NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution:

Parameter	Description	Value
$x(0)$	First term	2
$x(19)$	20^{th} term	-112
$y(n)$	sum upto n^{th} term	

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)\,u(n) \tag{1}$$

By refereing (**??**)

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{2}$$

Taking the inverse Z-transform by contour integration by refering (**??**),

$$y(n) = x(0)\,[(n+1)u(n)] + \frac{d}{2}\,[n(n+1)u(n)] \tag{3}$$

$$= \frac{n+1}{2}\,\{2x(0) + nd\}\,u(n) \tag{4}$$

Therefore,

$$y(4) = 5x(0) + 10d \tag{5}$$

$$y(9) = 10x(0) + 45d \tag{6}$$

Given,

$$\sum_{n=0}^{4} x(n) = \frac{1}{4}\sum_{n=5}^{9} x(n) \tag{7}$$

Simplifying:

$$y(4) = \frac{1}{4}\,(y(9) - y(4)) \tag{8}$$

$$\implies 5x(0) + 10d = \frac{1}{4}\,(5x(0) + 35d) \tag{9}$$

$$x(0) = \frac{-d}{3} \tag{10}$$

$$\implies d = -6 \tag{11}$$

From (15) and Table 1

$$\implies x(n) = (2 - 6n)\,u(n) \tag{12}$$

From (18):

$$x(19) = x(0) + 19d \tag{13}$$

$$= -112 \tag{14}$$

From (18) :

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{6z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{15}$$

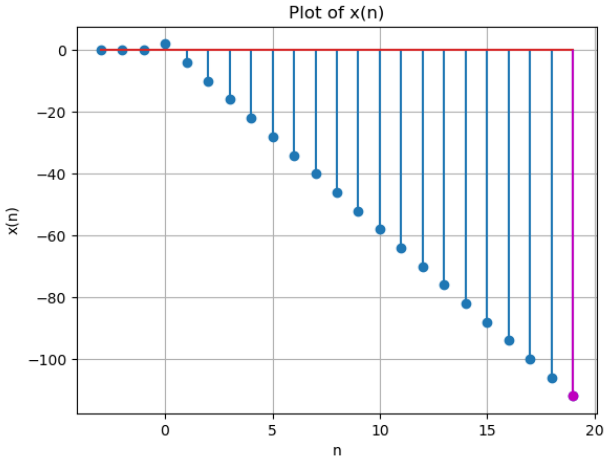

Fig. 1. graph of x(n) = 2 - 6n