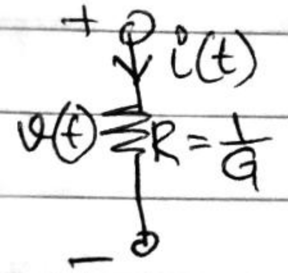
RECAP: CIRCUIT THEORY AND ANALYSES

Circuit Elements and their constitutive relations (relating branch voltages & current)

Resistor



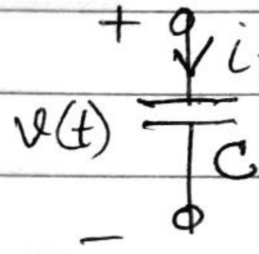
$v(t) = R \cdot i(t)$

$i(t) = G\, v(t)$

Units: $R \rightarrow \Omega$

$G \rightarrow \mho$ (Siemens)

Capacitor



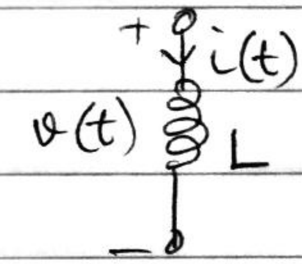
$i(t) = C \dfrac{dv(t)}{dt}$

or

$v(t) - v(0) = \dfrac{1}{C} \int_0^t i(\tau)\, d\tau$

Unit: Farad

Inductor



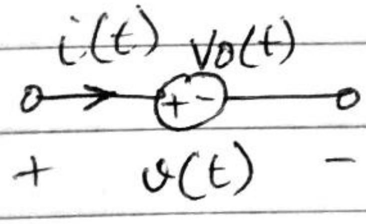
$v(t) = L \dfrac{di(t)}{dt}$

or

$i(t) - i(0) = \dfrac{1}{L} \int_0^t v(\tau)\, d\tau$

Unit: Henry

Ideal Independent Voltage source

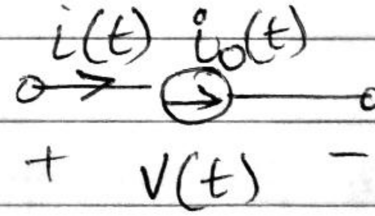


$v(t) = V_0(t)$

independent of $i(t)$

Units: Volts

Ideal Independent Current source



$i(t) = i_0(t)$

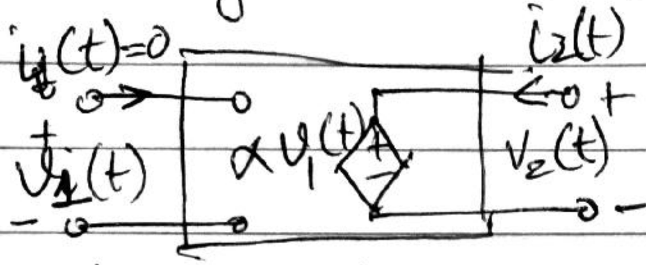
independent of $v(t)$

Units: Amperes

Dependent sources

- Used mostly in modeling active devices & simulation testbenches.

(1) Voltage-Controlled Voltage source (VCVS)

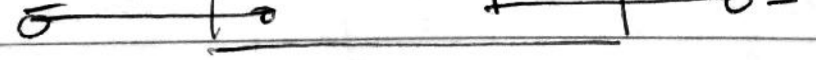


$v_2(t) = \alpha v_1(t)$
independent of $i_2(t)$

Eg.: Op-Amp modeled with a VCVS

$\alpha \rightarrow$ Voltage gain (Unitless)

(2) Voltage-controlled current source (VCCS)

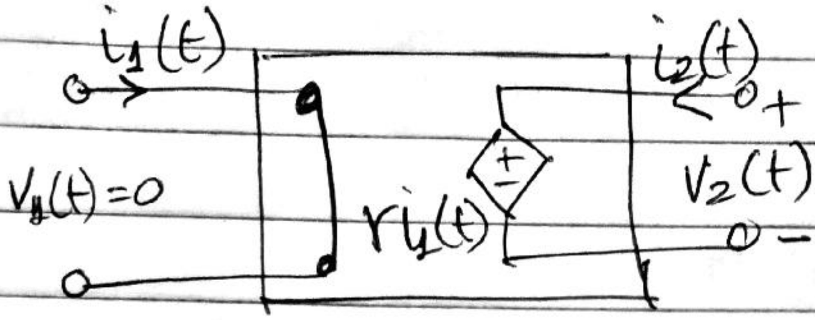


$i_2(t) = g \cdot v_1(t)$ independent of $v_2(t)$

$g \rightarrow$ transconductance (Unit: siemens)

Eg. Transistor modeled with a VCCS

③ Current-controlled Voltage source (CCVS)

