Instituto Politécnico Nacional
Escuela Superior de Cómputo

EDOL no Homogeneas de segundo orden

Materia: Ecuaciones Diferenciales

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Ejercicio 1

Priego Merino Saeed

Ecuacion

$$y'' - y' + \frac{5\,y}{4} = e^{\frac{x}{2}}\ \cos{(x)}$$

Calcular:

$$4\,y'' - 4\,y' + 5\,y = 4\,e^{\frac{x}{2}}\ \cos{(x)}$$

Ecuación lineal con coeficientes constantes

$$4\,\lambda^2 - 4\,\lambda + 5 = 0$$

Hallamos las raicez:

$$4\,\lambda^2 - 4\,\lambda + 5 \to\ \lambda_{1,2} = \pm i + \frac{1}{2}$$

$$\mathrm{k} = 1$$

$$\tau : C_1\,e^{\frac{x}{2}}\ \mathrm{sen}\,(x) + C\,e^{\frac{x}{2}}\ \cos{(x)}$$

Solucion general:

$$\overline{y} = C_1\,e^{\frac{x}{2}}\ \mathrm{sen}\,(x) + C\,e^{\frac{x}{2}}\ \cos{(x)}$$

Método de variación de parámetros

$$\overline{y} = e^{\frac{x}{2}}\ \mathrm{sen}\,(x)\ C_1\,(x) + e^{\frac{x}{2}}\ \cos{(x)}\ C\,(x)$$

Sistema:

$$\begin{cases} C'\,(x)\ y_1 + C_1'\,(x)\ y_2 = 0 \\ C'\,(x)\ y_1' + C_1'\,(x)\ y_2' = \dfrac{f\,(x)}{a_0} \end{cases}$$

$$y_1 = e^{\frac{x}{2}}\ \cos{(x)}$$

$$y_2 = e^{\frac{x}{2}}\ \mathrm{sen}\,(x)$$

$$y_1' = -\frac{e^{\frac{x}{2}}\ (2\ \mathrm{sen}\,(x) - \cos{(x)})}{2}$$

$$y_2' = \frac{e^{\frac{x}{2}}\ (\mathrm{sen}\,(x) + 2\ \cos{(x)})}{2}$$

$$a_0\,y'' = 4 \quad f\,(x) = 4\,e^{\frac{x}{2}}\ \cos{(x)}$$

$$\begin{cases} e^{\frac{x}{2}}\ C'(x)\ \cos{(x)} + e^{\frac{x}{2}}\ C_1'(x)\ \mathrm{sen}\,(x) = 0 \\ -\dfrac{e^{\frac{x}{2}}\ C'(x)\ (2\ \mathrm{sen}\,(x) - \cos{(x)})}{2} + \dfrac{e^{\frac{x}{2}}\ C_1'(x)\ (\mathrm{sen}\,(x) + 2\ \cos{(x)})}{2} = \dfrac{4\,e^{\frac{x}{2}}\ \cos{(x)}}{4} \end{cases}$$

Cramer:

$$\begin{vmatrix} e^{\frac{x}{2}}\ \cos{(x)} \\ e^{\frac{x}{2}}\ \mathrm{sen}\,(x) \\ -\dfrac{e^{\frac{x}{2}}\ (2\ \mathrm{sen}\,(x) - \cos{(x)})}{2} \\ \dfrac{e^{\frac{x}{2}}\ (\mathrm{sen}\,(x) + 2\ \cos{(x)})}{2} \end{vmatrix}$$

$$\left|\begin{matrix} 0 \\ e^{\frac{x}{2}}\ \operatorname{sen}(x) \\ e^{\frac{x}{2}}\ \cos(x) \\ \frac{e^{\frac{x}{2}}\ (\operatorname{sen}(x)+2\ \cos(x))}{2} \end{matrix}\right|$$

$$\left|\begin{matrix} e^{\frac{x}{2}}\ \cos(x) \\ 0 \\ -\frac{e^{\frac{x}{2}}\ (2\ \operatorname{sen}(x)-\cos(x))}{2} \\ e^{\frac{x}{2}}\ \cos(x) \end{matrix}\right|$$

