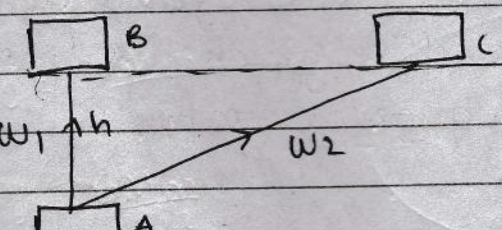
GRAVITATION FIELD

A region of space throughout which the non-contact force (force from distance) acts on different entities like mass, charge, magnetic poles etc. is called field of force.

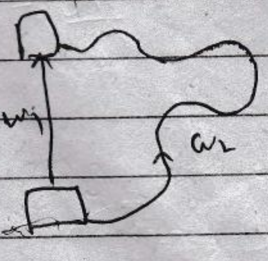
There are different entities (detectors) for different fields.

Gravitational field strength at a point inside a gravitational field is defined as gravitational force per unit mass at that point.

All entities in a field obey the inverse square law, which states that $F \propto 1/d^2$.
These forces are conservative in nature.
The forces don't depend on path followed.



No matter what be the path, the energy gained by the object is mgh regardless of distance. This is conservative.



In this case, the object is moving at ground against friction.
Here friction is non conservative as $w_1 \neq w_2$.

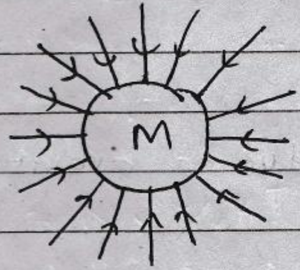
$$F = \frac{GMm}{r^2}$$

$\therefore$ $F/m = GM/r^2$

$\therefore$ $\varepsilon_g = GM/r^2$

This is gravitational field intensity, at a point situated at a distance r from the ~~large mass M.~~ mass M.

Gravitational force is always attractive in nature as negative mass doesn't exist.

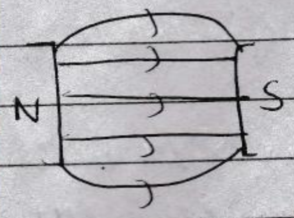


Gravitational field lines
- Imaginary path
- Never intersect each other.

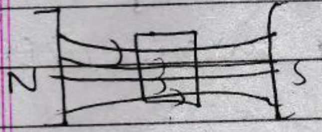
Dense lines of gravitational force suggests strong gravitational field.

Point mass means total mass of spherical object is assumed to concentrate at a point. (the center)
The distance is always measured from the centre of mass of an object.

The value of F_g betn two unit masses which are separated by a unit distance is equal to 6.67×10^{-11} N.
This is called universal gravitational constant. (G).
This force is very small and can't overcome frictional/electrostatic force of attraction so it's not noticable for small objects.



These are magnetic lines of force in vacuum. Now if we put a magnetic object in the middle, it provides easy path for lines of force and they concentrate



$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11})\times(6.0\times10^{24})}{(6400\times1000)^2} \approx 9.8\ ms^{-2}$$

Acceleration due to gravity (g)

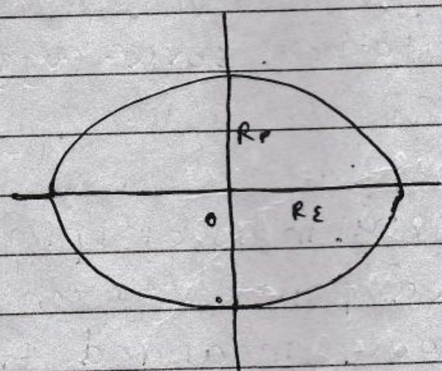
Rate of change in velocity of a body due to gravitational force of attraction.

