

# MATH 502, HOMEWORK ASSIGNMENTS

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ABSTRACT. Midterm Exam I for Math 502

## 1. MIDTERM EXAM I FOR MATH 502

**Attention:** This exam is for you to do independently. Communication to any other person, except the teacher, about this exam is disallowed. You can consult your notes, previous homework and textbooks. Each problem is 10 points.

Please note the closing time at the canvas link. Give yourself enough time before the closing time to do this exam.

————— **Exam problems:** —————

A1. Let  $\mathbf{x}, \mathbf{y}$  be two vectors in  $\mathbb{R}^n$ .

a) Compute  $\mathbf{x}\mathbf{y}^T$ .

b) Compute  $\det(\mathbf{x}\mathbf{y}^T)$ .

A2.  $A$  is a square matrix. Show that  $A^{-1}$  exists if and only if  $A$ 's column vectors are linearly independent.

A3. Rotate the standard  $(x_1, x_2)$  Cartesian coordinate system by  $60^\circ$  to get a new coordinate system. Find the relation of the coordinate of the same vector under these two coordinate systems.

A4. Given  $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$ .

a) Show that  $A$  is an orthogonal matrix.

b) Can the columns of  $A$  form an orthonormal basis of  $\mathbb{R}^3$ ? Why?

c) Find the  $x$ 's of the equation

$$\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = x_1 \mathbf{u}_1 + x_2 \mathbf{u}_2 + x_3 \mathbf{u}_3$$

where  $\mathbf{u}_j$  is the  $j$ -th column of  $A$ .

A5 Let  $\mathbf{u}, \mathbf{v}$  be two vectors in an inner product space  $V$ . Show that  $\|\mathbf{u} + \mathbf{v}\|^2 \leq (\|\mathbf{u}\| + \|\mathbf{v}\|)^2$ .

B1 Solve the following ODEs.

a)  $tx' - 2x = t^3$ .

b)  $x'' + 4x' - 12x = t^2$

c)  $x'' + 4x' + 4x = e^t$ .

d)  $x'' - 4x' + 5x = e^{2t} \cos t$ .

B2 A container with  $100m^3$  volume of salty water in it. Assume that salt in the water is always well mixed. Water flows out of the container at the rate of  $20m^3/hour$ . In the meanwhile, salty water containing  $0.1kg/m^3$  of salt is flows into the container at the rate of  $20m^3/hour$ . At the time  $t = 0$  hour, the container has 5 kg of salt in it. How much salt is in the container at time  $t$ ?

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