S 06

Data Analytics for Data Scientists

Design of Experiments (DoE)

Suggested solutions for Exercise 06: Effect size & Power analysis

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Suggested solution 01

Determining the sample size with G*Power

For a research project the sample size is to be determined.
In an experiment a new teaching method is to be tested against the old teaching method.
The variable to be measured is the **grade point average in the final exam**.

The project has the following characteristics

- Randomized controlled trial (RCT):
  Compare the new teaching method against the old one
- Analysis with t-test[1]
- Significance level $\alpha = 5\%$
- Cohen: Level of $\beta = 20\% \rightarrow$ Power $= 1 - \beta = 80\%$

Important difference

- From empirical studies, expert interviews, and based on theoretical considerations, a difference of **one grade** unit is considered to be important.

Other measurements

- Validated studies have shown that the standard deviation of exam grades in a comparable population has the value $\sigma_0 = 1.5$ grade units.

Procedure

First step
Determine effect size d that is relevant for the study.

Second step
How large does the total sample have to be for

- significance level $\alpha = 5\%$
- power $= 1 - \beta = 80\%$
- effect size d relevant for the study

to obtain a significant t-test if the change in the grade at the final exam is in fact one grade or more?

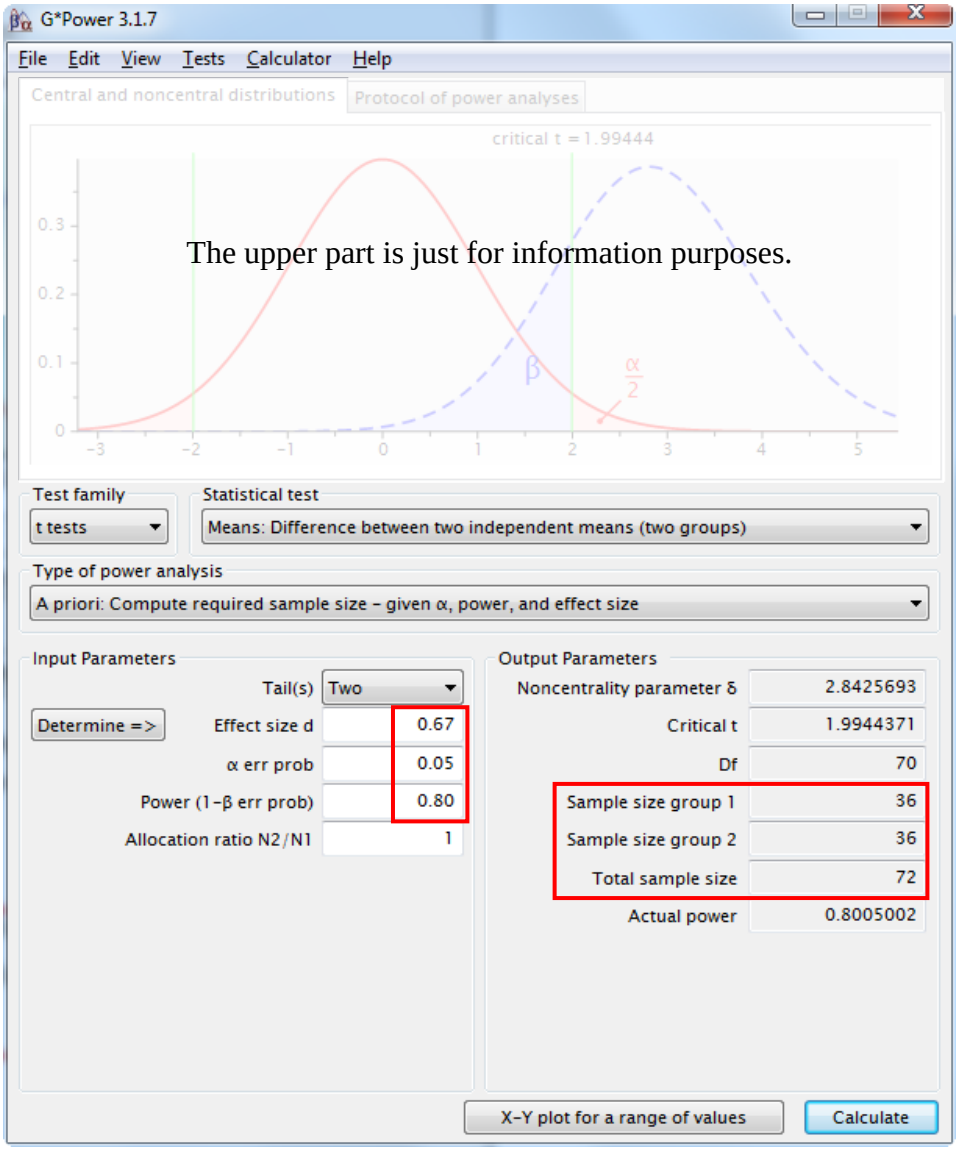
Use G*Power to calculate the sample size.

