Technical Report on Simulation Software in Robotics Term Project

Chengze Song, Bibek Poudel

December 14, 2024

1 Introduction

Our project is the simulation of a rollercoaster-cart over a track. We have considered different environmental parameters such as gravity, friction, and air resistance. The goal is to simulate the motion of the cart over the track and to analyze the forces acting on the cart. Our roller coaster simulation model is felxible for the user/designers as they can tweak all the parameters to test different real world scenarios. We have used Matlab/Simulink for the project. The simulation results and project details are discussed in the following sections.

2 Problem Statement

This project aims to simulate the motion of a rollercoaster-cart over a track. The cart is subject to gravity, friction, and air resistance. The cart is subjected to a initial thrust by a PMLSM motor which provides thrust for 4 sec over a horizontal track. The goal is to analyze the forces acting on the cart and to simulate the motion of the cart over the track. The track is composed of different sections with varying slopes(Horizontal, and Inclined). The simulation results will be used to analyze the forces acting on the cart and determine whether the thrust provided by the motor is sufficient to move the cart over the track.

2.1 System and Environment

Our system of interest is a rollercoaster-cart moving over a track. Our environment includes gravity, friction, roller-coaster track and air resistance

2.2 Desired Inputs and Outputs

The rollercoaster-cart system operates as an integrated model comprising two interconnected subsystems within a dynamic environment influenced by gravity, friction, and air resistance. The first subsystem, a PMLSM (Permanent Magnet

Linear Synchronous Motor), receives inputs such as the desired rollercoaster position over time and motor parameters (e.g., maximum thrust, stator resistance, and moving part mass). It outputs thrust over time and the cart's velocity at a specific horizontal section of the track. The second subsystem represents the rollercoaster itself, taking inputs like the track profile, environmental factors, and motor-provided thrust. It outputs the cart's velocity, position, forces (gravitational, frictional, and air resistance), and determines if the thrust suffices for motion. Additionally, it calculates the cart's acceleration, velocity, and position iteratively using feedback loops to solve ordinary differential equations for system updates. Together, these subsystems create a closed-loop mechanism to model and analyze rollercoaster dynamics effectively.

First Subsystem(PMLSM Motor):
Inputs:

- The desired behaviour of the position of the rollercoaster as a function of time.
- The parameters of the motor For example:
  - $F_{\max}$: 2000 N (Maximum thrust generated by the motor).
  - PM: 0.65 Wb (Permanent magnet flux linkage).
  - L_d: 0.01 H (Inductance along the d-axis).
  - L_q: 0.01 H (Inductance along the q-axis).
  - L_0: 2×10^{-4} H (Zero-sequence axis inductance).
  - R_s: 0.6 ohm (Stator resistance).
  - Mass: 1000 kg (Mass of the moving part).
  - Pitch: 0.1 m (Polar pitch or distance between poles).
  - N_p: $\pi/\text{Pitch}\,\text{rad/m}$ (Electrical pole-pair constant).

Outputs:

- Thrust provided by the motor as a function of time
- The final velocity of the cart at the horizontal section of the track

Second Subsystem(Roller Coaster):
Inputs:

- Track Profile i.e the length of the track and slope of the track
- Environmental parameters such as gravity, friction, and air resistance
- The thrust provided by the motor

Outputs:

- Velocity and position of the cart over the track

- Forces acting on the cart (gravitational force, frictional force, air resistance)
- Analysis of whether the thrust provided by the motor is sufficient to move the cart over the track as a boolaean variable (0 and 1)
- Acceleraton, velocity, and position of the cart at each time step which is then provided as a feedback to solve the ordinary differential equations to get the next state of the system. These parameters were used for the final animation of the system.

