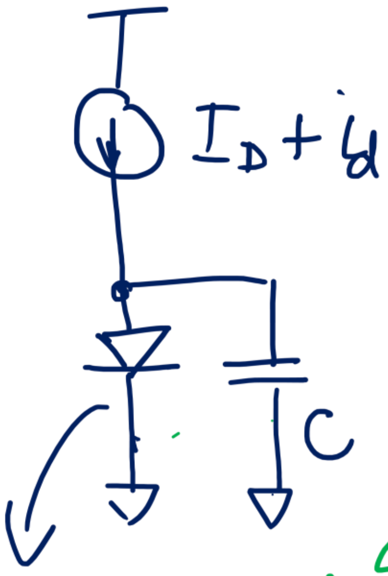
Small-Signal Modeling
Current Mirrors

12 June 2025

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

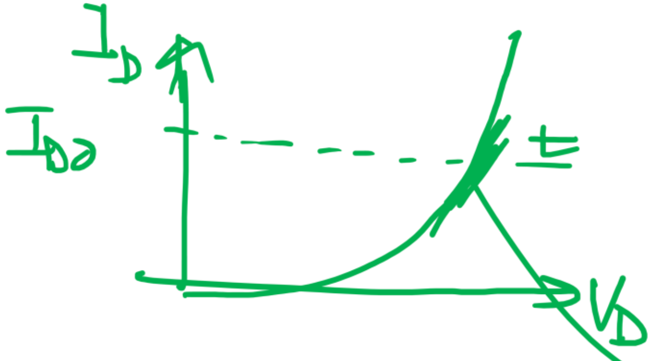
$$|x| \ll 1$$



$$I_{DO} + i_d = I_S \cdot e^{\frac{V_D + v_d}{V_T}} = \overbrace{I_S \cdot e^{V_D/V_T}}^{I_{DO}} \cdot e^{v_d/V_T}$$

$$= I_{DO}\left(1 + \frac{v_d}{V_T}\right) \Rightarrow i_d = I_{DO} \cdot \frac{v_d}{V_T}$$

$$\boxed{v_d = \underbrace{\frac{V_T}{I_{DO}}}_{r_d} \cdot i_d}$$

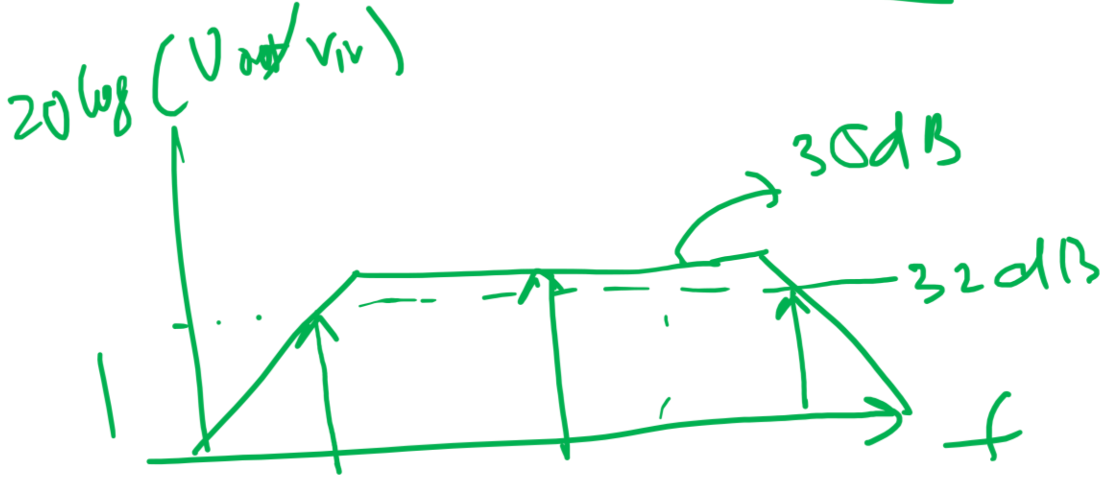


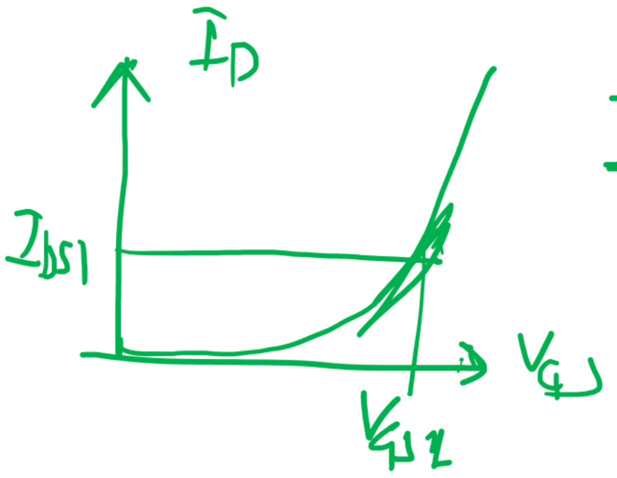
$$I_D \approx I_S \cdot e^{V_D/V_T}$$

$$\approx I_S\left(1 + \frac{V_D}{V_T} + \cancel{\ldots}\right) \ldots$$

v_d/V_T	$I_D = I_s \cdot e^{v_d/V_T}$	$I_D' \Rightarrow I_s\left(1 + v_d/V_T\right)$	error %
0.01			0%
0.1			0.4%
1			26.1%
10			79%

