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Mathematical Modeling of Javelin Throw Dynamics

Project Report

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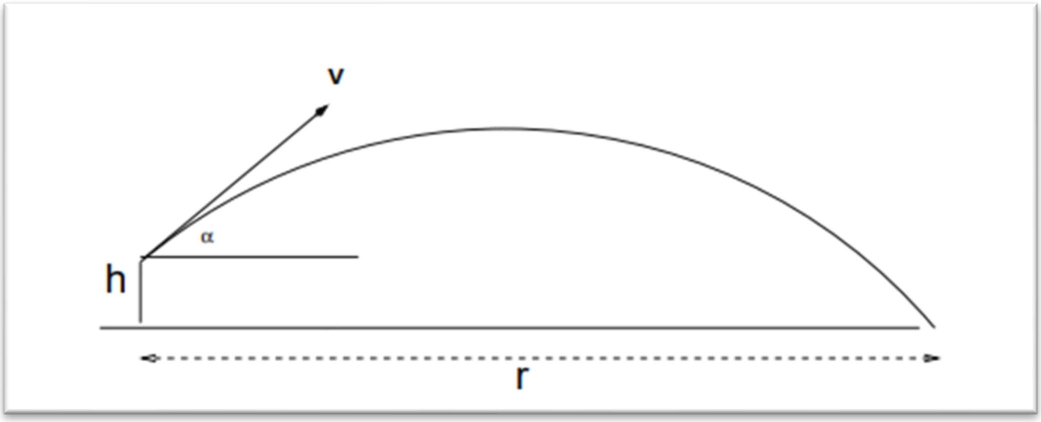
Model Design Idea



The mathematical model for the javelin throw is designed based on the principles of physics, specifically the laws of projectile motion. The primary concept in this model is the representation of the javelin as a particle. This is a common approach in physics known as the **“particle model”**, where an object is treated as if all its mass is concentrated at a single point - its center of mass.

Assumptions are made to further simplify the model:

1. **Neglecting Air Resistance**: In reality, air resistance plays a significant role in the trajectory of a javelin. However, including air resistance would complicate the model significantly as it is a non-linear force dependent on the speed and orientation of the javelin. By neglecting air resistance, we can use simple kinematic equations to describe the motion of the javelin.

2. **Launching from Ground Level**: We assume that the javelin is launched from ground level (initial vertical position ‘h’ is zero), this simplifies the equations of motion and is a reasonable approximation for a typical javelin throw.

These assumptions allow us to derive a simple yet reasonably accurate mathematical model for the javelin throw. It’s important to note that these assumptions limit the accuracy. The model is most accurate for conditions that closely match our assumptions. For example, it would be less accurate for a javelin throw in a high wind condition (where air resistance cannot be neglected) or if the javelin is launched from a significant height above the ground.

In the next section, we will use these assumptions to derive the equations of motion for the javelin throw.

Mathematical Derivations

To derive a mathematical model relating terminal distance to the javelin's launching speed and angle, utilize **the equations of projectile motion** following these steps:

1. **Initial Velocity Components**: When a javelin is launched, it has both horizontal and vertical components to its velocity.
    - The horizontal component (v_{ox}) is given by $\boldsymbol{v_o cos\ (\theta)}$**,** where v_o is the launching speed and $\boldsymbol{\theta}$ is the firing angle.
    - The vertical component $\left(v_{oy}\right)$ is given by $\boldsymbol{v_o sin\ (\theta)}$**.**
2. **Time to Reach Maximum Height**: The javelin reaches its maximum height when its vertical velocity becomes zero. Using the equation of motion:

$$\boldsymbol{v = u + at}$$

where $\boldsymbol{u}$ is initial velocity, $\boldsymbol{v}$ is final velocity, $\boldsymbol{a}$ is acceleration, and $\boldsymbol{t}$ is time

, and considering the vertical motion only, we can find the time taken to reach maximum height (T_{max}). Since the javelin is thrown upwards, the acceleration due to gravity acts against its motion, hence a= $-g$ (where g is acceleration due to gravity), and v=0 when it reaches maximum height.

$$0 = v_o\, sin(\theta) - g.\ t_{max}$$

$$t_{max} = \frac{v_{oy}}{g}$$

3. **Maximum Height Reached**: Using the kinematic equation $\boldsymbol{s = ut + \frac{1}{2}at^2}$, we can find the maximum height (h_{max}) reached by the javelin. Again, the javelin reaches its maximum height when its final velocity is zero, we use $\boldsymbol{v}$ **= 0** in this equation.

$$h_{max} = v_o\, sin(\theta)\,.\, t_{max} + \frac{1}{2}(-g).\,(t_{max})^2$$

$$h_{max} = \ v_o\, sin(\theta)\,.\frac{v_o\, sin(\theta)}{g} - \frac{1}{2}g(\frac{v_o\, sin(\theta)}{g})^2$$

$$h_{max} = \frac{1}{2g}(v_o\, sin(\theta))^2$$

$$h_{max} = \frac{{v_{oy}}^2}{2g}$$

4. **Time of Flight**: Since the time to reach maximum height is the same as the time taken to fall back to the ground, the total time of flight (t_{flight}) is twice the time taken to reach maximum height.

$$t_{flight} = 2\, t_{max}$$

5. **Horizontal Range**: The horizontal range (R) is simply the horizontal component of velocity multiplied by the total time of flight ($t$flight).

$$R = v_o\, cos(\theta)\,.\,t_{flight}$$

6. **Final Equation**: Simplifying the expression for horizontal range (R), we arrive at the final equation relating terminal distance to launching speed and launch angle, which is

$$R = \; v_o\, cos(\theta)\,.\,2\frac{v_{oy}}{g}$$

$$R = \; v_o\, cos(\theta)\,.\,2\frac{v_o sin\,(\theta)}{g}$$

$$\boldsymbol{R = \frac{{v_o}^2\, sin\,(2\theta)}{g}}$$

This equation encapsulates the relationship between the terminal distance, the initial velocity (launching speed), and the launch angle of the javelin when air resistance is neglected and the javelin is launched from ground level.

Optimum Firing Angle Calculation

In the previous section, we derived the equation for the range (R) of a projectile launched with an initial speed (v_o) at an angle (θ):

$$R = \frac{{v_o}^2 \; sin\;(2\theta)}{g}$$

This equation is a fundamental result from the physics of projectile motion and forms the basis for our analysis in this section.

Now, we will use this range equation to determine the optimum firing angle that yields the longest throwing distance for a given launching speed. This is a critical aspect of javelin throw performance, as the choice of firing angle can significantly impact the terminal distance achieved by the javelin.

To find the angle that maximizes the range, we take the derivative of the range with respect to the angle and set it equal to zero:

$$\frac{dR}{d\theta} = \frac{v^2}{g}\cos(2\theta) = 0$$

Solving for θ gives:

$$\cos(2\theta) = 0$$

This implies that:

$$2\theta = \frac{\pi}{2}$$

Therefore, the optimum firing angle θ that maximizes the range is:

$$\boldsymbol{\theta = 4\pi}$$

In our simplified model, the optimum firing angle for maximum range is **45 degrees**. This assumes no air resistance. However, in reality, factors like air resistance and the athlete's technique can affect the optimal angle. So, while our model provides a theoretical basis for understanding the impact of the firing angle on javelin throw performance, it should be used as a guide, not a definitive answer. Further studies and coaching are needed to determine the most effective angle for each athlete.

References

Chudinov, P. (2021). Study of the projectile motion in the air using simple analytical formulas. *arXiv preprint arXiv:2103.11111*.

1.8: Projectile Motion - Physics LibreTexts