Accelerometers

1 INTRODUCTION

In this laboratory we will be using acceleration data to measure velocities and distances.

2 LEARNING OBJECTIVES

At the conclusion of this activity you should be able to:

- Collect acceleration data using the Physics Toolbox Sensor Suite and your smartphone.
- Use acceleration data to estimate the height from which a phone is dropped.
- Use acceleration data to estimate the height of an elevator ride.

3 BACKGROUND

3.1 ACCELERATION, VELOCITY, AND POSITION

We know from Classical Mechanics that we can move between position, velocity, and acceleration by repeatedly taking time derivatives of the position of an object. Similarly, starting from acceleration, we can take subsequent integrals with respect to time to obtain velocity and position respectively.

Description	**Differential Form**	**Integral Form**
Position	$\vec{x}$	$\vec{x} = \int \vec{v}\,dt$
Velocity	$\vec{v} = \frac{d\vec{x}}{dt}$	$\vec{v} = \int \vec{a}\,dt$
Acceleration	$\vec{a} = \frac{d^2\vec{x}}{dt^2} = \frac{d\vec{v}}{dt}$	$\vec{a}$

3.2 NUMERICAL INTEGRATION

Numerical integration is the computational process by which the area under a curve can be estimated[1]. There are many ways to estimate the area under a curve. One common method, that is also provided by `NumPy`, is the Trapezoidal Rule[2].

Let's take for example the curve $y = x^2$. Suppose we want to numerically integrate between $x = 0$ and $x = 1$. This is an easy calculation analytically:

$$A = \int_0^1 x^2 dx = \frac{1}{3}. \tag{3.1}$$

To calculate this integral using the trapezoidal rule, we might do the following. First, we sample the space between $x = 0$ and $x = 1$ at regular intervals, say at $x \in [0, 0.25, 0.5, 0.75, 1]$. Next, we evaluate the function at each of our chosen x values:

x	0	0.25	0.5	0.75	1
y	0	0.0625	0.25	0.5625	1

Finally, we calculate the area of the trapezoids under the curve that are defined by the heights a and b (the calculated y values) and the length of the base d (the space between chosen x values). Specifically, the area of the first trapezoid, between $x = 0$ and 0.25 is given by:

$$A_{0\to 0.25} = \left(\frac{a+b}{2}\right) d = \left(\frac{0.0625+0}{2}\right) 0.25 = 0.0078125. \tag{3.2}$$

The subsequent area calculations are summarized in Figure 3.1.

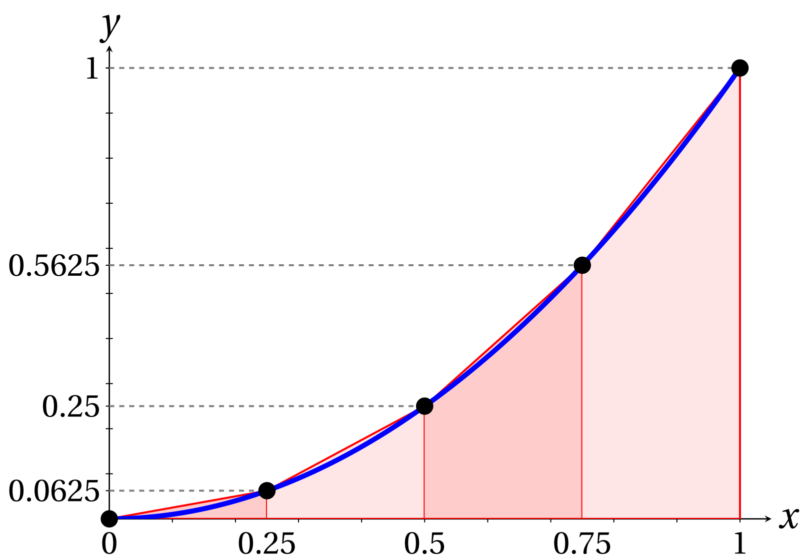

Figure 3.1: Trapezoids used to estimate the area under the curve $y = x^2$ (shown in blue). The area shaded in red represents the area of each trapezoid used to calculate the area under the curve. It is clear from the figure that the trapezoids tend to over-estimate the area under the curve for each interval.

Once the area of each trapezoid has been calculated, the total area under the curve is found by addition:

$$A_{tot} = A_1 + A_2 + A_3 + A_4 = 0.34375. \tag{3.3}$$

We can see that the area calculated using this method is slightly larger than the value calculated analytically. Of course, if more trapezoids are chosen, say 100 of them, we can improve our result:

$$A_{tot} = A_1 + A_2 + \cdots + A_{100} = 0.33335. \tag{3.4}$$

3.3 Numerical Integration and Acceleration

When it comes to determining velocity and distance from acceleration measurements, we use the same general process described in the previous section.

The `NumPy` package in Python provides a function for calculating an integral using the trapezoid rule. The function is called `trapz()`[3]. `trapz()` takes, as a required argument, the data that should be integrated and an optional argument of "x" values that should be used to sample the data. In our example above we would provide the list of y-values to the function and a list of x-values.

