

$$1.- u = (1, 2), v = (4, 1)$$

$$\|u\| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$$

$$\|v\| = \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$$

$$u \cdot v = (1, 2) \cdot (4, 1) = 4 + 2 = 6$$

$$d(u, v) = u - v = (1, 2) - (4, 1) = (1-4, 2-1) = (-3, 1)$$

$$d = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\|u\| = \sqrt{5}$$

$$\|v\| = \sqrt{17}$$

$$u \cdot v = 6$$

$$d = \sqrt{10}$$

①

$$2.- u = (-1, 2), v = (2, 3)$$

$$\|u\| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$$

$$\|v\| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$u \cdot v = (-1, 2) \cdot (2, 3) = -2 + 6 = 4$$

$$d = u - v = (-1, 2) - (2, 3) = (-2-1, 2-3) = (-3, -1)$$

$$d = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\|u\| = \sqrt{5}$$

$$\|v\| = \sqrt{13}$$

$$u \cdot v = 4$$

$$d = \sqrt{10}$$

②

$$3.- u = (2, 1, 1), v = (3, 2, -1)$$

$$\|u\| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\|v\| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$u \cdot v = (2, 1, 1) \cdot (3, 2, -1) = 6 + 2 - 1 = 7$$

$$d = u - v = (2, 1, 1) - (3, 2, -1) = (2-3, 1-2, 1+1) = (-1, -1, 2)$$

$$d = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\|u\| = \sqrt{6}$$

$$\|v\| = \sqrt{14}$$

$$u \cdot v = 7$$

$$d = \sqrt{6}$$

③

$$4.- u = (1, -1, 2), v = (2, 3, 1)$$

$$\|u\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\|v\| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4+9+1} = \sqrt{14}$$

$$u \cdot v = (1, -1, 2) \cdot (2, 3, 1) = 2 - 3 + 2 = 1$$

$$d = (1, -1, 2) - (2, 3, 1) = (1-2, -1-3, 2-1) = (-1, -4, 1)$$

$$d = \sqrt{1^2 + 4^2 + 1^2} = \sqrt{1+16+1} = \sqrt{18}$$

$$\begin{aligned} \|u\| &= \sqrt{6} \\ \|v\| &= \sqrt{14} \\ u \cdot v &= 1 \\ d &= \sqrt{18} \end{aligned} \quad (4)$$

$$5.- u = (1, -2, 0, 1), v = (1, 1, -1, 0)$$

$$\|u\| = \sqrt{1^2 + 2^2 + 0^2 + 1^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$\|v\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$u \cdot v = (1, -2, 0, 1) \cdot (1, 1, -1, 0) = 1 - 2 + 0 + 0 = -1$$

$$d = (1, -2, 0, 1) - (1, 1, -1, 0) = (1-1, -2-1, 0+1, 1-0) = (0, -3, 1, 1)$$

$$d = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{9+1+1} = \sqrt{11}$$

$$\begin{aligned} \|u\| &= \sqrt{6} \\ \|v\| &= \sqrt{3} \\ u \cdot v &= -1 \\ d &= \sqrt{11} \end{aligned} \quad (5)$$

$$6.- u = (1, -2, 2, 0), v = (2, -1, 0, 2)$$

$$\|u\| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\|v\| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$u \cdot v = (1, -2, 2, 0) \cdot (2, -1, 0, 2) = 2 + 2 = 4$$

$$d = (1, -2, 2, 0) - (2, -1, 0, 2) = (1-2, -2+1, 2-0, 0-2) = (-1, -1, 2, -2)$$

$$d = \sqrt{1^2 + 1^2 + 2^2 + 2^2} = \sqrt{1+1+4+4} = \sqrt{10}$$

$$\begin{aligned} \|u\| &= 3 \\ \|v\| &= 3 \\ u \cdot v &= 4 \\ d &= \sqrt{10} \end{aligned} \quad (6)$$

