Instituto Politécnico Nacional
Escuela Superior de Cómputo

Ecuaciones diferenciales lineales no homogéneas con coeficientes constantes Método de variación de parámetros

Materia: Ecuaciones Diferenciales

Integrantes:

Saeed Priego Merino
Diaz Torres Jonathan Samuel
Arellano Millan Gabriel
Ocaña Castro Hector
Lopez Chavez Moises
Vazquez Blancas Cesar Said

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Ejercicio 1

Priego Merino Saeed

Ecuacion

$$y'' + y = \operatorname{sen}(x)$$

Calcular:

$$y'' + y = \operatorname{sen}(x)$$

Ecuacion lineal con coeficientes constantes:

$$\mathrm{a}_0\, y^{(\mathrm{n})} + \mathrm{a}_1\, y^{(\mathrm{n}-1)} + \ldots + \mathrm{a}_{\mathrm{n}-1}\, y' + \mathrm{a}_{\mathrm{n}}\, y = f(x)$$

$$\lambda^2 + 1 = 0$$

hallamos las raicez

$$\lambda^2 + 1 \rightarrow\ \lambda_{1,2} = \pm i$$

$$\mathrm{k} = 1$$

$$\tau : C_1\ \operatorname{sen}(x) + C\ \cos(x)$$

y:

$$P_{\mathrm{k}-1}(x)\,,\ Q_{\mathrm{k}-1}(x)\ \rightarrow\ C_1 + \ldots + C_{\mathrm{k}}\, x^{\mathrm{k}-1}$$

Solucion general:

$$\overline{y} = C_1\ \operatorname{sen}(x) + C\ \cos(x)$$

Método de coeficientes indeterminados

$$y_{\mathrm{i}} = x^{\mathrm{s}}\, e^{\alpha\, x}\, (R_{\mathrm{m}}(x)\ \cos\beta\, x + T_{\mathrm{m}}(x)\ \operatorname{sen}\beta\, x)$$

Solución particular:

$$\alpha + \beta\, i = i\ \rightarrow\ \mathrm{s} = 1$$

$$(y = x\ (b\ \operatorname{sen}(x) + a\ \cos(x)))'_x$$

Calcular derivadas:

$$y_0'' = (-B\, x - 2\, A)\ \operatorname{sen}(x) + (2\, B - A\, x)\ \cos(x)$$

Susutituir:

$$2\, B\ \cos(x) - 2\, A\ \operatorname{sen}(x) = \operatorname{sen}(x)$$

Encontrar coeficientes:

$$\begin{cases} -2\, A = 1 \\ 2\, B = 0 \end{cases} = \begin{cases} A = -\dfrac{1}{2} \\ B = 0 \end{cases}$$

Solucion particular

$$y_0 = -\frac{x\ \cos(x)}{2}$$

Ejercicio 7

Diaz Torres Jonathan Samuel

Ecuacion

$$y'' - y' - 2\,y = 3\,e^{2\,x} - x^2$$

Calcular:

$$y'' - y' - 2\,y = 3\,e^{2\,x} - x^2$$

Ecuacion lineal con coeficientes constantes:

$$\mathrm{a}_0\,y^{(\mathrm{n})} + \mathrm{a}_1\,y^{(\mathrm{n}-1)} + \ldots + \mathrm{a}_{\mathrm{n}-1}\,y' + \mathrm{a}_{\mathrm{n}}\,y = f\,(x)$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)\;(\lambda + 1) = 0$$

hallamos las raicez

$$\lambda - 2 \rightarrow\; \lambda_1 = 2$$

$$\mathrm{k} = 1$$

$$\tau : C\,e^{2\,x}$$

$$\lambda + 1 \rightarrow\; \lambda_2 = -1$$

$$\mathrm{k} = 1$$

$$\tau : \frac{C_1}{e^x}$$

y:

$$P_{\mathrm{k}-1}\,(x)\,,\; Q_{\mathrm{k}-1}\,(x)\;\rightarrow\; C_1 + \ldots + C_{\mathrm{k}}\,x^{\mathrm{k}-1}$$