$$= \frac{I_D - I_D'}{I_D} \times 100$$

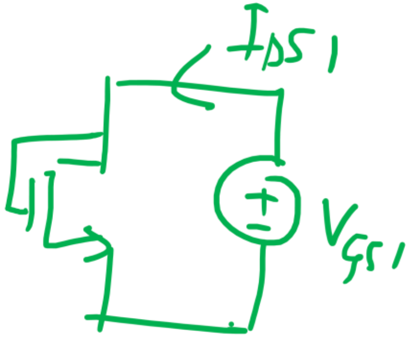




$$I_{Dsat} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_T)^2$$

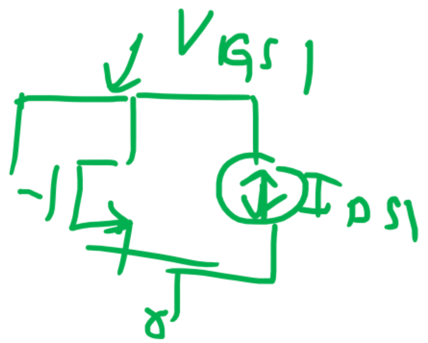
$$\frac{\partial I_{DS}}{\partial V_{GS}} = g_m = 2(\quad) = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)$$

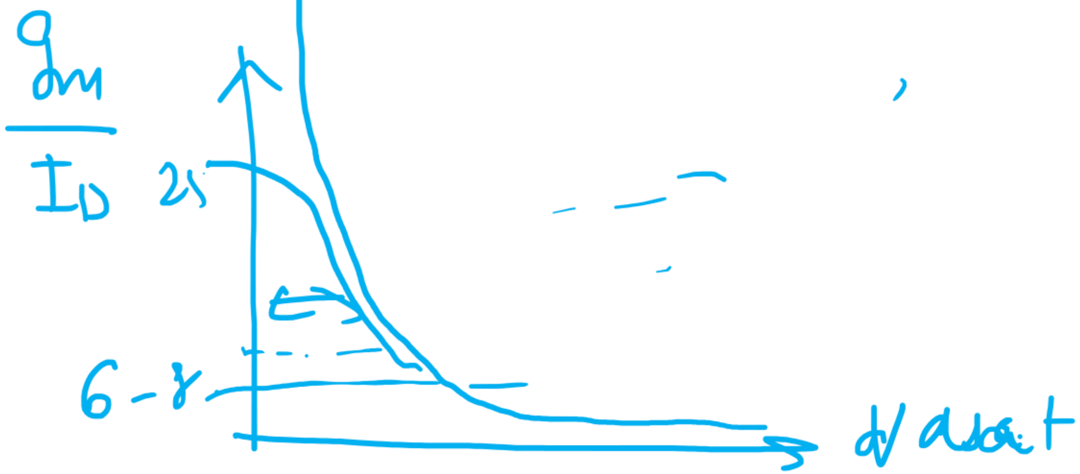
$$= \sqrt{I_D\, \mu_n C_{ox} \times 2 \times \frac{W}{L}}$$



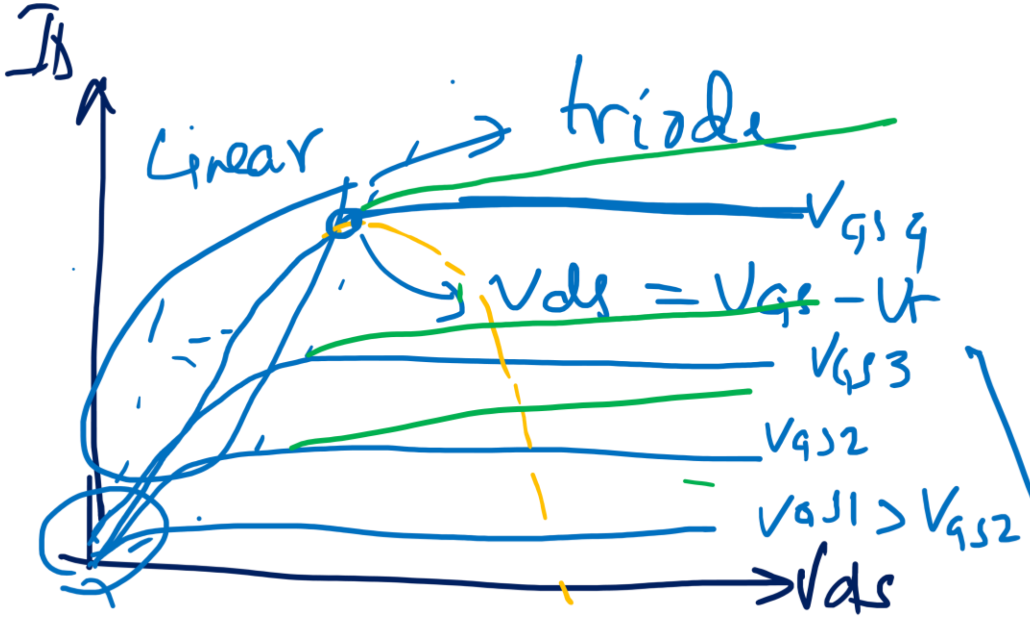
$$g_m = \frac{2 \times I_D}{(V_{GS}-V_T)} = \frac{2 I_D}{V_{dsat}} = \boxed{\frac{g_m}{I_D} = \frac{2}{V_{dsat}}}$$

