1. Min $-2x_1 - x_2$
   such that

$$x_1 \quad - x_2 \leq 2$$
$$x_1 \quad + x_2 \ \leq 6$$
$$x_1, x_2 \geq 0$$

a) Solve using the simplex method.

Switch to maximize by multiplying objective by -1

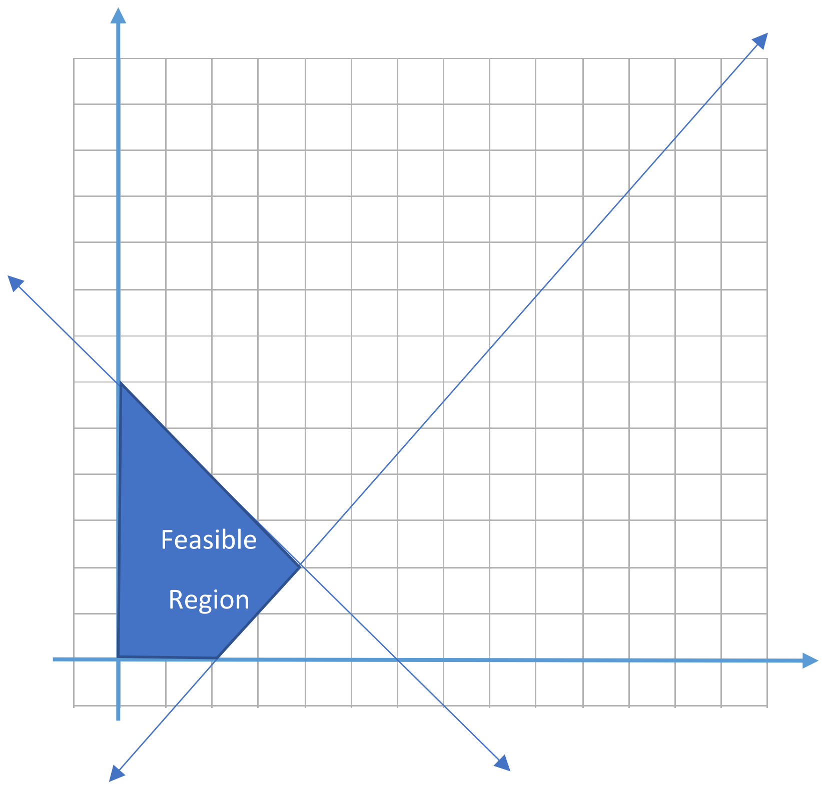
	x_1	x_2	s_1	s_2	RHS	Ratio
s_1	1	-1	1	0	2	2
s_2	1	1	0	1	6	6
z	-2	-1	0	0	0	

	x_1	x_2	s_1	s_2	RHS	Ratio
x_1	1	-1	1	0	2	
s_2	0	2	-1	1	4	2
z	0	-3	2	0	4	

	x_1	x_2	s_1	s_2	RHS
x_1	1	0	1/2	1/2	4
x_2	0	1	-1/2	1/2	2
z	0	0	.5	1.5	10

Final solution at (3,2) with optimal objective function value of -10 (in the original minimization problem.

b) Solve using the graphical method.



Ext Pt	Obj Val
(0,0)	0
(2,0)	-4
(4,2)	-10
(0,6)	-6

2. Min $2x_1 + 3x_2$

such that

$2x_1 \quad + x_2 \leq 600$

$x_1 \quad + x_2 \geq 350$

$x_1 \geq 125$

$x_1, x_2 \geq 0$

a) Solve using the simplex method.

	x_1	x_2	s_1	s_2	s_3	y_1	y_2	y_3	RHS	Ratio
y_1	2	1	1	0	0	1	0	0	600	300
y_2	1	1	0	-1	0	0	1	0	350	350
y_3	1	0	0	0	-1	0	0	1	125	125
	0	0	0	0	0	1	1	1		
z	-4	-2	-1	1	1	0	0	0	-1075	

	x_1	x_2	s_1	s_2	s_3	y_1	y_2	y_3	RHS	Ratio
y_1	0	1	1	0	2	1	0	-2	350	175
y_2	0	1	0	-1	1	0	1	-1	225	225
x_1	1	0	0	0	-1	0	0	1	125	
z	0	-2	-1	1	-3	0	0	4	-575	

	x_1	x_2	s_1	s_2	s_3	y_1	y_2	y_3	RHS	Ratio
s_3	0	1/2	1/2	0	1	1/2	0	-1	175	350
y_2	0	1/2	-1/2	-1	0	-1/2	1	0	50	100
x_1	1	1/2	1/2	0	0	1/2	0	0	300	600
	0	-1/2	1/2	1	0	3/2	0	1	-50	

