b) $P_{JB}(A=1, B=1, Y=0) = P(A=1, B=1 \mid Y=0) \cdot P(Y=0)$

$P_{JB}(A=1, B=1, Y=1) = P(A=1, B=1 \mid Y=1) P(Y=1)$

ne uităm la tabele

$= \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$

$= 0 \cdot \frac{2}{5} = 0$ ✓

$\Rightarrow Y_{JB} = 0$.

45, 46, 47 a, b, 52, ~~1 prob~~ de la arbori de decizie

4 probleme de la Clasificare Bayesiană

27.

A	B	C	Y
0	0	1	0
0	1	0	0
1	1	0	0
0	0	1	1
1	1	1	1
1	0	0	1
1	1	0	1

Clasificare$(A=0, B=0, C=1)$ cu Bayes Naiv.

~~Rez:~~

Rez: $Y_{map} = \underset{y \in \{0,1\}}{\text{maxarg}} \; P(Y=y \mid A=0, B=0, C=1)$

$\overset{TB}{=} \underset{y \in \{0,1\}}{\text{maxarg}} \; \frac{P(A=0, B=0, C=1 \mid Y=y) \cdot P(Y=y)}{P(A=1, B=1, C=1)}$

↳ const > 0 în rap cu Y

$= \underset{y \in \{0,1\}}{\text{maxarg}} \; P(A=0, B=0, C=1 \mid Y=1) \cdot P(Y=y)$

$\overset{ip\ ind}{=} P(A=0 \mid Y=1) \cdot P(B=0 \mid Y=1) \cdot P(C=1 \mid Y=1) \cdot P(Y=y)$

$P_0 = P(A=0|Y=0)\cdot P(B=0|Y=0)\cdot P(C=1|Y=0)\cdot P(Y=0)$

$\overset{MLE}{=} \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{3}{7} = \frac{2}{63} = 0{,}0317$

$P_1 = P(A=0|Y=1)\cdot P(B=0|Y=1)\cdot P(C=1|Y=1)\cdot P(Y=1)$

$\overset{MLE}{=} \frac{1}{4}\cdot\frac{2}{2}\cdot\frac{1}{2} = \frac{1}{8} = 0{,}125$

$P_0 < P_1 \Rightarrow y_{NB} = 1$

3.26. a) $X = \{Outlook=sunny, Temp=cold, Humidity=high, Wind=strong\}$

$YT_{map} = \underset{y=\{Yes,No\}}{\text{maxarg}}\ P(YT=y \mid O=s, T=c, H=h, W=s)$

$\overset{F.B.}{=} \underset{y=\{Yes,No\}}{\text{maxarg}}\ \frac{P(O=s, T=c, H=h, W=s \mid YT=y)\, P(YT=y)}{P(O=s, T=c, H=h, W=s)}$

→ const. în rap. cu y

$= \underset{y=\{Yes,No\}}{\text{maxarg}}\ P(O=s, T=c, H=h, W=s \mid YT=y)\cdot P(YT=y)$

$\overset{ind}{=} \underset{y=\{Yes,No\}}{\text{maxarg}}\ P(O=s|YT=y)\cdot P(T=c|YT=y)\cdot P(H=h|YT=y)\cdot P(W=s|YT=y)\cdot P(YT=y)$

$P_{yes} \overset{MLE}{=} \frac{2}{9}\cdot\frac{3}{9}\cdot\frac{3}{9}\cdot\frac{3}{9} = \frac{6}{729} = 0{,}0082$

$= P(O=s|YT=Yes)\cdot P(T=c|YT=Yes)\cdot P(H=h|YT=Yes)\cdot P(W=s|YT=Yes)\cdot P(Y=Yes)$

$$P_{NO} = \frac{3}{5} \cdot \frac{1}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{36}{625} = 0,0576$$

$$= P(O=s \mid Y=NO) \cdot P(T=c \mid Y=NO) \cdot P(H=h \mid Y=NO) \cdot P(W=s \mid Y=N$$

$$\cdot P(Y=NO)$$

$P_{yes} < P_{NO} \Rightarrow Y_{NB} = NO$

- b)

O	T	H	W	ET
o	h	h	w	Y
r	m	h	w	Y
r	c	n	w	Y
o	c	n	s	Y
s	c	n	w	Y
r	m	n	w	Y
s	m	n	s	Y
o	h	m	s	Y
o	h	n	w	Y
s	h	h	w	N
s	h	h	s	N
r	c	n	s	N
s	m	h	w	N
r	m	h	s	N

NB: 9 estimari
1+4+4

ET: $\begin{pmatrix} Y & N \\ \frac{9}{14} & \frac{5}{14} \end{pmatrix}$

MLE

O	T	H	W	ET
o	h	h	w	Y
r	m	h	w	Y
r	c	n	w	Y
o	c	n	s	Y
s	c	n	w	Y
r	m	n	w	Y
s	m	n	s	Y
o	h	m	s	Y
o	h	n	w	Y

MLE MLE MLE MLE

O	T	H	W	ET
s	h	h	w	N
s	h	h	s	N
r	c	n	s	N
s	m	h	w	N
r	m	h	s	N

MLE MLE MLE MLE

B1. $Y: HiKe \in \{T, F\}$ – merg / nu merg în drum

$X_1: Sunny \in \{T, F\}$ și

$X_2: Windy \in \{T, F\}$

$P(HiKe) = 0,5$

$P(Sunny \mid HiKe) = 0,8$

$P(Sunny \mid \overline{HiKe}) = 0,7$

$P(Windy \mid HiKe) = 0,4$

$P(Windy \mid \overline{HiKe}) = 0,5$

PP independente.

a) $P(Sunny = T, Windy = T, HiKe = T)$?

