Social Network Analysis MATH-517: Homework 2

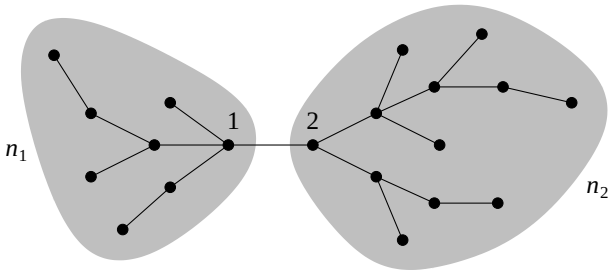
1. Consider a connected k-regular undirected network (i.e., a network in which every node has degree k).

   (i) Show that the uniform vector $\mathbf{1} = (1,1,1,\ldots)$ is an eigenvector of the adjacency matrix with eigen-value k.
   In a connected network there is only one eigenvector with all elements positive and hence the eigen-vector $\mathbf{1}$ gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every node.

   (ii) Find the Katz centralities of all nodes in the network as a function of k.

   (iii) You should find that, like the eigenvector centrality, the Katz centralities of all nodes are the same. Name a centrality measure that could give different centralities for different nodes in a regular network.

2. Consider an undirected tree of n nodes. A particular edge in the tree joins nodes 1 and 2 and divides the tree into two disjoint regions of n_1 and n_2 nodes as sketched here:



   Show that the closeness centralities C_1 and C_2 of the two nodes, defined according to Eq. (7.21, are related by

$$\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} + \frac{n_2}{n}.$$

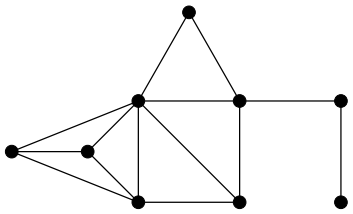
3. Consider an undirected tree of n nodes. Suppose that a particular node in the tree has degree k, so that its removal would divide the tree into k disjoint regions, and suppose that the sizes of those regions are $n_1 \ldots n_k$.

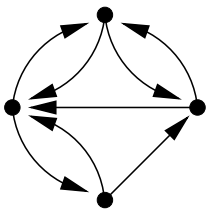
   (i) Show that the unnormalized betweenness centrality x of the node, defined by $x_i = \sum_{st} n_{st}^i / g_{st}$, where n_{st}^i is the number of paths from s to t through i and g_{st} is the total number of paths from s to t, is

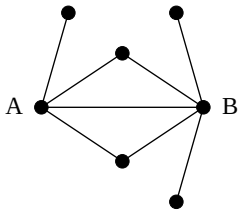
$$x = n^2 - \sum_{m=1}^{k} n_m^2.$$

   (ii) Hence or otherwise calculate the betweenness of the ith node from the end of a "line graph" of n nodes, i.e., n nodes in a row like this:

4. Consider these three networks:







(i) Find a 3-core in the first network.

(ii) What is the reciprocity of the second network?

(iii) What is the cosine similarity of nodes A and B in the third network?

5. Consider the following studies

(ii) In a survey of couples in the city of San Francisco in 1992, Catania *et al.* recorded, among other things, the ethnicity of interviewees and calculated the fraction of couples whose members were from each possible pairing of ethnic groups. The fractions were as follows:

		Women				
		Black	Hispanic	White	Other	Total
Men	Black		0.016	0.035	0.013	0.323
	Hispanic	0.012	0.157	0.058	0.019	0.247
	White	0.013	0.023	0.306	0.035	0.377
	Other	0.005	0.007	0.024	0.016	0.053
	Total	0.289	0.204	0.423	0.084	

Assuming the couples interviewed to be a representative sample of the edges in the undirected network of relationships for the community studied, and treating the nodes as being of four types—black, hispanic, white, and other—what are the numbers e_r and a_r that appear in Eq.(7.58) for each type? Using these numbers calculate the modularity of the network with respect to ethnicity.

6. a. Review and execute all examples from chapter 1 and chapter 2 of the following book "Network Science with Python and NetworkX Quick Start Guide "By Edward L. Platt b. Modify the examples in chapters 3-7(at least one example per chapter by providing your own dataset.

Deliverables:

1.Word document with answers Questions 1-5 and screenshots of generated networks for a Question 6.

Name your file FirstInitialLastName_HW2.docx

2.YourFirstInitialLastNameChapter3_<example#>.py

References:

1. Mark Newman, Networks 2ed, Chapter 7

2. Edward L Platt, Network Science with Python and NetworkX, Chapter3-7