GATE EE23 27

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question: Which of the following statement(s) is/are true?

a) If an LTI system is causal, it is stable.
b) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
c) If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.
d) If the impulse response $0 < |h[n]| < 1$ for all n, then the LTI system is stable.

Solution:

1)

$$\text{Assume} \quad h[n] = 2^n u[n] \tag{1}$$

$$y[n] = h[n] * x[n] \tag{2}$$

$$Y[Z] = H[Z]\,X[Z] \tag{3}$$

$$= \left(\frac{1}{1-2Z^{-1}}\right) X[Z] \tag{4}$$

$$H[Z] = \frac{1}{1-2Z^{-1}} \quad |Z| \geq 2 \tag{5}$$

Tis system is causal but not stable because ROC is not in unit circle.

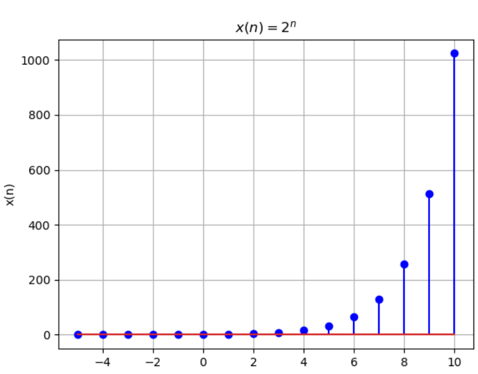

Fig. 1. graph of $x(n) = 2^n u[n]$

Therefore, this statement does not hold true.

2)

$$\text{Assume} \quad h[n] = \begin{cases} \frac{1}{3} & 0 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$y[n] = h[n] * u[n] \tag{7}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot u[n-k] \tag{8}$$

$$= \sum_{k=0}^{n} \left(\frac{1}{3}\right) \tag{9}$$

Therefore, in this example, the system's response to a step input is indeed 0 for $n < 0$, fulfilling condition for stability.
Therefore, this statement is true.

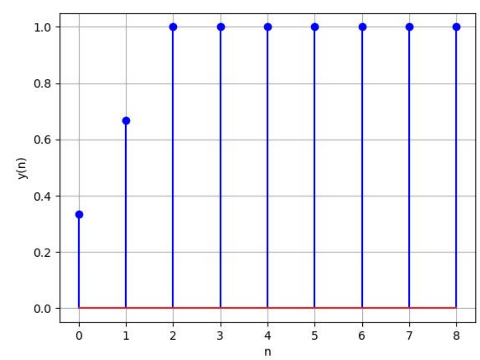

Fig. 2. graph of $y[n]$

3)

$$\text{Assume} \quad h[n] = \begin{cases} 1 & n = 0 \\ -2 & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$y[n] = h[n] * u[n] \tag{11}$$

$$y[n] = \begin{cases} 1 & n = 0 \\ -1 & n \geq 1 \end{cases} \tag{12}$$

The input response is finite but the output response in not BIBO stable.
Therefore, this statement does not hold true.

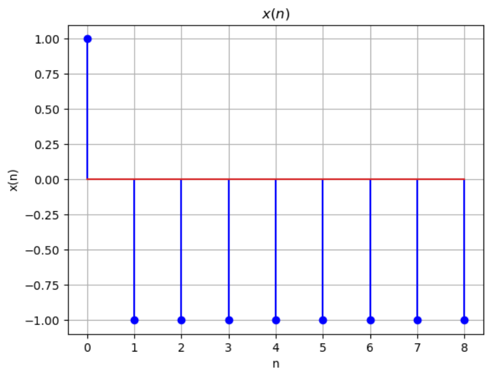

Fig. 3. graph of $y[n]$

4)

$$\text{Assume} \quad h[n] = \frac{1}{2}u[n] \tag{13}$$

$$g[n] = \sum_{n=0}^{\infty} |h[n]| \tag{14}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} \tag{15}$$

$$\implies \sum_{n=0}^{\infty} |h[n]| \to \infty \tag{16}$$

Hence it is unstable.

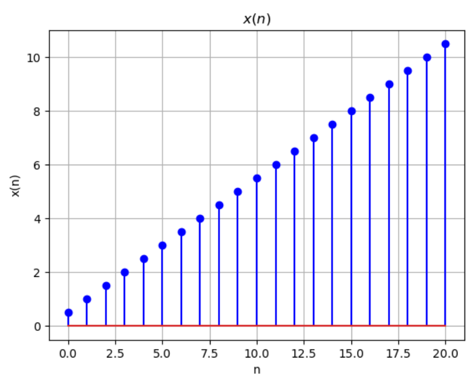

Fig. 4. graph of $g[n]$

Therefore, this statement does not hold true. So, the answer is option (B).