MATH 503: Mathematical Statistics
Dr. Kimberly F. Sellers, Instructor
Homework 4

1. Let $X_1, X_2, \ldots, X_n$ represent a random sample from the discrete distribution having the pmf

$$f(x;\theta) = \begin{cases} \theta^x(1-\theta)^{1-x} & x = 0, 1; 0 < \theta < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   Show that $Y_1 = \sum_{i=1}^{n} X_i$ is a complete sufficient statistic for θ. Find the unique function of Y_1 that is the UMVUE of θ.

2. Show that the first order statistic $X_{(1)}$ of a random sample of size n from the distribution having pdf $f(x;\theta) = e^{-(x-\theta)}$, $\theta < x < \infty$, $-\infty < \theta < \infty$, zero elsewhere, is a complete sufficient statistic for θ. Find the unique function of this statistic which is the UMVUE of θ.

3. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, and $\theta > 0$.

   (a) Show that the geometric mean, $(X_1 X_2 \cdots X_n)^{1/n}$ of the sample is a complete sufficient statistic for θ.

   (b) Find the MLE of θ. Note that it is a function of this geometric mean.

4. Let $X_1, X_2, \ldots, X_n$, $n > 2$, be a random sample from a binomial distribution $\mathrm{b}(1, \theta)$.

   (a) Show that $Y_1 = X_1 + X_2 + \ldots + X_n$ is a complete sufficient statistic for θ.

   (b) Find the function $\phi(Y_1)$ which is the UMVUE of θ.

5. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution that is $\mathrm{N}(0, \sigma^2 = \theta)$.

   (a) Show that $Y = \sum_{i=1}^{n} X_i^2$ is a complete sufficient statistic for θ.

   (b) Find the UMVUE of θ^2.

6. Let $X_1, \ldots, X_n$ are iid $\mathrm{N}(\mu, 1)$ random variables. Find the MVUE of $\theta = \mu^2$.