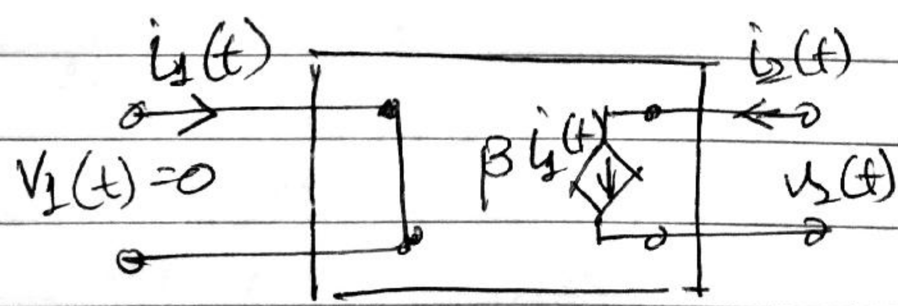


$V_2(t) = r \cdot i_1(t)$ independent of $i_2(t)$

$r \rightarrow$ transimpedance. (Unit: Ω)

Eg: Transimpedance amplifier modeled with (CCVS)

④ Current-controlled Current source. (CCCS)

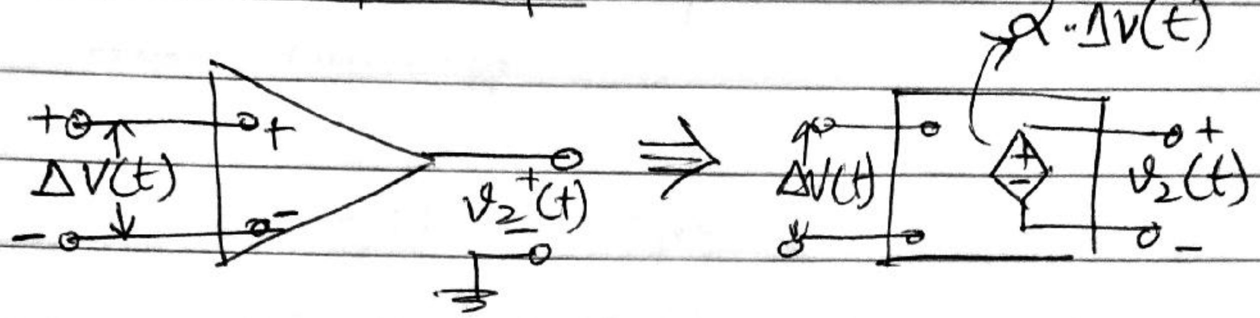


$i_2(t) = \beta i_1(t)$ independent of $v_2(t)$

$\beta \rightarrow$ current gain (Unit less)

Eg: modeling current amplifier.

Ideal Op-Amp.



The linear circuit elements and their network satisfy

SUPERPOSITION (LINEARITY) PRINCIPLE

If $i'(t)$ & $v'(t)$ satisfy the consitutive relation of an element or system, and $i''(t)$ & $v''(t)$ also satisfy the same Then, the element or the system is said to be linear if.

$i(t) = a i'(t) + b i''(t)$ & $v(t) = a v'(t) + b v''(t)$

also satisfy the consitutive relation for any a & b.

Rim

linear circuits & systems also satisfy

TIME-INVARIANCE PRINCIPLE

If $i(t)$ & $v(t)$ satisfy the constitutive relations, then $i(t-T)$ & $v(t-T)$ also satisfies for any value of T

Circuits composed entirely of linear time-invariant elements (except Independent sources), form Linear Time-Invariant (LTI) systems.

ANALYTICAL TOOLS FOR ANALYZING, UNDERSTANDING & DESIGNING LTI SYSTEMS.

KIRCHOFF'S LAWS.

KIRCHOFF'S CURRENT LAW (KCL):

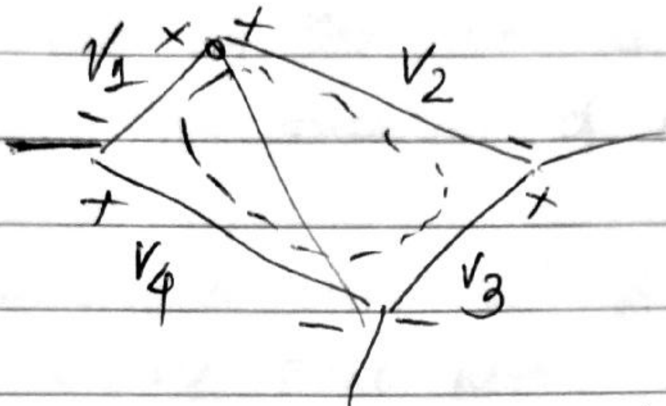
Algebraic sum of the currents entering/leaving a circuit node is zero.

$$i_1 + i_2 - i_3 + i_4 = 0$$

$$\Sigma i_i = 0$$

KIRCHOFF'S VOLTAGE LAW (KVL)

Algebraic sum of the directed voltage drops around any circuit mesh is zero.



$\sum v_k = 0 \Rightarrow v_1 - v_2 - v_3 + v_4 = 0$

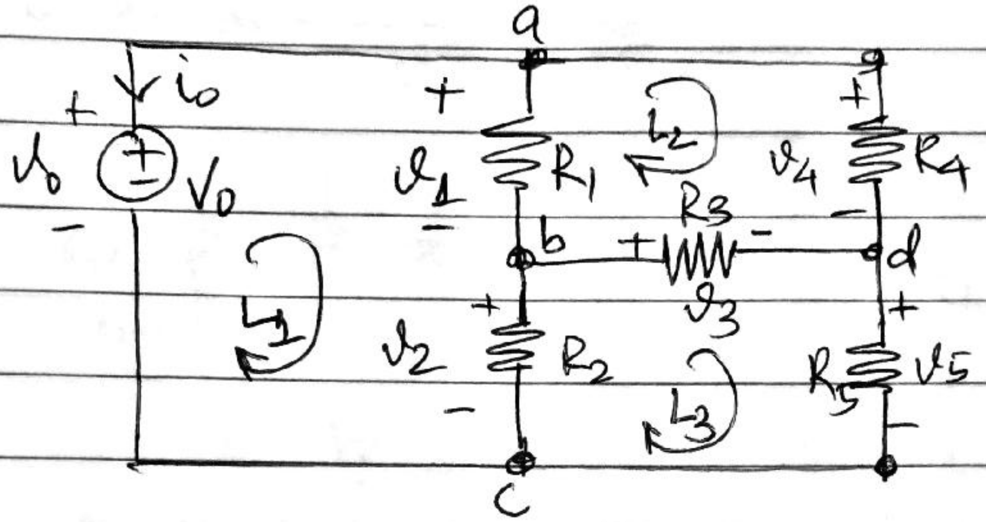
NOTE: KCL & KVL are derived from Maxwell's equation with the assumption that there is no radiation.

Together, Kirchoff's laws & the constitutive relations provide a set of 2N independent equations for N voltages & N currents associated with N branches of the circuit.

Several techniques are available for reducing the number of unknowns.

The first such procedure:

KCL, KVL Example



$v_0 \ldots v_5,\ i_0 \ldots i_5 \leftarrow$ 12 unknowns

① Element relationships (v, i)

$v_0 = V_0$, $v_3 = i_3 R_3$, $v_1 = i_1 R_1$

$v_4 = i_4 R_4$, $v_2 = i_2 R_2$, $v_5 = i_5 R_5$

— 6 eqns

② KCL at the nodes — redundant

a: $i_0 + i_1 + i_4 = 0$; b: $i_2 + i_3 - i_1 = 0$

d: $i_5 - i_3 - i_4 = 0$; c: $-i_0 - i_2 - i_5 = 0$

— 3 independent equations.

③ KVL for loops:

L1: $-v_0 + v_1 + v_2 = 0$; L2: $v_1 + v_3 - v_4 = 0$

L3: $v_3 + v_5 - v_2 = 0 \leftarrow$ 3 independent eqns

12 eqns, 12 unknowns, solve UGLY!

NODE EQUATIONS PROCEDURE

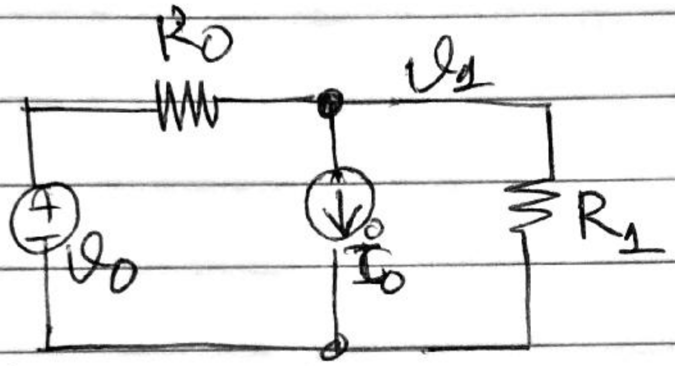
1). Pick a reference node.
The resulting eqns. will be simplest if the chosen node is one ~~of the~~ that is common to largest number of branches.

2) Assign a node voltage variable to every other node, except the only one of two nodes connected by an ideal voltage source (independent or dependent) need be assigned a node voltage variable.

3) Write KCL for all assigned node using the constitutive relations. (If one or more ideal voltage sources are connected, write KCL enclosing the voltage source.)

The node equations lead to as many equations as there is unknown which is very much less than 2x the number of branches.

EXAMPLE



1) chose this to be the reference node since most elements share this node.

2) Name all the voltage nodes. Only v_1 since the other node is in series with a independent source.

3) Write KCL for node v_1:

$$(v_1 - v_0)/R_0 + i_0 + \frac{v_1}{R_1} = 0$$

$$\Rightarrow v_1 = \frac{R_1}{R_1+R_o}\left(v_o - i_o \cdot R_o\right)$$

Using super position:

① $i_o = 0$ [open]

$$v_1' = \frac{R_1}{R_1+R_o} v_o \qquad — ①$$

② $v_o = 0$ [short]

$$v_1'' = -\frac{R_1 \cdot R_o}{R_1+R_o} i_o \qquad — ②$$

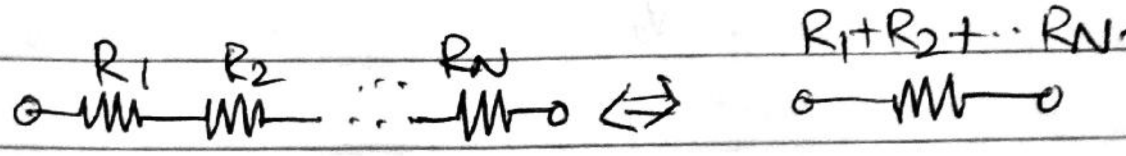
By super position, $v_1 = v_1'' + v_1'$

$$\Rightarrow \boxed{v_1 = \frac{R_1}{R_1+R_o}\left(v_o - i_o R_o\right)}$$

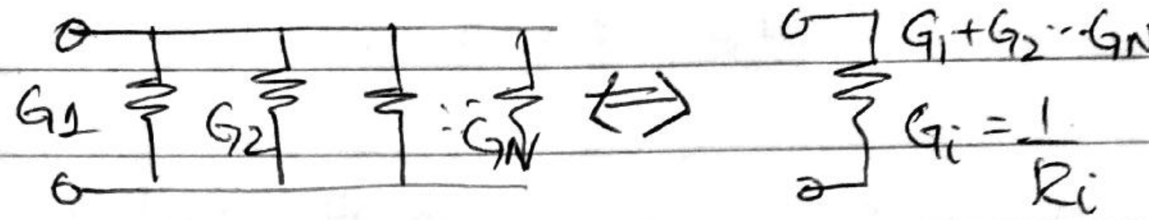
Side Note:

Simplification methods:

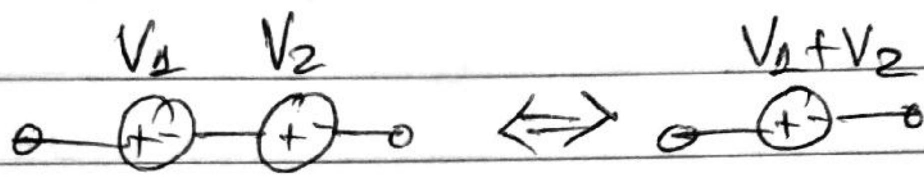
(1)



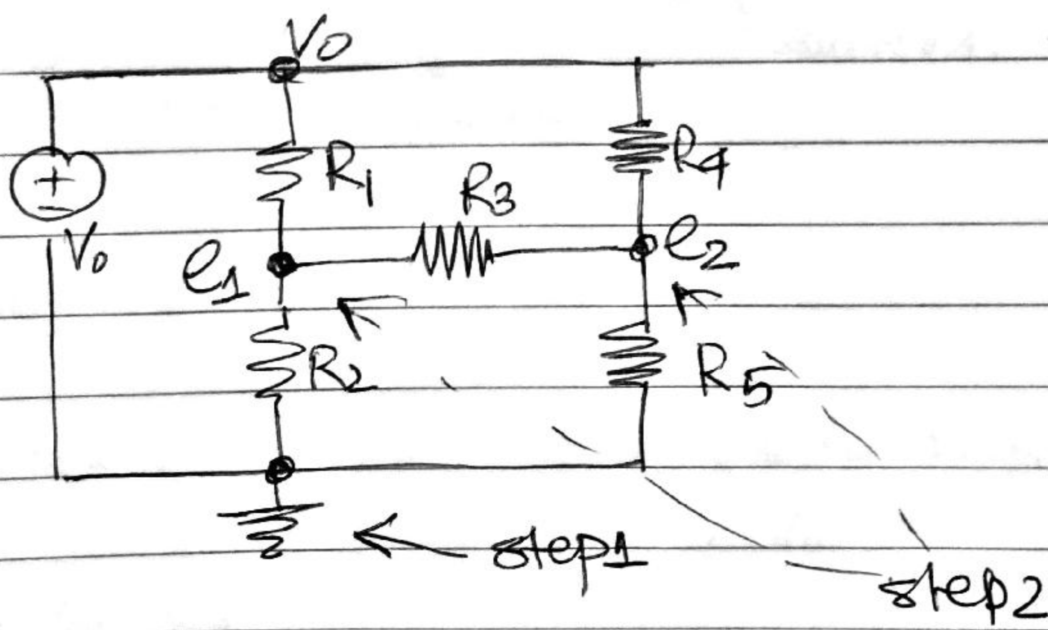
(2)



(3)



SOLVE PREVIOUS EXAMPLE USING NODE METHOD:



KCL @ e_1:

$$(e_1-V_0)G_1+(e_1-e_2)G_3+e_1G_2=0$$

KCL @ e_2:

$$(e_2-e_1)G_3+(e_2-V_0)G_4+e_2G_5=0$$

Moving constants to RHS & collecting unknowns :

$$(G_1+G_2+G_3)\cdot e_1 - G_3\cdot e_2 = G_1\cdot V_0$$
$$-G_3\cdot e_1 + (G_3+G_4+G_5)\cdot e_2 = G_4\cdot V_0$$

In matrix form:

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1 V_0 \\ G_4\cdot V_0 \end{bmatrix}$$

$$\underbrace{G}_{\text{Conductance matrix}} \cdot \underbrace{\tilde{e}}_{\text{unknown}} = \underbrace{\tilde{v}}_{\text{sources.}}$$

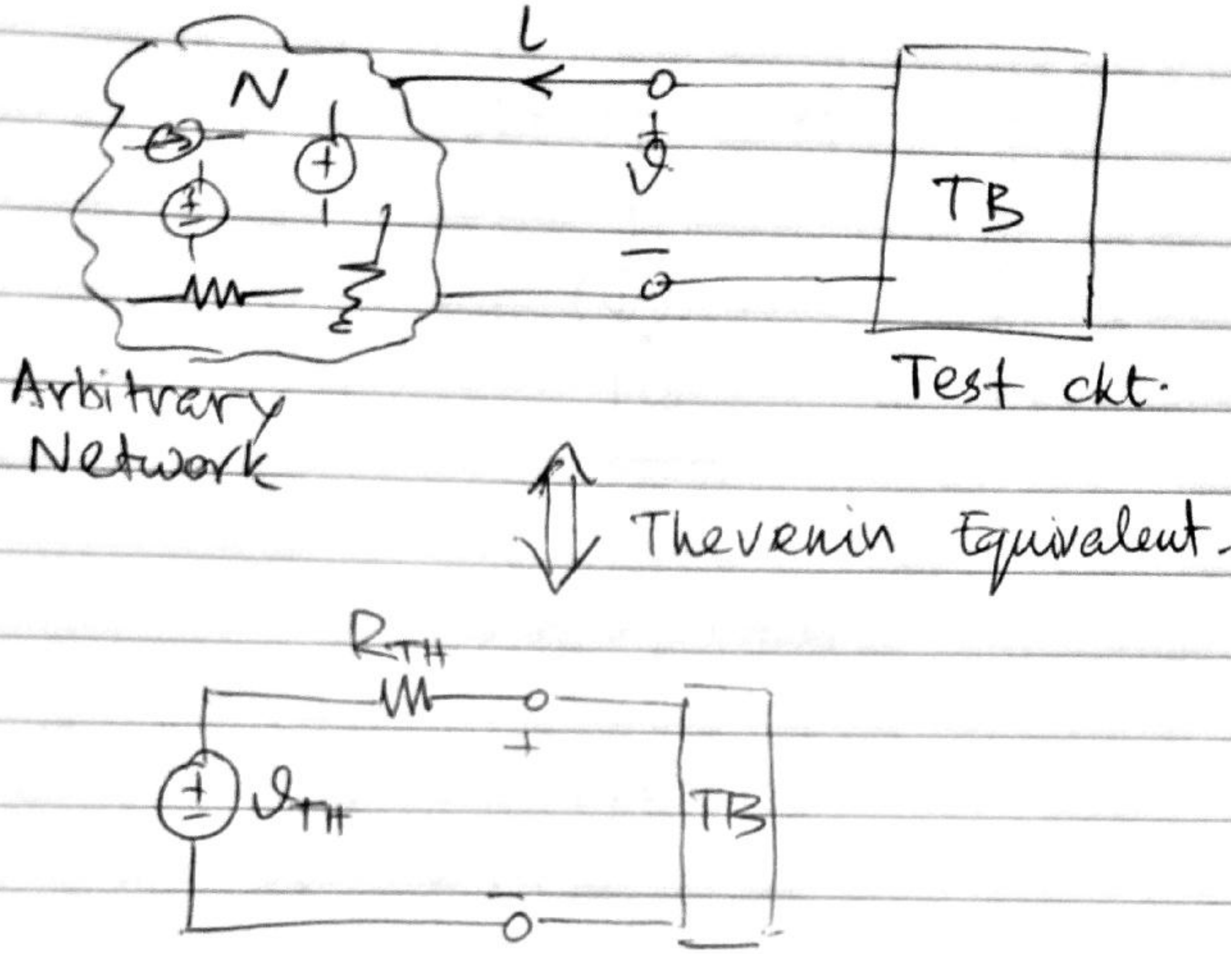
Solve : $\bar{e} = G^{-1}\cdot\tilde{v}$

Exercise : $G_1, G_5 = 1/8.2K$, $G_2, G_4 = 1/3.9K$
$G_3 = 1/1.5K$, $V_0 = 3V$

Find e_1, e_2

Solve above using python.

THEVENIN METHOD

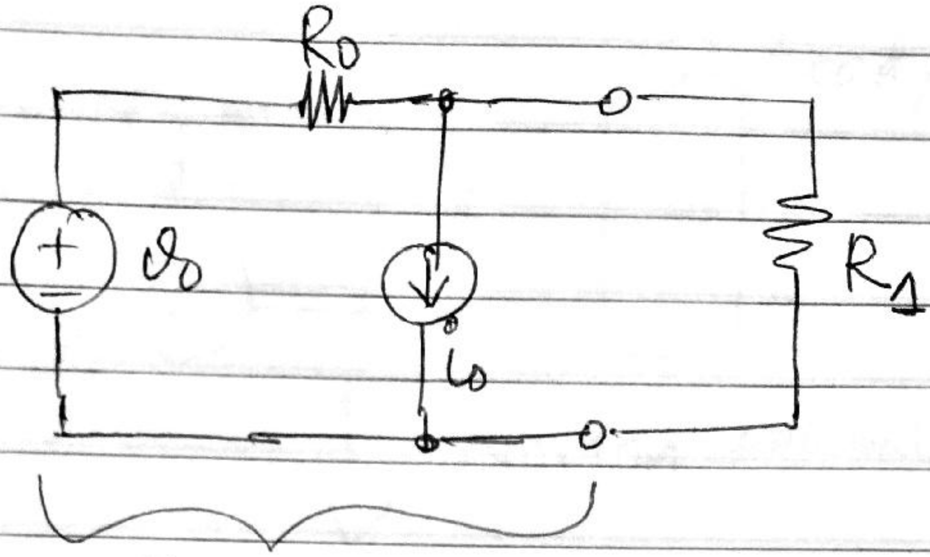


Method :

① v_{TH} is the open ckt. voltage at the two terminals.

② R_{TH} is the impedance of the two terminals when all independent sources inside the network is zero ie all voltage sources short & all current sources open.

Example

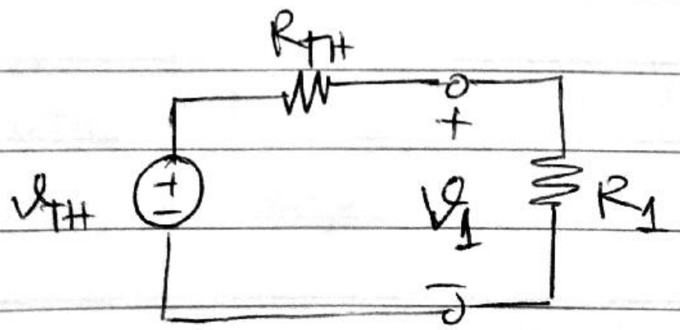


Thevenin equivalent.

① v_{TH} : by superposition

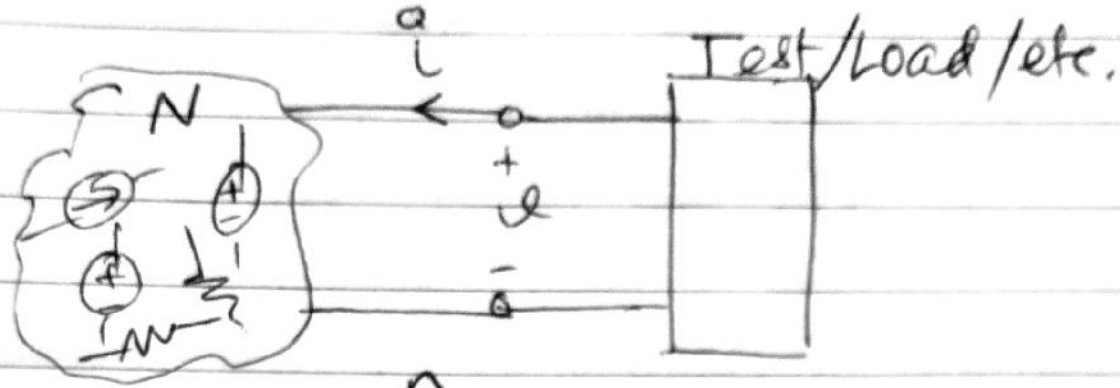
$v_{TH} = v_0 - i_0 R_0.$

② $R_{TH} = R_0$

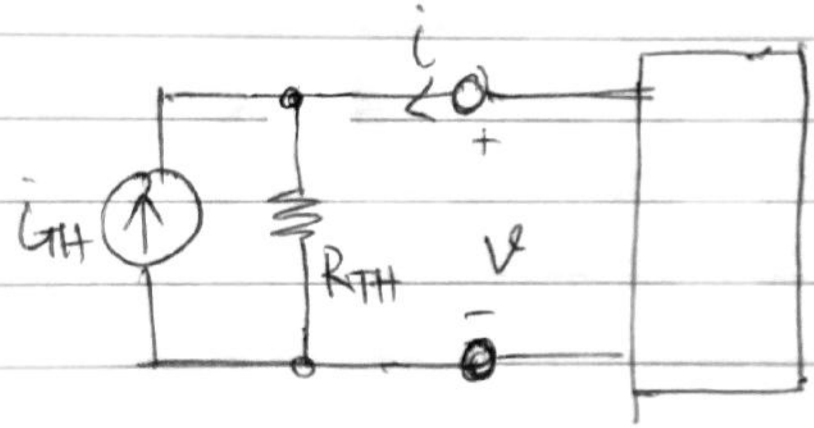


$$v_1 = \frac{R_1}{R_1 + R_{TH}} v_{TH} = \frac{R_1}{R_1 + R_0}(v_0 - i_0 R_0)$$

NORTON METHOD



⇕ Norton equivalent.



Method:

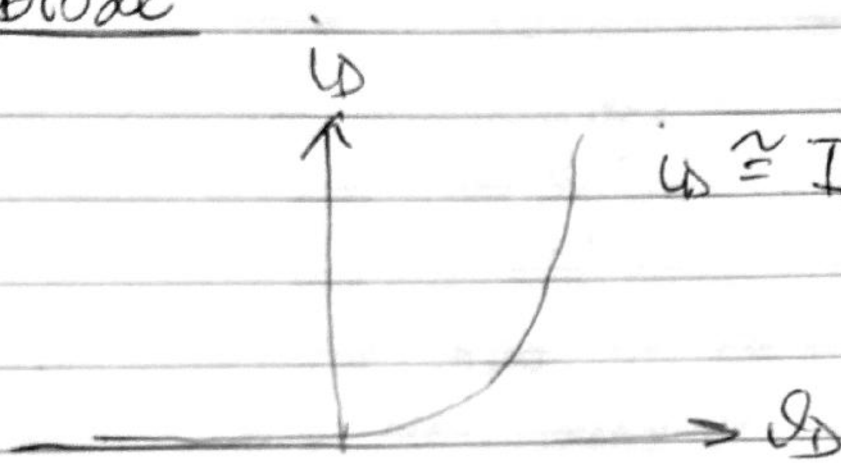
① i_{TH} : Find the short-circuit current by shorting the two terminals.

② R_{TH} : Same method as Thevenin

ANALYSIS OF NON-LINEAR CKTS.

Non-linear elements:

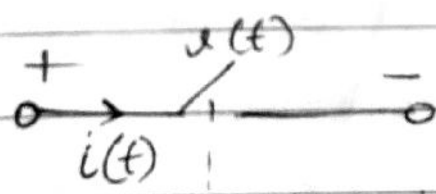
Diode



$$i_D \cong I_S e^{v_D/V_T}$$

$V_T = kT/q$

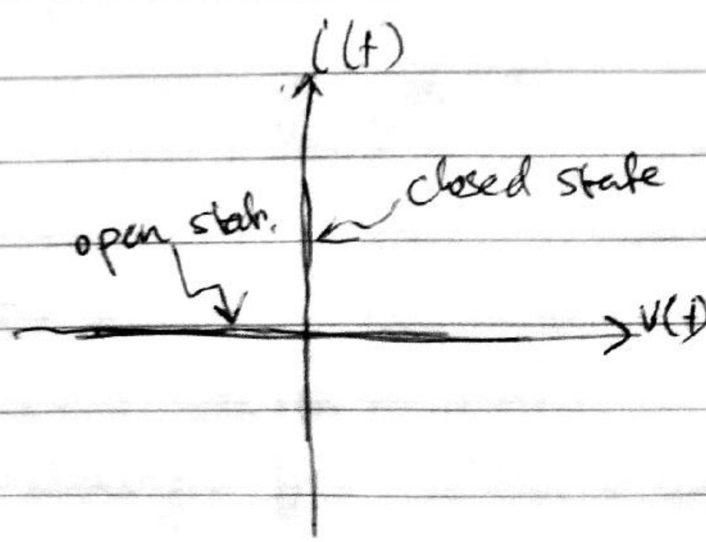
Ideal switch



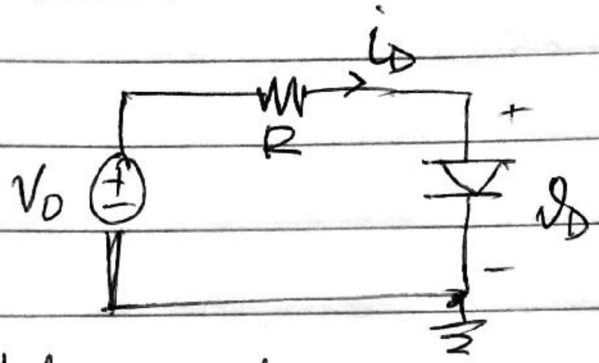
control ← independent f(t)

$i(t) = 0$: open state

$v(t) = 0$: close state



Analytical solution

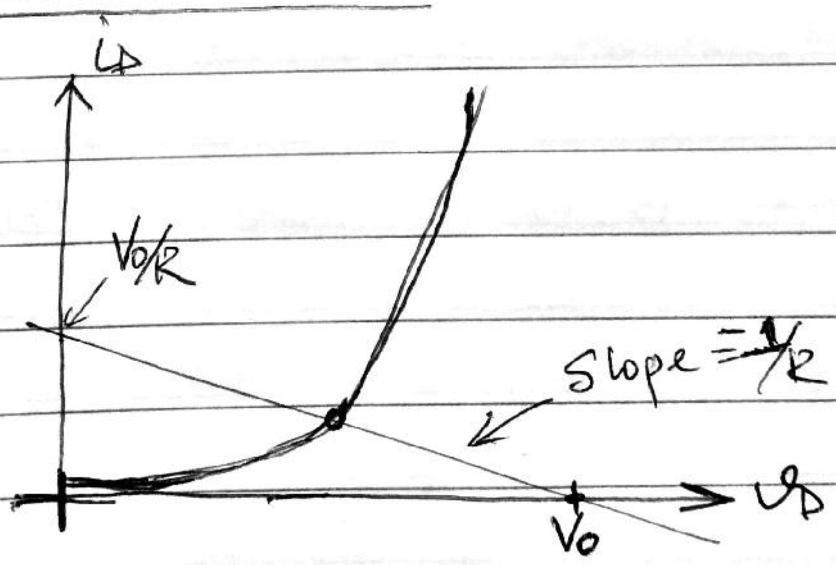


Node equation

$$\frac{V_0 - v_D}{R} - I_0 \cdot e^{v_D/V_T} = 0$$

This equation has to be solved iteratively. Not very insightful.

