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① $\begin{cases} x_1' = (x_2)^2 \\ x_2' = -2(x_1)^3 x_2 \end{cases}$

a) $F(t, x_1, x_2) = x_1^4 + x_2^2$ integrală primă

b) Reducerea dimensiunii sistemului.

a) $\frac{\partial F}{\partial t} + \frac{\partial F}{\partial x_1} \cdot f_1(t,x) + \frac{\partial F}{\partial x_2} f_2(t,x) = 0 + 4(x_1)^3 \cdot (x_2)^2 + 2x_2 \cdot (-2(x_1)^3 x_2)$

$= 4x_1^3 x_2^2 - 4x_1^3 x_2^2 = 0 \Rightarrow$

$\Rightarrow$ F integrală primă $\Rightarrow$ $\underline{x_1^4 + x_2^2 = C_1}$, $C_1 \in \mathbb{R}$

b) $x_2^2 = C_1 - x_1^4 \Rightarrow \underline{x_1' = C_1 - x_1^4}$.

ec. cu var. separabile.
(am redus de la 2 dimensiuni (x_1, x_2) la o ec. scalară pt x_1, deducă din 1)

② Fie sistemul $\begin{cases} x' = 4x - y \\ y' = 3x + y - z \\ z' = x + z \end{cases}$

Se cere S_A.

$$A = \begin{pmatrix} 4 & -1 & 0 \\ 3 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\det(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} 4-\lambda & -1 & 0 \\ 3 & 1-\lambda & -1 \\ 1 & 0 & 1-\lambda \end{vmatrix} = 0.$$

$\Rightarrow (4-\lambda)(1-\lambda)^2 + 0 + 1 - 0 - 0 + 3(1-\lambda) = 0$

$(4-\lambda)(1-2\lambda+\lambda^2) + 1 + 3 - 3\lambda = 0$

$4 - \underline{8\lambda} + 4\lambda^2 - \underline{\lambda} + 2\lambda^2 - \lambda^3 + 4 - \underline{3\lambda} = 0$

$-\lambda^3 + 6\lambda^2 - 12\lambda + 8 = 0$

$(-\lambda+2)^3 = 0 \Rightarrow \boxed{\lambda_1 = 2, \ m_1 = 3} \Rightarrow$

$\Rightarrow$ determinăm $p_0, p_1, p_2 \in \mathbb{R}^3$, nu toți nuli a.î.

$$\varphi(t) = \underline{(p_0 + p_1 t + p_2 t^2) \cdot e^{\lambda t}} \quad \text{să verifica } X' = AX \Rightarrow X' = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\Rightarrow \varphi'(t) = A\varphi(t) \Rightarrow (p_1 + 2p_2 t) e^{2t} + (p_0 + p_1 t + p_2 t^2) e^{2t} \cdot 2 = A \cdot (p_0 + p_1 t + p_2 t^2) e^{2t}$$

$$/ : e^{2t} \Rightarrow (p_1 + 2p_2 t) + (p_0 + p_1 t + p_2 t^2) 2 = Ap_0 + Ap_1 t + Ap_2 t^2 \Rightarrow$$

$$\Rightarrow (p_1 + 2p_0) + (2p_2 + 2p_1) t + 2p_2 t^2 = Ap_0 + Ap_1 t + Ap_2 t^2 \Rightarrow$$

Identif. coef $\Rightarrow$
$$\begin{cases} p_1 + 2p_0 = Ap_0 \Rightarrow p_1 = (A - 2I_3) p_0 \Rightarrow \\ 2p_2 + 2p_1 = Ap_1 \Rightarrow 2p_2 = (A - 2I_3) p_1 \\ 2p_2 = Ap_2 \qquad 0_{\mathbb{R}^3} = (A - 2I_3) p_2 \end{cases}$$

$$\Rightarrow \underbrace{(A - 2I_3) p_1}_{2p_2} = (A - 2I_3)^2 p_0 \Rightarrow 2p_2 = (A - 2I_3)^2 p_0 \Rightarrow$$

$$\Rightarrow 2 \underbrace{(A - 2I_3) p_2}_{= 0_{\mathbb{R}^3}} = (A - 2I_3)^3 p_0 \Rightarrow (A - 2I_3)^3 p_0 = 0_{\mathbb{R}^3}$$

