

$$\sigma_2(\langle A, B \rangle) = \pi_2(2^{\emptyset + \sum_{x \in A \cup B} \{x\}})$$

$$\pi_2(B) = \{h(x) \mid x \in B\}$$

$$h(\emptyset) = \emptyset$$

$$\{h(a) \mid (a \in A) \bigwedge (h(a) = a)\} = A$$

$$h(X) = \emptyset \cup \bigcup_{x \in X} \{2^{\{h(x)\}}\}$$

where h defines a mutually unambiguous mapping of generalized unoriented hereditarily finite sets into generalized ultra-unoriented hereditarily finite sets.

In this case:

$$A^{(+2)} \subseteq A^{(+1)}$$

Together with generalized unoriented finite sets, generalized ultra-unoriented finite sets can be mapped mutually unambiguously to inherited finite sets.

The classical mathematical model works with justified (grounded) sets, but some non-classical models correspond to other structures. While the structure of hereditarily finite sets is acyclic and involves various hierarchies, other (e.g., cholarchical [65]) structures consist of elements where each element is a part and is a whole (composed of parts). Some such structures can be visualized as periodic or cyclic structures. Another example of structures that may not fit within the framework of classical mathematical models are non-trivially automorphic structures, since such models adhere to the abstraction of identity. However, despite this, some of these “non-classical” structures can (under certain conditions) be represented by classical mathematical models. First of all, among such structures we will be interested in enumerable structures, i.e., such structures that can be enumerated.

Let us consider the representation of $A^{(+3)}$ in generalized hereditarily finite sets of the class of countably non-identically-equal generalized hereditarily finite sets on the alphabet \dot{A} , which can be given according to the expressions:

$$\tau_3(B) \stackrel{def}{=} B$$

$$\rho_3(\langle A, B \rangle) = C \times (A \cup B)^*$$

$$\sigma_3(\langle A, B \rangle) = D \times \pi_3(2^{\emptyset + \sum_{x \in A \cup B} \{x\}})$$

$$\pi_3(B) = B$$

$$a_{2k+1} = \begin{cases} \dot{a}_k \mid k \leq |\dot{A}| \\ d(2 * k + 1) \mid k > |\dot{A}| \end{cases}$$

$$a_{2k} = d(2 * k)$$

$$C \cup D = E$$

$$E = \{a_{2k} \mid (a_{2k} \in \dot{A}) \bigwedge (k \in \mathbb{N})\}$$

$$E \subseteq A$$

If

$$C \cap D = \emptyset$$

then the representations of oriented sets in $A^{(+3)}$ will not intersect with the representations of unoriented sets in $A^{(+3)}$

Fulfilled:

$$A^{(+3)} \subseteq A^{(+1)}$$

Together with generalized finite sets, representations of countable non-identically-equal generalized hereditarily finite sets can be mapped one-to-one onto hereditarily finite sets.

The representation of $A^{(+4)}$ is analogous to $A^{(+2)}$.

The representation of classes of unorient mixed countably non-identically-equal generalized hereditarily finite sets can be given according to the expressions:

$$\tau_5(A) = (2^{\emptyset + \sum_{x \in A} \{x\}})$$

$$\rho_5 = \rho_3$$

$$\sigma_5 = \sigma_3$$

$$\pi_5 = \pi_3$$

The representation of classes of orient mixed countably non-identically-equal generalized hereditarily finite sets can be given according to the expressions:

$$\tau_6(A) = A^{(*1)}$$

$$\rho_6 = \rho_3$$

$$\sigma_6 = \sigma_3$$

$$\pi_6 = \pi_3$$

Consider the representation of: $A^{(+6)}$ in generalized hereditarily finite sets of the class of countably nonidentically-equal generalized hereditarily finite sets on the alphabet \dot{A} with codes \ddot{A} , which can be given according to expressions:

$$\tau_7(B) \stackrel{def}{=} B$$

$$\rho_7(\langle A, B \rangle) = C \times (A \cup B)^*$$

$$\sigma_7(\langle A, B \rangle) = D \times \pi_7(2^{\emptyset + \sum_{x \in A \cup B} \{x\}})$$

$$\pi_7(B) = B$$

$$a_{2k+1} = \begin{cases} \dot{a}_k \mid k \leq |\dot{A}| \\ d(2 * k + 1) \mid k > |\dot{A}| \end{cases}$$

$$a_{2k} = d(2 * k)$$

$$C \cup D = E$$

$$E = \{a_{2k} \mid (a_{2k} \in \dot{A}) \bigwedge (k \in \mathbb{N})\}$$

$$E \subseteq A$$

where

$$\ddot{A} \cap \mathbb{N} \neq \emptyset$$

If a number in \ddot{A} is the Ackermann coding of the corresponding set in the representation of

enumerably selffounded generalized hereditarily finite sets by hereditarily finite sets, then such a set is self-founded, i.e. this set is considered as an element of the alphabet at the same position in \dot{A} as its code \ddot{A} .

