# Bibliographies Main LaTeX Document 1.0.0

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# 1 Mathematics

# 1.1 How to Learn our Mathematics

So you want to learn math. Fantastic! Math is a wonderful but grueling subject, which is why here we make sure you can have all the resources at your disposal to make sure you either get that A in your class, make math really easy or make sure you really, really know your math to become a mathematician. But say you're our general audience, you're most likely an an Engineering student. Do you really need to learn about topology or abstract algebra? Nope. So this is how to use our math and our suggested guide. Enjoy!

# 1.1.1 Engineering

# MechE/Aero/Astro/ChemE/Civil/CompE

- Basic Algebra
- Pre-Calculus
- Methods of Proof Yes proof techniques for an engineer, it will help with understanding proofs in Linear algebra and Multivariable Calculus
- Single Variable Calculus
- Multivariable Calculus
- Linear Algebra
- Ordinary Differential Equations
- Statistics for Engineering and the Sciences
- Partial Differential Equations Yes you can learn PDE without Real Analysis

# Nuclear/Electrical/ECE

- Basic Algebra
- Pre-Calculus
- Methods of Proof Yes proof techniques for an engineer, it will help understanding proofs in Linear algebra and Multivariable Calculus
- Single Variable Calculus
- Multivariable Calculus
- Linear Algebra
- Ordinary Differential Equations

- Statistics for Engineering and the Sciences
- Partial Differential Equations Yes you can learn PDE without Real Analysis
- Complex Analysis

#### 1.1.2 Sciences

# **Physics**

- Basic Algebra
- Pre-Calculus
- Methods of Proof
- Single Variable Calculus
- Multivariable Calculus
- Calculus of Variations
- Linear Algebra
- Ordinary Differential Equations
- Partial Differential Equations
- Tensor Calculus
- Statistics for Engineering and the Sciences
- Set Theory
- Real Analysis
- Complex Analysis
- Topology
- Differential Geometry

# Chemistry/Biology

- Pre-Calculus
- Methods of Proof
- Single Variable Calculus
- Multivariable Calculus
- Statistics for Engineering and the Sciences

- Linear Algebra Not necessary as far as I'm aware, but a very helpful field of mathematics, and some techniques are used in Ordinary Differential Equations
- Ordinary Differential Equations
- Partial Differential Equations Only if you're into computations

# Computer Science

- Pre-Calculus
- Methods of Proof
- Single Variable Calculus
- Statistics
- Discrete Mathematics
- Linear Algebra

#### 1.1.3 Mathematics

- Basic Algebra
- Pre-Calculus
- Methods of Proof
- Set Theory
- Single Variable Calculus
- Multivariable Calculus
- Linear Algebra
- Ordinary Differential Equations
- Statistics & Probability
- Real Analysis
- Partial Differential Equations
- Complex Analysis
- Topology
- Tensor Calculus
- Differential Geometry
- Calculus of Variations

# 1.2 Basic Algebra

Basic (or elementary) algebra extends arithmetic by introducing symbols known as variables that do not represent a specific number but any number to be inserted later. The goal of algebra is to manipulate expressions that involve these variables in order to study general relationships. For example, the equation can be used to express that the area of any rectangle is equal to its length times its width - replacing and with specific measurements will find the area of an actual rectangle. Using algebra, this equation can be changed into , which tells us that the length of any rectangle is equal to its area divided by its width. The ability to easily manipulate algebraic equations in a variety of ways is essential to studying more complex mathematical techniques.

Basic algebra should be distinguished from algebra in general, which is a branch of mathematics that manipulates symbols in the context of more complex structures with different properties than ordinary numbers; this more advanced field is sometimes called modern (or abstract) algebra.

**Prerequisites:** Readers who wish to study basic algebra must have mastered arithmetic. They should have the basic mathematical facts (one-digit addition, subtraction.

multiplication, and division) memorized. If these have not been memorized, readers should practice these math facts using flashcards until they can be recited automatically. The most common reason for difficulty in learning algebra is not having a sufficiently strong foundation in basic arithmetic.

#### Where to Start:

Readers should obtain a mathematically-rigorous introductory textbook appropriate to their current level. Textbooks must be read chapter-by-chapter, and it is extremely important to work as many problems found in this text as possible. Just as you can only achieve fluency in a language by speaking it frequently, you can only achieve proficiency in algebra by using it to solve a large number of problems. It may be helpful to purchase additional textbooks that provide additional problems or alternative explanations; there are also online tutorials and videos that might be helpful.

By the end of a study of basic algebra, it is very important that readers be able to manipulate algebraic expressions and equations with fluency. Readers who go on to study higher math will have to simplify and solve algebraic equations while applying more complex techniques, and unless it is second nature, readers will struggle with the algebra instead of learning the new techniques.

After mastering algebra, the next goal is calculus, which is the mathematical foundation of science and its laws. It is possible to go from algebra directly into calculus, but readers may benefit from studying precalculus first - the idea behind this is to cover important topics in trigonometry and analytic geometry beforehand so that students can focus exclusively on the calculus. Going on to study calculus will enable readers to begin learning the physical sciences and engineering

#### **Books:**

- Blitzer, Robert F. Introductory Algebra for College Students. Pearson: 2012, 6th ed. BA:1 (first half of a good introductory textbook, covers the basics has a solutions manual available)
- Blitzer, Robert F. Intermediate Algebra for College Students. Pearson: 2012, 6th ed. BA:2 (second part of an introduction to algebra, has a solutions manual available)
- Bullock, Gregory. Algebra in Words: A Guide of Hints, Strategies and Simple Explanations. Acute Books: 2014. BA:3 (conceptual explanations for topics, great supplement to rigorous text)
- Bullock, Gregory. Algebra in Words 2: MORE Hints, Strategies and Simple Explanations. Acute Books: 2014. BA:4
- Gelfand, Israel M. and Shen, Alexander. **Algebra**. Birkhäuser: 2002. **BA:5** (a highly-recommended introductory text)
- Huettenmueller, Rhonda. **Algebra DeMYSTiFieD**. McGraw-Hill Professional: 2010, 2nd ed. **BA:6** (a good introductory text)
- Huettenmueller, Rhonda. College Algebra DeMYSTiFieD. McGraw-Hill Professional: 2013, 2nd ed. BA:7 (more advanced introductory text)
- Kelley, W. Michael. **The Humongous Book of Algebra Problems**. ALPHA: 2008 **BA:8** (illustrates the important techniques of basic algebra through many series of example problems)
- Larson, Ron. College Algebra. Brooks Cole: 2013, 9th ed. BA:9 (an introductory text recommended for more advanced readers)
- McMullen, Chris. Algebra Essentials Practice Workbook with Answers. BA:10 (contains basic explanations of key topics and many practice problems to solve)
- Rappaport, Josh. Algebra Survival Guide: A Conversational Guide for the Thoroughly Befuddled. Singing Turtle Press: 2000, 1st ed. BA:11 (clear, simple explanation of the basics - there is a workbook available filled with practice problems)
- Selby, Peter H. and Slavin, Steve. **Practical Algebra: A Self-Teaching Guide**. John Wiley & Sons: 1991, 2nd ed. **BA:12** (might be useful for those looking for an algebra refresher)
- Sterling, Mary Jane. **Algebra I for Dummies**. For Dummies: 2010, 2nd ed. **BA:13** (has an accompanying workbook)

• Sterling, Mary Jane. **Algebra II for Dummies**. For Dummies: 2006, 1st ed. **BA:14** (has an accompanying workbook and book of practice problems)

#### **Articles:**

#### Videos:

- Delaware's "College Algebra" lectures (UM -Kansas City)
- KhanAcademy -Algebra basics
- KhanAcademy -Algebra I
- KhanAcademy Algebra II
- Leonard's "Intermediate Algebra" lectures (good discussion of some algebra concepts with worked examples)
- Ron Cox, "Basic Algebra Part 1 and 2"

#### Other Online Sources:

- algebrafree.com (contains worksheets with practice problems and explanations for basic concepts videos seem to be missing, however)
- artofproblemsolving.com's textbooks (a set of textbooks designed to supplement the standard primary school math curriculum for students entering math competitions)
- Chen and Duong's "Elementary Mathematics" lecture notes (Macquarie University)
- Coolmath.com guides (simple explanations of selected topics with worked examples)
- Kubota's College Algebra course notes
- math.com's algebra practice (work on the fundamentals of algebra with some good practice problems)
- $\bullet$  mathopolis.com Algebra 1 Math Skills Practice (choose a topic and get practice problems)
- mathopolis.com Algebra 2 Math Skills Practice (choose a topic and get practice problems)
- Paul's Online Math Notes: Algebra Practice Problems (practice problems with solutions)
- Purplemath modules (short explanations of algebra topics with problems to solve)

- West Texas A&M's VirtualMathLab College Algebra (tutorials on important topics in algebra, graphing and several important functions; includes practice tests with answer keys see also the beginning algebra and intermediate algebra pages)
- List of interactive math websites at /r/learnmath
- /r/math
- /r/learnmath
- /r/mathbooks
- /r/askmath

**Subtopics:** 

Lectures:

Problem Sets:

Exams:

Other Online Sources:

## 1.3 Pre-Calculus

Precalculus encompasses mathematical knowledge useful to those who have taken high school algebra and are preparing to learn university-level calculus. Although readers who have taken algebra *can* move straight into calculus, it is recommended to learn the important background topics from precalculus ahead of time so that readers can focus exclusively on the concepts of calculus. The topics that should be studied in precalculus course can be grouped into three subjects: algebra, trigonometry, and analytic geometry (which should cover the definition of a function).

## Prerequisites:

Readers should have a solid grasp of arithmetic before attempting to prepare for calculus. It is very important to have basic arithmetic facts memorized - if you struggle with arithmetic, you'll struggle with algebra. If you struggle with algebra, you'll struggle with calculus and so on until you decide that you are someone who is "not good at math". But there are no people who are inherently bad at math, only those who lack sufficient preparation. If you don't have the arithmetic tables memorized, it is very important to get a deck of flashcards and practice until they're automatic.

Readers should also have a basic familiarity with algebra. You should understand the basic rules of algebra and be able to manipulate equations, but may still need to write down every step in solving an equation. Algebra should be practiced diligently alongside the newer topics in trigonometry and analytic geometry. By the time you finish precalculus, you should be able to do algebra quickly and easily.

#### Where to Start:

Readers should obtain a precalculus textbook and work through each of the important topics chapter-by-chapter, solving as many problems as possible at the end of each section. As these books can be pricy, readers may want to purchase older editions, which will be far less expensive. A good preparation for calculus involves three topics - algebra, trigonometry, and analytic geometry. Standard precalculus texts do not focus on algebra, so if more practice with algebra is needed, it is recommended that you also pick up a supplementary text (see also the Basic Algebra section). Readers who are new to mathematics may find some textbook explanations difficult - use supplemental videos and online materials to get additional information on topics you find difficult. But as with any mathematical technique, the only way to learn is by solving many problems - be sure to work as many problems as possible from your textbook.

