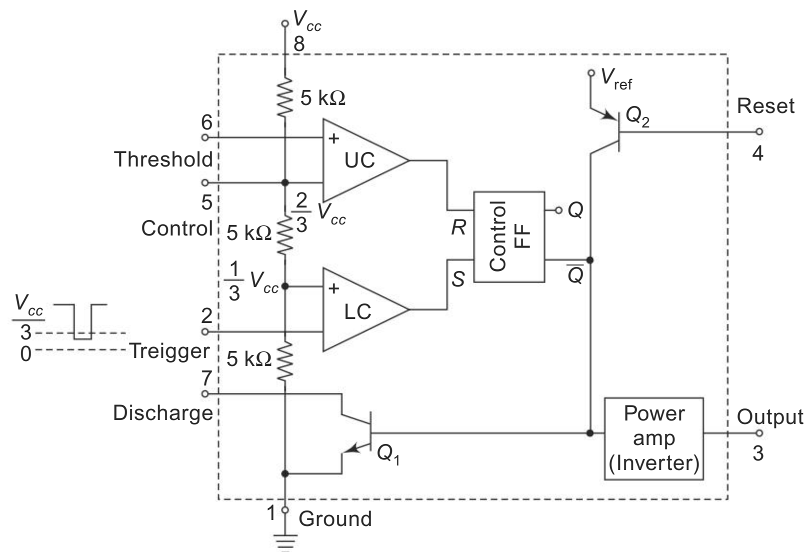


Fig. 14.41 Functional Block diagram of IC 555 Timer

alarm and timer circuits, traffic light controllers, etc. The 555 timer can provide time delay, ranging from micorseconds to hours.

Figure 14.41 shows the functional block diagram of 555 IC timer. The positive dc power supply terminal is connected to pin 8V_{CC} and negative terminal is connected to pin 1 (*Gnd*). The ground pin acts as a common ground for all voltage references while using the IC. The *output* (pin 3) can assume a HIGH level (typically 0.5 V less than V_{CC}) or a LOW level (approximately 0.1 V).

Two comparators, namely, upper comparator (UC) and lower comparator (LC) are used in the circuit. Three 5 kΩ internal resistors provide a potential divider arrangement. It provides a voltage of (2/3) V_{CC} to the (–) teminal of the upper comparator and (1/3) V_{CC} to the (+) input terminal of the lower comparator. A *control* voltage input terminal (pin 5) accepts a modulation control input voltage applied externally. Pin 5 is connected to ground through a bypassing capacitor of 0.1 μF. It bypasses the noise or ripple from the supply. The (+) input terminal of the *UC* is called the *threshold terminal* (pin 6) and the (–) input terminal of the *LC* is the *trigger terminal* (pin 2). The operation of the IC can be summarized as shown in Table 14.1.

Table 14.1 State of Operation of IC 555

Sl. No.	*Trigger (pin 2)*	*Threshold (pin 6)*	*Output state (pin 3)*	*Discharge state (pin 7)*
1.	Below (1/3)V_{CC}	Below (2/3)V_{CC}	High	Open
2.	Below (1/3)V_{CC}	Above (2/3)V_{CC}	Last state remains	Last state remains
3.	Above (1/3)V_{CC}	Below (2/3)V_{CC}	Last state remains	Last state remains
4.	Above (1/3)V_{CC}	Above (2/3)V_{CC}	Low	Ground

The standby (stable) state makes the output $\overline{Q}$ of flip-flop (FF) HIGH. This makes the output of inverting power amplifier LOW. When a negative going trigger pulse is applied to pin 2, as the negative edge of the trigger passes through $(1/3)V_{CC}$, the output of the lower comparator becomes HIGH and it sets the control FF making $Q = 1$ and $\overline{Q} = 0$. When the threshold voltage at pin 6 exceeds $(2/3)V_{CC}$, the output of upper comparator goes HIGH. This action resets the control FF with $Q = 0$ and $\overline{Q} = 1$.

The *reset* terminal (pin 4) allows the resetting of the timer by grounding the pin 4 or reducing its voltage level below 0.4 V. This makes the *output* (pin 3) low overriding the operation of lower comparator. When not used, the Reset terminal is connected to V_{CC}. Transistor Q_2 isolates the reset input from the *FF* and transistor Q_1. The reference voltage V_{ref} is made available internally from V_{CC}. Transistor Q_1 acts as a *discharge* transistor. When *output* (pin 3) is high, Q_1 is OFF making the *discharge* terminal (pin 7) *open*. When the output is low, Q_1 is forward-biased to ON condition. Then the *Discharge* terminal appears as a short circuit to ground.

14.16.1 Waveform Generator

The monolithic integrated circuit 555 timer can be used as square-wave generator, linear saw-tooth generator, pulse generator and time delay generator. The 555 timer circuit to generate square waveform (in astable mode) and triangular waveform is shown in Fig. 14.42. The capacitor C_t charges through the resistances R_1 and R_2 and discharges through the resistance R_2 only. The duty cycle can be controlled by the ratio of resistances R_1 and R_2. Here, the capacitor charges and discharges between $(1/3)V_{CC}$ and $(2/3)V_{CC}$. The charging and discharging times are independent of supply voltage.

The charging time of the capacitor is $t_1 = 0.693\,(R_1 + R_2)\,C_1$

The discharging time of the capacitor is $t_2 = 0.693\,R_2C_t$

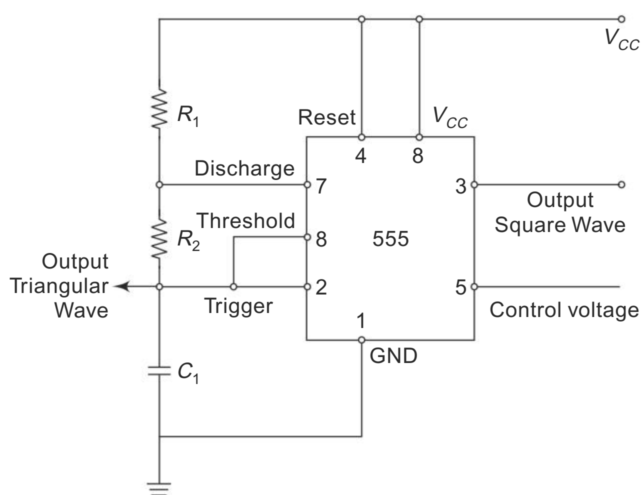


Fig. 14.42 Square and triangular waveforms generator using 555 Timer

Therefore, the time period of the waveform is given by

$$T_1 = t_1 + t_2 = 0.693\ (R_1 + 2R_2)C_t$$

The frequency of oscillation is given by

$$f = 1/T = 1.44/C_t\ (R_1 + 2R_2)$$

where R_1 and R_2 are in ohms and C_t is in Farads.

The 555 timer can also be used for the construction of Monostable multivibrator and Schmitt trigger.

14.16.2 Schmitt Trigger

The IC 555 timer can be used to function as a variable-threshold Schmitt trigger. Since the internal circuitry has a high input impedance and latching capability, the threshold voltage can be adjusted over a wide range with simultaneous open-collector/totempole outputs.

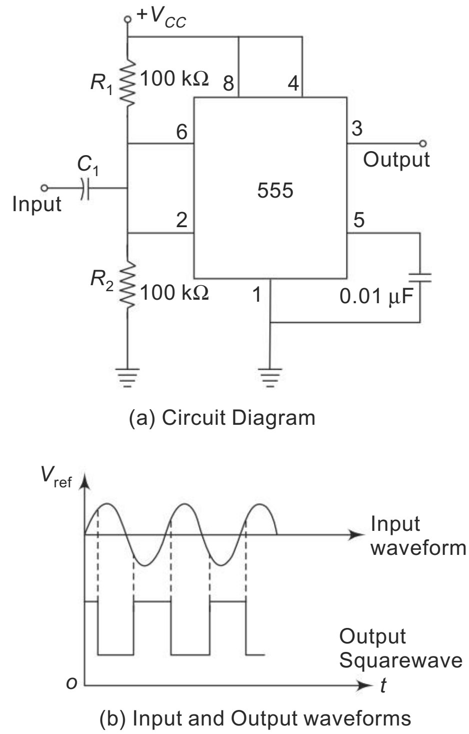


Fig. 14.43 Schmitt Trigger using IC 555 Timer

Another Schmitt trigger circuit is shown in Fig. 14.43 (a) where the two internal comparator inputs (pins 2 and 6) are connected together and externally biased at $(1/2)V_{CC}$ through R_1 and R_2. Since the upper comparator at pin 6 trip at $(2/3)V_{CC}$, and the lower comparator at $(1/3)V_{CC}$, the bias provided by R_1 and R_2 is centered within these two thresholds.

A sine-wave input of sufficient amplitude to exceed the reference levels causes the internal flip-flop to alternatively set and reset, generating a square wave output. As long as R_1 equals R_2, the 555 timer will automatically be biased for any supply voltage in the range of 5 to 16 V. From the curve shown in Fig. 14.42 (b), it is observed that there is a 180° phase shift.

Unlike a conventional multivibrator type of square wave generator that divides the input frequency by 2, the main advantage of Schmitt trigger is that it simply converts the sine-wave signal into square wave signal without division.

REVIEW QUESTIONS

1. What is meant by Integrated Circuit? Why is it preferred to discrete circuits?
2. List the limitation of ICs.
3. Classify ICs in terms of function, device and technology.
4. Explain the process of expitaxial growth in the fabrication of intergrated circuits.
5. Explain the process of diffusion.
6. What is meant by photolithography in IC fabrication and how is it carried out?
7. Describe, with proper diagrams, the steps involved in fabrication of IC transistor.
8. Draw the popular structures of monolithic diodes and explain them.
9. How are integrated resistors obtained in monolithic integrated circuits?
10. How are capacitors fabricated in ICs?
11. Explain the purpose served by SiO_2 layer during different fabrication processes.
12. Explain the steps involved in aluminium metallization.
13. What are the characteristics of an ideal operational amplifier?
14. Draw the schematic symbol of an op-amp and list the different terminals with their functions.
15. What are the different stages of an op-amp?
16. What are the desirable characteristics of the input stage of an op-amp?
17. Explain why a level-translator stage is required in an op-amp.
18. What is the last stage in an op-amp? Explain its function and desirable properties.
19. Define input offset voltage and explain why it exists in all op-amps.
20. What is the difference between Input Offset Current and Input Bias Current?
21. Define the common-mode rejection ratio (CMRR) and explain the significance of a relatively large value of CMRR.
22. For a given op-amp, CMRR = 2×10^5 and common-mode gain A_{cm} is 1, "determine" the differential gain A_d of the op-amp. [*Ans.* $A_d = 2 \times 10^5$]
23. What is meant by Slew rate in an op-amp?
24. Explain slew-rate limiting in an op-amp.
25. The 741C is used as an inverting amplifier with a gain of 10. The sinusoidal input signal has a variable frequency and maximum amplitude of 10 mV peak. What is the maximum frequency of the input at which the output will be undistorted? [*Ans.* f_{max} = 796 kHz]
26. An inverting amplifier using the 741C must have a flat response up to 25 kHz. The gain of the amplifier is 20. What maximum peak-to-peak input signal can be applied without distoring the output? [*Ans.* $V_{id} = 0.318\ V_{p-p}$]
27. Explain the term full power bandwidth in an op-amp.

28. Explain the term '*Gain-Bandwidth*' product.
29. Draw the equivalent circuit of an op-amp and explain the various parameters used in the equivalent circuit.
30. Draw the ideal voltage transfer characteristics of an op-amp and also explain why an op-amp behaves like this in an open-loop configuration.
31. Explain the three configurations of an op-amp and give the expressions for the output voltage in these configurations.
32. What are the limitations of open-loop op-amp configurations?
33. What is feedback? List two types of feedback.
34. Draw the inverting and non-inverting amplifier circuits of an op-amp in closed-loop configuration. Obtain the expressions for the closed-loop gain in these circuits.
35. For the inverting amplifier shown in Fig. 14.23, $R_1 = 470\ \Omega$ and $R_f = 4.7\ k\Omega$. Calculate the closed-loop voltage gain of the amplifier. [*Ans.* $A_f = -10$]
36. For the non-inverting amplifier shown in Fig. 14.24, $R_1 = 120\ \Omega$ and $R_f = 4.7\ k\Omega$. Calculate the closed-loop voltage gain of the amplifier. [*Ans.* $A_f = 39.2$]
37. Explain the *virtual ground* concept in an op-amp.
38. Explain how the bandwidth of an op-amp is changed with feedback.
39. What are the different sources of noise in an op-amp?
40. What is frequency response?
41. Draw the frequency response characteristics of a typical op-amp.
42. Briefly explain the need for compensating networks in op-amps.
43. How internal and external compensation is provided in an op-amp?
44. Explain how a basic inverting amplifier configuration of an op-amp is used as the following circuits (a) Sign changer (b) Scale changer.
45. What is a d.c. voltage follower? How will you simulate such a device using op-amp?
46. Draw the op-amp summing amplifier circuit and obtain an expression for the output voltage.
47. Explain the operation of inverting and noninverting amplifiers.
48. Explain the operation of an ideal integrator circuit with output waveforms.
49. For performing differentiation, integrator is preferred to differentiator. Explain why?
50. What is the basic function of a differentiator?
51. What are the limitations of an ideal differentiator?
52. What is the function of the capacitor in the basic integrator and differentiator?
53. What is the principle of a differentiator using op-amp? What are its drawbacks?
54. Draw the circuit diagram of an op-amp differentiator and derive an expression for the output in terms of the input.
55. Explain the difference between integrator and differentiator. List one application of each.
56. Write detailed notes on any five applications of op-amp.
57. What is a *precision diode*?
58. Draw a precision half-wave rectifier circuit and explain its operation.
59. Draw a half-wave rectifier circuit to rectify an ac voltage of 0.2 V. Explain the circuit diagram.
60. Draw an op-amp based comparator circuit and explain the operation.
61. What is meant by filter?

62. Classify filters.
63. What are the four main types of filters? Briefly explain with figures.
64. Explain how analog filters can be constructed using op-amp.
65. Explain how first-order low pass filter can be constructed using op-amp.
66. Explain how second-order low pass filter can be constructed using op-amp.
67. Explain how first-order high pass filter can be constructed using op-amp.
68. Explain how second-order high pass filter can be constructed using op-amp.
69. Explain how band-pass filter can be constructed using op-amp.
70. Explain how band-reject filter can be constructed using op-amp.
71. What is a comparator? How an op-amp can be used as a comparator?
72. Explain the operation of square wave generator using op-amp with capacitor and output voltage waveforms. How can you obtain a non-symmetrical square wave?
73. Draw the internal structure of IC 555 timer.
74. Derive the frequency of oscillatiuon of a square wave generator in astable mode using IC 555 timer.
75. Explain the operation of a Schmitt Trigger using IC 555 Timer. Show how Schmitt Trigger can be used for wave-shaping purposes.

DIGITAL ELECTRONICS

15

INTRODUCTION

A digital circuit is different from analog circuits. The term digital is derived from the way in which circuits perform operations by counting digits. Applications of digital electronics are not only confined to computer system but are applied in many diverse areas like telephony, data processing, radar navigation, military systems, medical instruments and consumer products. Digital technology has progressed from vacuum-tube circuits to integrated circuits and microprocessors. Digital circuits involve systems in which there are only two possible states. These states are typically represented by voltage levels. Other circuit conditions, such as current levels, open or closed switches, and ON or OFF lamps, can also represent the two states. In digital systems, the two states are used to represent numbers, symbols, alphabetic characters and other types of information.

15.1 NUMBER SYSTEM

In our daily life, decimal number system (0, 1, 2,..., 9) is commonly used even though there are many other number systems like binary, octal, hexadecimal, etc. For understanding the digital circuits and systems, students must be familiar with these number systems also. The following sections are dedicated to binary, octal and hexadecimal number systems.

15.1.1 Binary Numbers

The binary number system is simple because it is composed of only two digits, i.e. 0 and 1. Just as the decimal system, with its ten-digits, is a base-ten system, the binary system with its two digits is a base-two system. The position of 1 or 0 in a binary number indicates its "weight" within the number. The weight of each successively higher position (to the left) in a binary number is an increasing power of two.

For example, in decimal system,

$$139_{10} = 1 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$$

hundreds tens units Positional weights

Similarly, the binary numbers are also represented by positional weights. For example,

$$
\begin{aligned}
139_{10} &= 10001011_2 \\
&= 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
&= 128 + 0 + 0 + 0 + 8 + 0 + 2 + 1 \\
&= 139.
\end{aligned}
$$

In the digital system, each of the binary digits is called a bit and a group of bits having a significance is called a byte or word. The highest decimal number that can be represented by an n bits binary number is $2^n - 1$. Thus, with a 8-bit binary number the maximum decimal number that can be represented is $2^8 - 1 = 255$.

15.1.2 Decimal–Binary Conversion

One easier method of converting a decimal number into binary number is to divide progressively the decimal number by 2 until quotient of zero is obtained. Writing the remainders after each division in the reverse order, the binary number is obtained. This method is popularly known as *double-dabble* method. The procedure for converting the decimal to binary is explained below. Consider the decimal number 52; its binary equivalent can be obtained as follows.

52 divided by 2 = quotient 26 with a remainder of **0**
26 divided by 2 = quotient 13 with a remainder of **0**
13 divided by 2 = quotient 6 with a remainder of **1**
6 divided by 2 = quotient 3 with a remainder of **0**
3 divided by 2 = quotient 1 with a remainder of **1**
1 divided by 2 = quotient 0 with a remainder of **1**

Reading the remainders from bottom to top gives the binary equivalent. Thus, $52_{10} = 110100_2$.

If the decimal number is a fraction, its binary equivalent is obtained by multiplying the number continuously by 2, recording each time a carry in the integer position. The carries in the forward order gives the required binary number. For example, the decimal number 0.625_{10} can be expressed in binary as follows.

0.625 multiplied by 2	= **1**.250	: carry = **1**
0.250 × 2	= **0**.50	: carry = **0**
0.500 × 2	= **1**.00	: carry = **1**
0.000 × 2	= **0**.00	

As the product is zero, no further multiplication by two is possible. The binary equivalent is obtained by reading the carry terms from top to bottom. Thus, 0.625_{10} is 0.101_2. The binary equivalent of a decimal real number, which includes both integer and fractional parts, can be found using the same techniques, i.e. find the binary equivalent of the integer part and fractional part using respective methods and the combined number will give the binary equivalent. For example, $52.625_{10} = 110100.101_2$.

The conversion of a binary number into its decimal equivalent can also be carried out. In a binary number, digits from extreme right represent coefficient of the ascending powers of two, the starting power being zero. For example,

$$
\begin{array}{c}
1\ 0\ 1\ 1\ 1\ 0
\end{array}
\qquad
\begin{array}{rcr}
0 \times 2^0 &=& 0 \\
1 \times 2^1 &=& 2 \\
1 \times 2^2 &=& 4 \\
1 \times 2^3 &=& 8 \\
0 \times 2^4 &=& 0 \\
1 \times 2^5 &=& \underline{32} \\
 & & 46
\end{array}
$$

Therefore, $(101110)_2$ can be written as $(46)_{10}$. Conversion of fractions is also carried out in a similar manner. For example, the conversion of 0.1101 is illustrated below.

$$
\begin{array}{c}
0.1\ 1\ 0\ 1
\end{array}
\qquad
\begin{array}{rcr}
1 \times 2^{-4} &=& 0.0625 \\
0 \times 2^{-3} &=& 0.0000 \\
1 \times 2^{-2} &=& 0.2500 \\
1 \times 2^{-1} &=& \underline{0.5000} \\
 & & 0.8125
\end{array}
$$

Thus, 0.1101_2 is equivalent to 0.8125_{10}.

15.1.3 Octal Numbers

The octal number system uses the digits 0, 1, 2, 3, 4, 5, 6 and 7. The base or radix of this system is eight. Each significant position in an octal number has a positional weight. The least significant position has a weight of 8^0, i.e. 1, and the higher significant positions are, respectively, given weightage as the ascending powers of eight, i.e. 8^1, 8^2, 8^3, etc. The octal equivalent of a decimal number can be obtained by dividing the decimal number by 8 repeatedly, until a quotient of 0 is obtained. The procedure is exactly same as the double-dabble method explained earlier. The conversion from decimal to octal number is explained hereunder.

Example 15.1 Convert 12_{10} to an octal number.

Solution: The procedure is as follows.

12 divided by 8 = quotient 1 with a remainder of **4**

1 divided by 8 = quotient 0 with a remainder of **1**

Reading the remainders from bottom to top, the decimal number 12_{10} is equivalent to octal 14_8.

The conversion from octal to decimal number can be carried out by multiplying each significant digit of the octal number by the respective weights and adding the products. The following example illustrates the conversion from octal to decimal.

Example 15.2 Convert the following octal numbers to decimals.

(i) 444_8 (ii) 237_8 (iii)120_8

Solution: (i) $444_8 = 4 \times 8^2 + 4 \times 8^1 + 4 \times 8^0$

$= 4 \times 64 + 4 \times 8 + 4 \times 1$

$= 256 + 32 + 4$

$= 292_{10}$

(ii) $237_8 = 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$
$= 2 \times 64 + 3 \times 8 + 7 \times 1$
$= 128 + 24 + 7$
$= 159_{10}$

(iii) $120_8 = 1 \times 8^2 + 2 \times 8^1 + 0 \times 8^0$
$= 1 \times 64 + 2 \times 8 + 0 \times 1$
$= 64 + 16 + 0$
$= 80_{10}$

15.1.4 Octal–Binary Conversions

Conversion from octal to binary and vice-versa can be easily carried out. For obtaining the binary equivalent of an octal number, just replace each significant digit in the given number by its 3-bit binary equivalent. For example,

$$276_8 = 2 \quad 7 \quad 6$$
$$= 010 \quad 111 \quad 110$$

Thus, $276_8 = 010\ 111\ 110_2$. The reverse procedure is used for converting binary to octal, i.e. starting from the least significant bit, replace each group of 3 bits by their decimal equivalents. For example,

$$11011010101_2 = 11 \quad 011 \quad 010 \quad 101$$
$$= 3 \quad 3 \quad 2 \quad 5$$

Thus, $11011010101_2 = 3325_8$.

15.1.5 Hexadecimal Numbers

Hexadecimal number system has a radix of 16 and uses 16 symbols, namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, *A*, *B*, *C*, *D*, *E* and *F*. The symbols A, *B*, *C*, *D*, *E*, and *F* represent the decimals 10, 11, 12, 13, 14 and 15, respectively. Each significant position in hexadecimal number has a positional weight. The least significant position has a weight of 16^0, i.e. 1, and the higher significant positions are, respectively, given weightage as the ascending powers of sixteen, i.e. 16^1, 16^2, 16^3, etc. The hexadecimal equivalent of a decimal number can be obtained by dividing the decimal number by 16 repeatedly, until a quotient of 0 is obtained. The following example illustrates how the hexadecimal equivalent of a given decimal can be obtained.

Example 15.3 Convert 112_{10} and 253_{10} to hexadecimal numbers.

The procedure is as follows,

Solution: (i) 112 divided by 16 = quotient 7 with a remainder of **0**
7 divided by 16 = quotient 0 with a remainder of **7**

Reading the remainders from bottom to top, the decimal number 112_{10} is equivalent to hex 70_{16}. The hexadecimal 70_{16} can also be represented as 70 H.

(ii) 253 divided by 16 = quotient 15 with a remainder of **13**, i.e *D*
15 divided by 16 = quotient 0 with a remainder of **15**, i.e. *F*

Reading the remainders from bottom to top, the decimal number 253_{10} is equivalent to hex FD_{16}.

The conversion from hex to decimal number can be carried out by multiplying each significant digit of the hexadecimal number by the respective weights and adding the products. The following example illustrates the conversion from hex to decimal.

Example 15.4 Convert the following hexadecimal numbers to decimals.

(i) $4AB$ H (ii) $23F$ H

Solution: (i) $4AB_{16} = 4 \times 16^2 + A \times 16^1 + B \times 16^0$

$= 4 \times 16^2 + 10 \times 16^1 + 11 \times 16^0$

$= 4 \times 256 + 10 \times 16 + 11 \times 1$

$= 1024 + 160 + 11$

$= 1195_{10}$

(ii) $23F_{16} = 2 \times 16^2 + 3 \times 16^1 + F \times 16^0$

$= 2 \times 256 + 3 \times 16 + 15 \times 1$

$= 512 + 48 + 15$

$= 575_{10}$

15.1.6 Hexadecimal–Binary Conversions

Conversion from hex to binary and vice-versa can be easily carried out. For obtaining the binary equivalent of a hexadecimal number, replace each significant digit in the given number by its 4-bit binary equivalent. For example,

$$2A6_{16} = \quad 2 \quad A \quad 6$$
$$= \quad 0010 \quad 1010 \quad 0110$$

Thus, $2A6_{16}$ = 0010 1010 0110_2. The reverse procedure is used for converting binary to hex, i.e. starting from the least significant bit, replace each group of 4 bits by their decimal equivalents. For example,

$$11011010101_2 = \quad 110 \quad 1101 \quad 0101$$
$$= \quad 6 \quad D \quad 5$$

Thus, $11011010101_2 = 6D5_{16}$

15.1.7 Hexadecimal–Octal Conversions

Conversion between hexadecimal and octal numbers is sometimes required. To convert a hexadecimal number to octal, the following steps can be used.

(i) Convert the given hexadecimal number to equivalent binary.
(ii) Starting from the LSB, form groups of 3 bits.
(iii) Write the equivalent octal number for each group of 3 bits. For example,

$$24_{16} = 0010\ 0100_2$$
$$= 00\ 100\ 100_2$$
$$= 044_8$$

Thus, 24 in hexadecimal is equivalent to 44 in octal number system.

To convert an octal number to hexadecimal the steps are as follows.

(i) Convert the given octal number to equivalent binary.
(ii) Starting from the LSB, form groups of 4 bits.

(iii) Write the equivalent hexadecimal number for each group of 4 bits. For example,

$$24_8 = 010\ 100_2$$
$$= 01\ 0100_2$$
$$= 14_{16}$$

Thus, 24 in octal is equivalent to 14 in hexadecimal number system.

15.2 BINARY ARITHMETIC

The arithmetic rules for addition, subtraction, multiplication and division of binary numbers are given below:

	Addition	Subtraction	Multiplication	Division
(i)	0 + 0 = 0	0 – 0 = 0	0 × 0 = 0	0 ÷ 1 = 0
(ii)	0 + 1 = 1	1 – 0 = 1	0 × 1 = 0	1 ÷ 1 = 1
(iii)	1 + 0 = 1	1 – 1 = 0	1 × 0 = 0	0 ÷ 0 = not allowed
(iv)	1 + 1 = 10	10 – 1 = 1	1 × 1 = 1	1 ÷ 0 = not allowed

15.2.1 Binary Addition

Two binary numbers can be added in the same way as two decimal number are added. The addition is carried out from the least significant bits and proceeded to higher significant bits, adding the carry resulting from previous addition each time. Consider the addition of the binary number 1101 and 1111.

MSB LSB	Decimal
1101	13
1111	15
11100	28

The addition carried out above can be explained as follows.

Step 1: The least significant bits are added, i.e. 1 + 1 = 0 with a carry 1.

Step 2: The carry in the previous step is added to the next higher significant bits, i.e. 1 + 0 + 1 = 0 with a carry 1.

Step 3: The carry in the previous step is added to the next higher significant bits, i.e. 1 + 1 + 1 = 1 with a carry 1.

Step 4: The carry in the previous step is added to the most significant bit, i.e. 1 + 1 + 1 = 1 with a carry 1.

Thus the sum is 11100. The addition is also shown in decimal number system in order to compare the results.

15.2.2 Binary Subtraction

Binary subtraction is also carried out in the same way as decimal number are subtracted. The subtraction is carried out from the least significant bits and proceeded to higher significant bits. When a 1 is subtracted from a 0, a 1 is borrowed from immediate higher significant bit. The following problem explains the steps involved. Suppose that 1001 is subtracted from 1110.

MSB LSB	Decimal
1110	14
1001	9
0101	5

The steps are explained below.

Step 1: The least significant bits (1 column) are considered. 0 – 1 needs a borrow from the next higher significant bit (column 2 from the right). Thus, 0 – 1 results in a difference of **1** and borrow 1.

Step 2: The second column is taken. Since a 1 is given to the first column, now change the 1 here to 0. Thus the subtraction to be performed is 0 – 0 = **0**

Step 3: In the third column, the difference is given by 1 – 0 = **1**

Step 4: In the fourth column (MSB), the difference is given by 1 – 1 = **0**

Thus, the difference between the two binary numbers is 0101.

15.2.3 Binary Multiplication

Binary multiplication is rather simpler than the decimal multiplication. The procedure is same as that of decimal multiplication. The binary multiplication procedure is as shown.

Step 1: The least significant bit of the multiplier is taken. If the multiplier bit is 1, the multiplicand is copied as such and if the multiplier bit is 0, 0 is placed in all the bit positions.

Step 2: Next higher significant bit of the multiplier is taken and as in step 1, the product is written with a shift in left.

Step 3: Step 2 is repeated for all other higher significant bits and everytime a left shift is given.

Step 4: When all the bits in the multiplier have been taken into account, the product terms are added, which gives the actual product of the multiplier and the multiplicand. The following examples illustrates the multiplication procedure.

Example 15.5 Multiply the following binary numbers. (i) 1101 and 1100 (ii) 1000 and 101 (iii) 1111 and 1001

Solution: (i) 1101 × 1100 (ii) 1000 × 101

$$\begin{array}{rcccccccc}
 & & & & & 1 & 1 & 0 & 1 \\
 & & & & \times & 1 & 1 & 0 & 0 \\
\hline
 & & & & & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & \\
 & & & 1 & 1 & 0 & 1 & & \\
 & & 1 & 1 & 0 & 1 & & & \\
\hline
 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
\hline
\end{array}$$

$$\begin{array}{cccccc}
 & & 1 & 0 & 0 & 0 \\
 & \times & & 1 & 0 & 1 \\
\hline
 & & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & \\
1 & 0 & 0 & 0 & & \\
\hline
1 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}$$

(iii) 1111 × 1001

$$\begin{array}{r}
1\ 1\ 1\ 1 \\
\times\ 1\ 0\ 0\ 1 \\
\hline
1\ 1\ 1\ 1 \\
0\ 0\ 0\ 0\ \ \ \\
0\ 0\ 0\ 0\ \ \ \ \ \ \\
1\ 1\ 1\ 1\ \ \ \ \ \ \ \ \ \\
\hline
1\ 0\ 0\ 0\ 0\ 1\ 1\ 1
\end{array}$$

15.2.4 Binary Division

Division in binary follows the same procedure as division in decimal. Division by 0 is meaningless. An example is given below.

Example 15.6 Perform the following divisions: (i) 110 ÷ 10 (ii) 1111 ÷ 110

Solution: (i) 110 ÷ 10 (ii) 1111 ÷ 110

$$\begin{array}{r}
11_2 \\
10\overline{)110} \\
10\ \ \\
\hline
10 \\
10 \\
\hline
00
\end{array}
\qquad
\begin{array}{r}
3_{10} \\
2\overline{)6} \\
6 \\
\hline
0
\end{array}
\qquad
\begin{array}{r}
10.1_2 \\
110\overline{)1111.0} \\
110\ \ \ \ \ \\
\hline
110 \\
110 \\
\hline
000
\end{array}
\qquad
\begin{array}{r}
2.5_{10} \\
6\overline{)15.0} \\
12\ \ \ \\
\hline
30 \\
30 \\
\hline
00
\end{array}$$

15.3 1'S AND 2'S COMPLEMENTS

The usefulness of the complement numbers stems from the fact that subtraction of a number from another can be accomplished by adding the complement of the subtrahend to the minuend. The actual difference can be obtained with minor manipulations.

15.3.1 1's Complement Subtraction

Subtraction of binary numbers can be accomplished by using the 1's complement method, which allows us to subtract using only addition. The 1's complement of a binary number is found by simply changing all 1s to 0s and all 0s to 1s. To subtract a smaller number from a larger number, the 1's complement method is as follows:

1. Determine the 1's complement of the smaller number.
2. Add the 1's complement to the larger number.
3. Remove the carry and add it to the result. This carry is called ***end-around-carry***.

Example 15.7 Subtract 1010_2 from 1111_2 using 1's complement method. Show direct subtraction for comparison.

Solution: Direct subtraction 1's complement method

$$\begin{array}{r}
1\ 1\ 1\ 1 \\
-1\ 0\ 1\ 0 \\
0\ 1\ 0\ 1
\end{array}
\qquad
\begin{array}{rr}
 & 1\ 1\ 1\ 1 \\
\text{1's comp.} \rightarrow & 0\ 1\ 0\ 1 \\
\text{carry} & \mathbf{1}\ 0\ 1\ 0\ 0 \\
\text{add carry} & \mathbf{1} \\
 & 0\ 1\ 0\ 1
\end{array}$$

To subtract a larger number from a smaller one, the 1's complement method is as follows:

1. Determine the 1's complement of the larger number.
2. Add the 1's complement to the smaller number.
3. The answer has an opposite sign and is the 1's complement of the result. There is no carry.

Example 15.8 Subtract 1010_2 from 1000_2 using 1's complement method. Show direct subtraction for comparison.

Solution: Direct subtraction 1's complement method

$$
\begin{array}{r}
1\ 0\ 0\ 0 \\
-1\ 0\ 1\ 0 \\
\hline
-0\ 0\ 1\ 0
\end{array}
\qquad \text{1's comp.}\rightarrow \qquad
\begin{array}{r}
1\ 0\ 0\ 0 \\
0\ 1\ 0\ 1 \\
\hline
1\ 1\ 0\ 1
\end{array}
$$

No carry results and the answer is the 1's complement of 1101 and opposite in sign, i.e. – 0010.

The 1's complement method is particularly useful in arithmetic logic circuits because subtraction can be accomplished with an adder.

15.3.2 2's Complement Subtraction

The 2's complement of a binary number is found by adding 1 to its 1's complement. To subtract a smaller number from a larger one, the 2's complement method is applied as follows:

1. Determine the 2's complement of the smaller number.
2. Add the 2's complement to the larger number.
3. Discard the carry (there is always a carry in this case).

Example 15.9 Subtract 1010_2 from 1111_2 using 2's complement method. Show direct subtraction for comparison.

Solution: Direct subtraction 2's complement method

$$
\begin{array}{r}
1\ 1\ 1\ 1 \\
-1\ 0\ 1\ 0 \\
\hline
0\ 1\ 0\ 1
\end{array}
\qquad \begin{array}{r} \\ \text{2's comp.}\rightarrow \\ \text{carry} \end{array} \qquad
\begin{array}{r}
1\ \ 1\ \ 11 \\
0\ \ 1\ \ 10 \\
\hline
\mathbf{1}\ 0\ \ 1\ \ 01
\end{array}
$$

The carry is discarded. Thus, the answer is 0101_2.

To subtract a larger number from a smaller one, the 2's complement method is as follows:

1. Determine the 2's complement of the larger number.
2. Add the 2's complement to the smaller number.
3. There is no carry. The result is in 2's complement form and is negative.
4. To get an answer in true form, take the 2's complement and change the sign.

Example 15.10 Subtract 1010_2 from 1000_2 using 2's complement method. Show direct subtraction for comparison.

Solution:

Direct subtraction		2's complement method
1 0 0 0		1 0 0 0
−1 0 1 0	2's comp.→	0 1 1 0
0 0 1 0	no carry	1 1 1 0

No carry results. Thus, the difference is negative and the answer is the 2's complement of 1110_2, i.e. 0010_2.

Both 1's and 2's complement methods of subtraction may seem more complex compared to direct subtraction. However, they both have distinct advantages when implemented with logic circuits because they allow subtraction to be done using addition. Both 1's and the 2's complements of a binary number are relatively easy to accomplish with logic circuits; and the 2's complement has an advantage over the 1's complement in that an end-around-carry operation does not have to be performed.

15.4 BINARY CODED DECIMAL

When a computer is handling numbers in binary but in group of four digits, the number system is called Binary Coded Decimal (BCD). Combinations of binary digits that represent numbers, letters, or symbols are digital codes. The 8421 code is a type of binary coded decimal code. It has four bits and represents the decimal digits 0 through 9. The designation 8421 indicates the binary weights of the four bits. The case of conversion between 8421 code numbers and the familiar decimal numbers is the main advantage of this code. To express any decimal number in BCD, simply replace each decimal digit by the appropriate four-bit code. Table 15.1 gives the binary and BCD for the decimal numbers 0 through 15.

Table 15.1 Decimal numbers, equivalent binary and BCD

Decimal number	*Binary number*	*Binary coded decimal (8421)*
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	0001 0000
11	1011	0001 0001
12	1100	0001 0010
13	1101	0001 0011
14	1110	0001 0100
15	1111	0001 0101

15.4.1 BCD Addition

BCD is a numerical code, and many applications require that arithmetic operations be performed. Addition is the most important operation because the other three operations like subtraction, multiplication and division can be accomplished using addition. The rule for adding two BCD numbers is given below.

1. Add the two numbers, using the rules for binary addition.
2. If a four-bit sum is equal to or less than 9, it is a valid BCD number.
3. If a four-bit sum is greater than 9, or if a carry-out of the group is generated, it is an invalid result. Add 6 (0110_2) to the four-bit sum in order to skip the six invalid states and return the code to 8421. If a carry results when 6 is added, simply add the carry to the next four-bit group.

Example 15.11 Add the following BCD numbers:

(i)	1 0 0 1		9
	+ 0 1 0 0		+ 4
	1 1 0 1	Invalid BCD number	13_{10}
	+ 0 1 1 0	Add 6	
	0 0 0 1 0 0 1 1	Valid BCD number	
	↓ ↓		
	1 3		
(ii)	0 0 0 1 1 0 0 1		19
	+ 0 0 0 1 0 1 0 0		+ 14
	0 0 1 0 1 1 0 1	Right group is invalid	33_{10}
	+ 0 1 1 0	Add 6	
	0 0 1 1 0 0 1 1	Valid BCD number	
	↓ ↓		
	3 3		

15.5 BOOLEAN ALGEBRA

Boolean algebra is a set of rules, laws, and theorems by which logical operations can be expressed mathematically. It is a convenient and systematic way of expressing and analyzing the operation of digital circuits and systems. In Boolean algebra, a variable can be either a zero or a one. The binary digits are utilized to represent the two levels that occur within digital logic circuits. A binary 1 will represent a HIGH level and a binary 0 will represent a LOW level. The complement of a variable is represented by a 'bar' over the letter; for example, the complement of A is represented by $\overline{A}$.

15.5.1 Boolean Addition and Multiplication

Boolean addition involves variables having values of either a binary 1 or a 0. The basic rules for Boolean addition are as follows:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

The Boolean addition is the same as the logical OR operation. The multiplication rules for Boolean algebra are the same as the binary multiplication rules, discussed in Section 15.2.3.

$$0.0 = 0$$
$$0.1 = 0$$
$$1.0 = 0$$
$$1.1 = 1$$

The Boolean multiplication is the same as the logical AND operation. The three basic properties of Boolean algebra are commutativity, associativity and distributivity.