7- $u = (0, 1, -1, 1, 2)$, $v = (0, 1, -2, 1, 1)$

$$\|u\| = \sqrt{1^2 + 1^2 + 1^2 + 2^2} = \sqrt{1+1+1+4} = \sqrt{7}$$

$$\|v\| = \sqrt{1^2 + 2^2 + 1^2 + 1^2} = \sqrt{1+1+1+4} = \sqrt{7}$$

$$u \cdot v = (0, 1, -1, 1, 2) \cdot (0, 1, -2, 1, 1) = 1 + 2 + 1 + 2 = 6$$

$$d = (0, 1, -1, 1, 2) - (0, 1, -2, 1, 1) = (0, 0, 1, 0, 1)$$

$$\|d\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$\|u\| = \sqrt{7}$
 $\|v\| = \sqrt{7}$
 $u \cdot v = 6$
 $d = \sqrt{2}$

(7)

8- $u = (1, -1, 0, 1, 1)$, $v = (0, 1, -2, 2, 1)$

$$\|u\| = \sqrt{1^2 + 1^2 + 1^2 + 1^2} = \sqrt{4} = 2$$

$$\|v\| = \sqrt{1^2 + 2^2 + 2^2 + 1^2} = \sqrt{1+4+4+1} = \sqrt{10}$$

$$u \cdot v = 0 - 1 + 0 + 2 + 1 = 2$$

$$d = (1, -2, 2, -1, 0) \quad \|d\| = \sqrt{1^2 + 2^2 + 2^2 + 1^2} = \sqrt{1+4+4+1} = \sqrt{10}$$

$\|u\| = 2$
 $\|v\| = \sqrt{10}$
 $u \cdot v = 2$
 $d = \sqrt{10}$

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9- $v = (5, 3, -2)$

$$\|v\| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{25+9+4} = \sqrt{38}$$

$$\hat{v} = \frac{(5, 3, -2)}{\sqrt{38}} = \left(\frac{5}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{-2}{\sqrt{38}} \right)$$

$\|v\| = \sqrt{38}$
 $\hat{v} = \left(\frac{5}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{-2}{\sqrt{38}} \right)$

(9)

10- $v = (1, -2, 1)$

$$\|v\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$\hat{v} = \frac{(1, -2, 1)}{\sqrt{6}} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$\|v\| = \sqrt{6}$
 $\hat{v} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$

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11 -- $v = (-1, 1, 2)$

$$\|v\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$$

$$\hat{v} = \frac{(-1, 1, 2)}{\sqrt{6}} = \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$\|v\| = \sqrt{6}$$

$$\hat{v} = \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

12 -- $v = (0, 2, -1)$

$$\|v\| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$\hat{v} = \frac{(0, 2, -1)}{\sqrt{5}} = \left(0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$$

$$\|v\| = \sqrt{5}$$

$$\hat{v} = \left(0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$$

13 -- $v = (8, 8, 6)$

$$u = (4, 4, 3)$$

$$u = (-2, -2, -\frac{6}{4})$$

$$u = (-16, -16, -12)$$

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14 -- $\|c(2, 2, -1)\| = 3$

$$\|1(2, 2, -1)\|$$

$$\|(2, 2, -1)\| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

$$c = 1$$

(14)

15. $u = (2, 2), v = (-3, 3)$

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{(2, 2) \cdot (-3, 3)}{\sqrt{2+4} \sqrt{9+9}} = \frac{-6+6}{\sqrt{6} \sqrt{18}} = \frac{0}{\sqrt{6} \sqrt{18}} =$$

$$\arccos(0) = 90^\circ$$

$$\theta = 90^\circ \quad (15)$$

16. $u = (1, -1), v = (0, 1)$

$$\cos \theta = \frac{(1, -1) \cdot (0, 1)}{\sqrt{1+1} \sqrt{1}} = \frac{0-1}{\sqrt{2} \cdot 1} = -\frac{1}{\sqrt{2}} = \arccos\left(-\frac{1}{\sqrt{2}}\right)$$

