Escuela Superior de Cómputo

Ingeniería en Sistemas Computacionales

Matemáticas avanzadas para la Ingeniería

Integral definida mediante una trayectoria

Grupo: 4CV2
Profesor: Zárate Cárdenas Alejandro

Equipo:
Arellano Millán Gabriel
Gómez Tovar Yoshua Oziel
Herrera Tovar Karla Elena
Vazquez Blancas César Said
Zarco Sosa Kevin

29 de Mayo de 2024

1 Ejercicio 1

$$z = 0, \quad y \quad z = 1 + 2i$$

$$Z = z1 + (z2 - z1)t$$

Evaluamos

$$z = 0 + (1 + 2i - 0)t$$

$$z = 1 + 2i$$

$$0 <= t <= 1$$

2 Ejercicio 3

$$z = 4 + 2i \quad z = 3 + 5i$$

$$Z = z1 + (z2 - z1)t$$

Evaluamos

$$z = 4 + 2i + (3 + 5i - 4 - 2i)t$$

$$z = 4 + 2i + (-1 + 3i)t$$

$$0 <= t <= 1$$

3 Ejercicio 5

$$z1 = -4i \quad z2 = -7 + 38i$$

$$Z = z1 + (z2 - z1)t$$

Evaluamos

$$z = -4i + (-7 + 38i + 4i)t$$

$$z = -4i + (-7 + 42i)t$$

$$z = -4i + \frac{(-7 + 42i)}{7}t$$

$$z = -4i + (-1 + 6i)t$$

$$0 <= t <= 7$$

4 Ejercicio 7

$$(1 + 2i)t \quad 0 <= t <= 3$$

Segmento de recta de 0 a 3

5 Ejercicio 9

$$(1 - i - 2e^i t)$$

Semicirculo inferior (radio 2, centro 1-i)

6 Ejercicio11

$$t + 3(t^2)i \quad -1 <= t <= 2$$

Parabola $y = 3x^2$ de (-1,3) a (2,12)

7 Ejercicio 13

$$cost + 2isent \quad -\pi <= t <= \pi$$

Elipse $4x^2 + y^2 = 4$

8 Ejercicio 15

$$|z - 3 + 4i| = 4 \qquad z = z0 + re^{it}$$

$$\begin{aligned} z - 3 + 4i = z - (3 - 4i) \\ z_0 = 3 - 4i \\ r = 4 \\ z = 3 - 4i + 4e^{it} \quad \text{para} \quad 0 \leq t \leq 2\pi \end{aligned}$$

9 Ejercicio 17

$y = \frac{1}{x}$ desde (1,1) hasta $(3, \frac{1}{3})$
$x(t) = t$ para 1¡=t¡=3

$$\begin{aligned} y = \frac{1}{x} = y(t) = \frac{1}{x(t)} = \frac{1}{t} \\ x(t) = t \qquad y(t) = \frac{i}{t} \\ z(t) = x(t) + iy(t) = t + \frac{i}{t} \\ z(t) = t + \frac{i}{t} \qquad 1 <= t <= 3 \end{aligned}$$

10 Ejercicio 19

$$x^2 + 4y^2 = 4 \qquad x(t) = t$$

$$z = z0 + re^{it}$$
$$z = x^2 + 4y^2 + 4re^{it}$$
$$\frac{x^2}{2} + \frac{y^2}{1} = 1 \qquad x = acos(t) \quad y = bsen(t)$$
$$x = 2cos(t) \quad y = sen(t)$$
$$2cos(t) + isen(t) \qquad 0 <= t <= 2\pi$$

