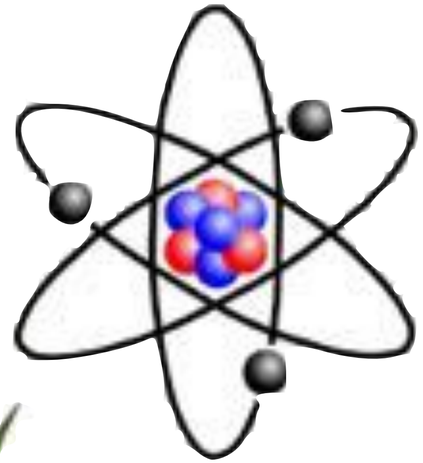
set Notation

Elements

Foundations, building blocks of sets



Lithium

Can be anything

Messi

Google

Aspirin

Structured

Numbers

→ 0 1 2 3 4

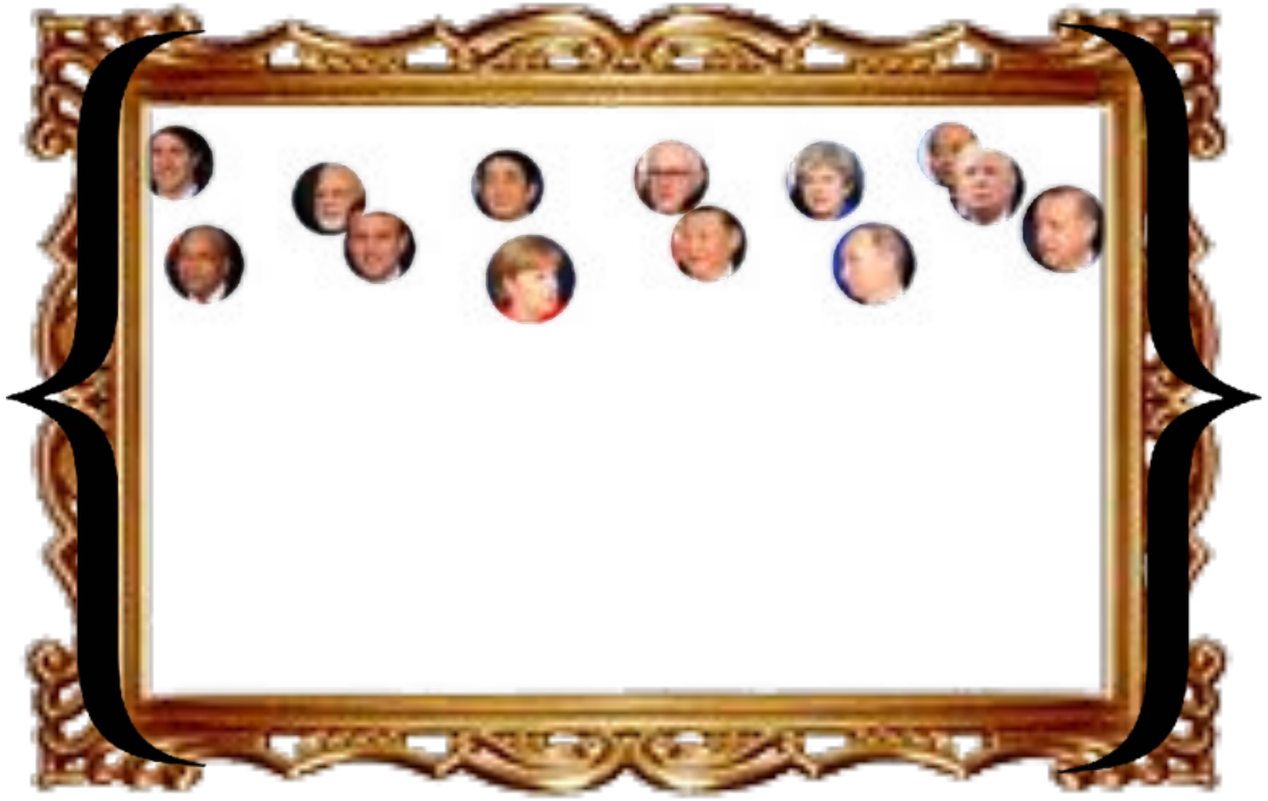
Elements to Sets

Beyond individual elements

“Bigger picture”

Set Collection of elements

Define specify elements



Specification

Explicit

- **Coin** { heads, tails }

- **Bits** { 0, 1 }
- **Die** { 1, 2, 3, 4, 5, 6 }

Implicit

- **Digits** { 0, 1, …, 9 }
- **Letters** { a, b, …, z }
- **Days** { Monday, …., Sunday}

Descriptive {four-letter words} = {love , like, dear ,…}

More

Compact ✅

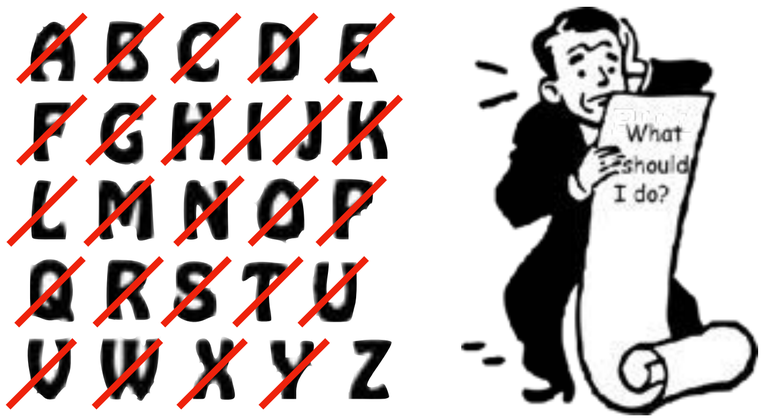
Expressive ✅

Ambiguous ⚠️

Common Sets

Integers	{ …, -2, -1, 0, 1, 2, … }	$\mathbb{Z}$
Naturals	{ 0, 1, 2, … }	$\mathbb{N}$
Positives	{1, 2, 3,… }	$\mathbb{P}$
Rationals	{ integer ratios m/n, n≠0 }	$\mathbb{Q}$
Reals	{ … Google … }	$\mathbb{R}$

Mnemonic



Zahl - number

CONVENIENTTION

Sets	UPPER CASE	A
Elements	lower case	a

Membership

If element x is in a set A, it is a member of, or belongs to A, denoted $x \in A$

$0 \in \{0,1\}$ $1 \in \{0,1\}$ $\pi \in \mathbb{R}$

Equivalently, A contains x, written $A \ni x$

$\{0,1\} \ni 0$ $\{0,1\} \ni 1$ $\mathbb{R} \ni \pi$

If x is not in A, then x is not a member, or does not belong to A, denoted $x \notin A$

$2 \notin \{0,1\}$ $\pi \notin \mathbb{Q}$

Equivalently, A does not contain x, $A \not\ni x$

$\{0,1\} \not\ni 2$ $\mathbb{Q} \not\ni \pi$

Set of States

United

$\in$

$\notin$

$\in$

$\notin$

$\in$

$\notin$

$\in$

$\notin$

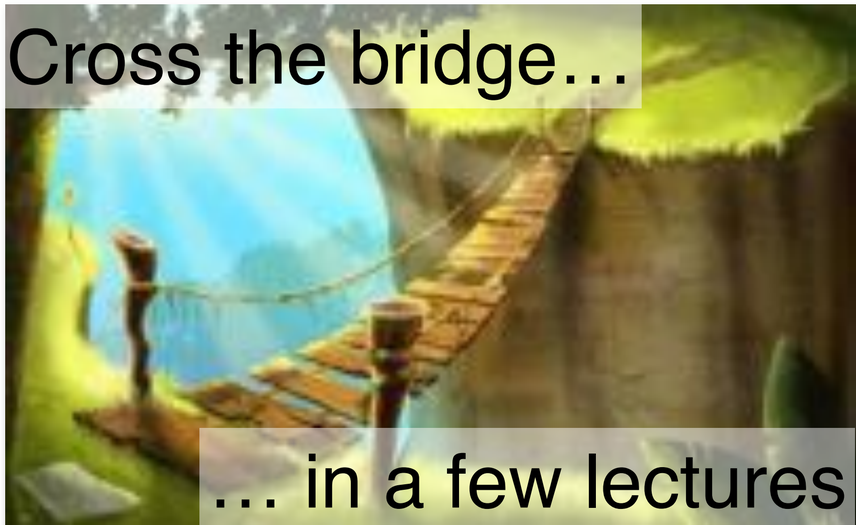
Doesn’t Matter

Order $\{0, 1\} = \{1, 0\}$

Repetition $\{0,1\} = \{0,1,1,1\}$

Order matters: use ordered tuples $(0,1) \neq (1,0)$

Repetition matters: use multisets, or bags



Special Sets

Empty set — contains no elements — $\varnothing$ or $\{\,\}$ — $\forall x,\ x \notin \varnothing$ — $\forall$ - All, every

Universal set — all possible elements — Ω — $\forall x,\ x \in \Omega$

Ω lets us consider only relevant elements

$\Omega = \mathbb{Z}$ integers

"prime"

Means 2, 3, 5,…

Not

amazon Prime

Ω depends on application

temperature $\Omega = \mathbb{R}$

text $\Omega = \{\text{words}\}$

Only one $\varnothing$ — set with no elements

Sets

Set Definition

{…} or **set({…})**

```
Set1 = {1,2}
print(Set1)
{1,2}
```

```
Set2 = set({2,3})
print(Set2)
{2,3}
```

For empty set use only **set()** or **set({})**

```
Empty1 = set()
type(Empty1)
set
print(Empty1)
set{}
```

```
Empty2 = set({})
type(Empty2)
set
print(Empty2)
set{}
```

```
NotASet = {}
type(NotASet)
dict
```

{} is not an empty set

Membership

∈ - **in**

```
Furniture = {‘desk’,’chair’}
’desk’ in Furniture
True
’bed’ in Furniture
False
```

∉ - **not in**

```
Furniture = {‘desk’,’chair’}
’desk’ not in Furniture
False
’bed’ not in Furniture
True
```

Testing if Empty, Size

Test empty

`not`

```
S = {1,2}
not S
False
```

```
T = set()
not T
True
```

Size

`len()`

```
print(len(S))
2
```

```
print(len(T))
0
```

Check if size is 0

`len()==0`

```
print(len(S)==0)
False
```

```
print(len(T)==0)
True
```

Basic Sets

ol

New toy~~x~~s

Intervals

Divisors/multiples

Useful later

Sets within Sets

Specify a set within a universe, or any other set

$\{ x \in A \mid \ldots \}$ = { elements x in A such that …} or $\{ x \in A : \ldots \}$

$\mathbb{N} = \{ x \in \mathbb{Z} \mid x \geq 0 \}$ $\mathbb{P} = \{ x \in \mathbb{N} \mid x > 0 \}$

Solutions to equations

$\{ x \in \mathbb{R} \mid x^2 \geq 0 \} = \mathbb{R}$ $\{ x \in \mathbb{R} : x^2 = 1 \} = \{ -1, 1 \}$

$\{ x \in \mathbb{R} \mid x^2 = 0 \} = \{ 0 \}$ ← a single-element set is a singleton

