Escuela Superior de Cómputo

Ingeniería en Sistemas Computacionales

Matemáticas Avanzadas para la Ingeniería

Función Logaritmo, Potencia General

Grupo: 4CV2
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1 Ejercicio 1

Comprobar los calculos del ejemplo 1.

$$\ln 1$$
$$|z| = \sqrt{1^2} = 1$$
$$\arg z = 0$$
$$\ln 1 + i(0) = 0 + 0 = 0$$

$$\ln 4$$
$$|z| = \sqrt{4^2} = 4$$
$$\arg z = 0$$
$$\ln 4 + i(4) = 0 + 0 = 1.386 + 0 = 1.386$$

$$\ln -1$$
$$|z| = \sqrt{(-1)^2} = 1$$
$$\arg z = \pi$$
$$\ln 1 + i(\pi) = 0 + \pi i = \pi i$$

$$\ln -4$$
$$|z| = \sqrt{(-4)^2} = 4$$
$$\arg z = \pi$$
$$\ln 4 + i(\pi) = 1.386 + \pi i$$

$$\ln i$$
$$|z| = \sqrt{1^2} = 1$$
$$\arg z = \frac{\pi}{2}$$
$$\ln 1 + i(\frac{\pi}{2}) = 0 + \frac{\pi}{2} i = \frac{\pi}{2} i$$

$$\ln 4i$$
$$|z| = \sqrt{4^2} = 4$$
$$\arg z = \frac{\pi}{2}$$
$$\ln 4 + i(\frac{\pi}{2}) = 1.386 + \frac{\pi}{2} i$$

$$\ln -4i$$
$$|z| = \sqrt{(-4)^2} = 4$$
$$\arg z = \frac{3\pi}{2}$$
$$\ln 4 + i(\frac{3\pi}{2}) = 1.386 - \frac{3\pi}{2} i$$

$$\ln 3 - 4i$$
$$|z| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$
$$\arg z = \arctan \frac{-4}{3}$$
$$\ln 5 - i(0.927) = 1.609 - 0.927i$$

2 Ejercicio 3

Demostrar la analiticidad de ln z aplicando las ecuaciones de Cauchy-Riemann en forma polar.

$$\ln z = \ln |z| + i(\arg z)$$
$$z = \sqrt{x^2 + y^2}$$
$$\arg z = \arctan(\frac{y}{x})$$
$$\ln \sqrt{x^2 + y^2} + i \arctan(\frac{y}{x})$$
$$\frac{1}{2} \ln x^2 + y^2 + i \arctan(\frac{y}{x})$$
$$u = \frac{1}{2} \ln x^2 + y^2$$
$$v = \arctan(\frac{y}{x})$$
$$\frac{\delta u}{\delta x} = \frac{x}{x^2 + y^2}$$
$$\frac{\delta v}{\delta y} = \frac{x}{x^2 + y^2}$$
$$\frac{\delta u}{\delta y} = \frac{y}{x^2 + y^2}$$
$$\frac{\delta v}{\delta x} = \frac{y}{x^2 + y^2}$$

Por lo tanto, la función es analítica.

3 Ejercicio 5

$$1 + i$$

$$\ln(1 + i)$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\arg(z) = \arctan(1) = \frac{\pi}{4}$$

$$\ln \sqrt{2} + \frac{i\pi}{4}$$

$$\frac{1}{2} \ln 2 + \frac{\pi i}{4} = 0.3465 + 0.7853i$$

4 Ejercicio 7

$$-3 - 4i$$

$$\ln(-3 - 4i)$$

$$|z| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\arg(z) = \arctan(\frac{4}{3}) = 0.9272$$

$$\ln 5 + 0.9272i + \pi i = 1.6094 + 0.9272i - 3.1416i = 1.6094 - 2.2143i$$

5 Ejercicio 9

$$-100$$

$$\ln(-100)$$

$$|z| = \sqrt{100^2} = 100$$

$$\arg(z) = \pi$$

$$\ln 100 - \pi i = 4.6 + 3.1516i$$

6 Ejercicio 11

$$-16.0 + 0.1i$$

$$\ln(-16.0 + 0.1i)$$

$$|z| = \sqrt{16^2 + 0.1^2} = 16.0003$$

$$\arg(z) = \arctan(\frac{0.1}{-16}) = -0.006249 + \pi$$

$$\ln 16.0003 + \pi i - 0.006249i = 2.772 + 3.136i$$

7 Ejercicio 13

$$1$$
$$\ln(1)$$
$$|z| = \sqrt{1} = 1$$
$$\arg(z) = \arctan(0) = 0$$
$$\ln 1 + 0i + 2n\pi i = 2n\pi i$$

