BITS Pilani
Pilani Campus

Artificial & Computational Intelligence

DSECLZG557

M4 : Knowledge Representation using Logics

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BITS -WILP

Course Plan

M1 Introduction to AI

2val

M2 Problem Solving Agent using Search

T V False

Propositional Logic

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

Towards Predicate Logic First order logic

1. All courses are offered and interesting → 1. Multiple var-val pair 2. Quantification

2. All offered courses are interesting

More expressable logic – FOL / Predicate Logic

3. Some of the courses are offered and interesting [Atleast one of the offered courses is interesting]

4. Some of the offered courses are interesting

Towards Predicate Logic → function definition

function(Val_1, Val_2 ... Val_n)

1. All courses are offered and interesting

offer(x) Interesting(x) common Noun / action verb obj / entity

All offered courses are interesting

Predicate fn (v_1, v_2 -- v_n) ⇓ tuples / arg.

O(x) I(x)

Unary Predicate — 1 arg

Some of the courses are offered and interesting [Atleast one of the offered courses is interesting]

Some of the offered courses are interesting innovate achieve lead

A is teaching ~~subject~~ Course AcI for the students S.

(2) Teaching (A, AcI, S)

(3) Seller sells ~~the~~ ~~All~~ some product to customer.

$\forall$ $\boxed{\forall p \; Sell(S, P, C)}$ $\rightarrow$ all the product

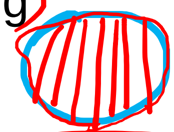
$\exists$ $\exists p \; Sell(S, P, C)$ $\rightarrow$ some of the product

existential Quantification

Towards Predicate Logic

$\forall x\, O(x) \wedge I(x)$

All courses are offered and interesting

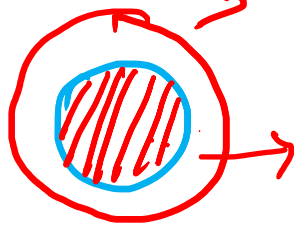


2 All offered courses are interesting

$\forall x\, O(x) \rightarrow I(x)$

10 → 10

implication law



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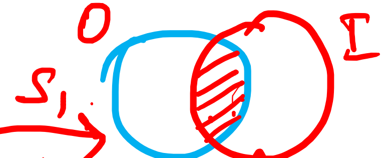
3 Some of the courses are offered and interesting [Atleast one of the offered courses is interesting]

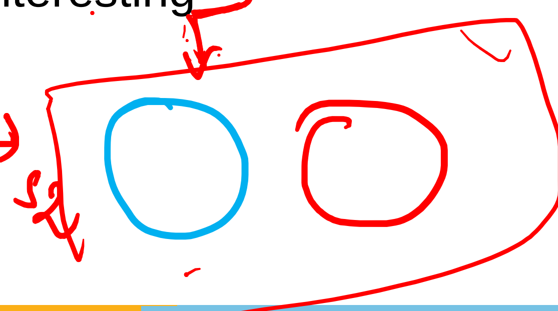


$\rightarrow \exists x\, [O(x) \wedge I(x)]$

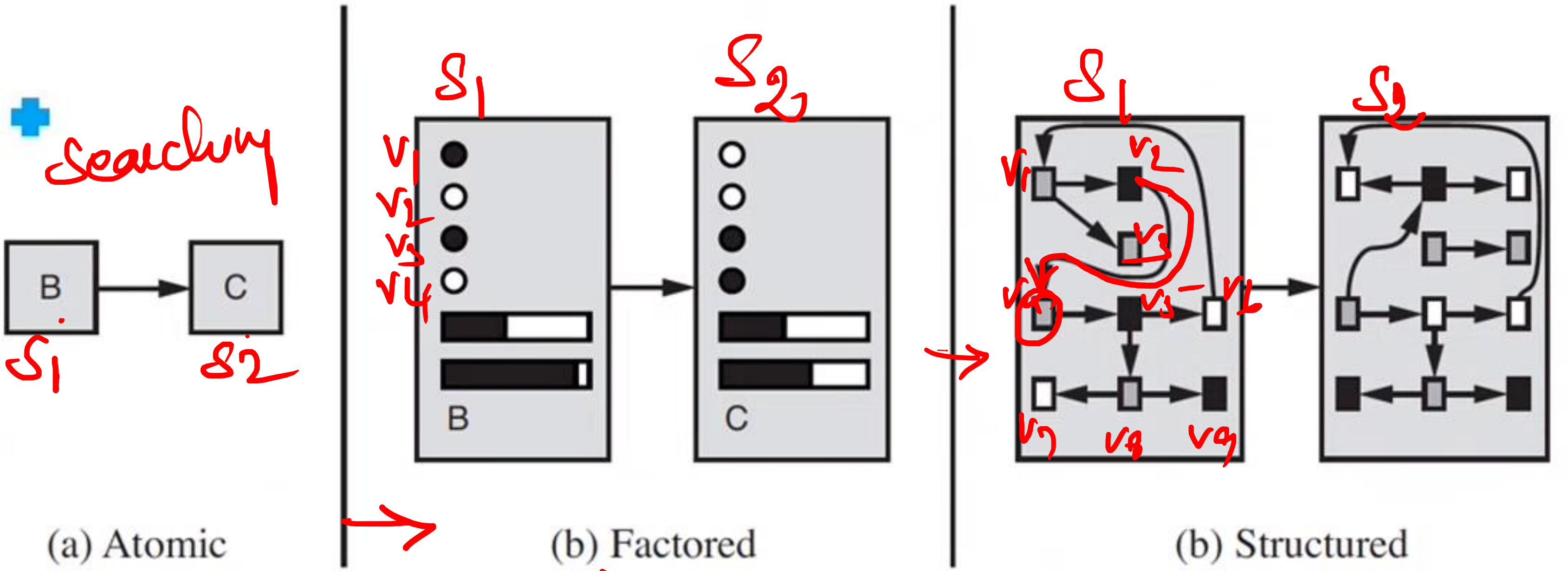
4 Some of the offered courses are interesting

$\exists x\, O(x) \rightarrow I(x)$

None



Towards Predicate Logic



(a) Atomic (b) Factored (b) Structured

PL resolution

Predicate logic

Predicate Logic

Squares neighboring the wumpus are smelly

Objects: squares, wumpus

Unary Relation (properties of an object): smelly N-ary

Relation (between objects): neighboring

Function: -

Primary difference between propositional and first-order logic lies in "ontological commitment" – the assumption about the nature of reality.

