Hyperbolic geometry of complex networks: models of network growth and embeddings of real networks

Summary of the doctoral (PhD) dissertation of Bianka Kovács

Hidden geometric structures underlying complex networks gain increasing attention in recent studies of network theory. The assumption of an analogy between the hyperbolic geometry and the connection rules of real-world networks yielded two natural applications: the generation of real-like artificial graphs using distance-dependent connection probabilities between nodes arranged in a hyperbolic space, and the hyperbolic embedding of real networks, where the aim is to find such a hyperbolic arrangement of the network nodes that well reflect the connection structure of the given network through the hyperbolic distances between the node position vectors. My PhD dissertation contributes to the fields of both hyperbolic network generation models and hyperbolic node embeddings.

It is well-known that hyperbolic network models, including the popularity-similarity optimization (PSO) model of network growth, are able to reproduce realistic network features given by the small-world property, a scale-free degree distribution and a high average clustering coefficient. In addition, as I have shown in my dissertation, by allowing in the PSO model the formation of new links between previously appeared nodes too and not only between the newly appearing node and the older ones, and by also simulating the disappearance of connections, the dependence of the average internal degree of the subgraphs spanning between the nodes of large enough degree on the degree threshold can be made well adjustable. Thus, a densification of such subgraphs towards the larger degree thresholds becomes achievable, which is also claimed to be a frequent property of real networks. Moreover, following some rudimentary observations from the literature, I confirmed in my dissertation via a detailed numerical study that strong community structures – which are often present in real networks too – can emerge from the PSO model in a large region of the parameter space, even in spite of the fact that this was not an intention at the construction of the model.

After the application of three- or higher-dimensional hyperbolic spaces has become more and more popular in the hyperbolic embedding methods, I explored in my dissertation the implications of increasing the number of dimensions of the hyperbolic space above 2 in the PSO model. The examination of the obtained dPSO model revealed that the increase in the number of dimensions d limits the achievable maximal strength of the clustering and the emergent community structure of angularly uniform dPSO networks having a given degree decay exponent. Besides, I used the dPSO model in my dissertation for transforming d-dimensional Euclidean node embeddings into hyperbolic ones, also utilizing the freedom in the radial ordering of equal-degree nodes described in the dissertation.

Finally, my dissertation presents the hyperbolic embedding algorithms that I developed for embedding not only undirected graphs but also directed networks in hyperbolic spaces of any number of dimensions, avoiding the assumption of any specific hyperbolic network model as the generator of the network to be embedded and using novel, exponential measures of the topological proximity and distance. According to numerous measurements carried out on real networks, both the methods that create hyperbolic embeddings from Euclidean node arrangements with my new, model-independent conversion and the algorithm that embeds networks directly in the hyperbolic space perform relatively well in mapping accuracy, graph reconstruction and also greedy routing at the same time.