More than anything else, the key to getting prepared for a college-level calculus is being able to manipulate algebraic expressions with fluency. Readers who struggle with symbols and equations, won't be focused on learning the underlying concepts in calculus. So it is very important to be comfortable with algebra before starting calculus. And the only way to do this is to practice algebra correctly - play with the algebra - until it feels natural. You should be able to look at equations like " $2/3 \times 9 = 5$ " and see how these numbers move from one side to the other to end up with "x = 21". Practice your algebra diligently

and you will set yourself up for success in calculus.

Trigonometry is encountered in calculus primarily because of its importance in physics and higher math - it is not essential to the concepts of elementary calculus, but will be encountered in problems and examples. Readers should understand what trigonometric ratios are and be able to explain what sine, cosine, and tangent mean using a right triangle inscribed within a unit circle. It will also be helpful to learn how to simplify trigonometric expressions using the most important identities.

In basic Algebraic Geometry, algebraic equations are studied by graphing them in the Cartesian coordinate system. Readers should at least learn the definition of a function, how to graph a function, how to interpret and work with graphs, and the functions associated with the conic sections (e.g. the parabola). There are a standard set of graphs and functions used in calculus as examples such as parabolas, hyperbolas, and the trigonometric functions, and readers should become familiar enough with these to be able to draw them on a graph from their algebraic form.

#### **Books:**

- Axler, Sheldon. **Algebra and Trigonometry**. Wiley: 2011, 1st ed. (contains worked solutions to problems, recommended for self-study; note that text and Axler's Precalculus cover nearly-identical content with slightly different focus)
- Axler, Sheldon. Precalculus: A Prelude to Calculus. Wiley: 2012, 2nd ed. (contains worked solutions to problems, recommended for selfstudy)
- Kelley, W. Michael. **The Humongous Book of Algebra Problems**. ALPHA: 2008 (illustrates the important techniques of basic algebra through many series of example problems)
- Kelley, W. Michael. **The Humongous Book of Trigonometry Problems**. ALPHA: 2008 (explains basic trigonometry through example problems)
- Kuang, Yang and Kase, Elleyne. **Pre-Calculus for Dummies**. For Dummies: 2012, 2nd ed. (Not as detailed as a textbook, but covers the essentials has an accompanying workbook of problems)
- Larson, Ron. **Precalculus**. Cengage Learning: 2010, '008 ed. (good, detailed coverage of trigonometry, analytic geometry, and functions with useful examples possibly the best precalculus text to use)
- McKeague, Charles P. and Turner, Mark D. **Trigonometry**. Brooks Cole: 2012, 7th ed. (an in-depth textbook covering all the essentials of trigonometry with clear explanations and examples)

- Neill, Hugh. Trigonometry A Complete Introduction: A Teach Yourself Guide. McGraw-Hill: 2013, 2nd ed. (many students have trouble with trigonometry this supplementary text may be helpful)
- Selby, Peter H. and Slavin, Steve. Practical Algebra: A Self-Teaching Guide. John Wiley & Sons: 1991, 2nd ed. (might be useful for those looking for an algebra refresher)
- Simmons, George F. Precalculus Mathematics in a Nutshell: Geometry, Algebra, Trigonometry. Wipf & Stock Publishers: 2003. (might be useful as a refresher course in precalculus mathematics, but not for those who have never studied these concepts)
- Sterling, Mary Jane. **Pre-Calculus for Dummies**. For Dummies: 2014, 1st ed. (1,001 extra problems covering the important topics, with solutions)
- Stewart, James; Redlin, Lothar; and Watson, Saleem. **Precalculus:** Mathematics for Calculus. Brooks Cole: 2011, 6th ed. (good coverage of topics, but explanations are a bit terse for the intended audience and might go into too much detail in some areas if you use Stewart, work lots of problems but don't get bogged down)

#### Articles:

#### Videos:

- Delaware's "College Algebra" lectures (UM Kansas City)
- KhanAcademy Algebra basics
- KhanAcademy Algebra I
- KhanAcademy Algebra II
- KhanAcademy Trigonometry
- KhanAcademy Precalculus (these topics are actually more advanced than needed for elementary calculus, but interesting nonetheless and they will be useful after calculus)
- Leonard's "Intermediate Algebra" lectures (good discussion of some algebra concepts with worked examples)
- midnight tutor's "Trigonometry: The Essentials that You Need for Calculus" (comments)
- Ron Cox, "Basic Algebra Part 1 and 2"

#### Other Online Sources:

- Chen and Duong's "Elementary Mathematics" lecture notes (Macquarie University)
- coolmath.com's pre-calculus pages
- Joyce's "Dave's Short Trig Course" (Clark University)
- Lamar University's "Trig Cheat Sheet"
- math.com's algebra practice (brush up on the fundamentals of algebra with some good practice problems)
- mathisfun.com's "Introduction to Trigonometry" page
- mathisfun.com's "Random Trigonometry Problems" (generates trigonometry problems to practice)
- Mueller's "Exploring Precalculus" page (has a few good conceptual explanations of functions and rates of change, and the "Am I Ready for Calculus?" page is a good read)
- New Planet School's YouTube videos on trigonometry
- Paul's Online Math Notes: Algebra Practice Problems (practice problems with solutions)
- Purplemath modules (short explanations of algebra and trigonometry topics with problems to solve)
- themathpage.com's "Topics in Precalculus" (focuses on functions, has somewhat terse explanations but includes review questions)
- West Texas A&M's VirtualMathLab College Algebra (tutorials on important topics in algebra, graphing and several important functions; includes practice tests with answer keys see also the beginning algebra and intermediate algebra pages)
- List of interactive math websites at /r/learnmath
- /r/math
- /r/learnmath
- /r/mathbooks
- /r/askmath

# **Subtopics:**

#### 1.4 Methods of Proof

# Preliminary:

Proof is essential to the structure of mathematics; it provides mathematical statements with a certainty that is impossible in virtually every other field of intellectual inquiry. A valid proof provides an absolute link between established axioms and truths of mathematics and a new piece of mathematical knowledge known as a theorem. Proficiency with these techniques is a prerequisite to the study of higher mathematics. This bibliography covers the basic methods that are used to contruct a proof of a theorem, while proof theory, computer-assisted proof, and other topics in mathematical logic are outside its scope.

#### Prerequisites:

Readers can study methods of proof without any prior knowledge. However, familiarity with basic propositional and first order logic may be helpful, since proofs are essentially informal arguments with an underlying formal logical structure. For example, one of the basic proof techniques is proving the contrapositive rather than the original statement of a theorem, and readers who have studied logic will immediately understand why the contrapositive is logically equivalent to the conditional statement itself. Many introductory proof textbooks will contain these aspects of formal logic, so a separate study is not strictly necessary.

It is difficult to demonstrate the methods of proof without having something to prove, and so different introductory texts will typically assume (or explain) some background mathematical knowledge. Readers should check that the sources they use do not assume too much knowledge beyond their current level; however this will not usually pose an insurmountable problem for those familiar with elementary mathematics and algebra.

#### Where to Start:

Readers wanting to learn how to construct proofs should obtain an introductory textbook. Proof techniques should be learned in two steps: first understand how the strategy works, then use that technique to prove simple mathematical statements until the proof strategy becomes second nature. For example, to understand proof by contradiction you must first understand the idea behind the technique - statements can only be true or false, so if you can demonstrate that it is impossible for a statement to be false by deriving a contradicton, then the statement must be true - then practice it by proving statements; the classic example of proof by contradiction is the proof that the square root of two is irrational: if you assume that the square root of two is a reduced fraction a/b, you can show that a,b must have the factor 2 in common, which contradicts the assumption that a/b is a reduced fraction, and therefore the square root of two must be irrational. Choose many simple mathematical statements and practice using each strategy several times.

Readers who complete a study of proof methods should understand the conditional structure of theorems, understand how to write concise proofs, and know the following proof methods: direct proof, proof by contradiction, proving the contrapositive, proof by exhaustion (cases), existence and uniqueness proofs,

universal and existential quantifiers and counterexamples, proving biconditional statements, and mathematical induction. After completing this study, readers will be prepared to study formal mathematics, although it is advisible to study basic math through elementary calculus before beginning work on pure mathematics. Good places to start are real analysis, discrete mathematics, or number theory.

#### **Books:**

- Chartrand, Gary; Polimeni, Albert D.; and Zhang, Ping. Mathematical Proofs: A Transition to Advanced Mathematics. Pearson: 2012, 3rd ed. (many recommendations)
- Hammack, Richard. **Book of Proof**. Self-published: 2013, revised edition. (available online here)
- Houston, Kevin. How to Think Like a Mathematician: A Companion to Undergraduate Mathematics. Cambridge University Press: 2009, 1st ed.
- Polya, G. and Conway, John H. How to Solve It: A New Aspect of Mathematical Method. Princeton University Press: 2014, reprint edition. (a classic on mathematical problem solving, but not specifically proof techniques good as a supplementary text)
- Solow, Daniel. How to Read and Do Proofs: An Introduction to Mathematical Thought Processes. Wiley: 2013, 6th ed. (highly recommended)
- Taylor, John and Garnier, Rowan. **Understanding Mathematical Proof**. Chapman and Hall/CRC: 2014, 1st ed.
- Velleman, Daniel J. **How to Prove It**. Cambridge University Press: 2006, 2nd ed.
- Wolf, Robert S. Proof, Logic, and Conjecture: The Mathematician's Toolbox. W. H. Freeman: 1998.

