NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution

Sr. No.	Parameter	Value/Formula
1.	First term ($x(1)$)	2
2.	20^{th} term	-112
3.	U(z)	$\frac{1}{1-z^{-1}}$
4.	x(n)	(x(0) + nd)u(n)
5.	common difference	d

TABLE 0
Input data

$$x_1 + x_2 + x_3 + x_4 + x_5 = \frac{1}{4}[x_6 + x_7 + x_8 + x_9 + x_{10}] \tag{1}$$

$$[x(0)+d+x(0)+2d+x(0)+3d+x(0)+4d+x(0)+5d)] = \frac{1}{4}[x(0)+6d+x(0)+7d+x(0)+8d+x(0)+9d+x(0)+10d]$$

Simplifying:

$$5x(0) + 15d = \frac{1}{4}(5x(0) + 40d) \tag{2}$$

$$\implies x(0) = \frac{-4d}{3} \tag{3}$$

given x(1) = x(0) + d = 2

$$2 = \frac{-4d}{3} + d \tag{4}$$

$$\implies d = -6 \tag{5}$$

$$\implies x(0) = 8 \tag{6}$$

$$x(20) = x(0) + 20d \tag{7}$$

$$= 8 + 20(-6) = -112 \tag{8}$$

From (5) and (6):

$$\implies x(n) = (8 - 6n)u(n) \tag{9}$$

For the sequence $x_n = (8 - 6n)u(n)$ when $n > 0$, we can write:

$$X(z) = \sum_{n=1}^{\infty}(8 - 6n)z^{-n} \tag{10}$$

$$X(z) = \sum_{n=1}^{\infty} 8z^{-n} - \sum_{n=1}^{\infty} 6nz^{-n} \tag{11}$$

$$X(z) = 8U(z) + 6(-z)\frac{d}{dz}U(z) \tag{12}$$

$$X(z) = \frac{8}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \tag{13}$$

$$ROC : |z| > 1$$

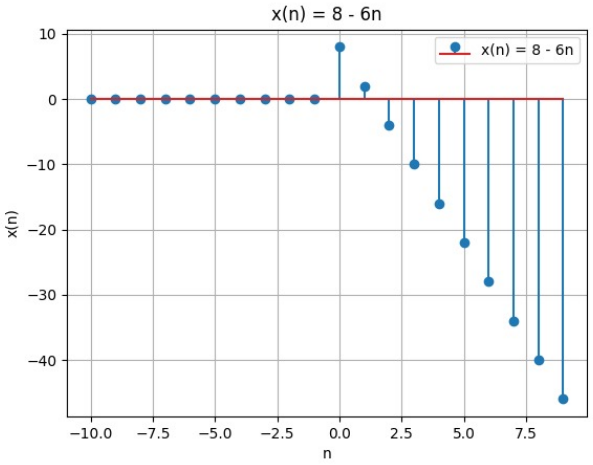

Fig. 0. graph of x(n) = 8 - 6n