MATH 503: Mathematical Statistics
Dr. Kimberly F. Sellers, Instructor
Homework 1

1. If C_1 and C_2 are independent events, show that C_1' and C_2 are also independent.

2. Find the constant c so that
$$p(x) = c\left(\frac{2}{3}\right)^x, \qquad x = 1, 2, 3, \ldots, \qquad \text{zero elsewhere}$$
is a pmf.

3. Determine the value of c that makes
$$f(x) = c\sin(x), \qquad 0 < x < \frac{\pi}{2}$$
a pdf.

4. Let X have a pmf $p(x) = \frac{1}{3}$, $x = 1, 2, 3$, zero elsewhere. Find the pmf of $Y = 2X + 1$.

5. Let X have a pdf $f(x) = \frac{x^2}{9}$, $0 < x < 3$, zero elsewhere. Find the pdf of $Y = X^3$.

6. Let $f(x) = 2x, 0 < x < 1$, zero elsewhere, be the pdf of X.
   (a) Compute $E(1/X)$.
   (b) Find the cdf and the pdf of $Y = 1/X$.
   (c) Compute $E(Y)$ and compare the result with the answer obtained in Part (a).

7. Let X be a random variable with a pdf $f(x)$ and mgf $M(t)$. Suppose f is symmetric about 0, i.e. $f(-x) = f(x)$. Show that $M(-t) = M(t)$.

8. Let X_1 and X_2 be two independent random variables. Suppose that X_1 and $Y = X_1 + X_2$ have Poisson distributions with means μ_1 and $\mu > \mu_1$, respectively. Find the distribution of X_2.

9. Suppose X is a random variable with the pdf $f(x)$ which is symmetric about 0, i.e. $f(-x) = f(x)$. Show that $F(-x) = 1 - F(x)$, for all x in the support of X.

10. Let X_n have a gamma distribution with parameter $\alpha = n$ and β, where β is not a function of n. Let $Y_n = X_n/n$. Find the limiting distribution of Y_n.

11. The Pareto distribution, with parameters α and β, has pdf

$$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \qquad \alpha < x < \infty, \qquad \alpha > 0, \qquad \beta > 0.$$

   (a) Verify that $f(x)$ is a pdf.

   (b) Derive the mean of this distribution.