1.- $T: R^2 \to R^2$, $T(v_1, v_2) = (v_1, v_1 + 2v_2)$, $v = (2,-3)$
$w = (4, 12)$

$T(\underset{v_1}{2}, \underset{v_2}{-3}) = (2, 2+2(-3)) = (2, 2-6) = (2,-4)$

$T(v) = (v_1, v_1 + 2v_2) = (4, 12)$

$$\begin{aligned} v_1 \quad &= 4 \\ v_1 + 2v_2 &= 12 \end{aligned} \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 1 & 2 & 12 \end{array}\right] \xrightarrow{e_2 - e_1} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 2 & 8 \end{array}\right] \xrightarrow{e_2/2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 4 \end{array}\right]$$

$$\boxed{\begin{array}{l} a)\ (2,-4) \\ b)\ (4, 4) \end{array}}$$

1

2.- $T: R^2 \to R^2$, $T(v_1, v_2) = (v_1 + v_2, 2v_2)$ $v = (4,-1)$
$w = (8, 4)$

$T(4,-1) = (4-1, 2(-1)) = (3,-2)$

$$\begin{aligned} v_1 + v_2 &= 8 \\ 2v_2 &= 4 \end{aligned} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 2 & 4 \end{array}\right] \xrightarrow{e_2/2} \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{e_1 - e_2} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right]$$

$$\boxed{\begin{array}{l} a)\ (3,-2) \\ b)\ (6, 2) \end{array}}$$

2

3. $T: R^3 \to R^3$, $T(v_1, v_2, v_3) = (0, v_1 + v_2, v_2 + v_3)$
$v = (-3, 2, 5)$, $w = (0, 2, 5)$

$T(\underset{v_1}{-3}, \underset{v_2}{2}, \underset{v_3}{5}) = (0, -3+2, 2+5) = (0,-1,7)$

$$\begin{aligned} 0 + 0 + 0 &= 0 \\ v_1 + v_2 + 0 &= 2 \\ 0 + v_2 + v_3 &= 5 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 5 \end{array}\right]$$

3

$$\boxed{\begin{array}{l} a)\ (0,-1,7) \\ b)\ \text{Infinitas soluciones} \end{array}}$$

$4.- T: R^3 \to R^3, \ T(v_1, v_2, v_3) = (v_1 + v_2, v_2 + v_3, v_3)$

$v = (-2, 1, 2), \ w = (0, 1, 2)$

$T(-2, 1, 2) = (-2+1, 1+2, 2) = (-1, 3, 2)$

$\ \ \ \ \ v_1 \ \ v_2 \ \ v_3$

$$\left[\begin{array}{r|r} v_1 + v_2 + 0 & -2 \\ v_2 + v_3 & 1 \\ v_3 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{e_1 - e_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{c} e_2 - e_3 \\ \to \\ e_1 + e_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

