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ADMM-based Distributed Electric Vehicle Charging Optimization Algorithm

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***Abstract*—As an increasing number of electric vehicles, the coordinated charging algorithms attract significant attention to reduce the operational costs and facilitate the scalability. This paper derives a distributed electric vehicle charging algorithm based on the alternating direction method of multipliers framework. Taking the total operational cost of electric vehicles as well as battery cell constraints into consideration, optimal charging currents of electric vehicles are obtained without central coordinators. The numerical simulation is conducted to demonstrate the effectiveness of the proposed distributed optimal charging strategy.**

***Index Terms*—distributed optimization, electric vehicles, alternating direction method of multipliers, battery cells**

I. Introduction

To reduce greenhouse gas emissions, electric vehicles (EVs) are widely promoted which can cut back on carbon dioxide emissions [1], [2]. However, the EV proliferation leads to the difficulty of coordinated charging in order to minimize operational costs [3]. In addition, a dramatic increase in the number of electric vehicles may cause grid congestion in Smart Grid[4], [5]. As a result, EV charging optimization problems have attracted considerable attention.

Recently, in Smart Grid, numerous coordinate charging algorithms have been developed [6–10]. The approaches to the optimal charging problems of EVs can be roughly divided into two control schemes including centralized and distributed control strategies [6]. In [7], according to the receding horizon optimization, a centralized charge-discharge control scheme for EVs was proposed. Likewise, taking the interests of customers into consideration, the central optimal charging problem of EVs was discussed in [8]. Compared with centralized charging strategies [7] and [8], electric vehicles charge in a distributed manner can bring benefits including fast calculation and scalability. The decentralized EV charging optimization problem with inequality constraints was considered in [11]. On the basis of the alternating direction method of multipliers (ADMM), the work [12] proposed a distributed algorithm with respect to the power flow problem involving the charging of EVs. The ADMM method is employed and extended to different kinds of optimization problems with linear constraints because it can handle large-scale problems [13]. In [14], combing the consensus theory and ADMM, a decentralized optimal charging protocol with a control center was developed to minimize the power supply cost with network constraints. However, a central coordinator was employed to exchange the information between electric vehicles under the proposed decentralized control scheme in [11], [12], and [14]. If the central coordinator is attacked or destroyed, the aforementioned protocols are not applicable, leading to a huge loss of the operation cost. In contrast, fully distributed EV charging schemes without central coordinators can overcome this drawback. Meanwhile, a fully decentralized algorithm based on the dual-consensus version of the alternating direction method of multipliers (DC-ADMM) also has advantages of privacy protection to a certain extent [15]. Therefore, it is vital to study the electric vehicle charging optimization schemes in a fully distributed manner.

On the other hand, rechargeable batteries have garnered significant attention since they are widely used for electric vehicles [16], [17]. There is a broad range of literature to design reasonable charging strategies on the basis of battery models including electrochemical models [18] and equivalent

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circuit models [19], [20]. In contrast to electrochemical models, equivalent circuit models have the advantages of simple structure, low computational burden, and flexibility. In practice, batteries are series-connected to provide necessary high voltage [19]. Due to various types of battery packs of electric vehicles, the parameters of EVs are different. It is needed to note that the aforementioned strategies [8–14] rarely consider different parameters of battery packs for EVs. The user-involved optimal charging control strategies were proposed for series-connected batteries cells [19], [20]. For the sake of secure operation, charging current restrictions, state-of-charge (SOC) constraints, and terminal voltage limitations were taken into consideration simultaneously. Although battery cells with different capacities and initial SOCs were considered, the developed algorithm is applicable for only a single electric vehicle.

Inspired by the above discussions, the distributed optimal electric vehicle charging schedule based on the ADMM is developed in this paper. The main contributions are summarized as follows:

1) Compared with the works [8], [12], and [13], a distributed ADMM-based method without central coordinators is employed to obtain the optimal charging current for electric vehicles. In addition, the proposed strategy has the advantage of fast calculation because the complex computing load can be distributed to each electric vehicle.

2) By employing the equivalent circuit model (ECM) of a battery cell for an electric vehicle, the charging currents are selected as the optimization variables rather than the charging power, which is different from the aforementioned works [7], [8], and [15]. Thus, the constraints of terminal voltages can be taken into consideration to avoid damaging the batteries. Moreover, the charging model and constraints of an electric vehicle considered in [19] are extended to a group of electric vehicles.

The rest of the paper is organized as follows. In section II, the model and constraints of battery cells for EVs are provided. In section III, the distributed ADMM-based optimal algorithm is proposed. A simulation example is given in section IV. Section V concludes this paper.

Notation: Let $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ be an n-dimensional column vector, where $\mathbb{R}^n$ denote n-dimensional vectors of real numbers and T denotes the transpose symbol. Let $\mathbf{1}_n$ denotes the column vector of length n with all elements being 1. For two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{p \times q}$, $A \oplus B = diag\{A, B\} \in \mathbb{R}^{(m+p)\times(n+q)}$ denote the matrix direct sum.

II. Problem Formulation

A. EV Battery Model

In this paper, the EV charging problem is considered in a distribution network for customers denoted by $\mathbb{N} := \{1, \dots, i, \dots, N\}$. The corresponding communication topology is expressed by the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. As the graph is undirected, the node ν_j can exchange information with its neighbors bilaterally.

For each EV, the battery cells charge separately by using the corresponding charger modules such that the battery pack can be charged fully. The equivalent circuit model [19] is employed to describe the dynamics of battery cells. As for the ith $(1 \le i \le N)$ EV, the model of the jth $(1 \le j \le n)$ cell can be defined by

$$\begin{aligned} \mathrm{SOC}_{i,j}(t+1) &= \mathrm{SOC}_{i,j}(t) + b_{i,j} I_{B_{i,j}}(t) \\ V_{B_{i,j}}(t) &= f_{i,j} + y_{i,j} I_{B_{i,j}}(t) \end{aligned} \tag{1}$$

time step

with $b_{i,j} = \mu\Delta/Q_{i,j}$, where $Q_{i,j}$ denotes the capacity of the jth cell, μ represents the Coulombic efficiency and Δ is the temporal resolution. $\mathrm{SOC}_{i,j}(t)$, $I_{B_{i,j}}(t)$, and $V_{B_{i,j}}(t)$ are the SOC, charging current, and terminal voltage of the jth battery cell, respectively. $f_{i,j}$ and $y_{i,j}$ represent the open-circuit voltage $V_{\mathrm{OC}_{i,j}}$ and the internal resistance $R_{i,j}$ of the jth cell.

Each EV has a set of arbitrary parameters, including the arrival time index $a_i \in S$, intended departure time index $d_i \in S$, minimum SOC $SOC_i^{\min}$, maximum SOC $SOC_i^{\max}$, maximum allowed charge current $I_{B_i}^{\max}$, and minimum charge current $I_{B_i}^{\min}$, where $S = \{1, \dots, t, \dots, l\}$. Then, the charging currents of electric vehicles are denoted as

$$I_B = \left[I_{B_1}^T, I_{B_2}^T, \dots, I_{B_N}^T\right]^T \in \mathbb{R}^{Nnl} \tag{2}$$

with

$$\begin{aligned} I_{B_i} &= \left[I_{B_{i,1}}^T, I_{B_{i,2}}^T, \dots, I_{B_{i,n}}^T\right]^T \in \mathbb{R}^{nl} \\ I_{B_{i,j}} &= \left[I_{B_{i,j}}(1), I_{B_{i,j}}(2), \dots, I_{B_{i,j}}(l)\right]^T \in \mathbb{R}^l. \end{aligned}$$

Besides, a diagonal matrix $D_i \in \mathbb{R}^{l \times l}$ is defined to limit the ith EV charging duration, where the diagonal entry is 1 if the EV is charging within the predefined period $[a_i, d_i]$ and 0 otherwise.

B. EV Battery Constraints

For the sake of secure operation, charging current restrictions, state-of-charge (SOC) constraints, and terminal voltage limitations for the jth battery cell of the ith EV were taken into consideration simultaneously.

The first constraint is the restriction of the charging current. It is vital to preset a suitable range such that the charging current of the jth battery cell maintains in the threshold, otherwise the battery will cause damage to the EV. The charging current restrictions are defined as

$$I_{B_i}^{\min} \le I_{B_{i,j}} \le I_{B_i}^{\max}. \tag{3}$$

The minimum charge currents for electric vehicles are denoted as

$$I_B^{\min} = \left[I_{B_1}^{\min}\mathbf{1}_{nl}^T, I_{B_2}^{\min}\mathbf{1}_{nl}^T, \dots, I_{B_N}^{\min}\mathbf{1}_{nl}^T\right]^T \in \mathbb{R}^{Nnl}. \tag{4}$$

Accordingly, the maximum allowed charge currents are described as

$$I_B^{\max} = \left[I_{B_1}^{\max}\mathbf{1}_{nl}^T, I_{B_2}^{\max}\mathbf{1}_{nl}^T, \dots, I_{B_N}^{\max}\mathbf{1}_{nl}^T\right]^T \in \mathbb{R}^{Nnl}, \tag{5}$$

where $\mathbf{1}_{nl}$ denotes the column vector of length nl with all elements being 1. Then, (4) can be simplified as

$$I_B^{\min} \leq I_B \leq I_B^{\max}. \tag{6}$$

The second constraint is respect to the SOC of battery cells for EVs. In order to avoid overcharging, the bound of the jth battery cell for the ith EV is represented as

$$SOC_i^{\min} \leq SOC_{i,j} \leq SOC_i^{\max}. \tag{7}$$

Here, we define the SOC of electric vehicles as

$$SOC = \left[SOC_1^T, SOC_2^T, \dots, SOC_N^T\right]^T \in \mathbb{R}^{Nnl} \tag{8}$$

with $SOC_i = \left[SOC_{i,1}^T, SOC_{i,2}^T, \dots, SOC_{i,n}^T\right]^T \in \mathbb{R}^{nl}$ and $SOC_{i,j} = \left[SOC_{i,j}(1), SOC_{i,j}(2), \dots, SOC_{i,j}(l)\right]^T \in \mathbb{R}^{l}$. Then, from (1), the SOC constraints can be briefly rewritten as

$$SOC^{\min} \leq \mathrm{SOC}^{in} + HI_B \leq SOC^{\max} \tag{9}$$

with

$$H = H_1 \oplus H_2 \oplus \cdots \oplus H_N$$
$$H_i = B_{i,1}D_i \oplus B_{i,2}D_i \oplus \cdots \oplus B_{i,n}D_i \in \mathbb{R}^{ln \times ln},$$

where $H \in \mathbb{R}^{Nnl \times Nnl}$, and $B_{i,j} = [b_{mn}] \in \mathbb{R}^{l \times l}$ represents a matrix satisfying $b_{mn} = b_{i,j}$ for $m \geq n$ and $b_{mn} = 0$ elsewhere. Here, $B_{i,1}D_i \oplus B_{i,2}D_i = diag\{B_{i,1}D_i, B_{i,2}D_i\}$. The upper bounds of SOC are

$$SOC^{\max} = \left[SOC_1^{\max}\mathbf{1}_{nl}^T, SOC_2^{\max}\mathbf{1}_{nl}^T, \dots, SOC_N^{\max}\mathbf{1}_{nl}^T\right]^T. \tag{10}$$

Accordingly, the lower bounds of SOC are

$$SOC^{\min} = \left[SOC_1^{\min}\mathbf{1}_{nl}^T, SOC_2^{\min}\mathbf{1}_{nl}^T, \dots, SOC_N^{\min}\mathbf{1}_{nl}^T\right]^T. \tag{11}$$

Similarly, the initial SOC vector can be denoted as

$$SOC^{in} = \left[SOC_1^{in\,T}, SOC_2^{in\,T}, \dots, SOC_N^{in\,T}\right]^T \in \mathbb{R}^{Nnl}, \tag{12}$$

where $SOC_i^{in} = \left[SOC_{i,1}^{in}\mathbf{1}_l^T, SOC_{i,2}^{in}\mathbf{1}_l^T, \dots, SOC_{i,n}^{in}\mathbf{1}_l^T\right]^T \in \mathbb{R}^{nl}$.

The third constraint is the limit of terminal voltage for the ith EV. Based on (1), with the internal resistance $y_{i,j}$ and open-circuit voltage $f_{i,j}$, the terminal voltage of the battery should remain in the suitable range which is denoted as

$$V_B^{\min} \leq f + yI_B \leq V_B^{\max}, \tag{13}$$

where

$$f = \left[f_1^T, f_2^T, \dots, f_N^T\right]^T \in \mathbb{R}^{Nnl}$$

with $f_i = \left[f_{i,1}\mathbf{1}_l^T, f_{i,2}\mathbf{1}_l^T, \dots, f_{i,n}\mathbf{1}_l^T\right]^T \in \mathbb{R}^{nl}$ representing the open-circuit voltage. Likewise,

$$y = y_1 \oplus y_2 \oplus \cdots \oplus y_N \in \mathbb{R}^{Nnl \times Nnl} \tag{14}$$

with

$$y_i = y_{i,1}I_{l\times l} \oplus y_{i,2}I_{l\times l} \oplus \cdots \oplus y_{i,n}I_{l\times l} \in \mathbb{R}^{nl \times nl}$$

representing the internal resistance for the EVs, where $I_{l\times l}$ denotes the $l \times l$ identity matrix. The minimum terminal voltages for electric vehicles are

$$V_B^{\min} = \left[V_{B_1}^{\min}\mathbf{1}_{nl}^T, I_{B_2}^{\min}\mathbf{1}_{nl}^T, \dots, V_{B_N}^{\min}\mathbf{1}_{nl}^T\right]^T \in \mathbb{R}^{Nnl}. \tag{15}$$

Accordingly, the maximum allowed terminal voltages are set as

$$V_B^{\max} = \left[V_{B_1}^{\max}\mathbf{1}_{nl}^T, V_{B_2}^{\max}\mathbf{1}_{nl}^T, \dots, V_{B_N}^{\max}\mathbf{1}_{nl}^T\right]^T \in \mathbb{R}^{Nnl}. \tag{16}$$

From (1), the power delivered to the ith EV is represented by $P_{EV,i}(t)$ for all $t \in \mathbb{S}$

$$P_{EV,i}(t) = \sum_{j=1}^{n} I_{B_{i,j}}(t)V_{B_{i,j}}(t). \tag{17}$$

C. Operational Costs

With the above background, this paper seeks to minimize the operational costs with respect to all customers of electric vehicles $i \in N$. Hence, the optimization problem is to minimize the following cost function with the optimization variables I_{Bi}

$$\sum_{i\in N} F_i(I_{Bi}) = \sum_{i=1}^{N}\left(\sum_{t=1}^{\ell}(\Delta\eta P_{EV,i}(t))\right)$$
$$= \sum_{i=1}^{N}\sum_{t=1}^{\ell}\sum_{j=1}^{n}\Delta\eta I_{B_{i,j}}(t)V_{B_{i,j}}(t), \tag{18}$$

where η represents the electricity price. It is subject to battery constraints for all electric vehicles including charging current restrictions (6), SOC constraints (9), and terminal voltage limitations (13). Therefore, the optimization problem with inequality constraints can be formulated as

$$\begin{aligned} \min_{I_B} \quad & F(I_B) = \sum_{i=1}^{N} F_i(I_{Bi}) \\ \text{subject to} \quad & I_B^{\min} \leq I_B \leq I_B^{\max} \\ & SOC^{\min} \leq \mathrm{SOC}^{in} + HI_B \leq SOC^{\max} \\ & V_B^{\min} \leq f + yI_B \leq V_B^{\max}. \end{aligned} \tag{19}$$

To simplify the presentation of optimization problem, the combined set of constraints can be represented as

$$\Omega = \{I_B : AI_B \geq C\}, \tag{20}$$

where

$$A = \begin{bmatrix} I & -I & H^T & -H^T & y^T & -y^T \end{bmatrix}^T \in \mathbb{R}^{6Nnl\times Nnl}$$

with $I \in \mathbb{R}^{Nnl\times Nnl}$. The vector $C \in \mathbb{R}^{6Nnl}$ can be represented as

$$C = \begin{bmatrix} I_B^{\min\,T} & -I_B^{\max\,T} & \sigma^{\min\,T} & -\sigma^{\max\,T} & \delta^{\min\,T} & -\delta^{\max\,T} \end{bmatrix}^T \tag{21}$$

where $\sigma^{\min} = SOC^{\min} - \mathrm{SOC}^{in}$, $\sigma^{\max} = SOC^{\max} - \mathrm{SOC}^{in}$, $\delta^{\min} = V_B^{\min} - f$, and $\delta^{\max} = V_B^{\max} - f$. Therefore, the optimization problem can be further written as

$$\begin{aligned} &\min_{I_B} \quad F(I_B) \\ &\text{subject to} \quad I_B \in \Omega \end{aligned} \tag{22}$$

III. Distributed Algorithm for EV Battery Charging

Based on the consensus theory and the ADMM algorithm, a distributed optimization algorithm is proposed to solve the optimization problem (22). In order to use the ADMM algorithm, the following indicator function is defined to transform the optimization problem (22).

$$g_1(S) = \begin{cases} 0, & \text{if } S \in \Omega \\ +\infty, & \text{otherwise.} \end{cases} \tag{23}$$

Hence, the optimization problem (22) can be directly transformed into the following form

$$\begin{aligned} &\min \quad F(I_B) + g_1(S) \\ &\text{subject to} \quad I_B - S = 0. \end{aligned} \tag{24}$$

S = Z (we

Then, the augmented Lagrangian function of problem (24) can be written as

$$\begin{aligned} L_\rho(I_B, S, \lambda) = F(I_B) + g_1(S) + \lambda^T(I_B - S) \\ + (\rho/2)\left\|I_B - S\right\|_2^2. \end{aligned} \tag{25}$$

In this paper, a distributed method is employed to solve the charging current I_B^{t+1} at $t+1$. According to the ADMM formulation and the augmented Lagrangian function (25), I_B^{t+1} can be obtained as follows

$$\begin{aligned} I_B^{t+1} &= \arg\min_{I_B} L_\rho\left(I_B, S^t, \lambda^t\right) \\ &= \arg\min_{I_B} \left(F(I_B) + (\rho/2)\left\|I_B - S^t + \lambda^t/\rho\right\|_2^2\right). \end{aligned} \tag{26}$$

Then, the problem (25) can be converted into the following form

$$\min \Phi(I_B) = \sum_{i=1}^{N} \Phi_i(I_{B_i}), \tag{27}$$

where

$$\Phi(I_B) = F(I_B) + (\rho/2)\left\|I_B - S^t + \lambda^t/\rho\right\|_2^2. \tag{28}$$

For the ith EV, $\Phi_i(I_{B_i})$ can be expressed as

$$\Phi_i(I_{B_i}) = F_i(I_{B_i}) + (\rho/2)\left\|I_{B_i} - S_i^t + \lambda_i^t/\rho\right\|_2^2. \tag{29}$$

In order to design the charging current I_{B_i} for electric vehicles with different parameters, an assumption that the parameters of each cell for an EV are identical is made. Specifically, the open-circuit voltage $f_{i,j}$ and internal resistance $y_{i,j}$ satisfy $f_{i,1} = f_{i,2} = \cdots = f_{i,n}$, $y_{i,1} = y_{i,2} = \cdots = y_{i,n}$, respectively. Then, the gradient of $\Phi_i(I_{B_i})$ can be expressed as

$$\beta_i = \frac{\partial \Phi_i(I_{B_i})}{\partial I_{B_i}} = \nabla F_i(I_{B_i}) + \rho\left(I_{B_i} - S_i^t + \lambda_i^t/\rho\right). \tag{30}$$

For further analysis, the operational cost for the ith EV can be rewritten as

$$F_i(I_{B_i}) = \frac{(I_{B_i} - \kappa_i)^2}{2\nu_i} + \omega_i, \tag{31}$$

where $\kappa_i = -f_i/2y_i$, $\nu_i = 1/2\Delta\eta y_i$, and $\omega_i = -\Delta\eta f_i^2/4y_i$. The formula (31) is equivalent to the cost function (18) which is defined earlier.

In the distribution network, the exchange of information can not be avoided because electric vehicles belong to different users. However, the power information contains some private data, which may not be allowed to exchange directly. Instead, the incremental cost is utilized for the exchange of information. Then, combining (30) and (31) yields

$$\begin{aligned} \beta_i(k+1) = &\frac{(\rho\nu_i + 1) I_{B_i} + \nu_i(\lambda_i^t - \rho S_i^t) - \kappa_i}{\nu_i} \\ &- \gamma \sum_{j=1}^{n} L_{i,j}\nu_i^{-1}\beta_j(k), \end{aligned} \tag{32}$$

where γ is the positive design parameter and $L_{i,j}$ is the element of the ith row and jth column in the Laplacian matrix. Moreover, by employing the incremental cost (32), the optimal charging current I_{B_i} can be obtained.

$$I_{B_i}(k+1) = I_{B_i}(k) - \zeta \sum_{j=1}^{n} L_{i,j}\beta_j(k), \tag{33}$$

where $\zeta > 0$ is a design parameter.

According to the ADMM algorithm, S^{t+1} is calculated by combining (25), (31), and (32).

$$\begin{aligned} S^{t+1} &= \arg\min_S L_\rho\left({I_B}^{t+1}, S, \lambda^t\right) \\ &= \arg\min_S \left\{g_1(S) + (\lambda)^T\left({I_B}^{t+1} - S\right) + \frac{\rho}{2}\left\|{I_B}^{t+1} - S\right\|_2^2\right\} \\ &= \arg\min_{S\in\Omega} (\rho/2)\left\|{I_B}^{t+1} - S + \lambda^t/\rho\right\|_2^2 \end{aligned} \tag{34}$$

The equation (34) ensures that the charging currents remain in the predefined constraints Ω. As a result, considering that the charging currents are positive with a set of constraints (19) in real applications, the specific form of S_i^{t+1} can be obtained

$$S_i^{t+1} = \begin{cases} I_i^{\min}, & \text{if } I_{B_i}^{t+1} + \lambda^t/\rho \le I_i^{\min} \\ I_{B_i}^{t+1} + \lambda^t/\rho, & \text{if } I_i^{\min} < I_{B_i}^{t+1} + \lambda^t/\rho < I_i^{\max} \\ I_i^{\max}, & \text{if } I_{B_i}^{t+1} + \lambda^t/\rho \ge I_i^{\max}, \end{cases} \tag{35}$$

where

$$\begin{aligned} I_i^{\min} &= \max\left\{I_{B_i}^{\min}, H^{-1}\sigma^{\min}, y^{-1}\delta^{\min}\right\}, \\ I_i^{\max} &= \min\left\{I_{B_i}^{\max}, H^{-1}\sigma^{\max}, y^{-1}\delta^{\max}\right\}. \end{aligned}$$

Then, from (33) and (35), it can be shown that

$$\lambda_i^{t+1} = \lambda_i^t + \rho\left(I_{B_i}^{t+1} - S_i^{t+1}\right) \tag{36}$$

In the sequel, the optimal solution can be obtained by employing (33), (35), and (36). In more detail, Algorithm 1 illustrates the proposed distributed algorithm based on the consensus theory and the ADMM algorithm.

Algorithm 1 Distributed ADMM algorithm

Initialization: Set $k = 0$, $t = 0$ and the initial charging currents I_B.

repeat

  Let $I_{B_i}(t+1) = I_{B_i}(k+1)$.

  repeat

    Update $I_{B_i}(k+1)$ based on (32) and (33).

    Let $k = k+1$.

  until $I_{B_i}(k+1) - I_{B_i}(k)$ satisfies the prescribed tolerance.

  Update S_i^{t+1} and λ_i^{t+1} with

$$S_i^{t+1} = \begin{cases} I_i^{\min}, & \text{if } I_{B_i}^{t+1} + \lambda^t/\rho \le I_i^{\min} \\ I_{B_i}^{t+1} + \lambda^t/\rho, & \text{if } I_i^{\min} < I_{B_i}^{t+1} + \lambda^t/\rho < I_i^{\max} \\ I_i^{\max}, & \text{if } I_{B_i}^{t+1} + \lambda^t/\rho \ge I_i^{\max} \end{cases} \tag{35}$$

$$\lambda_i^{t+1} = \lambda_i^t + \rho\left(I_{B_i}^{t+1} - S_i^{t+1}\right) \tag{36}$$

  Let $t = t+1$.

until the stopping criterion is satisfied, stop the outer loop and output I_B.

IV. Numerical Simulations

In this section, the numerical simulation is conducted to show the effectiveness of the proposed optimal charging algorithm.

Consider a group of electric vehicle including five electric vehicles with different parameters. The undirected communication topology is shown in Fig. 1. The initial charging currents of EVs are set as the same value with $I_{B_1}(0) = I_{B_2}(0) = I_{B_3}(0) = I_{B_4}(0) = I_{B_5}(0) = 6$. The upper and lower bounds with respect to charging currents, SOC, and terminal voltages are selected as in TABLE I. In addition, TABLE I also indicates the open-circuit voltage f_i and the internal resistance y_i of the battery cell for the ith EV.

Based on the communication graph and simulation parameters of electric vehicles, the optimal charging currents of five EVs are illustrated in Fig. 2. It is clear that the proposed optimal charging algorithm can guarantee that the charging currents always stay in the constrained region $I_B^{\min} \le I_B \le I_B^{\max}$. Fig. 3 is the terminal voltages of five electric vehicles. It can be seen that the voltages V_B always remain in $V_B^{\min} \le V_B \le V_B^{\max}$. From the above simulations results and analysis, we get that the optimal charging currents under the proposed optimal algorithm are obtained while the corresponding constraints are not violated.

V. Conclusion

In this paper, for the sake of secure operation and the reduction of the operation costs, a fully distributed optimal charging strategy with a set of battery constraints for electric vehicles is presented on the basis of a modified version of ADMM. The optimal charging currents of electric vehicles under the proposed optimal algorithm are obtained without central coordinators. The charging currents are selected as the optimization variables that can directly limit the terminal voltages of battery packs for multiple electric vehicles to avoid damaging the batteries. It should be noted that the open-circuit voltage and the internal resistance of battery cells are set as constants; while in practical applications, they are related to the SOC of the battery pack. In addition, the voltage constraint of distribution networks is also important with the increased

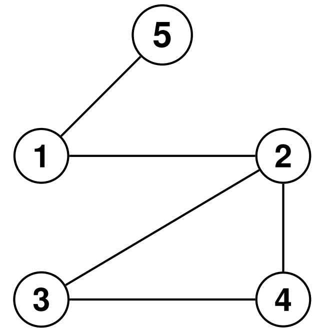

Fig. 1. The communication graph between electric vehicles.

TABLE I
SIMULATION PARAMETERS OF FIVE ELECTRIC VEHICLES

	EV1	EV2	EV3	EV4	EV5
y	0.04	0.05	0.08	0.06	0.03
f	3.8	4	4.2	4.1	3.6
κ	-47.5	-40	-26.3	-34.2	-60
ν	50	40	25	33.3	66.7
ω	-22.6	-20	-13.8	-18.8	-27
Range I_B	[0,12]	[0,15]	[0,15]	[0,15]	[0,20]
Range SOC	[0.1,1]	[0.1,1]	[0.1,1]	[0.1,1]	[0.1,1]
Range V_B	[3.2,5]	[3.2,5]	[3.2,5]	[3.2,5]	[3.2,5]

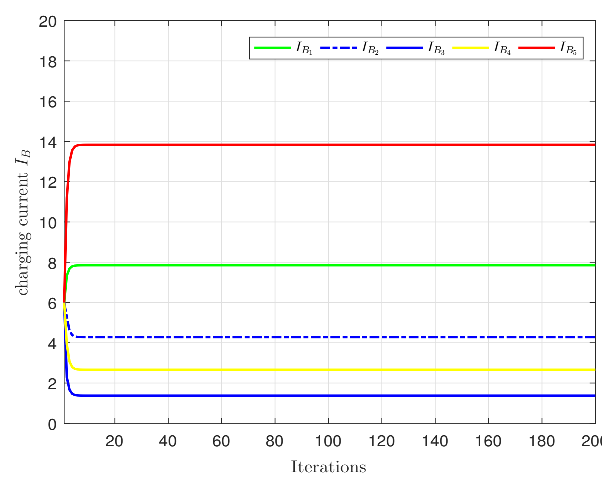

Fig. 2. The charging currents of five electric vehicles.

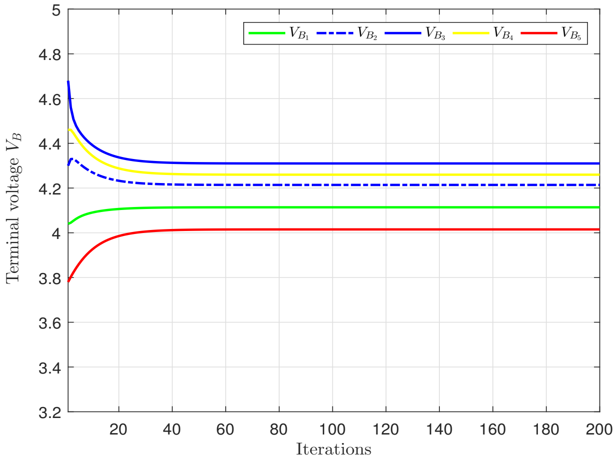

Fig. 3. The terminal voltages of five electric vehicles.

uptake of electric vehicles. As a result, it is meaningful and challenging to investigate the optimal charging problems with time-varying open-circuit voltages and internal resistances under the voltage constraints in distribution networks, which requires further investigation in the future.

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