

GATE ES22 13

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question: Assuming $s > 0$; Laplace transform for $f(x) = \sin(ax)$ is

- (A) $\frac{a}{s^2+a^2}$
- (B) $\frac{s}{s^2+a^2}$
- (C) $\frac{a}{s^2-a^2}$
- (D) $\frac{s}{s^2-a^2}$

Solution:

$$\mathcal{L}(\sin(ax)) = \int_0^{\infty} e^{-sx} \sin(ax) dx \quad (1)$$

$$= \int_0^{\infty} e^{-sx} \left(\frac{e^{iax} - e^{-iax}}{2i} \right) dx \quad (2)$$

$$= \frac{1}{2i} \int_0^{\infty} e^{-x(s-ia)} - e^{-x(s+ia)} dx \quad (3)$$

$$= \frac{1}{2i} \left(\frac{e^{-x(s-ia)}}{-(s-ia)} + \frac{e^{-x(s+ia)}}{-(s+ia)} \right) \Big|_0^{\infty} \quad (4)$$

$$= \frac{1}{2i} \left(\frac{1}{s-ia} - \frac{1}{s+ia} \right) \quad (5)$$

$$= \frac{a}{s^2 + a^2} \quad (6)$$

So, option (A) is correct.