200mV

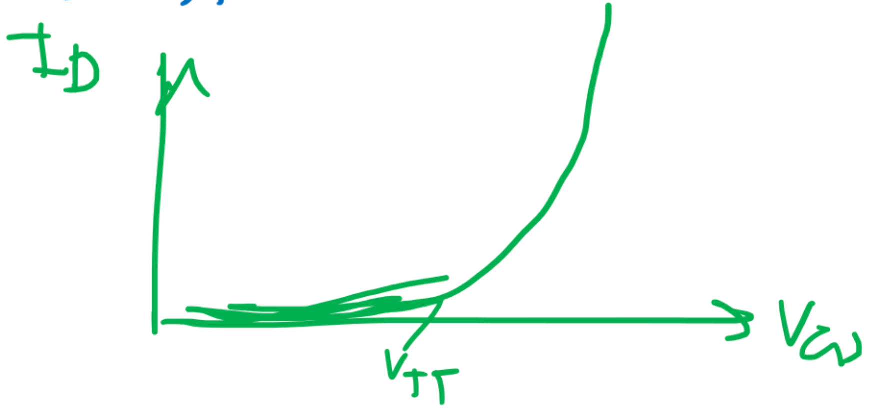




$$\frac{2}{200mV} = 10$$



$(V_{gs} - V_t) = V_{dsat}, V_{eff}, V_{on}, V_{xs}$



$\lambda \alpha \frac{I_D}{L}$

$$\boxed{I_{dsat} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{gs} - V_t)^2 (1 + \lambda V_{ds})}$$

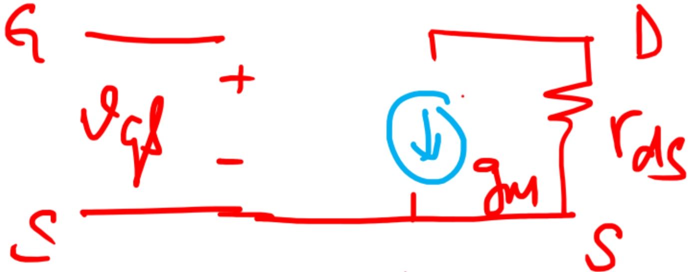
① $V_{gs} > V_T$, $V_{ds} \sim 0$

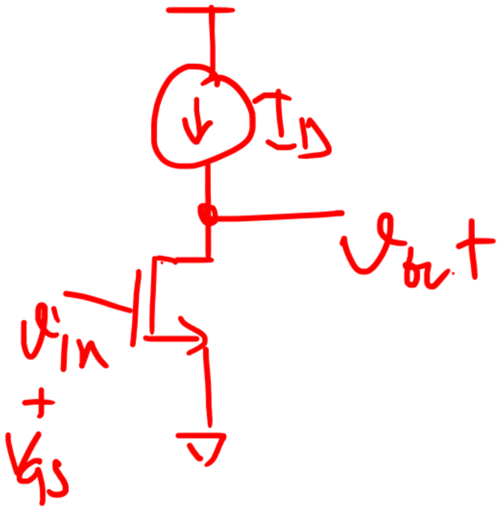
$$I_D = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t) . V_{ds}$$

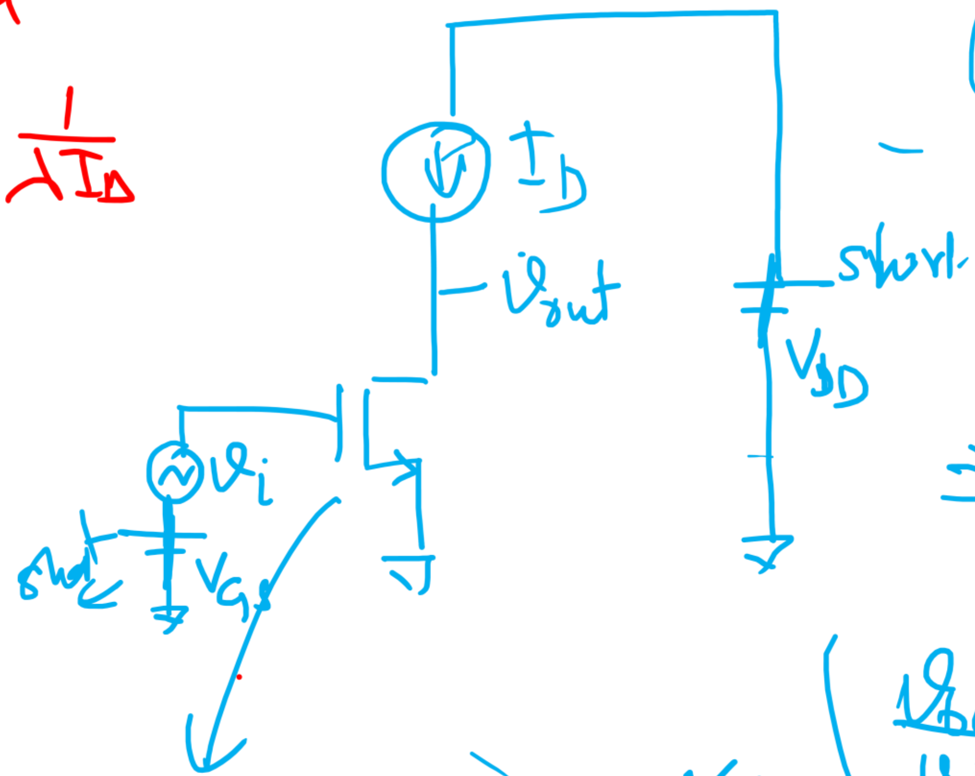
② triode: $V_{gs} > V_T$, $V_{ds} < (V_{gs} - V_t) \Rightarrow I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \left[2(V_{gs} - V_t) V_{ds} - V_{ds}^2 \right]$