[1] The grade scale is an ordinal scaled characteristic (grades vary from 1 to 6 with a half grade increment), which is why a t-test is generally unsuitable. However, for this task, you may use the t-test.

Suggested answers to the questions

The effect size d that is relevant for the study is

$$d = \frac{\text{Difference } \delta}{\sigma_0} = \frac{1}{1.5} = \frac{1}{\frac{3}{2}} = \frac{2}{3} = 0.67$$



The total sample size is 72 students.

With a sample size of 36 students in the class taught with the new teaching method and a sample size of 36 students in the class taught with the old method, a grade difference of at least one grade in the final exam can be tested significantly with a t-test.

Suggested solution 02

Determining the sample size with R

The task is the same as Task 01.

Use R to determine the sample size.

Describe the results – insert also the R-code, the R-output and if necessary, R-plots.

Suggested answers to the questions

```
library(pwr)
pwr.t.test(d = 0.67, power = 0.80, sig.level = 0.05)
```

```
     Two-sample t test power calculation

              n = 35.95537
              d = 0.67
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

number in each group = 35.95537 ≈ 36 → **Sample size n = 2 x 36 = 72**

The total sample size is 72 students.
With a sample size of 36 students in the class taught with the new teaching method and a sample size of 36 students in the class taught with the old method, a grade difference of at least one grade in the final exam can be tested significantly with a t-test.

Suggested solution 03

Determining the sample size for an analysis of variance

The results of an RCT study are to be evaluated with an analysis of variance (ANOVA).

A fictitious study on the effects of memory training serves as an example.

- Dependent variable (**DV**)
  Assessment of the memory capacity on a scale of 1 to 100
- Independent variable (**IV**)
  Memory training received "Yes" or "No"

How large must the total sample be for
  significance level $\alpha = 5\%$
  power $= 1 - \beta = 80\%$
  effect size *medium*
to obtain a significant analysis of variance?

The f-value for the effect size can be found in Cohen (1992) in *Table 1* – see below.

Compare the result with the example for the new medicine in the script.

Describe the results – insert also the R-code, the R-output and if necessary, R-plots.

Use this R function (→ details for example here bit.ly/pwr-anova-test)

```
pwr.anova.test(f = NULL, k = NULL, power = NULL, sig.level = NULL)
```

Table 1
ES Indexes and Their Values for Small, Medium, and Large Effects

Test	ES index	Effect size		
		Small	Medium	Large
1. m_A vs. m_B for independent means	$d = \frac{m_A - m_B}{\sigma}$	.20	.50	.80
2. Significance of product–moment r	r	.10	.30	.50
3. r_A vs. r_B for independent rs	$q = z_A - z_B$ where z = Fisher's z	.10	.30	.50
4. $P = .5$ and the sign test	$g = P - .50$	.05	.15	.25
5. P_A vs. P_B for independent proportions	$h = \phi_A - \phi_B$ where ϕ = arcsine transformation	.20	.50	.80
6. Chi-square for goodness of fit and contingency	$w = \sqrt{\sum_{i=1}^{k} \frac{(P_{1i} - P_{0i})^2}{P_{0i}}}$	.10	.30	.50
7. One-way analysis of variance	$f = \frac{\sigma_m}{\sigma}$	.10	.25	.40
8. Multiple and multiple partial correlation	$f^2 = \frac{R^2}{1 - R^2}$	.02	.15	.35

Suggested answers to the questions

```
library(pwr)
pwr.anova.test(f=0.25,k=2,power=0.80,sig.level=0.05)
```

```
     Balanced one-way analysis of variance power calculation

              k = 2
              n = 63.76561
              f = 0.25
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

number in each group = 63.76561 ≈ 64 → **Sample size n = 2 x 64 = 128**

With a sample size of n = 128, a difference in memory capacity can be tested significantly with an analysis of variance with α = 5% and a power of 80%, if the effect size in the population is assumed to be medium.

The result is identical with the example of the medicine to lower blood pressure in the script, because the ANOVA in the simplest case, i.e. with two groups, corresponds to the t-test.