Dacă considerăm aplicația liniară:

$$f : \mathbb{R}^3 \to \mathbb{R}^3$$

$$f(v) = (A - 2I_3)^3 v,$$

atunci $p_0 \in \ker(f) = \{ v \mid f(v) = 0 \}$.

$$(A - 2I_3)^3 v = 0.$$

$$A - 2I_3 = \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}$$

$$(A - 2I_3)^2 = \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$(A - 2I_3)^3 = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_3 \Rightarrow$$

$\Rightarrow f(v) = O_3 v = 0_{\mathbb{R}^3}$, $\forall v \in \mathbb{R}^3$ $\Rightarrow \ker((A - 2I_3)^3) = \mathbb{R}^3$, dar $p_0 \in \ker((A - 2I_3)^2)$ $\Rightarrow$ pt p_0 luăm elem. unei baze în $\mathbb{R}^3$

Luăm $p_0 \in \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

I) $p_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow p_1 = (A - 2I_3)p_0 = \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$p_2 = \frac{1}{2}(A - 2I_3)p_1 = \frac{1}{2} \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} =$

$= \frac{1}{2} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} \Rightarrow$

$\Rightarrow \varphi_1(t) = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} t + \begin{pmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} t^2 \right) \cdot e^{2t}$

$$\boxed{\varphi_1(t) = \begin{pmatrix} (1 + 2t + \frac{1}{2}t^2)e^{2t} \\ (3t + t^2)e^{2t} \\ (t + \frac{1}{2}t^2)e^{2t} \end{pmatrix}}$$

II) $p_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow p_1 = \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

$p_2 = \frac{1}{2} \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -1 \\ -\frac{1}{2} \end{pmatrix}$

$\varphi_2(t) = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} t + \begin{pmatrix} \frac{1}{2} \\ -1 \\ -\frac{1}{2} \end{pmatrix} t^2 \right) e^{2t} \Rightarrow$

$$\boxed{\varphi_2(t) = \begin{pmatrix} (-t + \frac{1}{2}t^2)e^{2t} \\ (1 - t - t^2)e^{2t} \\ -\frac{1}{2}t^2 e^{2t} \end{pmatrix}}$$

III) $p_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow p_1 = \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

$p_2 = \frac{1}{2} \begin{pmatrix} 2 & -1 & 0 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix}$

$\varphi_3(t) = \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} t + \begin{pmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} t^2 \right) e^{2t} \Rightarrow \boxed{\varphi_3(t) = \begin{pmatrix} \frac{1}{2}t^2 e^{2t} \\ (-t + t^2)e^{2t} \\ (1 - t + \frac{1}{2}t^2)e^{2t} \end{pmatrix}}$

$S_A = \{ c_1\varphi_1 + c_2\varphi_2 + c_3\varphi_3 \mid c_1, c_2, c_3 \in \mathbb{R} \}.$

verificăm că $\det(\varphi_1, \varphi_2, \varphi_3) \neq 0$

Avem
$$e^{2t}\cdot e^{2t} e^{2t} \begin{vmatrix} 1+2t+\frac{1}{2}t^2 & -t+\frac{1}{2}t^2 & \frac{1}{2}t^2 \\ 3t+t^2 & 1-t-t^2 & -t+t^2 \\ t+\frac{1}{2}t^2 & -\frac{1}{2}t^2 & 1-t+\frac{1}{2}t^2 \end{vmatrix} = \quad \begin{matrix} L1 \leftarrow L1 - L3 \\ L2 \leftarrow L2 - L3\cdot 2 \end{matrix}$$

$$= e^{6t} \begin{vmatrix} 1+t & -t+t^2 & t^2+t-1 \\ t & 1-t & -1+t \\ t+\frac{1}{2}t^2 & -\frac{1}{2}t^2 & 1-t+\frac{1}{2}t^2 \end{vmatrix} \neq 0$$
(temă!).