11 Ejercicio 21

$f(z) = az + b$,C es l segmento de recta desde -1-i hasta 1+i

$$z = z1 + (z2 - z1)t$$

$$z = -1 - i + (1 + i + 1 + i)t$$
$$z = -1 - i + (2 + 2i)t$$
$$z = -1 - i + (\frac{2 + 2i}{2})t$$
$$z = -1 - i + (1 + i)t \qquad 0 <= t <= 2$$
$$dz = (1 + i)dt$$
$$z = -1 - i + t + ti$$
$$\int f(Z)dz = \int_0^2 (a(-1 - i + t + ti) + b)(1 + i)dt$$
$$= a(1 + i)\int_0^2 -1 - i + (1 + i)t + bdt$$
$$= a + ai(-t - ti\frac{t^2}{2} + \frac{it^2}{2} + bt)|_0^2$$
$$= a + ai(\frac{1}{2}(t^2 - 2t + 2bt) + \frac{i}{2}(t^2 - 2t))$$
$$= abt + i(at^2 - 2at + abt)|_0 2$$
$$= ab(2) + i(a(2)^2 - 2a(2) + ab(2) - ab(0) + i(a(0)^2 - 2a(0) + ab(0)))$$
$$= 2ab + i(4a - 4a + 2ab)$$
$$= 2ab + 2abi$$
$$= 2ab(1 + i)$$

12 Ejercicio 23

$f(z) = z^3$C es la semicircunferencia —z—=2 de -2i a 2i en el semiplano derecho

$$z = z0 + re^{it} = z = 2e^{i(\frac{3\pi}{2}+t)}$$
$$dz = 2e^{i(\frac{3\pi}{2}+t)}idt$$
$$\int_0^\pi (2e^{i(\frac{3\pi}{2}+t)})2e^{i(\frac{3\pi}{2}+t)}idt$$
$$4i\int_0^\pi e^{i(\frac{3\pi}{2}+t)})e^{i(\frac{3\pi}{2}+t)}$$
$$4i\int cos2udu + \int sen(2u)du$$
$$4i(\frac{1}{2}sen(2(\frac{3\pi}{2}+x)) - i\frac{1}{2}cos(2(\frac{3\pi}{2}+x)))$$
$$= 2cos(2x+3\pi) + 2isen(2x+3\pi)|_0^\pi$$
$$= 2cos(5\pi) + 2isen(5\pi) - (2cos(3\pi) + 2isen(3\pi))$$
$$= 0$$

13 Ejercicio 25

$f(Z) = 2z^4 - z^4$ C es la circunferencia unitaria

$$z(t) = e^{it} \qquad 2\pi <= t <= 0$$

$$dz = e^{it} idt$$

$$\int_{2\pi}^{0} 2(e^{it}) - (e^{it})^{-}4 = \int_{2\pi}^{0} 2e^{4it} - e^{-4it}$$

$$2\int_{2\pi}^{0} e^{4it}dt - \int_{2\pi}^{0} e^{-4it}dt$$

$$u = 4it \qquad w = -4it$$

$$du = 4idt \qquad dw = -4idt$$

$$\frac{du}{4i} = dt \qquad \frac{dw}{-4i} = dt$$

$$2\int_{2\pi}^{0} \frac{e^u du}{4i} - \int_{2\pi}^{0} \frac{e^w dw}{-4i}$$

$$\frac{2}{4i}\int_{2\pi}^{0} e^u du + \frac{1}{4i}\int_{2\pi} 0e^w dw$$

$$\frac{2e^{4it}}{4i} + \frac{e^{-4it}}{4i}|_{2\pi}^{0}$$

$$\frac{2e^{4i0}}{4i} + \frac{e^{-4i0}}{4i} - (\frac{2e^{4i2\pi}}{4i} + \frac{e^{-4i2\pi}}{4i})$$

$$\frac{2}{4i} + \frac{1}{4i} - \frac{2}{4i} - \frac{1}{4i} = 0$$

14 Ejercicio 27

$$f(z) = lmz \qquad |z| = r$$

$$z = re^{it} \qquad 2\pi <= 0 <= 0$$

$$dz = e^{it}i$$

$$lm(z) = rcost + risent$$

$$lm(Z) = risent$$

$$\int_{2\pi}^{0} risent(re^{it}i)dt$$

$$\int_{2\pi}^{0} -r^2 sente^{it}dt$$

$$r^2 \int_{2\pi}^{0} sente^{it}dt$$

$$-r^2(\frac{1}{2}((-sen^2(t)) - 2cos^2(t)) + \frac{i}{4}(sen(2t) + 2t - 4costsent))|_{2\pi}^{0}$$

