

① Escribe los vectores renglón y columna

$$1.. \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

1

Vectores renglón:  $(0, -2)$  y  $(1, -3)$

Vectores columna:  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$$2.. \begin{bmatrix} 6 & s & -1 \end{bmatrix}$$

Vectores renglón:  $(6, s, -1)$

Vectores columna:  $\begin{bmatrix} 6 \\ s \\ -1 \end{bmatrix}$

2

$$3.. \begin{bmatrix} 4 & 3 & 1 \\ 1 & -4 & 0 \end{bmatrix}$$

3

Vectores renglón:  $(4, 3, 1)$  y  $(1, -4, 0)$

Vectores columna:  $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$4 - \begin{bmatrix} 0 & 2 & -3 \\ 3 & 1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$$

(4)

Vectores columnas:  $\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$

Vectores renglón:  $(0, 2, -3), (3, 1, 0), (-2, -1, 2)$

2) - Determinación de una base para un espacio renglón y rango

$$5 - \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} e_2/2 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$W = \{(1, 0), (0, 1)\}$$

$$W = \{(1, 0), (0, 1)\}$$

(5)

$$\text{Rango} = 2$$

$$6 - \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

$$W = \{(0, 1, -2)\}$$

(6)

$$\text{Rango} = 1$$

$$7.- \begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix} \xrightarrow{e_2 - 4e_1} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 14 & -7 \end{bmatrix} \xrightarrow{e_2 / 14} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

$$\xrightarrow{e_1 + 3e_2} \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

$$\boxed{\vec{w} = \{(1, 0, \frac{1}{2}), (0, 1, -\frac{1}{2})\}}$$

$$\text{rango} = 2$$

$$8.. \begin{bmatrix} 2 & -3 & 1 \\ 5 & 10 & 6 \\ 8 & -7 & 5 \end{bmatrix} \xrightarrow{e_2 \times 2} \begin{bmatrix} 2 & -3 & 1 \\ 10 & 20 & 12 \\ 8 & -7 & 5 \end{bmatrix} \xrightarrow{e_2 - 5e_1} \xrightarrow{e_3 - 4e_1}$$

$$\begin{bmatrix} 2 & -3 & 1 \\ 0 & 35 & 7 \\ 0 & 5 & 1 \end{bmatrix} \xrightarrow{e_2 / 7} \begin{bmatrix} 2 & -3 & 1 \\ 0 & 5 & 1 \\ 0 & 5 & 1 \end{bmatrix} \xrightarrow{e_2 - e_3} \begin{bmatrix} 2 & -3 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{e_1 - e_2} \begin{bmatrix} 2 & -8 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 / 2} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2 / 5} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{e_1 + 4e_2} \begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 0 \end{bmatrix} \boxed{\vec{w} = \{(1, 0, \frac{4}{5}), (0, 1, \frac{1}{5})\}} \quad \text{rango} = 2$$

$$9. \begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix} \xrightarrow{e_2 \times 2} \begin{bmatrix} -2 & -4 & 4 & 5 \\ 6 & 12 & -12 & -8 \\ -2 & -4 & 4 & 9 \end{bmatrix} \xrightarrow{e_2 + 3e_1} \begin{bmatrix} -2 & -4 & 4 & 5 \\ 6 & 12 & -12 & -8 \\ -2 & -4 & 4 & 9 \end{bmatrix} \xrightarrow{e_3 - e_1} \begin{bmatrix} -2 & -4 & 4 & 5 \\ 6 & 12 & -12 & -8 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -4 & 4 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xrightarrow{e_2 / 7} \begin{bmatrix} -2 & -4 & 4 & 5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xrightarrow{e_3 - e_2} \begin{bmatrix} -2 & -4 & 4 & 5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - 5e_2}$$

$$\begin{bmatrix} -2 & -4 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 + e_2} \begin{bmatrix} -1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\vec{w} = \{(1, 2, -2, 0), (0, 0, 0, 1)\}$  ⑨

rango = 2

$$10. \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 2 & -1 & 2 & 0 & 1 \\ 5 & 2 & 2 & 1 & -1 \\ 4 & 0 & 2 & 2 & 1 \\ 2 & -2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_3 \times 4} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 4 & -2 & 4 & 0 & 2 \\ 20 & 8 & 8 & 4 & -4 \\ 4 & 0 & 2 & 2 & 1 \\ 4 & -4 & 0 & 0 & 2 \end{bmatrix} \xrightarrow{e_2 - e_1} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 20 & 8 & 8 & 4 & -4 \\ 4 & 0 & 2 & 2 & 1 \\ 4 & -4 & 0 & 0 & 2 \end{bmatrix} \xrightarrow{e_3 - 5e_1} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & -4 & -2 & -3 & 1 \end{bmatrix} \xrightarrow{e_4 - e_1} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & -6 & 3 & -1 & 1 \end{bmatrix} \xrightarrow{e_5 - e_1}$$

$$\begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & -6 & 3 & -1 & 1 \end{bmatrix} \xrightarrow{e_3 + 4e_2} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & -6 & 3 & -1 & 1 \end{bmatrix} \xrightarrow{e_5 - 2e_3} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & -6 & 3 & -1 & 1 \end{bmatrix} \xrightarrow{e_5 + e_3} \begin{bmatrix} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[ \begin{array}{cccc|c} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -20 & -6 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 6 & -23 & -5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} e_3 + 23e_4 \\ e_3 + 5e_5 \end{array}$$

$$\left[ \begin{array}{cccc|c} 4 & 0 & 2 & 3 & 1 \\ 0 & -2 & 2 & -3 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} e_2 - 2e_3 \\ e_2 + 3e_4 \\ e_2 + e_5 \end{array} \rightarrow \left[ \begin{array}{cccc|c} 4 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} e_1 - 2e_3 \\ e_1 - 3e_4 \\ e_1 - e_5 \end{array}$$