Predicate Logic – Sample Modelling

3x3 9

1. "Squares neighboring the wumpus are smelly"

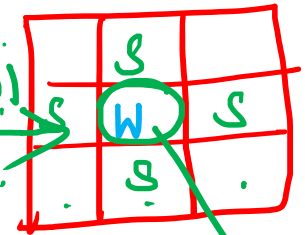
$\boxed{\forall x,y}\ Neighbour(x,y) \land Wumpus(y) \Rightarrow Smelly(x)$

$\forall x,y\ N(x,y) \land W(y) \Rightarrow S(x)$

Neigh loc (a1,b1)

Wo loc a2b2

a1b1



a2b2

$\{W_{x,y} \to S_{x+1,y} \land S_{x,y+1} \land S_{x-1,y} \land S_{x,y-1}\}$

Order of quantifiers is important

Predicate Logic – Sample Modelling

X
All

2. "Everybody loves somebody" → $\forall x, \exists y \ Loves(x, y)$

$\forall x \ \exists y \ Loves(x, y)$

everyone → All

someony → There exis

3. "There is someone who is loved by everyone"

$\exists y \ \forall x \ Loves(x, y)$

$\exists y \ \forall x \ Loves(x, y)$

every

Order of quantifiers is important

2 ways (forward chaining)
(backward ") PL

PL
TT
Then
DPLL

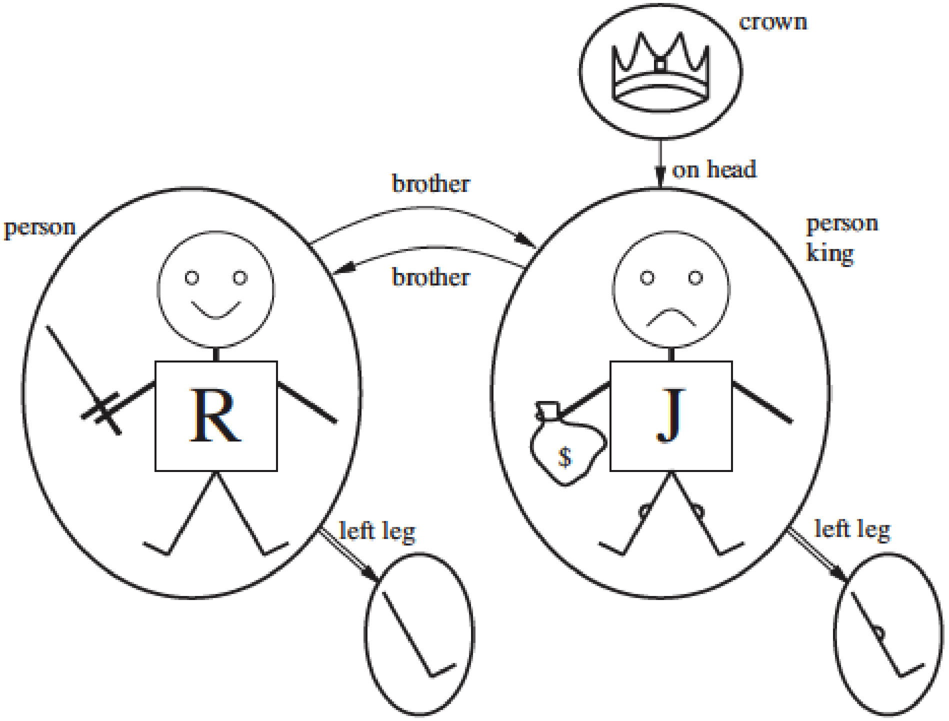
Convert the Predicate into poopportion

Predicate Logic – Sample Modelling

Brother(Richard, John) ∧ Brother(John, Richard)

King(Richard) ∨ King(John)

¬King(Richard) ⟹ King(John)

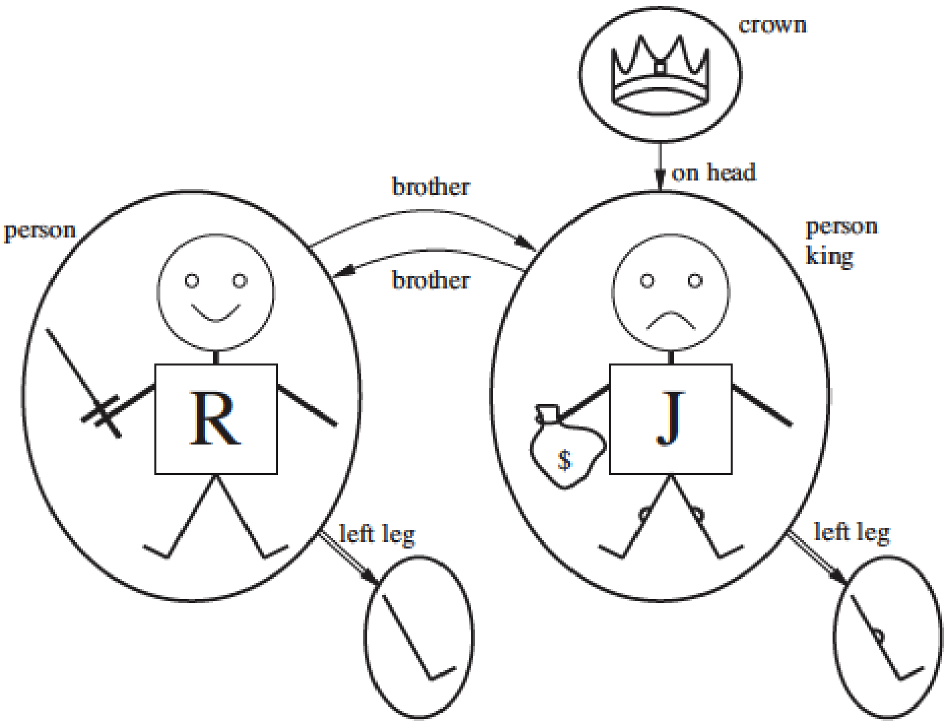


Unification & Lifting

Brother(Richard, John) ∧ Brother(John, Richard)

King(Richard) ∨ King(John)

¬King(Richard) ⟹ King(John)



Predicate Logic – Sample Modelling

Quantifiers

∀ & ∃

2 persons.

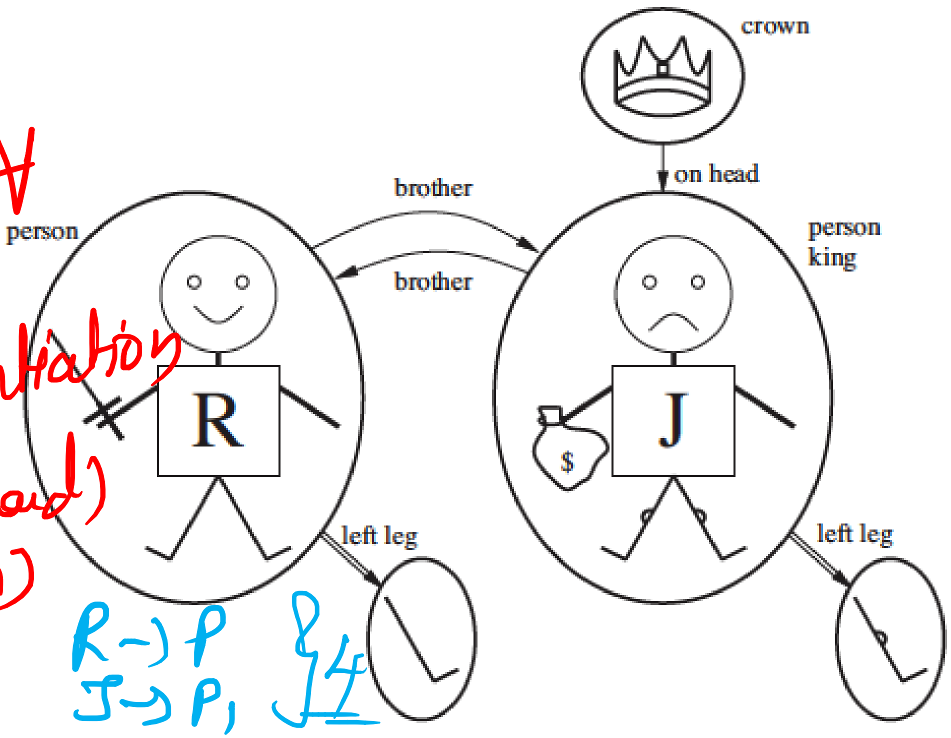
Brother(Richard, John) ∧ Brother(John, Richard)

King(Richard) ∨ King(John)

¬King(Richard) ⟹ King(John)

Removal of ∀

Universal Instantiation



"All Kings are persons"

$\forall x\ King(x) \Rightarrow Person(x)$

King(Richard) → Person(Richard)

King(John) → Person(John)

R → P

J → P1

"King John has a crown on his head"

$\exists x\ Crown(x)\ \ AOnHead(x, John)$

substitute temp var

Crown(Fe) AOnhead(Fe, John) → Existential Instantiation

Ground Term: A term with no variables. E.g., King(Richard)

Predicate Logic – Sample Modelling Quantifiers

Brother(Richard, John) ∧ Brother(John, Richard)

King(Richard) ∨ King(John)

¬King(Richard) ⟹ King(John)

"All Kings are persons"

$\forall x \ King(x) \Longrightarrow Person(x)$

"King John has a crown on his head"

$\exists x \ Crown(x) \ \land OnHead(x, John)$

Ground Term: A term with no variables. E.g., King(Richard)

Predicate Logic – Inference

1. Substitute for Quantifiers
2. Convert into Propositional Logic
3. Apply inference tech

$\forall x\ King(x) \Longrightarrow Person(x)$

Richard the Lionheart is a king ⟹ Richard the Lionheart is a person

King John is a king ⟹ King John is a person

$\exists x\ Crown(x)\ \ \mathrm{A} OnHead(x, John)$

Crown(C_1) A OnHead(C_1, John) ||C1 is imputed assumed fact

Forward Chaining

- Consider the following problem:

KB

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

fact

fact

Q:

- We will prove that West is a criminal (West) → Query

country (Nono)

American (West)

American(x) ∧ hostile(y) ∧ weapon(z) ∧

sells (x, z, y) → Criminal (x)

Forward Chaining

has some missiles x → West

P KB

- "All of its missiles were sold to it by Colonel West"

$$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

- Missiles are weapons

$$Missile(x) \Rightarrow Weapon(x)$$

- Hostile means enemy

$$Enemy(x, America) \Rightarrow Hostile(x)$$

- "West, who is American"

$$American(West)$$

- "The country Nono, an enemy of America"

$$Enemy(Nono, America)$$

Forward Chaining

- First, we will represent the facts in First Order Definite Clauses

" … it is a crime for an American to sell weapons to hostile nations"

$$American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \Rightarrow Criminal(x)$$

"Nono … has some missiles"

$$\exists x\ Owns(Nono, x) \land Missile(x)$$ M_1

is transformed into two definite clauses by Existential Instantiation

$$Owns(Nono, M_1)$$

$$Missile(M_1)$$

Forward Chaining

Query: Criminal(West)

starts from fact

4 Rule system

4 facts

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1) f1

Owns(Nono, M1) f2

American (West) f3

Enemy (Nono, America) f4

Forward Chaining

- Consider the following problem:

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- We will prove that West is a criminal

Algorithm:

1. Start from the facts
2. Trigger all rules whose premises are satisfied
3. Add the conclusion to known facts
4. Repeat the steps till no new facts are added or the query is answered

Forward Chaining

Q : criminal(west)

L.H.S R.H.S fact

4

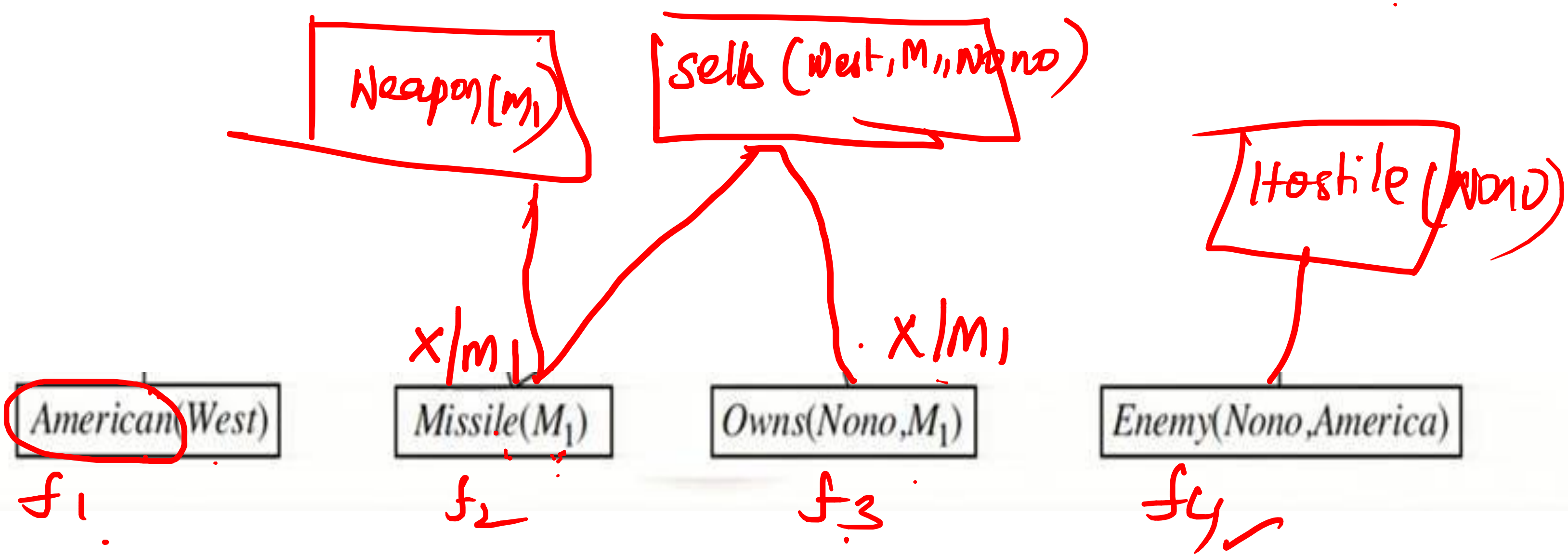
1. $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

f2 f3

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)

→ Mis(M1) ∧ Owns(Nono, M1) → R.H.S sells(West, M1, Nono) fact

→ mis(M1) → weapon(M1) fac



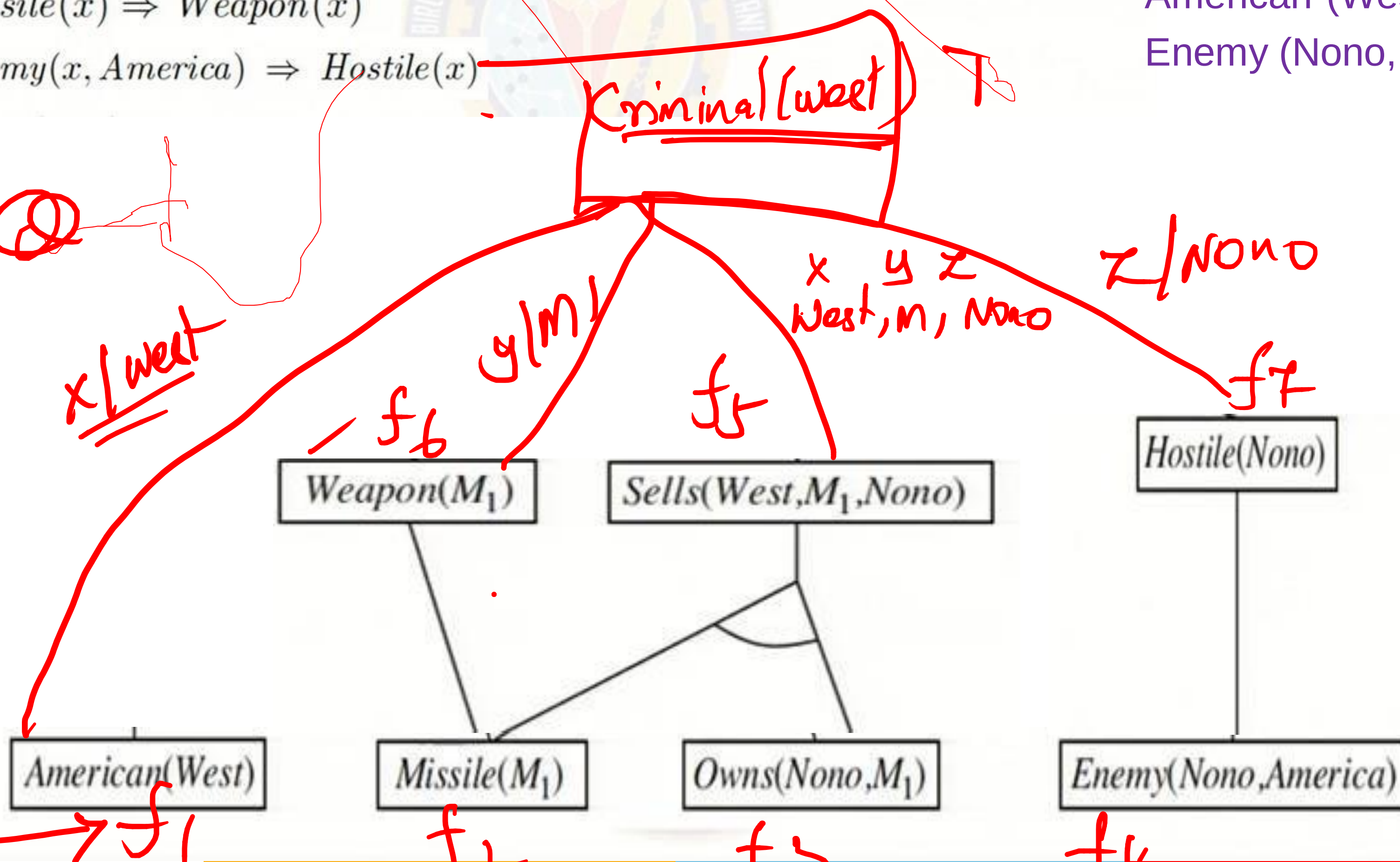
fact oriented approach

Forward Chaining

L.H.S R.H.S

1. $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1) f1
Owns(Nono, M1) f2
American (West) f3
Enemy (Nono, America) f4



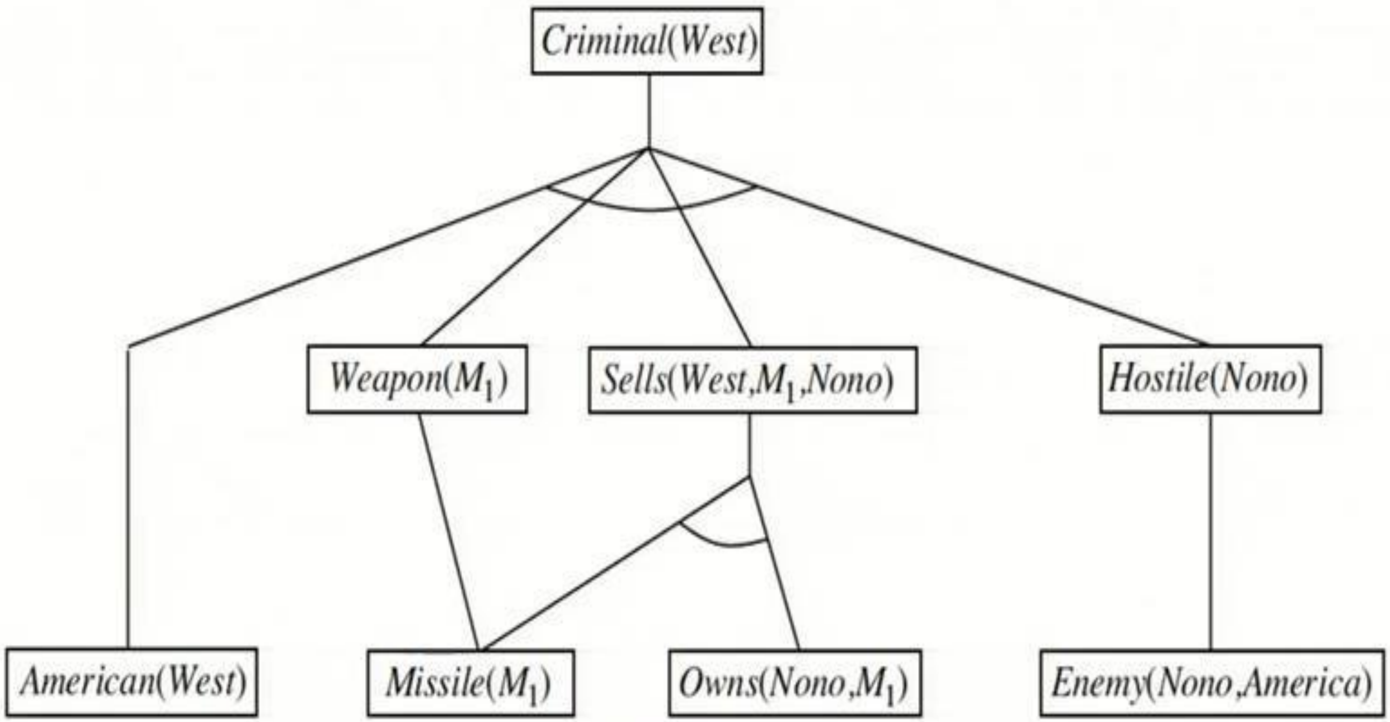
Forward Chaining

① $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

② $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

③ $Missile(x) \Rightarrow Weapon(x)$

④ $Enemy(x,America) \Rightarrow Hostile(x)$



Forward Chaining

Algorithm:

1. Start from the facts - Conjunct Ordering
2. Trigger all rules whose premises are satisfied - Pattern Matching
3. Add the conclusion to known facts – **Irrelevant Facts**
4. Repeat the steps till no new facts are added or the query is answered – Redundant Rule Matching

Backward Chaining → Goal oriented

Q

Algorithm:

1. Form Definite Clause
2. Start from the Goals
3. Search through rules to find the fact that support the proof
4. If it stops in the fact which is to be proved → Empty Set- LHS

Divide & Conquer Strategy

Substitution by Unification

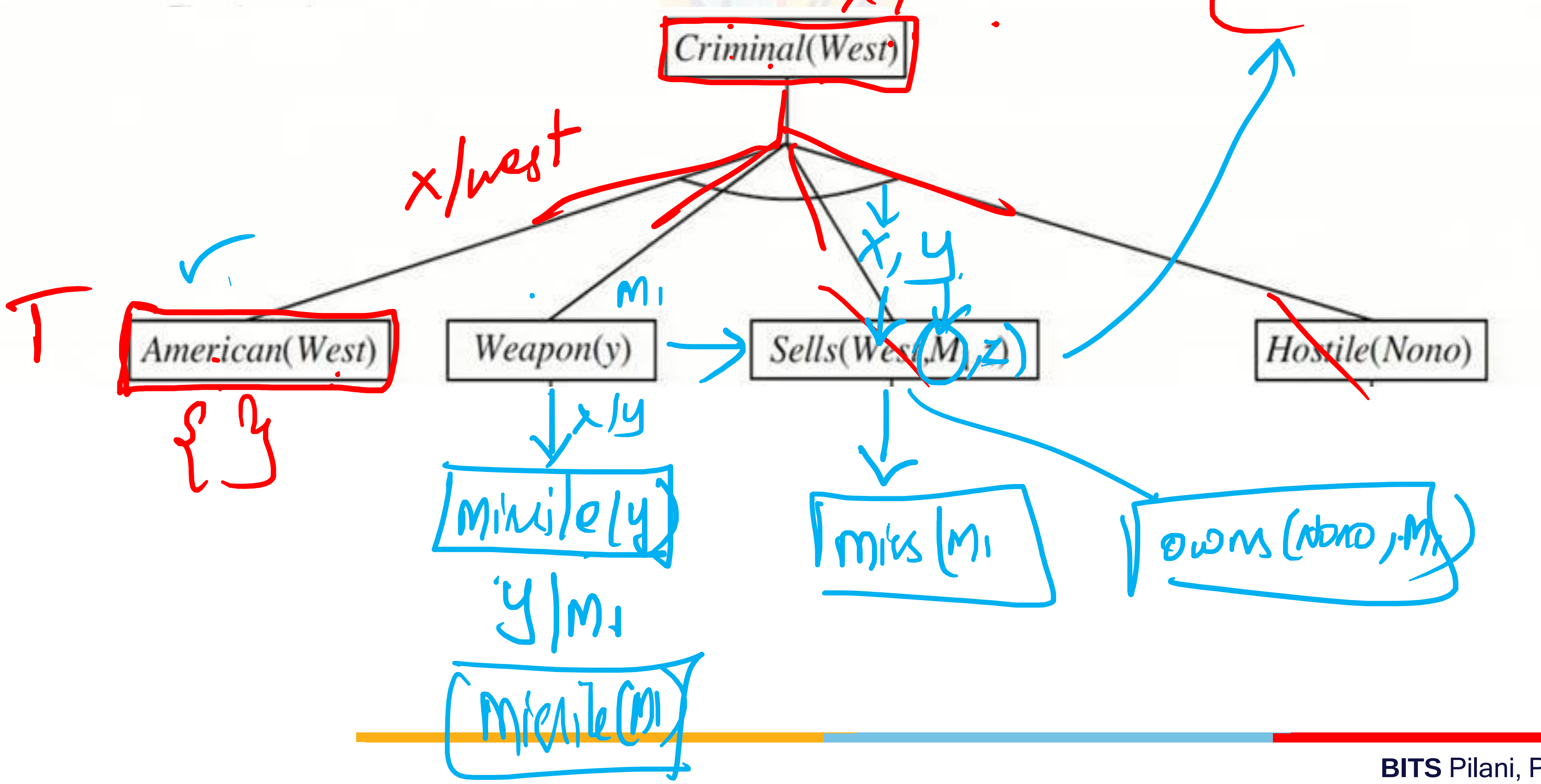
Backward Chaining

Q.

