GATE ES22 13

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question: Assuming $s > 0$; Laplace transform for $f(x) = sin(ax)$ is

(A) $\frac{a}{s^2+a^2}$
(B) $\frac{s}{s^2+a^2}$
(C) $\frac{a}{s^2-a^2}$
(D) $\frac{s}{s^2-a^2}$

Solution:

$$\text{We can write} \quad \sin(ax) = \frac{e^{ax} - e^{-ax}}{2i} \tag{1}$$

From (1)

$$\mathcal{L}(f(x)) = \int_{-\infty}^{\infty} e^{-sx} f(x)\, dx \tag{2}$$

$$\mathcal{L}(\sin(ax)) = \int_{0}^{\infty} e^{-sx} \left(\frac{e^{iax} - e^{-iax}}{2i}\right) dx \tag{3}$$

$$= \frac{1}{2i} \int_{0}^{\infty} e^{-x(s-ia)} - e^{-x(s+ia)} dx \tag{4}$$

$$= \frac{1}{2i} \left(\frac{e^{-x(s-ia)}}{-(s-ia)} + \frac{e^{-x(s+ia)}}{-(s+ia)}\right)_{0}^{\infty} \tag{5}$$

$$= \frac{1}{2i} \left(\frac{1}{s-ia} - \frac{1}{s+ia}\right) \tag{6}$$

$$= \frac{a}{s^2 + a^2} \tag{7}$$

So, option (A) is correct.