Small-signal model

$$g_m = \frac{2I_D}{V_{ov}}, \quad r_{ds} = \frac{1}{\lambda I_D}$$

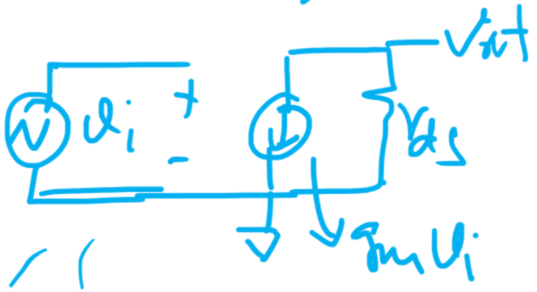






① Ind. sources = 0

⇒



$$\left(\frac{v_{out}}{v_i}\right) = -g_m \cdot r_{ds}$$

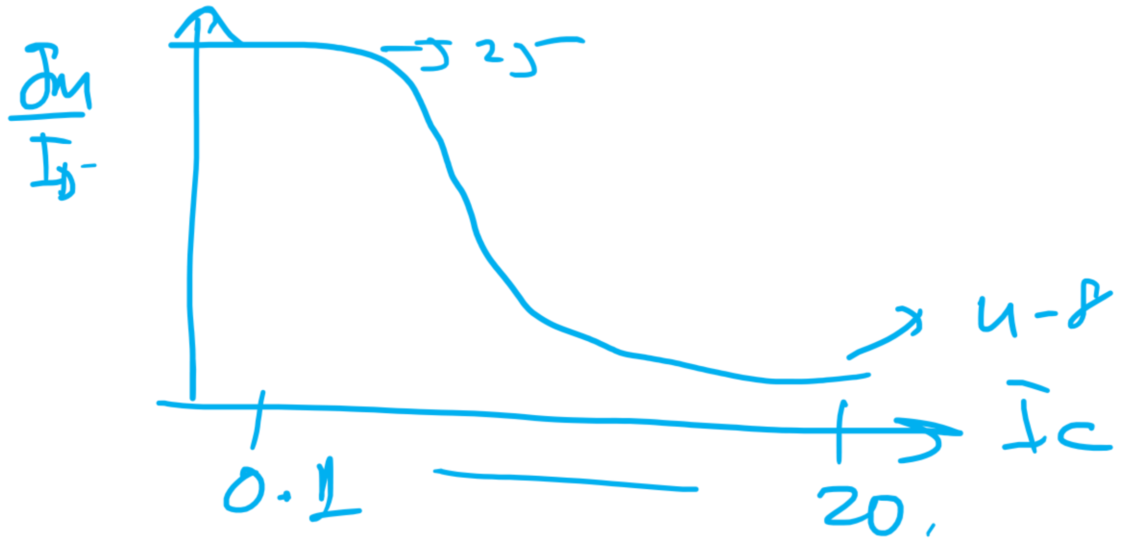
$$\frac{2\cancel{I_D}}{V_{dsat}} \times \frac{1}{\lambda \cancel{I_D}} \quad \frac{L}{I_D}$$

