Homework 2 :: MATH 504 :: Due Tuesday, September 20th, 11:59 pm

Your homework submission must be a single pdf called "LASTNAME-hw2.pdf" with your solutions to all theory problem to receive full credit. All answers must be typed in Latex. Submission should be done on Canvas.

1. Consider the quadratic function

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 5x_3^2 + 2x_1x_2 - 2x_1x_3 + 4x_2x_3 + x_1 - x_2.$$

   (a) Choose a matrix A and vector b so that with $x = (x_1, x_2, x_3)$, $f(x) = x^T Ax + b \cdot x$.
   (b) Choose another matrix B, such that $A \neq B$ and $B = B^T$ so that $f(x) = x^T Bx + b^T x$.

2. Use the Spectral Decomposition Theorem and determine the eigenvalue decomposition of the matrix A, given by

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

3. Find the linear and quadratic approximation of the following function

$$f(x) = \exp(x_1^2 + x_2^2 + x_3^2)$$

   at $\bar{x} = (0, 0, 0)^\intercal$

4. Determine whether the following quadratic function has a min, max, or saddle point. Explain why?

$$f(x_1, x_2) = 2x_1^2 - x_2^2 - x_1x_2 + 5x_2 - 1$$