Modeling the Human Arm as a Lever

1 INTRODUCTION

Complicated systems are often studied by developing a simplified version – a model – that captures the important features of the real system. One example of a complicated system is the human body. The intricate system of bones and joints with connective tissue, muscles, and tendons is a science in and of itself. From a physics point of view, the limbs of the human body can be simplified to a system of levers actuated by forces exerted by the connected muscles.

In this lab, you will develop an experiment – using your own arm – to study the forces involved in the operation of the human arm.

2 LEARNING GOALS

- Develop a mathematical model of a system.
- Study torque and equilibrium.
- To understand the physics of the human arm.

3 BACKGROUND

3.1 HOOKE'S LAW

In the 1600's, an English scientist named Robert Hooke developed a model to describe the force exerted by an extended (or compressed) spring. The model is now known as *Hooke's law*. For many springs, the force $\vec{F}$ exerted by a spring is proportional to the displacement $\vec{x}$ from its equilibrium position. Mathematically, Hooke's law is given by:

$$\vec{F} = -k\vec{x}. \tag{3.1}$$

The constant of proportionality, k, known as the *spring constant*, describes the strength of a given spring. The minus sign in Equation 3.1 indicates that the force exerted by the spring is opposite the direction of the displacement. For this reason, the force exerted by a spring is often called a *restorative force*.

3.2 TORQUE

The ability of a force to rotate a body about some fixed point is called torque, $\vec{\tau}$. The general expression for torque is given by:

$$\vec{\tau} = \vec{r} \times \vec{F}. \tag{3.2}$$

Torque is the cross-product of the displacement from the point around which a body rotates $\vec{r}$ and the applied force $\vec{F}$. The cross product can also be written in terms of the magnitude of the vectors and the sine of the angle ϕ between them:

$$\vec{\tau} = |\vec{r}||\vec{F}|\sin\phi. \tag{3.3}$$

Torque has units of force times distance; Newton-meters ($N \cdot m$) in SI units.

3.3 LEVERS AND MECHANICAL ADVANTAGE

Levers are simple machines that consist of a rigid beam and a fixed hinge or fulcrum. Levers are separated into three classes depending on the position of the load to be lifted (or output force), input force, and fulcrum. A force some distance from the fulcrum (point of rotation) produces a torque based on Equation 3.2.

The mechanical advantage, MA, of a lever is defined to be the ratio of the output force to the input force:

$$\text{MA} = \frac{|\vec{F_{out}}|}{|\vec{F_{in}}|}. \tag{3.4}$$

Systems where the mechanical advantage is greater than one allow a weight to be lifted with a smaller input force. When the mechanical advantage is less than one, the output is less than the input force. This type of lever loses in maximum lifting force but offers advantages in speed or displacement.

3.4 ROTATIONAL EQUILIBRIUM

Rotational equilibrium is achieved when the net torque around a specified point is zero. That is:

$$\sum_i \vec{\tau}_i = 0 \tag{3.5}$$

A system that is in rotational equilibrium is either stationary or rotating with a constant angular velocity.

3.5 PHYSIOLOGY OF THE HUMAN ARM

Two muscles are involved in the rotation of the lower-arm about the elbow; the biceps brachii and the brachialis (see Figure 3.1). The common usage of “biceps”generally refers to the biceps brachii - the big, two-headed, muscle on the humerus (upper arm bone) that we can see. This muscle; however, is not the strongest flexor of the elbow. The dominant flexor of the elbow is the deeper brachialis that connects the shaft of the humerus to the ulna (longest of the two bones in the forearm).

You will notice that when you flex your biceps, the muscle tenses. When the forearm is rotated, so the fist points away from the body, the observed size of the biceps decreases significantly. Flexing the elbow is predominantly controlled by the brachialis.