a) $(-1, 3, 2)$
b) $(1, -1, 2)$

4

$5.- T: R^2 \to R^2, \ T(x_1, x_2) = (x_1 + 2x_2, -x_1 - x_2)$

$T(u+v) = T(x_1 + 2x_2, -x_1 - x_2)$

$= T((u_1 + v_1) + 2(u_2 + v_2), -(u_1 + v_1) - (u_2 + v_2))$

$= ((u_1 + 2u_2) + (v_1 + 2v_2), (-u_1 - u_2) + (-v_1 - v_2))$

$= (u_1 + 2u_2, -u_1 - u_2) + (v_1 + 2v_2, -v_1 - v_2)$

$= T(u) + T(v)$

$T(cv) = T(cx_1 + 2cx_2, -cx_1, -cx_2)$

$= (cv_1 + 2cv_2, -cv_1 - cv_2)$

$= (c(v_1 + 2v_2), c(-v_1 - v_2))$

$= c(v_1 + 2v_2, -v_1 - v_2)$

$= cT(v)$

Es transformación Lineal

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

5

6.- $T: R^2 \to R^2$, $T(x_1, x_2) = (x_1 + 3, x_2)$

$T(-1(0,1)) = T(0,-1)$

$= (0+3, -1)$

$= (3, -1)$

$T(cv) \neq cT(v)$

No es una transformación lineal

6

$-1(T(0,1)) = -1(0+3, 1)$

$-1(3, 1)$

$(-3, -1)$

7.- $T: R^2 \to R^2$, $T(x, y) = (x - 2y, 2y - x)$

$T(u+v) = T(x - 2y, 2y - x)$

$((u_1 + v_1) - 2(u_2 + v_2), 2(u_2, v_2) - (u_1 + v_1))$

$= ((u_1 - 2u_2) + (v_1 - 2v_2), (2u_2 - u_1) + (2v_2 - v_1))$

$= (u_1 - 2u_2, 2u_2 - u_1) + (v_1 - 2v_2, 2v_2 - v_1)$

$= T(u) + T(v)$

$T(cu) = T(cx - 2cy, 2cy - cx)$
$= (cu_1 - 2cu_2, 2cu_2 - cu_1)$
$= (c(u_1 - 2u_2), c(2u_2 - u_1))$
$= c(u_1 - 2u_2, 2v_2 - v_1)$
$= c\,T(u)$

(7)

Es una transformación lineal

$$\begin{bmatrix} 1 & -2 \\ -1 & +2 \end{bmatrix}$$

10.. $T: R^2 \to R^2,\ T(x,y) = (|x|, |y|)$

$T(-1(3,4)) = T(-3,-4)$
$= (|-3|, |-4|)$
$= (3,4)$

$-1\,T(3,4) = T(|3|, |4|) \cdot -1$
$-1(3,4)$
$= (-3,-4)$

$T(cu) \neq cT(u)$

(10)

No es una transformación lineal

12.. $T: R^3 \to R^3,\ T(x,y,z) = (z,y,x)$

1- $T(u+v) = T(z,y,x)$
$= ((u_3+v_3), (u_2+v_2), (u_1+v_1))$
$= (u_3, u_2, u_1) + (v_3, v_2, v_1)$
$= T(u) + T(v)$

$T(cu) = T(cz, cy, cx)$
$= (cu_3, cu_2, cu_1)$
$c(u_3, u_2, u_1)$
$c\,T(u)$

Es una transformación Lineal

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(12)

13

$T(2,0) = (1,1) \rightarrow T(1,0) = (1/2, 1/2)$

$T(0,3) = (3,3) \rightarrow T(0,1) = (1, 1)$

Determine $T(1,1)$ y $T(0,1)$

$T(1,1) = 1(T(1,0) + T(0,1)$

$T(1,1) = 1(1/2, 1/2) + 1(1,1)$

$T(1,1) = (1/2, 1/2) + (1,1)$

$T(1,1) = (3/2, 3/2)$

$T(0,1) = 0(T(1,0) + 1(T(0,1))$

$T(0,1) = 0(1/2, 1/2) + 1(1,1)$

$T(0,1) = 0 + (1,1)$

$T(0,1) = (1,1)$

$$\boxed{\begin{aligned} T(1,1) &= (3/2, 3/2) \\ T(0,1) &= (1,1) \end{aligned}}$$ 13

14

$T(1,1,1) = 1$

$T(1,1,0) = 2$

$T(1,0,0) = 3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$T(0,1,1) = 0(3) + 1(-1) + 1(-1)$

$= 0 - 1 - 1$

$= -2$

$$\boxed{T(0,1,1) = -2}$$ 14

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(15) $T(1,1) = (2,3)$

$T(2,-1) = (1,0)$

$$\left[\begin{array}{cc|cc} 1 & 1 & 2 & 3 \\ 2 & -1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & 2 & 3 \\ 0 & -3 & -3 & -6 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

Determinar $T(0,-1)$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & 2 \end{array}\right]$$

