Instituto Politécnico Nacional
Escuela Superior de Cómputo

Matematicas Avanzadas para la Ingenieria

SERIE DE TAYLOR

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1.- $\mathbf{e}^{-z}, 0$

$$\begin{aligned} f(z) &= e^{-z} & f(0) &= e^0 = 1 \\ f'(z) &= -e^{-z} & f'(0) &= -e^0 = -1 \\ f''(z) &= e^{-z} & f''(0) &= e^0 = 1 \\ f'''(z) &= -e^{-z} & f'''(0) &= -e^0 = -1 \end{aligned}$$

$$\sum_{m=0}^{\infty} \frac{f^{(m)}(z_0)}{m!}(z-z_0)^m$$

$$1 - z + \frac{z^2}{2} - \frac{z^3}{3!} + - \cdots \quad R = \infty$$

3.- $\sin \pi z, 0$

$$\begin{aligned} f(z) &= \sin \pi z & f(0) &= 0 \\ f'(z) &= \pi \cos \pi z & f'(0) &= \pi \\ f''(z) &= -\pi^2 \sin \pi z & f''(0) &= 0 \\ f'''(z) &= -\pi^3 \cos \pi z & f'''(0) &= -\pi^3 \end{aligned}$$

$$\pi z - \frac{\pi^3 z^3}{3!} + \frac{\pi^5 z^5}{5!} - \cdots \quad R = \infty$$

5.- $\sin z, \pi/2$

$$\begin{aligned} f(z) &= \sin z & f(\pi/2) &= 1 \\ f'(z) &= \cos z & f'(\pi/2) &= 0 \\ f''(z) &= -\sin z & f''(\pi/2) &= -1 \\ f'''(z) &= -\cos z & f'''(\pi/2) &= 0 \\ f^{(4)}(z) &= \sin z & f^{(4)}(\pi/2) &= 1 \end{aligned}$$

$$1 - \frac{(z-\pi/2)^2}{2!} + \frac{(z-\pi/2)^4}{4!} - \cdots \quad R = \infty$$

7.- $\cdot\frac{1}{1-z}, -1$

$$f(z) = \frac{1}{1-z} \qquad f(-1) = \frac{1}{2}$$
$$f'(z) = \frac{1}{(1-z)^2} \qquad f'(-1) = \frac{1}{4}$$
$$f''(z) = \frac{1}{(1-z)^3} \qquad f''(-1) = \frac{1}{8}$$
$$f'''(z) = \frac{1}{(1-z)^4} \qquad f'''(-1) = \frac{1}{16}$$
$$f^{(4)}(z) = \frac{1}{(1-z)^5} \qquad f^{(4)}(-1) = \frac{1}{32}$$

$$\frac{1}{2} + \frac{(z+1)}{4} + \frac{(z+1)^2}{8} + \frac{(z+1)^3}{16} + \cdots \quad R = 2$$

9.- $\ln z, 1$

$$f(z) = \ln z \qquad f(1) = 0$$
$$f'(z) = \frac{1}{z} \qquad f'(1) = 1$$
$$f''(z) = -\frac{1}{z^2} \qquad f''(1) = -1$$
$$f'''(z) = \frac{1}{z^3} \qquad f'''(1) = 1$$
$$f^{(4)}(z) = -\frac{1}{z^4} \qquad f^{(4)}(1) = -1$$

$$(z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \cdots \quad R = 1$$

11.- $\mathbf{z}^5, -1$

$$
\begin{aligned}
f(z) &= z^5 & f(-1) &= -1\\
f'(z) &= 5z^4 & f'(-1) &= 5\\
f''(z) &= 20z^3 & f''(-1) &= -20\\
f'''(z) &= 60z^2 & f'''(-1) &= 60\\
f^{(4)}(z) &= 120z & f^{(4)}(-1) &= -120\\
f^{(5)}(z) &= 120 & f^{(5)}(-1) &= 120\\
f^{(6)}(z) &= 0 & f^{(6)}(-1) &= 0
\end{aligned}
$$

$$
\begin{aligned}
&-1 + 5(z+1) - 10(z+1)^2 + 10(z+1)^3\\
&-5(z+1)^4 + (z+1)^5
\end{aligned}
$$

