Introduction to Operations Research (MATH 615)

Stuart Price

August 29th, 2022

Outline

- Syllabus
- Operations Research
- Complexity
- Optimization
- Feedback

Syllabus

- **Objective**: The goal of this course is to introduce the students to some of the key tools of Operations Research, with a focus on the mathematical formulation and modeling of practical problems.

  In the first half of the semester, we will cover the formulation, solution, and interpretation of Linear Programs and Network Models.

  The second half of the course will include decision analysis, simulations, and meta-heuristics such as simulated annealing, tabu search, and genetic programming.

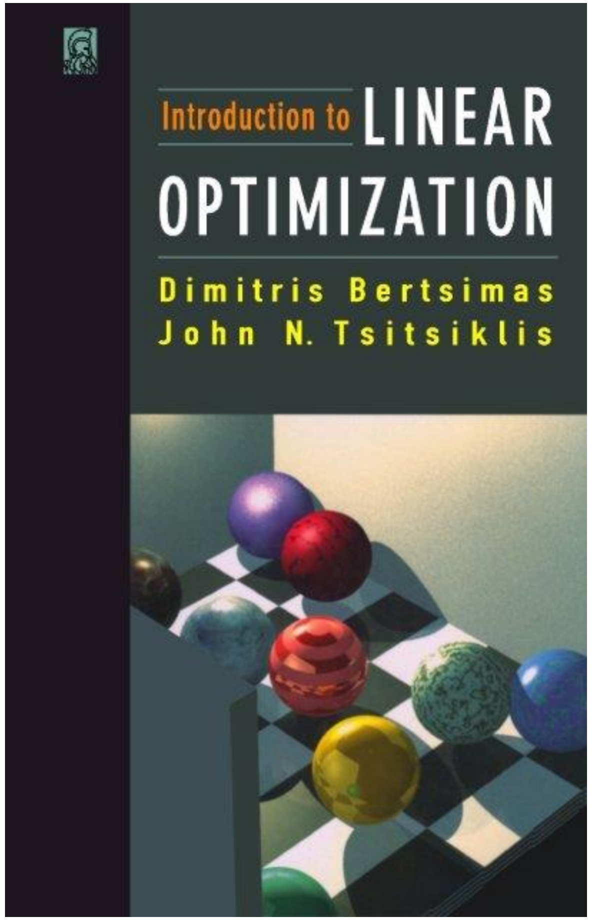
Contact Information

- **Instructor**: Stuart Price
- **Email**: stp42@georgetown.edu

- **TA**: TBD
- **TA Email**: TBD

- **Office Hours**: TBD

Recommended Text



- Bertsimas, Dimitris, and John N. Tsitsiklis. *Introduction to linear optimization*. Vol. 6. Belmont, MA: Athena Scientific, 1997
- Covers material for the first half of the course

Grades and Grading

Participation	**10%**
Midterm Exam	**25%**
Final Project	**25%**
Homework	**40%**

Grading scheme:

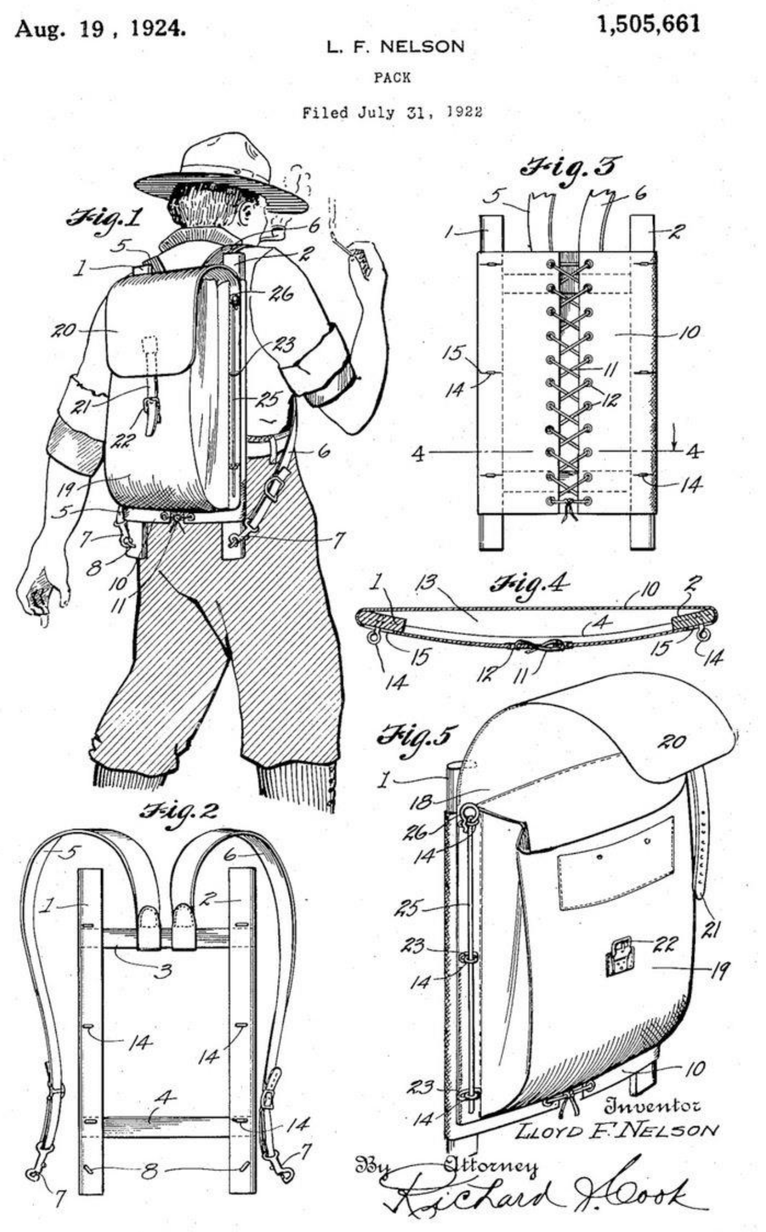
- A 93+
- A- 90-92
- B+ 87-89
- B 83-86
- B- 78-82
- C 70-77
- D Below 70

Coursework

- **Homework**: You are allowed to collaborate on homework, but every student must submit a unique (i.e. no cut and paste) copy of their own work. Physical copies are preferred for most assignments, though if you are unable to make it to class a pdf can be submitted by Canvas.