#### **Articles:**

- Jensen-Vallin, Jacqueline A. "Notes for a Course on Proofs". JIBLM 27 (2012). (filled with good, simple mathematical statements to prove)
- Taylor, Ron. "Introduction to Proof". JIBLM 4 (2007). (a short introduction to proof)

#### Videos:

• Shillito's Introduction to Higher Mathematics videos, "Lecture 4: Proof Techniques"

• Shillito's Introduction to Higher Mathematics videos, "Lecture 7: More Proof Techniques"

# Other Online Sources:

- Aboutabl's "Methods of Proof" notes
- Binegar's Analysis lecture notes (lectures 1-4) (Oklahoma State)
- California State University "Notes on Methods of Proof"
- Chakrabarty's "Proofs by Contradiction and by Mathematical Induction" notes (Dartmouth) (explanation and examples of direct proof, indirect proof, and induction)
- Coursera's "Introduction to Mathematical Thinking" (Stanford)
- Cusick's "How to Write Proofs" (Fresno State)
- Gallian's "Advice for Students Learning Proofs" (Minnesota-Duluth)
- Hagen's "Tips for Proofs" (Virginia Tech)
- Hayes' "Proof by Induction" (UCLA) (good overview of theory behind induction proofs)
- Hefferon's "Introduction to Proofs, an Inquiry-Based Approach" (St Michael's College) (learn proofs by working through selected proofs)
- Heil's "Writing Proofs" (Georgia Tech)
- Henning's "An Introduction to Logic and Proof Techniques" (KwaZulu-Natal)
- Hsu's "Writing Proofs" (San Jose State)
- Hutchings' "Introduction to Mathematical Arguments" (Berkeley)
- Lee's "Some Remarks on Writing Mathematical Proofs" (University of Washington)
- Meyer's "Mathematics for Computer Science" notes, chapter 1 (MIT)
- Pitman's "Notes on Proof Techniques (Other Than Induction)" (Cortland)
- Sundstrom's "Mathematical Reasoning: Writing and Proof" (Grand Valley State) (an open textbook on proofs, contains a good list of proof guidelines in the appendix)
- Wilde's "Math Camp Notes: Basic Proof Techniques" (South Florida) (good summary with a few mathematical statements to practice proofs)
- /r/mathriddles (a good place to find mathematical statements to prove)

- /r/math
- $\bullet$  /r/learnmath
- $\bullet$  /r/mathbooks
- $\bullet$  /r/askmath

# **Subtopics:**

# 1.5 Single Variable Calculus

Calculus is a set of mathematical techniques based on applying the idea of limit to functions, which makes it possible to study the rate at which a function changes at one specific instant rather than just its average rate of change over a finite period of time. The techniques of calculus are the foundation of physical science, and so it is no coincidence that calculus and modern physics were born simultaneously through the work of Sir Isaac Newton and his contemporaries.

# Prerequisites:

Readers who wish to learn elementary calculus must have an understanding of arithmetic and basic algebra (manipulating algebraic expressions and solving algebraic equations). It is helpful but not necessary to be familiar with trigonometry (sine, cosine, and tangent as ratios within the unit circle and their application to geometry) and analytic geometry (parabolas, hyperbolas, conic sections, and other related functions) - these can be learned while studying the calculus.

It is important to note that *learning this topic is not nearly as difficult as its "scary" reputation might suggest*. Do not be put off by the word "calculus" - all readers who have a good grasp of basic math and basic algebra will be able to learn its techniques. Understanding the ideas behind the techniques will require you to solve many problems, think about the concepts, and eventually study theorems, but anyone can learn calculus itself. Readers should think of elementary calculus as being merely the basic grammar of science.

## Where to Start:

Readers who wish to study calculus should pick a good introductory textbook and work through it chapter-by-chapter. These books tend to be very expensive, so readers may wish to choose a cheaper, older edition for self-study. It is very important to solve as many problems given in each section as possible - this is not just to test your reading; working (and sometimes struggling) with these problems is a necessary part of gaining proficiency in the techniques of calculus. Success will come with practice, and practice means solving problems.

At the end of a study of elementary calculus, readers should understand functions, limits, continuity, derivatives, and integrals, and should also be familiar with trigonometric, exponential, and logarithmic functions as well as sequences and series. This will prepare the reader to go on to study the mathematical laws of the physical sciences. Readers who wish to learn mathematics in more depth may wish to study analysis next, which covers the theorems and proofs behind calculus in far more depth. However, this will require an understanding of basic logic and the techniques needed to constructing proofs.

Readers who wish to study the physical sciences or engineering will discover that elementary calculus is only the first set of techniques they must master - the next steps are to learn multivariable calculus and differential equations. Multivariable calculus extends the techniques of calculus to functions of many variables (for example, one can find the volume of a geometric shape by integrating over the interior of the three-dimensional figure). This should culminate in a study of calculus applied to vector spaces, also known as vector calculus.

In the study of differential equations, readers will learn how to find functions that solve equations containing derivatives - and most of the universe's rules are written in the form of differential equations.

#### Books:

- Kleppner, Daniel and Ramsey, Norman. Quick Calculus: A Self-Teaching Guide. John Wiley & Sons: 1985, 2nd ed. (a fun first tour through calculus a good way to get a basic familiarity with the concepts, but should be followed with a more rigorous text like Larson)
- Kline, Morris. Calculus: An Intuitive and Physical Approach.

  Dover Publications: 1998, 2nd ed. (focuses on intuition and the connection between calculus and science this might be a good secondary text to help you understand why calculus is so useful)
- Larson, Ron. Calculus. Brooks Cole: 2013, 10th ed. (An alternative to Stewart that seems to be popular with students and has many problems to solve this might be the best place to start)
- Stewart, James. Calculus. Cengage Learning: 2012, 7th ed. (the nearly-ubiquitous calculus text used in university courses, this might not be as useful as other books for self-study)
- Thompson, Silvanus Phillips. Calculus Made Easy. CreateSpace Independent Publishing Platform: 2011, 2nd reprint ed. (a conceptual explanation of calculus that can help you prepare for a textbook or provide a useful supplement; older edition available for free on Project Gutenberg)

# Articles: Videos:

- Delaware's "Calculus I" lectures (UMKC) (very good series of lectures that take time to explain the important techniques and concepts in depth, highly recommended for beginners)
- Jerison's "Single Variable Calculus" lectures (MIT) (great lectures on the important topics; you might want to watch these after working through related sections of your textbook)
- Khan Academy's videos on calculus (a good place to look for a discussion of individual topics)
- Leonard's "Calculus 1" lectures (explains the "nuts and bolts" of various techniques, a good place to look for worked examples)
- Leonard's "Calculus 2" lectures
- patrickJMT's YouTube channel (a good place to look for worked examples covering specific topics)

- Strang's "Big Picture of Calculus" (MIT) (a short lecture explaining the concept of differentiating and integrating functions)
- 3Blue1brown Essence of Calculus

#### Other Online Sources:

- Paul's Online Notes (Lamar) (notes are broken up by topic, includes worked examples a useful resource)
- Calculus 1 Coursera (a free, open course in elementary calculus)
- List of interactive math websites at /r/learnmath
- /r/math
- /r/learnmath
- /r/mathbooks
- /r/askmath

# **Subtopics:**

- Multivariable Calculus
- Linear Algebra

# 1.6 Multivariable Calculus

"Multivariable calculus (also known as multivariate calculus) is the extension of calculus in one variable to calculus with functions of several variables: the differentiation and integration of functions involving multiple variables, rather than just one." -Wikipedia

#### Prerequisites:

- Basic Algebra
- Pre-Calculus
- Single Variable Calculus

#### Where to Start:

Readers who wish to study Multivariable calculus should pick a good introductory textbook and work through it chapter-by-chapter. These books tend to be very expensive, so readers may wish to choose a cheaper, older edition for self-study. It is very important to solve as many problems given in each section as possible - this is not just to test your reading; working (and sometimes struggling) with these problems is a necessary part of gaining proficiency in the techniques of calculus. Success will come with practice, and practice means solving problems.

At the end of a study of Multivariable calculus, readers should understand Limits and Continuity, Partial Differentiation, Multiple Integration, and the Fundamental Theorem of Calculus in Three Dimensions. This will prepare the reader to go on to study the mathematical laws of the physical sciences. Readers who wish to learn mathematics in more depth may wish to study analysis next, which covers the theorems and proofs behind calculus in far more depth. However, this will require an understanding of basic logic and the techniques needed to constructing proofs.

# Books:

- Vector Calculus, Linear Algebra and Differential Forms: A Unified Approach
- Div, Grad, Curl, and All That: An Informal Text on Vector Calculus A great secondary and primary book, though I would use this as a secondary book, teaches you in depth about what you need to take out of multivariable calculus.
- Multivariable Mathematics: Linear Algebra, Multivariable Calculus, and Manifolds Our recommended book. Good problems, a healthy explanation on what you need to know and adds in manifolds)
- Advanced Calculus of Several Variables (Dover Books on Mathematics)
- Calculus of Several Variables (Undergraduate Texts in Mathematics)

# Videos:

- Professor Leonard
- MIT OCW

# Problems & Exams

- CLP UBC
- MIT
- Swatchmore
- Duke
- Lamar
- Brunel
- Dartmouth
- Berkeley
- Princeton
- $\bullet$  Harvard
- Bates
- $\bullet$  UMD
- $\bullet$  MIT
- ICTM
- Exster

# Other Online Sources:

- Paul's Online Notes
- Khan Academy
- $\bullet$  /r/learnmath

# **Subtopics:**

- Tensor Calculus
- Calculus of Variations
- Vector Calculus

# 1.7 Linear Algebra

"Linear algebra is the branch of mathematics concerning linear equations such as linear functions such as and their representations through matrices and vector spaces. Linear algebra is central to almost all areas of mathematics. For instance, linear algebra is fundamental in modern presentations of geometry, including for defining basic objects such as lines, planes and rotations. Also, functional analysis may be basically viewed as the application of linear algebra to spaces of functions. Linear algebra is also used in most sciences and engineering areas, because it allows modeling many natural phenomena, and efficiently computing with such models. For nonlinear systems, which cannot be modeled with linear algebra, linear algebra is often used as a first-order approximation." -Wikipedia

#### **Prerequisites:**

• Basic Algebra

#### **Books:**

- Linear Algebra Problem Book (Dolciani Mathematical Expositions) Problem book for Linear Algebra. Highly recommended as a supplement.
- Introduction to Linear Algebra, Fifth Edition by Gilbert Strang *Professor* of MIT's OCW Linear Algebra Lecture series
- Linear Algebra and Its Applications
- Linear Algebra Done Wrong You'll get a good grip with Linear Algebra using this
- Linear Algebra by Stephen H. Friedberg A great introductory book
- Introduction to Applied Linear Algebra Vectors, Matrices, and Least Squares
- Linear Algebra Done Right (Undergraduate Texts in Mathematics) \*"A great book for math/physics undergrads who have already experienced matrix-centric linear algebra and would like to delve into the more abstract theory of finite-dimensional vector spaces and inner product spaces. Very clear cut with rewarding, but easy exercises." u/UglyMousanova19
- Linear Algebra As an Introduction to Abstract Mathematics
- Linear Algebra for Applications MAS201
- Advanced Linear Algebra

#### Lectures:

• MIT OCW

- James Hablin
- CCNY

# Other Online Sources:

- $\bullet$  Khan Academy
- Pauls Notes
- 3Blue1Brown

#### **Problem Sets**

- MIT Problem sets
- MIT Problem set 2
- MIT Problem set 3
- UC Davis
- PDX
- UPENN
- Puget Problems and Solutions

# Exams

- Boston College Exams
- MIT Exams
- Bates
- Brandeis
- $\bullet$  Brandeis
- Berkeley
- Pitt

# Captain's Log

- Added more online sources (11/28/19)
- Added Exams, Problems and solutions (11/28/19)

# 1.8 Ordinary Differential Equations

"A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two." -Wikipedia

# Prerequisites:

- Single Variable Calculus
- Multivariable Calculus
- Linear Algebra

#### **Books:**

• Ordinary Differential Equations (Dover Books on Mathematics) (Yup one book. Most Diff Eq textbooks are badly written, utilizing methods of solving Differential Equations and showing what the solutions are rather than the fundamental theorems and basis on where and why Differential Equations come from and why they do what they do. This is the only recommendation we can solely give out at this current time.)