$T(0,-1) = 0(1,1) - 1(1,2)$

$T(0,-1) = (-1,-2)$

(15)

$$\boxed{T(0,-1) = (-1,-2)}$$

(16) $T(1,-1) = (2,-3)$

$T(0,2) = (0,8)$

$$\left[\begin{array}{cc|cc} 1 & -1 & 2 & -3 \\ 0 & 2 & 0 & 8 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & -1 & 2 & -3 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

Determinar $T(2,4)$

$$\to \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$T(2,4) = 2(2,1) + 4(0,4)$

$T(2,4) = (4,2) + (0,16)$

$T(2,4) = (4,18)$

$T(1,2) = (2,9)$

(16)

$$\boxed{\begin{array}{l} T(2,4) = (4,18) \\ T(1,2) = (2,9) \end{array}}$$

(17) $A = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 0 & 0 \end{bmatrix}$ $V = (6,1,1)$, $w = (3,5)$

a) $2 \times 3 \;\therefore\; \mathbb{R}^3 \to \mathbb{R}^2$

b) $\begin{bmatrix} 0 & 1 & 2 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -12 \end{bmatrix} \to (3,-12)$

$0 + 1 + 2 = 3$

$-12 = -12$

c) $\left[\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ -2 & 0 & 0 & 5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 1 & 0 & 0 & -5/2 \end{array}\right] \rightarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5/2 \\ 0 & 1 & 2 & 3 \end{array}\right]$ $\begin{array}{l} x = -5/2 \\ y+2z = 3 \end{array}$ $\begin{array}{l} x = -5/2 \\ y = 3-2z \end{array}$

$(-5/2, 3-2z, z)$ 17

$\boxed{\begin{array}{l} a)\ \mathbb{R}^3 \rightarrow \mathbb{R}^2 \\ b)\ (3, -12) \\ c)\ (-5/2, 3-2z, z) \end{array}}$

18 $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ $V = (5, 2, 2)$, $w = (4, 2)$

a) 2×3 $\mathbb{R}^3 \rightarrow \mathbb{R}^2$

b) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 7 \\ 7 \end{bmatrix} \rightarrow (7, 7)$

$5 + 4 - 2 = 9 - 2 = 7$

$5 + 0 + 2 = 7$

c) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 1 & 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -2 & 2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -1 & 1 & -1 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{l} x + y = 2 \\ \quad y - z = 1 \end{array}\right.$ $\begin{array}{l} x = 2-y \\ z = y-1 \end{array}$

$(2-y, y, y-1)$

$\boxed{\begin{array}{l} a)\ \mathbb{R}^3 \rightarrow \mathbb{R}^2 \\ b)\ (7, 7) \\ c)\ (2-y, y, y-1) \end{array}}$ 18

(19)

$A = [1 \quad 1]$ $v = (2, 3)$ $w = 4$

a) 1×2 $\mathbb{R}^2 \to \mathbb{R}^1$

b) $\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \end{bmatrix} = 5$

$2 + 3 = 5$

c) $[1 \quad 1 \mid 4]$ $x + y = 4$ $x = 4 - y$ $(4 - y, y)$

(19)

a) $\mathbb{R}^2 \to \mathbb{R}^1$
b) 5
c) $(4 - y, y)$

(20)

$A = [2 \quad -1]$ $v = (1, 2)$ $w = -1$

a) 1×2 $\mathbb{R}^2 \to \mathbb{R}^1$

b) $\begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$

$2 - 2 = 0$

c) $[2 \quad -1 \mid -1]$ $2x - y = -1$ $y - 2x = 1$ $y = 2x+$

(20)

a) $\mathbb{R}^2 \to \mathbb{R}^1$
b) 0
c) $(x, 2x + 1)$

(21)

$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $v = (2, 1, -5)$, $w = (6, 4, 2)$

a) $3 \times 3 \therefore \mathbb{R}^3 \to \mathbb{R}^3$

a) $\mathbb{R}^3 \to \mathbb{R}^3$

b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}$

$2 + 1 - 5 = 3 - 5 = -2$

$0 + 1 - 5 = -4 = -4$

$0 + 0 + (-5) = -5$

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c) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$

