Homework 6

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1. Problems 8.D:1

1. Give an example as requested in Note 8.2 to show that a sampling plan where $\pi_i = \pi_j$ for all u_i, u_j is not necessarily simple random sampling.

we must differentiate between simple random sampling, where each set of units (or sample) has equal probability of selection, and equal probability sampling, where each unit has equal probability but not each set.

If we order all units and assign them IDs, and then randomly select a valid ID, p, and then select every k^{th} ID after that, or each $p+ik$, we have equal probability for units but not for sets. If $k=2$, then $\{1, 3, 5, \ldots\}$ is possible but $\{1, 2, 3, \ldots\}$ is not. So it is not SRS.

2. Problems 9.A: 1, 2, 3

1. Is s an unbiased estimator of S? why?

Recall that s^2 is the sampling variance and S^2 is the population variance. In Lemma 8.6 from the notes, we see that $E(s^2) = S^2$.

we know that s is an unbiased estimator of S if $E(s) = S$.

s is not an unbiased estimator of S because the square-root function is non-linear.

Expectation is not commutative with non-linear functions.

$s = \sqrt{s^2}$. Since s is a r.v., $Var(s) > 0$.

$$Var(s) = E(s^2) - E(s)^2 = S^2 - E(s)^2 > 0.$$

so $S^2 > E(s)^2 \Rightarrow S > E(s)$.

2. Find an unbiased estimator of σ^2.

we know that $S^2 = \frac{N}{N-1}\sigma^2$, where

$$\sigma^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N} \quad \text{and} \quad S^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}.$$

we have an unbiased estimator of S^2;

$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$. Therefore, if we multiply by the sam inverse factor $\frac{N-1}{N}$, to get $\frac{N-1}{N}s^2$, then we have $E(\frac{N-1}{N}s^2) = \frac{N-1}{N}E(s^2) = \frac{N-1}{N}S^2 = \sigma^2$.

3. If $y_i = 0$ or 1, show that $s^2 = \frac{n}{n-1}\hat{p}(1-\hat{p})$.

We know $s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$. From theorem 6.1

we know that $\sigma^2 = p(1-p)$. We know $S^2 = \frac{N}{N-1}\sigma^2$.

and so $S^2 = \frac{N}{N-1}p(1-p)$.

we then can consider an estimator of S^2, which

is σ^2. ~~It then~~ we know $E(\hat{p}) = p$.

So $E(s^2) = \frac{n}{n-1}E(\hat{p}(1-\hat{p})) \Rightarrow \frac{N}{N-1}p(1-p)$.

3. Problems 10.A:1

In example 10.3, why are the details of cases $n = 5, 6, 7, 8$ not given?

Intuitively, we see that if N is population size, and n is sample size, then as n increases but remains ~~much~~ fairly lower than N, the sampling distribution approaches Normal. However, as n increases further and approaches N, the sampling distribution approaches the probability distribution of the underlying r.v. when $n = N$, the sampling and prob distributions are the same

This phenomenon is shown in Example 10.2.