(1) $A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix} \rightarrow \begin{vmatrix} \lambda-2 & -1 \\ -5 & \lambda+2 \end{vmatrix} = (\lambda-2)(\lambda+2)-(5)$

$$\begin{array}{r} \lambda-2 \\ \lambda+2 \\ \hline 2\lambda-4 \\ -2\lambda \\ \hline \end{array} \rightarrow \lambda^2-4$$

$= \lambda^2-4-5 = \lambda^2-9$

$\lambda_1 = 3 \qquad \lambda_2 = -3$

$\begin{bmatrix} 3-2 & -1 \\ -5 & -3+2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 \\ -5 & 5 \end{bmatrix} \xrightarrow{e_2+5e_1} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \qquad x_2 = t$

$x_1 - x_2 = 0$

$x_1 = t$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} t \\ t \end{pmatrix} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$V_{\lambda_1} = (1, 1)$

$\begin{bmatrix} -3-2 & -1 \\ -5 & -3+2 \end{bmatrix} \rightarrow \begin{bmatrix} -5 & -1 \\ -5 & -1 \end{bmatrix} \xrightarrow{e_1-e_2} \begin{bmatrix} -5 & -1 \\ 0 & 0 \end{bmatrix} \xrightarrow{e_1/-5}$

$\begin{bmatrix} 1 & 1/5 \\ 0 & 0 \end{bmatrix} \qquad x_1 + 1/5\, x_2 = 0 \qquad x_2 = t$

$x_1 = -1/5\, t$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1/5\, t \\ t \end{pmatrix} = t\begin{pmatrix} -1/5 \\ 1 \end{pmatrix} \qquad V_{\lambda_2} = (-1/5, 1)$

$$\boxed{\begin{array}{l} V_{\lambda_1} = (1, 1) \\ V_{\lambda_2} = (-1/5, 1) \end{array}} \; (1) \qquad \boxed{\begin{array}{l} \lambda_1 = 3 \\ \lambda_2 = -3 \end{array}} \; (1)$$

② $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \to \begin{vmatrix} \lambda-2 & -1 \\ 4 & \lambda+2 \end{vmatrix}$ $(\lambda-2)(\lambda+2)-(-4)$

$= \lambda^2 \cancel{-4+4} = \lambda^2$ $\lambda_1 = 0$

$\lambda_2 = 0$

$$\begin{array}{r} \lambda - 2 \\ \lambda + 2 \\ \hline 2\cancel{\lambda} - 4 \\ \lambda^2 - \cancel{2\lambda} \\ \hline \lambda^2 - 4 \end{array}$$

$\begin{bmatrix} 0-2 & -1 \\ 4 & 0+2 \end{bmatrix} \to \begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix} \xrightarrow{e_2 + 2e_1} \begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}$

$e_1/-2 \to \begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix}$ $x_1 + 1/2\, x_2 = 0$

$x_2 = t$

$x_1 = -1/2\, t$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1/2\, t \\ t \end{pmatrix} = t \begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$ ②

$\boxed{\lambda_{1,2} = 0}$ ② $\boxed{V_{\lambda_1,\lambda_2} = (-1/2\,,\ 1)}$

③ $A = \begin{bmatrix} 9 & 4 & -3 \\ -2 & 0 & 6 \\ -1 & -4 & 11 \end{bmatrix} \to \begin{bmatrix} \lambda-9 & -4 & 3 \\ 2 & \lambda & -6 \\ 1 & 4 & \lambda-11 \end{bmatrix}$

$$\begin{array}{r} \lambda - 9 \\ \lambda - 11 \\ \hline -11\lambda + 99 \\ \lambda^2 - 9\lambda \\ \hline \lambda^2 - 20\lambda + 99 \end{array}$$

$$\left| \begin{array}{ccccc} \lambda-9 & -4 & 3 & \lambda-9 & -4 \\ 2 & \lambda & -6 & 2 & \lambda \\ 1 & 4 & \lambda-11 & 1 & 4 \end{array} \right|$$

$= (\lambda-9)(\lambda)(\lambda-11) + 24 + 24)$

$-(\cancel{3\lambda} - \cancel{24}\lambda + 216 - \cancel{8}\lambda + 88)$

$= (48 + \lambda^3 - 20\lambda^2 + 99\lambda)$

$-(29\lambda + 304)$

$= 48 + \cancel{\lambda^3} - 2\cancel{0\lambda^2} + \cancel{99}\lambda + \cancel{29}\lambda - 304$