21

a) $\mathbb{R}^3 \rightarrow \mathbb{R}^3$
b) $(-2, -4, -5)$
c) $(2, 2, 2)$

22

$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ $v = (8, 4)$ $w = (5, 2)$

a) $2 \times 2 \therefore \mathbb{R}^2 \rightarrow \mathbb{R}^2$

b) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 20 \\ 4 \end{bmatrix}$

$16 + 4 = 20$
$0 + 4 = 4$

c) $\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3/2 \\ 0 & 1 & 2 \end{array}\right]$

22

a) $\mathbb{R}^2 \rightarrow \mathbb{R}^2$
b) $(20, 4)$
c) $(3/2, 2)$

23

$A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \\ 1 & 1 \end{bmatrix}$ $v = (2, 2)$ $w = (4, -5, 0)$

a) 3×2 $\mathbb{R}^2 \rightarrow \mathbb{R}^3$

b) $\begin{bmatrix} 4 & 0 \\ 0 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 8 \\ 10 \\ 4 \end{bmatrix}$

$8 + 0$
$0 + 10$
$2 + 2 = 4$

23

a) $\mathbb{R}^2 \rightarrow \mathbb{R}^3$
b) $(8, 10, 4)$
c) $(1, -1)$

c) $\left[\begin{array}{cc|c} 4 & 0 & 4 \\ 0 & 5 & -5 \\ 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$

24) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$ $\quad v = (1, 2) \quad w = (2, -5, 12)$

24

a) $\mathbb{R}^2 \rightarrow \mathbb{R}^3$

b) $(1, -2, 5)$

c) $(2, 5)$

a) 3×2 $\quad \mathbb{R}^2 \rightarrow \mathbb{R}^3$

b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$

$1 + 0 = 1$

$0 - 2 = -2$

$1 + 4 = 5$

c) $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & -1 & -5 \\ 1 & 2 & 12 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 1 & 2 & 12 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$

(27).- $T: \mathbb{R}^4 \to \mathbb{R}^3$ $T(w,x,y,z) = (2w+4x+6y+5z, -w-2x+2y, 8y+4z)$

$$\begin{bmatrix} 2 & 4 & 6 & 5 \\ -1 & -2 & 2 & 0 \\ 0 & 0 & 8 & 4 \end{bmatrix} \to \begin{bmatrix} 2 & 4 & 6 & 5 \\ -2 & -4 & 4 & 0 \\ 0 & 0 & 8 & 4 \end{bmatrix} \to \begin{bmatrix} 2 & 4 & 6 & 5 \\ 0 & 0 & 10 & 5 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 4 & -4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$w + 2x - 2y = 0$ $\qquad w = -2x + 2y$

$2y + z = 0$ $\qquad z = -2y$

$(-2x + 2y, x, y, -2y)$

$(-2x, x, 0, 0)$ $\quad (2y, 0, y, -2y)$

a) $\{(-2,1,0,0),(2,0,1,-2)\}$ (27)

b) $\{(1,2,-2,0),(0,0,2,1)\}$

(28) $T: \mathbb{R}^3 \to \mathbb{R}^3$, $T(x,y,z) = (x+2y, y+2z, z+2x)$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & -4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 9 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (28)

$$\to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ $\quad x = 0$, $y = 0$, $z = 0$

a) $\{0\}$

b) $\{(1,0,0),(0,0,1),(0,1,0)\}$

29) $T: \mathbb{R}^3 \to \mathbb{R}^3 \quad T(x,y,z) = (x, y+2z, z)$ (29)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} x=0 \\ y=0 \\ z=0 \end{matrix}$$

$\boxed{\begin{array}{l} a)\ \{0\} \\ b)\ \{(1,0,0),(0,1,0) \\ \quad (0,0,1)\} \end{array}}$

30) $T: \mathbb{R}^3 \to \mathbb{R}^3, \ T(x,y,z) = (x+y, y+z, x-z)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\begin{array}{l} a)\ \{0\} \\ b)\ \{(1,0,0),(0,1, \\ \quad ,(0,0,1)\} \end{array}}$

