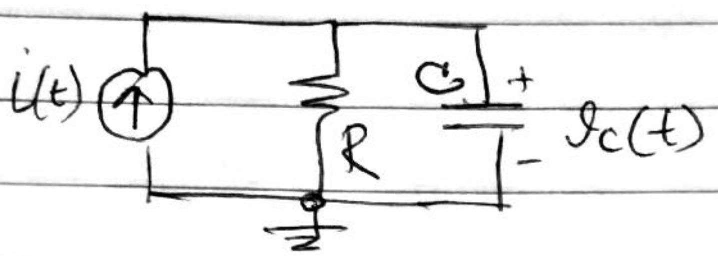
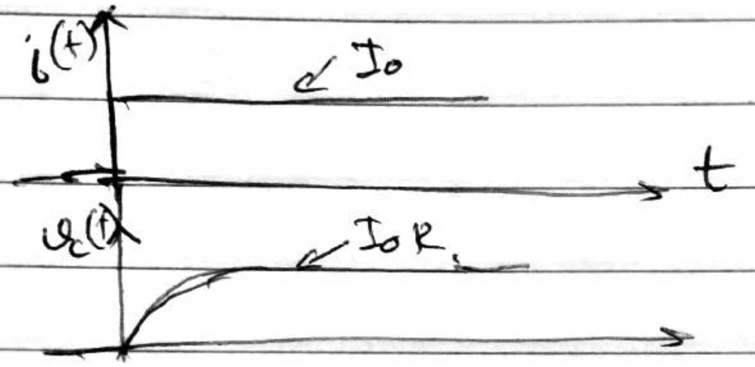
DYNAMIC CIRCUIT ANALYSIS

PARALLEL RC CIRCUIT, STEP INPUT





Node Analysis results in

$$i(t) = v_c/R + C\frac{dv_c}{dt}$$

Or, rewriting,

$$\frac{dv_c}{dt} + \frac{v_c}{RC} = \frac{i(t)}{C}.$$

Linear diff eqn with constant coeff.
Solution contains two parts:

1) homogenous solution &
2) particular solution

The complete solution is the summation of two

Homogenous solution:

$$\frac{dv_{CH}}{dt} + \frac{v_{CH}}{RC} = 0$$

Choosing $v_{CH} = Ae^{st}$ & substituting above & solving we get

$$s = -1/RC$$ ← characteristic eqn.

$\therefore$ $v_{CH} = Ae^{-t/RC}$ ← dimension of time called the time constant.

Particular solution

$$I_0 = \frac{v_{CP}}{R} + C\frac{dv_{CP}}{dt}$$

$\because$ I_0 is constant for $t > 0$, one acceptable solution:

$$v_{CP} = K$$

On substituting in above eqn.:

$$v_{CP} = I_0 R$$

∴ Complete solution is

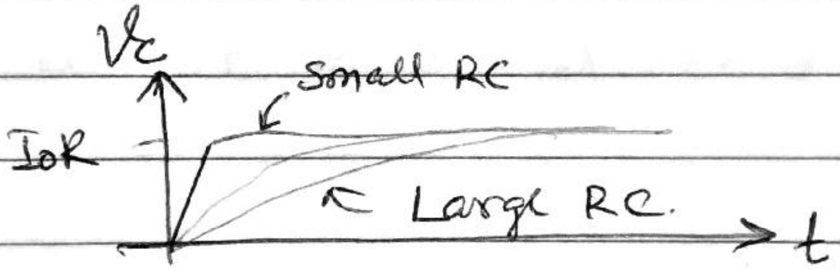
$$v_C = A e^{-t/RC} + I_o R$$

A is derived from initial cond. ie $t=0$

$$\Rightarrow A = -I_o R$$

$$\therefore v_C = -I_o R e^{-t/RC} + I_o R$$

$$\boxed{v_C = I_o R \left(1 - e^{-t/RC}\right)}$$



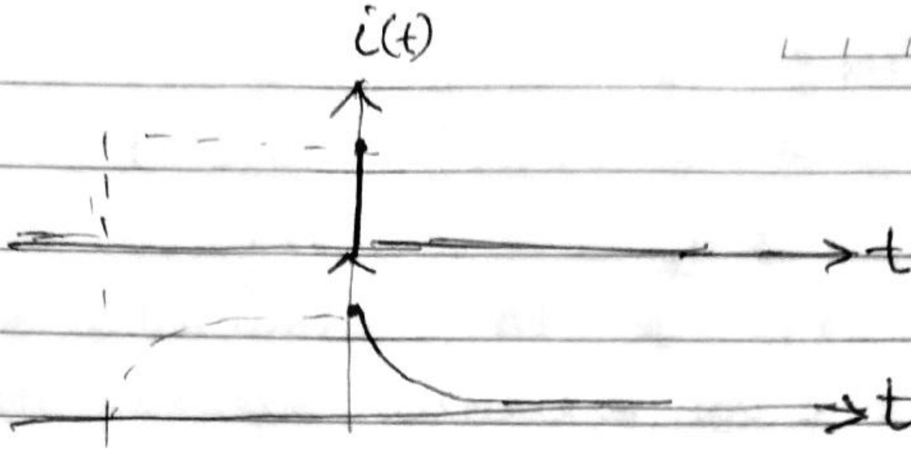
Discharge Transient

— Homogenous solution with initial condition $v_C = I_o R$.

$$\Rightarrow v_{CH} = A e^{-t/RC}$$

∴ $v_{CH} = I_o R$ when $t=0$

$$\boxed{v_C = I R_o \cdot e^{-t/RC}}$$



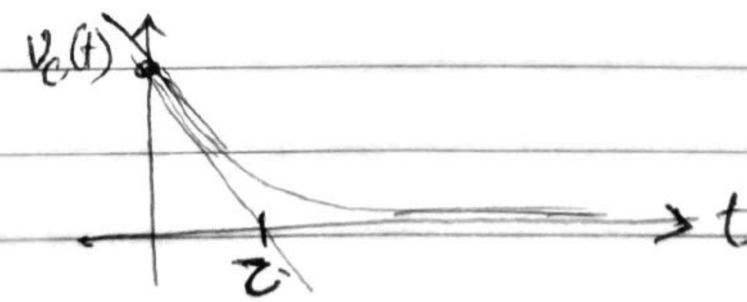
General form: $\boxed{v_c = v_c(0)\, e^{-t/RC}}$

Some properties of $e^{-t/RC}$

▶ For $x = Ae^{-t/\tau}$

the initial slope at $t = 0$

$$\left.\frac{dx}{dt}\right|_{t=0} = -\frac{A}{\tau}$$



▶ At $t = \tau$, $x(t=\tau) = \frac{A}{e}$

* $t_{90\%} - t_{10\%} = RC \ln\left(\frac{0.9}{0.1}\right) = 2.2\tau$

▶ At $t = 5\tau$ $\quad x(5\tau) = \dfrac{A}{e^5} \cong 0.006A$

In other words, ~~at~~ $t = 5\tau$, the output is assumed to be fully settled in steady state.

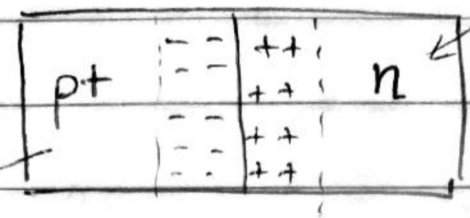
Similar Analysis can be made for the Thevenin equivalent.

So for a square-wave input, the two conditional analysis results can be used.

REVIEW IC DEVICES

Diode:

mildly doped

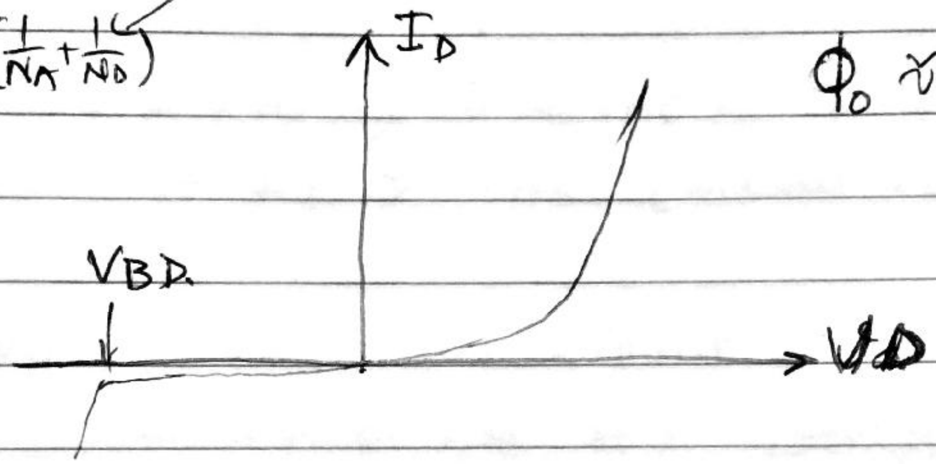


heavily dope

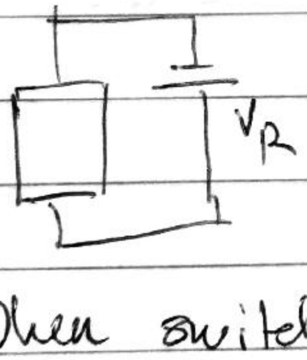
$$I_D \cong I_S e^{V_D/V_T} \leftarrow KT/q$$

$\propto A_d\left(\frac{1}{N_A}+\frac{1}{N_D}\right)$

$\phi_0 \approx 0.9V$



Reverse bias junction Capacitance.



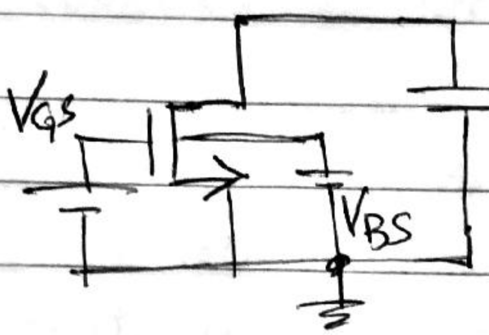
$$C_j = \frac{C_{j0}}{\sqrt{1+V_R/\phi_0}} \leftarrow \text{non-linear}$$

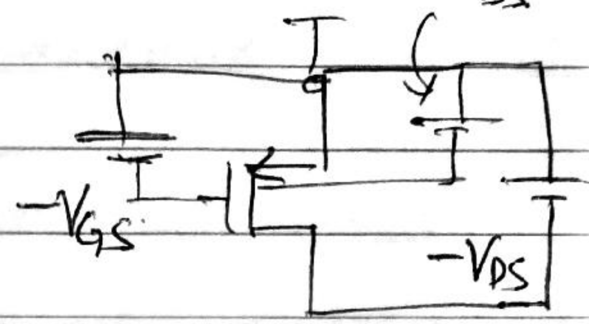
$\rightarrow f(V_R)$

When switching between V_1 & V_2 it can be approximated with

$$C_{j\text{-}av} = \frac{2C_{j0}\,\phi_0\left[\sqrt{1+V_2/\phi_0} - \sqrt{1+V_1/\phi_0}\right]}{V_2 - V_1}$$

MOS BIASING & OPERATION





* $V_{GS} > V_{TO}$, $0 < V_{DS} < V_{Dsat}$
  — $I_D \propto V_{DS}$ [Linear region]

* $V_{GS} > V_{TO}$, $V_{DS} > V_{Dsat}$
  — $I_D \propto (V_{GS} - V_T)^2$ [Saturation region]

* $V_{GS} < V_{TO}$
  — subthreshold region OR cut-off

  $I_D \propto e^{V_{GS}}$ $\quad$ $I_D \approx 0$

To determine if it's in subthreshold we need to know the inversion coefficient IC given by

$$IC = I_D / I_S$$

where $I_S = \dfrac{2 \mu C_{ox} V_T^2}{K} \cdot \dfrac{W}{L}$

where,
$V_T = kT/q$
$K \approx 0.7$

For IC < 0.1, its in weak inversion or sub-threshold operation

Moderate Inversion: $0.1 < IC < 10$

Strong Inversion: $IC > 10$.