#### **Articles:**

• Paul's Online Notes

#### Videos:

• Professor Leonard

#### Other Online Sources:

• MIT OCW

# Problems & Exams

- MIT
- Stanford
- UCLA Problems
- Oxford
- ResearchGate
- Swathmore College
- MIT Exams

- PITT
- $\bullet$  CMU
- UCDavis
- ullet Waterloo
- Purdue
- $\bullet$  UNL
- BU
- $\bullet$  UCCS

# **Subtopics:**

• Partial Differential Equations

# 1.9 Set Theory

Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used to define nearly all mathematical objects.

#### Prerequisites:

Since set theory is a fundamental topic it doesn't require any specific prerequisites. But, some mathematical maturity in the reader is needed to appreciate the relevance of some of the definitions and theorems. Depending on the set theory course, you may need first a course where you learn to write proofs. Or, that may be the emphasis of the set theory course itself.

#### Where to Start:

Readers who want to learn set theory should start with an introductory text book. A list of excellent choices is presented below. But, readers should be aware that there are a lot of books that teach set theory in "naive approach". For a more complete understanding of the subject, an "axiomatic approach" must complement the naive approach. Since set theory is a first year undergraduate course, some readers might find set theory different to topics they have faced in high school. The sudden focus on proofs on seemingly "trivial" topics might be tedious for readers, but it is necessary to understand that rigorous proofs of fundamental results ensures integrity of mathematics as a whole. Readers might find taking written notes more helpful in understanding the subject.

## **Books:**

- Book of Proof
- Elements of Set Theory
- Naive Set Theory \*Stands the test of time. We recommend this to learn from
- The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings (Springer Monographs in Mathematics)
- Set Theory (Studies in Logic: Mathematical Logic and Foundations) *THE* graduate book to learn from
- Classic Set Theory: For Guided Independent Study (Chapman & Hall Mathematics S) *Another graduate book*

#### Videos:

• Random Lectures

## Other Online Sources:

• Kanamori

This excellent paper by Kanamori talks about the historical development of set theory, what problems in mathematics gave rise to some of the ideas in set theory, and where the specific constructions came from. It's worth reading, or at least skimming

• Frederique A short lecture notes on set theory by Frederique.

# 1.10 Real Analysis

"In mathematics, real analysis is the branch of mathematical analysis that studies the behavior of real numbers, sequences and series of real numbers, and real-valued functions. Some particular properties of real-valued sequences and functions that real analysis studies include convergence, limits, continuity, smoothness, differentiability and integrability." - Wikipedia.

# Prerequisites:

- Single Variable Calculus
- Multivariable Calculus
- Methods of Proof

#### **Books:**

- A First Course in Real Analysis Our recommendation for analysis
- Walter Rudin's Principles of Mathematical Analysis The god level book of analysis
- Real Analysis and Applications
- Real Mathematical Analysis by Pugh.
- Tao, Analysis I
- Elementary Analysis Calculus
- Buck, Advanced Calculus

#### How to Learn:

- MIT Full course
- Trinity Lecture Notes
- University Of Louisville More notes
- University of Hawaii Problems and Solutions
- Temple University *Problems*
- University of Georgia Exams
- University of New Mexico Exams
- MSU Exams
- WUSTL Exams
- UCDavis Exams

• UCSD Exams

#### Lectures:

- Harvey Mudd
- Bethel University
- IIT Madras
- Scripps College

# **Subtopics:**

• Complex Analysis

This was posted by a user, whom I've banned due to being active participant in a quarantined community.

George Bergman's companion exercises to Rudin's textbook for Chapters 1-7.

Roger Cooke's solutions manual for Rudin's analysis

Tom Apostol's textbook

I find that Rudin is to Analysis textbooks what C++ is to programming languages. A little difficult at first, but with so many auxiliary sources that it becomes one of the best texts to learn from in spite of this.

# 1.11 Partial Differential Equations

In mathematics, a partial differential equation is a differential equation that contains beforehand unknown multivariable functions and their partial derivatives. PDEs are used to formulate problems involving functions of several variables, and are either solved by hand, or used to create a computer model. -Wikipedia

#### Prerequisites:

- Multivariable Calculus
- ODE
- Real Analysis

#### **Books:**

- Partial Differential Equations for Scientists and Engineers (Dover Books on Mathematics) (A great starter book)
- Partial Differential Equations: Second Edition (Graduate Studies in Mathematics) 2nd Edition
- Partial Differential Equations: An Introduction 2nd Edition
- Partial Differential Equations (Applied Mathematical Sciences) (v. 1) 4th Edition
- Partial Differential Equations: Methods and Applications (by McOwen is a good book, shorter and easier than Evans (and also making use of distributions)).

#### Videos:

- Lamar
- ICTP
- MIT OCW

# Other Online Sources:

- UCLA
- Leipzig
- Unkown
- Princeton
- UCSB
- PSU
- UTAH
- Homework
- Resources

# 1.12 Calculus of Variations

"Calculus of variations is a field of mathematical analysis that uses variations, which are small changes in functions and functionals, to find maxima and minima of functionals: mappings from a set of functions to the real numbers.-Wikipedia"

# Prerequisites:

- Multi-variable Calculus
- ODE
- PDE

# Books:

- Calculus of Variations (Dover Books on Mathematics) We reccomend using this textbook until hitting Landau in Physics.
- Applied Mathematical Methods in Theoretical Physics 2nd Edition
- Calculus of Variations (Dover Books on Mathematics)
- Calculus of Variations: An Introduction to the One-Dimensional Theory with Examples and Exercises (Texts in Applied Mathematics)

#### **Articles:**

- Lecture Notes
- Lecture Notes

## Videos:

• (Due to the nature of this field being a subset of Optimization Theory, there aren't any full online courses at this time)

# 1.13 Topology

### **Brief Explanation**

"In mathematics, topology is concerned with the properties of space that are preserved under continuous deformations, such as stretching, twisting, crumpling and bending, but not tearing or gluing. An n-dimensional topological space is a space with certain properties of connectedness and compactness. - Wikipedia"

# Prerequisites:

- Multivariable Calculus
- Set Theory

#### **Books:**

- Topology (2nd Edition) The best written book for topology hands down
- General Topology (Dover Books on Mathematics) Covers some information not in Munkres
- A First Course in Topology: Continuity and Dimension A great introduction book
- Elements of Combinatorial and Differential Topology (Graduate Studies in Mathematics, Vol. 74) A different approach, using more geometric methods
- Milnor, Topology from a Differentiable Viewpoint Assumes no pre-requiste knowledge

# Articles

- Curlie
- Notes
- Glossary

#### Problems & Exams

- PDRMI
- STEMZ
- iitk
- PSU
- Arizona
- Topology Exams

# Videos:

- $\bullet$  MIT OCW
- $\bullet$  African Institute for Mathematical Sciences (South Africa)
- ICTP

# Subtopics

• Algebraic Topology

# Captain's Log

• Added more problems (11/29/2019)

# 1.14 Differential Geometry

"Differential geometry is a mathematical discipline that uses the techniques of differential calculus, integral calculus, linear algebra and multilinear algebra to study problems in geometry." -Wikipedia

# Prerequisites:

- Multivariable Calculus
- Linear Algebra
- Topology

#### Books:

- Vector Analysis (Undergraduate Texts in Mathematics) Our recommendation for an intro book
- Geometric Integration Theory (Cornerstones)
- Introduction to Smooth Manifolds (Graduate Texts in Mathematics)
- Differential Geometry (Wiley Classics Library) A great intro book
- Elementary Differential Geometry Not the greatest book, but does things right

## Exams

- Math 405/538 Differential Geometry Final Exam
- Math 352
- Final Exam of Differential Geometry I
- Final Exam, Math 421 Differential Geometry: Curves and Surfaces in R3
- Math 421

## Videos:

• Differential Geometry 1 | NJ Wildberger

# Other Online Sources:

- MIT OCW
- ICTP
- The WE-Heraeus International Winter School on Gravity and Light

#### 1.15 Discrete Mathematics

"Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics – such as integers, graphs, and statements in logic – do not vary smoothly in this way, but have distinct, separated values. Discrete mathematics therefore excludes topics in "continuous mathematics" such as calculus or Euclidean geometry. Discrete objects can often be enumerated by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets (finite sets or sets with the same cardinality as the natural numbers). However, there is no exact definition of the term "discrete mathematics." Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions."

#### Prerequisites:

- Basic Algebra
- Pre-Calculus

#### **Books:**

- Oscar Levin Discrete Mathematics: An Open Introduction
- Ronald Graham Concrete Mathematics: A Foundation for Computer Science
- Discrete Mathematics with Applications Susanna S. Epp

#### Articles:

- Yale Notes
- IITK
- Stanford Notes
- Arkansas Tech
- Northwestern
- Warwick
- UPenn
- Common Mistakes

#### Videos:

• TheTrevTutor

- $\bullet$  Trefor Bazett
- MIT

# Problems and Exams:

- University of North Florida
- Syracuse University

# 1.16 Complex Analysis

"Complex analysis, traditionally known as the theory of functions of a complex variable, is the branch of mathematical analysis that investigates functions of complex numbers. It is useful in many branches of mathematics, including algebraic geometry, number theory, analytic combinatorics, applied mathematics; as well as in physics, including the branches of hydrodynamics, thermodynamics, and particularly quantum mechanics. By extension, use of complex analysis also has applications in engineering fields such as nuclear, aerospace, mechanical and electrical engineering"" -Wikipedia

# Prerequisites:

- Methods of Proof
- Multivariable Calculus
- Linear Algebra
- Set Theory
- Real Analysis

#### Books:

- Ahlfors "The classic". Terse, but very elegant. I studied out of it when
  I was frustrated with my course notes, and it made me much happier in
  terms of conceptual clarity. Somewhat light on examples and exercises,
  though the ones that are there are very good.
- Stein Awesome book. Has great coverage of applications to number theory, and very good problems.
- Gamelin Much less demanding of the reader, lots of nice examples (of the kinds of problems that are usually on complex analysis exams).