$\{ x \in \mathbb{R} \mid x^2 = -1 \} = \varnothing$ $\{ x \in \mathbb{C} \mid x^2 = -1 \} = \{ i, -i \}$

Integer Intervals

$\{m,\ldots, n\} = \{i \in \mathbb{Z} \mid m \leq i \leq n\}$

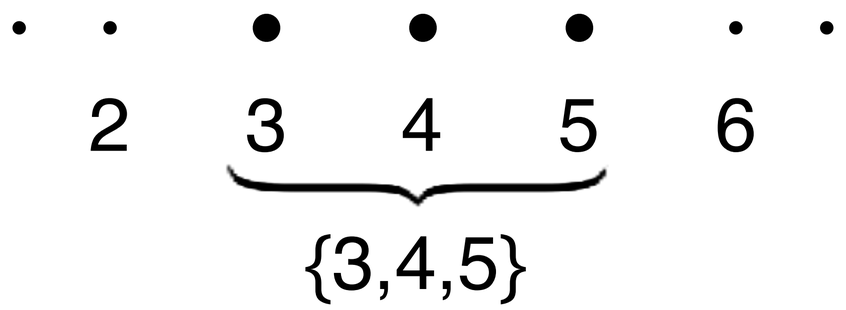
Integers from m to n, inclusive

$\{3,\ldots,5\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 5\} = \{3,4,5\}$

$\{3,\ldots,4\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 4\} = \{3,4\}$

$\{3,\ldots,3\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 3\} = \{3\}$

$\{3,\ldots,2\} = \{i \in \mathbb{Z} \mid 3 \leq i \leq 2\} = \varnothing$



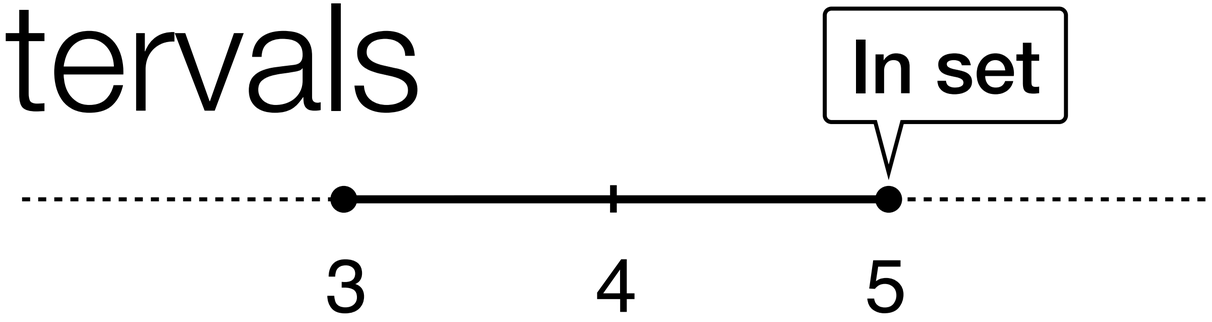
CONVENIENTTION

$[n] = \{1\ldots,n\}$

Real Intervals

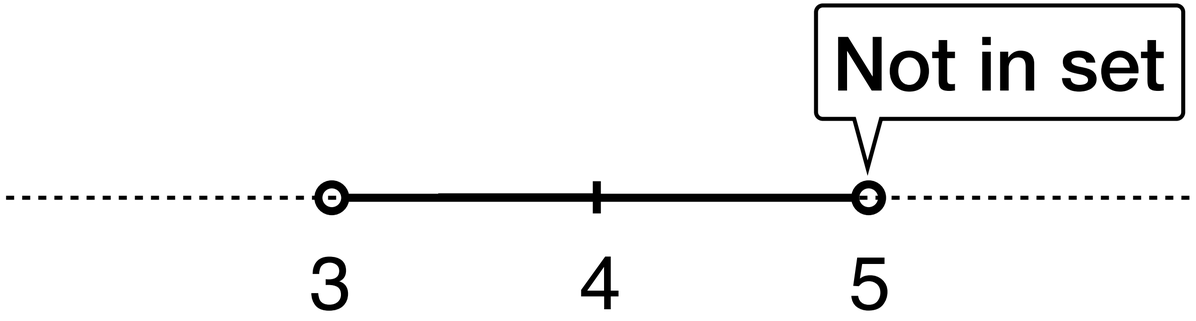
In set

$[a,b]$ $\{x \in \mathbb{R} \mid a \leq x \leq b\}$ $[3,5]$

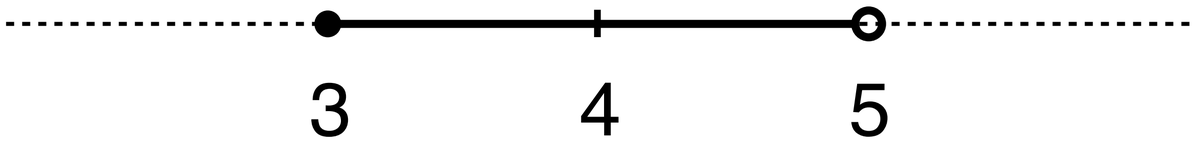


Not in set

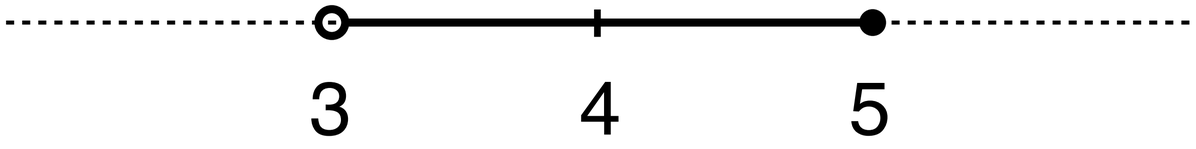
(a,b) $\{x \in \mathbb{R} \mid a < x < b\}$ $(3,5)$



$[a,b)$ $\{x \in \mathbb{R} \mid a \leq x < b\}$ $[3,5)$



$(a,b]$ $\{x \in \mathbb{R} \mid a < x \leq b\}$ $(3,5]$



$[3,3] = \{3\}$ singleton

$[3,2] = [3,3) = (3,3] = \varnothing$

Divisibility

$m, n \in \mathbb{Z}$

if $n = c \cdot m$ for some $c \in \mathbb{Z}$, we say that n is a multiple of m, or m divides n, and write $m \mid n$

$6 = 2 \cdot 3 \rightarrow 3 \mid 6$

n c m → m n

$-8 = (-2) \cdot 4 \rightarrow 4 \mid -8$

n c m → m n

$0 = 0 \cdot (-2) \rightarrow -2 \mid 0$

If no such c exists, m does not divide n, or n is not a multiple of m, denoted $m \nmid n$

There is no $c \in \mathbb{Z}$ such that $4 = c \cdot 3$ $\rightarrow 3 \nmid 4$

$0 \nmid n$ for any $n \neq 0$

Multiples

$3 \mid ?$	$\{\ldots, -6, -3, 0, 3, 6, \ldots\}$
$1 \mid ?$	$\mathbb{Z}$
$0 \mid ?$	0

Divisors

$? \mid 4$	$\pm 1, \pm 2, \pm 4$	
$? \mid 0$	$\mathbb{Z}$	
$? \mid$ any $n \neq 0$	± 1	$\pm n$

Set of Multiples

$m \in \mathbb{Z}$

${}_m\mathbb{Z} \stackrel{\text{def}}{=} \{ i \in \mathbb{Z} : m \mid i \}$

integer multiples of m

${}_2\mathbb{Z} = \{\ldots, -4, -2, 0, 2, 4, \ldots\} \stackrel{\text{def}}{=} \mathbb{E}$

Even numbers

${}_1\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} = \mathbb{Z}$

${}_0\mathbb{Z} = \{0\}$

$m \in \mathbb{Z}, n \in \mathbb{P}$

${}_m[n] \stackrel{\text{def}}{=} \{ i \in [n] : m \mid i \}$

multiples of m in {1..n}

${}_3[13] = \{ i \in \{1,\ldots,13\} : 3 \mid i \} = \{3, 6, 9, 12\}$

${}_7[13] = \{7\}$

${}_1[13] = [13]$

${}_{14}[13] = {}_0[13] = \varnothing$

Will use shortly

Intervals, Multiples

{0,..,n-1}

`range(n)`

note: [n] = {1…n}

```
print(set(range(3)))
{0,1,2}
```

Returns type **`range`**

{m,..,n-1}

`range(m, n)`

To print, convert to set

```
print(set(range(2, 5)))
{2,3,4}
```

{m, m+d, m+2d, … } < n-1

`range(m, n, d)`

```
print(set(range(2, 12, 3)))
{8,2,11,5}
```

Basic Sets

ol
New to~~y~~s

Intervals {m,..,n} [n] [a,b)

