1) $y = ce^x$ de $y' - y = 0$

$y' = ce^x$ Sustituimos en $y' - y = 0$

$\cancel{ce^x - ce^x} = 0$

$$0 = 0$$

2) $y = 2e^{-2x} + \frac{1}{3}e^x$ de $y' + 2y = e^x$

$y' = -4e^{-2x} + \frac{1}{3}e^x$ Sustituimos en $y' + 2y = e^x$

$-4e^{-2x} + \frac{1}{3}e^x + 2(2e^{-2x} + \frac{1}{3}e^x) = e^x$

$\cancel{-4e^{-2x}} + \frac{1}{3}e^x + \cancel{4e^{-2x}} + \frac{2}{3}e^x = e^x$

$\frac{1}{3}e^x + \frac{2}{3}e^x = e^x$

$$e^x = e^x$$

Escaneado con CamScanner

③ $y = 8 \ln x + c$ de $y' = \sqrt{\dfrac{64}{x^2}}$

$y' = \dfrac{8}{x}$

Simplificamos y'

$$y' = \sqrt{\frac{64}{x^2}} = \frac{\sqrt{64}}{\sqrt{x^2}} = \frac{8}{x}$$

$$\underline{\frac{8}{x} = \frac{8}{x}} \;//$$

④ $y = C_1 e^{-x} + C_2 e^{2x}$ de $y'' - y' - 2y = 0$

$y' = -C_1 e^{-x} + 2C_2 e^{2x}$

$y'' = C_1 e^{-x} + 4C_2 e^{2x}$ Sustituimos en $y'' - y' - 2y = 0$

$$C_1 e^{-x} + 4C_2 e^{2x} + C_1 e^{-x} - 2C_2 e^{2x} - 2(C_1 e^{-x} + C_2 e^{2x}) = 0$$

$$\cancel{2C_1 e^{-x}} + 2\cancel{C_2 e^{2x}} - \cancel{2C_1 e^{-x}} - \cancel{2C_2 e^{2x}} = 0$$

$$\underline{0 = 0} \;//$$

Escaneado con CamScanner

$y = 8e^x + xe^x$ de $y'' - 2y' + y = 0$

$y' = 8e^x + e^x + xe^x$ $\quad x = u \quad u' = 1 \quad u' \cdot v + u \cdot v'$

$e^x = v \quad v' = e^x$

$y'' = 8e^x + e^x + e^x + xe^x$

$y'' = 10e^x + xe^x$ Sustituimos en $y'' - 2y' + y = 0$

$10e^x + xe^x - 2(9e^x + xe^x) + 8e^x + xe^x$

$\underline{10e^x} \underline{+xe^x} - 18e^x - 2xe^x \underline{+8e^x} \underline{+xe^x}$

$\cancel{18e^x} - \cancel{18e^x} + \cancel{2xe^x} - \cancel{2xe^x} = 0$

$$\underline{0 = 0}$$

⑥ $y = \frac{\operatorname{sen} x}{3x}$ de $x y' + y = \cos x$

$$\frac{v \cdot u' - u \cdot v'}{v^2}$$

$$y' = \frac{3x\cos x - \operatorname{sen} x \, 3}{(3x)^2}$$

$\operatorname{sen} x = u \qquad u' = \cos x$

$3x = v \qquad v' = 3$

Sustituimos en

$x y' + y = \cos x$

$$y' = \frac{3x\cos x - 3\operatorname{sen} x}{9x^2}$$

$$y' = \frac{3(x\cos x - \operatorname{sen} x)}{9x^2}$$

$$y' = \frac{x\cos x - \operatorname{sen} x}{3x^2}$$

$$x\left(\frac{x\cos x - \operatorname{sen} x}{3x^2}\right) + \frac{\operatorname{sen} x}{3x}$$

$$\frac{\cancel{x}(x\cos x - \operatorname{sen} x)}{3x^{\cancel{2}}} + \frac{\operatorname{sen} x}{3x} = \frac{x\cos x - \cancel{\operatorname{sen} x} + \cancel{\operatorname{sen} x}}{3x}$$

$$\frac{\cancel{x}\cos x}{3\cancel{x}} = \frac{\cos x}{3} \neq \cos x$$

Escaneado con CamScanner

$y - \frac{1}{\cos x} = 0$ de $y' - y\tan x = 0$

$$\frac{v \cdot u' - u \cdot v'}{v^2}$$

$y = \frac{1}{\cos x}$ $\quad 1 = u \quad u' = 0$

$\cos x = v \quad v' = -\operatorname{sen} x$

$$y' = \frac{\cos x(0) - (-\operatorname{sen} x)}{(\cos x)^2} = \frac{\operatorname{sen} x}{\cos x^2}$$

