

NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

QUESTION

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112

SOLUTION

Parameter	Value
First term ($x(1)$)	2
20 th term	-112

TABLE 0
INPUT DATA

$$x_1 + x_2 + x_3 + x_4 + x_5 = \frac{1}{4}[x_6 + x_7 + x_8 + x_9 + x_{10}] \quad (1)$$

Let the common difference as d

$$[x(0) + d + x(0) + 2d + x(0) + 3d + x(0) + 4d + x(0) + 5d] = \frac{1}{4}[x(0) + 6d + x(0) + 7d + x(0) + 8d + x(0) + 9d + x(0) + 10d]$$

Simplifying:

$$5x(0) + 15d = \frac{1}{4}(5x(0) + 40d) \quad (2)$$

$$20x(0) + 60d = 5x(0) + 40d \quad (3)$$

$$15x(0) + 20d = 0 \quad (4)$$

$$3x(0) + 4d = 0 \quad (5)$$

$$\Rightarrow x(0) = \frac{-4d}{3} \quad (6) \quad 2)$$

$$\text{given } x(1) = x(0) + d = 2$$

$$\Rightarrow 2 = \frac{-4d}{3} + d \quad (7)$$

$$\Rightarrow 2 = \frac{-d}{3} \quad (8)$$

$$\Rightarrow d = -6 \quad (9)$$

$$\Rightarrow x(0) = 8 \quad (10)$$

$$x(20) = x(0) + 20d \quad (11)$$

$$= 8 + 20(-6) = -112 \quad (12)$$

$$x(0) = 8 \quad \text{and} \quad d = -6$$

$$x(n) = x(0) + nd \quad (13)$$

$$\Rightarrow x(n) = 8 + (n)(-6) \quad (14)$$

$$\Rightarrow x(n) = 8 - 6n \quad (15)$$

The Z-transform of a sequence $x(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \quad (16)$$

For the sequence $x_n = 8 - 6n$ when $n > 0$, we can write:

$$X(z) = \sum_{n=1}^{\infty} (8 - 6n)z^{-n} \quad (17)$$

$$X(z) = \sum_{n=1}^{\infty} 8z^{-n} - \sum_{n=1}^{\infty} 6nz^{-n} \quad (18)$$

1)

$$\sum_{n=1}^{\infty} 8z^{-n} = 8 \sum_{n=1}^{\infty} z^{-n} \quad (19)$$

$$= 8(z^{-1} + z^{-2} + z^{-3} + \dots) \quad (20)$$

$$= \frac{8z^{-1}}{1 - z^{-1}} \quad (21)$$

$$\sum_{n=1}^{\infty} 6nz^{-n} = 6 \sum_{n=1}^{\infty} nz^{-n} \quad (22)$$

$$= 6(1 \cdot z^{-1} + 2 \cdot z^{-2} + 3 \cdot z^{-3} + \dots) \quad (23)$$

This is AGP.

$$S = 1 \cdot z^{-1} + 2 \cdot z^{-2} + 3 \cdot z^{-3} + \dots \quad (24)$$

$$- z^{-1}S = 1 \cdot z^{-2} + 2 \cdot z^{-3} + 3 \cdot z^{-4} + \dots \quad (25)$$

$$(1 - z^{-1}S) = z^{-1} + z^{-2} + z^{-3} + z^{-4} + \dots \quad (26)$$

$$(1 - z^{-1})S = \frac{z^{-1}}{1 - z^{-1}} \quad (27)$$

$$S = \frac{z^{-1}}{(1 - z^{-1})^2} \quad (28)$$

$$\Rightarrow \sum_{n=1}^{\infty} 6nz^{-n} = \frac{6z^{-1}}{(1 - z^{-1})^2} \quad (29)$$

$$X(z) = \frac{8z^{-1}}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \quad (30)$$

The function $f(n) = 8 - 6n$ using step function is defined as follows:

$$f(n) = \begin{cases} 8 - 6n, & \text{if } n \geq 0 \\ 0, & \text{if } n < 0 \end{cases} \quad (31)$$

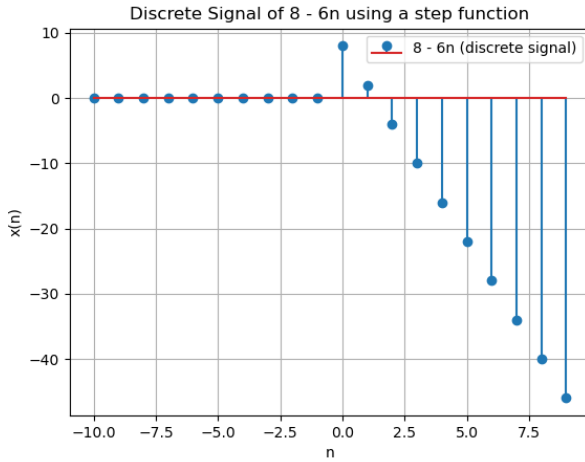


Fig. 2. graph of $x(n) = 8 - 6n$

Given that $n > 0$, let's analyze the ROC for the Z-transform of the sequence $x(n) = 8 - 6n$:

$$X(z) = \sum_{n=1}^{\infty} (8 - 6n) \cdot z^{-n} \quad (32)$$

For convergence, the series

$$\sum_{n=1}^{\infty} (8 - 6n) \cdot |z|^{-n} \quad (33)$$

must converge. This implies that $|z|$ should be greater than the magnitude of the terms

$(8 - 6n)$. The terms $(8 - 6n)$ decrease in magnitude as n increases, so the ROC is the set of complex numbers z such that $|z|$ is greater than the magnitude of the terms for all n .

In this case, the ROC is the region outside the circle with radius equal to the magnitude of the first term $|8|$ in the series, i.e.; outside the circle $|z| > 8$.

So, the Region of Convergence (ROC) for the Z-transform of the sequence $x[n] = 8 - 6n$ with $n > 0$ is the exterior of the circle $|z| > 8$.