



Artificial & Computational Intelligence

AIMLCZG57

M5 : Probabilistic Representation and Reasoning

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Course Plan

- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI

Probabilistic Representation and Reasoning

A. Inference using full joint distribution

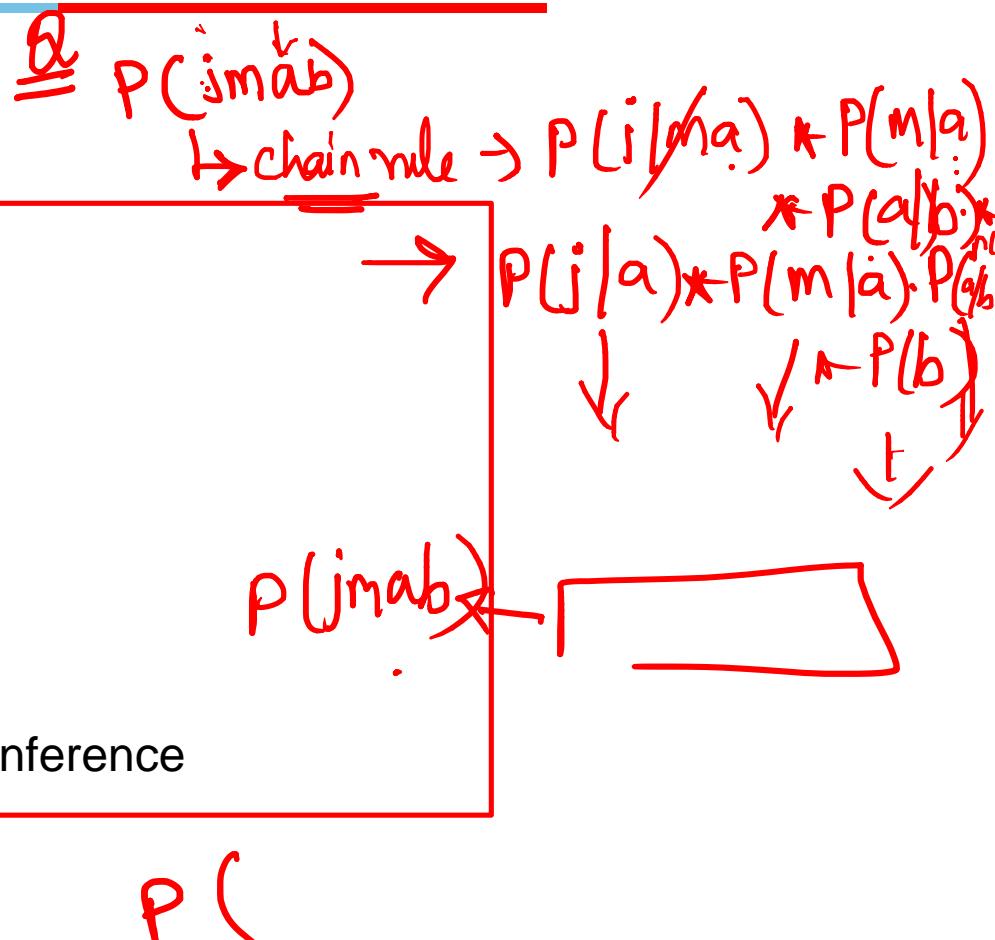
B. Bayesian Networks

I. Knowledge Representation

II. Conditional Independence

III. Exact Inference

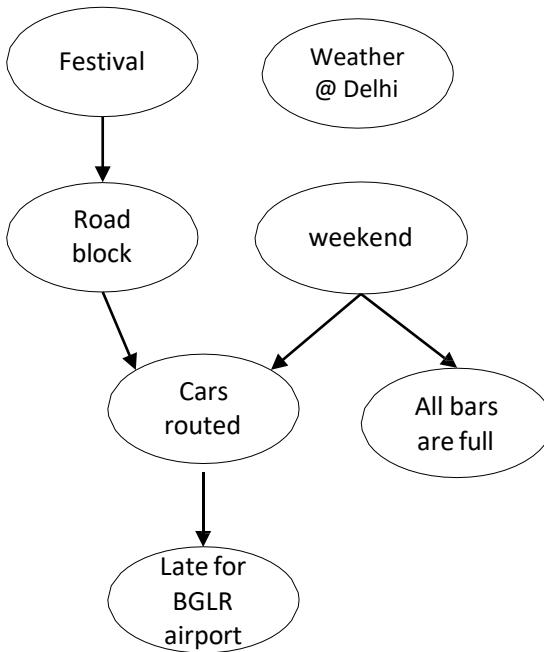
IV. Introduction to Approximate Inference



Inferences in Bayesian Nets

Enumeration

Examples



$$\underline{P(L | W \cap R \cap F \cap C)}$$

1. Calculate the probability that arrival at airport was delayed during a weekend but there was no road block or festival and car was not routed anywhere.

2. What is the probability that it is a festival season given cars where routed? $P(F|c)$ } cond'n prob

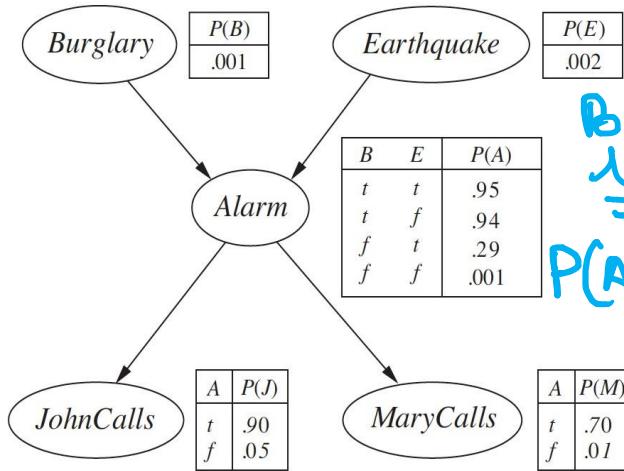
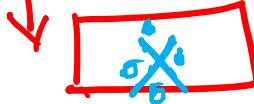
3. What is the probability that car arrived late at airport given it's a festival day? $P(L|F)$

Examples

Q1 $P(B|JM)$ → Joint prob dist'n
Line → Chain

2. What is the probability that Burglary happened given John & Mary called the police

$$= P(A) + P(\neg A) = 1$$



$$\hookrightarrow \frac{P(B|JM)}{P(JM)} + \frac{P(\neg B|JM)}{P(JM)} = 1$$

$$P(AB) = \frac{P(ANB)}{P(B)}$$

$$\Rightarrow \frac{P(AB)}{P(B)}$$

$$P(B|JM) = \frac{P(B, JM)}{P(J, M)}$$

$$P(B|JM) = \frac{\sum_{A, E} P(J, M, A, B, E)}{\sum_{A, B, E} P(J, M, A, B, E)}$$

$$\boxed{P(JM) = P(BJM) + P(\neg BJM)}$$

let

$$\alpha = \frac{1}{P(JM)}$$

$$\alpha = \frac{1}{P(BJM) + P(\neg BJM)} \rightarrow ①$$

$$P(B|JM) \Rightarrow \frac{P(BJM)}{P(JM)} \Rightarrow \frac{P(AB)}{P(B)}$$

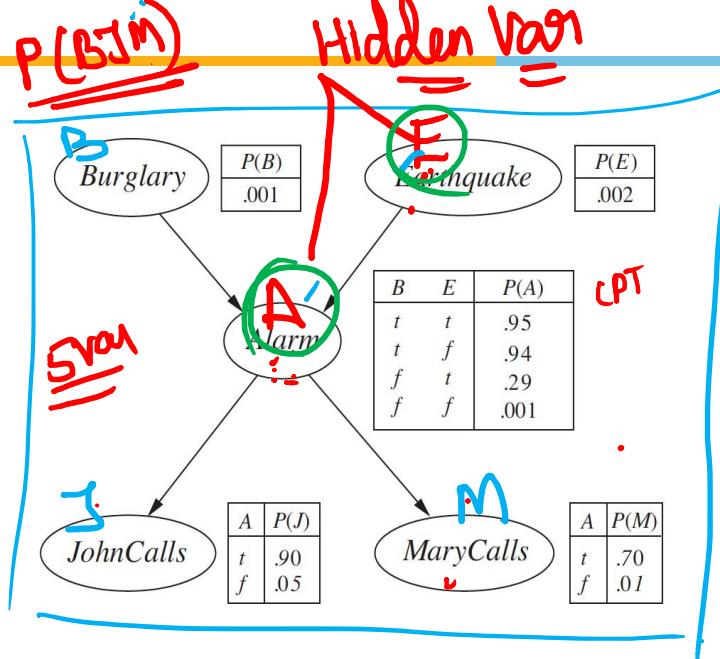
$$\Rightarrow \frac{P(BJM)}{P(BJM) + P(\neg BJM)} \rightarrow ①$$

$$P(BJM) \Rightarrow 0.5 \Rightarrow \\ P(\neg BJM) \Rightarrow 0.5$$

Examples

~~$P(BJM)$~~ $\Rightarrow \exists \forall \alpha$ ~~$P(\neg BJM)$~~ $\Rightarrow P(BJM) \Rightarrow P(jm@b@)$

2. What is the probability that Burglary happened given John & Mary called the police



$$P(B | JM) = \frac{P(B, JM)}{P(J, M)}$$

$$P(B | JM) = \frac{\sum_{A, E} P(J, M, A, B, E)}{\sum_{A, B, E} P(J, M, A, B, E)}$$



addit S

$$v_1 + v_2 + v_3 + v_4 \quad P(BJM) \Rightarrow v_1 + v_2 + v_3 + v_4 \Rightarrow \text{Marginalization}$$

$$P(B | JM) + P(\neg B | JM) = 1$$

$$\frac{P(BJM)}{P(JM)} + \frac{P(\neg BJM)}{P(JM)} = 1$$

$$\frac{1}{P(JM)} [P(BJM) + P(\neg BJM)] = 1$$

E	A	$P(BJM)$
T	T	- +
T	F	- +
F	T	- +
F	F	- -

$$\text{let } \alpha = \frac{1}{P(JM)}$$

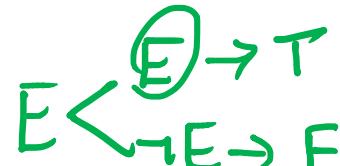
$$\alpha = \frac{1}{P(BJM) + P(\neg BJM)} \rightarrow ①$$

$$P(B|JM) = \sum_{A,E} P(J, M, A, B, E)$$

$$\sum_{AE} P(J|MABE) * P(M|ABE) * P(A|BE) * P(B|E) * P(E)$$

$$\sum_{AE} P(J|A) * P(M|A) * P(A|BE) * P(B) * P(E)$$

\sum_{AE} → Remove / Expand the summation



$$\sum_A P(J|A) * P(M|A) * P(A|BE) * P(B) * P(E) +$$

$$P(J|A) * P(M|A) * P(A|B\bar{E}) * P(B) * P(\bar{E}) \rightarrow A \dashv A$$

$$(P(J|A) * P(M|A) * P(A|BE) * P(B) * P(E) + P(J|\bar{A}) * P(M|\bar{A}) * P(\bar{A}|BE) * P(B) * P(E) + \dots)$$

$$P(J|A) * P(M|A) * P(A|B\bar{E}) * P(B) * P(\bar{E}) +$$

$$P(J|\bar{A}) * P(M|\bar{A}) * P(\bar{A}|B\bar{E}) * P(B) * P(\bar{E}) + \dots$$

