NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution

Parameter	Value/Formula	description
$x(0)$	2	First term
x(19)	-112	20^{th} term
x(n)	(x(0) + nd)u(n)	$(n+1)^{th}$ term of AP
d	-6	common difference

TABLE 0
Input data

$$x_0 + x_1 + x_2 + x_3 + x_4 = \frac{1}{4}[x_5 + x_6 + x_7 + x_8 + x_9] \tag{1}$$

$$[x(0)+x(0)+d+x(0)+2d+x(0)+3d+x(0)+4d)] =$$

$$\frac{1}{4}[x(0)+5d+x(0)+6d+x(0)+7d+x(0)+8d+x(0)+9d]$$

Simplifying:

$$5x(0) + 10d = \frac{1}{4}(5x(0) + 35d) \tag{2}$$

$$\implies x(0) = \frac{-d}{3} \tag{3}$$

$$\implies d = -6 \tag{4}$$

$$x(19) = x(0) + 19d \tag{5}$$

$$= 2 + 19(-6) = -112 \tag{6}$$

From (4) and Table 0:

$$\implies x(n) = (2 - 6n)u(n) \tag{7}$$

The Z-transform of x(n) :

$$X(z) = \sum_{n=1}^{\infty}(2 - 6n)z^{-n} \tag{8}$$

$$X(z) = \sum_{n=1}^{\infty}2z^{-n} - \sum_{n=1}^{\infty}6nz^{-n} \tag{9}$$

$$X(z) = 2U(z) + 6(-z)\frac{d}{dz}U(z) \tag{10}$$

$$X(z) = \frac{2}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \tag{11}$$

$$ROC : |z| > 1$$

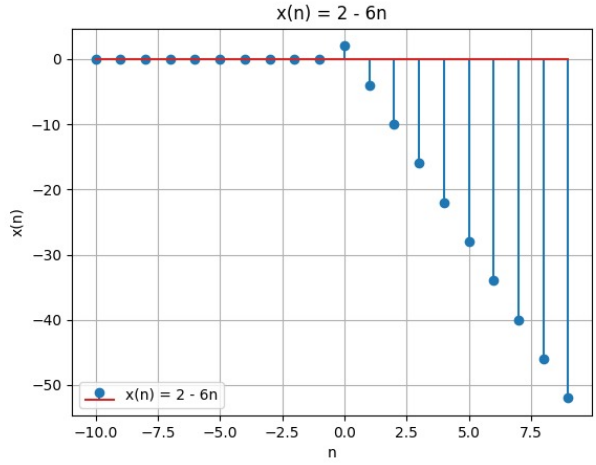

Fig. 0. graph of x(n) = 2 - 6n