To ensure the enumerability of self-founded generalized hereditarily finite sets, it is required to ensure that they are all finitely mutually-founded, that is, that they are not infinitely mutually-founded. In this case, the extensional closure will be reduced to a finite structure and there will be an algorithm for comparing these structures provided they are reduced to canonical form.

In general, it is not possible at this stage to canonize the representation of a structure that is a union of structures of all sets of a given class. To canonize the representation of such structures requires a separate investigation of the conditions under which this may be possible. The embedding of structures in hereditarily finite sets and natural numbers (Fig. 1), using Ackermann coding, gives a structure isomorphic to the Rado graph [62] which is universal for any graph, i.e. it allows us to isomorphically enclose any graph and its supergraphs. Stability is one of the important properties of Rado Graph has. There are known studies of universal uncountable structures and corresponding theories including studies of the property of their stability [60]. As for uncountable structures [31] associated with operational semantics [16], [55] one direction of research is to study an approach based on the use of decision procedures without the presence of enumeration procedures as well as the use of Büchi automata [61] and their hierarchy.

Using the considered classes of structures, one can represent arbitrary finite graph, pseudograph, multigraph, metagraph, hypergraph structures, including abstract simplicial complexes [28], their combinations and others.

The applied value of the considered classes is the possibility of algorithmic construction of canonical forms (representations) of knowledge structures.

IV. Similarity, proximity, other attribute and invariants of structures of meaning space and corresponding models

The analysis of structural properties implies consideration of topological relations and relations of similarity (similarity and analogies) and difference. These relations can be algorithmically realized within the framework of the knowledge specification model in accordance with the knowledge integration model. The knowledge specification model, by considering finite structures, allows for the decidability of the corresponding analysis algorithms.

As for the similarity relations, they can be formed from property detection relations or non-detection relations. A property detection relation itself can be a similarity relation when the property itself is revealed in its specification (i.e., the relation is reflexive); as a rule, such relations are transitive. If a property is not revealed in its specification, but is revealed in other structures, then all these structures can be united into a class of (pairwise) similar structures. In the first case, such relations can be reduced to the identification of full or partial embeddings, and morphisms: isomorphic, homomorphic or homeomorphic embedding.

The composition of two binary similarity relations is a binary similarity relation.

The union of two *ij*-similarity relations is a *ij*-similarity relation.

The intersection of two *ij*-similarity relations is a *ij*-similarity relation.

The difference of a *ij*-similarity relation and a *ij*-difference relation is a *ij*-similarity relation.

The difference of two *ij*-similarity relations, the first of which is a subset of the second, is a *ij*-difference relation.

The difference of a *ij*-difference relation and a second relation is a *ij*-difference relation.

The difference of two *ij*-difference relations is a *ij*-difference relation.

The union of two *ij*-difference relations is a *ij*-difference relation.

The intersection of two *ij*-difference relations is a *ij*-difference relation.

In the second case, similarity can be formed as an equivalence relation or a partial order relation on the set of structures for which no enumerated embeddings are identified (in the first case).

Topological properties [47] can be investigated through the consideration of operations on structures that correspond to topological closure.

Among the topological similarities we can distinguish: similarity on the inclusion of closures, similarity on the inclusion of derived sets, similarity on the inclusion of touch points and others.

Among topological similarities we can distinguish: similarity by equivalence of closures, similarity by equivalence of derived sets, similarity by equivalence of touch points and others.

In addition to search operations for extensional closures, we can identify closure operations on the class of automorphic elements of the structure.

It is also possible to identify search operations by a single (simple) pattern (reverse homomorphism) in extensional closures distancedefined neighborhood of a concept, and