(30)

31) $$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$ (31)

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$\boxed{\begin{array}{l} \ker(T) = \{(0,0)\} \\ null(T) = 0 \\ gen(T) = \{(1,0),(0,1)\} \\ rango(T) = 2 \end{array}}$

32) $$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$\boxed{\begin{array}{l} \ker(T) = \{(0,0)\} \\ null(T) = 0 \\ gen(T) = \{(1,0),(0,1)\} \\ rango(T) = 2 \end{array}}$ (32)

(33) $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 3 \\ 2 & 2 & 0 \\ 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

$\to \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{aligned} 2x + 2y &= 0 \\ y - 3z &= 0 \end{aligned}$ $\begin{aligned} 2x &= -2y \\ -3z &= -y \end{aligned}$ $\begin{aligned} x &= -y \\ z &= +y/3 \end{aligned}$

$(-y, y, +y/3)$

$\boxed{\begin{aligned} &\text{Kernel}(T) = (-y, y, y/3) \\ &\text{null}(T) = 1 \\ &\text{rango}(T) = \{(2,2,0), (0,1,-3)\} \\ &\text{rango}(T) = 2 \end{aligned}}$ (33)

(34) $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ (34)

$\to \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{\begin{aligned} &\text{Kernel}(T) = \{0\} \\ &\text{null}(T) = 0 \\ &\text{rango}(T) = \{(1,0,0), (0,1,0), (0,0,1)\} \\ &\text{rango}(T) = 3 \end{aligned}}$

(35) $T: \mathbb{R}^5 \to \mathbb{R}^3$ $\text{null}(T) = 2$

$\dim(T) = \text{null}(T) + \text{rango}(T)$

$5 = 2 + \text{rango}(T)$

$\text{rango}(T) = 5 - 2$

$\boxed{\text{rango}(T) = 3}$ (35)

(36) $T: \mathbb{R}^5 \to \mathbb{R}^3$ $\text{null}(T) = 4$

$\dim(T) = \text{null}(T) + \text{rango}(T)$

$5 = 4 + \text{rango}(T)$

$\text{rango}(T) = 5 - 4$

$\boxed{\text{rango}(T) = 1}$ (36)

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(37) $T: P_4 \to \mathbb{R}^5$ rango(T) = 3

$5 = \text{null}(T) + 3$

$\text{null}(T) = 5 - 3$

$\boxed{\text{null}(T) = 2}$ (37)

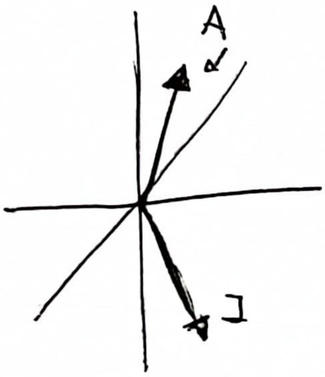
(38) $T: M_{2,2} \to M_{2,2}$ rango(T) = 3

$\text{null}(T) = \dim(T) - \text{rango}(T)$

$\text{null}(T) = 4 - 3$

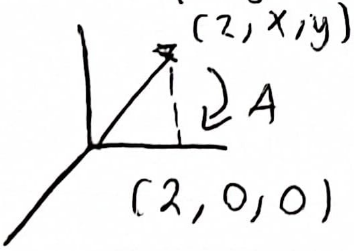
$\boxed{\text{null}(T) = 1}$ (38)

(39) $T: R^3 \to R^3$ reflexión en plano xy, Encontrar A^2



$$\boxed{A^2 \begin{cases} A \text{ si es impar} \\ I \text{ si es par} \end{cases}}$$ (39)

(40) $T: R^3 \to R^3$, proyección sobre el plano xy Encontrar A^2



$$\boxed{A^n = A \ \forall n \in \mathbb{N}}$$ (40)

(42) $T: P^3 \to P^3$, operador diferencial D, A^2

$A_T^2 = T \circ T = \dfrac{d^2}{dt^2}$

$p(x) = (a_0, a_1, a_2, a_3)$

$$\boxed{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$ (42)

