INSTITUTO POLITÉCNICO NACIONAL
Escuela Superior de Cómputo
Academia de Formación Básica

ESCOM

2º examen parcial de Álgebra Lineal

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Resuelve los siguientes ejercicios de manera clara y ordenada, recuerda que se califica el procedimiento. Cada ejercicio vale 2 puntos.

1. En el espacio M de matrices 2x2 se tiene el conjunto de matrices con la forma $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$. Demuestra que es un subespacio vectorial de M.

2. Comprueba si el conjunto de polinomios $\boldsymbol{B} = \{t^2 + 2t + 1,\ t + 1,\ 1\}$ es una base del espacio vectorial $\mathbf{P_2}$. Escribe el polinomio $(2t + 1)^2$ como combinación lineal de los vectores en **B**. θ

3. Dado el producto interno $\langle u|v\rangle = u_1v_1 + 2u_2v_2 + u_3v_3$, determina la distancia entre los vectores (1,2,-1) y (2,4,3), así como la norma de v. θ

4. Utiliza el método de Gram-Schmidt para determinar la base ortonormal asociada a la base $B = \{(1,2,3), (2,-1,0), (0,1,-1)\}$.

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a)

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 0 & 2c \end{pmatrix} \checkmark = \begin{pmatrix} a & b \\ 0 & 2c \end{pmatrix}$$

$$d\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} da & db \\ 0 & dc \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \checkmark$$

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las 2 se cumplen $\therefore$ $\boxed{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \text{ es subespacio de } M}$

2 $B = \{t^2 + 2t + 1,\ t+1,\ 1\}$ $\quad (2t+1)^2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{matrix} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix} \qquad |B| \neq 0$$

$$\begin{array}{r} 2t+1 \\ 2t+1 \\ \hline 2t+1 \\ 4t^2+2t \\ \hline 4t^2+4t+1 \end{array}$$

$|B| = 1 + 0 + 0 - (0 + 0 + 0)$ $\qquad |B| = 1 \neq 0$ $\checkmark$

$|B| = 1$

$\boxed{\therefore\ B \text{ es base}}$ 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 2 & 1 & 0 & 4 \\ 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow[e_3 - e_1]{e_2 - 2e_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 1 & 1 & -3 \end{array}\right] \xrightarrow{e_3 - e_2}$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$4t^2+4t+1 = 4(t^2+2t+1)-4(t+1)+1(1)$

$4t^2+4t+1 = 4t^2+8t+\cancel{4}-\cancel{4t}-\cancel{4}+1$

$4t^2+4t+1 = 4t^2+4t+1$

$\vec{w} = 4t^2+4t+1 \qquad \vec{v} = t^2+2t+1 \quad \vec{u} = t+1 \quad \vec{x} = 1$

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$\boxed{\vec{w} = 4\vec{v} - 4\vec{u} + 1\vec{x} \quad \text{combinación lineal: } 4, -4, 1}$

3 $\langle u|v\rangle = u_1v_1 + 2u_2v_2 + u_3v_3 \quad u=(1,2,-1) \quad v=(2,4,3)$

$u-v = (1,2,-1)-(2,4,3) = (1,2,-1)+(-2,-4,-3)$

$= (1-2, 2-4, -1, -3) = (-1,-2,-4)$

$\|u-v\| = \sqrt{\underset{u_1}{(-1}, \underset{u_2}{-2}, \underset{u_3}{-4)}\underset{v_1}{(-1}, \underset{v_2}{-2}, \underset{v_3}{-4)}} = \sqrt{(-1)(-1)+2(-2)(-2)+(-4)(-4)} = \sqrt{1+2(4)+16} = \sqrt{1+8+16} = \sqrt{25}$