R.H.S fact LCM

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Backward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
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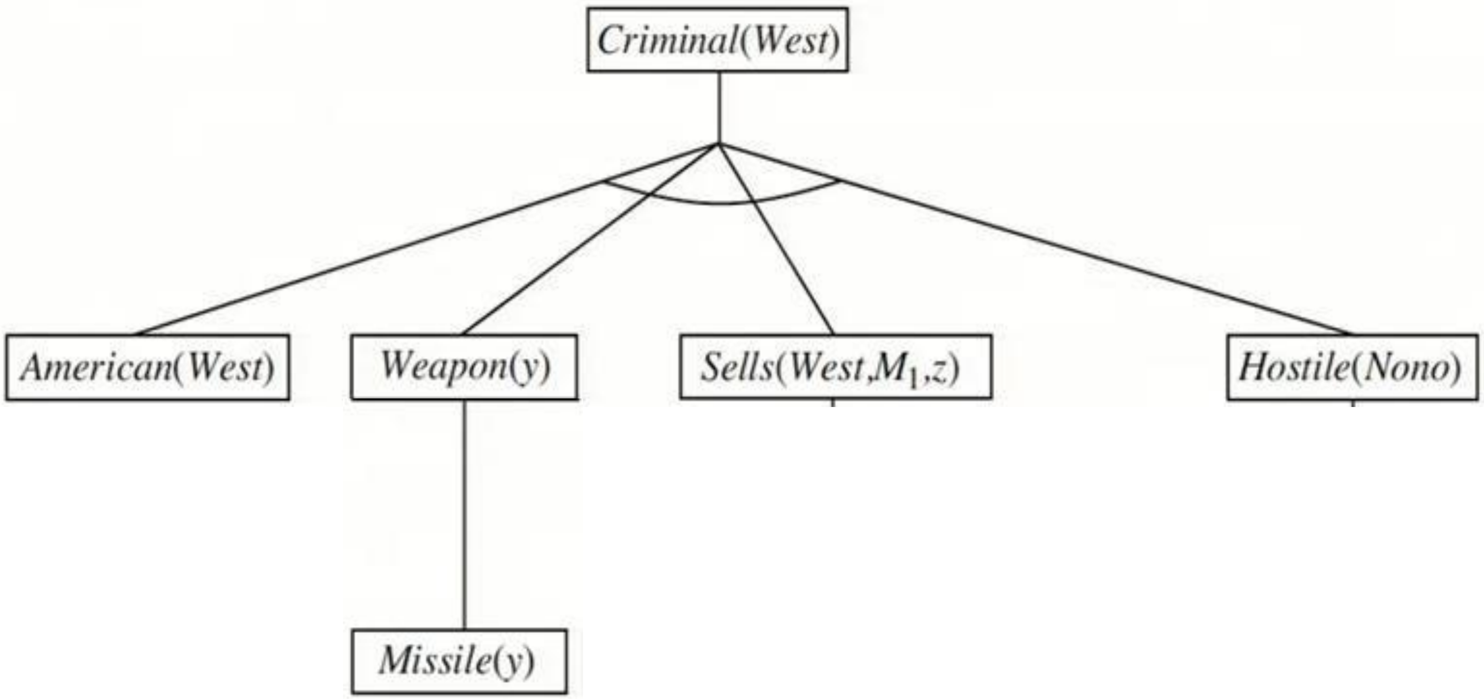
Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Backward Chaining

1. $American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$
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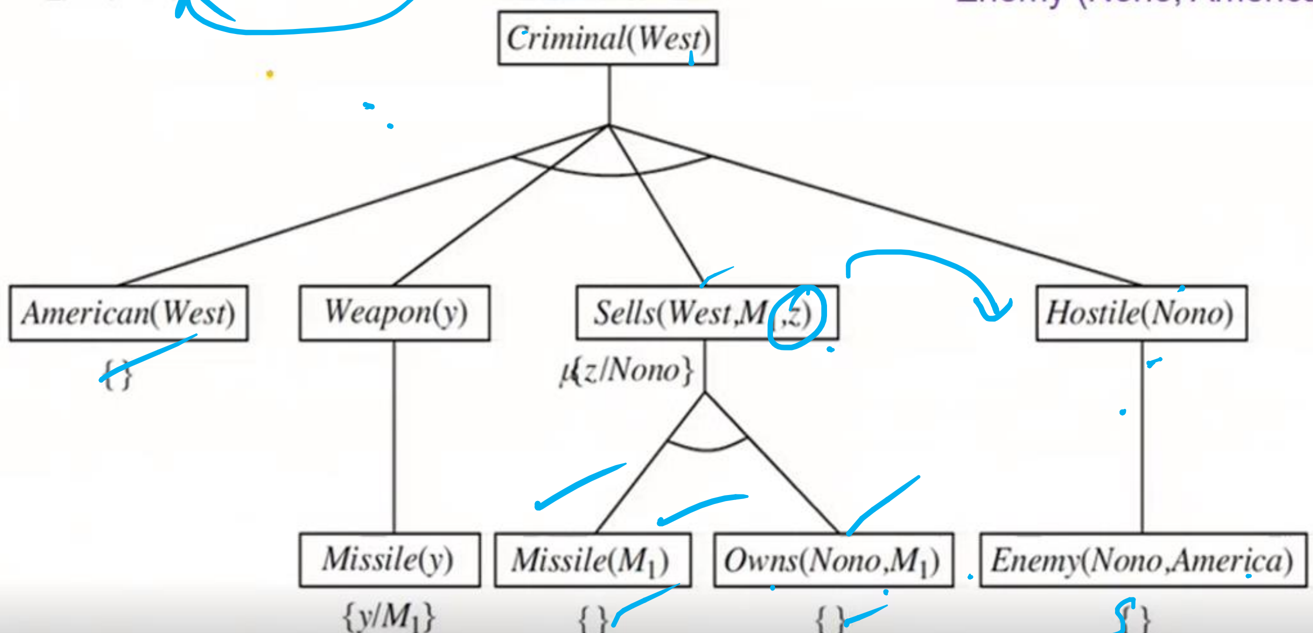