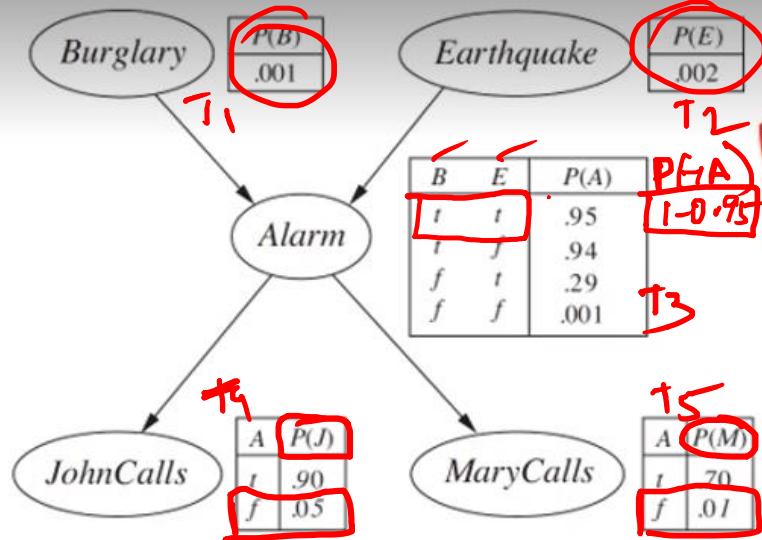
(4)

$$P(J|A) * P(M|A) * P(A|B\bar{E}) * P(B) * P(\bar{E}) + P(J|\bar{A}) * P(M|\bar{A}) * P(\bar{A}|B\bar{E}) * P(B) * P(\bar{E})$$

$$P(\bar{J}|B\bar{E}M)$$

$$P(B\bar{J}M) = \sqrt{ }$$

$$P(B|JM) = \frac{P(B|M)}{P(B|J)*P(B|M)}$$



$$\begin{aligned}
 P(B|J,M) &= \frac{P(J,M,A,B,E)}{\sum_{A,B,E} P(J,M,A,B,E)} \\
 &= \frac{\sum_{A,E} P(J,M,A,B,E)}{\sum_{A,B,E} P(J,M,A,B,E)} \\
 &= \frac{\sum_{A,E} P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)}{\sum_{A,B,E} P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} \\
 &= \frac{\sum_{A,E} P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)}{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} + \frac{\sum_{A,E} P(J|A) \cdot P(M|A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)}{P(J|A) \cdot P(M|A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)} \\
 &= \frac{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)}{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} + \frac{P(J|A) \cdot P(M|A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)}{P(J|A) \cdot P(M|A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)} \\
 &= \frac{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)}{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} + \frac{P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)}{P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)}
 \end{aligned}$$

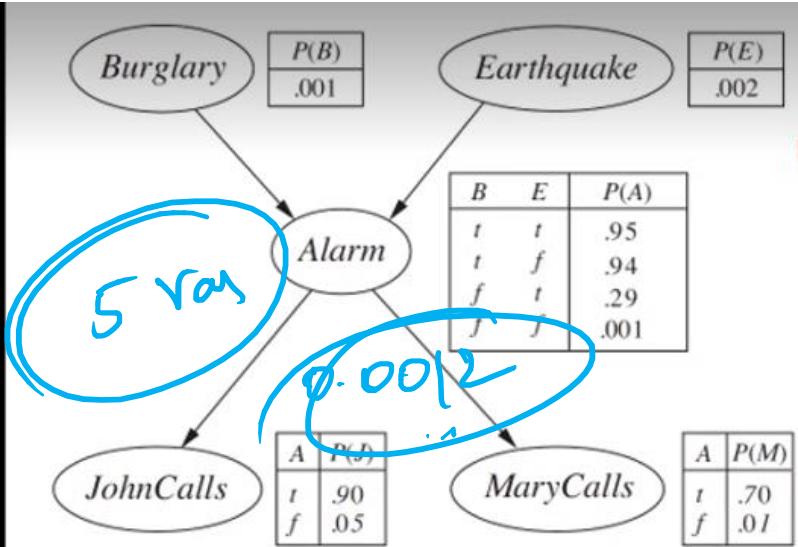
2. What is the probability that Burglary happened given John & Mary called the police

$$P(B|J,M) = \frac{P(B, J, M)}{P(J, M)}$$

$$P(B|J,M) = \frac{\sum_{A,E} P(J, M, A, B, E)}{\sum_{A,B,E} P(J, M, A, B, E)}$$

$$\underline{P(B|J,M)} \rightarrow \underline{P(\neg B|J,M)}$$

$$\begin{aligned}
 &= \frac{[P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)] + [P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)]}{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} \\
 &= \frac{[P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)] + [P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B,\neg E) \cdot P(\neg B) \cdot P(\neg E)]}{P(J|A) \cdot P(M|A) \cdot P(A|B,E) \cdot P(B) \cdot P(E)} \\
 &= \frac{0.90 \cdot 0.05 \cdot 0.01 \cdot 0.001 + 0.05 \cdot 0.01 \cdot 0.002}{0.90 \cdot 0.05 \cdot 0.01 \cdot 0.001 + 0.05 \cdot 0.01 \cdot 0.002} \Rightarrow \boxed{0.001}
 \end{aligned}$$



2. What is the probability that Burglary happened given John & Mary called the police

$$P(B | J, M) = \frac{P(B, J, M)}{P(J, M)}$$

$$P(B | J, M) = \frac{\sum_{A, E} P(J, M, A, B, E)}{\sum_{A, B, E} P(J, M, A, B, E)}$$

$$\cancel{P(B | J, M) + P(\neg B | J, M)}$$

$$= \cancel{\left[P(J|A) \cdot P(M|A) \cdot P(A|B, E) \cdot P(B) \cdot P(E) \right]} + \cancel{\left[P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B, E) \cdot P(B) \cdot P(E) \right]} + \cancel{\left[P(J|A) \cdot P(M|A) \cdot P(A|\neg B, E) \cdot P(B) \cdot P(\neg E) \right]} + \cancel{\left[P(J|\neg A) \cdot P(M|\neg A) \cdot P(A|\neg B, E) \cdot P(B) \cdot P(\neg E) \right]}$$

$$\begin{aligned}
 P(B | J, M) &= \sum_{A, E} P(J | M, A, B, E) \cdot P(M | A, B, E) \\
 &= \sum_{A, E} P(J | A) \cdot P(M | A) \cdot P(A | B, E) \cdot P(B) \cdot P(E) \\
 &= \sum_{A, E} \{ P(J | A) \cdot P(M | A) \cdot P(A | B, E) \cdot P(B) \cdot P(E) \}
 \end{aligned}$$

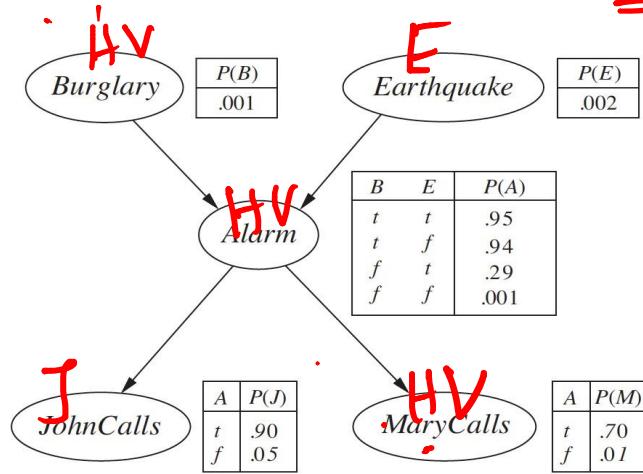
$$\begin{aligned}
 &= \sum_A \{ P(J | A) \cdot P(M | A) \cdot P(A | B, E) \cdot P(B) \cdot P(E) \} \\
 &\quad + \sum_A \{ P(J | \neg A) \cdot P(M | \neg A) \cdot P(A | \neg B, E) \cdot P(B) \cdot P(E) \} \\
 &\quad + \sum_A \{ P(J | A) \cdot P(M | A) \cdot P(A | \neg B, E) \cdot P(B) \cdot P(\neg E) \} \\
 &\quad + \sum_A \{ P(J | \neg A) \cdot P(M | \neg A) \cdot P(A | \neg B, E) \cdot P(B) \cdot P(\neg E) \}
 \end{aligned}$$

Examples

H|W

$$P(J|E) \Rightarrow \text{Bayes rule} \Rightarrow \frac{P(J|E)}{P(E)}$$

3. What is the probability that John calls given earthquake occurred?



$$\frac{P(J|E)}{P(J|E) + P(\neg J|E)}$$

$$\sum_{MAB} P(J|MAB)$$

$$P(J|\neg A) \rightarrow P(J|a)$$

$$P(J|E) = \frac{P(J, E)}{P(E)}$$

$$P(J|E) = \frac{\sum_{M, A, B} P(J, M, A, B, E)}{\sum_{J, M, A, B} P(J, M, A, B, E)}$$