$$\theta = 135^\circ \quad (16)$$

19. $u = (10, -5, 15), v = (-2, 1, -3)$

$$\cos \theta = \frac{(10, -5, 15) \cdot (-2, 1, -3)}{\sqrt{100+25+225} \sqrt{4+1+9}} = \frac{-20-5-45}{\sqrt{350} \sqrt{14}} = \frac{-70}{70} = -1$$

$$\theta = \arccos(-1) = 180^\circ$$

$$\theta = 180^\circ \quad (19)$$

20. $u = (1, 0, -3, 0), v = (2, -2, 1, 1)$

$$\cos \theta = \frac{(2+0-3+0)}{\sqrt{1+9} \sqrt{4+4+1+1}} = \frac{-1}{\sqrt{10} \sqrt{10}} = -\frac{1}{10} \quad \theta = \arccos\left(-\frac{1}{10}\right)$$

$$\theta = \arccos\left(-\frac{1}{10}\right) \quad (20)$$

$$21. u = (0, -4, 3)$$

$$0 - 4y + 3z = 0$$

$$3z = 4y$$

$$z = \frac{4y}{3}$$

$$V = \left(x, y, \frac{4y}{3} \right) \quad (21)$$

$$22. u = (1, -1, 2)$$

$$x - y + 2z = 0$$

$$2z = -x + y$$

$$z = \frac{-x + y}{2}$$

$$V = \left(x, y, \frac{y - x}{2} \right) \quad (22)$$

$$23. u = (1, -2, 2, 1)$$

$$x - 2y + 2z + w = 0$$

$$w = -x + 2y - 2z$$

$$V = (x, y, z, -x + 2y - 2z)$$

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$$24. u = (0, 1, 2, -1)$$

$$y + 2z - w = 0$$

$$-w = -2z - y$$

$$w = 2z + y$$

$$V = (x, y, z, 2z + y) \quad (24)$$

$$25. - u = (2, -\frac{1}{2}, 1) \quad v = (\frac{3}{2}, 2, -1)$$

$$\langle u, v \rangle = u_1 v_1 + 2 u_2 v_2 + 3 u_3 v_3$$

$$\begin{aligned} \langle (2, -\frac{1}{2}, 1) | (\frac{3}{2}, 2, -1) \rangle &= 2(\frac{3}{2}) + 2(-\frac{1}{2})(2) + 3(1)(-1) \\ &= 3 + (-2) - 3 \\ &= \cancel{3} - \cancel{2} - 3 = -2 \end{aligned}$$

$$(2, -\frac{1}{2}, 1) - (\frac{3}{2}, 2, -1) = (2 - \frac{3}{2}, -\frac{1}{2} - 2, 1 + 1)$$

$$(\frac{1}{2}, -\frac{5}{2}, 2)$$

$$\begin{aligned} \langle (\frac{1}{2}, -\frac{5}{2}, 2) | (\frac{1}{2}, -\frac{5}{2}, 2) \rangle &= \frac{1}{2}(\frac{1}{2}) + 2(-\frac{5}{2})(-\frac{5}{2}) + 3(2)(2) \\ &= \frac{1}{4} + \frac{50}{4} + 12 = \frac{1}{4} + \frac{50}{4} + \frac{48}{4} \\ &= \frac{99}{4} \end{aligned}$$

$$\boxed{\langle u, v \rangle = -2}$$

$$\boxed{d = \sqrt{99/4}} \quad (25)$$

$$26. - u = (0, 3, \frac{1}{3}) \quad v = (\frac{4}{3}, 1, -3)$$

$$\langle u, v \rangle = 2 u_1 v_1 + u_2 v_2 + 2 u_3 v_3$$

$$\begin{aligned} \langle (0, 3, \frac{1}{3}) | (\frac{4}{3}, 1, -3) \rangle &= 2(0)(\frac{4}{3}) + 3(1) + 2(\frac{1}{3})(-3) \\ &= 0 + 3 - 2 = 1 \end{aligned}$$

