Projections

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Syllabus

- **Affine and perspective geometry, orthographic and axonometric projections, techniques for generating perspective views, vanishing points, stereographic projection.**

Projection

To see the object in different orientations

To realize 3D view in 2D plane

OVERVIEW

- Multiple transformations are needed for different projections.

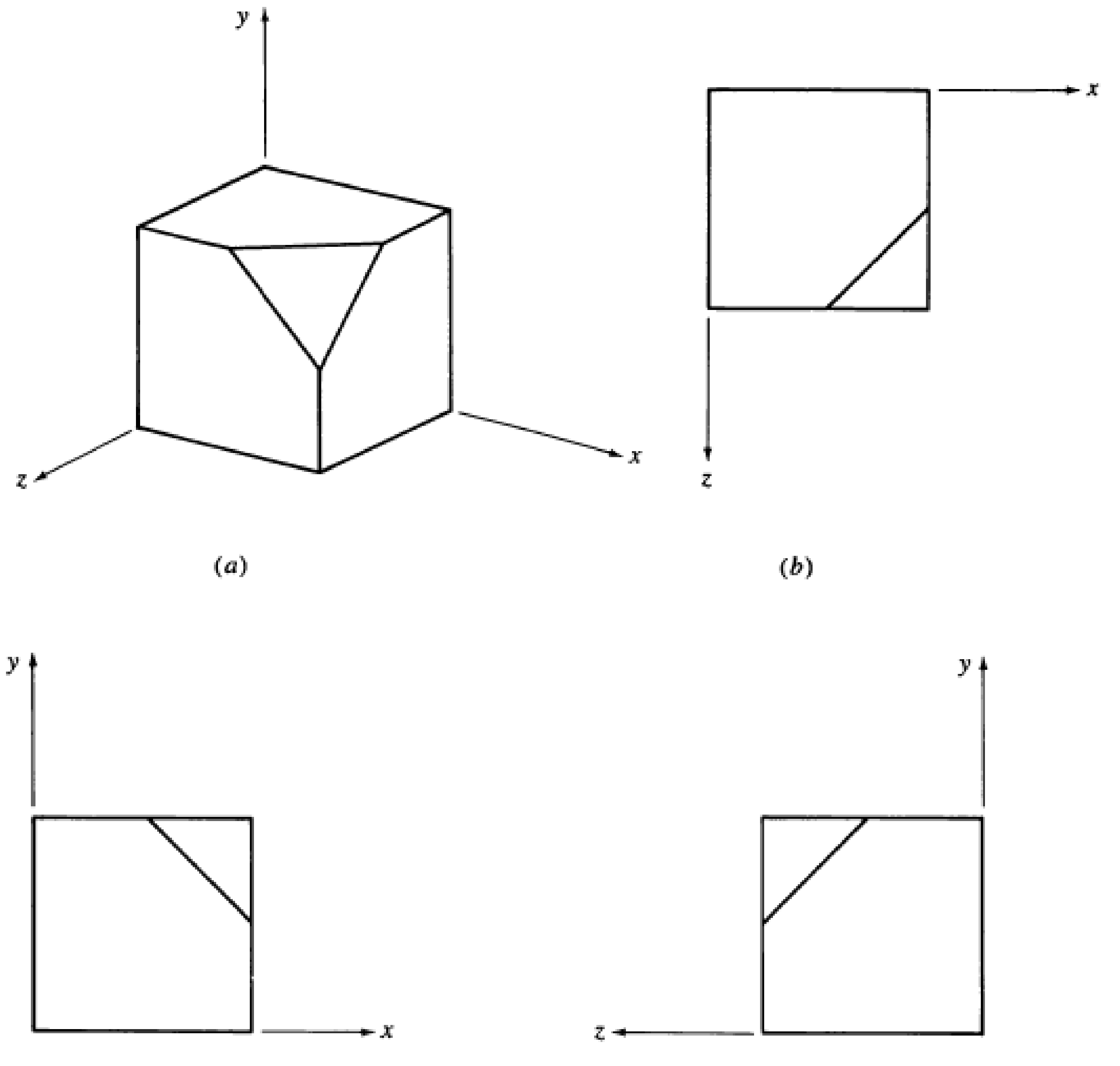
Projections

It is the process of converting a 3D object into a 2D object.

It is also defined as mapping or transformation of the object in projection plane or view plane.

The view plane is displayed surface.

Projection (Orthographic)

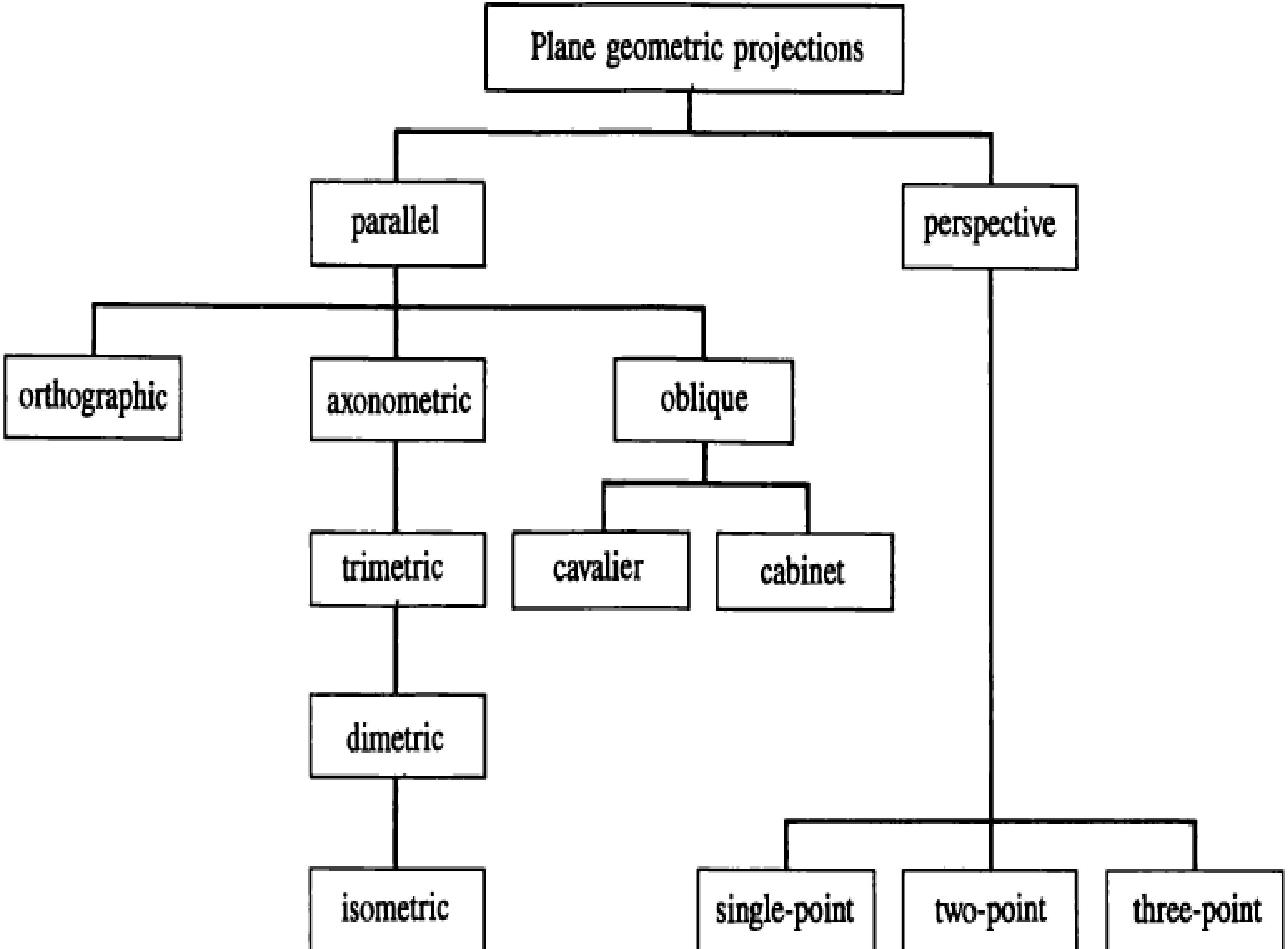


Projection

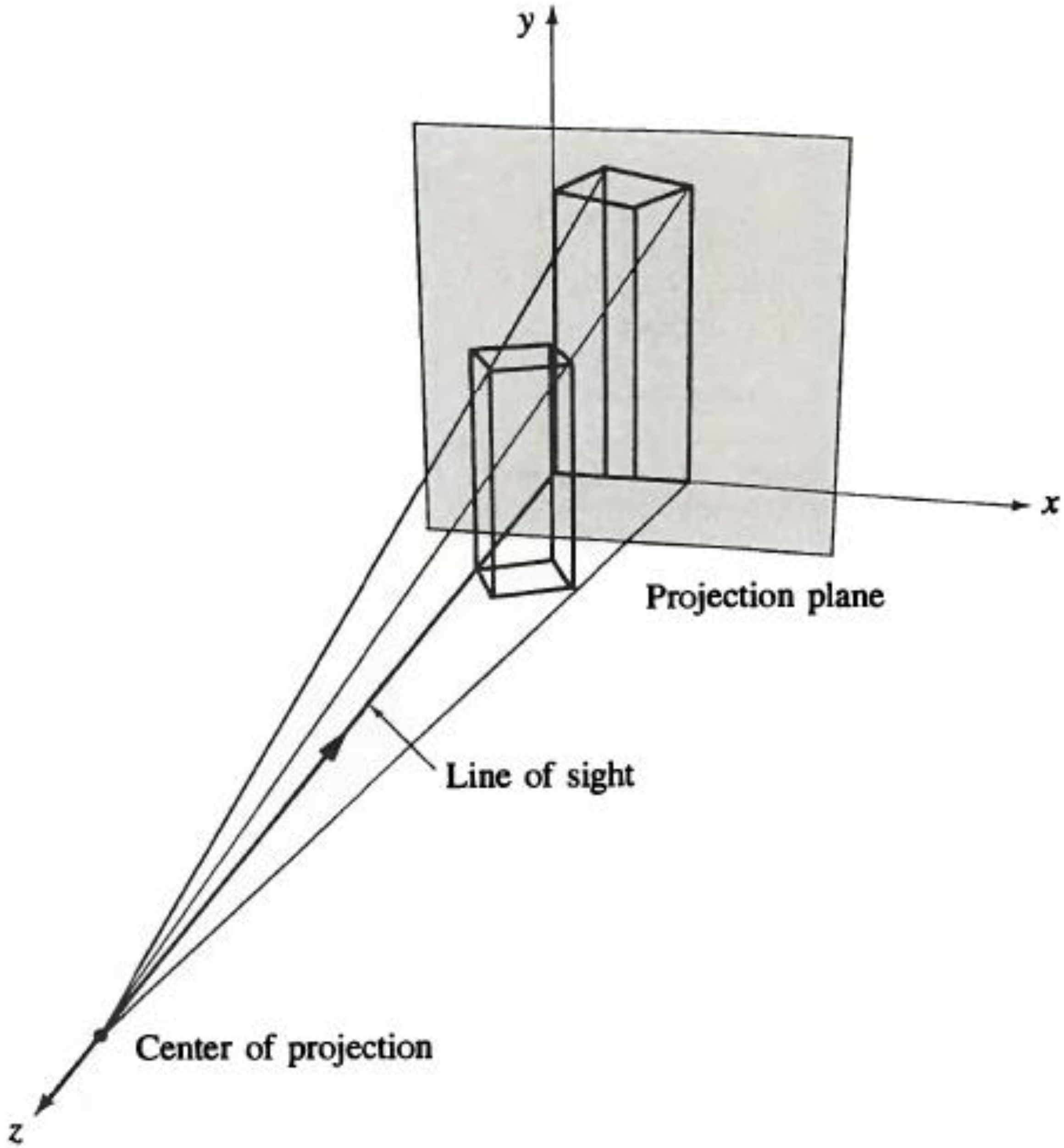
$$T(projZ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(projX) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(projY) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$


Plane geometric projections
parallel
perspective
orthographic
axonometric
oblique
trimetric
cavalier
cabinet
dimetric
isometric
single-point
two-point
three-point

Projections

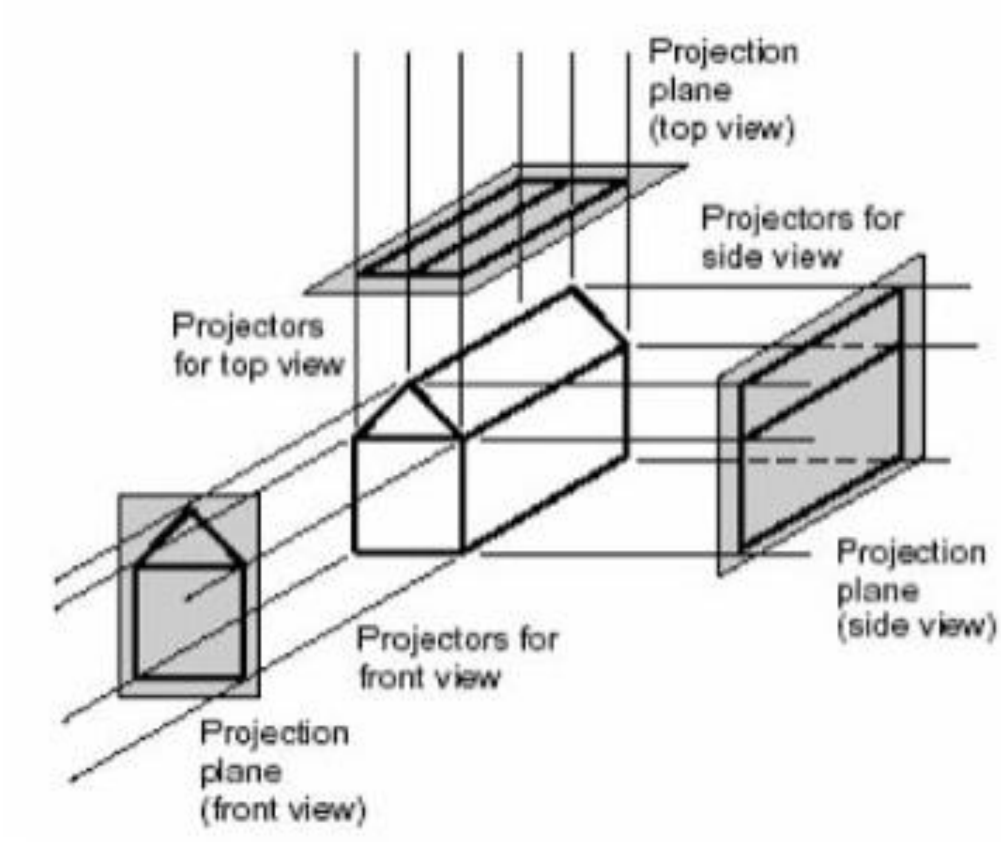


Parallel and perspective projections

In Parallel projectons rays are coming from source at infinite, hence projectors are parallel. Plane of projection may be in direction of plane normal or at an oblique angle.

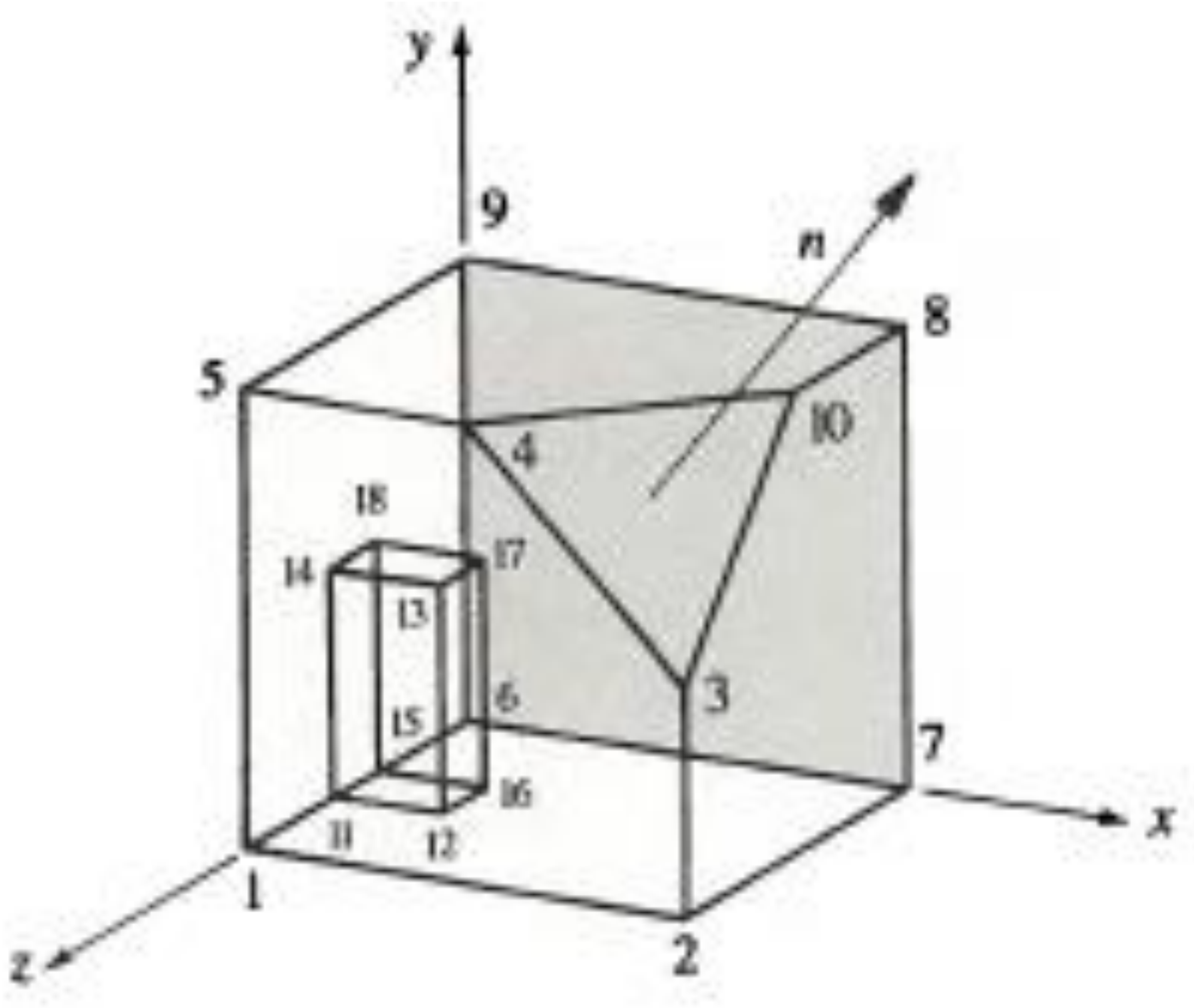
A **parallel projection** is a **projection** of an object in three-dimensional space onto a fixed plane, known as the **projection** plane or image plane, where the rays, known as lines of sight or **projection** lines, are **parallel** to each other. It is a basic tool in descriptive geometry.

