

We know the joint density is the product of the marginals.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2},$$

$$f_{X,Y}(x,y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2}.$$

$$W = 2X - Y.$$

$$P(W \leq a) = P(2X - Y \leq a) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2} dy dx$$

$$\stackrel{\text{Eq 6}}{=} F(a).$$

$$\frac{d}{da} F(a) = \frac{d}{da} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2} dy dx$$

$$= \int_{-\infty}^{\infty} \frac{d}{da} \int_{2x-a}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2} dy dx$$

$$= \int_{-\infty}^{\infty} \frac{d}{da} \int_{-\infty}^{2x-a} \frac{1}{\sqrt{2\pi}} e^{-(x^2+y^2)/2} dy dx$$

$$= \int_{-\infty}^{\infty} \left(-\frac{1}{\sqrt{2\pi}} e^{-(x^2+(2x-a)^2)/2} \right) \frac{d}{da} (2x-a) dx$$

$$= \int_{-\infty}^{\infty} -\frac{1}{\sqrt{2\pi}} e^{-(x^2+(2x-a)^2)/2} (-1) dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x^2+(2x-a)^2)/2} dx$$