we know the joint density is the product of the marginals.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2},$$

$$f_{X,Y}(x,y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2}.$$

$W = 2X - Y.$

$$P(W \le a) = P(2X - Y \le a) = \int_{-\infty}^{\infty}\int_{2x-a}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2}\, dy\, dx$$

$$= F(a).$$

$$\frac{d}{da} F(a) = \frac{d}{da} \int_{-\infty}^{\infty}\int_{2x-a}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2}\, dy\, dx$$

$$= \int_{-\infty}^{\infty} \frac{d}{da} \int_{2x-a}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2}\, dy\, dx$$

$$= \int_{-\infty}^{\infty} \frac{d}{da} - \int_{-\infty}^{2x-a} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2}\, dy\, dx$$

$$= \int_{-\infty}^{\infty} - \frac{1}{\sqrt{2\pi}} e^{-(x^2+(2x-a)^2)/2} \frac{d}{da}(2x-a)\, dx$$

$$= \int_{-\infty}^{\infty} - \frac{1}{\sqrt{2\pi}} e^{-(x^2+(2x-a)^2)/2} (-1)\, dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e$$

joint dist is $f_{X,Y}(x,y) = g(x)\,h(y)$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}\right)$$

$$= \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}.$$

$$E\left(\sqrt{x^2+y^2}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{x^2+y^2}\,\frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}\,dy\,dx$$

$$= \int_0^{2\pi}\int_0^{\infty} \sqrt{r^2}\,\frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\infty} \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}\, r^2\,dr\,d\theta.$$

$$\cancel{= \int_0^{2\pi}\int_0^{\infty} \frac{1}{\pi} e^{-u}\, u\,du\,d\theta} \qquad \cancel{u = r^2 \quad du = e^{-r^2/2\sigma^2} d} \qquad \cancel{dy = 2r\,dr \quad u = -}$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty} \left(\frac{r^2}{2\sigma^2}\right) e^{-\left(\frac{r^2}{2\sigma^2}\right)}\,dr\,d\theta$$

$u = \dfrac{r^2}{2\sigma^2}.$

$\cancel{\dfrac{du}{d\theta} = \dfrac{r}{\sigma^2}}$ $\quad \dfrac{du}{dr} = \dfrac{r}{\sigma^2}$

$r\,dr =$

$r^2\,dr$

$\frac{1}{\sigma^2}\,du = r\,dr.$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty}\left(\frac{r^2}{2\sigma^2}\right)e^{-\left(\frac{r^2}{2\sigma^2}\right)} dr\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty} u e^{-u} \frac{\sigma}{\sqrt{2}u^{1/2}} du\, d\theta$$

$$= \frac{\sigma}{\sqrt{2}\pi}\int_0^{2\pi}\int_0^{\infty} u^{1/2} e^{-u} du\, d\theta$$

$$= \frac{\sigma}{\sqrt{2}\pi}\int_0^{2\pi}\int_0^{\infty} w e^{-w^2} 2w\, dw\, d\theta$$

$$= \frac{2\sigma}{\sqrt{2}\pi}\int_0^{2\pi}\int_0^{\infty} w^2 e^{-w^2} dw\, d\theta$$

$$= \frac{\sigma}{\sqrt{2}\pi}\int_0^{2\pi} \frac{\sqrt{\pi}}{2} d\theta$$

$$= \sigma \frac{\sqrt{\pi}}{\pi}\frac{2}{\sqrt{2}}\int_0^{2\pi} d\theta$$

$$= \cancel{\sqrt{\frac{2}{\pi}}\sigma\, \theta \Big|_0^{2\pi} = \sqrt{\frac{2}{\pi}}(2\pi)}$$

$$= \sigma \frac{2\sqrt{\pi}}{\pi\sqrt{2}} (\theta)\Big|_0^{2\pi}$$

$$= \sigma \frac{2 \cdot 2\pi\sqrt{\pi}}{\pi\sqrt{2}}.$$

$$= 2\sqrt{2\pi}\,\sigma$$

$$r = \sqrt{2\sigma^2 u}$$

$$u = \frac{r^2}{2\sigma^2}$$

$$\frac{du}{dr} = \frac{r}{\sigma^2}$$

$$dr = \frac{\sigma^2}{r} du$$

$$dr = \frac{\sigma^2}{\sqrt{2\sigma^2 u}} du$$

$$= \frac{\sigma^2}{\sigma\sqrt{2u}} = \frac{\sigma}{\sqrt{2u}}$$

$$\cancel{\psi = u^{1/2}} \quad w = u^{1/2}$$

$$\cancel{u = \psi^2} \quad u = w^2$$

$$du = 2w\, dw$$

$$u = w \qquad dv = we^{-w^2}$$

$$du = dw \qquad v = -\frac{1}{2}e^{-w^2}$$

$$uv - \int v\, du$$

$$-\frac{1}{2}we^{-w^2}\Big|_0^{\infty} + \frac{1}{2}\int_0^{\infty} e^{-w^2} dw$$

$$(0-0) + \frac{1}{2}\cdot\frac{\sqrt{\pi}}{2}$$