$$P(J|E) + P(\neg J|E) = 1$$

$$\frac{P(J|E)}{P(E)} + \frac{P(\neg J|E)}{P(E)} = 1$$

$$\frac{1}{P(E)} [P(J|E) + P(\neg J|E)] = 1$$

$$P(J|E) + P(\neg J|E) = P(E)$$

Inferences in Bayesian Nets

Variable Elimination



Reduce Guaranteed Independent nodes

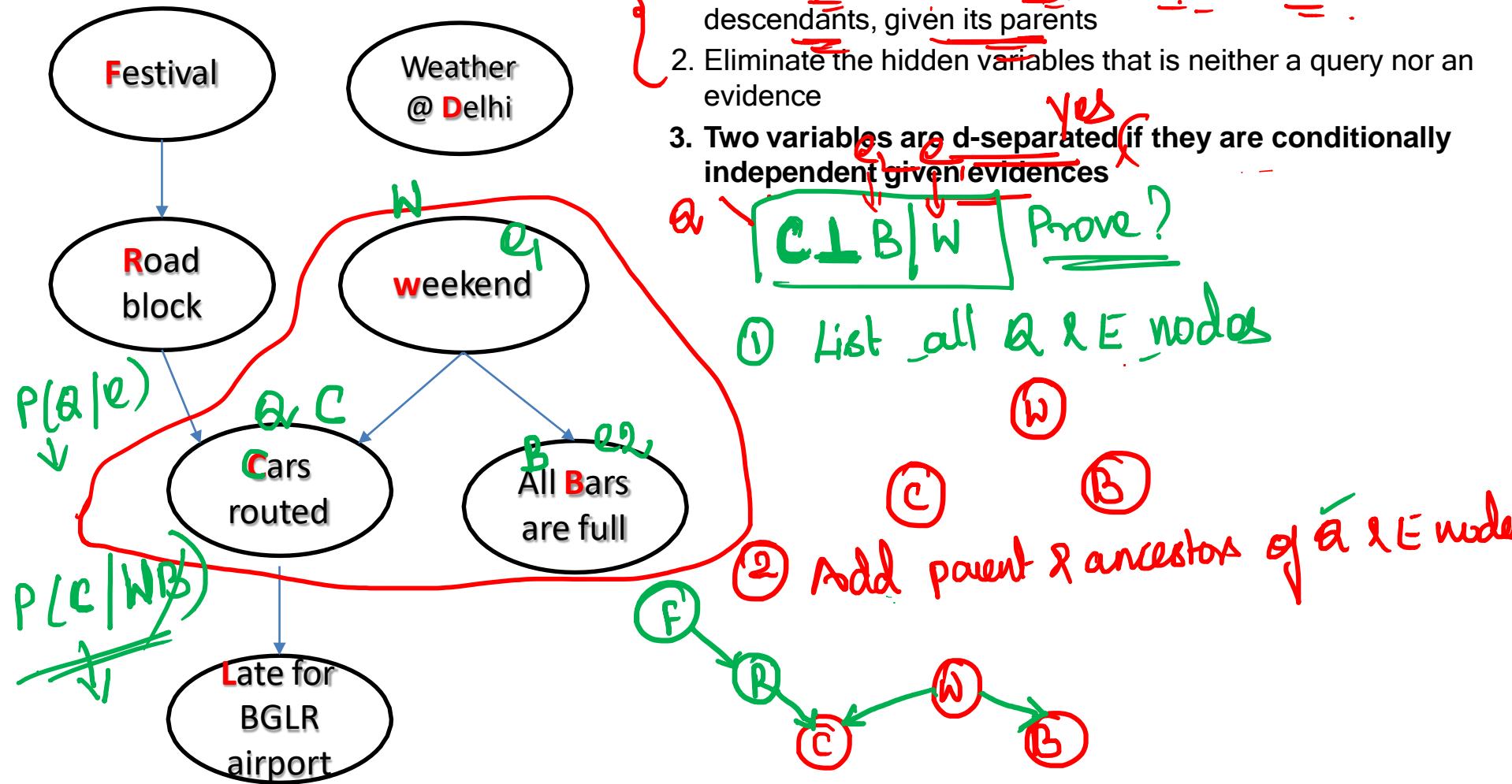
D-Connectedness Vs D-Separation

5 steps

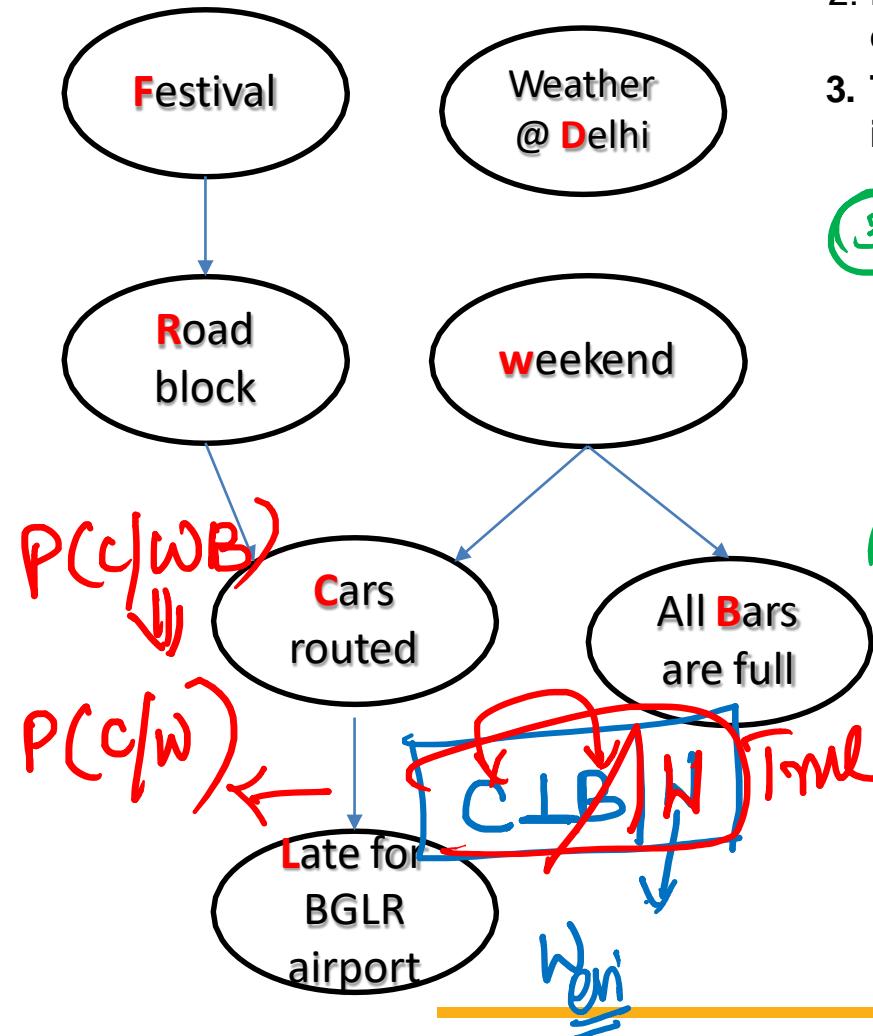
Alg

AI

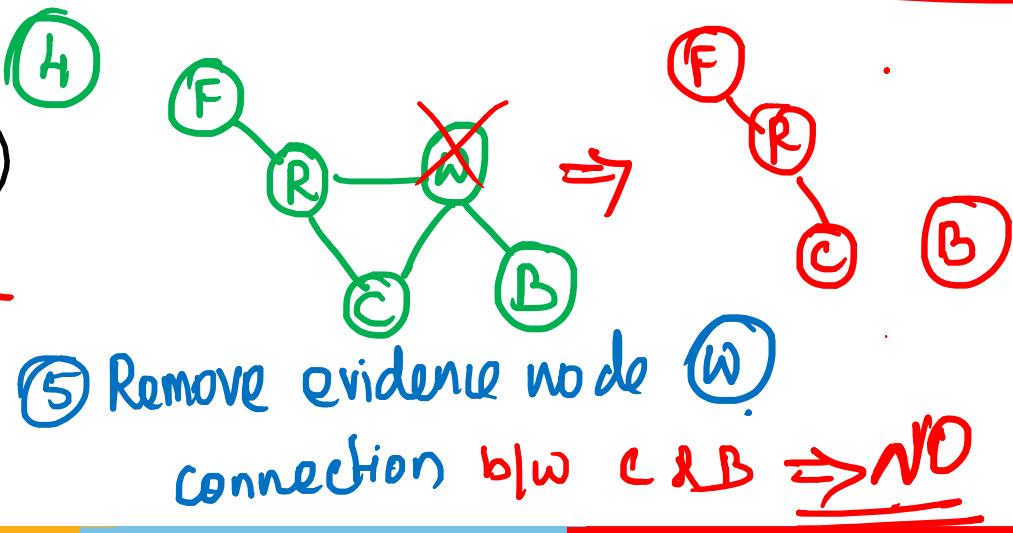
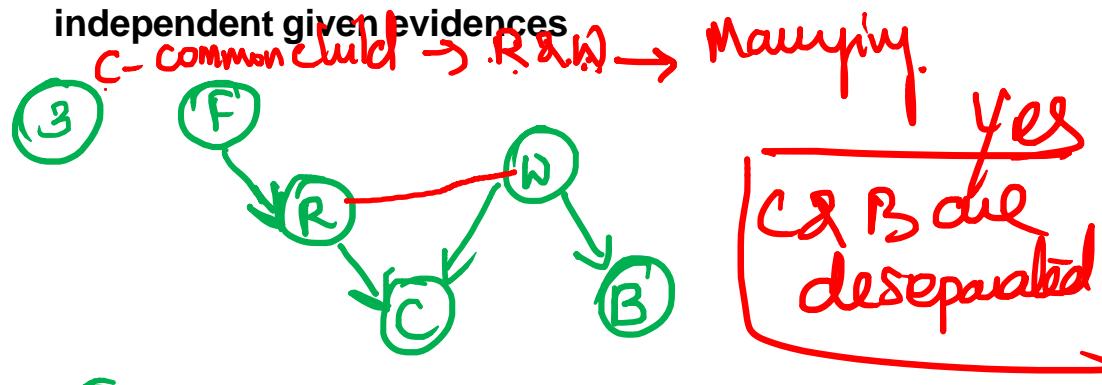
J/a



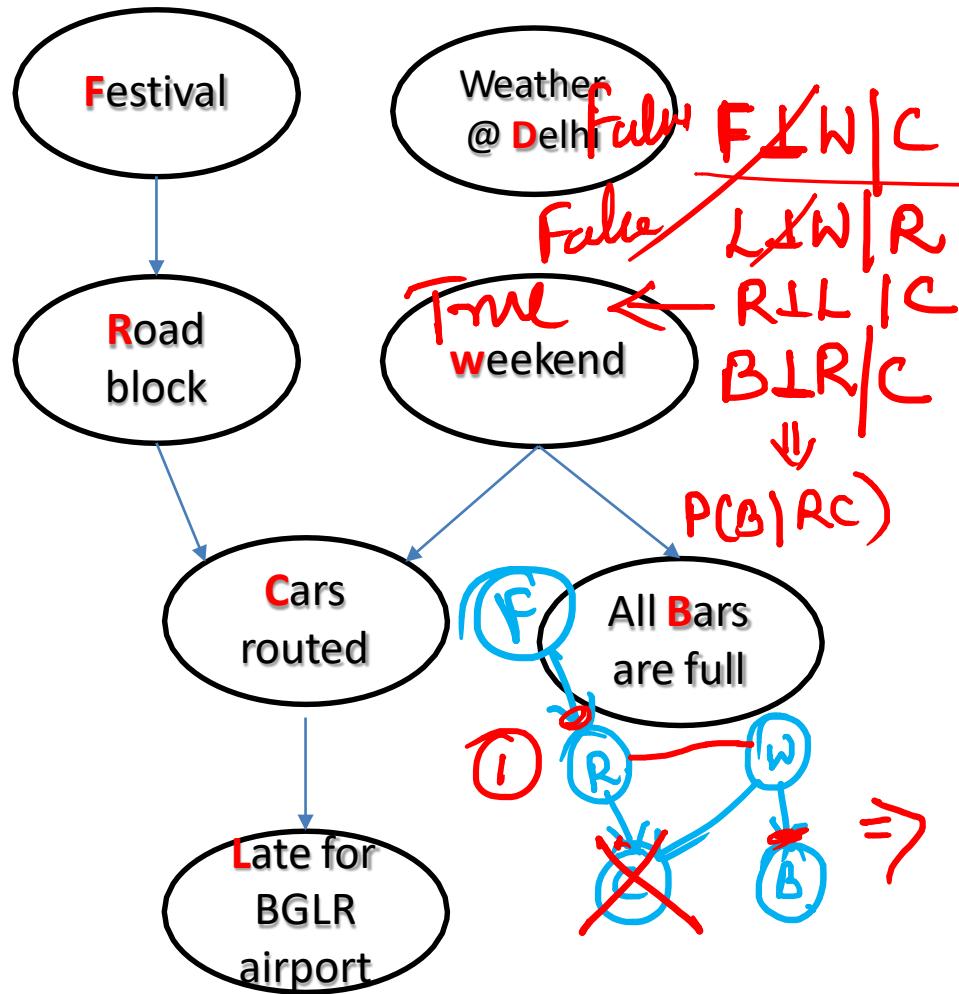
D-Connectedness Vs D-Separation



1. Each variable is conditionally independent of its non-descendants, given its parents
2. Eliminate the hidden variables that is neither a query nor an evidence
3. Two variables are d-separated if they are conditionally independent given evidences