Backward Chaining

1. $American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
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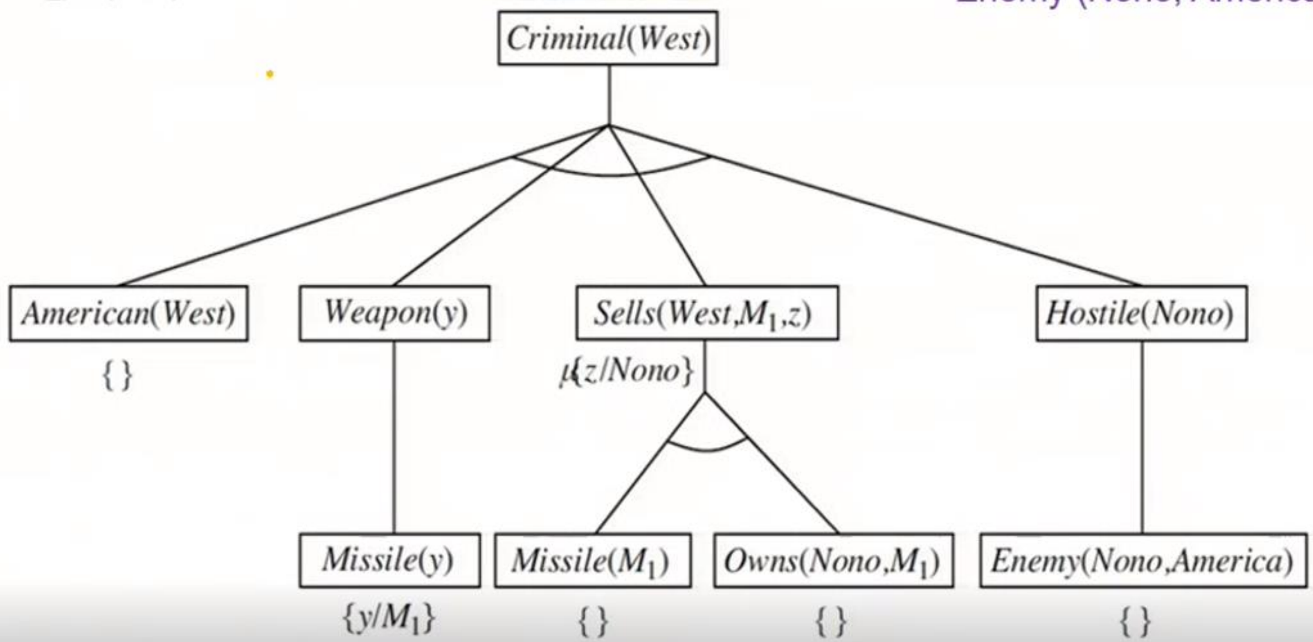
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Reasoning

Module 5:

Probabilistic Representation and Reasoning

A. Inference using full joint distribution

B. Bayesian Networks

   I. Knowledge Representation

   II. Conditional Independence

   III. Exact Inference

   IV. Introduction to Approximate Inference

Reasoning

- Monotonic Reasoning

- Non- Monotonic Reasoning

  Dependency Directed Backtracking: when a statement is deleted as “ no more valid”, other related statements have to be backtracked and they should be either deleted or new proofs have to be found for them. This is called dependency directed backtracking (DDB)

Reasoning

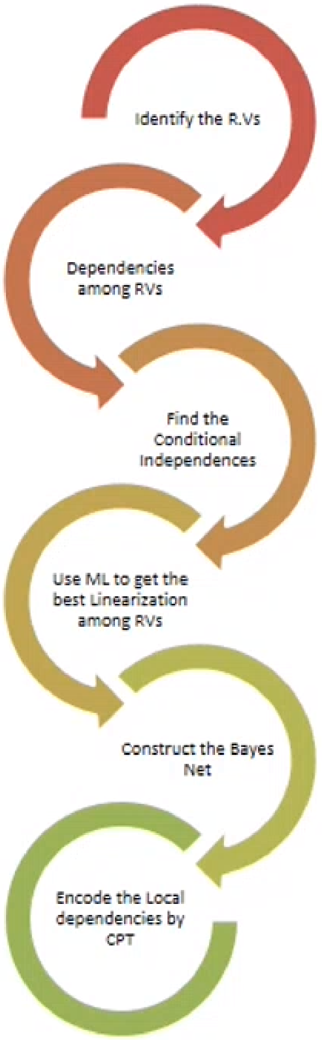
- Monotonic Reasoning
- Non- Monotonic Reasoning

Monotonic	Non-Monotonic
Consistent	Relaxed Consistency
Complete Knowledge	Incomplete Knowledge
Static	Dynamic
Discrete	Continuous & Learning Agent
Predicate Logic	Probabilistic Model

Bayesian Net???

Wumpus World Problem

- Wumpus Ghost traces of scent in the visited cell
- Earlier visited cell may become unsafe!!!
- **Problem**: Given the information that there is a possibility of apparition of Wumpus anywhere in the cave, AI agent needs to be safely travel with more caution!!



Uncertainty

You can reach Bangalore Airport from MG Road within 90 mins if you go by route A.

- There is uncertainty in this information due to partial observability and non determinism
- Agents should handle such uncertainty

Previous approaches like Logic represent all possible world states

Such approaches can't be used as multiple possible states need to be enumerated to handle the uncertainty in our information

Uncertainty

You can reach Bangalore Airport from MG Road within 90 mins if you go by route A.

Road Block	Festival Season	Weekend	Observation (20)	Prob
F	F	F	12	0.6
F	F	T	3	0.15
F	T	F	2	0.1
F	T	T	2	0.1
T	F	F	0	0
T	F	T	0	0
T	T	F	1	0.05
T	T	T	0	0
				=1

Probability Theory

Basics
Conditional Probability
Chain Rule
Independence
Conditional Independence
Belief Nets
Joint Probability distribution

Probability Basics - Refresher

Sample Space: Set of all possible outcomes.

- Ex: After tossing 2 coins, the set of all possible outcomes are
- {HH, HT, TH, TT}

Event: A subset of a sample space.

- An event of interest might be - {HH}

Probability Basics - Model

A fully specified **probability model** associates a numerical probability P(ω) with each possible world.

The **basic axioms**

1. Every possible world has a probability between 0 and 1
2. Sum of probabilities of possible worlds is 1 P(True) = 1
   P(False) = 0

3. P(a ∨ b) = P(a) + P(b) - P(a ∧ b)

E.g., P(HH) = 0.25; P(HT) = 0.25; P(TT) = 0.25, P(TH) = 0.25

$$0 \leq P(\omega) \leq 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1$$

Probability Basics – Exclusive / Exhaustive events

Mutually Exclusive Events:

- Two events (or propositions) are mutually exclusive or disjoint if they cannot both occur at the same time(be true).
- A clear example is the set of outcomes of a single coin toss, which can result in either heads or tails, but not both.

Exhaustive Events:

- A set of events is jointly or collectively exhaustive if at least one of the events must occur.
- E.g., when rolling a six-sided die, the events 1, 2, 3, 4, 5, and 6 are collectively exhaustive.

