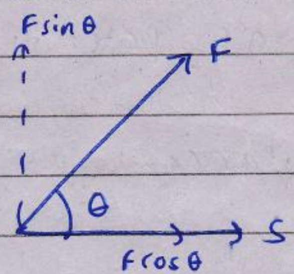
June 2022

WORK, ENERGY AND POWER

Work done is the product of force and displacement of force in the direction of force.

When,

i) $0° \leq \theta \leq 90°$, $\cos\theta \to +ve$, $W \to +ve$



Here F sinθ does no work as it is perpendicular to the displacement.

ii) $\theta = 90°$, $\cos\theta \to 0$, $W = 0$

iii) $90° \leq \theta \leq 180°$, $\cos\theta \to -ve$, $W \to -ve$.

Work done is the transformation of energy.

Work done by a variable force.

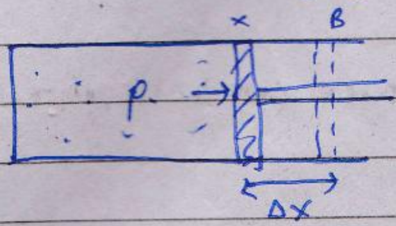
$F_{avg} = \frac{0 + F}{2} = \frac{F}{2}$

$W = F_{av} \times s = \frac{1}{2} Fe$ (When F changes linearly)

However this formula fails when ~~F changes unifor~~ rate of change of F is ununiform. i.e. presence of jerk, $\frac{da}{dt}$ in velocity.

$dW = F \cdot ds$ $W = \int_0^s F\, ds$

Work done by gas.



When the gas is slowly heated, it transfers that energy to the work it does to expand and move the piston.

$\Delta W = F \times \Delta x = PA\Delta x$ where P is ~~press~~ gas pressure and A is area of piston.

The piston moves as pressure inside tends to exceed pressure outside but doesn't exceed as piston actually moves due to tendency and lack of friction.

ΔV = change in volume. $= A \times \Delta x$

$\therefore \Delta W = P.\Delta V$

$dW = P\,dV$

$\therefore W = \int_{V_1}^{V_2} P\,dV$

Where Pressure can be expressed in terms of volume.

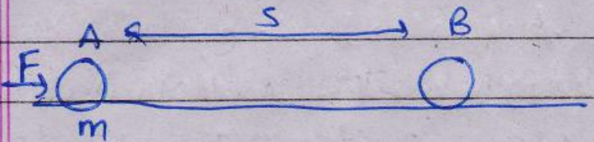
ENERGY

Energy is the ability/capacity to do work. May or may not be sufficient.

Kinetic energy is the energy possesed by a body by virtue of its motion.

Potential energy is the energy possesed by a body by virtue of its position (configuration

Derivation of KE



$u = 0$

Work done on body by constant net force F from A to B is, $W = Fs$, $= mas$.

$v^2 = u^2 + 2as$

or, $v^2 = 2as$

or, $as = \frac{1}{2}v^2$

$\therefore W = \frac{1}{2}mv^2$.

$v^2 = u^2 + 2as$

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mas$

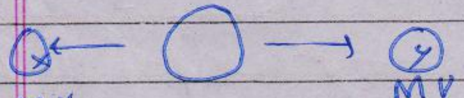
$\therefore W = \frac{1}{2}m(v^2 - u^2)$

$\therefore$ Gain in KE = work done from A to B.

$W = E_{KB} - E_{KA}$ ← work-energy theorem

Work done by a resultant / net force on a body is equal to the gain in KE of the body.

KE of exploding body



$mv + MV = 0$ $\quad mv = -MV$ $\quad \therefore P_x = -P_y$.

$E_x = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = P_x^2/2m$

$E_y = P_y^2/2M$

$\frac{E_y}{E_x} = \frac{P_y^2}{2M} \times \frac{2m}{P_x^2} = \frac{m}{M}$.

$\therefore E_x \propto \frac{1}{m_x}$, $E_y \propto \frac{1}{m_y}$.

$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{m}{M}$$

or, $\varepsilon_x = \frac{\varepsilon_y M}{m}$

or, $\varepsilon_x m = \varepsilon M - \varepsilon_x M$

or, $\varepsilon_x (m+M) = \varepsilon M$

$\therefore \varepsilon_x = \frac{\varepsilon M}{m+M}$ // $\varepsilon_y = \frac{\varepsilon m}{m+M}$ //

Potential Energy

Force is the cause of potential energy.
Energy possesed by a body by virtue of its position or its configuration.

Gravitational PE
→ Energy stored in a body because of gravitational force acting on the body in the gravitational field.
In gravitational field of earth, PE of body is joint property of body-earth system.

$F = mg$ → gravitational force.
The work done against gravitational force from low height to high height:

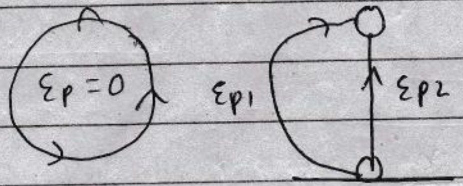
$W = Fs$

or, $W = F \Delta h$

$\therefore W = mg\Delta h$ //

Work done is equal to change in gravitational PE.
In uniform field, g is assumed constant.

In the grav. field work done by/on the body is independent of the path followed. Such a force is ~~o~~ called conservative force, work done round the trip is zero.



$E_{p1} = E_{p2}$.

For instance: Frictional force is non-conservative force.

Relation between force and PE

$\Delta E_p = \Delta W$

$\Delta E_p = -F\Delta h$

$\therefore F = -\dfrac{\Delta E_p}{\Delta h}$, force is negative PE gradient.

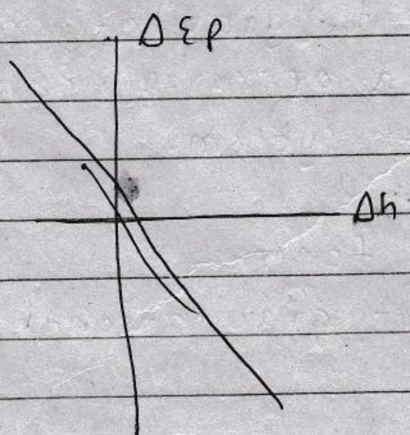
→ Potential is amount of work done per unit mass.

→ Potential energy is amount of work done for any mass.

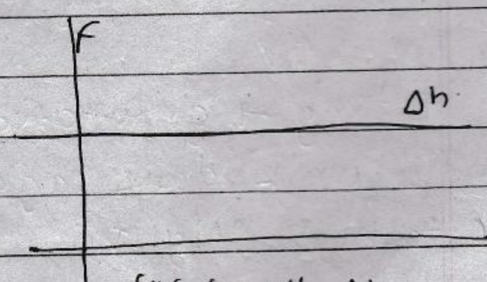
$F = -\dfrac{\Delta E_p}{\Delta h}$

or, $\dfrac{F}{m} = \dfrac{-\Delta E_p}{\Delta h \times m}$

$\therefore g = \dfrac{\Delta V}{\Delta h}$.



Assuming 'g' is constant,



for small Δh.

In the uniform field of conservative force, the force on an influenced particle is numerically equal to -ve potential energy gradient.

Principle of conservation of energy

- Energy can neither be created nor be destroyed, but can only be converted from one form to another.
- Energy transfer is movement of energy from one form to another
- Energy transformation is the change of energy from one form to another.

Conservation of Mechanical Energy

- Sum of kinetic energy and potential energy (mechanical energy) during fall remains constant.

Power

- Work done per unit time
- $P = W/t = Js^{-1}$ "

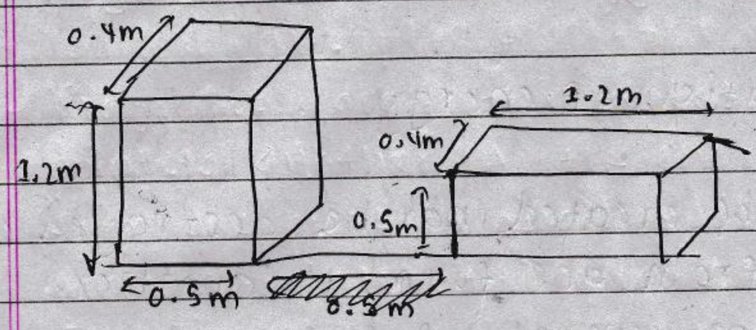
Efficiency

- Measure of how well a device or a system transfers energy into the form we want.
- $\text{Efficiency} = \dfrac{\text{Useful energy / work output / Power output}}{\text{Total energy / work input / power input}}$

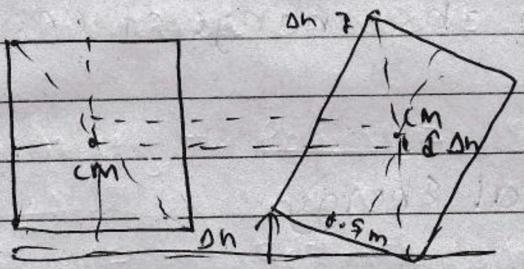
When energy is transferred, some converts into unwanted form called wasted energy.

~~When energy is transferred, some energy turns into unwanted forms. This is wasted energy.~~

#



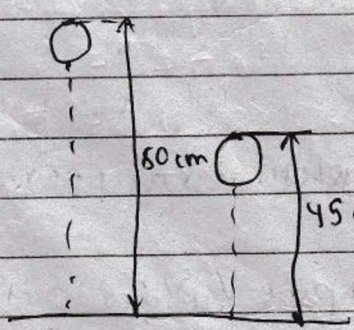
Minimum energy required to roll this. $W = 4000\ N$



We only need to roll block to highest point. Then it rolls by itself.
Min energy = increse in PE.

Diagonal becomes perpendicular.
Diagonal $= \sqrt{1.2^2 + 0.5^2} = 1.30\ m$
$\Delta h = \frac{1}{2}$ Diagonal $- \frac{1}{2}$ length $= \frac{1}{2}(1.3 - 1.2) = 0.05\ m$
$\therefore$ Min energy $= mg\Delta h = 4000 \times 0.05 = 200\ J$

A solid rubber ball has 5.0 cm diameter. It's released from rest. It falls vertically and bounces up.

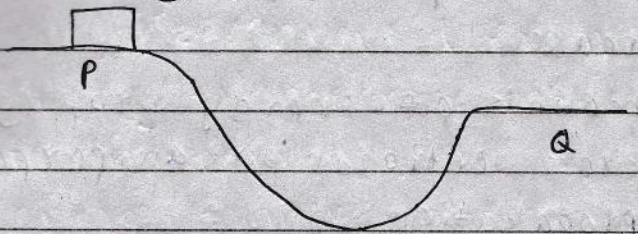


KE_{ball} before striking $= 0.75\ J$, what is KE just after it leaves ground.
$v = \sqrt{2gh} = 3.79\ ms^{-1}$
$\frac{1}{2}mv^2 = 0.75$
$\therefore m = 0.10\ kg$
$\frac{1}{2}mv^2 = mgh$ (h = ~~45 cm~~ 45 - 5 cm)
$\therefore KE = 0.385\ J$

Alternatively, we can use ratio of distances.

$$KE = \frac{45-8}{80-8} \times 0.75 = 0.385 \text{ J}$$

A trolley runs from P to Q along a track. At Q its PE is 50 kJ less than at P. At P KE = 5 kJ. Betn P and Q work trolley does against friction is 10 kJ. Find KE at Q.



Loss in PE = Gain in KE + work against friction

$50 = KE_Q - KE_P + 10$

$50 = KE_Q - 5 + 10$

$\therefore KE_Q = 45 \text{ kJ}$

A turbine has blades sweeping $2000 m^2$. Converts power in wind with 50% efficiency. What is electrical power if wind speed is $10 ms^{-1}$ when $\rho_{air} = 1.3 \text{ kg m}^{-3}$

Power wind = 2 Power output

$\pi r^2 = 2000$

$\therefore r = 25.2$ m

$\therefore 2\pi r = 158.5$ m

At $10 ms^{-1}$, $t = 15.85$ s

$\cancel{AdV =}$ $A.ds = V$

$A.\frac{ds}{dt} = \frac{m}{\rho . dt}$

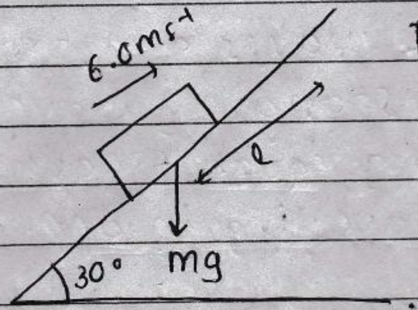
$A V \times \rho \times dt = m$

$\therefore m = 412100$ kg

$$P_{wind} = \frac{\text{Work wind}}{\text{Time}} = \frac{\frac{1}{2} m v^2}{15.85} = \frac{\frac{1}{2} \times 412100 \times 100}{15.85} = 1.3 \times 10^6 \text{ W}$$

$\therefore P_{out} = 6.5 \times 10^5$ W

WORK, ENERGY, POWER...



The percent of power from engine used to raise train is 40%. What is train power?

$M = 94000\ kg$

$KE = \frac{1}{2}mv^2 = 1692000\ J$

$PE = mgh = 94000 \times 9.81 \times h$

~~Gain in ME =~~

~~work done~~ by engine ~~= PE - KE~~

Power out $= mgh/t$

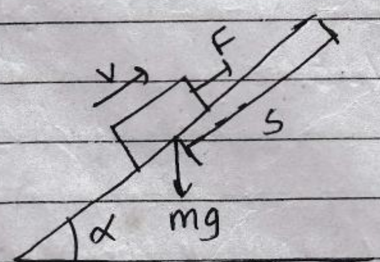
$P_{out} = mgl\sin\theta/t = mgv\sin\theta$

$\therefore P_{out} = 2766420\ W.$

40% of η = 2766420

~~$\therefore \eta = P_o$~~

$\therefore$ Power of engine $= 6916050\ W =$ ~~6.91 MW~~ $6.92\ MW$



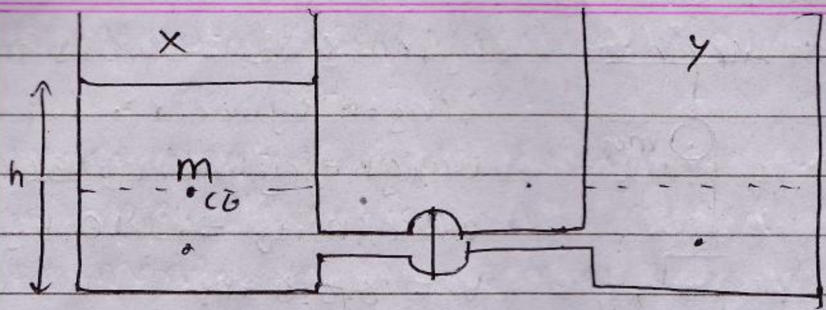
* Find efficiency of the process.

$W_{in} = Fs$

$W_{out} = mgh = mgs\sin\alpha$

$$\eta = \frac{W_{out}}{W_{in}} = \frac{mgs\sin\alpha}{Fs} = \frac{mg\sin\alpha}{F}$$

#



When value is opened, water moves from X to Y until depths are equal. How much PE is lost by water in this process.

$\cancel{PE}_{\cancel{lost}} = mgh - \cancel{mg}\,{}^{h}/_{2} \neq {}^{mgh}/_{2}$ J.

~~PE lost~~

Initial PE $= {}^{mgh}/_{2}$

Final PE $= \cancel{{}^{mgh}/_{4}}$ $\quad {}^{m}/_{2} \cdot g \cdot {}^{h}/_{4} + {}^{m}/_{2} \cdot g \cdot {}^{h}/_{4} = {}^{mgh}/_{4}$

$\therefore$ PE lost $= {}^{mgh}/_{2} - {}^{mgh}/_{4} = {}^{mgh}/_{4}$ J.

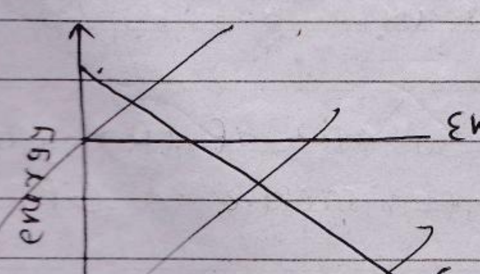
The forward motion of a boat is opposed by F, where $F = kv^2$. Effective power to maintain v is P. Relate k, P and v.

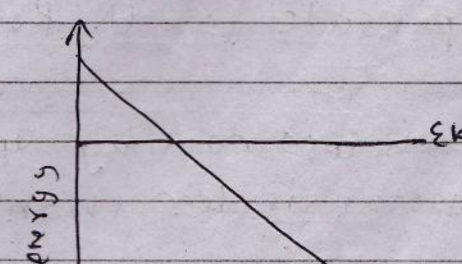
$P = Fv$

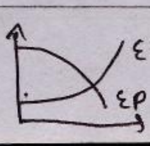
$P = kv^3$

$\therefore k = {}^{P}/_{v^3}$

Steel ball is falling in oil at constant speed. show variation of GPE and KE.

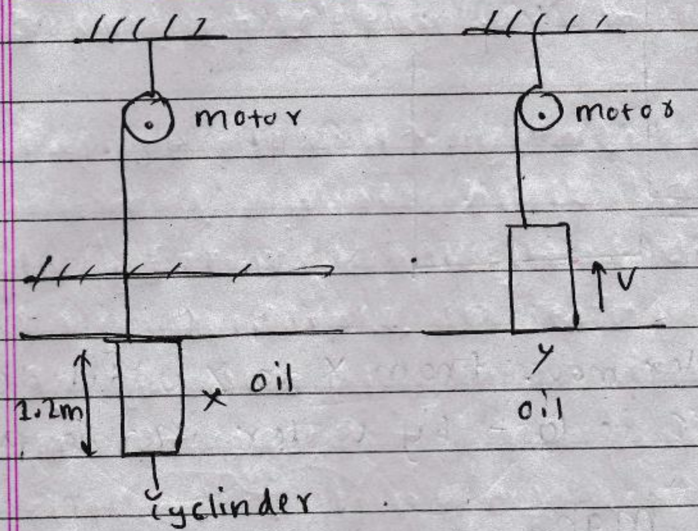






t in free fall.

Date ______
Page ______



Cylinder CSA = $0.018\ m^2$
Length cylinder = 1.2 m
Weight cylinder = 560N
oil density = $940\ kg\,m^{-3}$.
$v = 0.020\ ms^{-1}$.
Viscous force is negligible.

ρ cylinder = ?

$\rho = m/V = W/g/Al = 2640\ kg\,m^{-3}$.

For cylinder at X, calculate upthrust, and tension in wire.
Calculate power output of motor

$U = dgV = \cancel{1\mathcal{}}\ 940 \times 9.81 \times 1.2 \times 0.018 \approx 200\ N$

$T + U = W$

$\therefore T = 360\ N$.

Power $= FV = 360 \times 0.02 = 7.2\ W$.

State and explain variation in power output as cylinder is raised.

- Volume of liquid displaced decreases. So U decreases, and thus for V, motor does extra work to compensate for U. Power increases.

- Work is also done by upthrust so rate of energy output of motor is less than rate of increase in GPE.