Try it & Test



X	Y	Evidence Z	d-sep?
F	W	C ✓	No
L	W	R	No
R	L	C	Yes
B	R	C	No

$$\Rightarrow P(R | L, C) = P(R | L)$$

R & L are ~~dis~~-separated ie., conditionally independent given C

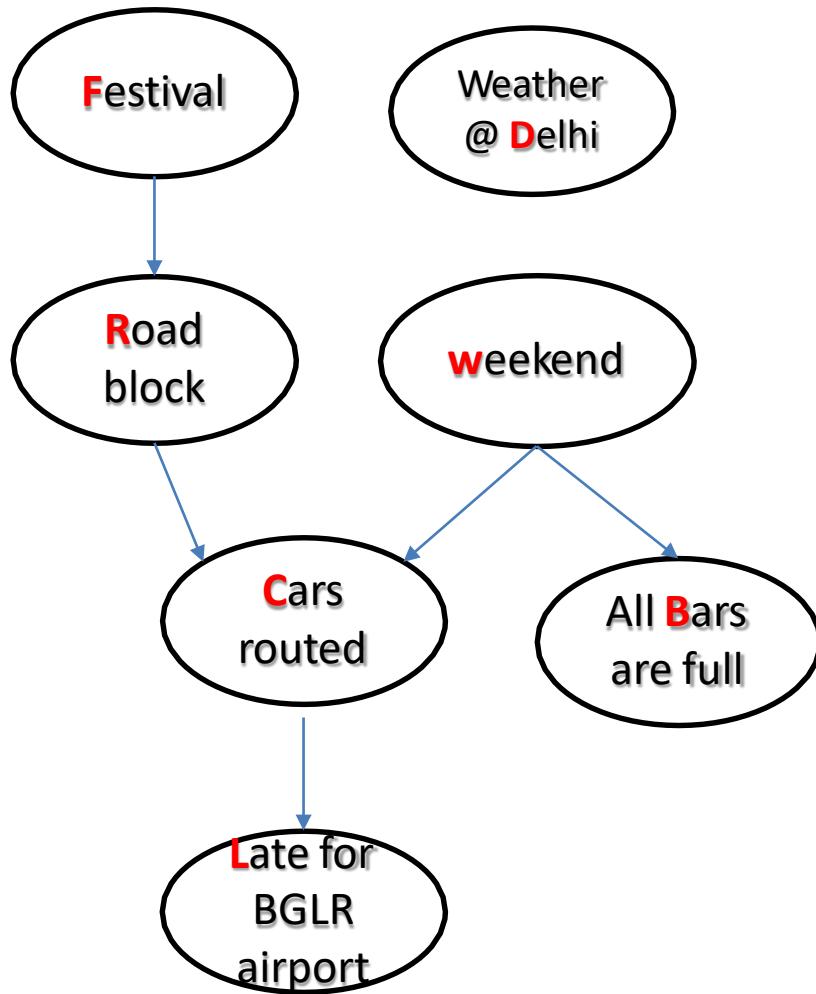
given C B L R I C

~~BXR/C~~

B & R are
not desperate

connect yes

Try it & Test

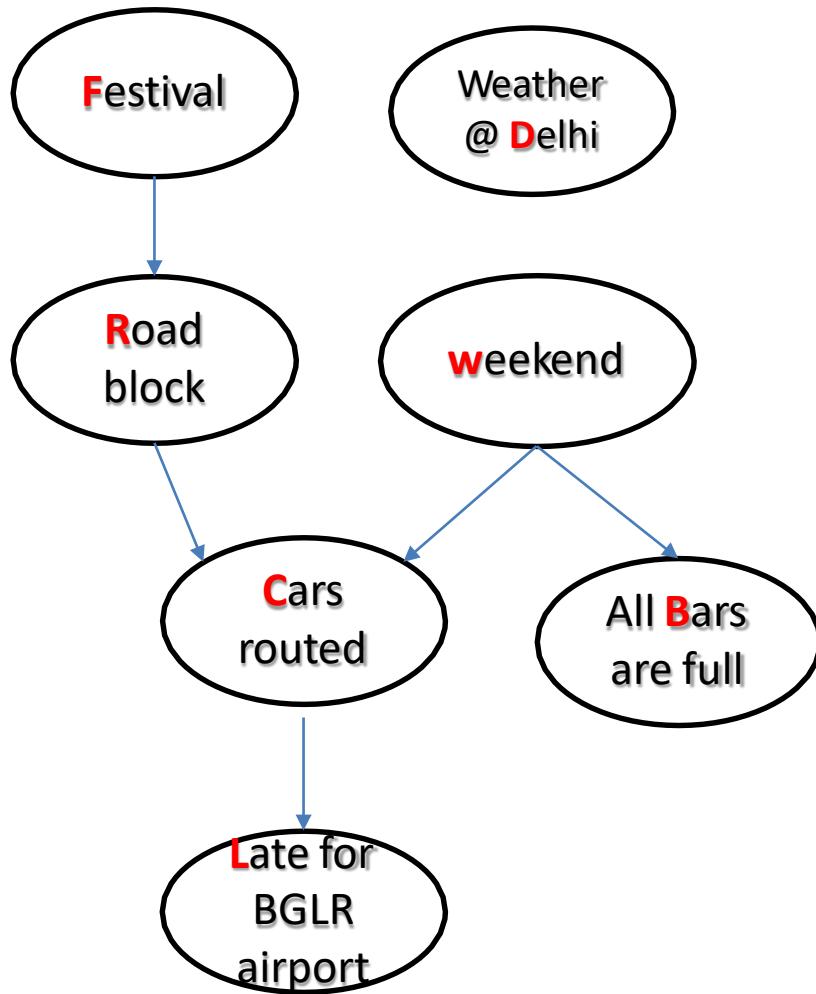


X	Y	Evidence Z	d-sep?
F	W	C	No
L	W	R	No
R	L	C	Yes
B	R	C	No

➤ $P(R | L, C) = P(R | L)$

R & L are d-separated ie., conditionally independent given C

Try it & Test

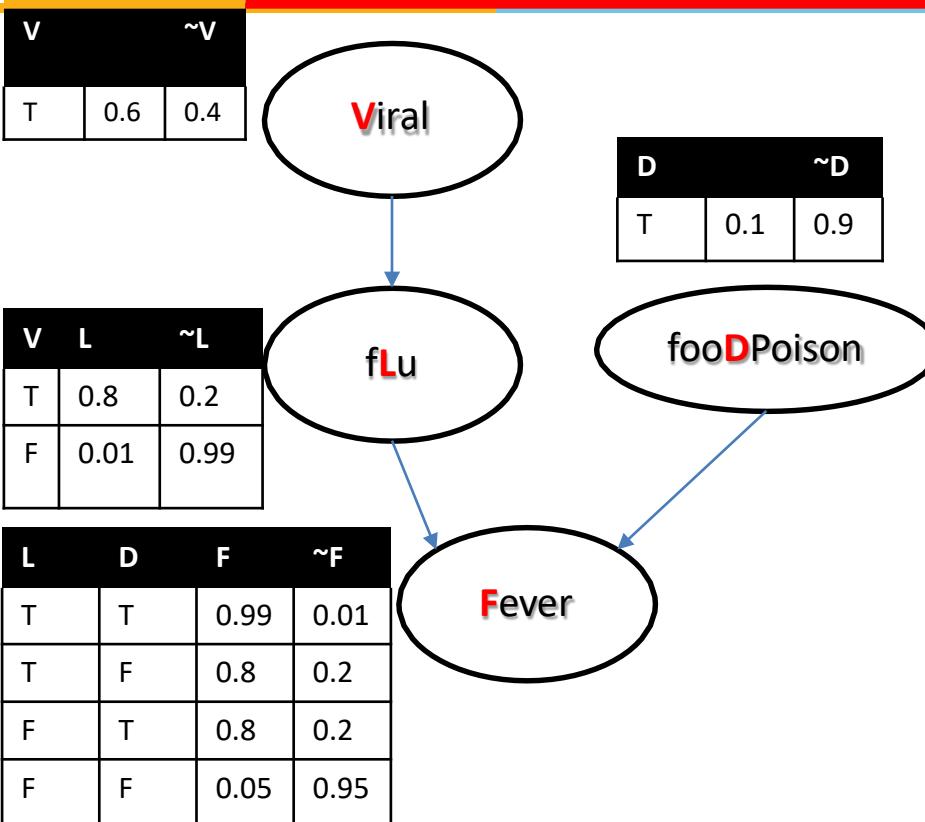


X	Y	Evidence Z	d-sep?
F	W	C	No
L	W	R	No
R	L	C	Yes
B	R	C	No

➤ $P(R | L, C) = P(R | L)$

R & L are d-separated ie., conditionally independent given C

D-Separation in Inference

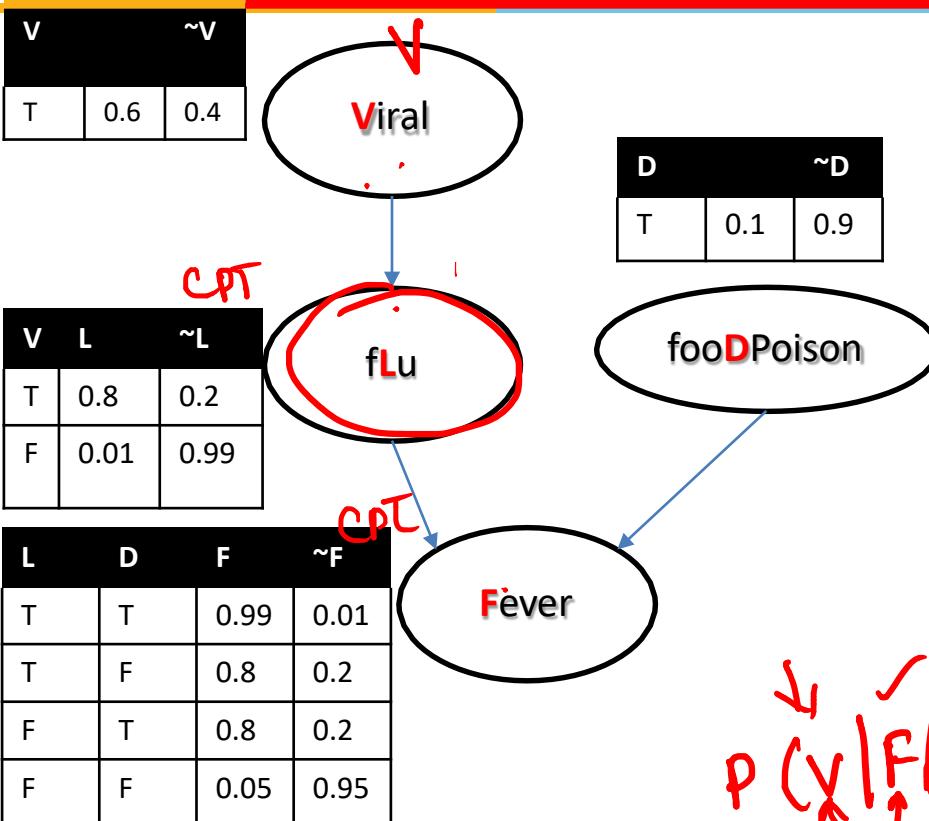


X	Y	Evidence Z	d-sep?
V	F	L	Yes
V	D	L	Yes

$P(V | F, L) \rightarrow V \perp F | L$,
 $P(V | D, L) \rightarrow V \perp D | L$