3 Relevance in Mechatronics and Robotics

Often Engineering design projects are tested using simulation software before the actual implementation. Engineering projects are complex, costly and margin of error is very low. Simulation software helps to test the design and analyze the performance of the system before the actual implementation. The simulation of the rollercoaster-cart over a track is relevant in the field of mechatronics and robotics as it helps to analyze the forces acting on the cart and determine whether the thrust provided by the motor is suhe cart over the track . The simulation results can be used to optimize the design of the rollercoaster-cart and to analyze the performance of the system. Furthermore, based on the simulation results, different parameters such as the g forces experienced by the cart, the maximum speed of the cart, and the energy consumption of the motor can be analyzed which can be used to optimize the design of the rollercoaster-cart. Over the years mechanical dynamics has been a part of mechatronics and robotics curriculum. Due to advancement in computing power, simulation software has become an integral part of the design process of dynamical systems. The simulation of the rollercoaster-cart over a track is a good example of how simulation software can be used to analyze the performance of a dynamical system and to optimize the design of the system. Simulation softwares can be used to model different design parameters for consideration and get the best possible design for the system without using any physical resources,which is cost-effective and time-saving.[1]

4 Models Used

4.1 Diagrams and Equations

The model of the rollercoaster-cart over a track is based on the following equations: The model of the rollercoaster-cart over a track is based on the following equations:

$$acceleration = \frac{F_{\text{net}}}{mass} \tag{1}$$

For Horizontal track :

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{friction}} \tag{2}$$

Where, $F_{\text{friction}} = \mu \cdot m \cdot g$ and μ is the coefficient of friction.

For Inclined track :

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{friction}} - F_{\text{gravity}} - F_{\text{air resistance}} \tag{3}$$

Where $F_{\text{gravity}} = \mu \cdot m \cdot g \cdot \sin(\theta)$ and $F_{\text{friction}} = \mu \cdot m \cdot g \cdot \cos(\theta)$

$$velocity = \int acceleration \cdot dt \tag{4}$$

The initial condition of the velocity integration is the final velocity of the part where the thrust is provided

$$position = \int velocity \cdot dt \tag{5}$$

$$F_{\text{gravity}} = \mu \cdot m \cdot g \cdot \sin(\theta) \tag{6}$$

$$F_{\text{friction}} = \mu \cdot m \cdot g \cdot \cos(\theta) \tag{7}$$

$$F_{\text{air resistance}} = 0.5 \cdot \rho \cdot A \cdot C_d \cdot v^2 \tag{8}$$

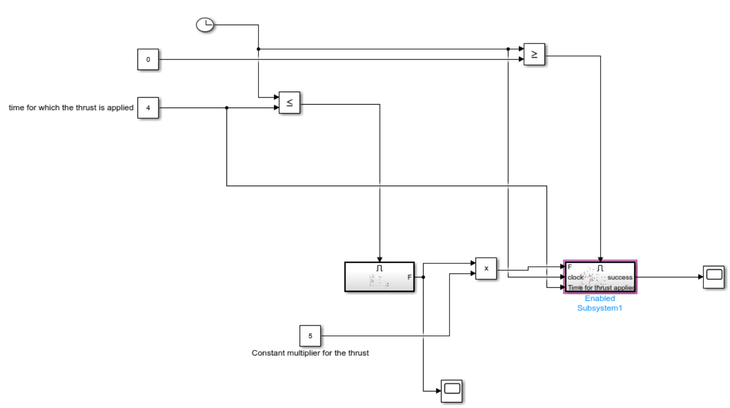

Figure 1: System Integration

System Integration: This diagram demonstrates the integration of the PMLSM motor and the rollercoaster model.

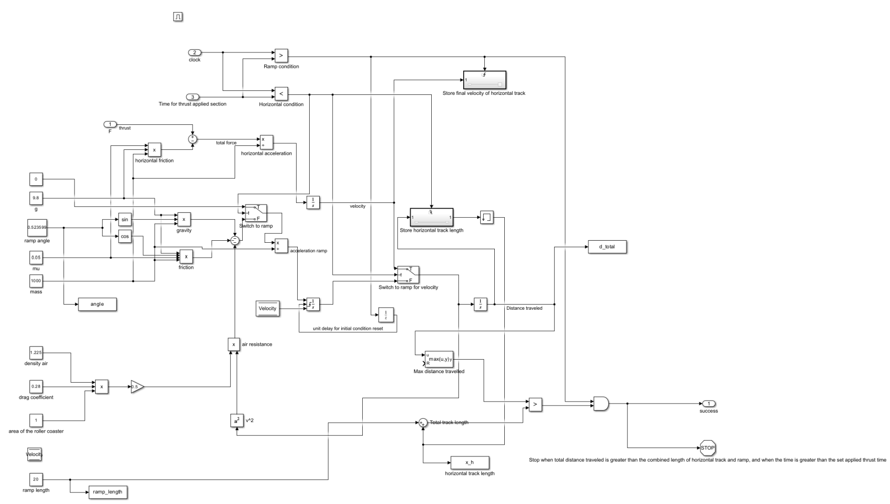

Figure 2: Roller Coaster System

Roller Coaster System: This diagram demonstrates the rollercoaster system. The rollercoaster system includes the thrust from the PMLSM motor, the rollercoaster model, and the environmental parameters such as gravity, friction, and air resistance. A switch block is used to provide thrust to the cart for specific time over a horizontal track. Then the cart moves over an inclined track. The paramters that can be changed in the simulation are :