Parallel and perspective projections



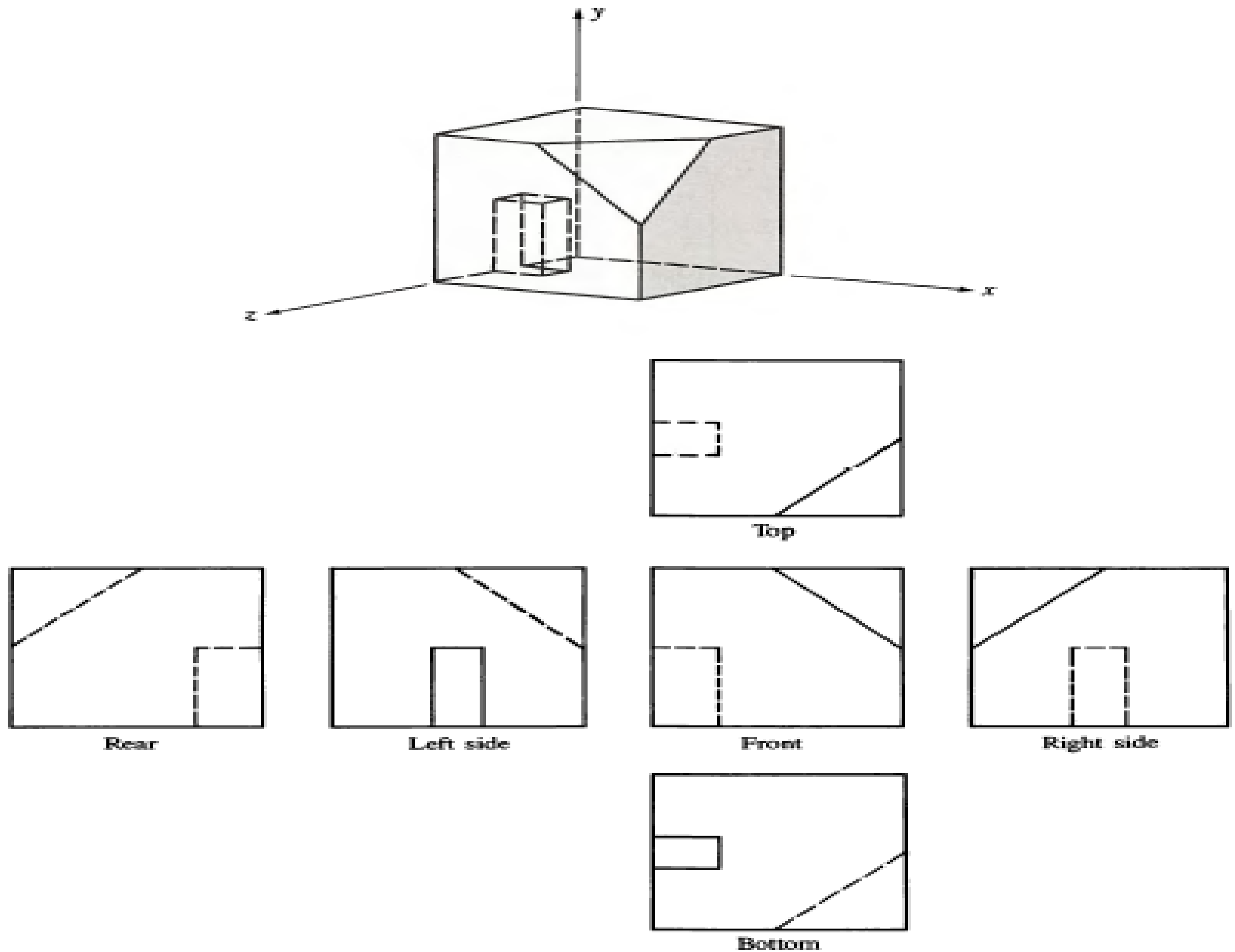
In Perspective projection, center of projection is at finite distance and projectors are non-parallel.

Projections(orthographic)

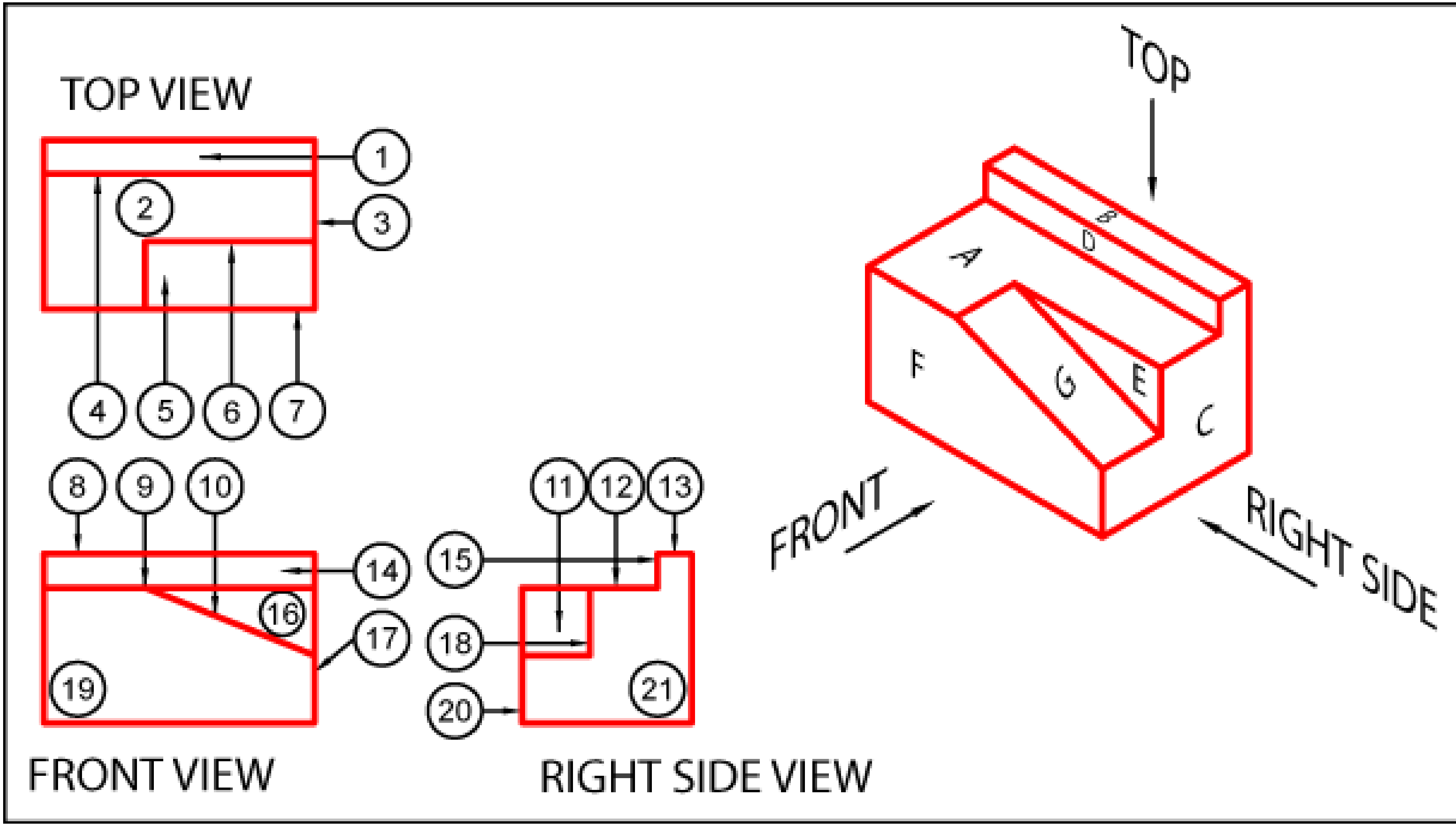


Projections(orthographic)

$$[\ X\ ] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0.5 & 1 & 1 \\ 0.5 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0 & 0 & 0.6 & 1 \\ 0.25 & 0 & 0.6 & 1 \\ 0.25 & 0.5 & 0.6 & 1 \\ 0 & 0.5 & 0.6 & 1 \\ 0 & 0 & 0.4 & 1 \\ 0.25 & 0 & 0.4 & 1 \\ 0.25 & 0.5 & 0.4 & 1 \\ 0 & 0.5 & 0.4 & 1 \end{bmatrix}$$


y
x
z
Top
Rear
Left side
Front
Right side
Bottom

Projections



Projections

What letter (identifying a surface on the 3-D isometric) matches what number (identifying a surface on the 2-D orthographic projection)?

Surface Number	Enter Corresponding Surface Letter	Check	Surface Number	Enter Corresponding Surface Letter	Check
1			2		
3			4		
5			6		
7			8		
9			10		
11			12		
13			14		
15			16		
17			18		
19			20		
21					

Axonometric Projections

Axonometric projection is a type of orthographic **projection** used for creating a pictorial **drawing** of an object, where the lines of sight are perpendicular to the plane of **projection**, and the object is rotated around one or more of its axes to reveal multiple sides.

Necessity of Axonometric Projections

A single orthographic projection fails to illustrate general 3D view of the shape of the object.

This limitation will be overcome in **axonometric projection by manipulating objects by rotations and translations such that at least three faces are shown.**
The transformed geometry is then projected from centre of projection in infinity onto one of the coordinate plane

Axonometric Projections

A face will be of true shape only when it is parallel to the plane of projection.

Ratio of modified shape with the original shape is called foreshortening factor

Axonometric projections

- Depending upon the foreshortening of the principal axis, there are following the three types axonometric projections:

1. **Isometric projections**
2. **Dimetric projections**
3. **Trimetric projections**

Axonometric Projections

$$[T] = [R_y][R_x][P_z]$$

$$= \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Axonometric Projections

$$[U][T] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} [T]$$

- [U] is unit vector in x y and Z directions
- [T] equivalent transformation matrix for axonometric projections

Final transform unit vectors

$$\begin{bmatrix} x_x^* & y_x^* & 0 & 1 \\ x_y^* & y_y^* & 0 & 1 \\ x_z^* & y_z^* & 0 & 1 \end{bmatrix}$$

$$f_x = \sqrt{x_x^{*2} + y_x^{*2}}$$

$$f_y = \sqrt{x_y^{*2} + y_y^{*2}}$$

$$f_z = \sqrt{x_z^{*2} + y_z^{*2}}$$

Fx, Fy and Fz are foreshortening factors in x, y and z directions

Foreshortening factors

$$f_x^2 = ({x_x^*}^2 + {y_x^*}^2) = \cos^2\phi + \sin^2\phi\sin^2\theta$$ --(1)

$$f_y^2 = ({x_y^*}^2 + {y_y^*}^2) = \cos^2\theta$$ ----(2)

$$f_z^2 = ({x_z^*}^2 + {y_z^*}^2) = \sin^2\phi + \cos^2\phi\sin^2\theta$$ ---(3)

Trimetric projections

The projection plane normal (projector) makes unequal angles with each principal axis (all three axes are unequally foreshortened)

Axonometric Projections

Consider the center illustration of Fig. 3–16 formed by a $\phi = 30°$ rotation about the y-axis, followed by a $\theta = 45°$ rotation about the x-axis, and then

$$[\,T\,] = [\,R_y\,][\,R_x\,][\,P_z\,]$$

$$= \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & \sqrt{2}/4 & 0 & 0 \\ 0 & \sqrt{2}/2 & 0 & 0 \\ 1/2 & -\sqrt{6}/4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Axonometric Projections

$$[\,U\,][\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & \sqrt{2}/4 & 0 & 0 \\ 0 & \sqrt{2}/2 & 0 & 0 \\ 1/2 & -\sqrt{6}/4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & \sqrt{2}/4 & 0 & 1 \\ 0 & \sqrt{2}/2 & 0 & 1 \\ 1/2 & -\sqrt{6}/4 & 0 & 1 \end{bmatrix}$$

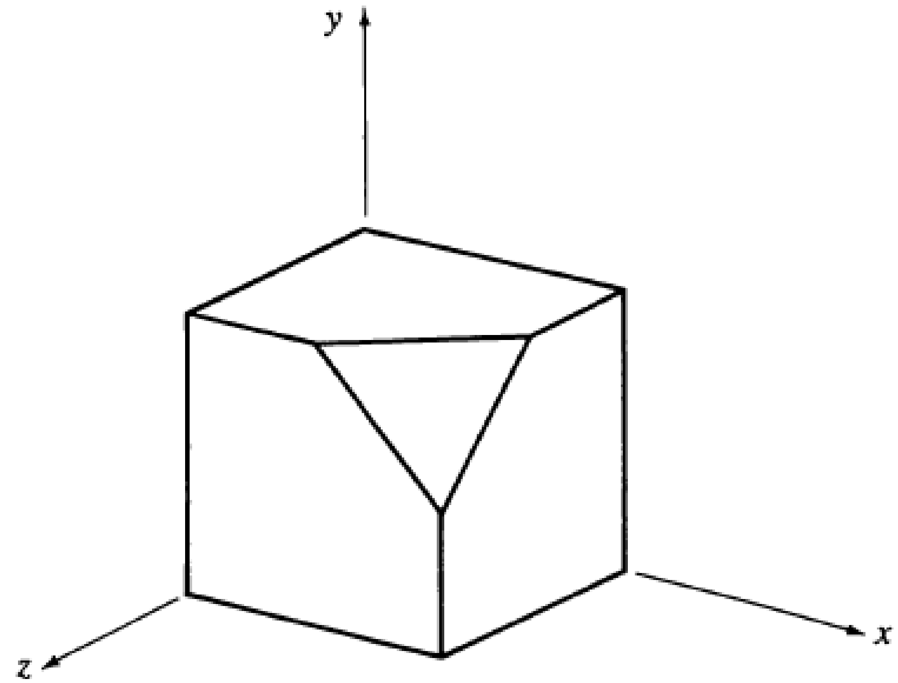
$$f_x = \sqrt{(\sqrt{3}/2)^2 + (\sqrt{2}/4)^2} = 0.935$$

$$f_y = \sqrt{2}/2 = 0.707$$

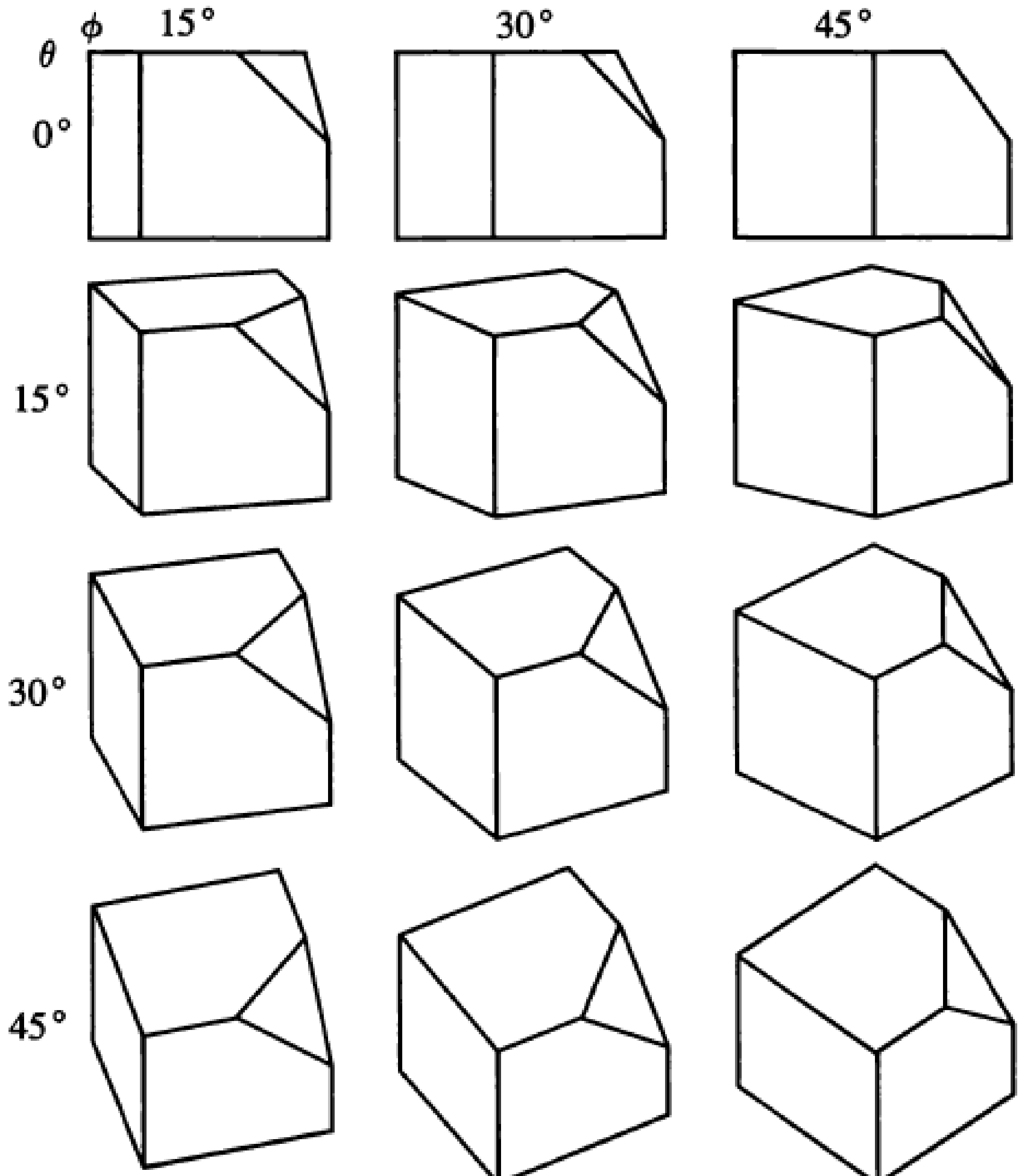
$$f_z = \sqrt{(1/2)^2 + (-\sqrt{6}/4)^2} = 0.791$$

