1.- Serie de Fourier de :

$f(x) = 4 \quad , -3 \leq x \leq 3$

$$a_0 = \frac{1}{3}\int_{-3}^{3} 4\,dx = \frac{1}{3}\,4\int_{-3}^{3} dx = \frac{4}{3}\left(x\Big|_{-3}^{3}\right)$$

$$\frac{4}{3}(3-(-3)) = \frac{4}{3}(3+3) = \frac{4}{3}(6)$$

$$= \frac{24}{3} = 8$$

$$a_n = \frac{1}{3}\int_{-3}^{3} 4\cos\frac{n\pi x}{3}\,dx$$

$$a_n = \frac{4}{3}\int_{-3}^{3}\cos\frac{n\pi x}{3}\,dx = \cancel{\frac{4}{3}\,\frac{3\operatorname{sen}\frac{n\pi x}{3}}{n\pi}} \rightarrow 0$$

$$b_n = \frac{4}{3}\int_{-3}^{3}\operatorname{sen}\frac{n\pi x}{3}\,dx = \frac{4}{3}\,\frac{3\cos\frac{n\pi x}{3}}{n\pi}\Bigg|_{-3}^{3}$$

$$\frac{4}{3}\left(\frac{3\cos\frac{3\pi n}{3}}{n\pi} - \frac{3\cos\frac{3\pi n}{3}}{n\pi}\right) 0$$

$$\frac{4}{3}(0) = 0$$

$$f(x) = \frac{a_0}{2} = \frac{8}{2} = 4$$

2.- Determinar la serie de Fourier compleja. Dibujar la señal $f(t)$ y el espectro de frecuencia

$$f(t) = \frac{3t}{4} \quad \text{Para } 0<t<8 \text{ con}$$

$$f(t+8) = f(t)$$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-i2\pi nt/T}\, dt$$

$$= \frac{1}{8}\int_0^4 \frac{3t}{4}\, e^{\frac{-i\pi nt}{4}}\, dt = \frac{3}{32}\int_0^4 t\, e^{\frac{-i\pi nt}{4}}$$

$$u = t \qquad du = dt$$

$$v = \frac{-4e^{\frac{-i\pi nt}{4}}}{i\pi n} \qquad dv = e^{\frac{-i\pi nt}{4}}$$

$$\frac{3}{32}\left(\frac{-t4e^{-\frac{i\pi nt}{4}}}{i\pi n}+\int\frac{4e^{-\frac{i\pi nt}{4}}}{i\pi n}dt\right)$$

$$\frac{3}{32}\left(\frac{-t4e^{-\frac{i\pi nt}{4}}}{i\pi n}+\frac{4}{i\pi n}\int e^{-\frac{i\pi nt}{4}}dt\right)$$

$$\frac{3}{32}\left(\frac{-t4e^{-\frac{i\pi nt}{4}}}{i\pi n}+\frac{4}{i\pi n}\left(\frac{-4e^{-\frac{i\pi nt}{4}}}{i\pi n}\right)\right)$$

$$\frac{3}{32}\left(\frac{-t4e^{-\frac{i\pi nt}{4}}}{i\pi n}-\frac{12e^{-\frac{i\pi nt}{4}}}{i^2\pi^2n^2}\right)$$

$$\frac{3}{32}\left(\frac{-t4e^{-\frac{i\pi nt}{4}}}{i\pi n}+\frac{12e^{-\frac{i\pi nt}{4}}}{\pi^2n^2}\Bigg|_0^8\right)$$

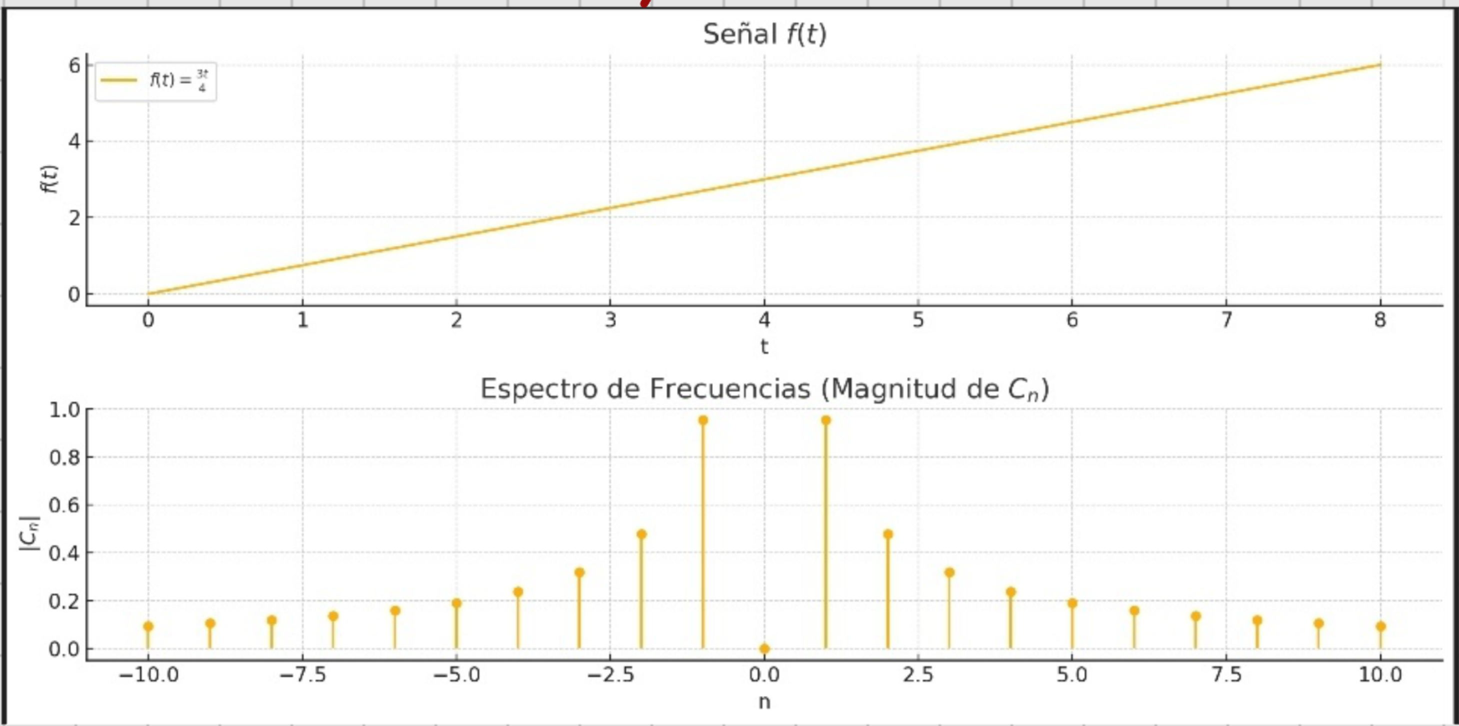
$$\frac{3}{32}\left(\frac{-32e^{-2i\pi n}}{i\pi n}+\frac{12e^{-2i\pi n}}{\pi^2n^2}-\left(\frac{-04e^{-\frac{i\pi n0}{4}}}{i\pi n}+\frac{12e^{-\frac{i\pi n0}{4}}}{\pi^2n^2}\right)\right)$$

$$\frac{3}{\cancel{32}}\left(\frac{-\cancel{32}e^{-2i\pi n}}{i\pi n}+\frac{12e^{-2i\pi n}}{\pi^2n^2}-\frac{12}{\pi^2n^2}\right)$$

$$\frac{3}{\cancel{32}}\left(-\frac{\cancel{32}(1)}{i\pi n} + \cancel{\frac{12}{n^2\pi^2} - \frac{12}{\pi^2 n^2}}\right)$$

$$\frac{3}{\cancel{32}}\left(-\frac{\cancel{32}}{i\pi n}\right) = -\frac{3}{i\pi n} = -\frac{3}{i\pi n} = \frac{3i}{\pi n}$$

$$= \sum_{n=\infty}^{\infty} \frac{3i}{\pi n} e^{\frac{in\pi t}{4}}$$



3) Encontrar la tranformada de fourier de:

$$f(t)\begin{cases}\cosh t, & -K\leq t\leq K\\ 0, & |t|>K\end{cases}$$

$$F(w)=\int_{-\infty}^{\infty}f(t)e^{-iwt}dt$$

$$F(w)=\int_{-K}^{K}\cosh t\, e^{-itw}dt=\int_{-K}^{K}\frac{e^{t}+e^{-t}}{2}e^{-itw}dt$$

$$=\frac{1}{2}\int_{-K}^{K}(e^{t}+e^{-t})e^{-itw}dt=\frac{1}{2}\int_{-K}^{K}e^{t}e^{-itw}+e^{-t}e^{-itw}dt$$

$$=\frac{1}{2}\int_{-K}^{K}e^{t-itw}+e^{-t-itw}=\frac{1}{2}\int_{-K}^{K}e^{(1-iw)t}+\int_{-K}^{K}e^{(-1-iw)t}$$

$$=\int e^{at}dt=\frac{e^{at}}{a}$$

$$\frac{1}{2}\left(\frac{e^{(1-iw)t}}{(1-iw)}+\frac{e^{(-1-iw)t}}{(-1-iw)}\Bigg|_{-K}^{K}\right)$$

$$\frac{1}{2}\left(\frac{e^{(1-iw)K}}{(1-iw)}+\frac{e^{(-1-iw)K}}{(-1-iw)}\right.$$

$$\left.-\frac{e^{(1-iw)-K}}{(1-iw)}-\frac{e^{(-1-iw)-K}}{(-1-iw)}\right)$$

$$\frac{1}{2}\left(\frac{e^{K-iwK}}{(1-iw)}+\frac{e^{-K-Kiw}}{(-1-iw)}-\frac{e^{-K+iwK}}{(1-iw)}\right.$$

$$\left.-\frac{e^{K+iwK}}{(-1-iw)}\right)$$

$$\frac{1}{2}\left(\frac{e^{K-iwK}-e^{-K+iwK}}{(1-iw)}+\frac{e^{-K-Kiw}-e^{K+iwK}}{(-1-iw)}\right)$$

9.- Considere $f(t)=t$, $-\pi<t<\pi$

con $T=2\pi$, cuya serie de fourier es

$$f(t)=t=2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\operatorname{Sen} nt$$

Utilizando Parseval

$$\frac{1}{\pi}\int_{-\pi}^{\pi}[f(t)]^2dt=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}t^2dt=\frac{1}{\pi}\left[\frac{t^3}{3}\right]_{-\pi}^{\pi}=$$

$$\frac{1}{\pi}\left[\frac{\pi^3}{3}-\frac{-\pi^3}{3}\right]=\frac{1}{\pi}\left[\frac{\pi^3}{3}+\frac{\pi^3}{3}\right]$$

$$=\frac{1}{\pi}\left[\frac{2\pi^3}{3}\right]=\frac{2\pi^{\cancel{3}}}{3\cancel{\pi}}=\frac{2\pi^2}{3}$$

$$\frac{2\pi^2}{3}=2^2\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{2\pi^2}{3}=4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{2\pi^2}{4(3)}=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$