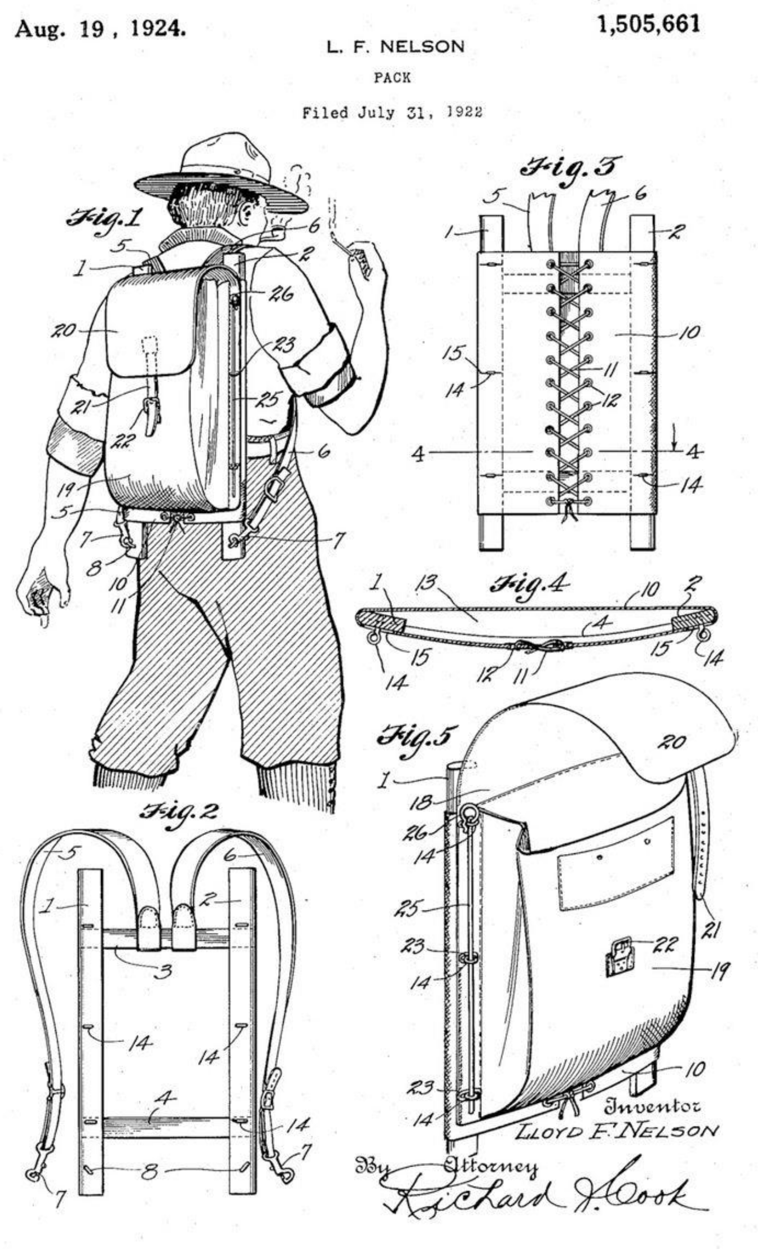
- **Midterm**: The midterm will cover the linear programming and network models section of the course (roughly speaking the first half of the course). It will be completed by each student working independently, though I have not decided between a take-home or in-class (timed) test given the circumstances.

