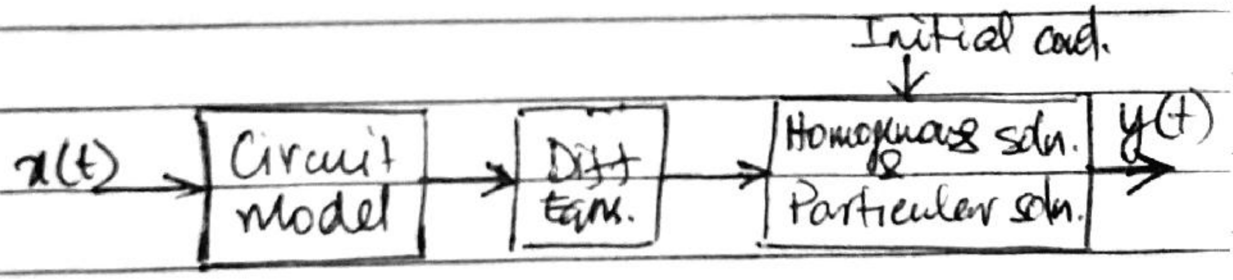
INTRODUCTION TO FREQUENCY ANALYSIS

Review of Dynamic Circuit Analysis Method. (For Linear circuits)

* simplify circuit if possible (series res, etc)
* Formulate the differential equation using nodal equation method.
* Solve the equations. Find the homogenous and particular soln.
* Use initial conditions to evaluate the constants in the homogenous solution



* This method is basic yet powerful.
* But math is complex when input is complex

So we look at a method which simplifies the analysis of dynamic systems considerably for certain domain of Linear & LTI systems.

The method involves steady state anlalysis of <u>sinusoidal</u> drive. for any arbitrary signal can be constructed from linear combination of <u>sinusoids</u>. Since we are analyzing linear systems we can solve the problem of arbitrary input with superposition of sinusoid responses.

Moreover, a large vareity of analog circuits (eg. audio amps, filters, etc) are characterized by sinusoid response.

Solution to linear dynamic circuits are greatly simplified by assuming a drive of the form $\underline{e^{st}}$.

Because the differential eqns. transforms to a set of algebraic equation.

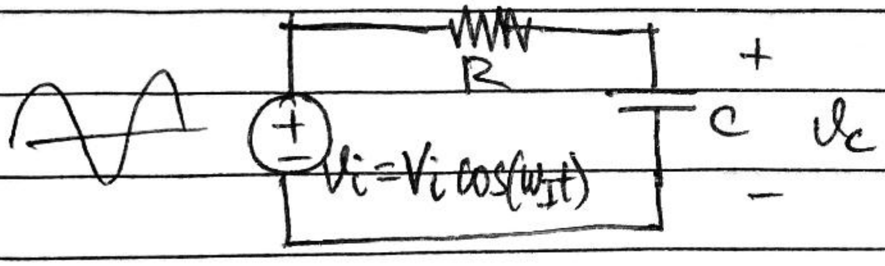
Why e^{st} ?

$$e^{st} \text{ (for } s=j\omega) \xRightarrow{\text{Euler Identity}} \underbrace{\cos(\omega t)}_{\text{real}} + j\underbrace{\sin(\omega t)}_{\text{imaginary}}$$

After finding the solution,

- the real part of the soln is due to $\cos(\omega t)$
- the imaginary part of the soln is due to $\sin(\omega t)$

EXAMPLE: FIRST-ORDER RC CIRCUIT



The differential equation for the ckt:

$$v_i = v_c + RC\frac{dv_c}{dt}$$

Solution: $v_c = v_{ch} + v_{cp}$

homogeneous → v_{ch}

particular → v_{cp}

On solving (with some trignometric stunt)

we get:

$$v_{ch} = K_1 e^{-t/RC}$$

&

$$v_{cp} = K_2 \sin(wt) + K_3 \cos(wt)$$

Nothing wrong with this method, just some math work involved which gets increasingly hard for complex ckts.

Alternatively:

The Euler relation

$$e^{jwt} = \underbrace{\cos(wt)}_{\text{wanted}} + j\underbrace{\sin(wt)}_{\text{unwanted}}$$

Let's use the stimulus

$$\tilde{v}_i = V_i e^{s_1 t} \quad \text{where } s_1 = jw_1$$

"~" signifies the input stimulus carries both the wanted (real) & unwanted (img)

The homogenous solution is the same.

