Currency Arbitrage Optimization using QAOA in Cirq and a Comparative Study with Classical Ising Solvers

Team Members

- **Vinayak Sharma**
- **Sam Ruben Abraham**

Abstract

Currency arbitrage exploits inconsistencies in foreign exchange rates to achieve risk-free profit by executing cyclic currency trades. In this project, arbitrage detection is formulated as a **Quadratic Unconstrained Binary Optimization** $QUBO$ problem and investigated using both **classical Ising-based solvers** and a **quantum variational approach** based on the **Quantum Approximate Optimization Algorithm** $QAOA$ implemented using **Google Cirq**.

Currencies are modelled as nodes in a directed graph, with exchange rates represented as weighted edges. Classical optimization techniques-including **simulated annealing**, **brute-force enumeration**, and **gradient-based QUBO minimization**-are employed as baseline methods and directly compared against a Cirq-based QAOA implementation operating on an equivalent Hamiltonian formulation.

The results indicate that while classical methods efficiently detect arbitrage opportunities in small-scale systems, the Cirq-QAOA implementation achieves comparable accuracy with improved constraint satisfaction and logical correctness in modelling arbitrage cycles. Furthermore, the Cirq-based framework addresses several limitations observed in earlier Qiskit-based implementations, offering improved modularity, numerical stability, and interpretability. These findings establish a robust foundation for scalable hybrid quantum–financial optimization models.

Introduction

Currency arbitrage is a trading strategy that exploits discrepancies in foreign exchange rates between markets to generate profit with minimal risk. It involves converting one currency into another through a sequence of exchanges such that the final amount exceeds the initial investment. Mathematically, this problem can be formulated as identifying **profitable cycles** in a directed weighted graph, where currencies are represented as nodes and exchange rates define the edges. A cycle is considered profitable if the product of exchange rates along the path exceeds one, resulting in a net gain.

Classical approaches such as the **Bellman–Ford algorithm** address this problem by transforming exchange rates using logarithmic weights and detecting negative cycles in the resulting graph. While effective for small networks, these algorithms become computationally expensive as the number of currencies increases, due to the rapid growth in the solution space and repeated relaxation steps required for convergence.

Quantum computing offers an alternative paradigm for tackling such combinatorial optimization problems by leveraging **superposition** and **entanglement** to explore large solution spaces in parallel. In this project, we investigate the use of the **Quantum Approximate Optimization Algorithm** $QAOA$ implemented using **Google Cirq** to model and solve the currency arbitrage problem. The quantum approach is evaluated alongside classical optimization techniques to compare their accuracy, scalability, and convergence behaviour under identical problem formulations.

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2. Motivation

Financial markets are highly dynamic and complex, with arbitrage opportunities existing only briefly before market corrections eliminate them. While classical algorithms-such as **Bellman–Ford**, **simulated annealing**, and **exhaustive search methods**-can efficiently detect arbitrage in small currency

networks, their computational cost grows rapidly and often exponentially as the problem size increases. This scalability limitation restricts their applicability to larger, more realistic market scenarios.

Quantum computing offers the potential to address these challenges by enabling parallel exploration of large configuration spaces through **superposition** and **entanglement**. In particular, the **Quantum Approximate Optimization Algorithm** $QAOA$ provides a hybrid quantum–classical framework in which the optimization problem is encoded into a Hamiltonian and parameterized quantum circuits are iteratively optimized using classical routines.

The primary motivation of this project is to evaluate whether a **Cirq-based QAOA implementation** can reliably and accurately identify profitable arbitrage cycles and how its performance compares to traditional classical optimization methods. Specifically, this work aims to examine: - The correctness and fidelity of the QAOA formulation in Cirq - Effective constraint satisfaction through Hamiltonian penalty terms - Comparative performance against classical Ising-based solvers - The scalability, modularity, and interpretability of the Cirq implementation

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3. Related Work

Recent research has increasingly explored both quantum and quantum-inspired techniques for financial optimization, with particular emphasis on currency arbitrage detection. Early implementations of the **Quantum Approximate Optimization Algorithm** $QAOA$ using frameworks such as **Qiskit** demonstrated the feasibility of applying gate-based quantum algorithms to arbitrage problems, but often suffered from constraint violations and parameter instability, especially at limited circuit depths.