13.- $\sin^2 z, 0$

$$
\begin{aligned}
f(z) &= \sin^2 z & f(0) &= 0\\
f'(z) &= \sin 2z & f'(0) &= 0\\
f''(z) &= 2\cos 2z & f''(0) &= 2\\
f'''(z) &= -4\sin 2z & f'''(0) &= 0\\
f^{(4)}(z) &= -8\cos 2z & f^{(4)}(0) &= -8\\
f^{(5)}(z) &= 16\sin 2z & f^{(5)}(0) &= 0\\
f^{(6)}(z) &= 32\cos 2z & f^{(6)}(0) &= 32
\end{aligned}
$$

$$
z^2 - \frac{2^3 z^4}{4!} + \frac{2^5 z^6}{6!} - \frac{2^7 z^8}{8!} + \cdots \quad R = \infty
$$

15.- $\cos(z - \pi/2), \pi/2$

$$
\begin{aligned}
f(z) &= \cos(z-\pi/2) & f(\pi/2) &= 1\\
f'(z) &= -\sin(z-\pi/2) & f'(\pi/2) &= 0\\
f''(z) &= -\cos(z-\pi/2) & f''(\pi/2) &= -1\\
f'''(z) &= \sin(z-\pi/2) & f'''(\pi/2) &= 0\\
f^{(4)}(z) &= \cos(z-\pi/2) & f^{(4)}(\pi/2) &= 1
\end{aligned}
$$

$$
1 - \frac{(z-\pi/2)^2}{2!} + \frac{(z-\pi/2)^4}{4!} - \cdots \quad R = \infty
$$

17.- Deducir 14 y 15 a partir de 12.Obtener 16 a partr de Teorema de Taylor

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \qquad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \qquad \cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\begin{aligned} e^{iz} - e^{-iz} &= \sum_{n=0}^{\infty} \frac{z^n i^n}{n!} - \sum_{n=0}^{\infty} \frac{-iz^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{z^n i^n - (-iz^n)}{n!} = \sum_{n=0}^{\infty} \frac{z^n i^n + iz^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{2i^n z^n}{n!} = \sum_{n=0}^{\infty} \frac{2z^{2n+1}}{(2n+1)!}(-1)^n \end{aligned}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\begin{aligned} e^{iz} + e^{-iz} &= \sum_{n=0}^{\infty} \frac{z^n i^n}{n!} + \sum_{n=0}^{\infty} \frac{z^n (-i)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{z^n i^n + z^n (-i)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{z^n (i^n + (-i)^n)}{n!} \\ &= \sum_{n=0}^{\infty} \frac{z^{2n} (i^{2n} + (-i)^{2n})}{(2n)!} \qquad = \sum_{n=0}^{\infty} \frac{z^{2n} ((-1)^n + (-1)^n)}{(2n)!} \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{2z^{2n}}{(2n)!} \end{aligned}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$
\begin{aligned}
e^z - e^{-z} &= \sum_{n=0}^{\infty} \frac{z^n}{n!} - \sum_{n=0}^{\infty} \frac{(-z)^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{z^n - (-z)^n}{n!} = \sum_{n=0}^{\infty} \frac{z^n + z^n}{n!} \qquad \text{(odd terms only)} \\
&= \sum_{n=0}^{\infty} \frac{2z^{2n+1}}{(2n+1)!} \\
\sinh z &= \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}
\end{aligned}
$$

$$
\begin{aligned}
e^z + e^{-z} &= \sum_{n=0}^{\infty} \frac{z^n}{n!} + \sum_{n=0}^{\infty} \frac{(-z)^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{z^n + (-z)^n}{n!} = \sum_{n=0}^{\infty} \frac{z^n - z^n}{n!} \qquad \text{(even terms only)} \\
&= \sum_{n=0}^{\infty} \frac{2z^{2n}}{(2n)!} \\
\cosh z &= \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}
\end{aligned}
$$

$$
\begin{aligned}
&\ln(1+z) \\
f(z) &= \ln(1+z) & f(0) &= 0 \\
f'(z) &= \frac{1}{1+z} & f'(0) &= 1 \\
f''(z) &= -\frac{1}{(1+z)^2} & f''(0) &= -1 \\
f'''(z) &= \frac{1}{(1+z)^3} & f'''(0) &= 1 \\
f^{(4)}(z) &= -\frac{1}{(1+z)^4} & f^{(4)}(0) &= -1
\end{aligned}
$$

