NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution

Sr. No.	Parameter	Value/Formula
1.	First term ($x(1)$)	2
2.	20^{th} term	-112
3.	U(z)	$\frac{z^{-1}}{1-z^{-1}}$
4.	x(n)	x(0) + nd

TABLE 0
Input data

$$x_1 + x_2 + x_3 + x_4 + x_5 = \frac{1}{4}[x_6 + x_7 + x_8 + x_9 + x_{10}] \tag{1}$$

Let the common difference as d

$$[x(0)+d+x(0)+2d+x(0)+3d+x(0)+4d+x(0)+5d)] = \frac{1}{4}[x(0)+6d+x(0)+7d+x(0)+8d+x(0)+9d+x(0)+10d]$$

Simplifying:

$$5x(0) + 15d = \frac{1}{4}(5x(0) + 40d) \tag{2}$$
$$20x(0) + 60d = 5x(0) + 40d \tag{3}$$
$$15x(0) + 20d = 0 \tag{4}$$
$$3x(0) + 4d = 0 \tag{5}$$
$$\implies x(0) = \frac{-4d}{3} \tag{6}$$

given x(1) = x(0) + d = 2

$$\implies 2 = \frac{-4d}{3} + d \tag{7}$$
$$\implies 2 = \frac{-d}{3} \tag{8}$$
$$\implies d = -6 \tag{9}$$
$$\implies x(0) = 8 \tag{10}$$

$$x(20) = x(0) + 20d \tag{11}$$
$$= 8 + 20(-6) = -112 \tag{12}$$

$$x(0) = 8 \quad \text{and} \quad d = -6$$

$$x(n) = x(0) + nd \tag{13}$$
$$\implies x(n) = 8 + (n)(-6) \tag{14}$$
$$\implies x(n) = 8 - 6n \tag{15}$$

The Z-transform of a sequence $x(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \tag{16}$$

For the sequence $x_n = 8 - 6n$ when $n > 0$, we can write:

$$X(z) = \sum_{n=1}^{\infty} (8 - 6n)z^{-n} \tag{17}$$
$$X(z) = \sum_{n=1}^{\infty} 8z^{-n} - \sum_{n=1}^{\infty} 6nz^{-n} \tag{18}$$
$$X(z) = 8U(z) + 6(-z)\frac{d}{dz}U(z) \tag{19}$$
$$X(z) = \frac{8}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \tag{20}$$

The function $f(n) = 8 - 6n$ using step function is defined as follows:

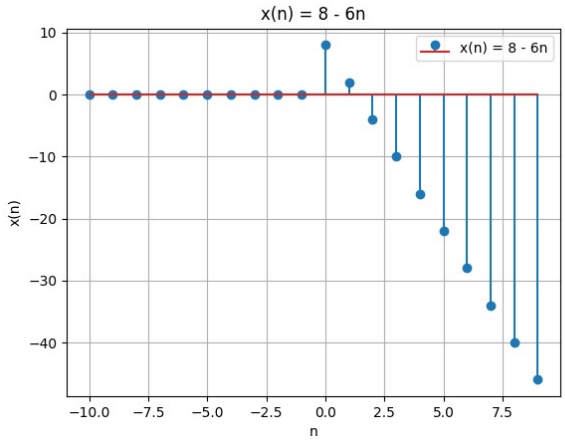

Fig. 0. graph of x(n) = 8 - 6n

$$x(n) = \begin{cases} 8 - 6n, & \text{if } n \geq 0 \\ 0, & \text{if } n < 0 \end{cases} \qquad (21)$$

Given that $n > 0$

$\therefore ROC : |z| > 1$