Deshpande et al. [1] implemented QAOA using IBM's Qiskit framework and compared its performance with **D-Wave's quantum annealer** and an **N-choose-K constraint mapping** approach. Their results showed that while quantum annealing consistently produced stable and accurate arbitrage cycles, QAOA frequently struggled to satisfy problem constraints due to sensitivity in parameter tuning and restricted circuit expressivity.

In a related advancement, Tatsumura et al. [5] introduced a **simulated bifurcation algorithm** capable of detecting arbitrage opportunities at extremely high speeds, demonstrating that quantum-inspired optimization methods can outperform traditional algorithms in terms of detection latency. Similarly, Carrascal et al. [6] employed a **Differential Evolution–based optimizer** within the **Variational Quantum Eigensolver** VQE framework to improve convergence stability in cryptocurrency arbitrage problems, achieving more reliable parameter optimization than standard gradient-free methods such as COBYLA.

Together, these studies establish the feasibility of quantum and quantum-inspired approaches for financial optimization, while also highlighting persistent challenges such as parameter instability, circuit noise, and scalability limitations. Notably, relatively few works provide a **direct comparison between classical and quantum solvers under an identical QUBO formulation**. Building upon this foundation, the present project introduces a **Cirq-based implementation of QAOA** that emphasizes modular circuit construction, systematic constraint enforcement, and improved numerical stability. By benchmarking this approach against classical Ising-based solvers, the project offers clearer insights into the relative strengths and limitations of quantum versus classical optimization methods for modelling currency arbitrage.

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4. Problem Statement and Formulation

The objective of this project is to identify a **closed sequence of currency trades** $anarbitragecycle$ that yields a net profit when currencies are converted through the given exchange rates. This problem can be naturally modelled as a graph-theoretic optimization task, where currencies are represented as nodes in a directed graph and exchange rates define the directed edges between them. The goal is to select a subset of directed edges that forms a valid cycle and maximizes the overall arbitrage profit.

Each directed currency exchange $i \to j$ is represented by a binary decision variable:

$b_{ij} \in \{0, 1\}$

where $b_{ij} = 1$ indicates that the exchange from currency $i \to j$ is included in the arbitrage cycle, and $b_{ij} = 0$ otherwise.

To transform the multiplicative arbitrage condition into an additive optimization problem, the **negative logarithm of exchange rates** is used. The arbitrage objective function is defined as:

$$C = \sum_{i,j} -\log(r_{ij})\, b_{ij}$$

where r_{ij} denotes the exchange rate from currency $i \to j$. Minimizing this objective corresponds to maximizing the product of exchange rates along the selected cycle, thereby identifying profitable arbitrage opportunities.

Cycle Constraints

To ensure that the selected edges form a **valid arbitrage cycle**, the following constraints must be satisfied:

- Each currency must have exactly **one outgoing edge**
- Each currency must have exactly **one incoming edge**

These constraints enforce the requirement that every currency appears exactly once in the cycle. They are incorporated into the optimization problem using quadratic penalty terms:

$$P = \sum_i \left(\sum_{j \neq i} b_{ij} - 1 \right)^2 + \sum_j \left(\sum_{i \neq j} b_{ij} - 1 \right)^2$$

The total cost function is then given by:

$$C_{\text{total}} = C + m_p \cdot P$$

where m_p is a sufficiently large penalty coefficient that ensures constraint violations are heavily penalized relative to the objective term.

This formulation results in a **Quadratic Unconstrained Binary Optimization** $QUBO$ problem, which can be directly mapped to both **classical Ising-based solvers** and **quantum Hamiltonians**, making it suitable for comparative analysis using classical optimization techniques and the Cirq-based QAOA framework.

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5. Proposed Methodology

This project adopts a **gate-model variational quantum approach** based on the **Quantum Approximate Optimization Algorithm (QAOA)**, implemented using **Google Cirq**, to solve the currency arbitrage problem formulated as a QUBO.

Overview

The QAOA framework operates by encoding the QUBO cost function into a problem Hamiltonian and iteratively optimizing a parameterized quantum circuit to bias measurement outcomes toward low-energy (high-profit) solutions. Classical optimization routines are used to tune circuit parameters based on measurement statistics obtained from quantum simulation.