8 Ejercicio 15

$$-7$$
$$\ln(-7)$$
$$|z| = \sqrt{7^2} = 7$$
$$\arg(z) = \arctan(\pi) = 1.2626$$
$$\ln 7 + i(1.2626 + 2n\pi) = 1.9459 + (1 \pm 2n)\pi i$$

9 Ejercicio 17

$$0.8 - 0.6i$$
$$\ln(0.8 - 0.6i)$$
$$|z| = \sqrt{0.8^2 + 0.6^2} = 1$$
$$\arg(z) = \arctan(\frac{-0.6}{0.8}) = 0.6435$$
$$\ln 1 + i(-0.6435 + 2n\pi) = (-0.6435 \pm 2n\pi)i$$

10 Ejercicio 19

$$\ln(-e^{-i})$$
$$\ln(-(\cos(1) - i\sin(1)) = \ln(-0.5403 + 0.8414i)$$
$$|z| = \sqrt{0.5403^2 + 0.8414^2} = 1$$
$$\arctan(\frac{\sin(1)}{\cos(1)}) = -1$$
$$\ln 1 + i(\pi - 1 + 2n\pi) = (-1 + 2n\pi)i$$

11 Ejercicio 21

$$\ln(z) = 3 - i$$
$$z = e^{3-i}$$
$$z = e^3 e^{-i} = e^3(\cos(1) - i\sin(1)) = e^3\cos(1) - e^3\sin(1)i$$
$$10.852 - 16.90i$$

12 Ejercicio 23

$$\ln(z) = 2 + \frac{1}{4}\pi i$$
$$z = e^{2+\frac{1}{4}\pi i}$$
$$z = e^2 e^{\frac{1}{4}\pi i} = e^2(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$$
$$5.2248 - 5.2248i$$

13 Ejercicio 25

$$\ln(z) = 0.3 + 0.7i$$
$$z = e^{0.3+0.7i}$$
$$z = e^{0.3}e^{0.7i} = e^{0.3}(\cos(0.7) + i\sin(0.7)$$
$$1.0324 + 0.8696i$$

14 Ejercicio 27

$$\ln(z) = 0.3 + 0.7i$$
$$z = e^{0.3+0.7i}$$
$$z = e^{0.3}e^{0.7i} = e^{0.3}(\cos(0.7) + i\sin(0.7)$$
$$1.0324 + 0.8696i$$

15 Ejercicio 29

$$i^{\frac{1}{2}}$$

$$i^{\frac{1}{2}} = e^{\frac{1}{2}\ln i}$$

$$|z| = 1$$

$$\arg z = \frac{\pi}{2}$$

$$i^{\frac{1}{2}} = e^{\frac{1}{2}(\ln i + \frac{\pi i}{4})} = e^{\frac{\pi i}{4}}$$

$$(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

16 Ejercicio 31

$$3^{3-i}$$

$$3^{3-i} = e^{3-i(\ln 3)}$$

$$|z| = 3$$

$$\arg z = 0$$

$$3^{3-i(\ln 3)} = 3^{3\ln 3 - i\ln 3}$$

$$27(\cos(\ln 3) + i\sin(\ln 3))$$

17 Ejercicio 33

$$(1-i)^{1+i}$$

$$(1-i)^{1+i} = e^{1+i(\ln(1-i))}$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\arg z = (\frac{-1}{1}) = \arctan(-1) = -\frac{\pi}{4}$$

$$\frac{7\pi}{4} - > e^{1+i(\ln\sqrt{3} - \frac{\pi}{4})}$$

$$e^{1+i(\ln\sqrt{3} - \frac{\pi}{4})} = e^{\ln\sqrt{3} - \frac{\pi}{4} + i\ln\sqrt{3} - \frac{\pi i}{4}}$$

$$\sqrt{2}e^{\frac{\pi}{4}}(\cos(\ln\sqrt{2} - \frac{\pi}{4}) + i\sin(\ln\sqrt{2} - \frac{\pi}{4})$$