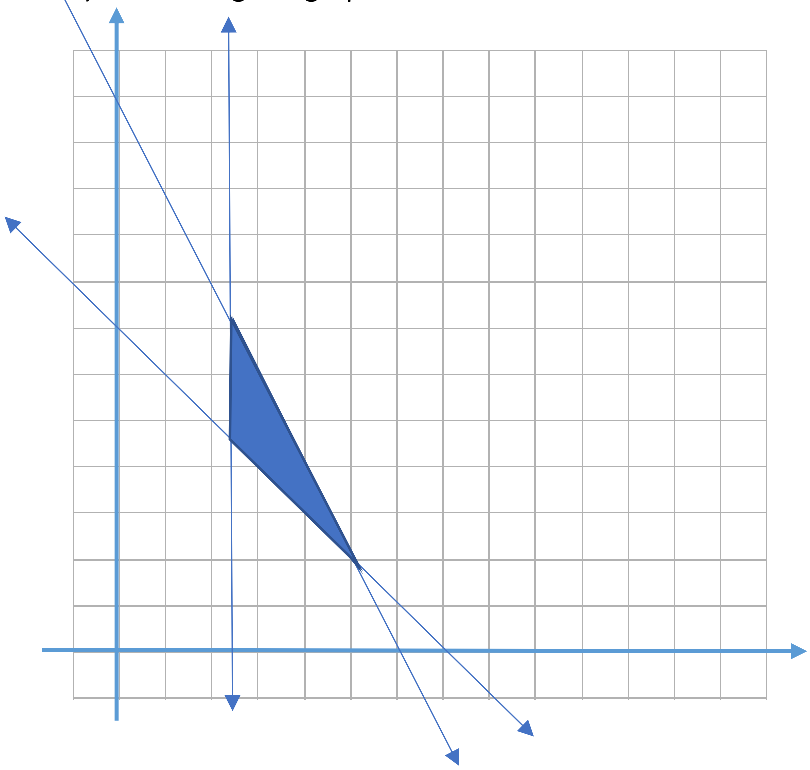
	x_1	x_2	s_1	s_2	s_3	y_1	y_2	y_3	RHS
s_3	0	0	1	1	1	1	-1	-1	125
x_2	0	1	-1	-2	0	-1	2	0	100
x_1	1	0	1	1	0	0	0	0	250
	0	0	0	0	0	1	1	1	0

Using this as a basic feasible solution we setup the simplex tableau for the second phase

	x_1	x_2	s_1	s_2	s_3	RHS
s_3	0	0	1	0	1	125
x_2	0	1	-1	-2	0	100
x_1	1	0	1	1	0	250
	2	3	0	0	0	
z	0	0	1	4	0	-800

Final solution (250, 100) with optimal objective function value of 800 (in the original minimization problem)

b) Solve using the graphical method.



Ext Pt	Obj Val
(125, 225)	925
(125, 350)	1300
(250, 100)	800

3. For the following justify all answers using the sensitivity analysis. For each change indicated state whether the optimal production levels stay the same or change and the new objective function value given the stated change. If any of the changes cannot be calculated with the given information indicate that the Linear Program must be resolved before answering. Show all work.

Max $63E + 95S + 135D$

Such that:

$1E +$	$1S + 1D \leq 200$	Fan Motors
$1E +$	$2S + 4D \leq 320$	Cooling Coils
$8E +$	$12S + 14D \leq 2400$	Hours

$E,S,D \geq 0$

Name	Final Value	Reduced Cost	Coefficient	Min_ Coef	Max_ Coef
Economy	80	0	63	47.5	75
Standard	120	0	95	87	126
Deluxe	0	-24	135	-Inf	159

Name	LHS	RHS	Shadow_ price	Min_ RHS	Max_ RHS
Fan Motors	200	200	31	160	280
Cooling Coils	320	320	32	200	400
Hours	2080	2400	0	2080	Inf

a.) Increase Cooling Coils by 40, Fan Motors by 40, and Hours by 500.

All of the changes are in the right hand side (RHS) of the constraints so we must first check if the add up to at most 100% of the allowable change thus 40/80+40/80+1/1E+30 = .5 + .5 + 0 = 1 which is the maximum allowable change at which we are guranteed that the shadow prices are valid. Because we are changing the RHS we are changing the feasible region and know that the optimal production level will change. We use the shadow prices to calculate the change in the objective function value at the optimal solution. Thus 40*31=1240 is the increase to the objective function because of 40 new fan motors. All together we have 40*31+40*32+500*0=2520 which is how much the objective function value will increase. We can calculate the old objective function value by 80*63+120*95+0*135=16,440 (from the information about the objective function at the optimal solution). And thus the new solution is 18,960.

b.) Decrease Cooling Coils and Fan Motors by 30.

Similar to last time we check the 100% rule for changes in the RHS and see 30/40+30/120=.75+.25=1 which means that the shadow prices are valid for the changes. Again because we are changing the RHS the production levels and the objective function value at the optimal solution will change. When we decrease the RHS we subtract the shadow price for each unit we decrease it by. Thus -30*31-30*32=-930-960=-1890 thus we've decreased the old optimal solution 16,440 by 1890 which gives us 16,440-1,890=14,550.

c.) Raise price of all models (Economy, Standard, and Deluxe) by $8.

If we raise the price we are changing the coefficients in the objective function. To check if this is a valid change we use the 100% rule for changes to the objective function coefficients. The allowable increases are 12, 31, and 24 respectively which is the valid range for changing a single coefficient at a time. Applying the 100% rule we see 8/12+8/31+8/24= .67+.258+.33 =1.258. This is more than the allowable change so we must **re-solve** the problem to know anything with certainty.

d.) Decrease the price of standard by 4 and increase the price of deluxe by 12.

We must first check that the 100% rule for changes in the coefficients is satisfied. 4/8+12/24=1 so the optimal production levels will stay the same, but since the prices change the objective function value will change. We can calculate this two ways:

Changing the coefficients and resolving with the optimal solution
80*63+ 120*(95-4)+0*(135+12)=5,040 + 10,920 + 0 = 15,960

Calculating the change from the old objective function
Old Profit –Decrease in Price*#units sold of that type+Increase in Price* #units sold of that type
16,440 – 4*120 +12 *0=16,440-480=15,960