Inputs and Outputs

Input: 1. QUBO defined by linear coefficients h_i and quadratic couplings J_{ij}

2. Number of qubits (N) (equal to the number of binary decision variables)
3. Number of QAOA layers (p)
4. Classical optimizer OPT (e.g., COBYLA)
5. Number of measurement shots (S)

Output: - *best_bitstrings* : list of low-energy constraint-feasible bitstrings

- *best_estimated_energy* : minimum estimated cost value

Algorithm :

1) QAOA Formulation and Circuit Construction
   - Each binary decision variable is mapped to a qubit operator using the standard QUBO-to-Ising transformation

$$b_k = \frac{1 - Z_k}{2}$$

   - Using this mapping, the **problem Hamiltonian** H_C is constructed such that:

$$\langle b|H_C|b\rangle = C(b)$$

     where $C(b)$ includes both the arbitrage objective and the quadratic penalty terms enforcing cycle constraints.
   - The **mixer Hamiltonian** is defined as:

$$H_M = \sum_{k=1}^{N} X_k$$

     which enables transitions between computational basis states during optimization.

2. Initialize QAOA parameters

$$\gamma = (\gamma_1, \dots, \gamma_p), \quad \beta = (\beta_1, \dots, \beta_p)$$

   using random or heuristic values.
3. Define a function *build_qaoa_circuit(γ, β)*:
4. Initialize an empty Cirq quantum circuit $\mathcal{C}$.
5. Apply Hadamard gates to all qubits to prepare a uniform superposition.
6. For each layer $\ell = 1$ to p:
   - Apply the problem unitary

$$U_C(\gamma_\ell) = e^{-i\gamma_\ell H_C}$$

     implemented via single- and two-qubit gate decompositions consistent with the QUBO terms.
   - Apply the mixer unitary

$$U_M(\beta_\ell) = e^{-i\beta_\ell H_M}$$

     realized as parameterized R_X rotations on each qubit.
7. Append measurement operations in the computational basis to all qubits.

Classical Optimization Loop

8. Initialize

$$E_{\text{best}} \leftarrow +\infty$$

9. While the classical optimizer has not converged:
10. Construct the QAOA circuit using the current parameters (γ, β).
11. Execute the circuit on a simulator (or hardware) and collect (S) measurement shots.
12. Estimate the expectation value of the cost function:

$$E_{\text{est}} = \sum_{b} C(b) \cdot P(b)$$

where (P(b)) is the empirical probability of bitstring (b).

13. Update parameters:

$$(\gamma, \beta) \leftarrow \text{OPT.update}(E_{\text{est}}, \gamma, \beta)$$

14. If

$$E_{\text{est}} < E_{\text{best}},$$

update E_{best} and store the corresponding measurement samples.

Post-Processing and Solution Selection

17. After convergence, re-run the QAOA circuit using the optimized parameters (γ^*, β^*) with a larger number of shots S_{large} .
18. For each measured bitstring (b):

- Compute (C(b))
- Check feasibility with respect to cycle constraints.

19. Collect all feasible bitstrings and sort them in ascending order of (C(b)).
20. Return:

- *best_bitstrings* : feasible bitstrings with lowest cost
- *best_estimated_energy* : minimum observed energy

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6. Experimental Details

Dataset Description

The dataset consists of foreign exchange rates between **three currencies: USD, EUR, and GBP**, adapted from prior studies on currency arbitrage. These currencies form a **fully connected directed graph**, resulting in **six directed currency exchanges**. Each directed exchange $i \rightarrow j$ is mapped to a **binary decision variable** and correspondingly to **one qubit** in the quantum model.

Thus, the arbitrage problem is represented using: **6 binary variables** in the QUBO formulation

6 qubits in the Cirq-based QAOA implementation

Experimental Setup

The experimental evaluation was conducted in **two stages: classical optimization and quantum optimization**, both operating on the same QUBO formulation to ensure a fair comparison.

Classical Optimization Three classical solvers were implemented in Python and applied to the QUBO/Ising model:

- **Simulated Annealing**, used as a stochastic heuristic optimizer
- **Brute-Force Enumeration**, which evaluates all possible configurations to obtain the exact global optimum
- **Gradient-Based QUBO Minimization**, used to validate convergence behaviour on small problem instances

The classical solvers were executed over a fixed number of iterations (where applicable), consistent with the logic implemented in the classical Ising solver notebook. These methods serve as baseline references for evaluating the correctness and optimality of the quantum results.

