Homework 5 Nathan Bick

Problems 6.B: 1

1. Prove Theorem 6.1. If y is an indicator variable assuming 0 with prob $1-p$ and 1 with prob p, then $\sigma^2 = p(1-p)$.

we see that y has a Bernoulli distribution. we know from def 6.6 that $\sigma^2 = V(y)$, or that we need to calculate the variance of y. Since Y is a Bernoulli random variable, we know $V(Y) = p(1-p)$ from prior homeworks.

Problems 7.B : 1, 2

1. show (7.5): $MSE(\hat{\theta}) = V(\hat{\theta}) + [Bias(\hat{\theta})]^2$.

By definition, we know $MSE(\hat{\theta}) = E((\hat{\theta} - \theta)^2)$.

$$E[(\hat{\theta} - \theta)^2] = E[(\hat{\theta} - E(\hat{\theta}) + E(\hat{\theta}) - \theta)^2]$$

$$= E[(\hat{\theta} - E(\hat{\theta}))^2 + 2(\hat{\theta} - E(\hat{\theta}))(\overset{E(\hat{\theta}) - \hat{\theta}}{\cancel{\hat{\theta} - E(\hat{\theta})}}) + \cancel{E(\hat{\theta})} + (E(\hat{\theta}) - \theta)^2]$$

$$= E[(\hat{\theta} - E(\hat{\theta}))^2] + 2E[(\hat{\theta} - E(\hat{\theta}))(\overset{E(\hat{\theta} - \hat{\theta})}{\cancel{\hat{\theta} - E(\hat{\theta})}})] + E[(E(\hat{\theta}) - \theta)^2]$$

Note $E[\hat{\theta} - E(\hat{\theta})] = E(\hat{\theta}) - E[E(\hat{\theta})] = E(\hat{\theta}) - E(\hat{\theta}) = 0$.

so we get $E[(\hat{\theta} - E(\hat{\theta}))^2] + (E(\hat{\theta}) - \theta)^2$

$$= Var(\hat{\theta}) + (Bias(\hat{\theta}))^2.$$

2. If $MSE(\hat{\theta}) \neq V(\hat{\theta})$, is $\hat{\theta}$ an unbiased estimator of θ? Why?

we know from (7.5) that $MSE(\hat{\theta}) = V(\hat{\theta}) + [Bias(\hat{\theta})]^2$.

$[Bias(\hat{\theta})]^2 \geq 0$ means that we have two cases:

i $(Bias(\hat{\theta}))^2 = 0$. Then $MSE(\hat{\theta}) = V(\hat{\theta})$.

ii $(Bias(\hat{\theta}))^2 > 0$. Then $MSE(\hat{\theta}) \neq V(\hat{\theta})$.

so if $MSE(\hat{\theta}) \neq V(\hat{\theta})$, then the bias is not zero, and so by definition $\hat{\theta}$