$$(0, 3, \frac{1}{3}) - (\frac{4}{3}, 1, -3) = (-\frac{4}{3}, 2, +\frac{10}{3})$$

$$\begin{aligned} \langle (-\frac{4}{3}, 2, \frac{10}{3}) | (-\frac{4}{3}, 2, \frac{10}{3}) \rangle &= \sqrt{2(-\frac{4}{3})(-\frac{4}{3}) + 2(2) + 2(\frac{10}{3})(\frac{10}{3})} \\ &= \sqrt{\frac{32}{9} + 4 + \frac{200}{9}} \\ &= \sqrt{\frac{32}{9} + \frac{36}{9} + \frac{200}{9}} = \sqrt{\frac{268}{9}} \end{aligned}$$

$$\boxed{\langle u, v \rangle = 1}$$

$$\boxed{d = \sqrt{\frac{268}{9}}} \quad (26)$$

$$9 \overline{) 268} \\ \underline{88} \\ 88$$

$$31 - u = (2, 4) \quad v = (1, -5)$$

$$\text{proj}_v u = \left(\frac{\langle u, v \rangle}{\langle v, v \rangle} \vec{v} \right)$$

$$\text{proj}_v u = \frac{(2, 4)(1, -5)}{(1, -5)(1, -5)} (1, -5) = \frac{2 - 20}{1 + 25} (1, -5)$$

$$\text{proj}_v u = \frac{-18}{26} (1, -5) = -\frac{9}{13} (1, -5) = \left(-\frac{9}{13}, \frac{45}{13} \right) \quad (31)$$

$$\boxed{\text{proj}_v u = \left(-\frac{9}{13}, \frac{45}{13} \right)}$$

$$32 - u = (2, 3) \quad v = (0, 4)$$

$$\text{proj}_v u = \frac{(2, 3)(0, 4)}{(0, 4)(0, 4)} (0, 4) = \frac{0 + 12}{0 + 16} (0, 4) = \frac{12}{16} (0, 4)$$

$$\text{proj}_v u = \frac{6}{8} (0, 4) = \frac{3}{4} (0, 4) = \left(0, \frac{12}{4} \right) = (0, 3) \quad (32)$$

$$\boxed{\text{proj}_v u = (0, 3)}$$

$$33 - u = (1, 2), \quad v = (2, 5)$$

$$\text{proj}_v u = \frac{(1, 2)(2, 5)}{(2, 5)(2, 5)} (2, 5) = \frac{2 + 10}{4 + 25} (2, 5) = \frac{12}{29} (2, 5)$$

$$\text{proj}_v u = \left(\frac{24}{29}, \frac{60}{29} \right)$$

$$\boxed{\text{proj}_v u = \left(\frac{24}{29}, \frac{60}{29} \right)} \quad (33)$$

$$34.. u = (2, 5), v = (0, 5)$$

$$\text{proj}_v u = \frac{(2, 5)(0, 5)}{(0, 5)(0, 5)} (0, 5) = \frac{0 + 25}{0 + 25} (0, 5) = 1(0, 5)$$

$$\text{proj}_v u = (0, 5)$$

$$\text{proj}_v u = (0, 5)$$

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$$35.. u = (0, -1, 2), v = (3, 2, 4)$$

$$\text{proj}_v u = \frac{(0, -1, 2)(3, 2, 4)}{(3, 2, 4)(3, 2, 4)} (3, 2, 4) = \frac{0 - 2 + 8}{9 + 4 + 16} = \frac{6}{29}$$

$$\frac{6}{29} (3, 2, 4) = \left(\frac{18}{29}, \frac{12}{29}, \frac{24}{29} \right)$$

$$\text{proj}_v u = \left(\frac{18}{29}, \frac{12}{29}, \frac{24}{29} \right)$$

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$$36.. u = (1, 2, -1), v = (0, 2, 3)$$

$$\text{proj}_v u = \frac{(1, 2, -1)(0, 2, 3)}{(0, 2, 3)(0, 2, 3)} (0, 2, 3) = \frac{0 + 4 - 3}{0 + 4 + 9} (0, 2, 3)$$

