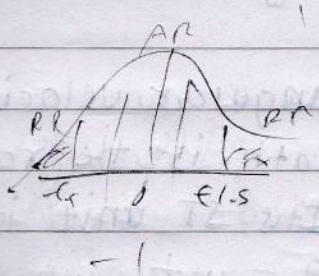
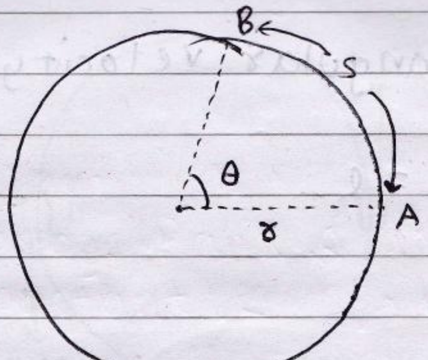
MOTION IN A CIRCULAR PATH



- Wheels of a car or a bicycle
- Hands of clock

Angular displacement

6cm



The angle θ through which the object has moved is known as the angular displacement.
Angle between line joining initial and final position with center, in a circle.
SI of angular displacement is radian

Angular displacement, $\theta = l/r$, where l is arc length and r is radius.
When $l = 2\pi r$ (one complete turn)
$\theta = 2\pi r / r = 2\pi$ rad. (360°)

When arc length = radius = r
$\theta = r/r = 1$ radian
So, one radian can be defined as the angle subtended at the center of a circle by an arc of length equal to its radius.
1 radian $= 360/2\pi = 57.3^\circ$.

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Angular velocity (ω)

- Angular velocity of an object moving in circular path is the rate of change of angular displacement.
- Its SI unit is radian/s.
- $\theta = \omega t$, where θ = angular displacement

Relation between linear & angular velocity.

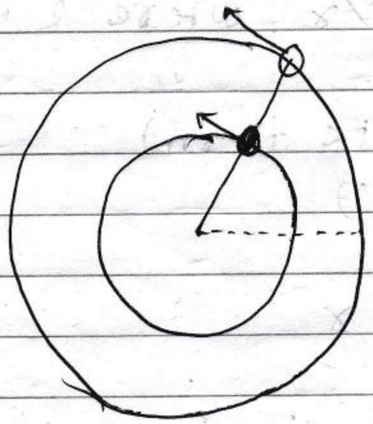
We have, $\theta = l/r$ and $r\theta = l$

$\frac{r\theta}{t} = \frac{l}{t}$

or $r\omega = v$

$\therefore v = \omega r$, [velocity = angular velocity × radius]

$\therefore v \propto r$, when ω is constant.



White particle has higher linear velocity but both have the same angular velocity.

If an object makes f complete turns in 1 seconds.

Then frequency (f) = f Hz

Angular displacement (θ) = $2\pi f^c$

1 complete turn in $1/f$ s. = T

This time taken to complete 1 turn is called Time Period (T)

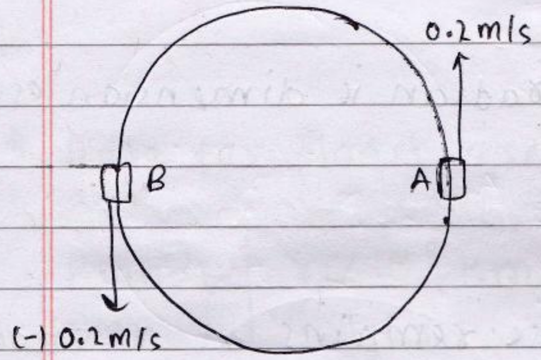
$\omega = 2\pi f/1 = 2\pi f = 2\pi/T$

Calculate angular velocity of second hand, minute hand and hour hand.

Second hand = $^{2\pi}/_{60}$ rad/s = $^{\pi}/_{30}$ rad/s

Minute hand = $^{2\pi}/_{3600}$ rad/s = $^{\pi}/_{1800}$ rad/s

Hour hand = $^{2\pi}/_{12 \times 3600}$ rad/s = $^{\pi}/_{21600}$ rad/s.



from B to A,

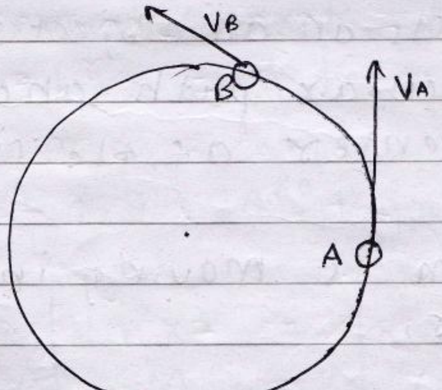
change in speed = 0

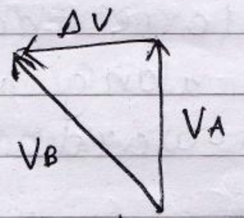
change in velocity = $0.2 - (-0.2)$

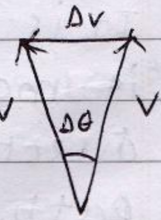
$= 0.4$ m/s

Acceleration of particle moving in circular path.

