BITS Pilani
Pilani Campus

Artificial & Computational Intelligence

AIML CZG557

M4 : Knowledge Representation using Logics

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Course Plan

innovate achieve lead

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

Propositional Logic

1. PL resolution.
2. TT Entilement
3. Theorem Proving Mechanism
4. DPLL Alg

Predicate Logic

Representation by Propositional Logic

innovate achieve lead

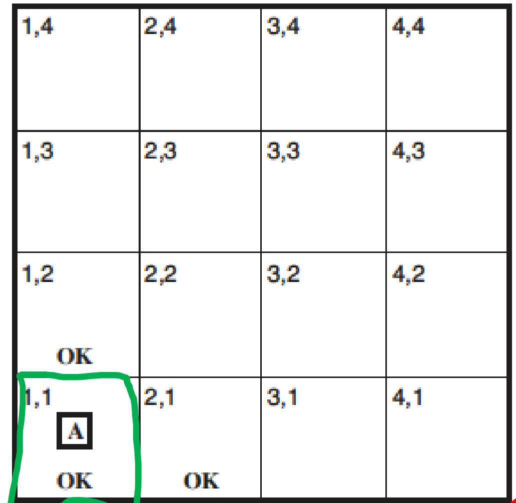
For each [x, y] location

$P_{x,y}$ is true if there is a pit in [x, y]

$W_{x,y}$ is true if there is a wumpus in [x, y]

$B_{x,y}$ is true if agent perceives a breeze in [x, y]

$S_{x,y}$ is true if agent perceives a stench in [x, y]



Premises

KB ----------------------- R is the sentence in KB

$R_1 : \neg P_{1,1}$ → fact1

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ → Rule1

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ Rule2

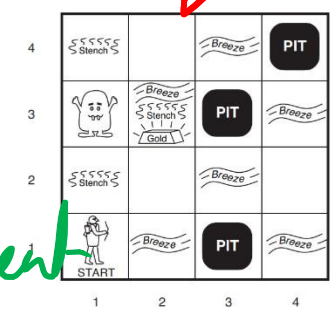
$R_4 : \neg B_{1,1}$ → fact2

$R_5 : B_{2,1}$ → fact3

3 fact + 2 Rule sys → KB

$\boxed{KB \vdash \neg P_{1,2}}$

⇑ TT Entailment



Query : $\neg P_{1,2}$ entailed by our KB?

= NO PIT LOC 1,2

Representation by Propositional Logic

For each [x, y] location

$P_{x,y}$ is true if there is a pit in [x, y]

$W_{x,y}$ is true if there is a wumpus in [x, y]

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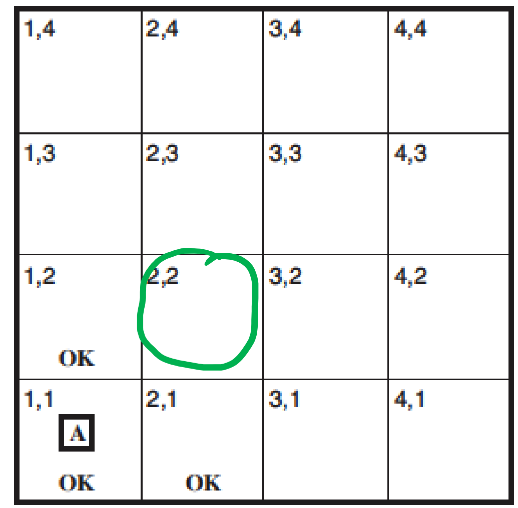


KB ----------------------- R is the sentence in KB

1

$R_1 : \neg P_{1,1}$

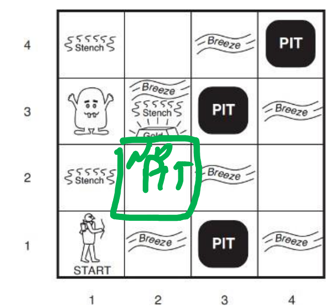
$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (\cancel{P_{1,1}} \vee P_{2,2} \vee P_{3,1})$

$R_4 : \neg \cancel{B_{1,1}}$

$R_5 : \cancel{B_{2,1}}$

2 3 4 5 6 7



Query : $\neg P_{1,2}$ entailed by our KB?