$$\frac{1}{13} (0, 2, 3) = \left(0, \frac{2}{13}, \frac{3}{13} \right)$$

$$\text{proj}_v u = \left(0, \frac{2}{13}, \frac{3}{13} \right)$$

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$$37.. B = \{(1, 1), (0, 2)\}$$

$$w_1 = (1, 1)$$

$$(1, 1)(-1, 1) = -1 + 1 = 0$$

$$w_2 = (0, 2) - \frac{(0, 2)(1, 1)}{(1, 1)(1, 1)} (1, 1) = (0, 2) - \frac{2}{2} (1, 1) = (0, 2) - (1, 1)$$

$$(-1, 1)$$

$$\|w_1\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\|w_2\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$B = (1, 1), (-1, 1)$$

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$$B' = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$38 - B = (3, 4), (1, 2)$$

$$w_1 = (3, 4)$$

$$\frac{3+8}{9+16} = \frac{11}{25} = \frac{\frac{6}{25} \cdot 30}{\frac{2}{5} \cdot 50} = \frac{3}{5}$$

$$w_2 = (1, 2) - \frac{(1, 2)(3, 4)}{(3, 4)(3, 4)} (3, 4) = (1, 2) - \frac{11}{25} (3, 4)$$

$$(1, 2) - \left(\frac{33}{25}, \frac{44}{25} \right) = \left(\frac{25}{25} - \frac{33}{25}, \frac{50}{25} - \frac{44}{25} \right) = \left(-\frac{8}{25}, \frac{6}{25} \right)$$

$$(3, 4) \left(-\frac{8}{25}, \frac{6}{25} \right) = -\frac{24}{25} + \frac{24}{25} = 0$$

$$\frac{-\frac{8}{25} \cdot 30}{\frac{2}{5} \cdot 50} = \frac{-40}{50} = -\frac{4}{5}$$

$$\|w_1\| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\|w_2\| = \sqrt{\left(\frac{8}{25}\right)^2 + \left(\frac{6}{25}\right)^2} = \sqrt{\frac{64}{625} + \frac{36}{625}} = \sqrt{\frac{100}{625}} = \frac{\sqrt{100}}{\sqrt{625}} = \frac{10}{25} = \frac{2}{5} \quad (38)$$

$$B = \left(\frac{3}{5}, \frac{4}{5} \right), \left(-\frac{4}{5}, \frac{3}{5} \right) \quad B = \left(\left(\frac{3}{5}, \frac{4}{5} \right), \left(-\frac{4}{5}, \frac{3}{5} \right) \right)$$

$$39 - B = \{(0, 3, 4), (1, 0, 0), (1, 1, 0)\}$$

$$w_1 = (0, 3, 4) \quad \frac{3}{9+16} = \frac{3}{25} \quad \frac{1}{1}$$

$$w_2 = (1, 0, 0)$$

$$w_3 = (1, 1, 0) - \frac{(1, 1, 0)(0, 3, 4)}{(0, 3, 4)(0, 3, 4)} (0, 3, 4) - \frac{(1, 1, 0)(1, 0, 0)}{(1, 0, 0)(1, 0, 0)} (1, 0, 0)$$

$$w_3 = (1, 1, 0) - \frac{3}{25} (0, 3, 4) + (-1, 0, 0)$$

$$\frac{29}{25} - \frac{9}{25} = \frac{16}{25}$$

$$w_3 = (1, 1, 0) + (0, -\frac{9}{25}, -\frac{12}{25}) + (-1, 0, 0)$$

$$w_3 = (0, \frac{16}{25}, -\frac{12}{25}) \quad (0, \frac{16}{25}, -\frac{12}{25}) (1, 0, 0) = (0, 0, 0)$$

$$(0, \frac{16}{25}, -\frac{12}{25}) (0, 3, 4) = 0 + \frac{48}{25} - \frac{48}{25} = 0$$