Commutative property The Boolean addition is commutative, i.e.

$$A + B = B + A$$

This says that the order in which the variables are ORed makes no difference. The Boolean algebra is also commutative over multiplication, i.e.

$$A \cdot B = B \cdot A$$

This states that the order in which the variables are ANDed makes no difference.

Associative property The associative property of addition is stated as follows:

$$A + (B + C) = (A + B) + C$$

The ORing of several variables results in the same regardless of the grouping of the variables. The associative law of multiplication is stated as follows:

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

This law tells us that it makes no difference in what order the variables are grouped when ANDing several variables.

Distributive property The Boolean addition is distributive over Boolean multiplication. That is,

$$A + BC = (A + B)\,(A + C)$$

Also, the Boolean multiplication is distributive over Boolean addition, i.e.,

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

The first distributive property states that ANDing several variables and ORing the result with a single variable is equivalent to ORing the single variable with each of the several variables and then ANDing the sums. The second distributive property states that ORing several variables and ANDing the result with a single variable is equivalent to ANDing the single variable with each of the several variables and then ORing the products. The other basic laws of Boolean algebra are given in Table 15.2.

Table 15.2 Laws of Boolean algebra

Sl. No.	*Boolean Law*
1	$A + 0 = A$
2	$A + 1 = 1$
3	$A \,.\, 0 = 0$
4	$A \,.\, 1 = A$
5	$A + A = A$
6	$A + \overline{A} = 1$
7	$A \,.\, A = A$
8	$A \,.\, \overline{A} = 0$
9	$(\overline{A})' = A$
10	$A + AB = A$
11	$A + \overline{A}B = A + B$
12	$AB + \overline{A}C + BC = AB + \overline{A}C$

The rules 1 to 9 can be very easily proved by perfect induction. The proofs for the rules 10, 11 and 12 are given as follows.

Example 15.12 Prove the following laws of Boolean algebra. (i) $A + AB = A$ (ii) $A + \overline{A}B = A + B$ (iii) $AB + \overline{A}C + BC = AB + \overline{A}C$

Solution: (i) $A + AB$ $= A\,(1 + B)$ distributive law

$= A \,.\, 1$ law 2

$= A$ law 4

(ii) $A + \overline{A}B$ $= (A + \overline{A}) \,.\, (A + B)$ distributive law

$= 1 \,.\, (A + B)$ law 6

$= A + B$ law 4

(iii) $AB + \overline{A}C + BC$ $= AB + \overline{A}C + BC1$

$= AB + \overline{A}C + BC\,(A + \overline{A})$

$= AB + \overline{A}C + ABC + \overline{A}BC$

$= AB\,(1 + C) + \overline{A}C\,(1 + B)$

$= AB + \overline{A}C$

The above property, i.e $AB + \overline{A}C + BC = AB + \overline{A}C$, is called *consensus theorem*.

15.5.2 De Morgan's Theorems

De Morgan proposed two theorems that are an important part of Boolean algebra. The first theorem is stated as follows. *The complement of a product is equal to the sum of the complements*. That is, if the variables are A and B, then,

$$\overline{AB} = \overline{A} + \overline{B}$$

The second theorem is stated as *the complement of a sum is equal to the product of the complements*. In equation form, this can be written as,

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

15.5.3 Algebraic Simplification of Logical Expressions

In the application of Boolean algebra one has to reduce a particular expression to its simplest form or change its form to a more convenient one in order to implement the expression most efficiently. The basic rules and laws of Boolean algebra are used to manipulate and simplify an expression. The algebraic simplification method requires a thorough knowledge of Boolean algebra and considerable practice in its application. Several examples are given below to illustrate the technique.

Example 15.13 Simplify the following expressions using Boolean algebra:

(a) $A + AB + A\overline{B}C$ (b) $(\overline{A} + B)\,C + ABC$ (c) $A\overline{B}C\,(BD + CDE) + A\overline{C}$

Solution: (a) $A + AB + A\overline{B}C$

Step 1: Apply rule 10 of Table 15.2, i.e $A + AB = A$. The expression simplifies to

$$A + A\overline{B}C$$

Step 2: Apply distributive property

$$(A + A)\,(A + \overline{B}C)$$
$$= A\,(A + \overline{B}C)$$

Step 3: Taking A as the common term,

$$A[1.\,(1 + \overline{B}C)]$$

Step 4: Apply rule 2 of Table 15.2, i.e. $1 + \overline{B}C = 1$

$$A.1 = A$$

Thus, the simplified expression is A

(b) $$(\overline{A} + B)\,C + ABC$$

Step 1: Apply distributive property

$$\overline{A}C + BC + ABC$$

Step 2: Taking BC as a common term,

$$\overline{A}C + BC\,(1 + A)$$

Step 3: Apply rule 2

$$\overline{A}C + BC.1$$

Step 4: Taking C as the common term,

$$C\,(\overline{A} + B)$$

Thus, the simplified expression is $C\,(\overline{A} + B)$

(c) $$A\overline{B}C\,(BD + CDE) + A\overline{C}$$

Step 1: Apply distributive property

$$A\overline{B}BCD + A\overline{B}CCDE + A\overline{C}$$

Step 2: Apply rules 8 and 7 to the first and second terms, respectively,

$$0 + A\overline{B}CDE + A\overline{C}$$

Step 3: Taking A as the common term,

$$A\,(\overline{B}CDE + \overline{C})$$

Step 4: Apply rule 11, i.e., $\overline{B}CDE + \overline{C} = \overline{B}DE + \overline{C}$

$$A\,(\overline{B}DE + \overline{C})$$

Thus, the simplified expression is $A\,(\overline{B}DE + \overline{C})$

15.6 LOGIC GATES

The basic elements that make up a digital system are called as logic gates. The most common logic gates are OR, AND, NOT, NAND and NOR gates. The NAND and NOR gates are called as universal gates. Exclusive-OR gate is another logic circuit which can be constructed using AND, OR and NOT gates.

15.6.1 OR Gate

The OR gate performs logical addition, commonly known as OR function. The OR gate has two or more inputs and only one output. The operation of OR gate is such that a high (1) on the output is produced when any of the inputs is high (1). The output is low (0) only when all the inputs are low (0).

As shown in Fig. 15.1, A and B represent the inputs and Y the output. Resistance R is the load resistance.

If $A = 0$ and $B = 0$ then $V_0 = 0$ and $Y = 0$.

If $A = 1$ and $B = 0$, diode D_1 will conduct and so the output $Y = 1$.

If $A = 0$ and $B = 1$, diode D_2 will conduct and the output $Y = 1$.

If $A = 1$ and $B = 1$, both the diodes will conduct and so the output $Y = 1$.

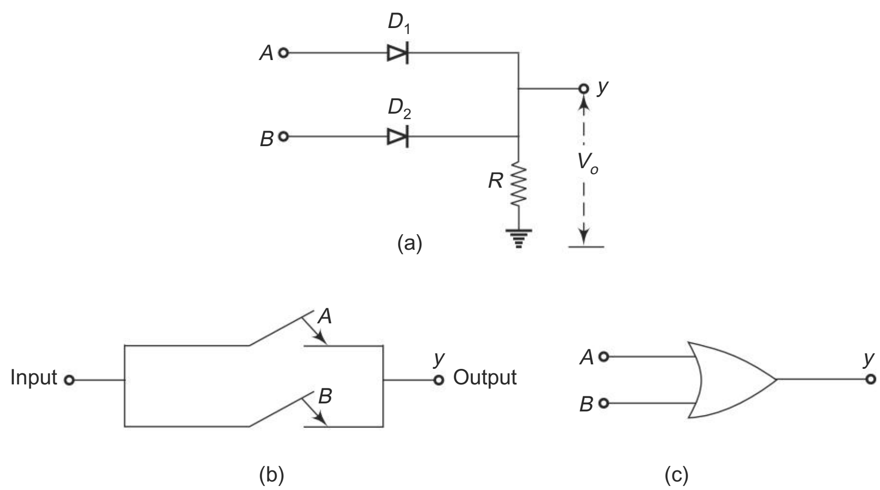


Fig. 15.1 (a) Circuit diagram of an OR gate, (b) Electrical equivalent of an OR gate, (c) Logic symbol

The electrical equivalent circuit of an OR gate is shown in Fig. 15.1(b) where switches A and B are connected in parallel with each other. If either A, B or both are closed, then the output will result. The logic symbol for OR gate is shown in Fig. 15.1(c). The logic operation of the two input OR gate is described in the truth table shown in Table 15.3.

Table 15.3 Truth table for the two-input OR gate

Input		*Output*
A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

15.6.2 AND Gate

The AND gate performs logical multiplication, commonly known as AND function. The AND gate has two or more inputs and a single output. The output of AND gate is high only when all the inputs are high. When any of the inputs is low, the output is low.

As shown in the Fig. 15.2(a), A and B represent the inputs and Y represents the output.

If $A = 0$ and $B = 0$, both diodes conduct as they are forward biased and the output $Y = 0$.

If $A = 0$ and $B = 1$, diode D_1 conducts and D_2 does not conduct, and again the output $Y = 0$.

If $A = 1$ and $B = 0$, diode D_1 does not conduct and D_2 conducts, and the output $Y = 0$.

If $A = 1$ and $B = 1$, both the diodes do not conduct as they are reverse based and so the output $Y = 1$.

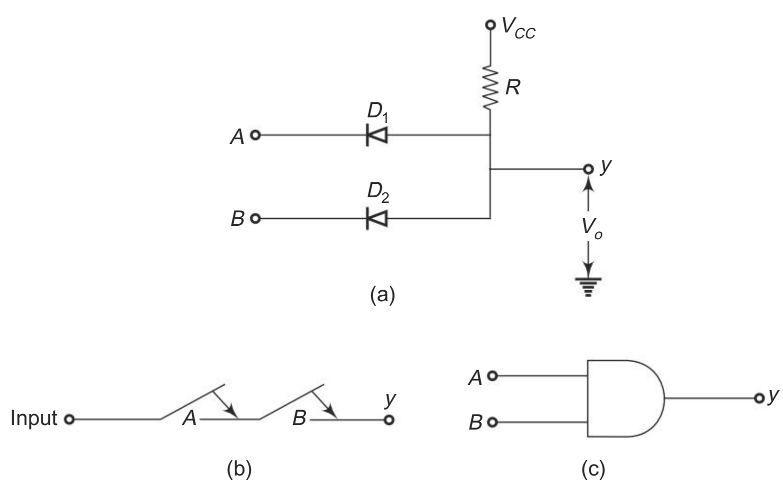


Fig. 15.2 (a) Circuit diagram of an AND gate, (b) Electrical equivalent of an AND gate, (c) Logic symbol

The electrical equivalent circuit of an AND gate is shown in Fig. 15.2 (b) where two switches *A* and *B* are connected in series. If both *A* and *B* are closed, then only output will result. Logic symbol of the AND gate is shown in Fig. 15.2(c). The logic operation of the two input AND gate is described in the truth table shown in Table 15.4.

Table 15.4 Truth table for a two-input AND gate

Input		*Output*
A	*B*	*Y*
0	0	0
0	1	0
1	0	0
1	1	1

15.6.3 NOT Gate (Inverter)

The NOT gate performs a basic logic function called inversion or complementation. The purpose of the gate is to change one logic level to opposite level. It has one input and one output. When a high level is applied to an inverter input, a low level will appear at its output and vice-versa. The operation of the circuit can be explained as follows. When a high voltage is applied to the base of the transistor, base current increases and the transistor is saturated. The transistor now acts as a closed switch and conducts heavily. Thus the output voltage is logic 0. On the other hand, when a low voltage is applied at the base, the transistor is cut-off due to very low or no base current. Now, the transistor can be considered as an open switch, with no current flowing through it. The output is now clamped to the supply voltage. The transistor when operated between cut-off and saturation will act as a switch.

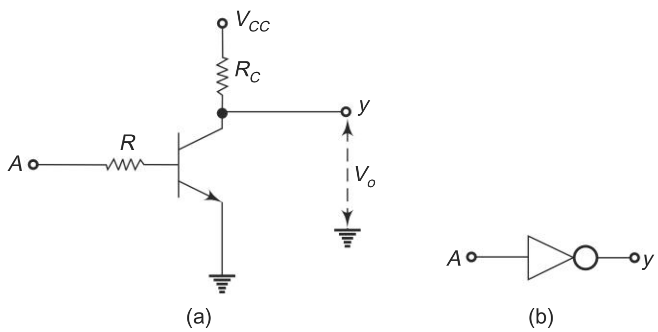


Fig. 15.3 (a) Circuit diagram of an INVERTER gate (b) Logic symbol

As shown in Fig. 15.3(a), *A* represents the input and *y* represents the output. If the input is high, the transistor is in ON state and the output is low. If the input is low, the transistor is in OFF state and the output is high. The symbol for the inverter is shown in Fig. 15.3(b). The truth table is given in Table 15.5.

Table 15.5 Truth table for an INVERTER

Input *A*	*Output* *Y*
0	1
1	0

15.6.4 NAND Gate

NAND is a contraction of NOT–AND. It has two or more inputs and only one output. When all the inputs are high, the output is low. If any of the inputs is low, the output is high. The logic symbol for the NAND gate is shown in Fig. 15.4.

The truth-table for the NAND gate is shown in Table 15.6.

Table 15.6 Truth table for NAND gate

Input *A*	*B*	*Output* *y*
0	0	1
0	1	1
1	0	1
1	1	0

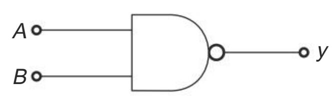


Fig. 15.4 Logic symbol for the NAND gate

The NAND gate is a very popular logic function because it is an universal function; that is, it can be used to construct an AND gate, an OR gate, and INVERTER or any combination of these functions. Figure 15.5 shows how NAND gates can be connected to realize various logic gates.

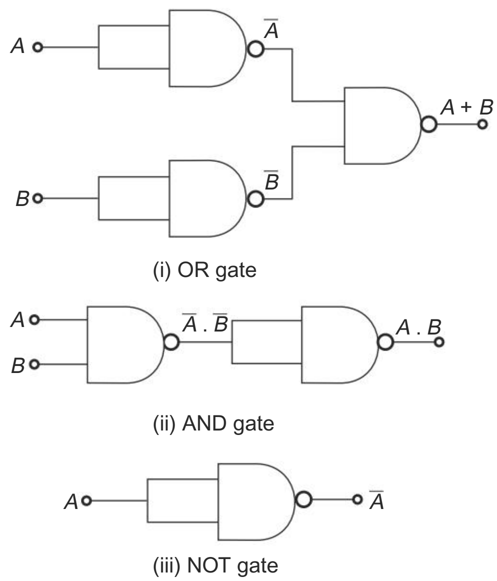


Fig. 15.5 NAND gates connected to realize (a) OR (b) AND and (c) NOT gates

15.6.5 NOR Gate

NOR is a contraction of NOT–OR. It has two or more inputs and only one output. Only when all the inputs are low, the output is high. If any of the inputs is high, the output is low. The logic symbol for the NOR gate is shown in Fig. 15.6.

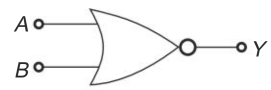


Fig. 15.6 Logic symbol for NOR gate

The truth-table for the NOR gate is shown in Table 15.7.

Table 15.7 Truth table for NOR gate

Input		*Output*
A	*B*	*Y*
0	0	1
0	1	0
1	0	0
1	1	0

The NOR gate is also a very popular logic function because it is also an universal function; that is, it can be used to construct an AND gate, an OR gate, and INVERTER or any combination of these functions. Figure 15.7 shows how NOR gates can be connected to realize various logic gates.

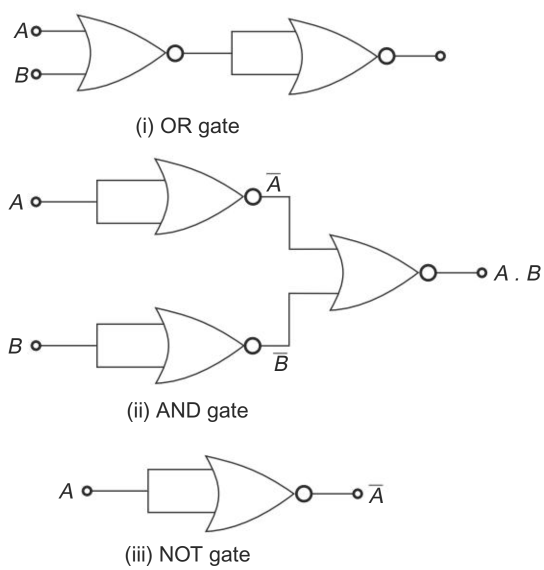
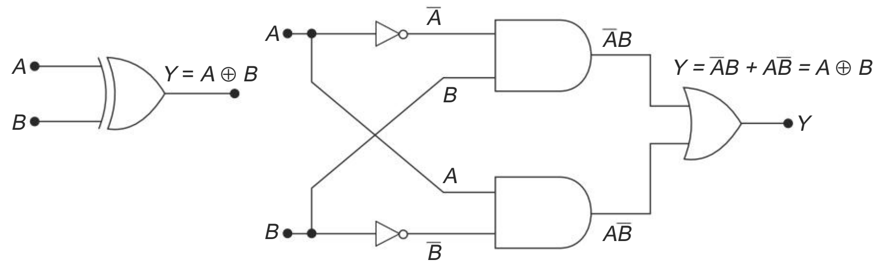


Fig. 15.7 NOR gates connected to realize (a) OR (b) AND and (c) NOT gates

15.6.6 Exclusive–OR (Ex-OR) Gate

An Exclusive-OR Gate is a gate with two or more inputs and one output. The output of a two-input Ex-OR gate assumes a HIGH state if one and only one input assumes a HIGH state. This is equivalent to saying that the output is HIGH if either input A or input B is HIGH exclusively, and low when both are 1 or 0 simultaneously.

The logic symbol for the Ex-OR gate is shown in Fig. 15.8 (a) and the truth table for the Ex-OR operation is given in Table 15.8.



(a) Logic symbol (b) Using AND-OR-NOT gates

Fig. 15.8 Ex-OR gate

Table 15.8 Truth table of a 2-input Ex-OR gate

Input		*Output*
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	0

The truth table of the Ex-OR gate shows that the output is HIGH when any one, but not all, of the inputs is at 1. This exclusive feature eliminates a similarity to the OR gate. The Ex-OR gate responds with a HIGH output only when an odd number of inputs is HIGH. When there is an even number of HIGH inputs, such as two or four, the output will always be LOW. From the truth table of a 2-input Ex-OR gate, the Ex-OR function can be written as $Y = \overline{A}B + A\overline{B} = A \oplus B$.

The above expression can be read as Y equals A Ex-OR B. Using the above expression, a 2-input Ex-OR gate can be implemented using basic gates like AND, OR and NOT gates as shown in Fig. 15.8.

The 2-input Ex-OR gate can also be implemented using NAND gates as shown in Fig. 15.9.

The main characteristic property of an Ex-OR gate is that it can perform modulo-2 addition. It should be noted that the same Ex-OR truth table applies when adding two *binary digits* (bits). A 2-input Ex-OR circuit is, therefore, sometimes called a *module-2-adder* or a *half-adder* without carry output. The name half-adder refers to the fact that possible carry-bit, resulting from an addition of two preceding bits, has not been taken into account. A full addition is performed by a second Ex-OR circuit

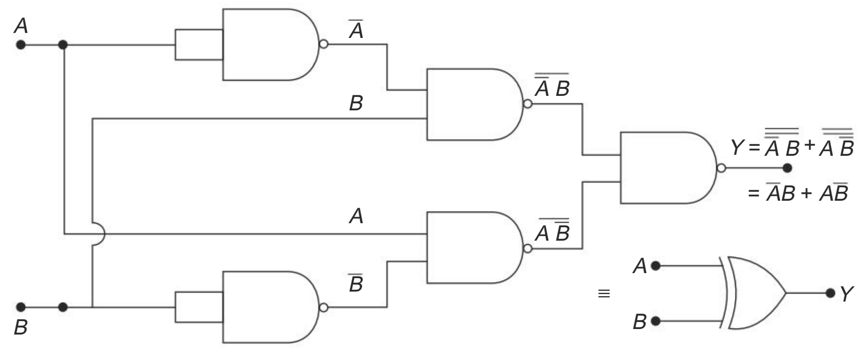


Fig. 15.9 Ex-OR gates using NAND gates

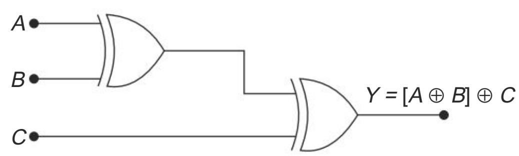


Fig. 15.10 Cascading of two Ex-OR circuits

with the output signal of the first circuit and the carry as input signals, as shown in Fig. 15.10.

The configuration of Fig. 15.10 is a cascading of two Ex-OR circuits resulting in an Ex-OR operation of three variables A, B, and C. Consequently, the sum output of a full adder for two bits is an Ex-OR operation of the 2 bits to be added and the carry of the preceding adding stage. The logic expression of the Ex-OR operation of three variables A, B, and C is given by

$$A \oplus B \oplus C = (A\overline{B} + \overline{A}B)\overline{C} + \overline{(A\overline{B} + \overline{A}B)}C$$
$$= (A\overline{B} + \overline{A}B)\overline{C} + (\overline{A}\,\overline{B} + AB)C$$
$$A \oplus B \oplus C = A\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C + ABC$$

In general, Ex-OR operation of n variables results in a logical 1 output if an *odd* number of the input variables are 1s. An Ex-OR operation of n variables can be obtained by cascading 2-input Ex-OR gates.

Another important property of an Ex-OR gate is that it can be used as a *controlled inverter*, i.e. by using an Ex-OR gate, a logic variable can be complemented or allowed to pass through it unchanged. This is done by using one Ex-OR input as a control input and the other as the logic variable input as shown in Fig. 15.11. When the control input is HIGH, the output $Y = \overline{A}$ and when the control input is LOW, the output $Y = A$.

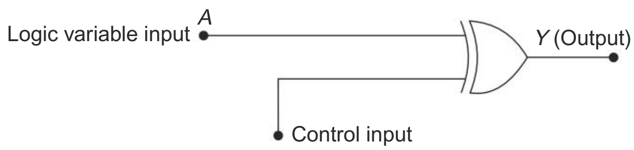


Fig. 15.11 Ex-OR gate as a controlled inverter

15.6.7 Exclusive-NOR (Ex-NOR) Gate

The exclusive-NOR gate, abbreviated Ex-NOR, is an Ex-OR gate, followed by an inverter. An Exclusive-NOR gate has two or more inputs and one output. The output of a two-input Ex-NOR gate assumes a HIGH state if both the inputs assume the same logic state or have an even number of 1s, and its output is LOW when the inputs assume different logic states or have an odd number of 1s. The logic symbol of Ex-NOR gate is shown in Fig. 15.12 and its truth table is given in Table 15.9. From the truth table, it is clear that the Ex-NOR output is the complement of the Ex-OR gate. The Boolean expression for the Ex-NOR gate is

$$Y = \overline{A \oplus B}$$

Read the above expression as "*Y* equals *A* Ex-NOR *B*". According to DeMorgan's theorem.

$$\begin{aligned}\overline{A \oplus B} &= \overline{\overline{A}B + A\overline{B}} \\ &= \overline{\overline{A}B}.\,\overline{A\overline{B}} \\ &= (A + \overline{B})\,(\overline{A} + B) \\ &= AB + \overline{A}\,\overline{B}\end{aligned}$$

Table 15.9 Truth table of 2-input Ex-NOR gate

Input		*Output*
A	*B*	$Y = \overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

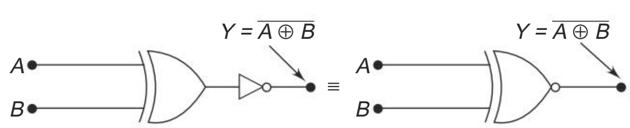


Fig. 15.12 Logic symbol of 2-input Ex-NOR gate

An important property of the Ex-NOR gate is that it can be used for bit comparison. The output of an Ex-NOR gate is 1 if both the inputs are similar, i.e. both are 0 or 1; otherwise, its output is 0. Hence, it can be used as a one-bit comparator. It is also called a *coincidence circuit*.

Another property of the Ex-NOR gate is that it can be used as an *even-parity checker*. The output of the Ex-NOR gate is 1 if the number of 1 s in its inputs is even; if the number of 1s is odd, the output is 0. Hence, it can be used as an even/odd parity checker. Hence, the 2-input Ex-NOR gate is immensely useful for bit comparison and parity checking.

Example 15.14 Realise the logic expression $Y = \overline{B}\,\overline{C} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B}$ using basic gates.

Solution: In the given expression, there are 3 product terms each with two variables which can be implemented using three 2-input AND gates, and the product terms can be OR operated together using a 3-input OR gate. The complemented form of individual variable can be obtained by 3 NOT gates. Thus, the circuit for the given expression is realised as shown in Fig. 15.13.

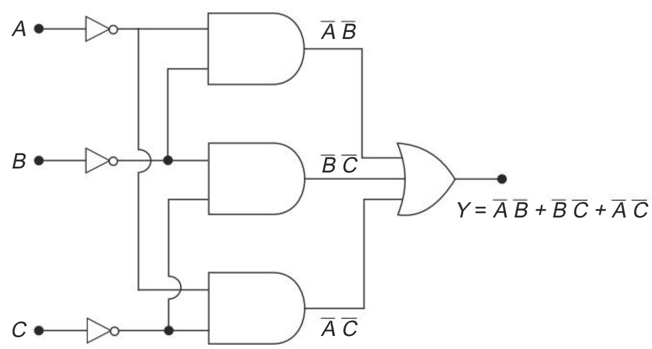


Fig. 15.13

Example 15.15 Realise the logic expression $Y = (A + B)\,(\overline{A} + C)\,(B + D)$ using basic gates.

Solution: In the given expression, there are 3 sum terms which can be implemented using three 2-input OR gates and their outputs are AND operated together by a 3-input AND gate. A NOT gate can be used to obtain the inverse of A. Now, the realized circuit is shown in Fig. 15.14.

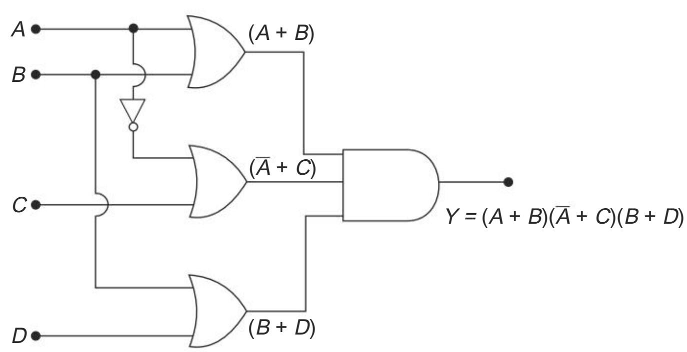


Fig. 15.14

Example 15.16 Implement $Y = \overline{AB} + A + (\overline{B + C})$ using NAND gates only.

Solution: The implementation of the given function is shown in Fig. 15.15.

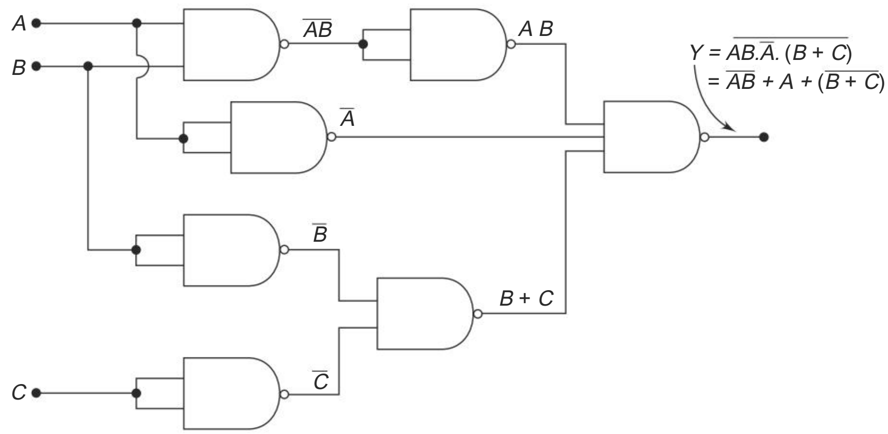


Fig. 15.15

Example 15.17 Simplify the logic circuit shown in Fig. 15.16(a)

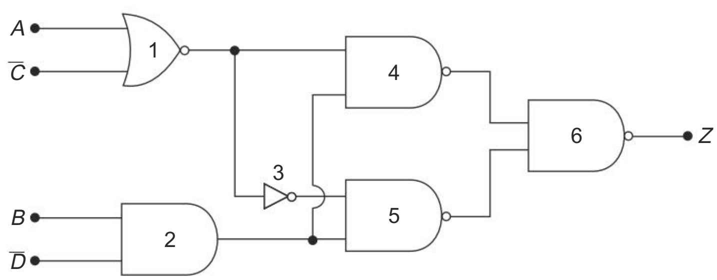


Fig. 15.16 (a)

Solution: From the given logic circuit, the expression for Z can be written as

$$Z = \overline{\overline{\overline{(\overline{A + \overline{C}}) \cdot B\overline{D}} \cdot \overline{(\overline{A + \overline{C}}) \cdot \overline{B\overline{D}}}]}}$$

$$= \overline{\overline{[(\overline{A + \overline{\overline{C}}})} + \overline{B\overline{D}}] \cdot \overline{(\overline{\overline{A + C}})} + \overline{\overline{B}\overline{D}}]}$$

$$= \overline{[(A + \overline{C}) + \overline{\overline{B\overline{D}}}] \cdot [(\overline{A + \overline{C}}) + \overline{\overline{B\overline{D}}}]}$$

$$= \overline{\overline{B\overline{D}} + (A + \overline{C})(\overline{A + \overline{C}})} \qquad [\therefore (A + B)(A + C) = A + BC]$$

$$= \overline{\overline{B\overline{D}}} \qquad [\because A\overline{A} = 0]$$

$$= B\overline{D}$$

Therefore, the above logic circuit can be simplified as shown in Fig. 15.16 (b).

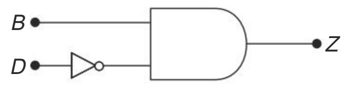


Fig. 15.16 (b)

Instead of using Boolean algebra, the logic circuit can be simplified directly as shown below. In the given logic circuit shown in Fig. 15.16 (a) the NAND gate (6) can be replaced by an OR gate with a bubble at its inputs as shown in Fig. 15.16 (c).

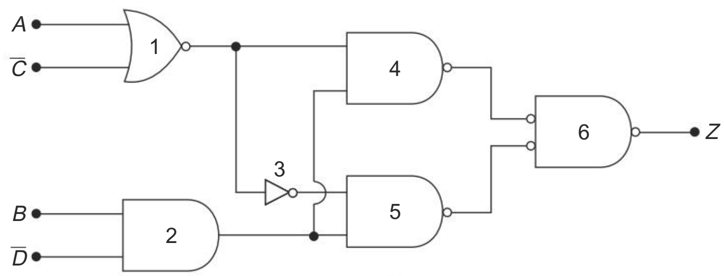


Fig. 15.16 (c)

Now, using $\bar{\bar{A}} = A$, the bubble at the outputs of gates 4 and 5 get cancelled with the bubble at the inputs of gate 6 as shown in Fig. 15.16 (d).

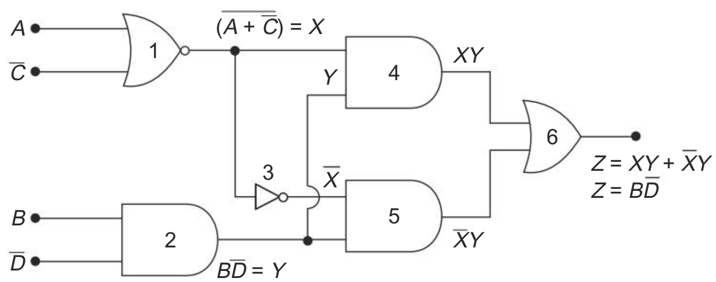


Fig. 15.16 (d)

In the above figure, if we assume the output of gate 1 $\overline{(A + \bar{C})} = X$ and the output of gate 2 $B\bar{D} = Y$, then the output of gate 4 is XY and the output of gate 5 is $\bar{X}Y$. If XY and $\bar{X}Y$ are OR operated in gate 6, then $XY = \bar{X}Y = Y(X + \bar{X}) = Y = B\bar{D}$. Therefore, the above circuit can be simplified as shown in Fig. 15.16 (e).

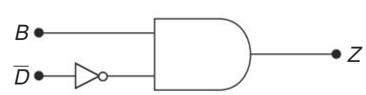


Fig. 15.16 (e)

Example 15.18 Realise (a) $Y = A + BC\bar{D}$ using NAND gates and (b) $Y = (A + C)(A + \bar{D})(A + B + \bar{C})$ using NOR gates.

Solution Using NAND gates

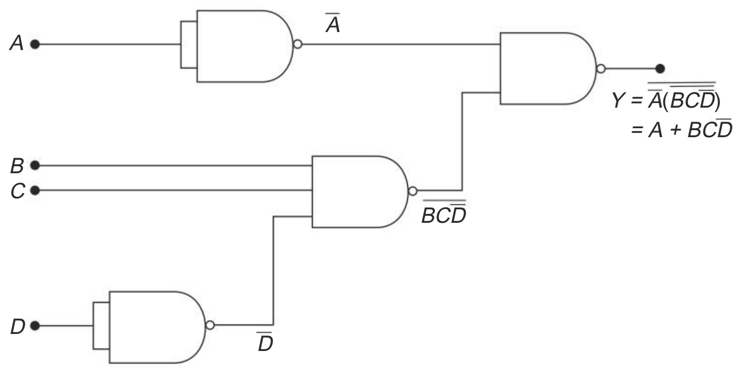


Fig. 15.17 (a)

(b) Using NOR gates

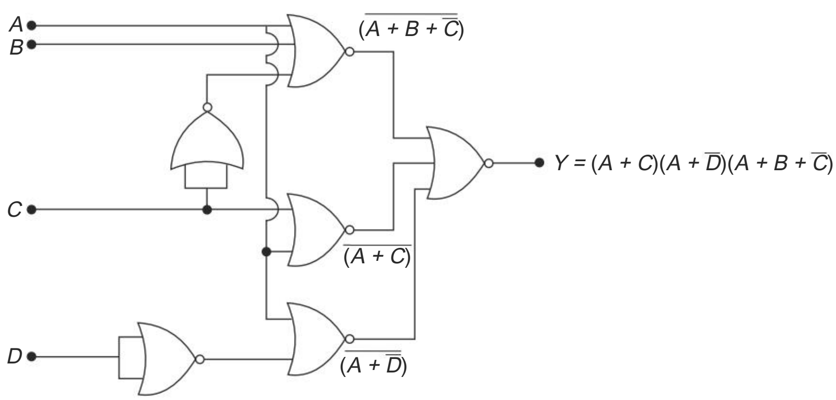


Fig. 15.17 (b)

15.7 COMBINATIONAL LOGIC DESIGN

The logic gates are the fundamental building blocks of the combinational logic circuit. When logic gates are connected together to produce a specified output for certain specified combinations of input variables, with no storage involved, the resulting network is called combinational logic. In combinational logic, the output level is at all times dependent on the combination of input levels. Basically, digital circuits are divided into (i) Combinational Circuits, and (ii) Sequential Circuits.

In combinational circuits, the outputs at any instant of time depend upon the inputs present at that instant of time. This means there is no memory in these circuits. There are other types of circuits in which the output at any instant of time depend upon the present inputs as well as past outputs. This means that there are elements used to store past information. Such circuits are known as sequential circuits.

Logical functions are expressed in terms of logical variables. Boolean algebraic theorems are used for the manipulation of logical expressions. A logical expression can be realized using the logical gates. The values assumed by the logical functions as well as the logical variables are in the binary form. Any arbitrary logical function can be expressed in the following forms.

(i) Sum of Products Form (SOP)

(ii) Product of Sums Form (POS)

For example, the logical expression, $Y = AB + A'C + BC$ is a sum of products expression and $Y = (A + B)(B + C')$ is in product of sums form. The sum-of-product form of logical expressions can be realized using AND–OR combination as shown in Fig. 15.18. This realization is known as two level realization. The first level consists of AND gates and the second level consists of OR gates.

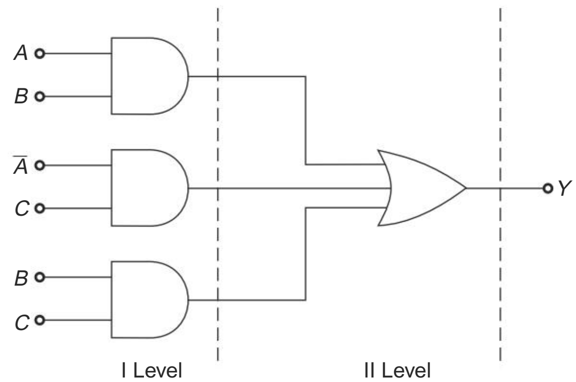


Fig. 15.18 AND-OR realization of $Y = AB + A'C + BC$

Consider the expression,

$$Y = (A + BC)(B + C'A)$$
$$= (A + B)(A + C)(B + C')(B + A)$$
$$= (A + B)(A + C)(B + C')$$

The above equation can be realized using OR–AND combination as shown in Fig. 15.19.

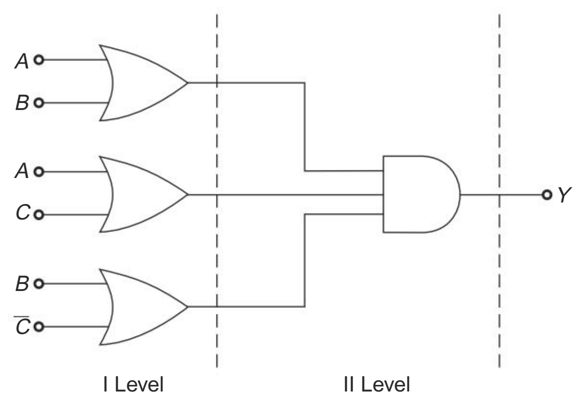


Fig. 15.19 OR-AND realization of $Y = (A + B)(A + C)(B + C')$

Each individual term in the standard SOP form is called as minterm and in the standard POS form as maxterm. An important characteristics of sum-of-products and product-of-sums forms is that the corresponding implementation is always a two-level gate network; hence, the maximum number of gates through which a signal must pass in going from an input to the output is two, excluding inversions.

