JOHNS HOPKINS UNIVERSITY, PHYSICS AND ASTRONOMY
GENERAL PHYSICS LABORATORY

Error Propagation Reference

The following rules are a collection of formulas from Taylor, J. R. (1996), *An Introduction to Error Analysis, 2nd Ed.*, Sausalito, CA: University Science Books. For a rigorous justification of each case, and examples of each, please read Chapter 3.

Assuming that q is some value calculated from the quantities $x,\ldots,w$, with associated uncertainties $\delta x,\ldots,\delta w$, then the following rules may be used to calculate the uncertainty δq.

SUMS AND DIFFERENCES

If

$$q = x+\ldots+z-(u+\ldots+w), \tag{0.1}$$

then

$$\delta q = \sqrt{(\delta x)^2+\ldots+(\delta z)^2+(\delta u)^2+\ldots+(\delta w)^2}. \tag{0.2}$$

PRODUCTS AND QUOTIENTS

If

$$q = \frac{x\times\ldots\times z}{u\times\ldots\times w}, \tag{0.3}$$

then

$$\delta q = |q|\sqrt{\left(\frac{\delta x}{x}\right)^2+\ldots+\left(\frac{\delta z}{z}\right)^2+\left(\frac{\delta u}{u}\right)^2+\ldots+\left(\frac{\delta w}{w}\right)^2}. \tag{0.4}$$

MEASURED QUANTITY TIMES EXACT NUMBER

If B is known exactly and

$$q = Bx, \tag{0.5}$$

then

$$\delta q = |B|\delta x. \tag{0.6}$$

Uncertainty in a Power

If n is an exact number and

$$q = x^n \tag{0.7}$$

then

$$\delta q = \left| \frac{nq}{x} \right| \delta x \tag{0.8}$$

Uncertainty in a Function of One Variable

If

$$q = q(x), \tag{0.9}$$

then

$$\delta q = \left| \frac{dq}{dx} \right| \delta x \tag{0.10}$$

General Formula for Error Propagation

If

$$q = q(x, \ldots, z) \tag{0.11}$$

then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x \right)^2 + \ldots + \left(\frac{\partial q}{\partial z} \delta z \right)^2}. \tag{0.12}$$