③ Fie sistemul
$$\begin{cases} x' = 2x+4y + t \\ y' = 4x+2y + e^t \end{cases}$$

Se cere soluția generală (mulțimea soluțiilor)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} t \\ e^t \end{pmatrix}}_{b(t)}$$

- integrăm sistemul liniar omogen asociat:

$$\begin{pmatrix} \bar{x}' \\ \bar{y}' \end{pmatrix} = A \underbrace{\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}}_{X} \Rightarrow X' = AX \quad (1)$$

Știm că (sem. 7) $\Rightarrow$ $\begin{cases} \varphi_1(t) = \begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix} \\ \varphi_2(t) = \begin{pmatrix} e^{6t} \\ e^{6t} \end{pmatrix} \end{cases}$

formează sist. fundam. de soluții pt (1)

$\rightarrow$ $\underline{\phi(t) = \text{coloane}\,(\varphi_1(t), \varphi_2(t))}$ = matricea fundam. de soluții

$$\phi(t) = \begin{pmatrix} e^{-2t} & e^{6t} \\ -e^{-2t} & e^{6t} \end{pmatrix}$$

$\phi'(t) = \text{coloane}\,(\varphi_1'(t), \varphi_2'(t))$
$= \text{coloane}\,(A\varphi_1(t), A\varphi_2(t))$
$= A\,\text{coloane}\,(\varphi_1(t), \varphi_2(t))$
$= A\phi(t)$

Știm că: $\phi'(t) = A\phi(t)$

$$S_A = \{\varphi(t) = \phi(t)c \mid c \in \mathbb{R}^2\}.$$

mult. sol pt (1)

- met. variației const: det $c : \mathbb{R} \to \mathbb{R}^2$ a.î.

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$\varphi(t) = \phi(t)C(t)$ se verifica sist:

$$X' = AX + b(t) \Rightarrow$$

$$\Rightarrow (\phi(t)C(t))' = A\phi(t)C(t) + b(t) \Rightarrow$$

$$\Rightarrow \underbrace{\phi'(t)}_{A(t)\phi(t)}C(t) + \phi(t)C'(t) = A\phi(t)\cancel{C(t)} + b(t)$$

$$\phi(t)C'(t) = b(t) \Rightarrow \boxed{C'(t) = (\phi(t))^{-1}b(t)}$$

$$\det\phi(t) = e^{-2t}e^{6t} + e^{-2t}e^{6t} = e^{4t} + e^{4t} = 2e^{4t} \neq 0 \quad \forall t \in \mathbb{R}$$

$$^{T}(\phi(t)) = \begin{pmatrix} e^{-2t} & -e^{-2t} \\ e^{6t} & e^{6t} \end{pmatrix} \Rightarrow (\phi(t))^{*} = \begin{pmatrix} e^{6t} & -e^{6t} \\ e^{-2t} & e^{-2t} \end{pmatrix}$$

$$(\phi(t))^{-1} = \frac{1}{\det\phi(t)}(\phi(t))^{*} = \frac{1}{2e^{4t}}\begin{pmatrix} e^{6t} & -e^{6t} \\ e^{-2t} & e^{-2t} \end{pmatrix} \Rightarrow$$

$$\Rightarrow C'(t) = \frac{1}{2e^{4t}}\begin{pmatrix} e^{6t} & -e^{6t} \\ e^{-2t} & e^{-2t} \end{pmatrix}\begin{pmatrix} t \\ e^{t} \end{pmatrix}$$

$$\begin{cases} C_1'(t) = \frac{1}{2e^{4t}}(te^{6t} - e^{7t}) \\ C_2'(t) = \frac{1}{2e^{4t}}(te^{-2t} + e^{-t}) \end{cases}$$

$$\begin{cases} C_1'(t) = \frac{1}{2}(te^{2t} - e^{3t}) \\ C_2'(t) = \frac{1}{2}(te^{-6t} + e^{-5t}) \end{cases}$$

$$C_1(t) = \frac{1}{2}\left(\int t\left(\frac{e^{2t}}{2}\right)'dt - \frac{e^{3t}}{3}\right) =$$

