Macroeconomics of Aggregate Fertility Outcomes

An Emergentist Perspective from Aoki's Statistical Macroeconomics

Biagio Rosso*

February 2024

Abstract

The paper develops a macroeconomic theory of aggregate fertility outcomes, with a substantive interest in high fertility traps in developing economies. High Fertility Traps (HFTs) are a recurrent feature across a number of low income and developing economies, and have well explored implications for poverty rates and economic stagnation. The mainstream literature explains aggregate fertility outcomes the result of *demand for fertility* for a representative household in the economy — in the neoclassical vernacular, as a market failure emerging on the back of a perverse allocation of average incentives. The policy recommendations associated to this view of the emergence HFTs are well known. In developing a macroeconomic model of fertility traps, this paper considers and addresses, substantively, two key neglected but policy-relevant aspects of the emergence of aggregate fertility outcomes: (i) the limited ability of economic analyses and policies focusing on average incentives in addressing undesirable aggregate fertility outcomes; (ii) how the role of aggregate or societal uncertainty on the value of different fertility regimes to the individual household might explain this puzzle faced by demand for fertility approaches. The paper does this by developing on and adapting in a novel way on the framework for bottom-up emergentist macroeconomic theory pioneered by Aoki and Yoshikawa, based on applications from statistical mechanics and Markov Chain Processes to link individual decision-making to complex and emergent outcomes at the macro-level. Based on this theoretical framework for modelling fertility outcomes, it is argued that (high) fertility traps obtain as aggregate long run dynamics even if average incentives are in line with the socially optimal fertility outcome. This flows primarily from uncertainty or the second moment of the distribution of preferences. A society which makes information on alternative choices more easily available is less prone to HFT formation or persistence compared to one in which policy fails to also reduce uncertainty, even if average behaviour is the same (or equalised through policy) across societies.

*(Author Details)

1 Introduction

This paper studies the emergence of aggregate fertility outcomes, and develops two key aspects of a theory of aggregate fertility outcomes as a macroeconomic phenomenon, as opposed to reducing them to an aggregation of micro-behaviours of individual agents popularised by demand for fertility models. An explicitly *macroeconomic* theory of aggregate fertility outcomes is envisioned to enable a study of the role of complexity and emergence in driving the link between individual preferences on fertility and aggregate fertility outcomes in the economy. In turn, with an eye to assessing policy-making failures, this allows to rationalise how and with what implications for reforming policy intervention the link between average incentives (as the usual target of policy interventions) and aggregate fertility outcomes is distorted.

The substantive focus is on high-fertility traps in developing economies. High-Fertility Traps (HFTs) are a stable feature of low income and developing economies and have well explored deleterious and mutually reinforcing effects on – or at the very least, a robust correlation with – poverty and economic stagnation (Wietzke, 2020; Dasgupta, 2005). Shedding light on the mechanisms responsible for the emergence of HFTs in low income and developing economies is thus of central interest to both development economics and for the ability of policy-makers to respond contemporary international development. While the main focus is on HFTs in developing economies, key results and discussion apply as well to the emergence of low-fertility traps in advanced economies since the 1980s – a phenomenon attracting increasing attention from scholarship and policy-making alike due to its repercussions, inter-alia, on the fiscal sustainability of existing pension funds and welfare systems.

The mainstream literature on endogenous demand for fertility indeed explains the phenomenon of HFTs as a market failure resulting from a conflict between individual and collective optimal fertility in the presence of externalities to households 'reproductive choices' (Ray, 1998; Bardhan and Udry, 1999). As explained in section 2, similar explanations result though underspecified and unsatisfactory on a number of grounds in accounting for features of policy-resilient high-fertility traps, prompting a theoretical reassessment that takes into consideration the neglected role of household heterogeneity and aggregate uncertainty in mediating the *amplification* of individual fertility preferences into aggregate high-fertility traps (Durlauf and Walker, 2001).

Add isomorphism to the diffusion of discrete behavioural regimes/incentive regimes within a finite population

Add distributional/statistical turn in modern macroeconomics. Aggregate outcomes as dynamics/stochastic motion of a measure/distribution of agents, and steady state as a stationary measure –¿ distributive issues are aggregate issues

The paper hence examines whether and under what conditions high-fertility traps can still form in the absence of conflict between individual and social optimality of fertility options in the average household incentive structure. In particular, two key neglected and policy-relevant aspects of the HFTs problem are considered. First, the limited ability of economic analyses and policies focusing on average incentives in addressing HFTs. Second, as a potential explanation, how the role of aggregate or societal uncertainty regarding the value of different fertility regimes to the individual household might explain this puzzle faced by demand for fertility approaches. To do this, the paper develops upon and adapts in a novel way the framework for micro-founded macroeconomic theory development pioneered by Aoki and Yoshikawa, itself based on applications from statistical mechanics and Markov Chain Process in continuous time to heterogenous economies (Aoyama et al., 2021; Aoki, 2007, 2002, 1996; Yoshikawa, 2003).

As hypothesised on the basis of the literature review and examined through the model, when household-level fertility decisions are undertaken in contexts marked by both social interaction and heterogeneity in relative preferences among households, resulting aggregate uncertainty (measured as dispersion in relative prefences) can be expected to lead to aggregate high-fertility traps even if no conflict subsists between optimal demand for fertility for the average household and socially optimal demand for (lower) fertility. The intention behind the contribution is two-fold.

First, on a substantive and policy-relevant level, developing an alternative analysis of HFTs in developing economies that takes into accounts aspects neglected in the available mainstream framework. Second, an applied-methodological contribution examining how the alternative framework for micro-founded macroeconomic theory development by Aoki and Yoshikawa, originally geared to examining more classical macroeconomic issues relating to productivity and economic cycles, can be extended to rethink problems in development economics by vindicating a macroeconomic approach as an alternative to the dominant microeconomic and programme-evaluation perspective. The rest of the paper is organised as follows. Section 2 provides motivates the research question and presents a literature review. Section 3 develops a hypothesis on the back of the review and justifies the chosen methodology. Section 4 introduces, analyses, and discusses take-homes from the developed model.

2 Background Literature

This section reviews some essential background in the literature and challenges faced by the mainstream approaches to theorising HFTs. The presentation of economic models in this section is kept informal, and emphasis is rather placed on conceptual issues. The subject of HFTs is of central interest to development economics, falling in particular within the purview of the field's interest in the nature and determinants of poverty traps, and in the historical interest in the concept of social coordination failures for explaining low development outcomes.