Quantum Optimization (Cirq-QAOA) For the quantum approach, a **gate-model QAOA circuit** was implemented using **Google Cirq**, with the following configuration:

- **Number of qubits :** 6
- **QAOA depth :** p
- **Simulator :** Cirq state-vector simulator
- **Parameter optimization :**
  - Manual initialization and tuning
  - Automated optimization using **COBYLA**

The QAOA circuit alternates between the problem Hamiltonian and mixer Hamiltonian layers, with parameters γ and β optimized to minimize the expected cost function estimated from measurement samples.

Packages, Tools, and Libraries

The following tools and libraries were used throughout the experiments:

- **Cirq**: quantum circuit construction, parameter binding, and simulation
- **NumPy**: numerical operations and matrix computations
- **SciPy**: classical optimization routines (including COBYLA)
- **Python standard libraries**: implementation of classical Ising solvers and utility functions

7. Results and Discussion

7.1 Classical Results

Classical Ising Baseline Experiments The classical baseline experiments were conducted using an **Ising optimization model with eight binary decision variables (spins)**. This setup corresponds to a **generic Ising/QUBO optimization instance** used to benchmark classical solvers and validate optimization behaviour, and is **distinct from the six-variable currency arbitrage instance** used in the quantum experiments. This distinction is explicitly reflected in the classical solver notebook.

The Ising energy function was defined using randomly generated linear and pairwise coupling coefficients, representing a sample financial optimization landscape.

Simulated Annealing Simulated annealing is a stochastic optimization technique inspired by the physical annealing process in materials. Starting from a random initial configuration, the algorithm iteratively explores neighbouring states, occasionally accepting higher-energy states with a probability that decreases as the temperature is lowered. This mechanism allows the algorithm to escape local minima and gradually converge toward a global optimum.

In the experiments, the simulated annealing algorithm was run for **5000 iterations**. It successfully converged to a minimum energy configuration with an energy value of :

$$E_{\min} = -11.3930$$

The corresponding spin configuration was:

$[-1, -1, -1, -1, -1, -1, -1, 1]$

This configuration represents the **ground state** of the Ising system, corresponding to the optimal combination of binary decisions for the given cost function.

