Homework 2 Nathan Bick

. Problems 3.B: 1 and 2

consider finite population $\mathcal{U} = \{U_1, U_2, U_3, U_4\}$. suppose the experiment is to select $n=2$ units without replacement. Samplespace S given by

$$S = \{(U_1, U_2), \ldots, (U_3, U_4)\}, \text{ six outcomes.}$$

If the prob of each outcome is $\frac{1}{6}$ find:

(a) $P(U_1 \text{ is selected}) = \frac{3}{6} = \frac{1}{2}$

(b) $P(U_2 \text{ is selected}) = \frac{3}{6} = \frac{1}{2}$

(c) $P(U_3 \text{ is selected}) = \frac{3}{6} = \frac{1}{2}$

(d) $P(U_4 \text{ is selected}) = \frac{3}{6} = \frac{1}{2}$

we know this because, for example

$$P(U_1 \text{ is selected}) = P(U_1, U_2) + P(U_1, U_3) + P(U_1, U_4)$$

that is, we sum the probabilities of any sample that includes U_1.

2. Consider the same setup as problem 1 but with

$$P(U_1, U_2) = \frac{1}{12} \qquad P(U_2, U_3) = \frac{4}{12}$$

$$P(U_1, U_3) = \frac{1}{12} \qquad P(U_2, U_4) = \frac{1}{12}$$

$$P(U_1, U_4) = \frac{4}{12} \qquad P(U_3, U_4) = \frac{1}{12}.$$

Answer a, b, c, d using these probs.

(a) $P(U_1 \text{ is selected}) = \frac{1}{12} + \frac{1}{12} + \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$

(b) $P(U_2 \text{ is selected}) = \frac{1}{12} + \frac{4}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$

(c) $P(U_3 \text{ is selected}) = \frac{1}{12} + \frac{4}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$

(d) $P(U_4 \text{ is selected}) = \frac{4}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{2}.$

2. Problems 3.C: 1 and 2

1. For any two events E, F, show $P(E)P(F|E) = P(F)P(E|F)$.

we know by ~~Bayes~~ definition that $P(E|F) = \frac{P(E \cap F)}{P(F)}$ if $P(F) \neq 0$.

So clearly we have $P(F)P(E|F) = P(E \cap F)$.

Similarly we have $P(F|E) = \frac{P(F \cap E)}{P(E)}$ if $P(E) \neq 0$.

But we know that $E \cap F = F \cap E$ so $P(E \cap F) = P(F \cap E)$.

so $P(F|E) = \frac{P(E \cap F)}{P(E)} \Rightarrow P(E)P(F|E) = P(E \cap F)$

Therefore $P(F)P(E|F) = P(E)P(F|E)$.

If $P(F) = 0$, $P(E) = 0$, then we have $0 \cdot P(F|E) = 0 \cdot P(E|F)$

$0 = 0$.

2. Show that if $P(E) = P(F) \neq 0$, then $P(F|E) = P(E|F)$.

From the previous problem we have that

$$P(F)P(E|F) = P(E)P(F|E).$$

If $P(F) = P(E) = a \neq 0$, then

$$aP(E|F) = aP(F|E) \Rightarrow P(E|F) = P(F|E).$$

Problems 3.D: 1, 2, and 5

1. Show that if $P(E|F) = P(E)$ then $P(F|E) = P(F)$.

If $P(E|F) = P(E)$, then E and F are independent. Then $P(E \cap F) = P(E)P(F)$.

So $$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{P(E)P(F)}{P(E)}$$

$$= P(F).$$

2. If E and F are independent events, does that imply that E and F^c are independent? Why? What about E^c and F^c? Why?

We know that $E \cap F$ and $E \cap F^c$ are disjoint.

so $E = (E \cap F) \cup (E \cap F^c)$ and

$P(E) = P(E \cap F) + P(E \cap F^c)$.

since E, F independent, then

$P(E) = P(E)P(F) + P(E \cap F^c)$

$$\Rightarrow P(E \cap F^c) = P(E) - P(E)P(F)$$
$$= P(E)(1 - P(F))$$
$$= P(E)P(F^c)$$

This means E, F^c are independent.

We can apply the same proof to independent events F^c and E to show that F^c and E^c are independent.

5. If E and F are independent and E and G are independent, does this imply that F and G are independent?

If E and F independent, then $P(E \cap F) = P(E)P(F)$, and if E and G are independent then $P(E \cap G) = P(E)P(G)$.

We could have the case where $F = G$.

Then still E and F, and E and G are independent, but clearly F and G are not independent.

So, this is not true.