Homework 10 :: MATH 504 :: Due Tuesday, November 15th, 11:59 pm

Your homework submission must be a single pdf called "LASTNAME-hw10.pdf" with your solutions to all theory problem to receive full credit. All answers must be typed in Latex.

1. Consider "Rosenbrock" function

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

With a starting point $[0, 0]^\mathsf{T}$, apply two iterations of Newton's method to minimize Rosenbrock function. Hint:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2. Let $S = \text{span}\{x_1, x_2, x_3\}$, where

$$x_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad x_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ -3 \end{bmatrix}, \qquad x_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}.$$

Find an orthonormal basis for S, using Gram-Schmidt algorithm.

3. Write a code and implement the Gauss-Newton Method on the last example given in the lecture, to find A, ω, and ϕ such that the resulting sinusoid

$$y = A \sin(\omega t + \phi)$$

best fits $(t_i, y_i), i = 1, 2, \ldots, 21$, with $t_1 = 0$ and $t_{21} = 10$ and y_i given roughly below.