D-Separation in Inference

Enumeration

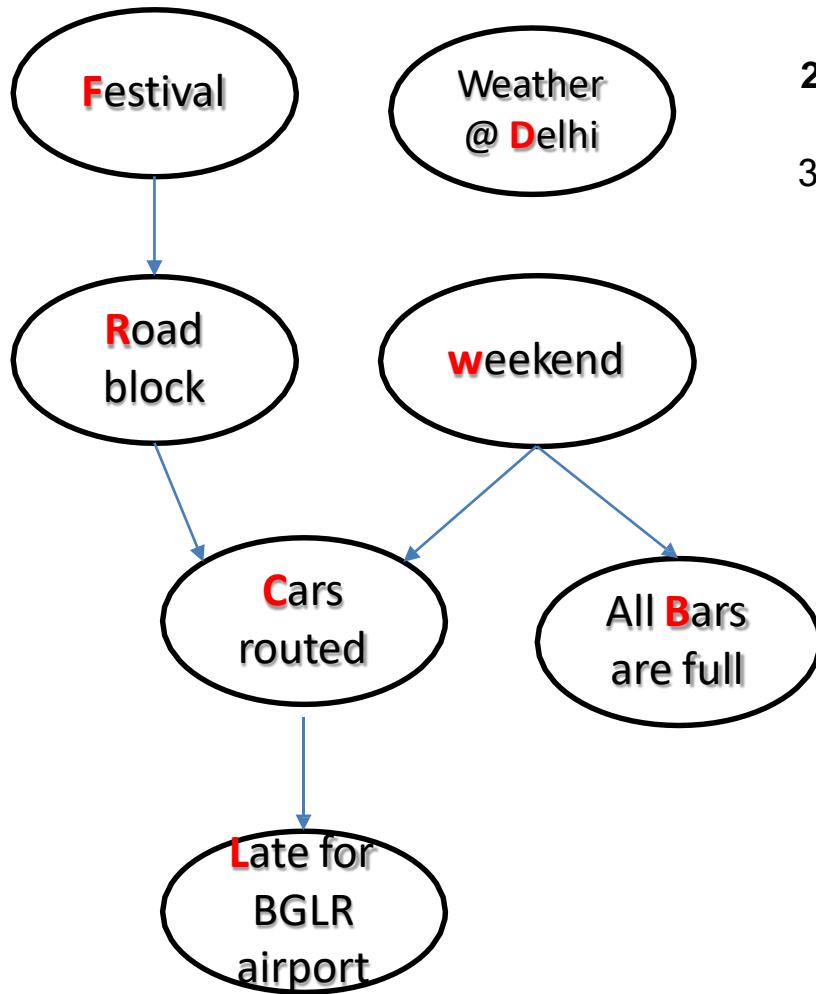


$P(X|FD)$

X	Y	Evidence Z	d-sep?
V	F	L	Yes
V	D	L	Yes

- $P(V | F, L)$
- $P(V | D, L)$

Variable Elimination



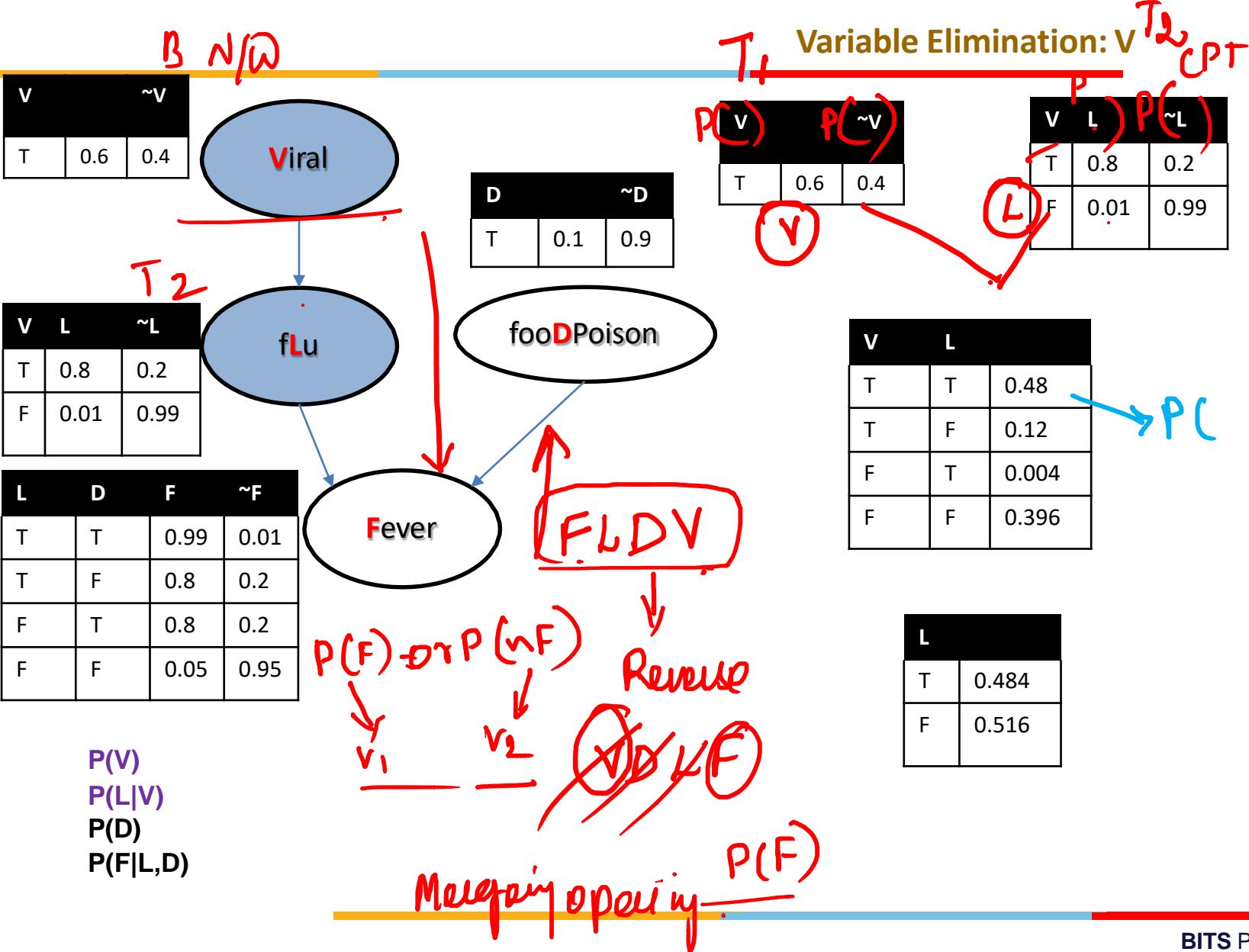
1. Each variable is conditionally independent of its non-descendants, given its parents
2. **Eliminate the hidden variables that is neither a query nor evidence**
3. Two variables are d-separated if they are conditionally independent given evidences

$$\begin{aligned}
 \mathbb{P}(B) &= \sum_{L, C, R, F} \mathbb{P}(L, C, B, W, R, F) \\
 &= \sum_L \sum_B \mathbb{P}(L|C) \cdot \mathbb{P}(B|W) \cdot \sum_W \mathbb{P}(C|W, R) \cdot \sum_R \mathbb{P}(R|F) \cdot \sum_F \mathbb{P}(F) \\
 &= \mathbb{P}(B|W)
 \end{aligned}$$

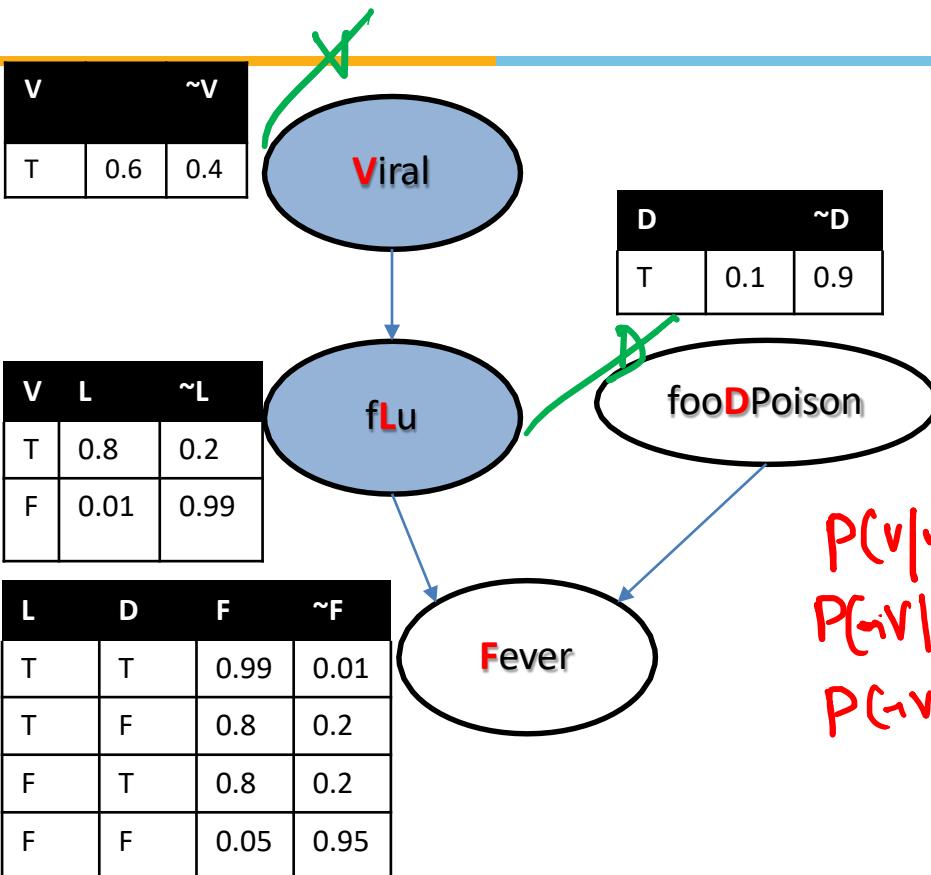
All other variables are hidden w.r.t to B as (L, C, R, F) are neither evidence nor query nor $(L, C, R, F) \in \text{Ancestors}(W, B)$

This is variable elimination example targeting irrelevant nodes

Inference



Inference



$P(V)$
 $P(L|V)$
 $P(D)$
 $P(F|L,D)$

Variable Elimination: V

$T \cap V$

$P(L)$

L

	V	$\sim V$
T	0.6	0.4

	V	L	$\sim L$
T	0.8	0.2	
F	0.01	0.99	

$$\begin{aligned}
 P(CVL) &\rightarrow P(v|L) \cdot P(\sim v) \\
 P(v|L) &\rightarrow 0.8 \times 0.6 \\
 P(\sim v|L) &\rightarrow 0.2 \times 0.6 \\
 P(v|\sim L) &\rightarrow 0.01 \times 0.4 \\
 P(\sim v|\sim L) &\rightarrow 0.99 \times 0.4
 \end{aligned}$$

	L
T	0.484
F	0.516

$$P(L) < T_F$$

Inference

V	$\sim V$
T	0.6
F	0.4



V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

$P(L)$
 $P(D)$
 $P(F|L,D)$

D	$\sim D$
T	0.1
F	0.9



Fever

$P(FLD)$

$P(F|LD)$

$P(LD)$

$P(D)$

$$0.99 \times 0.484 \times 0.1$$

T_1

Variable Elimination: L, D

L	$\sim L$
T	0.484
F	0.516

D	$\sim D$
T	0.1
F	0.9

L	D	$\sim F$	F
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

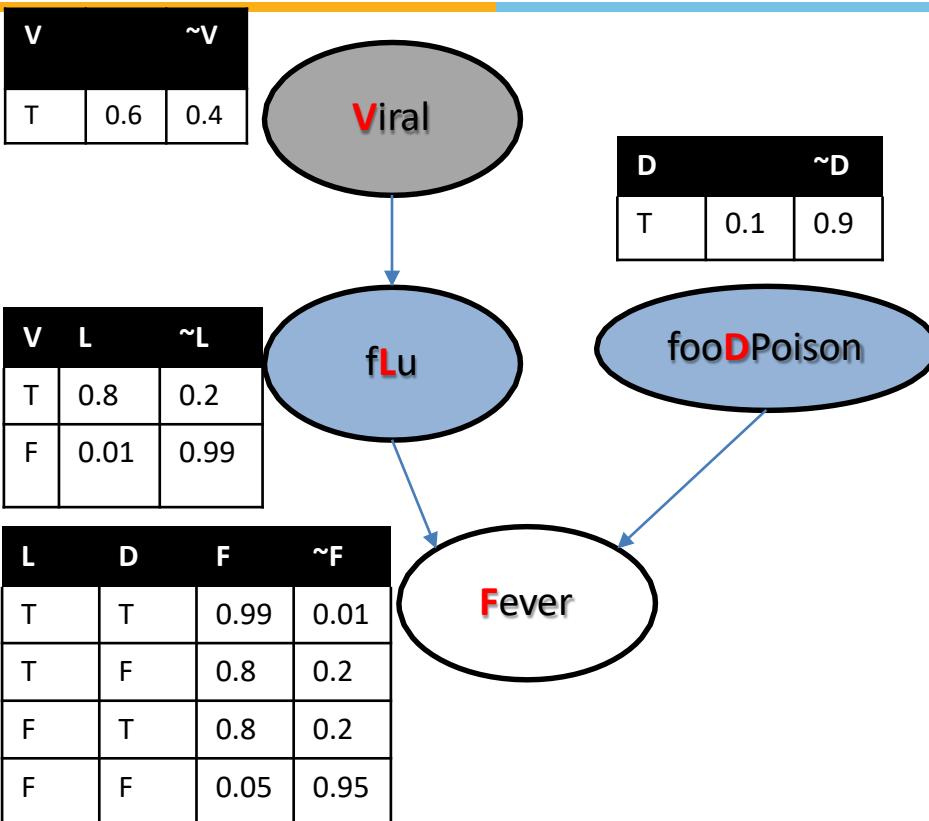
L	F	$\sim T$	T	0.048
T	T	T	T	0.34852
F	T	T	T	0.04128
F	F	T	T	0.02322
T	T	F	T	0.00048
T	F	F	T	0.087
F	T	F	T	0.01032
F	F	F	T	0.44118

