Instituto Politécnico Nacional

Escuela Superior de Cómputo

Matematicas Avanzadas para la Ingenieria

SERIE DE LAUREN T

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1.- $\mathbf{e}^z/z^2$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{z^n}{n!}\frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{z^n}{n!z^2} = \sum_{n=0}^{\infty} \frac{z^n z^{-2}}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{z^{n-2}}{n!} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + ...R = \infty$$

3.- cosh 2z/z

$$cosh = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$$

$$cosh\ 2z = \sum_{n=0}^{\infty} \frac{2z^{2n}}{(2n)!}(\frac{1}{z}) = \sum_{n=0}^{\infty} \frac{2^{2n}z^{2n}}{(2n)!z}$$

$$\sum_{n=0}^{\infty} \frac{2^{2n}z^{2n-1}}{(2n)!} = \sum_{n=0}^{\infty} \frac{2^{2n}}{(2n)!}z^{2n-1}$$

5.- 1/z(1+z^2)

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z} si|z| < 1$$

$$= \frac{1}{z}\frac{1}{(1+z^2)} =$$

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}\sum_{n=0}^{\infty} z^{2n} = \frac{1}{1-z^2}$$

$$\sum_{n=0}^{\infty}(-1)^n z^{2n}(\frac{1}{z}) = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{z}$$

$$\sum_{n=0}^{\infty}(-1)^n z^{2n-1} = \frac{1}{z} - z + z^3 - z^5 \cdots R = 1$$

7.- z cos(1/z)

$$cos\ z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} |z| < \infty$$

$$cos\ \frac{1}{z} = \sum_{n=0}^{\infty} (-1)^n \frac{(\frac{1}{z})^{2n}}{(2n)!}$$

$$(cos\ \frac{1}{z}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{z^{2n}(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{z^{2n}(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n z}{z^{2n}(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n z^{1-2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{1-2n} = z - \frac{1}{2z} + \frac{1}{24z^3} \cdots R = \infty$$

9.- 1/z$^6(1+z)^2$

$$9. - \frac{1}{z^6(1+z)^2}$$

$$\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n \qquad z < 1$$

$$\frac{\partial \frac{1}{1+z}}{\partial z} = \frac{-1}{(1+z)^2} = \sum_{n=0}^{\infty} (-1)^n n z^{n-1} \frac{dz}{z}$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} n z^{n-1} (\frac{1}{z^6})$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} n \frac{z^{n-1}}{z^6} = \sum_{n=0}^{\infty} (-1)^{n+1} n z^{n-1-6}$$

$$\sum_{n=0}^{\infty} (-1)^{n+1} n z^{n-7} = \frac{1}{z^6} - \frac{2}{z^5} + \frac{3}{z^4} - \frac{4}{z^3} ... R = 1$$

11.- $\mathbf{1/z^2} + 1, z0 = i$

$$11. - \frac{1}{z^2+1}, \qquad z_0 = i \qquad 0 < |z - z_0| < R$$

$$\frac{1}{(z-i)(z+i)} \qquad \begin{array}{c} w = z - i \\ z = w + i \end{array}$$

$$\frac{1}{(w+i-i)(w+i+i)} = \frac{1}{w(w+2i)}$$

$$\frac{1}{w}\frac{1}{w+2i} = \frac{1}{2i} \cdot \frac{1}{1+\frac{w}{2i}}$$

$$\sum_{n=0}^{\infty}(-1)^n(\frac{w}{2i})^n = \frac{1}{1+\frac{w}{2i}}$$

$$\sum_{n=0}^{\infty}(-1)^n(\frac{w^n}{(2i)^n})(\frac{1}{w})$$

$$\sum_{n=0}^{\infty}(-1)^n(\frac{w^n}{(2i)^n w}) = \sum_{n=0}^{\infty}(-1)^n(\frac{w^{n-1}}{(2i)^{n+1}})$$

$$|w| < 2i$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2i)^{n+1}}(z-1)^{n-1} \qquad |z - i| < 2$$

$$= \sum_{n=0}^{\infty}(\frac{i}{2})^{n+1}(z-1)^{n-1} \qquad R = 2$$

13.- cos z / (z-π)$^3, z0 = \pi$

$$13. - \frac{\cos z}{(z-\pi)^3} \qquad z_0 = \pi \qquad 0 < |z-\pi| < \infty$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} \qquad |z| < \infty$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(z-\pi)^{2n}}{(2n)!} (\frac{1}{(z-\pi)^3})$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (z-\pi)^{2n}}{(2n)!(z-\pi)^3} = \sum_{n=0}^{\infty} \frac{(-1)^n (z-\pi)^{2n-3}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n)!} (z-\pi)^{2n-3} \qquad R = \infty$$