$g = GM/r^2$

$F_g = \frac{GMm}{r^2}$

$\cancel{m}g = \frac{GM\cancel{m}}{r^2}$

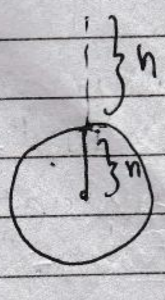
$\therefore g = \frac{GM}{r^2}$



$R_P < R_E$

$\therefore$ ~~G_P~~ $g_P > g_E$

$\frac{g_P}{g_E} = \left(\frac{R_E}{R_P}\right)^2$



$g' = \frac{GM}{(R+h)^2}$ $\quad g' = \frac{GM}{(R-n)^2}$

Here h is height from surface and n is depth from earth's surface towards the centre.

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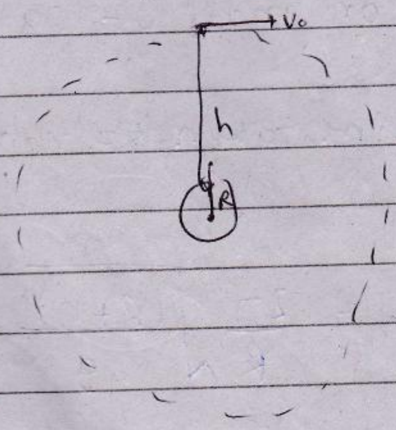
Geostationary Orbit

It is the orbit of satellite which is appeared as a rest object though it is revolving around the earth. The cause behind it is it's period equal to period taken by earth to make one compete rotation.

- Has a period of 24 hours.
- lies in equatorial plane (above the equator)
- Direction of revolution is as that of direction of spinning of earth (west to east).

So this satellite can be used as a permanent station for communication purpose.

Geosyncronous satellite just has a period of 24 hours but nothing else.



$$\frac{m v_0^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$\text{or, } v_0^2 = \frac{GM}{R+h}$$

$$\therefore v_0 = \sqrt{\frac{GM}{R+h}}$$

$$S = 2\pi(R+h)$$

$$S = v_0 T$$

$$\therefore T = \frac{S}{v_0}$$

$$T = \frac{2\pi(R+h)^{3/2}}{\sqrt{GM}}$$

$$\therefore T^2 = \frac{4\pi^2(R+h)^3}{GM}$$

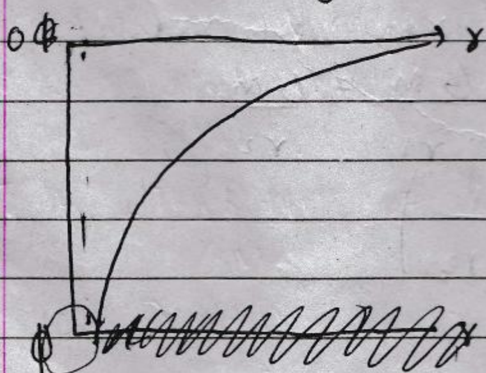
Gravitational potential (ϕ)

Test mass's gravitational field is negligible compared to the stationary mass.
Gravitational potential at a point in a gravitational field is amount of work done per unit mass in bringing the test mass from infinity to that point.

$\phi_p = -\frac{GM}{r}$ SI unit is $J kg^{-1}$

$E_p = \phi m$, where E_p is gravitational potential energy.
Here due to -ve sign, when r increases, ϕ_p also increases.
This is why E_p increases with increse in height.

Max. value of ϕ_p is 0 which happens at infinity.
Sign is -ve, as gravitational force is alwas attractive and thus system is doing work itself.



ϕ doesn't change linearly with 'r' when r is very high, ϕ doesn't really change with Δr, which indicates ϕ field is becoming weaker.