Divisors/multiples mln

Sets of multiples $_d[n]$

gram
set Visualization

Venn Diagram

Instagram

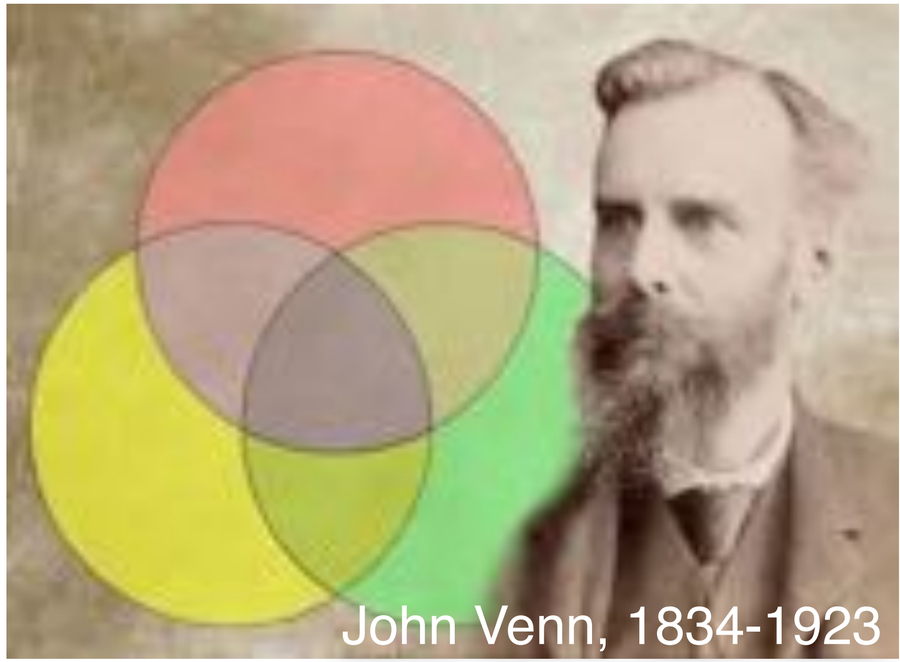
Visualize

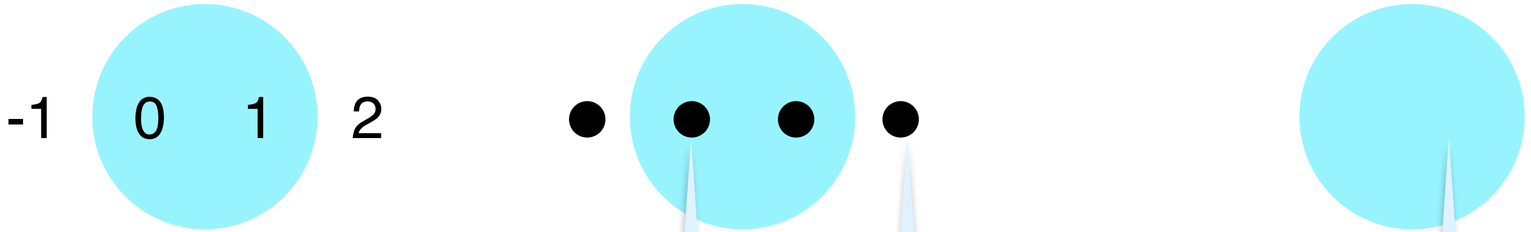
Sets

Regions

Elements

Points





{ 0, 1 }

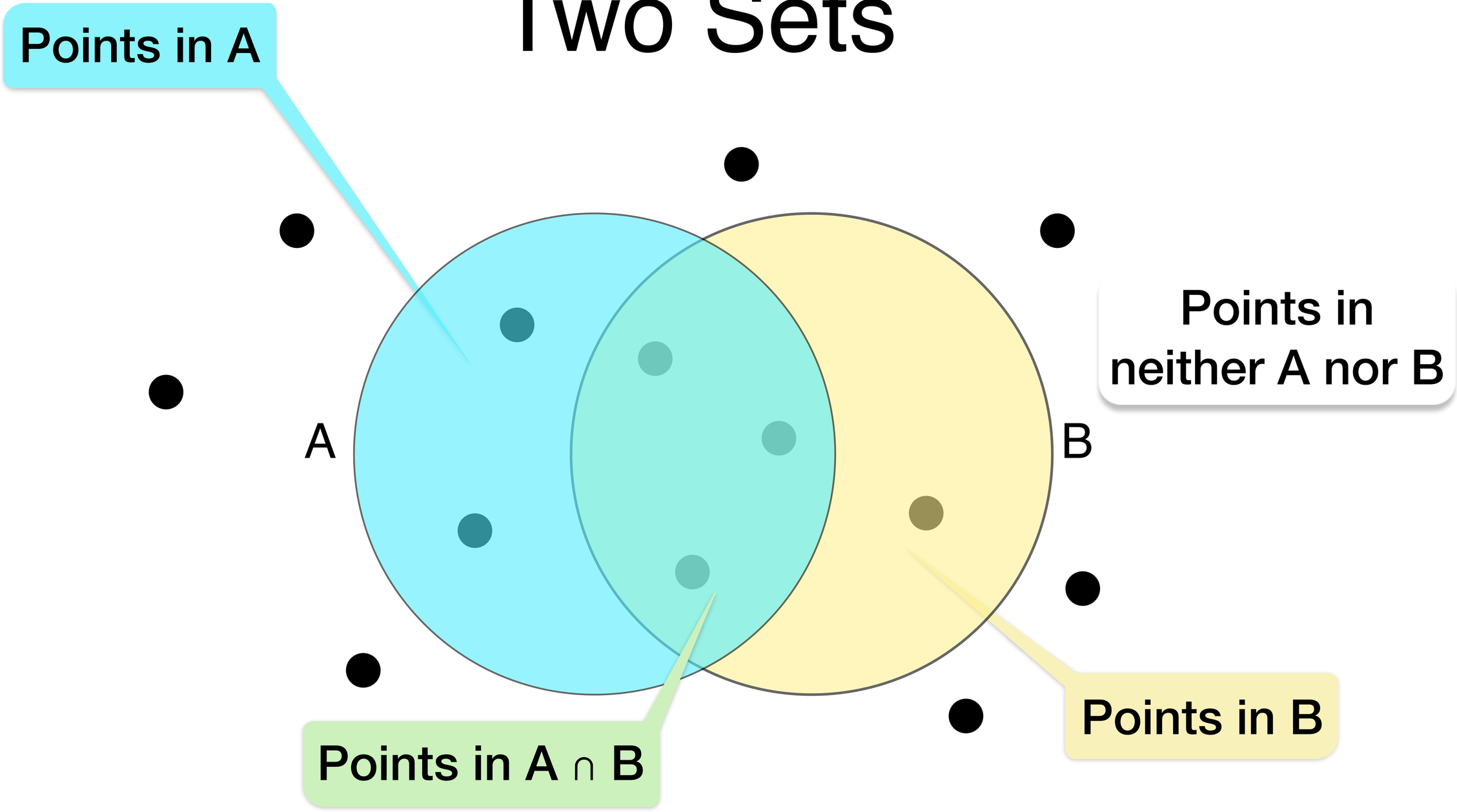
Elements in set

Elements not in set

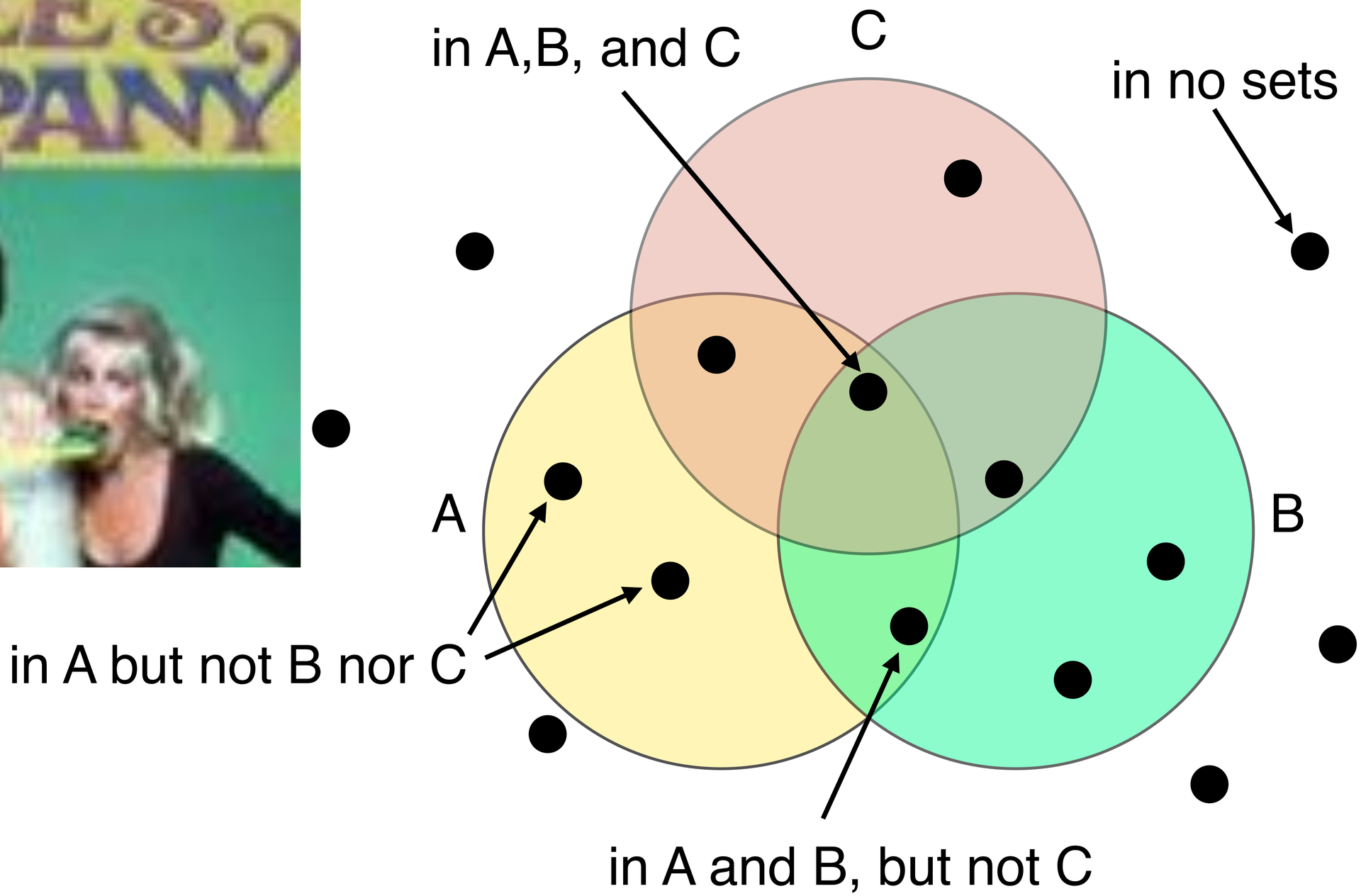
Elements in set

Elements not in set

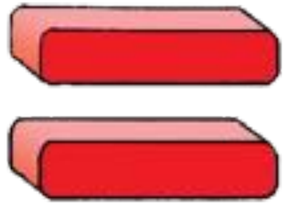
Two Sets


Points in A
A
B
Points in neither A nor B
Points in B
Points in A ∩ B

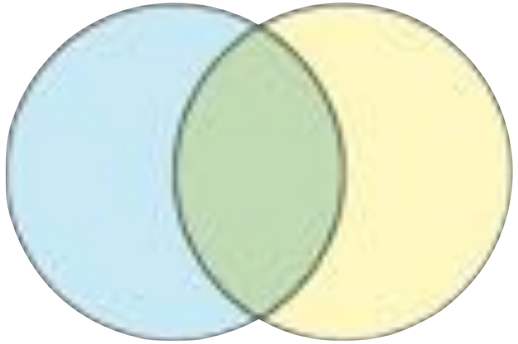
THREE'S
COMPANY

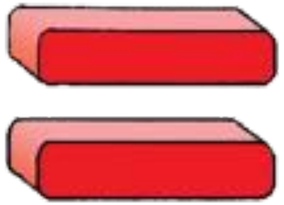

in A,B, and C
C
in no sets
A
B
in A but not B nor C
in A and B, but not C

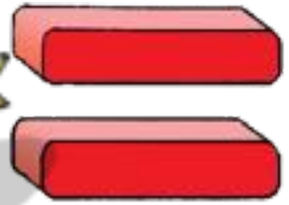
Why Venn



10^3





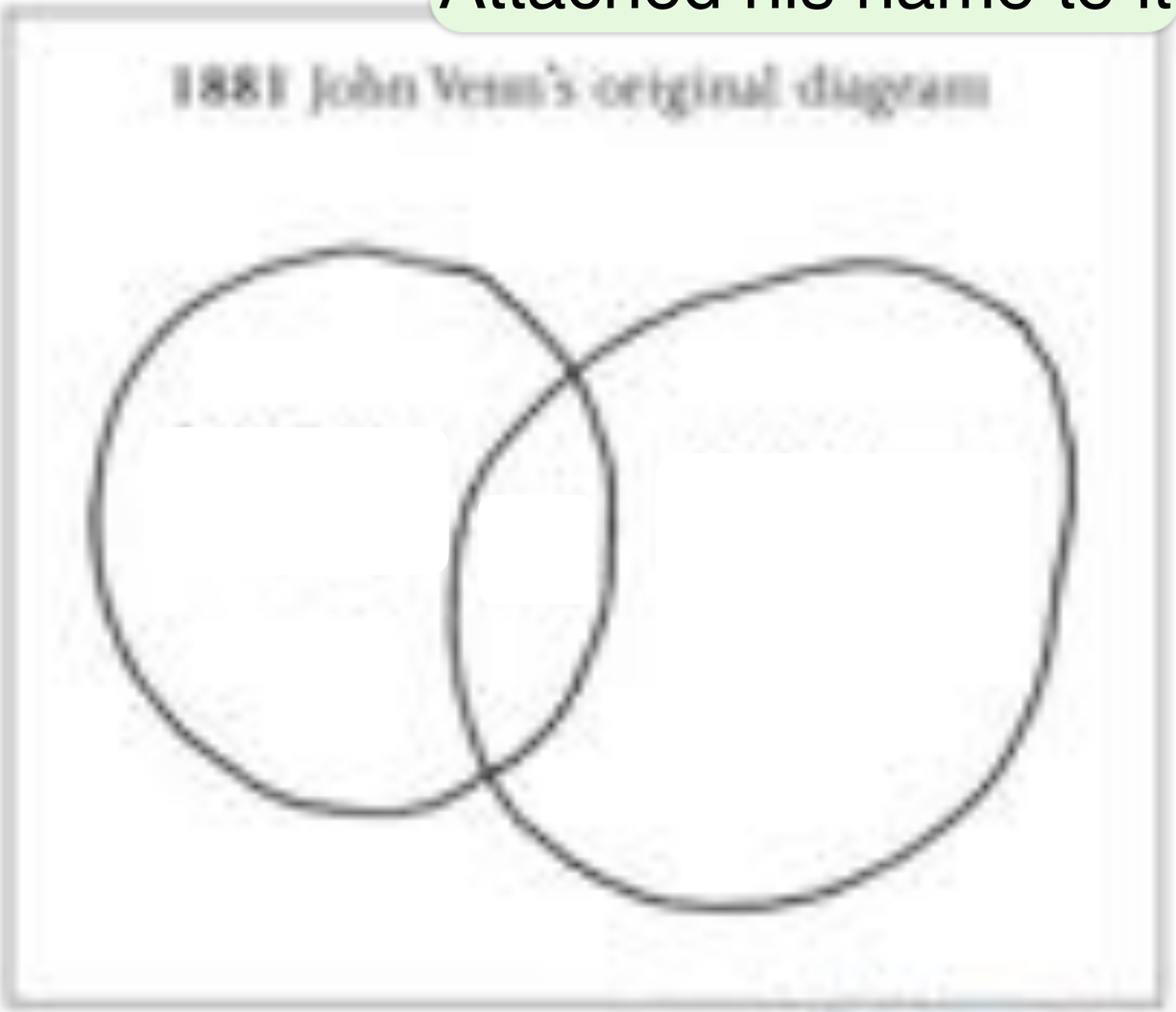


Visualize definitions & proofs

Gods of Science

Got something right

Attached his name to it



Venn Diagrams

Download matplotlib_venn package (link: notebook)

```
import matplotlib.pyplot as plt
import matplotlib_venn as venn
S = {1, 2, 3}
T = {0, 2, -1, 5}
venn.venn2([S, T], set_labels=('S','T'))
plt.show()
```

```
venn.venn3([S,T,U], set_labels=(’S’,’T’,'U'))
```

Venn Diagrams

Visualize sets

Help define, understand , and prove results about sets

Show in python (mostly for class)

Set Relations

set Relations

Relation Types

Human relations

Number relations

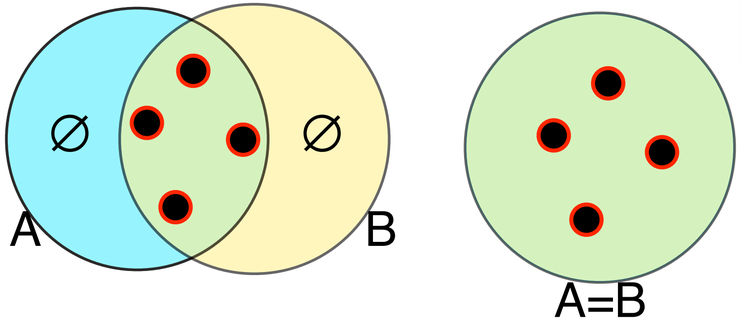
$=$ $\leq$ $<$

Generalize to sets

