

Homework 7 :: MATH 504 :: Due Tuesday, October 27th, 11:59 pm

Your homework submission must be a single pdf called “LASTNAME-hw7.pdf” with your solutions to all theory problem to receive full credit. All answers must be typed in Latex.

1. Consider the sets

$$C = \{(x, y) \mid \|x\|_2 \leq y\} \quad \text{and} \quad \hat{C} = \{(x, y) \mid \|x\|_2^2 \leq y\}.$$

Determine whether the sets C and \hat{C} are convex or not?

2. Consider the smooth (differentiable) functions $h : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R}^n \rightarrow \mathbb{R}$. Prove that the function

$$f = h \circ g : \mathbb{R}^n \rightarrow \mathbb{R}$$

where

$$f(x) = h(g(x)) \quad \text{and} \quad \text{dom } f = \{x \in \text{dom } g \mid g(x) \in \text{dom } h\}$$

is convex if one of the following conditions on h and g holds.

- (a) If h and g are convex functions, and h is nondecreasing, or
- (b) if h is convex and nonincreasing, and g is concave.