

$$\textcircled{1} (2x - 5y + 2) dx + (1 - 6y - 5x) dy = 0$$

$$M = 2x - 5y + 2 \quad N = 1 - 6y - 5x$$

$$\frac{\partial M}{\partial y} = -5$$

$$\frac{\partial N}{\partial x} = -5$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial F}{\partial x} = 2x - 5y + 2$$

$$\partial F = (2x - 5y + 2) dx$$

$$\int \partial F = \int (2x - 5y + 2) dx$$

$$F = \frac{\cancel{2}x^2}{\cancel{2}} - 5yx + 2x + f(y)$$

$$F = x^2 - 5yx + 2x + f(y)$$

$$\frac{\partial F}{\partial y} = (-5x + f'(y))$$

$$\cancel{-5x} + f'(y) = 1 - 6y - \cancel{5x}$$

$$f'(y) = 1 - 6y$$

$$\int f'(y) = \int (1 - 6y)$$

$$f(y) = y - \frac{6y^2}{2}$$

$$f(y) = y - 3y^2$$

$$f = x^2 - 5yx + 2x + y - 3y^2$$

①

$$x^2 + 2x - 3y^2 + y - 5yx = c$$

$$\left(1 - \frac{y}{x^2} e^{y/x}\right) dx + \left(1 + \frac{1}{x} e^{y/x}\right) dy = 0$$

$$M = 1 - \frac{y}{x^2} e^{y/x}$$

$$N = 1 + \frac{1}{x} e^{y/x}$$

$$\frac{\partial M}{\partial y} = 0 - \frac{e^{y/x} y + e^{y/x} x}{x^3} = -\frac{e^{y/x} y + e^{y/x} x}{x^3}$$

$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial y}$$

$$\frac{\partial N}{\partial x} = 0 - \frac{e^{y/x} y - e^{y/x} x}{x^3} = -\frac{e^{y/x} y + e^{y/x} x}{x^3}$$

$$\frac{\partial F}{\partial x} = 1 - \frac{y}{x} e^{y/x} \rightarrow \partial F = 1 - \frac{y}{x} e^{y/x} dx \rightarrow \int \partial F = \int 1 - \frac{y}{x} e^{y/x}$$

$$F = x - (-e^{y/x}) \rightarrow F = x + e^{y/x} + f(y)$$

$$\frac{\partial F}{\partial y} = \frac{e^{y/x}}{x} + f'(y)$$

$$f'(y) = 1$$

$$\int f'(y) = \int 1$$

$$f(y) = y$$

~~$$\frac{e^{y/x}}{x} + f'(y) = 1 + \frac{1}{x} e^{y/x}$$~~

$$F = x + e^{y/x} + y$$

(7)

$$e^{y/x} + y + x = C$$

$$(13) \left(\operatorname{sen} y + \frac{y}{x^2} \operatorname{sen} \frac{y}{x} \right) dx + \left(x \cos y - \frac{1}{x} \operatorname{sen} \frac{y}{x} \right) dy = 0$$

$$M = \operatorname{sen} y + \frac{y}{x^2} \operatorname{sen} \frac{y}{x} \quad N = x \cos y - \frac{1}{x} \operatorname{sen} \frac{y}{x}$$

$$\frac{\partial M}{\partial y} = \cos y + \frac{x \operatorname{sen} \frac{y}{x} + y \cos \frac{y}{x}}{x^3}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = \cos y + \frac{x \operatorname{sen} \frac{y}{x} + y \cos \frac{y}{x}}{x^3}$$

$$\frac{\partial F}{\partial x} = \operatorname{sen} y + \frac{y}{x^2} \operatorname{sen} \frac{y}{x} \Rightarrow dF = \operatorname{sen} y + \frac{y}{x^2} \operatorname{sen} \frac{y}{x} dx$$

$$\int dF = \int \operatorname{sen} y + \frac{y}{x^2} \operatorname{sen} \frac{y}{x} dx \Rightarrow F = \cos \frac{y}{x} + x \operatorname{sen} y + f(y)$$

$$\frac{\partial F}{\partial y} = \frac{-\operatorname{sen} \frac{y}{x}}{x} + x \cos y + f'(y)$$