Axonometric Projections

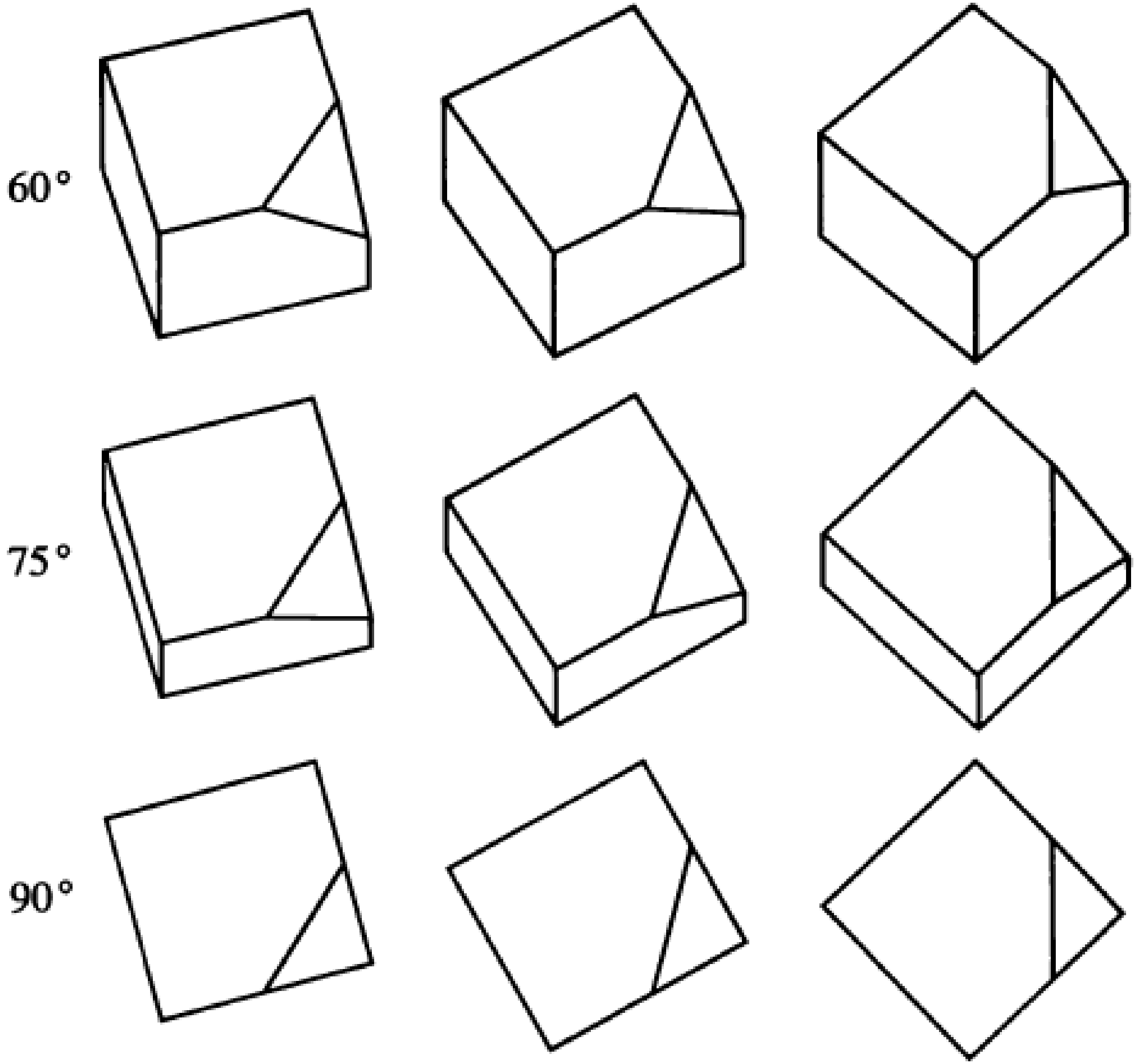
$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0.5 & 1 & 1 \\ 0.5 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \end{bmatrix}$$



Trimetric projections



Trimetric projections



Dimetric projections

The projection plane normal (projector) makes equal angles with two of three principal axes (two of three axes are equally foreshortened). Three different dimetric projections are shown below.

$$f_x^2 = f_y^2$$

Dimetric Projections

$$f_x^2 = f_y^2$$

From equation (1) and (2)

$$\cos^2\phi + \sin^2\phi\sin^2\theta = \cos^2\theta$$

Using the identities $\cos^2\phi = 1 - \sin^2\phi$ and $\cos^2\theta = 1 - \sin^2\theta$ yields

$$\sin^2\phi = \frac{\sin^2\theta}{1-\sin^2\theta} \qquad \text{--------(4)}$$

From Equation 3

$$f_z^2 = (x_z^{*2} + y_z^{*2}) = \sin^2\phi + \cos^2\phi\sin^2\theta$$

Dimetric Projections

$$2\sin^2\theta - 2\sin^4\theta - (1-\sin^2\theta)f_z^2 = 0$$

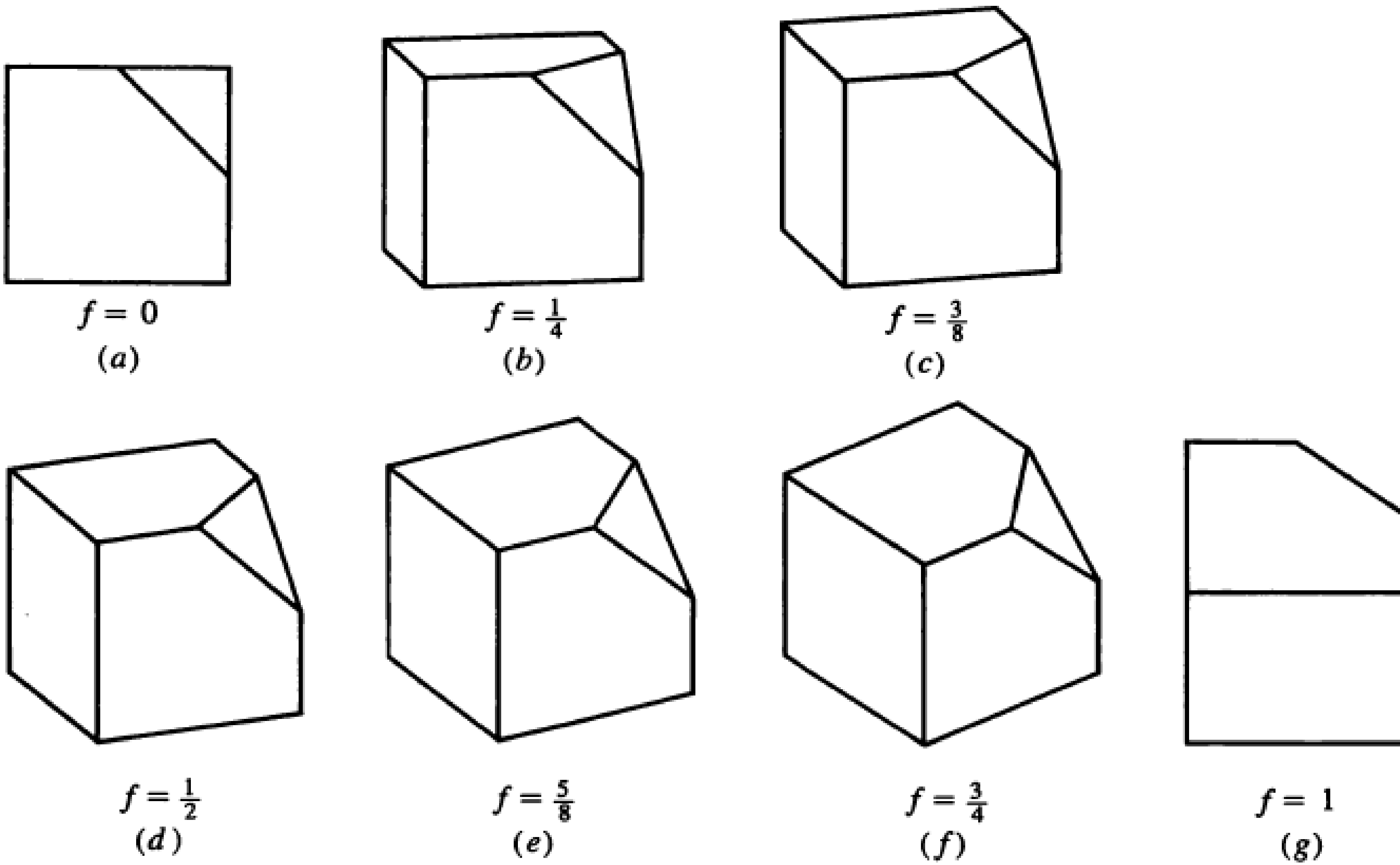
$$2\sin^4\theta - (2+f_z^2)\sin^2\theta + f_z^2 = 0$$

$$\sin^2\theta = f_z^2/2,\ 1$$

$$\theta = \sin^{-1}\left(\pm f_z/\sqrt{2}\right) \qquad \text{--------(5)}$$

$$\phi = \sin^{-1}\left(\pm f_z/\sqrt{2-f_z^2}\right) \qquad \text{--------(6)}$$

Dimetric Projections



Isometric projections

The projection plane normal makes equal angles with each(x, y, z) principal axis .

The three axes are equally foreshortened.

$$f_x^2 = f_y^2 = f_z^2 \quad \text{--------(7)}$$

Isometric Projections

$$\sin^2\phi = \frac{\sin^2\theta}{1-\sin^2\theta}$$

From equations 2 and 3

$$f_y^2 = ({x_y^*}^2 + {y_y^*}^2) = \cos^2\theta$$

$$f_z^2 = ({x_z^*}^2 + {y_z^*}^2) = \sin^2\phi + \cos^2\phi\sin^2\theta$$

$$\sin^2\phi = \frac{1-2\sin^2\theta}{1-\sin^2\theta} \qquad \text{--------(8)}$$

Isometric Projections

$$\sin^2\theta = 1/3$$

$$\sin\theta = \pm\sqrt{1/3}$$ ---------(9)

$$\theta = \pm 35.26^\circ$$

$$\sin^2\phi = \frac{1/3}{1-1/3} = 1/2$$ --------(10)

$$\phi = \pm 45^\circ$$

$$f = \sqrt{\cos^2\theta} = \sqrt{2/3} = 0.8165$$ -------(11)

Calculation of angle that a projected x-axis make with the horizontal is very important for the manual construction of isometric projection

$$[\,U_x^*\,] = [\,1 \quad 0 \quad 0 \quad 1\,]\begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [\,\cos\phi \quad \sin\phi\sin\theta \quad 0 \quad 1\,]$$

If angle between x-axis and horizontal is α

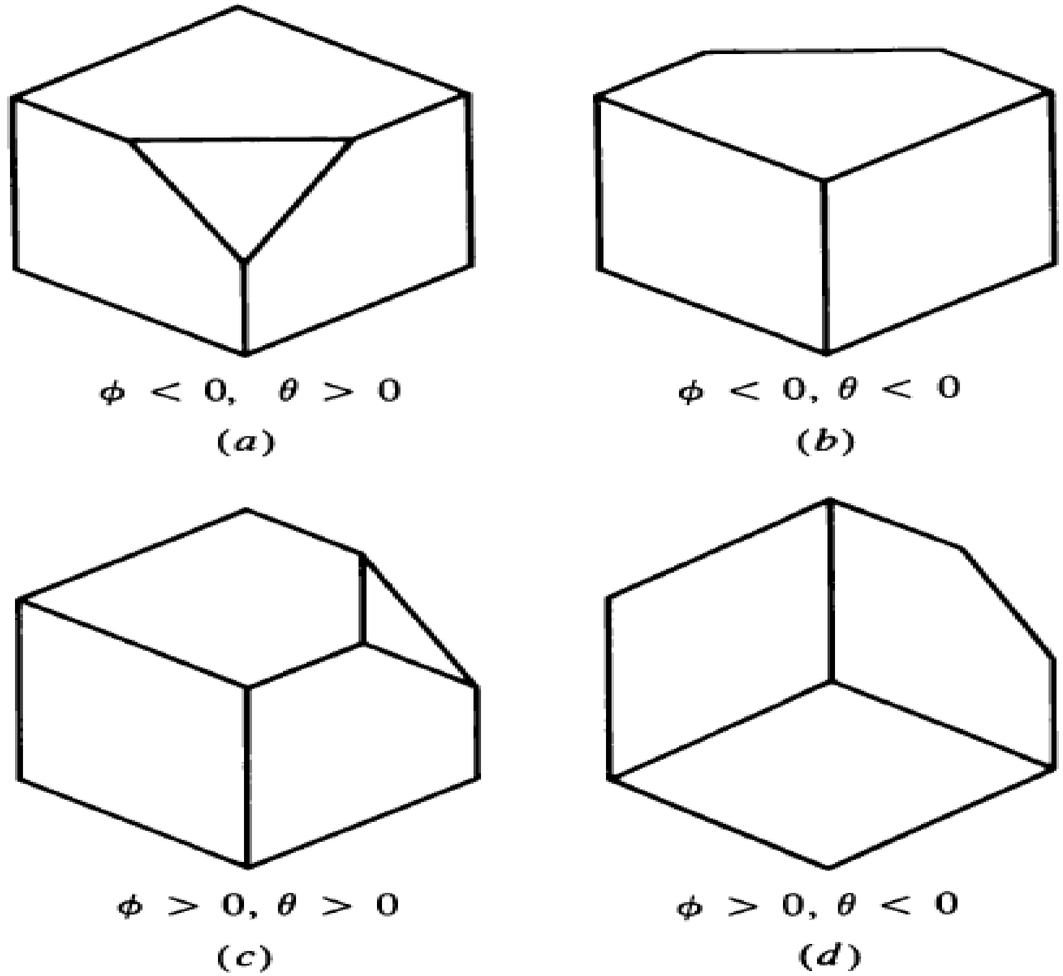
$$\tan\alpha = \frac{y_x^*}{x_x^*} = \frac{\sin\phi\sin\theta}{\cos\phi} = \pm\sin\theta$$

Using isometric case

$$\alpha = \tan^{-1}(\pm\sin 35.26439^\circ) = \pm 30^\circ$$

Common values are 30° and 60°

Isometric Projections



Four possible isometric projections with rotation angles $\phi = \pm 45°$, $\theta = \pm 35.26°$. (a) $\phi = -45°$, $\theta = +35.26°$; (b) $\phi = -45°$, $\theta = -35.26°$; (c) $\phi = +45°$, $\theta = +35.26°$; (d) $\phi = +45°$, $\theta = -35.26°$.

Oblique projections

In oblique projection the projector are coming from infinite and making Oblique angle with plane of projection

Oblique projections illustrate the general three-dimensional shape of the object. However, only faces of the object parallel to the plane of projection are shown at their true size and shape, i.e., angles and lengths are preserved for these faces only. In fact, the oblique projection of these faces is equivalent to an orthographic front view. Faces not parallel to the plane of projection are distorted.

Oblique projections

There two type of oblique projections (a) Cavalier, (b) Cabinet

A Cavalier projection is obtained when angle between the oblique projectors and plane of projection is 45 degree. Foreshortening factors in each principle direction are one. The resulting figure are thick which is corrected in the other oblique projection Cabinet

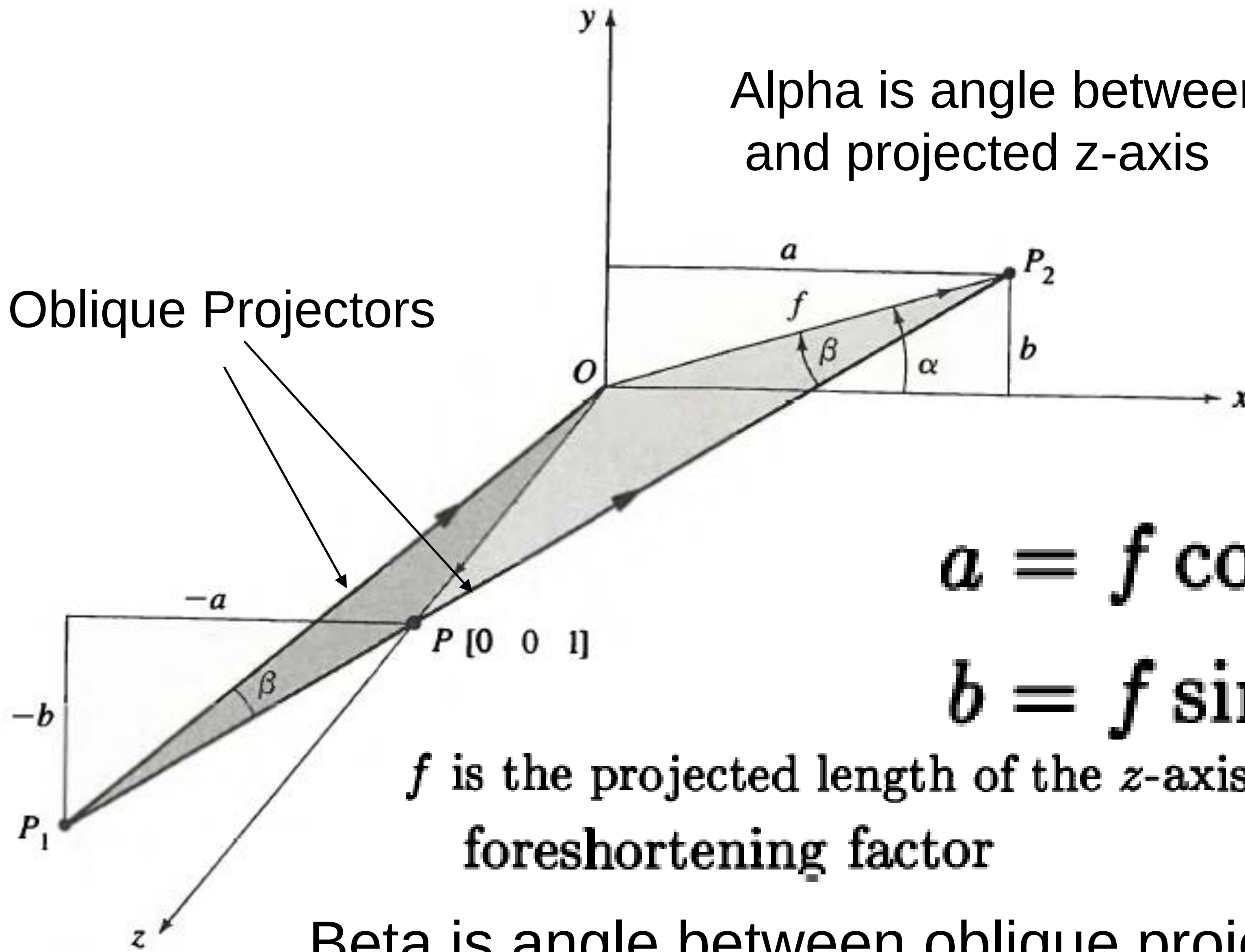
Oblique projections

An oblique projection for which the foreshortening factor for all edges perpendicular to the plane of projection is One-half is called cabinet projection.
Angle between projectors and plane of projection is cot-1(1/2) = 63.43 degree

Oblique projections

Alpha is angle between horizontal and projected z-axis

Oblique Projectors



$$a = f \cos \alpha$$

$$b = f \sin \alpha.$$

f is the projected length of the z-axis unit vector,

foreshortening factor

Beta is angle between oblique projectors and the plane of projection

Oblique projections

$$[\,T'\,] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & -b & 1 \end{bmatrix}$$

$$[\,T''\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oblique projections

$$[\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -f\cos\alpha & -f\sin\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where f is the projected length of the z-axis unit vector

If $f = 0$, $\beta = 90°$, then an orthographic projection results. If $f = 1$, the edges perpendicular to the projection plane are not foreshortened. This is the condition for a cavalier projection.

