NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution:

Parameter	Value/Formula	description
$x(0)$	2	First term
x(19)	-112	20^{th} term

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)\,u(n) \tag{1}$$

The corresponding Z-transform can be written as

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{2}$$

Y(n) is the sum of terms from 0 to n,

$$y(n) = x(n) * u(n) \tag{3}$$

$$y(n - p) = x(n) * u(n - p) \tag{4}$$

On taking Z-transform,

$$Y(z) = X(z)\,U(z) \tag{5}$$

$$Y(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}\right)\frac{1}{1 - z^{-1}} \tag{6}$$

$$Y(z) = \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3}\right) \tag{7}$$

On taking inverse Z-transform,

$$Y(n) = \oint_C Y(z)\,z^{n-1}dz \tag{8}$$

On using contour integral,

$$Y(n) = x(0)(n + 1) + \frac{n(n + 1)}{2}d \tag{9}$$

Given,

$$\sum_{n=0}^{4} x(n) = \frac{1}{4}\sum_{n=5}^{9} x(n) \tag{10}$$

Simplifying:

$$Y(4) = \frac{1}{4}(Y(9) - Y(4)) \tag{11}$$

$$5x(0) + 10d = \frac{1}{4}(5x(0) + 35d) \tag{12}$$

$$x(0) = \frac{-d}{3} \tag{13}$$

$$\implies d = -6 \tag{14}$$

From (14) and Table 1

$$x(19) = x(0) + 19d \tag{15}$$

$$= -112 \tag{16}$$

From (14) and Table 1:

$$\implies x(n) = (2 - 6n)\,u(n) \tag{17}$$

From (2) and (17) :

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{6z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{18}$$

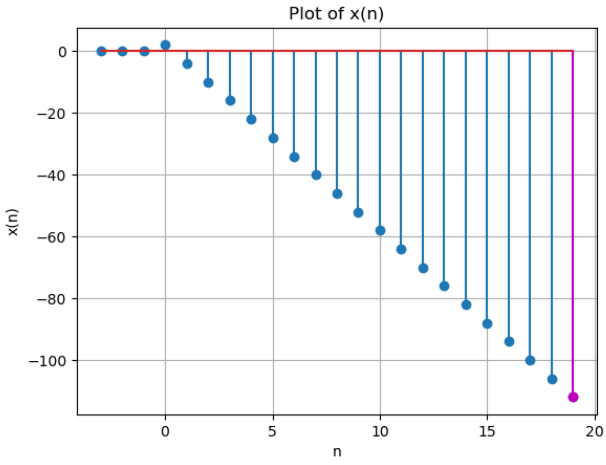

Fig. 1. graph of x(n) = 2 - 6n