L	F	$\sim T$	T	0.08928
T	T	T	T	0.37174
F	F	F	T	0.0108
F	F	F	T	0.52818

F	T	0.46102
F	F	0.53898

Inference

Variable Elimination: L



L	
T	0.484
F	0.516

L D F $\sim F$			
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

$P(L)$
 $P(D)$
 $P(F|L,D)$

Approximate Inferences in Bayesian Nets

Introduction

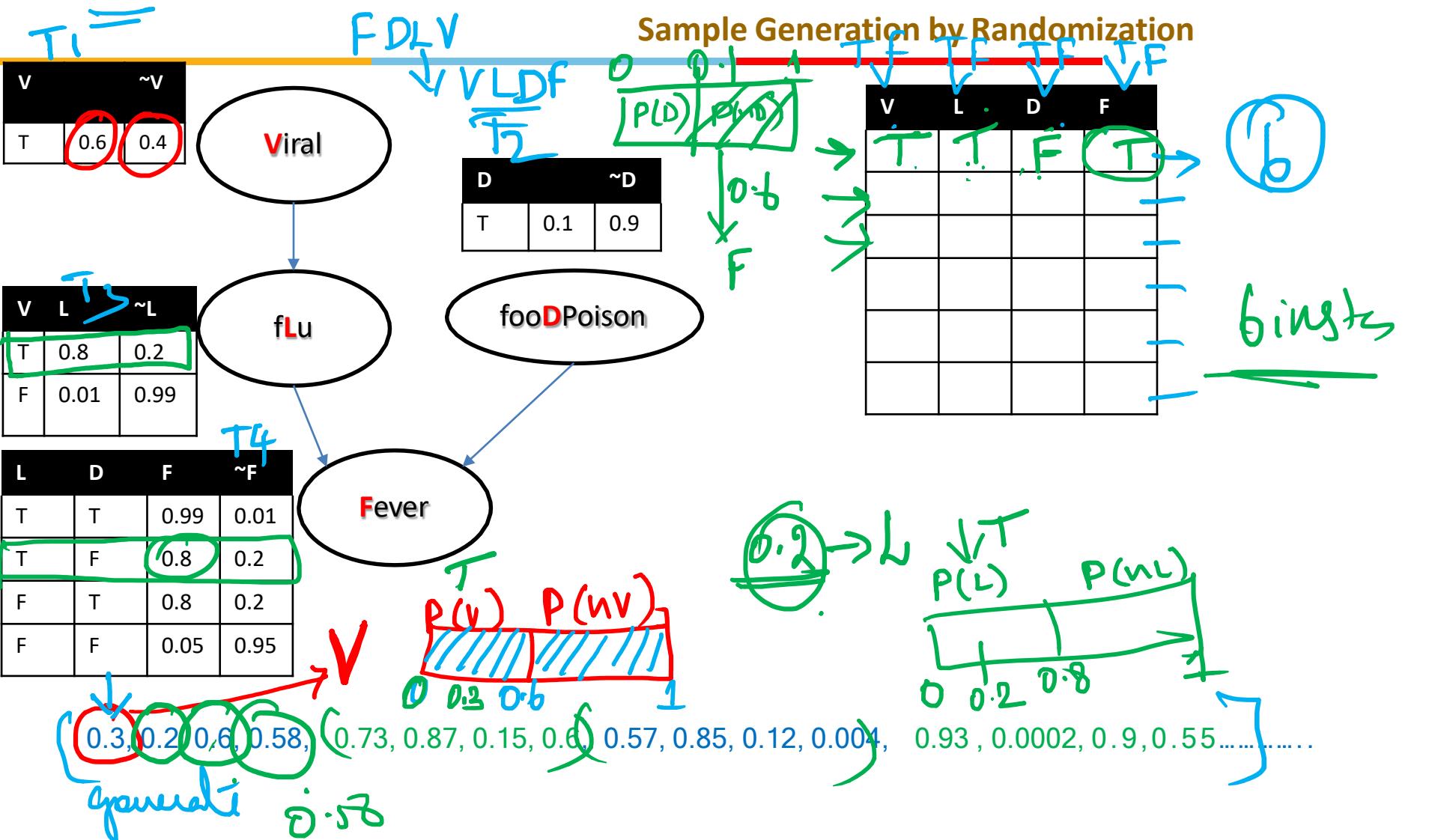
Direct Sampling

↳ 3 → Prior samp

↓
Posterior samp
Likelihood "

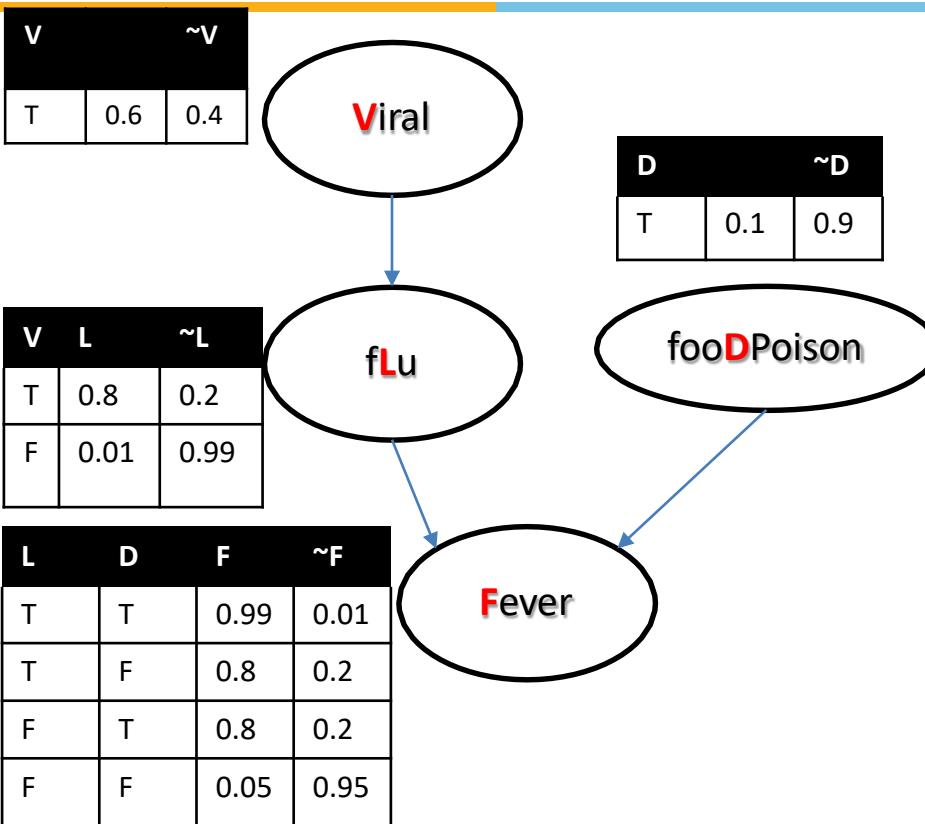
$$P(v) P(\sim v) \Rightarrow P(v) P(\sim v)$$

Prior Sampling



Prior Sampling

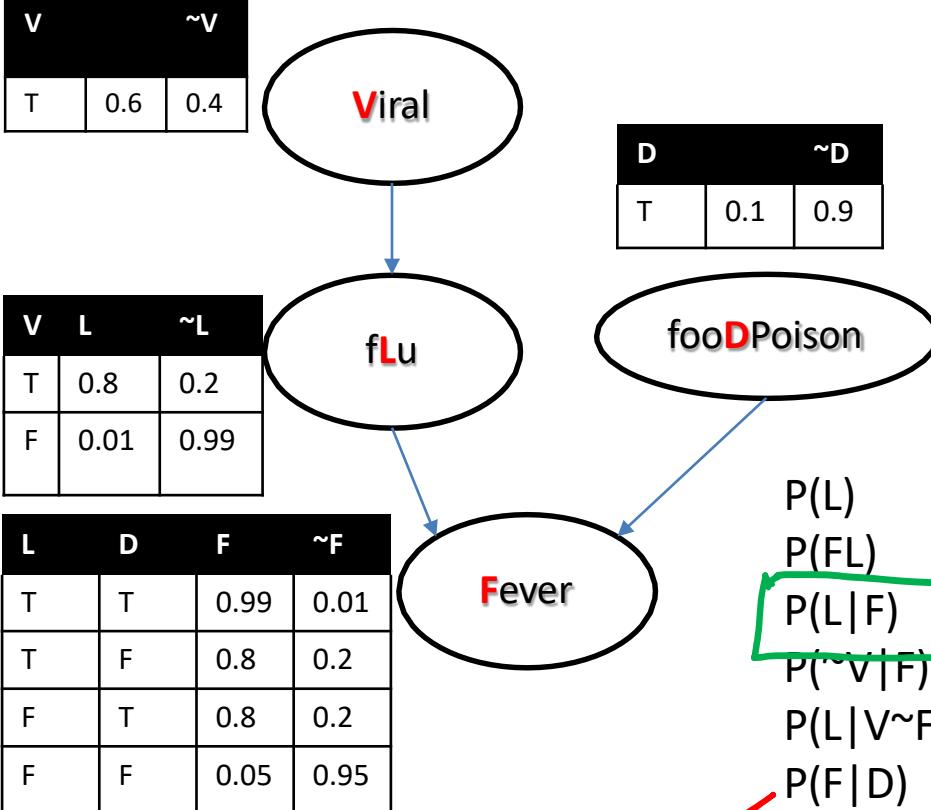
Sample Generation by Randomization



V	L	D	F
T	T	F	T
F	F	F	F
T	F	F	T
F	T	F	T
..			
.....			

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.55.....

Prior Sampling



$$P(F|D) = \frac{5}{0}$$

Inference

Table showing the joint probability distribution of variables **V, L, D, F** given evidence **E**.

V	L	D	F
T	T	F	T
F	F	F	F
T	F	F	T
F	T	F	T
T	F	F	T
T	F	F	F
F	F	F	T
T	F	F	F

Final state

Handwritten notes explaining the inference process:

① $P(L|F)$ \Rightarrow # matches w.r.t L

② \Rightarrow # matches w.r.t F

③ $\Rightarrow \frac{3}{5}$

Rejection Sampling

Sample Generation by Randomization

V	$\sim V$	
T	0.6	0.4



V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

fLu

D	$\sim D$	
T	0.1	0.9

fooDpoison

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

Fever

$$P(L) = \frac{3}{8}$$

$$P(F) = \frac{3}{8}$$

$$P(L|F) = \frac{3}{5}$$

$$P(\sim V|F) = \frac{2}{5}$$

$$P(L|V\sim F) = 0$$

$$P(F|D) = \text{?????}$$

D is fine



V	L	D	F
T	T	F	
.....			

~~rejected~~

Rejection Sampling

Sample Generation by Randomization

V	$\sim V$	
T	0.6	0.4



D	$\sim D$	
T	0.1	0.9

V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

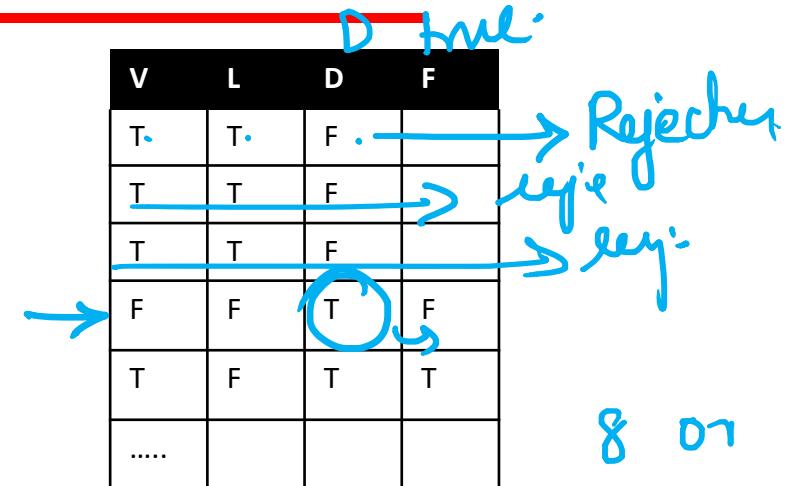


L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95



$$\begin{aligned}
 P(L) &= 3/8 \\
 P(F) &= 3/8 \\
 P(L|F) &= 3/5 \\
 P(\sim V|F) &= 2/5 \\
 P(L|V\sim F) &= 0 \\
 P(F|D) &= \text{?????}
 \end{aligned}$$

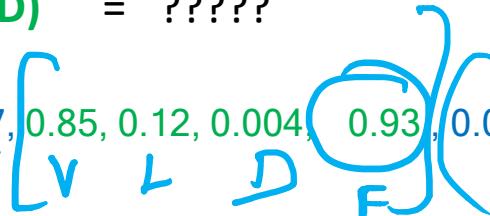
D true



V	L	D	F
T	T	F	
T	T	F	
T	T	F	
F	F	T	F
T	F	T	T
.....			

8 D1

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.555, 0.38.....



