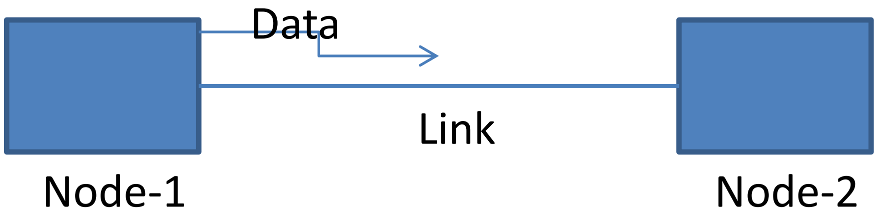
Physical Layer: Theory

Kameswari Chebrolu

Recap

- Nodes generate data (bits: 1’s and 0’s)
- Links carry signals in the form of electromagnetic waves
- Task on hand: Convert data into signals
  - Process termed: Encoding/Modulation
- First: Some Theory



Link Characteristics

- Data Rate: How many bits per second can be transferred on the link? (expressed in bps,kbps,Mbps,Gbps)
- Loss rate: What is the probability of packet error (or bit error) rate on the link?
- Delay: How much time does it take for the bits to reach other end?

Simple Encoding

- Data: 101111011

You Your Friend (Far Away)

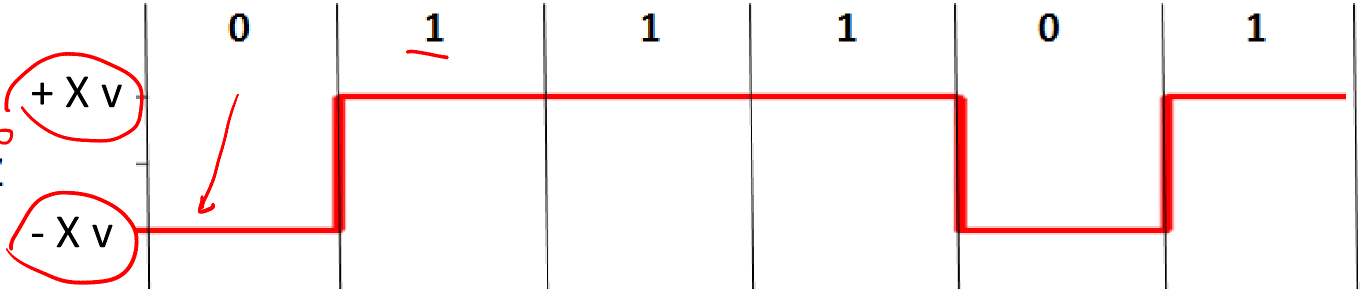
Wire Pair

~~I will follow the wire, reach other end and convey the data in person~~

- How would you send the data over the wires?

Non Return to Zero

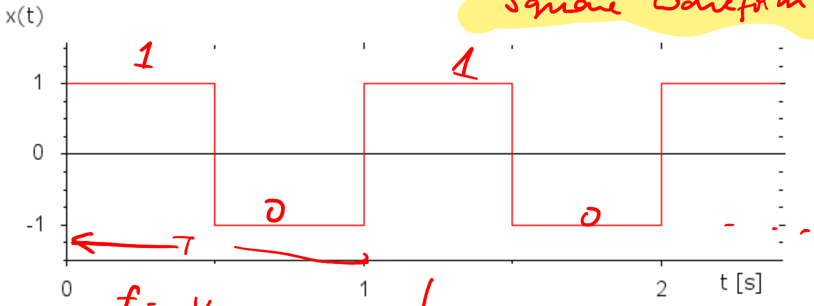
NRZ



Signals and Bandwidth

- Fourier Analysis shows that any signal can be decomposed to sinusoids of different amplitude, frequency and phases