$$-r^2(\frac{1}{2}(0+2)) + \frac{i}{4}(0 + 4\pi - 0)$$

$$-r^2(1 + \frac{4\pi i}{4} - 1) = -4r^2(\frac{\pi i}{4})$$

$$= -\pi r^2 i$$

15 Ejercico 29

$$f(z) = (z-1)^{-}1 + 2(z-1)^{-}2$$

$$|Z-1| = 4 \qquad 0 <= t <= 2\pi$$

$$z = z0 + re^{it} \qquad z = 1 + 4e^{it}$$

$$dz = e^{it}i$$

$$\int_0^{2\pi} (1 + 4e^{it} - 1)^-1 + 2(1 + 4e^{it} - 1)^-24e^{it}i$$

$$\int_0^{2\pi} (4e^{it})^-1 + 2(4e^{it})^-24e^{it}i$$

$$\int_0^{2\pi} \frac{4e^{it}i}{4e^{it}} + \frac{4e^{it}i}{32e^{it}}dt$$

$$\int_0^{2\pi} i + \frac{i}{8e^{it}} = \int_0^{2\pi} i(1 + \frac{1}{8e^{it}})$$

$$\int_0^{2\pi} i(1 + \frac{1}{8e^{it}}) = i(\int_0^{2\pi} 1dt + \int_0^{2\pi} \frac{1}{8e^{it}dt})$$

$$i(t + \frac{1}{8}\int_0^{2\pi} \frac{1}{e^{it}}dt$$

$$u = it \qquad du = idt \qquad \frac{du}{i} = dt$$

$$\frac{1}{8}\int_0^{2\pi} \frac{du}{e^u i} = \frac{1}{8i}\int_0^{\pi} \frac{du}{eu}$$

$$w = e^u \qquad dw = e^u du \qquad du = \frac{dw}{eu}$$

$$\frac{1}{8i}\int_0^{\pi} = \frac{1}{8i}(-\frac{1}{w})$$

$$\frac{1}{8i}(-\frac{1}{e^u}) = -\frac{1}{8ie^{it}}$$

$$it - \frac{i}{8ie^{it}}|_0^{2\pi}$$

$$it - \frac{1}{8e^{it}}|_0^{\Pi}$$

$$i(0) - \frac{1}{8e^0} - i(2\pi) + \frac{1}{8e^{2\pi}}$$

$$0 - \frac{1}{8} - 2i\pi + \frac{1}{8}$$

$$= -2\pi i$$

16 Ejercicio 31

$f(z) = e^{2z}$ C es el segmento vertical de πi hasta $2\pi i$

$$z = yi \qquad z = idt$$

$$z(t) = ti \qquad \pi i <= t <= 2\pi i$$

$$\int_{\pi i}^{2\pi i} e^{2ti} i = i \int_{\pi i}^{2\pi i} e^{2ti}$$

$$u = 2ti \qquad du = 2idt \qquad dt = \frac{du}{2i}$$

$$i \int_{\pi i}^{2\pi i} \frac{e^u du}{2i} = \frac{1}{2} \int_{\pi i}^{2\pi i} e^u du = \frac{1}{2} e^u |_{\pi i}^{2\pi i}$$

$$\frac{e^{2ti}}{2} |_{\pi i}^{2\pi i}$$

$$\frac{e^{2(2\pi i)}}{2} - \frac{e^{2\pi i}}{2} = \frac{e^{4\pi i}}{2} - \frac{e^{2\pi i}}{2}$$

$$\frac{1}{2} - \frac{1}{2} = 0$$

17 Ejercicio 33

f(z)=cosh3z desde $\pi \frac{i}{b}$ hasta 0

$$z(t) = (1-t)\frac{\pi i}{6} \qquad 0 <= t <= 1$$

$$dz = \frac{\pi i}{6} dt$$

$$\int_0^1 (cosh(\frac{3\pi i}{6} - \frac{3\pi i t}{6}))\frac{\pi i}{6} dt$$

$$\int_0^1 cosh(\frac{\pi i}{2} - \frac{\pi i t}{2})\frac{\pi i}{6} dt$$

$$\frac{\pi i}{6} \int_0^1 cosh(\frac{\pi i}{2} - \frac{\pi i t}{2}) dt$$

$$u = \frac{\pi i}{2} - \frac{\pi i t}{2} \qquad dv = -\frac{\pi i}{2} dt$$

$$-\frac{2\pi}{6\pi} \int_0^1 cosh(u) du$$

$$-\frac{1}{3} senh(u)|_0^1 = -\frac{1}{3} senh(\frac{\pi i}{2} - \frac{\pi i t}{2})$$