Solucion general:

$$\overline{y} = C\,e^{2\,x} + \frac{C_1}{e^x}$$

Método de coeficientes indeterminados

$$y_{\mathrm{i}} = x^{\mathrm{s}}\,e^{\alpha\,x}\,(R_{\mathrm{m}}\,(x)\,\cos\beta\,x + T_{\mathrm{m}}\,(x)\,\mathrm{sen}\,\beta\,x)$$

Solución particular:

$$\alpha + \beta\,i = 2\;\rightarrow\;\mathrm{s} = 1$$

$$\left(y = a\,x\,e^{2\,x}\right)'_x$$

Calcular derivadas:

$$y_0' = (2\,A\,x + A)\;e^{2\,x}$$

$$y_0'' = (4\,A\,x + 4\,A)\;e^{2\,x}$$

Susutituir:

$$3\,A\,e^{2\,x} = 3\,e^{2\,x}$$

Encontrar coeficientes:

$$3\,A = 3 \rightarrow A = 1$$

Solucion particular

$$y_0 = x\, e^{2\, x}$$

Solución particular:

$$\alpha + \beta\, i = 0 \;\rightarrow\; \mathrm{s} = 0$$

$$\left(y = a\, x^2 + b\, x + c\right)'_x$$

Calcular derivadas:

$$y_1' = 2\, A\, x + B$$

$$y_1'' = 2\, A$$

Sustituir:

$$-2\, A\, x^2 + (-2\, B - 2\, A)\; x - 2\, C - B + 2\, A = -x^2$$

Encontrar coeficientes:

$$\begin{cases} -2\, A = -1 \\ -2\, B - 2\, A = 0 \\ -2\, C - B + 2\, A = 0 \end{cases} = \begin{cases} A = \dfrac{1}{2} \\ B = -\dfrac{1}{2} \\ C = \dfrac{3}{4} \end{cases}$$

Solucion particular

$$y_1 = \frac{x^2}{2} - \frac{x}{2} + \frac{3}{4}$$

Solucion:

$$y_1 = x\, e^{2x} + \, x\frac{x^2}{2} - \frac{x}{2} + \frac{3}{4}$$

Ejercicio 13

Arellano Millan Gabriel

Ecuacion

$$y'' - 2\,y' + y = 4\,\cos(3\,x) - 2\,\mathrm{sen}\,(2\,x)$$

Calcular:

$$y'' - 2\,y' + y = 4\,\cos(3\,x) - 2\,\mathrm{sen}\,(2\,x)$$

Ecuacion lineal con coeficientes constantes:

$$\mathrm{a}_0\,y^{(\mathrm{n})} + \mathrm{a}_1\,y^{(\mathrm{n}-1)} + \ldots + \mathrm{a}_{\mathrm{n}-1}\,y' + \mathrm{a}_\mathrm{n}\,y = f(x)$$

$$\lambda^2 - 2\,\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

hallamos las raicez

$$(\lambda - 1)^2 \to\ \lambda_{1,2} = 1$$

$$\mathrm{k} = 2$$

$$\tau:\ (C_1\,x + C)\ e^x$$

y:

$$P_{\mathrm{k}-1}(x)\,,\ Q_{\mathrm{k}-1}(x)\ \to\ C_1 + \ldots + C_\mathrm{k}\,x^{\mathrm{k}-1}$$

Solucion general:

$$\overline{y} = (C_1\,x + C)\ e^x$$

Método de coeficientes indeterminados

$$y_\mathrm{i} = x^\mathrm{s}\,e^{\alpha\,x}\,(R_\mathrm{m}(x)\,\cos\beta\,x + T_\mathrm{m}(x)\,\mathrm{sen}\,\beta\,x)$$