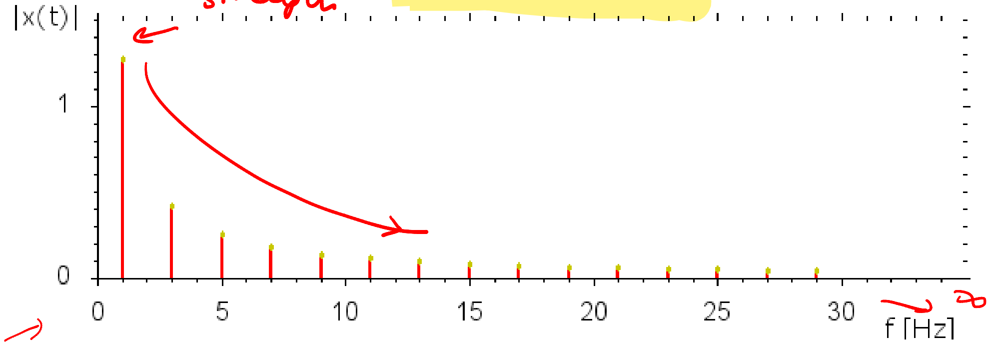
Square waveform



$f = 1/T$

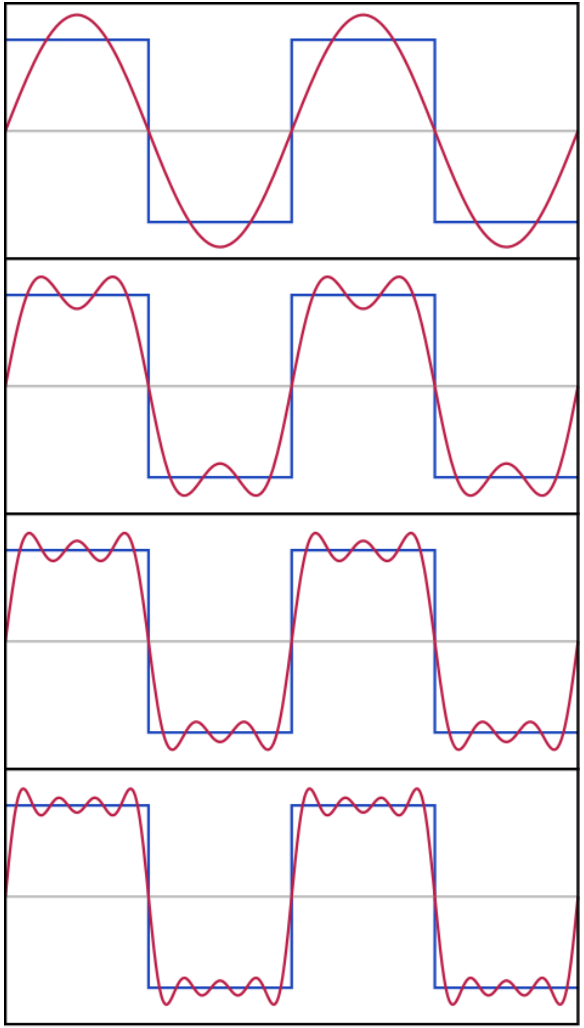
strength

Freq domain



$$s(t) = A \times \frac{4}{\pi} \times \sum_{k \text{ odd}, k=1}^{\infty} \frac{\sin(2\pi k f t)}{k}$$

f $3f$ $5f$ $7f$ ...