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$$\left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \quad \vec{w} = \{(1, 0, 0, 0, 0), (0, 1, 0, 0, 0), (0, 0, 1, 0, 0), (0, 0, 0, 1, 0), (0, 0, 0, 0, 1)\}$$

rango = 5

$$11 \cdot S = \{(1, 2, 4), (-1, 3, 4), (2, 3, 1)\}$$

$$\left[ \begin{array}{ccc} 1 & 2 & 4 \\ -1 & 3 & 4 \\ 2 & 3 & 1 \end{array} \right] \begin{array}{l} e_2 + e_1 \rightarrow e_2 \\ e_3 - 2e_1 \rightarrow e_3 \end{array} \rightarrow \left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & -1 & -7 \end{array} \right] \xrightarrow{e_3 \times 5} \left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & -5 & -35 \end{array} \right]$$

$$\left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & 0 & -27 \end{array} \right] \xrightarrow{e_3 \div -27} \left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{e_2 - 8e_3} \left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{array} \right] \quad (11)$$

$$\xrightarrow{e_2 / 5} \left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{e_1 - 2e_2} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \quad \vec{w} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

rango = 3

$$12.- S = \{(4, 2, -1), (1, 2, -8), (10, 1, 2)\}$$

$$\begin{bmatrix} 4 & 2 & -1 \\ 1 & 2 & -8 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{e_2 \times 4} \begin{bmatrix} 4 & 2 & -1 \\ 4 & 8 & -32 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{e_2 - e_1} \begin{bmatrix} 4 & 2 & -1 \\ 0 & 6 & -31 \\ 0 & 1 & 2 \end{bmatrix}$$

$$e_3 \times 6 \rightarrow \begin{bmatrix} 4 & 2 & -1 \\ 0 & 6 & -31 \\ 0 & 6 & 12 \end{bmatrix} \xrightarrow{e_3 - e_2} \begin{bmatrix} 4 & 2 & -1 \\ 0 & 6 & -31 \\ 0 & 0 & 43 \end{bmatrix} \xrightarrow[e_3 + 31e_3]{e_3/43} \boxed{12}$$

$$\begin{bmatrix} 4 & 2 & -1 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_2/6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \vec{w} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$13.- S = \{(4, 4, 8), (1, 1, 2), (1, 1, 1)\}$$

$$\begin{bmatrix} 4 & 4 & 8 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{e_1/4} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{e_2 - e_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{e_3 - e_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[e_1 - 2e_2]{e_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{w} = \{(1, 1, 0), (0, 0, 1)\} \quad \text{13}$$

$$14 - S = \{(1, 2, 2), (-1, 0, 0), (1, 1, 1)\}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ -1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{\begin{array}{l} e_2 + e_1 \\ e_3 - e_1 \end{array}} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{e_{212}} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{e_3 + e_2} \boxed{\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

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$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - 2e_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \boxed{\vec{W} = \{(1, 0, 0), (0, 1, 1)\}}$$

$$15 - S = \{(2, 9, -2, 53), (-3, 2, 3, -2), (8, -3, -8, 17), (0, -3, 0, 15)\}$$

$$\begin{bmatrix} 2 & 9 & -2 & 53 \\ -3 & 2 & 3 & -2 \\ 8 & -3 & -8 & 17 \\ 0 & -3 & 0 & 15 \end{bmatrix} \xrightarrow{e_2 \times 2} \begin{bmatrix} 2 & 9 & -2 & 53 \\ -6 & 4 & 6 & -4 \\ 8 & -3 & -8 & 17 \\ 0 & -3 & 0 & 15 \end{bmatrix} \xrightarrow{\begin{array}{l} e_2 + 3e_2 \\ e_3 - 4e_1 \end{array}} \begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & -3 & 0 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 31 & 0 & 155 \\ 0 & -39 & 0 & -195 \\ 0 & -3 & 0 & 15 \end{bmatrix} \xrightarrow{\begin{array}{l} e_2/31 \\ e_3 + 3e_1 \\ e_4/3 \end{array}} \begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & -1 & 0 & 5 \end{bmatrix} \xrightarrow{\begin{array}{l} e_3 - e_2 \\ e_4 + e_2 \end{array}} \begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \xrightarrow{e_4/10} \begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_2 - 5e_4} \begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 9 & -2 & 53 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\begin{array}{l} e_1 - 9e_2 \\ e_1 - 53e_4 \end{array}} \begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_1/2} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[ \begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\quad} W = \{(1, 0, -1, 0), (0, 1, 0, 0), (0, 0, 0, 1)\}$$

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$$16.- S = \{(6, -3, 6, 34), (3, -2, 3, 19), (8, 3, -9, 6), (-2, 0, 6, -5)\}$$

$$\left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 3 & -2 & 3 & 19 \\ 8 & 3 & -9 & 6 \\ -2 & 0 & 6 & -5 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 \times 2 \\ e_3 \times 3 \\ e_4 \times 3 \end{array}} \left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 6 & -4 & 6 & 38 \\ 24 & 9 & -27 & 18 \\ -6 & 0 & 18 & -15 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 - e_1 \\ e_3 - 4e_1 \\ e_4 + e_1 \end{array}}$$

$$\left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & -1 & 0 & 4 \\ 0 & 21 & -51 & -118 \\ 0 & -3 & 24 & 19 \end{array} \right] \xrightarrow{e_2 \rightarrow -1} \left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 21 & -51 & -118 \\ 0 & -3 & 24 & 19 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 - 21e_2 \\ e_4 + 3e_2 \end{array}}$$

$$\left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -51 & -34 \\ 0 & 0 & 24 & 7 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 \rightarrow -51 \\ e_4 \rightarrow 1 \end{array}} \left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 34/51 \\ 0 & 0 & 24 & 7 \end{array} \right] \xrightarrow{e_4 - 24e_3}$$

$$\left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 34/51 \\ 0 & 0 & 0 & -9 \end{array} \right] \xrightarrow{e_1 \rightarrow 9} \left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 34/51 \\ 0 & 0 & 0 & 1 \end{array} \right] \rightarrow$$