5 pre → 7 unique literals

TT – Entails Inference – Example

Agents based on Propositional logic, TT-Entail for inference from truth table

Q: $\boxed{\neg P_{1,2}}$ entailed by our KB?

Way – 1 :

T F T F T F T F T F

1. Get sufficient information $B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$

1 2 3 4 5 6 7

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

TT – Entails Inference – Example

Agents based on Propositional logic , TT-Entail for inference from truth table

¬ $P_{1,2}$ entailed by our KB?

Way – 1 :

7 $\quad 2^7$ combination => 128

1. Get sufficient information $B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$
2. Enumerate all models with combination of truth values to propositional symbols

KB -> 5 sent

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
1	2	3	4	5	6	7						
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

2^7

TT – Entails Inference – Example

Agents based on Propositional logic, TT-Entail for inference from truth table

$Q : \neg P_{2,2}$ → False → P

→ True.

$\neg P_{1,2}$ entailed by our KB?

Way – 1 :

1. Get sufficient information $B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$
2. Enumerate all models with combination of truth values to propositional symbols
3. In all the models, find those models where KB is true, i.e., every sentence R_1, R_2, R_3, R_4, R_5 are true
4. In those models where KB is true, find if query sentence $\neg P_{1,2}$ is true
5. If query sentence $\neg P_{1,2}$ is true in all models where KB is true, then it entails, otherwise it won't

$Q \Rightarrow P_{12}$ → True → Query is false

$Q \Rightarrow \neg P_{12}$ → False. → Query True!!

T T T T T TStatus

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

KB

TT – Entails Inference – Example

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

5 pre → <u>7</u> unique lit

$2^7 \Rightarrow \underline{128}$

Inference : Properties

1. Entailment : $\alpha \models \beta$
2. Equivalence : $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$
3. Validity
4. Satisfiability

Inference : Example – Theorem Proving (Self Study)

3rd

Propositional theorem proving - Proof by resolution

Logical Equivalence rules can be used as inference rules

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Inference : Example – Theorem Proving

1. **Modes Ponens**
2. AND Elimination

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) && \text{De Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

α : I walk in rain without the umbrella

β : I get wet

α → β **α**

β

Inference : Example – Theorem Proving

1. Modes Ponens
2. **AND Elimination**

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg \alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg \beta \Rightarrow \neg \alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg \alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
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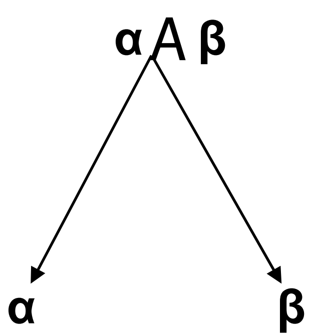
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

α : I walk in rain without the umbrella

β : I get wet



Inference : Example – Theorem Proving +

KB =

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

$(\alpha \land \beta) \equiv (\beta \land \alpha)$ commutativity of $\land$

$(\alpha \lor \beta) \equiv (\beta \lor \alpha)$ commutativity of $\lor$

$((\alpha \land \beta) \land \gamma) \equiv (\alpha \land (\beta \land \gamma))$ associativity of $\land$

$((\alpha \lor \beta) \lor \gamma) \equiv (\alpha \lor (\beta \lor \gamma))$ associativity of $\lor$

$\neg(\neg \alpha) \equiv \alpha$ double-negation elimination

$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$ contraposition

$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \lor \beta)$ implication elimination

$\boxed{(\alpha \Leftrightarrow \beta)} \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$ biconditional elimination

$\neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta)$ De Morgan

$\neg(\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta)$ De Morgan

$(\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \land \beta) \lor (\alpha \land \gamma))$ distributivity of $\land$ over $\lor$

$(\alpha \lor (\beta \land \gamma)) \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma))$ distributivity of $\lor$ over $\land$

10 → another combin

Query: $\neg P_{1,2}$. Can we prove if this sentence be entailed from KB using inference rules?_______________

$\Leftrightarrow \quad \Rightarrow \quad \rightarrow$

AI 1

Agent

$A_3 \to 50$

Refe =

$R_2: B_{11} \Leftrightarrow (P_{12} \lor P_{21})$

$R_6: B_{11} \to \cancel{(P_{12} \lor P_{21})} \land (P_{12} \lor P_{21}) \to B_{11}$

