TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES [21MAT31]

— TIE REVIEW TEAM

Module - 01

01. Find the Laplace Transform of the following:

(i) $\dfrac{\cos 2t - \cos 3t}{t}$ (ii) $\dfrac{\cos at - \cos bt}{t}$

(iii) $t^2 e^{2t} \cos 2t$ (iv) $\dfrac{1 - \cos t}{t^2}$

02. Evaluate $\int_0^\infty e^{-t} t^3 \sin t \, dt$ using Laplace Transforms

03. Find the Laplace Transform of the periodic function $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$

Hence show that $L\{f(t)\} = \dfrac{1}{s^2} \tanh\left(\dfrac{as}{2}\right)$

04. Find the inverse Laplace transforms of:

(i) $\dfrac{4s + 5}{(s-1)^2 (s+2)}$ (ii) $\dfrac{s + 3}{s^2 - 4s + 13}$

(iii) $\dfrac{s^2 + 4}{s(s+4)(s-4)}$ (iv) $\log\left[\dfrac{s^2 + 1}{s(s+1)}\right]$

05. Obtain the inverse Laplace transform using Convolution theorem:

(i) $L^{-1}\left\{\dfrac{1}{(s^2 + a^2)^2}\right\}$ (ii) $L^{-1}\left\{\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}$

(iii) $L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$ (iv) $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)^2}\right\}$

06. Solve the differential equation $y''+4y'+4y=e^{-t}$ given $y(0)=0,\ y'(0)=0$ using Laplace transforms.

Module - 02

01. Find the fourier series of $f(x)=|x|$ in $-\pi<x<\pi$. Hence deduce that $\dfrac{\pi^2}{8}=\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}$

02. Find the fourier series in $(-\pi,\pi)$ to represent $f(x)=x-x^2$. Hence deduce that

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

03. Obtain the fourier expansion of the function $f(x)=\dfrac{\pi-x}{2}$ $0<x<2\pi$. Hence deduce that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$$

04. Obtain the fourier series for the function

$$f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

Deduce that $\dfrac{\pi^2}{8}=\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}$

05. Find the cosine half range fourier series for $f(x)=x(l-x)$ in $0<x<l$.

06. Find the cosine half range Fourier series for $f(x) = x\sin x$ in $0 < x < \pi$. Deduce that

$$\frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \frac{1}{7\cdot9} + \cdots = \frac{\pi-2}{4}.$$

07. Obtain the sine half range series of

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$$

08. Obtain the constant term and first sine and cosine terms in the fourier expansion of y from the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

09. Express y as a Fourier series upto first harmonic given:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

10. The following table gives the variations of periodic current over a period. Show that there is a direct current part of 0.75 amp and obtain amplitude of the first harmonic.

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Module-03

01. Find the Fourier transform of the following:

(i) $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$ Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

(ii) $f(x) = e^{-|x|}$, hence evaluate $\int_0^\infty \frac{\cos t}{1+t^2}\,dt$

(iii) $f(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence

deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$

(iv) $f(x) = \begin{cases} 1-x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$, Hence evaluate

$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\,dx$, $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx$

(v) $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$, Find the fourier cosine transform.

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02. Find the fourier sine and cosine transform of $f(x) = e^{-ax}$, $a > 0$. Hence deduce that

$\int_0^\infty \frac{\cos mx}{a^2+x^2}\,dx = \frac{\pi}{2a} e^{-am}$

03. Find the fourier sine transform of the function $f(x) = e^{-|x|}$, hence evaluate

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx, \quad m>0$$

04. Solve the integral equation $\int_0^\infty f(x)\cos\alpha x\,dx = \begin{cases} 1-\alpha & \text{for } 0 \le \alpha \le 1 \\ 0 & \text{for } \alpha > 1 \end{cases}$ and hence deduce

the value of $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt$

05. Find the inverse fourier sine transform of

$$\frac{e^{-as}}{s}, \quad a>0$$

06. Find the z-transform of the following:

(i) $\cosh n\theta$ (ii) $\sinh n\theta$ (iii) $\cos(n\theta)$ and $\sin(n\theta)$

07. Find the inverse z-transform of:

(i) $\dfrac{2z^2+3z}{(z+2)(z-4)}$ (ii) $\dfrac{4z^2-2z}{z^3-5z^2+8z-4}$

08. Using z-transform, solve the following:

(i) $u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = 0$, $u_1 = 0$

(ii) $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$

(iii) $u_{n+2} - 4u_n = 0$ with $u_0 = 0$, $u_1 = 2$

Module - 04

01. Classify the following Partial differential equations:

(i) $u_{xx} + 4u_{xy} + 4u_{xy} - u_x + 2u_y = 0$

(ii) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$

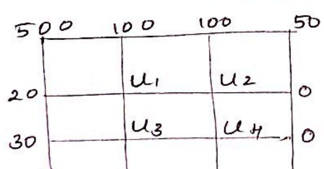
02. Solve numerically $u_{xx} = 0.0625\, u_{tt}$, subject to the conditions $u(0,t) = 0 = u(5,t)$, $u(x,0) = x^2(x-5)$ and $u_t(x,0) = 0$ by taking $h = 1$ for $0 \leq t \leq 1$.

03. Solve $u_{xx} = 32 u_t$ subject to the conditions $u(0,t) = 0$, $u(1,t) = t$ and $u(x,0) = 0$. Find the values of u upto $t = 5$ by schmidt's process taking $h = \frac{1}{4}$. Also extract the following values:

(a) $u(0.75, 4)$ (b) $u(0.5, 5)$ (c) $u(0.25, 4)$

04. Solve numerically the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to the conditions $u(0,t) = 0 = u(1,t)$, $t \geq 0$ and $u(x,0) = \sin(\pi x)$, $0 \leq x \leq 1$. Carryout computations for two levels taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$.

05. Solve $u_{xx} + u_{yy} = 0$ in the following square region with the boundary conditions as indicated:



Module - 05

01. Apply fourth order Runge-Kutta method to:

(i) find $y(0.1)$ for an IVP $y'' - x^2 y' - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$

(ii) find $y(0.2)$ for an IVP $y'' - xy' - y = 0$, $y(0) = 1$, $y'(0) = 0$

(iii) solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$, $y(0) = 1$, $y'(0) = 0$. Evaluate $y(0.2)$ to four decimal places.

02. Apply Milne's method to:

(i) find $y(0.8)$ given that $y'' = 1 - 2yy'$

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(ii) find $y(0.4)$ for the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6x = 0$ given that $y(0) = 1$, $y(0.1) = 1.03995$, $y(0.2) = 1.138036$, $y(0.3) = 1.29865$, $y'(0) = 0.1$, $y'(0.1) = 0.6955$, $y'(0.2) = 1.258$, $y'(0.3) = 1.873$.

03. Apply Milne's predictor-corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

04. Given the differential equation

$$2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$$ and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

05. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

06. Find the curves on which the functional $\int_0^1 [(y')^2 + 12xy]dx$ with $y(0)=0$ and $y(1)=1$ can be extremized.

07. Show that the geodesics on a plane are straight lines.

(or)

Prove that shortest distance between two points in a plane is along the straight line joining them.