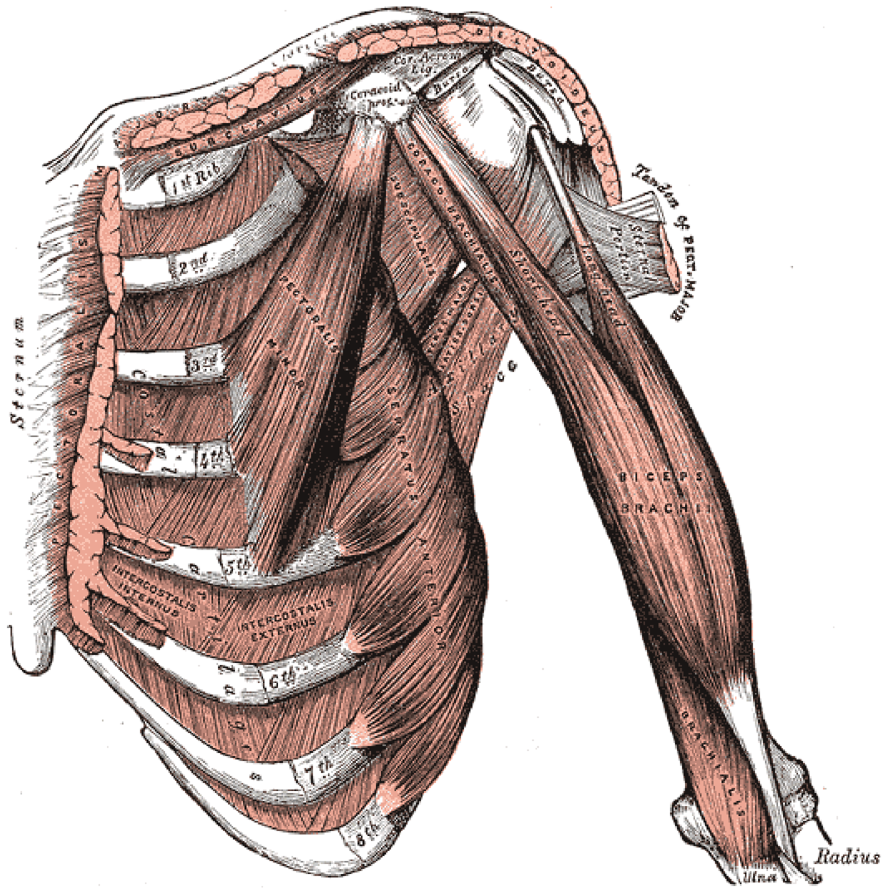

Figure 3.1: The two heads of the biceps brachii are the predominantly seen muscles running along the humerus bone of the upper arm. The brachialis attaches to the humerus beneath the biceps and can be seen connecting to the ulna near the elbow.
(Photo credit: http://en.wikipedia.org/wiki/Biceps)

When a muscle contracts, parts of the muscle tissue called sarcomers shorten. This process occurs simultaneously, across many layers of muscle tissue to generate a large force. The amount of force produced by a muscle depends on the cross-sectional area of the muscle. The *maximum* force that contracting muscle fibers can produce is in the range of $30-100 N/cm^2$[*].

4 SAFETY

A steel spring is used in this experiment to model the force exerted by a muscle. The energy stored in a spring can be extremely large. **To avoid possible injury:**

- **Ensure that all clamps are tight and that the spring is securely attached to your apparatus.**

[*] Int. Z angew. Physiol. einschl. Arbeitsphysiol. 26, 26-32 (1968)

- **Do not over-extend the spring**. Limit the maximum extension of the spring to 1.0 m.
- **Wear eye protection** when working with the spring.

5 Procedure

The goal of this lab is to develop a mathematical model that describes the physics of the human arm.

5.1 Characterize the Spring

The steel spring can be used to apply various loads to your arm.

The characteristics of the spring can be measured by suspending various masses from it. Use these data to generate a plot from which the spring constant k can be measured using Hooke's Law (see Equation 3.1). Include your plot and measured spring constant (with associated uncertainty) in your lab report.

5.2 Model the Arm as a Lever

Develop your own *mathematical* model of the human arm. The goal is to develop a mathematical model to estimate the force that is applied by the brachialis arm muscles.

How do I develop a Mathematical Model of an Arm?

- A good place to start your model is with a free-body diagram illustrating the forces on your skeleton when your hand is lifting a load.
- Torques depend on the product of a force with a distance (as shown in Equation 3.2). Identify the relevant lengths in your free-body diagram and measure the corresponding lengths on your own arm.

  Note: Measuring physiological distances on a living human is difficult. In most cases, the quantities of interest are buried beneath your skin. It is however, possible to *estimate* the values in question.
- The uncertainty assigned to any measured quantity should reflect the confidence in the measurement. If a distance is difficult to measure precisely, it will often be accompanied by a larger uncertainty estimate.