$$C'(x) = \frac{W_1}{W} = -\cos(x)\ \operatorname{sen}(x)$$

$$C_1'(x) = \frac{W_2}{W} = \cos^2(x)$$

$$\int -\cos(x)\ \operatorname{sen}(x)\ \mathrm{d}x$$

$$-\frac{\operatorname{sen}^2(x)}{2}$$

$$\int \cos^2(x)\ \mathrm{d}x$$

$$\frac{\operatorname{sen}(2\,x)}{2}$$

$$y = \frac{x\,e^{\frac{x}{2}}\ \operatorname{sen}(x)}{2}$$

Ejercicio 7

Diaz Torres Jonathan Samuel

Ecuacion

$$y'' + 2\,y' + y = \frac{1}{x\,e^{x}}$$

Calcular:

$$y'' + 2\,y' + y = \frac{1}{x\,e^{x}}$$

Ecuación lineal con coeficientes constantes

$$(\lambda + 1)^2 = 0$$

Hallamos las raicez:

$$(\lambda + 1)^2 \rightarrow \ \lambda_{1,2} = -1$$

$$\mathrm{k} = 2$$

$$\tau : \frac{C_1\,x + C}{e^{x}}$$

Solucion general:

$$\overline{y} = \frac{C_1\,x + C}{e^{x}}$$

Método de variación de parámetros

$$\overline{y} = \frac{x\,C_1\,(x) + C\,(x)}{e^{x}}$$

Sistema:

$$\begin{cases} C'\,(x)\ y_1 + C_1'\,(x)\ y_2 = 0 \\ C'\,(x)\ y_1' + C_1'\,(x)\ y_2' = \dfrac{f\,(x)}{a_0} \end{cases}$$

$$y_1 = \frac{1}{e^{x}}$$

$$y_2 = \frac{x}{e^{x}}$$

$$y_1' = -\frac{1}{e^{x}}$$

$$y_2' = -\frac{x-1}{e^{x}}$$

$$a_0\,y'' = 1 \quad f\,(x) = \frac{1}{x\,e^{x}}$$

$$\begin{cases} \dfrac{C'(x)}{e^{x}} + \dfrac{x\,C_1'(x)}{e^{x}} = 0 \\ -\dfrac{C'(x)}{e^{x}} - \dfrac{(x-1)\ C_1'(x)}{e^{x}} = \dfrac{1}{x\,e^{x}} \end{cases}$$

Cramer:

$$\begin{vmatrix} \frac{1}{e^x} \\ \frac{x}{e^x} \\ -\frac{1}{e^x} \\ -\frac{x-1}{e^x} \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ \frac{x}{e^x} \\ \frac{1}{x\,e^x} \\ -\frac{x-1}{e^x} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{e^x} \\ 0 \\ -\frac{1}{e^x} \\ \frac{1}{x\,e^x} \end{vmatrix}$$

$$\int -1 \,\mathrm{d}x$$

$$-1 = -x$$

$$\int \frac{1}{x} \,\mathrm{d}x$$

$$\ln\left(|x|\right)$$

$$y = \frac{x\,C_1\left(x\right) + C\left(x\right)}{e^x}$$

$$y = \frac{x\,\ln\left(x\right) + C_3\,x}{e^x}$$

Ejercicio 13

Arellano Millan Gabriel

Ecuacion

$$y'' - 9\,y' = 18\,x^2\,e^{9\,x}$$

Calcular:

$$y'' - 9\,y' = 18\,x^2\,e^{9\,x}$$

Ecuación lineal con coeficientes constantes

$$(\lambda - 9)\ \lambda = 0$$

Hallamos las raicez:

$$\lambda - 9 \rightarrow\ \lambda_1 = 9$$

$$\mathrm{k} = 1$$

$$\tau : C\,e^{9\,x}$$

$$\lambda \rightarrow\ \lambda_2 = 0$$

$$\mathrm{k} = 1$$

$$\tau : C_1$$

Solucion general:

$$\overline{y} = C\,e^{9\,x} + C_1$$

Método de variación de parámetros

$$\overline{y} = e^{9\,x}\,C\,(x) + C_1\,(x)$$

Sistema:

$$\begin{cases} C'\,(x)\ y_1 + C_1'\,(x)\ y_2 = 0 \\ C'\,(x)\ y_1' + C_1'\,(x)\ y_2' = \dfrac{f\,(x)}{a_0} \end{cases}$$

$$y_1 = e^{9\,x}$$

$$y_2 = 1$$

$$y_1' = 9\,e^{9\,x}$$

$$y_2' = 0$$

$$a_0\,y'' = 1 \quad f\,(x) = 18\,x^2\,e^{9\,x}$$

Cramer:

$$\begin{vmatrix} e^{9\,x} \\ 1 \\ 9\,e^{9\,x} \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ 1 \\ 18\,x^2\,e^{9\,x} \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} e^{9\,x} \\ 0 \\ 9\,e^{9\,x} \\ 18\,x^2\,e^{9\,x} \end{vmatrix}$$

$$\int 2\,x^2\;\mathrm{d}x$$

$$2\cdot\frac{x^3}{3}=\frac{2\,x^3}{3}$$

$$\int -2\,x^2\,e^{9\,x}\;\mathrm{d}x$$

$$-\frac{2\,x^2\,e^{9\,x}}{9}+\frac{4\,x\,e^{9\,x}}{81}-\frac{4\,e^{9\,x}}{729}+C$$

$$y=e^{9\,x}\,C\left(x\right)+C_1\left(x\right)$$

$$y=\frac{2\,x^3\,e^{9\,x}}{3}-\frac{2\,x^2\,e^{9\,x}}{9}+\frac{4\,x\,e^{9\,x}}{81}+C_2\,e^{9\,x}-\frac{4\,e^{9\,x}}{729}+C_3$$

Ejercicio 19

Ocaña Castro Hector Ecuacion

$$x^2\, y'' - x\, y' + 2\, y = x\, \ln(x)$$

Calcular:

$$x^2\, y'' - x\, y' + 2\, y = x\, \ln(x)$$

Ecuacion de Euler:

$$\mathrm{a}_0\, x^{\mathrm{n}} y^{(\mathrm{n})} + \mathrm{a}_1\, x^{\mathrm{n}-1}\, y^{(\mathrm{n})} + \ldots + \mathrm{a}_{\mathrm{n}-1}\, x\, y' + \mathrm{a}_{\mathrm{n}}\, y = f(x)$$

Sustitucion:

$$x = e^u$$

$$(\lambda - 1)\ \lambda - \lambda + 2 = 0$$

$$\lambda^2 - 2\,\lambda + 2 = 0$$

Ecuación lineal con coeficientes constantes

$$\lambda^2 - 2\,\lambda + 2 = 0$$

Hallamos las raicez:

$$\lambda^2 - 2\,\lambda + 2 \rightarrow\ \lambda_{1,2} = \pm i + 1$$

$$\mathrm{k} = 1$$

$$\tau : C_1\, e^u\ \mathrm{sen}(u) + C\, e^u\ \cos(u)$$

Solucion general:

$$\overline{y} = C_1\, e^u\ \mathrm{sen}(u) + C\, e^u\ \cos(u)$$

Método de variación de parámetros

$$\overline{y} = e^u\ \mathrm{sen}(u)\ C_1(u) + e^u\ \cos(u)\ C(u)$$

Sistema:

$$\begin{cases} C'(u)\ y_1 + C_1'(u)\ y_2 = 0 \\ C'(u)\ y_1' + C_1'(u)\ y_2' = \dfrac{f(u)}{a_0} \end{cases}$$

$$y_1 = e^u\ \cos(u)$$

$$y_2 = e^u\ \mathrm{sen}(u)$$

$$y_1' = -e^u\ (\mathrm{sen}(u) - \cos(u))$$

$$y_2' = e^u\ (\mathrm{sen}(u) + \cos(u))$$

$$a_0\, y'' = 1 \quad f(u) = u\, e^u$$

Cramer:

$$\begin{vmatrix} e^u\ \cos(u) \\ e^u\ \mathrm{sen}(u) \\ -e^u\ (\mathrm{sen}(u) - \cos(u)) \\ e^u\ (\mathrm{sen}(u) + \cos(u)) \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ e^{u}\ \operatorname{sen}(u) \\ u\, e^{u} \\ e^{u}\ (\operatorname{sen}(u)+\cos(u)) \end{vmatrix}$$

$$\begin{vmatrix} e^{u}\ \cos(u) \\ 0 \\ -e^{u}\ (\operatorname{sen}(u)-\cos(u)) \\ u\, e^{u} \end{vmatrix}$$

$$\int -u\ \operatorname{sen}(u)\ \mathrm{d}u$$

$$u\ \cos(u)-\operatorname{sen}(u)+C$$

$$\int u\ \cos(u)\ \mathrm{d}u$$

$$u\ \operatorname{sen}(u)+\cos(u)+C$$

$$y=e^{u}\ \operatorname{sen}(u)\ C_1(u)+e^{u}\ \cos(u)\ C(u)$$

$$y=x\ \ln(x)\ \operatorname{sen}^2(\ln(x))+C_3\, x\ \operatorname{sen}(\ln(x))+x\ \ln(x)\ \cos^2(\ln(x))+C_2\, x\ \cos(\ln(x))$$

Ejercicio 25

Lopez Chavez Moises

Ecuacion

$$y'' + 4\,y = 4\,\cos\left(2\,x\right)$$

Calcular:

$$y'' + 4\,y = 4\,\cos\left(2\,x\right)$$

Ecuación lineal con coeficientes constantes

$$\lambda^2 + 4 = 0$$

Hallamos las raicez:

$$\lambda^2 + 4 \to \ \lambda_{1,2} = \pm 2\,i$$

$$\mathrm{k} = 1$$

$$\tau : C_1\,\operatorname{sen}\left(2\,x\right) + C\,\cos\left(2\,x\right)$$

Solucion general:

$$\overline{y} = C_1\,\operatorname{sen}\left(2\,x\right) + C\,\cos\left(2\,x\right)$$

Método de variación de parámetros

$$\overline{y} = \operatorname{sen}\left(2\,x\right)\,C_1\left(x\right) + \cos\left(2\,x\right)\,C\left(x\right)$$

Sistema:

$$\begin{cases} C'\left(x\right)\,y_1 + C_1'\left(x\right)\,y_2 = 0 \\ C'\left(x\right)\,y_1' + C_1'\left(x\right)\,y_2' = \dfrac{f\left(x\right)}{a_0} \end{cases}$$

$$y_1 = \cos\left(2\,x\right)$$

$$y_2 = \operatorname{sen}\left(2\,x\right)$$

$$y_1' = -2\,\operatorname{sen}\left(2\,x\right)$$

$$y_2' = 2\,\cos\left(2\,x\right)$$

$$a_0\,y'' = 1 \quad f\left(x\right) = 4\,\cos\left(2\,x\right)$$

Cramer:

$$\begin{vmatrix} \cos\left(2\,x\right) \\ \operatorname{sen}\left(2\,x\right) \\ -2\,\operatorname{sen}\left(2\,x\right) \\ 2\,\cos\left(2\,x\right) \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ \operatorname{sen}\left(2\,x\right) \\ 4\,\cos\left(2\,x\right) \\ 2\,\cos\left(2\,x\right) \end{vmatrix}$$

$$\left|\begin{array}{c} \cos\left(2\,x\right) \\ 0 \\ -2\,\operatorname{sen}\left(2\,x\right) \\ 4\,\cos\left(2\,x\right) \end{array}\right|$$

$$\int -\frac{4\,\cos\left(2\,x\right)\,\operatorname{sen}\left(2\,x\right)}{2\,\operatorname{sen}^2\left(2\,x\right)+2\,\cos^2\left(2\,x\right)}\;\mathrm{d}x$$

$$-\frac{\operatorname{sen}^2\left(2\,x\right)}{2}+C$$

$$\int \frac{4\,\cos^2\left(2\,x\right)}{2\,\operatorname{sen}^2\left(2\,x\right)+2\,\cos^2\left(2\,x\right)}\;\mathrm{d}x$$

$$\frac{\operatorname{sen}\left(4\,x\right)}{4}+x+C$$

$$y=\operatorname{sen}\left(2\,x\right)\,C_1\left(x\right)+\cos\left(2\,x\right)\,C\left(x\right)$$

$$y=x\,\operatorname{sen}\left(2\,x\right)-\frac{pi}{2}\,\operatorname{sen}\left(2\,x\right)+,\cos\left(2\,x\right)$$

Ejercicio 31

Vazquez Blancas Cesar Said

Ecuacion

$$x^2\,y'' - 2\,y = 9\,x^3\,e^x$$

Calcular:

$$x^2\,y'' - 2\,y = 9\,x^3\,e^x$$

Ecuacion de Euler:

$$\mathrm{a}_0\,x^{\mathrm{n}} y^{(\mathrm{n})} + \mathrm{a}_1\,x^{\mathrm{n}-1}\,y^{(\mathrm{n})} + \ldots + \mathrm{a}_{\mathrm{n}-1}\,x\,y' + \mathrm{a}_{\mathrm{n}}\,y = f\,(x)$$

Sustitucion:

$$x = e^u$$

$$(\lambda - 1)\ \lambda - 2 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

Ecuación lineal con coeficientes constantes

$$(\lambda - 2)\ (\lambda + 1) = 0$$

Hallamos las raicez:

$$\lambda - 2 \rightarrow\ \lambda_1 = 2$$

$$\mathrm{k} = 1$$

$$\tau : C\,e^{2\,u}$$

$$\lambda + 1 \rightarrow\ \lambda_2 = -1$$

$$\mathrm{k} = 1$$

$$\tau : \frac{C_1}{e^u}$$

Solucion general:

$$\overline{y} = C\,e^{2\,u} + \frac{C_1}{e^u}$$

Método de variación de parámetros

$$\overline{y} = e^{2\,u}\,C\,(u) + \frac{C_1\,(u)}{e^u}$$

Sistema:

$$\begin{cases} C'\,(u)\ y_1 + C_1'\,(u)\ y_2 = 0 \\ C'\,(u)\ y_1' + C_1'\,(u)\ y_2' = \dfrac{f\,(u)}{a_0} \end{cases}$$

$$y_1 = e^{2\,u}$$

$$y_2 = \frac{1}{e^u}$$

$$y_1' = 2\,e^{2\,u}$$

$$y_2' = -\frac{1}{e^u}$$

$$a_0\,y'' = 1 \quad f\,(u) = 9\,e^{e^u + 3\,u}$$

Cramer:

$$\begin{vmatrix} e^{2\,u} \\ \frac{1}{e^u} \\ 2\,e^{2\,u} \\ -\frac{1}{e^u} \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ \frac{1}{e^u} \\ 9\,e^{e^u + 3\,u} \\ -\frac{1}{e^u} \end{vmatrix}$$

$$\begin{vmatrix} e^{2\,u} \\ 0 \\ 2\,e^{2\,u} \\ 9\,e^{e^u + 3\,u} \end{vmatrix}$$

$$\int 3\,e^{e^u + u}\,\mathrm{d}u$$

$$3\,e^{e^u} + C$$

$$\int -3\,e^{e^u + 4\,u}\,\mathrm{d}u$$

$$\left(-3\,e^{3\,u} + 9\,e^{2\,u} - 18\,e^u + 18\right)\,e^{e^u} + C$$

$$y = \frac{18\,e^x - C_3}{x} + 9\,x\,e^x - 18\,e^x + C_2\,x^2$$

$$y = \frac{18\,e^x - C_3}{x} + 9\,x\,e^x - 18\,e^x + C_2\,x^2$$

Cauchy:

$$\begin{cases} y = \dfrac{18\,e^x - C_3}{x} + 9\,x\,e^x - 18\,e^x + C_2\,x^2 \\ y' = -\dfrac{18\,e^x - C_3}{x^2} + 9\,x\,e^x + \dfrac{18\,e^x}{x} - 9\,e^x + 2\,C_2\,x \end{cases}$$

$$x = 1$$

$$y = 9\,e$$

$$y' = 1$$

$$\begin{cases} 9\,e = -C_3 + C_2 + 9\,e \\ 1 = C_3 + 2\,C_2 \end{cases} \to \begin{cases} C_2 = \dfrac{1}{3} \\ C_3 = \dfrac{1}{3} \end{cases}$$

Resultado:

$$y = \frac{18\,e^{x} - \frac{1}{3}}{x} + 9\,x\,e^{x} - 18\,e^{x} + \frac{x^{2}}{3}$$