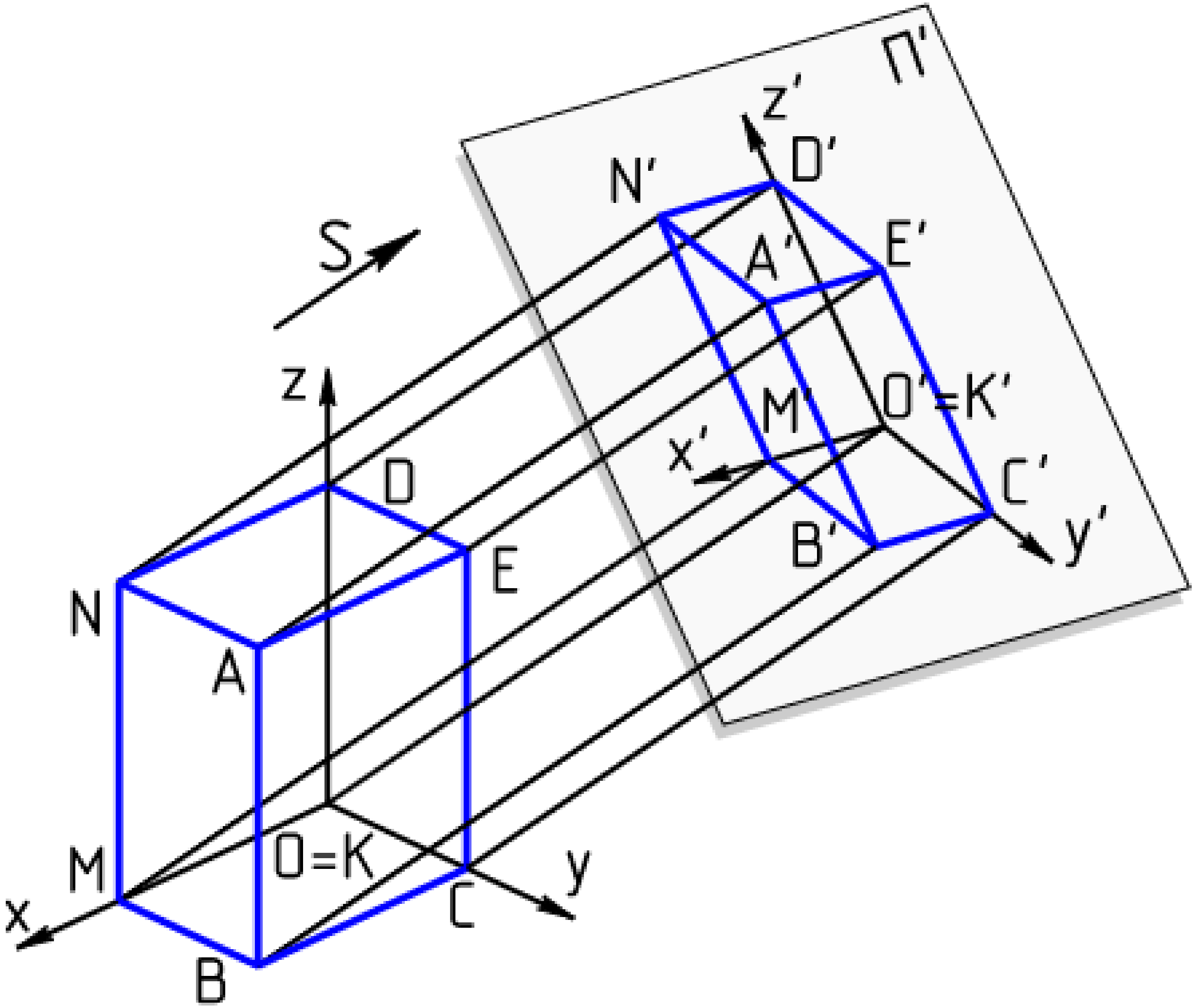
Oblique projections

$$\beta = \cot^{-1}(1) = 45^\circ$$

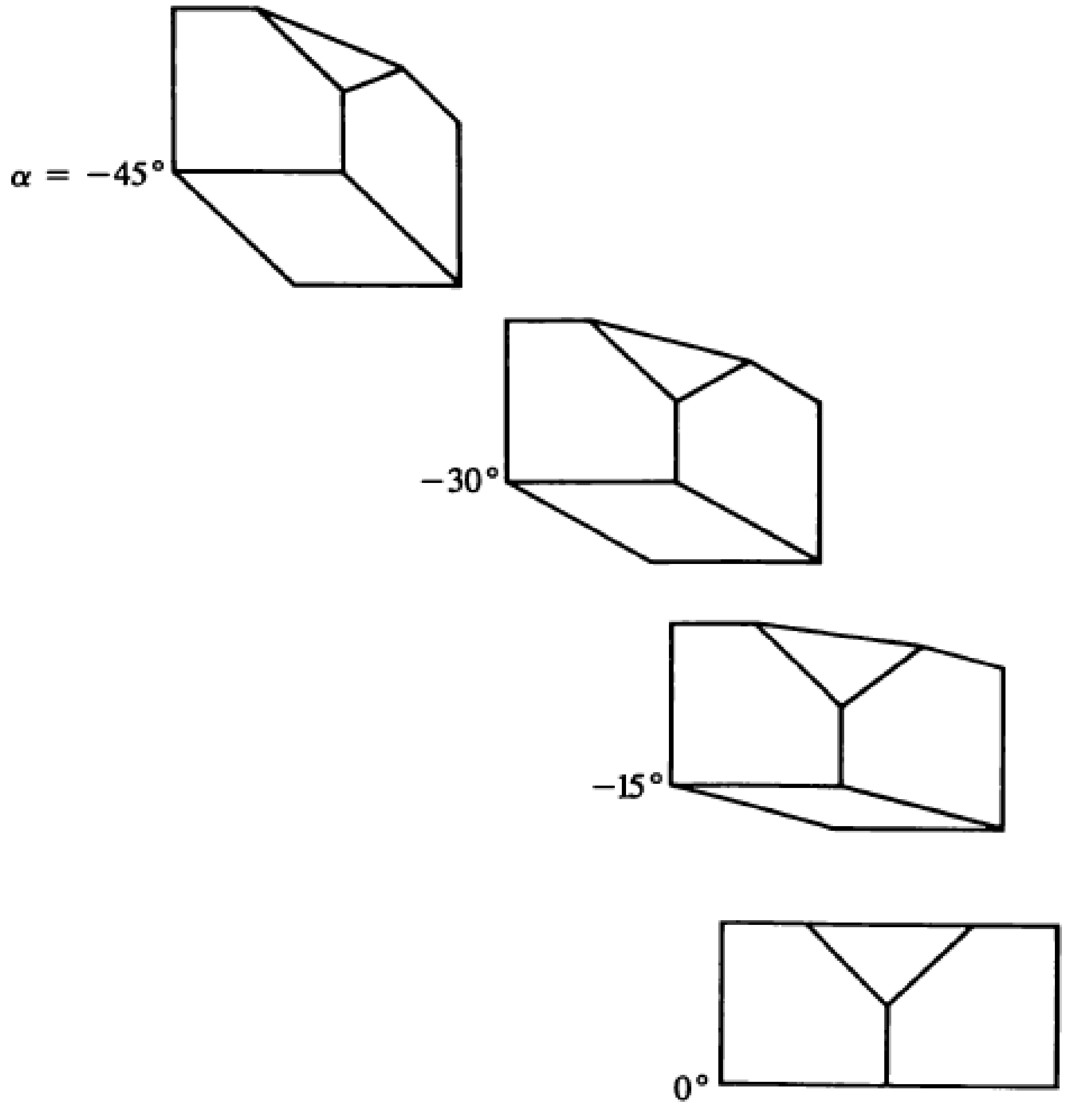
A cabinet projection is obtained when the foreshortening factor $f = 1/2$. Here

$$\beta = \cot^{-1}(1/2) = 63.435^\circ$$

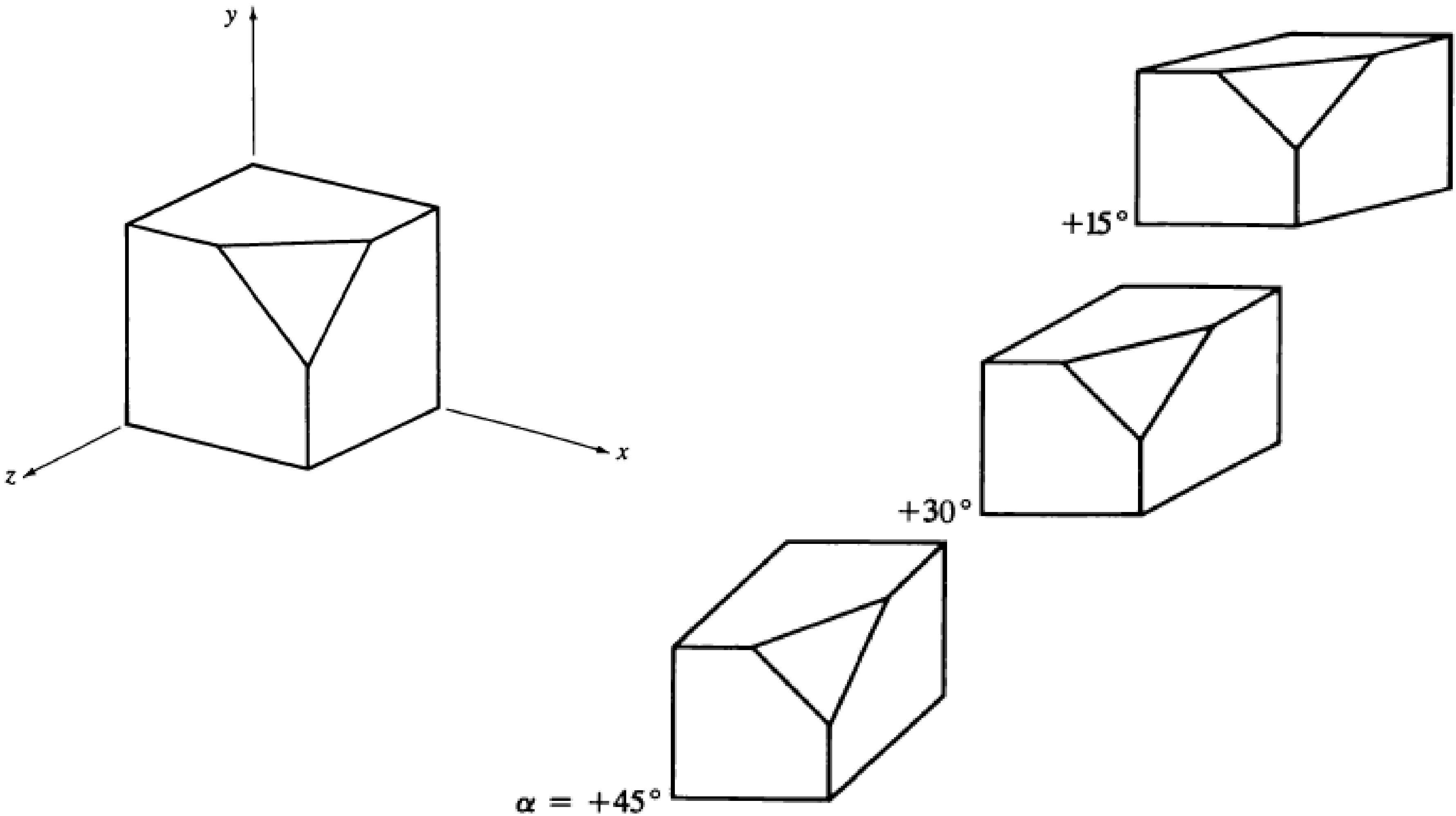
Oblique projections



Oblique projections

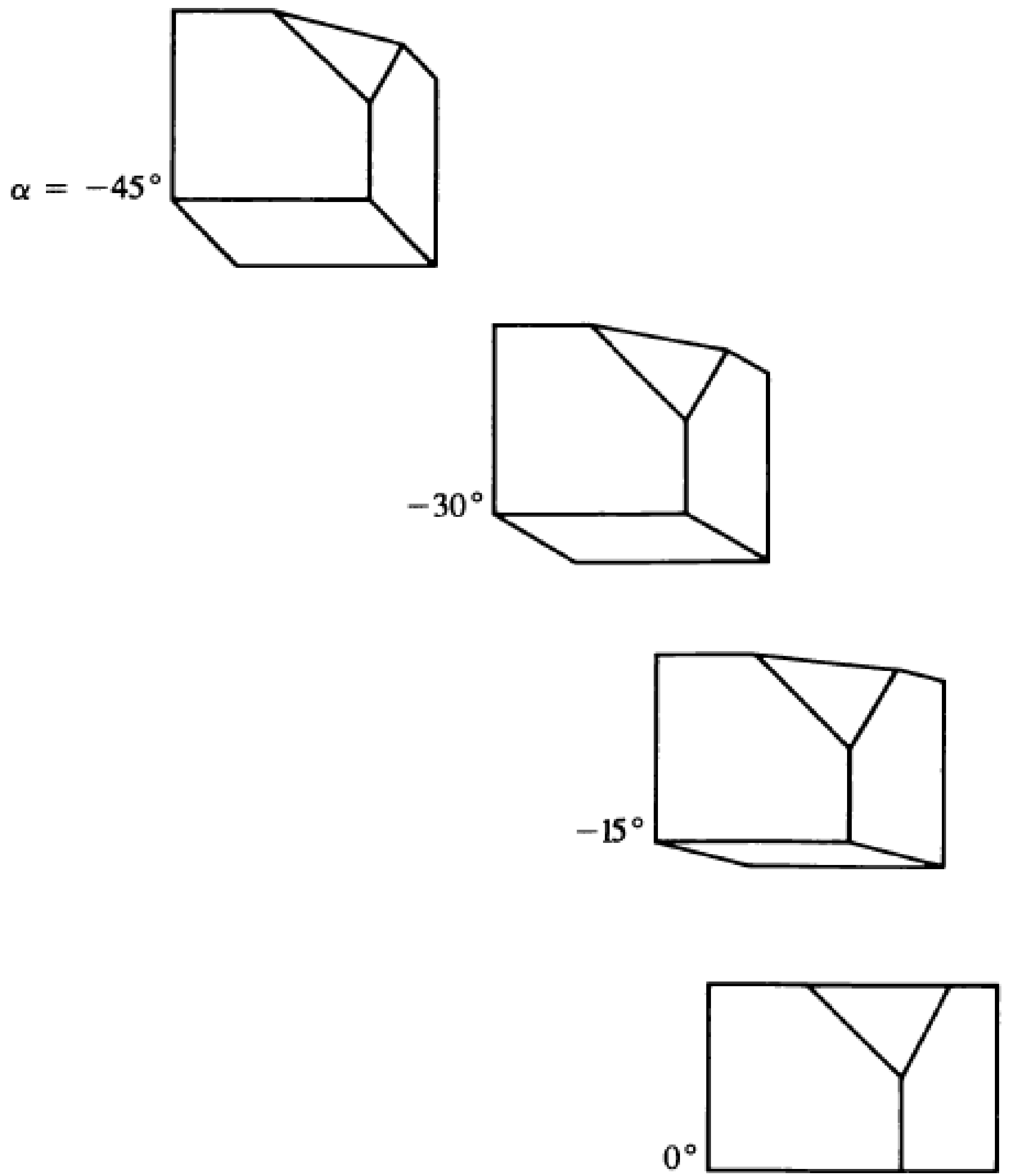


Oblique projections

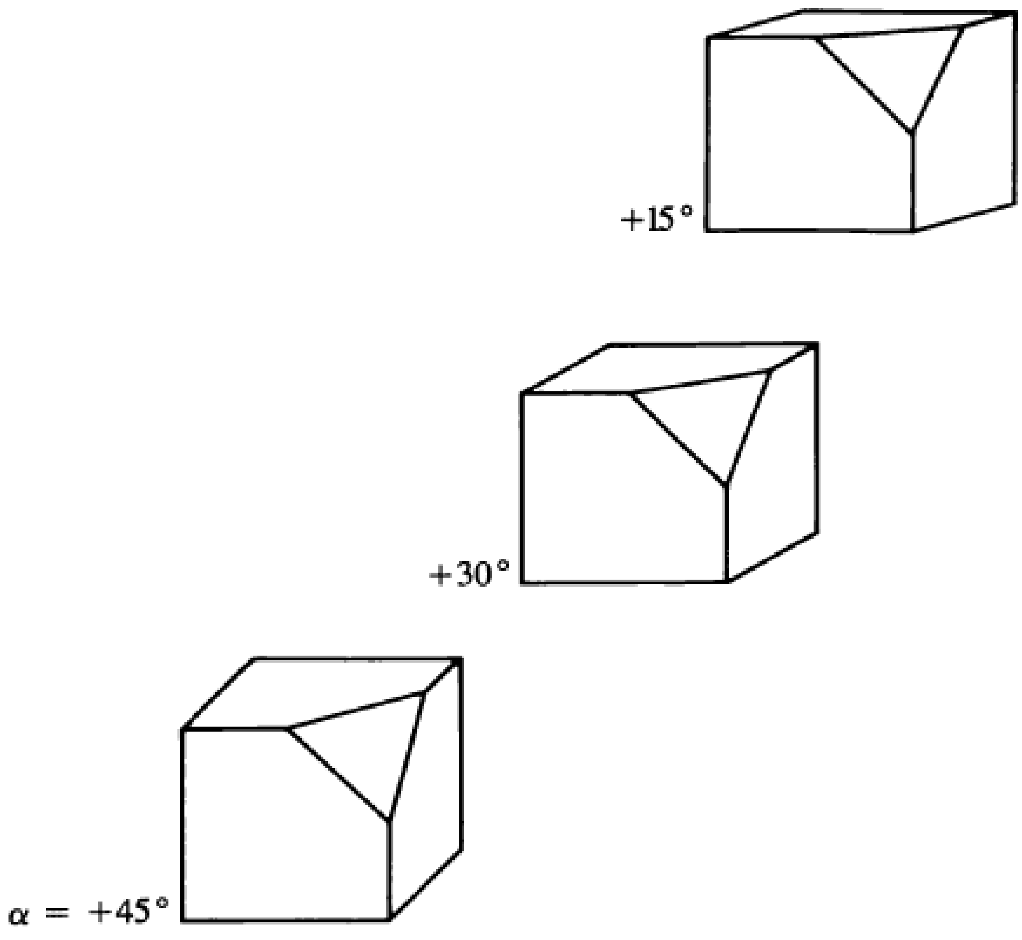


Cavalier projections. Top to bottom, $\alpha = -45°$ to $+45°$ at $15°$ intervals with $\beta = 45°$.