15.8 KARNAUGH MAP REPRESENTATION OF LOGICAL FUNCTIONS

Karnaugh map technique provides a systematic method for simplifying and manipulating Boolean expressions. In this technique, the information contained in a truth table or available in POS or SOP form is represented on Karnaugh map (K-map). In an n-variable K-map there are 2^n cells. Each cell corresponds to one of the combinations of n variables. Therefore, we see that for each row of the truth table, i.e. for each minterm and for each maxterm, there is one specific cell in the K-map. The variables have been designated as A, B, C and D, and the binary numbers formed by them are taken as AB, ABC, and $ABCD$ for two, three and four variables, respectively. The K-map for two, three and four variables are shown in Fig. 15.20.

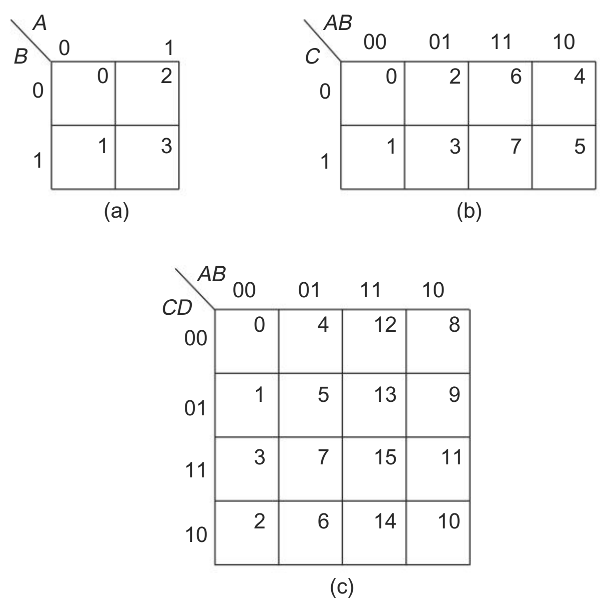


Fig. 15.20 Karnaugh maps: (a) Two-variable (b) Three-variable (c) Four-Variable

The entries in a truth table can be represented in a K-map as discussed below. Consider the truth table shown in Table 15.10.

Table 15.10 Truth table of a digital system

Inputs			*Output*
A	*B*	*C*	*Y*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The output Y can be written as,

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

The K-map for the above expression is shown below.

Variables		$\overline{AB}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$	0		1		1
C	1	1		1	

The value of the output variable Y (0 or 1) in each cell can be entered corresponding to its decimal or minterm or maxterm indentification.

Simplification is based on the principle of combining terms in adjacent cells. One can group 1s that are in adjacent cells according to the following rules by drawing a loop around those cells:

1. Adjacent cells are cells that differ by only a single variable. For example, the cells numbered 000 and 001 are adjacent to each other because they differ by only one bit. Similarly, the cells 011 and 111, 000 and 010, 100 and 110 are all adjacent cells.
2. The 1s in adjacent cells must be combined in groups of 1, 2, 4, 8 and so on.
3. Each group of 1s should be maximized to include the largest number of adjacent cells as possible in accordance with rule 2.
4. Every 1 on the map must be included in at least one group. There can be overlapping groups if they include common 1s.

Grouping is illustrated by the following example.

Example 15.19 Simplify the K-map shown below.

Variables		$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
		00	01	11	10
$\bar{C}$	0	1			1
C	1		1	1	

Solution: The adjacent cells that can be combined together are the cells 000 and 100 and the cell 011 and 111.

By combining the adjacent cells, we get

$$Y = (A' + A)\,B'C' + (A' + A)\,BC$$
$$= B'C' + BC$$

The above equation can be obtained from the K-map by using the following procedure.

(i) Identify adjacent ones, then see the values of the variables associated with these cells. Only one variable will be different and gets eliminated. Other variables will appear in ANDed form in the term; it will be in the uncomplemented form if it is 1 and in the complemented form if it is 0.

(ii) Determine the term corresponding to each group of adjacent ones. These terms are ORed to get simplified equation in SOP form.

Example 15.20 Simplify the K-map shown below.

Variables		$\overline{A}\,\overline{B}$	$\overline{A}B$	AB	$A\overline{B}$
		00	01	11	10
$\overline{C}\,\overline{D}$	00	1			1
$\overline{C}D$	01		1	1	
CD	11				
$C\overline{D}$	10				

Solution: In the above K-map, the following adjacent cells can be combined to form two pairs of adjacent 1s. Thus, the cell pairs are $\overline{B}\,\overline{C}\,\overline{D}$ and $B\overline{C}D$. The simplified functions is $Y = \overline{B}\,\overline{C}\,\overline{D} + B\overline{C}D$.

15.9 SOME COMMON COMBINATIONAL CIRCUITS

In this section, several types of MSI combinational logic functions are studied, including adders, multiplexers, demultiplexers and code converters.

15.9.1 Half Adder

Adder circuits form one of the main applications of the combinational logic circuits. Half adders and full adders are discussed in this section. A logic circuit for the addition of two one-bit numbers is referred to as an half adder. The half adder accepts two binary digits on its inputs and produces two binary digits on its outputs, a sum bit and a carry bit. The logical operation of the half adder is explained in Table 15.11.

Table 15.11 Truth table for half-adder

Inputs		*Outputs*	
X	*Y*	*Sum*	*Carry*
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

From the truth table, the logical expressions for the sum (S) and carry (C) outputs are given by

$$S = X'Y + XY' = X \oplus Y$$
$$C = XY$$

The implementation required for the half adder is apparent from the above two logical expressions. The sum output is generated with an exclusive-OR gate and the carry output is produced with an AND gate with X and Y on the inputs. The realization of an half adder using gates is shown in Fig. 15.21

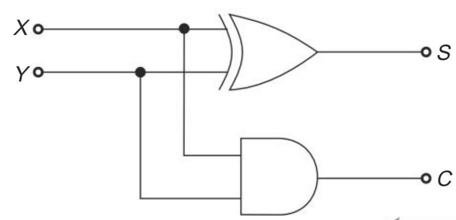


Fig. 15.21 Realization of a half-adder

15.9.2 Full Adder

An half adder has only two inputs and there is no provision to add a carry coming from the lower order bits when multibit addition is performed. To include the carry, a third input terminal is added and this circuit is used to add X_n, Y_n and C_{n-1}, where X_n and Y_n are the nth order bits of the numbers X and Y, respectively, and C_{n-1} is the carry generated from the addition of (n−1) order bits. This circuit is called the full adder circuit. The operation of a full adder is shown in Table 15.12.

Table 15.12 Truth table for full adder

Inputs			*Outputs*	
X_n	Y_n	C_{n-1}	*Sum*	C_n
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

From the truth table, the logical expressions for sum (S_n) and carry (C_n) outputs are given by

$$\begin{aligned} S_n &= \overline{X}_n \overline{Y}_n C_{n-1} + \overline{X}_n Y_n \overline{C_{n-1}} + X_n \overline{Y}_n \overline{C}_{n-1} + X_n Y_n C_{n-1} \\ &= X_n (\overline{Y}_n \overline{C}_{n-1} + Y_n \overline{C}_{n-1}) + \overline{X}_n (Y_n \overline{C}_{n-1} + \overline{Y}_n C_{n-1}) \\ &= X_n \oplus Y_n \oplus C_{n-1} \end{aligned}$$

$$\begin{aligned} C_n &= \overline{X}_n Y_n C_{n-1} + X_n \overline{Y}_n C_{n-1} + X_n Y_n \overline{C}_{n-1} + X_n Y_n C_{n-1} \\ &= Y_n C_{n-1} + X_n C_{n-1} + X_n Y_n \end{aligned}$$

The realization of a full adder circuit is shown in Fig. 15.22.

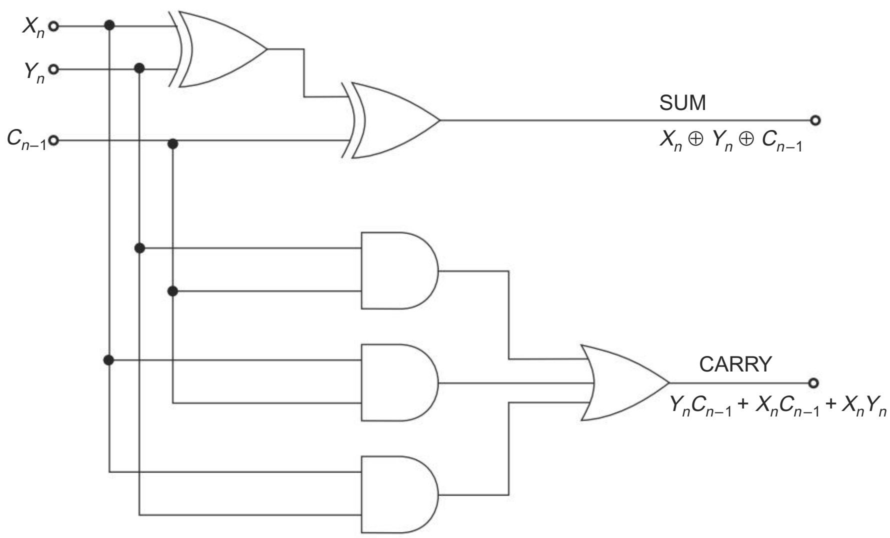


Fig. 15.22 Realization of a full adder circuit

15.9.3 Half Subtractor

A logic circuit for subtracting two bits is referred to as a half subtractor. The logical operation of a half subtractor can be understood from Table 15.13.

Table 15.13 Truth table for half subtractor

Inputs		*Outputs*	
X	Y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Here, X (minuend) and Y (subtrahend) are the two inputs, and D (difference) and B (borrow) are the two outputs. From the truth table, the logical expressions for D and B are obtained as

$$D = X'Y + XY' = X \oplus Y$$

$$B = X'Y$$

Figure 15.23 shows the realization of a half subtractor using logic gates.

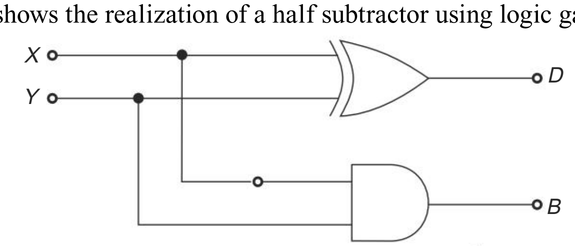


Fig. 15.23 Realization of a full subtractor

15.9.4 Full Subtractor

A full subtractor circuit performs multibit subtraction where a borrow from the previous bit position may be included. A full subtractor has three inputs, X_n (minuend), Y_n (subtrahend) and B_{n-1} (borrow from the previous stage) and two outputs, D_n (difference) and B_n (borrow).

Table 15.14 Truth table for full-subtractor

Input			Output	
X_n	Y_n	B_{n-1}	D_n	B_n
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

From the truth table, the output logical expressions can be written as

$$D_n = \bar{X}_n \bar{Y}_n B_{n-1} + \bar{X}_n Y_n \bar{B}_{n-1} + X_n \bar{Y}_n \bar{B}_{n-1} + X_n Y_n B_{n-1}$$

$$= X_n \oplus Y_n \oplus B_{n-1}$$

$$B_n = \bar{X}_n \bar{Y}_n B_{n-1} + \bar{X}_n Y_n \bar{B}_{n-1} + \bar{X}_n Y_n B_{n-1} + X_n Y_n B_{n-1}$$

$$= \bar{X}_n \bar{Y}_n B_{n-1} + \bar{X}_n Y_n \bar{B}_{n-1} + Y_n B_{n-1} (X_n + \bar{X}_n)$$

$$= \bar{X}_n (\bar{Y}_n B_{n-1} + Y_n \bar{B}_{n-1}) + Y_n B_{n-1}$$

$$= \bar{X}_n (Y_n \oplus B_{n-1}) + Y_n B_{n-1}$$

Figure 15.24 shows the realization of a full subtractor circuit.

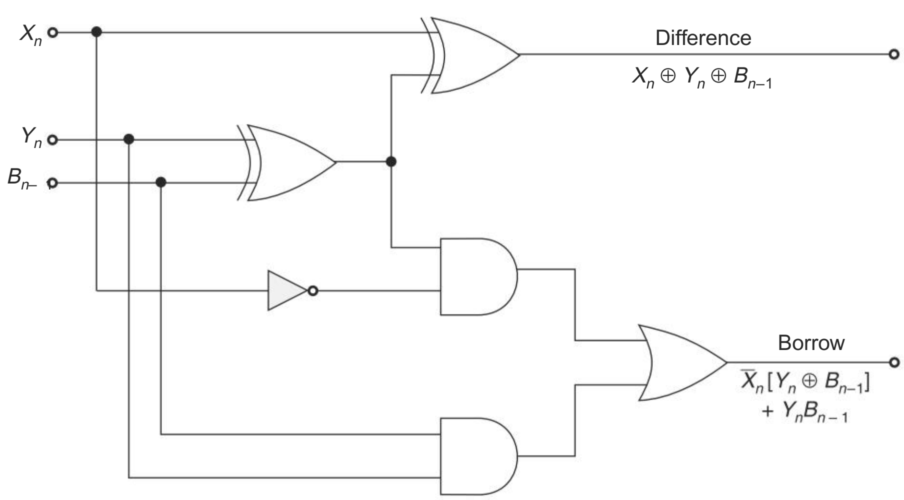


Fig. 15.24 Realization of a full subtractor

15.9.5 Multiplexer

The multiplexer (or data selector) is a logical circuit that gates one out of several inputs to a single output. The input selected is controlled by a set of select inputs. Figure 15.25 shows the block diagram of a multiplexer with n input lines and one output lines. For selecting one out of n inputs for connection to the output, a set of m ($2^m = n$) select signals is required.

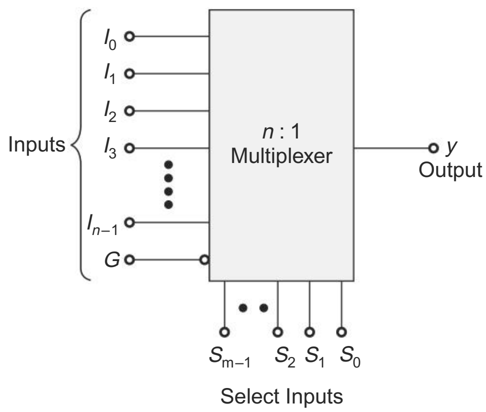


Fig. 15.25 Block diagram of a digital multiplexer

Depending upon the digital codes applied to the select inputs one out of n data sources is selected and transmitted to a single output channel, provided that the system is enabled. Truth table for 4–1 multiplexer is given in Table 15.15.

Table 15.15 Truth table for 4–1 line multiplexer

Select Inputs		*Output*
S_0	S_1	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

The logical output Y can be expressed as

$$Y = S_0'S_1'I_0 + S_0'S_1I_1 + S_0S_1'I_2 + S_0S_1I_3$$

The above equation can be realized using gates and is shown in Fig. 15.26.

15.9.6 Demultiplexer

The demultiplexer does the reverse operation of a multiplexer. It can be used to separate the multiplexed signals into individual signals. The block diagram of a demultiplexer is shown in Fig. 15.27. The select input code determines to which output the data input will be transmitted.

The number of output lines is n and the number of select lines is m, where $n = 2^m$. Figure 15.28 shows a 1–16 demultiplexer. The input data D is transmitted to one of

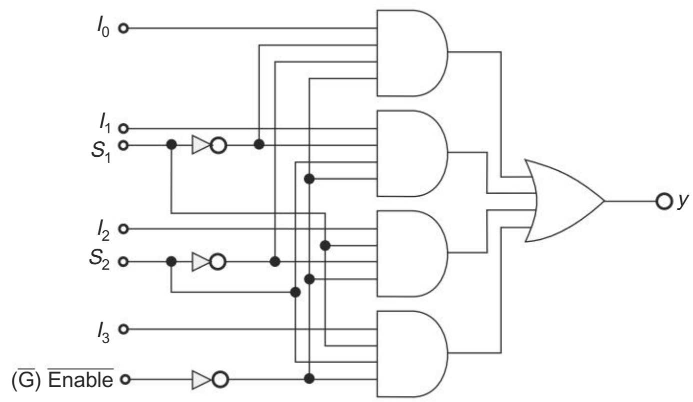


Fig. 15.26 A 4:1 line multiplexer with enable input

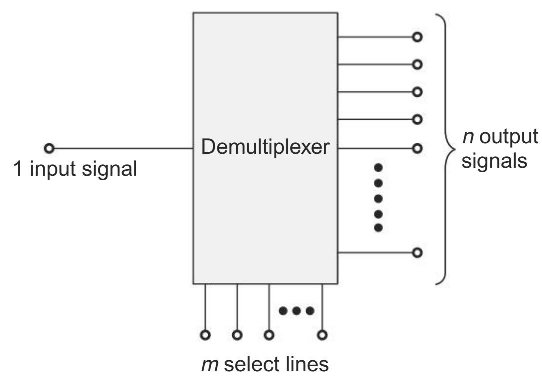


Fig. 15.27 Block diagram of a digital demultiplexer

the outputs Y_i by means of the select signals S_1, S_2, S_3, and S_4. When $S_4 = 0$, $S_3 = 0$, $S_2 = 0$ and $S_1 = 0$, data D is available at output Y_0, and for $S_4 = 1$, $S_3 = 1$, $S_2 = 1$ and $S_1 = 1$ data D is available at output Y_{15}. For other combinations of select signal lines, data D is available on the respective output lines.

15.9.7 Encoder

An encoder converts an active input signal into a coded output signal. Figure 15.29 shows the block diagram of an encoder. There are n input lines, only one of which is active.

Internal logic within the encoder converts this active input to a coded binary output with m bits. Figure 15.30 shows a common type encoder, the decimal to BCD encoder. For example, when the button 5 is pressed, the B and D OR gates have high inputs, therefore the output is $ABCD = 0101$.

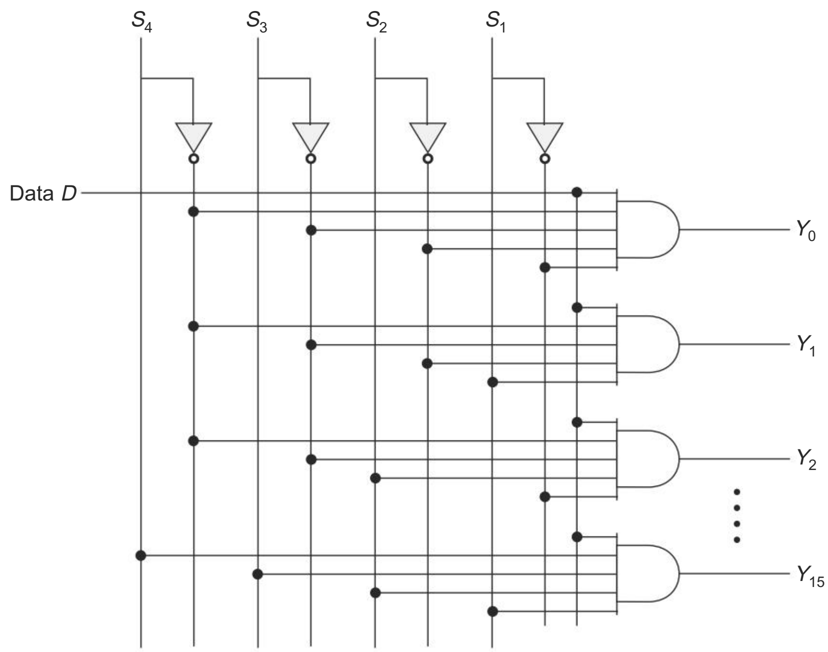


Fig. 15.28 1-to-16 demultiplexer

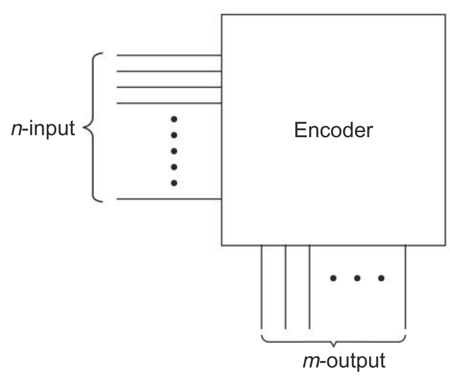


Fig. 15.29 Block diagram of an encoder

15.9.8 Decoder

A decoder is similar to demultiplexer, with one exception — there is no data input. The only inputs are the control bits *ABCD* as shown in Fig. 15.31. This logic circuit is called 1-of-16 decoder because only 1 of the 16 output lines is high. For instance, when *ABCD* is 0001, only the Y_1 AND gate has all inputs high, and the outputs of all other AND gates are low.

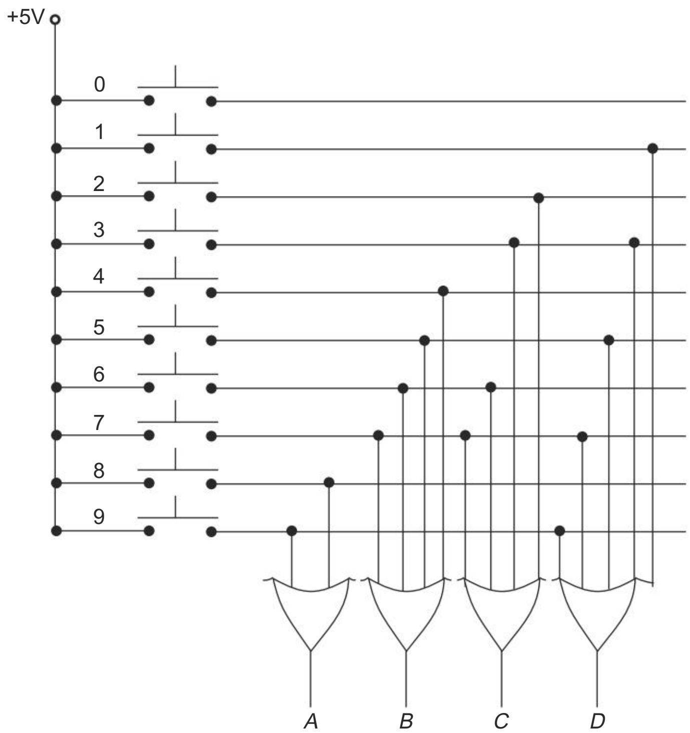


Fig. 15.30 Decimal-to-BCD encoder

15.9.9 Code Converters

Gray code The Gray code is an unweighted code, which means that there are no specific weights assigned to the bit positions. The Gray code exhibits only a single bit change from one code number to the other.

Binary to gray code conversion Table 15.16 gives a set of four bit binary numbers and their equivalent Gray code.

The most significant digit (left most) in the Gray code is the same as the corresponding digit in the binary number. Going from left to right, add each adjacent pair of binary digits to get the next Gray code digit. Neglect carries.

For example, consider the binary number 10110 to Gray code conversion.

1. The left most Gray digit is the same as the left most binary digit.

   10110 Binary
   1 Gray

2. Add the left most binary digit to the adjacent one

   10110 Binary
   11 Gray

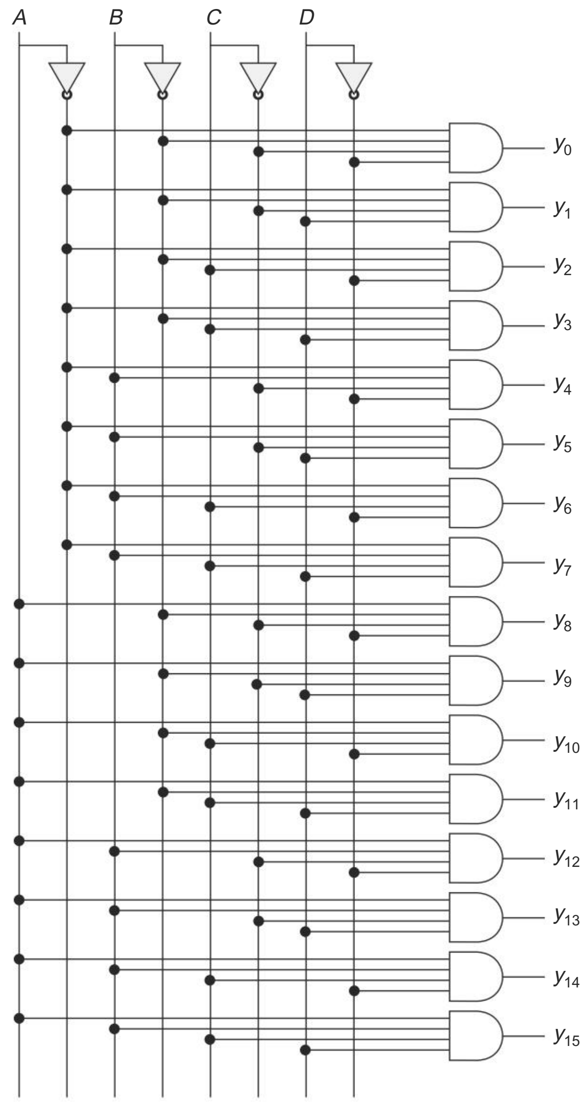


Fig. 15.31 1-of-16 decoder

3. Add the next adjacent pair

   10110 Binary
   111 Gray

4. Add the next adjacent pair and discard carry

   10110 Binary
   1110 Gray

5. Add the last adjacent pair

   10110 Binary
   11101 Gray

Table 15.16 Truth table for binary to Gray code conversion

Decimal	*Binary Numbers*	*Gray code*
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

The conversion is now complete and the Gray code is 11101. The logical expression for each of the Gray code bits can be obtained from the truth-table using algebraic simplification. The logical expressions for each bit of the 4-bit Gray code ($G_3G_2G_1G_0$) are given below.

$$G_3 = B_3$$
$$G_2 = B_3 \oplus B_2$$
$$G_1 = B_2 \oplus B_1$$
$$G_0 = B_1 \oplus B_0$$

where $B_3B_2B_1B_0$ is the given binary number. A binary-to-Gray code convertor is shown in Fig. 15.32(a).

Gray to binary conversion The most significant digit in the binary code is the same as the corresponding digit in the Gray code. Add each binary digit generated to the Gray digit in the next adjacent position. Neglect carries.

The conversion of the Gray code number 11011 to binary is given below.

1. The left most digits are the same.

   11011 Gray
   1 Binary

2. Add the last binary digit just generated to the Gray digit in the next position. Discard carry.

   11011 Gray
   10 Binary

3. Add the last binary digit generated to the next Gray digit.

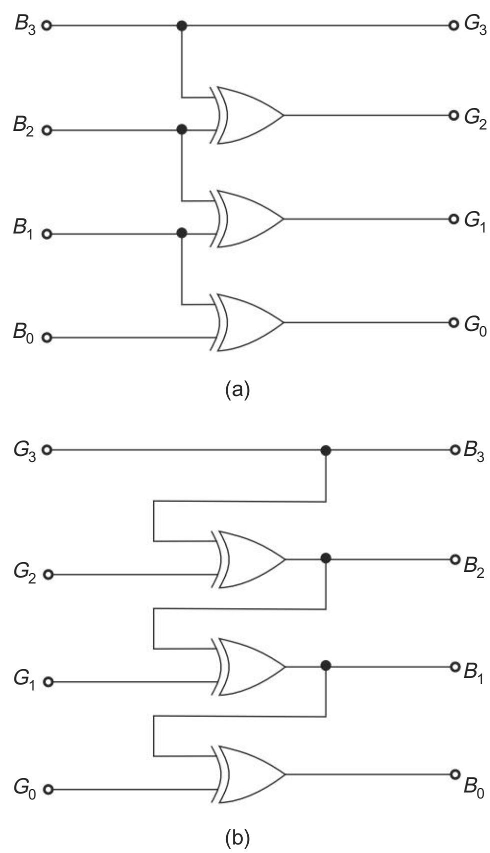


Fig. 15.32 (a) Binary-to-Gray code convertor, (b) Gray-to-binary convertor

11011	Gray
100	Binary

4. Add the last binary digit generated to the next Gray digit.

11011	Gray
1001	Binary

5. Add the last binary digit generated to the next Gray digit. Disregard carry.

11011	Gray
10010	Binary

The final binary number is 10010. The logical expression for each of the binary bits can be obtained from the truth-table using algebraic simplification. The logical expressions for each bit of the 4-bit Binary ($B_3B_2B_1B_0$) are given as follows.

$$B_3 = G_3$$
$$B_2 = G_3 \oplus G_2$$
$$B_1 = G_3 \oplus G_2 \oplus G_1$$
$$B_0 = G_3 \oplus G_2 \oplus G_1 \oplus G_0$$

where $G_3G_2G_1G_0$ is the given binary number. A Gray-to-binary code convertor is shown in Fig. 15.32 (b).

The Excess 3 code The Excess 3 code is got by adding 3 to each decimal digit and then converting the result to 4-bit binary. Excess 3 code is an unweighted code. The following Table 15.17 shows the conversion between decimal, *BCD* and Excess-3 codes.

Table 15.17 Conversion between decimal, BCD and Excess 3

Decimal	*BCD*	*Excess-3*
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

15.10 SEQUENTIAL CIRCUITS

A combinational circuit can be defined from the behavioural point of view as a circuit whose output is dependent on the inputs at that time instant, or can be defined from the constructional point of view as a circuit that does not contain any memory element.

A sequential circuit, on the other hand, can be defined from the behavioural point of view as a circuit whose output depends not only on the present inputs, but also on the past outputs, or can be defined from the constructional point of view as a circuit that contains at least one memory element. The block diagram of a sequential circuit is shown in Fig. 15.33.

Sequential circuits are classified into (i) asynchronous and (ii) synchronous circuits depending on the timing signals. A sequential circuit whose behaviour depends upon

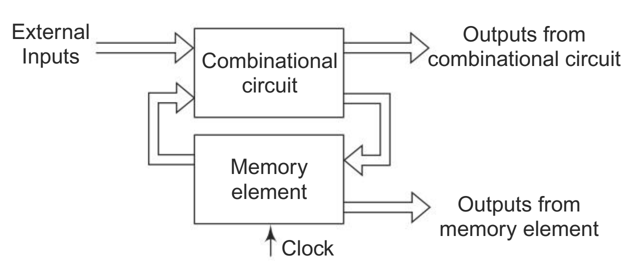


Fig. 15.33 Block diagram of sequential circuit

the sequence in which the input signals change is referred to as an asynchronous sequential circuit. The outputs will be affected whenever the inputs change.

A sequential circuit whose behaviour can be defined from the knowledge of its signals at discrete instants of time is referred to as a synchronous sequential circuit. In these circuits, the memory elements are affected only at discrete instants of time. The synchronisation is achieved by a timing signal known as system clock. The outputs are affected only with the application of a clock pulse.

15.10.1 Flip-Flops

A device that exhibits two stable states is extremely useful as a memory element in a binary system. Any electrical circuit that has this characteristic falls into the category of the devices commonly known as flip-flop (or bistable multivibrators).

RS flip-flops The most basic type of flip-flop is the reset/set flip-flop. This can be built using either two NOR gates or two NAND gates. Each flip-flop has two outputs, Q and Q'. When $Q = 1$ and $Q' = 0$, the flip-flop is said to be set. When $Q = 0$ and $Q' = 1$, the flip-flop is said to be reset. The truth-table for the RS flip-flop is given in Table 15.18.

Table 15.18 Truth table for RS flip-flop

R	S	Q
0	0	Last value
0	1	1 (set)
1	0	0 (reset)
1	1	Illegal

Consider the NOR gate flip-flop circuit shown in Fig. 15.34(a). When $R = 1$ and $S = 0$, the outputs $Q = 0$ and $Q' = 1$, therefore, the flip-flop is reset. When $R = 0$ and

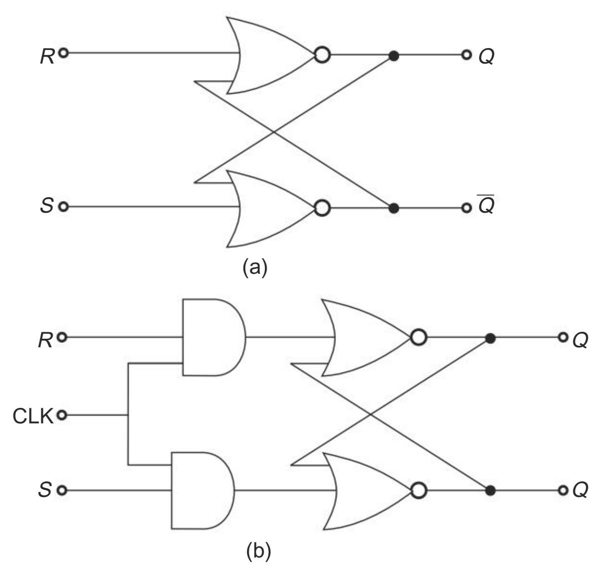


Fig. 15.34 (a) Realisation of RS flip-flop using NOR gates: (b) Clocked RS flip-flop

$S = 1$, the outputs $Q = 1$ and $Q' = 0$, therefore, the flip-flop is set. When the first condition is applied, i.e. $R = 0$ and $S = 0$, the flip-flop remains in its present state, i.e. Q remains unchanged. The condition $R = 1$ and $S = 1$ is forbidden as it violates the basic definition of a flip-flop taking both Q and Q' to the low state.

Clocked RS flip-flops Figure 15.34(b) shows the *RS* flip-flop that has a clock (CLK) input (Square wave). When the clock is low, the outputs will not change regardless of the conditions of the *R* and *S* inputs. When the clock input is high, the flip-flop will set if $R = 0$ and $S = 1$. When the clock input is high and $R = 1$ and $S = 0$, the flip-flop will reset. The truth-table of the clocked *RS* flip-flop is shown below in Table 15.19.

Table 15.19 Truth table for clocked RS flip-flop

CLK	*R*	*S*	*Q*
0	0	0	No change
0	0	1	No change
0	1	0	No change
0	1	1	No change
1	0	0	No change
1	0	1	1
1	1	0	0
1	1	1	Illegal

D flip-flops Figure 15.35 shows the logic symbol and the realisation of an edge triggered *D* flip-flop using NAND gates. The presence of a small triangle on the clock input indicates that the flip-flop is edge triggered.

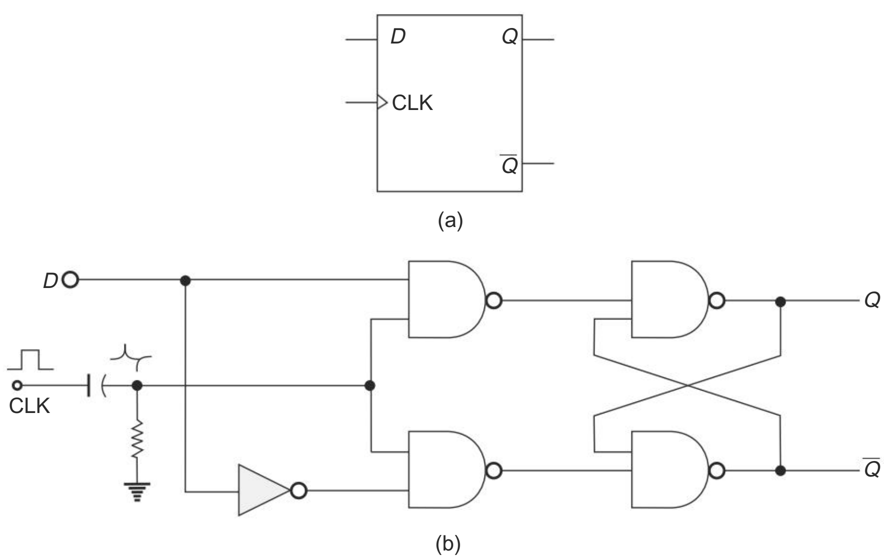


Fig. 15.35 (a) Logical symbol (b) NAND gates realisation of an edge-triggered D flip-flop

The x in the truth-table (Table 15.20) are called "don't cares" because if the clock is low, high or on its negative edge, the flip-flop is inactive. The outputs Q and Q' change only on the positive going edge of the incoming clock pulse. If $D = 0$ when the positive going clock appears, then $Q = 0$ and $Q' = 1$. If $D = 1$ when the positive going clock edge appears, then $Q = 1$ and $Q' = 0$. The data input and output are the same after the positive going pulse, i.e. the input data D is stored only on the positive going edge of the incoming clock pulse.

Table 15.20 Truth table for D flip-flop

Clock	*D*	*Q*
x	0	No change
x	1	No change
↑	0	0
↑	1	1

JK flip-flop Figure 15.36 shows the negative edge triggered JK flip-flop. The flip-flop is inactive when the clock is low, high, or on its positive going edge.

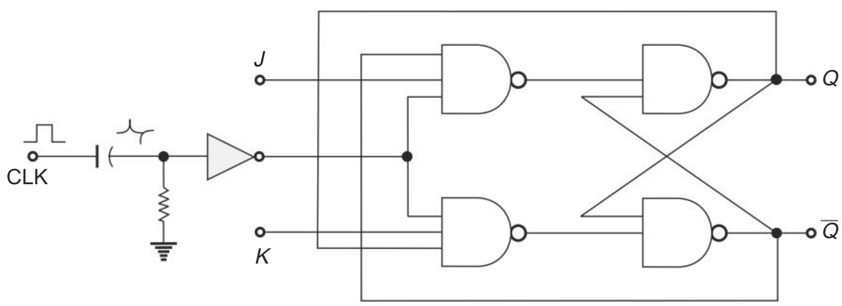


Fig. 15.36 Negative edge triggered JK flip-flop

When $J = 0$ and $K = 0$, the circuit is inactive. When $J = 0$ and $K = 1$, the negative going edge of the clock pulse puts the outputs at $Q = 0$ and $Q' = 1$. When $J = 1$ and $K = 0$, the negative going edge of the clock pulse puts the Q outputs at $Q = 1$ and $Q' = 0$. When $J = 1$ and $K = 1$, the outputs Q and Q' toggles or alternate with each negative going clock edge. (See Table 15.21).

Table 15.21 Truth table for JK flip-flop

Clock	*J*	*K*	*Q*
x	0	0	No change
x	0	1	No change
x	1	0	No change
x	1	1	No change
↓	0	0	No change
↓	0	1	0
↓	1	0	1
↓	1	1	$\overline{Q}$

T flip-flop This changes state with each clock pulse and hence, it acts as a toggle switch. If $J = K = 1$, then output is the complement of the previous state, so that the JK flip-flop is converted into a T flip-flop. The realisation of a T flip-flop from a JK flip-flop is shown in Fig. 15.37.

The truth-table for a positive edge-triggered T flip-flop is shown in Table 15.22.

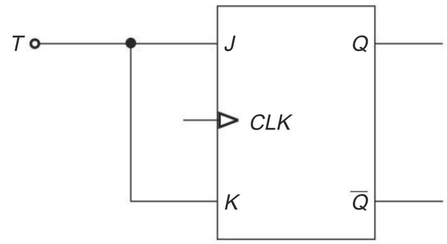


Fig. 15.37 Realisation of a T flip-flop from a JK flip-flop

Table 15.22 Truth table for positive edge triggered T flip-flop

Clock	*T*	Q_N
x	0	No change
x	1	No change
↑	0	No change
↑	1	$\overline{Q}_{N-1}$

15.10.2 Shift Registers

A register is a group of flip-flops that can be used to store a binary number. Registers find a variety of applications in digital systems including microprocessors. If the output of each flip-flop is connected to the input of the adjacent flip-flop, then the circuit is called a shift register. The name comes from the fact that each successive clock pulse moves or shifts the data bits one flip-flop to the left or to the right, depending on how the flip-flops are connected.