Rejection Sampling

V	$\sim V$	
T	0.6	0.4



V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

D	$\sim D$	
T	0.1	0.9



$$\begin{aligned}
 P(L) &= 3/8 \\
 P(FL) &= 3/8 \\
 P(L|F) &= 3/5 \\
 P(\sim V|F) &= 2/5 \\
 P(L|V\sim F) &= 0 \\
 P(F|D) &= 5/8
 \end{aligned}$$

Inference

V	L	D	F
T	T	T	T
F	F	T	F
T	F	T	T
F	T	T	T
T	T	T	T
T	F	T	F
F	F	T	T
T	F	T	F

8

8

5
8

$$P(a|e_1 e_2 \dots e_n e_{100}) \text{ rejected}$$

Rejection Sampling

Sample Generation by Randomization

V	$\sim V$
T	0.6
F	0.4



D	$\sim D$
T	0.1
F	0.9

V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

fLu



$$P(\sim V | F) = ?$$

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

Fever

V	L	D	F
T	T	F	T
F	F	F	
T	F	F	T
F	T	F	T
.....			

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.555,

Likelihood Weighing

Sample Generation by Randomization

V	$\sim V$
T	0.6
F	0.4



D	$\sim D$
T	0.1
F	0.9

V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

$$P(\sim F | D, \sim V)$$

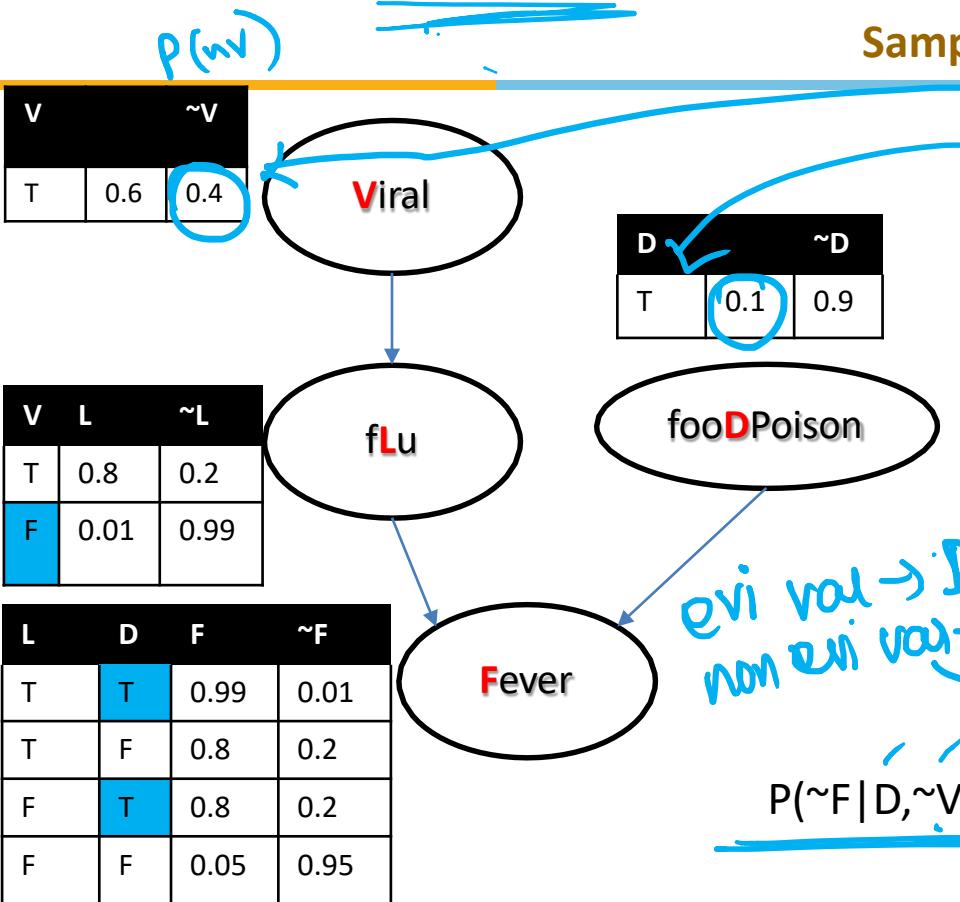
$e_1 e_2$
 T, F

V	L	D	F	wgt
F		T		
F		T		
F		T		
F		T		
F		T		
F		T		
F		T		

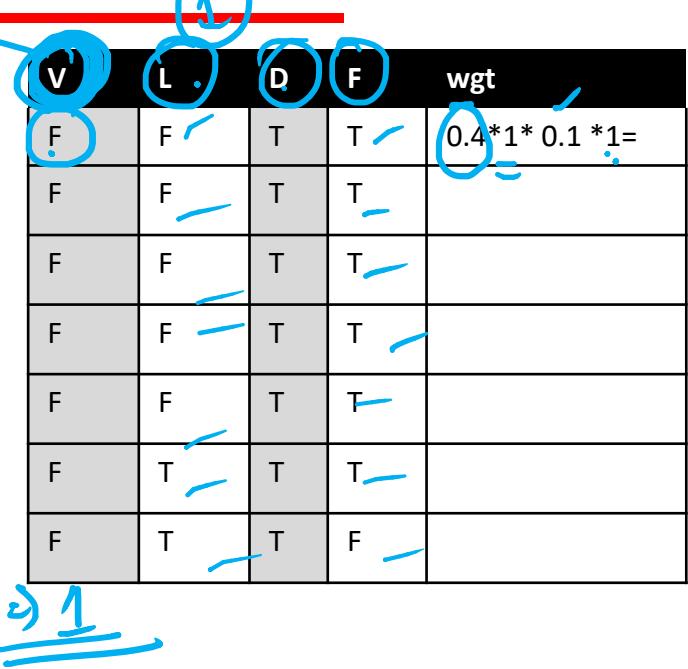
$$= 0.04 / 7 * 0.04$$

0.3, 0.2, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.99, ,.....

Likelihood Weighing



Sample Generation by Randomization



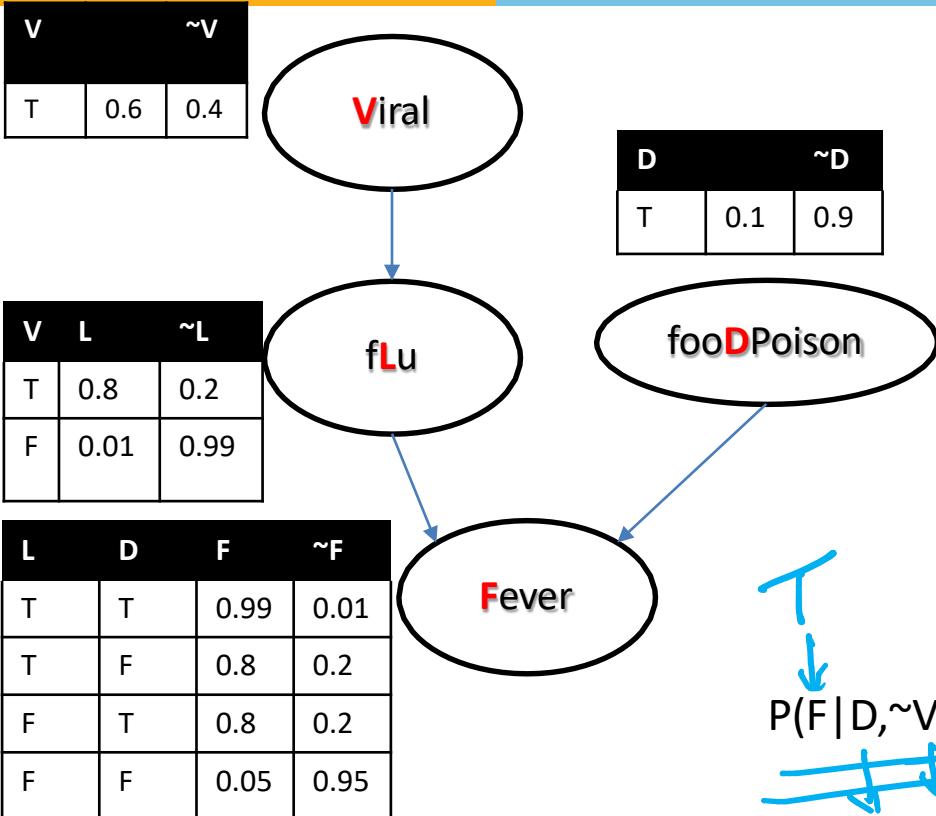
V	L	D	F	wgt
F	F	T	T	0.4*1*0.1*1=
F	F	T	T	
F	F	T	T	
F	F	T	T	
F	F	T	T	
F	T	T	T	
F	T	T	F	

evi val → DV
non evi val → LF
 \rightarrow wt → 1

$$= 0.04 / 7 * 0.04$$

$0.3, 0.2, \frac{F}{L}, \frac{F}{T}$, $0.58, 0.73, \frac{F}{T}, \frac{T}{F}, 0.87, 0.15, \frac{F}{T}, \frac{T}{F}$, $0.6, 0.57, \frac{F}{T}, \frac{T}{F}$, $0.85, 0.12, \frac{F}{T}, \frac{T}{F}$, $0.004, 0.93, \frac{T}{T}, \frac{T}{T}$, $0.0002, 0.99, \frac{T}{F}, \frac{F}{T}, \dots$

Likelihood Weighing



Inference

Handwritten note: 4 instances

V	L	D	F	wgt
F	F	T	F	$0.4 * 1 * 0.1 * 1 = 0.04$
F	T	T	T	$0.4 * 1 * 0.1 * 1 = 0.04$
F	F	T	T	$0.4 * 1 * 0.1 * 1 = 0.04$
F	F	T	F	$0.4 * 1 * 0.1 * 1 = 0.04$

$$\begin{aligned}
 & \frac{\text{w.r.t } Q}{\text{w.r.t } E} = \frac{0.04 + 0.04}{4 * 0.04} \\
 & = 0.04 + 0.04 / 4 * 0.04
 \end{aligned}$$

Likelihood Weighing

V	$\sim V$
T	0.6
F	0.4



D	$\sim D$
T	0.1
F	0.9

V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99



L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95



$$P(F | D, \sim L)$$

Inference

V	L	D	F	wgt
F	F	T	F	
F	F	T	T	
F	F	T	T	
T	F	T	F	

$$= 0.099 + 0.099 / (3 * 0.099 + 0.02)$$

Likelihood Weighing

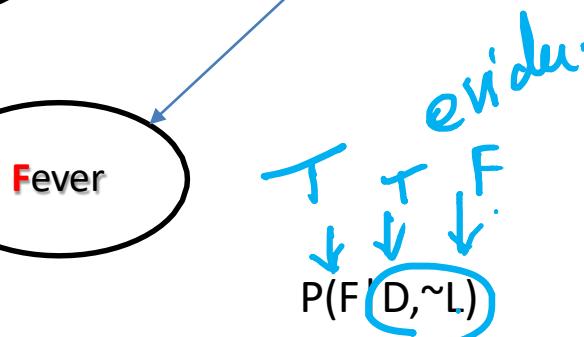
V	$\sim V$
T	0.6
F	0.4



D	$\sim D$
T	0.1
F	0.9

V	L	$\sim L$
T	0.8	0.2
F	0.01	0.99

L	D	F	$\sim F$
T	T	0.99	0.01
T	F	0.8	0.2
F	T	0.8	0.2
F	F	0.05	0.95

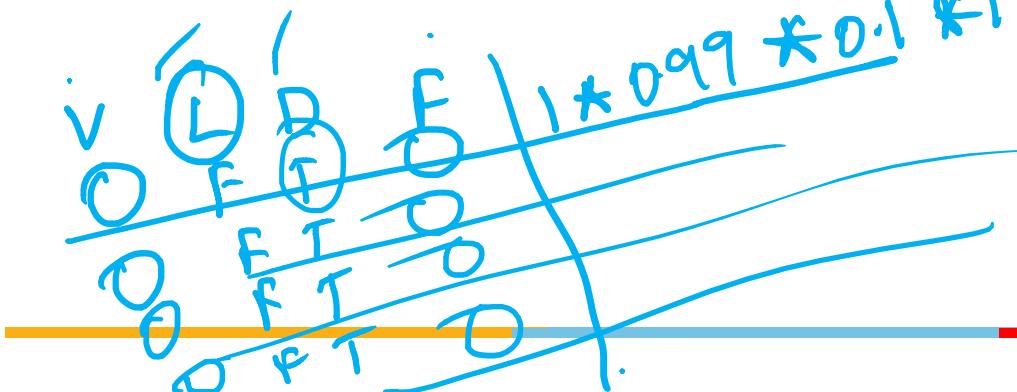


Inference

V	L	D	F	wgt
F	F	T	F	$1 * 0.99 * 0.1 * 1 =$
F	F	T	T	$1 * 0.99 * 0.1 * 1 =$
F	F	T	T	$1 * 0.99 * 0.1 * 1 =$
T	F	T	F	$1 * 0.2 * 0.1 * 1 =$