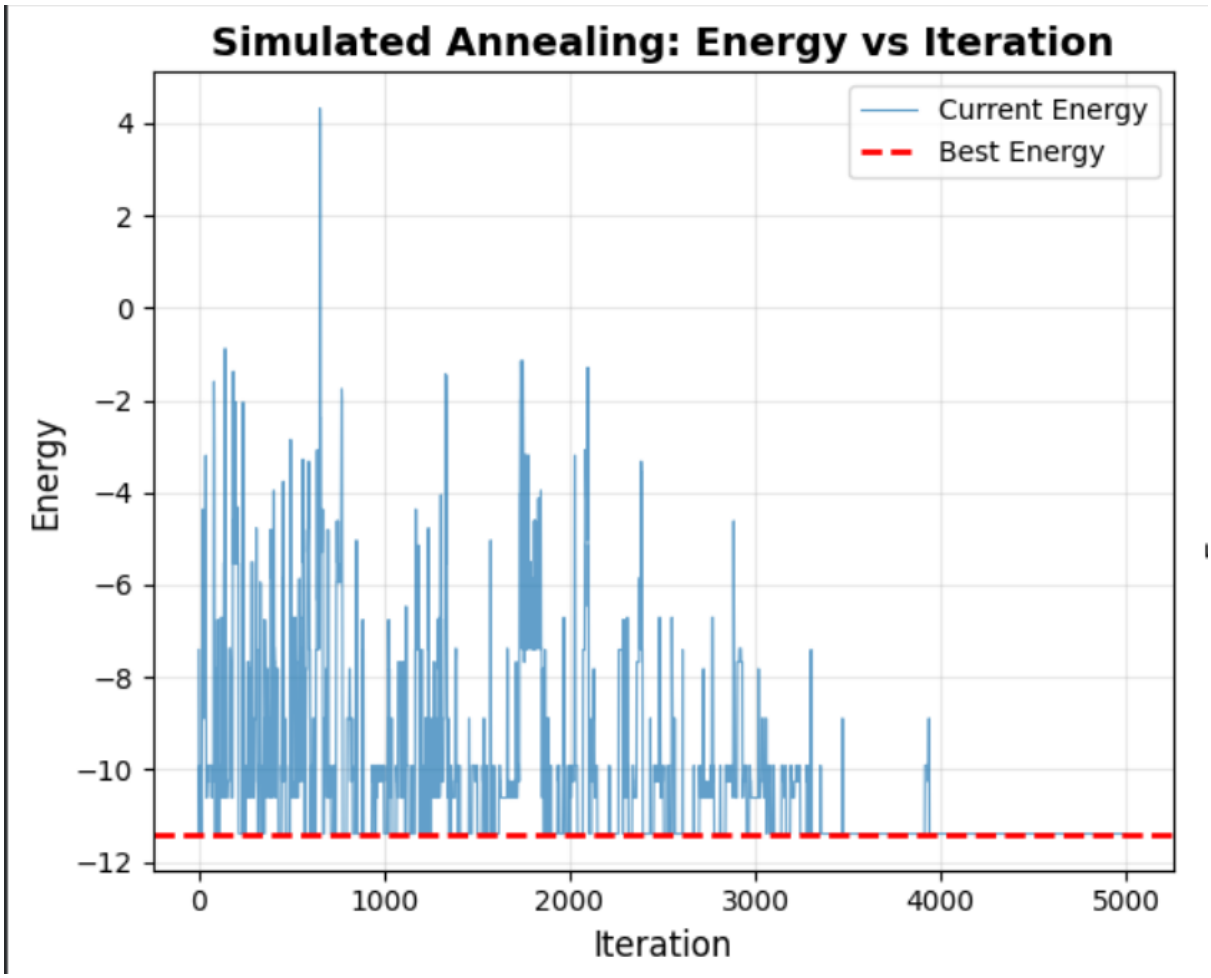

Figure 1: Simulated Annealing Energy vs Iterations

Brute-Force Enumeration To validate the simulated annealing results, a brute-force enumeration of all possible configurations was performed. For an eight-variable Ising model, this required evaluating: $2^8 = 256$ possible spin configurations. For each configuration, the total Ising energy (including all linear and coupling terms) was computed.

The brute-force solver identified the **same ground-state configuration** with an identical minimum energy of **−11.3930**, confirming that the simulated annealing algorithm successfully located the true global minimum.