$R_7: (P_{12} \lor P_{21}) \to B_{11}$

$R_8: \neg B_{11} \to \neg(P_{12} \lor P_{21})$

6

- Biconditional Elimination
- And Elimination
- Contraposition
- Modus Ponens
- Demorgans
- And Elimination

a (True)

$a \to b$ (True)

$b \to$ true

Inference : Example – Theorem Proving

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4 : \neg B_{1,1}$ fact → True

$R_5 : B_{2,1}$

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
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\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Query: $\neg P_{1,2}$ Can we prove if this sentence be entailed from KB using inference rules?_______________

a: $\neg B_{11}$

True

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_6 : (B_{1,1} \xRightarrow{\text{Part 1}} (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$ T — Biconditional Elimination (Part 2)

$R_7 : ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$ T — And Elimination

$R_8 : (\neg \overset{a}{B_{1,1}} \Rightarrow \neg \overset{b}{(P_{1,2}} \lor P_{2,1}))$ F a→b True — Contraposition

$R_9 : \neg (P_{1,2} \lor P_{2,1})$ ← Modus Ponens

$R_{10} : \neg P_{1,2} \land \neg P_{2,1}$ ← Demorgans

$R11: \neg P_{1,2}$ Pro — And Elimination

Inference : Example – Theorem Proving

Learnt infmm

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$
$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$
$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$
$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$
$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan}$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan}$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

Query: $\neg P_{1,2}$. Can we prove if this sentence be entailed from KB using inference rules?________________

(Assumed fact) n (Q) → KB

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ — Biconditional Elimination

$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ — And Elimination

$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ — Contraposition

$R_8 : (\neg B_{1,1} \Rightarrow \neg (P_{1,2} \vee P_{2,1}))$ — Modus Ponens

$R_9 : \neg (P_{1,2} \vee P_{2,1})$ — Demorgans

$R_{10} : \neg P_{1,2} \wedge \neg P_{2,1}$ — And Elimination

$R11: \neg P_{1,2}$

derived facts

→ R12 : P_{12}

$\neg(\neg P_{12}) \Rightarrow P_{12}$ → assumed fact

Propositional Logic

Proof by Contradiction

Derived fact

$R_{11}: \neg P_{12}$ $\quad$ $R_{12}: P_{12}$

$$\neg P_{12} \wedge P_{12} \equiv \text{False}$$

Inference

Assumed fact

Inference = FALSE

↓

Assumption is False

↓

Given Query is TRUE

Contradiction

False.

$\cancel{P_{12}}$

$\neg P_{12}$

Inference = TRUE

↓

Assumption is TRUE

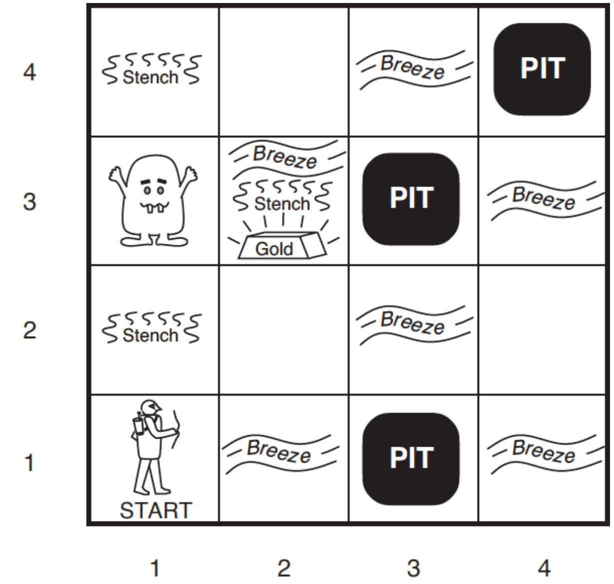
↓

Given Query is FALSE

PL-Resolution

Horn Clause

1. Definite Clause : A horn clause with exactly one positive literal
2. Fact : Definite clause with no negative literal / assertion
3. Rule
4. Inference by Chaining



Modus ponens & contraposition

⇓

Disjunctive syllogism

$R_1 - - - - R_5 \Rightarrow$ CNF Resolution

conjunction of disjunction

PL-Resolution : CNF conversion

Wumpus world Book example

exactly 2 laws

KB

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

$c_1 \quad c_2 \quad c_3$

$(A \vee \neg B) \wedge (A \vee B \vee \neg C) \wedge (\neg A)$ ✓

Eg:- clause → c_1 $\quad c_2 \quad c_3$

Conjunctive Normal Form :

