**Homework # 11**
Due 4/23

Below, you will have to evaluate integrals involving the normal pdf. For example,

$$\int_a^b \frac{1}{\sqrt{2\pi}} \exp[-x^2/2]dx \tag{1}$$

To evaluate these integrals, you can use the R function **pnorm**. For example,

$$\text{pnorm(b)} = \int_{-\infty}^b \frac{1}{\sqrt{2\pi}} \exp[-x^2/2]dx. \tag{2}$$

You are welcome to use a different resource to compute these intergrals (e.g. Python, MATLAB, web tool, etc.)

1. Reading: sections 8.1 – 8.3

2. Exercise 8.10 (prove everything yourself, don't quote Them 8.6, although you may use the authors' proof of the Theorem to help you), 8.11, 8.32 (calculate the value of the integral as well).

3. Let $U_1, U_2, U_3, \ldots$ be a sequence of r.v's . Show that if $U_n \to c$ in mean square, then $U_n \to c$ in probability.

4. Consider Chapter 5, Problem 1 from a previous hw. Let $X_c$ be the r.v. given by the pdf of the problem for a particular $c$. (In homework 5 you noted that $X_c \to 0$ in probability as $c \to \infty$, although we didn't call it that. Here you'll show that the convergence is actually much faster than that, and you'll show that not all r.v. converge that quickly.)

   (a) Show that $X_c \to 0$ as $c \to \infty$ in mean square and in probability.

   (b) Show that $E[X_c^{2k}] \to 0$ as $c \to \infty$ for any positive integer $k$.

   (c) Provide an example of a sequence of r.v. $Y_1, Y_2, Y_3, \ldots$ that converge in mean square to 0 but for which $E[Y^{2k}]$ does not converge to 0 for any integer $k > 1$.

5. If I flip a coin 1 trillion times, earning a dollar for every head and losing a dollar for every tail, how much can I reasonably expect to earn or owe after I'm done flipping? Roughly, what is the probability that I earn or loss more than 1 million dollars? What is the probability I earn or lose less than $10,000$ dollars? (You should compute values for the answers to these questions using the CLT approximation.)