- Angle of inclination
- Time for which the thrust is provided
- The environmental parameters such as gravity, coefficient of friction, and air resistance
- Length of the ramp
- Mass of the cart
- Density of air
- Drag coefficient

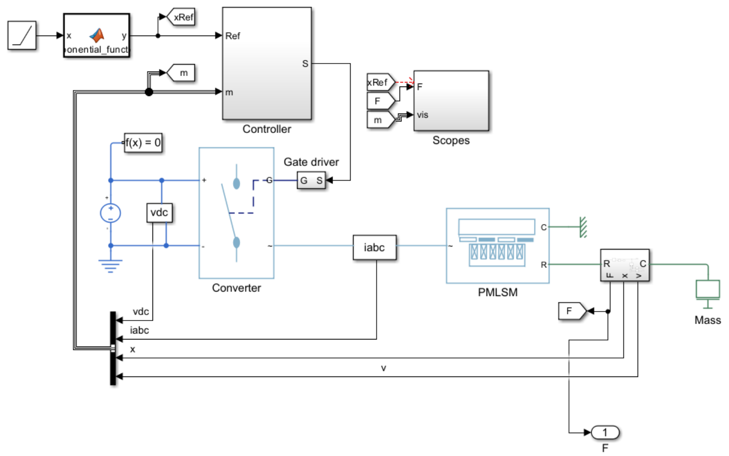

Figure 3: Motor Model

Motor Model: This diagram demonstrates position control in a three-phase Permanent Magnet Linear Synchronous Machine (PMLSM) drive using a PI-based cascade control structure. The control system includes an outer position loop, a speed loop, and two inner current loops. The PMLSM is powered by a controlled three-phase converter, with step references guiding the simulation. Results are visualized using the Scopes subsystem.

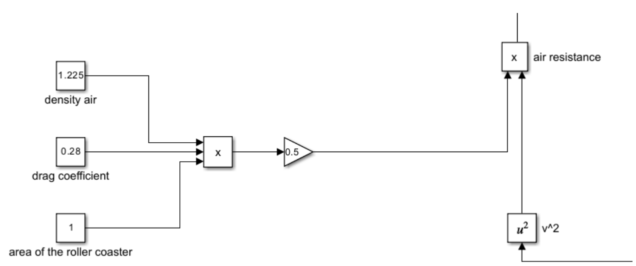

Figure 4: Air resistance model

Air Resistance Model: This diagram demonstrates the air resistance model. The air resistance is modeled as a function of the velocity of the cart. The air resistance is calculated using the formula $F_{\text{air resistance}} = 0.5 \cdot \rho \cdot A \cdot C_d \cdot v^2$ where ρ is the air density, A is the cross-sectional area of the cart, C_d is the drag coefficient, and v is the velocity of the cart.

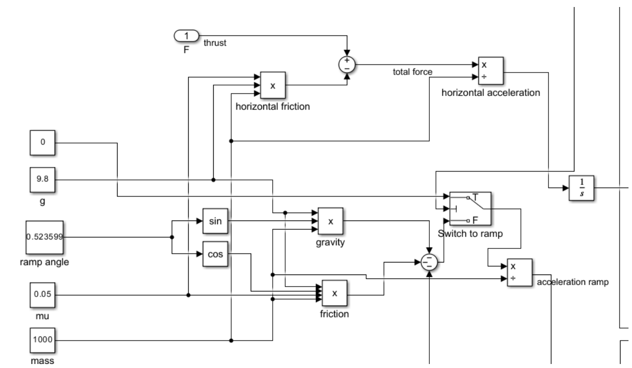

Figure 5: Friction and Gravity model

Friction and Gravity Model: This diagram demonstrates the friction and gravity model. The friction and gravity are modelled as a function of the angle of the track. The friction is calculated using the formula $F_{\text{friction}} = \mu{\cdot}m{\cdot}g{\cdot}\cos(\theta)$ and the gravity is calculated using the formula $F_{\text{gravity}} = \mu \cdot m \cdot g \cdot \sin(\theta)$ where m is the mass of the cart. g is the acceleration due to gravity, and θ is the angle of the track.

