Homework 9 :: MATH 504 :: Due Tuesday, November 8th, 11:59 pm

Your homework submission must be a single pdf called "LASTNAME-hw9.pdf" with your solutions to all theory problem to receive full credit. All answers must be typed in Latex.

Consider "Rosenbrock" function

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

This function is known as the banana function because of the shape of its level sets.

a. Prove that $x^* = [1, 1]^\mathsf{T}$ is the unique global minimizer of f over $\mathbb{R}^2$.

b. Write a method with signature function xsol = GradDescent (f, grad, x0)

   The input f and grad are function handles. The function f: $\mathbb{R}^N \to \mathbb{R}$ is an arbitrary objective function, and grad: $\mathbb{R}^N \to \mathbb{R}^N$ is its gradient. The method should minimize f using gradient descent, and terminate when the gradient of f is small. I suggest stopping when

$$\|\nabla f(x^k)\| < \|\nabla f(x^0)\| * 10^{-4}$$

   Test your algorithm on Rosenbrock function, and plot $\|x^k - x^*\|_2$ versus iteration numbers k for various fixed stepsize selection of $\alpha = 0.001, 0.05, 0.5$, and explain your observation.

c. Modify part b to create the new function xsol = GradDescentNesterov(f, grad, x0)
   This function should implement Nesterov's method, that is

$$\begin{aligned} x^k &= y^k - \alpha \nabla f(y^k) \\ \delta^{k+1} &= \tfrac{1+\sqrt{1+4(\delta^k)^2}}{2} \\ y^{k+1} &= x^k + \tfrac{\delta^k - 1}{\delta^{k+1}}(x^k - x^{k-1}) \end{aligned}$$

   The method is initialized with $x^0 = y^1$ and $\delta^1 = 1$, and the the first iteration has index $k = 1$. Test your algorithm on Rosenbrock function, and plot $\|x^k - x^*\|_2$ versus iteration numbers k for various fixed stepsize selection of $\alpha = 0.001, 0.05, 0.5$, and explain your observation.