

NCERT 11.9.2.3

EE23BTECH11043 - BHUVANESH SUNIL NEHETE*

Question:

In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

Solution:

Parameter	Value/Formula	description
$x(0)$	2	First term
$x(19)$	-112	20 th term

TABLE 1
INPUT DATA

General term can be written as

$$x(n) = (x(0) + nd)u(n) \quad (1)$$

For $n=0$ to $n=4$,

$$y(n) = x(n) * u(n) \quad (2)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (3)$$

$$Y(z) = \frac{x(0)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \quad (4)$$

For $n=5$ to $n=9$,

$$y(n+5) = x(n+5) * u(n) \quad (5)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (6)$$

$$Y(z) = \frac{(x(0) + 4d)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \quad (7)$$

$$(8)$$

On taking inverse Z-transform and substituting $n=4$

$$y(4) = 5x(0) + 10d \quad (9)$$

$$y(9) = 10x(0) + 45d \quad (10)$$

Given,

$$\sum_{n=0}^4 x(n) = \frac{1}{4} \sum_{n=5}^9 x(n) \quad (11)$$

Simplifying:

$$y(4) = \frac{1}{4} (y(9) - y(4)) \quad (12)$$

$$\Rightarrow 5x(0) + 10d = \frac{1}{4} (5x(0) + 35d) \quad (13)$$

$$x(0) = \frac{-d}{3} \quad (14)$$

$$\Rightarrow d = -6 \quad (15)$$

From (15) and Table 1

$$x(19) = x(0) + 19d \quad (16)$$

$$= -112 \quad (17)$$

From (15) and Table 1:

$$\Rightarrow x(n) = (2 - 6n)u(n) \quad (18)$$

From (18) :

$$X(z) = \frac{2}{1-z^{-1}} - \frac{6z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \quad (19)$$

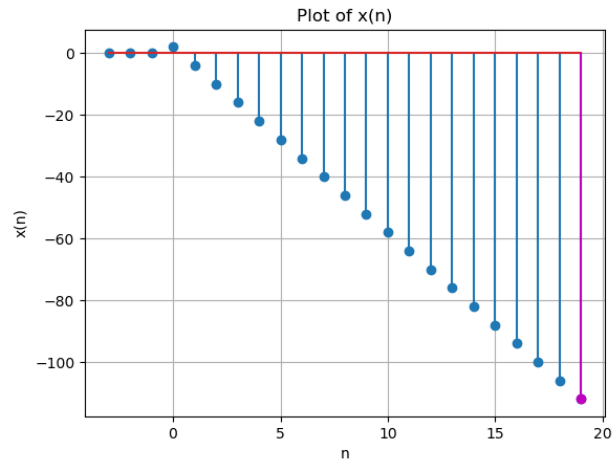


Fig. 1. graph of $x(n) = 2 - 6n$