MATH 502, CLASS 1

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Abstract. Part A: Today's topic :

- For $\mathbf{Ax} = \mathbf{0}$, only two possibilities: one solution $\mathbf{x} = 0$, or infinitely many solutions.
- Null space of A is defined and found.
- The relation of solutions of $A\mathbf{x} = \mathbf{0}$ and that of $A\mathbf{x} = \mathbf{b}$ is shown.
- Rank(A), and independent rows of A can be determined via echelon form.
- Solve many $A\mathbf{x} = \mathbf{x}$ for many different $\mathbf{b}$'s together. Matrix equation $AX = B$.
- Inverse matrix A^{-1} of A. What is it for?
- More matrix algebra.
- Matrix algebra applied to data science.

Part B. Ordinary differential equations.

- Solving inhomogeneous first order linear ODEs.
- Relation between solutions of inhomogeneous linear ODEs and that of homogeneous linear ODEs.
- Applications (or modeling and solving) first order linear ODEs.

Keywords:
Hwk: fill in this section

1. Part A: $A\mathbf{x} = \mathbf{b}$, $A\mathbf{x} = \mathbf{0}$, Matrix Equations

Recall:

Theorem 1.1. *The system of linear equations* $A\mathbf{x} = \mathbf{0}$*....*

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Why and when does $A\mathbf{x} = \mathbf{0}$ have infinite many solutions?
Ans: When there are more unknowns than **independent** equations.
Insight:

This lead to the following concepts
Rank of A is...

Independent rows of A.

1.1. **Null space of A.**

Definition 1.1. *The null space of a matrix A is...*

Example 1.1. Find the null space of ...

The relation of solutions of $A\mathbf{x} = \mathbf{0}$ and that of $A\mathbf{x} = \mathbf{b}$
Denote solutions of $A\mathbf{x} = \mathbf{0}$ by...
and that of ....

Theorem 1.2. *All solutions of*

Proof. □

1.2. **Solve many $A\mathbf{x} = \mathbf{b}$ simultaneously, and Matrix equation $AX = B$.**

Example 1.2. (From multiple $A\mathbf{x} = \mathbf{b}'s$ to a matrix eq $AX = B$)

1.3. **Inverse matrix A^{-1} of A.**

Definition 1.2. *A is a square matrix. Its....*

Remark 1.1. $A\mathbf{x} = \mathbf{b}$'s solution is....

Remark 1.2. Finding A^{-1} of A is ....

Example 1.3. Find A^{-1} for ....

Theorem 1.3.

$$(AB)^{-1} = ...$$

Proof. ..

□

Theorem 1.4. *(...) There is at most one A^{-1} for a A.*

Proof. ..

□

2. More matrix algebra

2.1. **Elements of a matrix.** Expressing matrices as $A = (a_{ij})$, $B = (b_{ij})$.

a_{ij} denotes.....

Then $i, j - th$ element of $A + B$ is ...

The $i, j - th$ element of AB is ...

Transpose of A is....

The $ij-$th element of A^T.

2.2. **Partition a matrix.** Partition a matrix by horizontal and vertical lines into blocks.

and you can....

Example 2.1.

Example 2.2.

2.3. **Matrix algebra applied to data science.** Data matrix: the columns in the matrix often represent measurements on the same feature.

Example 2.3.

3. Part B2: Ordinary Differential Equations

3.1. **Solving first order linear ODEs.** ..

The ODE:

Special case: Homogeneous

Solutions of homogeneous linear ODEs, leading to the idea of integral factor

Solving inhomogeneous linear ODEs.

Example 3.1.

Applications of first order linear ODEs.

Example 3.2. (Falling through air)

Example 3.3. (Heat transfer)

Example 3.4. (modeling via Flow rate eq. accretion-dilution problems)

Example 3.5. (modeling via Flow rate eq. From finance)

Example 3.6. (modeling via Flow rate eq. Planet climate, energy budget eq.)

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