$$= \frac{1}{2}\left(t\frac{e^{2t}}{2} - \frac{1}{2}\int e^{2t}dt - \frac{e^{3t}}{3}\right) \Rightarrow$$

$$\Rightarrow \boxed{C_1(t) = \frac{1}{2}\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4} - \frac{e^{3t}}{3}\right) + k_1}$$

$$C_2(t) = \frac{1}{2}\left(\int t\left(\frac{e^{-6t}}{-6}\right)'dt + \frac{e^{-5t}}{-5}\right) =$$

$$= \frac{1}{2}\left(\frac{te^{-6t}}{-6} + \frac{1}{6}\int e^{-6t}dt - \frac{e^{-5t}}{5}\right) \Rightarrow$$

$$\Rightarrow \boxed{C_2(t) = \frac{1}{2}\left(-\frac{te^{-6t}}{6} - \frac{e^{-6t}}{36} - \frac{e^{-5t}}{5}\right) + k_2}, \quad k_1, k_2 \in \mathbb{R}.$$

Sol. sist. din (3)

$$S_{A,b} = \left\{ X(t) = \begin{pmatrix} e^{-2t} & e^{6t} \\ -e^{-2t} & e^{6t} \end{pmatrix} \begin{pmatrix} \frac{1}{2}\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4} - \frac{e^{3t}}{3} \right) + K_1 \\ \frac{1}{2}\left(-\frac{te^{-6t}}{6} - \frac{e^{-6t}}{36} - \frac{e^{-5t}}{5} \right) + K_2 \end{pmatrix} \quad K_1, K_2 \in \mathbb{R} \right\}$$

Temă: Se cere soluția generală a sistemelor:

(4) $\begin{cases} x_1' = x_2 + 2e^t \\ x_2' = x_1 \end{cases}$ (5) $\begin{cases} x_1' = x_1 - x_2 + 2\sin t \\ x_2' = 2x_1 - x_2 \end{cases}$

(6) $\begin{cases} x_1' = -x_1 + x_2 - 2x_3 \\ x_2' = 4x_1 + x_2 \\ x_3' = 2x_1 + x_2 - x_3 \end{cases} + e^{-t}$

(7) Fie sistemul $\begin{cases} x_1' = x_2 \cdot 3t^2 \\ x_2' = x_1 \cdot 3t^2 \end{cases}$ (1)

a) Arătați că prin s.v. $t^3 = s$ se obține sistemul $\begin{cases} y_1' = y_2 \\ y_2' = y_1 \end{cases}$ (2)

b) Determinați mulț. sol. sist (2) și apoi soluțiile sistemului (1).
Precizați un sistem fundam. de soluții pt. (1).

(8) Fie sistemul $\begin{cases} x_1' = -\dfrac{x_1 + 2x_2}{t} + t\cos t \\ x_2' = \dfrac{3x_1 + 4x_2}{t} \end{cases}, t > 0$ (3)

a) Arătați că prin s.v. $t = e^s$ se obține un sistem (4) cu matrice constantă.

b) Determinați mulț. sol. sist. (4) apoi a sist (3).

(8) $t = e^s \Leftrightarrow s = \ln t$, $t > 0$.

$(t, x) \xrightarrow{\quad} (s, y)$

$\boxed{x(t) = y(s(t))}$

$x'(t) = y'(s) \cdot s'(t) = y'(s) \cdot \frac{1}{t} \Rightarrow t x' = y' \Rightarrow \begin{cases} t x_1' = y_1' \\ t x_2' = y_2' \end{cases}$

Din (3) $\Rightarrow \begin{cases} t x_1' = -x_1 - 2x_2 + t^2 \cos t \\ t x_2' = 3x_1 + 4x_2 . \end{cases}$

Aplicăm s.v $\Rightarrow \begin{cases} y_1' = -y_1 - 2y_2 + e^{2s} \cdot \cos e^s \\ y_2' = 3y_1 + 4y_2 \end{cases} \Rightarrow$

$$\Rightarrow \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix}}_{A} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \underbrace{\begin{pmatrix} e^{2s} \cos e^s \\ 0 \end{pmatrix}}_{b(s)}$$

