NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution:

Parameter	Value/Formula	description
$x(0)$	2	First term
x(19)	-112	20^{th} term

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)\,u(n) \tag{1}$$

For n=0 to n=4,

$$y(n) = x(n) * u(n) \tag{2}$$
$$\implies Y(z) = X(z)\,U(z) \tag{3}$$
$$Y(z) = \frac{x(0)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \tag{4}$$

For n=5 to n=9,

$$y(n+5) = x(n+5) * u(n) \tag{5}$$
$$\implies Y(z) = X(z)\,U(z) \tag{6}$$
$$Y(z) = \frac{(x(0)+4d)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \tag{7}$$
$$\tag{8}$$

On taking inverse Z-transform and substituting n=4

$$y(4) = 5x(0) + 10d \tag{9}$$
$$y(9) = 10x(0) + 45d \tag{10}$$

Given,

$$\sum_{n=0}^{4} x(n) = \frac{1}{4}\sum_{n=5}^{9} x(n) \tag{11}$$

Simplifying:

$$y(4) = \frac{1}{4}(y(9) - y(4)) \tag{12}$$
$$\implies 5x(0) + 10d = \frac{1}{4}(5x(0) + 35d) \tag{13}$$
$$x(0) = \frac{-d}{3} \tag{14}$$
$$\implies d = -6 \tag{15}$$

From (15) and Table 1

$$x(19) = x(0) + 19d \tag{16}$$
$$= -112 \tag{17}$$

From (15) and Table 1:

$$\implies x(n) = (2 - 6n)\,u(n) \tag{18}$$

From (18) :

$$X(z) = \frac{2}{1-z^{-1}} - \frac{6z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{19}$$

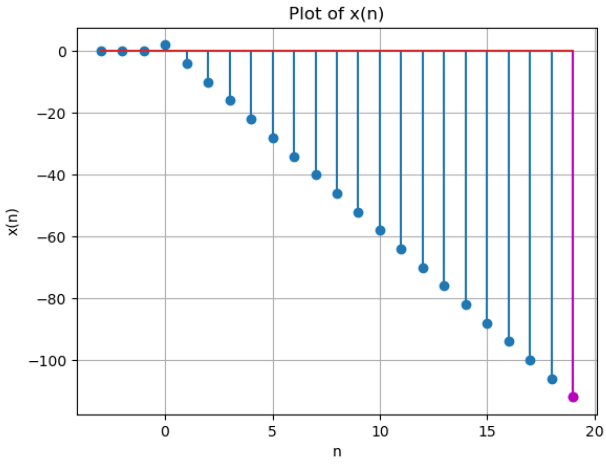

Fig. 1. graph of x(n) = 2 - 6n