$$
z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4}
$$

19.- Demostrar que cosh $\neq 0$

$$\begin{aligned}\cosh z &= \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \\ &= 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \cdots\end{aligned}$$

$$|z| \geq 0 \implies \cosh z \neq 0$$

21.- Demostrar que $(\sin z)' = \cos z$ y $(\cos z)' = -\sin z$

$$\begin{aligned}\sin z &= \sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+1}}{(2n+1)!} \\ (\sin z)' &= \left(\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+1}}{(2n+1)!}\right)' \\ &= \sum_{n=0}^{\infty} \frac{d}{dz}\left((-1)^n \frac{z^{2n+1}}{(2n+1)!}\right) \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{d}{dz}\left(\frac{z^{2n+1}}{(2n+1)!}\right) \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{(2n+1)z^{2n}}{(2n+1)!} \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{z^{2n}}{(2n)!} = \cos z\end{aligned}$$

$$\begin{aligned}(\cos z)' &= \left(\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n}}{(2n)!}\right)' \\ &= \sum_{n=0}^{\infty}(-1)^n \frac{d}{dz}\left(\frac{z^{2n}}{(2n)!}\right) \\ &= \sum_{n=1}^{\infty}(-1)^n \frac{2nz^{2n-1}}{(2n)!} \qquad = \sum_{n=1}^{\infty}(-1)^n \frac{z^{2n-1}}{(2n-1)!} \\ &= -\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+1}}{(2n+1)!} = -\sin z\end{aligned}$$

23.- f(z)=sen z / z esta indefinida en z=0, Definir f(0) de modo que f(z) se vuelva entera.

$$f(z) = \frac{\sin z}{z}$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\frac{\sin z}{z} = 1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \frac{z^6}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n+1)!}$$

$$f(0) = 1$$

f(0)= 1 la funcion se vuelve holomorfa o entera en todo el plano complejo

25.- Deducir 14 y 15 entre el seno y coseno trigonometricos e hiperbolicos a partir de 14 y 15

$$\sinh(iy) = i \sin(y)$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{(iy)^{2n+1}}{(2n+1)!} = i \sum_{n=0}^{\infty} \frac{y^{2n+1}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{iy^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{iy^{2n+1}}{(2n+1)!}$$

$$\cosh(iy) = \cos(y)$$

$$\cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(iy)^{2n}}{(2n)!}$$

27.-

$$27. - f(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

$$\int_0^z e^{-t^2} dt \qquad e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

$$\begin{aligned} e^{-t^2} &= 1 - t^2 + \frac{(-t^2)^2}{2!} + \frac{(-t^2)^3}{3!} + \cdots \\ &= 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} + \cdots \end{aligned}$$

$$\int_0^z \left(1dt - \int_0^z t^2 dt + \frac{1}{2!} \int_0^z t^4 dt - \frac{1}{3!} \int_0^z t^6 dt + \cdots \right)$$

$$t - \frac{t^3}{3} + \frac{t^5}{2!5} - \frac{t^7}{3!7} + \cdots \Big|_0^z$$

$$= \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{2!5} - \frac{z^7}{3!7} + \cdots \right) \qquad R = \infty$$

29.-

$$29. - S(z) = \int_0^z \sin t^2 \, dt$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots$$

$$\sin t^2 = t^2 - \frac{(t^2)^3}{3!} + \frac{(t^2)^5}{5!} - \frac{(t^2)^7}{7!} + \cdots$$
$$= t^2 - \frac{t^6}{3!} + \frac{t^{10}}{5!} - \frac{t^{14}}{7!} + \cdots$$

$$\int_0^z \left(t^2 dt - \frac{1}{3!} \int_0^z t^6 dt + \frac{1}{5!} \int_0^z t^{10} dt - \frac{1}{7!} \int_0^z t^{14} dt + \cdots \right)$$

$$\left. \frac{t^3}{3} - \frac{t^7}{3!7} + \frac{t^{11}}{5!11} - \frac{t^{15}}{7!15} + \cdots \right|_0^z$$

$$= \frac{z^3}{3} - \frac{z^7}{3!7} + \frac{z^{11}}{5!11} - \frac{z^{15}}{7!15} + \cdots \qquad R = \infty$$