- When particle is moving with steady speed in circular path, it is moving with variable velocity.
- Then, there must be acceleration.







$\overrightarrow{\Delta V} = -\overrightarrow{V_A} + \overrightarrow{V_B}$

$\therefore \overrightarrow{\Delta V} = \overrightarrow{V_B} - \overrightarrow{V_A}$

if $\Delta\theta$ is small this resembles an arc, so $\theta = l/r$

$\Delta\theta = \Delta V / V$

If A and B are very close to each other, then $\overrightarrow{\Delta V}$ is towards the center. ($\overrightarrow{\Delta V}$ is normal to $\overrightarrow{V_A}$).

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acceleration $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ [$\vec{a}$ and $\Delta \vec{v}$ have same directio (towards centre

$\Delta\theta = \frac{\Delta v}{v}$, for small change in velocity

$\therefore \Delta v = \Delta\theta . v$

$\therefore \vec{a} = v\frac{\Delta\theta}{\Delta t} = v\omega$ [$v = \omega r$]

$\therefore \vec{a} = v^2/r$, $\vec{a} = \omega^2 r$

+ SI unit is m/s^2. because radian is dimensionless

Centripetal force

+ Every object in the universe remains at rest or uniform motion in a straight line unless net force is applied.
+ When an object moves, at a given instant some force acts perpendicular to the velocity vector of the object which causes the object to move in circular motion. This is centripetal force.
+ It is inversely proportional to radius.
+ It is net force that acts on an object to keep it moving along a circular path and is directed towards the center of the circular path.
+ If an object of mass m is moving in a circular path of radius r,

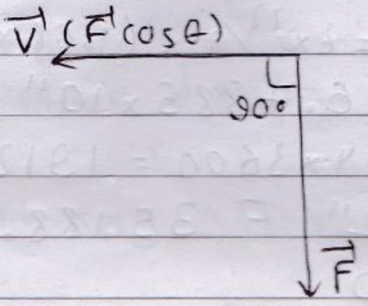
$F = ma$

$F = \frac{mv^2}{r}$, $m\omega^2 r$

centripetal force = tension on string.

$T = \frac{mv^2}{r}$

+ In solar system, centripretal force = gravitational force. $= GMm/r^2$



~~Resultant~~ Component of force along velocity will be $\vec{F}\cos\theta$

$F\cos 90 = 0$ ..

$\therefore$ $\vec{F}$ can't change magnitude of $\vec{v}$, but only direction

\# Data for plaret are given.

Planet	$r/10^8$ km	T/years
Venus	1.08	0.615
Neptune	45.0	

i) Calculate value of T for neptune.

$$T^2 = \frac{4\pi^2}{GM} r^3$$

or, $$\left(\frac{0.615}{12\times30\times24\times60\times60}\right)^2 = \frac{4\times(22/7)^2}{(6.67\times10^{-11})\times M} \times (1.08\times10^8\times1000)^3$$

or, $$(3.909\times10^{-16}) = \frac{5.923\times10^{11}}{M} \times 1.259\times10^{33}$$

$\therefore M = 1.9\times10^{60}$ kg ..

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\times(22/7)^2\times(45\times10^8\times1000)^3}{(6.67\times10^{-11})\times(1.9\times10^{60})}}$$

$= \sqrt{2.84\times10^{-11}} = 5.33\times10^{-6}$

$= 165.78$ years.

$= 166$ years ..

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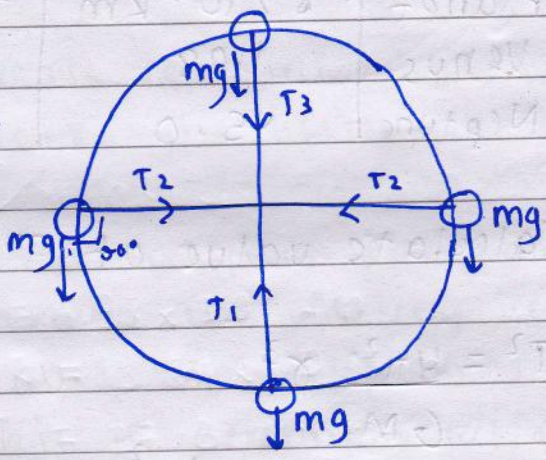
ii) Determine linear speed of venus in its orbit.