Registers are classified depending upon the way in which data is entered and retrieved. They are:

(1) Serial-in serial-out (SISO)
(2) Serial-in parallel-out (SIPO)
(3) Parallel-in serial-out (PISO)
(4) Parallel-in parallel-out (PIPO)

Serial-in serial-out Input data is applied one bit at a time to the first flip-flop in a chain and read out from the last flip-flop in a chain one bit at a time. Figure 15.38 shows four *D* flip-flops connected in serial-in serial-out fashion. The output *Q* of one flip-flop is connected to the *D* input of the next flip-flop. *CLK*, $\overline{PRE}$, and $\overline{CLR}$ signals are connected in parallel to all four flip-flops, so that they are all clocked, all set, or all reset at the same time. This shift register is known as the shift right register because when the clock pulse is given the data are shifted to the right.

When an active low signal, as shown in Fig. 15.38(a), is given to the $\overline{CLR}$ input, all the *Q* outputs are 0's. Next, assume that the data signal '1' is given as the input to the flip-flop *A*. When the next positive edge of the clock pulse occurs, the 1 on the *D* inputs of the *A* flip-flop will be transferred to the Q_A output. On the next rising edge of the clock signal, the 1 at Q_A is transferred to Q_B output. The *D* input for the

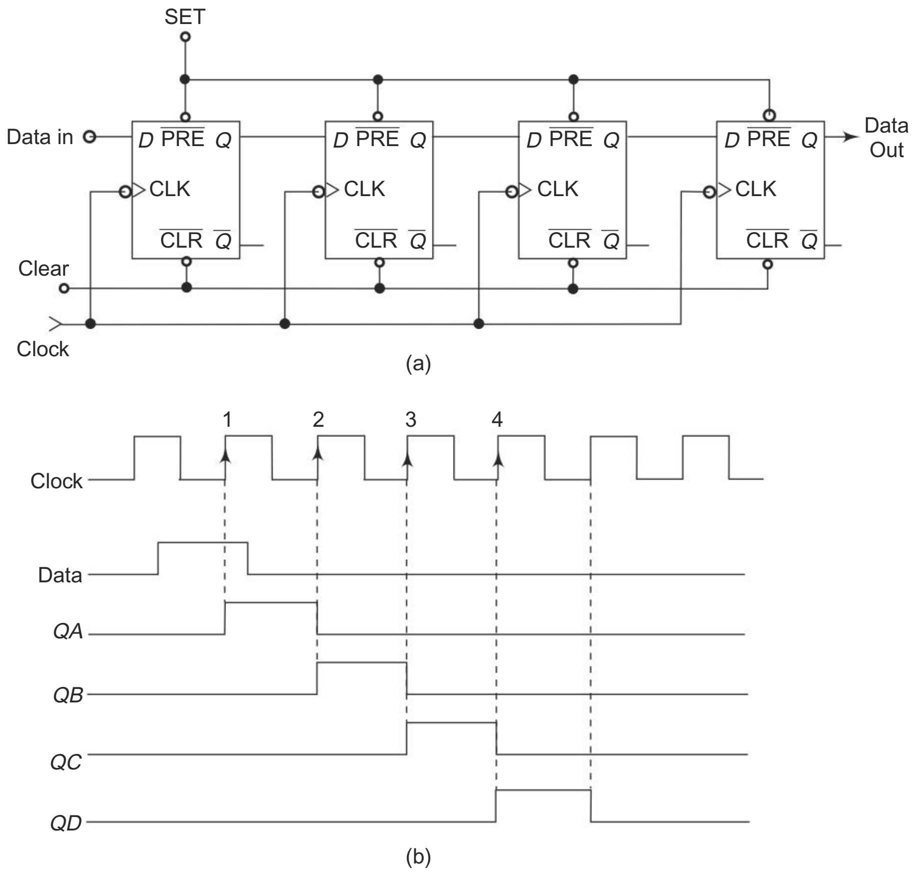


Fig. 15.38 (a) D flip-flops connected as a shift register, (b) Waveforms for shift register

flip-flop A is '0' and hence, the output Q_A is '0'. On the 3rd positive edge of the clock pulse Q_C becomes high and for the 4th positive edge of the clock pulse Q_D becomes high. The corresponding waveforms are shown in Fig. 15.38(b).

Serial-in parallel-out Input data is applied one bit at a time to the D input of the first flip-flop in a chain and read out from the Q outputs in parallel after a data word is all shifted in. A serial in—parallel out shift register is shown in Fig. 15.39.

Parallel-in serial-out In a parallel-in serial-out shift registers, the bits are entered simultaneously into their respective stages on parallel lines rather than on a bit by bit basis on one line as with serial data inputs. Figure 15.40 shows the PISO shift register. There are four input data lines, A, B, C and D, and a SHIFT/$\overline{LOAD}$ input that allows four bits for data to be entered into the shift register in a parallel fashion. When SHIFT/$\overline{LOAD}$ is low, gates $G1$, $G2$ and $G3$ are enabled, allowing each data bit to be applied to the D input of the respective flip-flops. When a clock pulse is applied, the flip-flops with $D = 1$ will SET and those with $D = 0$ will RESET, thereby storing all 4 bits simultaneously. When SHIFT/$\overline{LOAD}$ is high, gates $G4$, $G5$

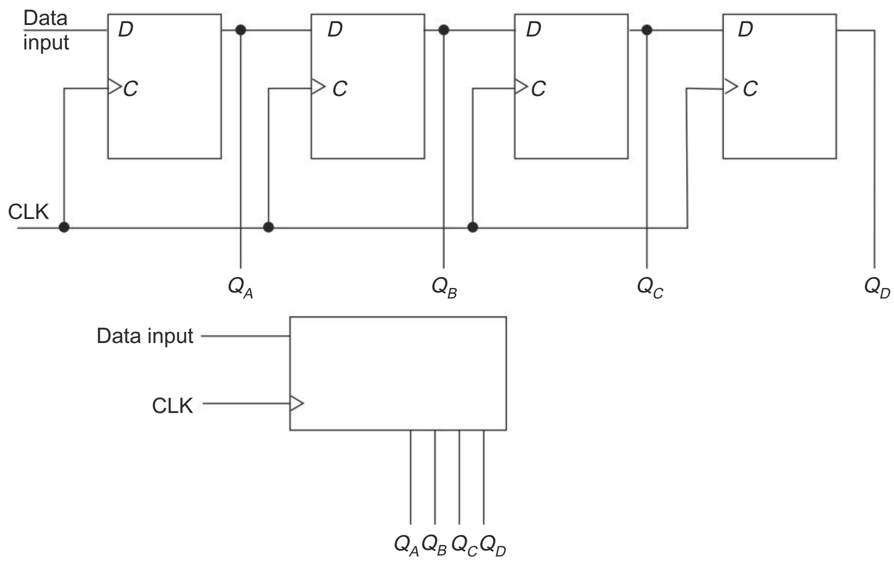
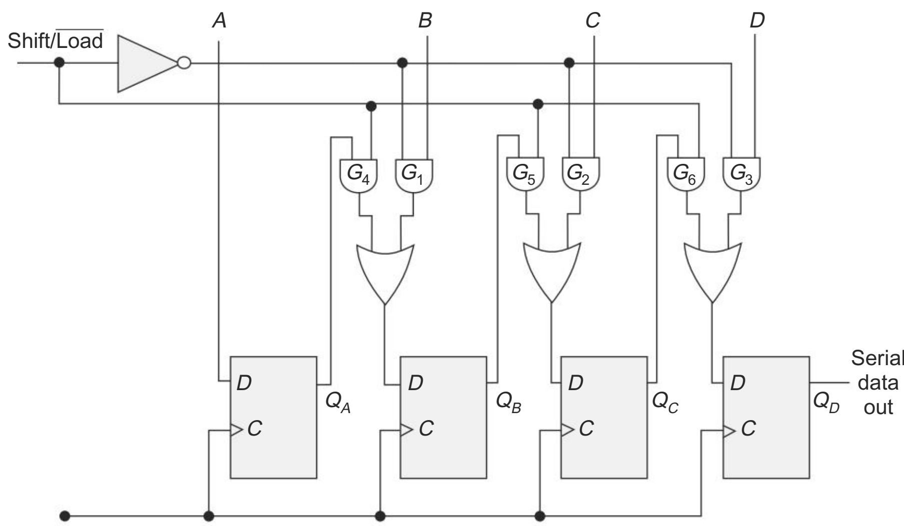
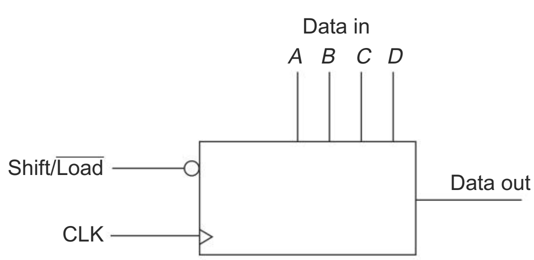


Fig. 15.39 A serial-in parallel-out shift register



(a) Logic diagram



(b) Logic symbol

Fig. 15.40 A four-bit parallel-in serial-out shift register and $G6$ are enabled, allowing the data bits to shift right from one stage to the next. The OR gates allow either the normal shifting operation or the parallel data entry operation, depending on which AND gates are enabled by the SHIFT/$\overline{LOAD}$ control signal.

Parallel-in parallel-out Data is entered into the PIPO shift register as in the previous PISO case. The difference is, the outputs are taken only from the Qs of all the flip-flops simultaneously.

15.10.3 Counters

A counter is one of the most important subsystems in a digital system. A counter circuit activated by a clock can be used to count the number of clock cycles. There are two types of counters:

(1) Synchronous counters
(2) Asynchronous counters.

The ripple counter is simple and requires less hardware. However, it has speed limitation. Each flip-flop is triggered by the previous flip-flop, and thus the counter has a cumulative settling time. These counters are called Serial or Asynchronous.

The speed of operation can be increased by using a parallel or synchronous counter but the hardware cost increases because of the extra circuitry introduced. In this type, every flip-flop is triggered by the clock in synchronism and the settling time is equal to the delay time of a single flip-flop.

15.10.4 Asynchronous Counters

Binary ripple counter Figure 15.41 shows the connection of JK flip-flops to act as a binary counter. For each flip-flop, the J and K inputs are connected to +5V. This means that each flip-flop will toggle when its clock input receives a negative going clock pulse. The MSB of the counter is Q_3 and the LSB of the counter is Q_0. Initially, the $\overline{CLR}$ input of all the flip-flops are tied to ground and hence all the Q outputs will be '0'. When the flip-flops are counting, the $\overline{CLR}$ input is tied to +5V, its inactive state. Figure 15.41 shows how the Q outputs of each flip-flop respond to each negative going clock edge. At each negative going clock edge, the count increases by 1.

Initially the count is 0000. For the first negative going edge of the clock input, the LSB flip-flop sets, increasing the count to 0001. The Q_0 output is connected to the clock input of the next most significant flip-flop. This high clock pulse does not cause the Q_1 output to change. However, the second negative going clock edge applied to the LSB flip-flop causes the Q_0 output to toggle from 1 to 0. This negative going clock edge causes the Q_1 output to go from 0 to 1, changing the count to 0010. The count continues until 1111 is reached. Then on the next negative going clock edge, all flip-flop outputs toggle back to 0 for a count of 0000.

Each flip-flop divides the incoming clock pulse frequency by a factor of 2. The frequency of the Q_0 output is one-half that of the clock input. Similarly, Q_1 output is one-fourth the frequency of the clock input, Q_2 output is one-eighth the frequency of the clock input and the Q_3 output is one-sixteenth the frequency of the clock input.

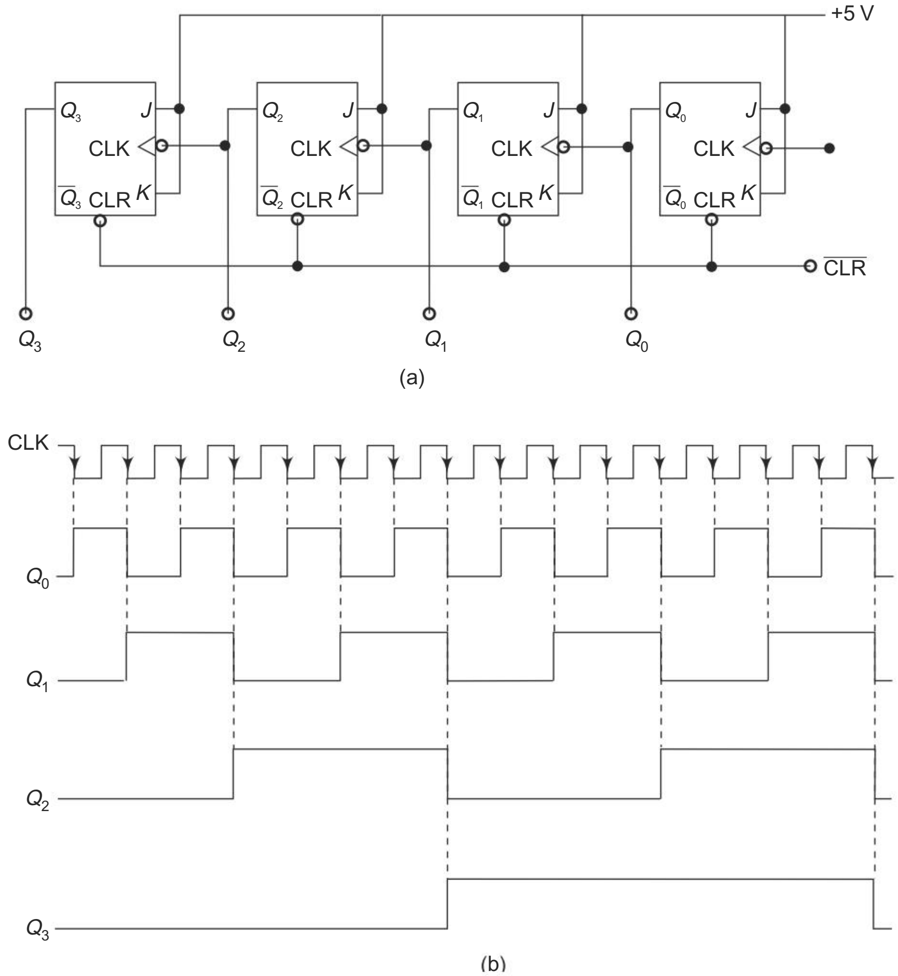
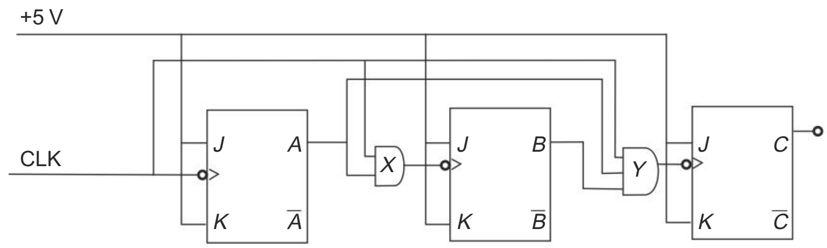


Fig. 15.41 (a) 4-bit ripple counter (b) Output waveforms at each flip-flop

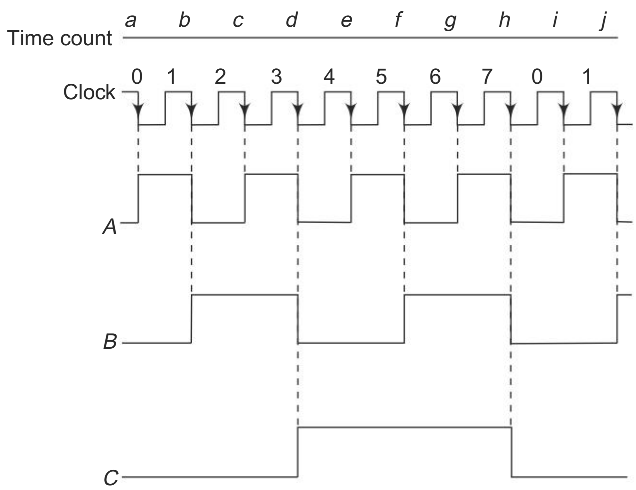
The counter is called a ripple counter because the output of one flip-flop is fed to the clock input of another.

15.10.5 Synchronous Counters

The delay time and hence, the settling time increase in the ripple counters are overcome in the synchronous counters. Figure 15.42 shows the circuit of a parallel (synchronous) binary counter. The J and K inputs of each flip-flop is tied to +5V, such that the flip-flop toggles for negative clock transition at its clock input. The AND gates are used to gate every second clock to flip-flop B, every fourth clock to flip-flop C, and so on. The clock is directly applied to flip-flop A. Since J and K inputs are tied to +5V, flip-flop A will change state for each negative clock transition. When *A* is high, AND gate X is enabled and the clock pulse is passed through the gate to the clock input of flip-flop *B*. Similarly, the AND gate Y is enabled and the clock



(a)



(b)

C	B	A	Count
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7
0	0	0	0

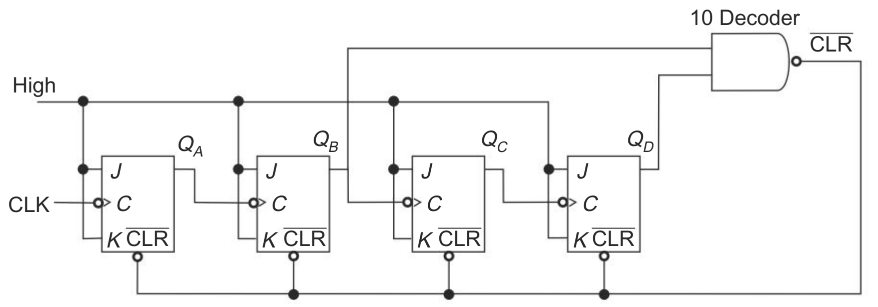
(c)

Fig. 15.42 (a) Mod-8 parallel binary counter (b) Waveform (c) Truth table

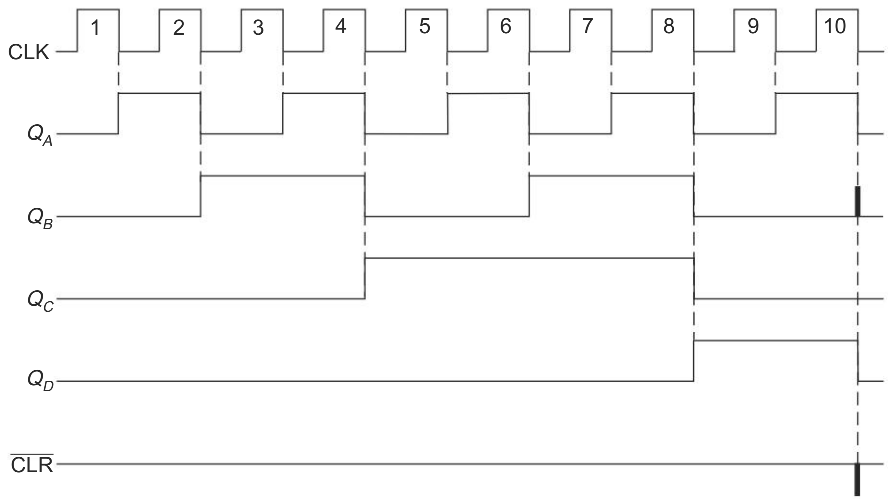
pulse is passed through the gate to the clock input of the flip-flop *C* when *A* and *B* are high. The counter counts from 000 to 111 in a synchronous manner.

15.10.6 Decade Counters

A decade counter requires four flip-flops. This is achieved by recycling after the count of 9 (1001) is reached. To decode the count 1010, a NAND gate is used and the



(a)



(b)

Fig. 15.43 (a) An asynchronously clocked decade counter with asynchronous recycling (b) waveform

output of the gate is connected to the clear ($\overline{CLR}$) inputs of the flip-flop as shown in the Fig. 15.43(a).

Only Q_B and Q_D are connected to the NAND gate inputs. This is an example of partial decoding, in which the two unique state ($Q_B = 1$ and $Q_D = 1$) are sufficient to decode the count 10_{10} because none of the other states have both Q_B and Q_D high at the same time. When the counter goes into count 1010, the decoding gate output goes low and asynchronously RESETS all the flip-flops.

15.11 A/D AND D/A CONVERTER CIRCUITS

Most physical quantities such as pressure, temperature, and flow are analog in nature. There are usually several steps in producing electrical signals which represent the values of these variables and in converting the electrical signals to a digital form that can be used for example, to drive an LED display or be stored in the memory of a microcomputer.

The first step involves a sensor which produces a current or voltage signal that is proportional to the amount of the physical pressure, temperature, or other variable. The signals from most sensors are quite small, so they must be amplified and perhaps filtered to remove unwanted noise. Amplification is usually done with some type of operational amplifier circuit. The final step is to convert the signal to a proportional binary word with an analog-to-digital (A/D) converter.

15.11.1 Digital-to-Analog Converters

Many systems accept a digital word as an input signal and translate or convert it to an analog voltage or current. These systems are called digital-to-analog converters. The digital word is presented in a variety of codes, the most common being pure binary or binary-coded-decimal.

A binary-weighted resistor D/A converter Figure 15.44 shows a circuit which will produce an output voltage proportional to the binary word applied to its inputs by the four switches. Since this converter has four data inputs, it is called a 4-bit converter. The circuit in Fig. 15.44 is just a 4-input op amp adder circuit. The noninverting input of the op amp is tied to ground, so that input will be at 0 V. There is feedback from the output of the op amp to the inverting input, so the op amp will work continually to hold this input at 0 V.

If none of the data switches are closed, there will be no current through any of the input resistors and no current through R_F. The output of the op amp, then, will be at the same voltage as the inverting input, 0 V.

Now, suppose the D_0 data switch is closed. This will apply a voltage of +5 V to one end of R_1. The other end of R_1 will be held at 0 V by the op amp, so R_1 has a voltage of 5 V across it and the current through R_1 is 0.05 mA. In order to hold the inverting input at 0 V, the op amp pulls this current through R_F. The voltage across R_F produced by this current will be 0.05 × 10 kΩ = 0.5 V. Since one end of R_F is at

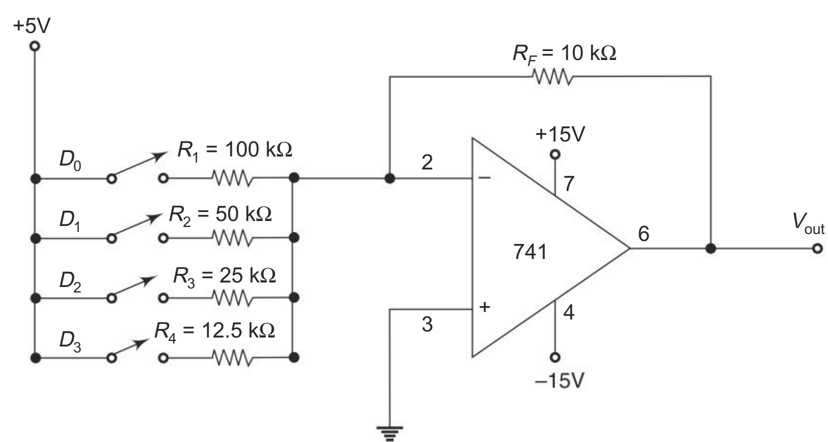


Fig. 15.44 Binary-weighted resistor D/A converter

0 V, the output of the op amp must be at −0.5 V in order to keep the current flowing from +5 V through R_1 to 0 V and on through R_F. Thus, closing D_0 produces a voltage of −0.5 V on the output of the converter circuit.

Suppose the D_1 data switch alone is closed. Since R_2 is only half the value of R_1, twice as much current, or 0.1 mA, will flow through R_2 to the summing point and on through R_F into the output of the op amp. In order to pull a current of 0.1 mA through R_F, the op amp will assert a voltage of −1 V on its output. The D_1 data switch then produces an output voltage of −1 V, or twice as much as that produced by the D_0 switch.

If D_2 alone is closed, a current of 0.2 mA, will flow through R_3 to the summing point an on through R_F into the output of the op amp. This current will produce a voltage of −2 V on the output of the op amp. Likewise, if switch D_3 is closed, a current of 0.4 mA, will flow into the summing point and on through R_F into the output of the op amp. This current will produce a voltage of −4 V on the output of the op amp.

Now, suppose that switches D_2 and D_3 are both closed. The 0.2 mA current through R_3 will combine with the 0.4 mA current through R_4 at the summing point to produce a total current of 0.6 mA. The output voltage is proportional to the sum of the currents produced by the closed switches and is −6 V. The value of the four currents are related to each other in the same way that the weights of binary digits are. Therefore, the output voltage will be proportional to the binary word applied to the data inputs. D_0 represents the least significant bit (LSB) because it produces the smallest current, and D_3 represents the most significant bit (MSB) because it produces the largest current.

Since there are four inputs, there are 16 possible input words, 0000 to 1111. These words produce output voltages ranging from 0 V for an input word of 0000 to −7.5 for an input word of 1111.

Ladder type D/A converter Figure 15.45 shows D/A converter using R–2R ladder network. The ladder used in this circuit is a current-splitting device, and thus the ratio of the resistors is more critical than their absolute value. It can be observed from the figure that at any of the ladder nodes the resistance is 2R looking to the left or the right or towards the switch. Hence, the current will split equally toward the left and right, and this happens at every node. Considering node $N-1$ and assuming the MSB turned ON, the voltage at that node will be $-V_R/3$. Since the gain of the operational amplifier to node N–1 is $-3R/2R$, the weight of the MSB becomes

$$V_o = -\left(\frac{V_R}{3}\right)\left(\frac{-3R}{2R}\right) = \frac{V_R}{2}$$

Similarly, when the second most significant bit is ON and all others are OFF, the output will $V_o = +\dfrac{V_R}{4}$, the third bit gives $+\dfrac{V_R}{8}$, and the LSB gives $+\dfrac{V_R}{2^N}$. The circuit uses a negative reference voltage and gives a positive analog output voltage. If negative binary numbers are to be converted, the sign-magnitude approach is used; an extra bit is added to the binary word to represent the sign, and this bit can be used to select the polarity of the reference voltage.

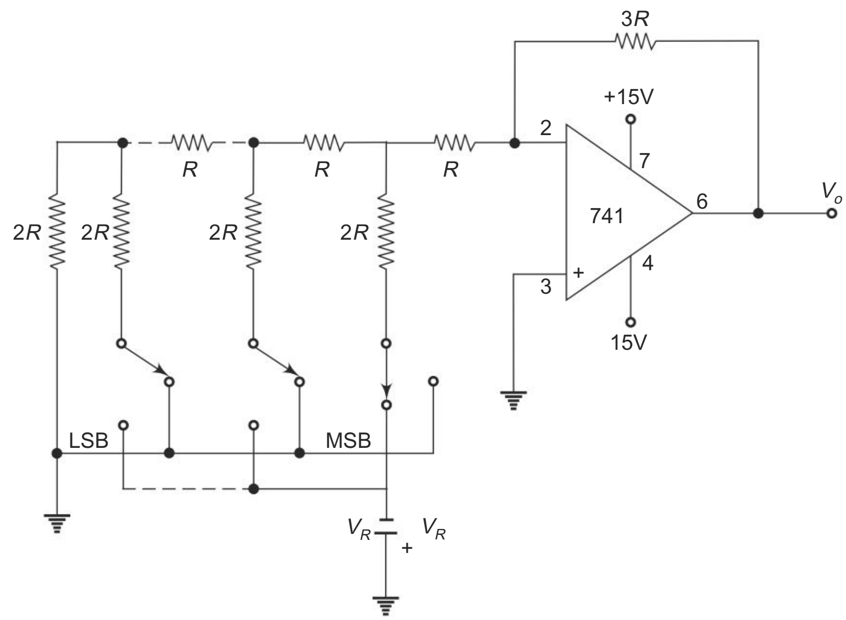


Fig. 15.45 D/A converter using *R*–2*R* ladder network

15.11.2 Analog-to-Digital Converters

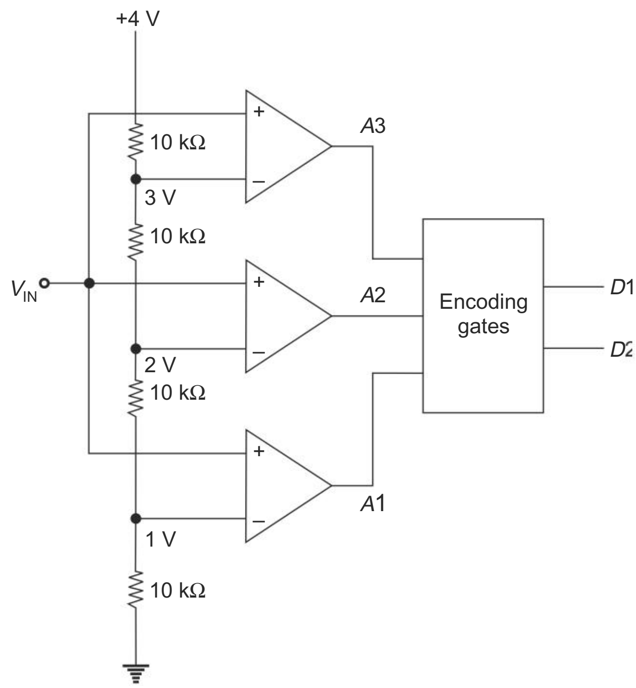
It is often required that data taken in a physical system be converted into digital form. Such data would normally appear in electrical analog form. For example, a temperature difference would be represented by the output of a thermocouple, the strain of a mechanical member would be represented by the electrical unbalance of a strain-gauge bridge, etc. The need therefore arises for a device that converts analog information into digital form. Two major characteristics of an A/D converter are *resolution and conversion time*.

The resolution of an A/D converter is determined by the number of bits in the output word. An 8-bit A/D converter, for example, will represent the value of an input voltage with one of 256 possible words. Another way of putting this is to say that it will resolve an input voltage to the nearest one of 256 levels.

The conversion time for an A/D converter is simply the time it takes the converter to produce a valid output word after it is given a 'start conversion' signal. When we refer to an A/D converter as 'high speed' or 'fast', we mean that it has a short conversion time.

There are many different ways to do an analog-to-digital conversion. The method chosen depends on the resolution, speed, and type of interfacing needed for a given application.

Parallel comparator or flash converter The simplest in concept and the fastest type of A/D converter is the parallel converter, or flash type shown in Fig. 15.46(a). The resistor voltage divider sets a sequence of voltages on the reference inputs of the comparators as shown. If the voltage on the '+' input of a comparator is



(a)

V_{IN} Volts	Comparator outputs A3	A2	A1	Binary outputs D1	D0
0 to 1	0	0	0	0	0
1.001 to 2	0	0	1	0	1
2.001 to 3	0	1	1	1	0
3.001 to 4	1	1	1	1	1

(b)

Fig. 15.46 (a) Circuit for flash type A/D converter, (b) Comparator output codes

greater than the reference voltage on its '–' input, the output of the comparator will be high. In the circuit, the voltage to be converted is applied to the + inputs of all the comparators in parallel, so the number of comparators that have high on their outputs indicates the amplitude of the input voltage. An input voltage of between 0 and 1 V, for example, is less than the reference voltage on any of the comparators, so none of the comparator outputs will be high. If the input voltage is between 1.001 and 2 V, only the A_1 output will be high. For an input voltage between 2.001 and 3 V, both the A_1 and A_2 outputs will be high. Finally, for an input voltage greater than 3V, all the comparator outputs will be high. Figure 15.46(b) summarizes the comparator output code that will be produced by input voltages between 0 and 4 V. The code produced on the outputs of the comparators is not binary, but with a simple gate circuit it can be converted to the binary equivalents shown in the rightmost column of Fig. 15.46(b).

To increase the resolution, more comparators can be added. Seven comparators are required for 3-bit resolution and 15 comparators are required for 4-bit resolution. An N-bit converter requires $2^N - 1$ comparators. The number of comparators needed increases rapidly as the desired number of bits increases. The main advantage of a parallel comparator type A/D converter is its speed. The binary word is present on the outputs after just the propagation delay time of the comparators plus the delay time of the encoding gates. This is why a parallel comparator type A/D converter is often called a flash converter.

Counter type A/D converter Figure 15.47 shows the block diagram of a counter type A/D converter. In this system, a continuous sequence of equally spaced pulses is passed through a gate. At the start of a conversion cycle, the counters are reset to 0, so the output of the D/A is at 0 V. A positive unknown voltage applied to the input of the converter will cause the output of the comparator to go high and enable the AND gate. This will let the clock pulses into the counter. Each clock pulse increments the counter by 1 and increases the voltage on the output of the D/A converter by one step. When the voltage on the output of the D/A converter passes the voltage on the unknown V input, the output of the comparator will go low and shut off the clock pulses to the counters. The count accumulated on the counters is proportional to the input voltage. The control circuitry then strobes the latches to transfer the count to the output and resets the counters to start another conversion cycle.

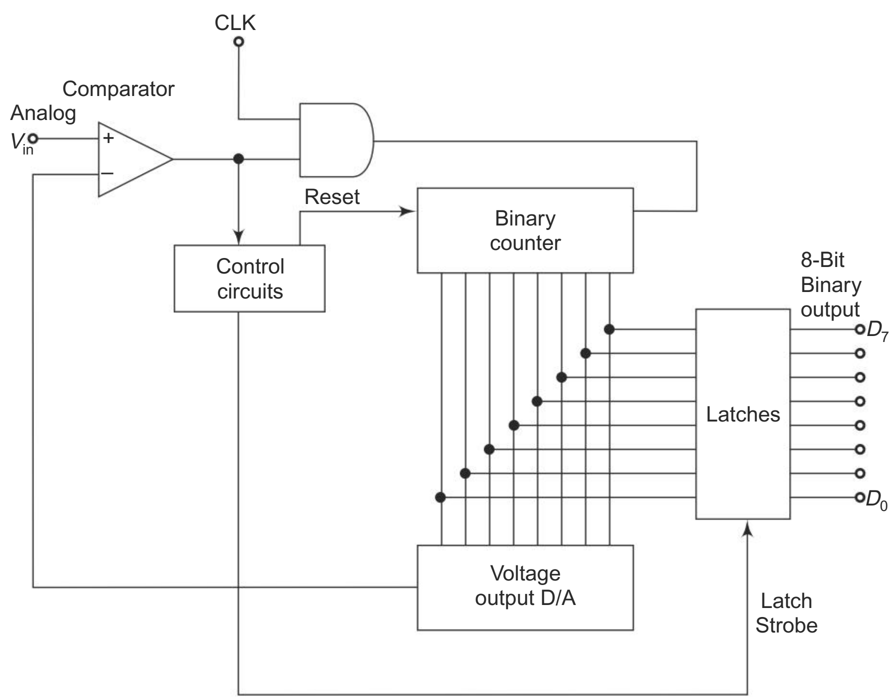


Fig. 15.47 Counter type A/D converter

The counter method is slower than flash type converter. The drawback of this type is that it requires a precision D/A converter. Another drawback of this type is that the counter has to start at 0 and count up until the D/A output passes the input voltage. For an 8-bit converter, then, conversion, may take as long as 255 clock cycles and for a 10-bit converter, a conversion may take as many as 1024 clock pulses.

Successive approximation A/D converters The successive approximation technique is another method to implement an A/D converter. Instead of a binary counter as shown in Fig. 15.47, a programmer is used. The programmer sets the most significant bit (MSB) to 1, with all other bits to 0, and the comparator compares the D/A output with the analog signal. If the D/A output is larger, the 1 is removed from the MSB, and it is tried in the next most significant bit. If the analog input is larger, the 1 remains in that bit. Thus a 1 is tried in each bit of the D/A decoder until, at the end of the process, the binary equivalent of the analog signal is obtained.

The successive approximation type A/D converter has the disadvantage that it requires a D/A converter, but it has the advantages of good resolution and relatively high speed.

15.12 LOGIC FAMILIES

Digital integrated circuits are classified as small-scale integration (SSI) with less than 12 gates on the same chip, medium-scale integration (MSI) from 12 to 100 gates per chip, large-scale integration (LSI) with more than 100 gates per chip, and very large-scale integration (VLSI) with more than 1000 gates per chip.

ICs can be manufactured using two basic techniques, namely, bipolar and metal-oxide semiconductor (MOS) technologies. Bipolar technology is preferred for SSI and MSI because it is faster. MOS technology dominates the LSI field because of increased density of MOSFETs in the same chip area. A digital family is a group of compatible devices with the same logic levels and supply voltages.

The major categories of the bipolar family are Diode−transistor logic (DTL), Transistor−transistor logic (TTL) and Emitter-coupled logic (ECL). The MOS category consists of PMOS−p channel MOSFET, NMOS−n channel MOSFET and CMOS−Complementary MOSFET families.

15.12.1 Diode–Transistor Logic (DTL)

DTL uses diodes and transistors. A DTL NAND gate may be implemented as shown in Fig. 15.48.

The operation of this positive NAND gate can be understood easily. If atleast one input is low, diode D connected to this input conducts and the voltage V_A at point A is low. Therefore, diodes D_1 and D_2 do not conduct, $I_B = 0$ and the transistor is off. This makes the output of transistor Q high and Y is in logic 1 state. When all the inputs are high (1) so that all input diodes D are cutoff, then V_A rise towards V_{CC}, and a base current I_B results. When I_B is sufficiently large, Q is driven into saturation and the output Y falls to its low (0) state. DTL has a fan-out of about 12 and can be further increased by replacing D_1 by a transistor. DTL gates are obsolete and are seldom used these days.

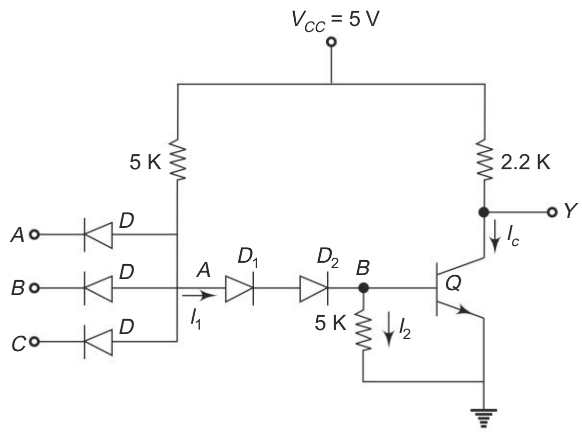


Fig. 15.48 An integrated positive DTL NAND gate

15.12.2 Transistor–Transistor Logic (TTL)

The fastest saturating logic circuit is the transistor–transistor logic shown in Fig. 15.49. TTL is fast, inexpensive, and easy to use. This switch uses a multiple-emitter transistor which can be easily and economically fabricated. The TTL circuit has the topology of the DTL circuit with the emitter junctions of Q_1 acting as the input diodes D of the DTL gate and the collector junction of Q_1 replacing diode D_1. The base-to-emitter diode of Q_2 is used in place of diode D_2 of the DTL gate, and both circuits have an output transistor.

Transistor Q_1 and 4-kΩ resistor act like a 2-input AND gate. The rest of the circuit inverts the signal so that the overall circuit acts like a 2-input NAND gate. The output transistors (Q_3 and Q_4) form a totem-pole connection; this kind of output stage is typical of most TTL devices. With a totem-pole output stage, either the upper or lower transistor is on. When Q_3 is on, the output is high and when Q_4 is on, the output is low.

The input voltages A and B are either low (ground) or high (+5 V). If A or B is low, the base of Q_1 is pulled down to approximately 0.7 V. This reduces the base voltage of Q_2 to almost zero. Therefore, Q_2 is cut off. With Q_2 open, Q_4 goes into cutoff, and the base of Q_3 is pulled high. Since Q_3 acts as an emitter follower, the Y output is pulled up to a high voltage.

On the other hand, when A and B are both high voltages, the emitter diodes of Q_1 stop conducting, and the collector diode goes into forward conduction. This forces Q_2 base to go high. In turn, Q_4 goes into saturation, producing a low output.

Without diode D_1 in the circuit, Q_3 will conduct slightly when the output is low. To prevent this, the diode is inserted; its voltage drop keeps the base–emitter diode of Q_3 reverse-biased. In this way, only Q_4 conducts when the output is low. Totem-pole transistors are used because they produce a low output impedance. Either Q_3 acts as an emitter follower (high output), or Q_4 is saturated (low output). The output voltage can change quickly from one state to the other because any stray output capacitance is rapidly charged or discharged through the low output impedance.

15.12.3 TTL Sub-families

High-speed TTL The circuit shown in Fig. 15.49 is called standard TTL. The internal time constants of the circuit can be lowered by decreasing the resistances. This decreases the propagation delay time; however, the small resistances increase the power dissipation. This design variation is known as high-speed TTL. A high speed TTL gate has a power dissipation of around 22 mW and a propagation delay time of approximately 6 ns, whereas a standard TTL gate has a power dissipation of about 10 mW and propagation delay time of approximately 10 ns.

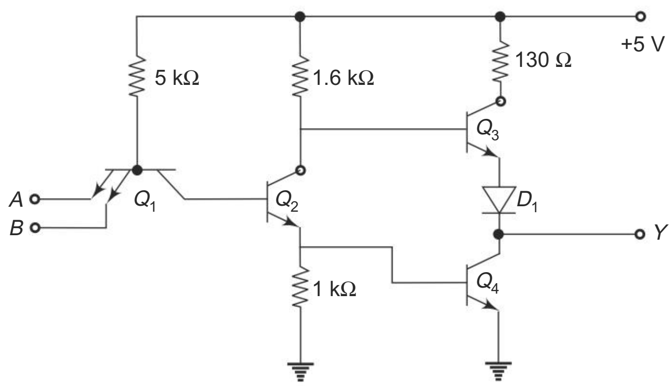


Fig. 15.49 Two-input TTL NAND gate

Low-power TTL By increasing the internal resistances, the power dissipation of TTL gates can be reduced. Devices of this type are called low-power TTL. These devices are slower than standard TTL because of the larger internal time constants. A low-power TTL gate has a power dissipation of 1 mW and a propagation delay time of about 35 ns.

Schottky TTL When a transistor is switched from saturation to cutoff, one has to wait for the extra carriers to flow out of the base. The delay is known as Saturation Delay time. This delay can be reduced by using Schottky TTL. A Schottky diode is fabricated along with each bipolar transistor of a TTL circuit, as shown in Fig. 15.50.

Because the Schottky diode has a forward voltage of only 0.25 to 0.4 V, it prevents the transistor from saturating fully. This eliminates saturation delay time and increases switching speed. Schottky TTL devices are very fast, capable of operating reliably at 100 MHz. The power dissipation is around 20 mW per gate and the propagation delay time is approximately 3 ns.

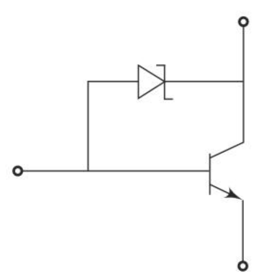


Fig. 15.50 Schottky diode prevents transistor saturation

Low-power schottky TTL By increasing internal resistances as well as using Schottky diodes, a compromise has been made between low power and high speed, which is referred to as low-power Schottky TTL. A low-power Schottky gate has a power dissipation of around 2 mW and a propagation delay time of approximately 10 ns.

15.12.4 Emitter-Coupled Logic (ECL)

Emitter-coupled logic has several other common names—current-mode logic (CML), current-steering logic, and non-saturating logic. The last term is the key to this type of circuit. The propagation delay time can be eliminated by operating transistors only in either the active or the cut-off regions rather than operating them in saturation or cut-off regions. The ECL devices are the fastest currently available. ECL has not proved as popular as TTL, primarily because it is more expensive. Superfast computers use ECL as do a number of the higher speed special-purpose computers.

The basic ECL configuration can be best understood by examining a particular inverter. Figure 15.51 shows an ECL inverter. The logic levels in this system are as follows: Binary 0 is represented by −1.55 V and binary 1 by −0.75 V. This is 'positive logic' since the more positive level, −0.75 V, is the binary 1.

The circuit's operation is based on a differential amplifier consisting of Q_3 and Q_4. When the input to Q_3 is at −1.55 V, Q_3 will be off and current will flow through R_3 and R_2. There will be a drop of about 0.8 V across R_2. So, figuring a base-emitter drop of 0.75 V for Q_1, the X output will be at −1.55 V.

Since Q_2 is cut-off by the −1.55 V input, very little current will flow through R_1, and the output X will be at the base-emitter drop voltage across Q_2. So the output will be at −0.75 V. When the input is at −0.75 V, transistor Q_3 will be on, Q_4 will be off, the X output will be at −0.75 V, and the X output will be at −1.55 V. The transistors are never saturated; they are either in their active region or off.

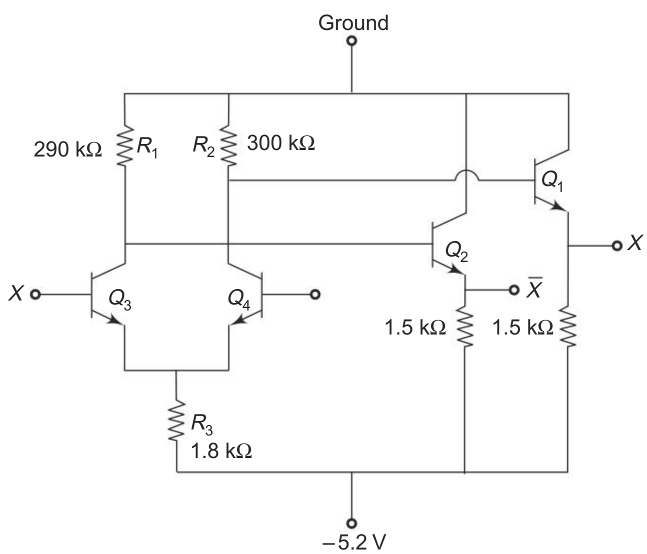
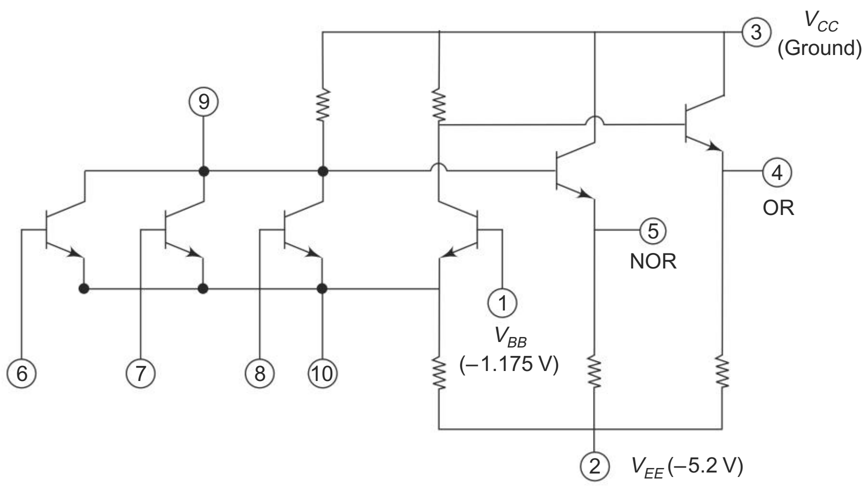


Fig. 15.51 Basic circuit of ECL inverter gate



Inputs			Outputs	
8	7	6	5	4
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

Fig. 15.52 Three-input ECL gate and its truth-table

A three-input ECL gate is shown in Fig. 15.52. This is a combined NOR and OR gate, depending on which output connection is used. Several generations of ECL circuits have evolved. In general, the circuits have become faster and require more power with each generation.

15.12.5 Integrated Injection Logic (IIL) Circuits

Integrated injection logic circuits represent an attempt to attain packing densities comparable to MOS circuits while using bipolar junction transistor technology. Standard bipolar technology is the fastest, however, it occupies larger space when compared to MOS circuits. This is due to the reasons that bipolar circuits require resistors, the transistors are larger and an isolation diffusion has to be provided. The problem can be overcome by the technique called **merging**, where the same transistor region is used as part of two or more devices. IIL is the most popular and commonly used merged technology.

The basic IIL gate and a possible semiconductor layout is shown in Fig. 15.53. Each gate has an injector transistor to feed current into the base. This logic circuit has single input and multiple outputs. As no standard symbol exists, the logic gate is represented by a rectangular box, with arrow to represent input and outputs.

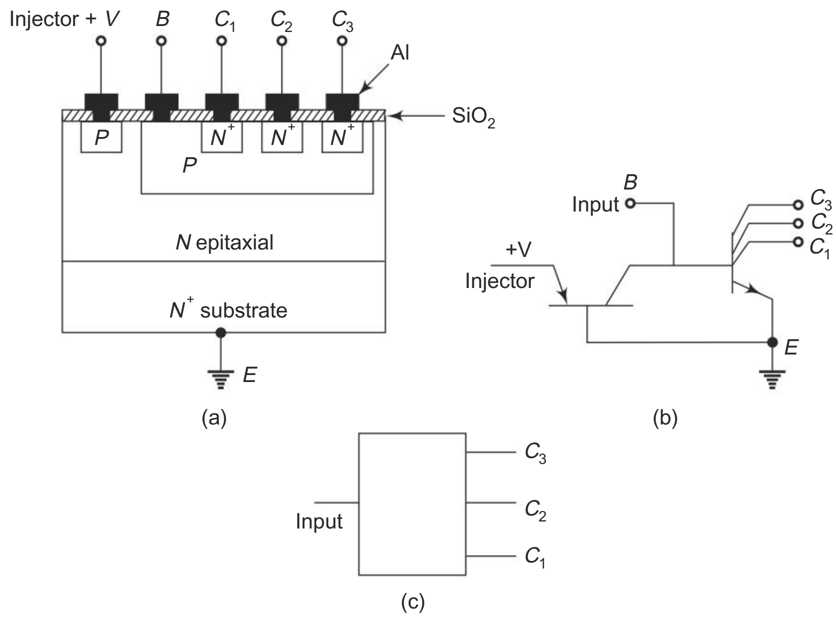


Fig. 15.53 (a) Structure of an IIL gate, (b) Layout and (c) Logic symbol

The configuration shown in Fig. 15.53 is actually an INVERTER with multiple outputs. Various other gates can be formed from this configuration. In Fig. 15.54, realizations for different logic gates are shown.

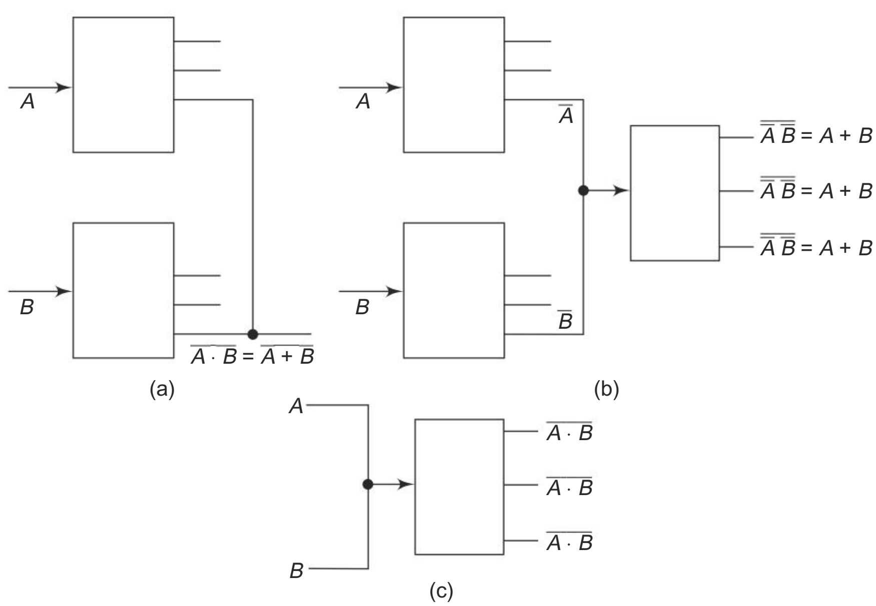


Fig. 15.54 IIL gates: (a) NOR, (b) OR, (c) NAND

The IIL outputs are open-collector outputs, and hence connecting them together forms a wired AND. Because the basic IIL gate has a single input and multiple outputs, design does not proceed along regular lines. The advantages of this technology has overcome this problem. IIL doesn't lend itself to chip interconnections as TTL, and seems primarily suited for large-scale integration.

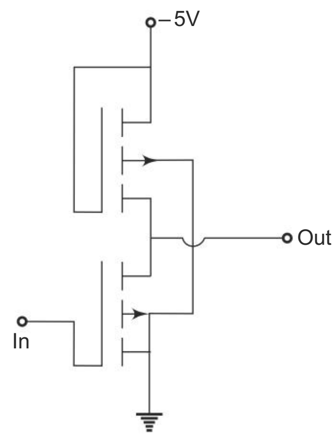


Fig. 15.55 A PMOS inverter

15.12.6 Metal Oxide Semiconductor (MOS) Circuits

The circuits described so far are all termed bipolar circuits and use conventional transistors. For large-scale integration, quite often a field-effect transistor (FET) is used. The FET devices are easier to manufacture, smaller in size and have small power dissipation.

As switching circuit, a FET can be used to form an inverter. A MOS INVERTER is shown in Fig. 15.55. With a logic 0, or ground input, the output of the circuit will have a −5 V output and with a −2 V or more negative input, the output will go to about 0 V. The FET on the top acts as a resistor.

When a P-type substrate with N-type doping for the source and drain is used, the N-channel MOSFET is formed. This type is called as NMOS device. Figure 15.56 shows a NMOS NOR gate.

The logic levels for this circuit are 0 to 1 V for a binary 0 and greater than +1.5 V for a binary 1. If any of the inputs A, B or C is a 1, the corresponding FET will conduct, causing the output to go to about +0.8 V or less. If all inputs are at +0.8 V or less, all the FETs will be off and the output will be at +5 V. Figure 15.57 shows a NMOS NAND gate.

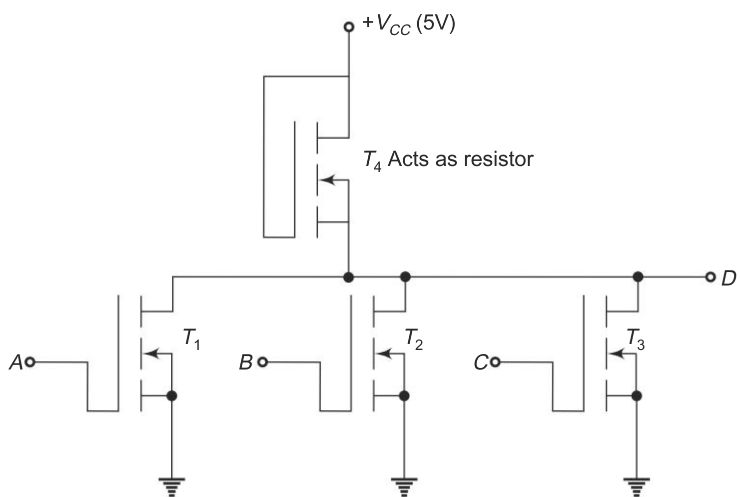


Fig. 15.56 Three-input NOR gate

CMOS *logic circuits* The complementary MOS (CMOS) circuits have very low power consumption and considerable resistance to noise. They are, however, slow. But large numbers of circuits can be placed on a single chip, the power supply voltage can vary over a large range, and the circuits are relatively economical to manufacture. The newest CMOS circuits have become relatively fast and are widely used for everything from electronic watches and calculators to microprocessors.

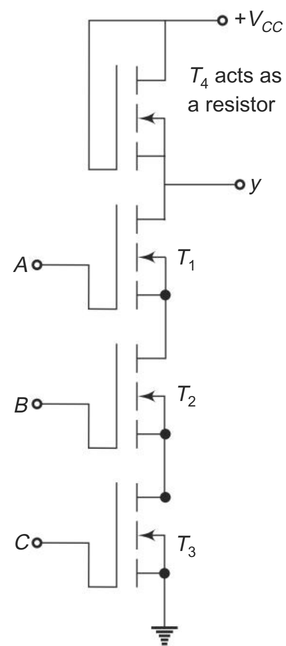


Fig. 15.57 Three-input NAND gate

In CMOS circuits both N- and P-channel field effect transistors are fabricated on the same substrate. The simplest form of CMOS integrated circuit consists of one N-channel and one P-channel MOS transistor, with both gate contacts tied together to form the input and both drain contacts tied together to form the output. Figure 15.58 shows the basic CMOS INVERTER.

When the voltage at the input is near ground level, the gate-to-source voltage of the P-channel transistor approaches the value of the supply voltage +V, and P-channel FET is turned on. A low-resistance path is created between the supply voltage and the output, while a high-resistance path exists between the output and ground because the N-channel FET is off. The output voltage will approach that of the supply voltage +V. When the input voltage is near +V, the P-channel FET is turned off and the N-channel FET is turned on. This makes the output voltage to approach the ground value.

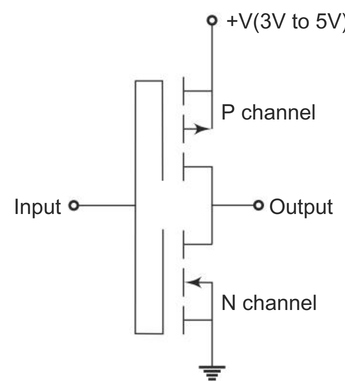


Fig. 15.58 CMOS Inverter

As one FET is always off and since both N-channel and P-channel FETs allow very low leakage current when off, the power consumption is very low in either state. A two-input NOR gate can be constructed using CMOS circuits as shown in Fig. 15.59. It can be seen that each additional input requires an additional P- and N-channel pair of MOSFETs.

15.12.7 Comparison of Logic Families

Logic families are normally compared based on the following parameters.

Fan-out: It is the maximum number of loads that the device can reliably drive. Here, the load and device refer to the logic gates of the same family.

Power dissipation: This is the power dissipated per gate.

Noise immunity: It is defined as the amount of noise voltage that causes unreliable operation. If noise immunity is 0.3 V, then as long as the noise voltages induced on connecting lines are less than 0.3 V, the device will work reliably.

Propagation delay: The time delay encountered when a transistor tries to come out of the saturation condition. When the base drive switches from high to low, a transistor cannot instantaneously come out of hard saturation; extra carriers must first flow out of the base region. The delay incurred in this process is called as the propagation delay.

Package density: It is the number of devices that can be fabricated per unit area of the chip.

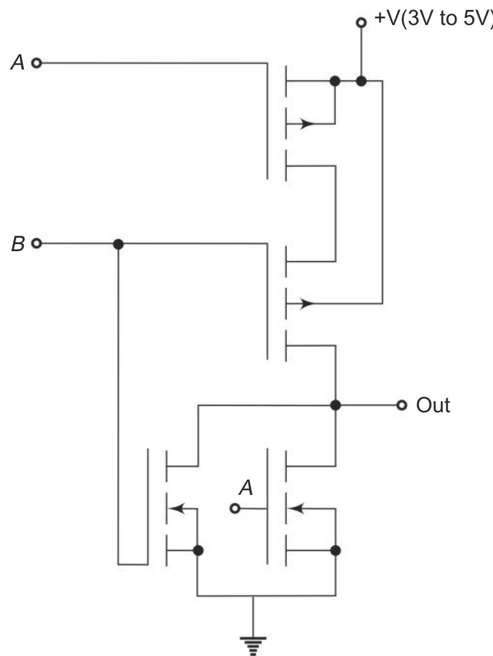


Fig. 15.59 CMOS NOR gate

Between the bipolar family and MOS family, in general, the bipolar circuits are faster and the MOS circuits provide high package density. Table 15.23 gives the values of the parameters discussed above, for different logic families.

Table 15.23 Comparison of major IC logic families

Parameter	*DTL*	*TTL*	*ECL*	*MOS*	*CMOS*
Basic gates	NAND	NAND	OR-NOR	NAND	NOR or NAND
Fan-out	8	10	25	20	> 50
Power dissipated per gate, mW	8 – 12	12 – 22	40 – 55	0.2 – 10	0.01
Noise immunity	Good	Very good	Good	Nominal	Very good
Propagation delay per gate, ns	30	12 – 6	4 – 1	300	70

REVIEW QUESTIONS

1. What is the base of the decimal number system?
2. Find the binary equivalent of each decimal number.
   (a) 238.112 (b) 36.002 (c) 1647 (d) 48.442

3. Find the decimal equivalent of each binary number.
   (a) 1101 (b) 100101 (c) 1011.011 (d) 1110.0011
4. Convert each decimal number to octal.
   (a) 16 (b) 45 (c) 270
5. Express each octal number in binary.
   (a) 7013 (b) 1234 (c) 72.66
6. What is the hexadecimal representation for 100001000010100101111001100010?
7. What binary number does $A4D6F_{16}$ represent?
8. Perform the indicated binary operations.
   (a) 110 + 011 (b) 111011.01 + 110001.11 (c) 110 − 010 (d) 11101 − 10101
   (e) 101 × 111 (f) 1110 ÷ 110
9. What is meant by 1's and 2's complement of a binary number?
10. Explain the rules for binary subtraction using 1's and 2's complement methods.
11. Subtract 1110 from 1001 using 1's complement and 2's complement methods.
12. What is a binary coded decimal?
13. What decimal number does the BCD sequence 0110 1110 1100 0010 1101 1011 0001 represent?
14. Explain how BCD addition is carried out.
15. What are the basic rules and properties of Boolean algebra?
16. State and explain De Morgan's theorems.
17. Determine whether or not the following equalities are correct:
   (a) $ABC + \overline{ABC} = A$
   (b) $A + BC + \overline{A}C = BC$
   (c) $A\,(\overline{A}BC + ABCD) = ABCD$
18. Using Boolean algebra techniques, simplify the following expressions as much as possible:
   (a) $A\,(A + B)$
   (b) $A\,(\overline{A} + AB)$
   (c) $A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C$
19. What are logic gates? What are the different types of logic gates?
20. What are called universal gates? Why they are called so?
21. Draw the logic diagram of an Ex-OR Gate and discuss its operation.
22. Give the action of the two input Ex-OR gate and construct it using NAND Gates.
23. What is an Ex-NOR Gate? Write its truth table.
24. Verify that the following operations are commutative and associative
   (a) AND (b) OR (C) Ex-OR
25. Design a logic circuit with four input variables that will produce a 1 output when any three input variables are 1s. Use K-map for simplifying the logical expression.
26. Implement the following logic expressions with logic gates:
   $Y = ABC + AB + BC$
   $Y = ABC\,(D + EF)$
27. What is a half adder? How it is realised using logic gates?
28. Design a full adder circuit using NAND gates.
29. What is a half subtracter? Realise a full subtracter using NAND gates only.
30. Determine the logic required to decode the binary number 1110 by producing a 'high' indication on the output.

31. What is a multiplexer? Explain the operation of 4-to-1 line multiplexer.
32. Explain how a digital demultiplexer can be realised using logic gates.
33. Draw a decimal-to-BCD encoder and explain its operation.
34. What is a decoder and how it is different from a demultiplexer?
35. What are the key features of Gray code and Excess-3 code?
36. Design a combinational logic circuit for binary to Gray code and Gray to binary code conversion.
37. Convert the following binary numbers to Gray codes.
    (a) 11001 (b) 1110101
38. Convert the following Gray codes to binary numbers.
    (a) 10010 (b) 111101
39. Design a binary to excess-3 code convertor using logic gates.
40. What is a sequential circuit? What is the main difference between combinational circuits and sequential circuits?
41. Show how a SR flip-flop can be constructed using NOR gates? Explain the different states of the SR flip-flop.
42. Explain the operation of D flip-flop, JK flip-flop and T flip-flop.
43. What are shift registers? What are the different types of shift registers?
44. What are binary counters? What are the two types of binary counters?
45. Implement an up-down counter using JK flip-flop, the up or down count being done based on a control signal.
46. What is a decade counter? Explain its operation using clock signal waveform.
47. Why A/D and D./A converters are needed?
48. What are the different types of D/A converters?
49. Explain the working of binary weighted resistor type D/A converter.
50. Explain the working of $R - 2R$ ladder type D/A converter.
51. What is meant by *resolution* of an A/D converter?
52. Explain the term *conversion time* of an A/D converter.
53. Explain the working of a *flash type* A/D converter.
54. Explain the working of a counter type A/D converter.
55. Explain the function of *successive approximation register* in A/D converter.
56. What are the different kinds of logic families available?
57. Explain how a DTL NAND gate works, with a suitable diagram.
58. What are the advantages of TTL circuits?
59. Draw a two-input TTL NAND gate and explain its operation.
60. What is a totem-pole arrangement and what is its advantage?
61. How a high-speed TTL can be realised?
62. Explain in brief the different subfamilies of TTL circuits.
63. What is a Schottky TTL circuit? What is its advantage?
64. What are the advantages of ECL circuit?
65. Draw the circuit of the basic ECL inverter circuit and explain its operation.
66. With a truth-table, explain how an ECL NOR/OR gate works.
67. What is an integrated injection logic?
68. What is the main advantage of IIL circuits?
69. Show how different logic gates can be realised using the basic IIL inverter.
70. What are called MOS circuits? What are the advantages of this family of circuits?

71. Explain how an inverter can be realised in MOS circuits?
72. Draw the NMOS circuits of NOR and NAND gates.
73. What is a CMOS circuit? Explain how a CMOS NOR gate functions, using a neat sketch.
74. What is meant by fan-out of a logic circuit?
75. What is meant by noise immunity of a logic circuit?
76. Compare the performance of different logic circuits, based on fan-out, noise immunity, package density, etc.

COMMUNICATION SYSTEMS

16

INTRODUCTION

Communication is a process of transfer of information bearing signals from one place to another. The equipment that transmits the information is the transmitter and the equipment that receives the information is the receiver. Channel is the medium through which the signal travels from the transmitter to the receiver. Telegraphy, telephony, facsimile, radio broadcast, TV transmission and computer communication are a few examples of communication services.

16.1 COMMUNICATION SYSTEM

The basic function of a communication system is to communicate a message. The block diagram of a communication system is shown in Fig. 16.1.

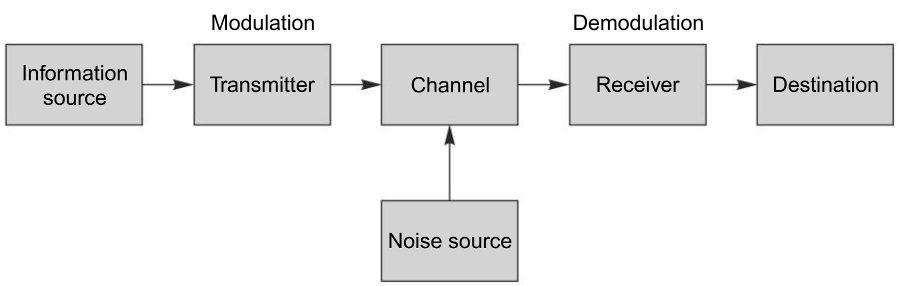


Fig. 16.1 Block Diagram of Communication System

The information to be transmitted is given by the information source. In most of the cases, the information will be non-electrical in nature. For example, audio signals in speech transmission and picture signals in television transmission. This information in the original form is converted into a corresponding electrical variation known as the message signal by using a transmitter. This message signal cannot be directly transmitted due to various reasons discussed in section 16.5. Hence this message signal is superimposed on a high frequency carrier signal before transmission. This process is referred to as modulation. After modulation, the modulated carrier is amplified by using power amplifiers in the transmitter and fed to the transmitting antenna.

Channel is a medium through which the signal travels from the transmitter to the receiver. There are various types of channels, such as the atmosphere for radio broadcasting, wires for line telegraphy and telephony and optical fibres for optical communication. As the signal gets propagated through the channel, it is attenuated by various mechanisms and affected by noise from the external source. Noise is an unwanted signal that interferes with the reception, of wanted signal. Noise is usually of random nature and in the design of communication system, careful attention should be paid to minimise the effect of noise on the reception of wanted signals.

At the receiving end, a weak modulated carrier that is transmitted from the transmitter is received. As the received signal power will be very small, it is first of all amplified to increase the power level and the process of demodulation is done to recover the original message signal from the modulated carrier. The recovered message signal is further amplified to drive the output transducer such as a loud speaker or a TV receiver.

Table 16.1 gives the electromagnetic spectrum used for various communication services.

Table 16.1 Electromagnetic Spectrum for Various Communication Services

Classification	*Frequency Range*	*Wavelength*	*Uses*
Very Low Frequencies (V.L.F.)	10–30 kHz	30–10 km	Long distance point to point communications
Low Frequencies (L.F.)	30–300 kHz	10–1 km	Long distance point to point communication, Marine, Navigation, Power Line Carrier communication and Broadcast
Medium Frequencies (M.F.)	300–3000 kHz	1000–100 metres	Power Line Carrier communication, Broadcast, Marine communications, Navigation and harbour telephone
High Frequencies (H.F.)	3–30 MHz	100–10 metres	Moderate and long range communication of all types, Broadcast
Very High Frequencies (V.H.F.)	30–300 MHz	10–1 metres	Short distance communications, TV and FM broadcast, Data communication, Mobile and Navigation systems
Ultra High Frequencies (U.H.F.)	300–3000 MHz	1–0.1 metres	Short distance communications, TV broadcast, Radar, Mobile, Navigation and Microwave relay systems
Super High Frequencies (S.H.F.)	3–30 GHz	10–1 cms	Radar, Microwave relay and navigation systems
Extremely High Frequencies (E.H.F.)	30–300 GHz	1–0.1 cm	Radar, Satellite, Mobile, Microwave relay and navigations

16.2 TELECOMMUNICATION SERVICES

Telegraphy This telecommunication service is used for transmission and reception of written texts. Teleprinters normally transmit signal at a speed of 50 bauds which occupies a bandwidth of 120 Hz. However recent teleprinters can operate upto 200 bauds. Teleprinters are interconnected through special exchanges called *telex* (Telegraph Exchange).

Telephony This is a communication service for the transmission of speech signal between two points. The speech signal is converted into the corresponding electrical signal by using a microphone and it is transmitted through a telephone line to a distant receiver where the original speech signal is reproduced by using a speaker. Human voice produces signals in the band of 300 Hz to 3.4 kHz. Hence a bandwidth of 4 kHz is allocated for a telephone channel. A human ear is highly sensitive to sound between 3 kHz and 4 kHz. As the female voice contains more energy in this frequency range, they are preferred as telephone operators and announcers.

Facsimile (FAX) This is a telecommunication service for the transmission and reception of picture information like photographs, drawing, weather maps, etc. The picture or any document to be transmitted is mounted on a cylinder and it is scanned by a photocell linked to the cylinder. The photocell produces an electrical analog signal as a voltage variation depending upon the intensity of the light and dark spots on the document. The electrical signal thus produced is converted into frequency variation and transmitted through a telephone line. At the receiving end, the frequency variations are converted back into corresponding voltage variations is given to a plotter for reconstructing the original picture or document. Thus a photocopy of the original picture is obtained at a distance. Similar to a telephone signal, the FAX message also occupies a bandwidth of 4 kHz.

The other communication services like radio broadcast, TV transmission and computer communication are discussed in the later sections of this chapter.

16.2.1 Transmission Paths

Transmission of messages can be either through bound media such as a pair of wires, coaxial cables, optical fibre cables, waveguides, etc. or through unbound media such as free space or atmosphere.

Line Communication Line communication refers to communication through pair of wires, coaxial cable and waveguides.

The *pair of wires or parallel-wire* is normally carried out using overhead lines on poles and the cables are normally buried under the ground. Buried cables have twisted pairs upto 4000 in numbers. The pairs are twisted to avoid crosstalk between subscribers. Such cables are used upto 500 kHz.

A *coaxial line* consists of a pair of concentric conductors with some dielectric filling the middle space, where the outer conductor is invariably grounded to act as an electrical shield. There may be a sheath around the outer conductor to prevent corrosion. A number of such coaxial lines are usually bunched inside a protective sleeve. A single coaxial line can be used to carry thousands of telephone channels. Coaxial lines are employed for higher frequencies upto 18 GHz.

A *waveguide* is used for signal transmission in the UHF range, and the SHF range, i.e. above 1 GHz. Here the signal gets propagated as an electromagnetic wave through a hollow pipe of rectangular or circular cross section. A waveguide acts as a high-pass filter which does not pass the signal below is cut-off frequency. A waveguide has a bandwidth which is in excess of 20% of its operating frequency. As an example, a waveguide operating at 5 GHz offers a bandwidth of 1 GHz which can accommodate 2,50,000 telephone links.

An *optical fibre* is a waveguide used for transmitting signals in the optical frequency range from 10^{13} to 10^{15} Hz. Signal transmission through an optical fibre is based on total internal reflection. Optical fibres are dealt with in detail in Section 16.16.

Radio Communication In radio communication, propagation of signals is through atmosphere or free space. Radio broadcasting, ground based microwave communication and satellite communication are a few examples of this type, which are discussed in detail in the later sections of this chapter.

Radio waves of electromagnetic waves in the frequency (LF) region and medium frequency (MF) region are normally used for radio broadcasting. They are reflected by the different layers such as D, E, F of the ionosphere at a height of 50 km to 400 km above the ground in the earth's atmosphere. Due to successive reflections from the ionosphere and the earth's surface, the radio waves travel for a long distance and increase the area of coverage of the broadcasting station.

Radio waves in the microwave frequency region above 1 GHz will penetrate the ionosphere and hence a satellite is required to reflect the signals towards the earth which forms the basis for satellite communication.

16.3 ANALOG AND DIGITAL SIGNALS

Signals are broadly classified into two types, viz. analog signal and digital signal.

Analog Signal The amplitude of an analog signal, i.e. voltage or current varies continuously with time, as shown in Fig. 16.2(a). The output signal from a microphone is an example for an analog signal.

Digital Signal A signal defined at discrete instants of time is called a discrete time signal. Discrete time signals are represented by sequences of samples. Each

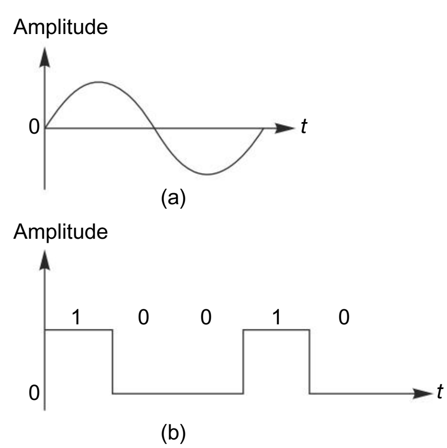


Fig. 16.2 (a) Analog Signal and (b) Digital Signal

sample of this discrete time signal is quantized, a process of taking round-off values and then coding, resulting in digital signals as shown in Fig. 16.2(b).

16.4 BASIC PRINCIPLES OF MODULATION

Modulation is the process of changing some parameter of a high frequency carrier signal in accordance with the instantaneous variations of the message signal.

The carrier signal has a constant amplitude and frequency. The function of a carrier signal is to carry the message signal and hence the name.

The message or modulating signals are low frequency audio signals which contain the information to be transmitted. Generally message signal ranges from 20 Hz to 20 kHz.

16.4.1 Need for Modulation

Modulation is an essential process in communication system to overcome the following difficulties in transmitting an unmodulated message signal.

Antenna Dimension When free space is used as communication media, messages are transmitted and received with the help of antennas. For effective transmission and reception, the dimension of the antenna should be of the order of quarter wavelength of the signal that is transmitted. If an audio frequency signal is directly transmitted, the required dimension of the antenna will be quite large so that it cannot be implemented in practice. For example, an audio frequency signal at 5 kHz requires a vertical antenna of height $\frac{\lambda}{4}$.

where $$\frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 5 \times 10^3} = 15{,}000 \text{ m.}$$

Hence modulation on a high frequency carrier should be done which transforms the frequency of the transmitted signal to the carrier frequency range thereby reducing the required dimension of the transmitting and receiving antennas.

Interference In the audio frequency range of 20 Hz–20 kHz, the programmes of different stations will get mixed up and will be inseparable in the common communication channel. Hence to reduce the interference, modulation of message signals from different stations are done on different carrier frequencies which transforms the modulated signal into different frequency slots.

Channel Characteristics Different communication channels will sustain signals over the different frequency ranges. A waveguide will sustain signals only in the microwave frequency range. An optical fibre will sustain signals only in the optical frequency range. Hence the low frequency message signal should be transformed into this desired frequency range for communication through the respective channel which requires modulation to be carried out on a carrier in the desired frequency range.

Ease of Radiation Since modulation translates the signals to higher frequencies, it becomes relatively easy to design and implement amplifiers and antenna systems.

Adjustment of Bandwidth Signal to noise ratio in the receiver is a function of the bandwidth of the modulated signal. Bandwidth can be adjusted by the modulation process resulting in the improvement of signal to noise ratio.

Shifting Signal Frequency to an Assigned Value Modulation process permits changing the signal frequency to a desired frequency band. This helps in effective utilization of the electromagnetic spectrum.

16.4.2 Types of Analog Modulation

The two types of analog modulation are (i) Amplitude Modulation and (ii) Angle Modulation. Angle modulation can be further classified as frequency modulation and phase modulation.