$= \lambda^3 - 20\lambda^2 + 128\lambda - 256 = 0$

$\lambda_1 = 4$ $\lambda_2 = 8$

$$\begin{bmatrix} 4-9 & -4 & 3 \\ 2 & 4 & -6 \\ 1 & 4 & 4-11 \end{bmatrix} \to \begin{bmatrix} -5 & -4 & 3 \\ 2 & 4 & -6 \\ 1 & 4 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -7 \\ 2 & 4 & -6 \\ -5 & -4 & 3 \end{bmatrix} \xrightarrow[e_3+5e_1]{e_2-2e_1}$$

$$\begin{bmatrix} 1 & 4 & -7 \\ 0 & -4 & 8 \\ 0 & 16 & -32 \end{bmatrix} \xrightarrow[e_3/2]{e_2/21} \begin{bmatrix} 1 & 4 & -7 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{e_3+e_2} \begin{bmatrix} 1 & 4 & -7 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1+4e_2}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2/-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 = 0$ $\qquad x_1 = -x_3$

$x_2 - 2x_3 = 0$ $\qquad x_2 = 2x_3$

$x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -t \\ 2t \\ t \end{pmatrix} = t \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 8-9 & -4 & 3 \\ 2 & 8 & -6 \\ 1 & 4 & 8-11 \end{bmatrix} \to \begin{bmatrix} -1 & -4 & 3 \\ 2 & 8 & -6 \\ 1 & 4 & -3 \end{bmatrix} \xrightarrow[e_3-e_1]{e_2-2e_1} \begin{bmatrix} 1 & 4 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$e_1/-1$

$x_1 + 4x_2 - 3x_3 = 0$ $\qquad x_2 = t$

$x_3 = s$

$x_1 = -4t + 3s$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4t+3s \\ t \\ s \end{pmatrix} = s\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$$

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$$\boxed{\begin{aligned} V_{\lambda_1} &= (-1, 2, 1) \\ V_{\lambda_2} &= (3, 0, 1) \\ V_{\lambda_3} &= (-4, 1, 0) \end{aligned}}$$

$$\boxed{\begin{aligned} \lambda_1 &= 4 \\ \lambda_2 &= 8 \end{aligned}}$$

3

(4)

$$A = \begin{bmatrix} -4 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \begin{vmatrix} \lambda+4 & -1 & -2 \\ 0 & \lambda-1 & -1 \\ 0 & 0 & \lambda-3 \end{vmatrix}$$

$\lambda$

$$\left|\begin{matrix} \lambda+4 & -1 & -2 \\ 0 & \lambda-1 & -1 \\ 0 & 0 & \lambda-3 \end{matrix}\;\begin{matrix} \lambda+4 & -1 \\ 0 & \lambda-1 \\ 0 & 0 \end{matrix}\right| = (\lambda+4)(\lambda-1)(\lambda-3)$$

$\lambda_1 = -4 \quad \lambda_2 = 1 \quad \lambda_3 = 3$

$$\begin{bmatrix} -4+4 & -1 & -2 \\ 0 & -4-1 & -1 \\ 0 & 0 & -4-3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -1 & -2 \\ 0 & -5 & -1 \\ 0 & 0 & -7 \end{bmatrix} \xrightarrow{e_3/-7} \begin{bmatrix} 0 & -1 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_2/-5}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_2 = 0$
$x_3 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 1+4 & -1 & -2 \\ 0 & 1-1 & -1 \\ 0 & 0 & 1-3 \end{bmatrix} = \begin{bmatrix} 5 & -1 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow[e_3/-2 \\ e_3-e_2]{e_2/-1} \begin{bmatrix} 5 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 1/5\, x_2 = 0$
$x_3 = 0$

$x_1 = 1/5\, x_2 \qquad x_2 = t$
$x_3 = 0$
$x_1 = 1/5\, t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/5\, t \\ t \\ 0 \end{pmatrix} = t \begin{pmatrix} 1/5 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 3+4 & -1 & -2 \\ 0 & 3-1 & -1 \\ 0 & 0 & 3-3 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 - 2e_2}$$

$x_2 = t$

$$\begin{bmatrix} 7 & -5 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[e_2/2]{e_1/7} \begin{bmatrix} 1 & -5/7 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 5/7\, x_2 = 0$

$x_2 - 1/2\, x_3 = 0$

$x_1 = 5/7\, t$

$x_3 = 2x_2$

4

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5/7\, t \\ t \\ 2t \end{pmatrix} = t \begin{pmatrix} 5/7 \\ 1 \\ 2 \end{pmatrix}$$

$V_{\lambda_1} = (1, 0, 0)$

$V_{\lambda_2} = (1/5, 1, 0)$

$V_{\lambda_3} = (5/7, 1, 2)$

$\lambda_1 = -4$

$\lambda_2 = 1$

$\lambda_3 = 3$

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5) $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-2 & 0 & -1 \\ 0 & \lambda-3 & -4 \\ 0 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda-2 & 0 \\ 0 & \lambda-3 \\ 0 & 0 \end{matrix}$

$(\lambda-2)(\lambda-3)(\lambda-1)$

$\lambda_1 = 2 \quad \lambda_2 = 3 \quad \lambda_3 = 1$

$\begin{bmatrix} 2-2 & 0 & -1 \\ 0 & 2-3 & -4 \\ 0 & 0 & 2-1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[e_1+e_3]{e_2+4e_3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$a = 0$
$y = 0$
$z = 0$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad V_{\lambda_1} = (1, 0, 0)$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow[\substack{e_2+4e_3 \\ e_1+e_2}]{e_2/2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{matrix} x = 0 \\ z = 0 \\ 0 = 0 \end{matrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix} \qquad V_{\lambda_3} = [0, 1, 0)$

$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} x + z = 0 & x = -z \\ y + 2z = 0 & y = -2z \end{matrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ -2z \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

$\boxed{\begin{matrix} V_{\lambda_1} = (1, 0, 0) \\ V_{\lambda_2} = (0, 1, 0) \\ V_{\lambda_3} = (-1, -2, 1) \\ \lambda_1 = 2 \quad \lambda_2 = 3 \quad \lambda_3 = 1 \end{matrix}}$

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6. $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 1 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-1 & 0 & -4 \\ 0 & \lambda-1 & +2 \\ -1 & 0 & \lambda+2 \end{bmatrix} \begin{matrix} \lambda-1 & 0 \\ 0 & \lambda-1 \\ -1 & 0 \end{matrix}$

$(\lambda-1)(\lambda-1)(\lambda+2) - (4\lambda-4) = \lambda^3 - 3\lambda + 2 - 4\lambda + 4$

$\lambda^3 - 7\lambda + 6$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -3$

$$\begin{array}{r} \lambda - 1 \\ \lambda - 1 \\ \hline -\lambda \quad 1 \\ \lambda^2 - \lambda \\ \hline \lambda^2 - 2\lambda + 1 \end{array} \qquad \begin{array}{r} \lambda^2 - 2\lambda + 1 \\ \lambda + 2 \\ \hline 2\lambda^2 - 4\lambda + 2 \\ \lambda^3 - \cancel{2\lambda^2} \quad \lambda \\ \hline \lambda^3 - 3\lambda + 2 \end{array}$$

$\begin{bmatrix} 0 & 0 & -4 \\ 0 & 0 & 2 \\ -1 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} z = 0 \\ 0 = 0 \\ x = 0 \end{matrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad V_{\lambda_1} = (0, 1, 0)$

$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ -1 & 0 & 4 \end{bmatrix} \xrightarrow{e_3 + e_1} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x - 4z = 0 & x = 4z \\ y + 2z = 0 & y = -2z \end{matrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4z \\ -2z \\ z \end{bmatrix} = z \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \rightarrow V_{\lambda_2} = (4, -2, 1)$

$$\begin{bmatrix} -4 & 0 & -4 \\ 0 & -4 & 2 \\ -1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x + z = 0$
$y + 1/2 z = 0$
$x = -z$
$y = 1/2 z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ 1/2 z \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ 1/2 \\ 1 \end{bmatrix}$$

$V_{\lambda_3} = (-1, 1/2, 1)$

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$$\boxed{\begin{array}{l} V_{\lambda_1} = (0, 1, 0) \\ V_{\lambda_2} = (4, -2, 1) \\ V_{\lambda_3} = (-1, 1/2, 1) \\ \lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda = -3 \end{array}}$$

