A Pseudo Cross Bilateral Filter for Image Denoising Based on Laplacian Pyramid

Wu Meng , Zhang Huisheng, Huang He
College of Electronic Information
Northwestern Polytechnic University
Xi'an, China

Abstract—**To attain the goal of preserving the edges and removing the noise for the images, a pseudo cross bilateral filter based on Laplacian pyramid is proposed. The proposed algorithm firstly decomposes the image and its filtered version into several levels via Laplacian pyramid, in which the filtered version is obtained by adaptive Wiener filter. At each level of the decomposed images, a pseudo cross bilateral filter is then applied regarding the filtered version as the "real" image. To optimize and determine the parameters for the algorithm, the noise variance estimation at each level and its relationship with the photometric spread parameter in bilateral filter are discussed. The experimental results indicate the proposed algorithm is superior to adaptive Wiener filter and the traditional bilateral filter in both objective and subjective assessment.**

Keywords- image denoising; anisotropic filtering; bilateral filter; Laplacian pyramid; Wiener filter

I. INTRODUCTION

With the prevalent application of imaging devices to the aviation, medical analysis and other fields, a great multitude of images have been acquired. How to extract details like edges successfully from these images contaminated by the noise existing in the process of their generation and acquisition is of vital importance. As an open problem, image denoising, then, plays an indispensable role in image processing and computer vision.

In past, many efforts were devoted to the noise suppression or removal in the spatial domain. In particular, Gaussian low-pass filter performs better than averaging filter by computing a weighted average of pixels in the neighborhood of the processing pixel. These two filters, however, blur the edges while suppressing the noise due to their isotropy. To retain the edges in denoising, a nonlinear robust estimation[1], bilateral filter, was proposed by Tomasi and Manduchi[2] in 1998. Since its birth, it has been mainly applied to noise removal[2-3] and other fields like photograph enhancement[4], dynamic filter closely related to anisotropic diffusion, range compression[5] and so on. When applied to the denoising, the bilateral filter presents comparable performance while it is non-iterative, stable and simple. In view of this advantage, considerable research has been conducted to solve its defect of long running time[5-7]. Different from other references, Ref. [3] tries to apply bilateral filter in wavelet domain to improve its denoising performance, which however fails in the real image denoising because the parameters of bilateral filter have to be estimated according the Signal-Noise-Ratio (SNR) of the contaminated image. To the same end of improving denoising performance, this paper, proposes a novel image denoising algorithm based on a pseudo cross bilateral filter and also discusses the parameters selection for the bilateral filter. Experiments indicate the proposed algorithm herein can improve the denoising effects compared with previous algorithms.

Supported by Aviation Science Funds (05F53032)

978-1-4244-3531-9/08/$25.00©2008 IEEE

II. BILATERAL FILTER

Given an image I , a low-pass domain filter, like Gaussian low-pass filter and averaging filter, is defined by

$$\hat{I}_p = \frac{1}{k_d(p)} \sum_{q \in S} c(p-q) I_q \tag{1}$$

where $\hat{I}$ is the filtered image, $c(p-q)$ is the geometric similarity between the central pixel p and a nearby pixel q in the neighborhood, and $k_d(p) = \sum_{q \in S} c(p-q)$ is the normalization factor.

Spatial similarity is assumed in the neighborhood of the processing pixel in Eq.(1). Nevertheless, in reality, huge differences exist where edges occur in the images. Considering this, bilateral filter is devised by[1-2]

$$\hat{I}_p = \frac{1}{k(p)} \sum_{q \in S} c(p-q) s(I_p - I_q) I_q \tag{2}$$

where $s(I_p - I_q)$ is the photometric similarity between the central pixel I_p and a nearby pixel I_q of the neighborhood, and $k(p) = \sum_{q \in S} c(p-q) s(I_p \text{-} I_q)$ is the normalization factor.

Typically geometric and photometric similarity is defined using a metric based on a shift-invariant Gaussian function of the Euclidean distance.

$$c(p-q) = e^{-1/2(\|p-q\|/\sigma_g)^2}, \quad s(p-q) = e^{-1/2(\|I_p - I_q\|/\sigma_p)^2} \tag{3}$$

where σ_g is the geometric spread parameter and σ_p is the photometric spread parameter.

Eq.(2) reveals that bilateral filter takes into account both geometric similarity and photometric similarity of neighboring pixels by computing a weighted average of pixel values which decays with both space and intensity difference from the central pixel. Due to its intensity-selective filtering, bilateral filter avoids edges diffusing but effectively suppresses the noise.

III. PSEUDO CROSS BILATERAL FILTER BASED ON LAPLACIAN PYRAMID

A. Laplacian pyramid

Given an original image I, the proposed algorithm firstly decomposes the image via Gaussian-Laplacian pyramid[8-9]. The Gaussian pyramid is computed as follows. The original image is convolved with a Gaussian kernel $\omega(m,n)$, and downsampled by 2. This process is repeated iteratively to bottom-up construct a set of levels of Gaussian pyramid.

$$G_l = \begin{cases} I, & l = 0 \\ \sum_{m=-2}^{2}\sum_{n=-2}^{2}\omega(m,n)G_{l-1}(2i+m,2j+n), & l \le N \end{cases} \tag{4}$$

where N is the decomposition level, and the separable Gaussian kernel $\omega(m,n)$ is expressed by

$$\omega(m,n) = \omega(m)\omega(n) \tag{5}$$

In this paper, $\omega(m,n)$ is a 5×5 array, and $\omega(n) = \{0.6, 0.25, -0.05, 0.25, 0.6\}$.

To produce Laplacian pyramid (LP), the Gaussian image G_{l+1} is up-sampled by interpolating zeroes between pixels, and filtered through the Gaussian kernel defined above. The Laplacian pyramid is the difference between image G_l and the expanded image of G_{l+1}.

$$L_l = \begin{cases} G_l - Expand(G_{l+1}), & l < N \\ G_N, & l = N \end{cases} \tag{6}$$

where $Expand$ is $G_{l,1}(i,j) = 4\sum_{m=-2}^{2}\sum_{n=-2}^{2}\omega(m,n)G_l(\frac{i-m}{2},\frac{j-n}{2})$.

The original image can be reconstructed by expanding the top level G_N and adding it to L_{N-1}, repeating this process until the bottom level G_0 is recovered.

$$G_l = L_l + Expand(G_{l+1}), \quad 0 \le l < N \tag{7}$$

The reasons why the Laplacian pyramid is used here is that it decomposes the original image into a set of bandpass filtered images, which represent the features of the image at various scales. As the scale increases, distinctive image features such as edges extracted appear enhanced, and the amounts of noise decreases rapidly, thereby it is more suitable to perform the improved bilateral filter at different levels of Laplacian pyramid with different sizes of window. In addition, the translation-invariant property of Laplacian pyramid can reduce the aliasing effect in reconstruction.

B. Pseudo cross bilateral filtering

The cross bilateral filter[4] was initially exploited in image enhancement, which combined two photos taken with and without the flash in dark ambiances to generate a single enhanced image, in which the intensity difference of the flash-image is used as the photometric similarity function.

$$\hat{I}_p^{nf} = \frac{1}{k(p)}\sum_{q\in S}c(p-q)s(I_p^f - I_q^f)I_q^{nf} \tag{8}$$

where I^f and I^{nf} are the flash image and the no-flash picture, and $k(p) = \sum_{q\in S}c(p-q)s(I_p^f - I_q^f)$ is the normalization factor.

In this paper, the denoising procedure is performed in the framework of Laplacian pyramid. To better suppress the noise on various levels, a technique similar to the cross-bilateral filter mentioned above is applied to the decomposed levels of the noisy image, in which the photometric similarity function is supplied by the appropriate levels of the original image and its filtered version via Laplacian pyramid. This technique, however, is called pseudo cross bilateral filtering for the sake of using the filtered image as the “real” image. In addition, the window size of the pseudo cross bilateral filter varies adaptively with the decomposed level. The higher the decomposed level, the smaller the window size.

$$\hat{L}_p = \frac{1}{k(p)}\sum_{q\in S}c(p-q)s(L_p - L_q^w)L_q \tag{9}$$

where L and L^w are the corresponding levels of the original image I and its filtered version $\hat{I}^w$, and $k(p) = \sum_{q\in S}c(p-q)s(L_p - L_q^w)$ is the normalization factor.

Note that the filtered image is obtained by the adaptive Wiener filter[10] over a 3×3 neighborhood S_0.

$$\hat{I}_p^w = \mu + \frac{v^2 - \sigma_0^2}{v^2}(I_p - \mu) \tag{10}$$

where $\mu = \frac{1}{9}\sum_{q\in S_0}I_q$ is the local mean, $v^2 = \frac{1}{9}\sum_{q\in S_0}I_q^2 - \mu^2$ is the variance, and σ_0 is the noise standard deviation of the image.

C. Parameters determination and the workflow

1) Noise variance estimation and parameters selection of *bilateral filter*

In order to optimize the algorithm, the noise standard deviation of each level of Laplacian pyramid need be estimated. The block "Noise Estimation"[11] is introduced to calculate the noise standard deviation of an original image I .

$$\sigma_0 \approx \sqrt{\frac{\pi}{2}} \frac{1}{6CR} \sum_p \left| H * I_p \right| \tag{11}$$

where $C \times R$ is the dimension of the original image , and $H = (1 \quad -2 \quad 1; \quad -2 \quad 4 \quad -2; \quad 1 \quad -2 \quad 1)$. The noise variance of the level k image in Laplacian pyramid is computed by[9]

$$(\sigma_L^k)^2 = \frac{1}{(4\pi\sigma^2)^k} \sigma_0^2 - \frac{1}{(4\pi\sigma^2)^{k+1}} \sigma_0^2 \tag{12}$$

where σ is the standard deviation of Gaussian kernel in Eq.(5).

To determine two parameters of bilateral filter, an easy approach is adopted that the optimum parameter is obtained by fixing the other parameters in terms of the highest SNR. After amounts of experiments based on various standard test images, it is found that photometric spread parameter σ_p is linearly proportional to noise standard deviation of the processing image while the geometric spread parameter σ_g is less sensitive to the algorithm than the photometric spread parameter σ_p . Tab. 1 gives the SNR values of one image corresponding to different noise variances and multiples of noise variance σ_0 when σ_g is set to 3. It indicates that the SNR gets the peak value when $\sigma_p \approx 2\sigma_0$, which confirms that photometric spread parameter σ_p is linearly proportional to noise variance. Similar experiments are conducted on σ_g for a fixed σ_p and the results show that σ_g has a slighter influence on SNR value with different noise variances.

TABLE I. SNR results for different noise variances

$n\sigma_0$ / σ_0	1.5	2	2.5	3	3.5	4	4.5	5
10	18.71	18.94	18.66	18.16	17.57	16.94	16.31	15.70
20	14.21	15.00	14.91	14.39	13.72	13.06	12.46	11.95
30	11.39	12.36	12.33	11.89	11.38	10.94	10.58	10.30

2) The workflow of the proposed algorithm

The process of the proposed algorithm is illustrated in Fig.1. Given an original image, its filtered version is obtained by adaptive Wiener filter. Then, the original image and its filtered version are decomposed into a set of levels via Laplacian pyramid. The pseudo cross bilateral filters with different window sizes are applied to the different levels of the image. To the end, the denoised image is achieved by reconstructing the appropriate levels of the pyramid.

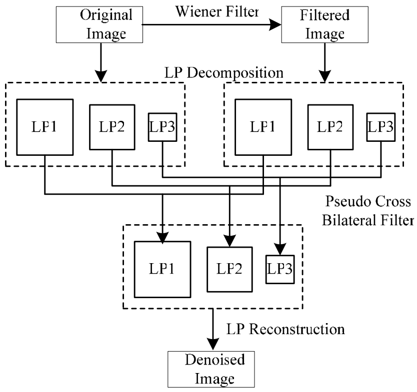

Figure.1 The workflow of the proposed algorithm

IV. EXPERIMENTS

Toward objective estimation of the proposed algorithm, standard test images corrupted by zero mean additive Gaussian noise with different variances are used. The performance of the proposed algorithm is compared with adaptive Wiener filter and traditional bilateral filter. The decomposition level of Laplacian pyramid N is 3. The window size of the pseudo cross bilateral filter is correspondingly 5×5 , 7×7 and 9×9 . The two parameters for bilateral filter in k level of Laplacian pyramid are $\sigma_g = 3$, and $\sigma_p = 2\sigma_L^k$ .The objective evaluation is presented in terms of SNR in dB.

TABLE II. Comparison of SNR results for test images

σ_0	Lena			Peppers		
	Adaptive Wiener	*Bilateral Filter*	*Our Algorithm*	*Adaptive Wiener*	*Bilateral Filter*	*Our Algorithm*
10	19.05	18.71	19.47	20.54	20.26	20.98
15	16.57	16.61	17.63	18.01	18.10	19.00
20	14.47	15.09	16.09	15.74	16.48	17.28
25	12.74	13.95	14.84	13.93	15.10	15.84
30	11.32	12.95	13.74	12.37	13.90	14.56

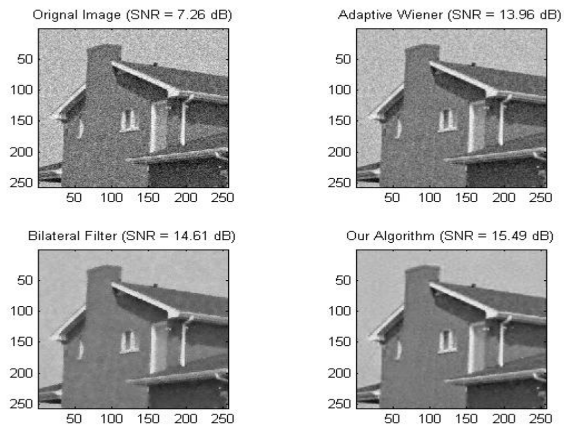

Figure 2. Comparison of visual results

The results of the proposed algorithm for 2 test images—Lena and peppers, are given in Tab.2. It can be seen that the proposed algorithm has better performances than the other two methods. In particular, when $\sigma_0=15$, the SNR of the proposed algorithm surpasses that of bilateral filter around 1.0 dB.

Besides the objective comparison, subjective comparison results of these three denoising algorithms are shown in Fig.2. It is obviously seen that the proposed algorithm outperforms the others. Compared with other algorithms, the proposed algorithm performs better on preserving edges and suppressing the noise on the wall of "house" image.

V. CONCLUSION

In this paper, a novel, simple and efficient image denoising algorithm is proposed. The main contributions of this paper consist of

1) A multiresolution framework via Laplacian pyramid distributes the noise at different levels for better controlling the suppression of noise;

2) A pseudo cross bilateral filter is constructed at the decomposed levels of the original image in which the appropriate levels of the filtered version through adaptive Wiener filter is regarded as the "real" image, thereby improving the performance;

3) The principles of two fine-tuning parameters of bilateral filter are discussed.

Compared with adaptive Wiener filter and traditional bilateral filter, the proposed algorithm represents significant SNR improvement and better visual results. Considering the computational complexity of bilateral filter, the future research is to apply acceleration methods of bilateral filter to the proposed algorithm.

REFERENCES

[1] D. Barash, "A fundamental relationship between bilateral filtering, adaptive smoothing, and the nonlinear diffusion equation", IEEE Trans. Pattern Anal. and Mach. Intell., vol. 24, pp. 844-847, Jun 2002.

[2] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images" Proc. 6th Int. Conf. Compu. Vis., 1998, pp. 839-846.

[3] Cai Chao, Ding Mingyue, Zhou Chengping and Zhang Tianxu, "Bilateral filtering in the wavelet domain", Acta Electronica Sinica, vol. 32, pp. 128-131, 2004. (in Chinese)

[4] E. Eisemann and F. Durand, "Flash photography enhancement via intrinsic relighting", ACM Trans. Graphics, vol. 23, pp. 673-678, 2004.

[5] F. Durand and J. Dorsey, "Fast bilateral filtering for the display of high dynamic range images", ACM Trans. Graphics, vol. 21, pp. 257-266, 2002

[6] Tuan D. Pham, Lucas J and van Vliet, "Separable bilateral filtering for fast video preprocessing", Proc. IEEE Int. Conf. Multimedia & Expo, 2005, pp. 454-457.

[7] Sylvain Paris and Fredo Durand, "A fast approximation of the bilateral filter using a signal processing approach", Proc. European Conf. Computer Vision, 2006, pp. 568-580.

[8] P. J. Burt and E. H. Adelson, "The Laplacian pyramid as a compact image code", IEEE Trans. Communications, vol. 31, pp. 532–540,1983.

[9] Liu Yanli, Wang Jin, Chen Xi, Guo Yanwen and Peng Qunsheng, "A robust and fast non-local means algorithm for image denoising", Journal of Computer Science and Technology, vol. 23, pp. 270-279, 2008.

[10] Tammana, G.A and Zheng, Y.F, "Image denoising using multi-resolution coefficient support based empirical Wiener filtering", IEEE Int. Conf. Image Processing (ICIP'2006), 2006, pp. 2613-2616.

[11] Markus Friebe and Andre Kaup, "Low complexity two classes gauss weighting filter for noise reduction in mobile received analog TV signals", European Signal Processing Conf., 2005, pp. 325-328.