- **Final Project**: TBD



Knapsack Problem

Object	Weight	Value
A	2	11
B	5	20
C	5	15
D	7	28
E	8	36
F	13	52



Knapsack Problem

Max Weight: 20

Object	Weight	Value
A	2	11
B	5	20
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D	7	28
E	8	36
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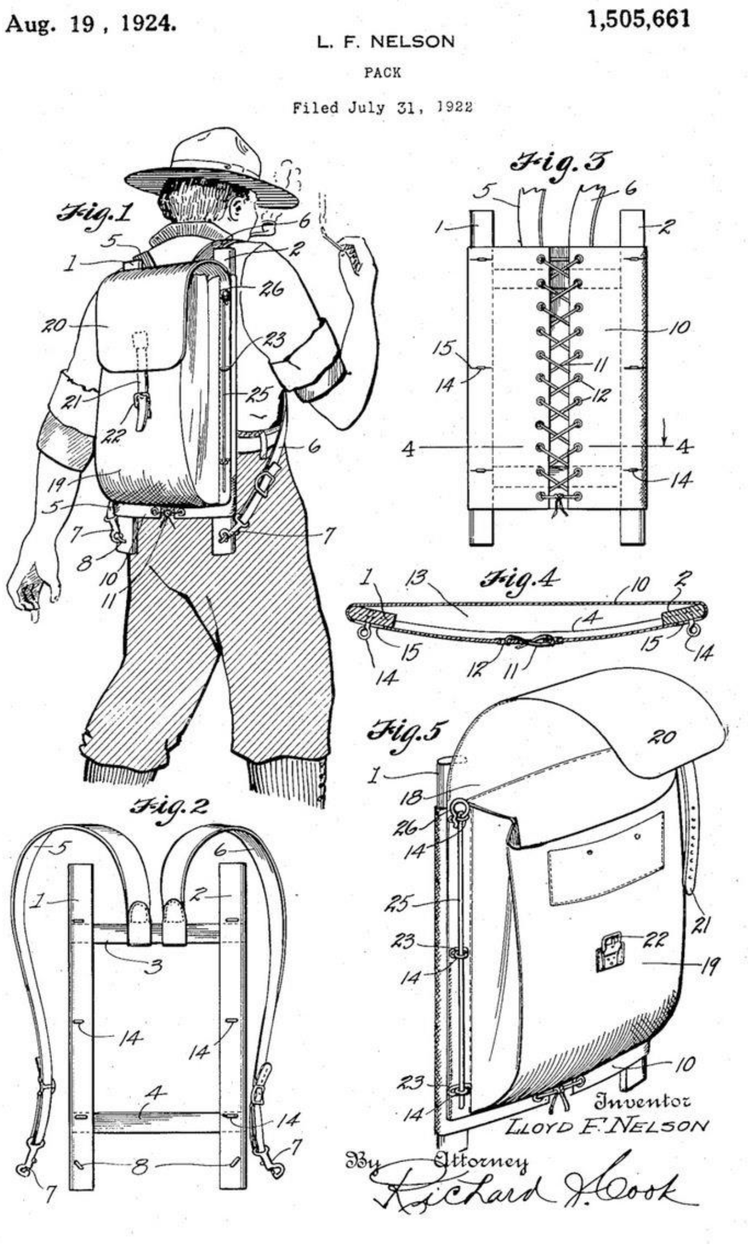
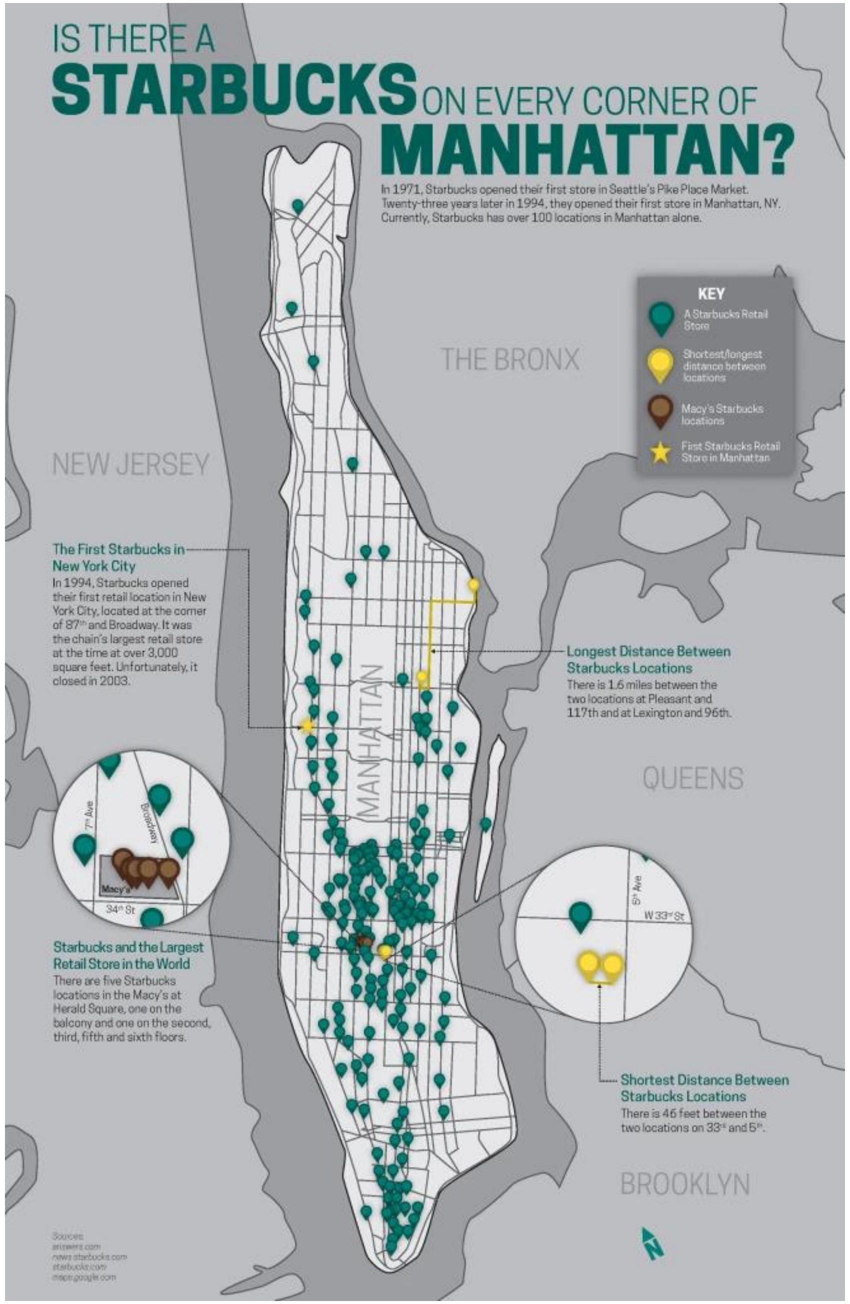


Knapsack Problem

Max Weight: 20

Object	Weight	Value	V/W
A	2	11	5.5
B	5	20	4
C	5	15	3
D	7	28	4
E	8	36	4.5
F	13	52	4



Where to open a new location?

- What is the objective?
- What are limiting factors?

Image credit:

(https://catiepeterson.com/portfolio/starbucks-in-manhattan/)

Why do airlines overbook flights?

- What is their objective?
- What are other objectives?
- How does time matter?

How can a grocery store cut down on food waste?

- What are other objectives?
- What information do I need?
- What decisions are being made?

