NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution

Parameter	Value/Formula	description
$x(0)$	2	First term
x(19)	-112	20^{th} term
x(n)	(x(0) + nd)u(n)	$(n+1)^{th}$ term of AP
d	-6	common difference

TABLE 0
Input data

General term can be written as

$$x(n) = (x(0) + nd)\,u(n) \tag{1}$$

The corresponding Z-transform can be written as

$$X(Z) = \frac{x(0)}{1 - Z^{-1}} + \frac{dZ^{-1}}{(1 - Z^{-1})^2} \tag{2}$$

Given,

$$\sum_{n=0}^{4} x(n) = \frac{1}{4}\sum_{n=5}^{9} x(n) \tag{3}$$

Simplifying:

$$5x(0) + 10d = \frac{1}{4}(5x(0) + 35d) \tag{4}$$

$$\implies x(0) = \frac{-d}{3} \tag{5}$$

$$\implies d = -6 \tag{6}$$

From (6) and Table 0

$$x(19) = x(0) + 19d \tag{7}$$

$$= 2 + 19(-6) = -112 \tag{8}$$

From (6) and Table 0:

$$\implies x(n) = (2 - 6n)u(n) \tag{9}$$

From (2) and (9) :

$$X(Z) = \frac{2}{1 - Z^{-1}} - \frac{6Z^{-1}}{(1 - Z^{-1})^2} \tag{10}$$

$$ROC : |Z| > 1$$

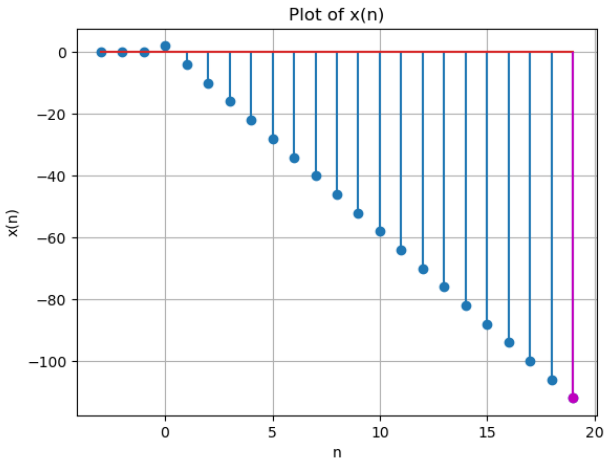

Fig. 1. graph of x(n) = 2 - 6n