Graphical method

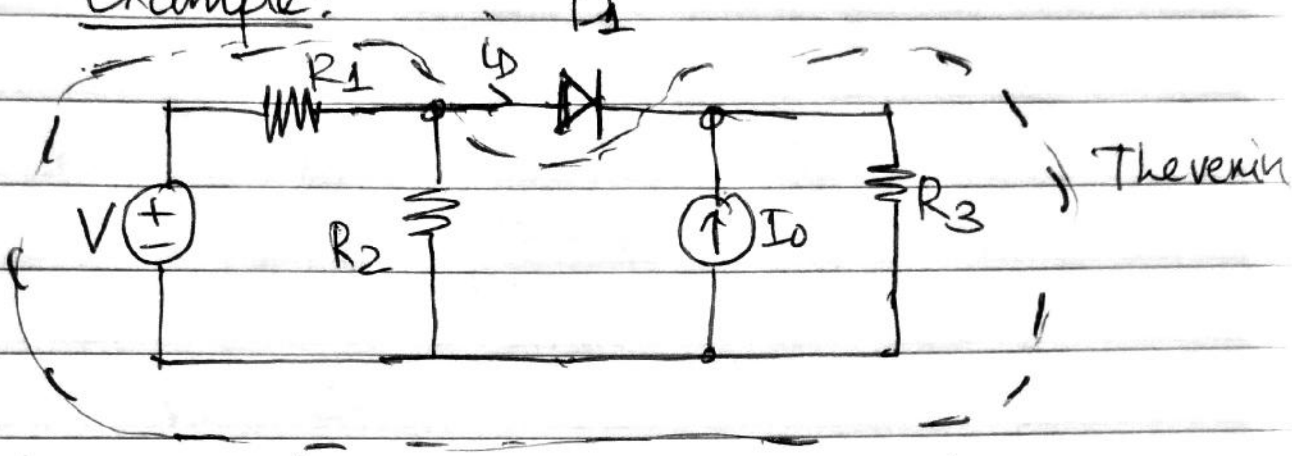


Incremental Analysis

To be covered during MOS small signal modeling. Do Python Problem

Using Linear Analysis Toolbox to solve Non-linear problems

Example:



* Node analysis cannot be used since a non-linear element is present.

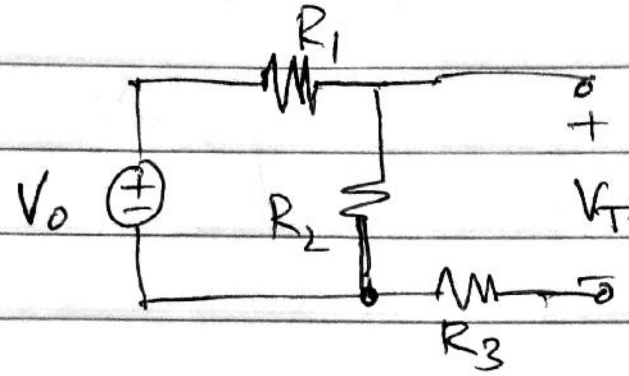
→ Thevenin/Norton equivalent of the ckt. enclosed inside the dotted circle.

Which one Norton or Thevenin?

→ Since we have a previous example for a way to solve for i_D, we do a Thevenin's equivalent.

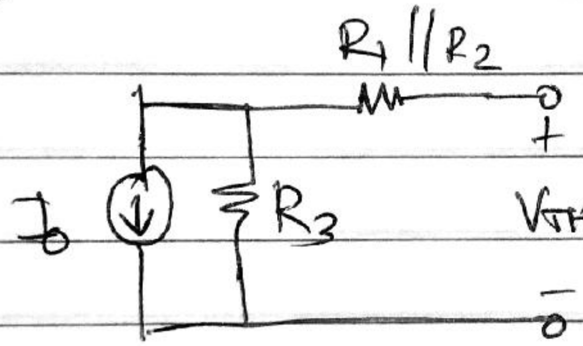
For V_{TH} (by super position)

① $I_0 = 0$



$$V_{TH}' = \frac{V_0 \times R_2}{R_1 + R_2}$$

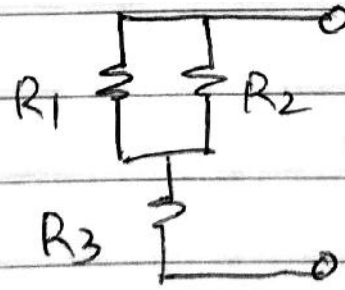
② $V_0 = 0$



$$V_{TH}'' = -I_0 R_3$$

$$\therefore \quad V_{TH} = \frac{V_0 \cdot R_2}{R_1 + R_2} - I_0 R_3$$

R_{TH}



$\leftarrow R_{TH} = (R_1 \| R_2) + R_3$

Simplified ckt.