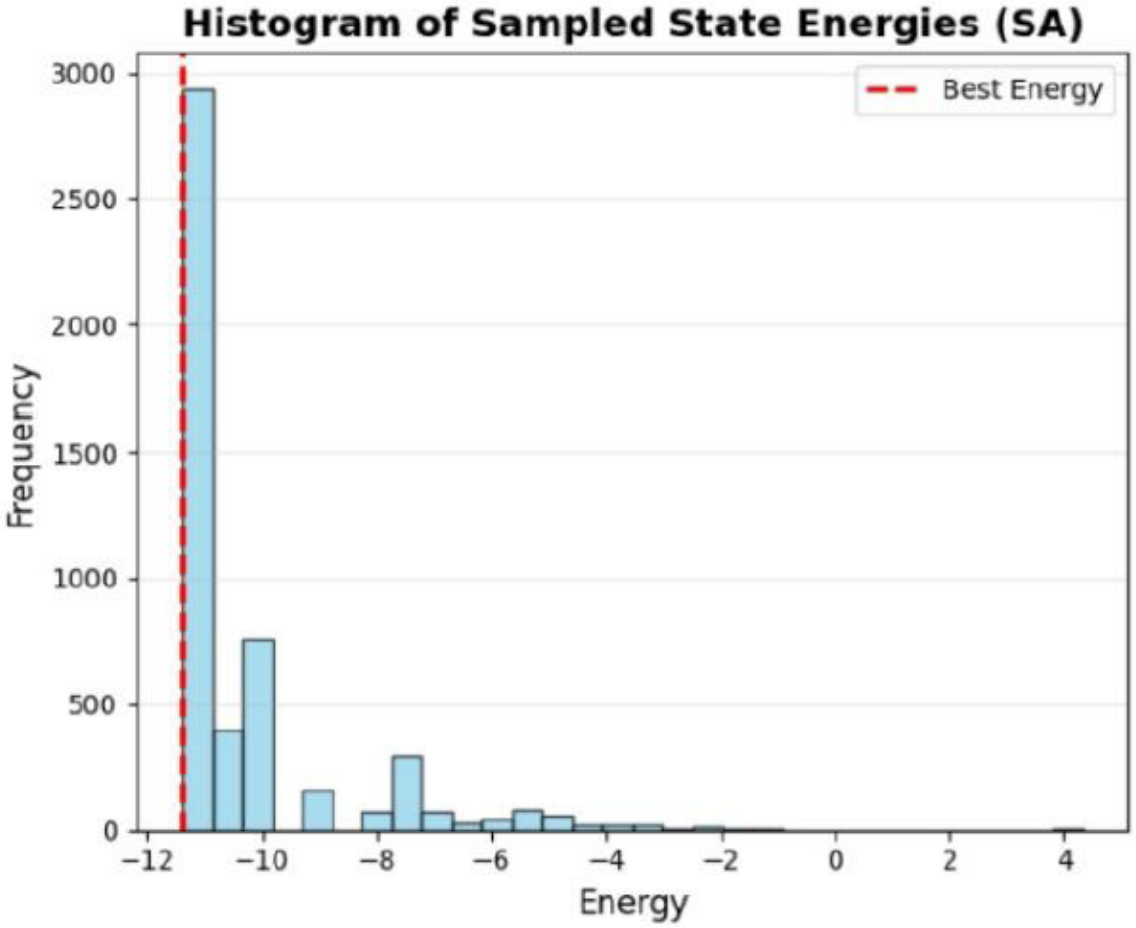

Figure 2: Histogram of Sampled Energies (SA)