The literature on poverty and inequality, from which the notion of high-fertility trap is drawn, indeed highlights a series of plausible causal chains between high fertility traps and the persistence of poverty (Wiezke, 2020; Dasgupta, 1998; 1995; cf. Barro, 1991). For example, where children plausibly constitute a form of savings or insurance or a productive asset as discussed in week two, individually optimal reproductive choices might come at the costs of a socially suboptimal rate of accumulation of human capital, insofar as a higher average number of children lowers the household resources expendable on the child's education and upbringing (Thomas and Duncan, 2007; Dasgupta, 1998; Becker and Lewis, 1974). In terms of household-level wealth and consumption dynamics, it is suggested that the lifecycle ability to earn wealth and incomes might be similarly impacted by childbearing and rearing costs associated to high-fertility regimes, insofar as reduced household resources affect nutritional intake, networking, and access to private healthcare where these are in turn sourced and traded through incomplete markets (Were et al., 2020; Banerjee and Duflo, 2011; Dasgupta, 1995). The potential for HFTs to result and kickstart spirals of deteriorating household asset positions, resources, and consumpiton makes the analysis of HFTs particularly important with developing economies in which empirical tests and case studies ordinarily reject the hypothesis of complete asset and insurance markets (Li et al., 2020). Finally, again within the literature on poverty examined in week two, the positive contribution of high fertility rates to the formation of poverty traps and precariousness of livelihoods might also result from the external costs that childbearing places on the wider community and on the local environmental resource base through the intensification of resource use (Ibid., Lee and Miller, 1990; cf. Cleaver and Sheiber, 1994 on Sub-Saharan Africa.). Now, while the average fertility rate has declined over the last fifty years globally, it remains substantially high in a number of low income and developing economies — 33 countries exhibited a per-capita fertility rate above 5 in 2010, chiefly in Sub Saharan and Sahelian Africa (World Bank, 2010) — with incentive-based policies undertaken in recent decades to redress HFTs showing only limited success (Shultz, 2007; Pitt and al., 1999). If persisting and incentive-resilient high fertility is of immediate concern to poverty traps, based on a preliminary appreciation of the above channels highighted in the literature, then it becomes important to examine the under what conditions (policy-resilient) HFTs can be expected to form. As we shall see briefly, mainstream neoclassical answers result unsatisfactory, prompting a theoretical reassessment based on complexity macroeconomics.

Explanations for high-fertility traps in the mainstream literature typically draw from 'demand for fertility' theory pioneered within rational choice theory (Becker, 1960; Becker and Lewis, 1974; cf. Ray, 1998; Bardhan and Udry, 1999). Given behavioural optimality of household choices (demand for fertility), high-fertility traps are essentially explained as a problem of equilibrium selection out of multiple (Pareto-ordered) fertility equilibria on which society fails to converge upon when each household is individually optimising according to the above general framework (Ray, 1998; Dasgupta, 2000; 1998; 1995; cf. Cooper and John, 1988). Three more specific proposed logic stand out in the demand for fertility literature. One strand of literature focuses on the labour market or pecuniary externalities associated to widespread high fertility (Bardhan and Udry, 1999; Galor and Weil, 1999; Basu and Van, 1988). Accordingly, when the overall fertility rate is high, the resulting excessive supply of labour depresses market wages received by the household, making it the best response for each household to bear a high number of children to prop up family incomes, which in turn keeps up the fertility rate (Ibid.). Equivalently, even if wages would rise if all were to choose to have a lesser number of children, it is not rational to do so for any household contingently on others opting to have high number of children. The second strand, similarly emphasising a social coordination failure but distinct from the former, explains high-fertility traps on the basis of the fact that the costs of rearing children might not be fully internalised by the household, for example if communal fosterage structures or organisation of property rights over resources are such that the private benefits from childbearing exceed the private costs of raising them (Ray, 1998). In this stronger version of coordination failure, social coordination on the low-fertility would systematically fail insofar as it is never individually optimal for an household to have a low number of children, even contingently on other households doing so (in fact, low-fertility is never a strategic equilibrium in this version). A final, and oft-complementary explanation to those above for the selection of socially suboptimal high fertility equilibria is cultural norms and conformity to expectations, which, as forms of strategic complementarities (Dasgupta, 2003, 1998; Bardhan and Udry, 1999). In this sense, where the utility of childbearing to a household increases in the average number of children, a society may be stuck in a socially suboptimal high-fertility equilibrium due to conformity to the social norm being the individually optimal response (Ibid.).

While explanations of high-fertility traps as social coordination failures due to pecuniary or non-pecuniary externalities may seem theoretically appealing, they fall short of adequately addressing the persistence of policy-resilient high-fertility traps with which are concerned on a number of levels, prompting a revision of explanations for phenomenon. First, they do not appear to plausibly explain high fertility traps in low-income economies, such as Kenya or Bangladesh, where the subsistence of pecuniary or non-pecuniary externalities to childbearing of the above sort has been questioned in empirical research (Lee and Miller, 2000; 1990). On a similar vein, they also appear inadequate for comparatively accounting for the emergence and specificity of similar high-fertility trap problems to contexts, such as South Asia (e.g. 1980s-1990s Bangladesh) and South-East Asia (1980s-1990s Laos), characterised by different familiar and child-fosterage structures (Ibid.; Dommaraju and Tan, 2014). Furthermore, a purely individual-choice based explanation on its own appears also inconsistent with the limited effectiveness or even perverse effect observed for incentive-correcting policies on fertility rates (rationalised by explanations attributing high fertility traps to perverse average incentive structures), such as an improvements in access to credit among women in Bangladesh (Shultz, 2007; Pitt et al, 1999).

3 Hypothesis and Methodology

3.1 Social Interactions under Heterogeneity and Uncertainty

On the face of it, these extant issues preliminarily suggest that high-fertility traps might then be not necessarily or primarily representative of a conflict between individual and collective optimality in reproductive choices. In a sense, that (policy-resilient) high-fertility traps should be able to emerge on the basis of weaker conditions than the perverse incentive structure emphasised in the above literature. How to provide an explanation for this possibility?

Criticism to demand for fertility explanations of high-fertility traps highlight some critical points to build upon to approximate a better or at least more complete explanation for the phenomenon of policy-resilient high fertility traps. One significant problem with the above approach, likely underlying the above shortcomings, is that aggregate fertility outcomes are modelled and explained on the basis of the choices — even where interdependent and subject to non-pecuniary external effects in the form of social interactions — of households homogeneous relative to preferences, constraints, access to information, and timings at which they make their fertility decisions (Deaton, 2005; Ray, 1998; Birdsall, 1988; Bardhan, 1988; Strauss and Thomas, 2007). Yet again, households considerably differ in all these respects: different earning capabilities and capacity for consumption smoothing, parental education levels, and allocation of decision-making power between man and woman (or other relatives in extended households) represent salient axes of variation unaccounted for in the above representative-household models (Bardhan and Udry, 1999).

In complexity-characterised contexts such as low-income economies where aggregate fertility outcomes are the decentralised product of large numbers of socially interacting (i.e. external effects) households idiosyncratically differing in the optimisation problems they face and their access to information, reducing aggregate outcomes to a specific incentive structure at the individual choice level involves a compositional fallacy (Aoki and Yoshikawa, 2007; Durlauf, 1997). It risks — as the criticism presented to demand for fertility models gestures at — underspecifying the conditions under which high-fertility traps can be expected to form and hence successfully intervened upon. Adequate explanations should then plausibly take into consideration the implications of complexity and sequential decision-making for the formation of macro-attractors in the economy such as high-fertility traps (Aoyama et al., 2021; cf. Durlauf et al., 1997). On this backbone, the hypothesis examined is that, in contexts where reproductive choices occur among a large number of heterogeneous and interacting households, aggregate uncertainty suffices for the formation of high-fertility traps even where conflicts between individual and social optimality does not characterise mean household preferences: in other words, that high-fertility traps are plausibly explained by a mismatch between micro and macro behaviour in the presence of uncertainty, even where the incentives are correctly allocated on average.