43 $T_1: \mathbb{R}^2 \to \mathbb{R}^3,\ T_1(x,y) = (x, x+y, y)$
$T_2: \mathbb{R}^3 \to \mathbb{R}^2,\ T_2(x,y,z) = (0,y)$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\boxed{A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ A = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}$$ 43

44 $T_1: \mathbb{R} \to \mathbb{R}^2,\ T_1(x) = (x, 3x)$
$T_2: \mathbb{R}^2 \to \mathbb{R},\ T_2(x,y) = (y+2x)$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$$

$2 \times 1 \qquad 1 \times 2$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} \to \begin{bmatrix} 5 \end{bmatrix}$$

$1 \times 2 \qquad 2 \times 1$

$2 + 3 = 5$

44

$$\boxed{A' = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \quad A = [5]}$$

(45) $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x,y) = (0,y)$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\text{Det} = 0 \therefore$

(45) $\boxed{\text{No tiene inversa}}$

(46) $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x,y) = (x\cos\theta - y\,\text{sen}\,\theta,\ x\,\text{sen}\,\theta + y\cos\theta)$

$$\begin{bmatrix} x\cos\theta & -y\,\text{sen}\,\theta \\ x\,\text{sen}\,\theta & y\cos\theta \end{bmatrix}$$

(47) $T: \mathbb{R}^2 \to \mathbb{R}^2\quad T(x,y) = (x,-y)$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right]$$

(47) $\boxed{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \to (x,-y)}$

(48) $T: \mathbb{R}^3 \to \mathbb{R}^2,\ T(x,y,z) = (x+y,\ y-z)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

dimensiones diferentes $\therefore$

(48) $\boxed{\text{No tiene inversa}}$

(49) $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $x = 0$, $y = 0$ null(T) = 0 $\therefore$ es uno a uno

$\mathbb{R}^2 \to \mathbb{R}^2$ dim = 2

rango(T) = 2 = dim = 2 $\therefore$ es sobre

$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ Det(T) = 6 $\neq 0$ $\therefore$ es invertible

$\boxed{\text{a) uno a uno} \quad \text{b) sobre} \quad \text{c) invertible}}$

(49)

(50) $A = \begin{bmatrix} 1 & 1/4 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $x = 0$, $y = 0$ null(T) = 0 $\therefore$ es uno a uno

$\mathbb{R}^2 \to \mathbb{R}^2 \to$ dim = 2

rango(T) = 2 = dim(T) $\therefore$ es sobre

Det(T) = 1 $\neq 0$ $\therefore$ es invertible

$\boxed{\text{a) uno a uno} \quad \text{b) sobre} \quad \text{c) invertible}}$

(51) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $x = 0$, $y + z = 0$ $x = 0$, $y = -z$ (51)

$(x, -z, z) \neq 0$ 2×3

$\mathbb{R}^3 \to \mathbb{R}^2$ dim(T) = 2

rango(T) = 2 = dim(T) = 2 $\therefore$ es sobre

dimensiones diferentes $\therefore$ no es invertible

$\boxed{\text{a) no uno a uno} \quad \text{b) sobre} \quad \text{c) No invertible}}$

(52) $A = \begin{bmatrix} 4 & 0 & 7 \\ 5 & 5 & 1 \\ 0 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 4 & 0 & 7 \\ 20 & 20 & 4 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 4 & 0 & 7 \\ 0 & 20 & -31 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 4 & 0 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ null = 0

rango(T) = 3 = dim(T)

Det(T) $\neq 0$

(52) $\boxed{\text{a) uno a uno} \quad \text{b) sobre} \quad \text{c) Invertible}}$

(S3) $T: \mathbb{R}^2 \to \mathbb{R}^3$ $T(x,y) = (-x, y, x+y)$, $v = (0,1)$

$T(1,0) = (-1,0,1)$
$T(0,1) = (0,1,1)$

$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$B = \{ \overset{v_1}{(1,1)}, \overset{v_2}{(1,-1)} \}$ $B' = \{ \overset{w_1}{(0,1,0)}, \overset{w_2}{(0,0,1)}, \overset{w_3}{(1,0,0)} \}$

