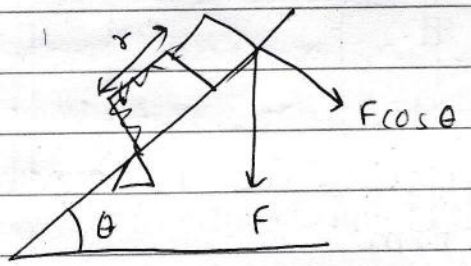
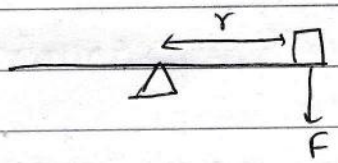
MOMENT OF FORCE

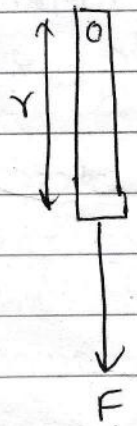
→ Turning effect of force.
→ Depends on magnitude of force and its position & direction.
→ Moment of force about a point / pivot / axis is defined as the product of force and perpendicular distance from the point to the line of action of force (⊥ drawn from surface).
→ Represented by M or τ and its unit is Nm.
→ The force and distance must be perpendicular.



Moment = Frcosθ



Moment = Fr

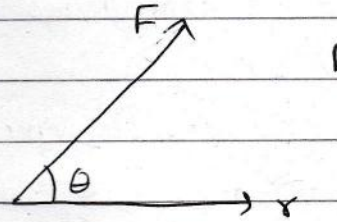


Moment = 0. (Fx0) = 0
This implies F produces no turning effect on the body.

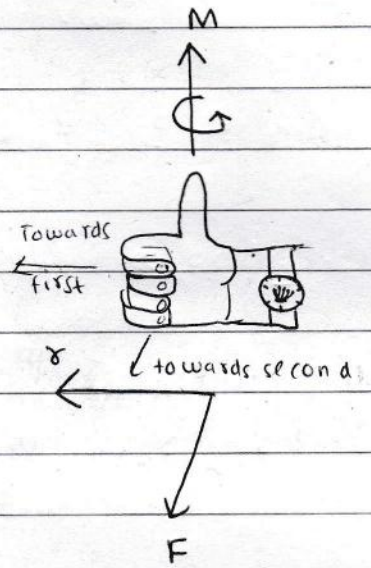
MOMENT OF FORCE....

→ Moment of a force is a vector quantity with the vector formulation as
$\vec{M} = \vec{r} \times \vec{F}$ [cross product]

→ Direction is remembered using right hand rule.



$M = Fr\sin\theta.$

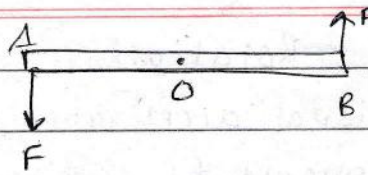


This is right hand thumb rule. Fingers must point along first vector and when curling, fingers must point towards second.

Weight produces no moment about the centre of gravity but produces moment on any other point.

Couple and moment of a couple.

→ Two forces acting on a body at the same time are said to form a couple when the forces are parallel to each other, equal in magnitude, opposite in direction and must be separated by a perpendicular distance, different lines of action.



The resultant force on rod is 0 and there is no resultant linear acceleration.

However, there is presence of rotational acceleration.

→ The effect of a couple on a body is only the rotational motion and not translational / linear.

→ Total moment / turning effect of a couple is called torque of a couple denoted by tau (τ)

Moment about O by force F at A

$M_1 = F \times OA$ → Anticlockwise.

Moment about O by force F at B;

$M_2 = F \times OB$ → Anticlockwise

Total moment of the couple (τ) = $M_1 + M_2$

$\tau = F \times OA + F \times OB$ [Same anticlockwise direction]

$\therefore \tau = F(OA + OB)$

$\therefore \tau = F.AB$ //

τ = force x perpendicular distance between forces.

Equilibrium

→ A body is said to be in equilibrium if it does not have acceleration of translation (linear) or rotation.

→ $a_{rotational} = \frac{dw}{dt}$

Translation	Rotation
→ Linear acceleration zero.	→ Rotational acceleration zero.
→ V should be constant.	→ Δw should be constant.

Static equilibrium is when ~~v is 0~~ V and w are 0 but dynamic equilibrium is when v and w are not 0 but a constant.

For body to be in equilibrium, resultant force and torque acting on a body must be zero.

$\sum F = 0 \quad \sum F_x = \sum F_y = 0$

Clockwise moment = anticlockwise moment. [about same pt]

Principle of moments

→ When a body is in equilibrium under the action of number of forces, the sum of clockwise moments about any point is equal to the sum of anticlockwise moments about the same point.

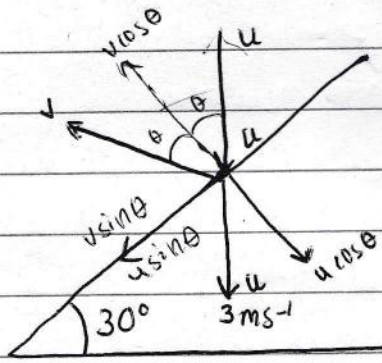
→ Sum of moments of all forces about any point is zero

QUESTIONS

1. The inclined plane is 1.5 m square and rain falls on it with $3ms^{-1}$. The volume of rain collected per min is $2.5 \times 10^{-2} m^3$ and density of water is $10^3 kg\,m^{-3}$. Assuming that velocity becomes 0 after collision find vertical force exerted on roof. The pressure on the roof. If instead the rain were metal spheres that collided elastically with same velocity and same mass per unit time, what would be force & pressure on roof? Angle of slope is 30°.

$\therefore F = \frac{mdv}{dt} = \frac{m}{dt} \times dv = \frac{2.5\times10^{-2}\times10^{3}\times3}{60} = 1.25\ N$

$\therefore P = \frac{F}{A} = \frac{1.25\cos30^\circ}{1.5^2} = 0.48\ Pa$ [1.5 m square means it a square with side 1.5m]



During elastic collision, angle of incidence and reflection are same. i.e. 30°.

$\Delta v = v\cos\theta - (-u\cos\theta)$

$= 2u\cos\theta$

$\therefore F = \frac{mdv}{dt} = \frac{25}{60} \times 2\times3\times\cos30^\circ = 2.16\ N$

$\therefore P = \frac{F}{A} = \frac{2.16}{1.5^2} = 0.96\ Pa$