3.2 Methodology and Rationale

To examine the case for the formulated theoretical hypothesis, the paper adapts and presents a simple formal stochastic model developed by Aoki and Yoshikawa micro-foundational approach to macroeconomic complexity and aggregate uncertainty traps (Aoyama et al., 2021; Aoki, 2007; Yoshikawa, 2003), to study the changes in the probability distribution of fertility outcomes in a large population of households — heterogenous in the optimisation problems they face and the timing at which they make decisions — when faced with discrete choices on whether to remain/become high fertility or low fertility households. The process of decision-making is modelled as a continuous-time Markov Chain, governed by a partial differential equation that describes the evolution of the probability distribution over the a state variable identified with the fraction of high-fertility households (Ibid.).

As standard, solving the model involves specifying the transition rates at which households change from one fertility-level choice to the other, and determining the steady state probability distribution of high-fertility traps, akin to log-likelihood function maximisation (Aoki, 2007; Durlauf and Walker, 2001). In particular, we are interested in the correspondence and lack thereof between the features of such macro distribution (i.e. multiple fertility equilibria/high-fertility traps) and the micro-level preferences of the average household in the presence of aggregate uncertainty. More specifically, it is examined whether highfertility traps can result as a macroequilibrium under large uncertainty even where the social optimal low fertility rate coincides with mean household preferences. This methodology is especially suited to examining our hypothesis: contrary to the examined representative household models, the probabilistic approach is specifically designed to extend to the analysis of decision making in complex contexts marked by interaction among heterogenous agents (Aoki and Yoshikawa, 2007), while allowing to retain a role for and assess (as is relevant to our hypothesis) the external effects and spillovers from reproductive choices postulated in rational choice literature by building them into the specified transitions rates for households choices as state-dependent multipliers on the probability of a transition from high to low fertility and vice-versa (Ibid.). Contrary to a comparable approach followed by Durlauf and Walker (2001), and differentiating the angle taken in this paper, the specific methodological choice to address the problem through the Aokian version of the model is based on the fact that it allows incorporate and study directly the role of heterogeneity-driven *aggregate uncertainty* in addition to that of social interactions, measuring it plausibly as the variance of population distribution of relative preferences over own fertility regimes.

4 A Model of Equilibrium Aggregate Fertility Outcomes

4.1 A Concept of Statistical Rational Expectations Equilibrium (SREE)

Before introducing the model and the solution concept employed to study its equilibrium, it is worth introducing informally the concept of equilibrium adopted in this paper. In particular, how it develops key elements in Aoki and Yoshikawa's *Stochastic Equilibrium* framework (2007) by interfacing it with the modern, rational expectation approach to equilibrium definitions in macroeconomic models. The reason for this is that it clarifies the model development in the previous sections, and generalises beyond the present application to the study of aggregate fertility outcomes (Rosso, forthcoming). The idea is that the resulting *Statistical Rational Expectations Equilibrium* (SREE) can be viewed as fundamentally weakening the concept of rational expectations equilibrium to a probabilistic setting in which rationality of agents' expectations holds in a *maximum-likelihood* sense.

Stationary SREE

– Agents take an aggregate state x as given / form expectations on x.

– Given such beliefs about x, agents optimal behaviour induces a law of motion on the space of measures over the aggregate state. I.e. a law of motion $H_x : \mathcal{M} \to \mathcal{M}$. The fixed point of such law of motion is an invariant distribution over the aggregate state $\mathcal{M}* = H_x\mathcal{M}*$. H is a complex operator, aggregating individual optimal behaviour as a fucntion of expectations on x.

– Aggregate state x is such that $\mathcal{M}$ peaks at x. In other words, x is the most probable aggregate state when individual perceive it/believe it to be the aggregate state. I.e. the most probable state is understood as the state such that,

Statistically rational expectations involve a perceived a law of motion on the aggregate state which, when taken as given by agents, results in a probability distribution over state laws of motion consistent with the expected law of motion being the most probable

The concept of equilibrium employed in the paper formalises key ideas in the work by Aoki and Yoshikawa on stochastic equilibrium in models with

4.2 Methodology 2

In all that follows, we employ the notation and procedures from the general framework developed by Aoki (2007). To examine the hypothesis through the model, we first adapt the above framework and key mathematical objects to characterise the economy under study (section 4.1). This will involve specifying the probabilistic distribution of preferences over discrete choices in the presence of heterogeneity and social interaction, and the resulting aggregate uncertainty measure of interest to later analysis. The construed distribution of preferences (in line with specifications employed in RUM-based dynamic optimisation models) will capture the micro-behavioural or incentive structure familiar from the demand for fertility literature. Based on this, we specify the PDE describing the associated continuous Markov Chain, and derive the steady state probability distribution of fertility outcomes. Then, in the analysis section (4.2) arguing that, when when uncertainty is large, multiple fertility equilibria/high-fertility traps emerging even when the mean household preference does not involve multiple or high-fertility equilibria (cf. Aoki et al., 2003 on the general logic of uncertainty traps).

4.3 Preferences for Fertility in the Heterogeneous Economy

We assume there are $N = \{1, ..., N\} \subset \mathbb{N}$ households faced with renewable discrete choices on their planned fertility rates, along a (continuous) sequence of points in time $t \in T \equiv \mathbb{R}_+$: a high-fertility behavioural option h and a low fertility option l. We denote as $x = n/N$ the fraction of households opting for a high fertility regime, and hence as $1 - x$ that of low-fertility households. Random variable x is our state variable of interest (lying by construction in a set $X \equiv [0, 1]$). This state variable will be microfounded by the probability distribution for fertility outcomes based on the model of household optimal fertility choices given below, and is in turned assumed to affect the latter to capture social interaction effects/externalities to individual demand for fertility.

To model the distribution of preferences underlying the aggregate fertility process with mean x, we let each household compare the expected returns from either choice $u^h = u(h, x)$ and $u^l = u(l, x)$, where utility is a real-valued function $u : \{h, l\} \times [0, 1] \to \mathbb{R}_+$ of the household's fertility choice and of the aggregate state, at any period t. Based on this simple model of preference, the discrete optimisation problem yielding the fertility choice or demand for fertility of any household i is

$$V_i(x) = \max_{j \in \{h,l\}} u(j, x) = \max\{u^h, u^l\}. \tag{1}$$

Hence a household chooses (by direct comparison of the utilities under either choice) the high fertility regime if and only if $V_i(x) = u^h \geq u^l$.[1] The low fertility regime is chosen, instead, if $V_i(x) = u^l > u^h$. To capture heterogeneity in the relative preferences across households, we treat

[1] The weak equality incorporates a tie-breaking assumption to ensure well-defined choices under indifference. Specifically, we are assuming that a household chooses high fertility whenever it is indifferent between the two options.

the latter as random variables. In particular, we let $\omega(x)$ denote the probability that a household chooses high fertility, as a function of the aggregate state. Making use of (1), this variable is thus defined by

$$\omega(x) = P(V_i(x) = u^h) = P(u^h - u^l \geq 0|x). \tag{2}$$

The probability that high-fertility is better compared to low-fertility for any household here depends on the fraction x of high-fertility households so to capture social interaction and externalities to individual choices needed to examine the hypothesis of interest. This state-dependent probability multiplier, which measures the probability that any optimising household will prefer high fertility as a function of the proportion of households choosing high-fertility, is key to provide the micro-foundations for the Markov transition rates defining the evolution of the probability distribution over high and low fertilty regimes in the population (as well as its aggregate moments of interest).

To derive an explicit expression for $\omega(x)$ and study the steady-state probability distribution of fertility outcomes in the next section, we assume that, conditional on the proportion of current highly fertile households x, the random incentive structure captured by difference $v = u^h - u^l$ follows a normal distribution $v \sim \phi(v|x) = \mathcal{N}(\mu(x), \sigma_v^2)$ in the population of households with unknown mean $\mu(x)$ and exogenously fixed variance σ_v^2. That is, for all Borel sets B in $\sigma(v)$, the probability of the event $v \in B$ conditional on event $X = x \in 1, 2, ..., N$ in the information set induced by the state $\sigma(X)$, viewed as a Borel function on $\sigma(X)$ and as a probability measure on $\sigma(v)$ is:

$$P(V \in B|\sigma(X))(x) = \int_B \frac{1}{\sqrt{2\pi\sigma_v^2}} e^{-\frac{1}{2}\frac{|v-\mu(x)|}{\sigma_v}} dv$$

It is important to stress that the mean of such distribution of relative preferences must be a function of the aggregate fertility state: were this a constant, then the distribution of preferences would not be affected by the current proportion of high-fertility households, contrary to the assumption of externalities or socially interactive components to households' decision-making on fertility. Hence we assume $\mu(x)$ to be a non-constant function of x. **see Appendix**

Hence we assume $\mu(x)$ to be a non-constant function of x. We also assume it to be continuous and differentiable in x (this involves no restriction on our model, see below). Based on this parametrisation, following the procedure in Aoki (2007), an expression for $\omega(x)$ can be found as follows. To get the intuition behind the derivation, computing $\omega(x)$ involves correcting the evaluation of probability mass of the *standard normal* distribution weakly below its mean (i.e. 0), depending on whether the unknown, potentially non-zero mean of the distribution of fertility preferences conditional on x – i.e. $v|x \sim \mathcal{N}(\mu(x), \sigma_v^2)$ falls to the right or left of 0. Accordingly, specialising (2) based on the normality assumption:

$$\begin{aligned}
\omega(x) &= 1 - \Phi(0|x) \\
= \omega(x) &= \frac{1}{2}\left(1 + \frac{2}{\sqrt{\pi}} \int_0^k e^{-z^2} dz\right) \\
= \omega(x) &= \frac{1}{2}\left(1 + \tanh\left(\frac{2k}{\sqrt{\pi}}\right)\right), \qquad (3)
\end{aligned}$$

where $k = \mu(x)/\sqrt{2\sigma_v}$, and z is the score $z = (v - \mu(x))/\sigma_v$, and the last step uses the approximation for the correction term given in Ingber (1982):

$$\frac{2}{\sqrt{\pi}} \int_0^k e^{-z^2} dz \approx \tanh\left(\frac{2k}{\sqrt{\pi}}\right).$$

Now, by the definition of $\tanh(x)$ and of k, we arrive at the standard closed expression modelling the state-dependent probability multiplier (cf. Aoki, 2007) in the presence of externalities/social interaction $\omega(x)$ — in our case, of high-fertility being preferred to low-fertility depending on the proportion of high-fertility households x:

$$\omega(x) = \frac{e^{\zeta\mu(x)}}{e^{\zeta\mu(x)} + e^{-\zeta\mu(x)}}, \quad \text{where} \quad \zeta = \sqrt{\frac{2}{\pi}}\sigma_v^{-1} \tag{4}$$

As in Aoki (2007) we anticipated in section 4.1 , parameter ζ enters the model and is interpretable as an inverse measure of aggregate uncertainty or dispersion in the societal distribution of preferences over own fertility regime (Ibid.) around the mean $\mu(x)$. Indeed, when the variance of the distribution of fertility preferences σ_v is high, so that households are more *uncertain* on average regarding the returns from fertility choices, the inverse-of-uncertainty parameter takes small values, and vice-versa. The role of this parameter, which is absent in rational choice analyses of mean incentive structure by assumption of a representative agent, is key to delivering on our hypothesis on this average uncertainty driving the formation and persistence of HFTs despite correctly allocated average incentives (i.e. measured in the model by a low, or negative value of $\mu(x)$). The next section takes up this point by analysing the long run equilibrium of the modelled economy.

4.4 Equilibrium Emergence of Aggregate Fertility Outcomes

Having specified the distribution of heterogeneous fertility preferences, and derived the probability $\omega(x)$ a household chooses a high fertility regime depending on the aggregate fertility state, the aggregate uncertainty measure ζ, we will now specifying the Markov transition rates for the aggregate fertility process process and deriving its steady-state (or long run equilibrium) distribution. In this economy, the process of household sequential choice-making, in continuous time, on own fertility underlies the evolution of the probability distribution of the economy over the state variable x. This process is naturally modelled as a Continuous Time Markov Chain (CTMC).

4.4.1 Distribution over the Aggregate State

Our key interest lies in the **probability distribution of the economy over the aggregate (high) fertility state**, proxying for the probability of aggregate high-fertility outcomes. To formalise this object and its dynamics, consider any point in time $t \in T = \mathbb{R}_+$.

Need to introduce stochastic process for the aggregate state more explicitly – we treat is as exogenous in developing key definition but this of course results from aggregation of agent choices. We then can define the the probability distribution function over the aggregate state in any period. Rework the below correctly. Effectively, key idea is that we are constructing the state as an ergodic Markov process with endogenous transition rates / probability transition kernel

We construct a time-dependent probability density function $f : N \times T \to [0, 1]$ which lists the probability $P(n, t)$, where $P : N \times T \to [0, 1]$, that the number of households opting for high-fertility at time t is any one of the $N+1$ admissible values $n \in N$, for all such values. In particular, given finiteness of the support of the distribution, this function is a vector-valued function:

$$f(\{n\}_{n\in N},t) = (f_1(\{n\}_{n\in N},t),...,f_N(\{n\}_{n\in N},t))' = (P(0,t),...,P(N,t))', \tag{5}$$

where we assume $P(n,t)$ to be differentiable with respect to time for all $n \in N$.

Following Aoki (2007), in any time interval we assume that only *one* household can switch from h to l and vice-versa. To make this more concrete, consider an economy distributed at time t so that there is a $p(n,t)$ probability that n households are highly fertile in that period. In any such defined time interval, based on the choice process modelled by (1)-(4), the number of highly fertile households can increase or decrease by 1, respectively becoming $(n+1)$ and $(n-1)$. For any state in the support $\{0,1,...,N\}$, the rates at which such transitions respectively occur are given by:

$$\tau^{+}_{n,n+1} = N(1-x)\omega(x),$$
$$\tau^{-}_{n,n-1} = Nx\omega(x).$$

As modelled in the equations, these transition rates respectively depend on two variables. First, the number of households currently opting for low-fertility $N(1-x)$ or high-fertility x, cast in terms of our state variable. Second, The economy-wide probability that a household chooses to be highly fertile $\omega(x)$ or lowly fertile $(1-\omega(x))$, both dependent on the aggregate state via $\mu(x)$ in equation (4).

Based on this formalisation of the probability distribution of the economy over the aggregate fertility states and of transition rates, we will now focus on the analysis of dynamics in the system and of the steady state of the economy. The steady state of the economy consists of a statistical or stochastic equilibrium (cf. Yoshikawa, 2003). That is, the equilibrium will be a time-invariant probability distribution of the economy over the high fertility state.

4.4.2 Dynamics of the Measure over Aggregate Fertility Outcomes

The dynamics of the probability distribution over (continuous) time are modelled, as standard, as the rate of change or first derivative of the density function in (5) with respect to time. Accordingly, dynamics are modelled by a $n+1$-vector-valued function (yielding the gradient of the probability distribution when evaluated at t):

$$D_t f(n,t) = (D_t f_0(\{n\}_{n\in N},t),...,D_t f_N(\{n\}_{n\in N},t))'.$$

Making use of the definition in (5), then:

$$D_t f(n,t) = \left(\frac{\partial P(0,t)}{\partial t}, \frac{\partial P(1,t)}{\partial t},...,\frac{\partial P(N,t)}{\partial t}\right)'. \tag{6}$$

Hence, the dynamics of the distribution of the economy over the aggregate high fertility state state (a continuous time Markov Chain) are pinned down by a set of $n+1$ partial derivatives, each of them modelling how the probability of any particular high-fertility aggregate outcome $P(n,t)$ changes over time.

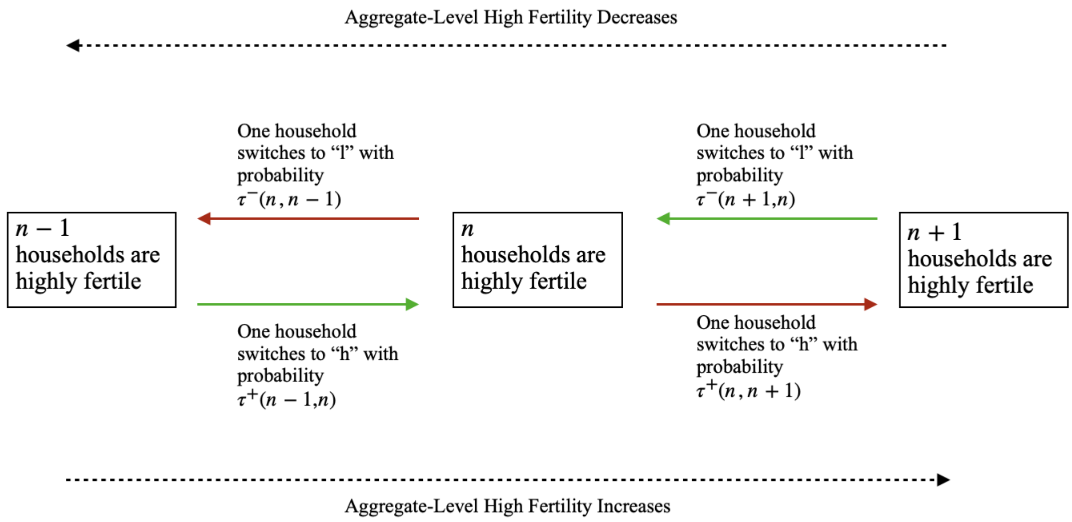

Fig.1 Partial dynamics at aggregate state n. Red arrows are probability fluxes out of the state, green arrows are probability fluxes into the state. Source: Author's own.

We will now model these partial rates of change via Partial Differential Equations (PDE) based on the preference-dependent transition rates derived earlier. The approach taken here is different from the original one in Aoki (2007), and valuably relaxes the form of the PDEs governing transitional dynamics outside of the steady state. Consider any admissible state n within the finite support N. As visualised in Fig.1, the net change in probability over the defined time interval is the net results of probability flows out of the state ($n \to n+1$ and $n \to n-1$), and into the state ($n+1 \to n$ and $n-1 \to n$). Accordingly, each partial derivative in (6) obeys PDE in a set of $N+1$ PDEs of the form:

$$\frac{\partial P(n,t)}{\partial t} = P(n+1,t)\tau^{-}_{n+1,n} + P(n-1,t)\tau^{+}_{n-1,n} - P(n,t)\tau^{+}_{n,n+1} - P(n,t)\tau^{-}_{n,n-1}. \quad (7)$$

Essentially, the PDE models the partial rate of change of the distribution over N at any n as the difference between the transition rates out of n into the $n+1$ and $n-1$, weighted by the probability that $n+1$ and $n-1$ occur and the transition rates into state n, also weighted by the probability n occurs. To simplify analysis of the steady state of the distribution, Aoki (2007) restricts the second and fourth term on the right hand side of (7) to zero.[2] This paper does not make this (demanding) restriction on transitional dynamics. Nonetheless, as outlined below, we are able to obtain the same structure underlying the steady state by suitably exploiting the recursivity of the PDEs governing aggregate dynamics.

In particular, consider the PDE in the set in (7) associated to state $n-1$:

$$\frac{\partial P(n-1,t)}{\partial t} = P(n,t)\tau^{-}_{n,n-1} + P(n-2,t)\tau^{+}_{n-1,n-1} - P(n-1,t)\tau^{+}_{n-1,n} - P(n-1,t)\tau^{-}_{n-1,n-2}$$

Exploiting this, the PDE in (7) can be rewritten recursively as:

$$\frac{\partial P(n,t)}{\partial t} = P(n+1,t)\tau^{-}_{n+1,n} - P(n,t)\tau^{+}_{n,n+1} - \frac{\partial P(n-1,t)}{\partial t} + P(n-2,t)\tau^{+}_{n-2,n-1} - P(n-1,t)\tau^{-}_{n-1,n-2}.$$

Reiterating the same argument for the PDE governing partial dynamics at $n-2$, $n-3$, and onwards sufficient recursions lead to the following model for Partial dynamics at n on which analysis of the

[2]To visualise this in figure 1, it corresponds to requiring the flows between n and $n-1$ to balance to 0 at all t.

steady state dynamics/statistical equilibrium will be based (alternative to the one based on Aoki's (2007) restriction):[3]

$$\frac{\partial P(n,t)}{\partial t} = -\sum_{k=1}^{n} \frac{\partial P(n-k,t)}{\partial t} + P(n+1,t)\tau^{-}_{n+1,n} - P(n,t)\tau^{+}_{n,n+1}, \qquad \forall n. \tag{8}$$

4.4.3 Stationary Equilibrium Fertility Traps

To examine our hypothesis on HFTs, we assume these emerge as long-run outcomes of dynamics in the economy captured by the steady-state of the economy, and hence we focus on the steady-state of the probability distribution of the economy over aggregate high-fertility $f(n,t) = (P(1,t), P(2,t), ..., P(N,t))$, subject to a boundary condition $P(0,t) = a \in \mathbb{R}_+$.

This steady state is easily found, by setting to zero the left hand side of the corresponding PDE in (7), which describes the rate of change in the respective probability $P(n,t)$, for all high-fertility outcomes $n \in N$:

$$\frac{\partial P(n,t)}{\partial t} = 0. \tag{9}$$

Dropping time dependence (as the steady state is time-invariant by definition), we can substitute condition (9) in the recursive representation we obtained in (8), in order to pin down the steady state as the probability distribution $f(n)$ satisfying the $N+1$ equations:

$$P(n+1) = \frac{\tau^{+}_{n,n+1}}{\tau^{-}_{n+1,n}} P(n) \qquad \forall n \tag{10}$$

together with boundary condition:

$$P(0) = a$$

Noting the recursive structure at the steady-state of the equations describing the steady state, then we must likewise have:

$$P(n) = \frac{\tau^{+}_{n-1,n}}{\tau^{-}_{n,n-1}} P(n-1) \qquad \forall n.$$

Following the step proposed in Aoki (2007), we can exploit this recursion to derive, by substitution of the above equation in (10), the stationary probability distribution of a high-fertility trap with exactly n households choosing a high fertility regime, as follows:

$$P(n) = \frac{\tau^{+}_{n-1,n}}{\tau^{-}_{n,n-1}} \frac{\tau^{+}_{n-2,n-1}}{\tau^{-}_{n-1,n-2}} P(n-2)$$

$$= \frac{\tau^{+}_{n-1,n}}{\tau^{-}_{n,n-1}} \frac{\tau^{+}_{n-2,n-1}}{\tau^{-}_{n-1,n-2}} ... \frac{\tau^{+}_{n-j,n-j+1}}{\tau^{-}_{n-j+1,n-j}} P(n-j) \qquad \forall n, j \leq n.$$

Setting $j = 0$, and using $P(0) = a$, we obtain the steady-state probability distribution of the economy over each of $n \in N$ high fertility aggregate states:

$$P(n) = a \prod_{i=1}^{n} \frac{\tau^{+}_{n-i,n}}{\tau^{-}_{n,n-i}}, \qquad \forall n \in N, \tag{11}$$

[3]in addition to the recursion, the derivation uses the fact that the transition rate $\tau^{-}(0,-1) = 0$ by definition of the transition rate, and the boundary condition/state boundary on the measures over the aggregate state $p(-1,t) = 0$ as $-1 \neq N$.

where the transition rates are computed according to the definition on p.7. As Aoki (2007) argues for the more general case of Markov Chains, since there are many possible ways in which a complex and interacting context of N households can lead to such outcome (i.e. we do not care about the identity of the high fertility households), the probability distribution of a high-fertility trap with n highly-fertile household should be adjusted to account for all possible households combinations consistent with such outcome. From standard combinatorics, we multiply the (vector valued) solution in (11) by the number of ways we can select n households from the set N. Hence the steady state probability distribution (statistical equilibrium) is pinned down by:

$$\hat{P}(n) = a\frac{N!}{n!(N-n)!}\prod_{i=1}^{n}\frac{\tau^{+}_{n-i,n}}{\tau^{-}_{n,n-i}}, \qquad \forall n \in N \tag{12}$$

The above combinatorial/complexity feature absent in representative household models. Combined with the micro-founded uncertainty ζ resulting from household heterogeneity, this delivers on our hypothesis in section (3).

To show this, we rewrite the above probability distribution of high-fertility traps, with an arbitrary number n of households choosing a high fertility regime, in logarithmic form (a likelihood function). To do this, we make use of the micro-foundational specification of transition rates on p.7. The resulting expression is consistent with a Log-Likelihood function representation. In line with Maximum Likelihood Estimation theory, we model the outcome of the economy as the one maximising the log-likelihood function – i.e. the one consistent with the aggregate choice structure on fertility in (12) being the most likely to occur.

ADDED: key idea is recognising that with a finite number of households, n and x are simultaneously determined. This is the point of statistical rational expectations. Given beliefs about x in equilibrium, the implied measure over the aggregate state is peaking at such forecast x.

– **Idea**

– Assume n/N or x is the equilibrium of the economy (i.e. a fixed point of the law of motion $G_x : X -- > X$).

– Then, with SREE, it must be the case that such outcome/law of motion is/matches the most probable one when individual expect it/believe it to be the law of motion (i.e. if it is not, then it is not a SRE Equilibrium – it is not supported by statistically rational expectations). THINK ABOUT INFINITELY REPEATING... individual could not rationally expect x when they would observe every tim they do some other x results at least – where would they put their bucks?

– If this is the case, then it must be the case that x enters both the LHS and RHS. I.e. we set to thd outcome x over which the measure is defined the expectation entering the law of motion defining the stationary measure. The maximum probability is *then* attained at the x maximising the RHS.

– We could proceed, similarly to classical rational expectations, the other way around and suppose that individuals have some belief or forecast x. Then by restriction of the solution concept / assumption of SREE, it must be the case that belief in x induces the highest probability that x occurs out of all possible state configurations. (Cf. with capital stock path in closing decentralised RCK model). But if this is the case, then at the maximum of the likelihood function (RHS) with respect to x, we must necessarily be at a maximum of the likelihood that x is the aggregate outcome. I.e. similalry to above.

– In this analytical/closed form case, we can use this notion to directly solve for SREEs of the economy, and quantify the impact of uncertainty on the micro-macro transmission mechanism.

Accordingly, taking the logarithm of (12) and making use of the definition of transition rates, the Log-Likelihood function of a high-fertility trap with n housholds opting for high fertility is:

$$\ln(P(x)) = \ln\left(\frac{1!}{x!(1-x)!}\right) + \int_{[0,n/N]} \ln\frac{\omega(x)}{1-\omega(x)}dx \tag{13}$$

In the above equation, the number n of high-fertility households in the binomial coefficient has been replaced, as standard, by the corresponding state variable x (and the numerator normalised to one accordingly); the summation instead was replaced by the corresponding integral on the support $[0, n/N]$ to account for the continuous nature of the latter (cf. Aoki, 2007). Based on (13), by Stirling's formula and the expression derived for the state-dependent probability $\omega(x)$ of high fertility being preferred to low fertility given social interactions and externalities, we finally obtain a micro-founded Log-Likelihood function (14). Equation (14), equivalent to (13), models the log-likelihood of an *n-household sized high fertility trap* as function of moments of the distribution of preferences derived in section 4.1:

$$\ln(P(x)) = \zeta\left(2\int_{[0,n/N]} \mu(x)dx - \zeta^{-1}(x\ln x + (1-x)\ln x)\right). \tag{14}$$

To study the above and bring it to bear on our question, it is important to reflect on the duality of variable x in the present context. The fraction x of highly fertile households enters the left hand side as a (proportion) variable on which a probability measure $P(x)$ is defined, while it enters the RHS as a parameter of the (log) likelihood function of a high fertility trap of size x.

Maximisation of the log likelihood function, by standard, selects the value of the parameter that induces the highest likelihood that a data-generating process modelled on the basis of such parameter outputs the available or simulated data (in our case, the simulated HFT of size x). Because of the noted parameter-data duality of x, maximisation of the log-likelihood in this context by construction selects the size of the fertility trap with the highest probability of occuring in the population. In this sense, we refer to the solution to the log-likelihood maximisation problem as a probabilistic macro-equilibrium for the economic system: no other outcome, upon consideration of the proposed data generating process via household preferences condensed in equation (13), occurs with higher probability. The solution to the problem is the proportion x^* easily found by differentiation of the left hand side to yield the first order condition:

$$2\mu(x^*) - \zeta^{-1}\ln\frac{x^*}{1-x^*} = 0 \tag{15}$$

Now, there is a key intuition linking the above condition to the distribution of incentives we micro-founded earlier, and thereby allowing to examine our hypothesis relative to the macro-equilibrium of the system relative to the diffusion of high fertility traps. Namely, that the number and stability of critical points of the conditional mean function $\mu(x)$ captures the probabilistic counterpart to the incentive structures underlying high-fertility equilibria or lack thereof in the examined literature on demand for fertility. This function, we recall from p.6, is the population mean of the difference in household-level utility/preferences for alternative fertility regimes, hence modelling average household preferences/incentives for high fertility, under external effects from the high-fertility choice of other households via dependence on the economy wide high fertility households proportion x. For example, when $\mu(x) > 0$, on average there is a preference for high fertility, implying x rises, and vice versa for $\mu(x) < 0$. A unique stable zero at x s.t $\mu(x) = 0$ then implies a unique micro-behavioural equilibrium for the average agent such that the agent has no incentive to deviate from its current choice. This equilibrium can correspond to an aggregate high-fertility or low-fertility outcome depending on whether parameter x is respectively high or low. Multiple stable zeros (a collection of x such that $\mu(x) = 0$ holds for each of theinstead capture the 'perverse' incentive structure or potential for coordination failure emphasised in rational choice explanations (recall the literature review).

From the derived condition for x^* to constitute a macro-equilibrium (i.e. for there to be a sticky, high-fertility trap involving some proportion sized x^* of the economy), it follows immediately that such proportion x^* will coincide with the micro-behavioural equilibrium individuated by the indifference condition in average conditional preferences $\mu(x) = 0$ only when parameter ζ is high enough, corresponding — recall— to low uncertainty induced by households heterogeneity. In fact, we have that, at the macro-equilibrium x^*:

$$\lim_{\zeta \to \infty} 2\mu(x^*) - \zeta^{-1} \ln \frac{x^*}{1 - x^*} = 2\mu(x^*)$$

.

Hence the condition for a macro-equilibrium tends to the condition for a micro-equilibrium only as uncertainty shrinks to zero. In other words, the conditional preferences or incentives of the average agent drive or constitute a representation of aggregate behaviour (the macro-equilibrium) only in a limiting sense. The requisite condition, in particular, is no uncertainty or no dispersion in society relative to preferences on fertility. Technically, this is the case because, under no uncertainty condition, the distribution of preferences collapses to a spike at $\mu(x)$, so that the economy can be summarised by a single representative agent. Conversely, heterogeneity-induced uncertainty is high, the solution to $\mu(x) = 0$— or the micro equilibrium identified by the distribution of fertility incentives — will not generally constitute a macro-equilibrium.

In this sense, to answer our narrowed-down question, high-fertility traps or multiple fertility equilibria still form at the aggregate level even when highfertility does not coincide with the behavioural implications of mean household fertility preferences, even in the presence of external effects: no perverse incentive structure is necessary, with heterogeneity-induced uncertainty alone sufficing to the formation of high-fertility traps in defiance of average microeconomic rationality. Before concluding, we should also consider that the complementary explanation offered by a complexity-based approach also appears to cohere with several of the corollary critical issues we raised against the adequacy of purely incentive based demand for fertility models. For example, it appears to provide a possible explanation for the limited effectiveness of incentive-based policies such as micro-credit which we contested. Since these elements, in our model, would be easily interpreted as a modification of the conditional mean function $\mu(x)$ to alter the micro-equilibrium identified by incentives, they are unlikely to succeed in dislodging the macro-equilibrium from a high-fertility trap if uncertainty is high and the two do not coincide as per our explanation of the phenomenon.

5 Conclusion

This theoretical paper examined explanations for the conditions under which we can expect high-fertility traps to form. The subject draws critically from and was motivated in the relation to the course material studied in week two on the correlates of poverty and the feedback mechanisms, including socially adverse reproductive choices, leading to the emergence of poverty traps — especially, the extensive work of Dasgupta. Building on such literature, rational choice or 'demand for fertility' analyses of the phenomenon were assessed and found unsatisfactory on a number of levels, with evidence suggesting that more complete explanations should take into consideration and explain the possibility that high-fertility traps form even in the absence of perverse incentive structures.

The paper, building on the alternative literature on social interaction and household heterogeneity, justified and adopted a heterogenous agent-based approach to model, on the blueprint of an analogous procedure and model set out by Aoki (2007), the paradoxical formation of aggregate high-fertility traps in economies in which the average incentive structure is in line with lower aggregate fertility outcomes. As hypothesised, in contexts marked by social interaction among finite numbers of heterogeneous households (so that individual heterogeneity does not was out in the aggregate), the resulting uncertainty or social dispersion of preferences is a most critical factor, sufficing indeed to the formation of (low) high-fertility traps even when mean household incentives do not individuate (low) high-fertility as an individually optimal behaviour. This result strengthens our understanding of low and high fertility traps by relaxing the conditions under which they can be expected from canonic ones in the market failure approach to explaining fertility traps (public good provision failure, coordinaton failure, externalities) to a milder requirement of substantial heterogeneity in preferences in the population.

While a key sufficient condition, market failures in collective delivery of fertility outcomes through decentralised decision making is not a necessary pre-condition for the persistence of fertility traps. In turn, shifting attention from policies attempting to correct average incentives in the population to policies attempting to compress dispersion in social preferences. While the paper has not explicitly modelled the uptake and implications of alternative policies to the mainstream average incentive-targeting ones (a future research direction), policies that seriously engage with the distributional underpinnings of the outcomes seem particularly apt: improved informational flows at the extremes of the distribution of preferences and re-distributive measures, in particular, might be critical to reducing spread around the representative agent normally targeted by average incentive-restructuring policy. Put more prosaically, mainstream policies targeting average incentive structures might be premature because they only work in the limit: first, uncertainty must critically be reduced.

Bibliography

Aoki, M. (2007). 'A New Stochastic Framework for Macroeconomics: some illustrative examples' Complexity Hints for Economic Policy (Eds. Salzano, M. and Colander, D.). Springer.

Aoki, M. (2002). Modeling Aggregate Behaviour and Fluctuations in Economics. Cambridge: Cambridge University Press.

Aoki, M. (1996). New Approaches to Economic Modeling. Cambridge: Cambridge University Press.

Aoki, M. and Yoshikawa, H. (2007). Reconstructing Macroeconomics. Cambridge: Cambridge University Press.

Aoki, M. and Yoshikawa, H. (2003). 'Uncertainty, Policy Ineffectiveness, and Long Stagnation in the Macroeconomy' Working Paper 316. Stern Business School. New York.

Aoyama, H., Aruka, Y., and Yoshikawa, H. (2021). Complexity, Heterogeneity, and the Methods of Statistical Physics in Economics: Essays in Memory of Masanao Aoki. Springer.

Banerjee, A.V. and Duflo, E. (2011). Poor Economics. Hatchette. Bardhan, P. And Udry, C. (1999). Development Microeconomics. Oxford: Oxford University Press.

Barro, Robert J. 1991. 'Economic Growth in a Cross-Section of Countries' Quarterly Journal of Economics. 106. pp. 407–443.

Bardhan, P. (1988). 'Alternative Approaches to Development Economics' Handbook of Development Economics 1A. Amsterdam.

Basu, K, and Van, P.H. (1998). 'The Economics of Child Labor' American Economic Review. 88.

Basu, K. (1998). 'Comment on P. Dasgupta, The Economics of Poverty in PoorCountries' The Scandinavian Journal of Economics. 100(1). pp. 69-72.

Batliwala, S. and Reddy, A.K.N. (1994). 'Energy Consumption and Population' in Population: The Complex Reality. London: The Royal Society.

Becker, G., and Lewis, H.G. (1974). 'Interactions between Quantity and Quality of Children' in Economics of the Family (Ed. Shultz, T.N.). Chicago: University of Chicago Press.

Becker, G. (1960). 'An Economic Analysis of Fertility' in Demographic and Economic Change in Developed Countries (Ed. Becker, G.). Princeton: Princeton University Press.

Besley, T. (1995), 'How do Market Failures Justify Interventions in Rural Credit Markets' World Bank Research Observer 9(1).

Birdsall, N. (1998). 'Economic Approaches to Population Growth' in Handbook of Development Economics 1 (Eds. Srinivasan, T.N. and Chenery, H.). Amsterdam: North Holland.

Bongaarts, J. (2020). 'Trends in Fertility and Fertility Preferences in Sub- Saharan Africa: the Role of Education and Family Planning Programmes' Genus. 76(32).

Bongaarts, J. and Watkins, S. (1996). 'Social interactions and contemporary fertility transitions' Population and Development Review. 22(1). pp. 639–682.

Caldwell, J.C. (1976). 'Toward a Restatement of Demographic Transition Theory' Population and Development Review. 2(3,4). pp. 321-66.

Cleaver, K.M. and Shreiber, G.A. (1994). Reversing the Spiral: The Population, Agriculture, and Environment Nexus in Subsaharan Africa. World Bank. Washington DC.

Cooper, R. And John, A. (1988). ‘Coordinating Coordination Failures in Keynesian Models’ Quarterly Journal of Economics.

Dasgupta, P. (2000). ‘Reproductive Externalities and Fertility Behaviour’ European Economic Review. 44(1). pp. 619-644.

Dasgupta, P. (1998). ‘The Economics of Poverty in Poor Countries’ The Scandinavian Journal of Economics. 100(1). pp. 41-68.

Dasgupta, P. (1995). ‘The Population Problem: Theory and Evidence’ Journal of Economic Literature. 33(4). pp. 1879-1902.

Dasgupta, P. (1993). An Inquiry into Wellbeing and Destitution. Oxford: Oxford University Press.

Dasgupta, P. and Weale, M. (1992) ‘On Measuring the Quality of Life’ World Development 20. pp. 119-131.

Dommaraju, P., Tan, J. (2014). Households in Contemporary Southeast Asia. Journal of Comparative Family Studies, 45(4), 559–580.

Durlauf, S.N. (1997). ’Statistical mechanics approaches to socioeconomic behavior.’ in The Economy as an Evolving Complex System II (Eds. Arthur, W.B., Durlauf, S. Lane, D., and Park, M.). California: Addison- Wesley.

Durlauf, S.N. and Walker, J.R. (2001). ‘Social Interactions and Fertility Transitions’ Diffusion Processes and Fertility Transitions (Ed. Casterline, J.B.). Washington. pp. 115-37.

Ingber, L. ‘Statistical Mechanics of neocortical interactions’ Physica, D5. pp. 83-107.

Lee, R.D. and Miller, T. (2000). ‘Population Policy and Externalities to Childbearing’ The Annals of the American Academy of Political and Social Science. 510: World Population. pp. 17-32.

Lee, R.D., and Miller, T. (1990). ‘Population Growth, Externalities to Childbearing, and Fertility Policies in Developing Countries. World Bank Economic Review. 4(1). pp. 275-304.

Pritchett, L. (1994). ’Desired Fertility and the Impact of Population Policies’ Population and Development Review. 20(1). pp. 1-55.

Ray, D. (1998). Development Economics. Princeton: Princeton University Press.

Shultz, T.P. (2007). ‘Fertility in Developing Countries’ Economic Growth Center Discussion Papers (n. 953). Yale University.

Schultz, T.P. (1988). ‘Economic Demography and Development’ The State of Development Economics: Progress and Perspectives (Eds. Ranis, G. and Shultz, T.P.). Oxford: Blackwell. pp. 416-51.

Srinivasan, T.N and Chenery, H. (1988). Handbook of Development Economics 1A. Amsterdam: North Holland.

Strauss, J. And Thomas, D. (2007). ‘Human Resources: Empirical Modeling of Household and Family Decisions’ Handbook of Development Economics 3A (Eds. Behrman, J. And Srinivasan, T.N.). Amsterdam.

Were, J.M., Stranges, S. and Creed, I.F. (2020). 'Fertility is a key predictor of the double burden of malnutrition among women of child-bearing age in sub-Saharan Africa' Journal of Global Health. 10(2).

World Bank. (2010). Determinants and Consequences of High Fertility: A Synopsis of the Evidence. Washington, DC.

Yoshikawa, H. (2003). 'Stochastic Equilibria' The Japanese Economic Review. 54. pp. 1-27.