15.- z$^2 - 4/z - 1, 0 < |z-1| < R$

$$15. - \frac{z^2-4}{z-1} \qquad 0 < |z-1| < R$$

$$z + 1 - \frac{3}{z-1}$$

$$(z-1) + 1 - \frac{3}{(z-1-1)}$$

$$(z-1) + 1 - \frac{3}{(z-2)}$$

$$(z-1) + 2 - \frac{3}{(z-1)}$$

17.- 1/(z+i)$^2 - (z+i), z0 = -i$

$$17.-\frac{1}{(z+i)^2-(z+i)}, \qquad z_0=-i$$

$$\frac{1}{(z+i)^2-z-i}=\frac{1}{(z+i)^2}\frac{1}{-i-z} \qquad |z+i|<1$$

$$\sum_{n=0}^{\infty} z^n=\frac{1}{1-z}$$

$$\sum_{n=0}^{\infty}(z+i)^{n+1}=\frac{1}{1-(z+i)}=\frac{1}{1-z-i}$$

$$\sum_{n=0}^{\infty}(z+i)^{n+1}=\frac{1}{-z-i}$$

$$\frac{(z+i)^{n+1}}{(z+i)^2}=z+i^{n+1-2}=z+i^{n-1}$$

$$=-\sum_{n=0}^{\infty}(z+i)^{n-1} \qquad R=1$$

19.- 1/1-z$^2, 0 < z < 1$

$$19.-\frac{1}{1-z^2} \qquad 0\le z\le 1$$

$$\sum_{n=0}^{\infty} z^n=\frac{1}{1-z} \qquad |z|<1$$

$$\sum_{n=0}^{\infty} z^{2n}=\frac{1}{1-z^2} \qquad |z|<1\to\sum_{n=0}^{\infty} z^{2n}$$

21.- 1/1-z^2, $0 < |z-1| < 2$

$$21.\frac{1}{1-z^2} \qquad 0 < |z-1| < 2$$

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$$

$$\frac{1}{1+z}\sum_{n=0}^{\infty} z^n = \frac{1}{1-z} \qquad \frac{1}{(1-z)(1+z)}$$

$$|z-1| < 2$$

$$\sum_{n=0}^{\infty}(z-1)^{n-1} = \frac{1}{1-(z-1)} = \frac{1}{1-z+1}$$
$$= \frac{1}{1-z}$$

$$\sum_{n=0}^{\infty}\frac{(z-1)^{n-1}}{1+z} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2^{n+1}}(z-1)^{n-1}$$

$$\sum_{n=0}^{\infty}\frac{(z-1)^{n-1}}{1+z} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2^{n+1}}(z-1)^{n-1}$$

23.-3z$^2 - 6z + 2/z^3 - 3z^2 + 2z, 1 < |z| < 2$

$$23.\frac{3z^2-6z+2}{z^3-3z^2+2z}, \qquad 1<|z|<2$$

$$= \frac{1}{z} + \frac{1}{-1+z} + \frac{1}{-2+z}$$

$$\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} = \frac{1}{1-z}, \qquad 1<|z|<2$$

$$\frac{1}{1-z} = -(-\sum_{n=0}^{\infty} \frac{1}{z^{n+1}}) = \sum_{n=0}^{\infty} \frac{1}{z^{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} = \frac{-1}{1-z} \Rightarrow \frac{1}{-1+z}$$

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z} = -\sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}} = \frac{1}{2+z}$$

$$= \frac{1}{z} + \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} - \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

25.-1/1-z$^3, z0 = 0, z = z0$

$$25.\frac{1}{1-z^3}, \qquad z_0 = 0 \qquad z = z_0|z| < 1$$

$$\sum_{n=0}^{\infty} z^{3n} = \frac{1}{1-z^3}$$

$$-\frac{1}{z^3(1-\frac{1}{z^3})} = \frac{-1}{z^3}\frac{1}{(1-\frac{1}{z^3})}$$

$$\sum_{n=0}^{\infty} \frac{1}{z^{3n}} = \frac{1}{1-\frac{1}{z^3}}$$

$$-\frac{1}{z^3}\sum_{n=0}^{\infty} \frac{1}{z^{3n}} = -\sum_{n=0}^{\infty} \frac{1}{z^{3n}}(\frac{1}{z^3}) z > 1$$

$$= -\sum_{n=0}^{\infty} \frac{1}{z^{3n+3}}$$

$$\sum_{n=0}^{\infty} z^{3n} \qquad |z|<1, \qquad -\sum_{n=0}^{\infty} \frac{1}{z^{3n+3}} \qquad |z|>1$$

