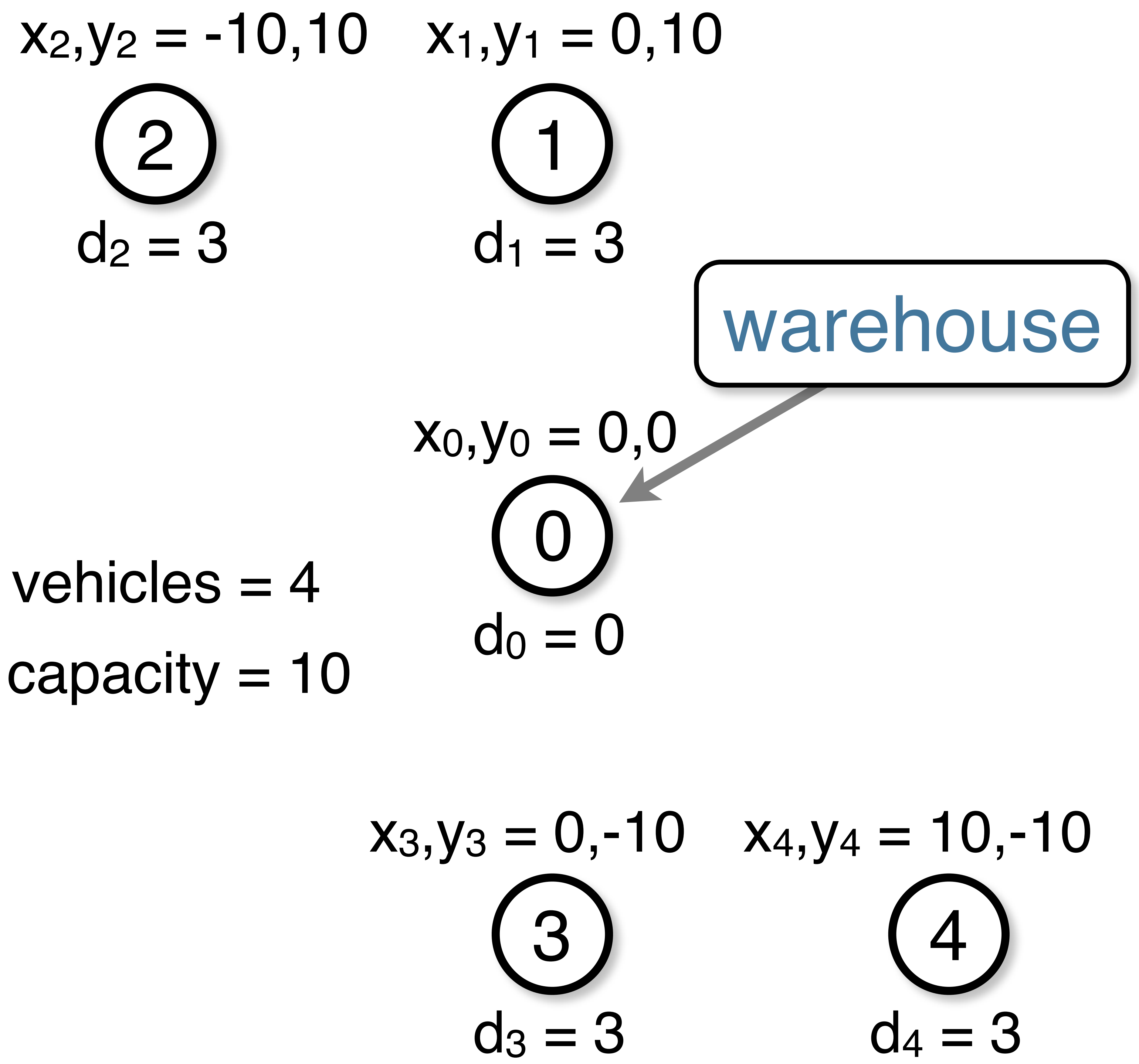
Discrete Optimization

Assignments: Vehicle Routing

The Vehicle Routing Problem (VRP)

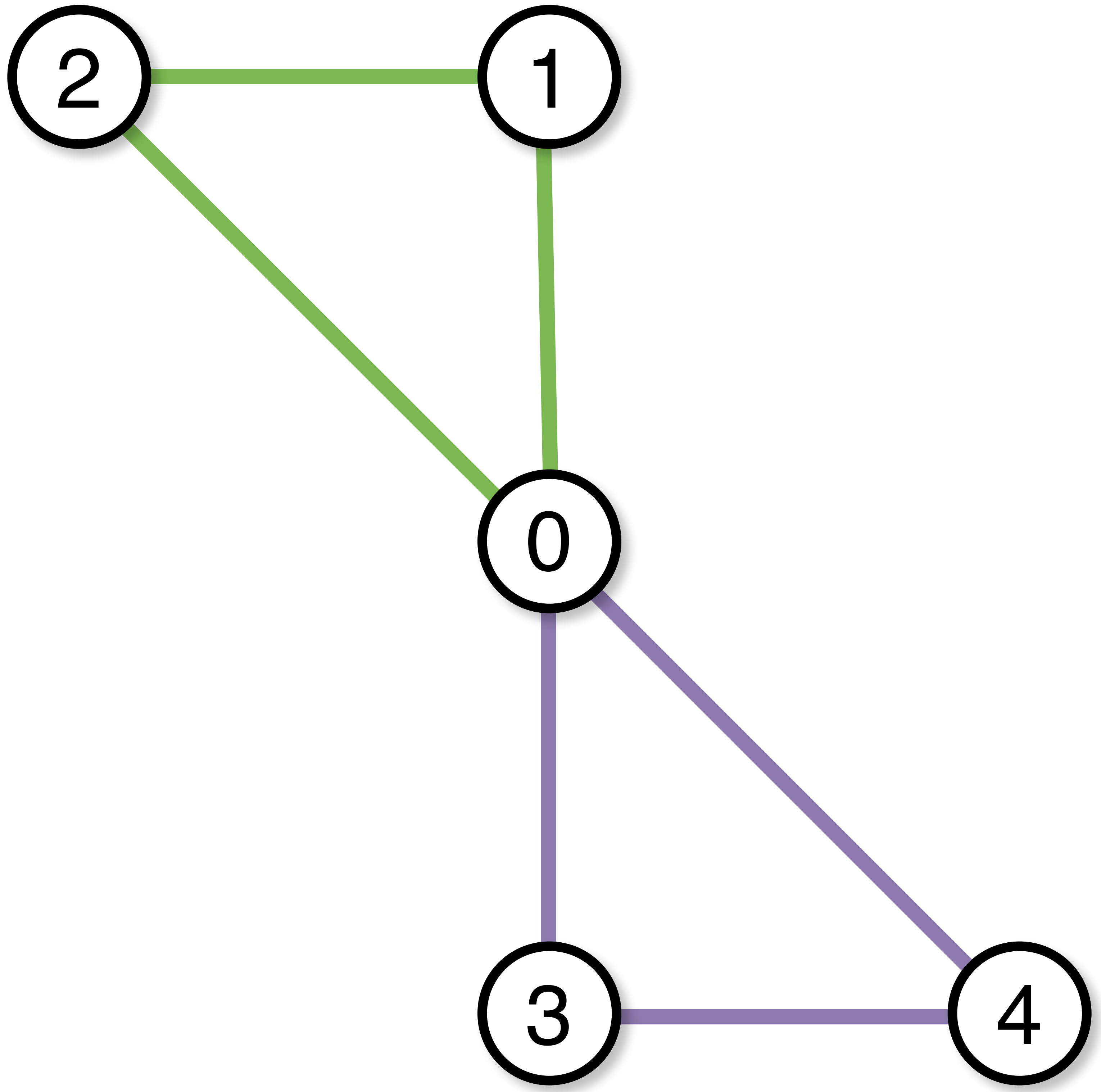
- Many variants
  - This is the Capacitated VRP - CVRP
- Like the Traveling Salesman Problem
  - on steroids...