All men are created equal

generalize = of numbers

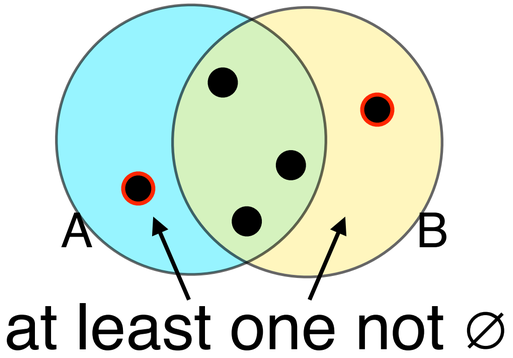
Sets A and B are equal, denoted A = B, if they have exactly the same elements



$\{0,1\} = \{1,0\}$

All sets are not created equal

If A and B are not equal, they are different, denoted A ≠ B



$\{0,1\} \neq \{1,2\}$

Equality QUIZ

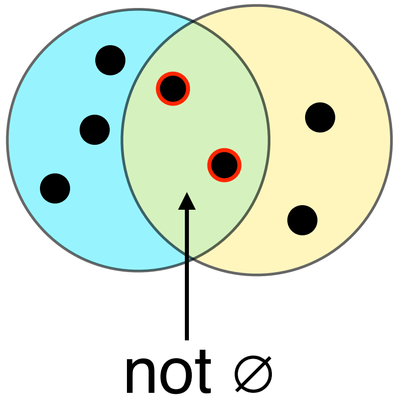
What does set equality have in common with trust?

=	All elements must be identical	$\{1,2,4\} = \{4,1,2\}$
≠	One different element enough	$\{1,2,4\} \neq \{1,2,4,8\}$

Intersection

Two sets intersect if they share at least one common element

$\exists x$
$x \in A \wedge x \in B$

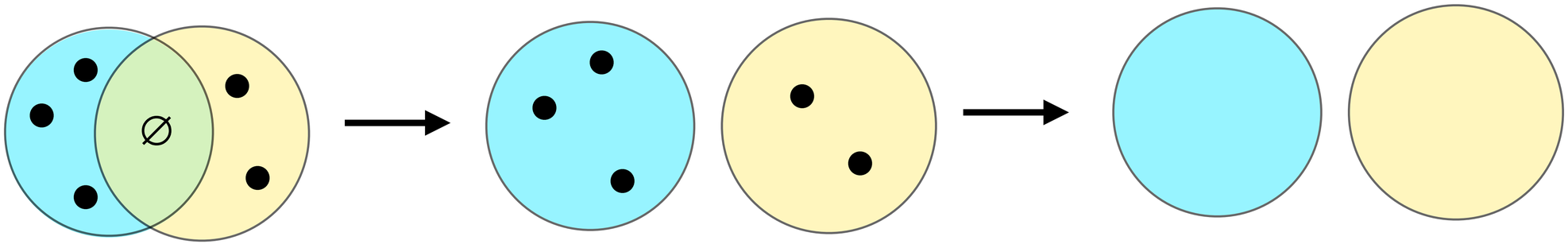


{0,1} {1,2} (1) [3,4] [2,5] (3.5,..)

Two sets are disjoint if they share no elements

$\neg \exists x$
$x \in A \wedge x \in B$

{0,1} {2,3} [3,4] (4,5]

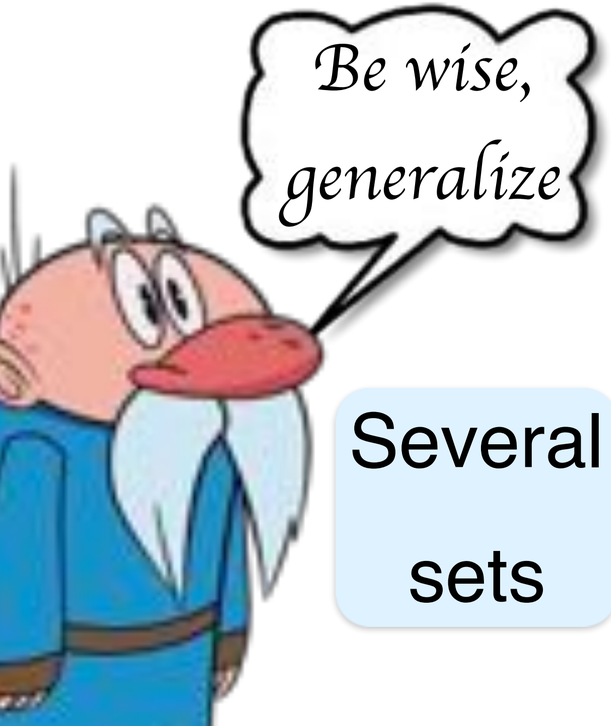


Intersection

$\varnothing$ disjoint from any set

Non-empty Ω intersects every set

A set intersects itself iff it is non-empty



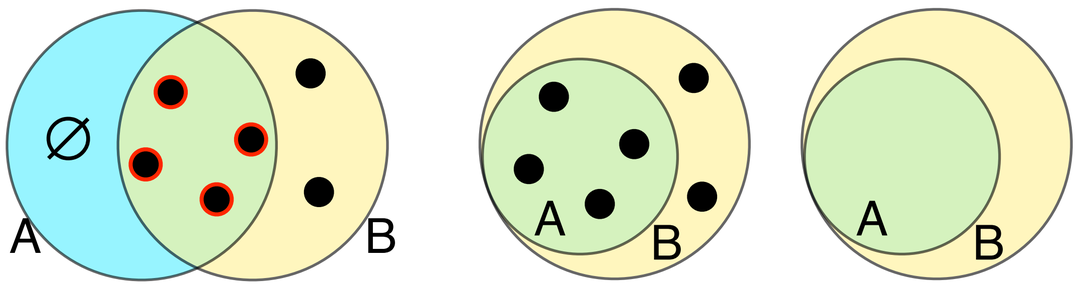
Several sets

intersect if all share a common element

mutually disjoint if every two are disjoint

Subsets

generalize $\leqslant$

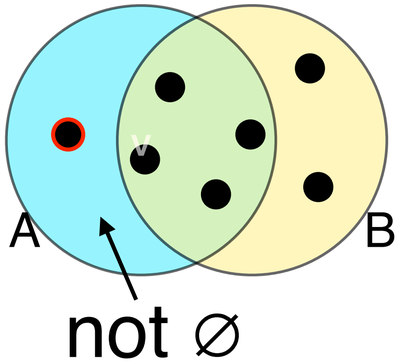


If every element in A is also in B, then A is a subset of B, denoted $A \subseteq B$

$\{0\} \subseteq \{0,1\}$ $\{0\} \subseteq \{0\}$

Equivalently, B is a superset of, or contains, A, denoted $B \supseteq A$ $\{0,1\} \supseteq \{0\}$

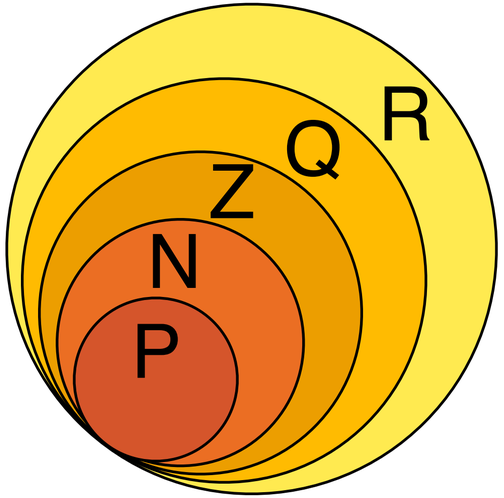
If A has an element that's not in B, then A is not a subset of B, denoted $A \nsubseteq B$, or $B \nsupseteq A$



$\{0,1\} \nsubseteq \{1,2\}$ $\{1,2\} \nsupseteq \{0,1\}$

Subsets

$P \subseteq N \subseteq Z \subseteq Q \subseteq R$

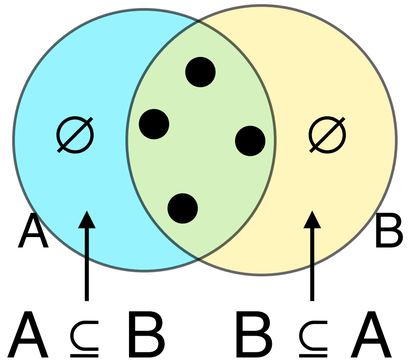


$\varnothing \subseteq A \subseteq A \subseteq \Omega$

$A \subseteq B$ and $B \subseteq C \rightarrow A \subseteq C$

$\subseteq$ is transitive

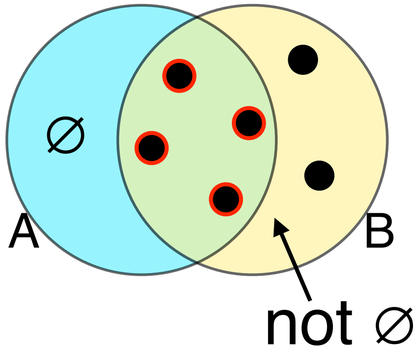
$A \subseteq B$ and $B \subseteq A \rightarrow A = B$



Strict Subsets

generalize <

If $A \subseteq B$ and $A \neq B$, A is a strict subset of B, denoted $A \subset B$, and B is a strict superset of A, denoted $B \supset A$



$\{ 0 \} \subset \{ 0, 1 \}$ $\{ 0,1 \} \supset \{ 0 \}$

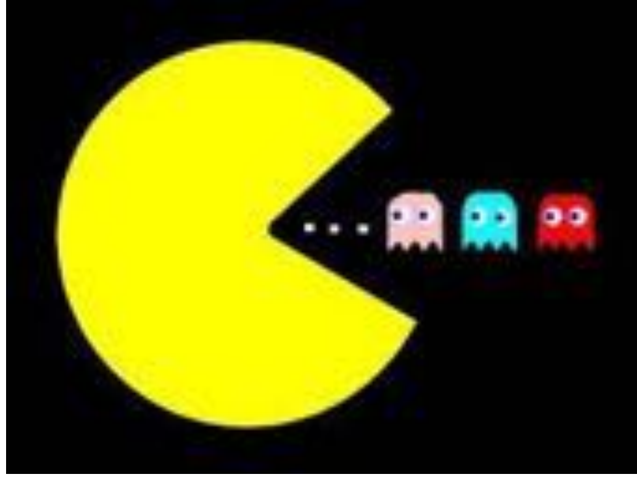
If A is not a strict subset of B, we write $A \not\subset B$ or $B \not\supset A$

Two possible reasons

$A \not\subseteq B$ $\{ 0 \} \not\subset \{ 1 \}$

$A = B$ $\{ 0 \} \not\subset \{ 0 \}$

belongs to $\in$ **vs.** $\subseteq$ subset of



$\in$ Relation between an element and a set

$x \in A$: element x belongs to, or is contained in, set A

{0,1} has two elements: 0 and 1 $\boxed{0 \in \{0,1\}}$ $\boxed{\{0\} \notin \{0,1\}}$

$\subseteq$ Relation between two sets

$A \subseteq B$: set A is a subset of set B

{0,1} two elements: 0 and 1 {0} one elt: 0 $\boxed{\{0\} \subseteq \{0,1\}}$

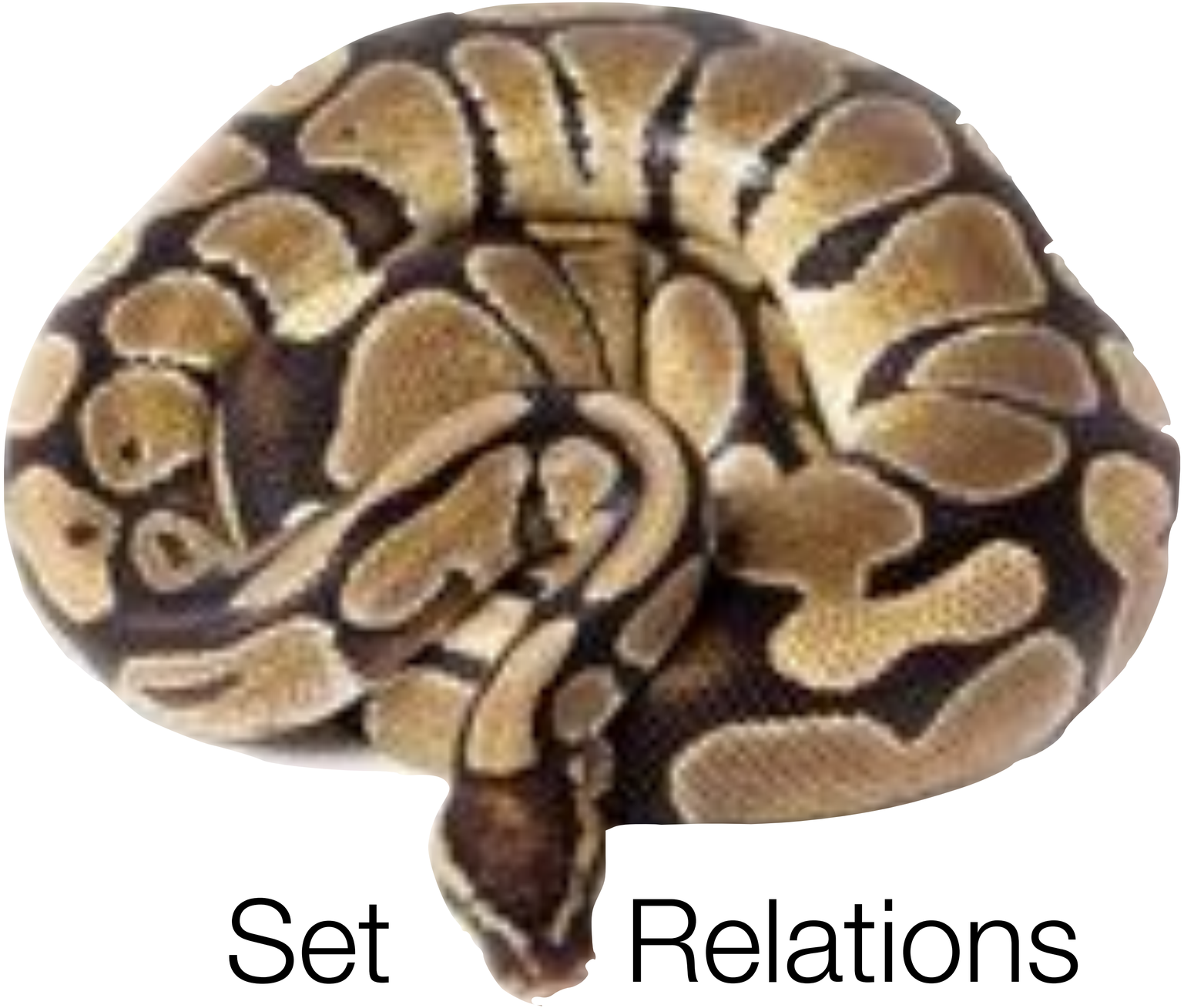
0 is an element of {0,1}, but 0 is not a set $\boxed{0 \not\subseteq \{0,1\}}$

$\mathcal{P}$ $\mathcal{U}$ $\mathcal{Z}$ $\mathcal{Z}$ $\mathcal{L}$ $\mathcal{E}$

Are there sets A and B such that
A is both an **element** and a **subset** of B?

$A \in B$ and $A \subseteq B$

Next lecture



Set Relations

Equality, Inequality, Disjoint

= ==

```
S1 = {0,1}
S2 = set({0,1})
S3 = {1,0,1}
T  = {0,2}

S1 == T
False

S1 == S2
True

S1 == S3
True
```

≠ !=

```
S1 != S2
False

S1 != T
True
```

disjoint **isdisjoint**

```
S1.isdisjoint(T)
False
S1.isdisjoint({2})
True
```

Subsets and Supersets

```
zero = {0}
```

```
zplus = {0,1}
```

```
zminus = {0,-1}
```

⊆ **<= or issubset**

```
zminus <= zplus
False
zminus.issubset(zero)
True
```

⊂ **<**

```
zplus < plus
False
zero < zminus
True
```

⊇ **>= or issuperset**

```
zplus >= zminus
False
zplus.issuperset(zero)
True
```

⊃ **>**

```
zminus > zminus
False
zplus > zero
True
```

Set Relations

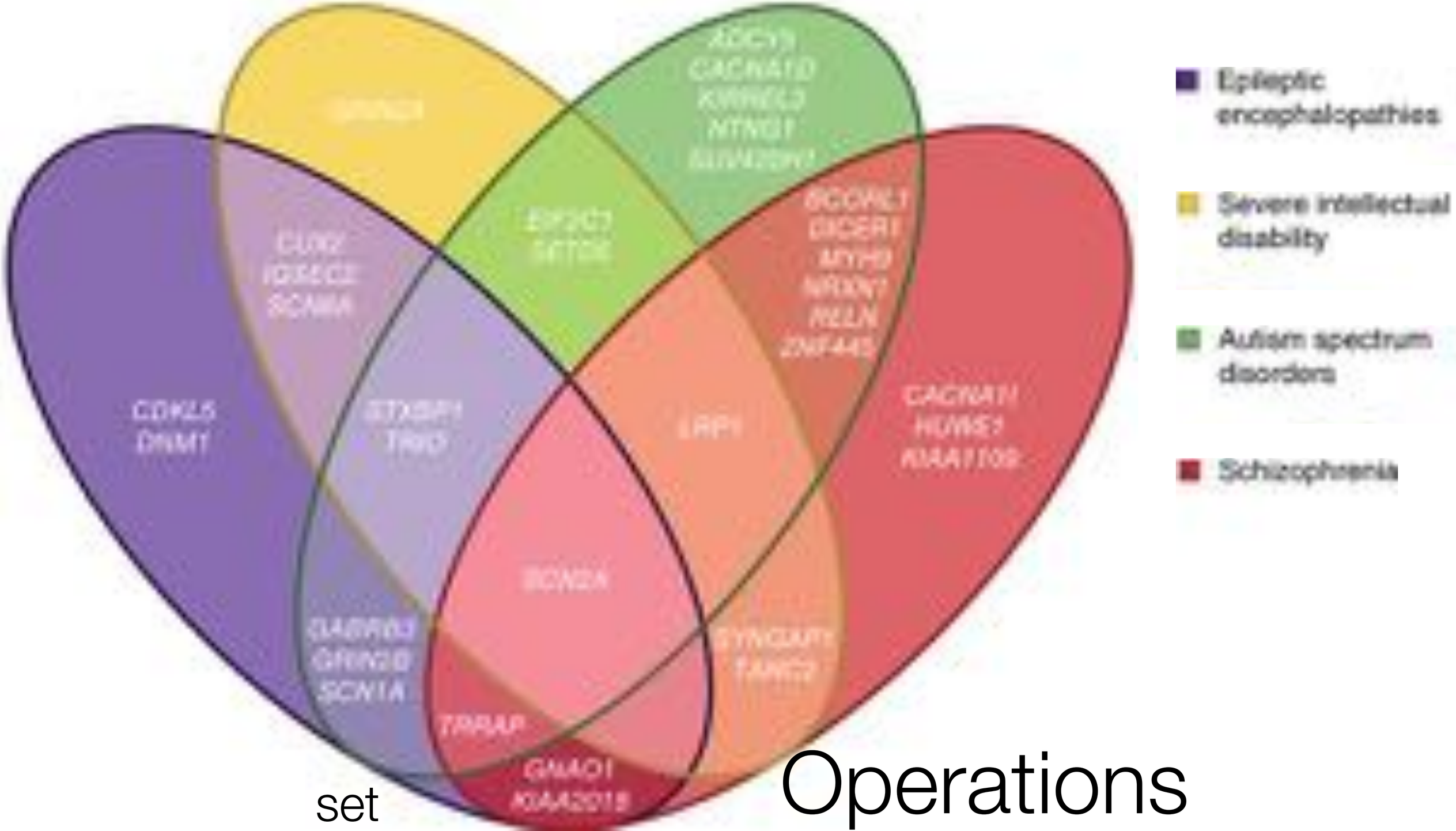
Equality and inequality $=$ $\neq$

Intersection and disjointness

Subsets and Supersets $\subseteq$ $\subset$ $\supseteq$ $\supset$ negations

Python `==` `!=` `isdisjoint` `<=` `<` `>=` `>`

Set Operations



set Operations

$\mathcal{P}$ $\mathcal{U}$ $\mathcal{Z}$ $\mathcal{Z}$ $\mathcal{L}$ $\mathcal{E}$

Are there sets A and B such that
A is both an **element** and a **subset** of B?

$A \in B$

$A \subseteq B$

Yes!

$\varnothing \subseteq$ any set

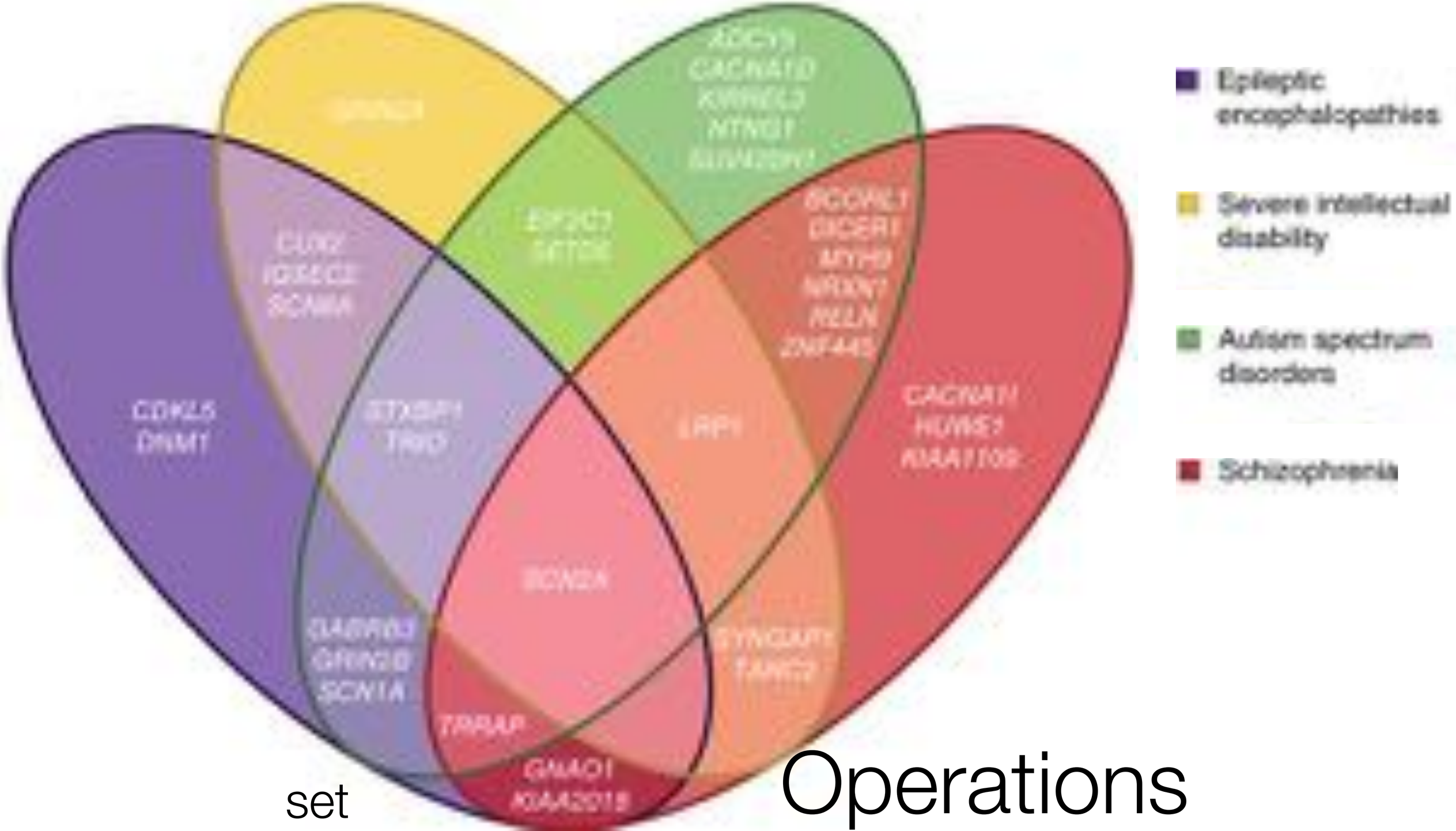
$A = \varnothing$

need $\varnothing \in B$

$B = \{\varnothing\}$

$\varnothing \in \{\varnothing\}$ and $\varnothing \subseteq \{\varnothing\}$

Also solutions with nonempty sets, but this is simplest



set

Operations

Plan

Again generalize numbers

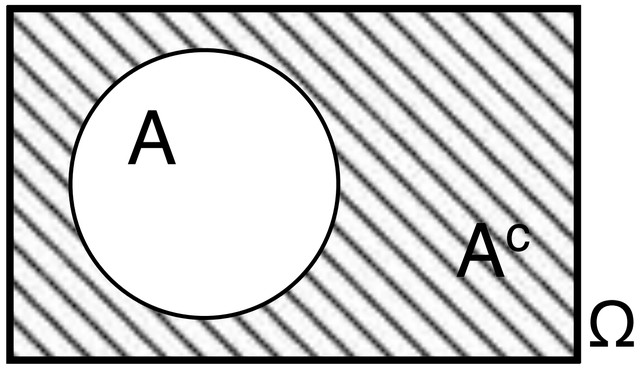
	Relations			Operations		
Number	$=$	$\leq$	$<$	$+$	$-$	$\times$
Set	$=$	$\subseteq$	$\subset$	$\cup$	$-$	$\times$
				This lecture		Next lecture
				$\cap$		

Complement

Recall: Universal set Ω contains all elements

The complement A^c of A is the set of Ω elements not in A

$A^c = \{ x \in \Omega : x \notin A \}$



$\Omega = \{0,1\}$ $\{0\}^c = \{1\}$ $\{0,1\}^c = \varnothing$ $\varnothing^c = \{0,1\}$

$\Omega = \{0,1,2\}$ $\{0\}^c = \{1,2\}$ ← A^c depends on both A and Ω

$\Omega = \mathbb{Z}$ $\{\ldots,-2,-1\}^c = \mathbb{N}$

$\mathbb{E}$ - even, $\mathbb{O}$ - odd $\mathbb{E}^c = \mathbb{O}$

Set Identities

Relations that hold for all sets

$\varnothing^c = \Omega$

$\Omega^c = \varnothing$

A and A^c are disjoint

$(A^c)^c = A$

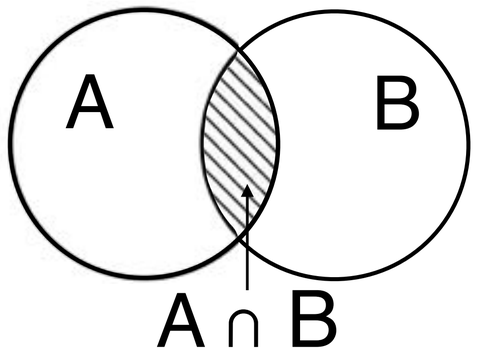
"involution"