$$\sim k \cdot \frac{2 \cdot L}{V_{dsat} \cdot I_D}$$

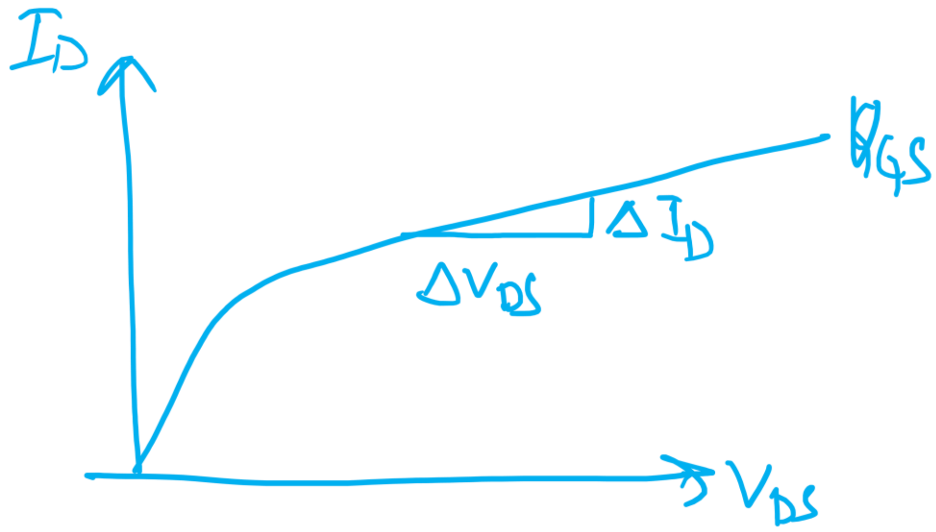
$V_{GS} \leq V_T - nV_T$ $\underline{\underline{IC}}$ – Inversion

$$I_D = I_{DS} \cdot e^{V_{GS}/nV_T}$$

$$g_m = \frac{\partial I_D}{\partial V_g} = \frac{I_{D0}}{nV_T}$$



$$\frac{g_m}{I_D} = \frac{1}{1.5 \times 25\,mV} \cong 25$$



$$I_{DS} = \underbrace{\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2}(1 + \lambda V_{DS})$$

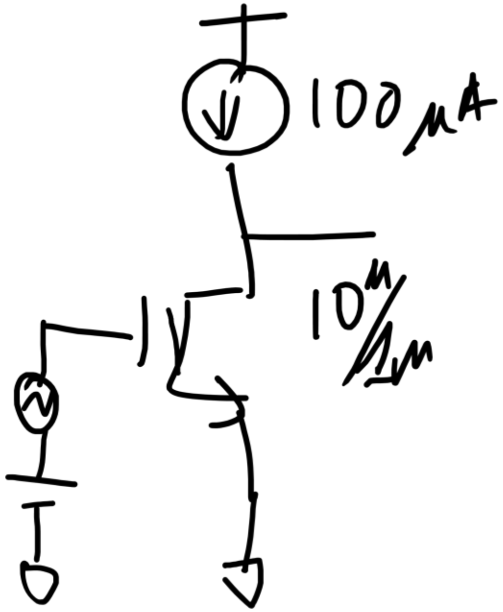
$$\frac{\partial I_{DS}}{\partial V_{DS}} = I_{Dsat} \cdot \lambda$$

$$\boxed{g_{ds} = \lambda \cdot I_{dsat}}$$

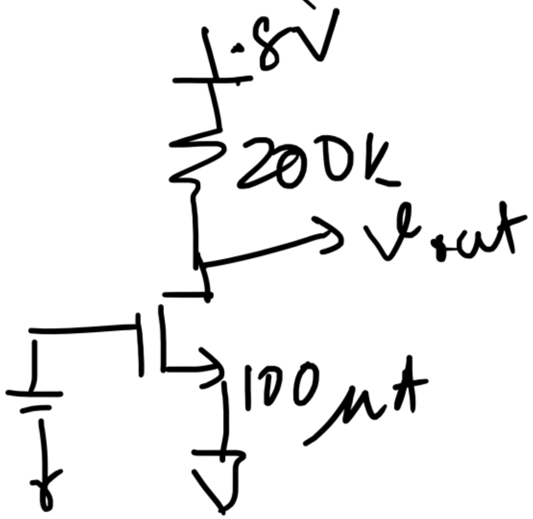
$$\boxed{r_{ds} = \frac{1}{\lambda \cdot I_D}}$$

$$\lambda \propto \frac{1}{L}$$

$$\text{gain} = -\frac{2\cancel{I_D}}{V_{ov}} \times \frac{1}{\lambda \cancel{I_D}} \quad V_{ov} = 200\,mV, \ \lambda = 0.05 \quad = -\frac{2}{0.2 \times 0.05} = \underline{-200}$$



