1.- Sean $z = x + iy$. Encuentre la expresión

a. $\operatorname{Im}(2z + 4\bar{z} - 4i)$

$2(x+iy) + 4(x-iy) - 4i$

$2x + 2iy + 4x - 4iy - 4i$

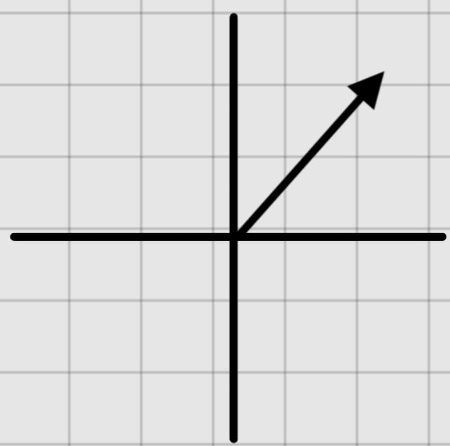
$6x - 2iy - 4i$

$\operatorname{Im}(2z + 4\bar{z} - 4i) = -2y - 4$

2.- Escriba

a) En forma polar el número complejo

$z = 1 + i$



$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\arctan(1) = \pi/4$

$\sqrt{2}\left(\cos \pi/4 + i \operatorname{sen} \pi/4\right)$

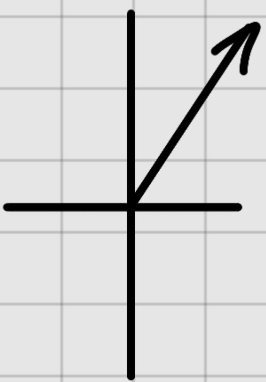
b) En forma rectangular $(a+ib)$ el número polar $z = 6(\cos \pi/8 + i \operatorname{sen} \pi/8)$

$$6(0.92387 + i0.38268)$$

$$z = 5.5432 + i2.2961$$

3.- Calcular la potencia $(1+\sqrt{3}i)^9$

$$|z| = \sqrt{x^2+y^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$



$$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \arctan(\sqrt{3}) = \pi/3$$

$$2^9(\cos 9(\pi/3) + i \operatorname{sen} 9(\pi/3))$$

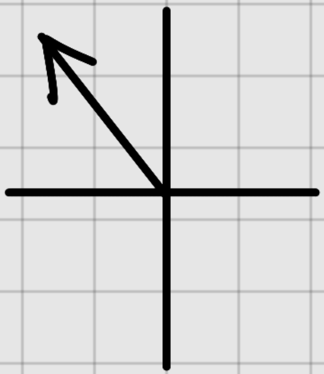
$$512(\cos(3\pi) + i \operatorname{sen}(3\pi))$$

$$= -512 + 0 = -512$$

$$-512$$

4.- Calcular las raicez $(-1+\sqrt{3}\,i)^{1/2}$

$n=2 \quad r=\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=\sqrt{4}=2$



$\theta = \arctan(-\sqrt{3}) = -\pi/3$

$K=0$

$$2^{1/2}\left[\cos\left(\frac{-\pi/3+2\cancel{0}\pi}{2}\right)+ i\,\mathrm{sen}\left(\frac{-\pi/3+2\cancel{0}\pi}{2}\right)\right.$$

$$\sqrt{2}\left[\cos(-\pi/6)+i\,\mathrm{sen}(-\pi/6)\right]$$

$$\sqrt{2}\left[\cos(-\pi/6)+i\,\mathrm{sen}(-\pi/6)\right]=-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$$

$$\sqrt{2}\left[\cos\left(\frac{-\pi/3+2\pi}{2}\right)+ i\,\mathrm{sen}\left(\frac{-\pi/3+2\pi}{2}\right)\right]$$

$$\sqrt{2}\left[\cos(5\pi/6)+i\,\mathrm{sen}(5\pi/6)\right]$$

$$-\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$$

$$K=0 \rightarrow -\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$$

$$K=1 \rightarrow -\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$$

5.- Encontrar $f(3+i)$ donde el mapeo $f(z) = z^2 + 2z$

$$(3+i)^2 + 2(3+i) = 9+6i-1+6+2i$$

$$= 14+8i$$

$$u = Re\{z^2+2z\} = 14$$

$$v = Im\{z^2+2z\} = 8$$