Circumference of orbit $= 2\pi r = 2 \times 22/7 \times (1.08 \times 10^{11})$
$= 6.7885 \times 10^{11}$ m

Time taken $= 0.615 \times 12 \times 30 \times 24 \times 3600 = 19128960$ s

$\therefore$ Linear speed $= \frac{6.7885 \times 10^{11}}{19128960} = 35488.07$ m/s
$= 35.4$ km/s
$= 35$ km/s

Object rotating in vertical plane tied on a rope



i) At lowest point
Net force towards center = centripetal force.

$T_1 - mg = mv^2/r$
$\therefore T_1 = mv^2/r + mg$

ii) At highest point
Net force towards center = centripetal force.
$\cancel{T_3}$
$T_3 + mg = mv^2/r$
$\therefore T_3 = mv^2/r - mg$

iii) At mid of vertical plane
Net force towards center = centripetal force

$mg = T_2 \cos 90°$ $\quad \therefore mg = 0$

$\therefore T_2 = \frac{mv^2}{r}$

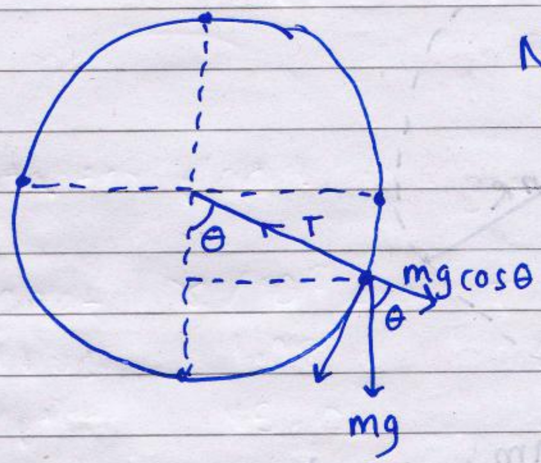
For object to be barely in circular path,

$T_3 = 0$

or, $mg = mv^2/r$

$\therefore v_{min} = \sqrt{rg}$

iv) At any other point



Net force towards centre = centripetal force.

$T - mg\cos\theta = mv^2/r$

$\therefore T = mv^2/r + mg\cos\theta$

Lowest tension will be experienced at highest pt

Highest tension will be experienced at lowest pt.

Car moving on a curved road

\# Earth has radius R and density ρ. Moon has mass m, as shown. The moon makes one complete orbit in T time. Show that ρ is given by.

$$\rho = \frac{3\pi n^3}{GT^2}$$

Centripetal force $= GMm/(nR)^2$

$$\frac{\cancel{m}v^2}{nR} = \frac{GM\cancel{m}}{n^2R^2}$$

or, $v^2 = \dfrac{GM}{nR}$

or, $\left(\dfrac{2\pi nR}{T}\right)^2 = \dfrac{GM}{nR}$

or, $\dfrac{4\pi^2 (nR)^2}{T^2} = \dfrac{GM}{nR}$

or, $\dfrac{4\pi^2}{T^2} = \dfrac{GM}{n^3R^3}$

or, $\dfrac{4\pi n^3}{GT^2} = \dfrac{M}{\pi R^3}$

or, $\dfrac{4\pi n^3}{\frac{4}{3}GT^2} = \dfrac{M}{\frac{4}{3}\pi R^3}$

or, $\dfrac{3\pi n^3}{GT^2} = \dfrac{M}{V}$

$\therefore \rho = \dfrac{3\pi n^3}{GT^2}$

$R = 6.38 \times 10^3$ km

$nR = 3.84 \times 10^5$ km

$T = 27.3$ days

$\rho = ?$

$$\rho = \frac{3\pi n^3}{GT^2}$$

$$= \frac{3 \times 22/7 \times \left(\frac{3.84 \times 10^8}{6.38 \times 10^6}\right)^3}{(6.67 \times 10^{-11}) \times (27.3 \times 24 \times 3600)^2}$$

$$= \frac{66/7 \times 218037.7225}{371.089}$$

$= 5539.86$

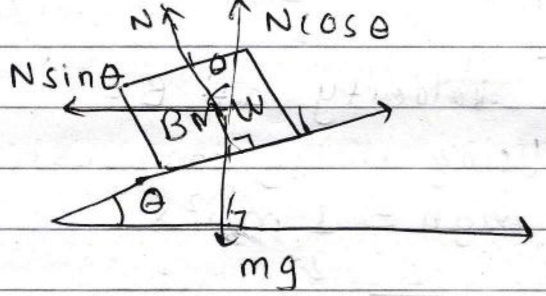
$= 5540 \text{ kgm}^{-3}$

CIRCULAR MOTION...