Oblique projections

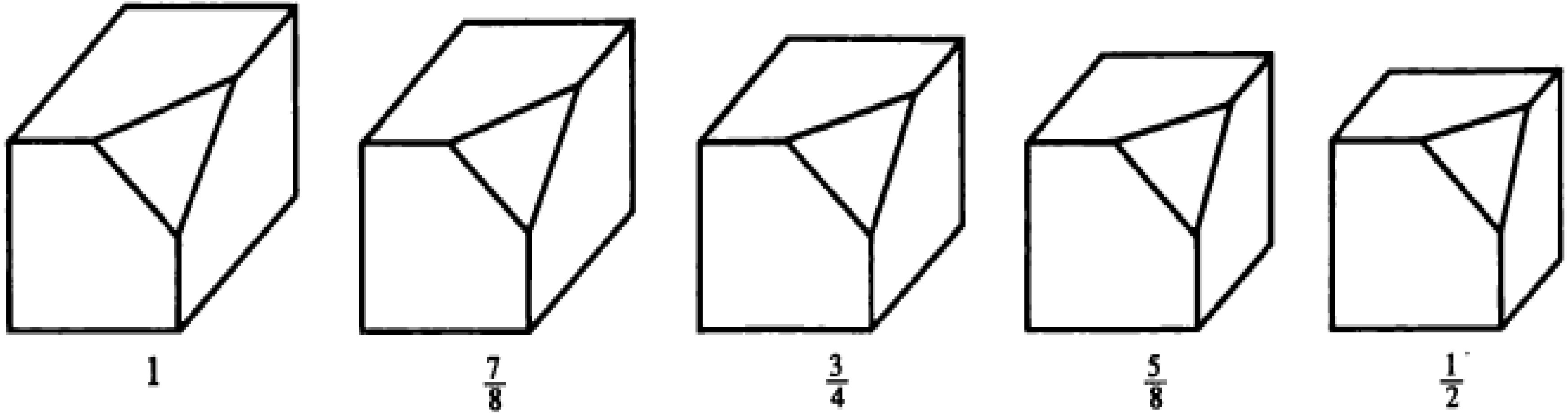


Oblique projections

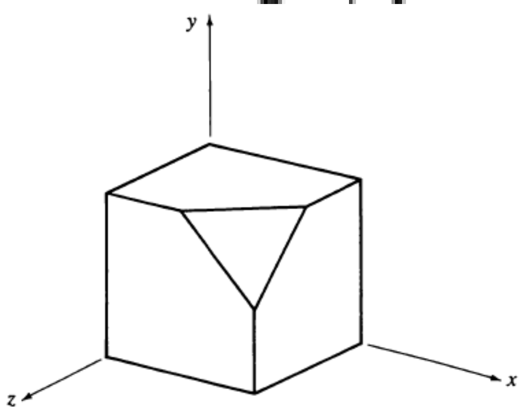


Cabinet projections. Top to bottom, $\alpha = -45°$ to $+45°$ at $15°$ intervals with $f = 0.5$.

Oblique projections



Oblique projections. Left to right, $f = 1,\ 7/8,\ 3/4,\ 5/8,\ 1/2$, with $\alpha = 45^\circ$.



Perspective projection

When any of the first three elements of the fourth column of general 4X4 homogeneous coordinate transformation matrix is non-zero. It is transformation of
One three space to another three space
Incontras to the parallel transformation , the perspective transformation parallel lines converge, object size reduced with increasing distance from center of projection

Perspective projection

Non-uniform foreshortening of lines in the object as function of orientation and distance of object the center of projection, but shape of the object is not preserved

x y z 1

$\downarrow$

x y z px+qx+rz+1

$$[T] = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix}$$

Perspective projection

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x & y & z & (px + qy + rz + 1) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{x}{px + qy + rz + 1} & \dfrac{y}{px + qy + rz + 1} & \dfrac{z}{px + qy + rz + 1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x^* & y^* & z^* & 1 \end{bmatrix}$$

Type of perspective projection

1. Single point perspective projection
2. Two point perspective projection
3. Three point perspective projection

Perspective projections pros and cons:

+ Size varies inversely with distance - looks realistic
– Distance and angles are not (in general) preserved
– Parallel lines do not (in general) remain parallel

Types of perspective projection

If the distance between the Centre of Projection (COP) to the projection plane is finite, the projectors are not parallel to each other. This kind of projections is called perspective projection. Perspective projections are further subdivided by the number of vanishing points, thus called 1-point, 2-point, or 3-point perspectives.

- One-point perspective — simplest to draw
- Two-point perspective — gives better impression of depth
- Three-point perspective — most difficult to draw

Single point perspective projection

$$[\,x \quad y \quad z \quad 1\,]\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix} = [\,x \quad y \quad z \quad rz+1\,]$$

$$[\,x^* \quad y^* \quad z^* \quad 1\,] = \left[\,\frac{x}{rz+1} \quad \frac{y}{rz+1} \quad \frac{z}{rz+1} \quad 1\,\right]$$

$$[\,T\,] = [\,P_r\,][\,P_z\,]$$

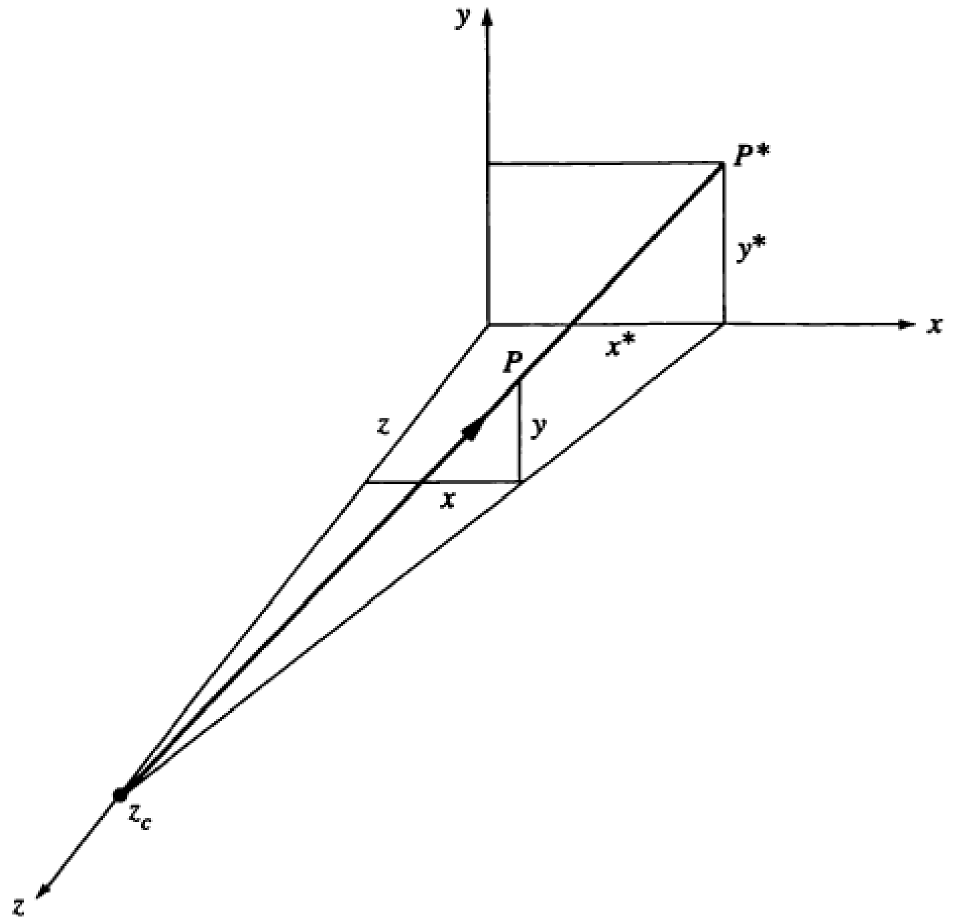
$$= \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & r\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Perspective projection

$$[\,x \quad y \quad z \quad 1\,]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} = [\,x \quad y \quad 0 \quad rz+1\,]$$

$$[\,x^* \quad y^* \quad z^* \quad 1\,] = \left[\,\frac{x}{rz+1} \quad \frac{y}{rz+1} \quad 0 \quad 1\,\right]$$

Perspective projection



three-dimensional point P onto a $z = z^* = 0$

$$\frac{x^*}{z_c} = \frac{x}{z_c - z}$$

$$x^* = \frac{x}{1 - \frac{z}{z_c}}$$

$$\frac{y^*}{\sqrt{x^{*2} + z_c^2}} = \frac{y}{\sqrt{x^2 + (z_c - z)^2}}$$

$$y^* = \frac{y}{1 - \frac{z}{z_c}}$$

Perspective projection

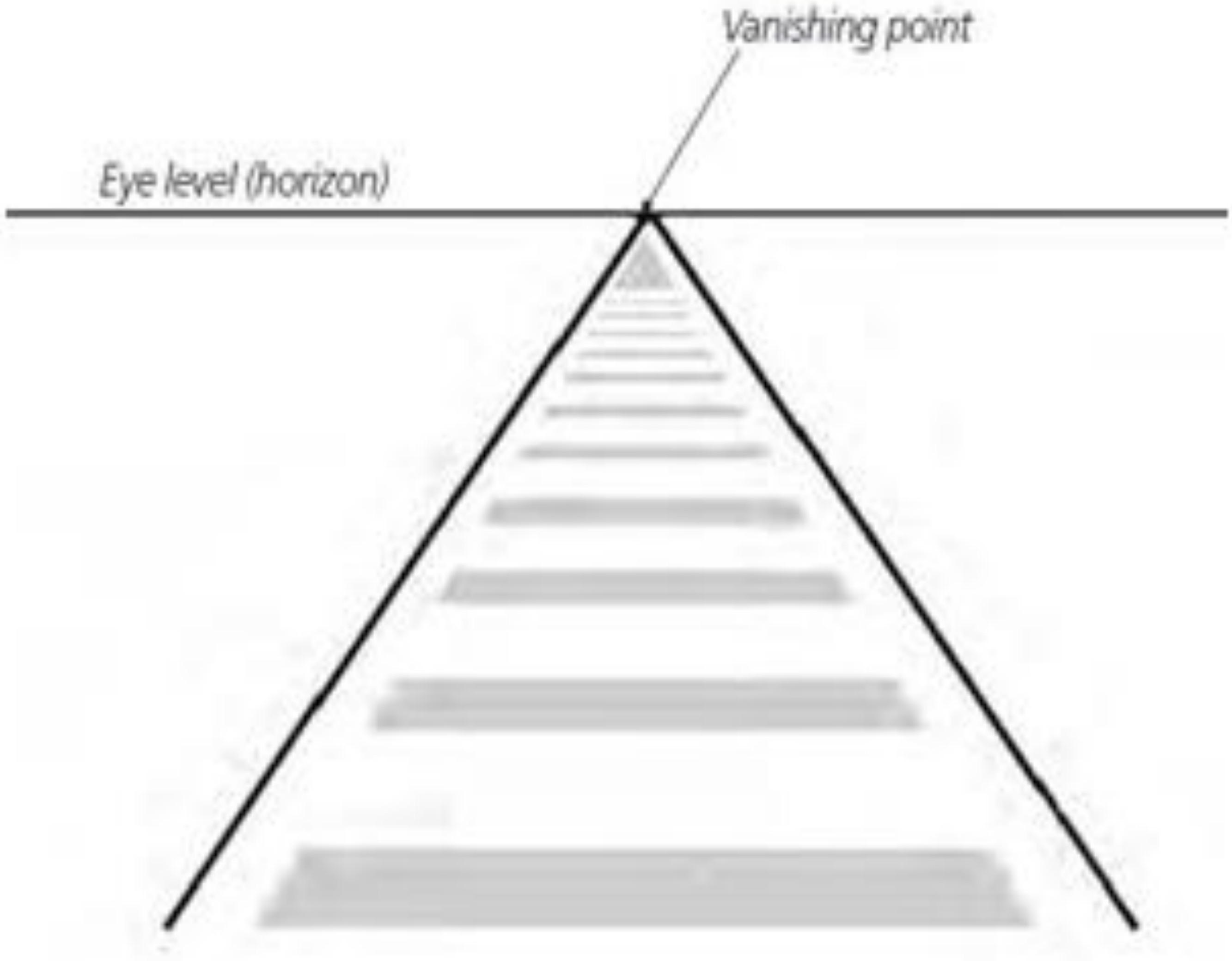
$$\begin{array}{c} A \\ B \end{array} \begin{bmatrix} 3 & 2 & 4 & 1 \\ 3 & 2 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 & 3 \\ 3 & 2 & 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.667 & 1.333 & 1 \\ 0.6 & 0.4 & 1.6 & 1 \end{bmatrix} \begin{array}{c} A' \\ B' \end{array}$$

Vanishing point

The points at which parallel lines receding from an observer seem to converge.
The point in linear perspective at which all imaginary lines of perspective converge.
The point at which a thing disappears or ceases to exist.

Vanishing point

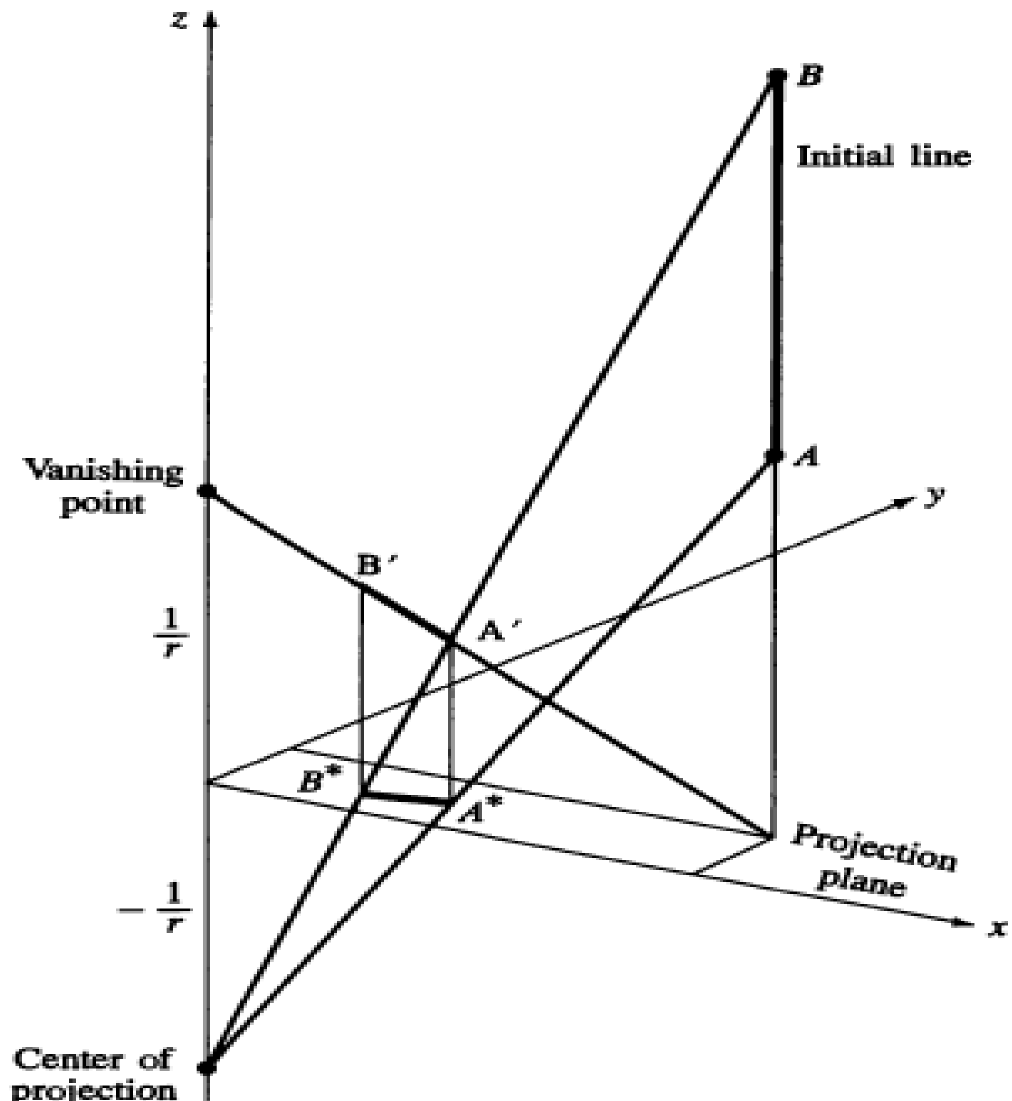


Perspective projection

$$\begin{array}{c} A \\ B \end{array} \begin{bmatrix} 3 & 2 & 4 & 1 \\ 3 & 2 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 & 3 \\ 3 & 2 & 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.667 & 1.333 & 1 \\ 0.6 & 0.4 & 1.6 & 1 \end{bmatrix} \begin{array}{c} A' \\ B' \end{array}$$

Perspective Transformation of Line Parallel to z-axis



Perspective projection

The parametric equation of the line segment $A'B'$ is

$$P(t) = [\, A' \,] + [\, B' - A' \,]t \quad 0 \leq t \leq 1$$

or $\quad P(t) = [\, 1 \quad 0.667 \quad 1.333 \quad 1 \,] + [\, -0.4 \quad -0.267 \quad 0.267 \quad 0 \,]t$

Intersection of this line with the $x = 0$, $y = 0$ and $z = 0$ planes yields

$$x(t) = 0 = 1 - 0.4t \qquad \rightarrow \qquad t = 2.50$$

$$y(t) = 0 = 0.667 - 0.267t \quad \rightarrow \quad t = 2.50$$

$$z(t) = 0 = 1.333 + 0.267t \quad \rightarrow \quad t = -5.0$$

Substituting $t = 2.5$ into the parametric equation of the line $A'B'$ yields

Perspective projection

$$z(2.5) = 1.333 + (0.267)(2.5) = 2.0$$

which represents the intersection of the line $A'B'$ with the z-axis at $z = +1/r$, the vanishing point. Now substituting $t = -5.0$ into the x and y component equations yields the intersection with the $z = 0$ plane, i.e.,

$$x(-5.0) = 1 - (0.4)(-5.0) = 3.0$$

$$y(-5.0) = 0.667 - (0.267)(-5.0) = 2.0$$

which is the same as the intersection of the line AB with the $z = 0$ plane.

