① Dados los vectores $u=(1,2)$ y $w=(1,-1)$ determina la combinación lineal adecuada, si existe, para obtener v

$V=(2,1)$

$\vec{V} = a\vec{U} + b\vec{W}$

$(2,1) = a(1,2) + b(1,-1)$

$(2,1) = (a, 2a) + (b,-b) \rightarrow (2,1) = (a+b, 2a-b)$

$$\begin{matrix} a+b=2 \\ 2a-b=1 \end{matrix} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & -1 & 1 \end{array}\right] \xrightarrow{E_2+E_1 \to e_2}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 0 & 3 \end{array}\right] \xrightarrow{e_2/3} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 0 & 1 \end{array}\right] \xrightarrow{E_1-E_2 \to e_1} \left[\begin{array}{cc|c} 0 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right]$$

$$\boxed{\begin{matrix} \vec{V} = 1\vec{U} + 1\vec{W} \\ a=1 \quad b=1 \end{matrix}}$$ 1

$V=(0,3)$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & -1 & 3 \end{array}\right] \xrightarrow{e_1+e_2 \to e_1} \left[\begin{array}{cc|c} 3 & 0 & 3 \\ 2 & -1 & 3 \end{array}\right] \xrightarrow{e_1/3}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 2 & -1 & 3 \end{array}\right] \xrightarrow{e_2-2e_1 \to e_2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$$

2

$$\boxed{\begin{matrix} \vec{V} = 1\vec{U} - 1\vec{W} \\ x=1;\ y=-1 \end{matrix}}$$

$(0,3) = 1(1,2) - 1(1,-1)$

$(0,3) = (1,2) - (1,-1)$

$(0,3) = (1-1, 2+1)$

$(0,3) = (0,3)$

$\vec{V} = (3,0)$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -1 & 0 \end{array}\right] \xrightarrow{e_1+e_2 \to e_1} \left[\begin{array}{cc|c} 3 & 0 & 3 \\ 2 & -1 & 0 \end{array}\right] \xrightarrow{e_1/3} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{e_2-2e_1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & -1 & -2 \end{array}\right] \xrightarrow{e_2/-1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

$$\boxed{\begin{array}{l} \vec{V} = 1\vec{u} + 2\vec{w} \\ x=1 \quad y=2 \end{array}}$$

3

$(3,0) = 1(1,2) + 2(1,-1)$

$(3,0) = (1,2) + (2,-2)$

$(3,0) = (1+2, 2-2) = (3,0)$ //

$\vec{V} = (1,-1)$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & -1 \end{array}\right] \xrightarrow{e_1+e_2 \to e_1} \left[\begin{array}{cc|c} 3 & 0 & 0 \\ 2 & -1 & -1 \end{array}\right] \xrightarrow{e_1/3} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 2 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & -1 \end{array}\right] \xrightarrow{e_2(-1)} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

$$\boxed{\begin{array}{l} \vec{V} = 0\vec{u} + 1\vec{w} \\ x=0, \; y=1 \end{array}}$$

4

$(1,-1) = 0(1,2) + 1(1,-1)$

$(1,-1) = (0,0) + (1,-1)$

$(1,-1) = (0+1, 0-1)$

$(1,-1) = (1,-1)$

Escaneado con CamScanner

(5) $\vec{v} = (-1, -2)$

$$\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 2 & -1 & -2 \end{array}\right] \xrightarrow{e_1 + e_2 \to e_1} \left[\begin{array}{cc|c} 3 & 0 & -3 \\ 2 & -1 & -2 \end{array}\right] \xrightarrow{e_1/3}$$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 2 & -1 & -2 \end{array}\right] \xrightarrow{e_2 - 2e_1} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 0 \end{array}\right]$$

$$\boxed{\begin{array}{c} \vec{V} = -1\vec{u} + 0\vec{w} \\ x = -1 \ , \ y = 0 \end{array}}$$

(5)