$(4/\pi)\,[\sin(2\pi f t)]$

$f \rightarrow$ fundamental freq

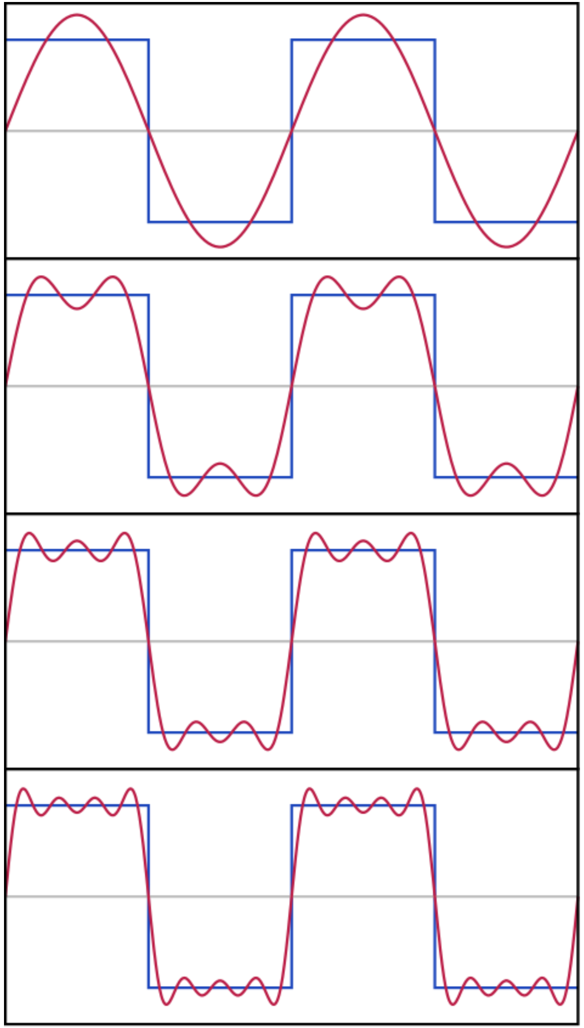
$= \frac{1}{T}$

$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t)]$

f, 3f

$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$

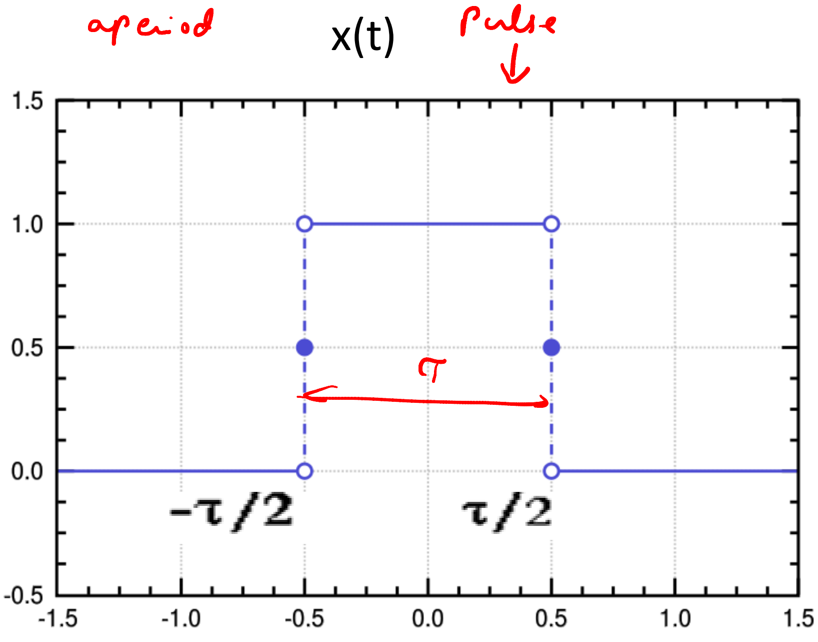


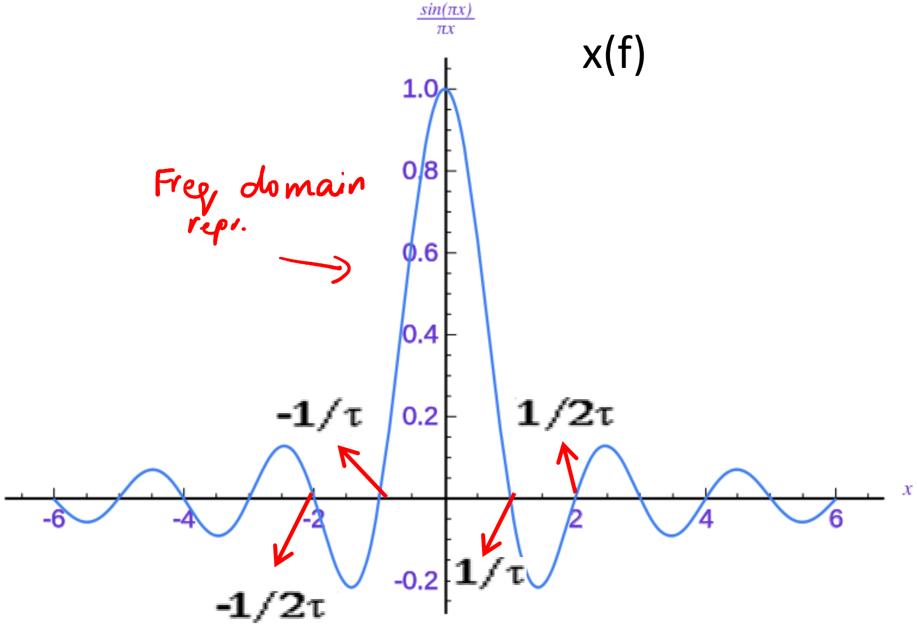
$$(4/\pi)\,[\sin(2\pi f t)]$$

$$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t)]$$

$$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$$

$$(4/\pi)\,[\sin(2\pi f t) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$$





earlier case, freq discrete: fourier series
now continuous: fourier transform

Fourier Transform

NRZ bandwidth

aperiodic.

Periodic or aperiodic?