$$\left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 34/51 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 - 34e_4 \\ e_3 \rightarrow 51 \end{array}} \left[ \begin{array}{cccc} 6 & -3 & 6 & 34 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 + 4e_4 \\ e_1 + 3e_2 \\ e_1 - 6e_3 \\ e_1 - 34e_4 \end{array}}$$

$$\left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \boxed{\vec{W} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}} \quad (16)$$

$$17 \dots S = \{(-3, 2, 5, 28), (-6, 1, -8, -1), (14, -10, 12, -10), (0, 5, 12, 50)\}$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ -6 & 1 & -8 & -1 \\ 14 & -10 & 12 & -10 \\ 0 & 5 & 12 & 50 \end{array} \right] \xrightarrow{e_3/12} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ -6 & 1 & -8 & -1 \\ 7 & -5 & 6 & -5 \\ 0 & 5 & 12 & 50 \end{array} \right] \xrightarrow[e_2 - e_1]{e_3 \times 3}$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & -3 & -18 & -57 \\ 21 & -15 & 18 & 15 \\ 0 & 5 & 12 & 50 \end{array} \right] \xrightarrow{e_3 + 7e_1} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & -3 & -18 & -57 \\ 0 & -1 & 53 & 211 \\ 0 & 5 & 12 & 50 \end{array} \right] \xrightarrow[e_2 / -3]{e_2 / -3}$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & -1 & 53 & 211 \\ 0 & 5 & 12 & 50 \end{array} \right] \xrightarrow{e_3 \times 3} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & -3 & 159 & 633 \\ 0 & 15 & 36 & 150 \end{array} \right] \xrightarrow[e_4 + 5e_2]{e_3 + e_2}$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & 0 & 165 & 652 \\ 0 & 0 & 6 & 55 \end{array} \right] \xrightarrow{e_4 + 6} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & 0 & 1 & 55/16 \\ 0 & 0 & 165 & 652 \end{array} \right] \xrightarrow{e_3 - 165e_4} e_3 - 165e_4$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & 0 & 1 & 55/16 \\ 0 & 0 & 0 & 5163/16 \end{array} \right] \xrightarrow{\begin{array}{l} e_4 \times 6 / 5163 \\ e_3 \times 6 \end{array}} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 - 6e_1 \\ e_2 - 19e_1 \end{array}} \left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 3 & 6 & 19 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad (17)$$

$$\left[ \begin{array}{cccc} -3 & 2 & 5 & 28 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} e_1 - 2e_2 \\ e_1 - 5e_3 \\ e_1 - 28e_4 \end{array}} \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \vec{w} = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$$

(18) -  $S = \{(2, 5, -3, -2), (-2, -3, 2, -5), (1, 3, -2, 2), (-1, -5, 3, 5)\}$

$$\left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -5 & 3 & 5 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 \times 2 \\ e_4 \times 2 \end{array}} \left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 2 & 6 & -4 & 4 \\ -2 & -10 & 6 & 10 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 + e_1 \\ e_3 - e_1 \\ e_4 + e_1 \end{array}}$$

$$\left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ 0 & 2 & -1 & -7 \\ 0 & 1 & -1 & 6 \\ 0 & -5 & 3 & 8 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 \times 2 \\ e_4 \times 2 \end{array}} \left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ 0 & 2 & -1 & -7 \\ 0 & 2 & -2 & 12 \\ 0 & -10 & 6 & 16 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 - e_2 \\ e_4 + 5e_2 \end{array}}$$

$$\left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ 0 & 2 & -1 & -7 \\ 0 & 0 & -1 & 19 \\ 0 & 0 & 1 & -19 \end{array} \right] \xrightarrow{e_4 + e_3} \left[ \begin{array}{cccc} 2 & 5 & -3 & -2 \\ 0 & 2 & -1 & -7 \\ 0 & 0 & -1 & 19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{e_2 - e_3}$$

$$\left[ \begin{array}{cccc} 2 & s & -3 & -2 \\ 0 & 2 & -1 & -7 \\ 0 & 0 & -1 & 19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\quad} \left[ \begin{array}{cccc} 2 & s & -3 & -2 \\ 0 & 2 & 0 & -26 \\ 0 & 0 & -1 & 19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow[e_2/2]{\quad} \left[ \begin{array}{cccc} 2 & s & -3 & -2 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow[e_3+1]{\quad}$$

$$\left[ \begin{array}{cccc} 2 & s & -3 & -2 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow[e_1+3e_3]{\quad} \left[ \begin{array}{cccc} 2 & s & 0 & -59 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow[e_1-5e_2]{\quad}$$

$$\left[ \begin{array}{cccc} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow[e_1/6]{\quad} \left[ \begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & -19 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

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$$\vec{W} = \{(1, 0, 0, 3), (0, 1, 0, -13), (0, 0, 1, -19)\}$$

$$19 - \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix} \xrightarrow{e_2 - 2e_1} \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \xrightarrow{e_2/4} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow{e_1 - e_2}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{e_1/2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\vec{w} = \{(1, 0), (0, 1)\}$   
rango = 2

(19)

$$20. [1, 2, 3]$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \xrightarrow{e_2/2} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \xrightarrow{e_3/3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(20)

$\vec{w} = \{(1), (1), (1)\}$   
rango = 3

$$21 - \begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \\ 4 & 1 \end{bmatrix} \xrightarrow{e_2 - 2e_1} \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 0 & 5 \end{bmatrix} \xrightarrow{e_3/5} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow{e_3 - e_2}$$

(21)