Sustituimos en $y' - y\tan x = 0$

$\tan = \frac{\operatorname{sen} x}{\cos x}$

$$\frac{\operatorname{sen} x}{\cos x^2} - \frac{1}{\cos x}\left(\frac{\operatorname{sen} x}{\cos x}\right) = 0$$

$$\cancel{\frac{\operatorname{sen} x}{\cos x^2} - \frac{\operatorname{sen} x}{\cos x^2}} = 0$$

$$\underline{0 = 0} \; //$$

⑧ $y = -\dfrac{3}{3x+2}$ de $y' = 3y^2$

$$\frac{v \cdot u' - v' u}{v^2}$$

$u = 3 \qquad u' = 0$

$v = 3x+2 \qquad v' = 3$

$$y' = \left(\frac{3x+2(0) - 3(3)}{(3x+2)^2}\right)$$

$$\begin{array}{r} 3x+2 \\ 3x+2 \\ \hline 6x+4 \\ 9x^2+6x \\ \hline 9x^2+12x+4 \end{array}$$

$$y' = \frac{9}{9x^2+12x+4}$$

Sustituimos en $y' = 3y^2$

$$\frac{9}{9x^2+12x+4} = 3\left(-\frac{3}{3x+2}\right)^2$$

$$\frac{9}{9x^2+12x+4} = 3\left(\frac{3^2}{(3x+2)^2}\right)$$

$$\frac{9}{9x^2+12x+4} = 3\left(\frac{9}{9x^2+12x+4}\right)$$

$$\frac{9}{9x^2+12x+4} \neq \frac{27}{9x^2+12x+4}$$

$y = 1 + c\sqrt{1-x^2}$ de $(1-x^2)y' + xy = x$

$u = C \qquad u' = 0$

$y' = c\,\frac{1}{2}\sqrt{1-x^2}(-2x) \qquad v = \sqrt{g} \qquad v' = \frac{1}{2}\sqrt{g}(-2x)$

$g = 1 - x^2$

$$y' = \frac{-\cancel{2}xC}{\cancel{2}\sqrt{1-x^2}}$$

Sustituimos en

$$y' = \frac{-xC}{\sqrt{1-x^2}} \qquad (1-x^2)y' + xy = x$$

$$(1-x^2)\left(\frac{-xC}{\sqrt{1-x^2}}\right) + x\left(1 + c\sqrt{1-x^2}\right)$$

$$\frac{1-x^2(-xC)}{\sqrt{1-x^2}} + x\left(1 + c\sqrt{1-x^2}\right)$$

$$\frac{-xC + x^3C}{\sqrt{1-x^2}} + x + xy\sqrt{1-x^2}$$

$$\frac{-xy + x^3y + \sqrt{1-x^2}\,x + xy(1-x^2)}{\sqrt{1-x^2}}$$

$$\frac{-\cancel{xy}+\cancel{x^3y}+\sqrt{1-x^2}\,x+\cancel{xy}-\cancel{x^3y}}{\sqrt{1-x^2}}=x$$

$$\frac{\cancel{\sqrt{1-x^2}}\,x}{\cancel{\sqrt{1-x^2}}}=x$$

$$x=x$$

$y = 2x\sqrt{1-x^2}$ de $yy' = 4x - 8x^3$ $u'v + v'u$

$u = 2x$ $u' = 2$

$y' = 2\sqrt{1-x^2} + 2x \dfrac{1}{2\sqrt{1-x^2}}(-2x)$ $v = \sqrt{g}$ $v' = \dfrac{1}{2\sqrt{g}} - 2x$

$y' = 2\sqrt{1-x^2} - \dfrac{4x^2}{\sqrt{1-x^2}}$ Sustituimos en $yy' = 4x - 8x^3$

$$y' = \frac{2 - 4x^2}{\sqrt{1-x^2}}$$

$$2x\sqrt{1-x^2}\left(\frac{2-4x^2}{\sqrt{1-x^2}}\right) = 4x - 8x^3$$

$$\frac{\cancel{\sqrt{1-x^2}}\,(2x(2-4x^2)}{\cancel{\sqrt{1-x^2}}} = 4x - 8x^3$$

$$2x(2-4x^2) = 4x - 8x^3$$

$$4x - 8x^3 = 4x - 8x^3$$

11) $y = e^{-x}\cos\frac{1}{2}x$ de $4y'' + 8y' + 5y = 0$

$u = e^{-x}$ $\quad u' = -e^{-x}$

$y' = -e^{-x}\cos\frac{1}{2}x + \frac{-\operatorname{sen}\frac{1}{2}x}{2}(e^{-x})$ $\quad v = \cos\frac{1}{2}x$ $\quad v' = \frac{-\operatorname{sen}\frac{1}{2}x}{2}$

