SMALL-SIGNAL MODELING

Incremental Analysis

$i_D = f(v_D)$ ← Non-linear

To linearize, take Taylor series of

$$f(V_D + v_d) = I_D + i_d$$

neglect higher-order terms

$$= f(V_D) + \left.\frac{\partial f(v_d)}{\partial v_d}\right|_{V_D} \times v_d + \cdots$$

or,

$$i_d = \underbrace{\left.\frac{\partial f(v_d)}{\partial v_d}\right|_{V_D} \times v_d}_{\text{Linear term}}$$

SMALL-SIGNAL MODEL OF DIODE

$$i_D = I_S \cdot e^{(V_D + v_d)/V_T} = I_S \cdot e^{V_D/V_T} \cdot e^{v_d/V_T}$$

$$= I_D \cdot e^{v_d/V_T}$$

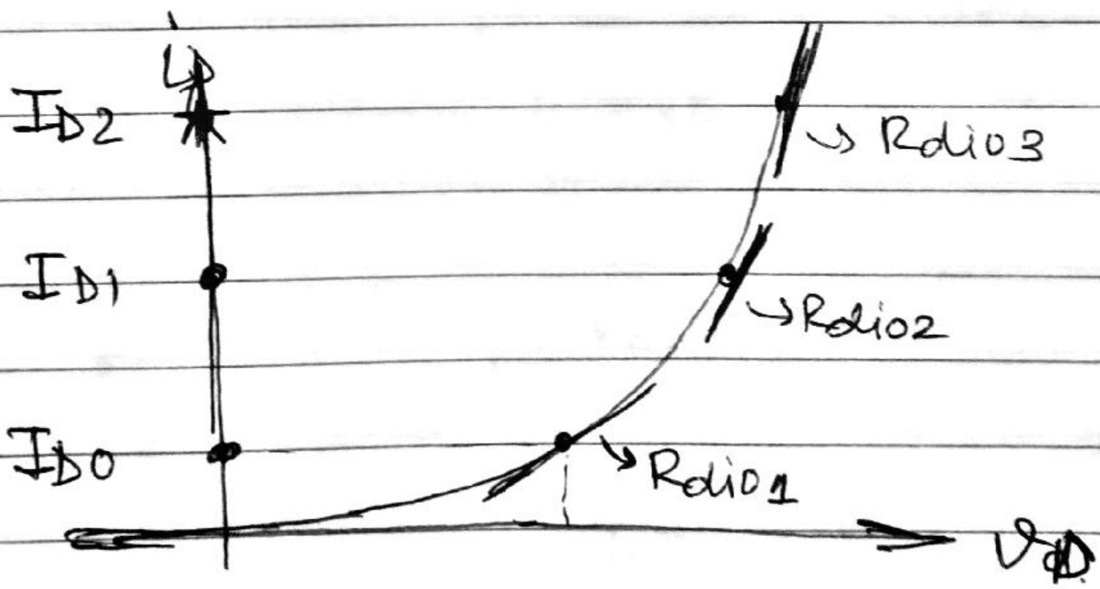
Taylor series of $e^{x} = 1 + x + \frac{x^2}{2!} + \dots$

for $|x| < 1$

$$\therefore i_D = \cancel{I_D} + i_d = \cancel{I_D} + \frac{I_D}{V_T} \cdot v_d$$

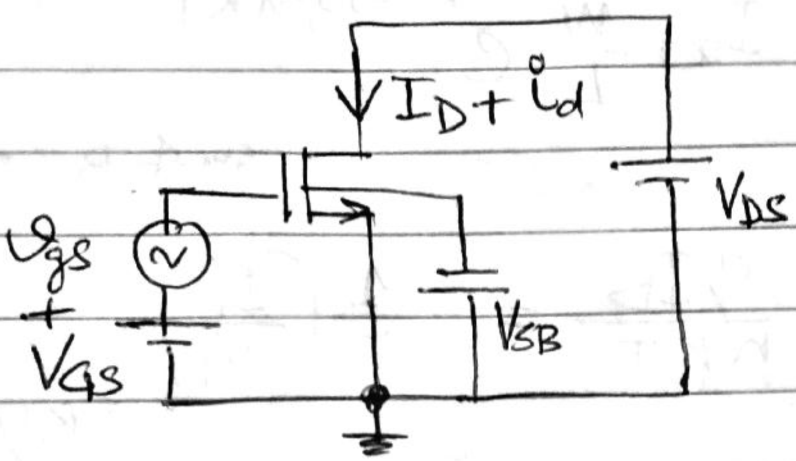
$$\therefore i_d \approx \frac{v_d}{V_T/I_D}$$