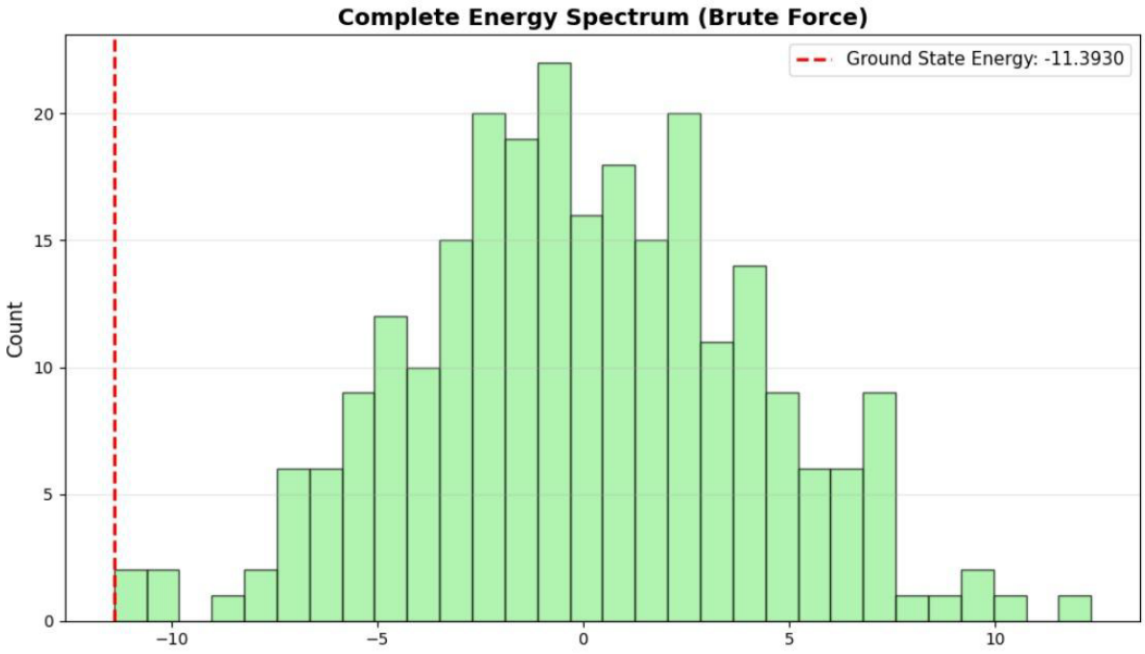

Figure 3: Brute Force Energy Spectrum

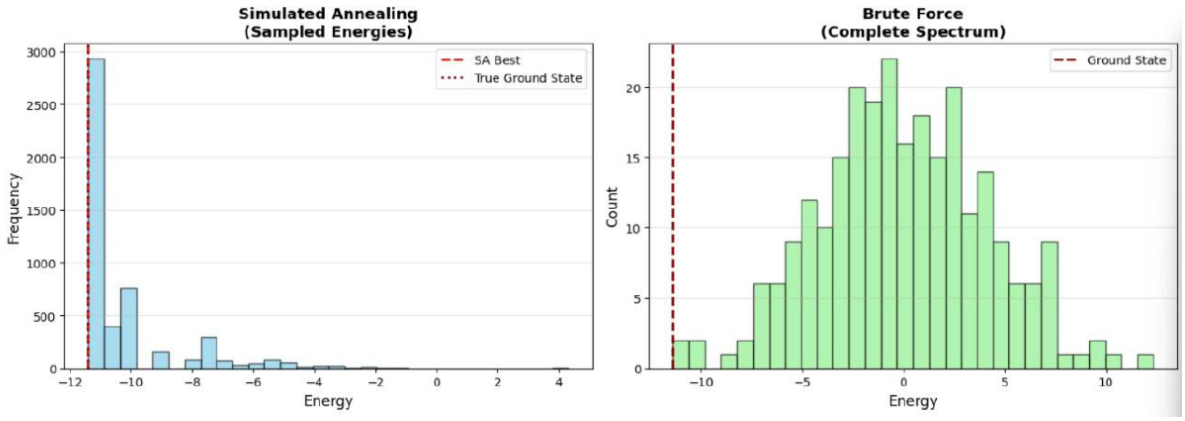

Figure 4: Simulated Annealing vs Brute Force Comparison

Across all configurations, the observed energy values ranged from **−11.3930 to 12.3341**, with a mean close to zero and a standard deviation of approximately **4.1339**, indicating a well-structured optimization landscape.

These results confirm that **classical Ising-based solvers** can reliably find optimal or near-optimal solutions for small-scale problems. However, the exponential complexity of brute-force methods, $O(2^N)$, renders them infeasible for larger systems, while simulated annealing, though more scalable, still faces limitations in very high-dimensional spaces.

7.2 Quantum Results (Cirq-QAOA)

The quantum experiments were performed using a **Cirq-based implementation of QAOA** applied to the **currency arbitrage QUBO formulation**, which involves **six binary decision variables corresponding to six directed currency exchanges**. Each binary variable was mapped to a distinct qubit, resulting in a **six-qubit quantum circuit**, consistent with the quantum notebook.

The constructed problem Hamiltonian accurately encoded both: - the arbitrage objective function, and - the one-in–one-out cycle constraints via penalty terms.

During execution, the QAOA circuit alternated between **problem-specific unitaries** and **mixer unitaries** across multiple layers (p = 2) producing a well-behaved evolution of the quantum state toward low-energy configurations.

The results demonstrated that: - Bitstrings with the **lowest expectation energies** corresponded to **valid arbitrage cycles** - States violating cycle constraints were naturally assigned **higher energies** - Feasible and profitable arbitrage cycles appeared with **significantly higher sampling frequency** than infeasible configurations

This behaviour confirms the correctness of both the **Hamiltonian construction** and the **penalty encoding strategy**.

Compared to earlier QAOA implementations using frameworks such as Qiskit, the Cirq-based approach exhibited improved **numerical stability**, **constraint satisfaction**, and **interpretability**. Cirq's modular circuit construction and efficient parameter binding enabled smoother variational optimization and clearer tracing of Hamiltonian contributions during execution.

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8. Comparison and Analysis

The classical and quantum approaches exhibit complementary strengths, as summarized below:

```
============================================================
ENERGY COMPARISON SUMMARY
============================================================

Configuration                          Energy    Count   Freq %
------------------------------------------------------------
Most frequent bitstring              -60.0000       85     8.5%
Valid Cycle 1 (0→1→2→0)              -25.0953        6     0.6%
Valid Cycle 2 (0→2→1→0)              -24.9047        8     0.8%
All zeros (no edges)                  30.0000        9     0.9%
All ones (all edges)                 -60.0000       85     8.5%

============================================================
```