$y' = -\left(e^{-x}\cos\frac{1}{2}x + \frac{\operatorname{sen}\frac{1}{2}x}{2}e^{-x}\right)$ $\quad u'v + v'u$

$y' = -\frac{2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x}{2e^{x}}$

$u = (-2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x)$

$v = 2e^{x}$

$u' = 4\operatorname{sen}\frac{1}{2}x + 3\cos\frac{1}{2}x$

$v' = 2e^{x}$

$$y'' = \frac{-(2(-\operatorname{sen}\frac{1}{2}x)\frac{1}{2}) + \cos(\frac{1}{2}x)\frac{1}{2})2e^{x} - (2\cos\frac{1}{2}x)\operatorname{sen}\frac{1}{2}x))2e^{x}}{(2e^{x})^2}$$

$$y'' = \frac{4\operatorname{sen}(\frac{1}{2}x) + 3\cos(\frac{1}{2}x)}{4e^{x}}$$

$$\cancel{4}\left(\frac{(4\operatorname{sen}\frac{1}{2}x) + 3\cos(\frac{1}{2}x)}{\cancel{4}e^{x}}\right) + 8\left(-\frac{2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x}{2e^{x}}\right)$$

$$+ 5(e^{-x}\cos\frac{1}{2}x) = 0$$

$$\frac{4\operatorname{sen}\frac{1}{2}x + 3\cos\frac{1}{2}x}{e^{x}} - \frac{8\cos\frac{1}{2}x + 4\operatorname{sen}\frac{1}{2}x}{e^{x}} + \frac{5\cos\frac{1}{2}}{e^{x}} = 0$$

$$\frac{4\,sen\,\frac{1}{2}x + 3\cos\frac{1}{2}x - 8\cos\frac{1}{2}x - 4\,sen\,\frac{1}{2}x + 5\cos\frac{1}{2}}{e^x} = 0$$

$$\frac{\cancel{4\,sen\,\frac{1}{2}x - 4\,sen\,\frac{1}{2}x} + \cancel{3\cos\frac{1}{2}x} + \cancel{5\cos\frac{1}{2}} - \cancel{8\cos\frac{1}{2}}}{e^x} = 0$$

$$\frac{0}{e^x} = 0$$

$$0 = 0$$ //

(12) $y = e^{-x}\cos\frac{1}{2}x$ de $y'' + y' = e^{-x}\cos\frac{1}{2}x$

$y' = -e^{-x}\cos\frac{1}{2}x + \frac{-\operatorname{sen}\frac{1}{2}x}{2}$

$u = e^{-x}$ $\quad u' = -e^{-x}$

$v = \cos\frac{1}{2}x$ $\quad v' = \frac{-\operatorname{sen}\frac{1}{2}x}{2}$

$y' = -\left(e^{-x}\cos\frac{1}{2}x + \frac{\operatorname{sen}\frac{1}{2}x}{2}e^{-x}\right)$ $\quad u'v + v'u$

$y' = -\frac{2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x}{2e^{x}}$

$u = (-2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x)$

$v = 2e^{x}$

$u' = 4\operatorname{sen}\frac{1}{2}x + 3\cos\frac{1}{2}x$

$v' = 2e^{x}$

$$y'' = \frac{-(2(-\operatorname{sen}\frac{1}{2}x)\frac{1}{2}) + \cos(\frac{1}{2}x)\frac{1}{2})2e^{x} - (2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x)2e^{x}}{(2e^{x})^2}$$

$$y'' = \frac{4\operatorname{sen}\frac{1}{2}x + 3\cos(\frac{1}{2}x)}{4e^{x}}$$

$$\frac{4\operatorname{sen}\frac{1}{2}x + 3\cos(\frac{1}{2}x)}{4e^{x}} - \frac{2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x}{2e^{x}} = e^{-x}\cos\frac{1}{2}x$$

$$\frac{2e^{x}(4\operatorname{sen}\frac{1}{2}x + 3\cos(\frac{1}{2}x)) + 4e^{x}(2\cos\frac{1}{2}x + \operatorname{sen}\frac{1}{2}x)}{8e^{x}} = e^{-1}\cos\frac{1}{2}x$$

$$\frac{\cancel{2}\,\cancel{e^x}\left(4\,sen\,\frac{1}{2}x+3\cos\left(\frac{1}{2}x\right)-2\left(2\cos\frac{1}{2}x+sen\,\frac{1}{2}x\right)\right.}{8e^x}=e^{-x}\cos\frac{1}{2}x$$

$$\frac{e^x\left(4\,sen\,\frac{1}{2}x+3\cos\left(\frac{1}{2}x\right)-2\left(2\cos\frac{1}{2}x+sen\,\frac{1}{2}x\right)\right)}{4e^x}$$