$V_T/I_D \rightarrow R_{dio}$

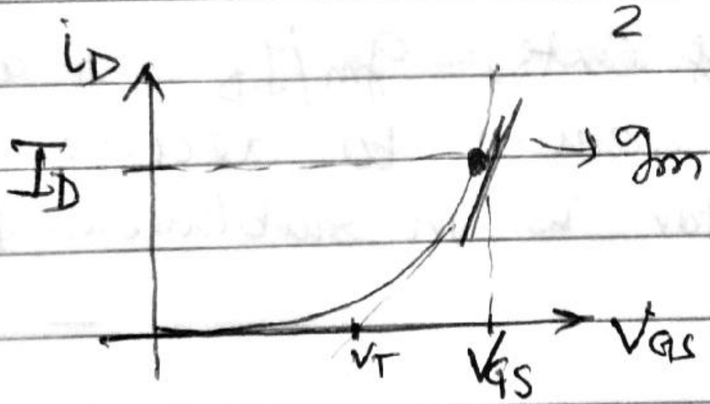


MOS Small-Signal Model.

Transconductance (g_m)



Saturation: $i_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (v_{GS} - V_{tn})^2$



$\rightarrow g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{V_{GS}}$

$$\left.\begin{aligned} g_m &= \mu_n C_{ox} \frac{W}{L} V_{eff} \\ g_m &= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \end{aligned}\right\} \rightarrow$$

Useful for analysis when W/L, I_D or V_{eff} is known.

$$\boxed{g_m = \frac{2 I_D}{V_{eff}}}$$

$\leftarrow$ Most useful for design since it separates I_D & headroom (V_{eff})

For subthreshold region:

$$I_D = I_{DO} \frac{W}{L} e^{qV_{eff}/nKT}$$

$$\therefore g_m = \frac{q I_D}{nKT} = \frac{1}{n}\left(\frac{I_D}{V_T}\right)$$

gm of BJT

KT/q

$$n = \frac{C_{ox} + C_{jo}}{C_{ox}} \approx 1.5$$

When designing ckts, g_m/I_D is a very useful parameter to decide if a transistor is in subthreshold or saturation

Subthreshold

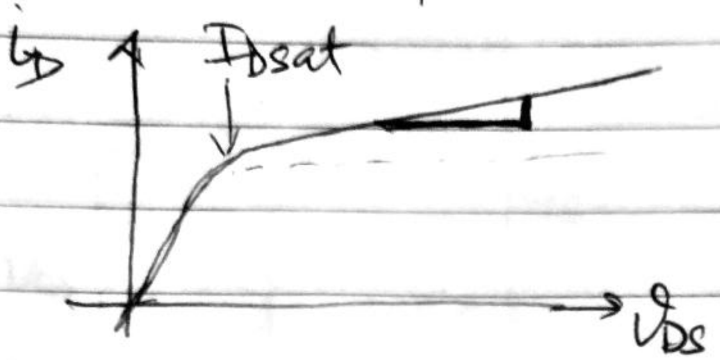
$$g_m/I_d = \frac{1}{n \cdot V_T} \simeq \frac{1}{1.5 \times 0.025} = 26$$

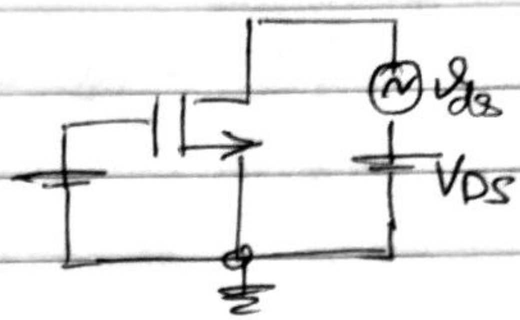
Using Inversion co-efficient $IC = I_D/I_S$

IC	0.1	0.5	1.0	5.0	10.0	20.0
g_m/I_d	28	22	19	11	8	6

deep subthreshold — moderate inversion — deep saturation

Output Impedance





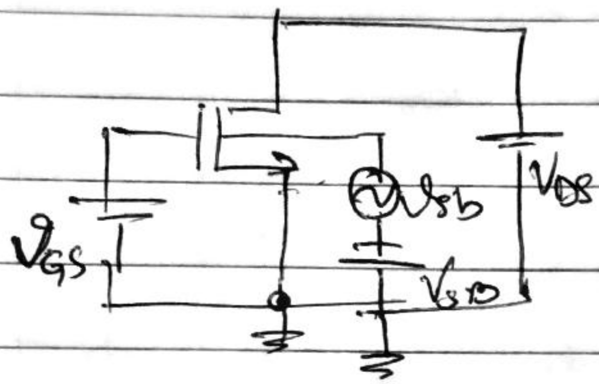
$$i_D = I_{dsat} \cdot [1 + \lambda(V_{DS} - V_{eff})]$$

$$g_{ds} = \frac{1}{r_{ds}} = \left.\frac{\partial I_{DS}}{\partial V_{DS}}\right|_{V_{DS}=V_{DSO}}$$

$$= \lambda I_{dsat} \simeq \lambda I_D$$

$$\therefore \boxed{r_{ds} \cong \frac{1}{\lambda I_D}}$$

Substrate transconductance (g_s, g_{mb})

