

### Homework 3 :: MATH 504 :: Due Thursday, September 29th, 11:59 pm

Your homework submission must be a single pdf called “LASTNAME-hw1.pdf” with your solutions to all theory problem to receive full credit. All answers must be typed in Latex.

Let

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

- a) Use the LU factorization to express  $A = LU$  where  $L$  is a lower triangular and  $U$  is an upper triangular matrices.
- b) Obtain the solution  $x^*$  of the system  $Ax = b$  using  $LU$  factorization of  $A$  together with forward and backward substitution.
- c) Use Jacobi method twice, starting with  $x^{(0)} = [1, 1, 0]^T$  to find an approximate to the solution. Report the error  $\|x^{(k)} - x^*\|_\infty$ ,  $k = 1, 2$ .
- d) Write a code for the Gauss-Seidel method, and apply it to find the solution of the system  $Ax = b$  with  $10^{-5}$  digits of accuracy. That is  $\|x^k - x^*\| \leq 10^{-5}$  where  $x^*$  is the solution of the system  $x^* = A^{-1}b$  and  $x^k$  is the  $k$ th iterate of Gauss-Seidel.