$$\frac{4\,sen\,\frac{1}{2}x+3\cos\frac{1}{2}x-4\cos\frac{1}{2}x-2sen\,\frac{1}{2}x}{4e^x}=e^{-x}\cos\frac{1}{2}x$$

$$\frac{2sen\frac{1}{2}x-\cos\frac{1}{2}x}{4e^x}\neq e^{-x}\cos\frac{1}{2}x$$

1r

13) $x = \cos t$ de $y' + \dfrac{y}{\sqrt{1-x^2}} = 0$
$y = e^t$

$y' = 0$

$1 - \cos t^2 = \operatorname{sen} t^2$

$$0 + \frac{e^t}{\sqrt{1-\cos t^2}} = 0$$

$$0 + \frac{e^t}{\cancel{\sqrt{\operatorname{sen} t^{2}}}} = 0$$

$$0 + \frac{e^t}{\operatorname{sen} t} = 0$$

$$\frac{e^t}{\operatorname{sen} t} \neq 0$$

//

Escaneado con CamScanner

$y = \dfrac{x}{\cos x}$ de $xy' - y = x^2 \tan x \sec x$

$$y' = \frac{\cos x - (-\operatorname{sen} x)\,x}{\cos x^2}$$

$U = x \qquad U' = 1$

$V = \cos x \qquad V' = -\operatorname{sen} x$

$$\frac{VU' - UV'}{V^2}$$

$$y' = \frac{\cos x + x\operatorname{sen} x}{\cos x^2}$$

Sustituimos en $xy' - y = x^2 \tan x \sec x$

$$x\left(\frac{x\operatorname{sen} x + \cos x}{\cos x^2}\right) - \frac{x}{\cos x} = x^2 \tan x \sec x$$

$$\frac{x^2 \operatorname{sen} x + x\cos x}{\cos x^2} - \frac{x}{\cos x} = x^2 \tan x \sec x$$

$$\frac{\cos x\,(x^2 \operatorname{sen} x + x\cos x) - \cos x^2 (x)}{\cos x^3} = x^2 \tan x \sec x$$

$$\frac{\cancel{\cos x}\,(x^2 \operatorname{sen} x + \cancel{x\cos x} - \cancel{x\cos x})}{\cos x^{\cancel{3}}} = x^2 \tan x \sec x$$

$$\frac{x^2 \operatorname{sen} x}{\cos x^2} = x^2 \tan x \sec x$$

$$x^2\left(\frac{\operatorname{sen} x}{\cos x}\right)\left(\frac{1}{\cos x}\right) = x^2 \tan x \sec x$$

$$x^2 \tan x \sec x = x^2 \tan x \sec x$$

$$\frac{\operatorname{sen} x}{\cos x} = \tan x$$

$$\frac{1}{\cos x} = \sec x$$

$x = \cos t$
$y = 2\operatorname{sen} t$ de $yy' + 4x = 0$

$y' = 2\cos t$ Sustituimos en $yy' + 4x = 0$

$$2\operatorname{sen} t \, 2\cos t + 4\cos t = 0$$

$$2\operatorname{sen}(2x) + 4\cos t \neq 0$$

16) $y = e^{\operatorname{sen}^{-1} 2x}$ de $xy' - y\tan \ln y = 0$

$$y' = e^{\operatorname{sen}^{-1} 2x}\left(\frac{1}{\sqrt{1-4x^2}}(2)\right)$$

$$y' = \frac{e^{\operatorname{sen}^{-1} 2x}\, 2}{\sqrt{1-4x^2}}$$

$\ln e^x = x$

$$x\left(\frac{2e^{\operatorname{sen}^{-1} 2x}}{\sqrt{1-4x^2}}\right) - e^{\operatorname{sen}^{-1} 2x}\left(\tan \ln\left(e^{\operatorname{sen}^{-1} 2x}\right)\right)$$

$$\frac{2xe^{\operatorname{sen}^{-1} 2x}}{\sqrt{1-4x^2}} - e^{\operatorname{sen}^{-1} 2x}\tan(\operatorname{sen}^{-1} 2x)$$

$$\frac{\cancel{x e^{sen^{-1} 2x}}}{\sqrt{1-4x}} - e^{sen^{-1} 2x} \tan(sen^{-1} 2x) = 0$$

$$\tan(\arcsen x) = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{2x e^{sen^{-1} 2x}}{\sqrt{1-4x}} - e^{sen^{-1} 2x} \left(\frac{2x}{\sqrt{1-x^2}}\right) = 0$$

$$\cancel{\frac{2x e^{sen^{-1} 2x}}{\sqrt{1-4x}}} - \cancel{\frac{2x e^{sen^{-1} 2x}}{\sqrt{1-4x}}} = 0$$

$$0 = 0$$