Operations Research

- Is the application of analytic methods and mathematical models to aid in real world decision making

- Very interdepartmental in nature sometime appearing in Math, Business, or Engineering departments, but working closely with the practitioners of the application including transportation, logistics, healthcare, telecom, and more

Courses Typical of an OR Degree

Optimization

- Mathematical Programming
  - Linear, Integer, MIP,…
- Non-linear Optimization
- Combinatorial Optimization

Probability and Statistics

- Stochastic Processes
- Statistical Inference
- Econometrics
- Causal Modeling

Machine Learning

- Optimization for Machine Learning
- Statistical Learning Theory

Modeling

- Game Theory
- Network Models
- Behavioral Economics
- Inventory Theory and Supply Chains
- Discrete Event Simulations

Problems Addressed by OR

- Routing
- Scheduling
- Network optimization
- Resource allocation
- Supply chain management
- Facility location
- Optimal search
- …

Common terms for Problems

- Traveling salesman problem (TSP)
- Multi-vehicle routing problem
- Knapsack problem
- Binning problem
- Assignment problem
- Scheduling problem
- Min Path
- Max Flow

Problem Solving

Decision Making

- Identify the problem
- Determine the set of alternatives
- Determine criterion of evaluation
- Evaluate the alternatives
- Choose a solution
- Implement the solution
- Evaluate the results to determine if things are satisfactory

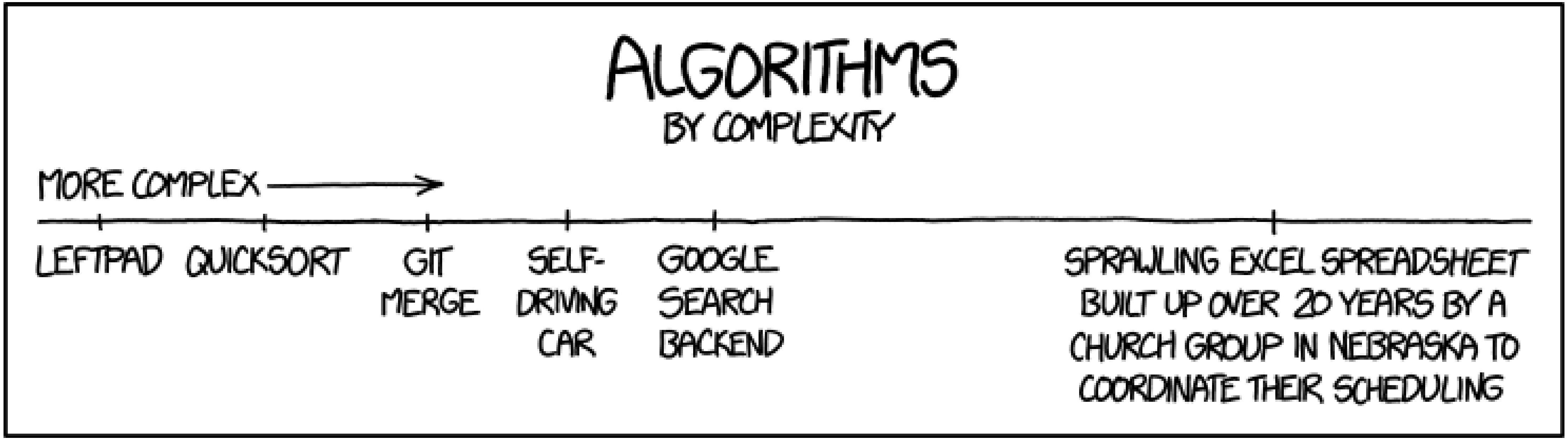
Structuring

Analyzing

Tools of OR

- Linear Programming
- Integer/ Mixed Integer Programming
- Dynamic Programming
- Simulation
- Stochastic Models
- Forecasting
- Meta-heuristics
- Game Theory
- …

Complexity


ALGORITHMS
BY COMPLEXITY
MORE COMPLEX
LEFTPAD
QUICKSORT
GIT MERGE
SELF-DRIVING CAR
GOOGLE SEARCH BACKEND
SPRAWLING EXCEL SPREADSHEET BUILT UP OVER 20 YEARS BY A CHURCH GROUP IN NEBRASKA TO COORDINATE THEIR SCHEDULING

A Quick Introduction to Big O notation

- Big O notation is used to describe the asymptotic behavior of algorithms with respect to their inputs

$$f(x) = O\big(g(x)\big)\ as\ x \to \infty\ \ iff$$
$$\exists M > 0, x_0 \in \mathbb{R} \ni |f(x)| \leq M|g(x)| \forall x \geq x_0$$