$$-\frac{1}{3} senh(\frac{\pi i}{2}) + \frac{1}{3} senh(0)$$

$$= -\frac{i}{3}$$

18 Ejercicio 35

$f(z) = Re(z^2)$ C es el cuadrado del problema 0,1,1+i,i

$$z = z1 + (z2 - z1)t$$

$$z = 0 + (1 - 0)t \qquad z1(t) = t \qquad 0 <= t <= 1$$

$$z2 = 1 + (1 + i - 1)t \qquad 1 + (i)t = z2(t) = 1 + it \qquad 0 <= t <= 1$$

$$z3 = 1 + i + (i - 1 - i) \qquad z3(t) = 1 + i - t \qquad 0 <= t <= 1$$

$$z4 = i + (0 - i)t \qquad z4(t) = i - it \qquad 0 <= t <= 1$$

$$z(t) = t \qquad dz = dt$$

$$z(t) = 1 + it \qquad dz = idt$$

$$z(t) = 1 + i \qquad dz = -dt$$

$$z(t) = i - it \qquad dz = -idt$$

$$Re(t)^2 = t^2$$

$$Re(1 + it)^2 = 1 + 2it - t^2 = 1 - t^2$$

$$Re(1 + i + -t)^2 = -2t - 2ti + t^2 + 2i = -2t + t^2$$

$$Re(i - it)^2 = -t^2 + 2t - 1 = -t^2 + 2t - 1$$

$$\int_0^1 t^2 dt + \int_0^1 1 - t^2 idt + \int_0^1 t^2 - 2t(-dt) + \int_0^1 -t^2 + 2t - 1(-dt)$$

resolvemos las primeras 2 integrales

$$\int_0^1 t^2 dt = \frac{t^3}{3}|_0^1 = \frac{0^3}{3} - \frac{1}{3} = -\frac{1}{3}$$

$$\int_0^1 1 - t^2 idt = \int_0^1 idt + \int_0^1 -t^2 idt$$

$$= i\int_0^1 dt - i\int_0^1 t^2 dt = it - \frac{it^3}{3}|_0^1$$

$$0 - 0 - i + \frac{i}{3} = -i + \frac{i}{3} = -\frac{2i}{3}$$

resolvemos las ultimas 2 integrales

$$\int_0^1 t^2 - 2t(-dt) = -\int_0^1 t^2 dt + \int_0^1 2tdt$$

$$= -\frac{t^3}{3} + \frac{2t^2}{2}|_0^1 = -0 - 0 + \frac{1}{3} - 1^2$$

$$= \frac{1}{3} - 1 = -\frac{2}{3}$$

$$\int_0^1 -t^2 + 2t - 1(-idt) =$$

$$9 \quad i\int_0^1 t^2 dt - i\int_0^1 2tdt + i\int_0^1 dt =$$

$$\frac{it^3}{3} - it^2 + it|_0^1 = 0 - 0 + 0$$

$$-\frac{i}{3} + i - i = \frac{-i}{3}$$

Sumamos el resultado de todas las integrales

$$-\frac{1}{3}-\frac{2}{3}-\frac{i}{3}-\frac{2i}{3}=-\frac{3}{3}-\frac{3i}{3}$$
$$=-1-i$$

19 Ejercicio 37

$\overline{z}$,Cla parabola $y=x^2$ desde 0 hasta 1+i

$$z(t)=t+it^2 \qquad 0<=t<=1$$
$$dz=1+2tidt \qquad \overline{z}=t-it^2$$
$$\int_0^1 t-it^2(1+2ti)dt$$
$$\int_0^1 t+2t^2i-it^2+2t^3dt$$
$$\int_0^1 tdt+i\int_0^1 t^2dt+2\int_0^1 t^3dt\frac{t^2}{2}|_0^1+i\frac{t^3}{3}|_0^1+\frac{t^4}{2}|_0^1$$
$$\frac{1}{2}-\frac{0}{2}+\frac{i}{3}-\frac{0}{3}+\frac{1}{2}-\frac{0}{2}$$
$$\frac{1}{2}+\frac{1}{2}+\frac{i}{3}=1+\frac{i}{3}$$