~~$P(HiKe) = P(\overline{HiKe})$~~

$P(A=1, B=1 \mid Y=y) \cdot P(Y=y) = P(A=1, B=1, Y=y)$

$\Rightarrow P(Sunny = T, Windy = T \mid HiKe = T)$

PPind $P(Sunny = T \mid HiKe = T) \cdot P(Windy = T \mid HiKe = T)$

$\cdot P(HiKe = T)$

$= 0,8 \cdot 0,4 \cdot 0,5 = 0,8 \cdot 0,2 = 0,16$

~~b)~~

29) 8 Atribute

Bayes Naiv - clasificare pt (1,0,0,1,1,1,1,0)

D - domeniu

$D_{map} = \underset{d \in \{P,S\}}{\max\arg} P(D=d \mid A=1, B=0, C=0, D=1, E=1, F=1, G=1, H=0)$

$\overset{FB}{=} \underset{d \in \{P,S\}}{\max\arg} \dfrac{P(A=1, B=0, C=0, D=1, E=1, F=1, G=1, H=0 \mid D=d) \cdot P(D=d)}{P(A=1, B=1, C=1, D=1, E=1, F=1, G=1, H=1)}$

↳ constantă în rap. cu D.

$= \underset{d \in \{P,S\}}{\max\arg} P(A=1 \dots H=0 \mid D=d) \cdot P(D=d)$

$\overset{ind}{=} \underset{d \in \{P,S\}}{\max\arg} P(A=1 \mid D=d) \cdot P(B=0 \mid D=d) \cdot P(C=0 \mid D=d) \cdot P(D=1 \mid D=d) \cdot$
$\cdot P(E=1 \mid D=d) \cdot P(\dots) \cdot P(H=0 \mid D=d) \cdot P(D=d)$

$P_P = P(A=1 \mid D=P) \cdot P(B=0 \mid D=P) \cdot \dots \cdot P(H=0 \mid D=P) \cdot P(D=P)$

$= \frac{2}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{6}{13}$

$= 0,00137$

$P_S = P(A=1 \mid D=S) \cdot P(B=0 \mid D=S) \dots \cdot P(H=0 \mid D=S) \cdot P(D=S)$

$= \frac{5}{7} \cdot \frac{2}{7} \cdot \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{2}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{6}{7} \cdot \frac{7}{13}$

$= 0,00001868$

$$\rho_1 = P(A=0 \mid C=1) \cdot P(B=1 \mid C=1) \cdot P(C=1)$$

$$\overset{MLE}{=} \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} = 0,4444 \cdot \frac{1}{2} = \underline{\underline{0,22222}}$$

$$c_0 < c_1 \Rightarrow c_{NB} = 1$$

d) Clasif $(A=0, B=1)$ cu Bayes Optimal.

$$c_{map} = \underset{c \in \{0,1\}}{\arg\max} \; P(C=c \mid A=0, B=1)$$

$$\overset{F.B}{=} \underset{c \in \{0,1\}}{\arg\max} \; \frac{P(A=0, B=1 \mid C=c) \cdot P(C=c)}{P(A=0, B=1)}$$

$\hookrightarrow$ const. în rap cu C

$$= \underset{c \in \{0,1\}}{\arg\max} \; P(A=0, B=1 \mid C=c) \cdot P(C=c)$$

$$c_0 = P(A=0, B=1 \mid C=0) \cdot P(C=0)$$

$$\overset{MLE}{=} \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} = 0,16666$$

$$c_1 = P(A=0, B=1 \mid C=1) \cdot P(C=1)$$

$$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} = 0,1666 \qquad ??$$

b)

A=0	0	$\frac{1}{3}$
A=1	$\frac{1}{3}$	$\frac{1}{3}$

pt prima parte.

	B=0	B=1
A=0	$\frac{1}{3}$	$\frac{1}{3}$
A=1	0	$\frac{1}{3}$

pt a doua parte

6 estimări pt Bayes Optimal.

c) Bayes Naiv - clasificare (A=0, B=1)

$C_{MAP} = \underset{c \in \{0,1\}}{\text{maxarg}} \; P(C=c \mid A=0, B=1)$

$\overset{FB}{=} \underset{c \in \{0,1\}}{\text{maxarg}} \; \frac{P(A=0, B=1 \mid C=c) \cdot P(C=c)}{P(A=0, B=1)}$ → constantă pe în rapport cu c

$= \underset{c \in \{0,1\}}{\text{maxarg}} \; P(A=0, B=1 \mid C=c) \cdot P(C=c)$

$\overset{\text{ipot ind}}{=} P(A=0 \mid C=c) \cdot P(B=1 \mid C=c) \cdot P(C=c)$

$P_0 = P(A=0 \mid C=0) \cdot P(B=1 \mid C=0) \cdot P(C=c)$

$\overset{MLE}{=} \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} = 0,2222 \cdot \frac{1}{2} = \underline{0,11111}$

$\Rightarrow p_P > p_S \Rightarrow y_{NB} = P$ ✓

P să fie clasificat clasa politică =

= ~~0,000...~~

= 0,00137

28. ~~A B C D~~

A	B	C	nr. ap
0	0	1	3
0	1	0	1
0	1	1	4
1	0	0	5
1	1	0	2
1	1	1	1

$\Rightarrow$

A	B	C	nr. ap
0	1	0	1
1	0	0	5
1	1	0	2
0	0	1	3
0	1	1	4
1	1	1	1

MLE: $C\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

MLE (C = 0): $A\begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

MLE (C = 0): $B\begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

MLE (C = 1): $A\begin{pmatrix} 0 & 1 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

MLE (C = 1): $B\begin{pmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

a) 5 estimări pt Bayes Naiv