# MATRICES & DIFFERENTIAL CALCULUS
## UNIT-I QUESTION BANK

1. Define rank of a matrix.
2. Find the rank of matrix A= $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ by using elementary row operations.
3. Reduce the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ into Echelon form and hence, find its rank.
4. Find the rank of matrix $A = \begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \end{bmatrix}$ by reducing into triangular form.
5. Find k, if the rank of $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & k \end{bmatrix}$ is (i) 2, (ii) 3.
6. Find all values of μ for which the rank of the matrix $A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is 3.
7. Solve $4x - 3y - 9z + 6w = 0, 2x + 3y + 3z + 6w = 0, 4x - 21y - 39z - 6w = 0$.
8. Determine k for which the system of equations $x - ky + z = 0; kx + 3y - kz = 0; 3x + y - z = 0$ has (i)only zero solution (ii) non-zero solution.
9. Test the consistency and solve the system of equations
   x + 2y + 2z = 2; 3x- 2y - z = 5; 2x -5y +3z = -4; x + 4y + 6z = 0.
10. Show that the system of equations $4x + 9y + 3z - 6 = 0; 2x + 3y + z - 2 = 0; 2x + 6y + 2z - 7 = 0$ is inconsistent.
11. For what values of k, the equations $x + y + z = 1; 2x + y + 4z = k; 4x + y + 10z = k^2$ have a solution and solve them completely in each case.
12. Find the values of λ and μ for which the system of equations x + y + z = 3, x + 2y + 2z = 6, x+ λ y + 3z = μ has (i) no solution (ii) a unique solution and (iii) infinite number of solutions.
13. Test for consistency and solve $\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$.
14. Show that the set $A = \{(1,0,0), (1,2,0), (1,2,3)\}$ is linearly independent.
15. Determine whether the vectors $(1\ \ 1\ \ 0\ \ 1), (1\ \ 1\ \ 1\ \ 1), (4\ \ 4\ \ 1\ \ 1), (1\ \ 0\ \ 0\ \ 1)$ are linearly dependent?
16. Apply Gauss elimination method to solve the equation $x + 4y - z = -5; x + y - 6z = -12; 3x - y - z = 4$.
17. Solve the following system of equations(if possible) using Gauss elimination method $\begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$.

18. Solve the following system of equations(if possible) using Gauss elimination method
$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}.$$

19. Using Gauss Jordan method, solve $x + \mathrm{y} + \mathrm{z} - 9 = 0; 2\mathrm{x} - 3\mathrm{y} + 4\mathrm{z} - 13 = 0; 3\mathrm{x} + 4\mathrm{y} + 5\mathrm{z} - 40 = 0.$

20. Using Gauss Jordan method, solve $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & 5 \\ 3 & -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 25 \\ 2 \end{bmatrix}.$

## MATRICES & DIFFERENTIAL CALCULUS
## UNIT-II QUESTION BANK

1. Define eigen values & eigen vectors.
2. Define Linear transformation, Orthogonal transformation.
3. Find the characteristic roots and characteristic vectors of $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.
4. Find Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.
5. Find Eigen values and Eigen vectors for the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$. Hence, find the eigen values of $A^5$.
6. Find the sum and product of the eigen values of the following matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$
7. If the sum of the eigenvalues of $A = \begin{bmatrix} 5 & 7 & 3 \\ -2 & k & 5 \\ 0 & 3 & 2 \end{bmatrix}$ is -10, then find k.
8. What are the characteristic roots of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 6 \end{bmatrix}$. Hence, find the characteristic roots of $A^{-1}$.
9. Let a 3x3 matrix A have eigen values 1, 2, -1. Find the trace of the matrix $B = A - A^{-1} + A^2$.
10. State Caley Hamilton theorem.
11. Verify Caley Hamilton theorem, and find the inverse of A= $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.
12. Verify Cayley Hamilton Theorem, hence find $A^{-1}$ for the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$
13. Using Cayley-Hamilton theorem, find $A^8$, if A= $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.
14. Write a matrix for the quadratic form $x^2 - 3xy - 2y^2$.
15. Write a symmetric matrix for the quadratic form $x^2 - 2y^2 + 3z^2$.
16. Reduce the quadratic form $x^2 - 3z^2 + 2xy - 3yz$ into canonical form. Also find it's nature.
17. Reduce the quadratic form Q=2($x^2$+xy+12), into sum of squares. Also find index, signature and nature.
18. Find the quadratic form relating to the symmetric matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$.
19. Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2$ $-2x_2x_3$ $+4x_1x_3$ to canonical form. Hence, find rank, index, signature, and nature.
20. **20.** Find the nature of the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ .

MATRICES & DIFFERENTIAL CALCULUS

UNIT-III QUESTION BANK

1. State the following Theorems
   a. Rolle's Theorem
   b. Lagrange's Mean value Theorem
   c. Cauchy's Mean value Theorem
2. Verify Rolle's Theorem for the following functions:
   a. $\frac{sinx}{e^x}$ $in\ [0,\ \pi]$
   b. $e^x(sinx - cosx)$ $in$ $[\pi/4, 5\pi/4]$
   c. $(x+2)^3(x-3)^4$ $in$ $[-2, 3]$
   d. $x(x+3)e^{-x/2}$ $in$ $[-3, 0]$
   e. $log\left[\frac{x^2+ab}{x(a+b)}\right]$ $in\ [a, b]$
3. It is given that the Rolle's Theorem holds for the function $f(x) = x^3 + bx^2 + cx,$ $1 \le x \le 2\ at\ the\ point\ x = \frac{4}{3}$. Find the values of b and c.
4. Verify Lagrange's Mean Value Theorem for the following functions:
   a. $x(x-1)(x-2)\ in\ \left[0, \frac{1}{2}\right]$
   b. $logx$ $in$ $[1, e]$
   c. $x^3 - 3x - 1$ $in$ $\left[-\frac{11}{7}, \frac{13}{7}\right]$
   d. $x^3 - 6x^2 + 11x - 6$ $in$ $[0,4]$
5. Prove the following by using Lagrange's Mean Value Theorem:
   a. $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$
   b. $\frac{b-a}{\sqrt{1-a^2}} < sin^{-1}b - sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$
   c. $|cosb - cosa| \le |b - a|$
6. Verify Cauchy's Mean Value Theorem for the following functions:
   a. $logx\ and\ \frac{1}{x}$ $in\ [1, e]$
   b. $e^{-x}\ and\ e^x$ $in\ [a, b]$
   c. $sinx\ and\ cosx$ $in\ [a, b]$
7. Find the Taylors's series for the following functions:
   a. $tanx\ about\ x = 0$
   b. $\log(1+x)$ $about\ x = 0$
   c. $sinhx$ $about\ x = 0$
   d. $tan^{-1}x$ $about$ $x = 0$
   e. $sinx\ in\ powers\ of\ (x - \frac{\pi}{2})$
   f. $2x^{3.} + 7x^2 + x - 6$ $in\ powers\ of$ $(x - 2)$
8. Find the radius of curvature of the following curves:
   a. $x^3 + y^3 - 3axy = 0$ $at$ $\left(\frac{3a}{2}, \frac{3a}{2}\right)$
   b. $x^2 + y^2 = a^2\ at\ (x, y)$
   c. $ay^2 = x^3$ $at$ $(a, a)$
   d. $xy^2 = a^3 - x^3$ $at\ (a,\ 0)$
   e. $\sqrt{x} + \sqrt{y} = \sqrt{a}$ $at$ $\left(\frac{a}{4}, \frac{a}{4}\right)$
9. Find the center of curvature of the following curves:
   a. $y^2 = 4ax$

b. $y = \tan x \; at \; (\frac{\pi}{2}, 1)$

10. Find the equation of circle of curvature of the following curves:

a. $\sqrt{x} + \sqrt{y} = \sqrt{a} \quad at \quad \left(\frac{a}{4}, \frac{a}{4}\right)$

b. $xy = 1 \; at \; (1,1)$

c. $y = e^x \; at \; (0,1)$

MDC UNIT-IV

Question Bank

(1) Evaluate $\underset{(x,y)\to(0,0)}{Lt} \frac{xy}{x^2+y^2}$

(2) Determine $\underset{(x,y)\to(1,1)}{\text{Lt}} \frac{x(y-1)}{y(x-1)}$

(3) Show that $\underset{(x,y)\to(2,1)}{Lt}(x^2+2x-y^2)=7$

(4) If $f(x,y)=\frac{x-y}{2x+y}$, show that $\underset{x\to 0}{\text{Lt}}[\underset{y\to 0}{\text{Lt}} f(x,y)] \neq \underset{y\to 0}{\text{Lt}}[\underset{x\to 0}{\text{Lt}} f(x,y)]$

(5) Evaluate first order partial derivatives for

(i) $z=x^3+y^3-3axy$

(ii) $z=x^2y-x\sin xy$

(6) If $u=\text{x}\log|xy|$ where $x^3+y^3+3xy=1$, find $\frac{du}{dx}$

(7) If $u=F(x-y,y-z,z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

(8) If $z=f(x,y), x=e^{2u}+e^{-2v}, y=e^{-2u}+e^{2v}$ then show that

$$\frac{\partial f}{\partial u}-\frac{\partial f}{\partial v}=2\left[x\frac{\partial f}{\partial x}-y\frac{\partial f}{\partial y}\right]$$

(9) Find $\frac{dF}{dt}$ at $t=0$ when $F(x,y)=x\cos y+e^x\sin y$

$$x=t^2+1, y=t^3+t$$

(10) If $u=x+3y^2-z^3, v=4x^2yz, \omega=2x^2-xy$ Evaluate $\frac{\partial(u,v,\omega)}{\partial(x,y,z)}$ at $(1,-1,0)$

(11) Show that the following function is continuous at the point (0,0)

$$f(x,y)=\begin{cases}\frac{2x^4+3y^4}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(12) Show that the following function is discontinuous at the $point$ (0,0)

$$f(x,y)=\begin{cases}\frac{x-y}{x+y}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(13) Show that the function $f(x,y)=\begin{cases}\frac{xy}{x^2+2y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

is not continuous at $(0,0)$ but its partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exists at $(0,0)$

(14) Show that the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{x-y}, (x,y) \neq (0,0) \\ 0 \quad , \ (x,y) = (0,0) \end{cases}$ Possess first order partial derivatives at (0,0) though it is not continuous at (0,0)

(15) If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

(16) If $f(x,y,z) = x^3 + xz^2 + y^3 + xyz$

$x = e^t, y = \cos t, z = t^3$, find the total derivative $\frac{df}{dt}$ at $t = 0$.

(17) If $u = x^2 - y^2, v = 2xy$ & $x = r\cos\theta, y = r\sin\theta$

find $\frac{\partial(u,v)}{\partial(r,\theta)}$

(18) If $u = x^2 + y^2 + z^2, v = xy + yz + zx, \omega = x + y + z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(19) Discuss the maxima and minima of $f(x,y) = x^3y^2(1 - x - y)$

(20) Examine the following function for extreme values

$f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

(21) Find the maximum value of $f(x,y,z) = x^2y^3z^4$ Subject to the condition

$2x + 3y + 4z = 18$

(22) Given $x + y + z = a$, Find the maximum value of $x^my^nz^p$

(23) Using Lagrange's method of multipliers find the maximum distance of the point $(3,4,12)$ from the Sphere $x^2 + y^2 + z^2 = 1$

(24) Find the shortest distance between the line $y = 10 - 2x$, the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(25) Find the maximum of minimum values of

(i) $x^3 + y^3 - 3axy$

(ii) $2(x^2 - y^2) - x^4 + y^4$

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**MDC UNIT-5**

**QUESTION BANK**

1. **Find** $\int_1^2 \int_3^4 (xy + e^y)$ **dydx**

2. **Find** $\int_0^5 \int_0^{x^2} x(x^2 + y^2)$ **dydx**

3. **Find** $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)$ **dydx**

4. **Evaluate:** $\int_0^1 \int_0^x e^{\frac{x}{y}}$ **dydx**

5. **Evaluate:** $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$

6. **Evaluate:** $\int\int xy$ **dxdy over the positive quadrant of the circle** $x^2+y^2 = a^2$

7. **Evaluate:** $\int\int (x+y)^2$ **dxdy over the area bounded by the ellipse** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

8. **Evaluate:** $\int\int xy(x+y)$ **dxdy over the area between** $y=x^2$ **and** $y=x$

9. **Evaluate:** $\int \int_A xy$ **dxdy ,where A is the domain bounded by x-axis and the curve** $x^2= 4ay$

10. **Evaluate:** $\int\int_R x^2$ **dxdy where R is the region in the first qauadrant bounded by the lines X=y, y=0, x=8 and the curve xy=16**

11. **Evaluate:** $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}$ **dxdy by changing to polar coordinates**

12. **Evaluate:** $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)$ **dydx by changing to polar coordinates**

13. **Evaluate:** $\int_0^2 \int_0^{\sqrt{(2x-x^2)}} \frac{xdxdy}{x^2+y^2}$ **by changing to polar coordinates**

14. **Evaluate:** $\int_0^{4a} \int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2}$ **dxdy by changing to polar coordinates**

15. **Evaluate:** $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)$ **dxdydz**

16. **Evaluate:** $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2)$ **dxdydz**

17. **Evaluate:** $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)$ **dxdydz**

18. **Evaluate:** $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz$ **dxdydz**

19. **Evaluate:** $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$

**20. Evaluate:** $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}$ **dzdydx**

**21. Evaluate:** $\int_0^{log2} \int_0^x \int_0^{x+logy} e^{x+y+z}$ **dxdydz**

**22. Evaluate:** $\int_1^e \int_1^{logy} \int_1^{e^x} logz$ **dzdxdy**