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Lower bound for comparison based sorting algorithms

Difficulty Level : Easy • Last Updated : 06 Jul, 2021

The problem of sorting can be viewed as following.

Input: A sequence of n numbers $\langle a_1, a_2, \dots, a_n \rangle$.

Output: A permutation (reordering) $\langle a'_1, a'_2, \dots, a'_n \rangle$ of the input sequence such that $a'_1 \leq a'_2 \leq \dots \leq a'_n$.

A sorting algorithm is comparison based if it uses comparison operators to find the order between two numbers. Comparison sorts can be viewed abstractly in terms of decision trees. A decision tree is a [full binary tree](#) that represents the comparisons between elements that are performed by a particular sorting algorithm operating on an input of a given size. The execution of the sorting algorithm corresponds to tracing a path from the root of the decision tree to a leaf. At each internal node, a comparison $a_i \leq a_j$ is made. The left subtree then dictates subsequent comparisons for $a_i \leq a_j$, and the right subtree dictates subsequent comparisons for $a_i > a_j$. When we come to a leaf, the sorting algorithm has established the ordering. So we can say following about the decision tree.

1) Each of the $n!$ permutations on n elements must appear as one of the leaves of the decision tree for the sorting algorithm to sort properly.

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After combining the above two facts, we get following relation.

$$n! \leq 2^x$$

Taking Log on both sides.

$$\log_2(n!) \leq x$$

Since $\log_2(n!) = \Theta(n \log n)$, we can say

$$x = \Omega(n \log_2 n)$$

Therefore, any comparison based sorting algorithm must make at least $n \log_2 n$ comparisons to sort the input array, and Heapsort and merge sort are asymptotically optimal comparison sorts.

References:

[Introduction to Algorithms, by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein](#)

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