$\underline{\underline{d=5}}$

$$= \langle v, v \rangle = \langle (2,4,3), (2,4,3) \rangle$$

$u_1 \quad u_2 \quad u_3 \qquad v_1 \quad v_2 \quad v_3$

$$= \sqrt{(2)(2) + 2(4)(4) + 3(3)} = \sqrt{4 + 32 + 9} = \sqrt{45}$$

$$\|v\| = \sqrt{45}$$

$u_1 \quad u_2 \quad u_3 \qquad v_1 \quad v_2 \quad v_3$ 6.7

$$\cos = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\langle (1,2,-1),(2,4,3) \rangle}{\sqrt{45}\ \sqrt{10}}$$

$$\sqrt{(1)(1) + 2(2)(2) + (-1)(-1)} = \sqrt{1 + 8 + 1} = \sqrt{10}$$

$$\cos\theta = \frac{(1)(2) + 2(2)(4) + (-1)(3)}{\sqrt{45}\ \sqrt{10}} = \frac{2 + 16 - 3}{\sqrt{45}\ \sqrt{10}} = \frac{15}{\sqrt{45}\sqrt{10}}$$

$$\theta = \cos^{-1}\left(\frac{15}{\sqrt{45}\sqrt{10}}\right) \approx 45^\circ$$

$$\boxed{\begin{array}{l} d = 5 \\ \|v\| = \sqrt{45} \\ \theta = 45^\circ \end{array}}$$

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$\{(1,2,3),(2,-1,0),(0,1,-1)\}$
$\quad u \qquad\qquad v \qquad\qquad w$

$(1,2,3)(2,-1,0) = (2-2+0 = 0$

$\vec{w_1} = (1,2,3)$ $\quad \dfrac{2-3}{1+4+9} = \dfrac{-1}{14} \quad \dfrac{0-1}{4+1} = \dfrac{-1}{5}$

$\vec{w_2} = (2,-1,0)$

$\vec{w_3} = (0,1,-1) - \dfrac{(0,1,-1)(1,2,3)}{(1,2,3)(1,2,3)}(1,2,3) -$

$\dfrac{(0,1,-1)(2,-1,0)}{(2,-1,0)(2,-1,0)} = (0,1,-1) + \dfrac{1}{14}(1,2,3) + \dfrac{1}{5}(2,-1,0)$

$(0,1,-1) + \left(\dfrac{1}{14}, \dfrac{2}{14}, \dfrac{3}{14}\right) + \left(\dfrac{2}{5}, -\dfrac{1}{5}, 0\right)$

$= \left(\dfrac{33}{70}, \dfrac{33}{35}, \dfrac{-11}{14}\right) \quad \vec{w_3} = \left(\dfrac{33}{70}, \dfrac{33}{35}, \dfrac{-11}{14}\right) + \dfrac{3}{14} = -\dfrac{11}{14}$

$\|w_1\| = \sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

$\vec{w_1} = \left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$

$\|w_2\| = \sqrt{2^2+1^2+0^2} = \sqrt{4+1} = \sqrt{5}$

$\vec{w_2} = \left(\dfrac{2}{\sqrt{5}}, -\dfrac{1}{\sqrt{5}}, 0\right)$

$\|w_1\| = \sqrt{14}$
$\|w_2\| = \sqrt{5}$
$\|w_3\| = \sqrt{\dfrac{121}{70}}$

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$$\|w_3\| = \sqrt{\frac{33}{70}^2 + \frac{33}{35}^2 + \frac{11}{14}^2}$$

$$\|w_3\| = \sqrt{\frac{1089}{4900} + \frac{1089}{1225} + \frac{121}{196}}$$

$$\sqrt{\frac{1089}{4900} + \frac{4356}{4900} + \frac{3025}{4900}} = \sqrt{\frac{8470}{4900}}$$

$$\sqrt{\frac{4235}{2450}} = \sqrt{\frac{847}{490}} = \sqrt{\frac{121}{70}}$$

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$$\boxed{B' = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right), \left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0 \right), \left(\frac{\frac{33}{70}}{\sqrt{\frac{121}{70}}}, \frac{\frac{33}{35}}{\sqrt{\frac{121}{70}}}, \frac{-\frac{11}{14}}{\sqrt{\frac{121}{70}}} \right)}$$