$T(1,1) = (-1,1,2)$
$T(1,-1) = (-1,-1,0)$ $\to \begin{bmatrix} -1 & 1 & 2 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [$

$-1(0,1,0) + 1(0,0,1) + 2(1,0,0) = (2,-1,1)$

$(0,-1,0) + (0,0,1) + (2,0,0)$

$-1(0,1,0) - 1(0,0,1) + 0(1,0,0)$

$(0,-1,0) + (0,0,-1) = (0,-1,-1)$

$\begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -2 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 0 & 1 \\ 0 & -2 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & -1/2 \end{array}\right]$$

$$\begin{bmatrix} -1 & -1 \\ 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{array}{l} a)\ (0,1,1) \\ b)\ (0,1,1) \end{array}}$$ (S3)

(34) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (2y, 0), \quad v = (-1, 3)$.

$$\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$0 + 6$

$B = \{(2,1), (-1,0)\}, \quad B' = \{(-1,0), (2,2)\}$

$T(2,1) = (2,0)$

$T(-1,0) = (0,0)$

$2(-1,0) + 0(2,2) = (-2,0)$

$0(-1,0) + 0(2,2) = (0,0)$

$$\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -1 \end{bmatrix} \to \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & -1 & -7 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 7 \end{array}\right]$$

(34)

$$\boxed{\begin{array}{l} a) = (6, 0) \\ b) = (6, 0) \end{array}}$$

$$\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$$

$-6(-1,0) + 0(2,2)$.

$(6,0) + (0,0) = (6,0)$

(55) $T: \mathbb{R}^2 \to \mathbb{R}^2$ $T(x,y) = (x-3y, y-x)$, $B' = ((1,-1),(1,1))$

$A = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$ $B' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \xrightarrow{e_2/2}$

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right]$ $P^{-1} \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 1/2 & -1/2 & 1 & 0 \\ 1/2 & 1/2 & 0 & 1 \end{array}\right] \xrightarrow[e_2 \times 2]{e_1 \times 2} \left[\begin{array}{cc|cc} 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & -1 & 2 & 0 \\ 0 & 2 & -2 & 2 \end{array}\right] \xrightarrow{e_1/2} \left[\begin{array}{cc|cc} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right]$ $-3/2 - 1/2 = -4/2$

$-3/2 + 1/2 = -2/2 = -1$

$1+2$

$1-2$

$\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(55)

$\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$ $\boxed{A = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}}$

$0+1$ $0-1=-1$

(56) $T: R^3 \to R \quad T(x,y,z) = (x+3y, 3x+y, -2z)$

$B' = \{(1,1,0), (1,-1,0), (0,0,1)\}$

$$\begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad B' \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$1/2 + 3/2 = 4/2 = 2 \qquad 1/2 - 3/2 = -2/2 = -1$

$3/2 + 1/2 = 4/2 = 2 \qquad 3/2 - 1/2 = 2/2 = 1$

$$\begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}}$$

56

57

$$P = \begin{bmatrix} 3 & -5 \\ 1 & -4 \end{bmatrix} \quad A = \begin{bmatrix} 18 & -19 \\ 11 & -12 \end{bmatrix} \quad A' = \begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix}$$

$$PA' = AP$$

$$\begin{bmatrix} 3 & -5 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 5 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 18 & -19 \\ 11 & -12 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 1 & -4 \end{bmatrix}$$

57

$15+20=35$
$-9-5=-14$

$$\begin{bmatrix} 35 & -14 \\ 21 & -7 \end{bmatrix} = \begin{bmatrix} 35 & -14 \\ 21 & -7 \end{bmatrix}$$

$\boxed{\text{Son similares}}$

58

$$P = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad A' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & -3 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & -3 \\ 2 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Son similares}}$ 58

