GATE EE23 27

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question: Which of the following statement(s) is/are true?

a) If an LTI system is causal, it is stable.
b) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
c) If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.
d) If the impulse response $0 < |h[n]| < 1$ for all n, then the LTI system is stable.

Solution:

1)

$$\text{Assume} \quad h(t) = e^{2t} \cdot u(t) \tag{1}$$

$$\mathcal{L}\{e^{2t}\} = \frac{1}{s-2} \quad Re(s) > 2 \tag{2}$$

This system is causal but not stable because ROC does not contian imaginary axis.

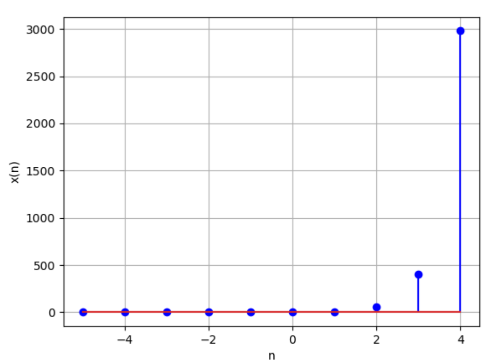

Fig. 1. graph of $h(t) = e^{2t} \cdot u(t)$

Therefore, this statement does not hold true.

2) For a causal system, its response to the step input should indeed be zero for $n < 0$, as the system hasn't yet "seen" any input before time $n = 0$.
Mathematically, the output of an LTI system $y[n]$ can be represented as the convolution of the input $u[n]$ with the system's impulse response $h[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} h[n]\, u[n-k] \tag{3}$$

Now applying step input,

$$y[n] = \sum_{k=0}^{\infty} h[n]\, u[n-k] \tag{4}$$

This is because $u[n-k]$ is zero for $n < k$, hence, the summation only starts from $k = 0$.
For $n < 0$, $u[n-k] = 0$ for all k, because $n - k < 0$ when $n < 0$. Therefore, $y[n] = 0$ for $n < 0$.
Therefore, this statement is true.

3)

$$\text{Assume} \quad h[n] = \begin{cases} n & \text{if } 0 \le n \le N \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$y[n] = h[n] * u[n] \tag{6}$$

$$y[n] = \sum_{k=0}^{n} h[k]\, u[n-k] \tag{7}$$

$$= \sum_{k=0}^{n} k \tag{8}$$

The input response is finite but the output rsponse in not BIBO stable.

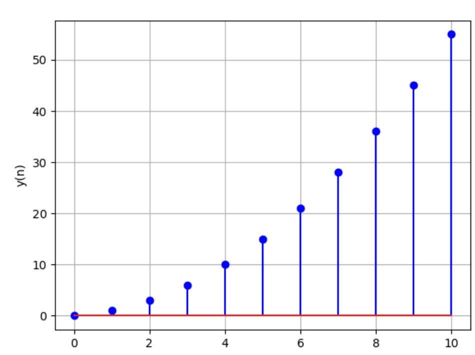

Fig. 3. graph of $y[n]$

Therefore, this statement does not hold true.

4)

$$\text{Assume} \quad h[n] = \frac{1}{2}u[n] \tag{9}$$

$$g[n] = \sum_{n=0}^{\infty} |h[n]| \tag{10}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} \tag{11}$$

$$\implies \sum_{n=0}^{\infty} |h[n]| \to \infty \tag{12}$$

Hence it is unstable.

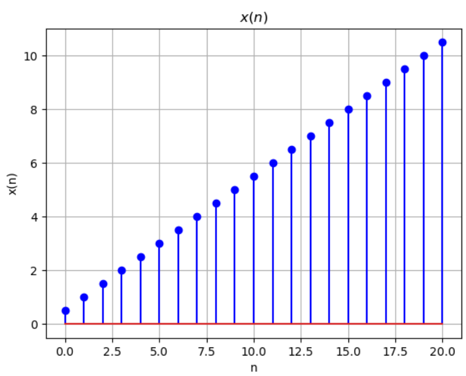

Fig. 4. graph of $g[n]$

Therefore, this statement does not hold true.

So, the answer is option (B).