Homework # 8
Due 3/26

1. Reading: Sections 6.5.
2. (a) Let X and Y be independent standard normals. Show that $aX + b$ for $a, b \in \mathbb{R}$ is normally distributed with mean b and variance a^2.

   (b) Continuing with the notation of (a), show that $aX + bY$ is normally distributed with mean 0 and variance $a^2 + b^2$.

   (c) Let $X_1, X_2, \ldots, X_n$ be iid standard normals. Use (b) to show that $\sum_{i=1}^{n} a_i X_i$ for $a_i \in \mathbb{R}$ is normal with mean 0 and variance $\sum_{i=1}^{n} a_i^2$. (This important result shows that a linear combinations of independent normals is normal.)

   (d) Given an example of two normally distributed r.v X and Y such that their sum $X + Y$ is not normal. Note that from (b) you must choose X and Y so that they are not independent.
3. Let X be $\mathcal{N}(\mu, \sigma^2)$. Show that $E[X] = \mu$ and $V[X] = \sigma^2$. (This result justifies referring to μ as the mean and σ^2 as the variance.)
4. **Example** 6.53 (feel free to use the authors' solution, but make sure you understand the steps)
5. Exercise 6.54, 6.55
6. Let X, Y be continuous r.v. and A a 2×2 invertible matrix. Define U, V by
$$\begin{pmatrix} U \\ V \end{pmatrix} = A \begin{pmatrix} X \\ Y \end{pmatrix} \tag{1}$$
   What is the Jacobian matrix $\partial(U,V)/\partial(X.Y)$? What is the Jacobian matrix $\partial(X,Y)/\partial(U,V)$? Let $f(x,y)$ be the joint pdf of X, Y, what is the joint pdf of U,V?