9 $A = \begin{bmatrix} 1 & -4 \\ -2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-1 & 4 \\ 2 & \lambda-8 \end{bmatrix} = (\lambda-1)(\lambda-8) - 8$

$$\begin{array}{r} \lambda - 1 \\ \lambda - 8 \\ \hline -8\lambda + 8 \\ \lambda^2 - \lambda \\ \hline \lambda^2 - 9\lambda + 8 \end{array}$$

$\lambda^2 - 9\lambda \cancel{+8-8}$
$\lambda^2 - 9\lambda = (\lambda - 3)(\lambda + 3)$
$\lambda(\lambda - 9)$
$\lambda_1 = 0 \quad \lambda_2 = 9$

$$\begin{bmatrix} -1 & 4 \\ 2 & -8 \end{bmatrix} \xrightarrow[e_1/-4]{e_2 - 2e_1} \begin{bmatrix} 1 & -4 \\ 0 & 0 \end{bmatrix}$$

$x - 4y = 0$
$x = +4y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4y \\ y \end{bmatrix} = y \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad V_{\lambda_1} = [4, 1]$$

$$\begin{bmatrix} 8 & 4 \\ 2 & 1 \end{bmatrix} \xrightarrow{e_2 - e_1/4} \begin{bmatrix} 8 & 4 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix} \rightarrow$$

$x + 1/2 y = 0$
$x = -1/2 y$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 y \\ y \end{bmatrix} = y \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \qquad V_{\lambda_2} = [-1/2, 1]$$

$$\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \qquad P^{-1} = \begin{bmatrix} 2/9 & 1/9 \\ -1/9 & 4/9 \end{bmatrix}$$

$$\begin{bmatrix} 2/9 & 1/9 \\ -1/9 & 4/9 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ -2 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 9 \end{bmatrix}$$

$-1/9 - 8/9 = -1$ $\qquad -4 + 4 = 0$

$+4/9 + 32/9 = 36/9 = 4$ $\qquad 1 + 8 = 9$

(11) $A = \begin{bmatrix} -2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} \lambda+2 & 1 & -3 \\ 0 & \lambda-1 & -2 \\ 0 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda+2 & 1 \\ 0 & \lambda-1 \\ 0 & 0 \end{matrix}$

$(\lambda-1)(\lambda-1)(\lambda+2)$

$\lambda_1 = 1 \quad \lambda_2 = 1 \quad \lambda_3 = -2$

$$\begin{bmatrix} 3 & 1 & -3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[e_1 - 3e_1]{e_2/-2} \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{e_1}{3}} \begin{bmatrix} 1 & 1/3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x + 1/3\,y = 0 \qquad x = -1/3\,y$

$z = 0 \qquad z = 0$

$y = y$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/3\,y \\ y \\ 0 \end{bmatrix} = y \begin{bmatrix} -1/3 \\ 1 \\ 0 \end{bmatrix}$$

$V_{\lambda_1} = (-1, 3, 0)$

$$\begin{bmatrix} 0 & 1 & -3 \\ 0 & -3 & -2 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow[e_2+2e_3 \\ e_1+3e_3]{e_3/-3} \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{e_2+3e_1} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$y=0$
$z=0$
$0=0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad V_{\lambda_2}=(1,0,0)$$

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$$\begin{bmatrix} -1 & -1 & 1 \\ 3 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{No es diagonalizable}}$

$$\begin{matrix} -1 & -1 & 1 & -1 & -1 \\ 3 & 3 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

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$$A=\begin{bmatrix} 3 & -2 & 2 \\ -2 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} \lambda-3 & 2 & -2 \\ 2 & \lambda & 1 \\ -2 & 1 & \lambda \end{bmatrix} \begin{matrix} \lambda-3 & 2 \\ 2 & \lambda \\ -2 & 1 \end{matrix}$$

$$\lambda^3-3\lambda^2-4-4-(4\lambda+\lambda-3+4\lambda)$$

$$\cancel{\lambda^3}-\cancel{3\lambda^2}\cancel{-8}\cancel{-4\lambda}\cancel{-\lambda}\cancel{+3}\cancel{-4\lambda}$$

$$\lambda^3-3\lambda^2-9\lambda-5=0 \qquad \lambda_2=-1 \quad \lambda_1=5$$

$$\begin{bmatrix} -4 & 2 & -2 \\ 2 & -1 & 1 \\ -2 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1/2 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x-1/2y+1/2z=0 \\ x=1/2y-1/2z \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2y-1/2z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & 1 \\ -2 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 \\ 2 & 5 & 1 \\ -2 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = 0$ $\quad x = 2z$

$y + z = 0$

$y = -z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -z \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\boxed{\text{Diagonizable}}$

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(13) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-1 & 0 & -2 \\ 0 & \lambda-1 & 0 \\ -2 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda-1 & 0 \\ 0 & \lambda-1 \\ -2 & 0 \end{matrix}$

$$(\lambda-1)(\lambda-1)(\lambda-1) - (4\lambda-4)$$

$$\lambda^3 - 3\lambda^2 + \cancel{3\lambda^2} - 1 - \cancel{4\lambda} + 4$$

$$\lambda^3 - 3\lambda^2 - \lambda + 3 \qquad (\lambda-3)(\lambda+1)(\lambda-1)$$

$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_3 = 3$

$$\begin{bmatrix} -2 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \xrightarrow[\substack{e_1/-2 \\ e_2/-2}]{e_3-e_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} x + z = 0 \\ y = 0 \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ 0 \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad V_{\lambda_1} = (-1, 0, 1)$$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \xrightarrow[e_3/-2]{e_1/-2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} z = 0 \\ 0 = 0 \\ x = 0 \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad V_{\lambda_2} = (0, 1, 0)$$

$$\begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \xrightarrow[\substack{e_1/2 \\ e_2/2}]{e_3+e_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} x - z = 0 \\ y = 0 \\ 0 = 0 \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ 0 \\ z \end{bmatrix} = z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \boxed{V_{\lambda_3} = (1, 0, 1)}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{matrix}$$

$Det = 1 - (-1)$

$Det = 2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix} \rightarrow \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & 0 & 3/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

$$\begin{bmatrix} 3/2 & 0 & 3/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\boxed{\text{Diagonicable}}$ 13

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(14) $A=\begin{bmatrix}2&-1&1\\-2&3&-2\\-1&1&0\end{bmatrix}\rightarrow\begin{bmatrix}\lambda-2&1&-1\\2&\lambda-3&2\\1&-1&\lambda\end{bmatrix}\begin{matrix}\lambda-2&1\\2&\lambda-3\\1&-1\end{matrix}$

$$(\lambda-2)(\lambda-3)(\lambda)+2+2-(-\lambda+3-2\lambda+4+2\lambda)$$

$$\lambda^3-5\lambda^2+6\lambda+4+\lambda-3+\cancel{2\lambda}-\cancel{4}-\cancel{2\lambda}$$

$$=\lambda^3-5\lambda^2+7\lambda-3=0$$

$$(\lambda+1)(\lambda-1)(\lambda-3)=(\lambda-1)^2(\lambda-3)$$

$\lambda_1=1 \qquad \lambda_2=3$

$$\begin{bmatrix}-1&1&-1\\2&-2&2\\1&-1&1\end{bmatrix}\xrightarrow[e_3+e_1\to e_3 \quad e_1/-1]{e_2+2e_1\to e_2}\begin{bmatrix}1&-1&1\\0&0&0\\0&0&0\end{bmatrix}\quad \begin{matrix}x-y+z=0\\x=-z+y\end{matrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}y-z\\y\\z\end{bmatrix}=y\begin{bmatrix}1\\1\\0\end{bmatrix}+z\begin{bmatrix}-1\\0\\1\end{bmatrix}\quad \begin{matrix}V_{\lambda_1}=(1,1,0)\\V_{\lambda_2}=(-1,0,1)\end{matrix}$$

$$\begin{bmatrix}1&1&-1\\2&0&2\\1&-1&3\end{bmatrix}\xrightarrow{e_2/2}\begin{bmatrix}1&1&-1\\1&0&1\\1&-1&3\end{bmatrix}\xrightarrow{e_3-e_2}\begin{bmatrix}1&1&-1\\1&0&1\\0&-1&2\end{bmatrix}$$

$$\begin{bmatrix}2&1&0\\1&0&1\\0&1&2\end{bmatrix}\rightarrow\begin{bmatrix}1&1&-1\\0&-1&2\\0&-1&2\end{bmatrix}\rightarrow\begin{bmatrix}1&1&-1\\0&-1&2\\0&0&0\end{bmatrix}\quad V_{\lambda_3}=(-1,2,$$

$$\begin{bmatrix}1&0&1\\0&1&-2\\0&0&0\end{bmatrix}\quad \begin{matrix}x+z=0\\y-2z=0\\x=-z\\y=2z\end{matrix}\qquad \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-z\\2z\\z\end{bmatrix}=z\begin{bmatrix}-1\\2\\1\end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{matrix}$$

$|A| = -1 - (2-1) \quad -1-(1-1)$

$|P| = -1 - 1 = -2 \quad -1+1$

$$\begin{bmatrix} -2 & -1 & 1 \\ 0 & 1 & -1 \\ -2 & -3 & +1 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & -2 \\ -1 & 1 & -3 \\ 1 & -1 & +1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ +1/2 & -1/2 & +3/2 \\ -1/2 & +1/2 & -1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ +1/2 & -1/2 & +3/2 \\ -1/2 & +1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1/2 & -1/2 & 3/2 \\ -3/2 & 3/2 & -3/2 \end{bmatrix}$$

$2 - 1 = 1 \quad 1 \quad -1/2 - 3/2 + 3/2 \quad \cancel{1/2} + 3/2 = \cancel{1/2}$

$-1 + 1 = 0$

$1/2 + 1 = 3/2 \quad -1/2 +$

$1 + 1 - 3/2 = 2 - 3/2 = +1/2$

$-1 - 1 + 1/2 = -3/2 \quad -1/2 - 1 =$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1/2 & -1/2 & 3/2 \\ -3/2 & 3/2 & -3/2 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\cancel{3/2} + 6/2 - \cancel{3/2} = 6/2 = 3$

$\boxed{\text{Diagonalizable}}$ 14

17) $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} \lambda & -2 \\ 0 & \lambda \end{bmatrix} = \lambda^2 = 0$ $V_{\lambda_1} = (1, 0)$

$\lambda = 0$

$\begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix}$ $-2y = 0 = y = 0$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = 0 + 0 = 0$

$\boxed{|P| = 0 \therefore \text{ no es diagonizable}}$ 17

18) $A = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda+1 & -2 \\ 0 & \lambda+1 \end{bmatrix} = (\lambda+1)(\lambda+1) = (\lambda+1)^2$

$\lambda_{1,2} = -1$ $\lambda_2 = -1$

$\begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $y = 0$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $|A| = 0$

$\boxed{|P| = 0 \therefore \text{ no es diagonicable}}$ 18

19) $\begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-3 & 0 & 0 \\ -1 & \lambda-3 & 0 \\ 0 & 0 & \lambda-3 \end{bmatrix} \begin{matrix} \lambda-3 & 0 \\ -1 & \lambda-3 \\ 0 & 0 \end{matrix}$

$(\lambda-3)^3$

$\lambda_1 = 3$ $\lambda_2 = 3$ $\lambda_3 = 3$

$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $x = 0$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ 19

$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{matrix}$ $|P| = 0$

$\boxed{|P| = 0 \therefore \text{ no es diagonizable}}$

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(20) $A = \begin{bmatrix} -2 & 3 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda+2 & -3 & -1 \\ 0 & \lambda-4 & -3 \\ 0 & 0 & \lambda+2 \end{bmatrix} \begin{matrix} \lambda+2 & -3 \\ 0 & \lambda-4 \\ 0 & 0 \end{matrix}$

$(\lambda+2)(\lambda-4)(\lambda+2)$

$\lambda_{1,2} = -2 \quad \lambda_3 = 4$

$\begin{bmatrix} 0 & -3 & -1 \\ 0 & -6 & -3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_2 - 2e_1} \begin{bmatrix} 0 & -3 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{matrix} y = 0 \\ z = 0 \\ 0 = 0 \end{matrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad V_{\lambda_1} = (1, 0, 0)$

$\begin{bmatrix} 6 & -3 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 6 & -3 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{matrix} x - 1/2\, y = 0 \\ z = 0 \end{matrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2\, y \\ 1 \\ 0 \end{bmatrix} = y \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$

$V_{\lambda_2} = (1, 2, 0)$

Hay 2 vectores
$\therefore$ no es diagonalizable

(20)

(21) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{vmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 2 \end{vmatrix} = (\lambda - 1)(\lambda - 2) \quad \lambda_1 = 1 \quad \lambda_2 = 2$

$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad y = 0 \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad x = 0 \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$PA = BP$

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

(21) $\boxed{\text{Similares}}$

(22)

$A = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

$\begin{bmatrix} \lambda - 5 & 0 \\ 0 & \lambda - 3 \end{bmatrix} = (\lambda - 5)(\lambda - 3)$ $\quad \lambda_1 = 5$, $\lambda_2 = 3$

$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \to \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad 0 = 0$, $y = 0$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad V_{\lambda_1} = (1, 0)$

$\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad x = 0$, $0 = 0$ $\quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$ $\quad V_{\lambda_2} = (0, 1)$

$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad P \cdot A = B P$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(22)

$$\begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \neq \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$$

$\boxed{\text{No son similares}}$

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$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$$\begin{bmatrix} \lambda-1 & -1 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda-1 & -1 \\ 0 & \lambda-1 \\ 0 & 0 \end{matrix} \quad (\lambda-1)(\lambda-1)(\lambda-1)$$

$$(\lambda_{1,2,3} = 1)$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} y = 0 \\ z = 0 \\ 0 = 0 \end{matrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-1 & -1 & 0 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda-1 & -1 \\ 0 & \lambda-1 \\ 0 & 0 \end{matrix} \quad (\lambda-1)\lambda-1(\lambda-1)$$

$$\lambda_{1,2,3} = 1$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} y = 0 \\ 0 = 0 \\ 0 = 0 \end{matrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{La matrices no son similares}}$$

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(24) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ , $B = \begin{bmatrix} 1 & -3 & -3 \\ 3 & -5 & -3 \\ -3 & 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda+2 & 0 \\ 0 & 0 & \lambda+2 \end{bmatrix} = (\lambda-1)(\lambda+2)^2$$

$\lambda_{1,2} = -2 \quad \lambda_3 = 1$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x = 0 \\ y = 1 \\ z = 1 \end{matrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \to \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} 0 = 0 \\ y = 0 \\ z = 0 \end{matrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$PA = BP$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -3 \\ 3 & -5 & -3 \\ -3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \neq \begin{bmatrix} 1 & -3 & -3 \\ 3 & -5 & -3 \\ -3 & 3 & 1 \end{bmatrix}$$

$\boxed{\text{No son similares}}$ (24)

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$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda-1 & 0 \\ -1 & 0 & \lambda-1 \end{bmatrix}$$

$(\lambda)(\lambda-1)(\lambda-1)-(\lambda-1)$

$\lambda^3-2\lambda^2\cancel{+\lambda}\cancel{-\lambda}+1$

$(\lambda^3-2\lambda^2+1)$

$\lambda = 1, \quad \lambda = \dfrac{1+\sqrt{5}}{2}, \quad \lambda = \dfrac{1-\sqrt{5}}{2}$

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es simétrica y ortogonal

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$$\begin{bmatrix} -3 & -1 \\ -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda+3 & 1 \\ 1 & \lambda+2 \end{bmatrix} = (\lambda+3)(\lambda+2)-1$$

$= \lambda^2+5\lambda+5$

$\lambda_1 = -\dfrac{\sqrt{5}+5}{2} \neq \lambda_2 = \dfrac{-5+\sqrt{5}}{2}$

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∴ es diagonizable y ortogonable

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$$\begin{bmatrix} 4 & 1 & 2 \\ 0 & -1 & 0 \\ 2 & 1 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda-4 & -1 & -2 \\ 0 & \lambda+1 & 0 \\ -2 & -1 & \lambda+5 \end{bmatrix} \begin{matrix} \lambda-4 & -1 \\ 0 & \lambda+1 \\ -2 & -1 \end{matrix}$$

$(\lambda-4)(\lambda+1)(\lambda+5)-(4\lambda+4)$

$\cancel{\lambda^3+2\lambda^2-19\lambda-20-4\lambda+4}$

$\lambda^3+2\lambda^2-23\lambda-24=0$

$\lambda_1 = -\dfrac{1+\sqrt{97}}{2} \neq \lambda_2 = \dfrac{-1+\sqrt{97}}{2}$

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no es diagonizable ortogonalmente