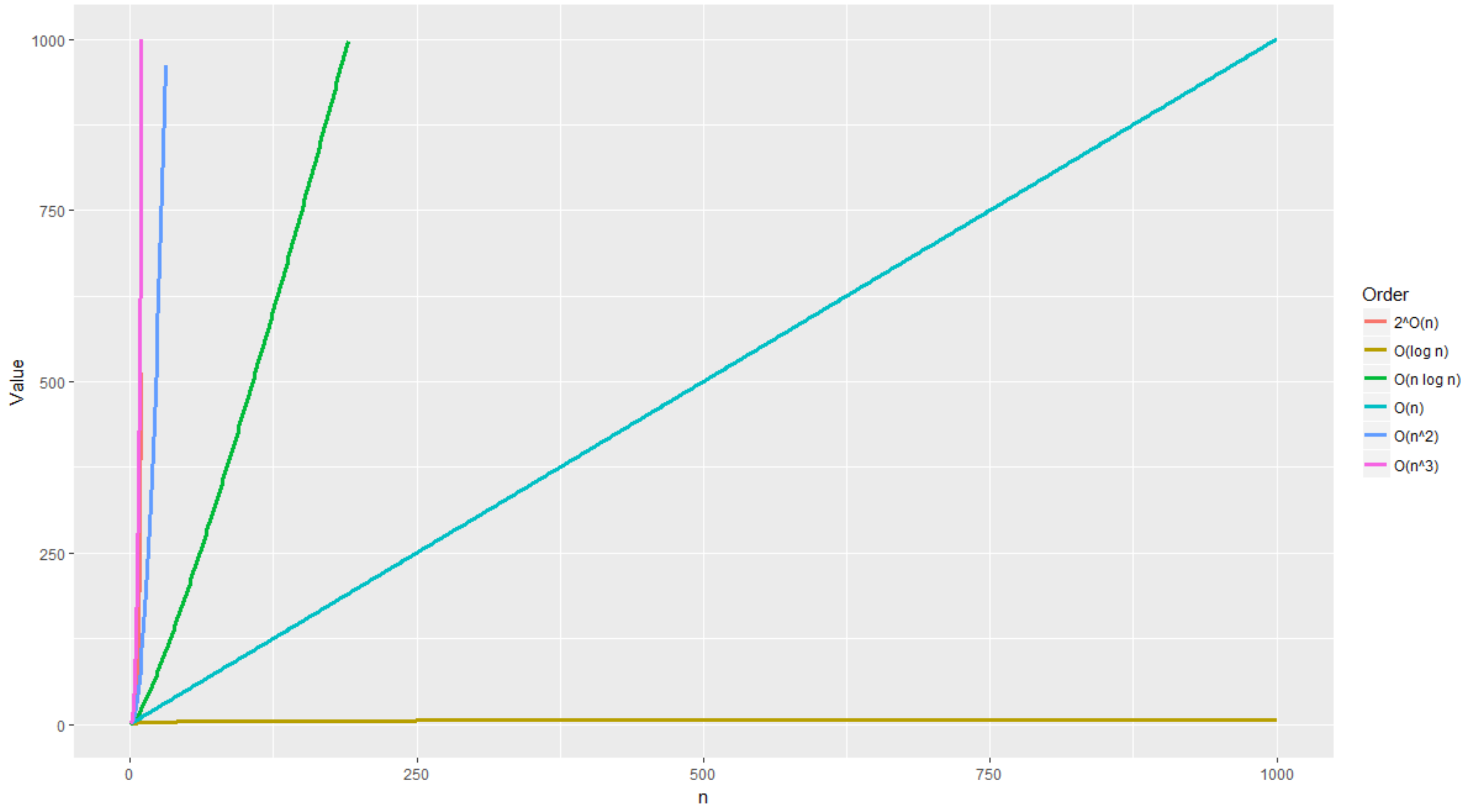
- Most often this is used to refer to the run time of the algorithm in question, but might also refer to space or communication

A Quick Introduction to Complexity

- Examples:

Run Time	Example Algorithms
$O(1)$	Even or Odd, Multiply by 0, Array Access, Insert into a list
$O(\log n)$	Binary Search
$O(n)$	Max in a list, Comparing Strings
$O(n \log n)$	Merge Sort
$O(n^2)$	Euclidean Algorithm for GCD
$O(n^{2.376})$	Coppersmith-Winograd Matrix multiplication, inversion
$O(n^3)$	Basic multiplication of nxn matrices, Basic matrix determinant
$2^{O(n)}$	Traveling salesman by dynamic programming
$n!$	Traveling salesman by brute force

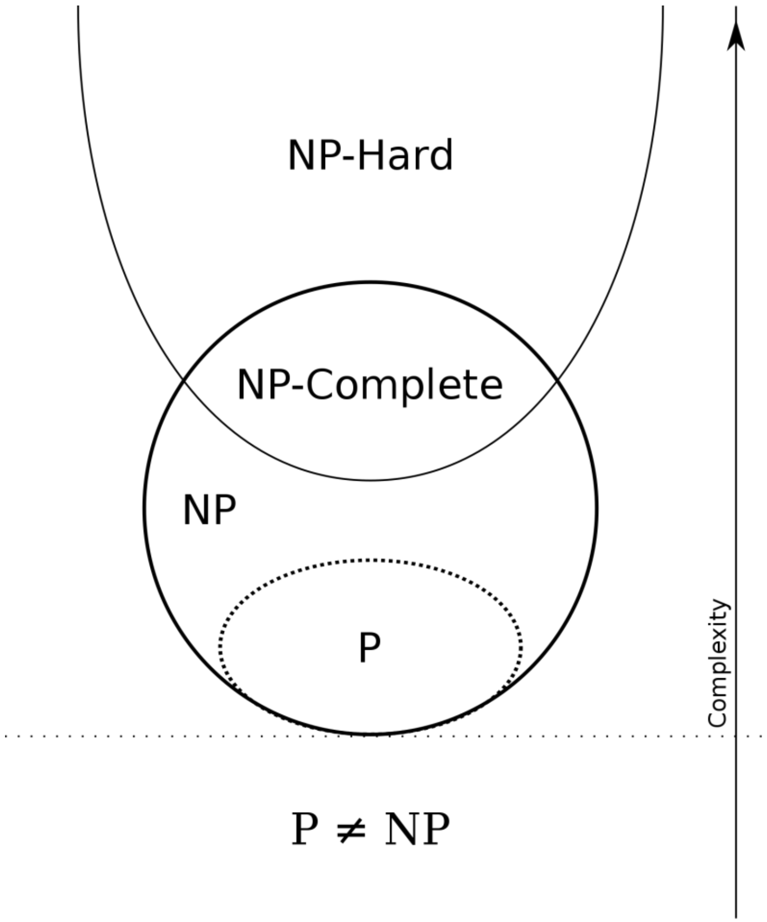
How quickly problems grow relative to size



