

$$b) P_{JB}(A=1, B=1, Y=0) = P(A=1, B=1 | Y=0) \cdot P(Y=0)$$

$$P_{JB}(A=1, B=1, Y=1) = P(A=1, B=1 | Y=1) \cdot P(Y=1)$$

Ne uitem la tabete

$$\begin{aligned} &= \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5} \\ &- 0 \cdot \frac{2}{5} = 0 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow Y_{JB} = 0.$$

45, 46, 47 a,b, 52, prob de la arbore do decizie

4 probleme de la Clasificare Bayesiană

A	B	C	Y
0	0	1	0
0	1	0	0
1	1	0	0
0	0	1	1
1	1	1	1
1	0	0	1
1	1	0	1

Clasificare ($A=0, B=0, C=1$)
cu Bayes Naiv.

rez;

$$\text{Rez: } Y_{\text{map}} = \max_{y \in \{0,1\}} P(Y=y | A=0, B=0, C=1)$$

$$\stackrel{\text{FB}}{=} \max_{y \in \{0,1\}} \frac{P(A=0, B=0 | Y=y) \cdot P(Y=y)}{P(A=1, B=1 | Y=1)}$$

↳ eerst > 0 in re

$$= \max_{y \in \{0,1\}} P(A=0, B=0 | Y=y) \cdot P(Y=y)$$

$$\stackrel{\text{PP}}{=} \frac{P(A=0 | Y=1) \cdot P(B=0 | Y=1) \cdot P(Y=1)}{P(A=1 | Y=1) \cdot P(B=1 | Y=1)}$$

$$P_0 = P(A=0|Y=0) \cdot P(B=0|Y=0) \cdot P(C=1|Y=0) \cdot P(Y=0)$$

$$\stackrel{\text{MLE}}{=} \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{7} = \frac{4}{63} = 0,0317$$

$$P_1 = P(A=0|Y=1) \cdot P(B=0|Y=1) \cdot P(C=1|Y=1) \cdot P(Y=1)$$

$$\stackrel{\text{MLE}}{=} \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16} = 0,0625$$

$$P_0 < P_1 \Rightarrow Y_{NB} = 1$$

3.26. a) $X = \{ \text{Outlook=sunny}, \text{Temp=cold}, \text{Humidity=high}, \text{Wind=strong} \}$

$$YT_{\text{map}} = \max_{y \in \{ \text{Yes}, \text{No} \}} P(YT=y | O=s, T=c, H=h, W=s)$$

$$\stackrel{\text{F.B.}}{=} \max_{y \in \{ \text{Yes}, \text{No} \}} \frac{P(O=s, T=c, H=h, W=s | YT=y) \cdot P(YT=y)}{P(O=s, T=c, H=h, W=s)}$$

const > 0 in arg max

$$= \max_{y \in \{ \text{Yes}, \text{No} \}} P(O=s, T=c, H=h, W=s | YT=y) \cdot P(YT=y)$$

$$\stackrel{\text{PP}}{=} \max_{y \in \{ \text{Yes}, \text{No} \}} P(O=s | YT=y) \cdot P(T=c | YT=y) \cdot P(H=h | YT=y) \cdot P(W=s | YT=y) \cdot P(YT=y)$$

$$P_{\text{Pos}}^{\text{MLE}} = \frac{2}{9} \cdot \frac{3}{9} \cdot \frac{3}{9} \cdot \frac{3}{9} = \frac{6}{729} = 0,0082$$

$$= P(O=s | YT=\text{Yes}) \cdot P(T=c | YT=\text{Yes}) \cdot P(H=h | YT=\text{Yes}) \cdot P(W=s | YT=\text{Yes})$$

$$P_{NO} = \frac{3}{5} \cdot \frac{1}{5} - \frac{4}{5} \cdot \frac{3}{5} = \frac{36}{625} = 0,0576$$

$$= P(O=S \mid T=NO) \cdot P(T=c \mid T=NO) \cdot P(H=h \mid T=NO) \cdot P(XH=s \mid T=NO) \\ \cdot P(YT=NO)$$

$$P_{\text{Yes}} < P_{\text{No}} \Rightarrow \underline{Y_{AB} = \text{No}}$$

	O	T	H	W	ET	
O		h	h	w	y	
R		m	h	w	yy	
C	c	n	n	w	yy	
S	c	n	n	s	yy	
R	c	n	n	w	yy	
S	v	n	n	s	yy	
R	v	m	m	s	yy	
O	o	n	n	w	yy	
S	h	h	w		N	
S	h	h	s		N	
R	c	n	s		N	
S	m	h	w		N	
R	m	h	s		(N)	

NB: 9 estimations
1 + 4 + 4

ET: $\begin{pmatrix} y \\ \frac{9}{14} \\ \frac{5}{14} \end{pmatrix}$

MSE

~~1~~) 8 Attribute

Bayes Naïv - klassifiziere pt (1, 0, 0, 1, 1, 1, 1, 0)

D-dominante

$$D_{\text{map}} = \max_{d \in \{P, S\}} P(D=d | A=1, B=0, C=0, D=1, E=1, F=1, G=1, H=0)$$

$$\stackrel{FB}{=} \max_{d \in \{P, S\}} \frac{P(A=1, B=0, C=0, D=1, E=1, F=1, G=1, H=0 | D=d) \cdot P(D=d)}{(P(A=1, B=1, C=1, D=1, E=1, F=1, G=1, H=1))}$$

↳ constants ≥ 0 in rep
on D.

$$= \max_{d \in \{P, S\}} P(A=1, B=0, C=0, D=1, E=1, F=1, G=1, H=0 | D=d) \cdot P(D=d)$$

$$PP = \max_{d \in \{P, S\}} P(A=1 | D=d) \cdot P(B=0 | D=d) \cdot P(C=0 | D=d) \cdot P(D=1 | D=d) \cdot \\ \cdot P(E=1 | D=d) \cdot P(F=1 | D=d) \cdot P(G=1 | D=d) \cdot P(H=0 | D=d) \cdot P(D=d)$$

$$PP = P(A=1 | D=P) \cdot P(B=0 | D=P) \cdot \dots \cdot P(H=0 | D=P) \cdot P(D=P) \\ = \frac{2}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{1}{6} \cdot \frac{8}{13} \\ = 0,00137$$

$$PS = P(A=1 | D=S) \cdot P(B=0 | D=S) \cdot \dots \cdot P(H=0 | D=S) \cdot P(D=S) \\ = \frac{5}{7} \cdot \frac{2}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{2}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{6}{7} \cdot \frac{4}{13} \\ = 0,00001868$$

$$\Rightarrow \text{if } p_p > p_s \Rightarrow \gamma_{NB} = p \quad \checkmark$$