To calculate the velocity from the measured acceleration, we would provide a list of measured accelerations and corresponding times. A call to the `trapz()` function might look something like this:

```
v = np.trapz(acceleration, time).
```

Keep in mind, that the function returns a single number, the integral of the acceleration over the time range provided.

We are typically interested in calculating the velocity for each time in our data file – the *instantaneous* velocity. This can be accomplished by using a `for` loop to calculate the integral from $t = 0$ up to the time of each row – for each row in the dataset:

$$v_{row} = \int_0^{t_{row}} a\,dt \tag{3.5}$$

3.4 Smart Phone Apps

Please see the "Physics Toolbox User Guide"[4] for detailed information on installing and using the Physics Toolbox App to make measurements.

Another app, "PhyPhox"[5] may also be used to collect accelerometer data with excellent results.

Both apps are available for free on both iOS and Android devices.

4 Procedure

> **!!! CAUTION !!!**
>
> You will be dropping your smartphone to make acceleration measurements in this lab. To avoid damaging your phone, only drop it onto a soft surface. In the laboratory we drop phones into boxes filled with packing peanuts. At home you might try dropping your phone onto a bed, couch, or pillows.
>
> To avoid damaging your phone, limit the height your phone drops to no more than 1 meter (roughly 3 feet).
>
> If you feel nervous about dropping your device, please check in with us – before continuing – to confirm that your setup will not cause damage. If you would rather NOT drop your phone, we can provide you with the data required to complete the assignment.

4.1 Phone Drop

Using accelerometer data collected with your smart phone, determine how far the phone falls when dropped.

- Using the Physics Toolbox Sensor Suite app, record acceleration data when your phone is dropped from a known height. Drop your phone into one of the "foam pits" in the laboratory to avoid damaging your phone.
- Measure the height of the drop using a meter stick or other conventional measuring technique.
- Using the accelerometer data collected with your phone, measure the height of the drop using integration.
- Quantitatively compare the measured heights.

4.2 Elevator Ride

- Using the Physics Toolbox Sensor Suite app, record acceleration data while riding an elevator in the Bloomberg building.
- Using the accelerometer data collected with your phone develop answers for at least two of the following questions:
  - What is the distance between two floors?
  - Is the spacing between each floor the same?
  - What is the maximum velocity of the elevator?
  - How long does it take for the elevator to reach its maximum speed?
  - Is the speed of the elevator the same when going up as going down?
  - What is the maximum acceleration of the elevator?
- Quantitatively compare your distance results with conventional measurements of the spacing between the Bloomberg building's floors made in the stairwell.

5 Lab Notebook

Your submission will be evaluated using the following rubric:

Lab Notebook Practices

- Lab Notebook Mechanics (6 points)
  - Relevant information *e.g.*: your name, your lab partner's name, date, *etc.* is present.
  - The notebook is organized and easy to read. Markdown cells are used for narrative text. Code cells are clearly organized and commented.
  - The ZIP file of the notebook is healthy and runs correctly.
- Data Analysis & Plots (6 points)
  - The notebook tells a scientific story; it is an accurate record of the work that you did.
  - The notebook should show evidence of trial and error. Keep a good record of your work – recording mistakes is useful.
  - Record rough data and plots that you used to verify that the analysis was on the right track. Final versions of plots should be well formatted and meet the plotting guidelines for the course.
  - Use models to identify trends that your data exhibit or other apparent relationships between your independent and dependent variables.
- Results and Comparison (6 points)
  - Clearly state the final result(s) of your experiment. Remember to quote your result with units and appropriate significant digits.
  - Final result plots are well formatted and meet the standards described in the Figure Formatting reference.
  - A useful comparison is made to a known/expected value or another similar result.
  - Choose the best available tools for your comparison (*e.g.* plots, pictures, discrepancy, significance of discrepancy, etc).
- Uncertainty and Error Propagation (6 points)
  - Identify the dominant source(s) of error in your experiment.
  - Support your conclusions with appropriate error estimates and error propagation calculations.
- Physical Interpretation (6 points)
  - Throughout the notebook, interpret the data, rough plots, and final results in terms of the underlying physics.
  - What are you able to conclude from your data? Clearly explain how you arrived at your conclusions from your experimental observations.
  - Reflect on how your experiments connects with the physics concepts you are studying.

REFERENCES

[1] See for example the Wikipedia entry on Numerical Integration: https://en.wikipedia.org/wiki/Numerical_integration.

[2] See the Wikipedia entry on the Trapezoidal Rule: https://en.wikipedia.org/wiki/Trapezoidal_rule.

[3] See the NumPy Trapz documentation page: https://numpy.org/doc/stable/reference/generated/numpy.trapz.html

[4] Physics Toolbox website: https://www.vieyrasoftware.net/

[5] PhyPhox website: https://phyphox.org/