Probability Basics - Propositions

Probabilistic assertions (Propositions)

- Usually not a particular world event but about a set of them
- E.g., two dice when rolled, a proposition φ can be "the sum of dice is 11"

For any proposition φ,

$P(\varphi) = P(\text{sum} = 11) \quad = P((5, 6)) + P((6, 5))$

$= 1/36 + 1/36 = 1/18$

Probability Basics – Unconditional/Prior

Unconditional / Prior probabilities:

Propositions like P(sum = 11) or P(two dices rolling equals) are called Unconditional or Prior probabilities

They refer to degree of belief in absence of any other information

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(a \wedge b) = P(a \mid b) P(b)$$

Probability Basics - Conditional

However, most of the time we have some information, we call it **evidence**

E.g., we can interested in two dice rolling a double (i.e., 1,1 or 2,2, etc)

When one die has rolled 5 and the other die is still spinning

Here, we not interested in unconditional probability of rolling a double

Instead, the **conditional** or **posterior** probability for rolling a double given the first die has rolled a 5

$P(doubles \mid Die_1 = 5)$ where | is pronounced “given”

E.g., if you are going for a dentist for a checkup, P(cavity) = 0.2

- If you have a toothache, then P(cavity | toothache) = 0.6

Independence

If we have two random variables, TimeToBnlrAirport and HyderabadWeather

P(TimeToBnlrAirport, HyderabadWeather)

To determine their relation, use the product rule

= P(TimeToBnlrAirport | HyderabadWeather) / P(HyderabadWeather)

However, we would argue that HyderabadWeather and TimeToBnlrAirport doesn't have any relation and hence

P(TimeToBnlrAirport | HyderabadWeather) = P(TimeToBnlrAirport)

This is called Independence or Marginal Independence

Independence between propositions a and b can be written as

$$P(a \mid b) = P(a) \quad \text{or} \quad P(b \mid a) = P(b) \quad \text{or} \quad P(a \wedge b) = P(a)P(b)$$

Bayes Rule

Using the product rule for propositions a and b

$$P(a \wedge b) = P(a \mid b)P(b) \qquad \text{and} \qquad P(a \wedge b) = P(b \mid a)P(a)$$

Equating the right hand sides and dividing by P(a)

$$P(b \mid a) = \frac{P(a \mid b)P(b)}{P(a)}$$

This is called the Bayes Rule

Conditional Independence

2 random variables A and B are conditionally independent given C iff

$P(a, b \mid c) = P(a \mid c)\, P(b \mid c)$ for all values a, b, c

More intuitive (equivalent) conditional formulation

- A and B are conditionally independent given C iff

  $P(a \mid b, c) = P(a \mid c)$ OR $P(b \mid a, c) = P(b \mid c)$, for all values a, b, c
- Intuitive interpretation:

  $P(a \mid b, c) = P(a \mid c)$ tells us that learning about b, given that we already know c, provides no change in our probability for a, i.e., b contains no information about a beyond what c provides

  $P(R \mid F, P) = P(R \mid P)$

Joint Probability Distributions

Instead of distribution over single variable, we can model distribution over multiple variables, separated by comma

E.g., **P**(A, B) = **P**(A | B) . **P**(B)

P(A, B) is the probability distribution over combination of all values of A and B

E.g., if A = Weather and B = Cavity

$$
\begin{aligned}
P(W = sunny \land C = true) &= P(W = sunny|C = true)\, P(C = true)\\
P(W = rain \land C = true) &= P(W = rain|C = true)\, P(C = true)\\
P(W = cloudy \land C = true) &= P(W = cloudy|C = true)\, P(C = true)\\
P(W = snow \land C = true) &= P(W = snow|C = true)\, P(C = true)\\
P(W = sunny \land C = false) &= P(W = sunny|C = false)\, P(C = false)\\
P(W = rain \land C = false) &= P(W = rain|C = false)\, P(C = false)\\
P(W = cloudy \land C = false) &= P(W = cloudy|C = false)\, P(C = false)\\
P(W = snow \land C = false) &= P(W = snow|C = false)\, P(C = false)\, .
\end{aligned}
$$

Probabilistic Inference

Computation of posterior probabilities given observed evidence, i.e., full joint probability distribution

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Query: P(cavity ∨ toothache)

0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28

Conditional Probability

Towards Chain Rule:

P(a | b) = P(a,b) / P(b)

P(a, b) = P(a | b) P(b)

P(a, b , c) = P(a , x)   where x= b,c

P(a,x)   = P(a | x) . P(x)

= P(a | bc) . P(b, c)

= P(a | bc) . P(b | c). P(c)

Hence : P(a,b,c) = P(a | bc) . P(b | c). P(c)

Chain Rule : Generalization

$$\mathbf{P(X_1, X_2, \ldots\ X_K) = \prod P(X_i \mid X_{i-1}, \ldots X_1)}$$

Where i = k to 1 (reverse)

Probability Theory

<u>Independence</u>

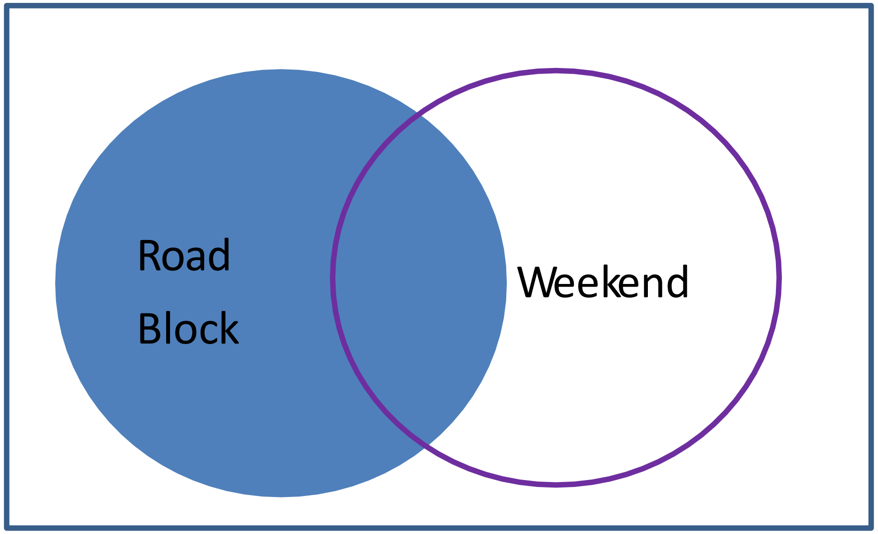
P(a | b) = P(a)

Implication:

P(a | b) = P(a,b) / P(b)

P(a) = P(a,b) / P(b)

P(a,b) = P(a) . P(b)



<u>Conditional Independence</u>

P(a | b c) = P(a | c)

Probability Theory

<u>Conditional Independence</u>

P(a | b c) = P(a | c)

Extension:

P(a b | c) = P(a | c) . P(b | c)

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1, #9

Next Session Plan:

- (Prerequisite Reading : Refresh the basics of probability , Bayes Theorem , Conditional Probability, Product Rule, Conditional Independence, Chain Rule)

- Inferences using Logic (Forward / Backward Chaining / DPLL algorithm)
- Bayesian Network
- Representation
- Inferences (Exact and approximate-only Direct sampling)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials