SIMPLE ENERGY RESOLUTION FOR DGS

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The following notations are going to be used:

- t_M - time the neutron leaves the moderator (will be set to 0 later)
- t_C - time the neutron arrives at the chopper
- t_S - time the neutron arrives at the sample
- t_D - time the neutron arrives at the detector (total time of flight)
- l_0 - distance from moderator to chopper
- l_1 - distance from chopper to sample
- l_2 - distance from sample to detector
- E_i - incident energy (constant from moderator to chopper and chopper to sample)
- E_f - final energy (from sample to detector)
- v_i, v_f - the corresponding velocities

The time neutrons arrive at the sample is given by

$$\begin{aligned} t_S &= t_M + \frac{l_0 + l_1}{v_i} && (1) \\ &= t_M + (t_C - t_M)\frac{l_0 + l_1}{l_0} && (2) \\ &= t_C \frac{l_0 + l_1}{l_0} - t_M \frac{l_1}{l_0} && (3) \end{aligned}$$

The energy transfer is given by

$$\begin{aligned} \hbar\omega &= E_i - E_f && (4) \\ &= \frac{m}{2}\left(v_i^2 - v_f^2\right) && (5) \\ &= \frac{m}{2}\left[\left(\frac{l_0}{t_C - t_M}\right)^2 - \left(\frac{l_2}{t_D - t_S}\right)^2\right] && (6) \\ &= \frac{m}{2}\left[\left(\frac{l_0}{t_C - t_M}\right)^2 - \left(\frac{l_2}{t_D - t_C\frac{l_0+l_1}{l_0} - t_M\frac{l_1}{l_0}}\right)^2\right] && (7) \end{aligned}$$

The energy resolution is given in terms of partial derivatives:

$$\Delta\hbar\omega = \sqrt{\sum_x \left(\frac{\partial\hbar\omega}{\partial t_x}\Delta t_x\right)^2} \qquad (8)$$

The Δt_x are uncertainties with respect to different time instances (moderator, chopper, detector).

$$
\begin{aligned}
\left(\frac{\Delta\hbar\omega}{E_i}\right)^2 &= \frac{1}{E_i^2}\left[\left(\frac{\partial\hbar\omega}{\partial t_M}\Delta t_M\right)^2+\left(\frac{\partial\hbar\omega}{\partial t_C}\Delta t_C\right)^2+\left(\frac{\partial\hbar\omega}{\partial t_D}\Delta t_D\right)^2\right] \quad (9)\\
&= \left[\frac{m}{2E_i}\left(\frac{2l_0^2}{(t_C-t_M)^3}+\frac{2l_2^2\left(\frac{l_1}{l_0}\right)}{\left(t_D-t_C\frac{l_0+l_1}{l_0}-t_M\frac{l_1}{l_0}\right)^3}\right)\Delta t_M\right]^2+\\
&\quad \left[\frac{m}{2E_i}\left(\frac{2l_0^2}{(t_C-t_M)^3}+\frac{2l_2^2\left(1+\frac{l_1}{l_0}\right)}{\left(t_D-t_C\frac{l_0+l_1}{l_0}-t_M\frac{l_1}{l_0}\right)^3}\right)\Delta t_C\right]^2+\\
&\quad \left[\frac{m}{2E_i}\left(\frac{2l_2^2}{\left(t_D-t_C\frac{l_0+l_1}{l_0}-t_M\frac{l_1}{l_0}\right)^3}\right)\Delta t_D\right]^2 \quad (10)
\end{aligned}
$$

We can now plug in $t_M = 0$. We are going to use

$$\frac{ml_0^2}{2t_C^2} = E_i \tag{11}$$

and

$$\frac{ml_2^2}{2\left(t_D-t_C\frac{l_0+l_1}{l_0}\right)^2} = E_f \tag{12}$$

Then:

$$
\begin{aligned}
\left(\frac{\Delta\hbar\omega}{E_i}\right)^2 &= \left[\frac{2\Delta t_M}{t_C} + 2\Delta t_M \left(\frac{ml_2^2}{2\left(t_D - t_C\frac{l_0+l_1}{l_0}\right)^2}\right)^{\frac{3}{2}} \frac{1}{E_i^{\frac{3}{2}}} \left(\frac{2E_i}{ml_2^2}\right)^{\frac{1}{2}} \frac{l_1}{l_0}\right]^2 + \\
&\left[\frac{2\Delta t_C}{t_C} + 2\Delta t_C \left(\frac{ml_2^2}{2\left(t_D - t_C\frac{l_0+l_1}{l_0}\right)^2}\right)^{\frac{3}{2}} \frac{1}{E_i^{\frac{3}{2}}} \left(\frac{2E_i}{ml_2^2}\right)^{\frac{1}{2}} \frac{l_0+l_1}{l_0}\right]^2 + \\
&\left[2\Delta t_D \left(\frac{ml_2^2}{2\left(t_D - t_C\frac{l_0+l_1}{l_0}\right)^2}\right)^{\frac{3}{2}} \frac{1}{E_i^{\frac{3}{2}}} \left(\frac{2E_i}{ml_2^2}\right)^{\frac{1}{2}} \frac{l_0}{l_0}\right]^2 \qquad (13) \\
&= \left[\frac{2\Delta t_M}{t_C} + 2\Delta t_M \left(\frac{E_f}{E_i}\right)^{\frac{3}{2}} \left(\frac{2E_i}{ml_0^2}\right)^{\frac{1}{2}} \frac{l_1}{l_2}\right]^2 + \\
&\left[\frac{2\Delta t_C}{t_C} + 2\Delta t_C \left(\frac{E_f}{E_i}\right)^{\frac{3}{2}} \left(\frac{2E_i}{ml_0^2}\right)^{\frac{1}{2}} \frac{l_0+l_1}{l_2}\right]^2 + \\
&\left[2\Delta t_D \left(\frac{E_f}{E_i}\right)^{\frac{3}{2}} \left(\frac{2E_i}{ml_0^2}\right)^{\frac{1}{2}} \frac{l_0}{l_2}\right]^2 \qquad (14) \\
&= \left\{\frac{2\Delta t_M}{t_C}\left[1 + \frac{l_1}{l_2}\left(\frac{E_f}{E_i}\right)^{\frac{3}{2}}\right]\right\}^2 + \\
&\left\{\frac{2\Delta t_C}{t_C}\left[1 + \frac{l_0+l_1}{l_2}\left(\frac{E_f}{E_i}\right)^{\frac{3}{2}}\right]\right\}^2 + \\
&\left\{\frac{2\Delta t_D}{t_C}\left[\frac{l_0}{l_2}\left(\frac{E_f}{E_i}\right)^{\frac{3}{2}}\right]\right\}^2 \qquad (15)
\end{aligned}
$$

If the time resolution of the detector is negligible compared to the other terms, $\Delta t_D = 0$.