20 Ejercicio 39

$f(z)=sen^2z$, C la semicircunferencia $|z|=\pi$ desde $-\pi i$ hasta πi en el semiplano derecho

$$z(t) = \pi e^{\frac{3\pi}{2}+t} \qquad 0 <= t <= \pi \qquad dz = \pi e^{i(\frac{3\pi}{2}+t)} idt$$

$$\int_0^\pi sent^2(\pi e^{i(\frac{3\pi}{2}+t)})\pi e^{i\frac{3\pi}{2}+t} idt$$

$$u = \pi e^{i(\frac{3\pi}{2}+t)} \qquad \frac{du}{e^{i(\frac{3\pi}{2}+t)}} = dt$$

$$\int_0^\pi \frac{sen^2(u)du\pi e^{i(\frac{3\pi}{2}+t)}i}{e^{i(\frac{3\pi}{2}+t)}i}dt$$

$$\int_0^\pi sen^2(2)du$$

$$\frac{e^{i(\frac{3\pi}{2}+t)}}{2} - \frac{sen(2\pi e^{i(\frac{3\pi}{2}+t)})}{4}|_0^\pi$$

$$\frac{\pi i}{2} - \frac{sen(2\pi i)}{4} + \frac{\pi i}{2} - \frac{sen(2\pi i)}{4}$$

$$\frac{2\pi i}{2} - \frac{sen(2\pi i)}{4}$$

$$(\pi i - \frac{1}{2}sen(2\pi i))$$

$$(\pi - \frac{1}{2}senh(2\pi))i$$

21 Ejercicio 41

Evaluar $\int_C ln(z^2)dz$desde 0 hasta 2+4i a lo largo de a) del segmento de recta,b)del ejex hasta 2 y luego verticalmente hasta 2+4i,c) de la parabola $y = x^2$

A

$$z = 0 + (2 + 4i - 0)t \qquad dz = 2 + 4idt$$

$$z = 2t + 4ti$$

$$lm(z^2) = 16t^2$$

$$\int_0^1 16t^2(2 + 4i)dt$$

$$\int_0^1 32t^2dt + \int_0^1 64t^2idt$$

$$32\int_0^1 t^2dt + 64\int_0^1 t^2dt$$

$$\frac{32t^3}{3} + \frac{64it^3}{3}|_0^1$$

$$\frac{32}{3} + \frac{64i}{3} + 0 + 0$$

$$= \frac{32}{3} + 64\frac{i}{3}$$

B

$$z1 = 2t \qquad \int_C lm(z) = 0$$

$$dz = 2dt$$

$$z2 = 2 + (2 + 4i - 2)t \qquad z2 = 2 + 4it \qquad 0 <= t <= 0$$

$$dz = 4idt$$

$$lm(2 + 4it)^2 = 16t$$

$$\int_0^1 16t(4i)dt = \int_0^1 64itdt$$

$$64i \int_0^1 tdt = \frac{64it^2}{2}|_0^1$$

$$\frac{64(1)i^2}{2} + \frac{64(0)i}{2} = \frac{64i}{2} = 32i$$

C

$$z = t + it^2 \qquad 0 <= t <= 1$$

$$dz = 1 + 2itdt$$

$$lm(z^2) = 2x^3$$

$$\int_0^2 2t^3(1 + 2it)dt$$

$$\int_0^2 2t^3dt + i \int_0^1 4t^4dt$$

$$\frac{2t^4}{4} + \frac{4t^5i}{5}|_0^2$$

$$\frac{2(2)^4}{4} + \frac{4(2)^5i}{5} - (\frac{2(0)^4}{4} + \frac{4(0)^5i}{5})$$

$$\frac{32}{4} + \frac{128i}{5} = 8 + 128\frac{i}{5}$$

22 Ejercicio 43

Evaluar $\int_C |z|dz$ desde A. hasta z=-1 Hasta B. z=i a lo largo de a) el segmento de recta AB,la semicircunferencia del plano derecho,c) la semicircunferencia del plano izquierdo