71) $T(x,y) = (x, 2y)$

$$\begin{bmatrix} 1 & 0 \\ 0 & ky \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

$\boxed{\text{Expansión vertical}}$ 71

72) $T(x,y) = (x+y, y)$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

$\boxed{\text{Deformación horizontal}}$ 72

73) $T(x,y) = (x, y+3x)$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

$\boxed{\text{Deformación vertical}}$ 73

74) $T(x,y) = (5x, y)$

$$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$$

$\boxed{\text{Expansión horizontal}}$ 74

75) $T(x,y) = (x+5y, y)$ 75

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

$\boxed{\text{Deformación horizontal}}$

76) $T(x,y) = (x, y+\frac{3}{2}x)$ 76

$$\begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ k & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

$\boxed{\text{Deformación vertical}}$

(81) $$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Producto de una reflexión en la recta $y = x$ y una expansión horizontal

(82) $$\begin{bmatrix} 1 & 0 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} 1 & 0 \\ K & 1 \end{bmatrix}$$

Producto de una deformación vertical y una expansión vertical

83) 45° alrededor del eje z

$$\begin{bmatrix} \cos 45° & -\text{sen}\,45° & 0 \\ \text{sen}\,45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$[\sqrt{2}, 0, 1] \qquad \boxed{\vec{w} = (\sqrt{2}, 0, 1)}$$ 83

84) 90° alrededor del eje x

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\text{sen}\,\theta \\ 0 & \text{sen}\,\theta & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\boxed{\vec{w} = (1, -1, -1)}$$ 84

85) 60° alrededor del eje x

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\text{sen}\,\theta \\ 0 & \text{sen}\,\theta & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{-1+\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} \end{bmatrix}$$

$$\boxed{\vec{w} = \left(1, \frac{-1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right)}$$ 85

86) 30° alrededor del eje y

$$\begin{bmatrix} \cos 30° & 0 & \text{sen}\,30° \\ 0 & 1 & 0 \\ -\text{sen}\,30° & 0 & \cos 30° \end{bmatrix} \rightarrow \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1+\sqrt{3}}{2} \\ -1 \\ \frac{\sqrt{3}-1}{2} \end{bmatrix}$$

$$\boxed{\vec{w} = \left(\frac{1+\sqrt{3}}{2}, -1, \frac{\sqrt{3}-1}{2}\right)}$$ 86

87) 60° alrededor del eje x seguida por 30° alrededor de eje z

$$\begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/4 & -\sqrt{3}/4 \\ 1/2 & \sqrt{3}/4 & -3/4 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \sqrt{3}/2 & -1/4 & +\sqrt{3}/2 \\ 1/2 & \sqrt{3}/4 & -3/4 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}}$$

87

88) 120° alrededor del eje y seguida por 45° alrededor del eje z

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & -1/2 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{4} & -\sqrt{2}/2 & \frac{\sqrt{6}}{4} \\ -\sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \\ -\sqrt{3}/2 & 0 & -1/2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -\sqrt{2}/4 & -\sqrt{2}/2 & \sqrt{6}/4 \\ -\sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \\ -\sqrt{3}/2 & 0 & -1/2 \end{bmatrix}}$$

88

(89) 30° alrededor del eje y seguida por 45° alrededor del eje z

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} \sqrt{6}/4 & -\sqrt{2}/2 & \sqrt{2}/4 \\ \sqrt{6}/4 & \sqrt{2}/2 & \sqrt{2}/4 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \sqrt{6}/4 & -\sqrt{2}/2 & \sqrt{2}/4 \\ \sqrt{6}/4 & \sqrt{2}/2 & \sqrt{2}/4 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix}}$$ (89)

(90) 60° alrededor del eje x seguida por 60° alrededor del eje z

$$\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/4 & 3/4 \\ \sqrt{3}/2 & 1/4 & -\sqrt{3}/4 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1/2 & -\sqrt{3}/4 & 3/4 \\ \sqrt{3}/2 & 1/4 & -\sqrt{3}/4 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}}$$ (90)