#### Articles:

- MIT Lecture Notes follows Alfhors
- UCDavis
- LSU
- lth

## Videos:

- Introduction to Complex Analysis
- MIT Complex Analysis 1968
- Dr.Gajendra Purohit

- Complex Analysis
- nptelhrd
- Winston Ou Complex Analysis
- MathStatsUNW
- Bethel University
- Steven Miller Complex Analysis

# Exams/Problems/Solutions:

- Occidental College
- OU
- Complex Analysis: Problems with solutions
- A Collection of Problems on Complex Analysis (Dover Books on Mathematics) AMAZON
- University of Manchester
- Random Problems
- ResearchGate
- UCLA Final Exam
- UPENN Final Exam
- UIC Final Exam

# **Subtopics:**

• Subtopic - Bibliography exists

## 1.17 Tensor Calculus

"In mathematics, tensor calculus, tensor analysis, or Ricci calculus is an extension of vector calculus to tensor fields (tensors that may vary over a manifold, e.g. in spacetime). Developed by Gregorio Ricci-Curbastro and his student Tullio Levi-Civita it was used by Albert Einstein to develop his theory of general relativity. Contrasted with the infinitesimal calculus, tensor calculus allows presentation of physics equations in a form that is independent of the choice of coordinates on the manifold. Tensor calculus has many real-life applications in physics and engineering, including elasticity, continuum mechanics, electromagnetism (see mathematical descriptions of the electromagnetic field), general relativity (see mathematics of general relativity) and quantum field theory." -Wikipedia

## Prerequisites:

- Multivariable Calculus
- Linear Algebra
- Differential Geometry
- Set Theory
- Real Analysis

## Books:

- Winitzki Linear Algebra via Exterior Products
- Schutz Geometrical Methods of Mathematical Physics 1st Edition
- Frankel The Geometry of Physics: An Introduction 3rd Edition
- Wald General Relativity
- Bishop & Goldberg Tensor Analysis on Manifolds (Dover Books on Mathematics)
- Greub Multilinear Algebra

#### **Articles:**

- Heidelberg
- UTexas
- MIT
- ARXIV
- TU

- Saint Mary's University Halifax
- ullet Research Gate
- $\bullet$  Israel Institute of Technology

# Videos:

- $\bullet \ \ {\rm MathTheBeautiful}$
- $\bullet$  MTB2
- eigenchris

Problems and Exams: Subtopics:

# 1.18 Functional Analysis

Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of vector spaces endowed with some kind of limit-related structure (e.g. inner product, norm, topology, etc.) and the linear functions defined on these spaces and respecting these structures in a suitable sense. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining continuous, unitary etc. operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations. -Wikipedia

#### Prerequisites:

- Linear Algebra
- Set Theory
- Real Analysis
- Topology
- Basic measure theory, esp Lebesgue measurable functions
- Lp spaces
- Basic Banach and Hilbert space theory

#### **Books:**

- Functional Analysis: An Introduction (Graduate Studies in Mathematics)
- Functional Analysis, Sobolev Spaces, and Partial Differential Equations by Haim Brezis
- A First Course in Functional Analysis by Orr Moshe Shalit
- A Course in Functional Analysis (Graduate Texts in Mathematics) 2nd Edition by John B Conway
- FUNCTIONAL ANALYSIS by Theo Buhler and Dietmar A. Salamon
- Elements of the Theory of Functions and Functional Analysis by A. N. Kolmogorov (Dover Books on Mathematics by)
- Lectures and Exercises on Functional Analysis by A. Ya. Helemskii
- Functional Analysis by Peter D. Lax
- $\bullet$  Introductory Functional Analysis with Applications by by E. Kreyszig
- Functional Analysis: An Introduction (Graduate Studies in Mathematics)

# Articles:

- $\bullet$  MIT
- MSU
- IIT Kanpur
- KIT
- LANCAS
- The Chinese University of Hong Kong
- Rhodes University
- London School of Economics

#### Videos:

- Basic Functional Analysis and Harmonic Analysis
- ullet IIT Kharagpur  $Book\ used$
- ICTP
- CSA
- University of Nottingham

# **Problems and Exams**

- MIT
- IITK
- BRIS UK
- University of Bergen
- PSU Exam
- Yale Exam
- Munchen Final Exam
- Random Exam
- Random Exam
- Google Search for exams
- Google Search for problem sets

# 1.19 Stochastic Calculus

"Stochastic calculus is a branch of mathematics that operates on stochastic processes. It allows a consistent theory of integration to be defined for integrals of stochastic processes with respect to stochastic processes. It is used to model systems that behave randomly." -Wikipedia

## Prerequisites:

- Real Analysis
- Measure Theory
- Discrete-time martingale theory
- Theories of convergence of stochastic processes
- Theory of continuous-time stochastic processes

#### **Books:**

- This list goes down in order of difficulty
- An Introduction to Stochastic Differential Equations
- Stochastic Calculus and Financial Applications (Stochastic Modelling and Applied Probability)
- Stochastic Differential Equations: An Introduction with Applications (Universitext)
- Stochastic Integration Theory (Oxford Graduate Texts in Mathematics)
- Stochastic Integration and Differential Equations
- Limit Theorems for Stochastic Processes

#### Exams:

- http://www-stat.wharton.upenn.edu/steele/Courses/955/Homework/SCFAFinal06.pdf
- Stochastic Calculus and Financial Applications Final Take Home Exam

#### Other Online Sources:

• MIT OCW

# 2 Physics

# 2.1 How to Learn our Physics

Quite different from the section of, "How to learn our Mathematics", this subsection rather focuses on the tools to be successful in the path of learning physics.

- General Physics I
  - Differential Calculus and Integral Calculus
- General Physics II
  - Multivariable Calculus and Vector Calculus
- Waves & Oscillations

# 2.2 General Physics I & II

Physics is the study of matter and energy, and seeks to understand how the universe works at its most fundamental level. The goal of physics is to come up with mathematical rules that can accurately predict and explain all of the various phenomena of our universe.

## Prerequisites:

Studying physics at the high-school or conceptual level requires a good understanding of basic math and algebra. University-level physics requires calculus, since the mathematical laws of physics involve instantaneous rates of change. Readers who wish to learn physics at this level must understand limits, derivatives, and integrals, and should eventually study linear algebra, multivariable calculus, and differential equations after moving on to more advanced subtopics.

#### Where to Start:

Readers who wish to start learning physics should begin by obtaining an introductory textbook, which will typically cover basic mechanics, electricity and magnetism, and a few selected topics in modern physics. Introductory textbooks can be roughly divided by depth and difficulty into high-school, conceptual (algebra-based), and university (calculus-based) levels. Readers who are familiar with elementary calculus should start with a university-level text. Those wishing to make a serious study of physics should first learn calculus and then study a university-level text. It is very important to study the chosen textbook methodically, chapter-by-chapter, and it is especially important to solve the problems found at the end of each section. There is no substitute for solving many problems on your own when it comes to understanding physics.

You may wish to supplement your textbook reading with conceptual readings (like the Feynman lectures on Physics) and lectures appropriate to your level. These may help you think about your reading, but cannot replace studying a textbook deeply and solving physics problems. Once you finish the introductory

text, you should be ready to move on to specialized subtopics - start with a more in-depth study of classical mechanics.

# **Books:**

- Giancoli, Douglas C. Physics: Principles with Applications. Pearson/Prentice Hall: 2004, 6th ed. (algebra-based introductory text)
- Halliday, David; Resnick, Robert; and Walker, Jearl. Fundamentals of Physics. Wiley: 2013, 10th ed. (a university-level (calculus-based) introductory text commonly used in college courses.)
- Kleppner, Daniel and Kolenkow, Robert. An Introduction to Mechanics. McGraw-Hill Science/Engineering/Math: 1973, 1st ed. (this book with the Purcell book below provides a very rigorous and challenging university-level introduction to physics; these two texts are sometimes used at MIT)
- Kuhn, Karl F. Basic Physics: A Self-Teaching Guide. Wiley: 1996, 2nd ed. (high-school level introductory text, may be useful for those wishing to get a basic overview of the subject)
- Purcell, Edward M. and Morin, David J. Electricity and Magnetism.
   Cambridge University Press: 2013, 3rd. ed. (this book along with the Kleppner book above provides a very rigorous and challenging university-level introduction to physics; these two texts are sometimes used at MIT)
- Young, Hugh D.; Freedman, Roger A.; Ford, A. Lewis. University Physics with Modern Physics. Addison-Wesley: 2011, 13th ed. (a university-level (calculus-based) introductory text commonly used in college courses.)

# Articles:

#### Videos:

- Feynman's The Character of Physical Law lectures
- Fullerton's High School Physics lectures
- KhanAcademy's physics videos (great resource for beginners)
- Lewin's Physics I lectures (MIT) (highly-recommended calculus-based lectures)
- Lewin's Physics II lectures (MIT)
- Lewin's Physics III lectures (MIT)
- Pomerantz's Introductory Physics (UCBerkeley)
- Shankar's General Physics 1 lectures (Yale)

- Shankar's General Physics 2 lectures (Yale)
- Stahler's Conceptual Physics lectures (UCBerkeley)

#### Other Online Sources:

- arxiv.org (a repository for research article preprints on several topics see what physics research looks like)
- Chicago undergraduate physics bibliography
- Duffy's Physics I notes (Boston University) (algebra-based course notes, has good explanations of important concepts mechanics, fluids, thermodynamics, waves)
- Duffy's Physics II notes (Boston University) (algebra-based course notes, has good explanations of important concepts electricity and magnetism, circuits, optics, modern physics)
- The Feynman Lectures on Physics (free to read online only)
- Fxsolver (a database of physics equations with built-in calculator that could be useful for self-studiers wanting to check solutions to worked problems)
- HyperPhysics (Georgia State) (explore a mind map of physics concepts)
- Luttermoser's General Physics I course notes (East Tennessee State) (a very well-organized set of notes with some excellent example problems, covers mechanics)
- Luttermoser's General Physics II course notes (East Tennessee State) (a very well-organized set of notes with some excellent example problems, covers electricity and magnetism)
- PhysicsForums (a great forum for physics students, contains helpful discussions and a list of free physics and math books)
- Physics StackExchange (a forum for physics questions you might find the answers to your questions here)
- Seville, Chapman. "How to Study Physics". (especially Ch. 5 on solving physics problems)
- 't Hooft, Gerard. "How to become a GOOD Theoretical Physicist". (An excellent overview of the subject, explains what must be learned to reach the highest levels of Physics, includes several reading lists)
- theoreticalminimum.com (A series of Stanford online courses maintained by Prof. Susskind at Stanford - this site will be especially useful to advanced students as they move on to study subtopics of general physics)

- The Physics Classroom (good, simple explanations of basic topics with some conceptual problems and interactives)
- ThinkQuest's Learn Physics Today online tutorial (archived) (guides your through the basics, includes problems to solve)
- WolframAlpha (A "computational knowledge engine" that can be used to check solutions to worked problems, e.g. kinematics problems)
- /r/physics
- /r/AskPhysics
- /r/physicsbooks

# **Subtopics:**

- Astronomy and Astrophysics
- AMO (Atomic, Molecular, Optical Physics)
- Biological Physics
- Classical Mechanics
- Chemical Physics
- Soft Condensed Matter Physics
- Hard Condensed Matter Physics
- Electrodynamics
- Experiments in Basic Physics
- High Energy Physics
- Mathematical Physics
- Nuclear Physics
- Optics and Waves
- Plasma Physics
- Quantum Mechanics
- Research Methods in Physics
- Solid-State Physics
- Special and General Relativity
- Statistical Mechanics

## 2.3 Waves and Oscillations

#### **Preliminary:**

This subsection will be quite small, due to two reasons; The course isn't taught at many schools, rather the course is wrapped up as a section in their Quantum Mechanics course, their Modern Physics course, Electrodynamics, or Classical Mechanics. Different Colleges and Universities teach their courses differently, so this subsection is to appease the general audience who have a separate course for Waves and Oscillations (or Vibrations). Users who do not, may continue onto the next subsection.

# Description:

## Prerequisites:

- General Physics I & II
- Electrostatics & Electromagnetism
- Multivariable Calculus
- Ordinary Differential Equations
- Parital Differential Equations

#### **Undergraduate Books:**

- R.A. Waldron Waves and Oscillations As far as I'm aware it's a fairly conceptual book (only 60 pages) but derives topics in W&O using PDE's and such. Seems like a decent book, and I've seen it recommend across a few forums and threads. Looking through the book, Chapter 6 on Network Theory seems outdated, but in general the book highlights, and derives the neccessary theories to understand the field.
- David Morin Waves and Oscillations Draft Harvard Scholar link, from the same Morin that has a Classical Mechanics Book out with Thompson. It's in draft format from a new book that he's writing. I haven't seen many PDE's rather n-th order Linear ODE's, I'd imagine it's quality just like his Classical Mechanics books.
- A.P French Vibrations and Waves MIT's book on said topic used in their version of the class at MIT, and possibly on OCW (will have to check on that). I'd imagine it's good enough, as it is used at MIT. Haven't done much checking on this book, but I recall his Introductory Physics book was pretty solid in any case.
- Howard Geogri The Physics of Waves Like a "textbook" textbook on the topic, as Georgri states, "a middle course between these two (ref: Optics by Hecht and Waves by Crawford)". Focuses on the mathematical formulations of 2D and 3D waves.

- M.I Rabinovich and D.I Trubetskov Oscillations and Waves: in Linear and Nonlinear Systems Russian/Soviet Era Textbook that \*will\* kick your ass. Has applications towards hydrodynamics and stochastic oscillations, in reference towards nonlinear oscillations and waves.
- Waves (Berkeley Physics Course, Vol. 3) Crawford Mentioned all across the forums and threads as the gold standard for learning waves. Probably the best book for the subject at the Undergraduate

#### Lectures:

• MIT OCW Resource Index for Physics III Resource Index links all of Videos, Notes, Problem Sets, Readings and In-Class Demonstrations

#### **Lecture Notes:**

Cal Poly Ponoma Cal Poly Notes on Oscillations and Mechanical Waves
 University of Texas University of Texas Oscillations and Waves - Richard Fitzpatrick

## 2.4 Classical Mechanics

Classical mechanics is the oldest subtopic within physics; it contains the ideas first discovered at the turn of the 17th century by Sir Isaac Newton, the father of modern physics. Classical mechanics is the study of the motion of "everyday things" - its goal is to use mathematical rules to predict the behavior of ordinary objects when acted upon by forces.

## Prerequisites:

- General Physics I & II
- Multivariable Calculus
- Ordinary Differential Equations

#### Where to Start:

Readers should start with a standard classical mechanics text, reading each chapter methodically and solving the problems found at the end of each chapter. As with general physics, there is no substitute for solving lots of problems - this is the only way to truly understand classical mechanics. Textbooks can be divided into undergraduate- and graduate-level; readers should start with undergraduate texts before attempting the more advanced works on the subject. Those who are self-studying and have just completed general physics should start by studying Taylor's book.

The study of classical mechanics begins with a review of Newtonian methods and concepts but at a deeper level, with new techniques and in more general or complex situations. Eventually readers will study the calculus of variations, a very important technique that makes new types of calculations possible and is very important in more advanced topics. Most basic texts will also have introductory sections on special relativity, in which you will discover that our principles of classical mechanics are only low-velocity approximations of the more general and far stranger rules of relativistic motion. Readers may wish to continue on to a more modern treatment of classical mechanics, which will require an understanding of differential geometry.

After completing a study of classical mechanics, readers trying to obtain a basic education in physics should move on to electrodynamics (which will require an understanding of multivariable calculus and vector calculus) or quantum mechanics (which requires linear algebra and, for some topics, multivariable calculus). It will become increasingly important to improve your knowledge of mathematical methods as you progress into more advanced subtopics.

#### Books:

- Undergraduate Books
- Taylor, John R. Classical Mechanics University Science Books: 2005. Taylor pretty much kills the competition in the Undergraduate area

- Marion, Jerry B. and Thornton, Stephen T. Classical Dynamics of Particles and Systems. Saunders College Publications: 1995, 4th ed. A very commonly-used undergraduate textbook
- Graduate Books
- Goldstein, Robert. Classical Mechanics. Addison-Wesley: 2001, 3rd ed. The standard graduate-level textbook
- Landau, L.D. Mechanics: Course of Theoretical Physics, Vol. 1. Butterworth-Heinemann: 1976, 3rd ed. The first volume in a classic series, great as a supplement to a graduate-level text
- Arnold, V.I. Mathematical Methods of Classical Mechanics. Springer: 1997, 2nd ed. Advanced classical mechanics written in a terse, mathematical style will be most appreciated by those coming from a background in mathematics
- José, Jorge V. and Saletan, Eugene J. Classical Dynamics: A Contemporary Approach. Cambridge University Press: 1998, 1st ed. An advanced graduate-level text that utilizes more modern techniques be warned, you will need a companion textbook in differential geometry to learn from this book
- Sussman, Gerald Jay and Wisdom, Jack. Structure and Interpretation of Classical Mechanics. MIT Press: 2001 Another choice for graduate-level classical mechanics, with the advantage of being freely available online

#### **Lecture Notes:**

- MIT OCW Classical Mechanics II
- MIT OCW Classical Mechanics III

## **Assignments:**

- MIT OCW Assignments for Classical Mechanics II
- MIT OCW Assignments for Classical Mechanics III

#### Videos:

- Susskind's lectures on Classical Mechanics (Stanford) (highly recommended)
- Linder's lectures on Classical Mechanics (NTNU)

#### Other Online Sources:

- Baez's notes on Classical Mechanics (UC Riverside) (advanced classical mechanics from the mathematician's perspective an excellent resource for those wishing to focus on a deep study of classical mechanics; includes books by the author)
- Golwala's "Lecture Notes on Classical Mechanics for Physics" (Caltech) (undergraduate to advanced undergraduate level notes written up as a text-book has an interesting pragmatic, problem-focused approach which may be helpful as a supplement)
- Tong's "Lectures on Classical Dynamics" (Cambridge) (a series of lecture notes forming a book written at the advanced undergraduate level)

# **Subtopics:**

# 2.5 Electrodynamics

Electrodynamics (or "Electricity and Magnetism", as it is sometimes called in introductory courses) is the study of the interaction between matter with electric charge and the electric and magnetic fields. Electric charges create these fields and also experience forces in their presence, and electrodynamics seeks to understand the mathematical laws governing this relationship.

#### Prerequisites:

Before studying electrodynamics in depth, readers should have completed a study of general physics by working through an university-level introductory text. Readers should also have completed a classical mechanics text, but this is not necessarily required; these two subtopics can be studied in parallel. Elementary calculus is required, and readers should also be familiar with vectors. Understanding vector and multivariable calculus is also recommended.

- Multivariable Calculus
- Vector Calculus
- Ordinary Differential Equations Grad Level
- Partial Differential Equations Grad Level

#### Where to Start:

Just as with general physics, readers who wish to study electrodynamics should begin by picking up an introductory textbook. This textbook should be read diligently, chapter-by-chapter, and readers should complete as many of the problems given at the end of each section as possible. Reading through the textbook will not suffice - readers will discover that they don't really understand the concepts until they've wrestled with a few tough problems. For those who are new to electrodynamics, having only worked through university-level general physics, the recommended textbook is Griffiths.

Eventually, readers will learn that the electric and magnetic fields are two aspects of the same field and that propagating electromagnetic fields (a.k.a light) travel at the same constant speed in all cases - even from the perspectives of two people moving at different velocities! Reconciling this strange fact with our ordinary notions of classical mechanics led to the theory of special relativity published by Einstein in 1905. Classical mechanics and electrodynamics form the foundation of a good physics education, so after completing electrodynamics, readers will be ready to study relativity, quantum mechanics, or any other advanced subtopic. But it is very important to study differential equations, linear algebra, and other mathematical methods in parallel with physics, since these become increasingly crucial as you move into more modern, advanced fields.