Solución particular:

$$\alpha + \beta\,i = 2\,i\ \to\ \mathrm{s} = 0$$

$$(y = b\ \mathrm{sen}\,(2\,x) + a\ \cos(2\,x))'_x$$

Calcular derivadas:

$$y_0' = 2\,B\ \cos(2\,x) - 2\,A\ \mathrm{sen}\,(2\,x)$$

$$y_0'' = -9\,B\ \mathrm{sen}\,(3\,x) - 9\,A\ \cos(3\,x)$$

Susutituir:

$$(6\,A - 8\,B)\ \mathrm{sen}\,(3\,x) + (-6\,B - 8\,A)\ \cos(3\,x) = 4\ \cos(3\,x)$$

Encontrar coeficientes:

$$\begin{cases} -6\,B - 8\,A = 4 \\ 6\,A - 8\,B = 0 \end{cases} = \begin{cases} A = -\dfrac{8}{25} \\ B = -\dfrac{6}{25} \end{cases}$$

Solucion particular

$$y_0 = -\frac{6\,\operatorname{sen}(3\,x)}{25} - \frac{8\,\cos(3\,x)}{25}$$

Solución particular:

$$\alpha + \beta\,i = 2\,i \;\rightarrow\; \mathrm{s} = 0$$

$$(y = b\,\operatorname{sen}(2\,x) + a\,\cos(2\,x))'_x$$

Calcular derivadas:

$$y_0' = 2\,B\,\cos(2\,x) - 2\,A\,\operatorname{sen}(2\,x)$$

$$y_1'' = -4\,B\,\operatorname{sen}(2\,x) - 4\,A\,\cos(2\,x)$$

Sustituir:

$$(4\,A - 3\,B)\,\operatorname{sen}(2\,x) + (-4\,B - 3\,A)\,\cos(2\,x) = -2\,\operatorname{sen}(2\,x)$$

Encontrar coeficientes:

$$\begin{cases} 4\,A - 3\,B = -2 \\ -4\,B - 3\,A = 0 \end{cases} = \begin{cases} A = -\dfrac{8}{25} \\ B = \dfrac{6}{25} \end{cases}$$

Solucion particular

$$y_1 = \frac{6\,\operatorname{sen}(2\,x)}{25} - \frac{8\,\cos(2\,x)}{25}$$

Solución particular:

$$y = -\frac{6\,\operatorname{sen}(3\,x)}{25} - \frac{8\,\cos(3\,x)}{25} + \frac{6\,\operatorname{sen}(2\,x)}{25} - \frac{8\,\cos(2\,x)}{25}$$

Ejercicio 19

Ocaña Castro Hector Ecuacion

$$y'' - 3\,y' - 9\,y = 4\,\cos(2\,x) - \frac{5}{e^x}$$

Calcular:

$$y'' - 3\,y' - 9\,y = 4\,\cos(2\,x) - \frac{5}{e^x}$$

Ecuacion lineal con coeficientes constantes:

$$\mathrm{a}_0\,y^{(\mathrm{n})} + \mathrm{a}_1\,y^{(\mathrm{n}-1)} + \ldots + \mathrm{a}_{\mathrm{n}-1}\,y' + \mathrm{a}_\mathrm{n}\,y = f(x)$$

$$\lambda^2 - 3\,\lambda - 9 = 0$$

hallamos las raicez

$$\lambda^2 - 3\,\lambda - 9 \to\ \lambda_1 = -\frac{3\,\sqrt{5} - 3}{2}$$

$$\mathrm{k} = 1$$

$$\tau : C\,e^{\left(\frac{3}{2} - \frac{3\,\sqrt{5}}{2}\right)\,x}$$

$$\to\ \lambda_2 = \frac{3\,\sqrt{5} + 3}{2}$$

$$\mathrm{k} = 1$$

$$\tau : C_1\,e^{\left(\frac{3\,\sqrt{5}}{2} + \frac{3}{2}\right)\,x}$$

y:

$$P_{\mathrm{k}-1}(x)\,,\ Q_{\mathrm{k}-1}(x)\ \to\ C_1 + \ldots + C_\mathrm{k}\,x^{\mathrm{k}-1}$$