Vehicle Routing



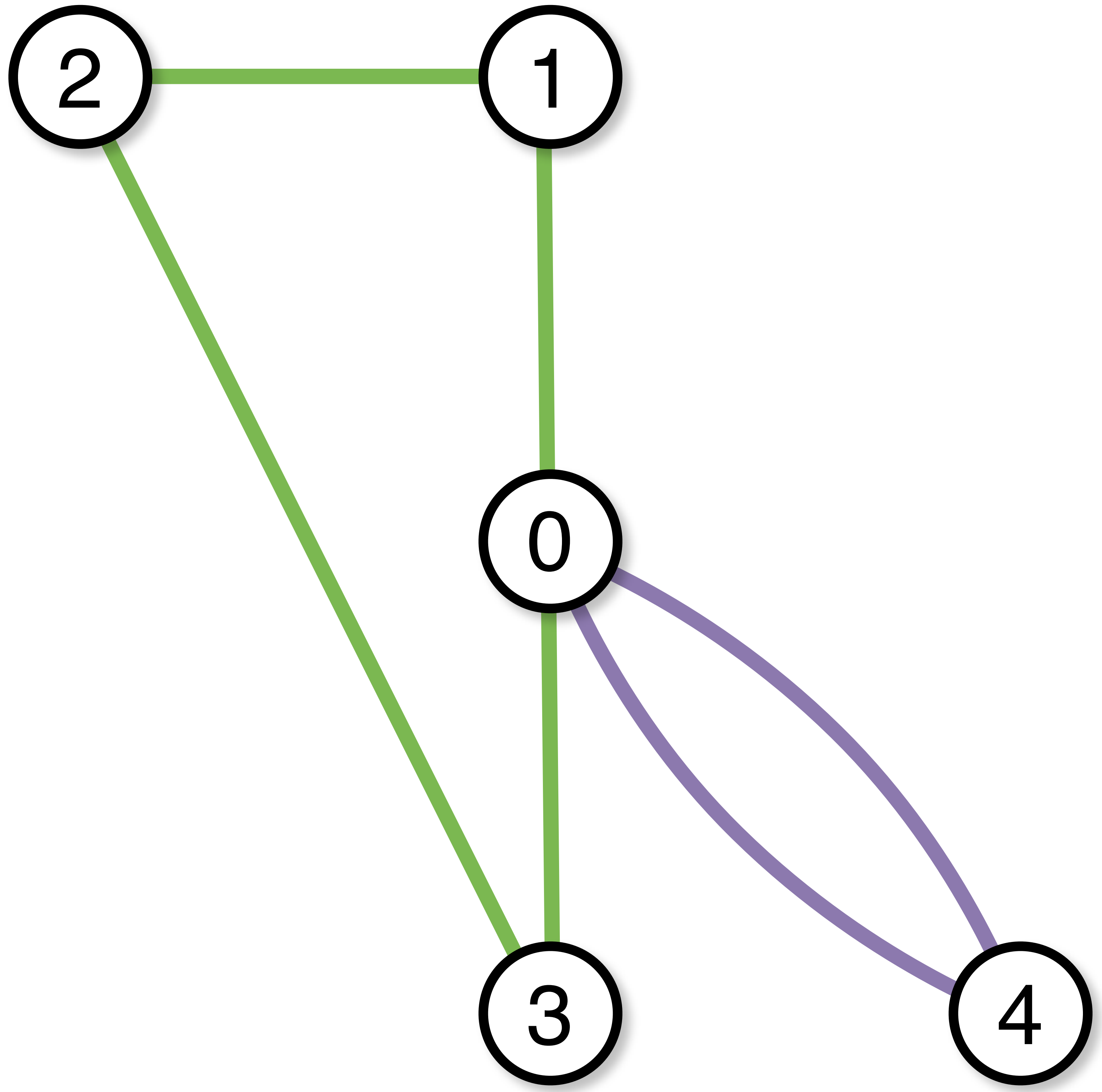
Vehicle Routing

cost = 68.3



vehicle 1 =
vehicle 2 =

cost = 80.6



vehicle 0 =
vehicle 1 =

Vehicle Routing

- n Locations, v Vehicles
- For each location,
  - demand d_i and location x_i, y_i
- The capacity of the vehicles c
- The sequence of deliveries of vehicle i, T_i

$$\text{minimize:} \quad \sum_{i \in V} \left(dist(0, T_{i,0}) + \sum_{\langle j,k \rangle \in T_i} dist(j,k) + dist(T_{i,|T_i|-1}, 0) \right)$$

$$\text{subject to:}$$

$$\sum_{j \in T_i} d_j \leq c \quad (i \in V)$$

$$\sum_{i \in V} (j \in T_i) = 1 \quad (j \in N \setminus \{0\})$$

Vehicle Routing Data Format

$$\text{minimize:} \quad \sum_{i \in V} \left(dist(0, T_{i,0}) + \sum_{\langle j,k \rangle \in T_i} dist(j,k) + dist(T_{i,|T_i|-1}, 0) \right)$$

subject to:

$$\sum_{j \in T_i} d_j \leq c \quad (i \in V)$$

$$\sum_{i \in V} (j \in T_i) = 1 \quad (j \in N \setminus \{0\})$$

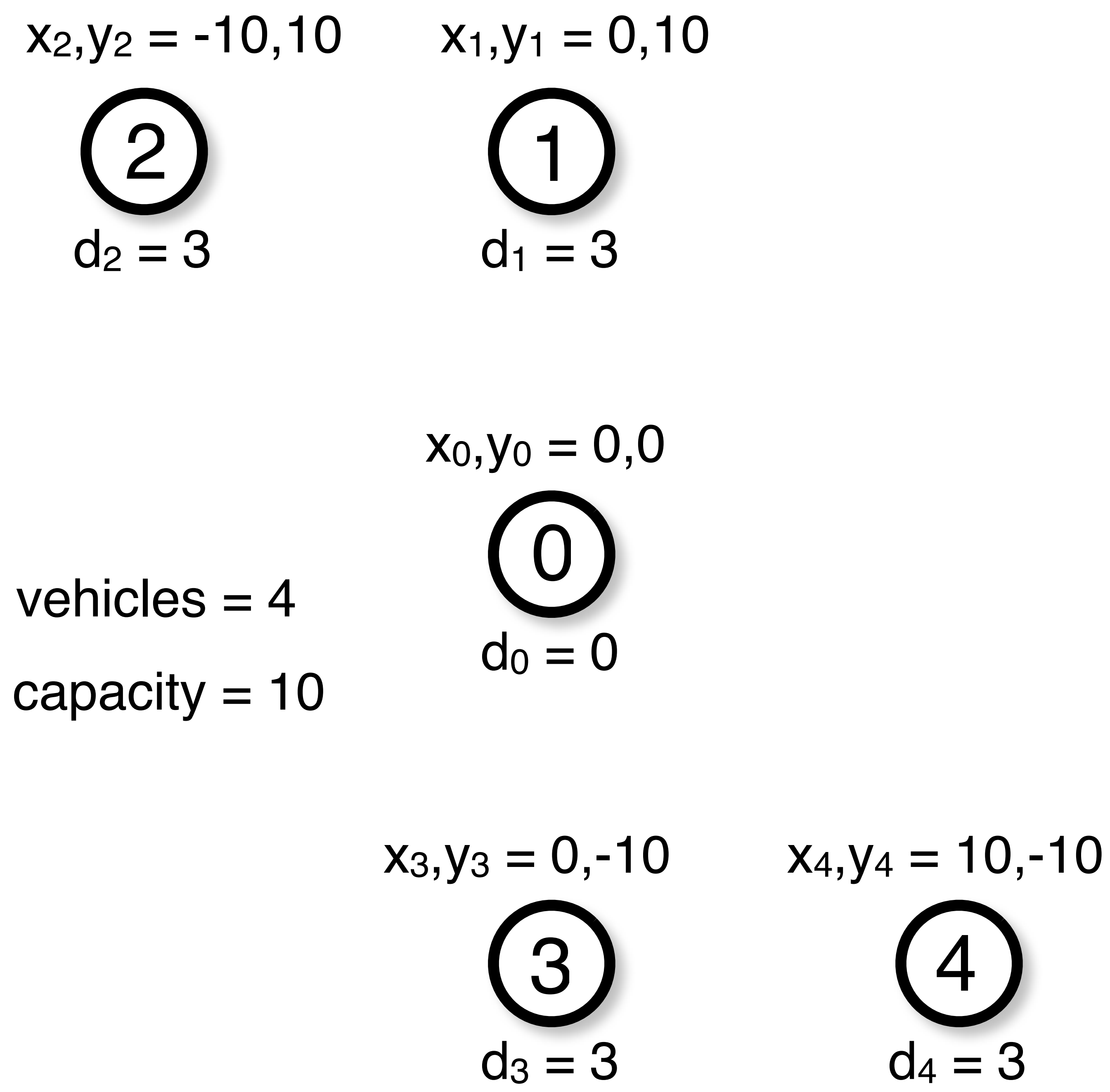
Input

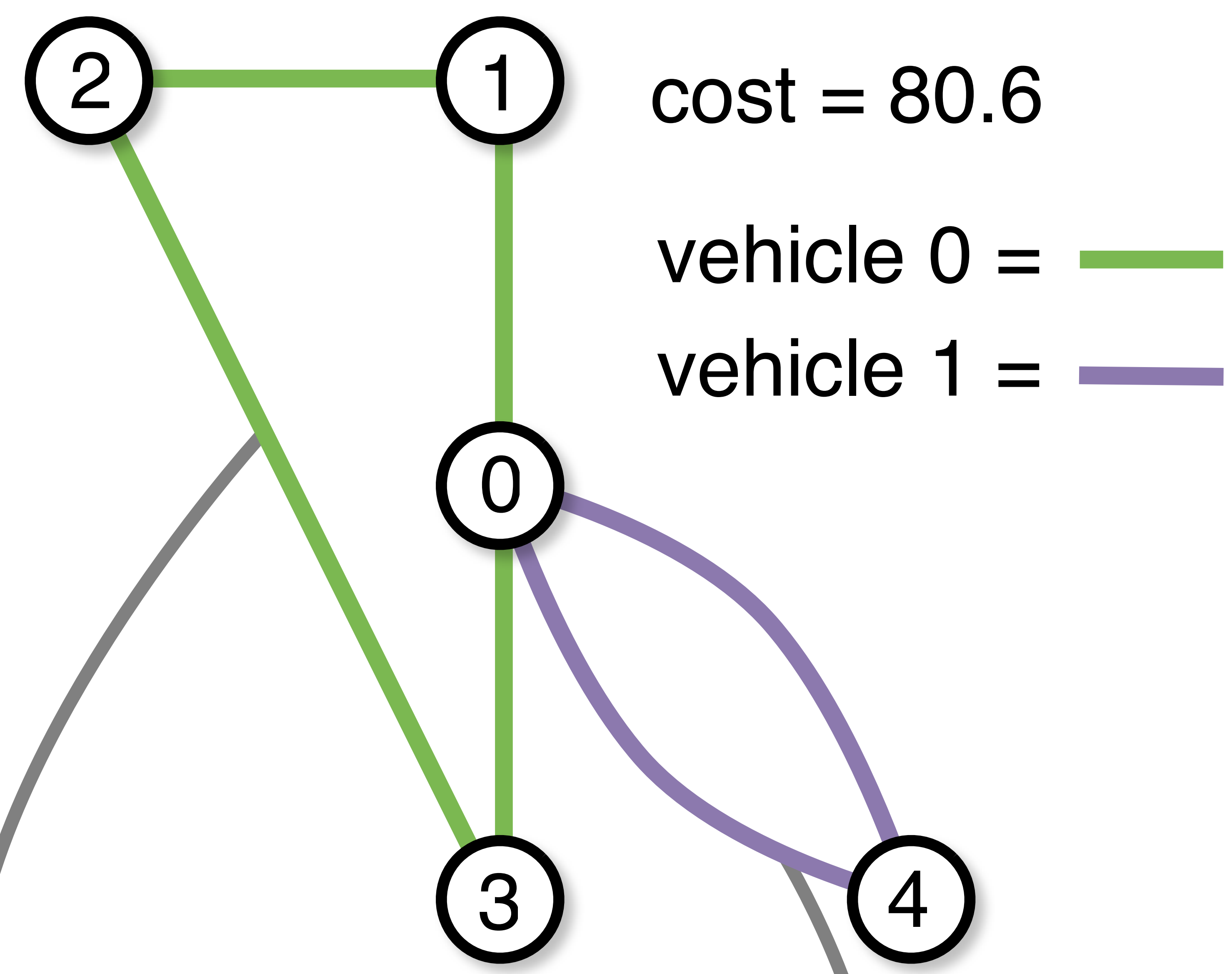
```
|N| |V| c
d_0 x_0 y_0
d_1 x_1 y_1
...
d_|N|-1 x_|N|-1 y_|N|-1
```

Output

```
obj opt
0 t_0_1 t_0_2 ... 0
0 t_1_1 t_1_2 ... 0
...
0 t_|V|-1_1 t_|V|-1_2 ... 0
```

Vehicle Routing Example





Input

```
5 4 10
0   0   0
3   0  10
3 -10  10
3   0 -10
3  10 -10
```

Output

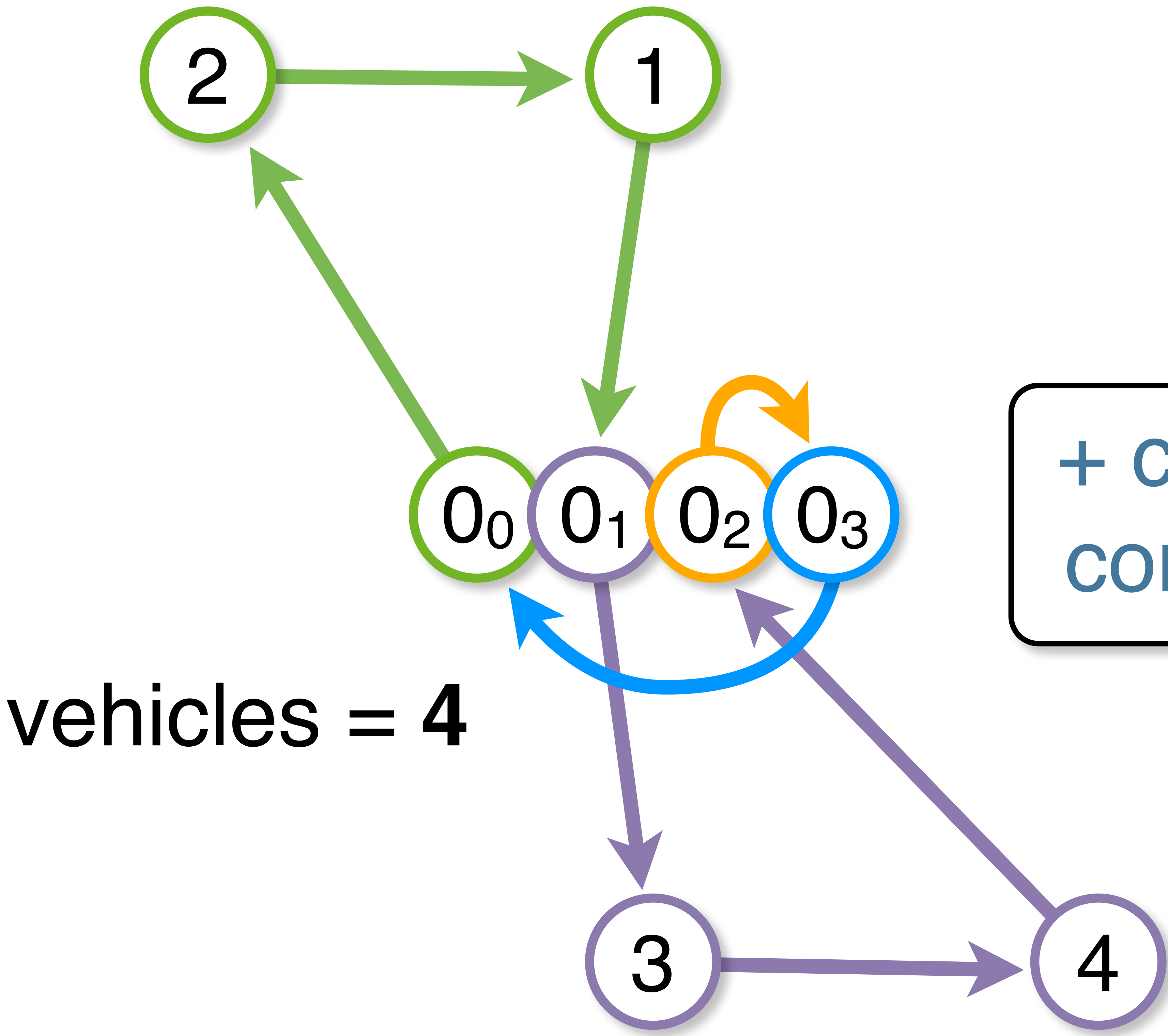
```
80.6 0
0 1 2 3 0
0 4 0
0 0
0 0
```

Getting Started

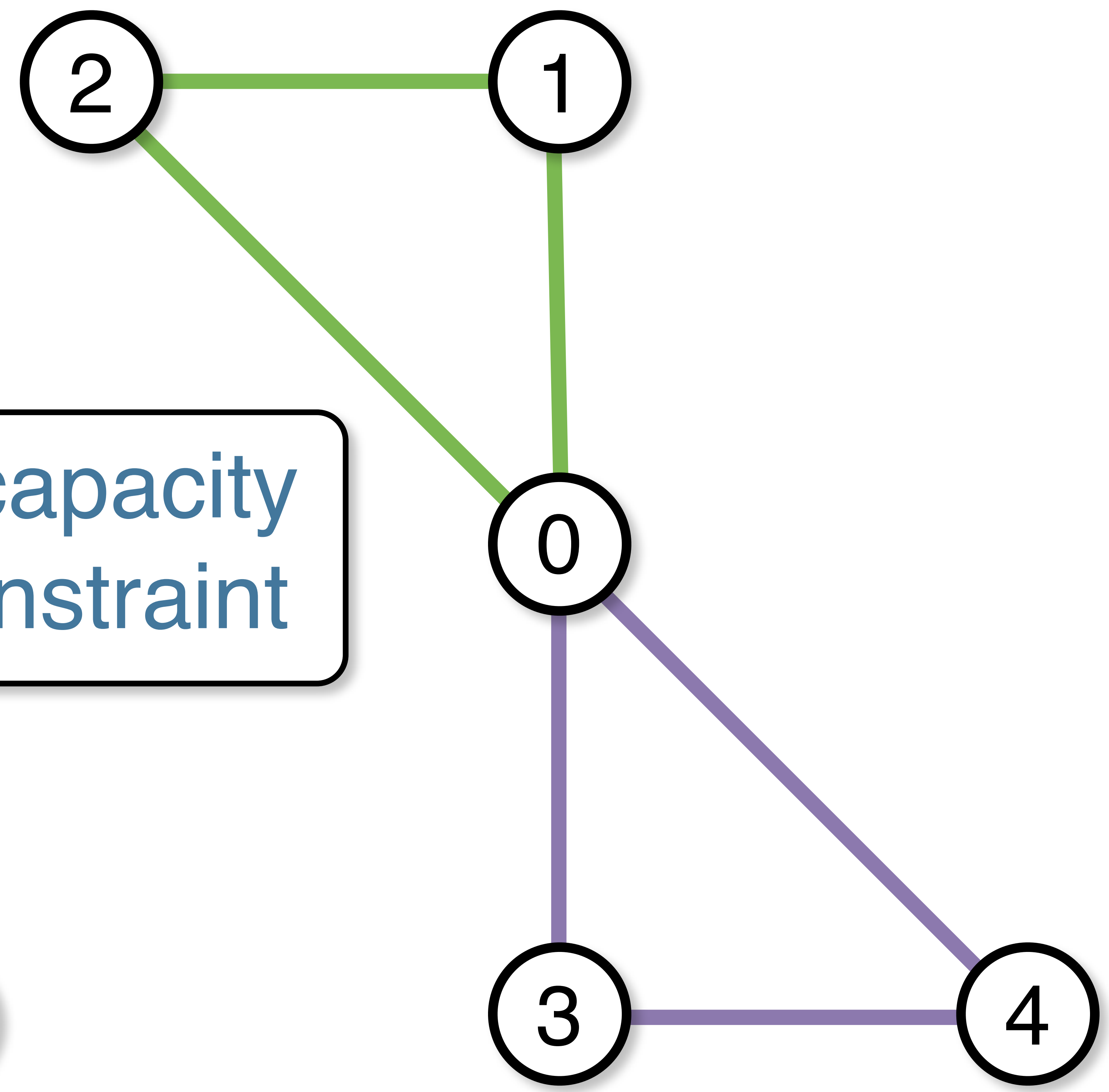
- This assignment is really hard.
  - Very close to a real world application.
- Three Models
  - CP
  - MIP
  - Local Search
- All connected to the TSP
  - Multi-Colored TSP

A CP Model

- One Big ***Circuit*** Constraint

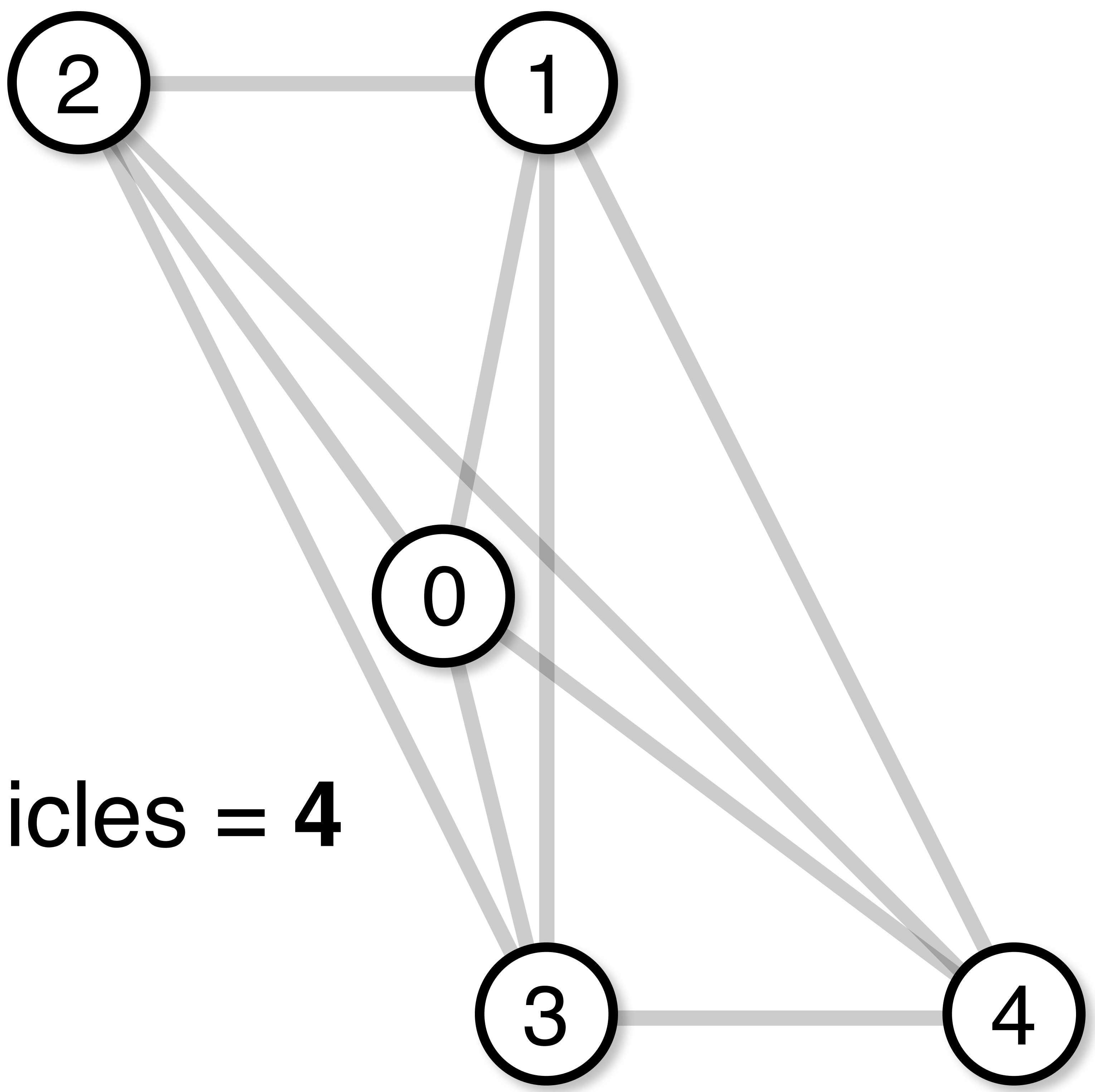


+ capacity constraint

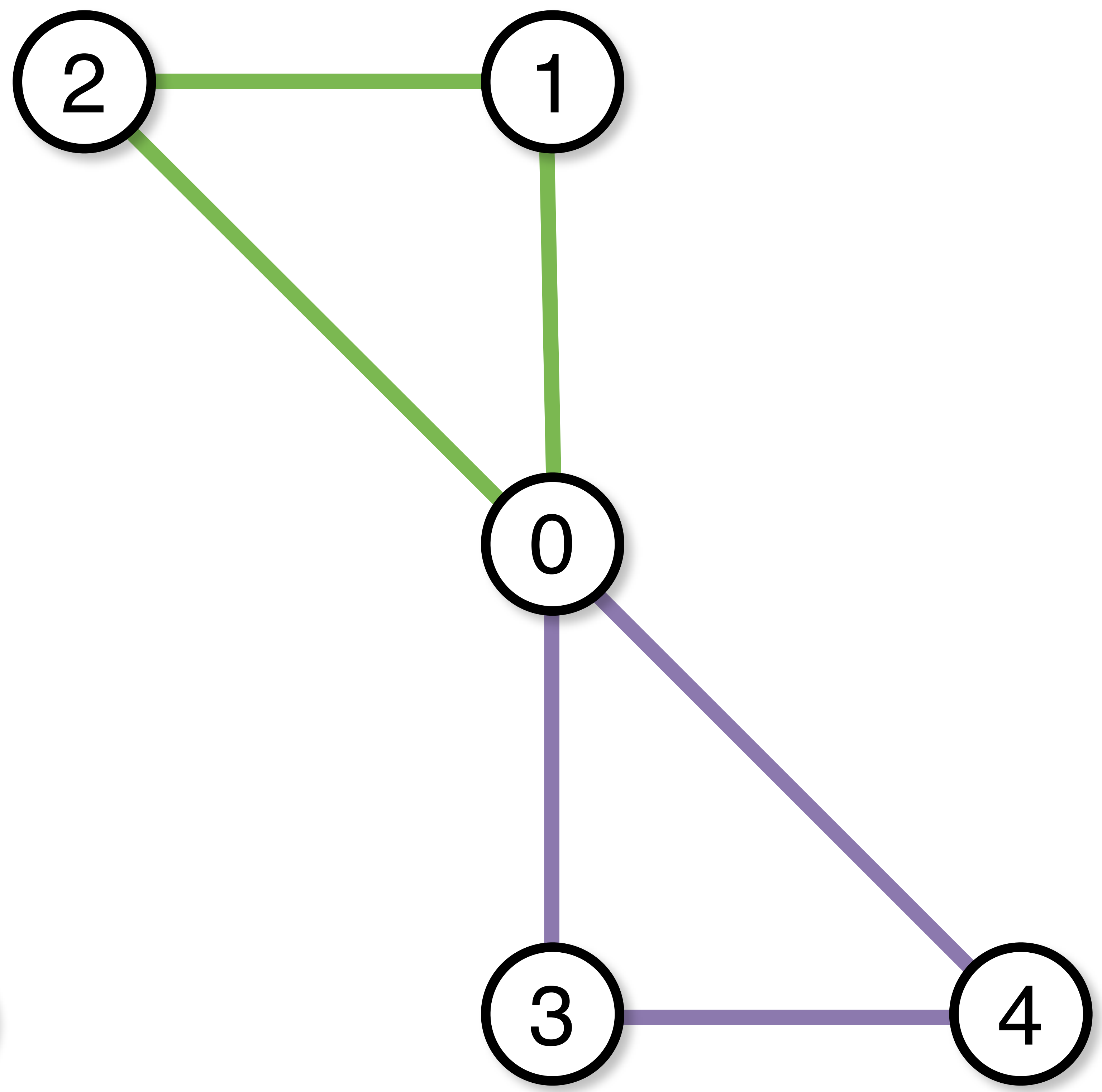


A MIP Model

- Go with the ***Flow*** (recall MIP TSP Model)

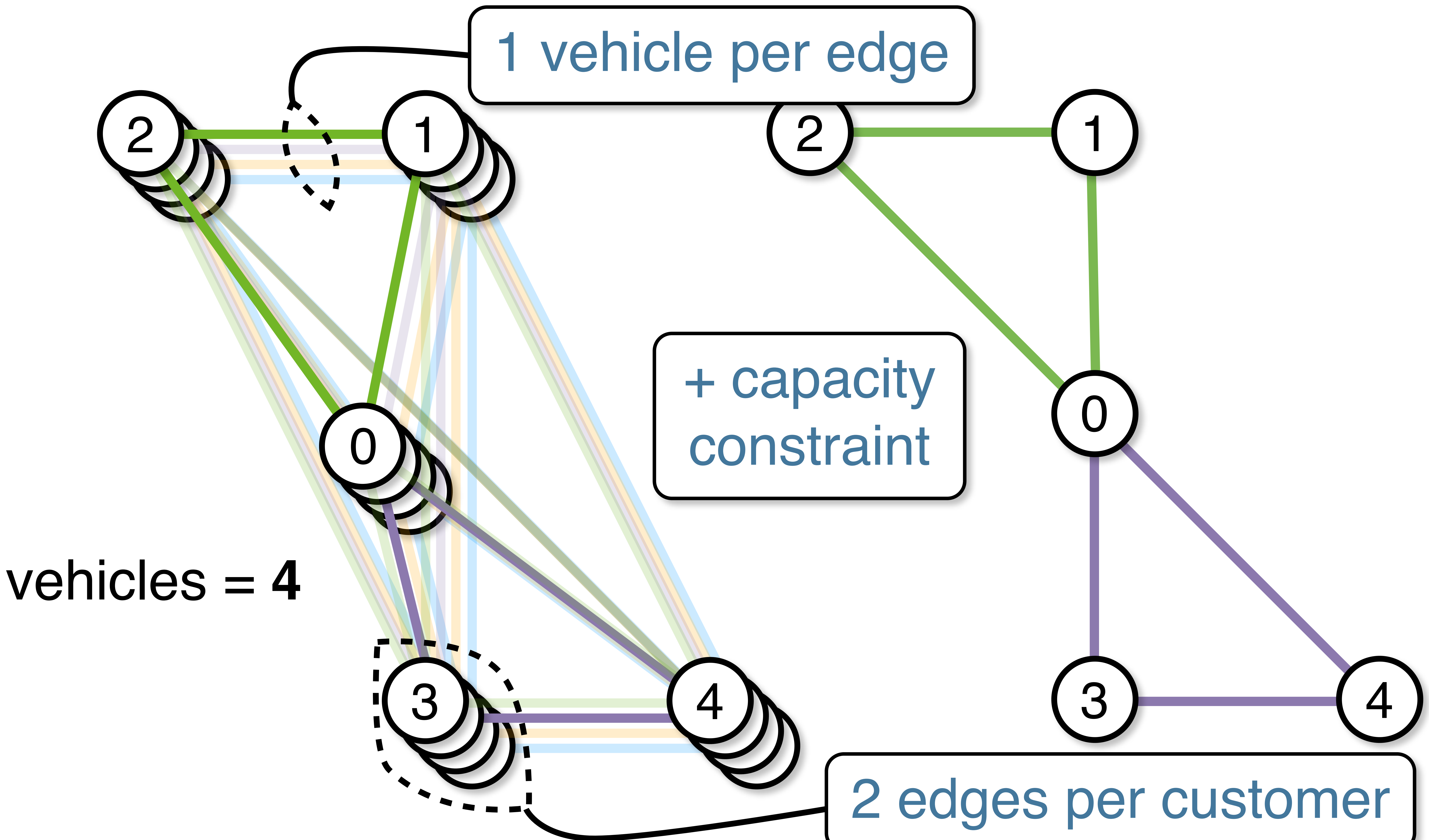


vehicles = **4**



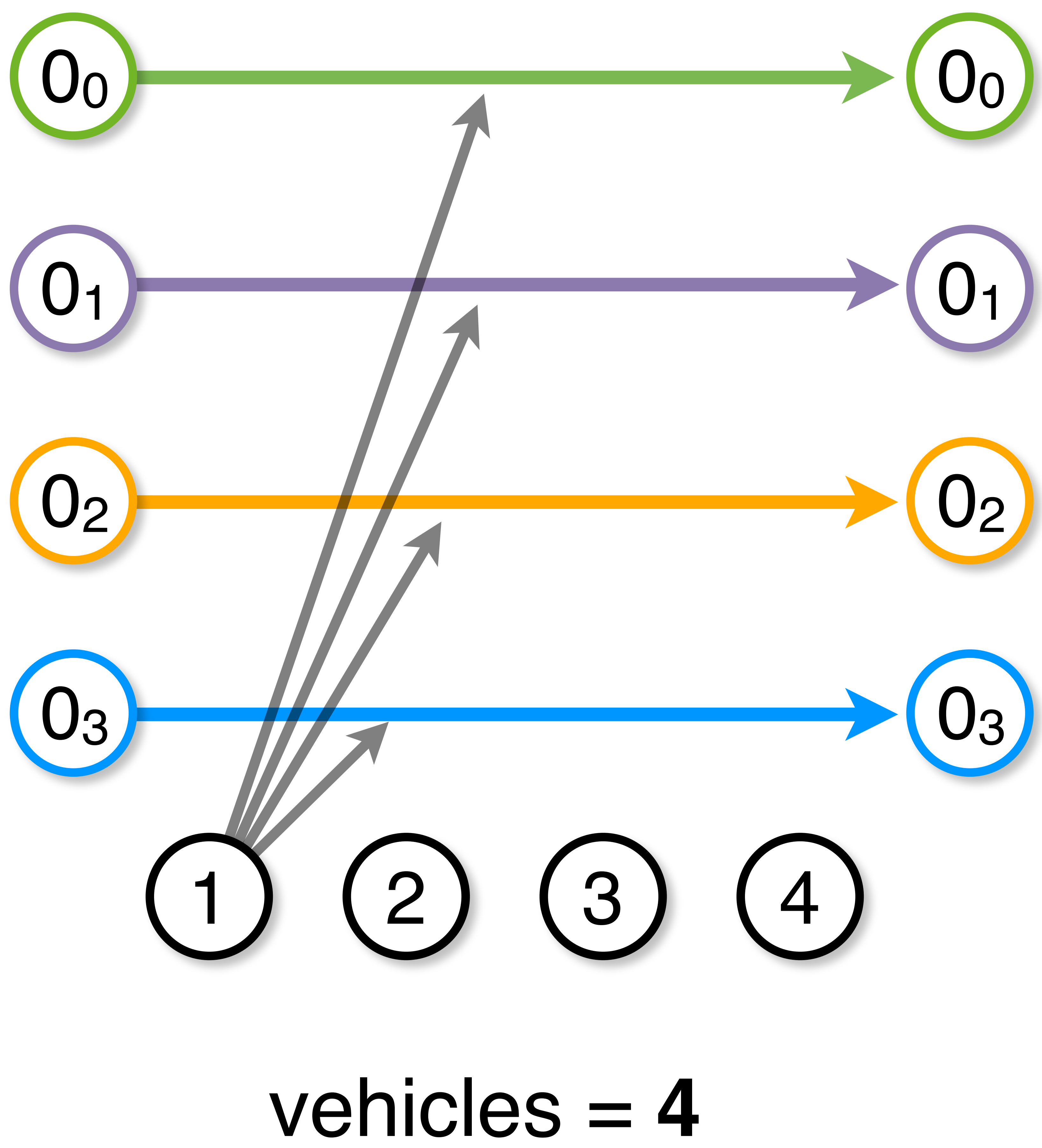
A MIP Model

- Go with the ***Flow*** (recall MIP TSP Model)



A Local Search Model

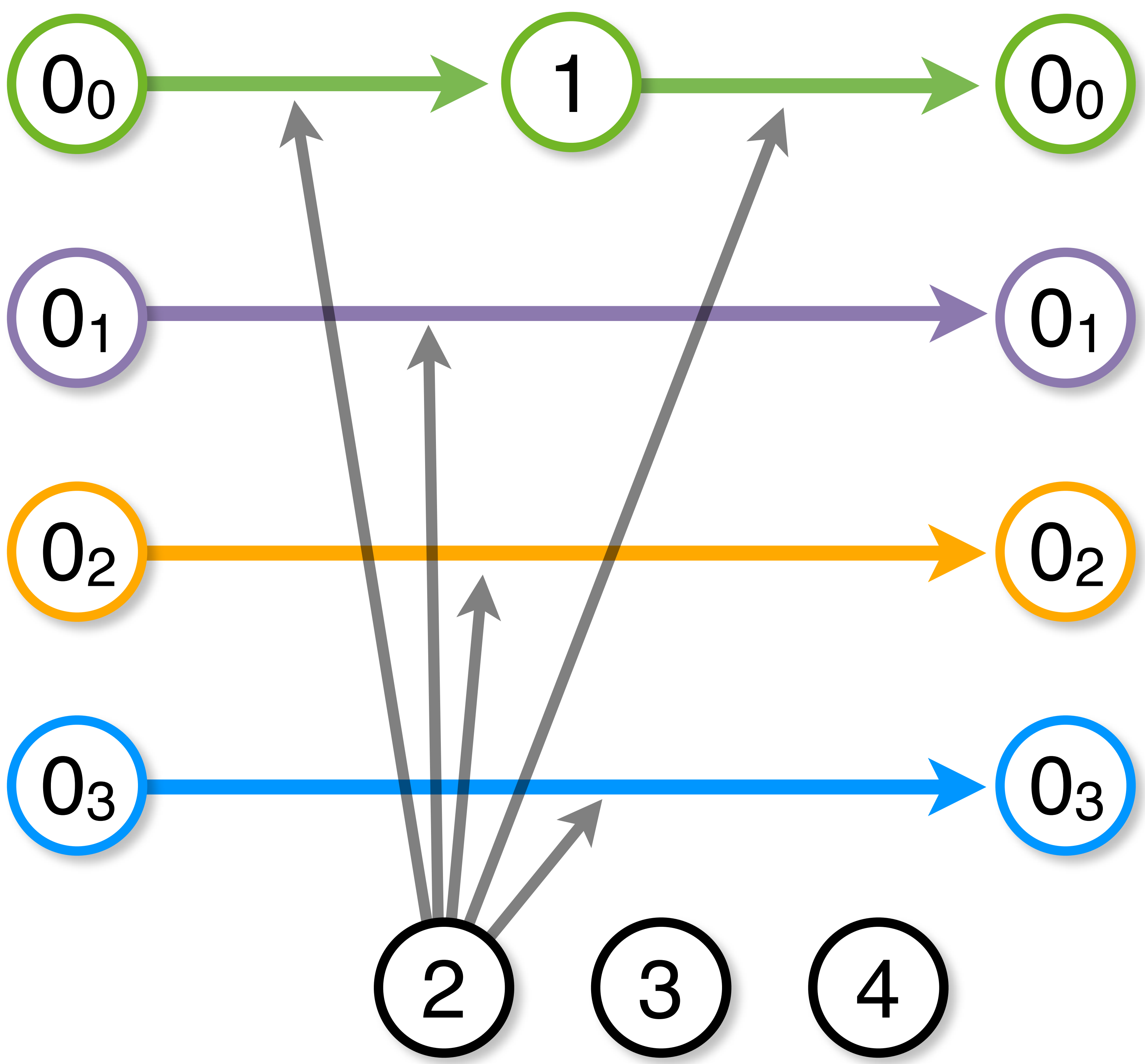
- Insert Customers



Insert customer 1

A Local Search Model

- Insert Customers

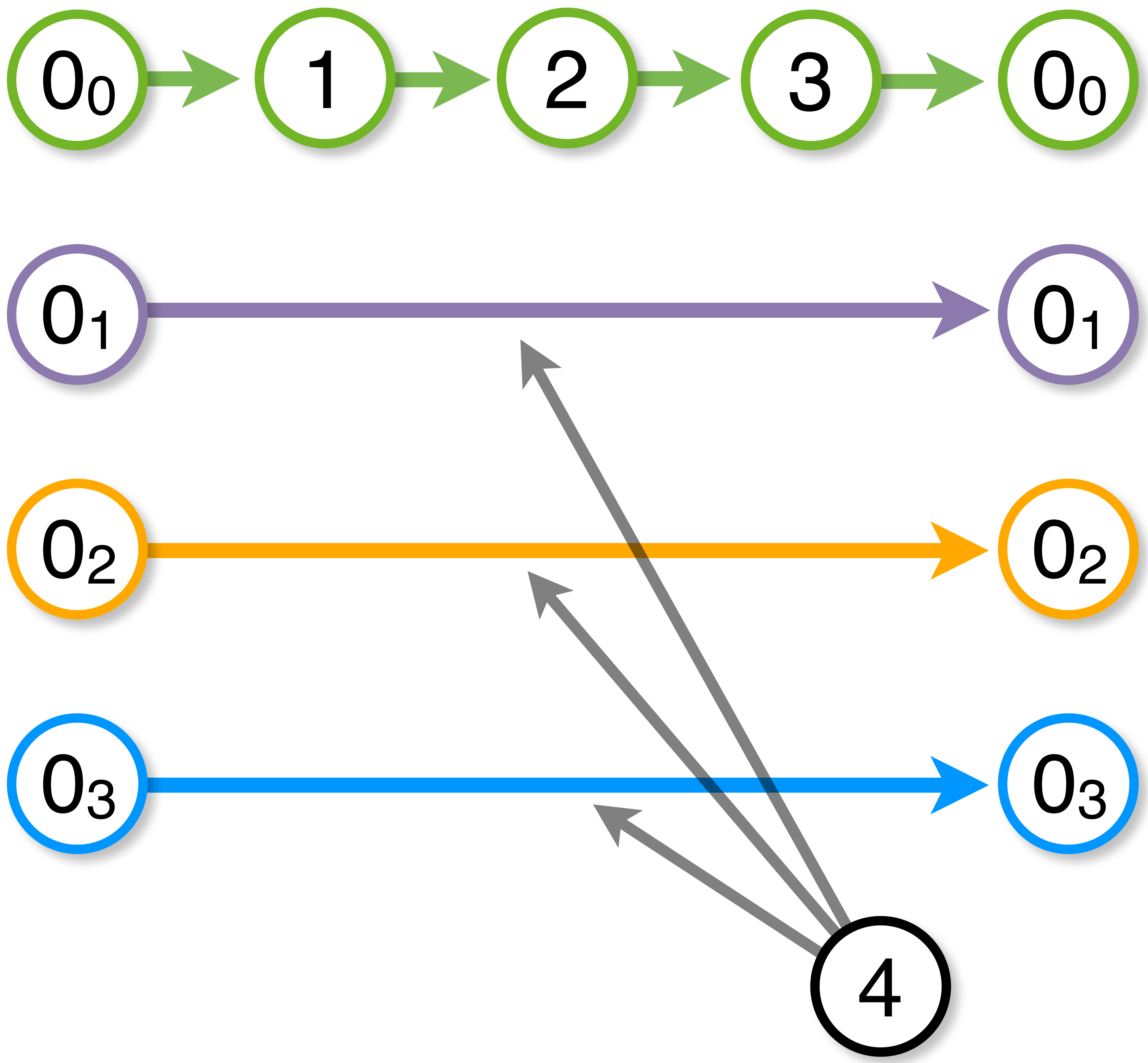


Insert customer 2

vehicles = **4**

A Local Search Model

- Insert Customers



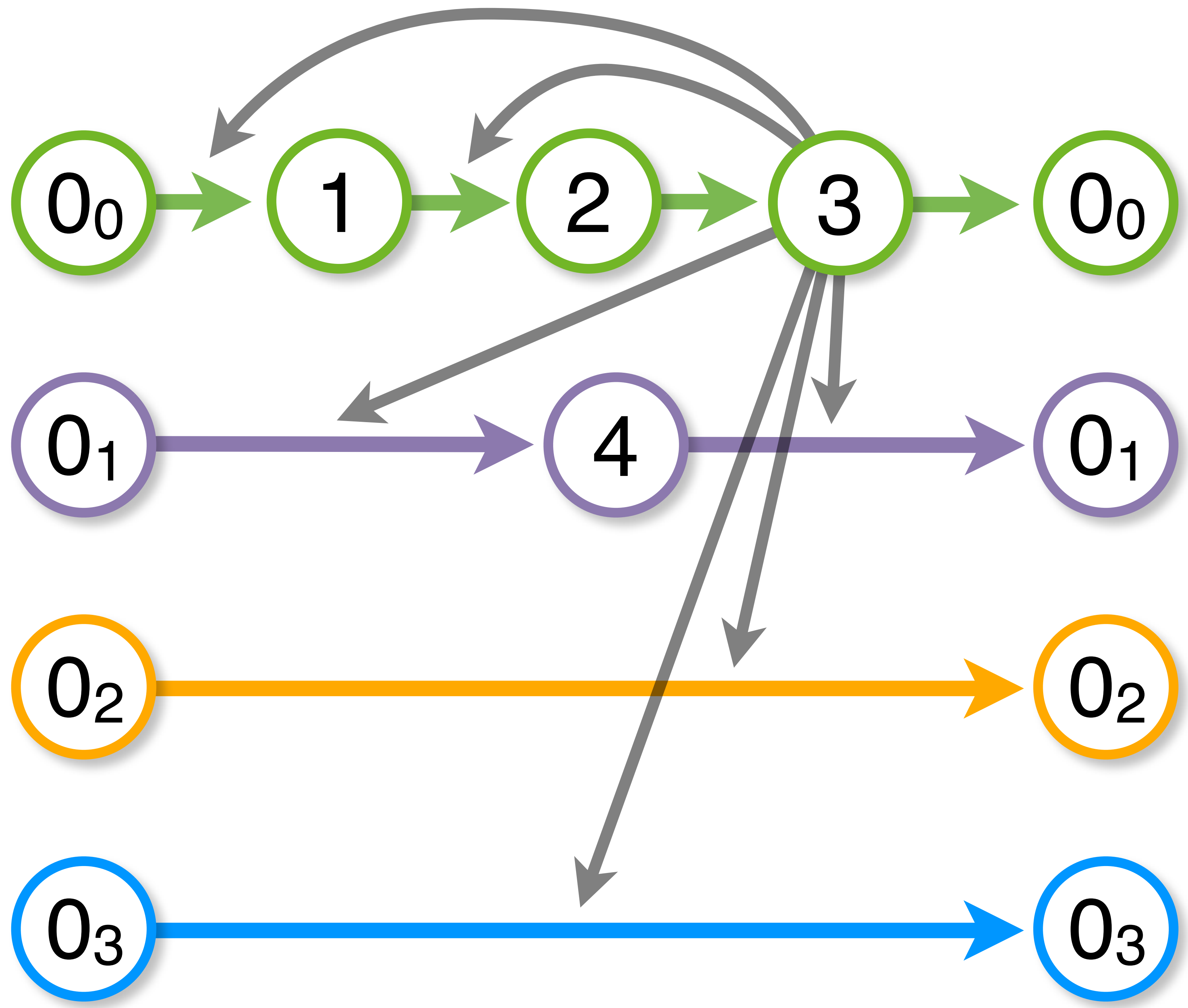
Insert customer 4

4 cannot fit in
to vehicle 0

vehicles = **4**

A Local Search Model

- Insert Customers

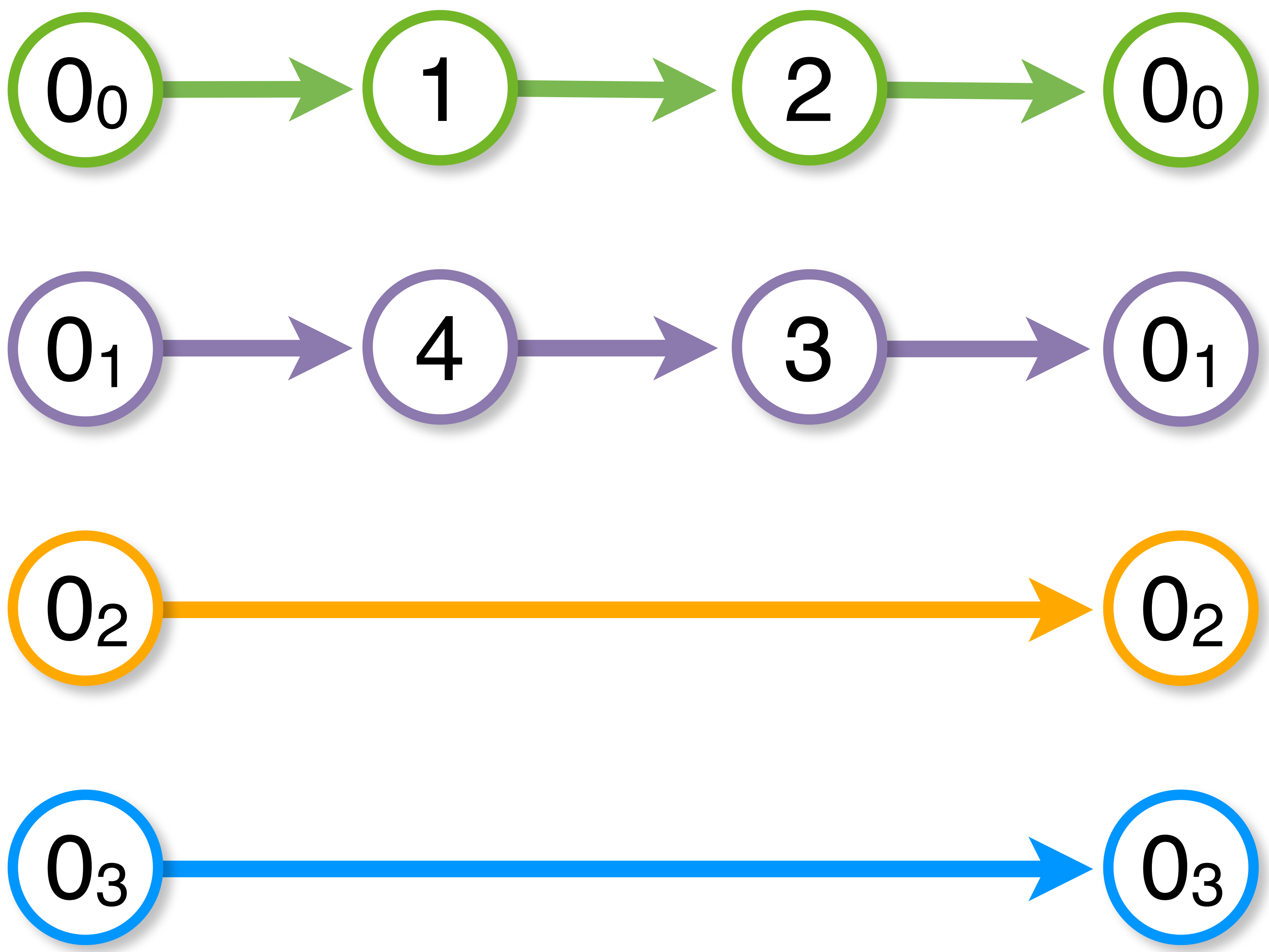


Reconsider customer 3

vehicles = **4**

A Local Search Model

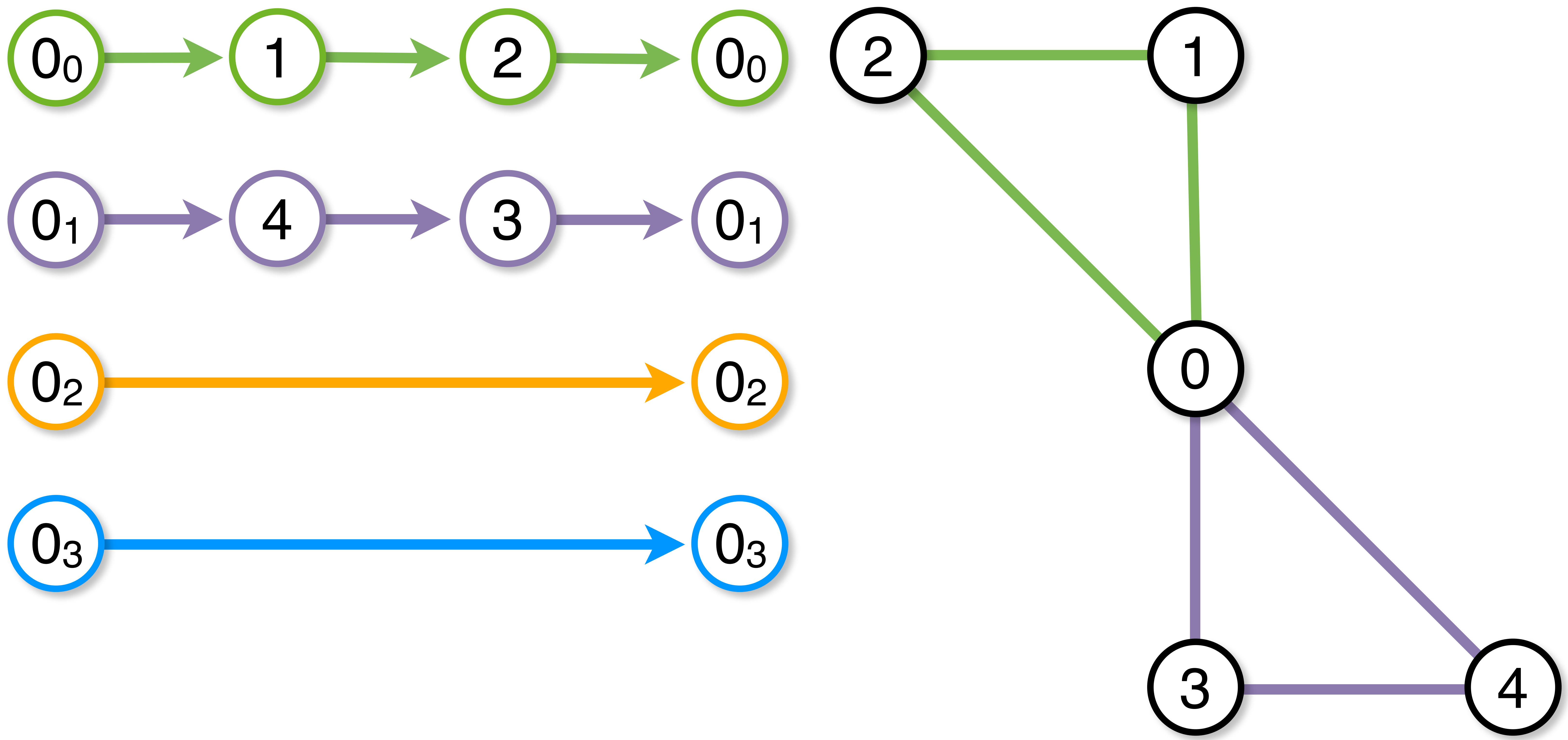
- Insert Customers



vehicles = **4**

A Local Search Model

- Insert Customers



vehicles = **4**

A Two-Stage Decomposition

- All of these methods consider the routing and customer assignment simultaneously
  - but we could break these into two steps

1. Assign the customers to vehicles
   - and ensure the capacity is satisfied
2. Solve a TSP for each vehicle (by any method)
   - CP
   - Local Search
   - MIP

- Decouples capacity constraint and routing objective

Packing isn't Easy

- Even with all these tips, its still tricky
- Consider
  - 4 customers of size 30
  - vehicles of capacity 40
- How many vehicles do we need to server these customers?
  - 4*30 = 120 is the total demand
  - total demand / vehicle capacity = 120 / 40 = 3
  - Looks like 3 vehicles will do!
  - Let us try it.

Packing isn't Easy

30

Houston, we have a problem.

Packing isn't Easy

- Luckily for you, the number of trucks in the assignment is fixed
  - However, you still need to find out how to pack them...
- This is a well known feasibility problem
  - called multi-knapsack
- Capacitated VRP is multi-knapsack and TSP combined

Assignment Tips

- Many approaches can work
  - Start off with the methods you like

- Reusing your TSP solver may be helpful

- Symmetries between vehicles

- *FAST* neighborhood computation

Have Fun!