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow{e_1 + e_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$\vec{w} = \{(1, 0), (0, 1)\}$   
rango = 2

$$22 - \begin{bmatrix} 4 & 20 & 31 \\ 6 & -5 & -6 \\ 2 & -11 & -16 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 & 2 \\ 20 & -5 & -11 \\ 31 & -6 & -16 \end{bmatrix} \xrightarrow{e_1/2} \begin{bmatrix} 2 & 3 & 1 \\ 20 & -5 & -11 \\ 62 & -12 & -32 \end{bmatrix} \xrightarrow{e_3 - 31e_1} \begin{bmatrix} 2 & 3 & 1 \\ 0 & -5 & 3 \\ 0 & 15 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & -5 & 3 \\ 0 & 15 & 9 \end{bmatrix} \xrightarrow{e_2 + 7} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 3 \\ 0 & 15 & 9 \end{bmatrix} \xrightarrow{e_3 - 3e_2} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{e_2 \times 3} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 15 & 9 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - e_2} \begin{bmatrix} 10 & 0 & -4 \\ 0 & 15 & 9 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1/2} \begin{bmatrix} 5 & 0 & -2 \\ 0 & 15 & 9 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2/15} \begin{bmatrix} 5 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1/5} \begin{bmatrix} 1 & 0 & -2/5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$\vec{w} = \{(1, 0, -2/5), (0, 1, 3)\}$  (22)

rango = 2

$$23 \cdot \begin{bmatrix} 2 & 7 & -3 & -6 \\ 7 & 14 & -6 & -3 \\ -2 & -4 & 1 & -2 \\ 2 & 4 & -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 7 & -2 & 2 \\ 7 & 14 & -4 & 4 \\ -3 & -6 & 1 & -2 \\ -6 & -3 & -2 & -2 \end{bmatrix} \xrightarrow{\begin{array}{l} e_2/2 \\ e_4/2 \end{array}} \begin{bmatrix} 2 & 7 & -2 & 2 \\ 2 & 7 & -2 & 2 \\ -3 & -6 & 1 & -2 \\ -6 & -3 & -2 & -2 \end{bmatrix} \xrightarrow{\begin{array}{l} e_1 - e_2 \\ e_4 - 2e_3 \end{array}}$$

$$\begin{bmatrix} 2 & 7 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ -3 & -6 & 1 & -2 \\ 0 & 9 & -4 & 2 \end{bmatrix} \xrightarrow{e_3 \times 2} \begin{bmatrix} 2 & 7 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ -6 & -12 & 2 & -4 \\ 0 & 9 & -4 & 2 \end{bmatrix} \xrightarrow{e_3 + 3e_1}$$

$$\begin{bmatrix} 2 & 7 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 9 & -4 & 2 \\ 0 & 9 & -4 & 2 \end{bmatrix} \xrightarrow{e_4 - e_3} \begin{bmatrix} 2 & 7 & -2 & 2 \\ 0 & 9 & -4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\begin{array}{l} e_1/2 \\ e_2/9 \end{array}}$$

$$\begin{bmatrix} 2 & 7 & -2 & 2 \\ 0 & 1 & -4/9 & 2/9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - 7e_2} \begin{bmatrix} 2 & 0 & 10/9 & 4/9 \\ 0 & 1 & -4/9 & 2/9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\vec{W} = \{(1, 0, 10/18, 2/9), (0, 1, -4/9, 2/9)\}$$

$$\text{rango} = 2$$

(23)

$$2S \cdot A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \xrightarrow{\quad} \left[ \begin{array}{cc|c} 2 & -1 & 0 \\ -6 & 3 & 0 \end{array} \right] \xrightarrow{e_2 + 3e_1} \left[ \begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$e_1/2 \rightarrow \left[ \begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad x - \frac{1}{2}y = 0 \\ x = \frac{1}{2}y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}y \\ \frac{1}{2}x \end{pmatrix} = x \left( \frac{1}{2} \right) + y \left( \frac{1}{2} \right)$$

$$N(A) = \left( \left[ \frac{1}{2} \right]; \left[ \frac{1}{2} \right] \right)$$

$$N(A) = \left\{ \left[ \begin{array}{c} \frac{1}{2} \\ 1 \end{array} \right] \right\}$$

2S

$$26 \cdot A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} e_{1/2} \begin{bmatrix} 1 & -1/2 \\ 1 & 3 \end{bmatrix} \xrightarrow{e_2 - e_1} \begin{bmatrix} 1 & -1/2 \\ 0 & 7/2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 7 \end{bmatrix} e_{2/7} \rightarrow \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} e_2 + e_1 \rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$N(A) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

(26)

$$27 \cdot A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$x + 2y + 3z$$

$$x = \begin{bmatrix} -2y - 3z \\ y \\ z \end{bmatrix}$$

$$\begin{aligned} x &= -2y - 3z \\ y &= \underline{-3z - x} \\ z &= \underline{\frac{-2y - x}{3}} \end{aligned}$$

$N(A) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}; \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$

(27)

$$28 - \begin{bmatrix} 1 & 4 & 2 \end{bmatrix}$$

$$x + 4y + 2z = x = -4y - 2z$$

(28)

$$x \begin{bmatrix} -4y - 2z \\ y \\ z \end{bmatrix}$$

$$N(A) = \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}; \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$29 - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix} e_1 - 2e_2 \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}$$

$$x + 3z \quad x = -3z$$

$$y = 0 \quad y = 0$$

$$x = \begin{bmatrix} -3z \\ 0 \\ z \end{bmatrix}$$

$$N(A) = \left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$30 - \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix} e_1 - 2e_2 \begin{bmatrix} 1 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x + 4y = 0 \quad x = -4y$$

$$z = 0 \quad x \begin{bmatrix} -4y \\ y \\ 0 \end{bmatrix}$$

$$N(A) = \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} \right\}$$

(30)

$$(31) \quad A = \begin{bmatrix} 3 & 2 & -3 \\ 2 & -1 & 4 \\ 4 & 3 & -2 \end{bmatrix} e_2 - 2e_1 \quad \begin{bmatrix} 1 & 2 & -3 \\ 0 & -5 & 10 \\ 0 & -5 & 10 \end{bmatrix}$$