Solucion general:

$$\overline{y} = C_1\,e^{\left(\frac{3\,\sqrt{5}}{2} + \frac{3}{2}\right)\,x} + C\,e^{\left(\frac{3}{2} - \frac{3\,\sqrt{5}}{2}\right)\,x}$$

Metodo de coeficientes indeterminados:

$$y_\mathrm{i} = x^\mathrm{s}\,e^{\alpha\,x}\,(R_\mathrm{m}(x)\,\cos\beta\,x + T_\mathrm{m}(x)\,\mathrm{sen}\,\beta\,x)$$

Solucion particular:

$$\alpha + \beta\,i = 2\,i\ \to\ \mathrm{s} = 0$$

$$(y = b\,\mathrm{sen}\,(2\,x) + a\,\cos(2\,x))'_x$$

Calcular derivadas:

$$y'_0 = 2\,B\,\cos(2\,x) - 2\,A\,\mathrm{sen}\,(2\,x)$$

$$y''_1 = -4\,B\,\mathrm{sen}\,(2\,x) - 4\,A\,\cos(2\,x)$$

Sustituir:

$$(6\,A - 13\,B)\,\mathrm{sen}\,(2\,x) + (-6\,B - 13\,A)\,\cos(2\,x) = 4\,\cos(2\,x)$$

Encontrar coeficientes:

$$\begin{cases} -6\,B-13\,A=4 \\ 6\,A-13\,B=0 \end{cases} = \begin{cases} A=-\dfrac{52}{205} \\ B=-\dfrac{24}{205} \end{cases}$$

Solucion particular

$$y_0=-\frac{24\,\operatorname{sen}\left(2\,x\right)}{205}-\frac{52\,\cos\left(2\,x\right)}{205}$$

Solucion particular:

$$\alpha+\beta\,i=-1\ \rightarrow\ \mathrm{s}=0$$

$$\left(y=a\,e^{-x}\right)_x'$$

Calcular derivadas:

$$y_1'=-\frac{A}{e^x}$$

$$y_1''=\frac{A}{e^x}$$

Sustituir:

$$-\frac{5\,A}{e^x}=-\frac{5}{e^x}$$

Encontrar coeficientes:

$$-5\,A=-5\rightarrow A=1$$

Solucion particular

$$y_1=\frac{1}{e^x}$$

Solucion:

$$y=-\frac{24\,\operatorname{sen}\left(2\,x\right)}{205}-\frac{52\,\cos\left(2\,x\right)}{205}+\frac{1}{e^x}$$

Ejercicio 25

Lopez Chavez Moises y Vazquez Blancas Cesar Said

Ecuacion

$$y'' - 3\,y' - 10\,y = 50\,\cos(5\,x) + 12\,e^{x} - \frac{7}{e^{2\,x}} + 20\,x$$

Calcular:

$$y'' - 3\,y' - 10\,y = 50\,\cos(5\,x) + 12\,e^{x} - \frac{7}{e^{2\,x}} + 20\,x$$

Ecuacion lineal con coeficientes constantes:

$$\mathrm{a}_0\,y^{(\mathrm{n})} + \mathrm{a}_1\,y^{(\mathrm{n}-1)} + \ldots + \mathrm{a}_{\mathrm{n}-1}\,y' + \mathrm{a}_{\mathrm{n}}\,y = f(x)$$

$$\lambda^2 - 3\,\lambda - 10 = 0$$

$$(\lambda - 5)\,(\lambda + 2) = 0$$

hallamos las raicez

$$\lambda - 5 \rightarrow\ \lambda_1 = 5$$

$$\mathrm{k} = 1$$

$$\tau : C\,e^{5\,x}$$

$$\lambda + 2 \rightarrow\ \lambda_2 = -2$$

$$\mathrm{k} = 1$$

$$\tau : \frac{C_1}{e^{2\,x}}$$

y:

$$P_{\mathrm{k}-1}(x)\,,\ Q_{\mathrm{k}-1}(x)\ \rightarrow\ C_1 + \ldots + C_{\mathrm{k}}\,x^{\mathrm{k}-1}$$