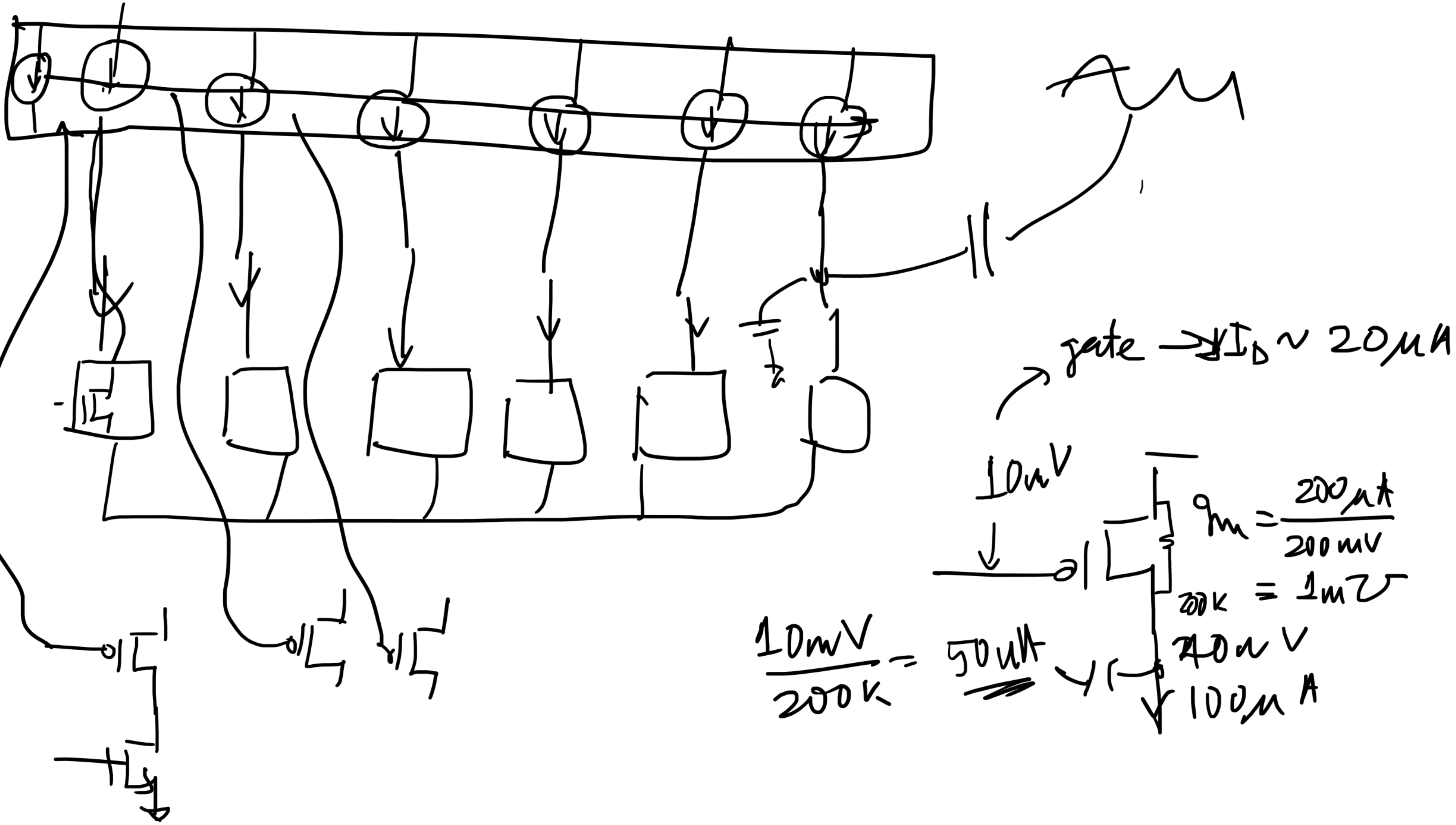
$$r_{ds} = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.1m} = \frac{1}{5 \times 10^{-6}} = \underline{\underline{200\,k\Omega}}$$