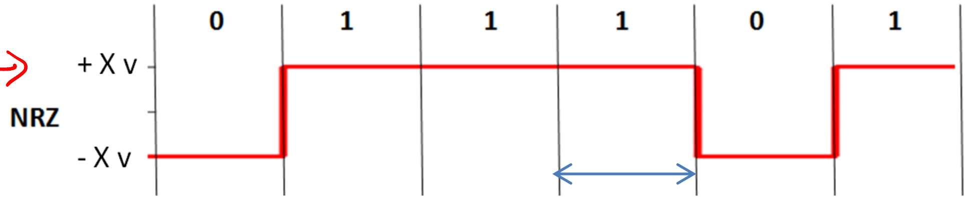
- Spectrum of a signal: Range of frequencies it contains
- Bandwidth: width of the spectrum

  infinity tbh

  effective BW

  – First Null Bandwidth = 1/T

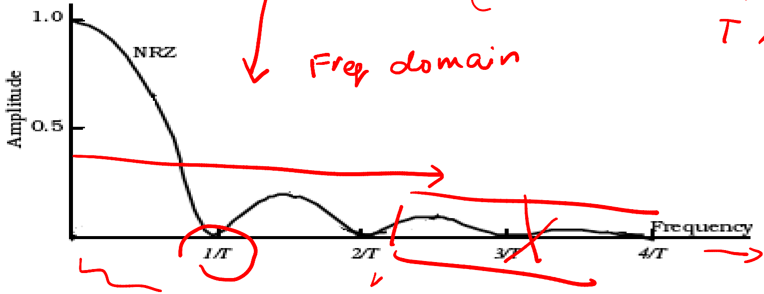
  Data Rate



T

1 bit/ T sec

Freq domain



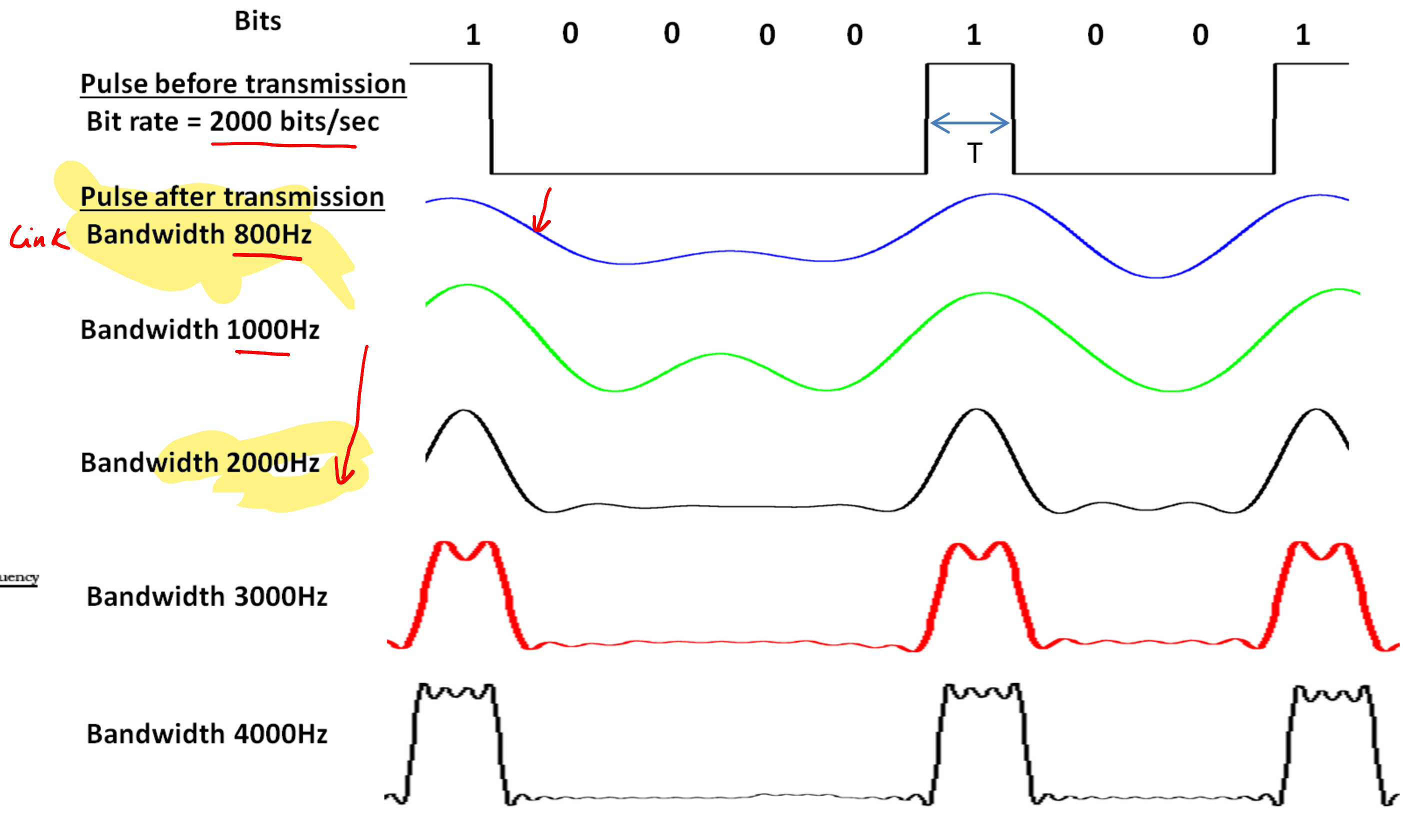
∞

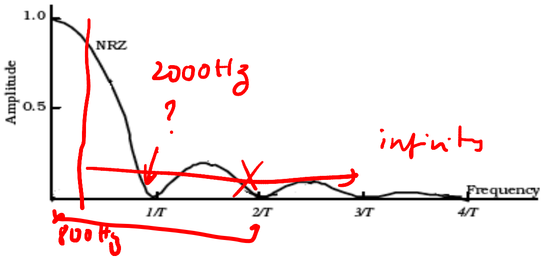
Spectrum of a random NRZ signal

each bit is of duuration T... we are sending 1 bit per T sec in the cable.

Link Bandwidth

- How much link bandwidth do I need to recover signal?