$A \subseteq B \rightarrow A^c \supseteq B^c$

Intersection

The intersection $A \cap B$ is the set of elements in both A and B

$A \cap B =$
$\{ x: x \in A \wedge x \in B \}$

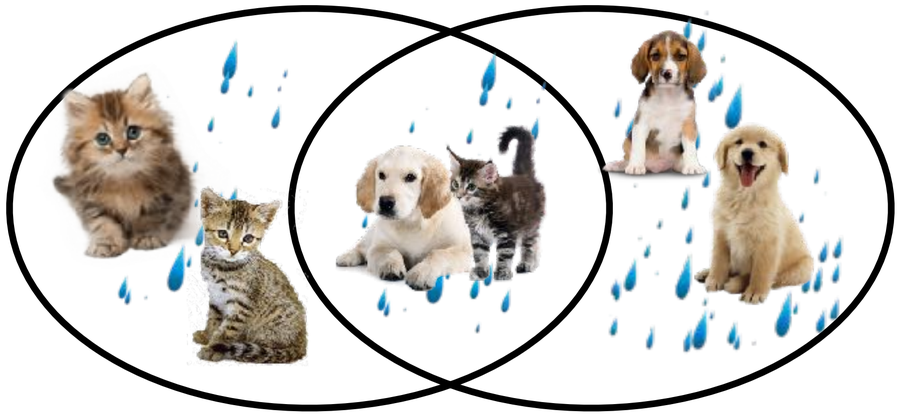


$\{0,1\} \cap \{1,3\} = \{1\}$

$\{0\} \cap \{1\} = \varnothing$

$[0,4) \cap [3,6] = [3,4)$

$[0,2] \cap (2,5] = \varnothing$

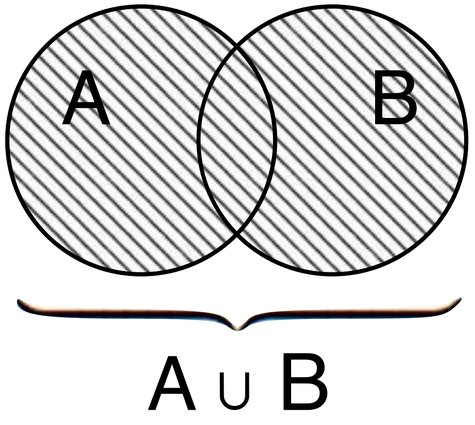


Raining cats and dogs

Union

The union $A \cup B$ is the collection of elements in A, B, or both

$$A \cup B = \{ x : x \in A \vee x \in B \}$$



$\{0,1\} \cup \{1,2\} = \{0,1,2\}$

$\{0,1\} \cup \{2\} = \{0,1,2\}$

$[0,2] \cup [1,3] = [0,3]$

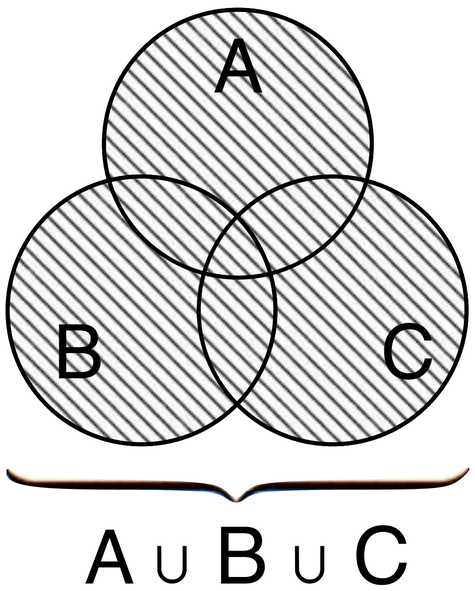
$(0,1) \cup \{1\} = (0,1]$

$\mathbb{E} \cup \mathbb{O} = \mathbb{N}$

Multiple Sets

$A \cup B \cup C = \{ x \in \Omega : x \in A \vee x \in B \vee x \in C \}$

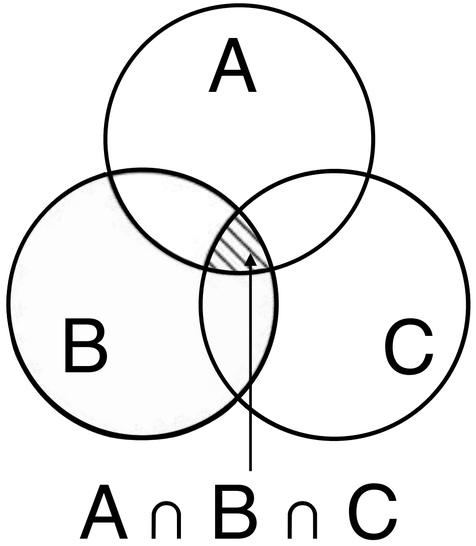
$\{0,1\} \cup \{1,2\} \cup \{2,3\} = \{0,1,2,3\}$



Generally $\bigcup_{i=1}^{t} A_i = \{x : \exists 1 \leq i \leq t, x \in A_i\}$

$\bigcup_{i=-\infty}^{\infty} \{i\} = \mathbb{Z}$

Similarly for intersection



Identities - One Set

Identity	$A \cap \Omega = A$	$A \cup \Omega = \Omega$
Universal bound	$A \cap \varnothing = \varnothing$	$A \cup \varnothing = A$
Idempotent	$A \cap A = A$	$A \cup A = A$
Complement	$A \cap A^c = \varnothing$	$A \cup A^c = \Omega$

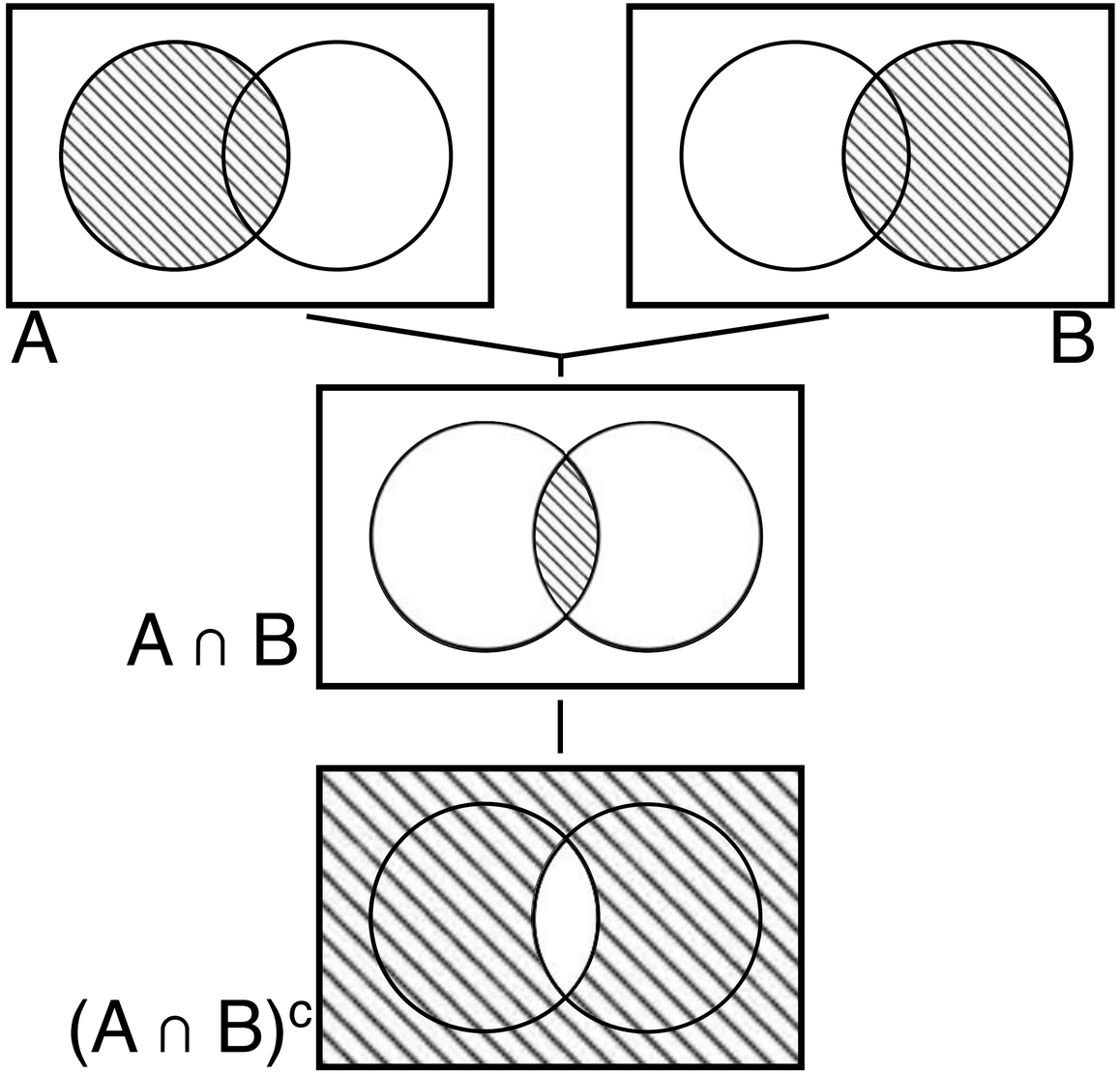
Often called laws

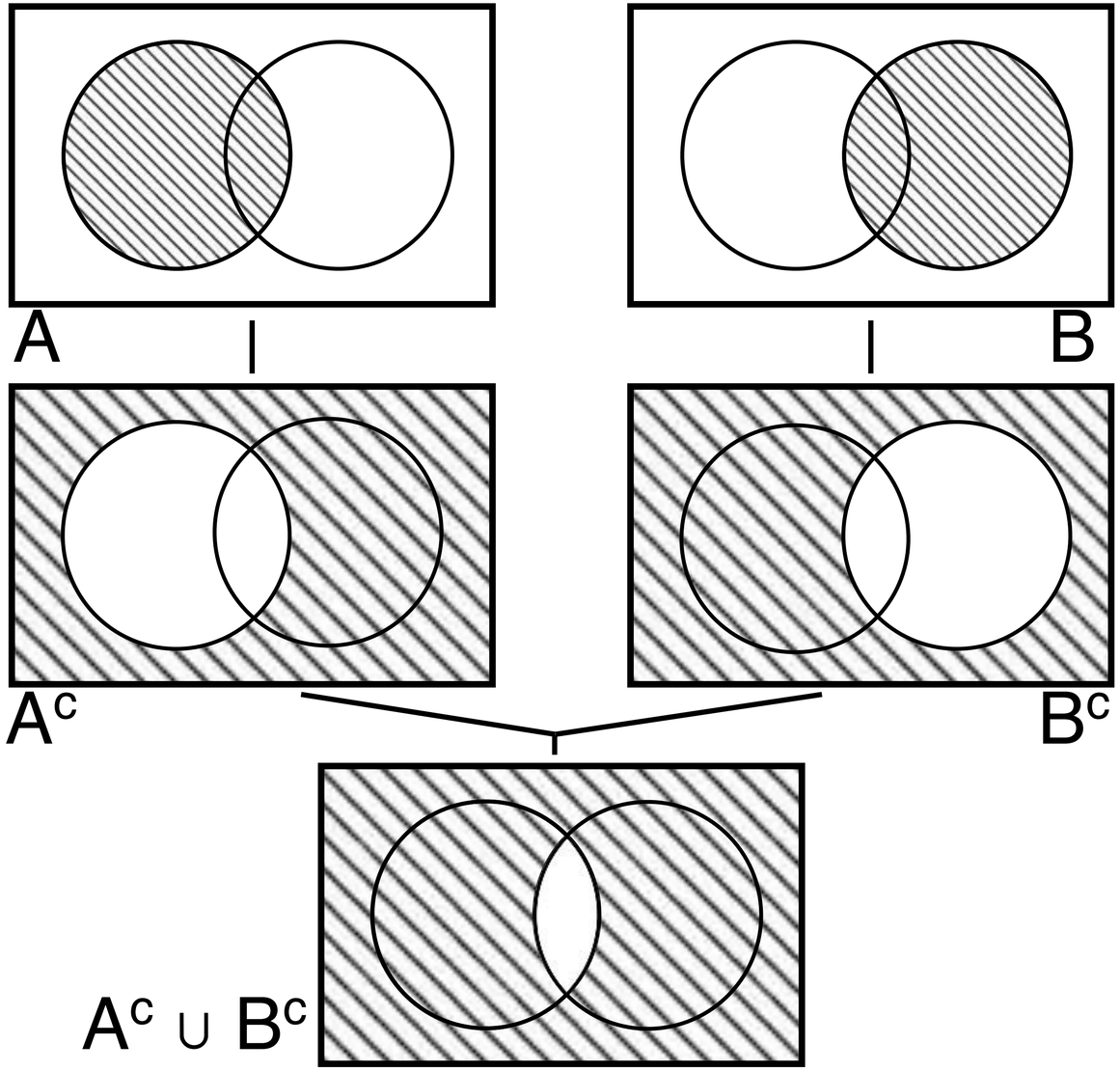
Laws - Two and Three Sets

Commutative	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De Morgan	$(A \cap B)^c = A^c \cup B^c$	$(A \cup B)^c = A^c \cap B^c$

