GATE EE23 27

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question: Which of the following statement(s) is/are true?

a) If an LTI system is causal, it is stable.
b) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
c) If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.
d) If the impulse response $0 < |h[n]| < 1$ for all n, then the LTI system is stable.

Solution:

1) Let's take an impulse signal $h[n] = 2^n u[n]$
Assume $y[n] = h[n] * x[n]$

$$Y[Z] = h[Z] X[Z] \tag{1}$$

$$Y[Z] = \left(\frac{1}{1 - 2Z^{-1}}\right) X[Z] \tag{2}$$

$$H[Z] = \frac{1}{1 - 2Z^{-1}} \quad |Z| \geq 2 \tag{3}$$

It is causal because it only depends on present or past inputs. But it is not stable because ROC is not in unit circle.

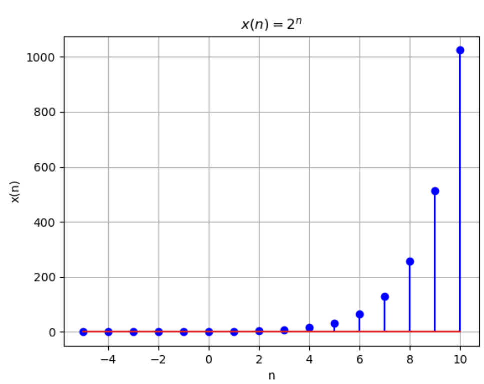

Fig. 1. graph of $x(n) = 2^n u[n]$

Therefore, this statement does not hold true.

2) In discrete-time signal processing, a system is considered causal if the output at any time depends only on the current and past inputs, not future inputs. This means that the response of the system to a step input should also be zero for negative time indices if the system is causal.
Conversely, if the response to a step input is non-zero for negative time indices, it implies that the system is anticipating future inputs, which violates the causality principle.
So, a discrete-time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
Therefore, this statement is true.

3) Let's take an impulse signal.

$$h[n] = \begin{cases} 1 & n = 0 \\ -2 & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Assume $y[n] = h[n] * u[n]$

$$y[n] = \begin{cases} 1 & n = 0 \\ 1 & n \geq 1 \end{cases} \tag{5}$$

The output never go back to zero. As per the condition for stability the output should be bounded for bounded input.

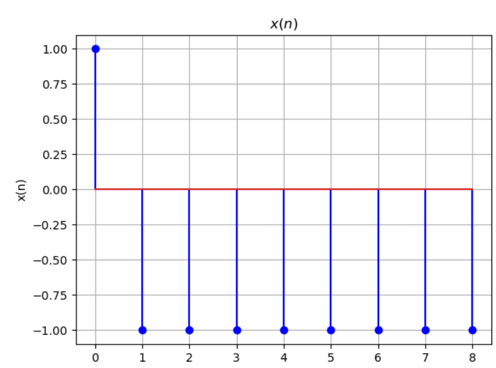

Fig. 1. graph of $y[n]$

Therefore, this statement does not hold true.

4) Let's take an impulse signal $h[n] = \frac{1}{2}u[n]$

$$g[n] = \sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \frac{1}{2} \tag{6}$$

$$\implies \sum_{n=0}^{\infty} |h[n]| \to \infty \tag{7}$$

Hence it is unstable.

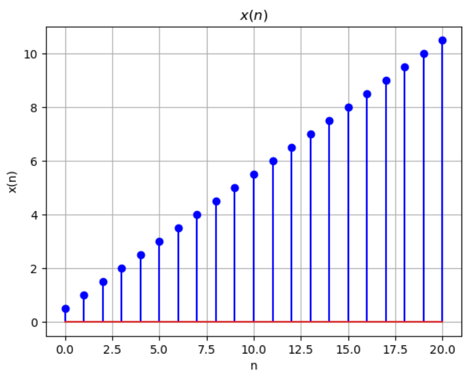

Fig. 1. graph of $g[n]$

Therefore, this statement does not hold true. So, the answer is option (B).