$$\left(\frac{1}{25}, \frac{1}{25} \right), \left(\frac{1}{25}, \frac{1}{25} \right) = 0$$

$$\|w_1\| = \sqrt{0^2 + 3^2 + 4^2} = \sqrt{0 + 9 + 16} = \sqrt{25} = 5$$

$$\|w_2\| = \sqrt{1^2} = 1$$

$$\|w_3\| = \sqrt{0 + \frac{16}{25} + \frac{12}{25}} = \sqrt{\frac{256}{625} + \frac{144}{625}} = \sqrt{\frac{400}{625}} = \frac{\sqrt{400}}{\sqrt{625}} = \frac{20}{25} = \frac{4}{5}$$

$$w_1 = (0, \frac{3}{5}, \frac{4}{5})$$

$$w_2 = (1, 0, 0)$$

$$w_3 = (0, \frac{4}{5}, -\frac{3}{5})$$

$$B = \left\{ \left(0, \frac{3}{5}, \frac{4}{5}\right), (1, 0, 0), \left(0, \frac{4}{5}, -\frac{3}{5}\right) \right\}$$

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$$40-B = \{(0, 0, 2), (0, 1, 1), (1, 1, 1)\}$$

$$w_1 = (0, 0, 2)$$

$$\frac{2}{4} = \frac{1}{2}$$

$$w_2 = (0, 1, 1) - \frac{(0, 1, 1)(0, 0, 2)}{(0, 0, 2)(0, 0, 2)} (0, 0, 2) = (0, 1, 1) - \frac{1}{2} (0, 0, 2)$$

$$w_2 = (0, 1, 1) + (0, 0, -1) = (0, 1, 0)$$

$$(0, 0, 2)(0, 1, 0)$$

$$w_2 = (0, 1, 0)$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{1}$$

$$0 + 0 + 0 = 0$$

$$w_3 = (1, 1, 1) - \frac{(1, 1, 1)(0, 0, 2)}{(0, 0, 2)(0, 0, 2)} (0, 0, 2) = \frac{(1, 1, 1)(0, 1, 0)}{(0, 1, 0)(0, 1, 0)} (0, 1, 0)$$

$$w_3 = (1, 1, 1) - \frac{1}{2} (0, 0, 2) + (0, -1, 0) = (1, 1, 1) + (0, 0, -1) + (0, -1, 0)$$

$$w_3 = (1, 0, 0)$$

$$(0, 0, 2)(1, 0, 0) = 0$$

$$(0, 1, 0)(1, 0, 0) = 0$$

$$\|w_1\| = \sqrt{2^2} = \sqrt{4} = 2$$

$$\|w_2\| = \sqrt{1^2} = \sqrt{1} = 1$$

$$\|w_3\| = \sqrt{1^2} = \sqrt{1} = 1$$

$$w_1 = (0, 0, \frac{2}{2}) = (0, 0, 1)$$

$$w_2 = (0, \frac{1}{1}, 0) = (0, 1, 0)$$

$$w_3 = (\frac{1}{1}, 0, 0) = (1, 0, 0)$$

$$B = \{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}$$

41- $B = \{(0, 2, -2), (1, 0, -2)\}$ $x = (-1, 4, -2)$

a) $\left[\begin{array}{cc|c} 0 & 1 & -1 \\ 2 & 0 & 4 \\ -2 & -2 & -2 \end{array} \right] \xrightarrow[e_3/2]{e_2/2} \left[\begin{array}{cc|c} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & -1 & -1 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ -1 & -1 & -1 \end{array} \right]$

$\xrightarrow[e_3+e_1]{e_3+e_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array} \right]$

$(-1, 4, -2) = 2(0, 2, -2) - 1(1, 0, -2)$
 $(-1, 4, -2) = (0, 4, -4) + (-1, 0, 2)$
 $(-1, 4, -2) = (-1, 4, -2)$

$$[x]_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad (9)$$

$$b) B = \{(0, 2, -2), (1, 0, -2)\} \quad \frac{4}{4+4} = \frac{4}{8} = \frac{1}{2}$$

$$w_1 = (0, 2, -2)$$

$$w_2 = (1, 0, -2) - \frac{(1, 0, -2) \cdot (0, 2, -2)}{(0, 2, -2) \cdot (0, 2, -2)} \cdot (0, 2, -2)$$

$$w_2 = (1, 0, -2) - \frac{1}{2} (0, 2, -2) = (1, 0, -2) + (0, -1, 1)$$

$$w_2 = (1, -1, 1)$$

$$(0, 2, -2) \cdot (1, -1, 1)$$

$$0 - 2 + 2 = 0$$

$$\|w_1\| = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8}$$

$$\|w_2\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$w_1 = \left(0, \frac{2}{\sqrt{8}}, -\frac{2}{\sqrt{8}}\right)$$

$$w_2 = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

$$\beta = \left\{ \left(0, \frac{2}{\sqrt{8}}, -\frac{2}{\sqrt{8}}\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) \right\}$$

$$42 - B = \{(-1, 2, 2), (1, 0, 0)\} \quad X = (-3, 4, 4)$$

$$a) \begin{bmatrix} -1 & 1 & -3 \\ 2 & 0 & 4 \\ 2 & 0 & 4 \end{bmatrix} \xrightarrow[e_1/2]{e_3 - e_1} \begin{bmatrix} -1 & 1 & -3 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{e_1 + e_2} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} (-3, 4, 4) &= 2(-1, 2, 2) - 1(1, 0, 0) \\ (-3, 4, 4) &= (-2, 4, 4) + (-1, 0, 0) \\ (-3, 4, 4) &= (-3, 4, 4) \end{aligned}$$

$$\boxed{[X]_{B'} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}} \quad (9)$$

$$\frac{\frac{2}{9}}{\frac{\sqrt{3}}{3}} = \frac{6}{9\sqrt{3}}$$

$$\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)(-1, 2, 2)$$

$$-\frac{8}{9} + \frac{4}{9} + \frac{4}{9} = 0$$

$$b) \quad \frac{2}{3\sqrt{8}} \quad \frac{-1}{1+4+4} = -\frac{1}{9} \quad \frac{\frac{8}{9}}{\frac{\sqrt{3}}{3}} = \frac{24}{9\sqrt{8}} = \frac{8}{3\sqrt{8}}$$

$$W_1 = (-1, 2, 2)$$

$$W_2 = (1, 0, 0) - \frac{(1, 0, 0)(-1, 2, 2)}{(-1, 2, 2)(-1, 2, 2)} = (1, 0, 0) + \frac{1}{9}(-1, 2, 2)$$

$$(1, 0, 0) + \left(-\frac{1}{9}, \frac{2}{9}, \frac{2}{9}\right) = \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right) \quad \frac{\frac{1}{\sqrt{8}}}{\frac{3}{3}} = \frac{3}{\sqrt{8}}$$

$$\|W_1\| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\|W_2\| = \sqrt{\left(\frac{8}{9}\right)^2 + \left(\frac{2}{9}\right)^2 + \left(\frac{2}{9}\right)^2} = \sqrt{\frac{64}{81} + \frac{4}{81} + \frac{4}{81}} = \sqrt{\frac{72}{81}} = \sqrt{\frac{8}{9}}$$

$$\frac{\sqrt{8}}{3} \quad W_1 = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \quad W_2 = \left(\frac{\sqrt{8}}{3}, \frac{2}{3\sqrt{8}}, \frac{2}{3\sqrt{8}}\right)$$

$$\frac{\frac{2}{9}}{\frac{\sqrt{3}}{3}} = \frac{6}{9\sqrt{3}}$$

$$\boxed{B' = \left\{ \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right), \left(\frac{\sqrt{8}}{3}, \frac{2}{3\sqrt{8}}, \frac{2}{3\sqrt{8}}\right) \right\}} \quad (42)$$