Homework 5 :: MATH 504 :: Due Tuesday, Oct 11th, 11:59 pm

Your homework submission must be a single pdf called "LASTNAME-hw5.pdf" with your solutions to all theory problem to receive full credit. All answers must be typed in Latex.

1. Create an 5×5 matrix A using the command hilb(5) in Matlab, or scipy.linalg.hilbert(5) in Python. Generate a random vector x, and compute $b = Ax$. Add a tiny amount of noise to b, call it $\hat{b}$. Then recover $\hat{x}$ from $A\hat{x} = \hat{b}$.

   How accurate is the recovered solution? Why did this happen? You don't need to provide any code or console output, just describe what you did and what you got in a few sentence.

2. (Coding) Construct any 3×3 invertible symmetric matrix with no entry equal to 0.

   a) Using the function **eig** in Matlab or equivalent in other programming languages to find the dominant eigenvalue $\lambda^*_{\max}$ and its corresponding eigenvector v^*.

   b) Use the Power Method to find the (approximate) dominant eigenvector $v^{(k)}$ and eigenvalue μ_k of this matrix for different stopping criteria

   $$\frac{\|v^{(k)} - v^*\|_2}{\|v^*\|_2} \leq \epsilon$$

   Record these data in the following table for given different ϵ values.

ϵ	iteration	$\lvert\mu_k - \lambda^*_{\max}\rvert$	$\frac{\|v^{(k)}-v^*\|_2}{\|v^*\|_2}$	$\frac{\|v^{(k)}-v^{(k-1)}\|_2}{\|v^{(k-1)}\|_2}$
10^{-3}				
10^{-6}				
10^{-9}				

   Note that in practice, we don't know the exact eigenvalues and eigenvectors. So the stopping criteria needs to be replaced by $\frac{\|v^{(k)}-v^{(k-1)}\|_2}{\|v^{(k-1)}\|_2} < \epsilon$.

3. (Coding) Build a connected network graph of 5 nodes, that is, a network with 5 pages. Determine the highest rated web page using the page rank approach discussed in the lecture.