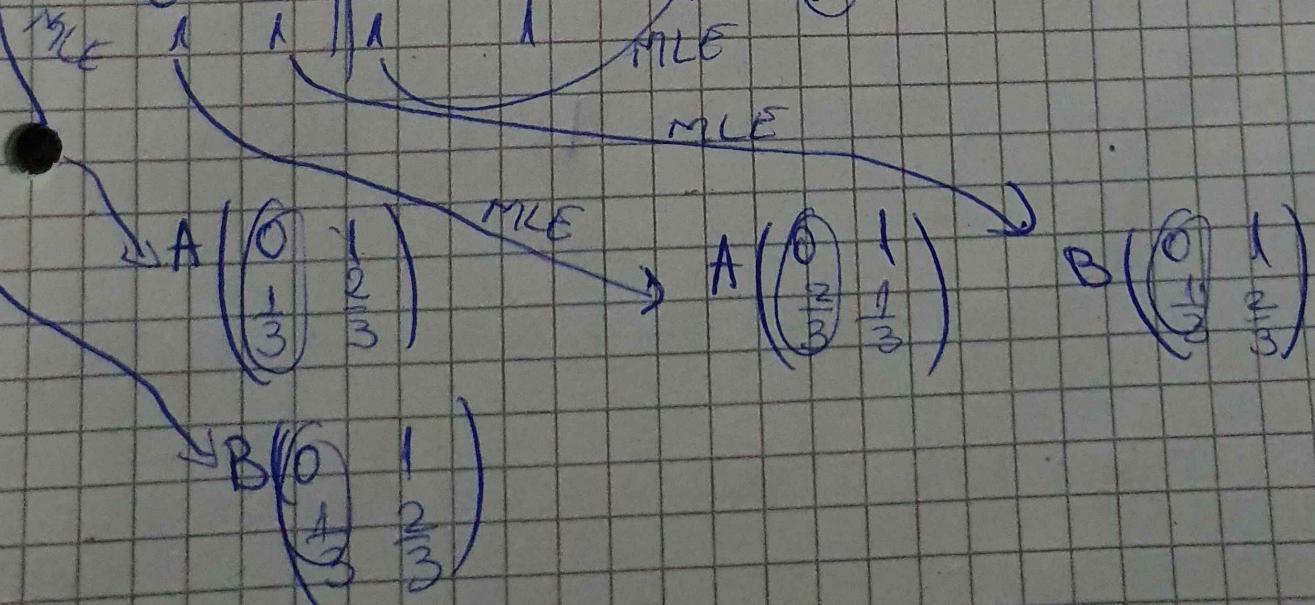
p_p is the classified elastic polyp =

$$= 0,0001777.$$

$$= 0,00137$$

	A	B	C	n.r. op
	0	0	1	3
	0	1	0	1
	0	1	1	4
	1	0	0	5
	1	1	0	2
	1	1	1	1

	A	B	C	n.r. op
	0	1	0	1
	1	0	0	5
	1	1	0	2
	0	0	1	3
	0	1	1	4
	1	1	1	1



a) 5 estimāri pt Bayes Natv

	$B=0$	$B=1$
$A=0$	(0)	($\frac{1}{3}$)
$A=1$	($\frac{1}{3}$)	($\frac{1}{3}$)

pt prima parte.

	$B=0$	$B=1$
$A=0$	($\frac{1}{3}$)	($\frac{1}{3}$)
$A=1$	(0)	($\frac{1}{3}$)

pt a doua parte

6 estimări pt Bayes Optimal.

c) Bayes Naïv - clasificare ($A=0, B=1$)

$$C_T = \max_{c \in \{0, 1\}} P(C=c | A=0, B=1)$$

$$= \max_{c \in \{0, 1\}} \frac{P(A=0, B=1 | C=c) \cdot P(C=c)}{P(A=0, B=1)}$$

$P(A=0, B=1)$ → constantă
pe n raport

$$= \max_{c \in \{0, 1\}} P(A=0, B=1 | C=c) \cdot P(C=c)$$

$$\stackrel{\text{PP:ind}}{=} P(A=0 | C=c) \cdot P(B=1 | C=c) \cdot P(C=c)$$

$$P_0 = P(A=0 | C=0) \cdot P(B=1 | C=0) \cdot P(C=0)$$

$$\text{MLE } \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} = 0,2222 \cdot \frac{1}{2} = \underline{0,1111}$$

$$P_1 = P(A=0 | C=1) \cdot P(B=1 | C=1) \cdot P(C=1)$$

$$\stackrel{MCE}{=} \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{4}{9} = 0,4444 \cdot \frac{1}{2} = \underline{\underline{0,22222}}$$

$$C_0 < C_1 \Rightarrow C_{NB} = 1$$

d) Classtf $(A=0, B=1)$ eu Bayes Optimal.

$$c_{\text{opt}} = \arg \max_{c \in \{0,1\}} P(C=c | A=0, B=1)$$

$$\stackrel{FB}{=} \arg \max_{c \in \{0,1\}} \frac{P(A=0, B=1 | c=c) \cdot P(c=c)}{P(A=0, B=1)}$$

\hookrightarrow const in c

$$= \arg \max_{c \in \{0,1\}} P(A=0, B=1 | c=c) \cdot P(c=c)$$

$$C_0 = P(A=0, B=1 | C=0) \cdot P(C=0)$$

$$\stackrel{MCE}{=} \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} = 0,16666$$

$$C_1 = P(A=0, B=1 | C=1) \cdot P(C=1)$$

$$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} = 0,1666 \quad ??$$

31. $Y : \text{Hike} \in \{\text{T}, \text{F}\}$ - morg / zu morg in drun

$X_1 : \text{Sunny} \in \{\text{T}, \text{F}\}$ +

$X_2 : \text{Windy} \in \{\text{T}, \text{F}\}$

$$P(\text{Hike}) = 0,5$$

$$P(\text{Sunny} | \text{Hike}) = 0,8$$

$$P(\text{Sunny} | \text{Hike}) = 0,7$$

$$P(\text{Windy} | \text{Hike}) = 0,4$$

$$P(\text{Windy} | \text{Hike}) = 0,5$$

pp. independent.

a) $P(\text{Sunny} = \text{T}, \text{Windy} = \text{T}, \text{Hike} = \text{T}) ?$

$$P(\text{Hike}) = P(\text{Hike})$$

$$P(A=1, B=1 | Y=y) \cdot P(Y=y) = P(A=1, B=1, Y=y)$$

$$\Rightarrow P(\text{Sunny} = \text{T}, \text{Windy} = \text{T} | \text{Hike} = \text{T})$$

