Homework # 10

Due 4/16

1. Reading: sections 7.2 – 7.5
2. Exercise 7.10, 7.37, 7.59b (and then use the moment generating function to compute the mean and variance of a Poisson r.v.).
3. Chapter 7 Problem 6 (use moment generating functions to prove the result)
4. Let $X_1, X_2, \ldots, X_n$ be iid random variables with mean 0 and variance σ^2. Compute $V[X_1 + X_2 + \cdots + X_n]$ and then use the Markov inequality to bound the probability $P((X_1 + X_2 + \cdots + X_n) > cn^r)$ for some $c > 0$ and $r > 0$. What does this bound say about the sum of the X_i as $n \to \infty$? If I flip a coin 1 trillion times, earning a dollar for every head and losing a dollar for every tail, how much can I reasonably expect to have or owe after I'm done flipping?
5. This problem demonstrates the important fact that normal r.v. are independent if their covariance is 0, but that this is not true in general, for non-normal r.v.
   (a) Assume that X_1 and X_2 are jointly distributed random variables with $\mathrm{Cov}(X_1, X_2) = 0$ then X_1 and X_2 are independent. (Hint: Explain why this means that the covariance matrix of X_1, X_2 is diagonal and then write down the resulting joint pdf.)
   (b) Show that this property does not hold in general by providing an example of jointly distributed r.v.'s X_1 and X_2 with $\mathrm{Cov}(X_1, X_2) = 0$ and with X_1 and X_2 not independent.