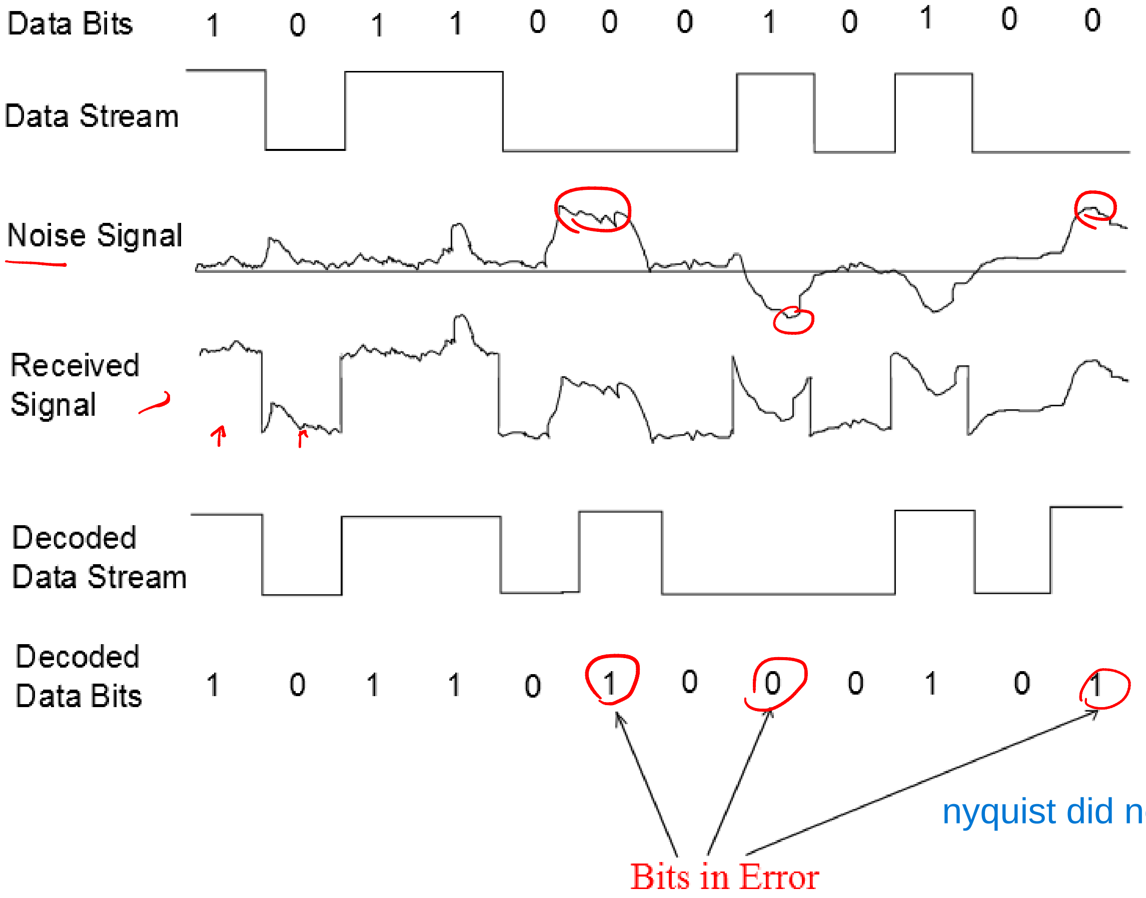
Random NRZ Signal Bandwidth

Nyquist Rate

- The number of independent pulses that could be put through a channel per unit time is limited to twice the bandwidth of the channel
- $f_p <= 2B$
  - f_p is pulse rate (number of pulses/sec)
  - B is bandwidth of the channel
- Example: Binary pulse with rate 2000 bps needs link bandwidth of at least 1000 Hz

least band width needed to recover that pulse.

Effect of Noise



2000bits/sec

1000 Hz

infinite Link BW

assume infinite band.width

nyquist did not consider noise.

Shannon's Theorem

- Provides an upper bound to the capacity (bps) of a link
- $C = B \log_2 (1 + S/N)$ bits/sec
  - C: capacity (bps), B: channel bandwidth (Hz), S/N: signal to noise ratio