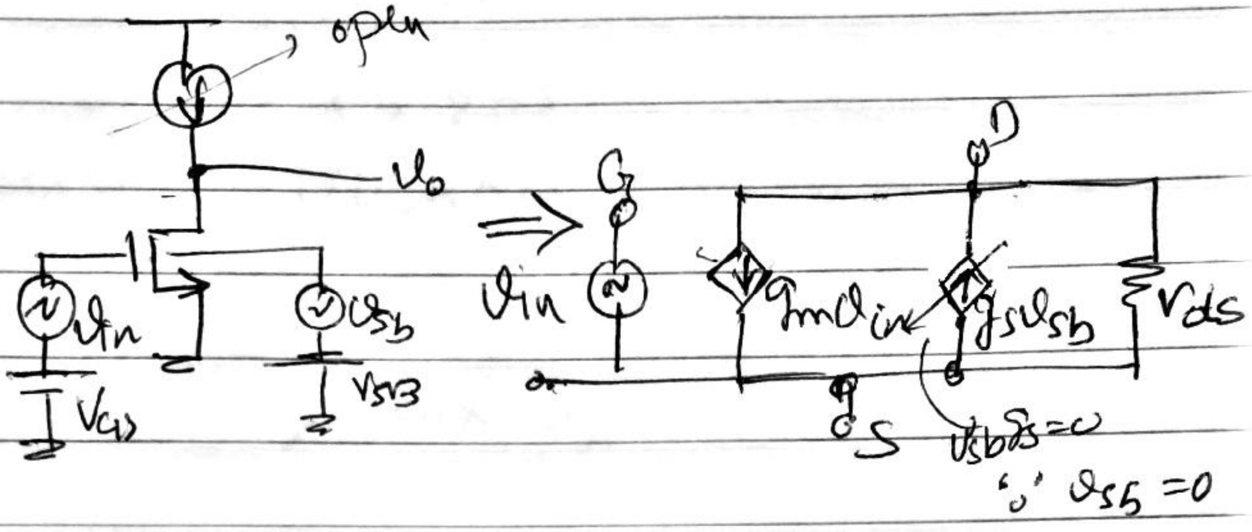


$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \left(V_{GS} - V_T(V_{SB})\right.$$

$$g_{mb}' = \left.\frac{\partial I_D}{\partial v_{SB}}\right|_{V_{SB}}$$

$$\boxed{g_{mb} = \frac{\gamma g_m}{2\sqrt{v_{SB} + 2|\phi_F|}}}$$

Low-frequency Small-signal Modeling



$$\boxed{A_i = \frac{v_o}{v_i} = g_m \cdot r_{ds}}$$

Problem 1.20 [Johns & Martin] 2nd Ed.

0.35μm NMOS to be biased with $I_D = 350\,\mu A$ with an intrinsic gain of $A_i = 35$. Find W, L

$$A_i = g_m \cdot r_{ds} = \frac{2 \cdot I_D}{V_{eff}} \times \frac{1}{\lambda I_D}$$

$$\Rightarrow 2/\lambda V_{eff} = 35$$

Given: $L = 0.35\,\mu m$ $\quad \lambda \cdot L = 0.16$

$$V_{eff} = \frac{2}{\lambda \cdot 35} = 0.125 = 125\ mV.$$

From Table 1.5 p53

$\mu_n C_{ox} = 190\ \mu A/V^2$

$$W/L = \frac{2 \cdot I_D}{\mu_n C_{ox} V_{eff}^2} = 235\ !!$$

Typically, Headroom is limited (ie, Vdsat is limited), a required gain decides ~~~~ L ~~~~.

Let's say Vdsat = 200 mV

$$\Rightarrow L = \frac{0.16 \times 35 \times 0.2}{2} = 0.56\ \mu m \qquad \left[\lambda = \frac{2}{35 \times V_{eff}} \quad \lambda \cdot L = 0.16 \right.$$

$$\frac{W}{L} = \frac{2 \times 350\ \mu A}{190\ \mu A/V^2 \times (0.2V)^2} = \underline{\underline{92}}$$