$$e_2 - e_3 \cdot \begin{bmatrix} 1 & 2 & -3 \\ 0 & -5 & 10 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2/(-5)} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - 2e_2}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} x + z = 0 \\ y - 2z = 0 \end{array} \quad \begin{array}{l} x = -z \\ y = 2z \end{array}$$

$$N(A) = \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\} \quad (31)$$

$$(32) \quad A = \begin{bmatrix} 3 & -6 & 21 \\ -2 & 4 & -14 \\ 1 & -2 & 7 \end{bmatrix} \xrightarrow{e_1/3} \begin{bmatrix} 1 & -2 & 7 \\ -1 & 2 & -7 \\ 1 & -2 & 7 \end{bmatrix} \xrightarrow{e_3 + e_2}$$

$$\begin{bmatrix} 1 & -2 & 7 \\ -1 & 2 & -7 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2 + e_1} \begin{bmatrix} 1 & -2 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x - 2y + 7z = 0 \quad x = 2y - 7z$$

$$N(A) = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}; \begin{bmatrix} -7 \\ 0 \\ 1 \end{bmatrix} \right\} \quad (32)$$

$$(33) \quad \left[ \begin{array}{cccc} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ -2 & -6 & 4 & -8 \end{array} \right] \xrightarrow{e_3 + 2e_1} \left[ \begin{array}{cccc} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{e_1 - 3e_2} \left[ \begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{aligned} x + z - 2w \\ y - z + 2w \end{aligned}$$

$$\begin{aligned} x &= 2w - z \\ y &= -2w + z \end{aligned}$$

(33)

$$N(A) = \left\{ \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}; \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$(34) \quad \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ -2 & -8 & -4 & -2 \end{array} \right] \xrightarrow{e_2 + 2e_1} \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{e_1 - 4e_2} \left[ \begin{array}{cccc} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{aligned} x &= 2z - 5w \\ y &= -z + w \end{aligned}$$

$$N(A) = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}; \begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(34)

$$35 \rightarrow \left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 2 & 1 & 0 & -2 \\ 3 & -2 & 1 & 1 \\ 0 & 6 & 2 & 0 \end{array} \right] \xrightarrow{e_4/2} \left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 2 & 1 & 0 & -2 \\ 3 & -2 & 1 & 1 \\ 0 & 3 & 1 & 0 \end{array} \right] \xrightarrow{e_3 \times 2}$$

$$\left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 2 & 1 & 0 & -2 \\ 6 & -4 & 2 & 2 \\ 0 & 3 & 1 & 0 \end{array} \right] \xrightarrow{e_2 - e_1} \left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & -5 & -3 & -3 \\ 6 & -22 & -7 & -1 \\ 0 & 3 & 1 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & 1 & 1/3 & 0 \\ 0 & -5 & -3 & -3 \\ 0 & -22 & -7 & -1 \end{array} \right] \xrightarrow{e_3 + 5e_2} \left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & -4/3 & -3 \\ 0 & 0 & 1/3 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & -4/3 & -3 \end{array} \right] \xrightarrow{e_4 + 4e_3} \left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 3 \end{array} \right] \xrightarrow{e_4/3}$$

$$\left[ \begin{array}{cccc} 2 & 6 & 3 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \begin{array}{l} x=0 \\ y=0 \\ z=0 \\ y=0 \end{array}$$

$$NCAJ = \left\{ \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \right\} \quad \text{35}$$

(36)

$$\left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 4 & 2 & 1 & 1 \\ 0 & 4 & 2 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} e_2 - 2e_1 \\ e_3 - 4e_1 \\ e_4 - 0 \end{array}} \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & -9 & -3 & -1 \\ 0 & -14 & -7 & -3 \\ 0 & 4 & 2 & 0 \end{array} \right] \xrightarrow{e_2/4} \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1/2 & 0 \\ 0 & -9 & -3 & -1 \\ 0 & -14 & -7 & -3 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & -9 & -3 & -1 \\ 0 & -14 & -7 & -3 \\ 0 & 1 & 1/2 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} e_3 + 9e_2 \\ e_4 + 14e_2 \end{array}} \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1/2 & 0 \\ 0 & -9 & -3 & -1 \\ 0 & -14 & -7 & -3 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 3/2 & -1 \\ 0 & 0 & 0 & -3 \end{array} \right] \xrightarrow{\begin{array}{l} \\ \\ e_3 - 1/2e_2 \\ \end{array}} \left[ \begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$N(A) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

36

$$1 - x = (5, -2)$$

$$x = (5, -2) = 5(1, 0) + (-2)(0, 1) = (5, 0) + (0, -2) =$$

$$(5, -2)$$

$$\boxed{[x]_s = \begin{bmatrix} 5 \\ -2 \end{bmatrix}} \quad 1$$

$$2 - x = (1, -3, 0)$$

$$x = (1, -3, 0) = 1(1, 0, 0) + (-3)(0, 1, 0) + 0(0, 0, 1)$$

$$(1, 0, 0) + (0, -3, 0) + (0, 0, 0) = (1, -3, 0)$$

$$\boxed{[x]_s = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}} \quad 2$$

$$3 - x = (7, -4, -1, 2)$$

$$x = (7, -4, -1, 2) = 7(1, 0, 0, 0) + -4(0, 1, 0, 0) + -1(0, 0, 1, 0) + 2(0, 0, 0, 1) = (7, 0, 0, 0) + (0, -4, 0, 0) + (0, 0, -1, 0) + (0, 0, 0, 2) = (7, -4, -1, 2)$$

$$\boxed{[x]_s = \begin{bmatrix} 7 \\ -4 \\ -1 \\ 2 \end{bmatrix}} \quad 3$$

$$y \cdot x = (-6, 12, -4, 9, -8)$$

$$x = -6(1, 0, 0, 0, 0) + 12(0, 1, 0, 0, 0) + -4(0, 0, 1, 0, 0) + 9(0, 0, 0, 1, 0) + -8(0, 0, 0, 0, 1) = (6, 0, 0, 0, 0) +$$
$$(0, 12, 0, 0, 0) + (0, 0, -4, 0, 0) + (0, 0, 0, 9, 0) + (0, 0, 0, 0, -8)$$
$$(-6, 12, -4, 9, -8)$$

