1.- Calcular $e^z$ (en la forma $u+iv$) y $|e^z|$ si $z$ es igual a $1+i$

$e^{1+i} = e^1 e^i = e(\cos(1) + i\,\text{sen}(1)) = e$

$= 1.4686 + i\,2.28$

$|e^z| = \sqrt{1.4686^2 + 2.28^2} = \sqrt{7.3551} =$

$2.712$

$e^z = 1.4686 + i\,2.28$

$|e^z| = 2.712$

1

2.- Calcular en la forma $u+iv$ lo siguiente $\cos(1.7 + 1.5i)$

$$\frac{e^{iz} + e^{-iz}}{2} = \frac{e^{i(1.7+1.5i)} + e^{-i(1.7+1.5i)}}{2}$$

$$= \frac{e^{1.7i-1.5} + e^{-1.7i+1.5}}{2} =$$

$$\frac{e^{-1.5}(\cos(1.7)+i\,\text{sen}(1.7))}{2} +$$

$$\frac{e^{1.5}(\cos(1.7)+i\,\text{sen}(1.7))}{2} =$$

$$= \frac{-0.02874+0.22127i-0.5774-4.444i}{2}$$

$$= -0.60614-4.22227i / 2$$

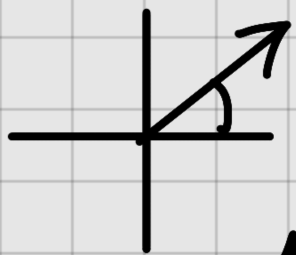
$$= -0.30307-2.11135i$$

2

## 3.- Calcular el valor principal de $\ln z$ si $z$ es igual a $1+i$

$\ln|z| + \arg(z)i$

$|z| = \sqrt{1^2+1^2} = \sqrt{2}$



$\arg z = \frac{1}{1} = 1$

$\ln\sqrt{2} + \arctan(1)i$

$\arg z = \arctan(1)$

$= 0.346557 + 0.785i$

$\arg z = 0.78$

3

4.- Integrar mediante el empleo de la trayectoria y comprobar el resultado mediante integración indefinida y sustitución de límite la función

$f(z) = az + b$ y $C$ el segmento de recta desde $-1-i$ hasta $1+i$

$z = z_1 + (z_2 - z_1)t$

$= -1-i+(1+i+1+i)t$

$= -1-i+(2+2i)t$

$= -1-i+(2+2i)t \quad 0 \leq t \leq 1$

$z = -1-i+(2+2i)t \quad dz = 2t + 2it$

$dz = 2+2i\,dt$

$$\int_0^1 (a(-1-i+2t+2it)+b)2+2i\,dt$$

$$\int_0^1 ((-a-ia+2at+2iat+b)\,2+2i\,dt$$

$$\int_0^1 ((-\cancel{2a}-\cancel{2ai}+\cancel{4at}+\cancel{4iat}+\cancel{2ab}-\cancel{2ia}$$
$$+\cancel{2a}+\cancel{4ati}-\cancel{4at}+2aib)\,dt$$

$$\int_0^1 (-4ai+8ati+2ab+2iab)\,dt$$

$$-4ai\int_0^1 dt+8ai\int_0^1 t\,dt+2ab\int_0^1 dt+$$

$$2ib\int_0^1 dt$$

$$-4ait+\frac{8ait^2}{2}+2abt+2iabt\Big|_0^1$$

$$-4ait+4ait^2+2abt+2iabt\Big|_0^1$$

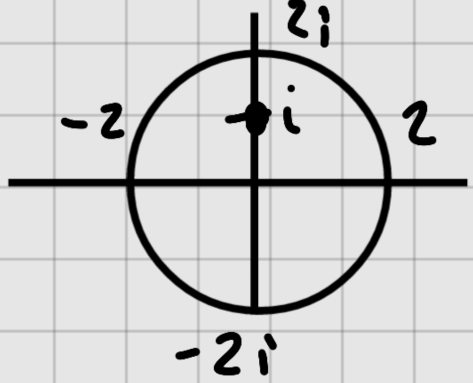
$$-\cancel{4ai}+\cancel{4ai}+2ab+2iab$$

$$=2ab(1+i)$$

4

5.- Calcular la siguiente integral siendo C la circunferencia $|z|=2$

$$\oint_C \frac{dz}{z-i}$$




$$\oint \frac{f(z)dz}{1-a}=2\pi i$$

$$a=i=\oint \frac{f(z)}{1-a}=2\pi i(1)$$

$$=f(z)\,1=2\pi i(1)$$

$$=2\pi i$$

5

6.- Encontrar la serie de Taylor con centro 0 $f(z) = e^z$ $\quad z_0 = 0$

$f(z) = e^z$ $\quad f(0) = e^0 = 1$

$f(z)' = e^z$ $\quad f(0) = e^0 = 1$

$f(z)'' = e^z$ $\quad f(0) = e^0 = 1$

$f(z)''' = e^z$ $\quad f(0) = e^0 = 1$

$$= z + \frac{z^2}{2} + \frac{z^3}{3!} + \ldots \quad R = \infty$$

6

---

7.- Desarrollar la serie de Laurent de

$0 \leq t \leq R$

$$f(z) = \frac{e^z}{z^2} = \qquad e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$f(z) = \frac{1}{z^2} \sum_{n=0}^{\infty} \frac{z^n}{n!} = \sum_{n=0}^{\infty} \frac{z^n}{n!} \frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{z^{n-2}}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{z^{n-2}}{n!} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \ldots \quad R = \infty$$

7

---