

NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

QUESTION

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

SOLUTION

Parameter	Value/Formula	description
x(0)	2	First term
x(19)	-112	20 th term
x(n)	(x(0) + nd)u(n)	(n + 1) th term of AP
d	-6	common difference

TABLE 0
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)u(n) \quad (1)$$

The corresponding Z-transform can be written as

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$S(n) = x(n) * u(n) \quad (3)$$

$$S(n - p) = x(n) * u(n - p) \quad (4)$$

On Z-transforming,

$$S(z) = X(z)U(z) \quad (5)$$

$$S(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \right) \frac{1}{1 - z^{-1}} \quad (6)$$

$$S(z) = \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \right) \quad (7)$$

On inverse Z-transforming,

$$S(n) = \oint_C S(z) z^{n-1} dz \quad (8)$$

$$\Rightarrow S(n) = \oint_C \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \right) z^{n-1} dz \quad (9)$$

$$\Rightarrow S(n) = \oint_C \frac{x(0)z^2(z-1) + dz^2}{(z-1)^3} z^{n-1} dz \quad (10)$$

Using the property,

$$f(z_0) = \oint_C \frac{f(z)}{z - z_0} dz \quad (11)$$

$$f'(z_0) = \oint_C \frac{f(z)}{(z - z_0)^2} dz \quad (12)$$

$$f''(z_0) = 2 \oint_C \frac{f(z)}{(z - z_0)^3} dz \quad (13)$$

On comparison,

$$f(z) = (x(0)z^2(z-1) + dz^2)z^{n-1} \quad (14)$$

$$z_0 = 1 \quad (15)$$

$$\Rightarrow S(n) = x(0)(n+1) + \frac{n(n+1)}{2}d \quad (16)$$

Given,

$$\sum_{n=0}^4 x(n) = \frac{1}{4} \sum_{n=5}^9 x(n) \quad (17)$$

Simplifying:

$$S(4) = \frac{1}{4}(S(9) - S(4)) \quad (18)$$

$$5x(0) + 10d = \frac{1}{4}(5x(0) + 35d) \quad (19)$$

$$\Rightarrow x(0) = \frac{-d}{3} \quad (20)$$

$$\Rightarrow d = -6 \quad (21)$$

From (21) and Table 0

$$x(19) = x(0) + 19d \quad (22)$$

$$= 2 + 19(-6) \quad (23)$$

$$= -112 \quad (24)$$

From (21) and Table 0:

$$\Rightarrow x(n) = (2 - 6n)u(n) \quad (25)$$

From (2) and (25):

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{6z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (26)$$

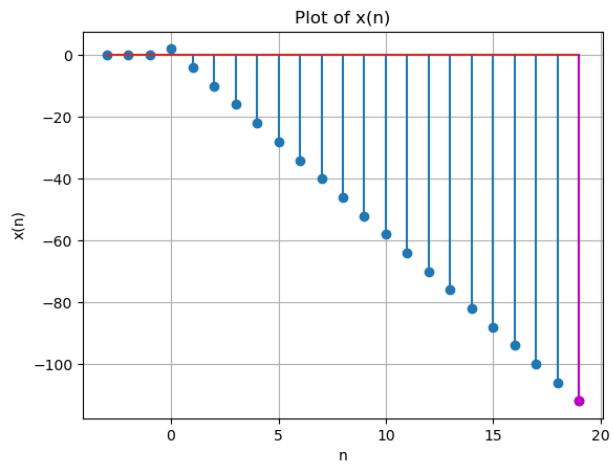


Fig. 1. graph of $x(n) = 2 - 6n$