$$[x]_S = \begin{bmatrix} -6 \\ 12 \\ -4 \\ 9 \\ -8 \end{bmatrix} \quad (4)$$

$$S \cdot B = \{(2, -1), (0, 1)\} \quad B^3 = \{(1, 0), (0, 1)\}$$

$$[x]_B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$x = 4(2, -1) + 1(0, 1) = (8, -4) + (0, 1) = (8, -3)$$

$$(8, -3) = 8(1, 0) + -3(0, 1) = (8, 0) + (0, -3) = (8, -3)$$

$$[x]_{B^3} = \begin{bmatrix} 8 \\ -3 \end{bmatrix} \quad (5)$$

$$6 \cdot B = \{(-1, 4), (4, -1)\}$$

$$[x]_B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$x = -2(-1, 4) + 3(4, -1) = (2, -8) + (12, -3) = (14, -11)$$

$$(14, -11) = 14(1, 0) + -11(0, 1) = ((4, 0) + (0, -11)) = (14, -11)$$

$$[x]_B = \begin{bmatrix} 14 \\ -11 \end{bmatrix} \quad (6)$$

$$7 \cdot B = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}; \quad [x]_B = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$x = 2(1, 0, 1) + 3(1, 1, 0) + 1(0, 1, 1) = (2, 0, 2) + (3, 3, 0) + (0, 1, 1) = (5, 4, 3)$$

$$(5, 4, 3) = (5(1, 0, 0) + 4(0, 1, 0) + 3(0, 0, 1)) = (5, 4, 3)$$

$$[x]_B = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} \quad (7)$$

$$8 \cdot B = \left\{ \left( \frac{3}{4}, \frac{5}{2}, \frac{3}{2} \right), (3, 4, \frac{7}{2}), \left( -\frac{3}{2}, 6, 2 \right) \right\} \quad [x]_B = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

$$x = 2 \left( \frac{3}{4}, \frac{5}{2}, \frac{3}{2} \right) + 0(3, 4, \frac{7}{2}) + 4 \left( -\frac{3}{2}, 6, 2 \right) = \frac{6}{4} = \frac{3}{2} \quad \frac{10}{2} = 5$$

$$x = \left( \frac{3}{2}, 5, 3 \right) + (0, 0, 0) + (-6, 24, 8) = \frac{12}{2} = 6 \quad \frac{0}{2} = 0$$

$$x = \left( -\frac{9}{2}, 29, 11 \right) \quad [x]_B = \begin{bmatrix} -9/2 \\ 29 \\ 11 \end{bmatrix} \quad (8) \quad \frac{3}{2} - \frac{12}{2} = \frac{9}{2}$$

$$9 \cdot B = \{(0,0,0,1), (0,0,1,1), (0,1,1,1), (1,1,1,1)\}$$

$$[x]_B = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \quad x = 1(0,0,0,1) + -2(0,0,1,1) + 3(0,1,1,1) \\ + -1(1,1,1,1) = \\ x = (0,0,0,1) + (0,0,-2,-2) + (0,3,3,3) \\ + (-1,-1,-1,-1) = (-1,2,0,1)$$

$$[x]_{B^1} = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad 9$$

$$\begin{array}{r} 17 \\ -8 \\ \hline 9 \end{array}$$

$$10 \cdot B = \{(4,0,7,3), (0,5,-1,-1), (-3,4,2,1), (0,1,5,0)\}$$

$$[x]_B = \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \quad x = -2(4,0,7,3) + 3(0,5,-1,-1) + 4(-3,4,2,1) \\ + 1(0,1,5,0) \\ x = (-8,0,-14,-6) + (0,15,-3,-3) + (-12, \\ 16,8,4) + (0,1,5,0) = (-20,32,-4,-5)$$

$$[x]_{B^1} = \begin{bmatrix} -20 \\ 32 \\ -4 \\ -5 \end{bmatrix} \quad 10$$

$$11 \cdot B^1 = \{(4, 0), (0, 3)\} \quad x = (12, 6)$$

$$\left[ \begin{array}{cc|c} 4 & 0 & 12 \\ 0 & 3 & 6 \end{array} \right] \xrightarrow{e_1/4} \left[ \begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array} \right]$$

$$x = 3(4, 0) + 2(0, 3) = (12, 0) + (0, 6) = (12, 6)$$

$$[x]_{B^1} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad (11)$$

$$12 \cdot B^1 = \{(-6, 7), (4, -3)\} \quad x = (-26, 32)$$

$$\left[ \begin{array}{cc|c} -6 & 4 & -26 \\ 7 & -3 & 32 \end{array} \right] \xrightarrow{e_1+2e_2} \left[ \begin{array}{cc|c} -3 & 2 & -13 \\ 7 & -3 & 32 \end{array} \right] \xrightarrow{e_2 \times 3} \left[ \begin{array}{cc|c} -3 & 2 & -13 \\ 21 & -9 & 96 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} -3 & 2 & -13 \\ 0 & 5 & 5 \end{array} \right] \xrightarrow{e_2/5} \left[ \begin{array}{cc|c} -3 & 2 & -13 \\ 0 & 1 & 1 \end{array} \right] \xrightarrow{e_1 - 2e_2} \left[ \begin{array}{cc|c} -3 & 0 & -15 \\ 0 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{e_1/-3} \left[ \begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \end{array} \right]$$

$$x = 5(-6, 7) + 1(4, -3) = (-30, 35) + (4, -3) = (-26, 32)$$

$$[x]_{B^1} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$B = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\} \quad x = (3, 19, 2)$$