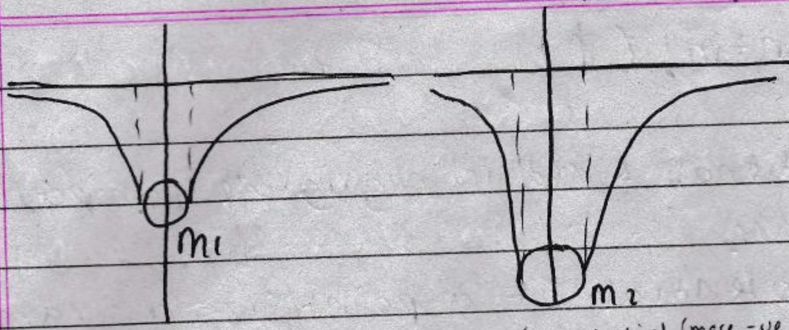
$dW = F.dx$

$dW = \frac{GM}{n^2} dx$

$W = \int_{\infty}^{n} \frac{GM}{n^2} dx$

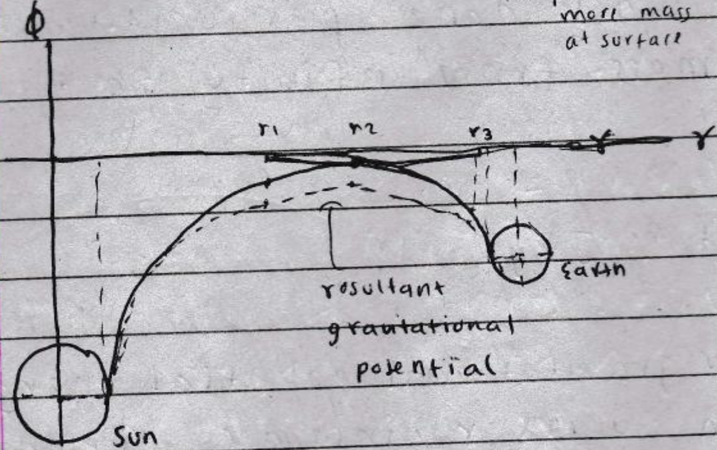
$\therefore W = -\frac{GM}{n}$

$\phi = -\frac{GM}{r}$.



Here $m_2 > m_1$, as value of ϕ is more negative in m_2 i.e. potential well is deeper

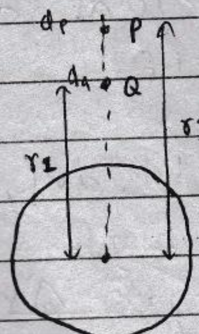
$$\phi = \frac{-GM}{r}$$



Resultant gravitational potential due to two masses.
r_2 is pt. where both masses contribute equal value of gravitational potential

We add the gravitational potentials in points r_1, r_2 & r_3 to find resultant.

Change in gravitational potential ($\Delta\phi$)



$$\Delta\phi = \phi_P - \phi_Q = -\frac{GM}{r_2} - \left(-\frac{GM}{r_1}\right)$$

$$= \cancel{\frac{GM}{r_2} - \frac{GM}{r_1}} \quad \frac{GM}{r_1} - \frac{GM}{r_2}$$

$$= GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

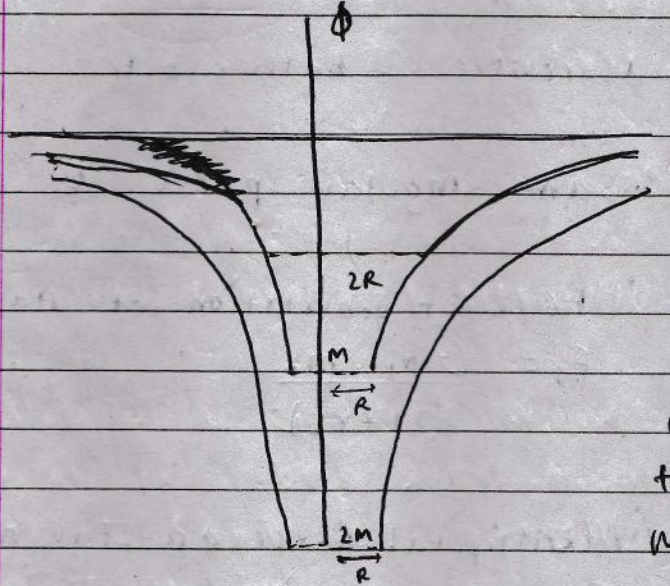
Gravitational Potential Energy (E_p)

Gravitational potential energy of a mass at a point inside the gravitational field is the amount of work done in bringing the mass from infinity to that point

$E_p = \phi . m$

SI unit is Joule (J)

It is also max. in infinity (0) and -ve in other places



When mass is doubled, the potential well is deeper and every ϕ pt. is doubled

When M is constant but R is doubled, at surface ϕ will be 1/2, but after that, it will overlap as M is constant.