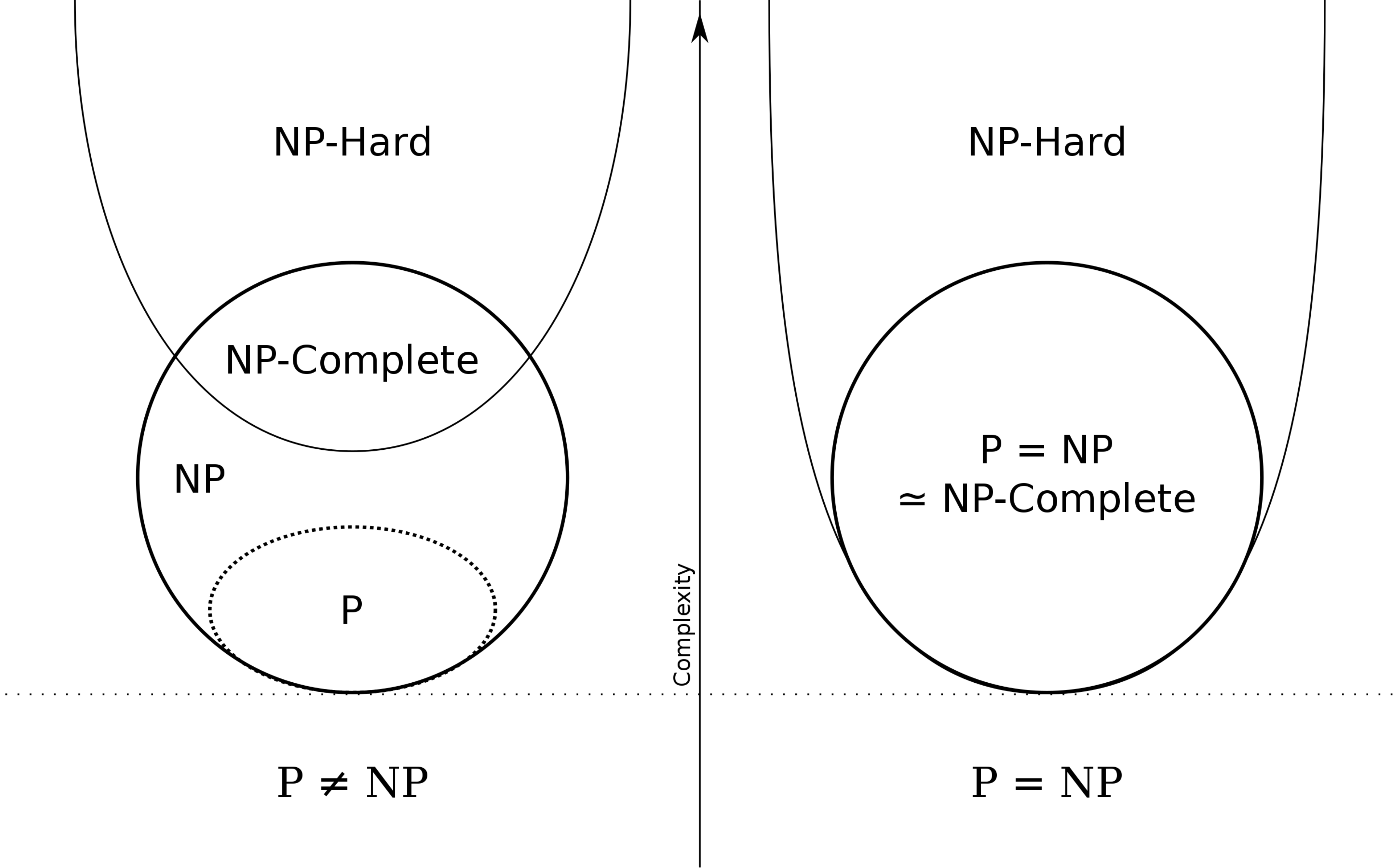
Runtimes

N	O(log n)	O(n)	O(n log n)	O(n^2)	O(n^2.376)	O(n^3)	2^O(n)
10	3.3s	10s	33s	1.7m	4m	16.7m	17.1m
100	6.6s	1.7m	11.1m	2.8h	15.7h	11.6d	4E22y
1,000	10s	16.7m	2.8h	11.6d	155.4d	31.7y	
10,000	13.3s	2.8h	1.5d	3.2y	101y	3E5y	
100,000	16.6s	1.2d	19.2d	317y	2E5y		
1,000,000	20s	11.6d	230.7d	3E5y			
10,000,000	23.3s	11.57d	7.4y				