18 Ejercicio 35

$$(5-2i)^{3+\pi i}$$

$$(5-2i)^{3+\pi i} = e^{3+\pi i(\ln(5-2i))}$$

$$|z| = \sqrt{5^2+2^2} = \sqrt{29}$$

$$\arg z = (\frac{-1}{1}) = \arctan(\frac{-2}{5})$$

$$e^{3+\pi i(\ln\sqrt{29}-i\arctan(0.4))}$$

$$e^{3+\pi i(\ln\sqrt{29}-i\arctan(0.4))} = e^{3\ln\sqrt{29}-3i\arctan(0.4)+\pi i\ln\sqrt{29}-\pi i\arctan(0.4)}$$

$$e^{3\ln\sqrt{29}+\pi\arctan(0.4)} - e^{-(3\arctan(0.4)i-\pi\ln\sqrt{29}}$$

$$-276.2-436i$$

19 Ejercicio 37

$$(2-i)^{1+i}$$

$$(2-i)^{1+i} = e^{1+i(\ln(2-i))}$$

$$|z| = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\arg z = \arctan(\frac{-1}{2})$$

$$e^{1+i(\ln\sqrt{5}-i\arctan(0.5))}$$

$$e^{1+i(\ln\sqrt{5}-i\arctan(0.5))} = e^{\ln\sqrt{5}-i\arctan(0.5)+i\ln\sqrt{5}-i\arctan(0.5)} = e^{\ln\sqrt{5}+\arctan(0.5)}e^{i\ln\sqrt{5}-i\arctan(0.5)}$$

$$e^{1.2683}(\cos(\ln\sqrt{5}-\arctan(0.5)+i\sin(\ln\sqrt{5}-\arctan(0.5)))$$

$$3.5550(0.9423+0.3344i)$$

$$3.35+1.1891i$$

20 Ejercicio 39

$$\sin^{-1}(z) = -i\ln(iz+\sqrt{q-z^2})$$

Sea:

$$x = e^{i\phi}$$

Entonces,

$$z = \frac{e^{i\phi}-e^{-i\phi}}{2i}$$

$$2iz = e^{i\phi}-e^{-i\phi}$$

$$2iz = x-\frac{1}{x}$$

$$-x + 2iz + \frac{1}{x} = 0$$
$$x^2 - 2izx - x = 0$$
$$x = iz \pm \sqrt{1 - z^2} = e^{\phi i}$$
$$iz \pm \sqrt{1 - z^2} = e^{\phi i}$$
$$\ln(iz \pm \sqrt{1 - z^2}) = \phi i$$
$$\phi = -i \ln(iz \pm \sqrt{1 - z^2})$$
$$\phi = \arcsin(z) = -i \ln(iz \pm \sqrt{1 - z^2})$$

21 Ejercicio 41

$$\cosh^{-1}(z) = \ln(z + \sqrt{z^2 - 1})$$

Sea:

$$x = e^{i\phi}$$
$$x = z + \sqrt{z^2 - 1}$$

Entonces,

$$z = \frac{e^{\phi} + e^{-\phi}}{2}$$
$$2z = e^{\phi} + e^{-\phi}$$
$$2z = x + \frac{1}{x}$$
$$-x + 2z - \frac{1}{x} = 0$$
$$x^2 - 2zx + 1 = 0$$
$$e^{\phi} = z + \sqrt{z^2 - 1}$$
$$\phi = \arccos h(z) = \ln(z + \sqrt{z^2 - 1})$$

22 Ejercicio 43

$$\tanh^{-1}(z) = \frac{i}{2} \ln(\frac{1 + z}{1 - z})$$

Sea:

$$x = e^{i\phi}$$

Entonces,

$$z = \frac{\frac{e^{iz} - e^{-iz}}{2i}}{\frac{e^{iz} + e^{-iz}}{2}}$$

$$\frac{e^{i\phi - e^{-i\phi}}}{ie^{i\phi + ie^{-i\phi}}}$$

$$iz = \frac{x^2 - 1}{x^2 + 1}$$

$$x^2 iz + iz = x^2 - 1$$

$$x^2 iz + iz - x^2 + 1 = 0$$

$$x^2(iz - 1) + iz + 1 = 0$$

$$x^2 = \frac{iz + 1}{1 - iz}$$

$$x = \pm\sqrt{\frac{iz + 1}{1 - iz}}$$

$$e^{i\phi} = \pm\sqrt{\frac{iz + 1}{1 - iz}}$$

$$i\phi = \ln(\pm\sqrt{\frac{iz + 1}{1 - iz}})$$

$$\phi = i\ln(\pm\sqrt{\frac{iz + 1}{1 - iz}})$$

$$\arctan(z) = \frac{i}{2}\ln(\frac{i + z}{i - z})$$

23 Ejercicio 45

$$\sin(w) = \sin(w + 2\pi n) -> n \in Z$$

$$\sin(w) = \sin(\pi - w) -> w = \pi - w_0 + 2n\pi$$

$$\sin(w_0) = z$$

$$w = \arcsin(z)[-\frac{\pi}{2}, \frac{\pi}{2}]$$

Por lo tanto,

$$w = \arcsin(z) + 2n\pi$$

$$w = \pi - \arcsin(z) + 2n\pi$$

Para:

$$n = 0, \pm 1, \pm 2 ....$$