Solucion general:

$$\overline{y} = C\,e^{5\,x} + \frac{C_1}{e^{2\,x}}$$

Metodo de coeficientes indeterminados:

$$y_{\mathrm{i}} = x^{\mathrm{s}}\,e^{\alpha\,x}\,(R_{\mathrm{m}}(x)\,\cos\beta\,x + T_{\mathrm{m}}(x)\,\mathrm{sen}\,\beta\,x)$$

Solucion particular:

$$\alpha + \beta\,i = 5\,i\ \rightarrow\ \mathrm{s} = 0$$

$$(y = b\,\mathrm{sen}(5\,x) + a\,\cos(5\,x))'_x$$

Calcular derivadas:

$$y_0' = 5\,B\,\cos(5\,x) - 5\,A\,\mathrm{sen}(5\,x)$$

$$y_0'' = -25\,B\,\mathrm{sen}(5\,x) - 25\,A\,\cos(5\,x)$$

Sustituir:

$$(15\,A - 35\,B)\,\mathrm{sen}(5\,x) + (-15\,B - 35\,A)\,\cos(5\,x) = 50\,\cos(5\,x)$$

Encontrar coeficientes:

$$\begin{cases} -15\,B - 35\,A = 50 \\ 15\,A - 35\,B = 0 \end{cases} = \begin{cases} A = -\frac{35}{29} \\ B = -\frac{15}{29} \end{cases}$$

Solucion particular

$$y_0 = -\frac{15\,\mathrm{sen}\,(5\,x)}{29} - \frac{35\,\cos(5\,x)}{29}$$

Solucion particular:

$$\alpha + \beta\,i = 1 \;\rightarrow\; \mathrm{s} = 0$$

$$(y = a\,e^{x})'_{x}$$

Calcular derivadas:

$$y_1' = A\,e^{x}$$

$$y_1'' = A\,e^{x}$$

Sustituir:

$$-12\,A\,e^{x} = 12\,e^{x}$$

Encontrar coeficientes:

$$-12\,A = 12 \rightarrow A = -1$$

Solucion particular

$$y_1 = -e^{x}$$

Solucion particular:

$$\alpha + \beta\,i = -2 \;\rightarrow\; \mathrm{s} = 1$$

$$\left(y = a\,x\,e^{-2\,x}\right)'_{x}$$

Calcular derivadas:

$$y_2' = -\frac{2\,A\,x - A}{e^{2\,x}}$$

$$y_2'' = \frac{4\,A\,x - 4\,A}{e^{2\,x}}$$

Sustituir:

$$-\frac{7\,A}{e^{2\,x}} = -\frac{7}{e^{2\,x}}$$

Encontrar coeficientes:

$$-7\,A = -7 \rightarrow A = 1$$

Solucion particular

$$y_2 = \frac{x}{e^{2\,x}}$$

Solucion particular:

$$\alpha + \beta\,i = 0 \;\rightarrow\; \mathrm{s} = 0$$

$$(y = a\,x + b)'_x$$

Calcular derivadas:

$$y_3' = A$$

$$y_3'' = 0$$

Sustituir:

$$-10\,A\,x - 10\,B - 3\,A = 20\,x$$

Encontrar coeficientes:

$$\begin{cases} -10\,A = 20 \\ -10\,B - 3\,A = 0 \end{cases} = \begin{cases} A = -2 \\ B = \frac{3}{5} \end{cases}$$

Solucion particular

$$y_3 = \frac{3}{5} - 2\,x$$

Solucion:

$$y = -\frac{15\,\operatorname{sen}(5\,x)}{29} - \frac{35\,\cos(5\,x)}{29} - e^x + \frac{x}{e^{2\,x}} - 2\,x + \frac{3}{5}$$