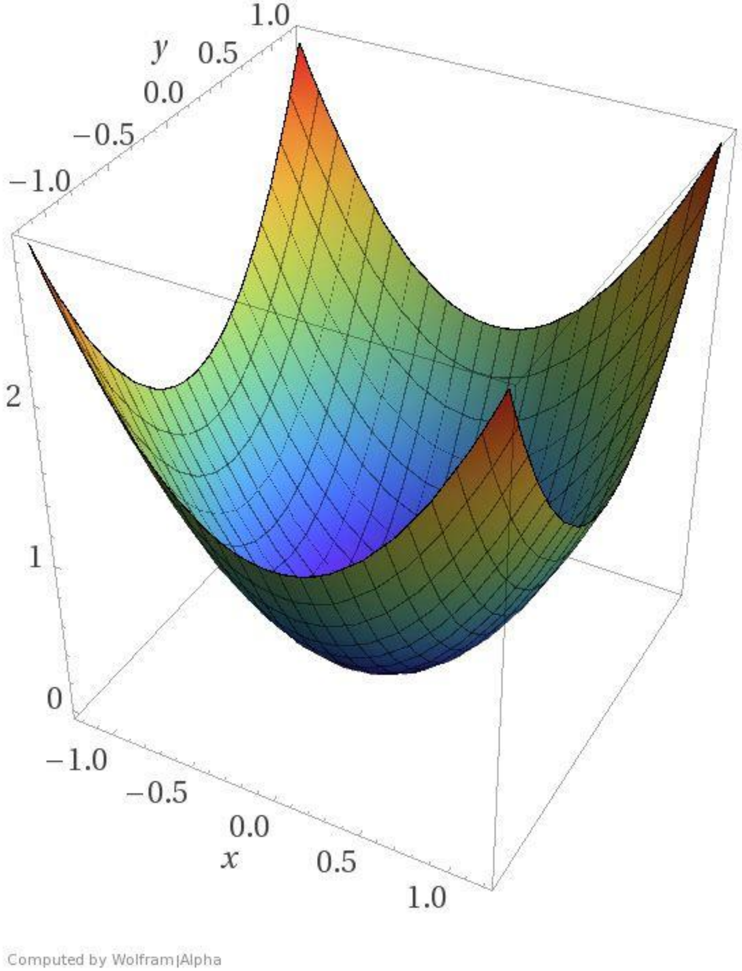
P and NP

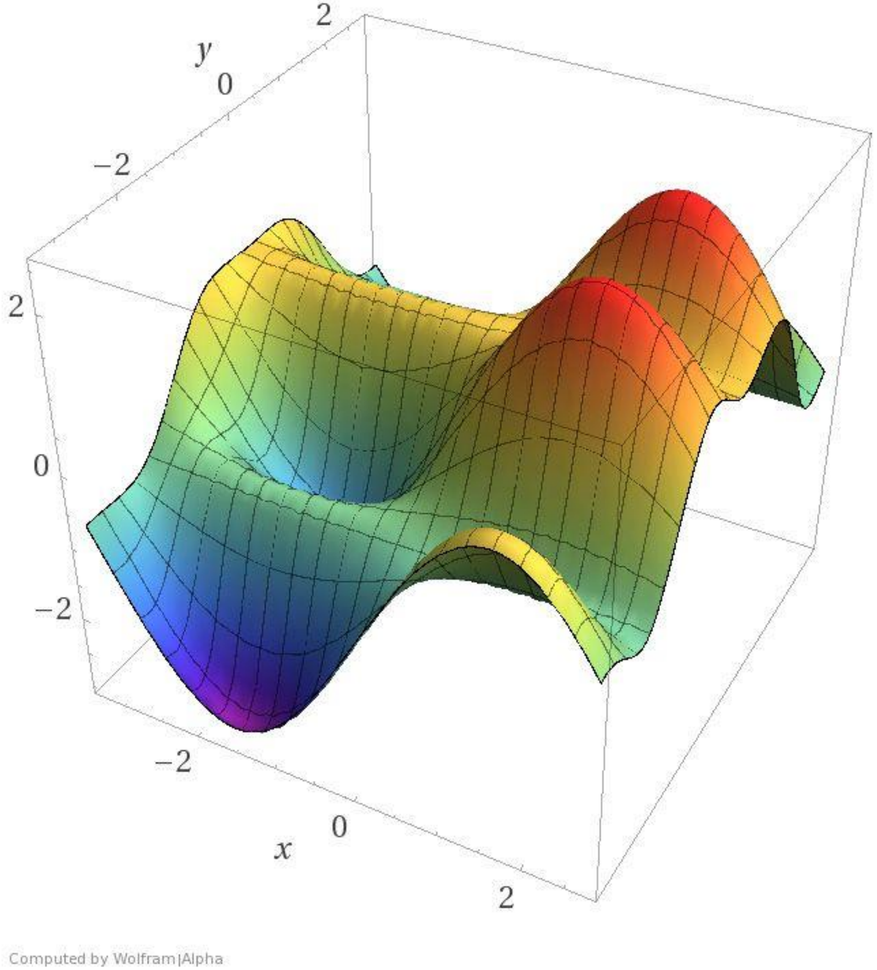


- P or PTIME (polynomial time) are the set problems that can be solved in $n^{O(1)}$ time
- NP problems are the set of problems that can be checked in polynomial time, though some might not be solvable in polynomial time
- NP-Complete are problems that all other NP can be reduced to in polynomial time


