Homework # 4

Due 3/5

1. Reading: Sections 5.5 – 5.6
2. Exercises: 5.54, 5.55 (A chi-squared distribution with one degree of freedom has density $f(x) = \frac{1}{\sqrt{2\pi x}} \exp[-x/2]$), 5.68.
3. Chapter 5, Problem 1. Also, describe what will happens to the density $f(x)$ as $c \to \infty$. Draw a graph of $f(x)$ representing the shape of the pdf for small, medium, and large values of c. Show,
$$\lim_{c\to\infty} P(|X| < a) = 1 \tag{1}$$
for any $a > 0$, where X is the r.v. with pdf $f(x)$. Explain what this limit shows.

   The two problems below demonstrate two important techniques involving normal r.v.

4. Exercise 5.69. This exercise involves an important technique that comes up in handling normal pdfs. Warm up by showing the following relations,
$$-\frac{x^2}{2} + x = \frac{-1}{2}(x-1)^2 + \frac{1}{2}, \tag{2}$$
To show the above relation, don't just expand the right hand side and verify equivalence, instead apply completing the square to the left to derive the right. Completing the square in the context of normal pdfs is the technique you need to know. Then show,
$$\int_{-\infty}^{\infty} \exp[-\frac{(x-1)^2}{2}]dx = \sqrt{2\pi} \tag{3}$$
Hint: view this as a piece of a normal pdf. Now you're warmed up and ready to do the exercise!
5. Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Define the r.v Y by $Y = (X-\mu)/\sigma$. Show that $Y \sim \mathcal{N}(0, 1)$. This problem shows how to rescale a normal so that it becomes a standard normal. The reason we say we are rescaling is because Y can be thought of as X under a

change of units. For example, consider computing temperature in Celsius (C) from Farenheit (F): $C = (F - 32)/(9/5)$.