A

$$z = -1 + (i+1)t \qquad , 0 <= t <= 1$$
$$z = -1 + ti + t \qquad dz = 1 + idt$$
$$|z| = \sqrt{(-1+t^2)t(t)^2} = \sqrt{1-2t+2t^2}$$
$$\int_0^1 (\sqrt{1-2t+2t^2})(1+i)dt$$
$$1 + i\int_0^1 (\sqrt{1-2t+2t^2})dt$$
$$u = 1-2t+2t^2 \qquad du = -2+4tdt \qquad \frac{du}{-2+4t} = dt$$
$$1 + i\int_0^1 \frac{\sqrt{u}du}{-2+4t}$$
$$\frac{\sqrt{2}}{8}(ln|-1+2t+\sqrt{2}\sqrt{1-2t+2t2}| - \sqrt{2}\sqrt{1}-2t+2t+2\sqrt{2}+\sqrt{-2t+2t})|_0^1$$
$$= 0.876558(1+i) =$$
$$0.876 + 0.876i$$

B

$$|z| = 1 \qquad 0 <= t <= 2\pi$$
$$z = e^{it} \qquad dz = e^{it}dt \qquad |z| = e^{it}$$
$$\int_0^{2\pi} e^{it}eitdt = 2\int_0^{2\pi} e^{it}dt$$
$$u = it \qquad du = idt \qquad \frac{du}{i} = dt$$
$$\frac{2i}{i}\int_0^{2\pi} e^u du = 2e^{it}|_0^{2\pi}$$
$$2e^{2\pi i} - 2e^0 = 2 - 2 = 0$$

La otra circunferencia es la misma por lo tanto para C es=0

23 Ejercicio 45

comprobar la expresion b para Kl,K,L=3z-z2,donde C es la mitad superior de la circunferencia unitaria desde 1 hasta-1

$$\int_C (k1f1(Z) + k2f2(z))dz$$
$$= \int_C k1f1(Z)dz + \int_C k2f2(z)dz$$
$$= k1\int_C f1(Z)dz + k2\int_C f2(z)dz$$

$$k1 + k2 = 3z - z^2 \qquad |z| = 1$$
$$z = e^{i(\theta))} \qquad [0, \pi]$$
$$3z - z^2 = 3e^{i\theta} - (e^{i\theta})^2 = 3i^{i\theta} - e^{2i\theta}$$

No afecta la linealidad de la integral

24 Ejercicio 47

Comprobar la expresion 8 para f(z)=z^2 donde C es el segmento de recta desde -1-i hasta 1+i

$$z^2 = 8t^2i - 8ti + 2i$$
$$dz = 2 + 2idt$$
$$= \int_0^1 8t^2i - 8ti + 2i(2 + 2i)dt$$
$$= \int_0^1 -4dt + \int_0^1 8t^2idt - \int_0^1 8tidt + \int_0^1 4idt$$
$$= -4t + \frac{8t^3i}{3} - 4t^2i + 4ti|_0^1$$
$$= 4 + \frac{8i}{3} - 4i + 4i - 0 + 0 - 0 + 0$$
$$= -4 + \frac{8i}{3}$$
$$= -4t + \frac{8t^3i}{3} - 4t^2i + 4ti|_1^0$$
$$= -0 + 0 - 0 + 04 + \frac{8i}{3} - 4i + 4i$$
$$= -4 + \frac{8i}{3} = -4 + \frac{8i}{3}$$

25 Ejrcicio 49

$$= \int_C e^z dz \quad 0 \quad a \quad 3+4i$$

$$z = 0 + (3+4i-0)t \qquad z = 3t+4+i \qquad 0 <= t <= 1$$

$$L = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5 = L$$

$$|e^z| = |e^{3+4ti}| = e^3 = M$$

$$|f(z)| <= M \qquad |\oint_C f(Z)dz| <= M$$

$$= 5e^3$$