∘ rez sist liniar omogen atașat:

$$\begin{pmatrix} \bar{y}_1' \\ \bar{y}_2' \end{pmatrix} = A \begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \end{pmatrix} \quad ; \quad \bar{y}' = A\bar{y}$$

$$\det(A - \lambda I_2) = 0 \Rightarrow \begin{vmatrix} -1-\lambda & -2 \\ 3 & 4-\lambda \end{vmatrix} = 0 \Rightarrow$$

$\Rightarrow (-1-\lambda)(4-\lambda) + 6 = 0$

$-4 + \lambda - 4\lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda_1 = 1, \ m_1 = 1$

$\lambda_2 = 2, \ m_2 = 1$

$\boxed{\lambda_1 = 1, m_1 = 1} \Rightarrow Au = \lambda_1 u, \quad u \neq 0_{\mathbb{R}^2}$

$$\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \Rightarrow \begin{cases} -u_1 - 2u_2 = u_1 \\ 3u_1 + 4u_2 = u_2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2u_2 = -2u_1 \\ 3u_2 = -3u_1 \end{cases} \Rightarrow u_2 = -u_1 \Rightarrow u = u_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \boxed{\varphi_1(s) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^s}$$

$\boxed{\lambda_2 = 2, m_1 = 1} \Rightarrow Au = \lambda_2 u, \quad u \neq 0_{\mathbb{R}^2}$

$$\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 2u_1 \\ 2u_2 \end{pmatrix} \Rightarrow \begin{cases} -u_1 - 2u_2 = 2u_1 \\ 3u_1 + 4u_2 = 2u_2 \end{cases}$$

$$\Rightarrow \begin{cases} 2u_2 = -3u_1 \\ 2u_2 = -3u_1 \end{cases} \Rightarrow u = \begin{pmatrix} u_1 \\ -\frac{3}{2}u_1 \end{pmatrix} = \frac{u_1}{2} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\Rightarrow \boxed{\varphi_2(s) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} e^{2s}}$$

$\bar{y}(s) = \phi(s)C$; $\phi(s) = \begin{pmatrix} e^s & 2e^{2s} \\ -e^s & -3e^{2s} \end{pmatrix}$

Met. var const $\Rightarrow$ $\boxed{y(s) = \phi(s)C(s)}$ $\Rightarrow$

$\xrightarrow{(y = Ay + b(s))}$ $\phi(s)C'(s) = b(s) \Rightarrow$

$\Rightarrow \begin{pmatrix} e^s & 2e^{2s} \\ -e^s & -3e^{2s} \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} e^{2s}\cos e^s \\ 0 \end{pmatrix}$

$\Rightarrow \begin{cases} e^s c_1' + 2e^{2s} c_2' = e^{2s}\cos(e^s) \\ -e^s c_1' - 3e^{2s} c_2' = 0 \end{cases}$ (sistem în c_1', c_2')

$-e^{2s} c_2' = e^{2s}\cos(e^s) \Rightarrow \boxed{c_2' = \cos(e^s)}$

$-e^s c_1' = 3e^{2s} c_2' \xrightarrow{(:e^s)} \boxed{c_1' = -3e^s \cdot \cos(e^s)}$

$c_1 = -3\int (e^s)' \cos(e^s)\,ds = -3\sin(e^s) + K_1$

$\boxed{c_1(s) = -3\sin(e^s) + K_1}$

$\boxed{c_2 = \int \cos(e^s)\,ds = \int_0^s \cos(e^u)\,du + K_2}$

$\boxed{c_2(s) = \int_0^s \cos(e^u)\,du + K_2}$

b) Sist. fundam de soluții pt partea omogenă la (3) este $\begin{cases} \psi_1(t) = \varphi_1(s(t)) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} t \\ \psi_2(t) = \varphi_2(s(t)) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} t^2. \end{cases}$

Temă: Avem $\Phi_1(t) = \begin{pmatrix} t & 2t^2 \\ -t & -3t^2 \end{pmatrix}$ matr. fundam de sol pt partea omogenă la (3)

Efectuați met var. const în sist. (3).