$(-1, -2) = -1(1, 2) + 0(1, -1)$

$(-1, -2) = (-1, -2) + (0, 0)$

$(-1, -2) = (-1, -2)$

(6) $\vec{v} = (1, -4)$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & -4 \end{array}\right] \xrightarrow{e_1 + e_2 \to e_1} \left[\begin{array}{cc|c} 3 & 0 & -3 \\ 2 & -1 & -4 \end{array}\right] \xrightarrow{e_1/3} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 2 & -1 & -4 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$$

$(1, -4) = -1(1, 2) + 2(1, -1)$

$(1, -4) = (-1, -2) + (2, -2)$

$(1, -4) = (-1 + 2, -2 - 2)$

$(1, -4) = (1, -4)$

$$\boxed{\begin{array}{c} \vec{V} = -1\vec{u} + 2\vec{w} \\ x = -1 \quad y = 2 \end{array}}$$

(6)

2) Determina la combinación lineal de cada v en términos de las u:

$v = (10,1,4) \quad u_1 = (2,3,5),\ u_2 = (1,2,4),\ u_3 = (-2,2,3)$

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 3 & 2 & 2 & 1 \\ 5 & 4 & 3 & 4 \end{array}\right] \xrightarrow[e_3 \times 2]{e_2 \times 2} \left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 6 & 4 & 4 & 2 \\ 10 & 8 & 6 & 8 \end{array}\right] \xrightarrow[e_3 - 5e_1 \to e_3]{e_2 - 3e_1 \to e_2}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 0 & 1 & 10 & -28 \\ 0 & 3 & 16 & -42 \end{array}\right] \xrightarrow{e_3 - 3e_2} \left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 0 & 1 & 10 & -28 \\ 0 & 0 & -14 & 42 \end{array}\right] \ e_3/-14$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 0 & 1 & 10 & -28 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{e_2 - 10e_3 \to e_2} \left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \ \begin{array}{l} e_1 - e_2 \\ +2e_3 \to e_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{e_1/2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$$\boxed{\begin{array}{l} \vec{V} = 1\vec{u}_1 + 2\vec{u}_2 - 3\vec{u}_3 \\ x = 1 \\ y = 2 \\ z = -3 \end{array}}$$

7

$(10,1,4) = 1(2,3,5) + 2(1,2,4)$
$\quad - 3(-2,2,3$

$(10,1,4) = (2,3,5) + (2,4,8) + (6,-6,-9)$

$(10,1,4) = (2+2+6,\ 3+4-6,\ 5+8-9)$

$(10,1,4) = (10,1,4)$

$\vec{v} = (-1, 7, 2) \quad u_1 = (1, 3, 5), u_2 = (2, -1, 3), u_3 = (-3, 2, -4)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right] \xrightarrow[e_3 - 5e_1 \to e_3]{e_2 - 3e_1 \to e_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & -7 & 11 & 7 \end{array}\right]$$

$$e_3 - e_2 \to e_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$0 \neq -3 \therefore$ No tiene solución

$\vec{U_1}, \vec{U_2}, \vec{U_3}$ no son combinación lineal de $\vec{V}$

2

$\vec{V} = (0, 5, 3, 0), u_1 = (1, 1, 2, 2), u_2 = (2, 3, 5, 6), u_3 = (-3, 1, -4, 2)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 1 & 3 & 1 & 5 \\ 2 & 5 & -4 & 3 \\ 2 & 6 & 2 & 0 \end{array}\right] \xrightarrow[\substack{e_3 - 2e_1 \to e_3 \\ e_4 - 2e_1 \to e_4}]{e_2 - e_1 \to e_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 8 & 0 \end{array}\right] \xrightarrow[e_4 - 2e_2 \to e_3]{e_3 - e_2 \to e_3} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & -10 \end{array}\right]$$

$0 \neq 10 \therefore$

$\vec{U_1}, \vec{U_2}, \vec{U_3}$ no son combinación lineal de $\vec{V}$

3

$\vec{V} = (2, 5, -4, 0) \quad u_1 = (1, 3, 2, 1), u_2(2, -2, -5, 4), u_3(2, -1, 3, 6)$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{array}\right] \xrightarrow[\substack{e_3 - 2e_1 \to e_3 \\ e_4 - e_1 \to e_4}]{e_2 - 3e_1 \to e_2} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & -2 & -4 & 2 \end{array}\right] \quad e_4 / -2 \to e_4$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 1 & 2 & -1 \end{array}\right] \xrightarrow{e_3 + 9e_4 \to e_4} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 17 & -17 \\ 0 & 1 & 2 & -1 \end{array}\right] \xrightarrow{e_3/17 \to e_3}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & -1 \end{array}\right] \xrightarrow{e_4 - 2e_3 \to e_4} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

$x + 2y + 2z = 2$

$-8y - 7z = -1$

$z = -1$

$y = 1$

$-8(1) - 7(-1) = -1$

$-8 + 7 = -1$

$x + 2(1) + 2(-1) = 2$

$x + \cancel{2 - 2} = 2 \qquad x = 2$

$\vec{V} = 2(1, 3, 2, 1) + 1(2, -2, -5, 4) - 1(2, -1, 3, 6)$

$\vec{V} = (2, 6, 4, 2) + (2, -2, -5, 4) + (-2, 1, -3, -6)$

$\vec{V} = (2 + \cancel{2 - 2}, 6 - 2 + 1, 4 - 5 - 3, 2 + 4 - 6)$

$\vec{V} = (2, 5, -4, 0)$

$\vec{V} = 2(1,$

4

$$\boxed{\begin{array}{c} \vec{V} = 2\vec{u}_1 + 1\vec{u}_2 - 1\vec{u}_3 \\ x = 2, y = 1, z = -1 \end{array}}$$

③.- De los siguientes conjuntos generadores, identifica cuales de ellos generan el espacio $\mathbb{R}^3$

$S = \{(4,7,3), (-1,2,6), (2,-3,5)\}$

$$\begin{bmatrix} 4 & -1 & 2 \\ 7 & 2 & -3 \\ 3 & 6 & 5 \end{bmatrix} \qquad \begin{vmatrix} 4 & -1 & 2 & 4 & -1 \\ 7 & 2 & -3 & 7 & 2 \\ 3 & 6 & 5 & 3 & 6 \end{vmatrix}$$

$84 + 49 = 133$

$12 \times 6 = 72$

$108 + 49 = 157$

$107 - 12 = 95$

$157 + 95 = 202$

$|A| = (40 + 9 + 84) - (12 - 72 - 35)$

$|A| = (133 + 95) = 228$

$\boxed{\therefore \text{ Es un conjunto generador de } \mathbb{R}^3}$ 1

$S = \{(6,7,6), (3,2,-4), (1,-3,2)\}$

$$\begin{bmatrix} 6 & 3 & 1 \\ 7 & 2 & -3 \\ 6 & -4 & 2 \end{bmatrix} \qquad \begin{vmatrix} 6 & 3 & 1 & 6 & 3 \\ 7 & 2 & -3 & 7 & 2 \\ 6 & -4 & 2 & 6 & -4 \end{vmatrix}$$

$18 \times 3 = 54$

$12 \times 6 = 72$

$72 + 42 + 12 = 126$

$126 + 58 = 184$

$|A| = (24 - 54 - 28) - (12 + 72 + 42)$

$|A| = -58 - 126 = -184$

$\boxed{\therefore \text{ Es un conjunto generador de } \mathbb{R}^3}$

4) Aplica la prueba de independencia lineal en cada uno de los siguientes conjuntos sobre su espacio correspondiente

$S = \{(-4,-3,4), (1,-2,3), (6,0,0)\}$

$$\left[\begin{array}{ccc|c} -4 & 1 & 6 & 0 \\ -3 & -2 & 0 & 0 \\ 4 & 3 & 0 & 0 \end{array}\right] \xrightarrow{e_2 \times 4} \left[\begin{array}{ccc|c} -4 & 1 & 6 & 0 \\ -12 & -8 & 0 & 0 \\ 4 & 3 & 0 & 0 \end{array}\right] \xrightarrow[e_3 + e_1 \to e_3]{e_2 - 3e_1 \to e_2}$$

$$\left[\begin{array}{ccc|c} -4 & 1 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 6 & 0 \end{array}\right] \xrightarrow{e_3 - 4e_2} \left[\begin{array}{ccc|c} -4 & 1 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{e_1 - e_2 - 6e_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\boxed{\therefore\ S \text{ es linealmente independiente}}$ //

$S = \{(1,0,0), (0,4,0), (0,0,-6), (1,5,-3)\}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 5 & 0 \\ 0 & 0 & -6 & -3 & 0 \end{array}\right] \quad e_2 -$$

$\boxed{S \text{ es linealmente dependiente}}$ //

$$S = \{(4,-3,6,2),(1,8,3,1),(3,-2,-1,0)\}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ -3 & 8 & -2 & 0 \\ 6 & 3 & -1 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow[e_3 \times 4 \quad e_4 \times 4]{e_2 \times 4} \left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ -12 & 32 & -8 & 0 \\ 24 & 12 & -4 & 0 \\ 8 & 4 & 0 & 0 \end{array}\right] \xrightarrow[e_3 - 6e_1 \quad e_4 - 2e_1]{e_2 + 3e_1}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ 0 & 35 & 1 & 0 \\ 0 & 6 & -22 & 0 \\ 0 & 2 & -6 & 0 \end{array}\right] \xrightarrow{e_4/2} \left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ 0 & 35 & 1 & 0 \\ 0 & 6 & -22 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow{e_3 - 6e_4}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ 0 & 35 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow[e_2 - e_3]{e_4 + 3e_3} \left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{S es linealmente independiente}}$

$S=\{(0,0,0,1),(0,0,1,1),(0,1,1,1),(1,1,1,1)\}$

$$\left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$z=0$

$y+z=0 \quad y+0=0, \ y=0$

$x+y+z=0 \quad x+0+0=0, \ x=0$

$w+x+y+z=0 \quad w=0, \quad w=0$

$\boxed{\text{S es linealmente independiente}}$ //

$A=\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \ B=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ C=\begin{bmatrix} -2 & 1 \\ 1 & 4 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -2 & 0 & 4 & 0 \end{array}\right] \xrightarrow{e_4+2e_1} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \ e_3-e_2 \to e_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{Es linealmente dependiente}}$ //

$$A=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ B\begin{bmatrix}0&1\\0&0\end{bmatrix}\ C=\begin{bmatrix}0&0\\1&0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{array}\right]$$

$x=0$
$y=0$
$z=0$
$x=0$

$\boxed{\text{Es linealmente independiente}}$

$$A=\begin{bmatrix}1&-1\\4&5\end{bmatrix},\ B=\begin{bmatrix}4&3\\-2&3\end{bmatrix}\ C=\begin{bmatrix}1&-8\\22&23\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&4&1&0\\-1&3&-8&0\\4&-2&22&0\\5&3&23&0\end{array}\right]\xrightarrow[\substack{e_3-4e_1\to e_3\\ e_5-5e_1\to e_5}]{e_2+e_1\to e_2}\left[\begin{array}{ccc|c}1&4&1&0\\0&7&-7&0\\0&-18&18&0\\0&-17&18&0\end{array}\right]\to$$

$$\left[\begin{array}{ccc|c}1&4&1&0\\0&1&-1&0\\0&-1&1&0\\0&-17&18&0\end{array}\right]\xrightarrow{e_3+e_2}\left[\begin{array}{ccc|c}1&4&1&0\\0&1&-1&0\\0&0&0&0\\0&-17&18&0\end{array}\right]\xrightarrow{e_4+17e_2\to e_4}$$

$$\left[\begin{array}{ccc|c}1&4&1&0\\0&1&-1&0\\0&0&1&0\\0&0&0&0\end{array}\right]\xrightarrow{e_2+e_3}\left[\begin{array}{ccc|c}1&4&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{array}\right]\xrightarrow{e_1-4e_2-e_3}$$

$$\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{array}\right]$$

$x=0$
$y=0$
$z=0$

$\boxed{\text{Es linealmente independiente}}$

(5).- Verifica si los siguientes conjuntos son base de sus respectivos espacios vectoriales

