NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

Solution:

Parameter	Value/Formula	description
$x(0)$	2	First term
x(19)	-112	20th term

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)\, u(n) \tag{1}$$

The corresponding Z-transform can be written as

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{2}$$

S(n) is the sum of terms from 0 to n,

$$S(n) = x(n) * u(n) \tag{3}$$
$$S(n - p) = x(n) * u(n - p) \tag{4}$$

On Z-transforming,

$$S(z) = X(z)\, U(z) \tag{5}$$
$$S(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}\right) \frac{1}{1 - z^{-1}} \tag{6}$$
$$S(z) = \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3}\right) \tag{7}$$

On inverse Z-transforming,

$$S(n) = \oint_C S(z)\, z^{n-1} dz \tag{8}$$
$$\implies S(n) = \oint_C \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3}\right) z^{n-1} dz \tag{9}$$
$$\implies S(n) = \oint_C \frac{x(0)\, z^2 (z - 1) + dz^2}{(z - 1)^3} z^{n-1} dz \tag{10}$$

Using the property,

$$f(z_0) = \oint_C \frac{f(z)}{z - z_0} dz \tag{11}$$
$$f'(z_0) = \oint_C \frac{f(z)}{(z - z_0)^2} dz \tag{12}$$
$$f''(z_0) = 2 \oint_C \frac{f(z)}{(z - z_0)^3} dz \tag{13}$$

On comparison of (10) and (13)

$$f''(z_0) = 2S(n) \tag{14}$$
$$f(z) = \left(x(0)\, z^2 (z - 1) + dz^2\right) z^{n-1} \tag{15}$$
$$z_0 = 1 \tag{16}$$
$$\implies S(n) = x(0)(n + 1) + \frac{n(n + 1)}{2} d \tag{17}$$

Given,

$$\sum_{n=0}^{4} x(n) = \frac{1}{4} \sum_{n=5}^{9} x(n) \tag{18}$$

Simplifying:

$$S(4) = \frac{1}{4} (S(9) - S(4)) \tag{19}$$
$$5x(0) + 10d = \frac{1}{4} (5x(0) + 35d) \tag{20}$$
$$x(0) = \frac{-d}{3} \tag{21}$$
$$\implies d = -6 \tag{22}$$

From (22) and Table 1

$$x(19) = x(0) + 19d \tag{23}$$
$$= -112 \tag{24}$$

From (22) and Table 1:

$$\implies x(n) = (2 - 6n)\, u(n) \tag{25}$$

From (2) and (25) :

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{6z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{26}$$

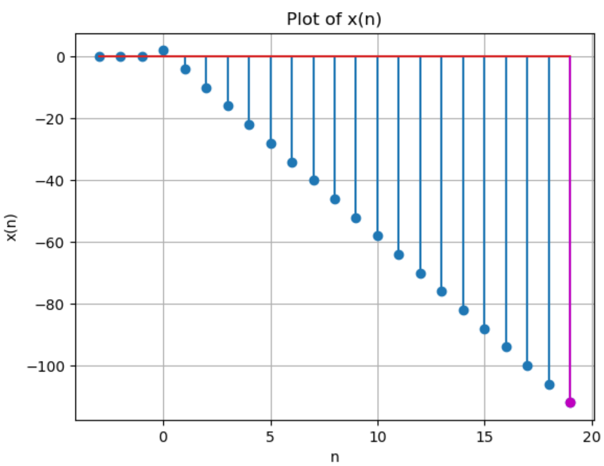

Fig. 1. graph of x(n) = 2 - 6n