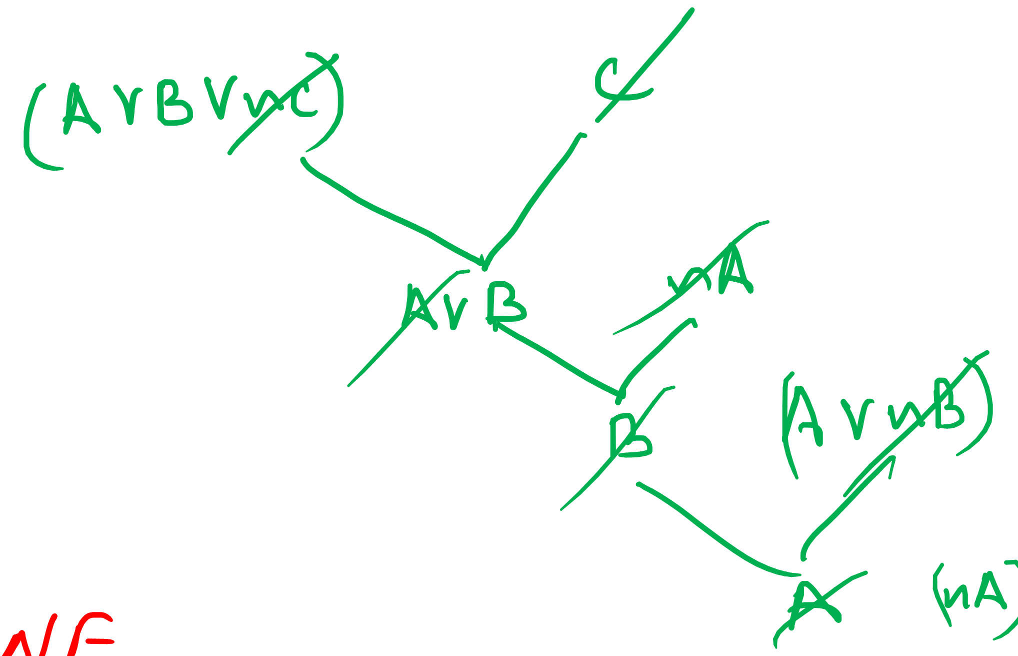
(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is 'C' true?

A B (disj) conj (disj) → CNF

Q : ¬C → Negate the Query ¬(¬C) → C



empty set { } ⇒ FALSE

PL-Resolution : CNF conversion

Wumpus world Book example

KB

$R_1 : \neg P_{1,1}$

$$\boxed{\begin{array}{l} \boxed{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})} \\ R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1}) \end{array}}$$

CNF conversion

H/W

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Conjunctive Normal Form :

(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is 'C' true?

Propositional Logic

Proof by Contradiction

Inference = FALSE ✓

↓

Assumption is False

↓

Given Query is TRUE

nc

Inference = TRUE

↓

Assumption is TRUE

↓

Given Query is FALSE

PL-Resolution : CNF conversion

Wumpus world Book example

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Conjunctive Normal Form :

(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is 'C' true?

PL-Resolution

$\neg B_{11}$

KB

- $R_1 : \neg P_{1,1}$
- ~~$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$~~
- ~~$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$~~
- $R_4 : \neg B_{1,1}$
- $R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

R_{13}: $\neg(\neg P_{12})$ P_{12}

- $R_6 : \neg B_{1,1} \vee P_{1,2} \vee P_{2,1}$
- $R_7 : \neg P_{1,2} \vee B_{1,1}$
- $R_8 : \neg P_{2,1} \vee B_{1,1}$
- $R_9 : \neg B_{2,1} \vee P_{1,1} \vee P_{2,2} \vee P_{3,1}$
- $R_{10} : \neg P_{1,1} \vee B_{2,1}$
- $R_{11} : \neg P_{2,2} \vee B_{2,1}$
- $R_{12} : \neg P_{3,1} \vee B_{2,1}$

X

derived fact

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$
$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$
$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$
$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$
$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan}$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan}$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