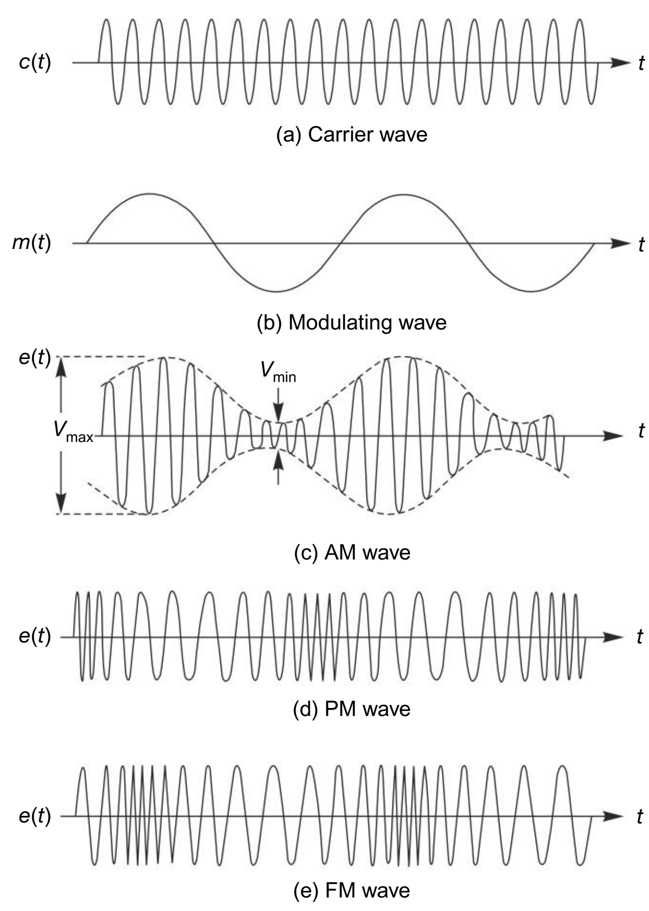


Fig. 16.3 AM, PM and FM Waves (a) Carrier Wave, (b) Sinusoidal Modulating Wave, (c) Amplitude Modulated Wave, (d) Phase-modulated Wave and (e) Frequency-modulated Wave

16.4.3 Amplitude Modulation

In amplitude modulation, the amplitude of the high frequency carrier signal is varied in accordance with the instantaneous value of the modulating signal as shown in Fig. 16.3 (c).

In AM the amplitude of the carrier wave is changed but its frequency remains the same.

The envelope of the modulated carrier is an exact replica of the audio frequency signal wave.

Let the carrier signal be represented by

$$c\,(t) = V_c \cos \omega_c t$$

where V_c is the constant amplitude of the carrier signal and the modulating signal is represented by

i.e. $$m\,(t) = V_m \cos \omega_m t$$

where V_m is the amplitude of the modulating signal.

The instantaneous voltage of the resulting amplitude modulated wave is represented by

$$e\,(t) = A \cos \omega_c t$$

The amplitude of the amplitude modulated wave, i.e.

$$A = V_c + m\,(t) = V_c + V_m \cos \omega_m t$$

Hence $$e(t) = (V_c + V_m \cos \omega_m t) \cos \omega_c t$$

$$= V_c \left[1 + \frac{V_m}{V_c} \cos \omega_m t\right] \cos \omega_c t$$

$$= V_c\,[1 + m_a \cos \omega_m t] \cos \omega_c t \qquad (16.1)$$

where $m_a = \dfrac{V_m}{V_c}$ is the modulation index (depth of modulation or percentage of modulation) for amplitude modulation.

From Fig. 16.3(c), $$V_m = \frac{V_{max} - V_{min}}{2}$$

$$V_c = V_{max} - V_m = V_{max} - \frac{V_{max} - V_{min}}{2}$$

$$= \frac{V_{max} + V_{min}}{2}$$

Therefore $$m_a = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

So, the value of m_a should lie between 0 and 1.

$$e\,(t) = V_c \cos \omega_c t + m_a V_c \cos \omega_c t \cos \omega_m t$$

$$= V_c \cos \omega_c t + \frac{m_a V_c}{2} [\cos\,(\omega_c + \omega_m)t + \cos\,(\omega_c - \omega_m)t] \qquad (16.2)$$

Hence the AM wave consists of three frequency components,

(i) f_c, the carrier frequency component.

(ii) $f_c + f_m$, the sum frequency component called as upper side band frequency component.

(iii) $f_c - f_m$, the difference frequency component called as lower side band frequency component.

Power Relation in the AM Wave The total power in the modulated wave is equal to the sum of the unmodulated carrier power and two side hand powers.

i.e.
$$P_t = P_c + P_{\text{LSB}} + P_{\text{USB}}$$
$$P_t = \frac{V_{\text{carr}}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{\text{USB}}^2}{R}$$

where all these voltages are rms values and R is the antenna resistance.

The unmodulated power is, $P_c = \dfrac{V_{\text{carr}}^2}{R} = \dfrac{(V_c/\sqrt{2})^2}{R} = \dfrac{V_c^2}{2R}$

Similarly, $P_{\text{LSB}} = P_{\text{USB}} = \dfrac{V_{SB}^2}{R} = \left[\dfrac{m_a V_c}{2\sqrt{2}}\right]^2 \dfrac{1}{R} = \dfrac{m_a^2 V_c^2}{8R}$

$$= \frac{m_a^2}{4}\frac{V_c^2}{2R} \tag{16.3}$$

Therefore
$$P_t = \frac{V_c^2}{2R} + \frac{m_a^2}{4}\frac{V_c^2}{2R} + \frac{m_c^2}{4}\frac{V_c^2}{2R} = P_c + \frac{m_a^2}{4}P_c + \frac{m_a^2}{4}P_c.$$

Therefore
$$\frac{P_t}{P_c} = 1 + \frac{m_a^2}{2} \tag{16.4}$$

The maximum power in the AM wave is $P_t = 1.5\,P_c$, when $m_a = 1$.

Current Calculations Let the rms value of the unmodulated current and the total modulated current of an AM transmitter be I_c and I_t respectively, then

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m_a^2}{2}.$$

Therefore
$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m_a^2}{2}}$$

Hence
$$I_t = I_c\sqrt{1 + \frac{m_a^2}{2}} \tag{16.5}$$

Modulation by Several Sine Waves Let V_1, V_2, V_3 etc. be the simultaneous modulating voltages. Then the total modulating voltage V_t is equal to the square root of the sum of the squares of the individual voltages, i.e.

$$V_t = \sqrt{V_1^2 + V_2^2 + V_3^2 + \ldots}$$

Dividing throughout by V_c, we obtain

$$\frac{V_t}{V_c} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \ldots}{V_c^2}}$$

$$\frac{V_t}{V_c} = \sqrt{\frac{V_1^2}{V_c^2} + \frac{V_2^2}{V_c^2} + \frac{V_3^2}{V_c^2}}$$

Hence the total modulation index is $m_{at} = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + \ldots}$ (16.6)

16.4.4 Angle Modulation

In angle modulation, the instantaneous angle of the carrier signal is varied in accordance with the instantaneous value of the modulating signal and its amplitude is kept constant. Two forms of angle modulation are (i) Frequency modulation and (ii) Phase modulation.

16.4.5 Frequency Modulation (FM)

In frequency modulation, the instantaneous frequency of the carrier is varied linearly with the variations of the message signal while the amplitude of the modulated carrier remains constant as shown in Fig. 16.3 (e).

The instantaneous frequency of the frequency modulated carrier is

$$f_i(t) = f_c + k_f m(t)$$

where $f_c(t)$ varies linearly with the base band signal, k_f is the frequency sensitivity constant (Hz/V) and f_c is the unmodulated carrier frequency.

Therefore $\omega_i(t) = 2\pi f_c + 2\pi k_f m(t)$

Then,
$$\theta_i(t) = \int \omega_i(t)\,dt$$
$$= 2\pi f_c \int dt + 2\pi k_f \int m(t)\,dt$$
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t)dt$$

So, the frequency modulated wave is represented by

$$e(t) = V_c \cos\left[2\pi f_c + 2\pi k_f \int m(t)\,dt\right]$$

When the modulating signal is $m(t) = V_m \cos \omega_m t$, then

$$e(t) = V_c \cos\left[2\pi f_c t + \frac{2\pi k_f V_m \sin \omega_m t}{\omega_m}\right]$$
$$= V_c \cos\left[2\pi f_c t + \frac{k_f V_m \sin \omega_m t}{f_m}\right]$$
$$= V_c \cos\left[2\pi f_c t + \frac{\delta \sin \omega_m t}{f_m}\right] \tag{16.7}$$

where $\delta (= k_f V_m)$ is defined as the peak (maximum frequency) deviation.

$$e(t) = V_c \cos[\omega_c t + m_f \sin \omega_m t]$$

Hence the modulation index for FM, m_f, is defined as

$$m_f = \frac{\delta}{f_m} = \frac{\text{maximum frequency deviation}}{\text{modulating frequency}} \tag{16.8}$$

The maximum value of frequency deviation (δ) is fixed at 75 kHz for commercial FM broadcasting. For normal band, the value of modulation index (m_f) is less than 1. For wide-band FM, m_f is greater than 1.

Unlike AM, where there are only the carrier and the two side-band components, the FM wave consists of a signal carrier frequency component and an infinite number of side frequency components. Hence the bandwidth required for transmission of FM signal is larger than for the AM signal. However, as the amplitude of the carrier is kept constant. FM signal is less affected by noise than the AM signal.

16.4.6 Phase Modulation (PM)

In phase modulation, the phase of the carrier is varied in accordance with the instantaneous value of the modulating signal, whereas the amplitude of the modulated carrier is kept constant, as shown in Fig. 16.3(d). The phase angle cannot change without affecting the frequency. So, phase modulation and frequency modulation are interrelated with each other.

In phase modulation, the instantaneous angle of the phase modulated carrier $\theta_i(t)$ is

$$\theta_i(t) = \omega_c t + k_p m(t)$$

where $\omega_c t$ is the angle of the unmodulated carrier and k_p is the phase sensitivity. $\theta_i(t)$ varies linearly with the band signal. Hence the phase modulated wave is represented as

$$e(t) = V_c \cos [\theta_i (t)]$$

$$= V_c \cos [\omega_c t + k_p m(t)]$$

Substituting $\quad m(t) = V_m \cos \omega_m(t)$

$$e (t) = V_c \cos [\omega_c t + k_p V_m \cos \omega_m t]$$

$$= V_c \cos [\omega_c t + m_p \cos \omega_m t] \tag{16.9}$$

where $m_p = k_p V_m$ is the modulation index for phase modulation, which is the maximum phase deviation of the phase modulated carrier.

Advantages of FM over AM

(i) The amplitude of the frequency modulated wave in FM is independent of the depth of modulation, whereas in AM, it is dependent on this parameter.

(ii) In AM, when the modulation index increases, the total transmitted power is increased. In FM, the total transmitted power is always same but the bandwidth is increased with the increased modulation index.

(iii) FM is much more immune to noise than AM and hence there is an increase in the Signal to Noise Ratio (SNR) in FM. This is because the FM receivers use amplitude limiters to remove amplitude variations.

(iv) By increasing frequency deviation, the noise can further be reduced in FM, whereas AM does not have this feature.

(v) As there is a guard band between FM stations, there is less adjacent channel interference in FM than in AM.

(vi) Since the FM transmitter operates in the upper VHF and UHF ranges, the space wave is used for propagation so that the radius of reception is limited to Line of Sight (LoS). Thus, it is possible to operate several independent transmitters on the same frequency with considerably less interference that would be possible with AM.

Disadvantages of FM over AM

(i) FM requires a much wider channel, perhaps 7 to 15 times as large as that needed by AM.
(ii) FM transmitting and receiving equipments are more complex and expensive.
(iii) Since reception is limited to line of sight, the area of reception for FM is much smaller than for AM.

Comparison of FM and PM

(i) In PM, the modulation index is proportional to the modulating voltage only. In FM, the modulation index is proportional to the modulating voltage and inverbely proportional to the modulating frequency.
(ii) FM is more immune to noise than FM.
(iii) PM and FM are indistinguishable for a single modulating frequency.

16.5 PULSE MODULATION TECHNIQUES

In order to transmit a large number of signals simultaneously through a single channel in an efficient manner, pulse modulation techniques are employed. Pulse modulation techniques yield better signal to noise ratios at the receiving end and hence they are highly immune to noise.

Here, a train of rectangular pulses is considered to be a carrier signal. In pulse modulation technique, the continuous waveform of the message signal is sampled at regular intervals. Information regarding the message signal is transmitted only at the sampling times. Hence for proper recovery of the message signal at the receiving end, the sampling rate should be greater than a specified value which is given by the sampling theorem.

16.5.1 Sampling Theorem

The sampling theorem states that if the sampling rate (f_s) in any pulse modulation system exceeds twice the maximum signal frequency (f_m), the original signal can be reconstructed at the receiving end with minimal distortion. Hence the condition, $f_s \geq 2 f_m$ should be satisfied in any pulse modulation system. For example, for a standard telephone channel which consists of audio frequency signals in the range of 300 Hz to 3400 Hz a sampling rate of 8000 samples per second is a worldwide standard.

16.5.2 Pulse Amplitude Modulation (PAM)

PAM is the simplest type of pulse modulation. In PAM, the amplitude of each pulse of the unmodulated pulse train is varied in accordance with the sample value of the modulating signal as shown in Fig. 16.4 (c).

16.5.3 Pulse Time Modulation (PTM)

In PTM, the amplitude of the pulses of the carrier pulse train is kept constant but their characteristics are varied and made proportional to the sampled signal amplitude at that instant. The variable timing characteristics may be pulse width or positions. leading to pulse width modulation or pulse position modulation.

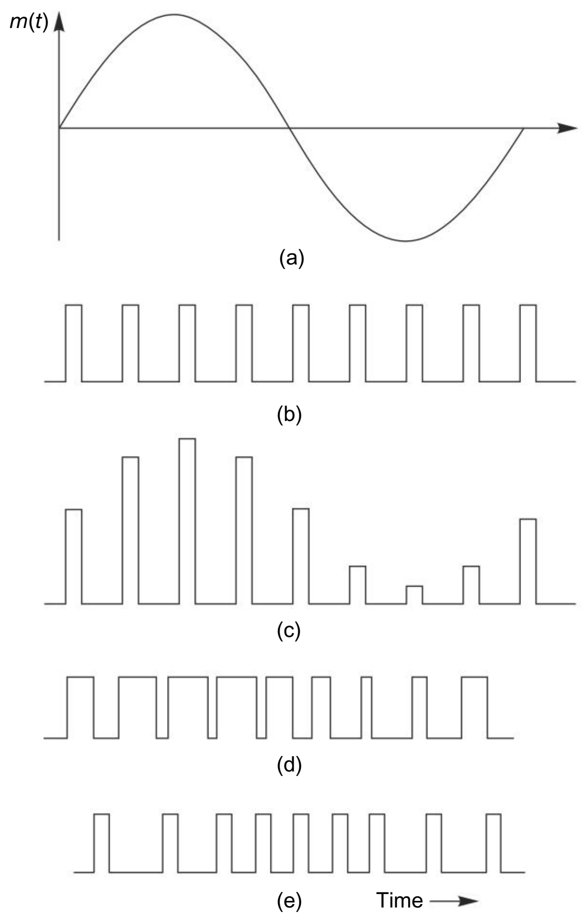


Fig. 16.4 Pulse Analog Modulation Techniques (a) Modulating Wave, (b) Pulse Carrier, (a) PAM Wave, (d) PWM Wave and (e) PPM Wave

Pulse Width Modulation (PWM) PWM is also called as Pulse Duration Modulation (PDM) or Pulse Length Modulation (PLM). In PLM, as shown in Fig. 16.4 (d), the amplitude and starting time of each pulse is made fixed, but the width of each pulse is fixed, but the width of each pulse is proportional to the amplitude of signal at that instant. The pulses of PWM are of varying width and therefore of varying power content. Even if synchronization between the transmitter and receiver fails, PWM still works whereas PPM does not.

Pulse Position Modulation (PPM) In Pulse Position Modulation as shown in Fig. 16.4 (e), the amplitude and width of the pulses are kept constant but the position of each pulse is varied in accordance with the instantaneous sampled value of the modulating wave.

Generation of PPM and PWM Each pulse of PWM has a leading edge and a trailing edge. In PWM the locations of the leading edges are fixed, whereas the trailing edges are position modulated. Hence, it is easy to obtain PPM from PWM by removing the leading edges and bodies of PWM pulses and keeping only the trailing edges.

16.6 PULSE DIGITIAL MODULATION-PULSE CODE MODULATION (PCM)

PCM is a digital modulation technique in which the analog message signal is converted into a digital signal before transmission. The essential operations in the transmission of a PCM system are sampling, quantizing and encoding as shown in Fig. 16.5(a).

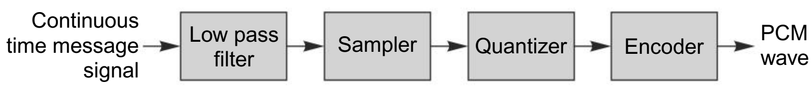


Fig. 16.5 (a) PCM Transmitter

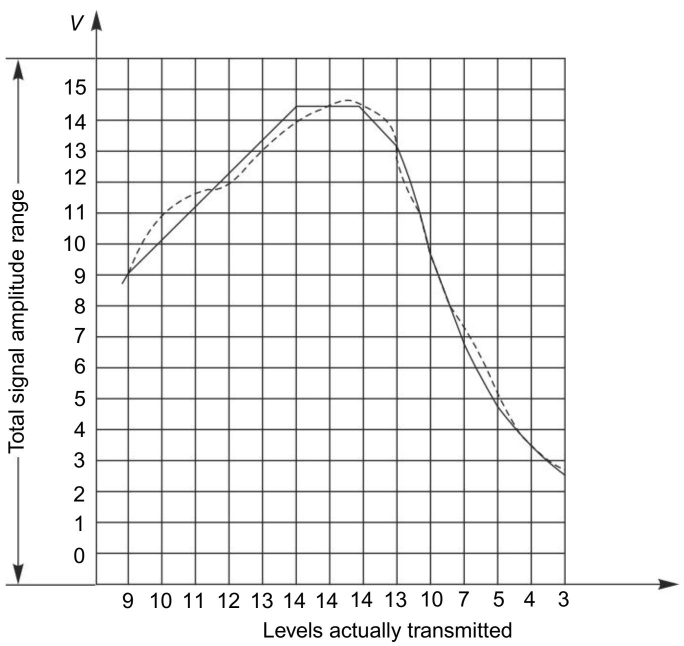


Fig. 16.5 (b) Quantization of Signal for PCM

Sampling The incoming message signal is sampled with a train of narrow rectangular pulses. The rate of sampling should be greater than twice the highest frequency component of the message signal in accordance with the sampling theorem.

Quantizing Quantizing is a process in which the sample value obtained at any sampling time is rounded off to the nearest standard (or quantum) level as shown in Fig. 16.5(b). For example, when the sample value is 6.8 V, it can be rounded off to 7 V.

Encoding This process translates the quantized values into a corresponding binary word by choosing an appropriate coding format. Thus, the quantized value 7 can be converted as the binary word 0111 by using the 8421 code. If the number of quantum levels is $16(2^4)$, four binary digits are required to represent a sample. This binary sequence is transmitted to the receiving end.

16.7 DIGITAL MODULATION TECHNIQUES

For transmitting digital signals over long distances, digital modulation techniques like Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK) and Phase Shift Keying (PSK) as shown in Fig. 16.6 are employed.

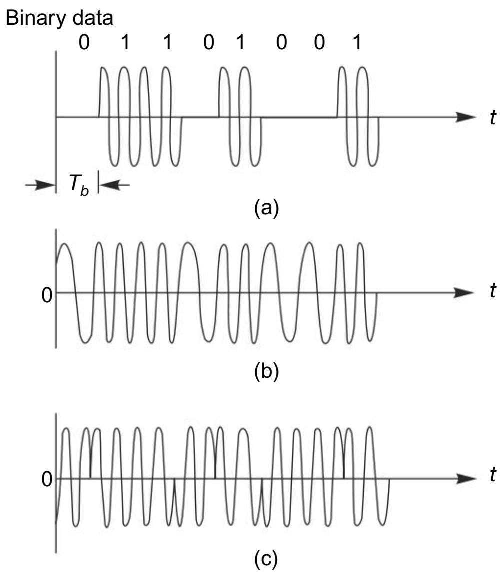


Fig. 16.6 Three Basic forms of Signaling Binary Information (a) Amplitude–shift Keying, (b) Phase-shift Keying and (c) Frequency-shift Keying

16.7.1 Amplitude Shift Keying

In ASK, binary 1 is transmitted as a carrier of certain amplitude and binary 0 is transmitted by a carrier of reduced or zero amplitude.

$$s(t) = A_c \sin \omega_c t \quad ; \ 1$$

16.7.2 Frequency Shift Keying

In FSK, binary 1 is transmitted as a carrier of particular frequency and binary 0 is transmitted as a carrier of different frequency.

$$s(t) = A_c \sin \omega_1 t \quad ; \ 1$$

$$= A_c \sin \omega_2 t \quad ; \ 0$$

16.7.3 Phase Shift Keying

In PSK, binary 1 is transmitted as a carrier of particular frequency and binary 0 is transmitted as a carrier of the same frequency but with a 180° phase shift.

$$s(t) = A_c \sin \omega_c t \quad ; \ 1$$
$$= A_c \sin (\omega_c t + \pi) = -A_c \sin \omega_c t \quad ; \ 0$$

16.8 DATA TRANSMISSION

The importance of data transmission has come into existence due to the development of computers. For the interconnection between a computer and other peripherals, a standard was made involving connection, signalling formats and signal levels. This became more useful when computer facilities began to use telephony for transmission requirements. Since the telephone was designed for voice communication, it was necessary to design a transmission circuit to send the digital data in the telephone lines.

16.8.1 Characteristics of Data Transmission

The telephone channel occupies a frequency range of 300 to 3400 Hz. When data is sent over telephone channels, the speed must be limited to ensure that the bandwidth required by data transmission will not exceed the telephone channel bandwidth. Faster the data transmitted, greater the bandwidth needed to accommodate it.

Data consists of pulse type energy represented by square wave signals. The repetition rate of rapid transition from one voltage level to another voltage level depends on binary representation of data word. For example, a 8-bit word 10101010 has the waveform as shown in Fig. 16.7.

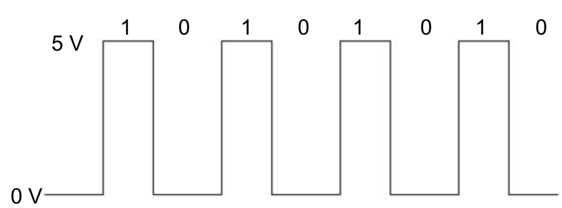


Fig. 16.7 Signal Level Representation for 10101010

16.8.2 Serial and Parallel Data Transmission

Data is commonly transferred between computers, and other peripherals. Such transfers are called parallel data transmission if a group of bits more over several lines at the same time, or several data transmission if the bits move one-by-one over a single line. Figure 16.8 illustrates parallel and serial data transmission.

In parallel transmission, each bit of a character travels on its own wire. A signal, called the strobe or clock, on an additional wire indicates the receiver when all the bits are present on their respective wires so that the values can be sampled. Computers and other digital system that are located near one another (within a few

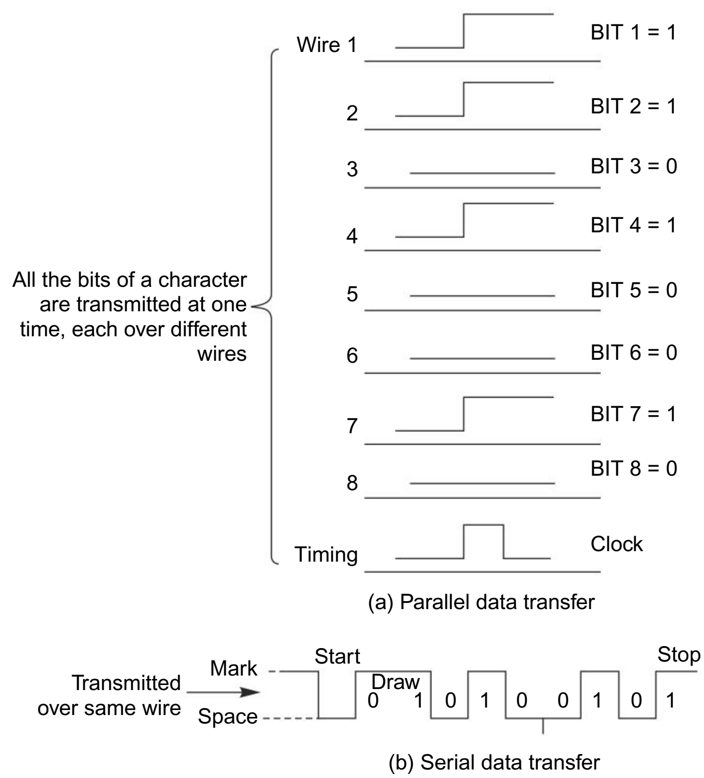


Fig. 16.8 Serial and Parallel Data Transfer

feet) usually use parallel transmission because it is much faster. As the distance between equipment increases, the multiple wires become more costly.

Serial transmission is used for long distances communications by single wire. The conversion from parallel to serial and vice-versa is accomplished with shift registers.

16.8.3 Modes of Transmission

Simplex Simplex is a data set which provides transmission in only one direction. Here the data set uses only one transmission channel so that no singnalling is available in the direction from the receiver to the transmitter. Though it is an economical method of data transfer, it has very limited applications and does not accommodate error correction and request for retransmission.

Half Duplex In half duplex, data transfer takes place in both directions by making flow in one direction at a particular time and data flow in opposite direction at another time. It requires only one transmission channel, the channel being bidirectional. Though it is economical, the speed of transmission is reduced because of necessity of sharing the source circuit for both directions.

Full Duplex In full duplex, operation permits transmission in both directions at the same time. Two circuit are required, one for each direction of transmission on the same channel.

16.8.4 Speed of Transmission

Signalling speed in transmission channels is called *baud rate* and is measured in bits per second (bps). Baud rate is equal to the maximum rate at which signal pulses are transmitted. It is different from the information bit rate which gives an idea of the encoding number of data bits in a single clock cycle.

Maximum signalling speed in bands is equal to twice the bandwidth of the channel. The Fig. 16.9 shows the different baud rates, i.e. variation in number of bits transmitted per second.

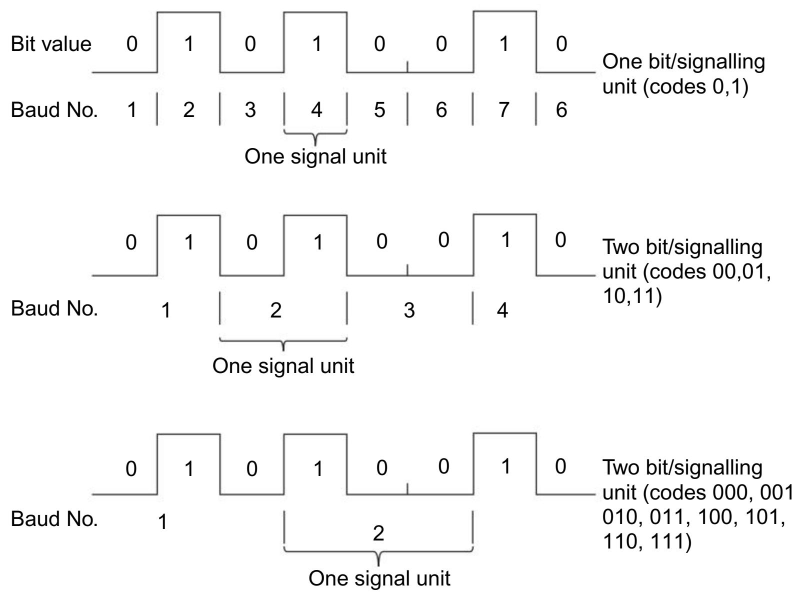


Fig. 16.9 Relationship of Bit Rate Baud and the Number of Bits per Signalling Unit

16.8.5 Effect of Noise

Noise is an unwanted signal. Noise corrupts the desired signal and makes it difficult to determine the signal value. There are many types of noise described as follows:

(i) *Impulse noise* It is caused by a sudden change in electrical activity such as an electric motor starting-up. During this starting-up period, the noise overwhelms the data signal and makes many data bits in a row useless.

(ii) *Frequency specific noise* This is caused by power line signals radiating energy. Any power lines, circuits to lamps and test equipment, and alternating current power wiring can act as antennas to transmit this noise. This noise affects the received data voltages and makes the communication system malfunction. This can be reduced by moving away the communication equipments from power lines.

(iii) *Wide band noise* It contains many frequency components and amplitude values. It is random in nature having the energy component extending over a wide range of frequencies. If the amplitude of the noise components at all frequencies within the bandwidth is equal, it is called as *white noise*.

The effect of noise on the data channel can be reduced by increasing the signal to noise represented as

$S/N = 2^{NR/\delta f} - 1$, where S/N = Signal to noise ratio, NR = Nyquist rate and δf = channel bandwidth.

Crosstalk Crosstalk is the reception of a portion of a signal from one channel to another channel in multiplexed systems. It occurs due to electromagnetic interaction between adjacent wires. The crosstalk can be reduced by using twisted-pair cables.

16.8.6 Asynchronous Data Transmission

In asynchronous data transmission, communication takes place between two devices of different speeds. Here the start bit and stop bit are introduced and the message is in between them. The transmitter transmits a start bit and sends the message block and ends with stop bit.

The receiver tries to recognize the start bit and accepts all the data until the stop bit. The message block with one start bit and one stop bit is known as a frame. Both transmitter and receiver are operated by a digital clock. If the speed of the clock varies, then bit timing drifts with each successive bit until the sampling of the last received in the character will be incorrect as shown in Fig. 16.10(a).

Figure 16.10(b) shows the error occurring when the received clock is slightly faster than transmission clock. Figure 16.10(c) shows the occurrence of error when the received clock is slower than the transmission clock.

16.8.7 Synchronous Data Transmission

In synchronous data transmission, the start and stop bits are used. Characters are sent in groups called block with special synchronization characters placed at the beginning of the block and within it, to ensure that enough 0 to 1/1 to 0 transitions occur for the receiver clock to remain accurate, as shown in Fig. 16.11. The control characters represent the block length and other related information.

The receiver gets sysnchronized by the synchronization pulses, then it gets the length of the block and then it receives the actual data bits or characters and special characters for the end of transmission. This forms a total frame. Error checking is done automatically on the entire block. If errors occur, the entire block is retransmitted.

16.9 MODEM

Modem is the contraction of Modulator-Demodulator. The normal digital signal cannot be directly transmitted through the telephone line, because telephone lines are designed for analog voice signals whereas data is normally represented by binary signals. Hence the data signals should be converted into a suitable form so that it is

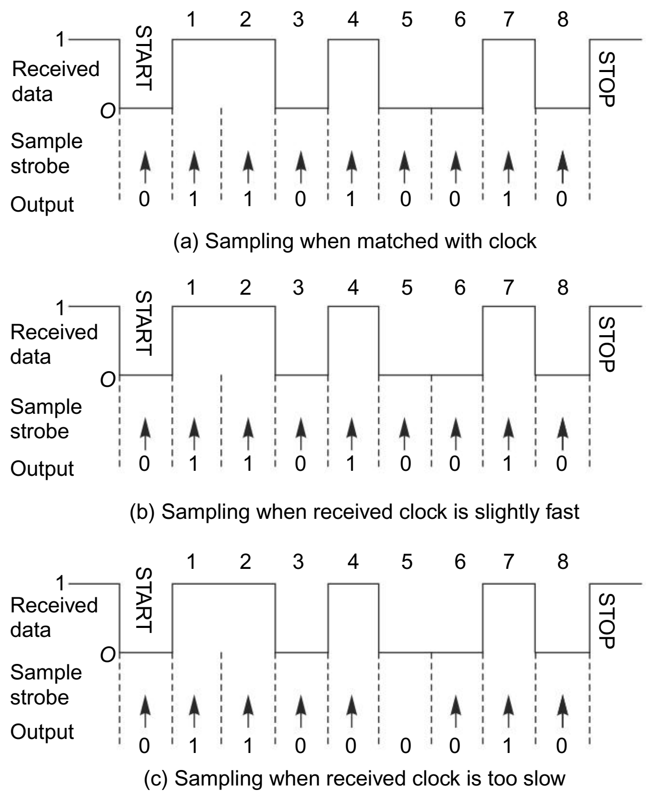


Fig. 16.10 Clocking of Asynchronous Data

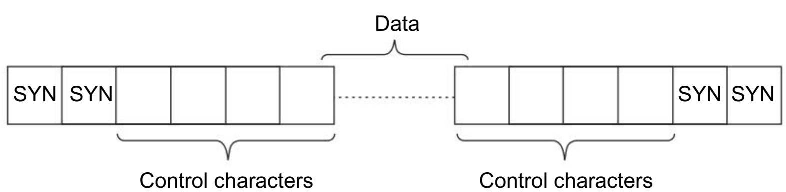


Fig. 16.11 Character Frame

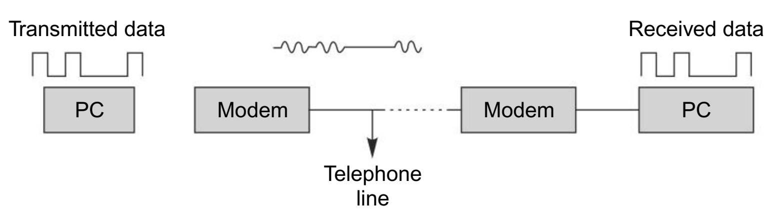


Fig. 16.12 Modem Converting Data Signals Compatible with Telephone Lines

compatible with phone line capabilities which is done by a modem as shown in Fig. 16.12. At the transmitting end, modem performs any one of the digital modulation techniques, i.e. ASK, FSK or PSK and the corresponding demodulation.

Functions of Modem (Transmitting end) Figure 16.13 shows a simplified block diagram of a modem used with RS-232 interconnection.

(i) Take the data from RS-232 interface.
(ii) Convert the data (0's and 1's) into appropriate tones (modulation process).
(iii) Perform line control and signalling to the other end of the phone line.
(iv) Send dialing signals if this modem is designed to dial without the user present.
(v) Have protection against line over voltage conditions and problems.

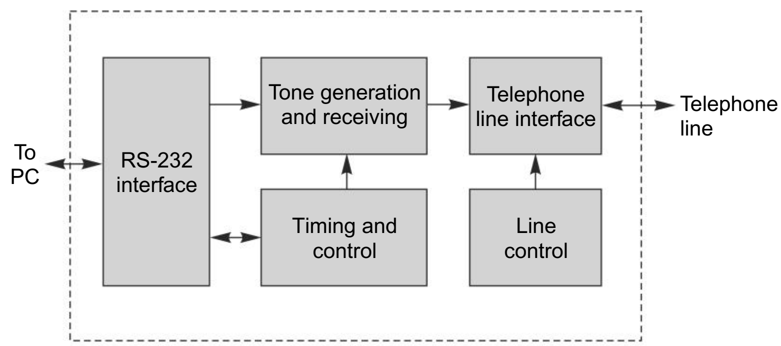


Fig. 16.13 Design of Modem

Functions of Modem (Receiving end)

(i) Receive tone from the phone line.
(ii) Demodulate these tones into 1's and 0's.
(iii) Put demodulated signal into RS-232 format and connect to RS-232 interface.
(iv) Perform line control and signalling.
(v) Have protection against line over voltage problems.
(vi) Adapt its receiving system to variations in received noise, distortion, signal levels and other imperfections, so that the data can be recovered from the received signals.

16.9.1 Operation of a Modem

Figure 16.14 shows the operation of a modem. A modem operates in simplex, half duplex and full duplex modes. The main activity of a modem is to send and receive digital data. As an example, consider a modem of FSK type that uses two frequencies to represent binary values. Let 1000 Hz be used for binary 0 and 2000 Hz for binary 1, and let these tones go over the phone lines. By design, the phone system can handle these tones reasonably well. The rate at which 1000 and 2000 Hz tones

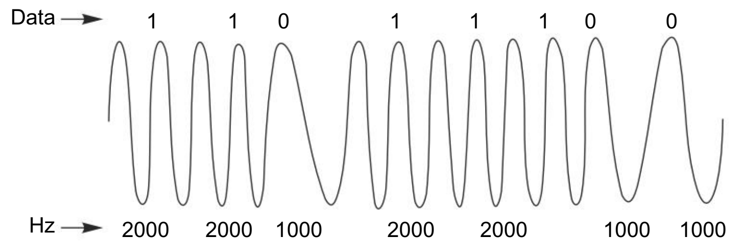


Fig. 16.14 Modem Working for Transferring the Data 11011100

are used to represent the data bits is the band value. In a 300 baud system, there is a new bit every 1/300 secs. If the bit has the same value as the preceding one, the tone is unchanged. If it is different, then the tone would be different.

Consider a stream of bits 11011100 as shown in Fig. 16.14. The phone line receives the following signals of 2000, 2000, 1000, 2000, 2000, 2000, 1000, 1000 Hz. If two bits in a row are same, then the tone is continuous for another bit period. Here the modem has done digital frequency modulation, i.e. FSK.

At the receiving end, the modem must be prepared to look for 1000 Hz and 2000 Hz signals despite their low signal levels, continuous variations in the level and addition of corrupting noise. To do this, a special technique is used at the receiving end to amplify the weak signal, separate the noise and demodulate the signal by using a filter circuit. The output of the filter circuit goes to the signal detector which senses the filter that has a the output at any instant of time. If the 1000 Hz filter gives an output, then a '0' must have been received. An output at the 2000 Hz filter indicates that a '1' has been received. The modem uses its timing circuitry to reformulate a string of 1's and 0's from the received tones, i.e. identical to the string of 1's and 0's that the equipment at the transmitting end generates.

Problem and Solution There is a problem that occurs if full duplex communication is needed. Suppose both sides of the link use same frequencies such as 1000 and 2000 Hz, the signals interfere with each other resulting in corruption of information. To overcome this, two solutions can be considered.

(i) The first method half duplex communication so that each side gets a turn to communicate but never simultaneously. Then, a single part of frequency can be used at both ends. When the receiving channel needs to have its turn, some protocol is used to indicate this. The data line is then turned around so that data communication can take place in the other direction, giving a turn around time of 100 to 500 ms.

(ii) The second method uses full duplex, where instead of single pair of tones, two different pairs are used. The first modem may use 1000/2000 Hz and the second modem uses 3000/4000 Hz. The modem that is transmitting 1000/2000 Hz must have a receiver and a filter set for 3000/4000 Hz and vice versa as shown in Fig. 16.15.

Coupling Methods There are two ways of connecting a modem to the line namely, (i) Acoustic coupling and (ii) Direct coupling.

