Rise Aware Travelling Salesman Problem

***Abstract*—**

***Index Terms*—Risk, Travelling Salemsan**

I. Introduction

This document is a model and instructions for LaTeX. Please observe the conference page limits.

II. Problem 1: Maximising the Conditional Value at Risk for a stochastic information map

Consider a graph $G(V, E)$. A cycle is defined as a path that starts and ends at the same vertex. We define a Hamiltonian cycle C on this graph as a cycle that includes all vertices, passing through each vertex once. We call a function f submodular if:

$$f(S) + f(T) \geq f(S \cup T) + f(S \cap T) \tag{1}$$

The submodular function gives diminishing modular returns. Let us now define S as the set of information $I(C)$ gained while travelling along C, the Hamiltonian cycle. We define $f(S, y)$ as a sumodular function with y as the random variable assosciated with variance in the information gained. We can see that $f(S, y)$ is a submodular function as the average information gained by travelling across edges must always be lesser than the average information gained from the edges seperately, that is

$$f(S \cup e) \leq f(S) + f(e) \tag{2}$$

We define the Value at Risk (VaR) as:

$$VaR_{\alpha} = \min_{\tau \in R} Pr[f(S, y) \leq \tau] \geq \alpha \tag{3}$$

and the Conditional Value at Risk (CVaR) as:

$$E_y[f(S, y) | f(S, y) \leq VaR_{\alpha}(S)] \tag{4}$$

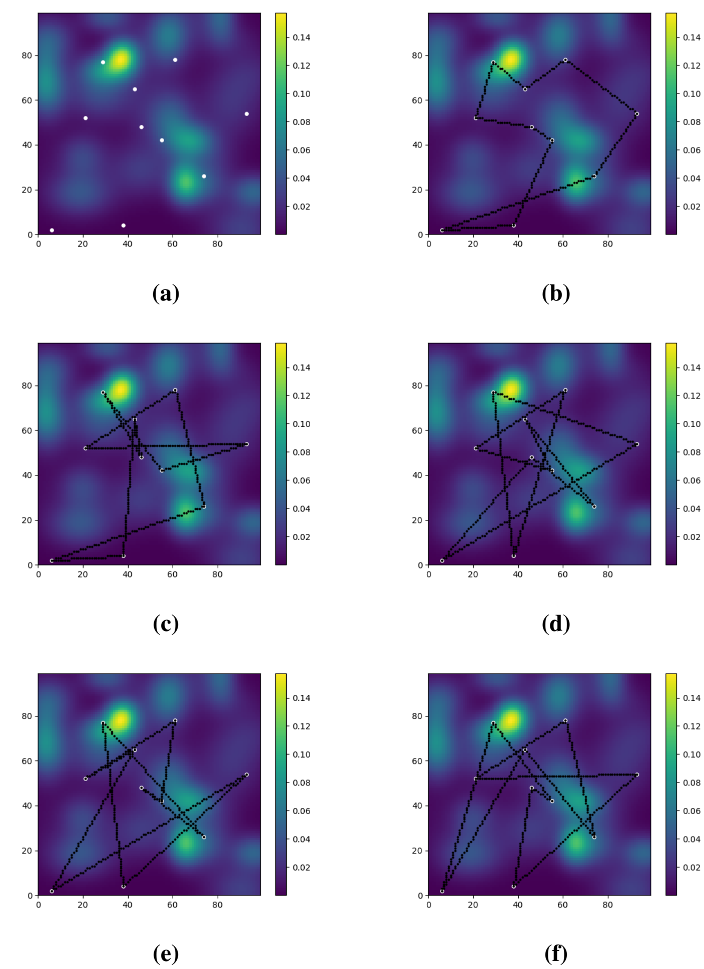
While optimizing the CVaR, we instead optimize the auxillary function $H(S, \tau)$ defined as:

$$H(S, \tau) = \tau - \frac{1}{\alpha} E[(\tau - f(S, y)^{+}] \tag{5}$$

Algorithm 1 Risk TSP - 1

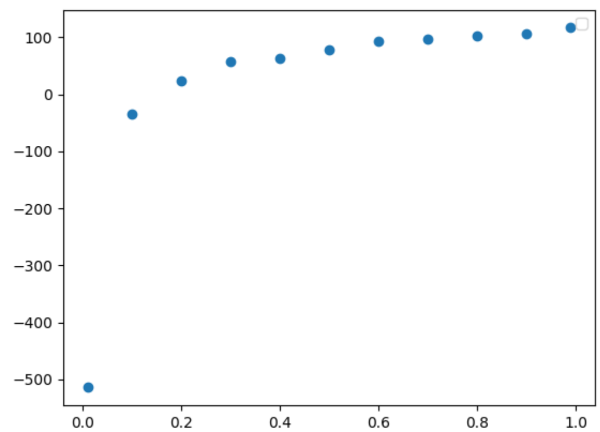
Input: $G(V, E), \tau_{max}, \alpha$, information map **M**
Output: H

while $\tau \leq \tau_{max}$ **do**
  $H_{all} = \emptyset$
  $f_{all} = \emptyset$
  C = $\emptyset$
  H = 0
  f = 0
  for $e \in E$ **do**
    for 100 samples **do**
      $H((S \cup e), y) = \tau - \frac{1}{\alpha} E[(\tau - f(S, y)^{+}])$
    $f_{all}.append(avg(f))$
    $H_{all}.append(avg(H))$
  $H = max(H_{all})$
  $f = max(f_{all})$
  $C.append(e_{max})$

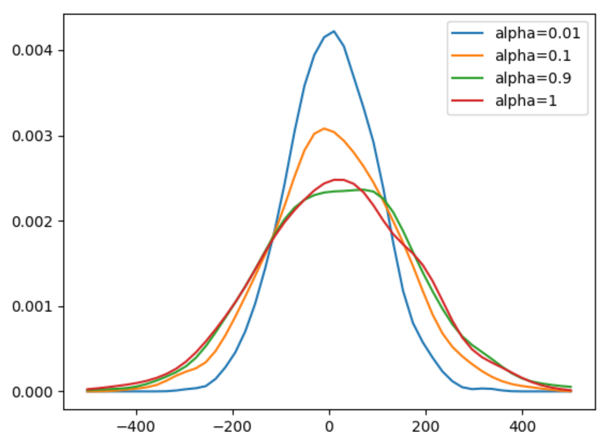


(a) (b) (c) (d) (e) (f)

Fig. 1



(a)



(b)

Fig. 2

III. Next To Do

Point discussed

- Implementation of stochastic risk for edges in graph $G(V, E)$, based on the Travelling Salesman formulation. We want to generate a Hamiltonan path with maximum reward given the expected risk along each edge (CVaR), and the operator threshold (α).
- Problem: $\max$ CVaR$_\alpha$ [w(S)], $s \in H$
- Each edge reward is a random variable, e.g., r_{eij} $N(10, 2)$
- Solution: CVaR minimization with greedy TSP as subroutine
- Adding to the above algorithm costs, defined by constraints such as distance/fuel/time. These costs are deterministic.
- Altering the costs to be expectations of stochastic values, and improving the above algorithm

IV. Conclusion and Future Work