27.-z$^2/1-z^4, z0=0$

$$27.-\frac{z^2}{1-z^4}, \qquad z_0=0$$

$$|z|<1$$

$$z^2\frac{1}{1-z^4}\sum_{n=0}^{\infty}z^{4n}=\frac{1}{1-z^4}$$

$$z^2\sum_{n=0}^{\infty}z^{4n}=\sum_{n=0}^{\infty}z^{4n}z^2=\sum_{n=0}^{\infty}z^{4n+2}$$

$$\frac{z^2}{z^4}\frac{1}{(1-\frac{1}{z^4})}=\frac{1}{z^2}\sum_{n=0}^{\infty}\frac{1}{z^n}=\frac{1}{1-\frac{1}{z}}$$

$$\frac{1}{z^2}\sum_{n=0}^{\infty}\frac{1}{z^{4n}}=\frac{1}{1-\frac{1}{z^n}}=\sum_{n=0}^{\infty}\frac{1}{z^{4n}}\frac{1}{z^2}$$

$$\sum_{n=0}^{\infty}\frac{1}{z^{4n}z^2}=\sum_{n=0}^{\infty}\frac{1}{z^{4n+2}} \qquad |z|>1$$

$$\sum_{n=0}^{\infty}z^{4n+2}, |z|<1, \qquad \sum_{n=0}^{\infty}\frac{1}{z^{4n+2}}, |z|>1$$

29.-1/z, z0=1

$$29. -\frac{1}{z} \qquad z_0 = 1 \qquad 0 < |z-1| < 1$$

$$\begin{aligned}
\sum_{n=0}^{\infty}(z-1)^n &= \frac{1}{1-(z-1)} = \frac{1}{1-z+1} \\
\sum_{n=0}^{\infty}(-1)^n(z-1)^n &= \frac{1}{z} \qquad z < 1 \\
\frac{1}{\frac{1}{z}} = \sum_{n=0}^{\infty}\frac{1}{(z-1)^{n+1}} &= \frac{1}{1-(\frac{1}{z}-1)} \\
= -\frac{1}{\frac{1}{z}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(z-1)^{n+1}} & \qquad |z-1| > 1
\end{aligned}$$

$$\begin{aligned}
\sum_{n=0}^{\infty}(-1)^n(z-1)^n, |z-1| < 1 \\
\sum_{n=0}^{\infty}\frac{(-1)^n}{(z-1)^{n+1}}, |z-1| > 1
\end{aligned}$$

31.-sen z/z+$\pi/2, z0 = -\pi/2$

$$31. - \frac{\sin z}{z + \frac{1}{2}\pi}, \qquad z_0 = -\frac{1}{2}\pi$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$0 < |z + \frac{1}{2}\pi| \leq 1$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(z + \frac{1}{2}\pi)^{2n}}{(2n)!} \left(\frac{1}{z + \frac{1}{2}\pi} \right)$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(z + \frac{1}{2}\pi)^{2n}}{(2n)!(z + \frac{1}{2}\pi)}$$

$$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(z + \frac{1}{2}\pi)^{2n-1}}{(2n)!} \qquad 0 < |z + \frac{1}{2}\pi| \leq 1$$

33.4z-1/z$^4 - 1, z0 = 0$

$$33. -\frac{4z-1}{z^4-1}, \qquad z_0 = 0$$

$$-1(\frac{4z-1}{z^4-1}) = \frac{1-4z}{1-z^4}$$
$$(1-4z)\sum_{n=0}^{\infty} z^{4n}, |z| < 1$$
$$\frac{(1-4z)}{-z^4}(\frac{1}{-\frac{1}{z^4}+1}) = (\frac{1}{1-\frac{1}{z^4}})$$
$$\sum_{n=0}^{\infty}\frac{1}{z^{4n}} = \frac{1}{1-\frac{1}{z^4}}$$
$$\frac{(1-4z)}{-z^4}\sum_{n=0}^{\infty}\frac{1}{z^{4n}}$$

$$1-4z(\frac{1}{1-z^4})\sum_{n=0}^{\infty} z^{4n} = \frac{1}{1-z^4}$$
$$-\sum_{n=0}^{\infty}\frac{(1-4z)}{z^4 z^{4n}} = -\sum_{n=0}^{\infty}\frac{(1-4z)}{z^{4n+4}}$$
$$(\frac{4}{z^3}-\frac{1}{z^4})\sum_{n=0}^{\infty}\frac{1}{z^{4n}}, |z| > 1$$

$$(1-4z)\sum_{n=0}^{\infty} z^{4n}, |z| < 1$$

35.

35. – *Sean* $\sum_{n=-\infty}^{\infty} a_n(z-z_0)^n$ y $\sum_{n=-\infty}^{\infty} c_n(z-z_0)^n$ las 2 series de Laurent de la misma función $f(z)$ en la misma corona. Ambas series se multiplican por $(z-z_0)^{-k-1}$ y se integra a lo largo del círculo con centro en z_0 en el interior de la corona. Como la serie converge uniformemente, es posible integrar término a término. Así se obtiene $2\pi i a_k = 2\pi i c_k$. Por tanto $a_k = c_k$ para todo $k = 0, \pm 1, ...$