$$\cancel{\frac{-\operatorname{sen} \frac{y}{x}}{x}} + \cancel{x \cos y} + f'(y) = \cancel{x \cos y} - \cancel{\frac{1}{x} \operatorname{sen} \frac{y}{x}}$$

$$f'(y) = 0 \quad f(y) = C$$

$$F = \cos \frac{y}{x} + x \operatorname{sen} y + C = C$$

$x \operatorname{sen} y + \cos \frac{y}{x} = C$

(13)

$$(2x \operatorname{sen} y + y e^{xy}) dx + (x \cos y + e^{xy}) dy = 0 \quad y(1) = 1$$

$$M = 2x \operatorname{sen} y + y e^{xy} \quad N = x \cos y + e^{xy}$$

$$\frac{\partial M}{\partial y} = 2x \cos y + e^{xy} + e^{xy} xy \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = \cos y + e^{xy} (y + x)$$

No es exacta

$$(2) \quad \left(\frac{-1-y^2}{x^2} - 1 \right) dx + \frac{2y}{x} dy = 0$$

$$M = \frac{-1-y^2}{x^2} - 1 \quad N = \frac{2y}{x}$$

$$y'(1) = 1$$

$$\frac{1}{x} = x^{-1} \Rightarrow -x^{-2} = -\frac{1}{x^2}$$

$$\frac{\partial M}{\partial y} = \frac{-2y}{x^2} \quad \frac{\partial N}{\partial x} = -\frac{2y}{x^2} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial F}{\partial y} = \frac{2y}{x} \rightarrow dF = \frac{2y}{x} dy \rightarrow \int dF = \int \frac{2y}{x} dy$$

$$F = \frac{1}{x} \frac{2y^2}{2} \Rightarrow F = \frac{y^2}{x} + f(x)$$

$$\frac{\partial F}{\partial x} = -\frac{y^2}{x^2} + f'(x) \rightarrow -\frac{y^2}{x^2} + f'(x) = \frac{-1-y^2}{x^2} - 1$$

$$f'(x) = -\frac{1}{x^2} - 1 \rightarrow \int f'(x) = \int -\frac{1}{x^2} - 1 \rightarrow f(x) = \frac{1}{x} - x + C$$

$$\frac{y^2}{x} + \frac{1}{x} - x + C = C$$

$$C = 4$$

$$\frac{2^2}{1} + \frac{1}{1} - 1 = C \rightarrow 4 + 1 - 1 = C$$

$$\frac{y^2}{x} + \frac{1}{x} - x = 4 \rightarrow \frac{y^2}{x} = 4 + x - \frac{1}{x}$$

$$y^2 = \left(4 + x - \frac{1}{x} \right) x$$

$$y^2 = 4x + x^2 - \frac{x}{x}$$

$$y^2 - x^2 + 1 = 4x$$

$$\rightarrow (y - \frac{1}{y})dx + (x + \frac{x}{y^2})dy = 0 \quad y^{-1} = -y^{-2} = -\frac{1}{y^2}$$

$$M = y - \frac{1}{y} \quad N = x + \frac{x}{y^2}$$

$$\frac{\partial M}{\partial y} = 1 - (-\frac{1}{y^2}) = 1 + \frac{1}{y^2} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$y^{-2} = \frac{y^{-1}}{-2}$$

$$\frac{\partial N}{\partial x} = x + \frac{x}{y^2} = 1 + \frac{1}{y^2}$$

$$\frac{\partial F}{\partial x} = y - \frac{1}{y} \rightarrow \partial F = y - \frac{1}{y} dx \rightarrow \int \partial F = \int y - \frac{1}{y} dx$$

$$F = yx - \frac{x}{y} + f(y)$$

$$\frac{\partial F}{\partial y} = x + \frac{x}{y^2} + f'(y) \rightarrow \cancel{x + \frac{x}{y^2}} + f'(y) = \cancel{x + \frac{x}{y^2}}$$

$$f'(y) = 0 \rightarrow \int f'(y) = \int 0 \rightarrow f = 0$$

$$xy - \frac{x}{y} - 0 = C$$

$$\boxed{xy - \frac{x}{y} = C} \quad (31)$$