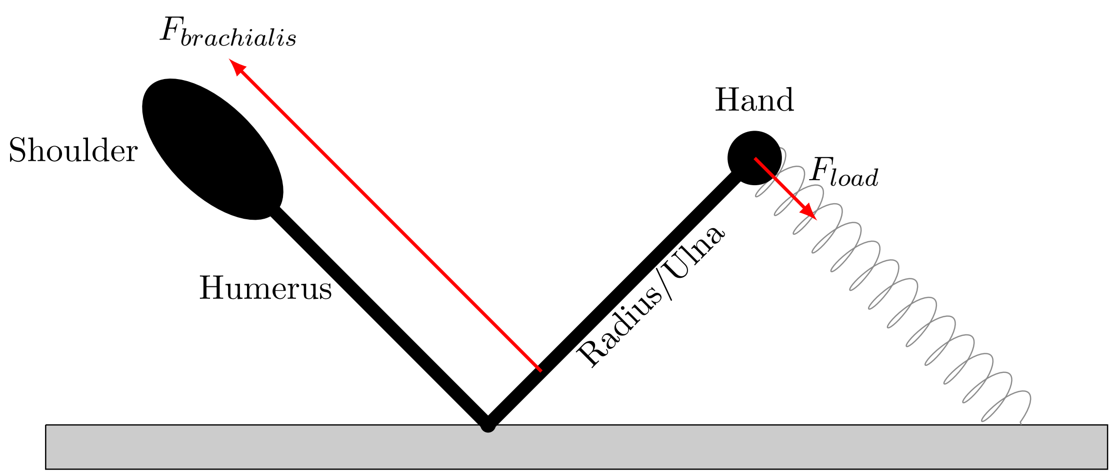

Figure 5.1: A cartoon of the basic forces in the lever system of the arm.

5.3 Measure the Force per Cross-Sectional Area of Your Arm

Combine your result from Section 5.1 and your mathematical model of the arm from Section 5.2 to design an experiment to estimate the force per unit area exerted by your brachialis muscle.

The following are some ideas to address with your experiment:

- What is the theoretical mechanical advantage of your arm model? Use Equation 3.4 (specifically the version derived in the pre-lab exercises) to calculate the mechanical advantage.

- For a given extension of the spring (*i.e.* a known force) how does your muscle react? What force is exerted by your brachialis muscle to flex the elbow? Present your data with a plot.
- The *maximum* force that muscle fibers can produce is in the range of $30-100 N/cm^2$. What muscle force per cross-sectional area do you observe? Compare your result to the expected value quantitatively.

6 Lab Report

Write a lab report to record and communicate your work. Your report should addresses the following ideas:

- Experiment Purpose and Design
  - In your own words, state the purpose of this experiment.
  - In a couple of sentences, describe how you will use the available equipment to make your measurements. Include pictures or diagrams as appropriate.
- Data Collection and Calculations
  - List the quantities you measured directly in this experiment.
  - For each quantity above, state the range of values over which you decided to make measurements. Describe how your measurements were distributed within that range. Justify your choice of measuring this way.
  - Demonstrate calculations where measured quantities are used to compute other quantities of interest.
  - Explain why you need to calculate each quantity listed above to arrive at your final result.
- Presentation and Analysis
  - Present the data you collected.
  - Analyze your data using insightful and well-formatted plot(s).
  - Using your plot(s), identify trends that your data exhibit or other apparent relationships between your independent and dependent variables.
  - If your data were fit with a trendline, interpret the results of the fit. What do you conclude from the results of the fit?
- Results
  - Identify and clearly state the final result(s) of your experiment.
- Uncertainty and Error Propagation
  - Demonstrate error propagation calculations used to quantify the uncertainty of your measurement.
  - Identify at least one source on random error in your experiment. Explain why each source of uncertainty can be considered random. If you are unable to identify a resonable source of random error, explain and justify this conclusion.

  - Identify at least one source of systematic error in your experiment. Explain why each source of uncertainty can be considered systematic. If you are unable to identify a reasonable source of systematic error, explain and justify this conclusion.
  - Describe actions you took or special procedures used while collecting data to minimize the impact of these sources of error on your final results.

- Discussion
  - Do your results agree with what you expected to see before you started the experiment?
  - Interpret your results. What are you able to conclude from your data? Describe how your conclusions relate to the physical principles being studied.
  - List at least one improvement you would make if you were to repeat the experiment. What effect do you think your improvement would have on your final results?

- Answers to Inline Questions
  - Include answers to specific inline questions as appropriate to the laboratory activity.