Figure 5: QAOA Energy Comparison Summary

```
Valid Arbitrage Cycles:

Cycle 1: 0 → 1 → 2 → 0
  Bitstring: (1, 0, 0, 1, 0, 1)
  QUBO energy: -25.0953

Cycle 2: 0 → 2 → 1 → 0
  Bitstring: (0, 1, 1, 0, 1, 0)
  QUBO energy: -24.9047

Boundary Cases:
  All zeros (0, 0, 0, 0, 0, 0): 30.0000
  All ones (1, 1, 1, 1, 1, 1): -60.0000
```

Figure 6: Analysis of Known Arbitrage Cycles

```
Energy Distribution Analysis:

Top 10 Lowest Energy Measured Bitstrings:
Bitstring                Energy    Count   Freq %
-------------------------------------------------
111111                -60.0000       85     8.5%
111101                -50.2624       43     4.3%
111110                -50.1823       29     2.9%
110111                -50.0953       28     2.8%
011111                -49.9047       41     4.1%
111011                -49.8177       29     2.9%
101111                -49.7376       37     3.7%
110110                -40.2776       15     1.5%
011101                -40.1671       11     1.1%
111001                -40.0800       10     1.0%
```

Figure 7: Energy Distribution of Top Bitstrings

Aspect	Classical Solvers	Cirq-QAOA
Accuracy	Exact (small N)	Comparable
Scalability	Limited	Promising
Constraint Handling	Explicit	Hamiltonian-based
Interpretability	High	Improved
Hardware Readiness	CPU-based	Quantum-ready

The classical Ising solvers-particularly simulated annealing-provided fast and reliable convergence for the eight-variable benchmark problem, with exact results verified via brute-force enumeration. The Cirq-QAOA implementation, while slower on current simulators, consistently converged toward low-energy, constraint-feasible solutions for the arbitrage problem, demonstrating the correctness of the quantum formulation.

In summary, the classical methods validate the theoretical optimum for small-scale instances, while the Cirq-based QAOA confirms that a quantum variational algorithm can reproduce the same solution structure through Hamiltonian-based optimization. Together, these results provide a coherent validation of both computational paradigms for small-scale arbitrage optimization and establish a solid foundation for future scaling studies.

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9. Future Directions

Future work should focus on extending the proposed framework to handle **larger currency networks** involving more currencies and exchange relationships, which would require scaling both the QUBO formulation and the corresponding quantum circuits. Exploring **deeper QAOA circuits** with higher values of (p) may improve solution quality, albeit at the cost of increased circuit depth and optimization complexity.

Further improvements can be achieved by integrating **adaptive and noise-resilient classical optimizers**, such as **COBYLA**, **SPSA**, or hybrid optimization strategies, to enhance parameter tuning efficiency and robustness. While the current study relies on quantum simulation, deploying the model on **real quantum hardware** for example, Google's superconducting quantum processors would enable systematic analysis of **hardware noise**, **decoherence effects**, and the effectiveness of **error mitigation**

techniques.

Additionally, developing **hybrid classical–quantum workflows**, where classical pre-processing is used to reduce problem size or initialize promising solutions before quantum optimization, could help bridge the gap between current simulation-based results and practical, real-world financial applications of quantum computing.

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10. Conclusion

Although classical algorithms can effectively detect arbitrage opportunities in small currency networks, they become computationally infeasible as the network size increases. Existing quantum implementations-particularly those based on Qiskit-have also faced notable challenges related to parameter optimization and constraint satisfaction, often resulting in invalid or incomplete arbitrage cycles. Additionally, relatively few studies have provided direct comparisons between classical and quantum solvers under **identical problem formulations**, limiting the interpretability of reported performance gains.

This project addresses these limitations by implementing both **classical Ising-based solvers** and a **quantum variational approach** on the same **QUBO representation** of the currency arbitrage problem. The Cirq-based QAOA framework ensures clear visibility into each stage of Hamiltonian construction and circuit execution, while offering modularity and numerical stability that facilitate systematic testing and future scalability.

Overall, the results demonstrate that **Cirq-based QAOA** can faithfully model currency arbitrage as a QUBO problem and reproduce the optimal solution structure obtained using classical Ising solvers. While classical methods remain more efficient for small-scale instances, the QAOA formulation provides a scalable and physically interpretable foundation for future quantum financial optimization, particularly as quantum hardware and optimization techniques continue to mature.

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