Lower bound for comparison based sorting algorithms

Difficulty Level : Easy • Last Updated : 06 Jul, 2021

The problem of sorting can be viewed as following.

Input: A sequence of n numbers $<a_1, a_2, \ldots, a_n>$.
Output: A permutation (reordering) $<a'_1, a'_2, \ldots, a'_n>$ of the input sequence such that $a'_1 <= a'_2 \ldots.. <= a'_n$.

A sorting algorithm is comparison based if it uses comparison operators to find the order between two numbers. Comparison sorts can be viewed abstractly in terms of decision trees. A decision tree is a full binary tree that represents the comparisons between elements that are performed by a particular sorting algorithm operating on an input of a given size. The execution of the sorting algorithm corresponds to tracing a path from the root of the decision tree to a leaf. At each internal node, a comparison $a_i <= a_j$ is made. The left subtree then dictates subsequent comparisons for $a_i <= a_j$, and the right subtree dictates subsequent comparisons for $a_i > a_j$. When we come to a leaf, the sorting algorithm has established the ordering. So we can say following about the decision tree.

1) Each of the n! permutations on n elements must appear as one of the leaves of the decision tree for the sorting algorithm to sort properly.

After combining the above two facts, we get following relation.

$$n! \le 2^x$$

Taking Log on both sides.

$$\log_2(n!) \le x$$

Since $\log_2(n!) = \Theta(n\text{Log}n)$, we can say

$$x = \Omega(n\text{Log}_2 n)$$

Therefore, any comparison based sorting algorithm must make at least $n\text{Log}_2 n$ comparisons to sort the input array, and Heapsort and merge sort are asymptotically optimal comparison sorts.

References:

Introduction to Algorithms, by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein

Previous

An Insertion Sort time complexity question

Next

Which sorting algorithm makes minimum number of memory writes?

RECOMMENDED ARTICLES

Page : 1 2 3

01 Can Run Time Complexity of a comparison-based sorting algorithm be less than N logN?
16, Jun 22

02 Asymptotic Analysis and comparison of sorting algorithms
28, Dec 16

03 Sorting without comparison of elements
12, Aug 18

05 Know Your Sorting Algorithm | Set 1 (Sorting Weapons used by Programming Languages)
26, Jun 16

06 Sorting objects using In-Place sorting algorithm
29, Oct 19

07 0/1 Knapsack using Least Cost Branch and Bound

26, Jun 16

08

10, Mar 21

Article Contributed By :

GeeksforGeeks

Vote for difficulty

Current difficulty : Easy

Easy Normal Medium Hard Expert

Improved By : sooda367

Article Tags : Merge Sort, Sorting

Practice Tags : Sorting, Merge Sort

Improve Article Report Issue

A-143, 9th Floor, Sovereign Corporate Tower,
Sector-136, Noida, Uttar Pradesh - 201305

feedback@geeksforgeeks.org