Coalescing - Brouillon

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Contre-exemple Opérateur :

$$d(x) \triangleq \nabla x$$

Formules :

$$\phi = \forall z : \nabla z \wedge d(\nabla z)$$

$$\widetilde{\phi} = \forall z : \nabla z \wedge \nabla\nabla z$$

Formules FOL :

$$\phi_{\text{FOL}} = \forall z : \boxed{\lambda z.\nabla z}(z) \wedge \boxed{d_{\nabla z}}(\boxed{\lambda z.\nabla z}(z))$$

$$\widetilde{\phi}_{\text{FOL}} = \forall z : \boxed{\lambda z.\nabla z}(z) \wedge \boxed{\lambda z.\nabla\nabla z}(z)$$

Modèle FOL $\mathcal{M}$:

$$\text{dom } \mathcal{M} = \{\text{tt}, \text{ff}\}$$

$$\mathcal{I}(\boxed{\lambda z.\nabla z}) : \left\{ \begin{array}{ccc} \text{tt} & \mapsto & \text{tt} \\ \text{ff} & \mapsto & \text{ff} \end{array} \right.$$

$$\mathcal{I}(\boxed{\lambda z.\nabla\nabla z}) : \left\{ \begin{array}{ccc} \text{tt} & \mapsto & \text{ff} \\ \text{ff} & \mapsto & \text{tt} \end{array} \right.$$

Complétion de $\mathcal{M}$ en $\mathcal{M}_d$:

$$\mathcal{I}_d(\boxed{d_\epsilon}) : a \mapsto [\![\boxed{\lambda x.\nabla x}(x)]\!]_{\mathcal{M}[x \mapsto a]}$$

$$\mathcal{I}_d(\boxed{\lambda z.\nabla z}) = \mathcal{I}(\boxed{\lambda z.\nabla z})$$

$$\mathcal{I}_d(\boxed{\lambda z.\nabla\nabla z}) = \mathcal{I}(\boxed{\lambda z.\nabla\nabla z})$$

Alors :

$$[\![\boxed{d_{\nabla z}}(\boxed{\lambda z.\nabla z}(z))]\!]_{\mathcal{M}_d} \neq [\![\boxed{\lambda z.\nabla\nabla z}(z)]\!]_{\mathcal{M}_d}$$

Car :

$$\begin{aligned}
[\![\boxed{d_{\nabla z}}(\boxed{\lambda z.\nabla z}(z))]\!]_{\mathcal{M}_d} &= [\![\boxed{\lambda x.\nabla x}(x)]\!]_{\mathcal{M}_d[x \mapsto [\![\boxed{\lambda z.\nabla z}(z))]\!]_{\mathcal{M}_d}} \\
&= [\![x]\!]_{\mathcal{M}_d[x \mapsto [\![\boxed{\lambda z.\nabla z}(z))]\!]_{\mathcal{M}_d}} \\
&= [\![\boxed{\lambda z.\nabla z}(z))]\!]_{\mathcal{M}_d} \\
&= [\![z]\!]_{\mathcal{M}_d}
\end{aligned}$$

Mais :

$$\begin{aligned}
[\![\boxed{\lambda z.\nabla\nabla z}(z)]\!]_{\mathcal{M}_d} &= \begin{cases} \mathrm{tt} & \text{si } [\![z]\!]_{\mathcal{M}_d} = \mathrm{ff} \\ \mathrm{ff} & \text{sinon} \end{cases} \\
&\neq [\![z]\!]_{\mathcal{M}_d}
\end{aligned}$$

Cela veut dire qu'il faut trouver une autre interprétation des $\boxed{d_{\vec{\epsilon}}}$ dans $\mathcal{M}_d$, pour pouvoir satisfaire la propriété voulue.