$$\left[ \begin{array}{ccc|c} 8 & 7 & 1 \\ 11 & 0 & 4 \\ 0 & 10 & 6 \end{array} \right] \xrightarrow{e_2 \times 8} \left[ \begin{array}{ccc|c} 8 & 7 & 1 \\ 88 & 0 & 32 \\ 0 & 10 & 6 \end{array} \right] \xrightarrow{e_2 - 11e_1} \left[ \begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & -77 & 21 & 119 \\ 0 & 10 & 6 & 2 \end{array} \right]$$

$$\xrightarrow{e_2/7} \left[ \begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & 11 & -3 & -17 \\ 0 & 110 & 66 & 22 \end{array} \right] \xrightarrow{e_3 - 10e_2} \left[ \begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & 11 & -3 & -17 \\ 0 & 0 & 96 & 192 \end{array} \right] \xrightarrow{e_3 / 96}$$

$$\left[ \begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & 11 & -3 & -17 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_2 + 3e_3} \left[ \begin{array}{ccc|c} 8 & 7 & 0 & 1 \\ 0 & 11 & 0 & -11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_2 / 11}$$

$$\left[ \begin{array}{ccc|c} 8 & 7 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_1 - 7e_2} \left[ \begin{array}{ccc|c} 8 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_1 / 8}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$x = 1(8, 11, 0) + -1(7, 0, 10) + 2(1, 4, 6) =$$

$$(8, 11, 0) + (-7, 0, -10) + (2, 8, 12) = (3, 19, 2)$$

$$\left[ \begin{array}{c} x \\ \hline B^1 \end{array} \right] = \left[ \begin{array}{c} 3 \\ 19 \\ 2 \end{array} \right]$$

$$\left[ \begin{array}{c} x \\ \hline B^1 \end{array} \right] = \left[ \begin{array}{c} 1 \\ -1 \\ 2 \end{array} \right] \quad (13)$$

$$14 - B' = \left\{ \left( \frac{3}{2}, 4, 1 \right), \left( \frac{3}{4}, \frac{5}{2}, 0 \right), \left( 1, \frac{1}{2}, 2 \right) \right\} \quad x = \left( 3, -\frac{1}{2}, 8 \right)$$

$$\left[ \begin{array}{ccc|c} \frac{3}{2} & \frac{3}{4} & 1 & 3 \\ 4 & \frac{5}{2} & \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 2 & 3 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ \frac{3}{2} & \frac{3}{4} & 1 & 3 \\ 4 & \frac{5}{2} & \frac{1}{2} & -\frac{1}{2} \end{array} \right] \xrightarrow{e_2 \times 2}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 3 & \frac{3}{2} & 2 & 6 \\ 4 & \frac{5}{2} & \frac{1}{2} & -\frac{1}{2} \end{array} \right] \xrightarrow{e_2 - 3e_1} \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & \frac{3}{2} & -4 & -18 \\ 4 & \frac{5}{2} & -\frac{1}{2} & -\frac{65}{2} \end{array} \right] \xrightarrow{e_3 - 4e_1} \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & \frac{3}{2} & -4 & -18 \\ 0 & \frac{5}{2} & -\frac{1}{2} & -\frac{65}{2} \end{array} \right] \xrightarrow{e_3 \times 2}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 3 & -8 & -36 \\ 0 & 5 & -15 & -65 \end{array} \right] \xrightarrow{e_3 \times 3} \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 3 & -8 & -36 \\ 0 & 15 & -45 & -195 \end{array} \right] \xrightarrow{e_3 - 5e_2}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 3 & -8 & -36 \\ 0 & 0 & -5 & -15 \end{array} \right] \xrightarrow{e_3 \rightarrow -5} \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 3 & -8 & -36 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{e_2 + 8e_3} \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 3 & 0 & -11 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{e_1 - 2e_3}$$

$$\xrightarrow{e_2 / 3} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$x = 2 \left( \frac{3}{2}, 4, 1 \right) + -4 \left( \frac{3}{4}, \frac{5}{2}, 0 \right) + 3 \left( 1, \frac{1}{2}, 2 \right) =$$

$$x = (3, 8, 2) + (-3, -10, 0) + (3, \frac{3}{2}, 6) = (3, -\frac{1}{2}, 8)$$

$$[x]_{B'} = \begin{bmatrix} 3 \\ -\frac{1}{2} \\ 8 \end{bmatrix}$$

$$[x]_{B'} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix} \boxed{14}$$

$$l_5 - B^1 = \{(4, 3, 3), (-11, 0, 11), (0, 9, 2)\} \times (11, 18, -7)$$

$$\left[ \begin{array}{ccc|c} 4 & -11 & 0 & 11 \\ 3 & 0 & 9 & 18 \\ 3 & 11 & 2 & -7 \end{array} \right] \xrightarrow{e_2 \times 3} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 3 & 11 & 2 & -7 \\ 4 & -11 & 0 & 11 \end{array} \right] \xrightarrow{e_2 - 3e_1} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 11 & 2 & -7 \\ 4 & -11 & 0 & 11 \end{array} \right] \xrightarrow{e_3 - 4e_1}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 11 & 2 & -7 \\ 0 & -11 & -12 & -13 \end{array} \right] \xrightarrow{e_3 + e_2} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 11 & 2 & -7 \\ 0 & 0 & -19 & -38 \end{array} \right] \xrightarrow{e_3 \times -1/19}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 11 & 2 & -7 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_2 + 7e_3} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$x = 0(4, 3, 3) + -1(-11, 0, 11) + 2(0, 9, 2)$$

$$x = (0, 0, 0) + (11, 0, -11) + (0, 18, 4) = (11, 18, -7)$$

$[x]_{B^1} =$	$\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$
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(15)