$\phi_R = -\frac{GM}{R}$, $\phi_{2R} = -\frac{GM}{2R}$ (At surface)

$\phi_R = -\frac{GM}{5R}$, $\phi_{2R} = -\frac{GM}{5R}$ (At $r = 5R$)

So, it will Overlap.

Every body in the universe attracts another body with a force that is directly proportional....

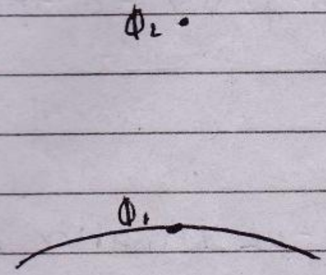
Newton's law of gravitation

Change in gravitational potential energy = Change in KE

or, $\varepsilon p_2 - \varepsilon p_1 = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

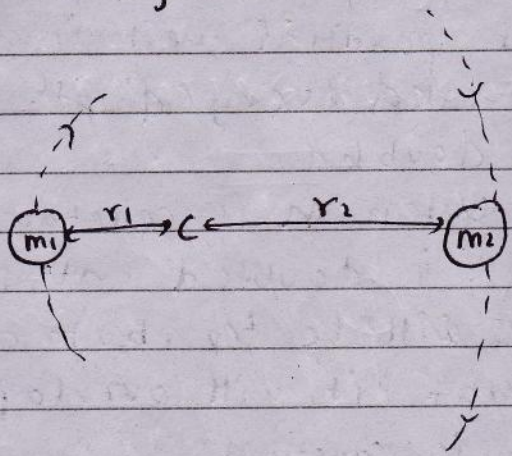
or $\Phi_2 m - \Phi_1 m = \frac{1}{2}m(v^2-u^2)$

$\therefore (\Phi_2 - \Phi_1) = \frac{1}{2}(v^2-u^2)$



To find velocity here, we can't use $v^2 = u^2 + 2as$ as acceleration is ununiform

A binary star where C is centre of both orbits.



Same angular speed ω

i) Force of gravitation betn stars

$F_g = \frac{G\, m_1 . m_2}{(r_1+r_2)^2}$

ii) Centripetal force on star m.

$F_c = m_1 . r_1 . \omega^2 \quad (m_1)$

$F_c = m_2 r_2 \omega^2 \quad (m_2)$

$\frac{F_c'}{F_c} = \frac{m_1 r_1 \omega^{2\prime}}{m_2 r_2 \omega^2}$

Centripetal force is provided by gravitational force.

or $m_2 r_2 = m_1 r_1$

$\therefore \frac{m_1}{m_2} = \frac{r_2}{r_1}$

$r_1 + r_2 = 3.2 \times 10^{11}$

or, $r_1 + 3r_1 = 3.2 \times 10^{11}$

$\frac{r_2}{r_1} = 3 \quad r_2 = 3r_1$

$\therefore r_1 = 8.0 \times 10^{10}$ m

$\therefore r_2 = \cancel{2.7 \times 10^{10}\ m}\ 2.4 \times 10^{11}$ m

$$\frac{G\,\cancel{m_1}.m_2}{(r_1+r_2)^2} = \cancel{m_1} r_1 \omega^2$$

$\therefore (6.51 \times 10^{-34})\, m_2 = 1.98 \times 10^{-4}$

$\therefore m_2 = 3.05 \times 10^{29}$ kg.

$\therefore m_1 = 9.15 \times 10^{29}$ kg