INSTITUTO POLITÉCNICO NACIONAL
Escuela Superior de Cómputo
Academia de Formación Básica

ESCOM

1er examen parcial de Álgebra Lineal

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Resuelve los siguientes ejercicios de manera clara y ordenada, recuerda que se califica el procedimiento. Cada ejercicio vale 2 puntos.

1. Representa en forma matricial el siguiente sistema de ecuaciones y resuélvelo mediante el método de Gauss-Jordan.

$$4x - y + z = 4$$
$$2y - z + 2x = 2$$
$$6x + 3z - 2y = 12$$

2. Determina la matriz inversa de $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

3. Utiliza propiedades de determinantes para calcular $\begin{vmatrix} 1 & -3 & 2 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ -1 & -2 & 0 & 1 & 0 \end{vmatrix}$

4. Dadas las matrices $C = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & -1 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 1 & 0 & 2 \\ -1 & 0 & -1 & 2 \\ 2 & 0 & 0 & 1 \end{pmatrix}$ y $E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, calcula $E^2 - 3DC$.

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D

$4x - y + z = 4$
$2y - z + 2x = 2$
$6x + 3z - 2y = 12$

$4x - y + z = 4$
$2x + 2y - z = 2$
$6x - 2y + 3z = 12$

$$\left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 2 & 2 & -1 & 2 \\ 6 & -2 & 3 & 12 \end{array}\right] \begin{array}{l} e_2 \times 2 \to e_2 \\ e_3 \times 2 \to e_3 \end{array} \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 4 & 4 & -2 & 4 \\ 12 & -4 & 6 & 24 \end{array}\right]$$

$$\begin{array}{l} e_2 - e_1 \to e_2 \\ e_3 - 3e_1 \to e_3 \end{array} \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 5 & -3 & 0 \\ 0 & -1 & 3 & 12 \end{array}\right] e_3 \times 5 \to \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 5 & -3 & 0 \\ 0 & -5 & 15 & 60 \end{array}\right]$$

$$e_3 + e_2 \to e_3 \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 12 & 60 \end{array}\right] e_3/12 \to \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$e_2 + 3e_1 \to e_2 \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 5 & 0 & 15 \\ 0 & 0 & 1 & 5 \end{array}\right] e_2/5 \to e_2 \left[\begin{array}{ccc|c} 4 & -1 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{array}{l} e_1 + e_2 - \\ e_3 \to e_1 \end{array} \left[\begin{array}{ccc|c} 4 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right] e_1/4 \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Vázquez Blancas
Cesa Said

$$R = \begin{array}{l} x = 1/2 \\ y = 3 \\ z = 5 \end{array}$$

1

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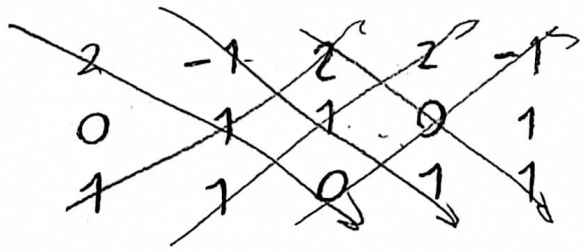
(2) Determina la inversa de $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \end{vmatrix} \qquad A^{-1} = \frac{C^T}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \end{vmatrix} \quad e_4 - 2e_1 \to e_4 \longrightarrow \quad \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{vmatrix}$$

$$|B| = 1 \begin{vmatrix} 2 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$



$$|B| = -1 - (2+2) = -1 - 4 = -5$$ ✓

$$|B| = c\,|A| \rightarrow -5 = 1\,|A| \qquad \underline{|A| = -5}$$

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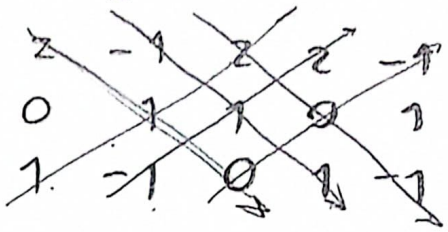
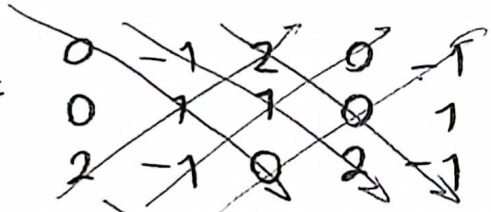
$$= \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & +6 & 4 & -4 \\ 1 & -1 & 1 & -1 \\ -2 & 2 & -2 & -3 \\ 0 & -3 & -2 & 2 \end{bmatrix} \quad \begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

$$_{1,1} = \begin{bmatrix} 2 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \rightarrow$$



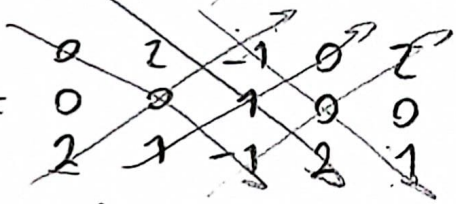
$_{1,1} = -1 - (2-2) = -1 - 0 = -1$

$_{1,2} =$



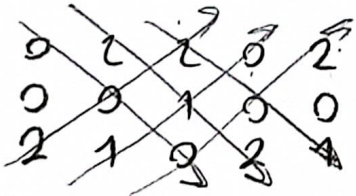
$= -2 - (4) = -2 - 4 = -6\ (-1) = 6$

$C_{1,4} =$



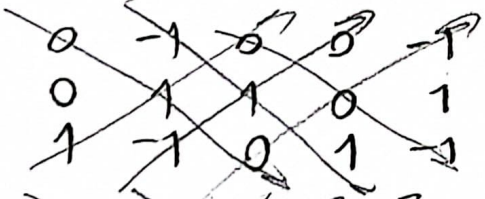
$= 4 - (0) = 4\ (-1) = -4$

$C_{1,3} =$



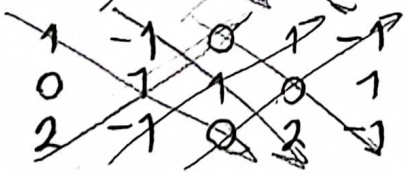
$= 4 = 4$

$C_{2,1} =$



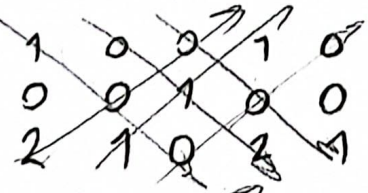
$= -1\ (-1) = 1$

$C_{2,2} =$



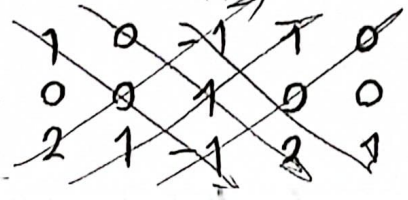
$= -2 - (-1) = -2 + 1 = -1$

$C_{2,3} =$



$= -1\ (-1) = 1 -$

$C_{2,4} =$



$= -1$

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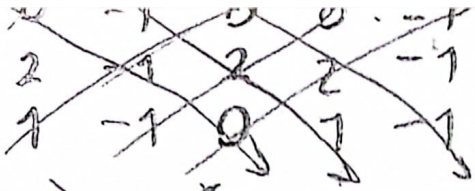
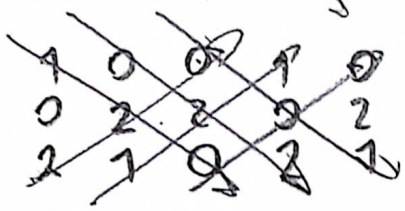
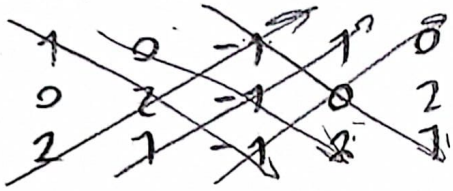
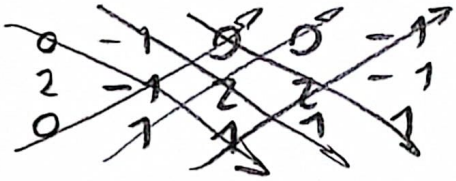
$C_{3,1} =$  $= -2$

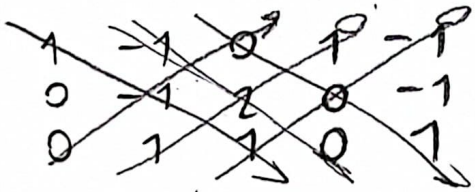
$C_{3,2} =$  $= -4 - (-2) = -4 + 2 = -2\,(-1) = 2$

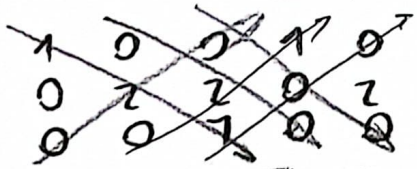
$C_{3,3} =$  $= -2$

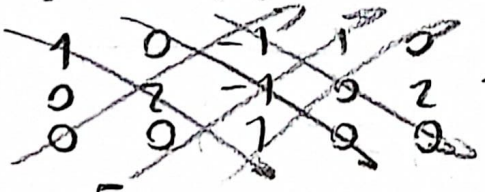
$C_{3,4} =$  $= -2 - (-4 - 1) = -2 - (-5)$

$-2 + 5 = 3\,(-1) = -3$

$C_{4,1} =$  $= -2 - (-2) = -2 + 2 = 0$

$C_{4,2} =$  $= -1 - (2) = -1 - 2 = -3$

$C_{4,3} =$  $= 2$

$C_{4,4} =$  $= 2$

$$C^T = \begin{bmatrix} -1 & 1 & -2 & 0 \\ 6 & -1 & 2 & -3 \\ 4 & 1 & -2 & -2 \\ -4 & -1 & -3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/5 & -1/5 & 2/5 & 0 \\ -6/5 & 1/5 & -2/5 & 3/5 \\ -4/5 & -1/5 & 2/5 & 2/5 \\ 4/5 & 1/5 & 3/5 & -2/5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/5 & -1/5 & 2/5 & 0 \\ 6/5 & 1/5 & \cdots & \cdots \end{bmatrix}$$

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Escaneado con CamScanner

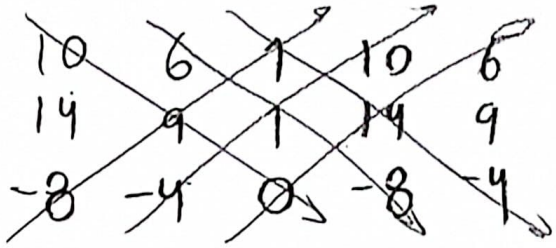
(3)

$$\begin{vmatrix} 1 & -3 & 2 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ -1 & -2 & 0 & 1 & 0 \end{vmatrix} \quad \begin{matrix} e_3 - e_1 \to e_3 \\ e_4 - 2e_1 \to e_4 \\ e_5 + e_1 \to e_5 \end{matrix}$$

$$\begin{vmatrix} 1 & -3 & 2 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 0 & 6 & -2 & 0 & 1 \\ 0 & 8 & -2 & 1 & 1 \\ 0 & -5 & 2 & 1 & 0 \end{vmatrix} \quad \begin{matrix} e_3 + 6e_2 \to e_3 \\ e_4 + 8e_2 \to e_4 \\ e_5 - 5e_2 \to e_5 \end{matrix} \quad \begin{vmatrix} 1 & -3 & 2 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 0 & 0 & 10 & 6 & 1 \\ 0 & 0 & 14 & 9 & 1 \\ 0 & 0 & -8 & -4 & 0 \end{vmatrix}$$

$|A| = 1(-1)\,|B|$

$$|B| = \begin{vmatrix} 10 & 6 & 1 \\ 14 & 9 & 1 \\ -8 & -4 & 0 \end{vmatrix}$$



$|B| = -48 - 56 - (-72 - 40)$

$|B| = -104 - (-112)$

$|B| = -104 + 112$??

$|B| = 8$ ← X

$|A| = 1(-1)(18)$

$|A| = -18$

3

Escaneado con CamScanner

$$C=\begin{pmatrix}1&0&1\\1&2&-1\\0&-1&0\\1&-1&0\end{pmatrix},\quad D=\begin{pmatrix}1&1&0&2\\-1&0&-1&2\\2&0&0&1\end{pmatrix}$$

$$E=\begin{pmatrix}2&0&0\\0&2&0\\1&1&1\end{pmatrix}\qquad E^2-3D(\qquad E^2$$

$$\begin{pmatrix}2&0&0\\0&2&0\\1&1&1\end{pmatrix}\left(\begin{pmatrix}2&0&0\\0&2&0\\1&1&1\end{pmatrix}\right.\quad\left(\begin{matrix}4&0&0\\0&4&0\\3&3&1\end{matrix}\right)$$

1.- 4 , 2.- 0 , 3.- 0 , 4.- 0 , 5.- 4 , 6.- 0 , 7.- 3

8.- 3 9.- 1

$$\begin{pmatrix}1&1&0&2\\-1&0&-1&2\\2&0&0&1\end{pmatrix}\left(\begin{pmatrix}1&0&1\\1&2&-1\\0&-1&0\\1&-1&0\end{pmatrix}\right.\quad\left(\begin{matrix}4&0&0\\1&-1&-1\\3&-1&2\end{matrix}\right)$$

$1+1+2=4$ $\quad -1+2=1$ $\quad 2+1=3$

$0+2+0-2=0$ $\quad 1-2=-1$ $\quad 0-1=-1$

$1-1=0$ $\quad -1=-1$ $\quad 2=2$

$$\begin{pmatrix}4&0&0\\0&4&0\\3&3&1\end{pmatrix}-\begin{pmatrix}12&0&0\\3&-3&-3\\9&-3&6\end{pmatrix}=\begin{pmatrix}4-12&0-0&0-0\\0-3&4+3&0+3\\3-9&3+3&1-6\end{pmatrix}$$

$$\begin{pmatrix}-8&0&0\\-3&7&3\\-1&6&-5\end{pmatrix}\quad 4$$

???

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