- For a car moving on curved road, centripetal force arises due to frictional force between road and tyres.
- If $v > \sqrt{\mu r g}$, where μ is coefficient of static friction, tokyo drift occurs.

Banking of road.

- If road is banked / tilted towards center, normal contact force can also help provide some more centripetal force.

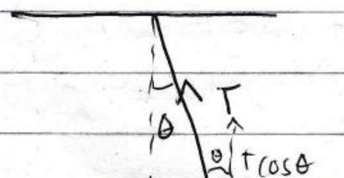


$\frac{mv^2}{r} = N\sin\theta$, $mg = N\cos\theta$

$$\frac{N\sin\theta}{N\cos\theta} = \frac{mv^2/r}{mg}$$

$\tan\theta = v^2/rg$

$\therefore$ ~~v must be high~~ higher v, means higher θ.

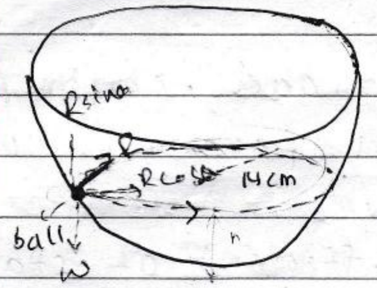


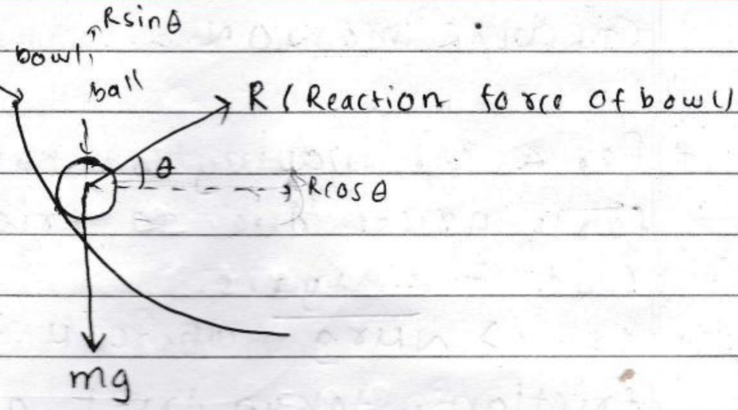
Conical pendulum, in which tension in string provides necessary centripetal force. ($T\sin\theta$)

$T\cos\theta = mg$ $\qquad$ $T\sin\theta = m\omega^2 r$

$T = \frac{mg}{\cos\theta}$ $\qquad$ $\therefore \tan\theta = \frac{\omega^2 r}{g}$

$\omega = 2\pi f, 2\pi/T$



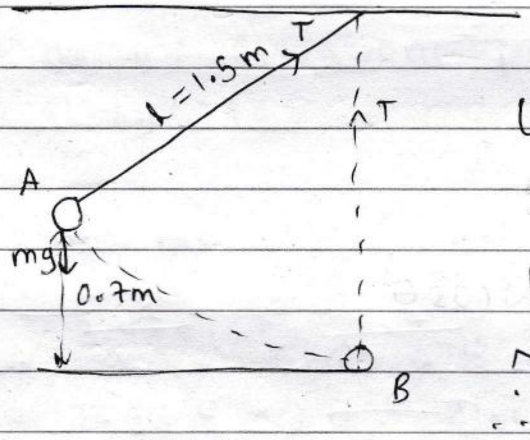


IF F is resultant force, $W = F\tan\theta$.

$R\sin\theta = mg$, $R\cos\theta = F$ $(mv^2/r, m\omega^2 r)$

$\frac{mg}{F} = \tan\theta$

$\therefore W = F\tan\theta$



Velocity at B = ?

Using energy conservation

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$\sqrt{\frac{2gh}{}} = v$

$\therefore v = \cancel{3.03\,m/s}$ 3.7 m/s

Find Tension on string when at point B.

$T = mv^2/r + mg$

$T = \frac{50}{1000} \times 3.7^2 / 1.5 + \frac{50}{1000} \times 9.81$

$\therefore T = \cancel{0.46\,N}$ 0.95 N