

## Homework # 2

Due 2/10

1. Reading:

- Sections 1.7, 1.8.
- Appendix A. (Appendix A discusses combinatorics but is, in my opinion, too terse. You may instead want to read the beginning of the Wikipedia article on combinations, <https://en.wikipedia.org/wiki/Combination>).

2. Exercises: 1.42, 1.44, 1.45 (you can restrict your answer to the case  $m = 2$ ), 1.52
3. Consider the outcome of a single die roll. Find two events  $A$  and  $B$  that are independent, but neither of which are the empty set or the whole sample space. Show that the events are independent.
4. Problem 5 (see equation (1.40) for a definition of independence of multiple sets). Intuitively, when we say two events are independent, we mean that knowing that one event occurs (i.e. an outcome occurs that is an element of the event) does not change the probability that the other event occurs. This intuition corresponds to the equation:  $P(A | B) = P(A)$ , meaning that conditioning on  $B$  occurring doesn't change the probability of  $A$  occurring. Show that if three events  $D, E, F$  are independent then  $P(D | E \cap F) = P(D)$  and explain intuitively, meaning just in non-technical words, why  $A$  is not independent of  $B \cap C$ . Then, explain intuitively why  $A$  is independent of  $B$  and why  $A$  is independent of  $C$ .
5. Problems: 7, 8 (ignore the expansion in Stirling's formula and set  $n = 300$ ), 9 (you can give the probability in any form you like, not necessarily the form shown in the problem), 10 (Hint: For each circuit there are 5 switches, which can be closed or open, making the sample space of size  $2^5 = 32$ . Computing the probability that current flows by summing over all 32 options is cumbersome, involving a sum of 32 terms. Instead, partition the space based on whether a particular line of flow, of which there are 3, is open and closed. This will lead to considering

8 subsets of the sample space, each of which is not hard to compute. Actually, even better, you can also partition the sample space into 4 disjoint sets, each of which is not hard to compute.).