PP1ad $P(\text{Sunny} = \text{T} | \text{Hike} = \text{T}) \cdot P(\text{Windy} = \text{T} | \text{Hike} = \text{T})$

$$\cdot P(\text{Hike} = \text{T})$$

$$= 0,8 \cdot 0,4 \cdot 0,5 = 0,8 \cdot 0,2 = 0,16$$

~~$P(X_1, X_2, Y) =$~~

			$P(X_1, X_2, Y) = P(X_1 Y) \cdot P(X_2 Y) \cdot P(Y)$	$P_{\text{NO}}(X_1, X_2)$	$P_{\text{NB}}(Y X_1, X_2)$
F	F	F	$0,3 \cdot 0,5 \cdot 0,5 = 0,075$	F	0,555
F	F	T	$0,2 \cdot 0,6 \cdot 0,5 = 0,06$	T	0,444
F	T	F	0,3 $\cdot 0,5 \cdot 0,5 = 0,075$	T	0,652
F	T	T	$0,2 \cdot 0,4 \cdot 0,5 = 0,04$	T	0,347
T	F	F	$0,7 \cdot 0,5 \cdot 0,5 = 0,175$	T	0,421
T	F	T	$0,8 \cdot 0,6 \cdot 0,5 = 0,24$	T	0,578
T	T	F	$0,7 \cdot 0,5 \cdot 0,5 = 0,175$	F	0,522
F	T	T	$0,8 \cdot 0,4 \cdot 0,5 = 0,16$	F	0,477

$$\text{err} = 0,06 + 0,06 + 0,04 + 0,175 + 0,16$$

$$= 0,435 \approx$$

$$33. Y = (A \wedge B) \vee \neg(B \vee C)$$

$$P(A=0) = 50\%, \rightarrow P(A=1) = 50\%.$$

$$P(B=0) = 50\%, \rightarrow P(B=1) = 50\%.$$

$$P(C=0) = 50\%, \rightarrow P(C=1) = 50\%.$$

variable binaria

a)

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1



A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1

A	B	C	Y
0	0	1	0
0	1	0	0
0	1	1	0
1	0	1	0
0	0	0	1

$$\text{I A: } \begin{pmatrix} 0 & 1 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

$$\text{B: } \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\text{C: } \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

$$\text{II A: } \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

$$\text{B: } \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\text{C: } \begin{pmatrix} 0 & 1 \\ \frac{3}{4} & 1 \end{pmatrix}$$

$$Y: \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

* estacionaria

$$b) \text{ i) } P_0 = P(A=0|Y=0) \cdot P(B=0|Y=0) \cdot P(C=0|Y=0) \cdot P(Y=0)$$

$$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{64}$$

$$P_1 = (P(A=0|Y=1)) \cdot P(B=0|Y=1) \cdot P(C=0|Y=1) \cdot P(Y=1)$$

$$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{64} \quad \text{Convenzione (1)}$$

$$2) P_0 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{9}{32} \cdot \frac{1}{2} = \frac{9}{64} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow Y_{NB} = 0$$

$$P_1 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{64}$$

$$3) P_0 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{64} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Convenzione (1)}$$

$$P_1 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{64}$$

$$4) P_0 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{9}{64} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow Y_{NB} = 1.$$

$$P_1 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{64}$$

$$5) P_0 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{64} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow Y_{NB} = 1$$

$$P_1 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{9}{64}$$

$$6) P_0 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{64} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow \text{Conv (1)}$$

$$P_1 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{64}$$

$$7) \left. \begin{array}{l} P_0 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{64} \\ P_1 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{9}{64} \end{array} \right\} \Rightarrow Y_{NB} = 1$$

$$8) \left. \begin{array}{l} P_0 = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{64} \\ P_1 = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{64} \end{array} \right\} \Rightarrow \text{Convenție (1)}$$

1.	1	✓
0	0	✓
0	1	-
0	0	✓
1	1	✓
0	1	-
1	1	✓
1	1	✓

$\text{err}_{NB} = \frac{2}{8} = \frac{1}{4} = 0,25$

c) ~~$\star \circ : A, B, C$~~

I Teat 8 variantele pt $Y=0$.
 II Teat 8 variantele pt $Y=1$.
 III $P(Y)$

(I pt că ultima prob
 e $1 - \text{suma celeritătii}$)

$$(I-1) + (II-1) + 1 = 7 + 7 + 1 = 15$$