Chapter 1

Sets and Relations

1.1 Cantor's Concept of a Set

A **set** S is any collection of definite, distinguishable objects of our intuition or of our intellect to be conceived as a whole. The objects are called the **elements** or **members** of S.

1.2 The Basis of Intuitive Set Theory

Membership relation: $x \in A$ if the object x is a member of the set A. If x is not a member of A then $x \notin A$. $x_1, x_2, \ldots, x_n \in A$ is shorthand for $x_1 \in A \wedge x_2 \in A \ldots x_n \in A$.
The intuitive principle of extension: Two sets are equal iff they have the same members.
Set equality: The equality of two sets X and Y will be denoted by $X = Y$ and inequality of X and Y by $X \neq Y$. Among the basic properties of this relation are:

$$X = X,$$
$$X = Y \Rightarrow Y = X,$$
$$X = Y \wedge Y = Z \Rightarrow X = Z,$$

for all sets X, Y, and Z.
unit set: a set $\{x\}$ whose sole member is x.
collection of sets: a set whose members are sets.
The intuitive principle of abstraction: A formula $P(x)$ defines a set A by the convention that the members of A are exactly those objects a such that $P(a)$ is a true statement, denoted by $A = \{x \mid P(x)\}$.
Note: $\{x \in A \mid P(x)\} := \{x \mid x \in A \wedge P(x)\}$. For a property P and function f we can write $\{f(x) \mid P(x)\} := \{y \mid \exists x \colon P(x) \wedge y = f(x)\}$.

1.3 Inclusion

If A and B are sets, then A is **included in** B iff each member of A is a member of B. Symbolized: $A \subseteq B$. We also say that A is a **subset** of B. Equivalently, B **includes** A, symbolized by $B \supseteq A$.
The set A is **properly included in** B (A is a **proper subset** of B / B **properly includes** A) iff $A \subseteq B$ and $A \neq B$.
Among the basic properties of the inclusion relation are

$$X \subseteq X;$$
$$X \subseteq Y \wedge Y \subseteq Z \Rightarrow X \subseteq Z;$$
$$X \subseteq Y \wedge Y \subseteq X \Rightarrow X = Y.$$

empty set: $\{x \in A \mid x \neq x\}$ for any set A is the set with no elements, symbolized by $\emptyset$.
power set: the set of all subsets of a given set. $\mathcal{P}(A) = \{B \mid B \subseteq A\}$ for a given set A.

1.4 Operations for Sets

union: for sets A and B, the set of all objects which are members of either A or B. $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$. (**sum**/**join**)
intersection: for sets A and B, the set of all objects which are members of both A and B. $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$. (**product**/**meet**)
disjoint: $A \cap B = \emptyset$ for sets A and B.
intersect: $A \cap B \neq \emptyset$ for sets A and B.
disjoint collection: for a collection of sets, each distinct pair of its member sets is disjoint.
partition: for a set X, a disjoint collection $\mathcal{A}$ of nonempty and distinct subsets of X such that each member of X is a members of some (exactly one) member of $\mathcal{A}$.
absolute complement of A: $\overline{A} = \{x \mid x \notin A\}$, the set of all members which are not in A.
relative complement of A with respect to X: $X - A = X \cap \overline{A} = \{x \in X \mid x \notin A\}$, the set of those members of X which are not members of A.
symmetric difference of A and B: $A + B = (A - B) \cup (B - A)$.
universal set: the set U such that all sets under consideration in a certain discussion are subsets of U.