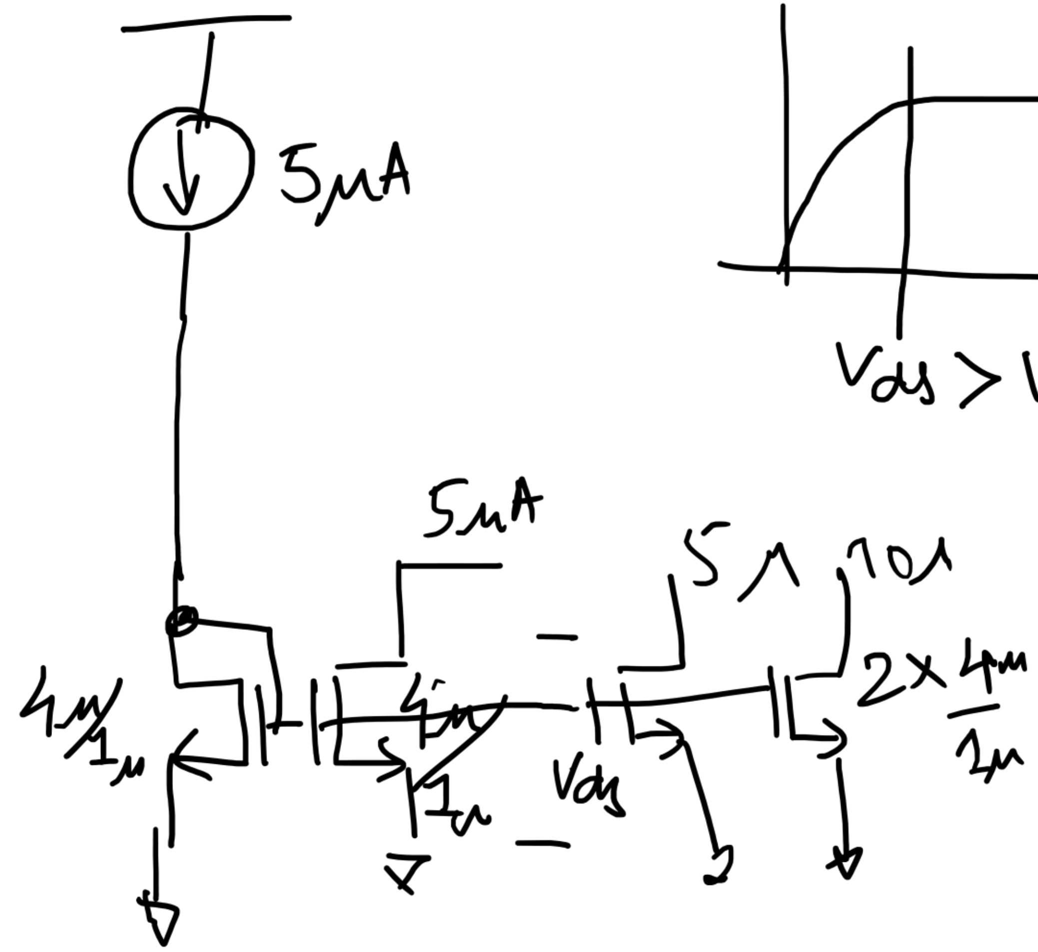
Current source

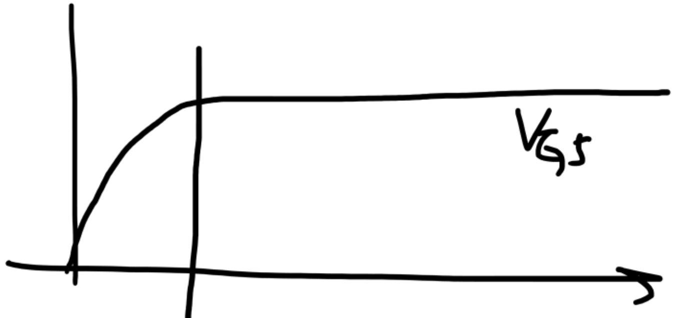
① Active load : decouples biasing & load res

② Biasing



gate → I_D ~ 20uA
10mV
$g_m = \frac{200uA}{200mV}$
200K
= 1m℧
20mV
100uA
$\frac{10mV}{200K}$ = 50uA