#### **Undergraduate Books:**

- Griffiths, David J. Introduction to Electrodynamics. Addison-Wesley: 2012, 4th ed. Undergraduate-level introductory text, Griffiths is commonly used across most Universities at the current moment. Loved by most, hated by the vocal minority; it's the best undergraduate Electrodynamics book that you can get
- Electricity and Magnetism 3rd Edition by Edward M. Purcell (Author), David J. Morin (Author) Written by Nobel Laureate and generally regarded to be chock full of theory, though it is a couple of decades old, so you'll find assumptions and experimental evidence to be quite indicative of the era it was written in

#### **Graduate Books:**

- Jackson, John David. Classical Electrodynamics. Wiley: 1998, 3rd ed. The standard yet somewhat-controversial graduate-level introductory text-book, this book is extremely comprehensive, with very challenging, sometimes overly-technical problems to work it may not be the best choice for self-study, but few good alternatives exist
- Landau, L.D. The Classical Theory of Fields. Butterworth-Heinemann: 1980, 4th ed. An outstanding supplement to a graduate-level textbook approaches the theory from the basic ideas about electromagnetic fields rather than building up from electrostatics, also discusses gravitational fields (general relativity)). Same Level as Jackson
- Panofsky, Wolfgang K.H. and Phillips, Melba. Classical Electricity and Magnetism. Dover Publications: 2005, 2nd ed. An alternative graduatelevel introductory textbook, not as thorough as Jackson but also not as hard on readers - and it's a much cheaper Dover book
- Static and Dynamic Electricity by William R. Smythe (Author) Thought Jackson was easy? Are you a massive masochist? Former reference for the field in the 20th century, Smythes' course caused Nobel Laureate Vernon Smith to switch majors

#### Lecture Notes:

- MIT OCW Undergraduate Electrodynamics / Requires Differential Equations
- David Tong Cambridge Lecture Notes
- Duke Graduate Electrodynamics
- Brown's "Lecture Notes for Physics 319" (Duke) An online text covering the second semester of a graduate-level course has good overview of mathematical methods (although some are unfinished), and a someone

- different focus than Jackson (which arguably has too much emphasis on boundary-value problems)
- Fitzpatrick's "Classical Electrodynamics" (UTexas Austin) An online text giving a good overview of a year-long graduate electrodynamics course
- Orfanidis' "Electromagnetic Waves and Antennas" (Rutgers) An online text focused on electromagnetic waves and many applications including transmission through solids, scattering, waveguides, antennas, and antenna arrays
- Perry's "Electrodynamics" (Cambridge) (A very short overview (or "cheat-sheet") of important graduate-level concepts in electrodynamics)
- Sparks' "Part A Electromagnetism" (Oxford) (Undergraduate-level notes, very concise and with good appendices on important mathematics)
- Wegner's "Classical Electrodynamics" (Heidelberg) (graduate-level online text that begins with a good overview of elementary concepts)

#### Videos:

- UNM Grad Level course
- ICTP Seemingly undergraduate, though it may be graduate
- jg394 (Jonathan Gardner)'s YouTube videos on electrodynamics (videos created as a companion to Griffith's introductory Electrodynamics text may be useful for beginners)

## **Assignments:**

• MIT OCW Undergraduate Electrodynamics / Requires Differential Equations

## Exams:

• MIT OCW Undergraduate Electrodynamics / Requires Differential Equations

#### Other Online Sources:

• KSU Landing Page with lecture notes and exams with solutions

## **Subtopics:**

• Quantum Electrodynamics

# 2.6 Mathematical Methods in Physics

# **Preliminary:**

Math methods is completely different than Mathematical Physics. Do not confuse either subject/field. Math Methods is not a field of physics, rather a field of internal instruction for physics majors.

Math Methods bridges the gap between Multivariable Calculus, Linear Algebra, Ordinary Differential Equations to complex mathematical areas which Physics Majors need to be fluent in, but not masters in. For example, most Physicists and/or majors do not need to be proficient in most areas of Real Analysis, Group Theory or Probability and Statistics. Some proficiency is required, but not to the level as Mathematicians and/or majors would need to be at. Math Methods essentially covers these areas to the degree of which you may require and not much afterwards.

In simple plain English, Math Methods takes out the bullshit and fluff that physicists don't require in their Mathematics.

## Prerequisites:

- Multivariable Calculus
- Linear Algebra
- Ordinary Differential Equations

#### Books:

- Mathematical Methods for Physicists: A Comprehensive Guide 7th Edition by George B. Arfken, Hans J. Weber, Frank E. Harris Covers Mathematics at the Graduate Level, does not do any area particularly in depth, but covers many areas widely and does it well. I personally used this textbook as I prefer it to Boas.
- Mathematical Methods in the Physical Sciences 3rd Edition by Mary L. Boas Most students find this a confusing bout the first time around. Looking back at it in your grad years, you'll find this a very good reference. Not good to teach yourself from as Boas makes jumps in explanations which aren't as clear learning through the first time.
- Mathematical Methods for Physics and Engineering: A Comprehensive Guide 3rd Edition by K. F. Riley (Author), M. P. Hobson (Author), S. J. Bence (Author) A book I've seen recommended across forums. An easier version of Boas and Arfken which is more hand-holdey and seems to be "just alright"
- Basic Training in Mathematics: A Fitness Program for Science Students by R. Shankar (Author) A book I've seen sometimes referenced due to the popularity of it's predecessor, though it doesn't cover as much material as Arfken or Boas. More like a bridge between simple engineering mathematics and physics mathematics.

# Videos:

- UC Irvine OCW
- $\bullet$  Carl Bender PSI Lectures Refers to the class as Mathematical Physics, though it is Mathematical Methods

# 2.7 Quantum Mechanics

#### **Preliminary:**

Quantum mechanics is the branch of physics that explains how the universe works at distances comparable to or smaller than the atom. Various observations made in the late 19th and early 20th centuries made it clear that physics at this distance scale cannot be described by ordinary classical physics. For example, in 1905 Albert Einstein explained an unusual aspect of the photoelectric effect (the effect behind the workings of solar cells): low-intensity, short-wavelength light was capable of knocking electrons out of a semiconductor material while high-intensity, long-wavelength light would not generate current in the material. Einstein realized that the light must contain energy "quanta" that would interact individually with electrons in the material, which was not consistent with the classical conception of light as a continuous wave that would gradually supply enough energy for these electrons to escape.

Quantum mechanics was developed to explain these strange phenomena of tiny things. It describes the dynamics of particles using quantized wavefunctions and expresses their observable values in terms of probabilities. Yet, amazingly, it still "corresponds" to classical mechanics at larger distances - it extends, but does not replace, our classical physics.

#### Prerequisites:

Readers should complete a study of general physics and classical mechanics before beginning work on quantum mechanics. In terms of mathematical experience, readers should be familiar with elementary calculus, linear algebra, and how to solve ordinary differential equations. For the more advanced standard problems, multivariable calculus and familiarity with solving partial differential equations will also be required, and a basic knowledge of electrodynamics will also be helpful.

# Where to Start:

Readers should begin by obtaining an introductory quantum mechanics textbook - for the beginner, Griffiths' text is probably the best choice. It is important to study each chapter in depth and work as many problems as possible at the end of each section. The core of a basic introduction to quantum physics is a study of canonical problems - free particles, potential wells, harmonic oscillators, and the Coulomb potential - readers should eventually be able to compute the basis wavefunctions for each of these standard potentials. And, just as with every other subtopic in physics, understanding is gradually developed as you solve many problems. After completing Griffiths, readers can move on to graduate-level texts like Shankar.

By the time you finish your initial study of quantum mechanics, you should understand the correspondence between the laws of classical and quantum mechanics, understand that Schrodinger's equation allows a derivation of the energy basis for wavefunctions, understand the time-dependence of wavefunctions, be able to compute expectation values for observable quantities, be able to find the energy levels and wavefunctions for basic potentials like the infinite square well, understand the quantum harmonic oscillator and ladder operators, under-

stand how to compute the electron energy levels in the Hydrogen atom, and be able to use perturbative methods to study small changes in quantum systems. Many of these concepts, particularly the harmonic oscillator and perturbation theory, are extremely important in more advanced quantum theory.

Quantum mechanics is just the first step in understanding how the universe works at very small scales and how our macroscopic world can be an emergent feature of the universe's most fundamental physics. It was quickly realized that ordinary quantum mechanics is incompatible with special relativity (it cannot describe the very small and the very fast). Quantum field theory developed from the need for a quantum theory that is consistent with special relativity and can describe processes in which particles are created or destroyed (as observed from radioactive decay or inelastic scattering within particle colliders). The next steps in understanding the most fundamental theories of physics are to study particle physics and quantum field theory, although this will require significant additional mathematical knowledge (e.g. complex analysis).

#### Books:

- Cohen-Tannoudji, Claude; Diu, Bernard; and Laloe, Frank. Quantum Mechanics. Wiley-VCH: 1992, 2 vols. (a good, comprehensive treatment of quantum mechanics might be possible for very ambitious beginners to study from this book alone, but Griffiths is still the best introduction)
- French, A.P. and Taylor, Edwin F. Introduction to Quantum Physics. W. W. Norton & Company: 1978, 1st ed. (might be a useful secondary text as a supplement to Griffiths)
- Griffiths, David J. Introduction to Quantum Mechanics. Pearson Prentice Hall: 2004, 2nd ed. (the best place for beginners to start any book by Griffiths is an excellent introductory text)
- McEvoy, J.P. and Zarate, Oscar. Introducing Quantum Theory: A
  Graphic Guide to Science's Most Puzzling Discovery. Icon Books:
  2003, 4th ed. (a fun, cartoon-based overview of the historical development
  and big ideas of quantum physics a good supplement to textbook study)
- Messiah, Albert. Quantum Mechanics. Dover Publications: 2014. (a comprehensive work best for those who have already completed a graduatelevel introductory textbook)
- Sakurai, J.J. and Napolitano, Jim J. Modern Quantum Mechanics. Addison-Wesley: 2010, 2nd ed. (the alternative to Shankar for graduate-level quantum mechanics, this book is not quite as popular or comprehensive)
- Shankar, R. Principles of Quantum Mechanics. Plenum Press: 2011, 2nd ed. (a widely-used advanced undergraduate- / graduate-level text, introduces math and concepts well)

# Articles: Videos:

- Adams' "Quantum Physics I" lectures (MIT)
- Balakrishnan's "Quantum Physics" lectures (IIT Madras)
- Beatty's "Quantum Physics" YouTube videos (contains good explanations of a couple of standard quantum problems including the infinite potential well)
- Binney's "Quantum Mechanics" lectures (Oxford)
- Susskind's "Modern Physics: Quantum Mechanics" lectures (Stanford) (basic overview of the subject, highly recommended)
- Susskind's "Quantum Mechanics" lectures (Stanford)
- Susskind's "Advanced Quantum Mechanics" lectures (Stanford)
- Theoretical Physics III/IV Quantum Mechanics: Follows the Townsend Text:

#### Other Online Sources:

- Adams, Evans, and Zwiebach's Quantum Physics I course (MIT) (undergraduate course, with videos and lecture notes available)
- Neumaier's "A theoretical physics FAQ" (University of Vienna) (a great list of commonly-asked questions, mostly concerning quantum theory)
- Shankar's "Fundamentals of Physics II" course (Yale) (starting with lecture 19, covers the experimental and conceptual basis of quantum mechanics; it is important for students to understand the important observations that led to the development of quantum physics)
- Walet's "Quantum Mechanics I" notes (Manchester) (good explanations to supplement a textbook)
- UCSD's Physics 130: Quantum Mechanics notes (a series of short discussions of many topics in quantum mechanics)
- van Veenendaal's "Quantum Mechanics 660/1" notes (Northern Illinois) (very concise graduate-level notes)
- /r/physics
- /r/AskPhysics

#### **Subtopics:**

# 2.8 Statistical Mechanics & Thermodynamics

## Description:

"Statistical mechanics is one of the pillars of modern physics. It is necessary for the fundamental study of any physical system that has many degrees of freedom. The approach is based on statistical methods, probability theory and the microscopic physical laws. It can be used to explain the thermodynamic behavior of large systems." -Wikipedia