NP-Hard
NP-Complete
NP
P
Complexity
P = NP
≃ NP-Complete
NP-Hard
P ≠ NP
P = NP

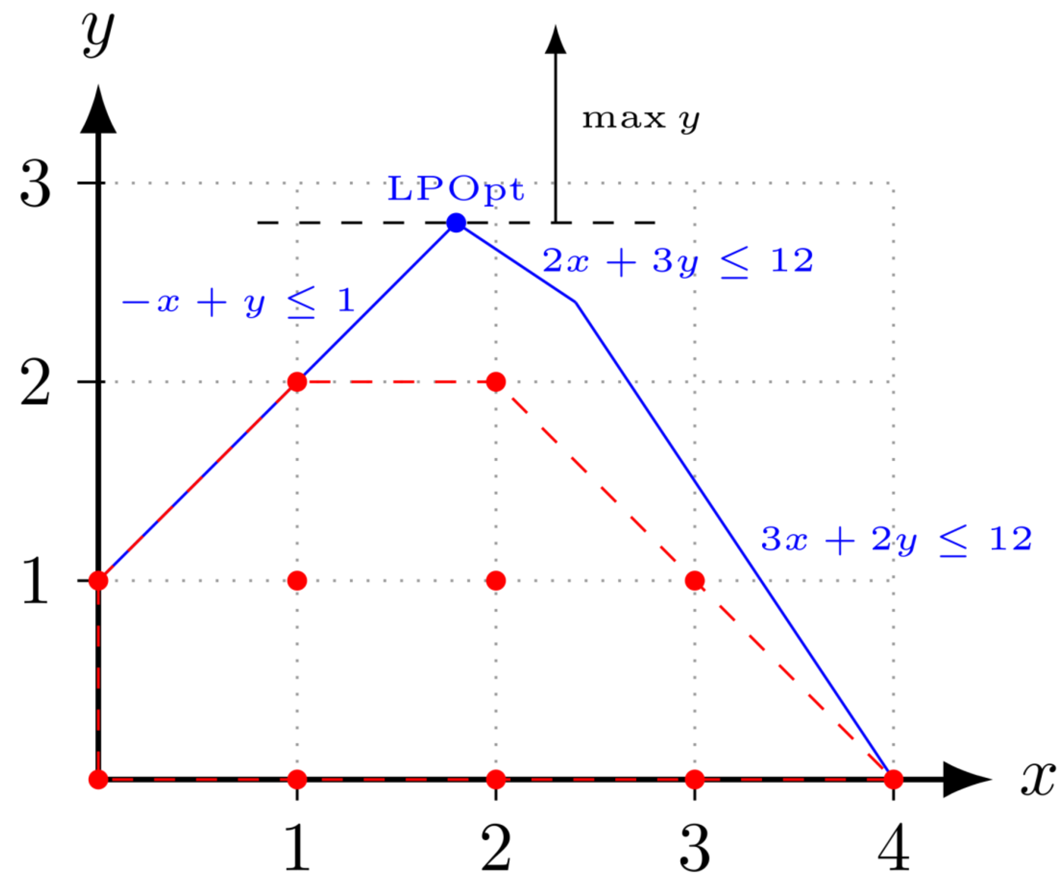
Optimization



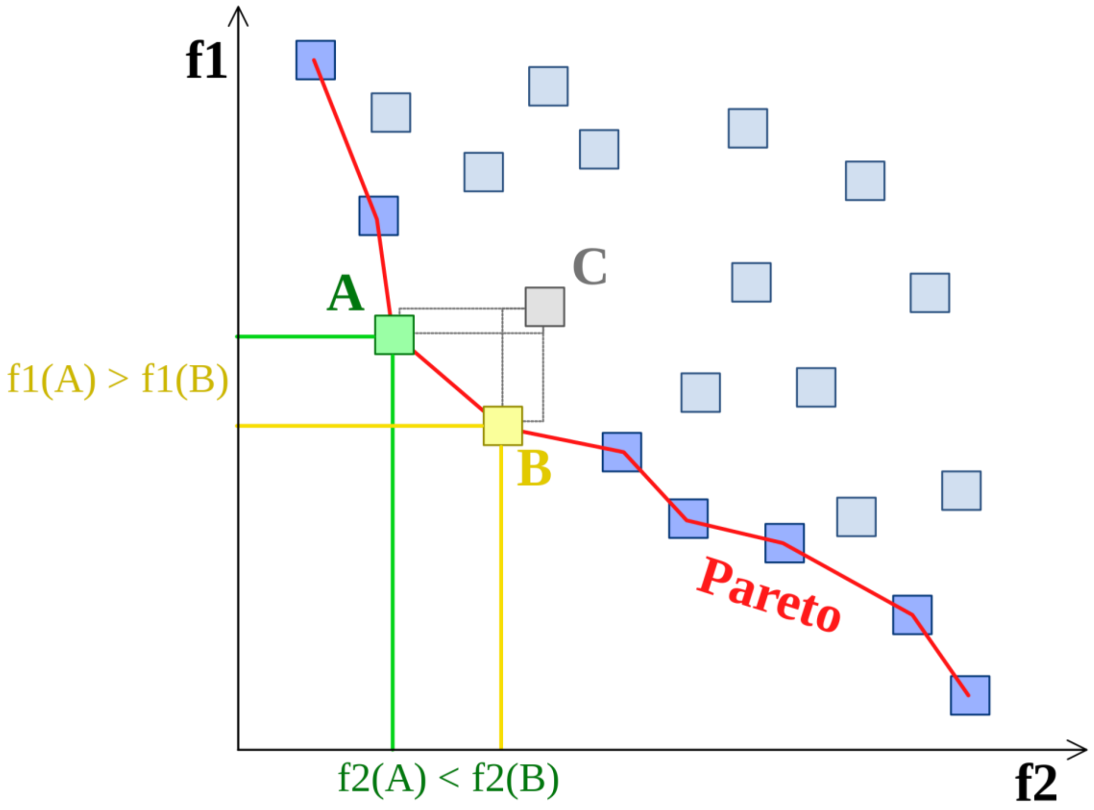


Types of Optimization

- Combinatorial optimization- optimization of an objective function from a discrete set of possible options
- Constrained optimization- optimization of an objective function from a constrained subset of the solu



Multi-Objective Optimization



- Often there are competing metrics or objectives when evaluating alternatives
- It is rare for a single solution to dominate in all metrics and so a Pareto-Frontier of solutions that are best for some combination of metrics is created

Some questions:

- We will be running some computer programs throughout the class to solve some problems. Both Python and R have packages/libraries to run the necessary algorithms:
  - Rate your familiarity with both Python and R on a scale between 1 (not at all familiar) and 10 (extremely comfortable/ use on a regular basis)
  - If you answered 1-3 for either language: if provided with working code that solves a problem, could you read and modify the code as necessary to solve similar problems?
  - Do you have a strong preference for examples and code in class to be presented in Python or R?

Some more questions:

- Do you have any conflicts for office hours (M-F after 5pm)? Please list any class conflicts as these are likely shared by several students.
- Given that homework will typically be available by Wednesday after class and due 10 days later, Saturday 11:59 pm. Do you have a preferred day/time for office hours?

Reminder

- Next Monday is Labor Day, but there is class on Tuesday (9/6) as the University will be operating on Monday schedule