

NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

Solution:

Parameter	Value/Formula	description
$x(0)$	2	First term
$x(19)$	-112	20 th term

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)u(n) \quad (1)$$

The corresponding Z-transform can be written as

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$Y(n)$ is the sum of terms from 0 to n ,

$$y(n) = x(n) * u(n) \quad (3)$$

$$y(n - p) = x(n) * u(n - p) \quad (4)$$

On taking Z-transform,

$$Y(z) = X(z)U(z) \quad (5)$$

$$Y(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \right) \frac{1}{1 - z^{-1}} \quad (6)$$

$$Y(z) = \left(\frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \right) \quad (7)$$

On taking inverse Z-transform,

$$Y(n) = \oint_C Y(z) z^{n-1} dz \quad (8)$$

On using contour integral,

$$Y(n) = x(0)(n+1) + \frac{n(n+1)}{2}d \quad (9)$$

Given,

$$\sum_{n=0}^4 x(n) = \frac{1}{4} \sum_{n=5}^9 x(n) \quad (10)$$

Simplifying:

$$Y(4) = \frac{1}{4} (S(9) - S(4)) \quad (11)$$

$$5x(0) + 10d = \frac{1}{4} (5x(0) + 35d) \quad (12)$$

$$x(0) = \frac{-d}{3} \quad (13)$$

$$\Rightarrow d = -6 \quad (14)$$

From (14) and Table 1

$$x(19) = x(0) + 19d \quad (15)$$

$$= -112 \quad (16)$$

From (14) and Table 1:

$$\Rightarrow x(n) = (2 - 6n)u(n) \quad (17)$$

From (2) and (17) :

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{6z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (18)$$

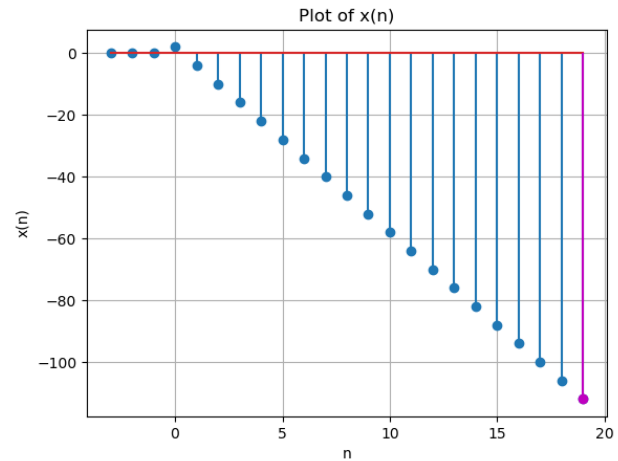


Fig. 1. graph of $x(n) = 2 - 6n$