# Preliminary:

I do want to say before a user starts this Bibliography, that this was one of the most difficult Bibs I've had to make in regards to the textbooks. For some reason, the textbooks pertaining to this field aren't highly regarded, nor are they usually well written. I have a hard time recommending any undergraduate textbook for Stat Mech or Thermodynamics:

- Kittel & Kroemer hasn't been updated in over 40 years and the publishers are still asking nearly \$150 for the book (at the time this bib was published). It is usually recommended in lieu of Schroeder.
- Schroeder is typically used for intro Statistical Mechanics, and in most forums, is usually disliked, wherein most users refer to Kittel & Kroemer as their preferred textbook. This begins a cycle where one users hates Kittel & Kroemer and recommends Schroeder, another user comes in and recommends Kittel & Kroemer and thus continues the cycle.
- Reif is known for it's usage for obscure notation, unnecessarily formality, and clarity issues. Some users state it is the best book, while others want to burn it in a fire.
- Herbert B. Callen: Published and not revised since 1985. "In the preface to this second edition, Callen described his 25-year-old postulatory approach to thermodynamics and statistical mechanics as "now widely accepted". In fact, by the time of his second edition, his approach was completely outdated, because it springs from nineteenth-century ideas of thermodynamics in which concepts such as entropy were not understood. This means that Callen simply postulated the core quantities such as entropy and temperature with essentially no context, and without providing any physical insight or analysis. It might all look streamlined, but his approach will give you no insight into the difficult and interesting questions of the subject. Callen described his approach as rendering the subject transparent and simple; but his approach comes across as obscure. For example, in the early part of the book, he insists on repeatedly writing'' 1/T1 = 1/T2'' for two temperatures that are ascertained to be equal, when anyone else would write "T1 = T2". And, for what he does write, the devil is often in the details that he tends to leave out. Even at the start, when Callen introduces the concept of work, he fails to say whether he is talking about the work done on the system, or by the system, leaving the reader to work that out for himself from some irrelevant comments

about the mechanical work term -P dV. Callen's incorrect renditions of the Taylor expansion in an appendix seem to suggest, rather oddly, that he didn't understand the difference between "dx" and " $\delta$  x". His book includes a 20-page postscript in which he makes claims about the role of symmetry in thermodynamics; but, as far as I can tell, this section says nothing useful at all. I suspect that the reason this book is as frequently cited as it is said to be lies in its being used as the basis for a course by many lecturers who never learned the subject themselves, and hence don't reseal that the book's approach is outdated. If you really want to learn the subject, use the modern statistical approach, in which entropy is defined to relate to numbers of configurations. As far as readability goes, Callen's writing tends to omit commas; but this can make his sentences tedious to read, since the reader ends up having to make two or three passes to decode what some sentences are saying. (If you use few commas yourself, study a typical sentence in Callen's book: "the intermediate states of the gas are non equilibrium states for which the enthalpy is not defined". Callen is not singling out a special set of non-equilibrium states here; instead, enthalpy is not defined for any non-equilibrium state. He should have included a single comma, by writing "the intermediate states of the gas are non-equilibrium states, for which the enthalpy is not defined".) " -Vijay Fafat - UCR

#### Prerequisites:

- Statistics and Probability
- Multivariable Calculus
- Linear Algebra
- Ordinary Differential Equations
- Classical Mechanics
- Mathematical Methods in Physics
- Electrodynamics
- Quantum Mechanics

#### **Undergraduate Books:**

- Thermal Physics (2nd Edition) Second Edition by Charles Kittel (Author), Herbert Kroemer (Author) Not a bad a book but considering that most Statistical Mechanics aren't very well written, it stands out from the few decent books
- Introduction to Thermodynamics and Statistical Mechanics 2nd Edition by Keith Stowe (Author) A very good book that has plenty of good explanations. Mathematics is a little less than what you would hope for,

- as some explanations and crucial calculations are left to the appendix. A more modern and meaningful approach to Stat Mech than most books
- An Introduction to Thermal Physics 1st Edition by Daniel V. Schroeder (Author) The replacement textbook from Kittel & Kroemer commonly used in most Universities.
- Fundamentals of Statistical and Thermal Physics by Frederick Reif (Author) A well known book among Physicists, usually known for its' use of obscure notation and being unnecessarily formal. Usually used for reference, as most information is not presented in a clear way
- Thermodynamics; Intro Thermostat 2E Clo 2nd Edition by Herbert B. Callen (Author) See Preliminary

#### Graduate Books:

- Statistical Physics: Volume 5 3rd Edition by L D Landau (Author), E. M. Lifshitz (Author) Landau & Lifshitz textbooks are typically considered the pinnacle textbooks of their fields, usually known for their clarity. Usual complaints are that it is very math heavy and challenging. Used to tighten up foundations and knowledge.
- Statistical Mechanics, 2nd Edition by Kerson Huang (Author) Mixed reviews from all over the place, after much deliberating and reading, I can't say that this book is a recommendation or not. It may either work for you or it may not.
- Statistical Mechanics by R K Pathria (Author) Unknown about how well written it is what most users think about this book. On the list do to the fact it is commonly used for some Graduate programs at R1 and R2 universities
- Statistical Physics of Particles 1st Edition by Mehran Kardar (Author)

  Used for Statistical Mechanics I at MIT
- Statistical Physics of Fields 1st Edition by Mehran Kardar (Author) *Used for Statistical Mechanics II at MIT*
- Statistical Physics: Theory of the Condensed State (Pt 2) by E.M. Lifshitz (Author), L. P. Pitaevskii (Author) Not written by Landau so the quality or difficulty may be up in the air. Has more applications to Condensed Matter Theory

## Assignments

- MIT OCW Undergraduate Statistical Physics I
- MIT OCW Undergraduate Statistical Physics II

- MIT OCW Graduate Statistical Mechanics I/Used in conjunction with Kardar Book I/Kardar Lecture I
- MIT OCW Graduate Statistical Mechanics II/Used in conjunction with Kardar Book II/Kardar Lecture II

#### **Lecture Notes:**

- MIT OCW Statistical Physics I
- MIT OCW Statistical Physics II
- MIT OCW Graduate Stat Mech I
- MIT OCW Graduate Stat Mech II
- Rochester Undergraduate Lecture Notes
- Stanford Undergraduate Statistical Mechanics
- Caltech Landing Page for all three terms
- UCSC Landing Page for Undergraduate Stat Mech & Thermo
- Rutgers Landing Page for Graduate Stat Mech for Rutgers
- University of Cambridge David Tong David Tongs' Lecture Notes are usually considered the best around
- University of California, San Diego Currently a Work in Progress, though David Tongs landing page refers to them directly
- MSU Graduate Statistical Mechanics/ Landing Page which has Lecture Notes, Problems and Solutions, and Midterms
- MSU Graduate Statistical Physics, course from 2007-2016

# Exams

- MIT OCW Statistical Physics I
- MIT OCW Grad Stat Mech I (Only Reviews, no actual tests)
- MIT OCW Grad Stat Mech II (Only Reviews, no actual tests)
- MSU Graduate Statistical Mechanics / Quizzes and Exams
- Rochester Homework/Midterms/Final Exam

# Lectures:

• Stanford - Leonard Susskind Undergraduate Statistical Mechanics

- Mark Ancliff As per the comments as Mark Ancliff states, this class is taught to non-native English speakers, the class may be a lower level than what you might expect, as he was making sure they were comfortable with all the basics in English.
- MIT OCW Grad Stat Mech I
- MIT OCW Grad Stat Mech II

# 2.9 Special Relativity

"Special Relativity is the generally accepted and experimentally confirmed physical theory regarding the relationship between space and time."

## Prerequisites:

Depending on the book:

- Multivariable Calculus
- Linear Algebra

#### **Books**

- Special Relativity (M.I.T. Introductory Physics) First Edition by A.P. French
- Special Relativity First Edition by Robert Resnick
- The Meaning of Relativity by Albert Einstein Helps make a connection between Special Relativity and General Relativity. A "mathy" introduction to Special Relativity compared to most books, requires a good grasp on Calculus and possibly of Linear Algebra
- SPECIAL RELATIVITY AND ITS EXPERIMENTAL FOUNDATION (Advanced Theoretical Physical Science) by Yuan Zhong Zhang An interesting read if you need to know the experimental basis of Special Relativity
- Special Relativity: An Introduction with 200 Problems and Solutions 2010th Edition by Michael Tsamparlis

#### **Article Notes**

• MIT - OCW Lecture Notes

#### Videos:

• Leonard Susskind - Stanford

# **Problems**

• MIT - OCW 9 Problem Sets/No Solutions

## Exams

• MIT - OCW Exams and Solutions

# **Subtopics:**

- Modern Physics
- General Relativity

# 2.10 General Relativity

#### Prerequisites:

- Differential Geometry
- Quantum Mechanics
- Special Relativity
- Electromagnetism

#### **Books**

- Carrol A fantastic book, my personal recommendation.
- Weinberg A really good tool for learning all the bits of General Relativity
- General Relativity and the Einstein Equations by Yvonne Choquet-Bruhat A rigourous introduction to GR and SR. "Topics are: 1. Lorentz Geometry 2. Special Relativity 3. General Relativity and Einstein's Equations 4. Schwarzchild spacetime and black holes 5. Cosmology 6. Local Cauchy Problem 7. Constraints 8. Other hyperbolic-elliptic well-posed systems 9. Relativistic fluids 10. Relativistic kinetic theory 11. Progressive waves 12. Global hyperbolicity and causality 13. Singularities 14. Stationary spacetimes and black holes 15. Global existance theorems and then 200 pages of appendices on functional analysis and field theory techniques" -u/Orion952
- A First Course in General Relativity by Schutz Also quite tough introduction to GR, but the overall winner of the best book to buy here. You should use this book while going through the rest of the bibliography.
- Gravitation Hardcover by Charles W. Misner (Author, Introduction), Kip S. Thorne (Author, Introduction), John Archibald Wheeler (Author), David I. Kaiser (Preface)
- Also see: Books for general relativity StackExchange, URL (version: 2013-09-18):

#### Lecture Notes

- Columbia University
- University of Bern
- Cambridge
- University of Leipzig
- Susskind Stanford

- Sean Carrol UC Santa Barbara
- MIT OCW
- University of McGill
- Syracuse

# Videos:

- Leonard Susskind Stanford
- Colorado School of Mines
- $\bullet$  UCI

# **Problems and Exams**

- UC Santa Cruz
- MIT OCW
- Solutions to Wald
- Sergei Winitzki
- Pamona College
- Trinity College Dublin

 ${\bf Subtopics:} 1 * {\bf Subtopic - Bibliography \ does \ not \ exist} \\ {\bf Captain's \ Log}$ 

 $\bullet$  3/25/2020: Susskind Lecture link broke, re-added proper link