The diff eqn. for the particular solution

$$\tilde{v}_i = V_i e^{s_1 t} = \tilde{v}_{cp} + RC \frac{d\tilde{v}_{cp}}{dt}$$

A reasonable assumption for $\tilde{v}_{cp}$

$$v_{cp} = V_c e^{st}$$

$$V_i e^{s_1 t} = V_c e^{st} + RC s V_c e^{st}$$

s must equal s_1 to satisfy the equation for all t.

∴ This where Magic happens!

∴ $e^{s_1 t}$ cannot be zero for positive values of t, it can be factored out. Resulting in,

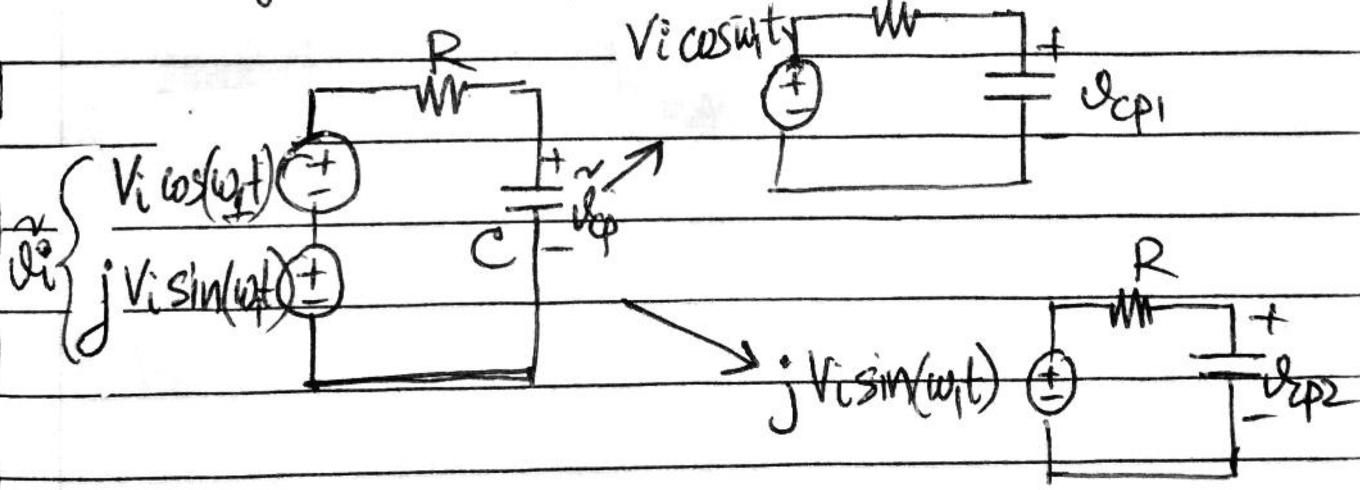
$$V_i = V_c + V_c RC s_1$$

So a differential equation gets converted to algebraic equation which simplifies the analysis dramatically.

The particular solution

$$\tilde{v}_{cp} = v_{cp1} + j\, v_{cp2} = \frac{V_i}{1 + j\omega_1 RC}\, e^{j\omega_1 t}$$

Diagramatically,



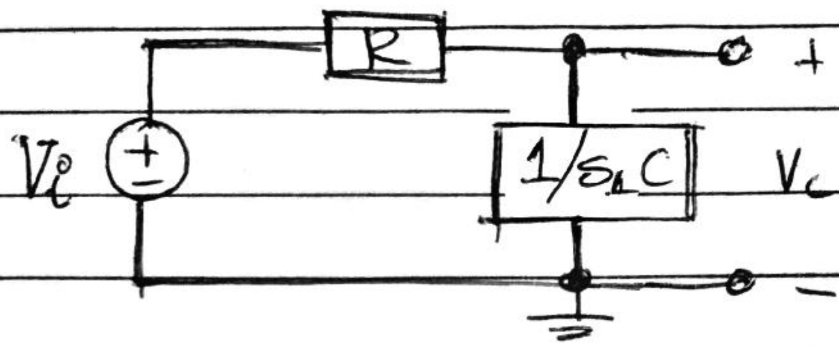
The real part of the particular soln.

$$v_{cp1} = \frac{V_i}{\sqrt{1 + (\omega_1 RC)^2}} \cos(\omega_1 t + \phi)$$

$$\phi = \tan^{-1}\left(\frac{-\omega RC}{1}\right)$$

which is the steady state solution to $V_i \cos(\omega_1 t)$.

A circuit interpretation of the above equation:



This is the basis of doing circuit analysis using Laplace transform (s-domain, frequency domain all diff names.).