R2 R3

Eliminate		$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$	$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
$\leftrightarrow$	Biconditional Elimination	a → b, b → a: $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$	$(B_{2,1} \Rightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})) \wedge ((P_{1,1} \vee P_{2,2} \vee P_{3,1}) \Rightarrow B_{2,1})$
$\rightarrow$	Implication Elimination	R6: $\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})$ S1 $\Rightarrow \neg B_{11} \vee P_{12} \vee P_{21}$ R6 $\neg(\mathbf{P_{1,2} \vee P_{2,1}}) \vee \mathbf{B_{1,1}}$ S2	$\neg B_{2,1} \vee (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ $\neg(\mathbf{P_{1,1} \vee P_{2,2} \vee P_{3,1}}) \vee \mathbf{B_{2,1}}$
Clause level $\neg$	De Morgan	$(\mathbf{\neg P_{1,2} \wedge \neg P_{2,1}}) \vee \mathbf{B_{1,1}}$ = con ↳ S3	$(\mathbf{\neg P_{1,1} \wedge \neg P_{2,2} \wedge \neg P_{3,1}}) \vee \mathbf{B_{2,1}}$
CNF Form	Distributivity of $\vee$ over $\wedge$	$(\mathbf{\neg P_{1,2} \vee B_{1,1}}) \wedge (\mathbf{\neg P_{2,1} \vee B_{1,1}})$ → CNF dis con dis True R7 True R8	$(\mathbf{\neg P_{1,1} \vee B_{2,1}}) \wedge (\mathbf{\neg P_{2,2} \vee B_{2,1}}) \wedge (\mathbf{\neg P_{3,1} \vee B_{2,1}})$

PL-Resolution

(Clause) unit Literal

Unit Resolution: Query: $\neg P_{1,2}$

$C_1 \rightarrow UL$

To find: Is there a pit in location (1,2) using the CNF obtained in previous slide

Inference = FALSE ✓

Assumption is False ✓

Given Query is TRUE

True

$\neg P_{12}$

R4: $\neg \cancel{B_{1,1}}$

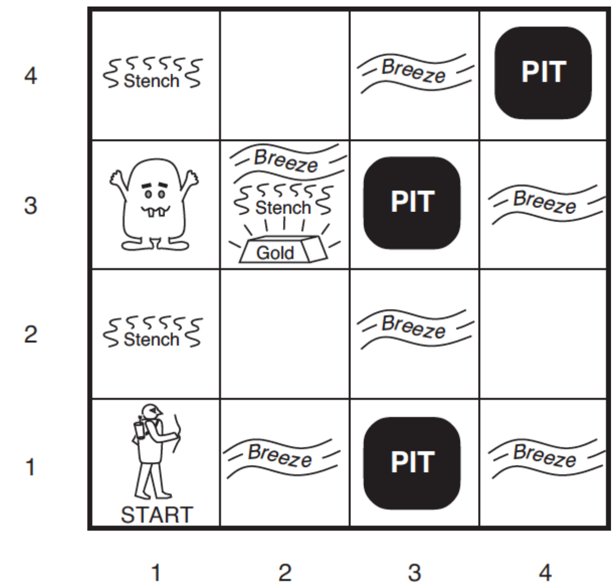
R7: $\neg P_{1,2} \vee \cancel{B_{1,1}}$

$\cancel{\neg P_{1,2}}$

R13: $\cancel{P_{1,2}}$

{ }

{ }



4th

DPLL Algorithm

True

Q + KB

still consistent one

In logic and computer science, the Davis–Putnam–Logemann–Loveland (**DPLL**) **algorithm** is a complete, backtracking-based search **algorithm** for deciding the satisfiability of propositional logic formulae in conjunctive normal form

DFS + backtracking

Improvements:

1. Early Termination
2. Pure Symbolic Heuristic
3. Unit Clause Heuristic

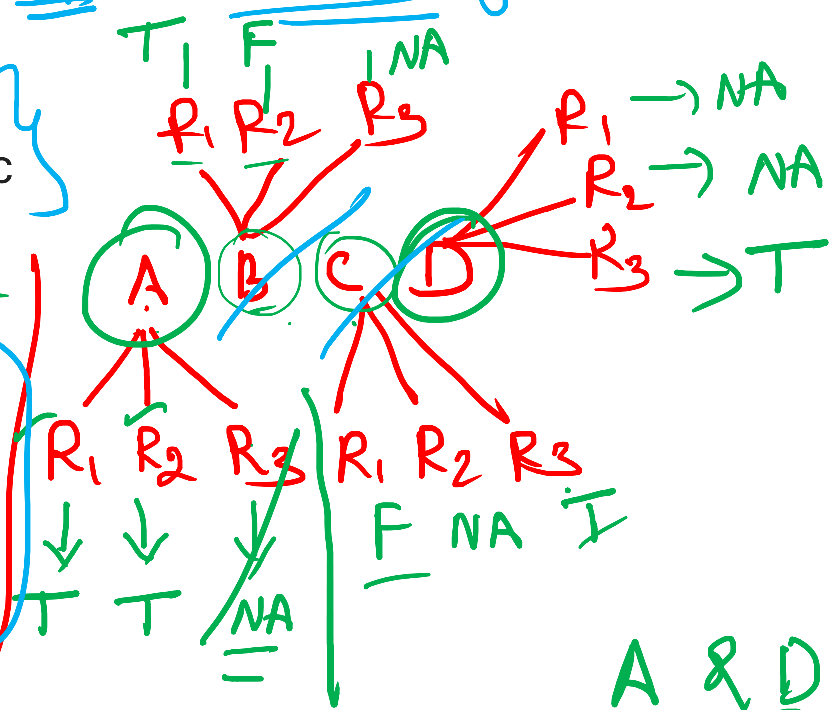
eg:- KB

$R_1 : A \vee B \vee \neg C$

$R_2 : \neg B \vee A$

$R_3 : D \vee C$

True



A → T

D → T

A & D → Pure symbol

True as

DPLL Algorithm

In logic and computer science, the Davis–Putnam–Logemann–Loveland (**DPLL**) **algorithm** is a complete, backtracking-based search **algorithm** for deciding the satisfiability of propositional logic formulae in conjunctive normal form

Improvements:

1. Early Termination
2. Pure Symbolic Heuristic
3. Unit Clause Heuristic

DPLL Algorithm

P_{12}, B_{11}, P_{21}

Randomly Pick

3

$2^3 \to 8$

KB + Q

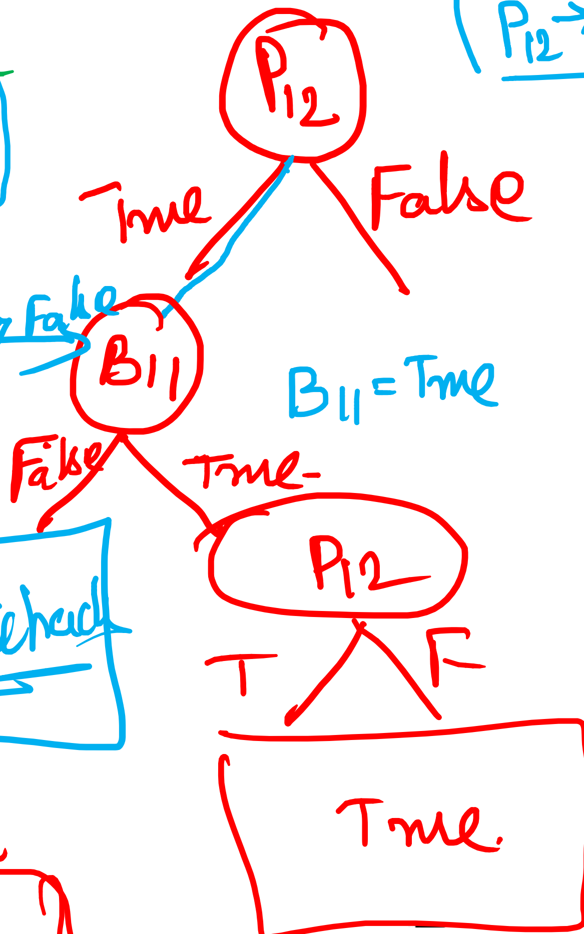
$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$\{P_{1,2}, B_{1,1}, P_{2,1}\}$

$P_{12} \to$ True

P_{12} B_{11}



KB $P_{12} \to$ False

$R_7 : \cancel{\neg} P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \cancel{\neg P_{1,2}} \vee \cancel{B_{1,1}} \to$ False