$S = \{(3,-2),(4,5)\}$ para $\mathbb{R}^2$

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \rightarrow \begin{vmatrix} 3 & 4 \\ -2 & 5 \end{vmatrix} \qquad |A| = 15-(-8) = 15+8 = 23$$

$$\boxed{\therefore S \text{ es base de } \mathbb{R}^2}$$

$S = \{(1,2),(1,-1)\}$ para $\mathbb{R}^2$

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \rightarrow \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \qquad |A| = -1-(2) = -1-2 = -3$$

$$\boxed{\therefore S \text{ es base de } \mathbb{R}^2}$$

$S = \{(1,5,3),(0,1,2),(0,0,6)$ para $\mathbb{R}^3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix} \rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 3 & 2 & 6 \end{vmatrix}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 6 & 3 & 2 \end{array}$$

$|A| = 6$

$$\boxed{\therefore S \text{ es base de } \mathbb{R}^3}$$

$S = \{(2,1,0),(0,-1,1)\}$ para $\mathbb{R}^3$

$$\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} x = 0 \\ x - y = 0 \end{array} \quad 0 - y = 0 \quad y = 0$$

$$y = 0$$

Es linealmente independiente

$$\left[\begin{array}{cc|c} 2 & 0 & x \\ 1 & -1 & y \\ 0 & 1 & z \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & \alpha \\ 1 & -1 & \beta \\ 0 & 1 & \varphi \end{array}\right] \xrightarrow{e_2 + e_3 \to e_2} \left[\begin{array}{cc|c} 1 & 0 & \alpha \\ 1 & 0 & \beta \\ 0 & 1 & \varphi \end{array}\right]$$

$$\xrightarrow{e_1 - e_2} \left[\begin{array}{cc|c} 0 & 0 & \alpha \\ 1 & 0 & \beta \\ 0 & 1 & \varphi \end{array}\right] \quad \begin{array}{l} \alpha = 0 \\ \beta = x \\ \varphi = y \end{array}$$

$\therefore$ No es conjunto generador de $\mathbb{R}^3$

$$\boxed{S \text{ no es base de } \mathbb{R}^3}$$

$S = \{(0,3,-2),(4,0,3),(-8,15,-16)\}$ para $\mathbb{R}^3$

$$\begin{vmatrix} 0 & 4 & -8 \\ 3 & 0 & 15 \\ -2 & 3 & -16 \end{vmatrix} \neq 0 \qquad \begin{array}{ccccc} 0 & 4 & -8 & 0 & 4 \\ 3 & 0 & 15 & 3 & 0 \\ -2 & 3 & -16 & -2 & 3 \end{array}$$

$$\begin{array}{r} -30 \\ \times\ 4 \\ \hline -120 \end{array} \qquad \begin{array}{r} 16 \\ \times\ 3 \\ \hline 48 \\ \times\ 4 \\ \hline 192 \end{array}$$

$|A| = -120 - 72 - (-192)$

$|A| = -192 + 192 = 0$

$\boxed{\therefore S \text{ no es base de } \mathbb{R}^3}$

$S = \{(0,0,0),(1,5,6),(6,2,1)\}$ Para $\mathbb{R}^3$

$$\begin{vmatrix} 0 & 1 & 6 \\ 0 & 5 & 2 \\ 0 & 6 & 1 \end{vmatrix} \neq 0 \qquad \begin{array}{ccccc} 0 & 1 & 6 & 0 & 1 \\ 0 & 5 & 2 & 0 & 5 \\ 0 & 6 & 1 & 0 & 6 \end{array}$$

$|A| = 0$

$\boxed{\therefore S \text{ no es base de } \mathbb{R}^3}$

$S = \{(-1, 2, 0, 0), (2, 0, -1, 0), (3, 0, 0, 4), (0, 0, 5, 0)\}$
para $\mathbb{R}^4$

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 4 & 0 \end{vmatrix} \neq 0 \rightarrow \begin{vmatrix} 1 & -2 & -3 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 4 & 0 \end{vmatrix} \xrightarrow{e_2/4}$$

$$\begin{vmatrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 4 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 3/2 & 5 \\ 0 & 0 & 4 & 0 \end{vmatrix}$$

$$|A| = 1(1)(-20) = -20 \times 4(-1) = 80$$

$$\boxed{\therefore S \text{ es conjunto y base de } \mathbb{R}^4}$$

$S = \{(1, 0, 0, 1), (0, 2, 0, 2), (1, 0, 1, 0), (0, 2, 2, 0)\}$ $\mathbb{R}^4$

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 2 & 0 & 0 \end{vmatrix} \neq 0 \quad e_4 - e_1 \rightarrow e_4 \quad \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 2 & -1 & 0 \end{vmatrix}$$

$$\begin{array}{ccccc} 2 & 0 & 2 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 2 & -1 & 0 & 2 & -1 \end{array}$$

$$-1(-(4-4)) = -1(-0) = 0$$

$$\boxed{\therefore S \text{ no es base de } \mathbb{R}^4}$$

$$S = \left\{ \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \right\}$$

$$\begin{vmatrix} 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 3 & 1 & 2 & 0 \end{vmatrix} \; e_4 \times 2 \to e_4 \; \begin{vmatrix} 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 6 & 2 & 4 & 0 \end{vmatrix} \xrightarrow{e_4 - 3e_1}$$

$$\begin{vmatrix} 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & -1 & 4 & 0 \end{vmatrix} \qquad \begin{array}{ccccc} 4 & 1 & 1 & 4 & 1 \\ 0 & 3 & 2 & 0 & 3 \\ -1 & 4 & 0 & -1 & 4 \end{array}$$

$$|A| = 2(-2-(-3+32)) = 2(-2-29) = 2(-31)$$

$$|A| = -62/2 = -31$$

$\boxed{S \text{ es base de } \mathbb{R}^{46}}$

$$S=\left\{\begin{bmatrix}1 & 2\\ -5 & 4\end{bmatrix},\begin{bmatrix}2 & -7\\ 6 & 2\end{bmatrix},\begin{bmatrix}4 & -9\\ 11 & 12\end{bmatrix},\begin{bmatrix}12 & -16\\ 17 & 42\end{bmatrix}\right\}$$

$$\begin{vmatrix}1 & 2 & 4 & 12\\ 2 & -7 & -9 & -16\\ -5 & 6 & 11 & 17\\ 4 & 2 & 12 & 42\end{vmatrix}\ \begin{matrix}e_2-2e_1\\ \longrightarrow\\ e_3+5e_1\\ e_4-4e_1\end{matrix}\ \begin{vmatrix}1 & 2 & 4 & 12\\ 0 & -11 & -17 & -40\\ 0 & 16 & 31 & 77\\ 0 & -6 & -4 & -6\end{vmatrix}\neq 0$$

$$\begin{vmatrix}-11 & -17 & -40 & -11 & -17\\ 16 & 31 & 77 & 16 & 31\\ -6 & -4 & -6 & -6 & -4\end{vmatrix}$$

$$|A|=2046+7854+2560$$
$$-(7440+3388+1632)$$

$$|A|=12460-12460$$

$$|A|=0$$

$$\begin{array}{r}31\\ \times\ 6\\ \hline 186\\ \times\ 11\\ \hline 186\\ 186\ \\ \hline 2046\end{array}\qquad \begin{array}{r}77\\ \times\ 17\\ \hline 539\\ 77\ \\ \hline 1309\end{array}\qquad \begin{array}{r}40\\ \times\ 16\\ \hline 240\\ 40\ \\ \hline 640\\ \times\ 4\\ \hline 2560\end{array}$$

$$\begin{array}{r}2046\\ 1309\\ 2560\\ \hline 5915\end{array}$$

$$\begin{array}{r}40\\ \times\ 31\\ \hline 40\\ 120\ \\ \hline 1240\\ \times\ 6\\ \hline 7440\end{array}\qquad \begin{array}{r}77\\ \times\ 11\\ \hline 77\\ 77\ \\ \hline 847\\ \times\ 4\\ \hline 3388\end{array}$$

$$\begin{array}{r}7440\\ 3388\\ 1632\\ \hline 12460\\ -\ 5919\\ \hline 6541\end{array}\qquad \begin{array}{r}17\\ \times\ 16\\ \hline 102\\ 17\ \\ \hline 272\\ \times\ 6\\ \hline 1632\end{array}$$

$$\boxed{\therefore S \text{ no es base}}$$