Final

Due 5/20

1. These problems aim to review partitioning of a sample space as a tool for computing probabilities
   (a) Let X, Y be independent, $\mathcal{N}(\mu, 1)$. Calculate $P(X > |Y|)$. You can leave your answer as the sum of integrals.
   (b) Chapter 1, Problem 17. As you prove the equation in the problem, be sure to explain how your arguments follow from the definition of a probability measure in Definition 1.13.

2. These problems aim to review computations with pdfs of continuous r.v.
   (a) Exercise 6.61
   (b) Chapter 6, Problem 19a.
   (c) Let $X \sim \mathcal{N}(0, \Sigma)$ where $X = (X_1, X_2, \ldots, X_n)$ and Σ is the $n \times n$ covariance matrix of X. Show that $Y = AX$ is also a multivariate normal for any $n \times n$ invertible matrix A and compute the mean and variance of Y.

3. List the unique properties of multivariate normals that we have proven through the course of the semester. Pick one of these results and prove it. (Try to choose a result with a proof that does not come directly to your mind. If you need to, you can look back at previous solutions.)

4. These problems aim to review CLT.
   (a) Consider the game in which we flip a fair coin repeatedly, winning 1 dollar when the coin lands heads, and losing 1 dollar when the coin lands tails. A fellow probabilist makes the following comment to you, "Since the coin is unbiased, the outcome with the highest probability is an equal number of heads and tails. Therefore, after many coin flips, we should not expect to have won or lost much money." What is your response?
   (b) Now suppose the coin is biased with probability p of landing heads. What is the minimum value of p with which

I can be roughly 90% certain that after 1 million flips, I will not owe any money? (You can use R or Python to calculate integrals involving the pdf of a normal.)