  - S/N often expressed in dB, $10\log_{10}(S/N)$
    - E.g. 30dB corresponds to a ratio of 1000
- Example: Data over telephone line calculation
- B = 3300Hz – 300Hz = 3000Hz; S/N = 1000 (30db); C ~ 30kbps

Implications

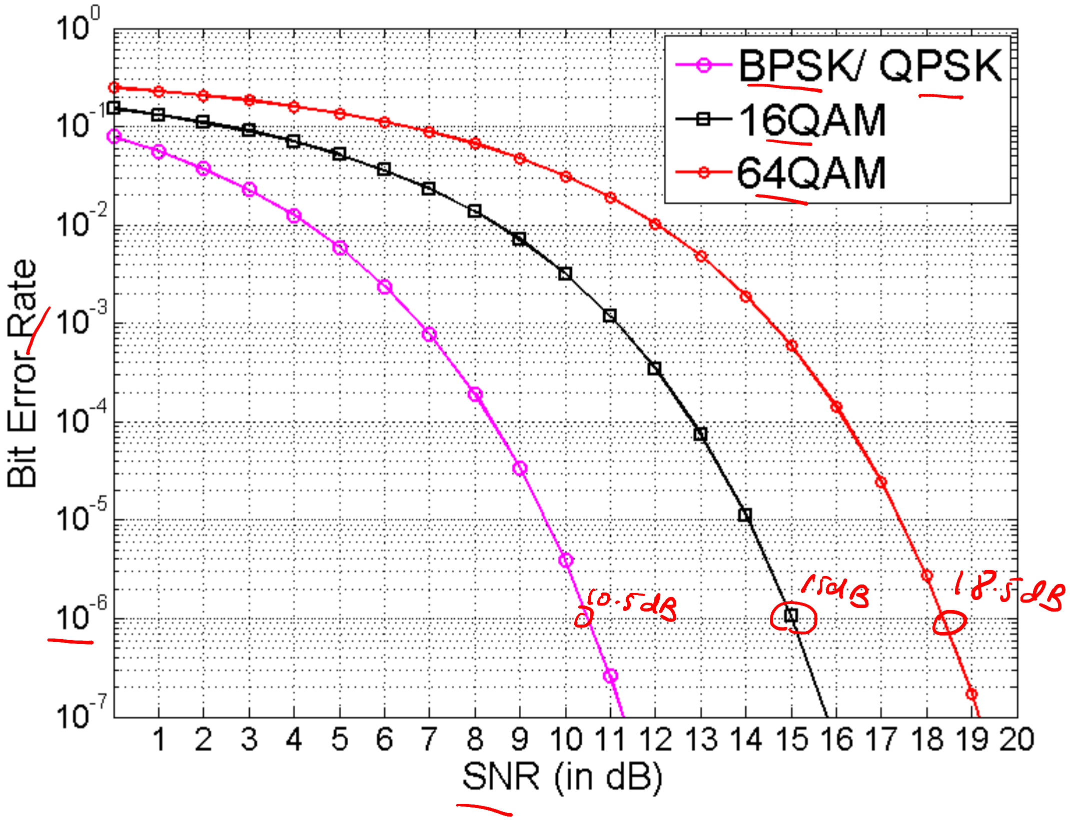
- If R < C, there exists a coding technique which permits transmission of data with arbitrary small error
  - The theorem does not specify how to reach this bound
  - Real systems rarely achieve this upper bound
  - E.g. WiFi: 64-QAM Modulation operating in 20Mhz bandwidth needs SNR of 27dB for packet error rate under 10% to provide 54Mbps
    - R is 54Mbps rate, B is 20Mhz band,, C is 124Mbps
- If R > C, the probability of error increases without bound