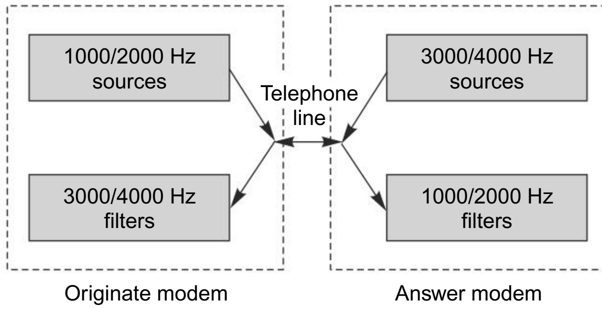


Fig. 16.15 A Full-duplex MODEM with a Set of Receiver and Filter

(i) Acoustic coupling In acoustic coupling as shown in Fig. 16.16, the modem generated tones go through the modem speaker into handset microphones. Tones coming in from phone lines go into the handset speaker and into the modem microphone. It has the disadvantages of being a large unit, having a slow performance, allowing the noise to get in. It cannot be used for automatic dialing.

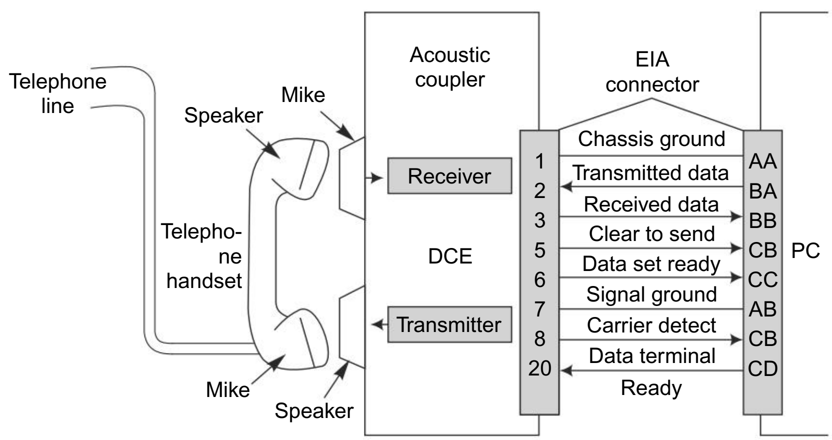


Fig. 16.16 Acoustic Coupling

(ii) Direct connection modem In a direct connection modem as shown in Fig. 16.17, the modem actually plugs into the phone line using the existing plug that normally goes into the phone. Most of these modems have a connector so that the phone can be connected and still be used as long as the modem is not in line. The direct connect modem can be used with any phone line that has the proper plug. Direct connect modems can dial automatically, answer and recognize the phone line tones such as the busy signal. A direct connect modem offers high performance at low cost.

Modem Data Transmission Speed Modems are generally classified according to the transmission speed. Low speed modem handles data rate upto 600 bps. Medium speed modem handle data rate from 600–2400 bps. High speed modems

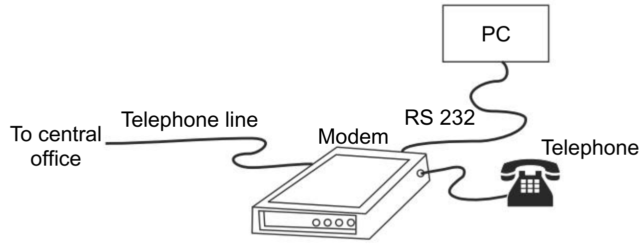


Fig. 16.17 Direct Connection Modem

handle data rate from 2400–10,800 bps. All modems operate within a single 300–3400 Hz telephone channel. As the speed increases beyond 19,000 bps, a wide modem is needed.

16.10 RADIO BROADCAST

16.10.1 AM Broadcast

AM broadcast services use the long wave band forms 150 to 285 kHz, medium wave band from 535 to 1605 kHz and short wave band upto 21.4 MHz. The mode of transmission is double side band full carrier, with an audio base band range of 5 kHz. Broadcast station frequency assignments are spaced at intervals of 10 kHz. The output power from the transmitter ranges from a few hundred watts for small local stations and to about 100 kilowatts for international transmitters.

Generation of AM Wave To generate an AM wave shown in Fig. 16.18(b), it is necessary to apply a series of current pulses of Fig. 16.18(a) to a tuned circuit. When a single current pulse is applied, it will initiate damped oscillations in the tuned circuit. The oscillation will have an initial amplitude proportional to the size of the current pulse and a decay rate dependent of the time constant of the circuit. When a train of pulses is fed to the tank circuit, each pulse will cause a complete sine wave proportional in amplitude to the size of this pulse. It will be followed by the next sine wave, proportional to the size of the next applied pulse, and so on. When at least ten pulses are applied per audio cycle at the output of the tank circuit, a good approximation of the AM wave will result if the original current pulses are made proportional to the modulating voltage. This process is known as *Flywheel Effect of Tuned Circuit*.

To generate a series of current pulses with their amplitudes proportional to the instantaneous values of the modulating voltage, a class C amplifier is used. The output current of a class C amplifier can be made proportional to the modulating voltage by applying the modulating voltage in series with any of the dc supply voltages of this amplifier. Accordingly, cathode (or emitter), grid (or base) and anode (or collector) modulation of a class C amplifier are all possible. If the output stage in a transmitter is plate-modulated (or collector-modulated) in a low power transmitter, the system is called *high level modulation*. If the modulation is applied either at the grid (base) or at the cathode (emitter) circuit, the system is called the *level modulation*.

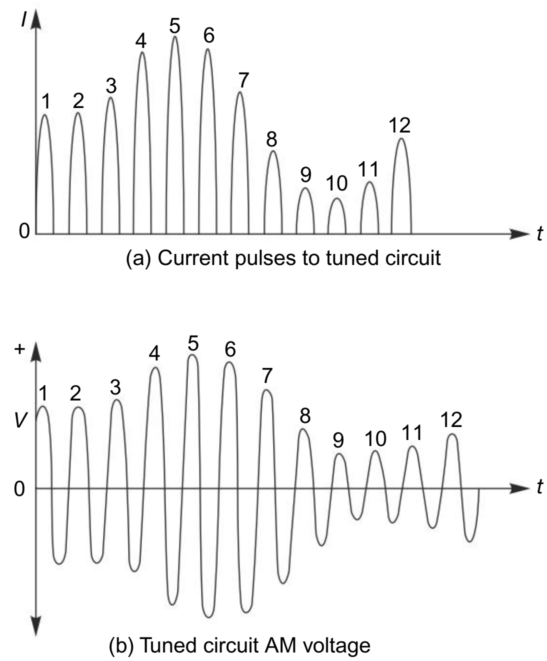


Fig. 16.18 Current Requirements for Generation of AM Wave (a) Current Pulses at the Input to a Tuned Circuit (b) AM Voltage Generated in the Tuned Circuit

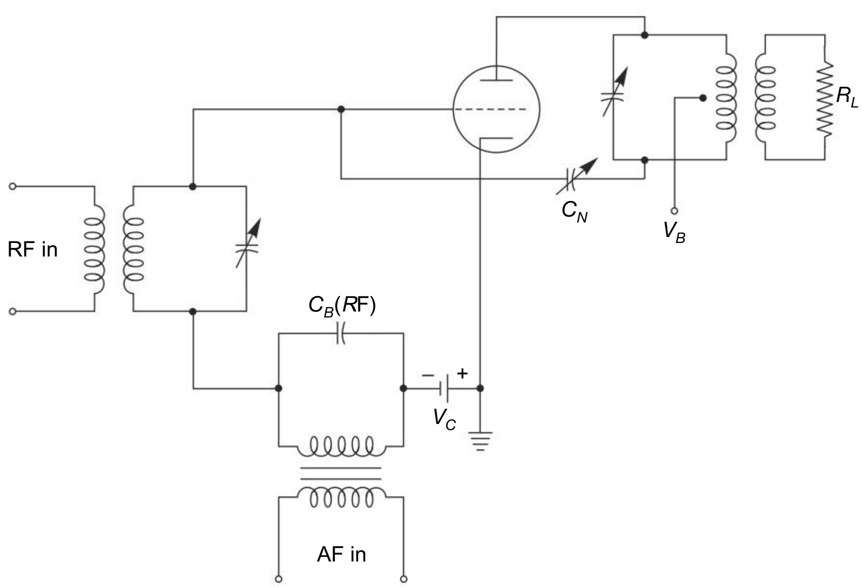


Fig. 16.19 Grid Modulated Class C Amplifier

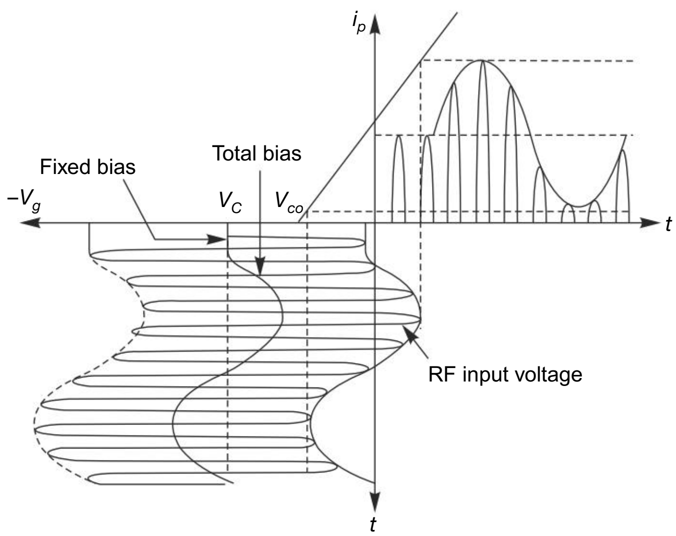


Fig. 16.20 Grid Voltage-Plate Current Waveforms for Grid-Modulated Class C Amplifier

Grid Modulated Class Amplifier A class *C* amplifier can be modulated by playing the modulating voltage in series with the grid bias as shown in Fig. 16.19. The modulating voltage is superimposed on the fixed negative grid bias of the amplifier. Hence the total grid bias is proportional to the amplitude of the modulating signal and varies at a rate equal to the modulating frequency as shown in Fig. 16. 20. The Radio Frequency (RF) carrier is applied to the input of the class *C* amplifier. The resulting plate current flows in pulses with the amplitude of each pulse being proportional to the instantaneous bias and therefore to the instantaneous bias and therefore to the instantaneous modulating voltage. The series of current pulses flow through a tuned circuit in the output of the amplifier which generates the required AM wave.

AM Transmitter Figure 16.21 shows a typical block diagram of an AM transmitter, where either low level or high level modulation is employed. As shown in the

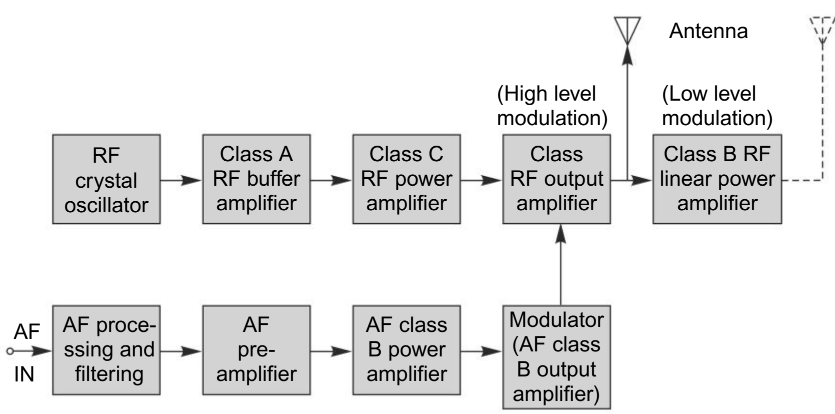


Fig. 16.21 AM Transmitter Block Diagram

block diagram the audio voltage is filtered so as to occupy the correct bandwidth and amplified by two or three stages of audio frequency amplifiers. The radio frequency carrier wave generated by the frequency stabilised crystal oscillator is amplified by two or three stages of amplifiers, to generate the required output power. For high level modulation, the amplified audio frequency signal is applied in series with the plate circuit of class *C* radio frequency output amplifier. The generated AM wave is directly given to the transmitting antenna.

For low level modulation, the amplified audio frequency signal is applied in series with the grid or cathode circuit of class *C* radio frequency amplifier and the generated AM wave is amplified by using a class *B* radio frequency linear power amplifier and fed to the transmitting antenna. Broadcast transmitters invariably use high level modulation because of the large amount of power requirements to be generated.

16.10.2 FM Transmitter

The block diagram of a directly modulated FM transmitter is shown in Fig. 16.22. The transmitter employs a reactance tube modulator to produce a frequency deviation in proportion to the signal amplitude at the output of the LC oscillator. The reactance tube modulator varies the total reactance of the resonant circuit of the LC oscillator thereby varying its output frequency. The resulting FM wave is passed through a number of frequency multiplier stages. These stages raise the centre frequency as well as the frequency deviation by the same factor. The frequency modulated wave is then amplified to the required level by class *C* power amplifier stages and transmitted.

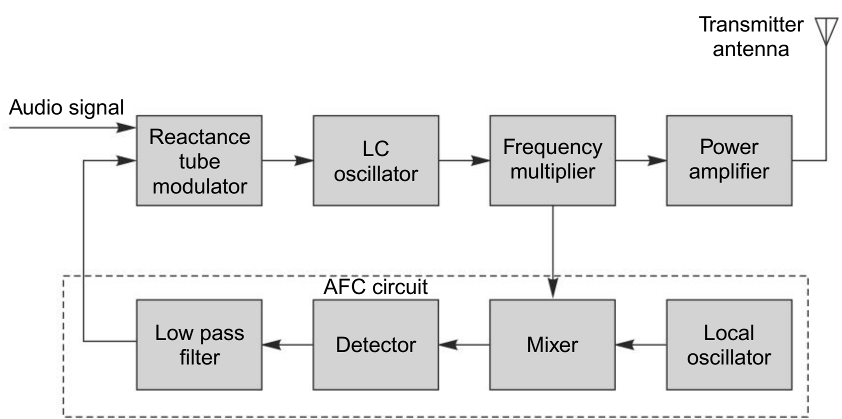


Fig. 16.22 Block Diagram of FM Transmitter

The Automatic Frequency Control (AFC) circuit maintains the entire frequency of the transmitter output at a fixed value if there is any drift in it due to changes in circuit parameters. Signal from the frequency multiplier is mixed with the local crystal oscillator output and the difference frequency is fed to a discriminator (FM demodulator). The discriminator gives a dc output according to the frequency shift with respect to the centre frequency and it is used to bring the centre frequency of LC oscillator output back to its original value.

Frequency Modulated radio (FM radio) uses the ultra-short wave band from 88 to 108 MHz. The channel spacing is 300 kHz. The FM reception is better than that of AM because is immune to noise and atmospheric disturbances.

16.10.3 Radio Receiver

The two types of radio receivers that have commercial significance are: (i) Tuned Radio-Frequency (TRF) receiver and (ii) Superheterodyne receiver. The superheterodyne receiver is used to a large extent now a days. The block diagrams of the basic superheterodyne receiver is shown in Fig. 16.23.

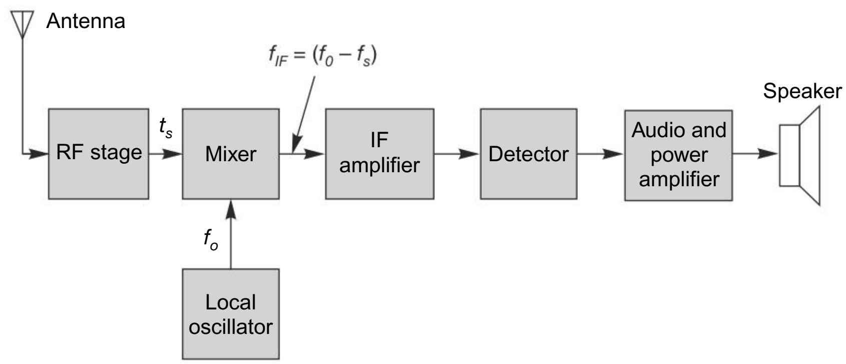


Fig. 16.23 Block Diagram of Superheterodyne Receiver

The modulated signal transmitted from the broadcast station is picked up by the antenna and fed to the RF stage where the signal is amplified. The signal voltage is combined with the output from a local oscillator in the mixer and converted into a signal of lower fixed frequency referred to as Intermediate Frequency (IF). The process of mixing the incoming signal with the local oscillator output is referred to as ***Superheterodyning***.

The signal at this intermediate frequency contains the same modulation as that of the original signal but it is at a lower frequency. By using ganged tuning the frequency of the local oscillator voltage is automatically adjusted so that the difference between the local oscillator voltage frequency (f_o) and the incoming signal frequency (f_s) is the fixed intermediate frequency ($f_{IF} = f_o - f_s$). The intermediate frequency is 455 kHz for AM receivers and 10.7 MHz for FM receivers.

The signal at intermediate frequency is amplified in the IF amplifier stage. If amplifier provides most of the gain and bandwidth requirements of the superheterodyne receiver. As the characteristics of the IF amplifiers are (adjacent channel rejection) and sensitivity (ability to receive weak signals) of the superheterodyne receiver will be uniform throughout the tuning range. The modulated carrier at the intermediate frequency is demodulated in the detector stage. Demodulation should correspond to modulation done at the transmitting end. The output of the detector is the message signal and it is amplified by using power amplifiers so that it can drive the output transducer (Speaker).

16.11 TELEVISION

For ages man has dreamt of viewing the scene and hearing the sound at a distance by transmitting the picture and sound from one place to another. It was made true during the year 1925–27 by J.L. Baird in London and by C.F. Jenkins in Washington, both working independently. They used mechanical system of scanning. As electronic vacucum tubes were developed use of the cathode ray tube for electronic scanning and writing purposes was thought of Regular television broadcasts were introduced in the late thirties. The inter carrier sound system, linking sound and picture together was introduced to enable easy tuning particularly on UHF channels which were allotted for TV broadcasting in 1952.

The Radio Corporation of America (RCA) developed a colour TV transmission system compatible with the monochrome system. Systems like NTSC (National Television Sub Committee) in USA. Sequential Colour and Memory (SECAM) in France and Phase Alternate Line (PAL) in Germany were the different colour TV systems available. All the three colour TV systems have survived to be accepted in different countries. The choice has been often affected by the monochrome system standard prevalent in the particular country.

Owing to the use of VHF-UHF frequencies for television broadcasting, direct reception of TV signals is limited to line of sight distances usually ranging from 75–150 km. For extending the broadcast service, relay stations that receive the signal via microwave links or coaxial cable and rebroadcast it to the extended regions are used. Due to the rapid strides made in satellite communication, TV has become a wide ranging and powerful medium of mass communication in this ever shrinking world. It has now become possible to have international programs with global coverage through geostationary satellites.

Since late seventies considerable research interest has been created in developing a television system that could have high definition and hence high quality of images. HDTV (High Definition Television) uses nearly twice as many scan lines as the standard TV and about 5 times the picture details in a wide screen format. In 1988, the Seoul Olympic games were broadcast throughout Japan in HDTV using INTELSAT-II and the direct broadcasting satellite (DBS). Attempts to set international HDTV standards have not been successful because of 30–25 frame rate difference in US, Japanese and European standards and the desire to provide compatibility with existing systems. Stereoscopic 3-D TV using holographic techniques can also be a reality in the coming decade.

16.11.1 Elements of TV System

Television means seeing the scene at a distance. A television system must faithfully reproduce the structural details, relative brightness, motion, sound and colour of the scene. The standard television picture has a ratio of 4 : 3 of width to height which is called as the *aspect ratio*.

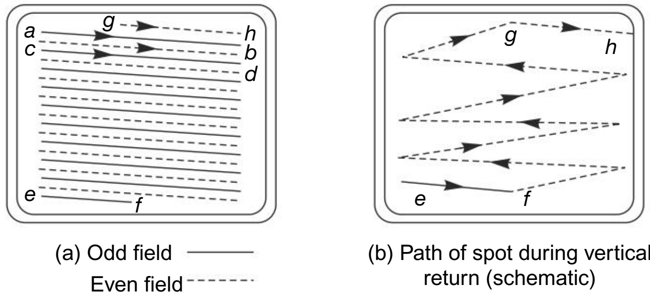


Fig. 16.24 Interlaced Scanning

The picture signal is generated by a TV camera and the sound signal by a microphone. The electron beam of the camera scans the image sequentially from left to right and from top to bottom. The supply frequency in India is 50 Hz and hence the scanning rate is kept 25 photo frames per second. The entire frame is scanned by horizontal scanning lines numbering 625. *Interlaced scanning* is employed where each frame is divided into two fields, i.e., odd field and field as shown in Fig. 16.24. Number of lines per field is 312.5. Therefore, horizontal scanning frequency is $625 \times 25 = 15625$ Hz. Vertical scanning frequency is 50 Hz. Interlaced scanning makes the human eye view the continuous motion pictures without flicker.

In order that the horizontal and vertical scanning circuits of the receiver are kept in synchronism, synchronizing pulses and blanking pulses are combined with the video signal. This signal is known as *composite video signal*. Composite video signals are amplitude modulated (AM) and sound signals are frequency modulated (FM).

TV Transmitter Television broadcasting station in India is assigned a bandwidth of 7 MHz for each channel, as shown in Fig. 16.25. Vestigial Sideband (VSB = Single Sideband + Trace(vestige) of other side band + Carrier) transmission is employed with the picture carrier 1.25 MHz above the low frequency end of the

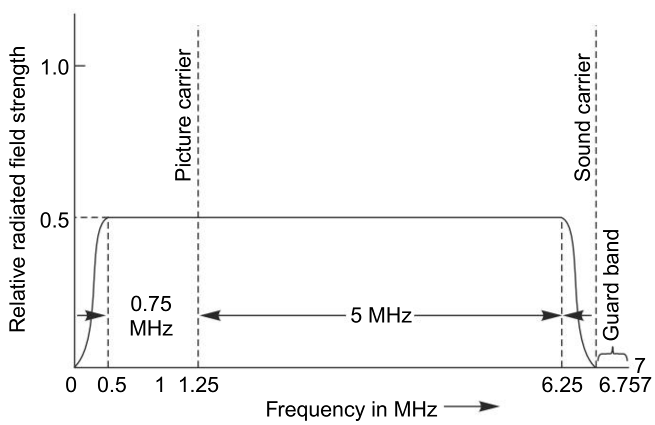


Fig. 16.25 TV Transmitter Channel

band. The sound carrier frequency is 5.5 MHz greater than the picture carrier. The nominal video bandwidth is 5 MHz. The television station are assigned channels in the frequency range from 54 to 216 MHz in the VHF band and 470 to 890 MHz in the UHF band. The colour system in India and several European countries is PAL.

The basic block diagram of a monochrome TV transmitter is shown in Fig. 16.26. A scene is focussed to the camera tube by optical means. The popularly used camera tubes are Image Orthicon, Vidicon and Plumbicon. They are scanned by an electronic beam, where the intensity is modulated by the brightness of the scene. A varying voltage is thus obtained. The camera tube that has photosensitive elements converts the optical image into the equivalent electrical signal. Each picture element is scanned in succession to convey the total information in the scene.

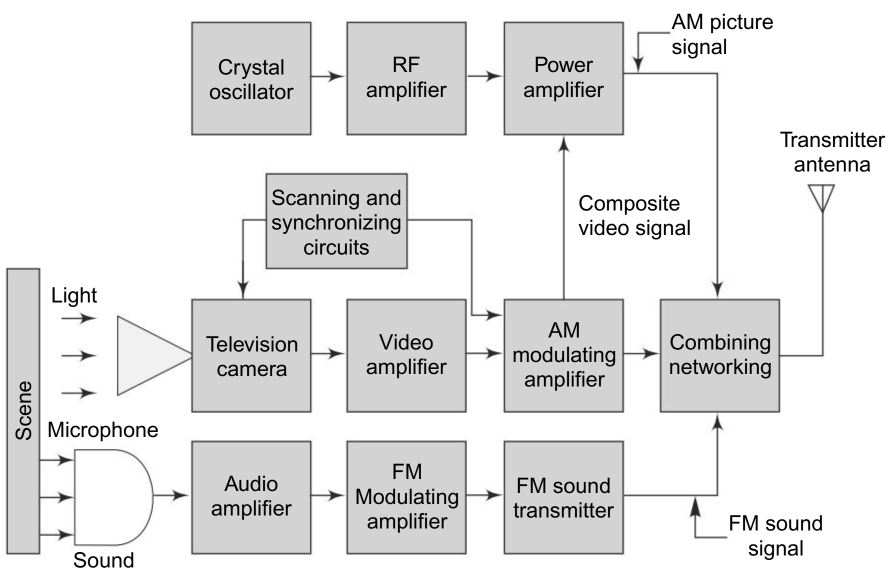


Fig. 16.26 Block Diagram of Monochrome TV Transmitter

The synchronizing or sync information is transmitted in addition to the picture information. The Black-and-White TV requires only the brightness or luminance whereas colour TV requires luminance and chrominance signals. The chrominance signal is assigned in the portions of total frequency spectrum where luminance signal does not use. Colour TV system and monochrome TV must be compatible, i.e., the chrominance signals must be coded in such a way that a satisfactory picture will be produced by a monochrome receiver and vice versa. Sound signals are transmitted along with picture signals.

Scanning and synchronizing circuits produce synchronizing and blanking pulses. Video signal, sync and blank pulses are combined in the AM modulating amplifier. The composite video signals is amplified by video amplifiers. Microphone picks up the sound signals. The sound signals are amplified by audio amplifiers and frequency modulated by the modulating amplifier. Then sound signals and the video

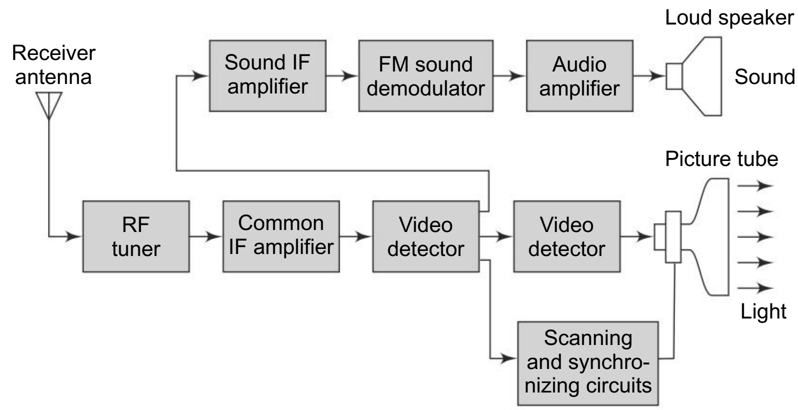


Fig. 16.27 Block Diagram of Black and White TV Receiver

signals are combined with the help of a combining network and then transmitted by the omnidirectional antenna, preferably Turnstile antenna.

TV Receiver The basic block diagram of a black and white TV receiver system is shown in Fig. 16.27.

The UHF antenna, e.g. Yagi-Uda, picks up the signal. The RF Tuner circuit selects the desired carrier frequency. The received signal is converted into IF frequency and amplified using IF amplifiers. The Video IF frequency is 38.9 MHz and Sound IF frequency is 33.4 MHz.

The video detector separates video, sync, blanking and sound signals. Video signals are amplified by the video amplifiers and given to the picture tube. The sync and blanking signals are also applied as control signals to the picture tube. Sound signals are separated and amplified by sound IF amplifiers. Then the sound demodulator demodulates the sound signals which are then amplified by the audio amplifier. The loudspeaker converts the output of the audio amplifier into sound signals.

Colour TV System A colour TV system is essentially the same as the monochrome TV system except that the additional chrominance information is also be sent along with the luminance signal. Red, green and blue (RGB) are the primary colours. The colour picture can be obtained by appropriately combining these primary colours. The luminance Y of a picture element is equal to 59% of green (G), 30% of red (R) and 11% of blue (B). The colour camera outputs are modified to obtain B – Y and R – Y (chrominance) signals.

16.12 MICROWAVE COMMUNICATION

Electromagnetic waves is the frequency range of 1 GHz to 30 GHz are referred to as ***microwaves***. As microwaves travel only on line-of-sight paths, the transmitter and receiver should be visible to each other. Hence, it is necessary to provide repeater stations in between the terminal stations at about 50 km intervals. As microwave communication offers a large transmission bandwidth, many thousands of telephone channels along with a few TV channels can be transmitted over the same route using the same facilities. Normally, carrier frequencies in the 3 to 12 GHz range are used for microwave communication. The transmitter output powers can be low because highly directional high gain antennas are used. Figure 16.28 shows the equipment needed to provide one channel of a ground based microwave system.

It consists of two terminal stations and one or more repeater stations. At the sending terminal several thousand telephone channels and one or two television channel(s) are frequency multiplexed to form the base band signal. The base band signal is allowed to frequency modulate an Intermediate Frequency (IF) carrier in the lower frequency range, which is then up converted to the microwave output frequency of 4 GHz. This signal is amplified and fed through a directional antenna towards a repeater station at a distance of about 50 kM.

At the repeater station, the signal is received on one antenna directed towards the originating station. The received signal is down converted to IF, amplified and up converted to a new frequency of 6 GHz. The frequency conversion is done so that the outgoing and incoming signals do not interfere with each other in the repeater stations. This signal is retransmitted towards the receiving terminal stations where it is down converted to the IF and demodulated to recover the base band signal. This base band signal is then demultiplexed to recover the individual telephone or television channel signals.

Presently, microwave communications are widely used for telephone networks, in broadcast and television systems and in several other communication applications by services, railways, etc.

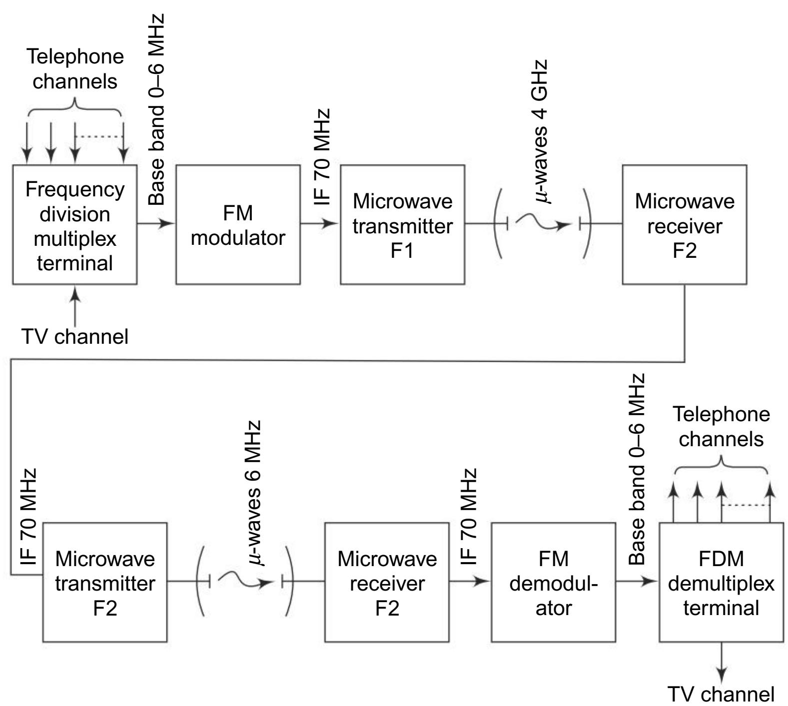


Fig. 16.28 A Microwave Relay System

16.13 SATELLITE COMMUNICATION

A satellite is a radio repeater, also called transponder, placed in the sky. A satellite system, as shown in Fig. 16.29 consists of a transponder and a minimum of two earth stations, one for transmission and another for reception. The transponder receives the signal from the transmitting earth station, frequency converts, amplifies and retransmits the signal towards the receiving earth station.

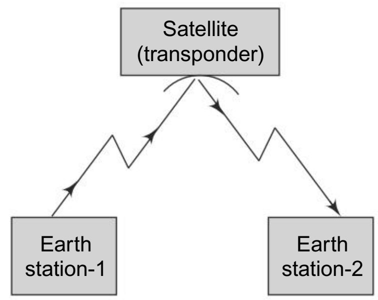


Fig. 16.29 Satellite System

The satellites are generally classified as passive and active types. A passive satellite simply reflects a signal back to earth and there are no gain devices on-board to amplify the signal. On the other hand, an active satellite receives, amplifies and retransmits the signal back towards earth.

Orbital Patterns Once launched, a satellite remains in orbit because the centrifugal force caused by its rotation around the earth is counter-balanced by the earth's gravitational pull. The closer to earth the satellite rotates, the greater the gravitational pull, and greater the velocity required to keep it from being pulled to earth.

Low altitude satellites that orbit close to the earth (150 to 500 km in height) travel at approximately 28,500 km/hr. At this speed, it takes approximately 90 minutes to rotate around the entire earth. Consequently, the time that the satellite is in the Line of Sight (LOS) of a particular earth station is 15 minutes or less per orbit.

Medium altitude satellites (9,500 to 19,000 km in height) have a rotation period of 5 to 12 hours and remain in LOS of a particular earth station for 2 to 4 hours per orbit.

High altitude, geosynchronous satellite is a satellite which is placed at a height of 35,786 km from the earth's surface and has an orbital velocity equal to that of the earth's. A geosynchronous satellite that lies on the earth's equatorial plane is called plane is called as the *geo-stationary satellite*. A geo-stationary satellite remains in a fixed position with respect to a given earth station and has 24 hours availability time. Three geostationary satellites spaced 120° apart can cover the whole world.

Based on the area of the coverage, the orbits are classified as *(i) Polar orbit. (ii) Inclined orbit and (iii) Equatorial orbit.* When the satellite rotates in an orbit above the equator, it is called an equatorial orbit. When the satellite rotates in an orbit that takes it over the north and south poles, it is called polar orbit. Any other orbital path is called an inclined orbit. 100% of the earth' s surface can be covered with a single satellite in a polar orbit. The satellite is rotating around the earth in a longitudinal orbit while the earth is rotating on a latitudinal axis. As a result, every location on earth lies within the radiation pattern of the satellite twice each day.

16.13.1 Satellite System

A satellite system, consists of three basic sections; the uplink (transmitting earth station), the satellite transponder and the down link (receiving earth station). Typical

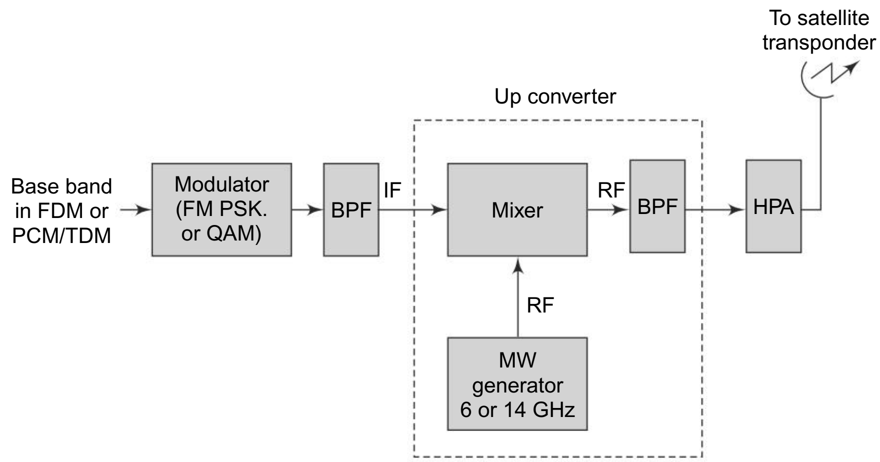


Fig. 16.30 Block Diagram of Earth Station Transmitter

frequencies for telecommunication services in a satellite system are 6/4 GHz and 14/12 GHz, where 6 GHz and 14 GHz represent uplink frequencies and 4 GHz and 12 GHz, the downlink frequencies.

Uplink Model The primary component of the uplink section of a satellite system is the earth station transmitter. Figure 16.30 is the block diagram representation of the earth station transmitter.

A typical earth station transmitter consists of an IF modulator, an IF-to-RF microwave upconverter, a high power amplifier (HPA) and a Band Pass Filter (BPF). The IF modulator converts the input base band signals to either an FM, or a PSK modulated intermediate frequency. The upconverter translates the IF to an appropriate RF carrier frequency. The HPA provides adequate output power to propagate the signal to the satellite transponder. More widely used HPAs are Klystrons and Travelling Wave Tube Amplifiers (TWTAs).

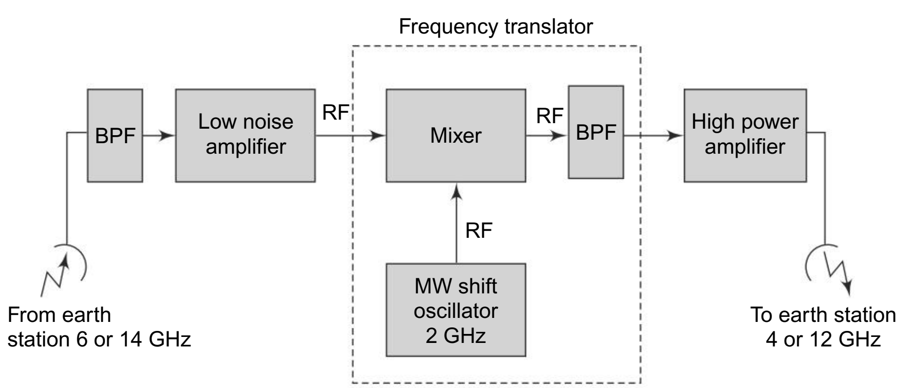


Fig. 16.31 Block Diagram of the Satellite Transponder

Satellite Transponder A satellite transponder shown in Fig. 16.31 consists of a Band Pass Filter (BPF), an input low noise amplifier (LNA), a frequency translator and a high power amplifier. The transponder is an RF-to-RF repeater.

Satellite Downlink Model An earth station receiver includes an input BPF, an LNA and an RF-to IF down converter. Figure 16.32 shows the block diagram of a typical earth station receiver. The input BPF restricts the input noise power to the LNA. The LNA normally used is a tunnel diode amplifier or a parametric amplifier. The RF to IF down converter is a mixer-BPF-combination which converts the received RF signal to an IF frequency.

In a satellite system where three or more earth stations wish to communicate with each other, any of the three methods of multiple accessing called Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) are required.

