Homework # 4

Due 2/25

1. Reading: Sections $2.5, 5.1 - 5.4$
2. Lett X be a discrete r.v., the setting of chapter 2. Below are some essential facts involving expected value and variance that you have encountered in the reading and previous exercises but which I want you to demonstrate yourself, in some cases for the second time, because of their importance. Don't quote theorems or results from the book, demonstrate each fact starting from the definition of expected value and variance.
   (a) Let $a, b \in \mathbb{R}$. Show $E[aX + b] = aE[X] + b$. (This result shows that we can pull constants out of the expectation.)
   (b) Let $g(x) : \mathbb{R} \to \mathbb{R}$ and $h(x) : \mathbb{R} \to \mathbb{R}$. Show $E[g(X) + h(X)] = E[g(X)] + E[h(X)]$. Here you can use the result, $E[g(X)] = \sum_{x \in \mathrm{Im}(x)} P(X = x)g(x)$. (This result shows that the expectation of a sum is the sum of the expectations.)
   (c) Let $a, b \in \mathbb{R}$. Show $V[aX + b] = a^2 V[X]$, where $V[X]$ is the variance of X. (Note the difference to (a))
   (d) Give an example of a discrete r.v. X such that $V[X+X^2] \neq V[X] + V[X^2]$. (This result shows that variance of a sum is **not** the sum of the variances.)
   (e) $V[X] = E[X^2] - E[X]^2$ where $V[X]$ is the variance of X.
3. Exercises 5.12, 5.30, 5.31, 5.32,
4. Using the change to polar coordinates described in Exercise 5.47. show that

$$\int_{-\infty}^{\infty} \exp[\frac{-x^2}{2}] = \sqrt{2\pi} \tag{1}$$

where $\exp[z] = e^z$. Then show that the pdf of a normal r.v. integrates to 1:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp[\frac{-(x-\mu)^2}{2\sigma^2}] = 1. \tag{2}$$

Hint: apply the change of variable $y = (x-\mu)/\sigma$ to the integral.