Error Rate

- What is the probability of bit error on a link?
- Function of received SNR and type of modulation
- For a given modulation, charts of SNR vs BER are often provided

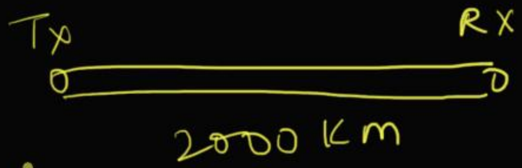
Bit Error Rate (BER)

- Packet Error Rate (PER) = 1-(1-BER)^N, where N is the size of packet in bits
  - BER 10^{-8}, PER = 0.008%
- Typical BERs:
  - Wireless ~ 10^{-6}, Twisted Pair ~ 10^{-8}, Fiber-optics ~ 10^{-9} to 10^{-12}



Propagation Delay

- Time required for a bit to propagate from beginning of link to end of the link
  - Depends on speed of light in the medium (S) & distance (D)
    - speed of light: $2* 10^8$ to $3 * 10^8$ m/s
  - Formula: D/S
- Note that Transmission delay(TD) is different from propagation delay
  - TD(sec) = Length of the packet (bits)/Data rate(bps)

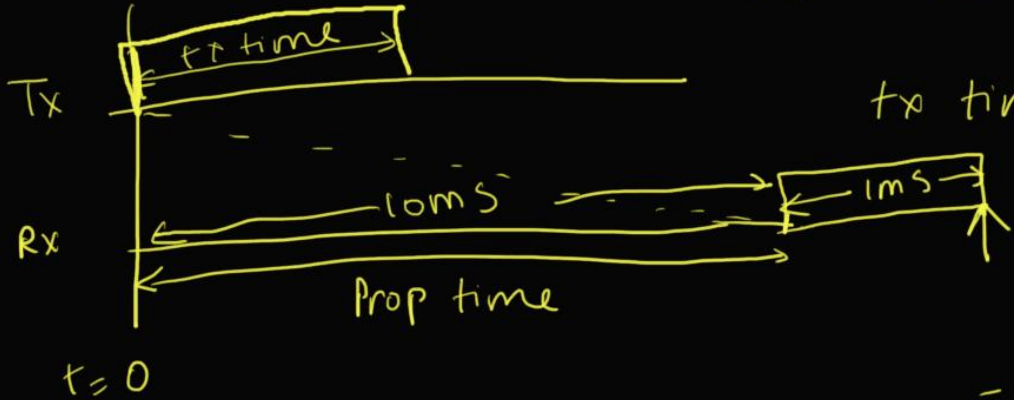


$D = 2000\text{ Km} = 2 \times 10^6 \text{ m}$

$S = 2 \times 10^8 \text{ m/s}$

$R = 1 \text{ Mbps} = 10^6 \text{ bps}$

$P = 1000 \text{ bits}$



$\text{tx time} = \frac{P}{R} = \frac{1000}{10^6} = 1\text{ ms}$

$\text{Prop time} = D/s$

$= \frac{2 \times 10^6}{2 \times 10^8} = 10\text{ ms}$