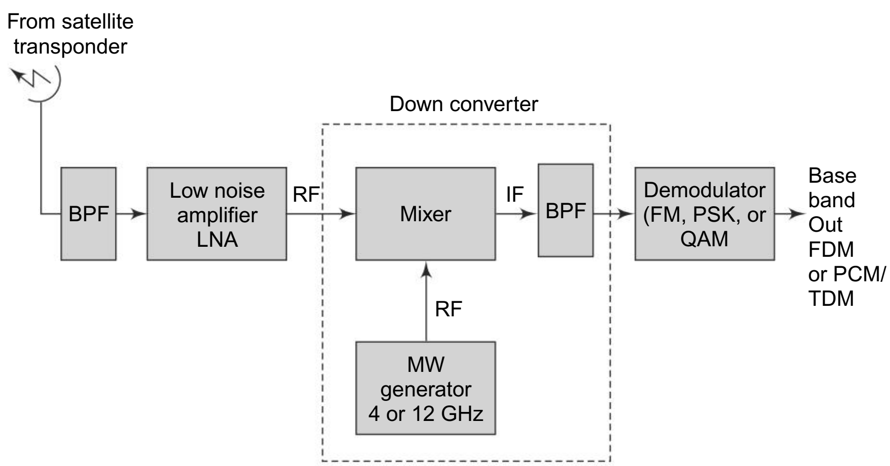


Fig. 16.32 Block Diagram of a Typical Earth Station Receiver

Communication Satellites USA based International Telecommunication Satellite Organisation has been launching INTELSATs (International Satellites) that provide telecommunication services to its 119 member countries throughout the world. Similarly, Indian Space Research Organisation (ISRO) of India has been launching INSATs (Indian Satellites) which provide communication services to the Indian region. The telecommunication includes not only telephone but also TV, digital transmission services, telegraphy, telex, video conferencing, video text, etc.

Advantages Satellite communication offers a number of features not readily available with other means of communication. Since a very large area of the earth's surface is visible from a satellite, the satellite can provide line of sight coverage for a large number of users. Hence the satellite system provides point to multipoint communication. The satellite offers telecommunication links which include telephone, TV, telegraphy, telex, FAX, video conferencing, video text, digital transmission services, etc. to remote communities in sparsely populated areas like north eastern states and hilly terrain like Ladakh, Himachal Pradesh, etc. Finally, the satellite system shows better face for search, rescue, mobile, meteorological, and navigational purposes.

With the advent of the geostationary satellites and low altitude satellites like Iridium and Global Positioning System (GPS), the world has been reduced to *global village*.

Disadvantages When communication is done through geostationary satellites, due to the large distance involved (approximately 75000 km), there is a large time delay of 250 milliseconds between the transmission and reception of a signal. A satellite once launched and placed in its orbit, the malfunctions in the satellite are highly difficult to correct. The initial cost involved is quite large.

16.14 RADAR SYSTEM

Radar is an acronym for Radio Detection and Ranging. It is an electromagnetic system for the detection and location of objects. The radar helps in extending one's senses for observing the environment. Sometimes, a radar can perform much better than human eyes. Radar can see through fog, mist, darkness and is capable of measuring the distance of the object from the point of observation, which human eyes cannot do. The developments in radar have been tremendous and its applications are very wide. However, the principle with which the radar operates is much simpler than how human eyes perceive vision.

A simple radar system consists of two antennae, one for transmitting electromagnetic signal and the other for receiving the echo signal from the objects which intercept the transmitted signal. It also consists of a source of electromagnetic signals and a receiver to process the echo signal. Assuming that a pulse of electromagnetic signal is transmitted at time t_0, and that the echo signal is received at time t_1, the distance from the point of transmission and the intercepting object (target) is given by,

$$R = \frac{c(t_1 - t_0)}{2} \tag{16.10}$$

where $(t_1 - t_0)$ is the time taken by the transmitted signal to travel to the target and return, c is the velocity of the electromagnetic pulse, which is same as the velocity of light and R is the distance or range of the target. The above equation can be used to determine the range of the target.

The Doppler's effect can be applied to determine whether the target is approaching or receding from the radar. When either the source of radiation or the observer of the radiation is in motion, there is an apparent shift in the frequency of radiation. This is called Doppler's effect. In the case of a moving target and a fixed radar system, the frequency of the echo signal will have a change; it increases if the target is coming towards the radar or decreases if the target goes away from radar.

Range Equation The radar range equation relates the range, i.e. the distance between the radar and the target, to the characteristics of the transmitter, receiver, antenna; target and environment. A simple form of range equation is derived here.

Let the power of the signal transmitted from the radar be P_t watts. If the gain of the transmitting antenna is G, the power density of the signal intercepted by the target at a distance R is given by,

$$\text{Power density from the antenna} = \frac{P_t G}{4\pi R^2} \tag{16.11}$$

The transmitted signal is intercepted by the target, however the signal is not intercepted by the entire surface area of the target. Let us call the surface of the target that intercepts the transmitted signal as 'radar cross-section of target σ'. The radar cross-section has units of area.

The signal after being intercepted by the target is scattered in all directions. A portion of this signal which reaches the radar back is called the 'echo'. The power density of the echo signal at the radar is

$$\text{Power density of echo} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$

The amount of power received by the antenna from this echo signal is directly proportional to the effective aperture area A_e of the antenna. Therefore, the power received by the receiving antenna is

$$P_r = \frac{P_t G}{4\pi R^2} \frac{\sigma A_e}{4\pi R^2} = \frac{P_t G \, \sigma A_e}{(4\pi)^2 R^4}$$

From this equation, it is clear that the received power is reduced as the range of the target increases. Let us define the maximum range, R_{max} of a target as that range beyond which the power of the received echo signal is negligibly small.

That is,

$$P_{r(\min)} = \frac{P_t G \, \sigma A_e}{(4\pi)^2 R_{\max}^4}$$

Rearranging the above equation, the maximum range of a target is obtained as

$$R_{\max} = \left[\frac{P_t G \, \sigma A_e}{(4\pi)^2 P_{r(\min)}} \right]^{1/4} \tag{16.12}$$

This gives the fundamental form of the radar equation.

A Simple Radar System The block diagram of a simple pulse radar is shown in Fig. 16.33.

Transmitter It consists of a high power oscillator and a modulator to generate a repetitive train of pulses.

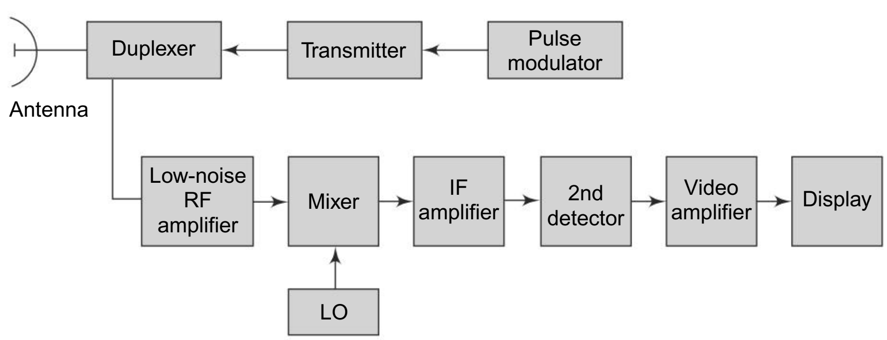


Fig. 16.33 Block Diagram of Simple Pulse Radar

Duplexer A duplexer is used whenever a single antenna is used for transmitting and receiving. The function of a duplexer is to protect the receiver from damage caused by the high power of the transmitter. It also channels the returned echo signals to the receiver and not to the transmitter.

RF amplifier This forms the first stage of the superheterodyne receiver. The purpose of this amplifier is to amplify the received echo signals to an acceptable level before processing it.

Mixer and Local Oscillator (LO) The mixer and local oscillator convert the RF signal to an intermediate frequency (IF) that is suitable for the receiver.

IF Amplifier and Second Detector The output from the mixer is an intermediate frequency of around 60 MHz for a typical air-surveillance radar. The IF amplifier increases the signal-to-noise ratio. i.e. it increases the signal power with respect to the noise power.

In the transmitter section, the continuous signal from the oscillator was modulated to form a train of pulses. The second detector is used to extract this pulse modulation information.

Display The ouput from the second dectector is amplified and is referred to as "raw data". This raw data is properly processed and applied to a display unit. The display unit can be any one of the following:

A-Scope It is a deflection-modulator CRT in which the vertical deflection is proportional to target echo strength and the horizontal coordinate is proportional to range.

B-Scope It is an intensity-modulator display with azimuth angle indicated by the horizontal coordinate and range by the vertical coordinate.

C-Scope It is an intensity-modulated display with azimuth angle indicated by the horizontal coordinate and elevation angle indicated by the vertical axis.

PPI or Plan Position Indicator It is sometimes called P-scope. It is an intensity-modulated circular display on which echo signals produced from reflecting objects are shown in plan position with range and azimuth angle displayed in polar coordinates, forming a map like display.

RHI or Range-Height Indicator It is an intensity modulated display with altitude as the vertical axis and range as the horizontal axis.

There are many other display devices with additional facilities like the display of alphanumeric characters, symbols, etc.

16.14.1 Types of Radars

A variety of radars have been developed and they are classified based on (i) the signal transmitted (ii) the target of interest and (iii) frequency of operation. Some of the commonly used radars are discussed as follows.

Continuous Wave (CW) Radar The radar transmitter transmits a continuous signal rather than a pulse one. The echo signal can be easily distinguished from the transmitted signal as there will be a shift in the frequency of the echo signal due to the Doppler effect. Also, the echo signal's strength is many times lower than the transmitted signal strength. The Doppler frequency shift is given by

$$f_d = \frac{2\, V_r f_o}{c} \tag{16.13}$$

where f_d = Doppler frequency shift
V_r = relative velocity of the target with respect to radar
f_o = transmitted frequency, and
c = velocity of light or propagation of electromagnetic signal
$= 3 \times 10^8$ m/s.

Using a CW radar, the relative velocity of a target can be determined. The drawback of a CW radar is that the range of the target cannot be determined.

Moving Target Indication (MTI) Radar The MTI radar transmits a train of electromagnetic pulses. This radar system is capable of distinguishing a moving target from the non-moving objects. The non-moving objects in atmosphere that intercept the transmitted signal and scatter it are called clutters. Since there will not be any Doppler frequency shift from fixed targets, as there is no relative motion, they are easily identified from the moving target. The echo from the moving target has varying frequencies due to Doppler effect.

Tracking Radar A tracking radar system locks on to a particular measures its coordinates and provides data from which the future course of the target can be predicted.

The tracking radar is similar to other radars except that the antenna beam of a tracking radar is made to look at the target always. This is achieved by a servomechanism activated by error signals. The error signal is obtained using several techniques, viz. sequential lobing, conical scan and simultaneous lobing.

Applications of Radar Radar systems find their applications on the ground, in the air, on the sea and in space. The ground based radars have been used in the detection, location and tracing of aircraft or space targets. The shipboard radars are used to navigate the ships and to locate buoys, shorelines and other ships. They are also used to observe aircraft. The airborne radars are used to detect land vehicles, ships and other aircraft. However, the principal application of airborne radars is for mapping of land storm avoidance and navigation. In space, radar is used for remote sensing purposes.

16.15 OPTICAL FIBRE COMMUNICATION

The principal motivations behind new communication systems are (i) to improve transmission fidelity, (ii) to increase the data rate (more information transmitted) and (iii) to increase the transmission distance between relay stations. Optical frequencies lie in the range 10^{14} Hz to 10^{15} Hz. The laser information carrying capacity is greater than the microwave system by a factor of 10^5. A fibre can carry approximately 10 million TV channels.

In optical fibre communication, electromagnetic waves in the optical frequency region is used as the carrier. The schematic block diagram of an optical fibre

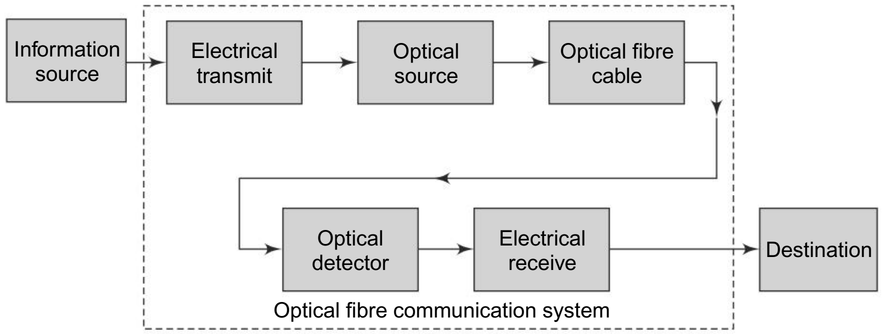


Fig. 16.34 Schematic Block Diagram of Optical Fibre Communication System

communication system is shown in Fig. 16.34. The message to be transmitted is converted into a suitable electrical form by the electrical transmit section. This electrical signal is allowed to modulate the light output from the optical source, which may be either a light emitting diode (LED) or an injection laser diode (ILD). The modulated light is launched into the optical fibre which is the communication channel linking the transmitter with the receiver. At the receiving end, the input optical signal is converted into suitable electrical variations by the optical detector, which may be either a PIN photodiode or Avalanche photodiode. These electrical variations are converted to the original message form in the electrical receive section and given to the destination.

Optical Fibre An optical fibre is a piece of very thin (hair-thin), highly pure glass, with an outside cladding of glass that is similar, but because of a slightly different chemical composition, has a different refractive index. As shown in Fig. 16.35, the simplest optical fibre consists of a central cylindrical core of constant refractive index n_1 and a concentric cladding surrounding the core of slightly lower refractive index n_2. An optic fibre cable is quite similar in appearance to the coaxial cable system. This type of fibre is called *step index fibre*, whose core diameter is in the range of 2 to 200 μm, as the refractive index makes a step change at the core-cladding interface. The refractive index profile which gives the variation of refractive index with distance along the cross section of the fibre may be defined as

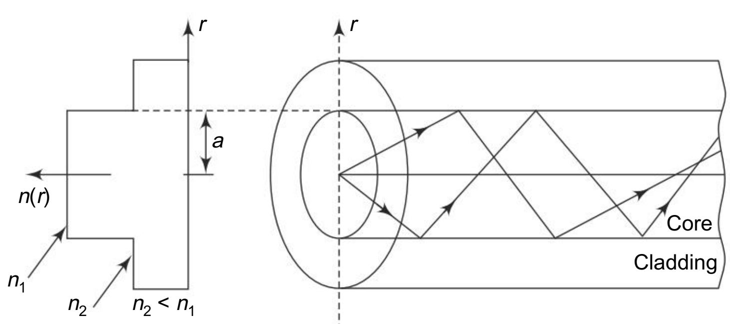


Fig. 16.35 Refractive Index Profile and Ray Transmission in Step Index Fibre

$$n(r) = n_1 \quad r < a \text{ (core)}$$
$$= n_2 \quad r \geq a \text{ (cladding)}$$

The refractive indices of core and cladding are related by the relative refractive index difference (Δ) between the core and the cladding by the relation

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$
$$= \frac{n_1 - n_2}{n_1}, \text{ where } n_1 = n_2 \tag{16.14}$$

As the core and the cladding are normally made of glass or plastic, the refractive indices n_1 and n_2 lie around 1.5. Step index fibre may be used for multimode or single-mode propagation.

If a light ray travelling in the core of higher refractive index is incident at the core-cladding interface with an angle of incidence, with respect to the normal, greater than the critical angle, it will be reflected back into the originating dielectric medium, i.e. core, with high efficiency (around 99%). This phenomenon is known as *total internal reflection*. In an optical fibre, transmission of light ray takes place by a series of total internal reflections at the core-cladding interface as shown in Fig. 16.36.

In the *graded-index* fibre, the refractive index gradually reduces from the centre to the outside of the fibre cross-section. The graded index fibres were initially easier to manufacture. Lower attenuations are possible with step index fibres.

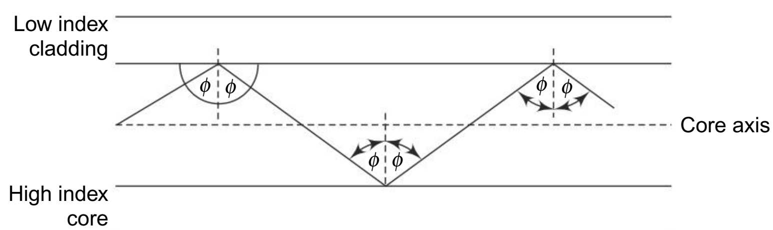


Fig. 16.36 Transmission of Light Ray in a Perfect Optical Fibre

Advantages of Optical Fibre Communication

(i) As an optical carrier in the range 10^{14} to 10^{15} Hz is used, the system has enormous potential bandwidth.

(ii) Optical fibres have very small diameters and hence they are of small and weight.

(iii) Optical fibres are fabricated from glass or a plastic polymer which are electrical insulators so that good electrical isolation in a hazardous environment is possible.

(iv) As optical fibre is a dielectric waveguide, it is free from electromagnetic interference (EMI), radio frequency interference (RFI), or switching transients giving electromagnetic pulses.

(v) As optical fibres do not radiate light, they provide a high degree of signal security and cross talk between parallel fibres is avoided.

(vi) Optical fibre cables exhibit very low attenuation compared with copper cables making them suitable for long-haul telecommunication applications. Hence, they are highly reliable and easy to maintain.

(vii) Optical fibres are manufactured with very high tensile strenths and they are flexible, compact and extremely rugged.

(viii) The glass from which the optical fibres are made is derived from sand which is a natural resource. So, in comparison with copper conductors, optical fibres offer the potential for low cost line communication.

(ix) Optical fibres offer high tolerance to temperature extremes as well as to liquids and corrosive gases and have longer life span.

Applications

(i) Computers Fibres are used because of their greater data handling capacities and higher memory densities. The fibres are used for mutual interfacing of the central processor, linking them to peripheral devices, data transmission within the mainframe. Hence reduced bit errors and free environmental interference are assured.

(ii) LAN In local area networks, fibres are used to link the computers in ring, star or bus topology. They find applications in office operations, private automatic brance exchanges, etc.

(iii) Industrial Electronics They are used in power plants, rail road networks and metal industry for data acquisition control and signal processing, as transmission is not affected by high energy fields. In medical electronics, by using fibres, noise free control signals are possible. In automobile manufacturing, fibres are used as sensors because of less weight.

(iv) Telecommunications The enormous bandwidth of optical fibre communication system finds their principle use in long distance communication for transmission of speech, video and digital data signals. In the near future all the, existing copper based trunk lines will be replaced by optical fibres.

16.15.1 Comparison of Conventional-Electrical and Optical Communication Systems

The Table 16.2 gives the comparison of conventional electrical and optical communication systems.

16.16 ISDN

Integrated Services Digital Network (ISDN) is defined as "A network that provides end to end digital connectivity between users to support a wide range of services including telephony (voice and music), data (telemetry, E-mail and alarm), Text telex, teletex and videotex) and image (Facsimile, TV conferencing, video phone)"

ISDN also provides supplementary services like Direct Dialing In, Call Wait and Call Hold, etc. In Direct Dialing In, the user can contact a person directly in an

Table 16.2 Comparison of Conventional-Electrical and Optical Communication Systems

Parameters	*Conventional-Electrical*	*Optical*
Carrier frequency	$10^4 - 10^{10}$ Hz	$10^{14} - 10^{15}$ Hz
Bandwidth	Smaller to medium	Larger about a million times greater than RF carrier
Source characteristic	Inherently coherent	Coherent and non-coherent (LED/LD)
Signal	Signal propagates as voltage or current	Signal propagates as wavefront of light
Modulation formats	AM/FM/PM/PCM are possible	Intensity modulation, phase, frequency polarisation are possible
Analog, Digital transmission of signals	Both are feasible	Both are feasible
Channel	Free space as a guided wave channel, coaxial cable and Metallic waveguides	Free space and fibres (dielectric)
Techniques for modulation and demodulation	Purely electronics	Either purely electronics or hybrid versions of electronic and optical methods
Detection Components	Purely electronics R, L, C	Opto-electronics methods Lenses, mirrors, beam splitters, gratings, prisms and R, L, C
Theoretical aspects	Linear System Theory (LST) and Statistical Theory of Communication (STC)	LST, STC, Optical diffraction and interference theory
Microminiaturisation	LSI, VLSI	Through Integrated Optics (IO) using thin film technology
Power density	Moderate	High
Transmitter antenna and receiver antenna size	Larger	Small
Attenuation	Coaxial cables have 10 dB/km	Optical fibres have 0.25 dB/km

office by dialing the extension number without the operator's intervention when the called person is connected to Private Branch Exchange (PBX).

The evolution of ISDN is based on two important technological developments, viz. (i) digital transmission and (ii) digital switching, Also, this evolution is driven by the need to provide economical voice communication and a variety of digital data services.

Architecture of ISDN Figure 16.37 shows the basic block diagram of ISDN architecture. It consists of common physical interface, ISDN central office, digital subscriber loop. ISDN channels and integrated digital network that are explained as follows.

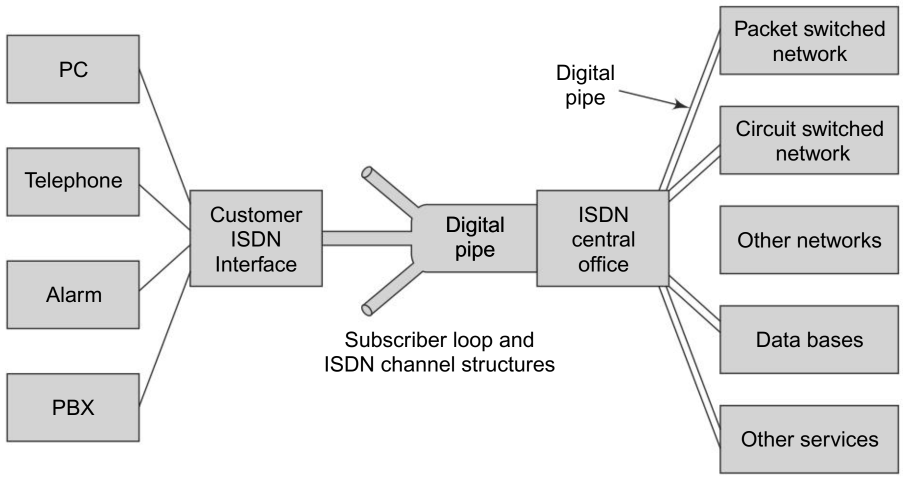


Fig. 16.37 Block Diagram of ISDN Architecture

Common Physical Interface In an ISDN, all devices such as Digital Telephone, Alarm, Computer terminal, Videotex, Facsimile, PBX and even a Local Area Network (LAN) can be connected to the transmission line using the common physical interface. It essentially provides a DTE-DCE connection, where DTE is Data Terminal Equipment and DCE is Data Communication Equipment.

Central Office It connects the numerous ISDN subscriber loop signals to the Integrated Digital Network (IDN). It provides subscribers the access to circuit switched networks, packet switched networks, databases and other services.

Digital Subscriber Loop and ISDN Channels The digital subscriber loop is the connection between common physical interface and the ISDN central office. In ISDN, this is one or two twisted pairs of copper cable or a fibre optic link that provides full duplex digital transmission. It has two different ISDN channel structures, viz, (i) Basic channel structure and (ii) Primary channel structure, Both the channel structures are constructed from the following channels:

(i) B-channel (64 kbps)
(ii) D-channel (16 or 64 kbps) and
(iii) H-channel (384, 1536 and 1920 kbps)

The *B-channel* is the basic user channel and used to carry digital data, PCM-enclosed digital voice or a mixture of lower rate digital data and digitized voice. Over a B-channel, one can set up circuit switched, packet switched or leased line connections.

The *D-channel* is used to carry either signalling information to control circuit switched calls or to carry low speed data (e.g. telemetry at 100 bps speed) over a packet switched connection when no signalling is required.

The *H-channel* is used to carry data at higher bit rates or speed. The applications which require high speed data transfer are: facsimile, video, high quality audio (music) and multiple information streams.

The *Basic channel structure* consists of two full duplex 64 kbps *B* channels and one full duplex 16 kbps *D* channel. So, the composition of Basic Service is $B + B + D$ or $2B + D$. The basic service allows individual, residential and office users to access simultaneous use of voice and several data applications such as packet switched data, control alarm service, fax, video-tex and so on.

The *Primary channel structure* is meant for users who need higher capacity applications such as offices with a digital PBX or a local area network. It consists of two different compositions of *B* and *D* channels (i) 30 *B* channels of 64 kbps each + 1 *D* channel of 64 kbps and (ii) 23 *B* channel of 64 kbps each + 1 *D* channel of 64 kbps. In addition, the primary channel structure can be constructed using *H*-channels instead of *B*-channels.

Broad Band ISDN A broad band ISDN or B-ISDN is a network of large bandwidth to carry data voice, image and video at a very high data rate. The proposed access rate for the B-ISDN is 150 Mbps. Such high data rate is adequate for image traffic and also allows the interconnection of high speed LANs. This high access rate also allows video broadcast traffic, video conferencing and many potential new applications.

Advantages of ISDN

Cost Due to rapid advancements in VLSI (Very Large Scale Integration) technology, the cost of digital switches is decreasing whereas the cost of analog switches has not changed for more than a decade.

Integrity The voice or data signals get attenuated as they travel because of losses in transmission medium. Hence amplifiers are placed along the transmission path in analog networks. Amplifiers amplify noise along with signals. In digital neworks, repeaters provided along the transmission path regenerate a new outgoing signal from the incoming distorted signal and hence noise is not additive. Therefore, the transmission efficiency is high in ISDN.

Capacity Utilisation Now a days, copper cables are replaced by fibre optic cables which offer very large bandwidth. Analog multiplexing techniques such as Frequency Division Multiplexing (FDM) cannot utilise this bandwidth effectively whereas digital multiplexing techniques such as Time Division Multiplexing (TDM) can utilise it effectively.

Security and Privacy In an analog network, the subscribers are connected through intermediate junction boxes and from these intermediate points, one can listen to conversation secretly and also make calls on other's account. Whereas ISDN provides end to end connectivity. i.e. the subscribers are directly connected to the exchange without any intermediate junction and hence security and privacy are enhanced.

Integration The various user services share the same physical transmission path at the same time in ISDN, whereas these integrated services are not possible in the case of analog networks.

Constraints in ISDN Implementation The main constraint in converting today's networks into digital networks is the modification of the existing local loop which is a pair of twisted copper wires providing full duplex transmission in analog networks. Since the local loop is optimised for human voice, it cannot transmit signals above 4 kHz.

16.17 INTERNET

Internet is a giant network of computers located all over the world that communicate with each other. Internet began in late 1960's as a project from ARPA (Advanced Research Project Agency) of USA department of Defence. It was an experimental network intended to allow scientists receiving research grants from ARPA to communicate with each other. In 1980's, NSF (National Science Foundation) of USA upgraded ARPA net as NSFnet. It was intended to link a dozen super computer centres around USA with high speed links. Later, regional networks also had links with NSFnet. Nowadays, Government organisations, research institutions and commercial organisations worldwide have begun offering access to the Internet.

Internet Organisation Internet is so large and complex that nobody knows how computers are connected to it and how many users are actually utilising it. There is a managing unit that assigns a number to the central or larger computer system that makes up the backbone of the Internet. Nobody is truly incharge of managing the Internet, because no system is keeping track of the entire organisation structure of the Internet. As shown in Fig. 16.38, data sent from remote PCs passes through the Local Internet System (LIS) where the data is converted into packets that can take any number of different routes to reach the destination PCs. Different parts of the same electronic message may travel over different internet routes to adjust for network traffic. The end users will not be aware of the data communication taking place in the internet.

Hardware and Software Any user wanting to communicate with the internet needs the required *hardware* and *software*.

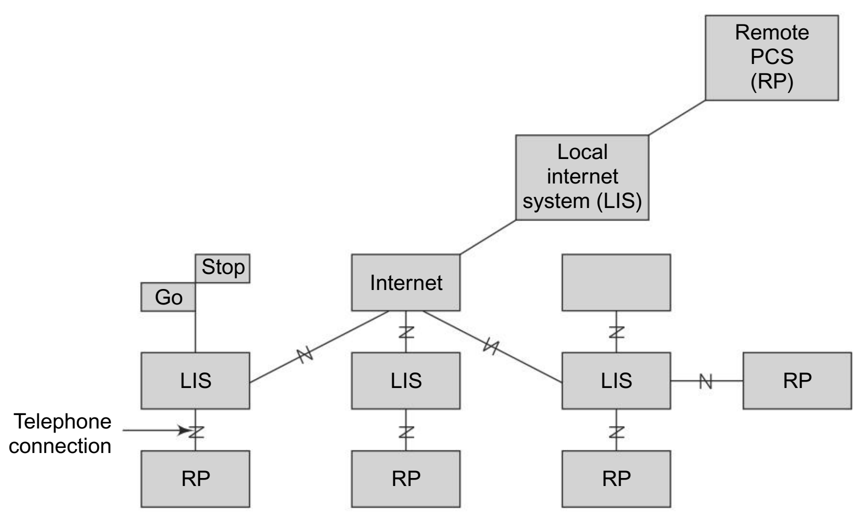


Fig. 16.38 Organisation of Internet

Hardware As shown in Fig. 16.39, a PC has been connected to the internet service provider through public switched telephone network with the help of a modem, which translates the digital signals from PCs to analog signals and transmits them over telephone lines. Any internet machine in its geographic area has an account.

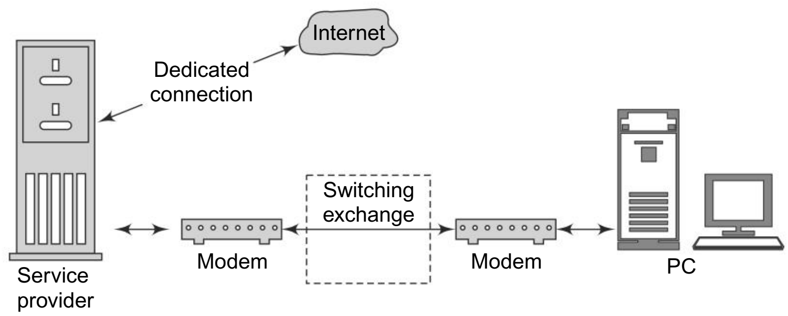


Fig. 16.39 Connecting a PC to the Internet Service

Software The type of tasks that can be performed on the internet will depend on the sort of terminal the communication software can emulate. When an account on the internet system is created, it will most likely be asked what kind of terminal the user has for processing. Most of the typical programs on internet machines need to emulate a VT-l00 terminal. VT-l00 is manufactured by Digital Equipment Corporation which has become an Industry Standard.

Some Functions of Internet

1. E-mail
2. Login to another computer
3. Download/Upload program on internet
4. Browsing on internet

E-mail (Electronic mail) Through E-mail, messages are sent electronically to a user across the country or around the world within a few seconds. It is the fastest, efficient and convenient method of information transfer. One can send E-mail to hundreds of people at a time. The user can also include copies of pertinent text from another person in his letter rather than reminding him what the other user has already said. E-Mail also has the facility of using the mailing list which automatically sends messages to everyone in the list.

Address format The internet provider allocates the address to the users. Like regular post office addresses, the Internet address has a name—the login ID—as an address, generally separated by @. You can break the address domain to computer's name followed by information about the computer's domain. A domain is a group to which the computers belong as listed below:

Domain Name	***Meaning***
.com	Commercial
.edu	Educational

.gov	Government body
.mil	Military
.org	Organisation

The following are a few examples of internet addresses.

user name @ aol.com	–User connect to American Online
user name @ appleline.apple.com	–User connect to Apple net
user name @ Attmail.com	–User connect to AT and T mail
user name @ host. .BITNET	–User connect to BITNET

Here.aol, .applelink, .attmail are the different groups that are service providers of internet. Messages can be sent to any internet user by using their addresses.

Each machine on Internet learns about other machines through Domain Name Server data base distributed throughout the Internet. When an E-mail is sent to another machine, the system looks up the machines' names in DNS (Domain Name Server) and retrieves the address. The E-mail is sent to the destination that accepts the incoming mail of that address. The route the E-mail takes is determined by routers. Routers are hardware devices that figure out the most efficient path for the data to take.

Reading E-mail When the system is logged in, the user might be greeted by "You have mail".

When the user presses the enter key, then he can read the message sent by somebody. E-mail can also be saved in a file. The user can also delete his/her mail if not necessary. Users can send the same message to multiple persons. Two persons can communicate in real time using E-mail. Users can send large files through E-mail in a few seconds.

Logging into Another Computer (Telnet) Internet gives the user, the capability of logging into another computer that permits it so that the services offered by that system can be utilised by the user. By logging into an internet system of another computer one can take advantage of the services of few of those systems without having to dial them directly. Other services that could be used by logging into a remote computer include data warehouses at universities and educational institutions around the world. Information can be obtained such as access to library card, catalogues, bibliographic data bases, and government publications.

The path through which the remote terminal has been accessed should be remembered while moving between multiple sites using telnet/rlogin. If not so, it will slow down the user's performance and cost more money.

Download/Upload program on Internet (FTP) For downloading and uploading programs the user needs to use File Transfer Protocol, a software, which makes exchanging data faster than any other method like E-mail.

Any type of data like weather maps, political data, IBM PC software, information about Internet available in the Internet can be downloaded using FTP. FTP works on the Client/Server principle. A client program enables the user to interact with a server in order to access information. Files that to be transferred are stored in FTP server. FTP client program interface allows the user to locate the file(s) to be transferred and initiate the transfer process. Files available publicity to download can be taken from

Share-ware-Software which can be used freely for a specified amount of period as trial.

Documents-Research papers, articles, etc.

Freeware-Fonts, Cliparts and games.

More than one file can be downloaded for the same command. Large files may be compressed so that they may consume less space and less money and can be downloaded easily.

Using Netnews Internet connects the user with many people all over the world having common topics of interest such as cinema, sports, politics, history, etc.

Netnews is used by the public who subscribe to news group. Information gets expired or automatically deleted after a specified period. Netnews is huge to measure and transmits 10 mega bytes of data per day.

Netnews groups are created depending upon RFD (Request for Discuss) or RFV (Request for Vote).

A few examples of the netnews are listed as follows.

Category	***Meaning***
bionet	Biology network
biz	Business
Comp	Computer based
news	Netnews covering network itself
talk	Talk about topics
Sci	Scientific topics
Misc	Miscellaneous

Netnews has commands that can help the user to find an article on a particular topic. It is possible for the Internet subscriber to read, save and post an article to the group.

Browsing on Internet To find information in Internet, the following are available.

(i) Who is	(ii) Finger	(iii) UU commands
(iv) Gopher	(v) Wals	(vi) World Wide Web.

(i) **Who is**–refers to identification of persons

(ii) **Finger**–provides information like user name, who has last logged in, how long his session had been idle, when he has viewed his last E- etc.

(iii) **UU Commands**–information could be received about machines connected to Internet by UUCP (Unix to Unix copy).

UU Hosts–information is received about a particular site who is responsible for it and what sort of UUCP connections it has with other sites

UU Where–displays the path between the machine, the user is currently us and the site machine.

(iv) Gopher–Gopher is a software protocol designed to search, retrieve and display documents from remote sites on internet.

(v) WAIS–Wide Area Information Service

WAIS is a software protocol having the capability of searching more than data line. The user has the provision of choosing the data for his requirement. Then he submits the query and gets the response for the query.

(vi) WWW: World Wide Web

WWW is the most important aspect of Internet and has accelerated the growth of Internet. It is an easy to use, point and click graphical interface. It is highly interactive having graphics, text, sound and animation, etc. It is used as a place, artgallery, library, community centre, school, publishing house, etc. There are search tools like web index and search engines to enable information searches and discover them more effectively.

REVIEW QUESTIONS

1. Draw the block diagram of a communication system and explain its operation.
2. Explain briefly some of the telecommunication services.
3. Distinguish between telegraphy and telephony.
4. What is Facsimile? How does it work?
5. What are the different transmission paths for communication?
6. What is the difference between analog and digital signals?
7. What is modulation?
8. Explain briefly the need for modulation.
9. What are the types of analog modulation?
10. Write down a general expression for AM wave.
11. Write the expression for modulation index in AM.
12. Write down a mathematical expression for a FM wave.
13. What is the advantage of FM over AM?
14. Compare FM and PM.
15. State sampling theorem.
16 Define the following terms:
    (i) Pulse Amplitude Modulation (PAM)
    (ii) Pulse Width Modulation (PWM)
    (iii) Pulse Time Modulation (PTM)
    (iv) Pulse Code Modulation (PCM)
17. Compare PAM and PWM.
18. Describe briefly some digital modulation schemes.
19. Define and describe pulse-position modulation.
20. In what way is pulse code modulation different from other modulation systems?
21. What is pulse width modulation? What other names does it have?
22. What is meant by Frequency-Shift Keying?
23. Differentiate FSK from PSK.
24. Explain two types of data transfer.
25. Explain the different modes of data transmission.
26. What is baud rate?
27. What are the different types of noises added to the original signals in data transmission? Explain them.
28. What is cross talk?

29. What are the two types of data transmission?
30. What is a modem? What does the work "modem" stand for?
31. Why must modems be used in pairs?
32. What are the functions of a transmitting modem?
33. What are the functions performed by the receiving modem?
34. How does an FM modem represent 1s and 0s? Explain the functions performed by a modem when receiving these signals.
35. What are the baud value ranges for low medium, and high-performance modems? What is the difference in the internal implementation?
36. Draw the block diagram arrangement of an AM broadcast transmitter and explain its operation.
37. Explain how AM wave is generated.
38. Explain the superheterodyne receiver with a neat block diagram.
39. Draw the block diagram of a FM radio transmitter and explain its operation.
40. What are the advantages of FM transmitter?
41. Why are AM systems preferred in broadcasting than FM systems?
42. What is aspect ratio?
43. What is composite video signal?
44. With the help of a.block diagram describe the working of a typical TV transmitter.
45. With the help of a block diagram describe the working of a typical TV receiver.
46. Name the various standards adopted internationally in Television transmission.
47. What is the range of microwave frequency?
48. Discuss briefly microwave communication system.
49. What is a satellite system?
50. What is an Orbit? Give its classifications.
51. List the three functions of a transponder.
52. What is the difference between geo-synchronous and geo-stationary satellites?
53. What are the advantages and disadvantages of satellite communication?
54. Give the block diagram of a satellite transponder and explain each block.
55. How is the distance from the point of transmission and the target determined by employing a radar system?
56. What is Doppler effect? How is it useful in radar?
57. Derive the basic radar range equation.
58. What is Pulse radar?
59. With the help of a block diagram explain the working of Pulse radar?
60. What are the types of radar? Explain them briefly.
61. List the applications of radar system.
62. Draw the block diagram of optical fibre communication system and explain it.
63. What are the advantages of optical fibre communication?
64. Explain the principle behind transmission of optical signals in an optical fibre.
65. Explain the structure of a step index fibre with refractive index profile.
66. Give some applications of optical fibre communication systems.
67. With a neat diagram explain the conceptual view of ISDN connection feature.
68. What are the services provided by ISDN?
69. What are the different channels in ISDN and mention their bandwidth?
70. What is the basic channel structure and primary channel structure in ISDN?
71. What are the advantages of ISDN?

72. What is the constraint in ISDN implementation?
73. What is Internet? Why do we need it?
74. What do you need to communicate with Internet?
75. What are the functions of Internet?
76. Distinguish between E-mail and facsimile.
77. How will you send an E-mail to another user?
78. How will you utilize the services of other computers in Internet?
79. How do you download/upload program in Internet?
80. What is netnews?
81. What is finger in Internet?
82. Explain WAIS, GOPHER and WWW in Internet.

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Part-II