4.2 System Parameters and State Variables

The state variables of the system are:

1. ***Position of the cart:*** The position of the cart is the distance traveled by the cart over the track. The position of the cart is calculated by integrating the velocity of the cart over time.

2. ***Velocity of the cart:*** The velocity of the cart is the speed of the cart at a specific point on the track. The velocity of the cart is calculated by integrating the acceleration of the cart over time.

3. ***Acceleration of the cart:*** The acceleration of the cart is the rate of change of velocity of the cart. The acceleration of the cart is calculated using Newton's second law of motion.

4. ***Forces acting on the cart:*** The forces acting on the cart include gravitational force, frictional force, and air resistance. The forces acting on the cart are calculated using the equations of motion.

4.3 Assumptions and Justifications

1. There is no air resistance in the horizontal section of the track : The linear synchonous motor uses feedback control system to provide thrust to the cart. Due to the feedback control system, the thrust compensates for the air resistance in the horizontal section of the track.

2. Coefficient of friction is constant throughout the track: The track is made of same material throughout the track. The coeffiecient of friction is constant throughout the track as the track is made of same material.

3. We have not considered the point of inclination during our numerical calculation. Sudden sharp kinks in the track can cause numerical instability during integration, by introducing the point of discontinuity leading to inaccurate simulation results.

4. The PI controllers have idealized behavior with no delay or non-linearities. This assumption simplifies initial control design and tuning. Any real-world imperfections can be addressed later during implementation.

5. The cart is assumed to be a point mass: The cart is assumed to be a point mass to simplify the analysis. The mass of the cart is concentrated at the center of mass of the cart

6. Mechanical losses such as friction, windage, and eddy currents are ignored or minimally considered.These effects are small compared to the overall system dynamics and can be incorporated later if needed.

7. For the sake of simplicity we have only considered the sliding friction and not the rolling friction.

4.4 Code Documentation

1. *Open the Matlab/Simulink model file:* The Matlab/Simulink model file contains the simulation of the rollercoaster-cart over a track. The model file contains the block diagram of the system and the parameters of the system.

2. *Run the simulation:* Run the simulation to simulate the motion of the cart over the track. The simulation results will be displayed in the scope block.

3. *Use scope to analyze:* Use the scope block to analyze the motion of the cart over the track. Based on the requirements there are different scopes. The scope block can be used to analyze the forces acting on the cart and to determine whether the thrust provided by the motor is sufficient to move the cart over the track, position, velocity, air resistance e.t.c.

4. *Change the parameters in simulation:* The parameters of the system such as the initial thrust provided by the motor, the track profile, and the environmental parameters can be changed in the simulation to analyze the performance of the system under different conditions.

5 Computational Results and Discussion

5.1 Results

The results of the simulation of the rollercoaster-cart over a track are shown below:

Scenario 1: The thrust provided by the motor is sufficient to move the cart to the top of the ramp:

Parameters settings:

1. Angle of inclination: 0.523599 rad
2. Time for which the thrust is provided: 4 sec
3. Density of the air: 1.225 kg/m^3
4. Drag constant: 0.28
5. Area of the cart subject to air resistance: 1 m^3
6. Length of the ramp: 20 m
7. Mass of the cart: 1000 kg
8. Friction constant: 0.05

Here is the output of the system:

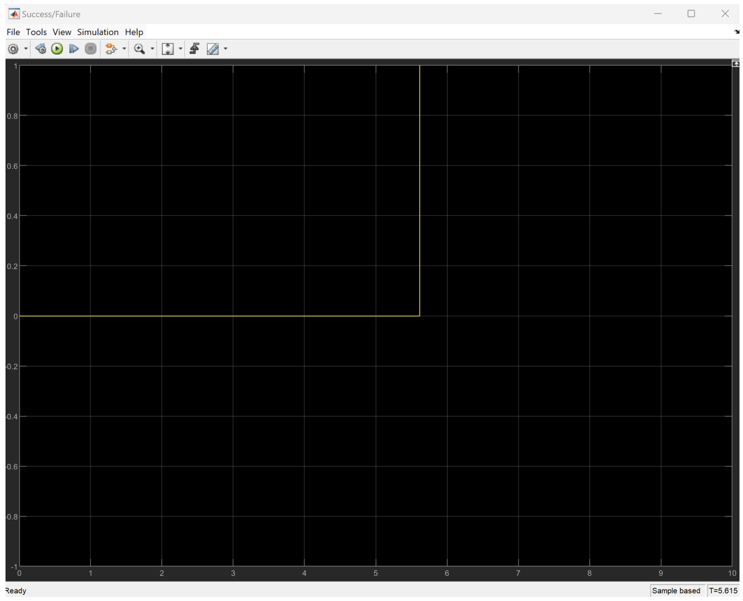

Figure 6: Boolean output for success

The figure shows that the boolean output jumps to 1 sometime after the simulation begins, which means that the cart successfully made to the top of the ramp.

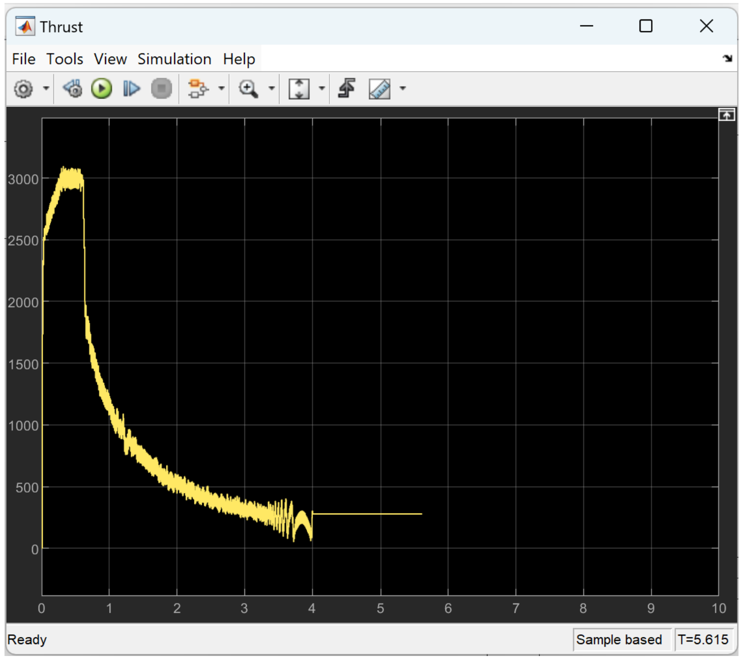

Figure 7: Thrust Provided by the motor

Scenario 2: The thrust provided by the motor is not sufficient to move the cart to the top of the ramp:

Parameters settings:

1. Angle of inclination: 0.523599 rad
2. Time for which the thrust is provided: 4 sec
3. Density of the air: 1.225 kg/m^3
4. Drag constant: 0.28
5. Area of the cart subject to air resistance: 1 m^3
6. Length of the ramp: 20 m
7. Mass of the cart: 5000 kg
8. Friction constant: 0.05

Here is the result of the simulation:

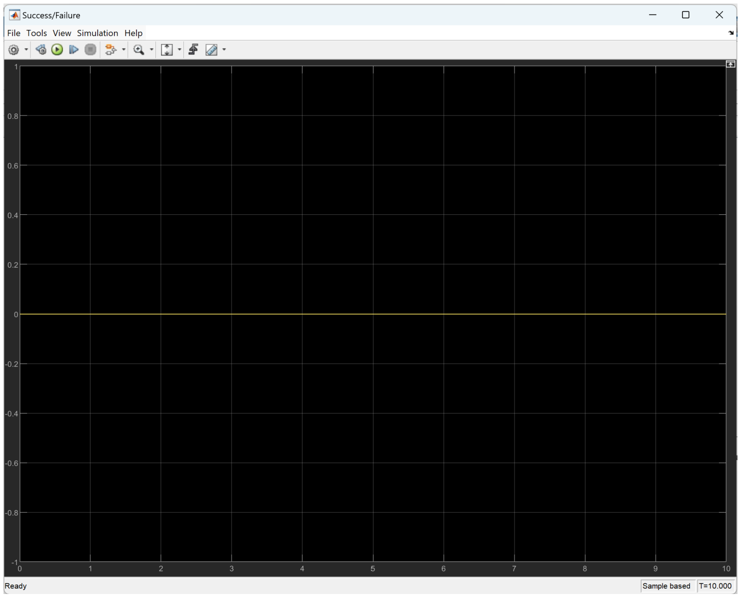

Figure 8: Failed to reach top

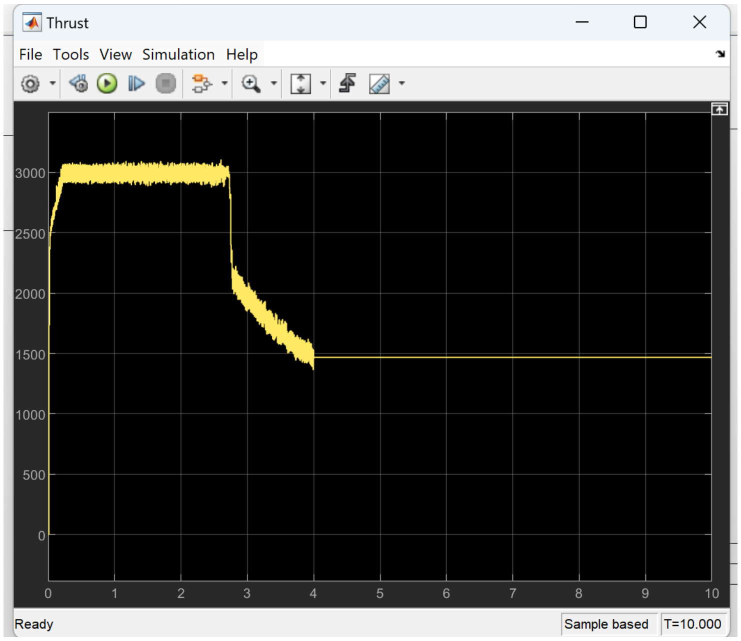

Figure 9: Thrust

With the above parameters, the roller coaster failed to reach the top of the ramp. As it is shown in the scope output graph, the boolean output remains False throughout the whole simulation. Also, since the mass is larger than the first scenario, the thrust is also increased by the system's feedback control in order to match the desired behavior of the cart on the horizontal part of the track.

Scenario 3: The thrust provided by the motor is not sufficient to move the cart to the top of the ramp:

Parameters settings:

1. Angle of inclination: 0.523599 rad
2. Time for which the thrust is provided: 4 sec
3. Density of the air: 1.225 kg/m^3
4. Drag constant: 0.28
5. Area of the cart subject to air resistance: 1 m^3
6. Length of the ramp: 50 m
7. Mass of the cart: 1000 kg
8. Friction constant: 0.05

Here is the result of the simulation:

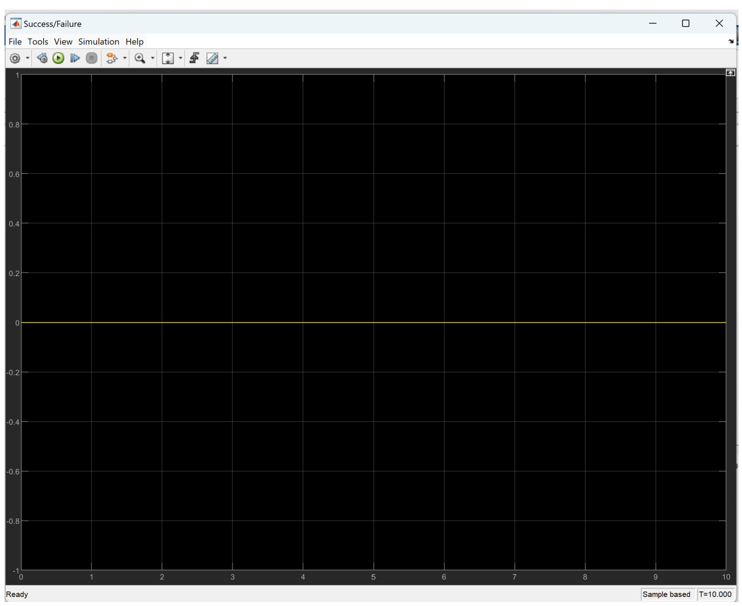

Figure 10: Faileure

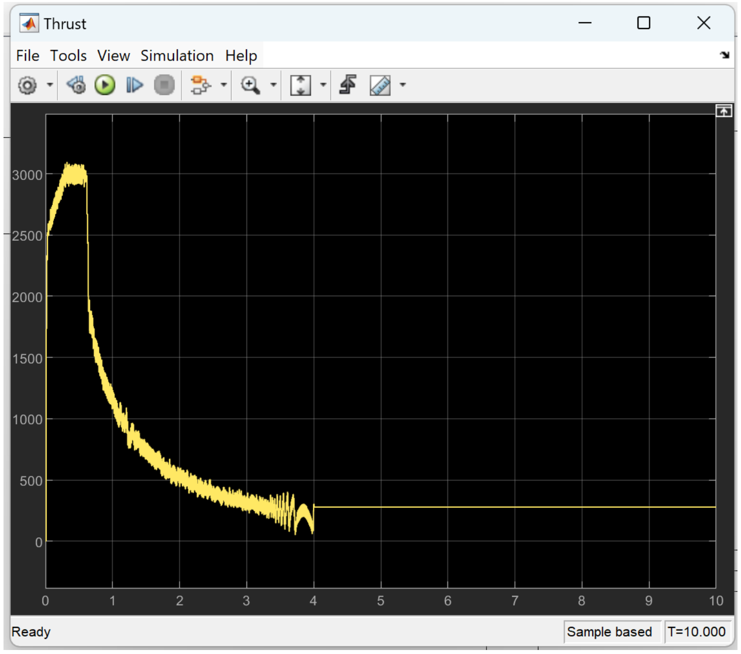

Figure 11: Thrust

With the above parameters, the roller coaster failed to reach the top of the ramp. As shown in the scope output graph, the boolean output remains False throughout the whole simulation.

6 Validation

The validation of the simulation results is performed using the **principle of conservation of energy**, which states that the total energy in a system remains constant unless acted upon by an external force. For the rollercoaster-cart system, the total mechanical energy is the sum of *kinetic energy (KE)*, *potential energy (PE)*, and the work done against friction and air resistance.

The key equations are:

1. **Kinetic Energy (KE):**
$$KE = \frac{1}{2}mv^2 \tag{9}$$
where m is the mass of the cart and v is its velocity.

2. **Potential Energy (PE):**
$$PE = mgh \tag{10}$$
where g is the gravitational acceleration, and h is the height relative to a reference point.

3. **Work done against friction and air resistance:**

$$W_{\text{loss}} = F_{\text{friction}} \cdot d + \int F_{\text{air resistance}} \cdot d \tag{11}$$

   where F_{friction} and $F_{\text{air resistance}}$ are the forces due to friction and air resistance, respectively, and d is the distance traveled.

Using these equations, we validate the simulation by ensuring that:

$$\text{Total Energy Input} = KE + PE + W_{\text{loss}} \tag{12}$$

6.1 Validation Procedure

1. **Initial State:** At the start of the simulation, the cart has an initial thrust energy imparted by the PMLSM motor. This energy is calculated as the work done by the motor over the time it is active:

$$E_{\text{motor}} = F_{\text{thrust}} \cdot d_{\text{thrust}} \tag{13}$$

2. **Energy Transitions:** As the cart moves, the thrust energy is converted into kinetic energy, potential energy (on inclined tracks), and energy lost due to friction and air resistance. At every time step, the simulation tracks these components.

3. **Validation Check:** At several points along the track, the total energy calculated from the simulation results is compared with the expected energy from the conservation of energy equation. Deviations are checked for numerical errors or unaccounted forces.

6.2 Observations

- On horizontal sections of the track, the change in kinetic energy corresponds closely with the work done by the thrust, friction, and air resistance.
- On inclined sections, the increase in potential energy matches the reduction in kinetic energy and the work done against gravity.
- The overall energy balance is maintained within acceptable tolerances, confirming the simulation's accuracy.

This validation demonstrates that the simulation adheres to fundamental physical laws, providing confidence in its results.

7 Discussion

Advantages and Disadvantages of Matlab/Simulink:

1. Advantages:
   (a) Matlab/Simulink is widely used in the industry for modeling and simulation of dynamical systems. The software is used in various fields such as control systems, robotics, and mechatronics.
   (b) The software provides a graphical user interface for modeling and simulation of dynamical systems. The user can drag and drop blocks to model the system and simulate the system.
   (c) The software provides a wide range of blocks for modeling different components of the system such as sensors, actuators, controllers, and plant models.
   (d) The software provides a wide range of solvers for simulating the system. The user can choose the appropriate solver based on the dynamics of the system.
   (e) The software provides a wide range of training options for learning how to use the software. The user can take online courses, attend workshops, and get certified in using the software.
2. Disadvantages:
   (a) The learning curve of Simulink is steep. It takes time to learn how to use the software effectively. The user needs to have a good understanding of the system dynamics and control theory to model the system accurately.
   (b) The software is expensive. The user needs to purchase a license to use the software. The cost of the software can be a barrier for small companies and individuals.
   (c) The software is not open-source. The user cannot modify the source code of the software. The user is limited to the features provided by the software.
   (d) The software is not platform-independent.

7.1 Limitations

The limitations of the simulation of the rollercoaster-cart over a track :

1. The simulation does not consider the effect of environmental factors such as temperature, humidity e.t.c. These factors can affect the friction.
2. The simulation does not consider the effect of vibrations on the cart. Vibrations can affect the motion of the cart over the track.
3. The simulation does not consider the whole rollercoaster track. It only considers a small section of the track. The simulation can be extended to consider the whole track.

8 Conclusion

In conclusion, the simulation of the rollercoaster-cart over a track has been successfully implemented using Matlab/Simulink, Python, and solidworks. The simulation results have been used to analyze the forces acting on the cart and to determine whether the thrust provided by the motor is sufficient to move the cart over the track. The simulation results coincides with the expectations. The results obtained from the simulation can be used as a reference for the development of the rollercoaster system in the real world. With the developement of computing power, we can simulate more complex systems and analyze the perfermance of the system before the actual implementation.

9 References

1. Teng, Rumin, et al. "Dynamic Modeling and Simulation of Roller Coaster." IEEE Xplore, 2010, ieeexplore.ieee.org/abstract/document/5620012. Accessed 21 Apr. 2021.

2. "PMLSM." MathWorks, www.mathworks.com/help/sps/ug/three-phase-pmlsm-drive.html. Accessed 4 Dec. 2024.

10 Appendix

List of all constants and parameters used in the simulation:

Motor Parameters:

- F_{max}: 2000 N (Maximum thrust generated by the motor).
- PM: 0.65 Wb (Permanent magnet flux linkage).
- L_d: 0.01 H (Inductance along the d-axis).
- L_q: 0.01 H (Inductance along the q-axis).
- L_0: 2×10^{-4} H (Zero-sequence axis inductance).
- R_s: 0.6 ohm (Stator resistance).
- Mass: 1000 kg (Mass of the moving part).
- Pitch: 0.1 m (Polar pitch or distance between poles).
- N_p: $\pi/\text{Pitch}\,\text{rad/m}$ (Electrical pole-pair constant).
- Ts: 5×10^{-5} s (Fundamental sample time)
- fsw: 2000 Hz (PMSM drive switching frequency)

- Tsc: 1×10^{-3} s (Sample time for inner control loop)
- Kp_{id}: 10 (Proportional gain id controller)
- Ki_{id}: 1250 (Integrator gain id controller)
- Kp_{iq}: 10 (Proportional gain iq controller)
- Ki_{iq}: 1250 (Integrator gain iq controller)
- Kp_{v}: 250 (Proportional gain velocity controller)
- Ki_{v}: 250 (Integrator gain velocity controller)
- Kp_{p}: 200 (Proportional gain position controller)
- Ki_{p}: 100 (Integrator gain position controller)

Track and Cart Parameters:

- Angle of inclination: 0.523599 rad
- Time for which the thrust is provided: 4 sec
- Density of the air: 1.225 kg/m^3
- Drag constant: 0.28
- Area of the cart subject to air resistance: 1 m^3
- Length of the ramp: 20 m
- Mass of the cart: 1000 kg
- Friction constant: 0.05