Projection of line $A'B'$ into the line A^*B^* in the $z = 0$ plane is given by

Perspective projection

$$\begin{array}{c} A' \\ B' \end{array} \begin{bmatrix} 1 & 0.667 & 1.333 & 1 \\ 0.6 & 0.4 & 1.6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.667 & 0 & 1 \\ 0.6 & 0.4 & 0 & 1 \end{bmatrix} \begin{array}{c} A^* \\ B^* \end{array}$$

Single point Perspective Transformation for a cube

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

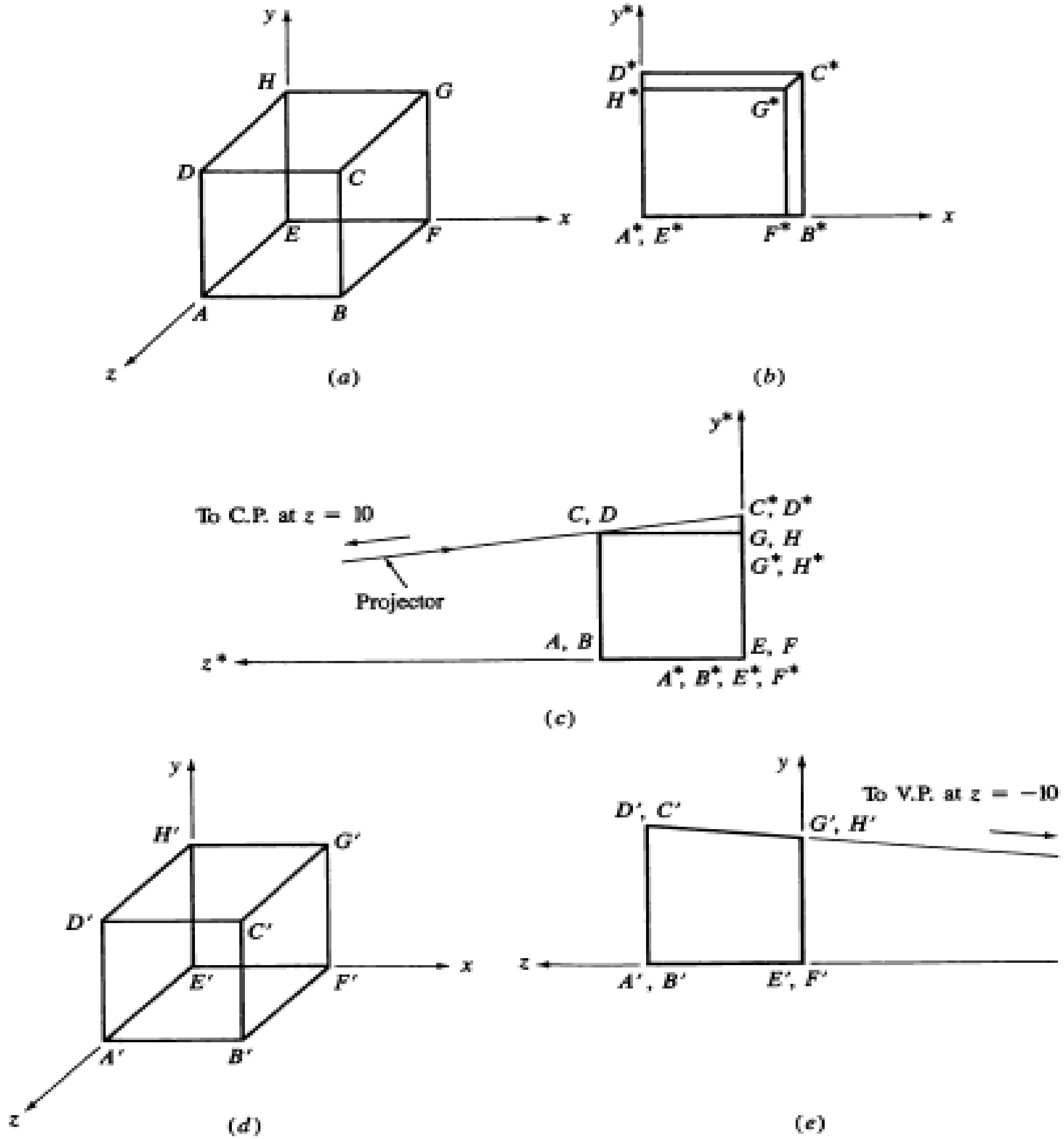
$$= \begin{bmatrix} 0 & 0 & 0 & 0.9 \\ 1 & 0 & 0 & 0.9 \\ 1 & 1 & 0 & 0.9 \\ 0 & 1 & 0 & 0.9 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1.11 & 0 & 0 & 1 \\ 1.11 & 1.11 & 0 & 1 \\ 0 & 1.11 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Single point Perspective Transformation for a cube

$$[X'] = [X][P_r] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0.9 \\ 1 & 0 & 1 & 0.9 \\ 1 & 1 & 1 & 0.9 \\ 0 & 1 & 1 & 0.9 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1.11 & 1 \\ 1.11 & 0 & 1.11 & 1 \\ 1.11 & 1.11 & 1.11 & 1 \\ 0 & 1.11 & 1.11 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Single point Perspective Transformation for a cube



Single-point perspective projection of a unit cube.

Two point Perspective projection rotation about a single Principal axis

Consider the cube shown in Fig. 3–35a rotated about the y-axis by $\phi = 60°$ to reveal the left-hand face and translated -2 units in y to reveal the top face projected onto the $z = 0$ plane from a center of projection at $z = z_c = 2.5$.

Using Eq. (3–38) with $\phi = 60°$, Eq. (3–47) with $z_c = 2.5$ and Eq. (3–14) with $n = l = 0$, $m = -2$ yields

$$[T] = [R_y][Tr][P_{rz}]$$

$$= \begin{bmatrix} 0.5 & 0 & -0.866 & 0 \\ 0 & 1 & 0 & 0 \\ 0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

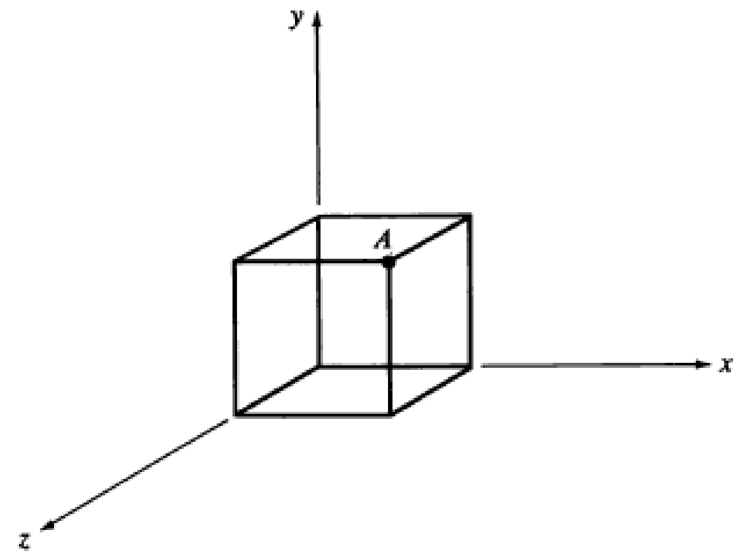
$$= \begin{bmatrix} 0.5 & 0 & 0 & 0.346 \\ 0 & 1 & 0 & 0 \\ 0.866 & 0 & 0 & -0.2 \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

Perspective projection

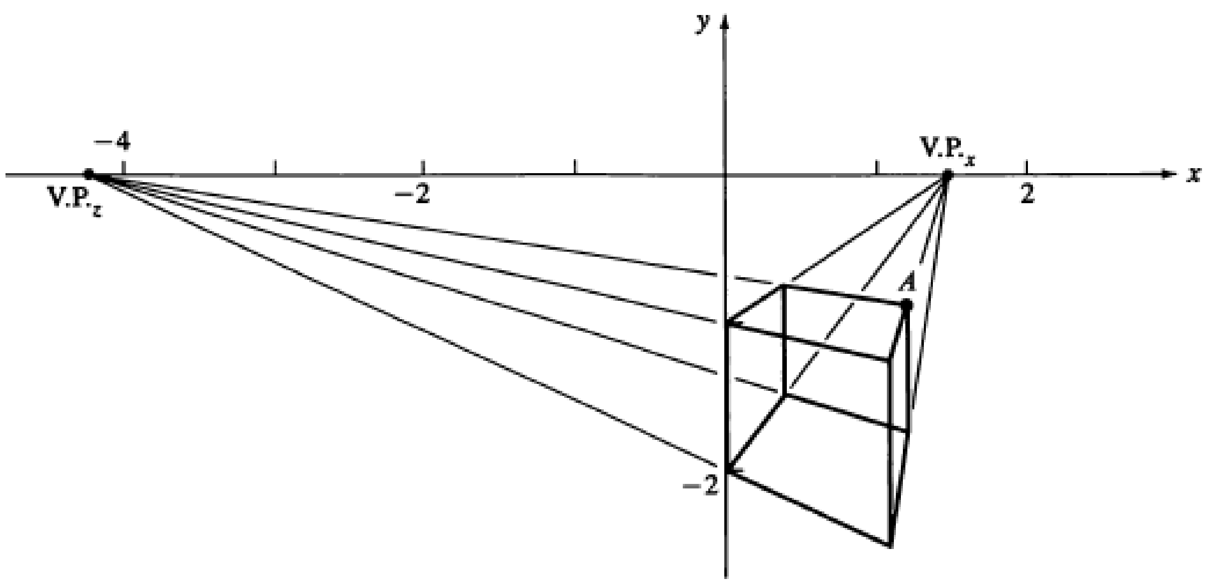
$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0.346 \\ 0 & 1 & 0 & 0 \\ 0.866 & 0 & 0 & -0.2 \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -2 & 0 & 0.8 \\ 1.366 & -2 & 0 & 1.146 \\ 1.366 & -1 & 0 & 1.146 \\ 0.866 & -1 & 0 & 0.8 \\ 0 & -2 & 0 & 1 \\ 0.5 & -2 & 0 & 1.346 \\ 0.5 & -1 & 0 & 1.346 \\ 0 & -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.083 & -2.5 & 0 & 1 \\ 1.192 & -1.745 & 0 & 1 \\ 1.192 & -0.872 & 0 & 1 \\ 1.083 & -1.25 & 0 & 1 \\ 0 & -2 & 0 & 1 \\ 0.371 & -1.485 & 0 & 1 \\ 0.371 & -0.743 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Perspective projection



(a)



Perspective projection

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & -0.612 & 0 & 0.755 \\ 0.366 & -0.966 & 0 & 0.614 \\ 0.366 & -0.259 & 0 & 0.331 \\ -0.5 & 0.095 & 0 & 0.472 \\ 0 & 0 & 0 & 1 \\ 0.866 & -0.354 & 0 & 0.859 \\ 0.866 & 0.354 & 0 & 0.576 \\ 0 & 0.707 & 0 & 0.717 \end{bmatrix} = \begin{bmatrix} -0.662 & -0.811 & 0 & 1 \\ 0.596 & -1.574 & 0 & 1 \\ 1.107 & -0.782 & 0 & 1 \\ -1.059 & 0.201 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1.009 & -0.412 & 0 & 1 \\ 1.504 & 0.614 & 0 & 1 \\ 0 & 0.986 & 0 & 1 \end{bmatrix}$$

The result is shown in Fig. 3–36.

Perspective projection

Similarly, a three-point perspective transformation is obtained by rotating about two or more of the principal axes and then performing a single-point perspective transformation. For example, rotation about the y-axis followed by rotation about the x-axis and a perspective projection onto the $z = 0$ plane from a center of projection at $z = z_c$ yields the concatenated transformation matrix

$$
\begin{aligned}
[\,T\,] &= [\,R_y\,][\,R_x\,][\,P_{rz}\,] \\
&= \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & \dfrac{\sin\phi\cos\theta}{z_c} \\ 0 & \cos\theta & 0 & -\dfrac{\sin\theta}{z_c} \\ \sin\phi & -\cos\phi\sin\theta & 0 & -\dfrac{\cos\phi\cos\theta}{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Perspective projection

Example 3–26 Principal Vanishing Points by Transformation

Recalling Ex. 3–24 the concatenated complete transformation was

$$[\,T\,] = \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transforming the points at infinity on the x-, y- and z-axes yields

$$[\,VP\,][\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6.142 & 2.5 & 0 & 1 \\ 0 & -2.5 & 0 & 1 \\ 2.04 & 2.5 & 0 & 1 \end{bmatrix}$$

These vanishing points are shown in Fig. 3–36.

Perspective projection

Example 3–24 Three-Point Perspective Projection with Rotation About Two Axes

Consider the cube shown in Fig. 3–35a rotated about the y-axis by $\phi = -30^\circ$, about the x-axis by $\theta = 45^\circ$ and projected onto the $z = 0$ plane from a center of projection at $z = z_c = 2.5$.