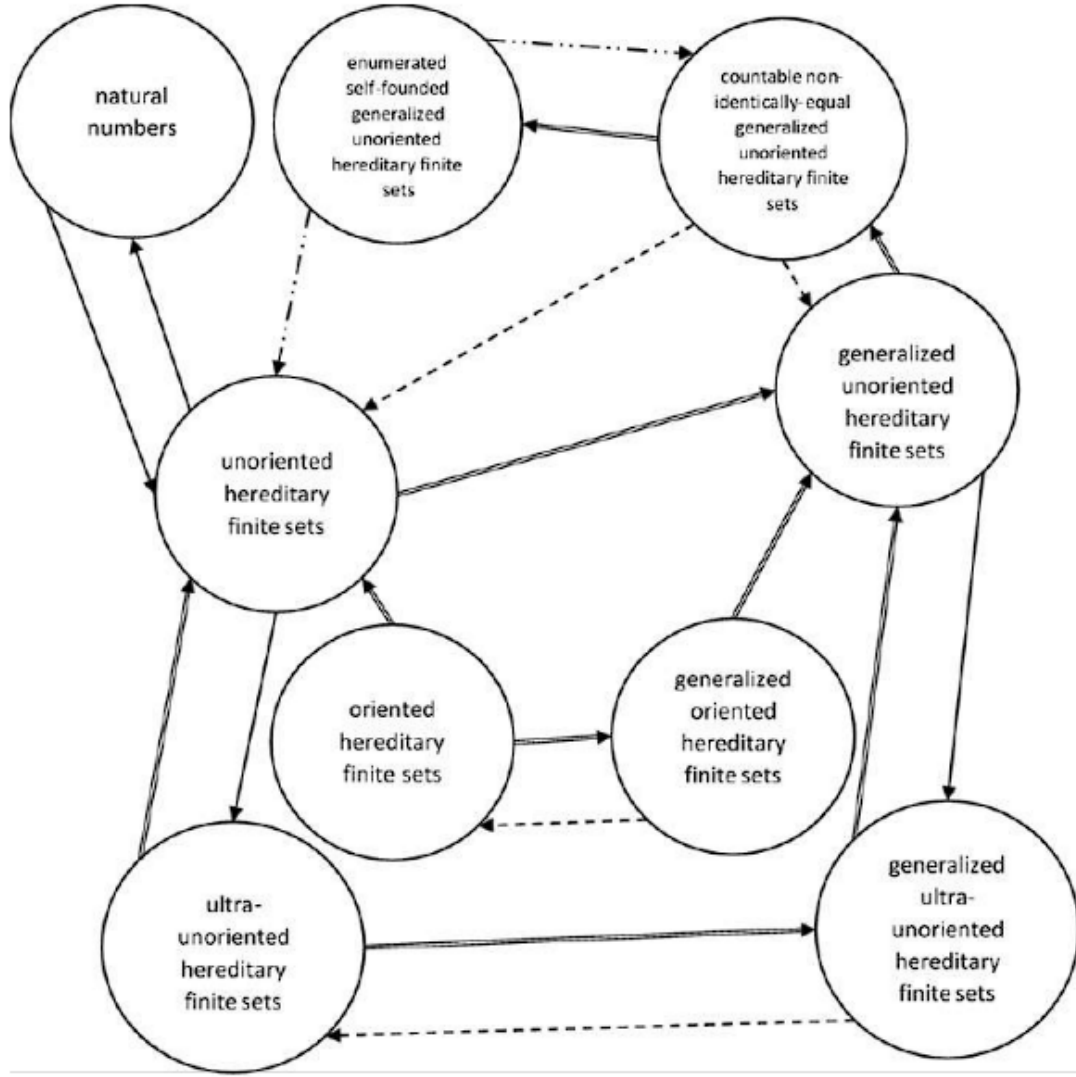


FIG. 1: General finite structures classes.

all elements incident to the concept must be all elements of the automorphism class in this neighborhood. Among such neighborhoods we can choose a maximal neighborhood. The corresponding neighborhood can be considered as a closed set if it is not included in any other such neighborhood. In this case, the search operation on the corresponding pattern can also be considered as a closure operation.

The number of mappings of a pseudograph to the ordinal scale distinguishable with precision up to the order of vertices does not exceed $|V|^{|V|}$.

The number of mappings of the pseudograph to the metric scale distinguishable with order-of-magnitude accuracy of pairwise distances does not exceed $(|V^2| - |V|)^{(|V^2| - |V|)}$ (k -ary generalized distances - $(|V^k| - |V|)^{(|V^k| - |V|)}$).

For finite pseudographs and other representations of G for which the smallest A is found:

$$G \in A^{(+*)}$$

a quantitative attribute model can be based on the following class of quantitative feature:

$$\mathbb{N}r_+^{A(+*)}$$

where

$$\mathbb{R} \subseteq \mathbb{N}r$$

All closures defined on finite structures are finite. Finite structures, including finite (extensional) closures, have the following (global) characteristics: length, width, graph dimension, neg-entropy, sets (sets) of local characteristics and others. Global characteristics can be considered as invariants [17], [68], on the basis of which characteristics similarity or proximity measures (metrics or pseudo-metrics) can be calculated. Local (conditional) numerical characteristics include: centrality, metrics [10] and others. Another example of numerical features are measures of scalar and coscalar product for introorthogonal sets considered in taxonomy management problems [].