10ms + 1ms

= 11ms ← fully receive the packet

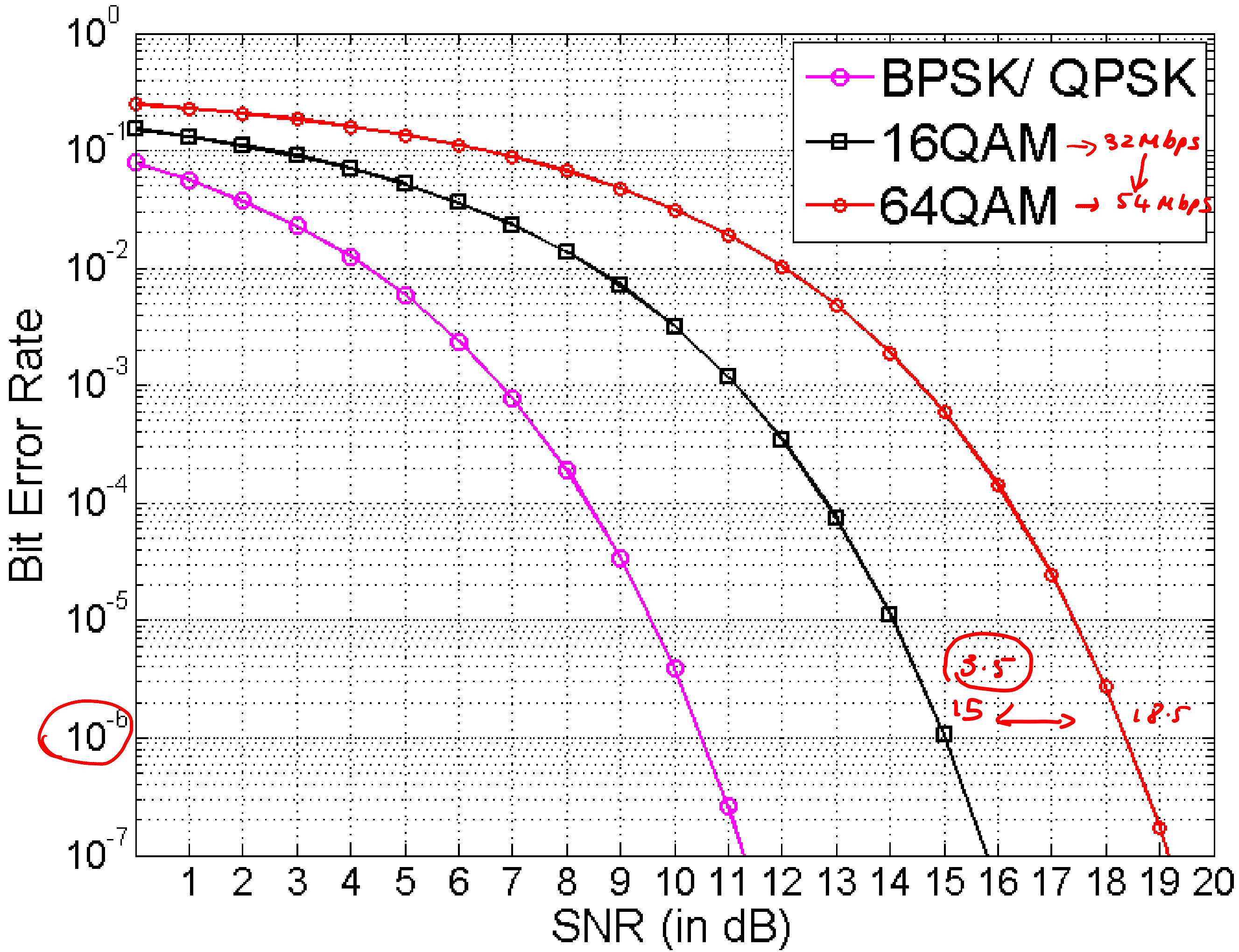
Goals of Modulation

- Bandwidth Efficiency: Data-rate/bandwidth-required, bps/Hz

  for recovery without errors.
  - Goal: Try to reach Shannon limit
  - Real Systems: Ranges from 0.001 to 16
- Power Efficiency: Energy per bit/N_o

  better SNR ratio
  - N_o is noise power spectral density
  - Goal: Minimize SNR required for a given BER
- Tradeoff bandwidth efficiency and power efficiency
  - Can achieve high BW only at the expense of more energy per bit

    means smaller bit intervals....unless u increase energy u cannot maintain same BER.



see lecture again , what mam said here ????

Summary

- Signals and frequency domain representation (bandwidth they occupy)
- How many bits per sec can be sent on a link?
  - Upper bounded by Shannon theorem
  - In reality, depends on medium and modulation
- What is the packet error rate?
  - Function of BER which is determined by SNR and modulation
- Signal corresponding to a bit takes time to propagate
  - Propagation delay is function of speed of light in medium and distance
- Goals of Modulation