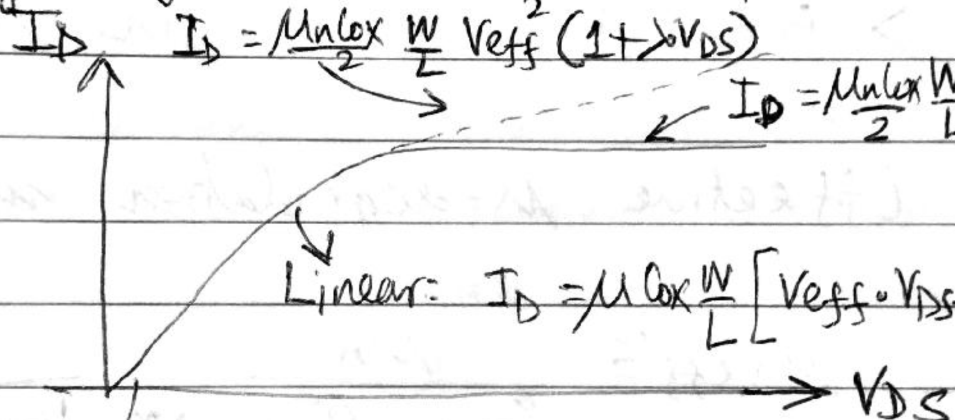
g_m/I_D is more appropriate design parameter to be discussed later.

Large-Signal Id-Vds Response

$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} V_{eff}^2 (1+\lambda V_{DS})$

$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} V_{eff}^2$

Linear: $I_D = \mu C_{ox} \frac{W}{L} \left[V_{eff} \cdot V_{DS} - \frac{V_{DS}^2}{2} \right]$



$I_D \cong \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn}) V_{DS}$

Linear resistor $R_{on} = \frac{V_{DS}}{I_D} = \frac{1}{\mu C_{ox} \frac{W}{L} V_{eff}}$

$\mu \rightarrow$ mobility of the channel

$\lambda \rightarrow$ channel length modulation coefficent

$V_{eff} \rightarrow V_{GS} - V_T \Rightarrow$ excess bias

also written as V_{on}, V_{dsat}

98, 138, 348

~~Dwsi~~

When $0.1 < IC < 10$ both strong & weak inversion expression over estimate g_m.

Accurate expression is using EKV:

$$g_m \approx \frac{K I_D}{U_T} \cdot \frac{2}{1 + \sqrt{1 + 4 \cdot IC}}$$

The current mechanism in subthreshold region is diffusion since channel is not formed.

$$I_D \cong I_{DO} \left(\frac{W}{L}\right) e^{(qV_{eff}/nKT)}$$

$$n = \frac{C_{ox} + C_{jo}}{C_{ox}} \approx 1.5$$

$$I_{DO} = (n-1) \mu_n C_{ox} \left(\frac{KT}{q}\right)^2$$

BODY EFFECT

In the previous sections, the large signal equations are valid under the assumption that the source & bulk are at the same potential i.e $V_{SB} = 0V$.

However, often case, that is not the case.

Typically called the body effect, the influence of the body potential on the channel is modelled as an increase in the threshold voltage, V_{tn}, with increasing source-to-body bias. For a NMOS,

$$\boxed{V_{tn} = V_{tno} + \gamma \cdot \left(\sqrt{V_{SB} + 2|\phi_F|} - \sqrt{|2\phi_F|}\right)}$$

where, $\phi_F = KT/q \, \ln(N_A/n_i)$, $\gamma = \dfrac{\sqrt{2qN_AK_s\epsilon_0}}{C_{ox}}$

n Short-channel effects

Mobility degradation

- Large electric field → effective mobility degradation

- Velocity saturation ~ 10^7 cm/s due to high lateral E-field

- In addition, vertical E-field $> 5\times10^6$ V/m result in μ-degradation

- Effective μ-degradation model:

$$\mu_{n\,eff} \cong \frac{\mu_n}{\left(\left[1 + (\theta V_{eff})^m\right]^{1/m}\right)}$$

θ, m → device parameters

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{eff}^2\left[\frac{1}{1+(\theta V_{eff})^m}\right]^{1/m}$$

Short channel effects: Hot Carrier

- High velocity electrons near the drain cause electron-hole pair leading to substrate current

- results in low output impedance & therefore problem in using them for current mirrors.

- another effect is the hot carriers entering the oxide & causing finite gate current.

- Causes shift in threshold voltage & reliabity.