$$16. B' = \{(9, -3, 15, 4), (3, 0, 0, 1), (0, -5, 6, 8), (3, -4, 2, -3)\}$$

$$x = (0, -20, 7, 15)$$

$$\left[ \begin{array}{cccc|c} 9 & 3 & 0 & 3 & 0 \\ -3 & 0 & -5 & -4 & -20 \\ 15 & 0 & 6 & 2 & 7 \\ 4 & 1 & 8 & -3 & 15 \end{array} \right] \xrightarrow{e_1/3} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ -3 & 0 & -5 & -4 & -20 \\ 15 & 0 & 6 & 2 & 7 \\ 4 & 1 & 8 & -3 & 15 \end{array} \right] \xrightarrow{e_2 + e_1 \rightarrow e_2} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -4 & -20 \\ 15 & 0 & 6 & 2 & 7 \\ 4 & 1 & 8 & -3 & 15 \end{array} \right] \xrightarrow{e_3 - 5e_1 \rightarrow e_3} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -4 & -20 \\ 0 & 0 & 6 & 2 & 7 \\ 4 & 1 & 8 & -3 & 15 \end{array} \right] \xrightarrow{e_4 \times 3 \rightarrow e_4} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -4 & -20 \\ 0 & 0 & 6 & 2 & 7 \\ 0 & 1 & 8 & -3 & 15 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & -5 & 6 & -3 & 7 \\ 0 & -1 & 24 & -13 & 95 \end{array} \right] \xrightarrow{e_3 + 5e_2} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 19 & -16 & 25 \end{array} \right] \xrightarrow{e_4 + e_3 \rightarrow e_4} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & -2 & 25 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & -34 & -68 \end{array} \right] \xrightarrow{e_4 / -34} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_3 + 18e_4} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_2 + 3e_4} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_1 - e_4} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & -5 & -3 & -20 \\ 0 & 0 & 19 & -18 & -93 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 3 & 1 & 0 & 0 & -2 \\ 0 & 1 & -5 & 0 & -14 \\ 0 & 0 & 19 & 0 & -57 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_3 / -19} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 0 & -2 \\ 0 & 1 & -5 & 0 & -14 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_2 + 5e_3} \left[ \begin{array}{cccc|c} 3 & 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{x = (-9, 3, -15, -4) + (3, 0, 0, 1) + }$$

$$\left[ \begin{array}{cccc|c} 3 & 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{e_1 - e_2 / 3} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{(0, -15, 18, 24), (6, -8, 4, -6), (0, -20, 7, 15)} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$[x]_S = \begin{bmatrix} -1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$17 - B = \{(1,0), (0,1)\}, B^{\circ} = \{(2,4), (1,3)\}$$

$$\left[ \begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{array} \right] \xrightarrow{e_2 - 2e_1} \left[ \begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array} \right] \xrightarrow{e_1 - e_2 \rightarrow e_1} \boxed{17}$$

$$\left[ \begin{array}{cc|cc} 2 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array} \right] \xrightarrow{e_1/2} \left[ \begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & -2 & 1 \end{array} \right]$$

$$P = \boxed{\begin{bmatrix} 3/2 & -1/2 \\ -2 & 1 \end{bmatrix}}$$

$$18 - B = \{(1,0), (0,1)\}, B^{\circ} = \{(1,1), (5,6)\}$$

$$\left[ \begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 1 & 6 & 0 & 1 \end{array} \right] \xrightarrow{e_2 - e_1} \left[ \begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array} \right] \xrightarrow{e_1 - 5e_2} \boxed{18}$$

$$\left[ \begin{array}{cc|cc} 1 & 0 & 6 & -5 \\ 0 & 1 & -1 & 1 \end{array} \right]$$

$$P = \boxed{\begin{bmatrix} 6 & -5 \\ -1 & 1 \end{bmatrix}}$$

$$19 - B = \{(2,4), (-1,3)\} B^{\circ} = \{(1,0), (0,1)\}$$

$$\left[ \begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & 4 & 3 \end{array} \right] \boxed{19}$$

$$P = \boxed{\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}}$$

$$20 - B = \{(1, 1), (1, 0)\}, B^3 = \{(1, 0), (0, 1)\}$$

$$\left[ \begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right]$$

$$P = \left[ \begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array} \right] \quad (20)$$

$$21 - B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$B^3 = \{(1, 0, 0), (0, 2, 8), (6, 0, 12)\}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 8 & 12 & 0 & 0 & 1 \end{array} \right] \xrightarrow{e_2/2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 8 & 12 & 0 & 0 & 1 \end{array} \right] \xrightarrow{e_3 - 8e_1} \dots$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 12 & 0 & -4 & 1 \end{array} \right] \xrightarrow{e_3/12} \left[ \begin{array}{ccc|ccc} 1 & 0 & 6 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 1/12 \end{array} \right] \xrightarrow{e_1 - 6e_3} \dots$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 1/12 \end{array} \right]$$

$$P = \left[ \begin{array}{ccc} 1 & 2 & -1/2 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/12 \end{array} \right] \quad (21)$$

$$22 \cdot B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$B' = \{(1, 3, -1), (2, 7, -4), (2, 9, -7)\}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{e_2 - 3e_1} \left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array} \right] \xrightarrow{e_3 + 2e_2} \left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right]$$

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array} \right] \xrightarrow{e_2 - 3e_3} \left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & 11 & -4 & -2 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array} \right] \xrightarrow{e_1 - 2e_3} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array} \right]$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array} \right] \quad P = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \quad 22$$

$$23 \cdot B = \{(3, 4, 0), (-2, -1, 1), (1, 0, -3)\}$$

$$B' = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & 1 \\ 0 & 1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -3 \end{array} \right] \quad P = \begin{bmatrix} 3 & -2 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & -3 \end{bmatrix} \quad 23$$

$$24 \cdot B = \{(1, 3, 2), (2, -1, 2), (5, 6, 1)\}$$

$$B' = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 5 \\ 0 & 1 & 0 & 3 & -1 & 6 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{array} \right] \quad P = \begin{bmatrix} 1 & 2 & 5 \\ 3 & -1 & 6 \\ 2 & 2 & 1 \end{bmatrix} \quad 24$$