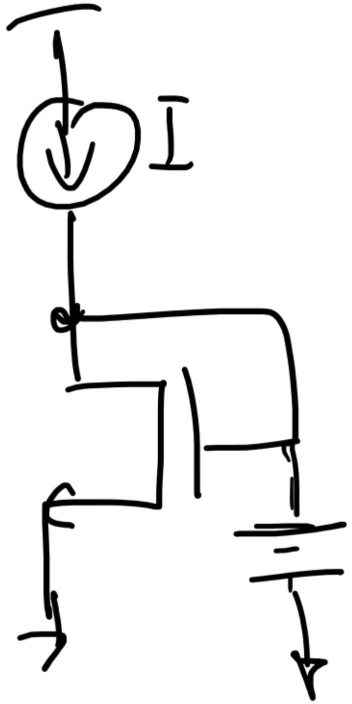


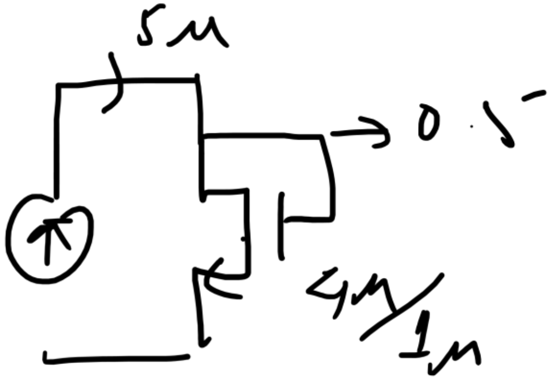


$V_{ds} > V_{gs} - V_T$

5uA $V_{GS} = 0.5$ $T = 27^\circ C$

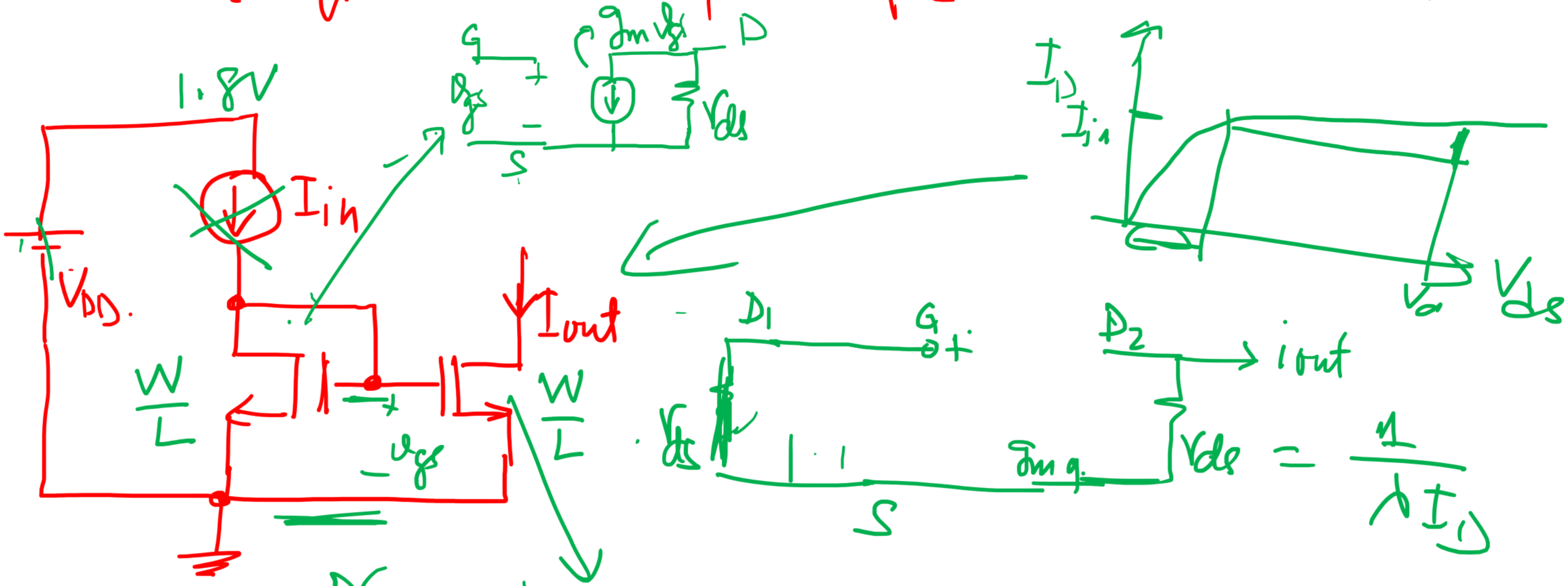
$V_{GS} = 0.4$ $T = 85^\circ C$





$$R = \frac{0.5}{5\mu A} = \underline{\underline{100\,k\Omega}}$$

Small-signal model of simple current mirror



$V_{dsat} = 200\,mV$

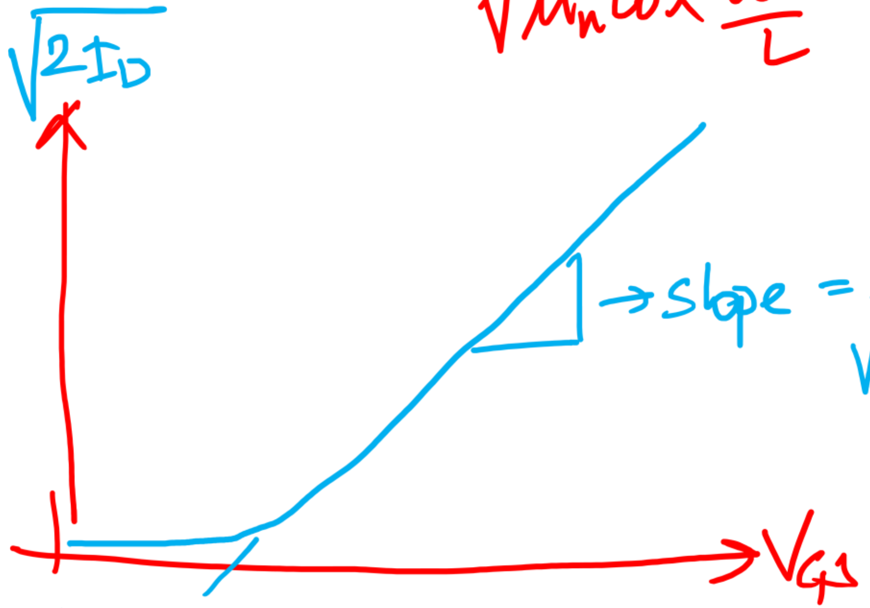
① Independent DC src = 0

② Subst. non-linear devices model.

Parameter Extraction

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_T)^2$$

$$V_{GS} = V_T + \frac{1}{\sqrt{\mu_n C_{ox}\frac{W}{L}}}\sqrt{2I_D}$$



$\rightarrow$ slope $= \frac{1}{\sqrt{\mu_n C_{ox}\frac{W}{L}}}\ \frac{\sqrt{A}}{V}$

$V_{T0} \sim 0.45V$

measured slope $= 0.028\times10^{-3}\ \frac{\sqrt{A}}{V}$

$$\Rightarrow \frac{1}{\sqrt{\mu_n C_{ox}\ 4}} = 0.028\times10^{-3}\ \frac{\sqrt{A}}{V}$$

$$\mu_n C_{ox} = \frac{1}{(0.028\times10^{-3}\times4)^2} = 79.7\ \frac{\mu A}{V^2}$$

$$C_{ox} = 8.78\times10^{-3}\ F/m^2 \quad \left(8.78\ \frac{fF}{\mu m^2}\right)$$

$$\mu_n = \frac{79.7\times10^{-6}}{8.78\times10^{-3}}\ \frac{A/V^2}{F/m^2} = \frac{m^2}{V\cdot s}$$

$$\boxed{\mu_n = 9.07\times10^{-3}\ \frac{m^2}{V\cdot s} = \frac{90\ cm^2}{V\cdot s}}$$