MATH 502, CLASS 1

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ABSTRACT. Part A: Systems of linear equations are written in matrix form $bfAx = \mathbf{b}$. They are solved by Gauss Elimination method, ie achieving echlon form via row operations. The advantage of such method is stated.

There are only three possibilities for the echlon form and hence 3 possibilities for solutions of $\mathbf{Ax} = \mathbf{b}$: no solution, one solution or infinitely many solutions.

For $\mathbf{Ax} = \mathbf{0}$, only two possibilities: one solution $\mathbf{x} = 0$, or infinitely many solutions.

Rank(A) and independent rows of A can be determined via echlon form.

Part B. Ordinary differential equations.

Ordinary Differential Equations (ODEs) are introduced. Solutions of an ODE are defined. Motivations for ODEs are stated. Geometric meaning of $y' = f(x, y)$ is presented. Separable ODEs are solved.

KEYWORDS:
Hwk: fill in this section

1. Part A: System of Linear Equations

1. Example of system of linear eqs

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2. Matrices and matrix multiplication

3. Matrix form of a system of linear eqs.

The beauty of matrix form:

Hwk1.1 Write the following system of eqs in matrix form.

$$\begin{cases} x_1 - 3x_2 + 2x_3 = 1, \\ 3x_1 - 2x_2 + 4_3 = 2, \\ 4x_1 + 2x_2 - 2x_3 = 3. \end{cases} \tag{1.1}$$

Hwk 1.2 Write the matrix equation

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ -2 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}. \tag{1.2}$$

back to a system of individual eqs.

4. **Solving** $A\mathbf{x} = \mathbf{b}$.

4.1 Systems of echlon forms are easy to solve.

Example 4.1.1.

Example 4.1.2.

Example 4.1.3.

4.2. **Gauss Elimination: Convert** $A\mathbf{x} = \mathbf{b}$ **to echlon form to solve it.**

4.2.2 Row operations:

4.2.2a.

4.2.2b.

4.2.2c.

4.2.3. These row operations do not change solutions.

Example 4.2.1 (Gauss elimination method, and its efficiency)

Hwk: Hwk 1.3 in the Chapter 1 of Module 1.

4.2.4 Three solution possibilities for $A\mathbf{x} = \mathbf{b}$.

4.2.4a When its echlon form is inconsistent.

4.2.4b When its echlon form is full ranked.

4,2,4c When else.

Theorem 1.1. *The system of linear equations* $A\mathbf{x} = \mathbf{b}$*....*

Theorem 1.2. *The system of linear equations* $A\mathbf{x} = \mathbf{0}$*....*

4.2.5 **Why and when** $A\mathbf{x} = \mathbf{0}$ may have infinite many solutions?

Ans: When there are more unknowns than independent equations.

Insight:

This lead to the following concepts

Rank of A

Independent rows of A.

4.4. Dependence and Independence of vectors.

2. Part B1: Ordinary Differential Equations

2.1. **A differential equation.** is .....

Definition 2.1. *A solution of an ODE* $y = f(x, y)$ *is a function* $y = y(x)$ *that makes the ODE true.*

Remark 2.1. Another way to say that $y(x)$ makes the ODE true is "$y(x)$ satisfies the ODE".

Example 2.1. $y(x) = e^{2x}$ is a solution of the ODE $y' = 2y$ because.....

The order of an ODE is...

Geometric meaning of $y' = f(x, y)$

An ODE typically have infinitely many solutions because

Definition 2.2. *A general solution of an ODE is....*

To fix a solution, we need to specify...

3. B1.2. Separable ODE

Definition 3.1. *An ODE of the form ...*

Solving separable ODEs

General strategy

Example 3.1.

Example 3.2. (From Geometry)

Example 3.3. (From orbital mechanics)

Example 3.4. (Physics: Falling in the air)

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