Using Eq. (3–62) yields

$$[T] = \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

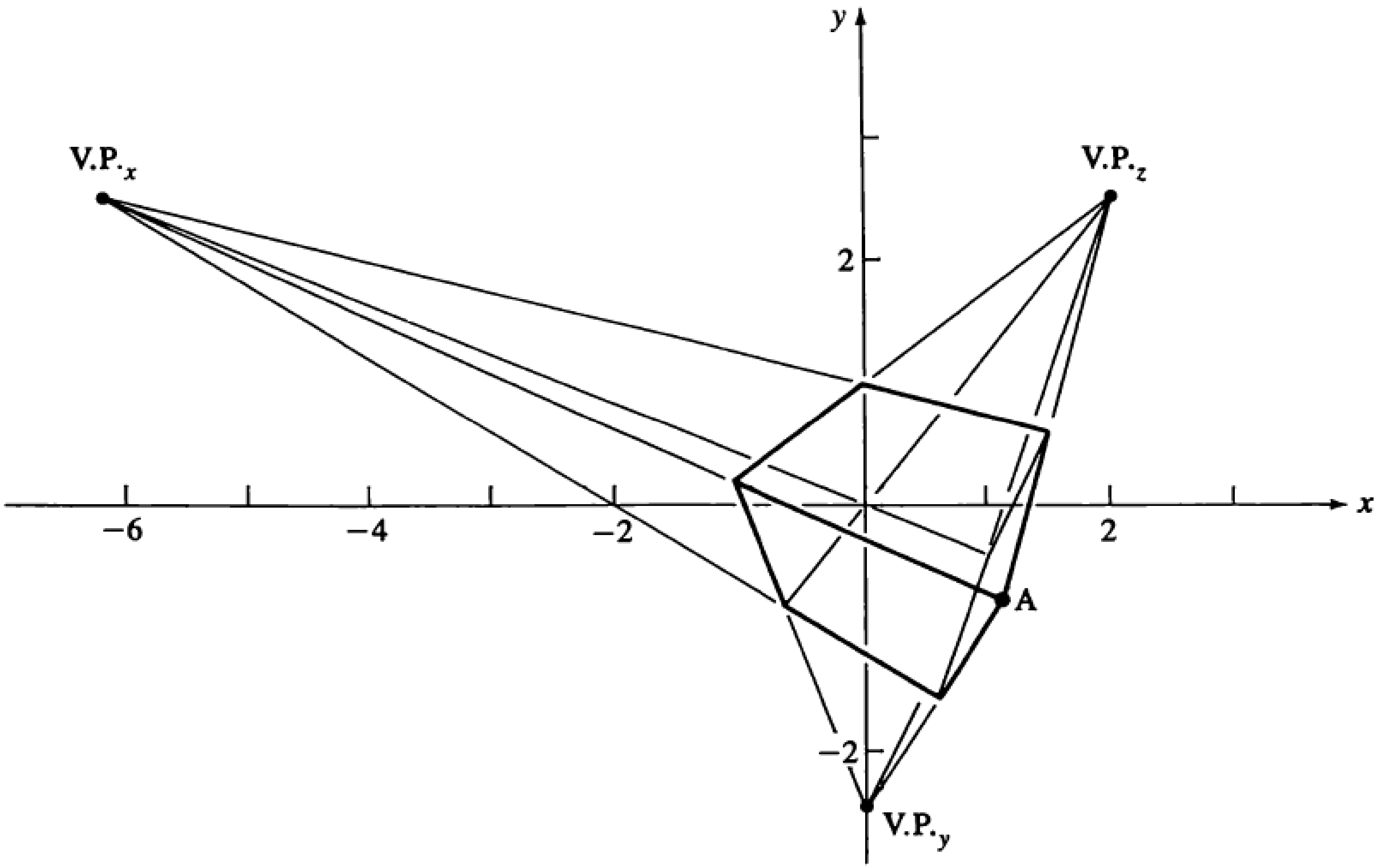
The transformed position vectors are

Perspective projection

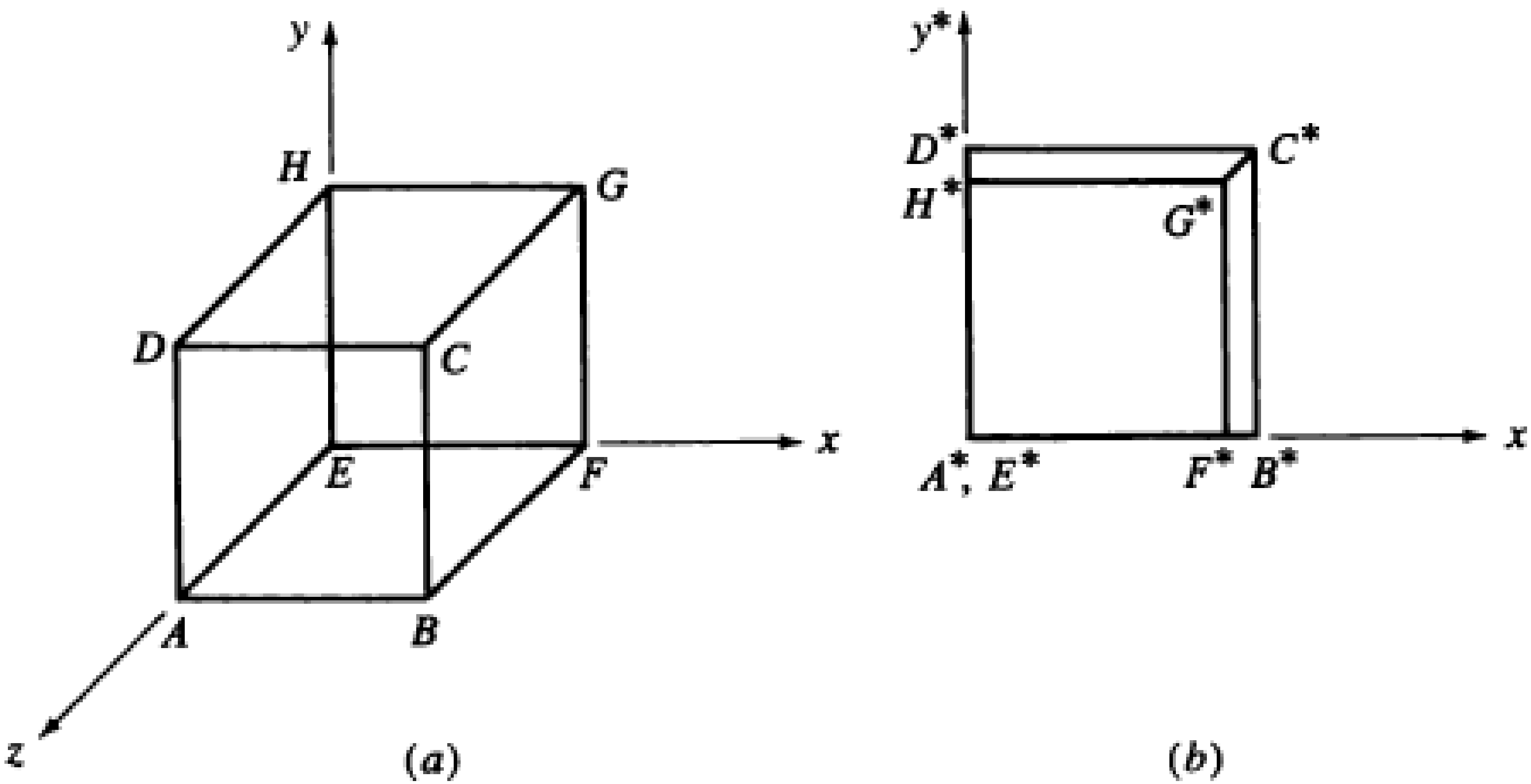
$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & -0.612 & 0 & 0.755 \\ 0.366 & -0.966 & 0 & 0.614 \\ 0.366 & -0.259 & 0 & 0.331 \\ -0.5 & 0.095 & 0 & 0.472 \\ 0 & 0 & 0 & 1 \\ 0.866 & -0.354 & 0 & 0.859 \\ 0.866 & 0.354 & 0 & 0.576 \\ 0 & 0.707 & 0 & 0.717 \end{bmatrix} = \begin{bmatrix} -0.662 & -0.811 & 0 & 1 \\ 0.596 & -1.574 & 0 & 1 \\ 1.107 & -0.782 & 0 & 1 \\ -1.059 & 0.201 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1.009 & -0.412 & 0 & 1 \\ 1.504 & 0.614 & 0 & 1 \\ 0 & 0.986 & 0 & 1 \end{bmatrix}$$

Perspective projection



Example single point Perspective projection for a cube on to z=0 plane for a unit cube from center of projection at z=10 on z-axis



$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

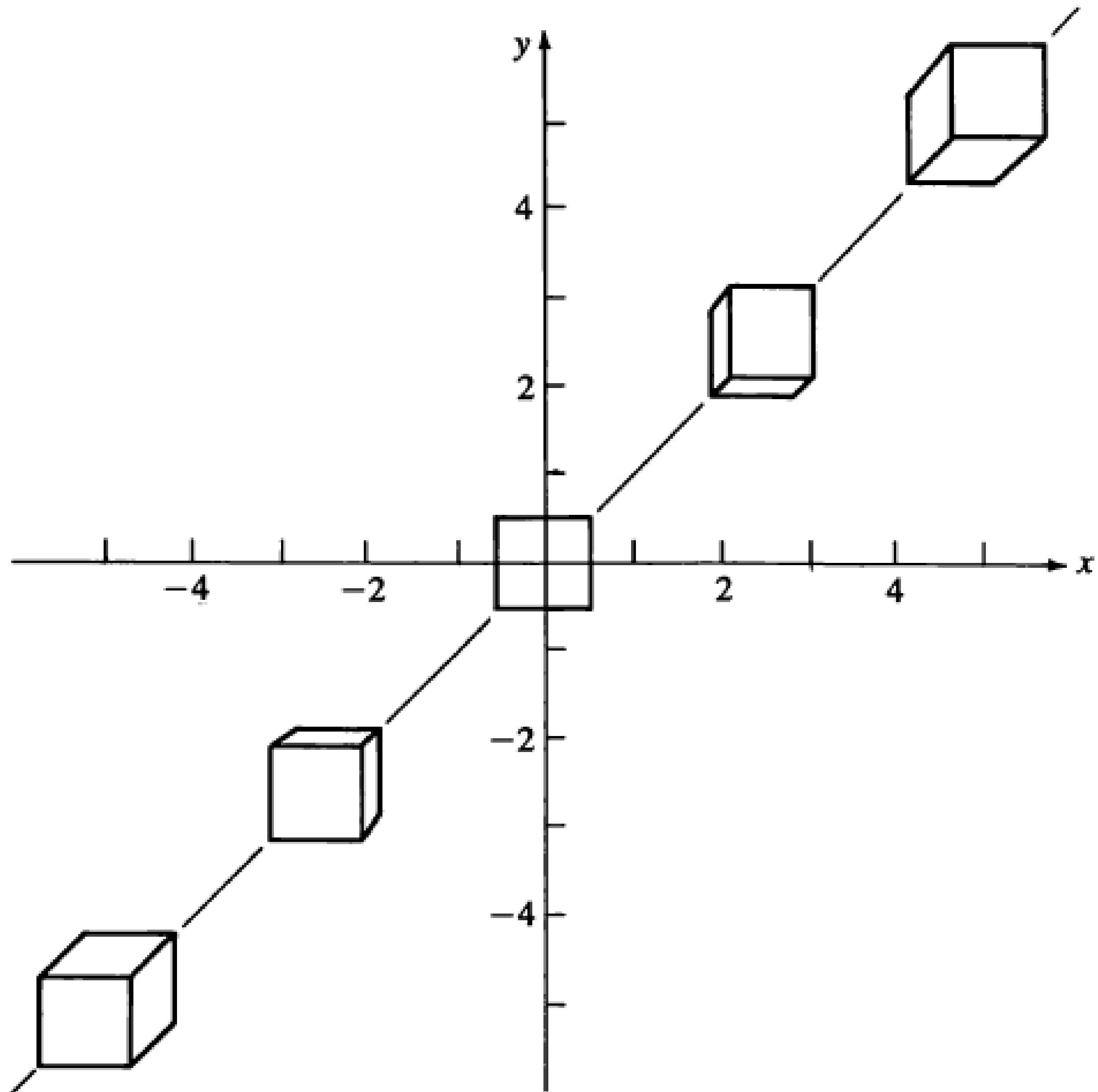
$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0.9 \\ 1 & 0 & 0 & 0.9 \\ 1 & 1 & 0 & 0.9 \\ 0 & 1 & 0 & 0.9 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1.11 & 0 & 0 & 1 \\ 1.11 & 1.11 & 0 & 1 \\ 0 & 1.11 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Techniques to generate perspective view

$$[\,T\,] = [\,Tr_{xyz}\,][\,P_{rz}\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ l & m & 0 & 1+rn \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ l & m & 0 & 1-n/z_c \end{bmatrix}$$



y
4
2
−2
−4
x
−4
−2
2
4

Rotation and perspective transformations

$$[T] = [R_y][P_{rz}] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & 0 & 0 & \dfrac{\sin\phi}{z_c} \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & 0 & -\dfrac{\cos\phi}{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

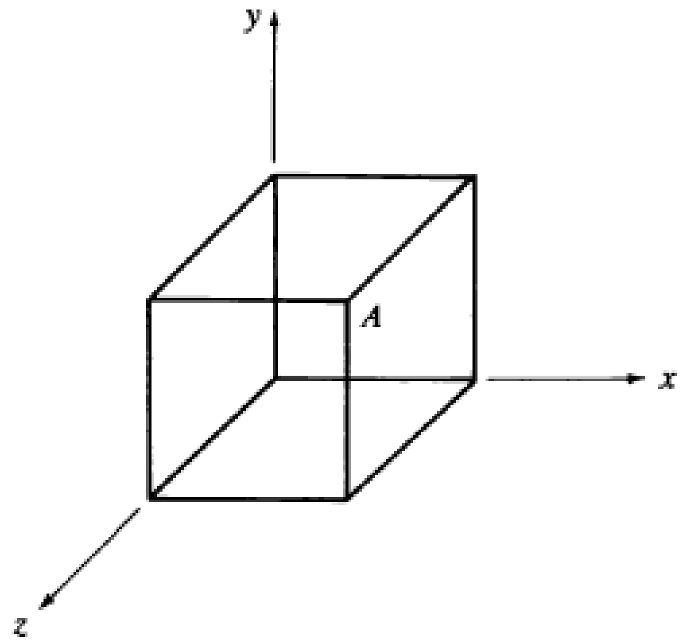
$$
[\,T\,] = [\,R_x\,][\,P_{rz}\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & -\dfrac{\sin\theta}{z_c} \\ 0 & -\sin\theta & 0 & -\dfrac{\cos\theta}{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

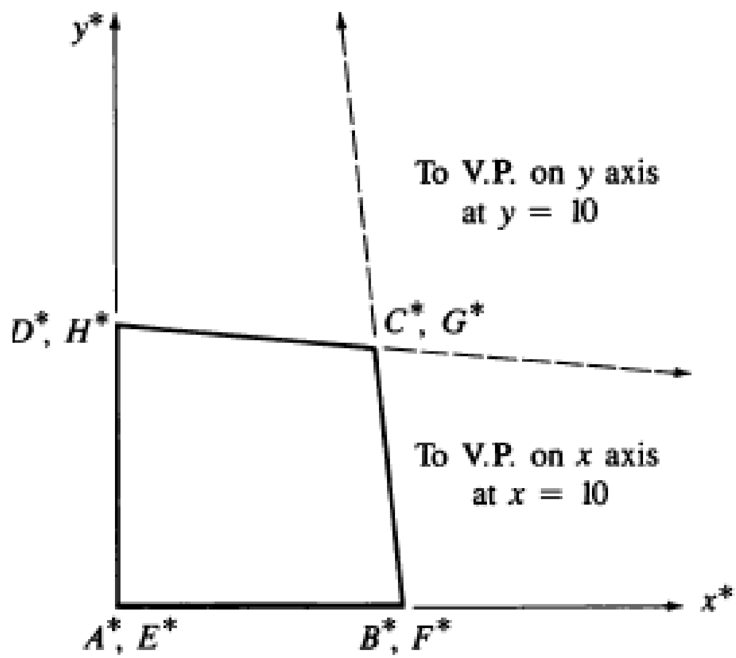
Example 2 point Perspective projection for a cube on to z=0 plane for a unit cube from centers of projection at x=-10, y=10 on z-axis

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1.1 \\ 1 & 1 & 0 & 1.2 \\ 0 & 1 & 0 & 1.1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1.1 \\ 1 & 1 & 0 & 1.2 \\ 0 & 1 & 0 & 1.1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.909 & 0 & 0 & 1 \\ 0.833 & 0.833 & 0 & 1 \\ 0 & 0.909 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.909 & 0 & 0 & 1 \\ 0.833 & 0.833 & 0 & 1 \\ 0 & 0.909 & 0 & 1 \end{bmatrix}$$



y
A
x
z



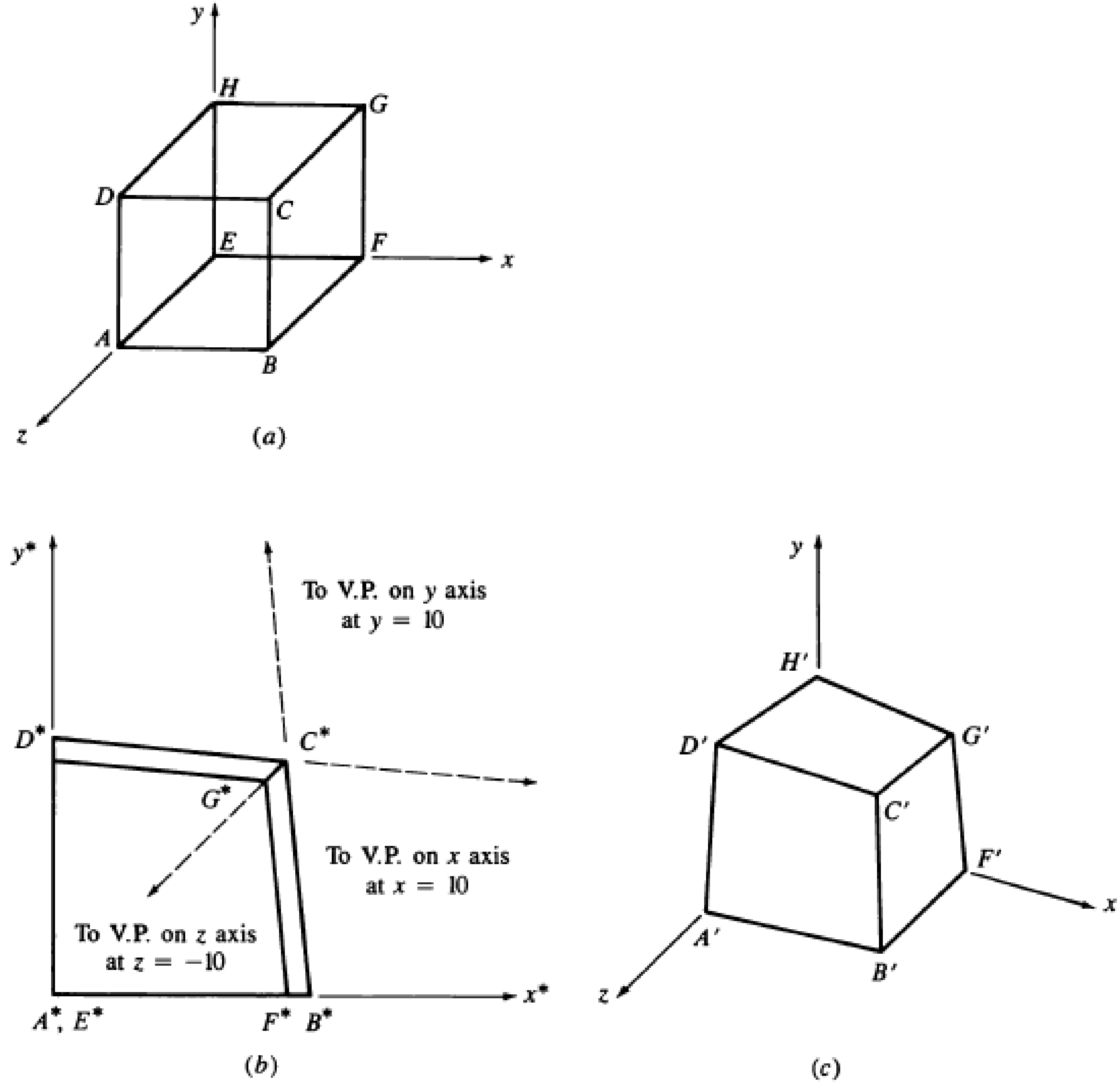
y*
To V.P. on y axis at y = 10
D*, H*
C*, G*
To V.P. on x axis at x = 10
x*
A*, E*
B*, F*

Example 3 point Perspective projection for a cube on to z=0 plane for a unit cube from centers of projection at x=10, y=10, z=-10 on z-axis

$$[\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\,X^*\,] = [\,X\,][\,T\,] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0.9 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1.1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1.1 \\ 1 & 1 & 0 & 1.2 \\ 0 & 1 & 0 & 1.1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0.909 & 0.909 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.909 & 0 & 0 & 1 \\ 0.833 & 0.833 & 0 & 1 \\ 0 & 0.909 & 0 & 1 \end{bmatrix}$$


y
H
G
D
C
E
F
x
A
B
z
(a)
y*
To V.P. on y axis
at y = 10
D*
C*
G*
To V.P. on x axis
at x = 10
To V.P. on z axis
at z = −10
A*, E*
F*
B*
x*
(b)
y
H′
D′
G′
C′
F′
x
A′
B′
z
(c)

Equivalent transformation matrix for rotation about y axis by phi and x-axis by theta and a perspective projection on z=0 from center of projection z=zc

$$[\,T\,] = [\,R_y\,][\,R_x\,][\,P_{rz}\,]$$

$$= \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & \dfrac{\sin\phi\cos\theta}{z_c} \\ 0 & \cos\theta & 0 & -\dfrac{\sin\theta}{z_c} \\ \sin\phi & -\cos\phi\sin\theta & 0 & -\dfrac{\cos\phi\cos\theta}{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equivalent transformation matrix for rotation about y axis by phi and x-axis by theta and a perspective projection on z=0 from center of projection z=zc

$$[\,T\,] = [\,R_y\,][\,R_x\,][\,P_{rz}\,]$$

$$= \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/z_c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & \dfrac{\sin\phi\cos\theta}{z_c} \\ 0 & \cos\theta & 0 & -\dfrac{\sin\theta}{z_c} \\ \sin\phi & -\cos\phi\sin\theta & 0 & -\dfrac{\cos\phi\cos\theta}{z_c} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equivalent transformation matrix for rotation about y axis by -30 degree and x-axis by 45 degree perspective projection on z=0 from center of projection z=2.5