$R_8 : \neg P_{2,1} \vee B_{1,1}$

KB

P_{12}	B_{11}	P_{12}
True	True	T
T	T	False

KB True $\to$ Consistent

$R_7 : \cancel{\neg P_{1,2}} \vee B_{1,1}$ (T) $\to$ True

$R_8 : \neg P_{2,1} \vee B_{1,1}$ (T) $\to$ True

T F

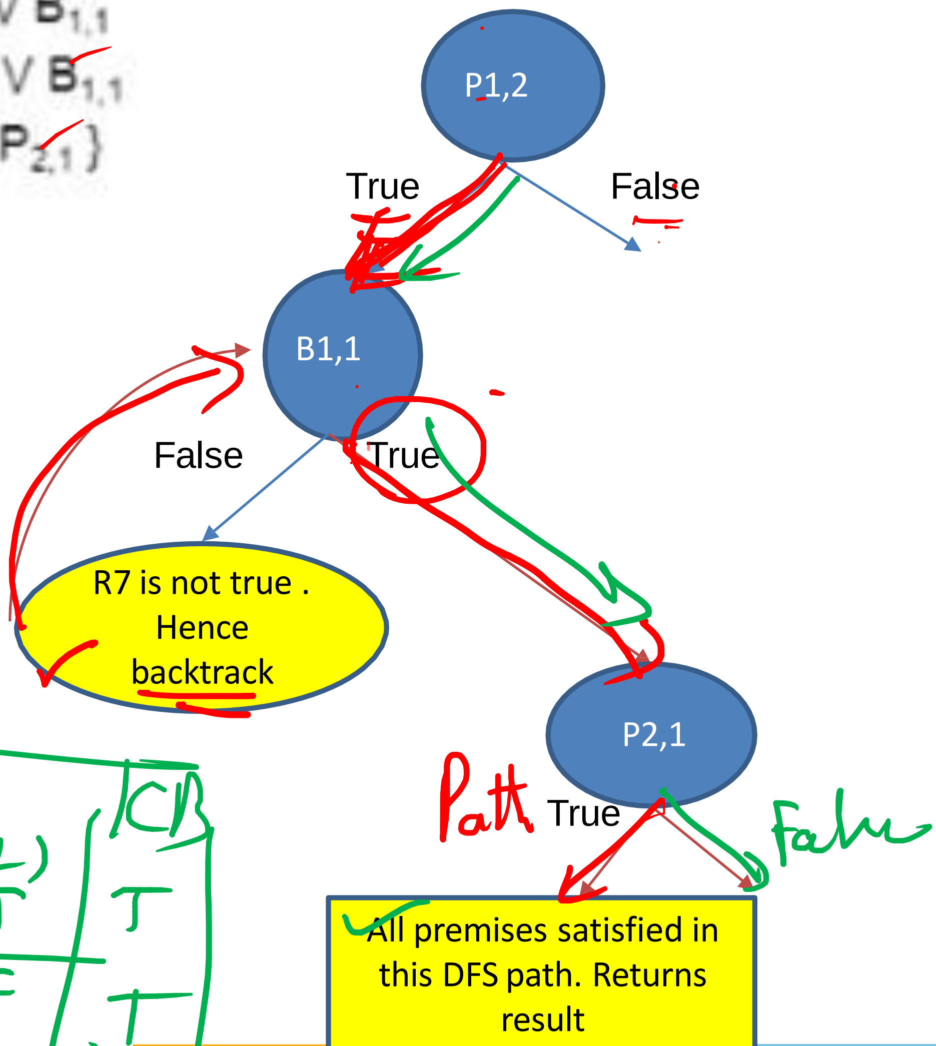
DPLL Algorithm

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$\{P_{1,2}, B_{1,1}, P_{2,1}\}$

P_{12}	B_{11}
T	F



P_{12}	B_{11}	P_{21}	CB
T	T	T	T
T	T	F	T

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1} \rightarrow$ Fal

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$\rightarrow R_7 : \neg P_{1,2} \vee B_{1,1}$ True

$R_8 : \neg P_{2,1} \vee B_{1,1} \rightarrow$ Tru

F

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1, #9

Next Session Plan:

- (Prerequisite Reading : Refresh the basics of probability , Bayes Theorem , Conditional Probability, Product Rule, Conditional Independence, Chain Rule)

- Inferences using Logic (Forward / Backward Chaining / DPLL algorithm)
- Bayesian Network
- Representation
- Inferences (Exact and approximate-only Direct sampling)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials