

# NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE\*

## QUESTION

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20<sup>th</sup> term is -112

## SOLUTION

Sr. No.	Parameter	Value
1.	First term (x(1))	2
2.	20 <sup>th</sup> term	-112

TABLE 0  
INPUT DATA

$$x_1 + x_2 + x_3 + x_4 + x_5 = \frac{1}{4}[x_6 + x_7 + x_8 + x_9 + x_{10}] \quad (1)$$

Let the common difference as  $d$

$$[x(0) + d + x(0) + 2d + x(0) + 3d + x(0) + 4d + x(0) + 5d] = \frac{1}{4}[x(0) + 6d + x(0) + 7d + x(0) + 8d + x(0) + 9d + x(0) + 10d]$$

Simplifying:

$$5x(0) + 15d = \frac{1}{4}(5x(0) + 40d) \quad (2)$$

$$20x(0) + 60d = 5x(0) + 40d \quad (3)$$

$$15x(0) + 20d = 0 \quad (4)$$

$$3x(0) + 4d = 0 \quad (5)$$

$$\Rightarrow x(0) = \frac{-4d}{3} \quad (6)$$

$$\text{given } x(1) = x(0) + d = 2$$

$$\Rightarrow 2 = \frac{-4d}{3} + d \quad (7)$$

$$\Rightarrow 2 = \frac{-d}{3} \quad (8)$$

$$\Rightarrow d = -6 \quad (9)$$

$$\Rightarrow x(0) = 8 \quad (10)$$

$$x(20) = x(0) + 20d \quad (11)$$

$$= 8 + 20(-6) = -112 \quad (12)$$

$$x(0) = 8 \quad \text{and} \quad d = -6$$

$$x(n) = x(0) + nd \quad (13)$$

$$\Rightarrow x(n) = 8 + (n)(-6) \quad (14)$$

$$\Rightarrow x(n) = 8 - 6n \quad (15)$$

The Z-transform of a sequence  $x(n)$  is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \quad (16)$$

For the sequence  $x_n = 8 - 6n$  when  $n > 0$ , we can write:

$$X(z) = \sum_{n=1}^{\infty} (8 - 6n)z^{-n} \quad (17)$$

$$X(z) = \sum_{n=1}^{\infty} 8z^{-n} - \sum_{n=1}^{\infty} 6nz^{-n} \quad (18)$$

$$X(z) = 8U(z) + 6(-z)\frac{d}{dz}U(z) \quad (19)$$

$$X(z) = \frac{8z^{-1}}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \quad (20)$$

The function  $f(n) = 8 - 6n$  using step function is defined as follows:

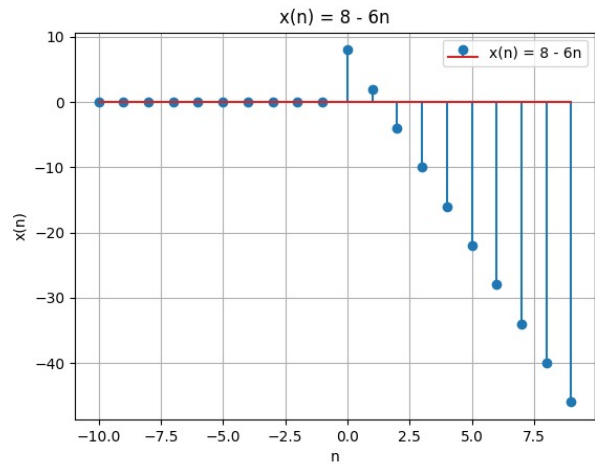


Fig. 0. graph of  $x(n) = 8 - 6n$

$$f(n) = \begin{cases} 8 - 6n, & \text{if } n \geq 0 \\ 0, & \text{if } n < 0 \end{cases} \quad (21)$$

Given that  $n > 0$

$$X(z) = \sum_{n=1}^{\infty} (8 - 6n) \cdot z^{-n} \quad (22)$$

$$ROC : |z| > 8$$