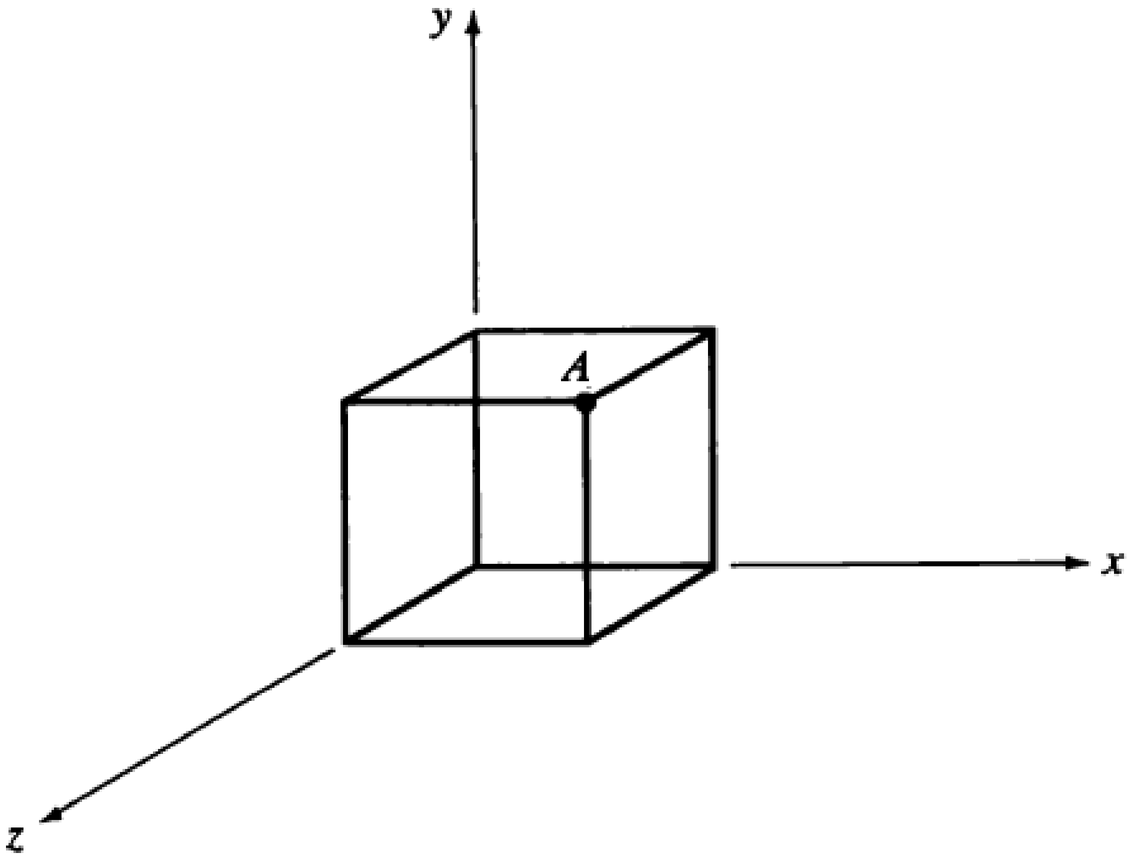
$$[\,T\,] = \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the transformed cube for the case discussed in previous page

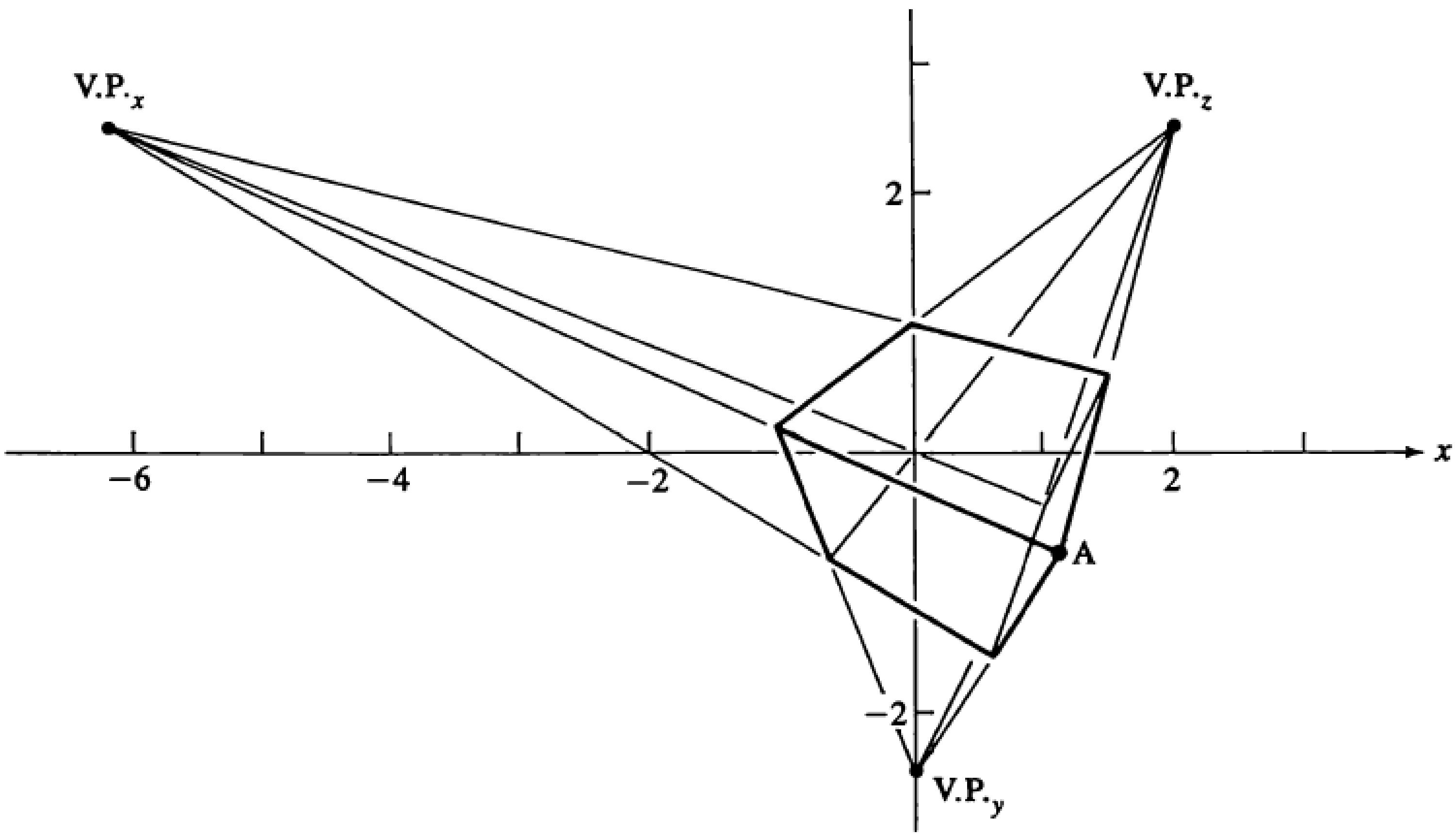
$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & -0.612 & 0 & 0.755 \\ 0.366 & -0.966 & 0 & 0.614 \\ 0.366 & -0.259 & 0 & 0.331 \\ -0.5 & 0.095 & 0 & 0.472 \\ 0 & 0 & 0 & 1 \\ 0.866 & -0.354 & 0 & 0.859 \\ 0.866 & 0.354 & 0 & 0.576 \\ 0 & 0.707 & 0 & 0.717 \end{bmatrix} = \begin{bmatrix} -0.662 & -0.811 & 0 & 1 \\ 0.596 & -1.574 & 0 & 1 \\ 1.107 & -0.782 & 0 & 1 \\ -1.059 & 0.201 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1.009 & -0.412 & 0 & 1 \\ 1.504 & 0.614 & 0 & 1 \\ 0 & 0.986 & 0 & 1 \end{bmatrix}$$

Find the transformed cube for the case discussed in previous page



Find the transformed cube for the case discussed in previous page



Same transformation for triangular prism

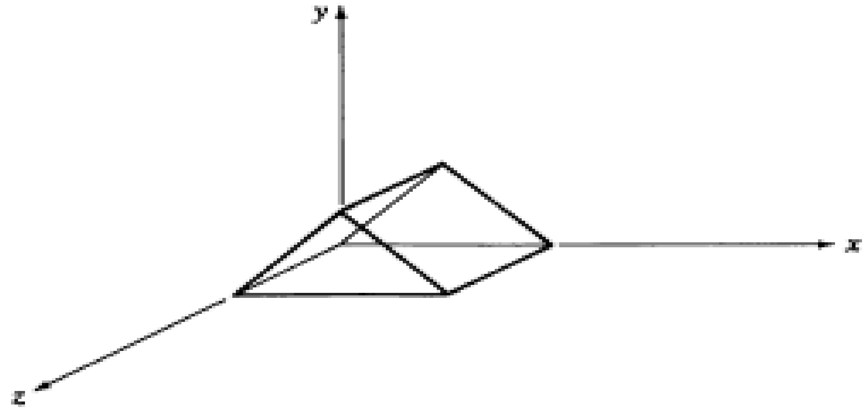
$$[X^*] = [X][T]$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0.5 & 0.5 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.354 & 0 & -0.141 \\ 0 & 0.707 & 0 & -0.283 \\ -0.5 & -0.612 & 0 & -0.245 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

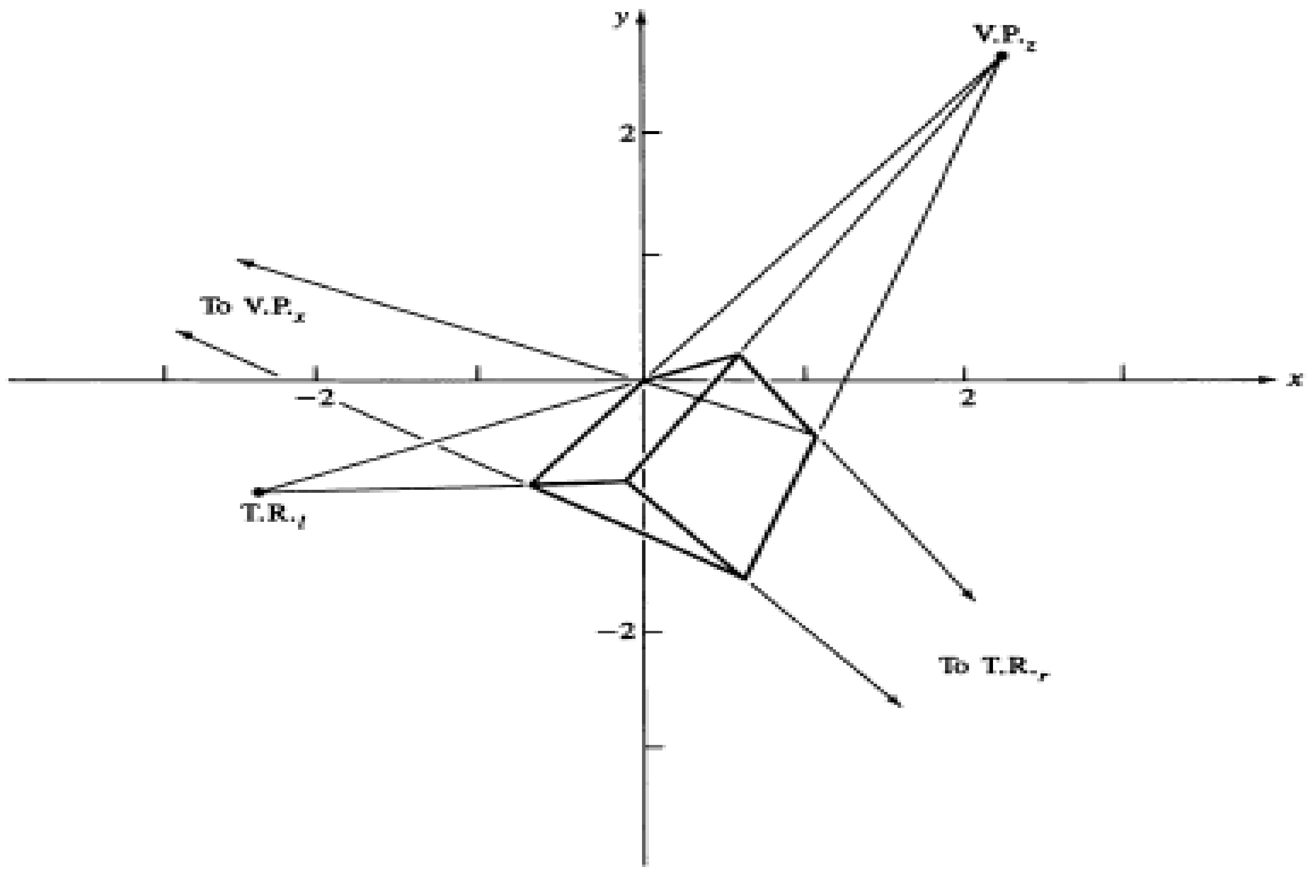
Triangular prism

$$= \begin{bmatrix} -0.5 & -0.612 & 0 & 0.755 \\ 0.366 & -0.966 & 0 & 0.614 \\ -0.067 & -0.436 & 0 & 0.543 \\ 0 & 0 & 0 & 1 \\ 0.866 & -0.354 & 0 & 0.859 \\ 0.433 & 0.177 & 0 & 0.788 \end{bmatrix}$$

$$= \begin{bmatrix} -0.662 & -0.811 & 0 & 1 \\ 0.596 & -1.574 & 0 & 1 \\ -0.123 & -0.802 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1.009 & -0.412 & 0 & 1 \\ 0.55 & 0.224 & 0 & 1 \end{bmatrix}$$



(a)



Characteristics Of Perspective Projection

1. Types of perspective projection include: one-point, two-point and three-point perspective projection.
2. Perspective projection cannot give the accurate view of object
3. Perspective projection represents the object in three dimensional way
4. Perspective projection forms a realistic picture of object
5. In perspective projection, the distance of the object from the center of projection is finite.
6. Projector in perspective projection is not parallel.
7. Perspective projection cannot preserve the relative proportion of an object
8. The lines of perspective projection are not parallel

Characteristics Of Parallel Projection

1. There are two types of parallel projection, that is, orthographic and oblique parallel projection.
2. Parallel projection can give the accurate view of object.
3. Parallel projection represents the object in a different way like telescope.
4. Parallel projection does not form realistic view of object.
5. In parallel projection, the distance of the object from the center of projection is infinite.
6. Projector in parallel projection is parallel.
7. Parallel projection can preserve the relative proportion of an object.
8. The lines of parallel projection are parallel.

BASIS OF DIFFERENCE	PERSPECTIVE PROJECTION	PARALLEL PROJECTION
Description	A perspective projection can be described as the projector lines (lines of sight) that converge at the center of projection, which results in many visual effects of an object.	A parallel projection is a projection of an object in three-dimensional space onto a fixed plane referred as the projection plane or image plane, where the rays, known as lines of sight or projection lines are parallel to each other.
Types	One-point perspective Projection. Two-point perspective projection. Three-point perspective projection.	Orthographic parallel projection Oblique parallel projection.
Accurate View Of Object	Perspective projection cannot give the accurate view of object.	Parallel projection can give the accurate view of object.
Object Representation	Perspective projection represents the object in three dimensional way.	Parallel projection represents the object in a different way like telescope.
Realistic View	Perspective projection forms a	Parallel projection does not

BASIS OF DIFFERENCE	PERSPECTIVE PROJECTION	PARALLEL PROJECTION
Distance Of The Object From The Center Of Projection	The distance of the object from the center of projection is finite.	In parallel projection, the distance of the object from the center of projection is infinite.
Projector	Projector in perspective projection is not parallel.	Projector in parallel projection is parallel.
Preservation Of Relative Portion Of An Object	Perspective projection cannot preserve the relative proportion of an object.	Parallel projection can preserve the relative proportion of an object.
Lines Of Projection	The lines of perspective projection are not parallel.	The lines of parallel projection are parallel.

What You Need To Know About Orthographic Projection

•Orthographic projection is a method of projection in which an object is depicted using parallel lines to project its outline on a plane. For example, an orthographic projection of a house typically consists of a top view or plan and a front view and one side view (front and side elevations).

• Orthographic projection can also be described as a two-dimensional graphic representation of an object in which the projecting lines are at right angles to the plane of the projection.

•Orthographic projection can also be referred to as orthogonal projection.

•It is a form of parallel projection

•In orthographic projection, the projector lines intersect the plane being projected on to at a perpendicular angle (thus, they are orthogonal).

•Typically, an orthographic projection drawing consists of three different views: a front view, a top view and a side view. Other names for these views are plan, elevation and section. Occasionally, more views are used for clarity. The side view is usually the right side, but if the left side is used, it is noted in the drawing.

•Orthographic projection can be subdivided into three categories: **isometric, diametric and trimetric projection.**

•An orthographic drawing is a clear, detailed way to represent the image of an object. It may be used by engineers, designers, architects and technical artists to help a manufacturer understand the specifics of a product that needs to be created.

•A lens providing an orthographic projection is referred to as an object-space telecentric lens.

•When principal planes or axes of an object are not parallel within the projection plane, but are rather tilted to reveal multiple sides of the object, the projection is referred to as axonometric projection.

What You Need To Know About Oblique Projection

•Oblique projection is a simple type of technical drawing of graphical projection used for producing two-dimensional images of three-dimensional objects.
•Oblique projection can also be described as a parallel projection in which the projection lines are not orthogonal to the projection line.
•Oblique projection is a form of parallel projection.
•Usually, the presence of one or more 90º angles within a pictorial image is usually a good indication that the perspective is oblique.
•In oblique projection, the projector lines form oblique angles (non-right angles) with the projection plane.
•There are two types of oblique projections: Cavalier and Cabinet. The Cavelier projection makes 45º angle with projection plane whereas In Cabinet projection, the projection of a line perpendicular to the view plane has the same length as the line itself.
•Usually when drawing, all the three axes of the oblique drawing are constructed in which one axis is horizontal and the other is vertical while the third axis is from 30º to 60º to the horizontal line. At least two views of orthographic projection are selected.
•Oblique projection is commonly used in technical drawing.
•Oblique drawings are also used in engineering and design. The object is drawn with the most distinguishing features facing towards the observer, showing true shape of these features.

BASIS OF COMPARISON	ORTHOGRAPHIC PROJECTION	OBLIQUE PROJECTION
Description	Orthographic projection can also be described as a two-dimensional graphic representation of an object in which the projecting lines are at right angles to the plane of the projection.	Oblique projection can also be described as a parallel projection in which the projection lines are not orthogonal to the projection line.
Form	It is a form of parallel projection	Oblique projection is a form of parallel projection.
Types	Isometric Projection Diametric Projection Trimetric projection	Cavalier Projection Cab inet Projection
Projector	The projector lines intersect	The projector lines form