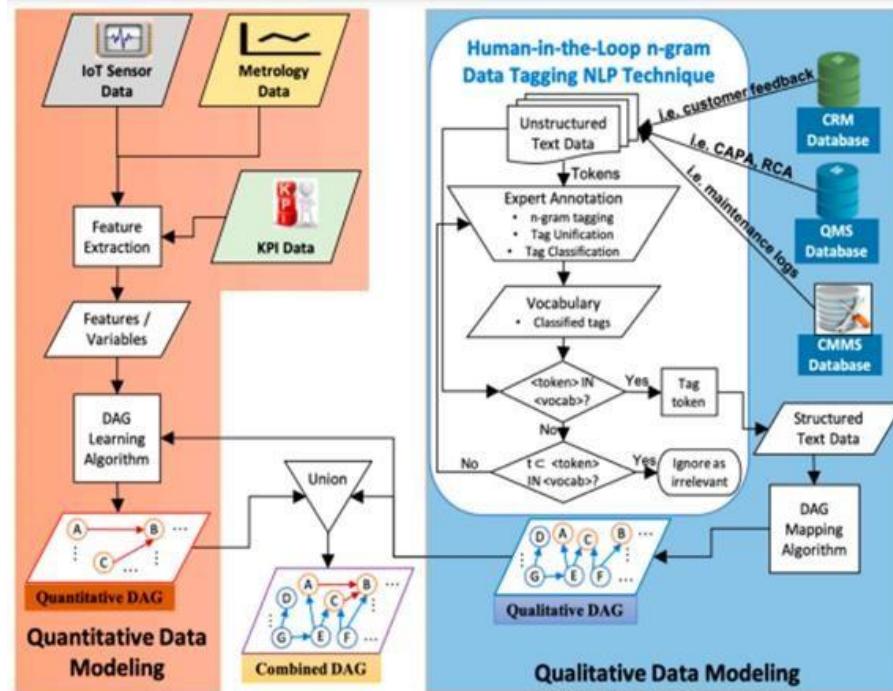
$$= 0.099 + 0.099 / (3 * 0.099 + 0.02)$$

random



Bayesian Network

Fault Diagnostic System

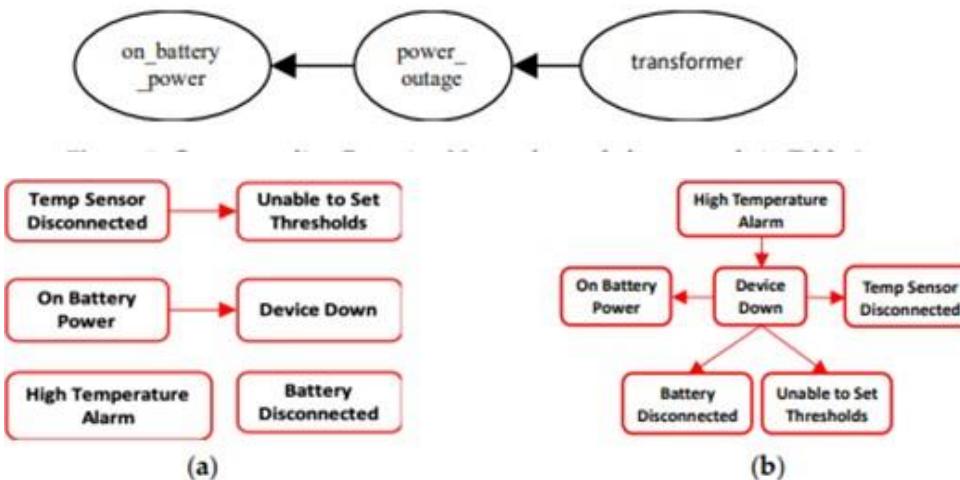


Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network

Fault Diagnostic System

Raw Data	Short Description		Resolution Notes		
	Symptom	Cause(s)	Link		
Classified Tags	on_battery_power	power_outage, transformer_fire	due_to		
BN Mapping	Child Variable	Child State	Parent Variable	Parent State	Ancestor Variable
	on_battery_power	yes	power_outage	yes	transformer
					Fire



Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network

Fault Diagnostic System

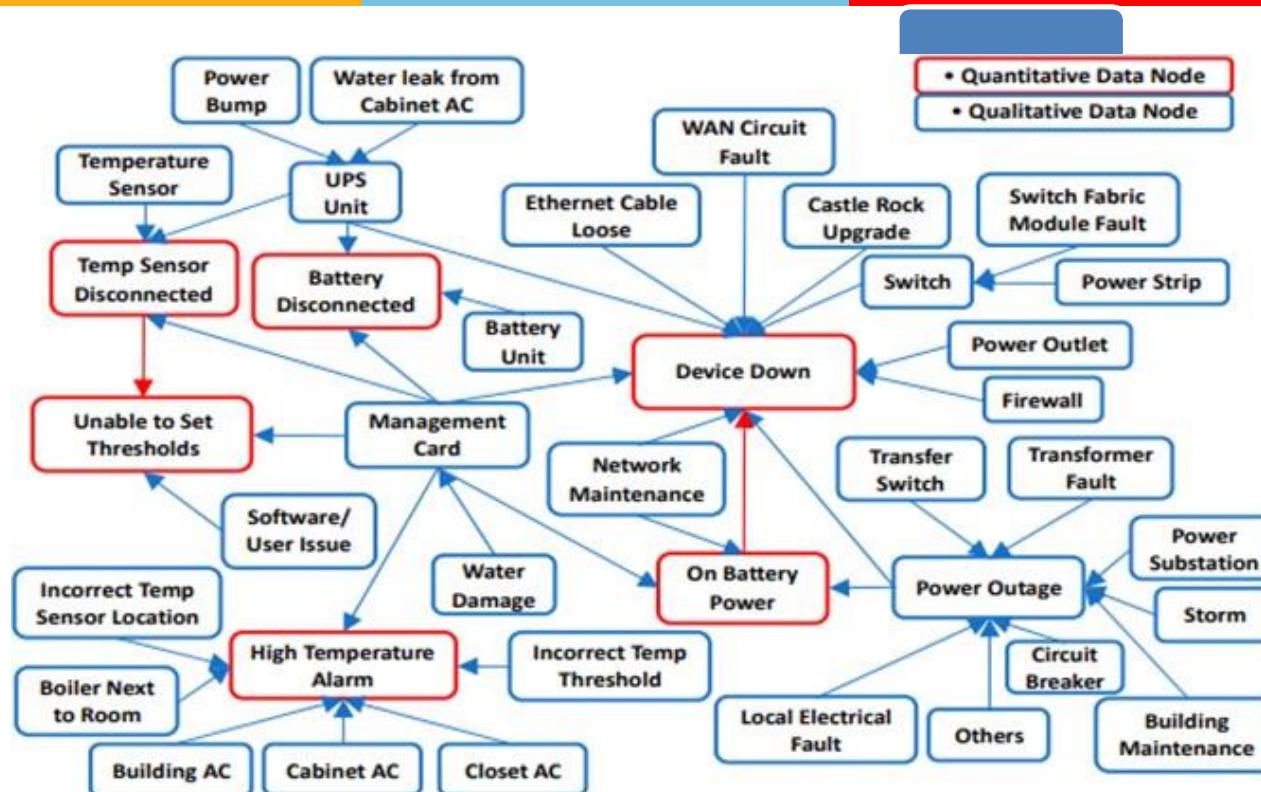
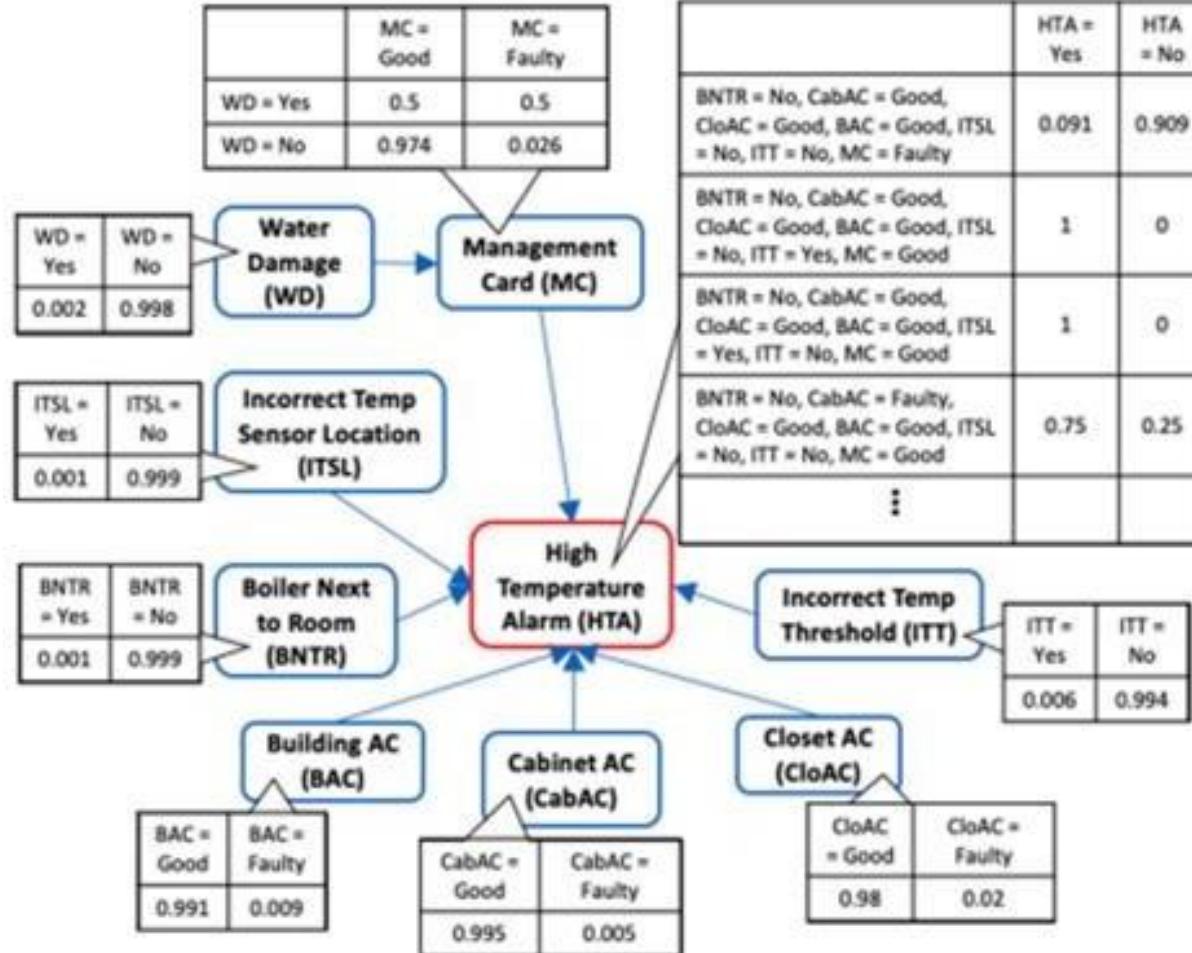


Figure 8. Fused Bayesian Network structure for top six occurring UPS messages.

Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network

Fault Diagnostic System



Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1, #9 Refer to the handout

Next Session Plan:

- Hidden Markov Models

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials