Inelastic Collisions

Conservation laws are some of the most powerful concepts in physics. In this lab we will explore the conservation of momentum and energy by studying *inelastic* collisions.

1 LEARNING OBJECTIVES

At the conclusion of this activity you should be able to:

- Identify the difference between elastic and inelastic collisions.
- Design an experiment to measure momentum and energy conservation.
- Use a photogate to measure the velocity of an object.

2 BACKGROUND

Conservation laws can be used to describe complicated physical systems. For example, collisions involving fundamental particles can be accurately reconstructed using the conservation of energy and momentum. At larger scales, conservation laws allow us to describe stellar and galactic dynamics.

Conservation laws require that the initial and final quantities of an observable are the same. The conservation of energy and momentum are often useful for solving many problems in general physics.

2.1 ERROR PROPAGATION

Read the document "Error Propagation Reference" that is available on the "Reference Material" link on our Blackboard site. Pay particular attention to the example that is given in Section 2 of that reference.

2.2 CONSERVATION OF MOMENTUM

Recall that the momentum[1] of a system is given by:

$$\vec{P} = m\vec{v}, \tag{2.1}$$

where m is the mass of an object and $\vec{v}$ is the velocity.

The total momentum $\vec{P}_T$ of a system of n discrete parts can be expressed as the sum of each of the parts:

$$\vec{P}_T = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n \tag{2.2}$$

When the sum of external forces acting on a system is zero, momentum is conserved. That is, the total initial momentum of the system $\vec{P}_i$ is equal to the total final momentum $\vec{P}_f$:

$$\vec{P}_i = \vec{P}_f. \tag{2.3}$$

2.3 Kinetic Energy

An object of mass m and velocity $\vec{v}$ has a kinetic energy[2] given by:

$$K = \frac{1}{2} m \vec{v}^2 \tag{2.4}$$

The total kinetic energy K_T of a system of n discrete parts can be expressed as the sum of each of the parts:

$$K_T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots + \frac{1}{2} m_n v_n^2. \tag{2.5}$$

2.4 Elastic Collisions

Elastic collisions[3] involve only conservative forces. In these collisions the total kinetic energy of the objects involved is conserved:

$$K_i = K_f, \tag{2.6}$$

where K_i and K_f are the total initial and final kinetic energies of the system respectively.

In an elastic collision, the total initial kinetic energy (sum of the parts) is equal to the total final kinetic energy. Examples of (nearly perfect) elastic collisions are those between billiard balls.

2.5 Inelastic Collisions

Inelastic collisions do not conserve energy[4]. The total initial kinetic energy of the system is not equal to the total final kinetic energy:

$$K_i \neq K_f \tag{2.7}$$

where K_i and K_f are the total initial and final kinetic energies respectively.

An example of an inelastic collision is a car crash. Automobiles are designed to collapse and deform to absorb as much energy as possible (maximize the value of F) in a collision to protect the passengers.

2.6 Fractional Energy Loss

Fractional energy loss F is one way to quantify the amount of energy "lost"in a collision. The fractional energy loss is given by:

$$F = \frac{|K_i - K_f|}{K_i} \tag{2.8}$$

where K_i is the total kinetic energy of the system *before* the collision and K_f is the total kinetic energy of the system *after* the collision.

In a perfectly elastic collision, the fractional energy loss is equal to zero.

Suppose we have two masses m_2 and m_1 that collide. Before the collision, m_2 is initially at rest and m_1 has velocity v_i. After the collision, the two masses stick together and travel with velocity v_f. In this specific case, the fractional energy loss can be written in terms of the ratio of initial and final velocities.

$$F = \frac{|K_i - K_f|}{K_i} \tag{2.9}$$

$$F = \frac{\left|\frac{1}{2} m_1 v_i^2 - \frac{1}{2}(m_1 + m_2) v_f^2\right|}{\frac{1}{2} m_1 v_i^2} \tag{2.10}$$

$$F = \left|1 - \left(\frac{m_1 + m_2}{m_1}\right)\left(\frac{v_f}{v_i}\right)^2\right| \tag{2.11}$$

3 Procedure

You will use gliders on an (approximately frictionless) air track[5] to study inelastic collisions.

The goal is to measure the fractional energy loss F during inelastic collisions between carts of different masses.

The velocity of each cart can be measured directly using Logger Pro and photogates. A photogate is an optical device that is used to accurately time events. When an object obstructs the beam of a photogate the timer starts, when the obstruction is removed, the timer stops. Based on the dimensions of the object that obstructs the photogate and the measured time, a velocity can be calculated. For more information on the photogate you are using see:
https://www.pasco.com/file_downloads/Downloads_Manuals/Photogate-Head-Manual-ME-9498A.pdf

Most of the work of photogate timing is handled by the Logger Pro DAQ software. In Logger Pro, open the file: `File ▸ Open... ▸ Probes & Sensors ▸ Photogates ▸ Collision Timer.cmbl`. This Logger Pro configuration file will be used throughout the experiment.

3.1 Experiment

- Setup inelastic collisions with a moving glider (Cart 1 with mass m_1) colliding with a stationary glider (Cart 2 with mass m_2). Vary m_1 and m_2 by adding additional masses to the carts. Note that mass must be added symmetrically to each cart to keep the cart balanced on the air track.
- For each mass configuration:
  - Compute the *predicted* fractional energy loss using the equation you derived in the Pre-lab Quiz that depends only on the cart masses.
  - Measure the initial velocity of cart 1 v_i and the final velocity of the cart 1 + cart 2 system v_f for several (at least 5) collisions. Calculate the average ratio of velocities $\overline{v_f/v_i}$. Use statistics to estimate the uncertainty associated with the average ratio of velocities $\sigma_{\overline{v_f/v_i}}$.
  - Use $\overline{v_f/v_i}$ and Equation 2.11 to obtain the *observed* fractional energy loss.
  - Use error propagation to obtain the uncertainty associated with the fractional energy loss for the series of collisions.
- Repeat the above steps for several different m_1 and m_2 mass configurations.

3.2 Suggestions for Analysis

- Visualize the collected data by making the plot described in the Pre-lab Quiz.
- What measured quantities are used to compute the *predicted* fractional energy loss? Propagate the uncertainty in these measured quantities to estimate the uncertainty in the *predicted* fractional energy loss.
- Repeat the previous step to estimate the uncertainty in the *observed* fractional energy loss.
- Use a linear model to quantitatively describe the relationship between the *observed* and *predicted* fractional energy losses.

4 Lab Notebook

Your submission will be evaluated using the following rubric:

- Lab Notebook Mechanics (6 points)
  - Relevant information *e.g.*: your name, your lab partner's name, date, *etc.* is present.
  - The notebook is organized and easy to read. Markdown cells are used for narrative text. Code cells are clearly organized and commented.
  - The ZIP file of the notebook is healthy and runs correctly.
- Data Analysis & Plots (6 points)
  - The notebook tells a scientific story; it is an accurate record of the work that you did.
  - The notebook should show evidence of trial and error. Keep a good record of your work – recording mistakes is useful.
  - Record rough data and plots that you used to verify that the analysis was on the right track. Final versions of plots should be well formatted and meet the plotting guidelines for the course.
  - Use models to identify trends that your data exhibit or other apparent relationships between your independent and dependent variables.
- Results and Comparison (6 points)
  - Clearly state the final result(s) of your experiment. Remember to quote your result with units and appropriate significant digits.
  - Final result plots are well formatted and meet the standards described in the Figure Formatting reference.
  - A useful comparison is made to a known/expected value or another similar result.
  - Choose the best available tools for your comparison (*e.g.* plots, pictures, discrepancy, significance of discrepancy, etc).
- Uncertainty and Error Propagation (6 points)
  - Identify the dominant source(s) of error in your experiment.
  - Support your conclusions with appropriate error estimates and error propagation calculations.

- Physical Interpretation (6 points)
  - Throughout the notebook, interpret the data, rough plots, and final results in terms of the underlying physics.
  - What are you able to conclude from your data? Clearly explain how you arrived at your conclusions from your experimental observations.
  - Reflect on how your experiments connects with the physics concepts you are studying.

REFERENCES

[1] Resnick, R., Halliday, D., Krane, K. S. (2002) *Physics, Vol. 1, 5th Edition*. Danvers, MA: John Wiley & Sons, Inc.. See Chapter 6-2 (pg. 121).

[2] Ibid. See Chapter 11-6 (pp. 239-243).

[3] Ibid. See Chapter 6-5 (pp. 126-130) and Chapter 11-8 (pp. 244-246).

[4] Ibid. See Chapter 6-1 (pp. 119-121) and Chapters 6-4 through 6-5 (pp. 124-130).

[5] PASCO 2.0m Air Track Manual: https://www.pasco.com/file_downloads/Downloads_Manuals/2-m-Air-Track-Manual-SF-9214.pdf