De Morgan's Law

$(A \cap B)^c = A^c \cup B^c$

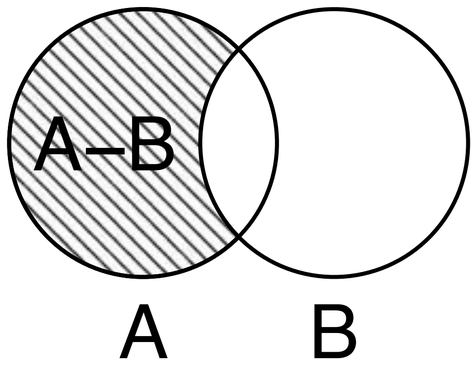




Set Difference

The difference $A-B$ is the set of elements in A but not in B

$$A-B = \{ x: x \in A \wedge x \notin B \}$$



$\{0,1\} - \{1\} = \{0\}$

$\{0,1\} - \{0,1,2\} = \varnothing$

$[1,3] - [2,4] = [1,2)$

$[1,3] - (1,3) = \{1,3\}$

Notetation

Also

$A \setminus B$

$A - B = A \cap B^c$

Symmetric Difference

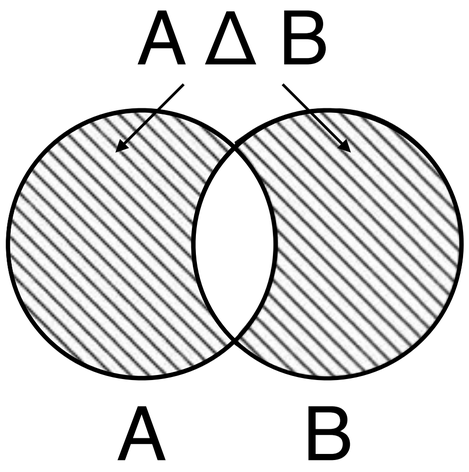
The symmetric difference of two sets is the set of elements in exactly one set

$$A \Delta B = \{x: x \in A \wedge x \notin B \ \vee \ x \notin A \wedge x \in B\}$$

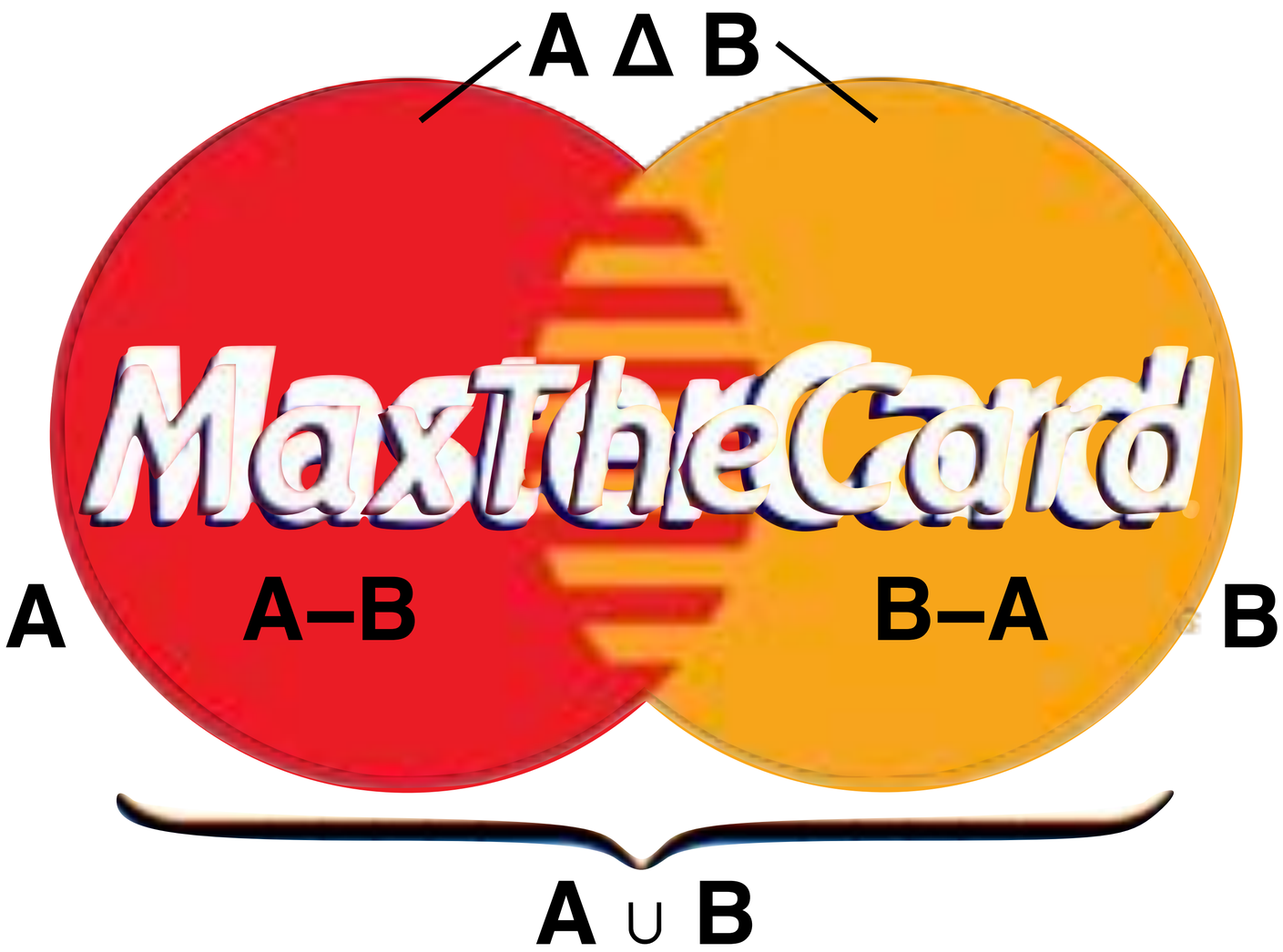
$\{0,1\} \Delta \{1,2\} = \{0,2\}$

$[0,2] \Delta [1,4] = [0,1) \cup (2,4]$

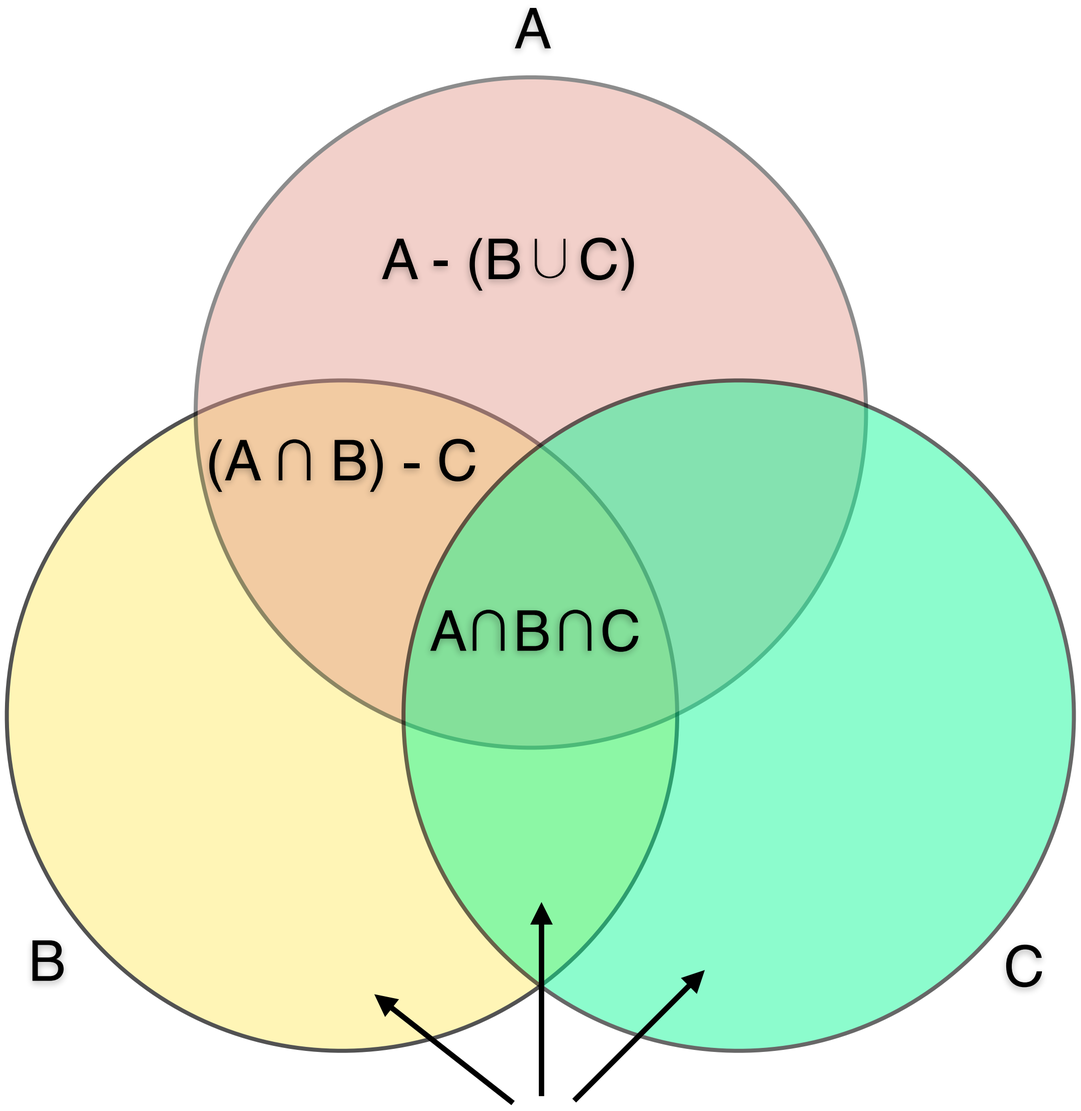
$A \Delta B = (A-B) \cup (B-A)$



Venn's Master Chart



3 Sets



3 regions combined: (B ∪ C) - A

set Operations

Union and Intersection

```
A = {1,2}
B = {2,3}
```

∪ | or **union**

```
A | B
{1, 2, 3}
C = A.union(B)
print(C)
{2, 1, 3}
```

∩ **&** or **intersection**

```
A & B
{2}
C = A.intersection(B)
print(C)
{2}
```

Set- and Symmetric-Difference

```
A = {1,2}
B = {2,3}
```

Set difference

`-` or **`difference`**

```
A - B
{1}
B.difference(A)
{3}
```

Symmetric difference

`^` or **`symmetric_difference`**

```
A ^ B
{3,1}
B.symmetric_difference(A)
{3,1}
```

Set Operations

Operations

- Complement A^c
- Union $\cup$
- Intersection $\cap$
- Set difference -
- Symmetric difference Δ

Laws

- Commutative
- Associative
- Distributive
- De Morgan

Python

- `|` or `union`
- `&` or `intersection`
- `-` or `difference`
- `^` or `symmetric_difference`

Cartesian Products

Cartesian Products

Philosopher, scientist, mathematician

Considered father of western philisophy

Cogito ergo sum

I think therefore I am

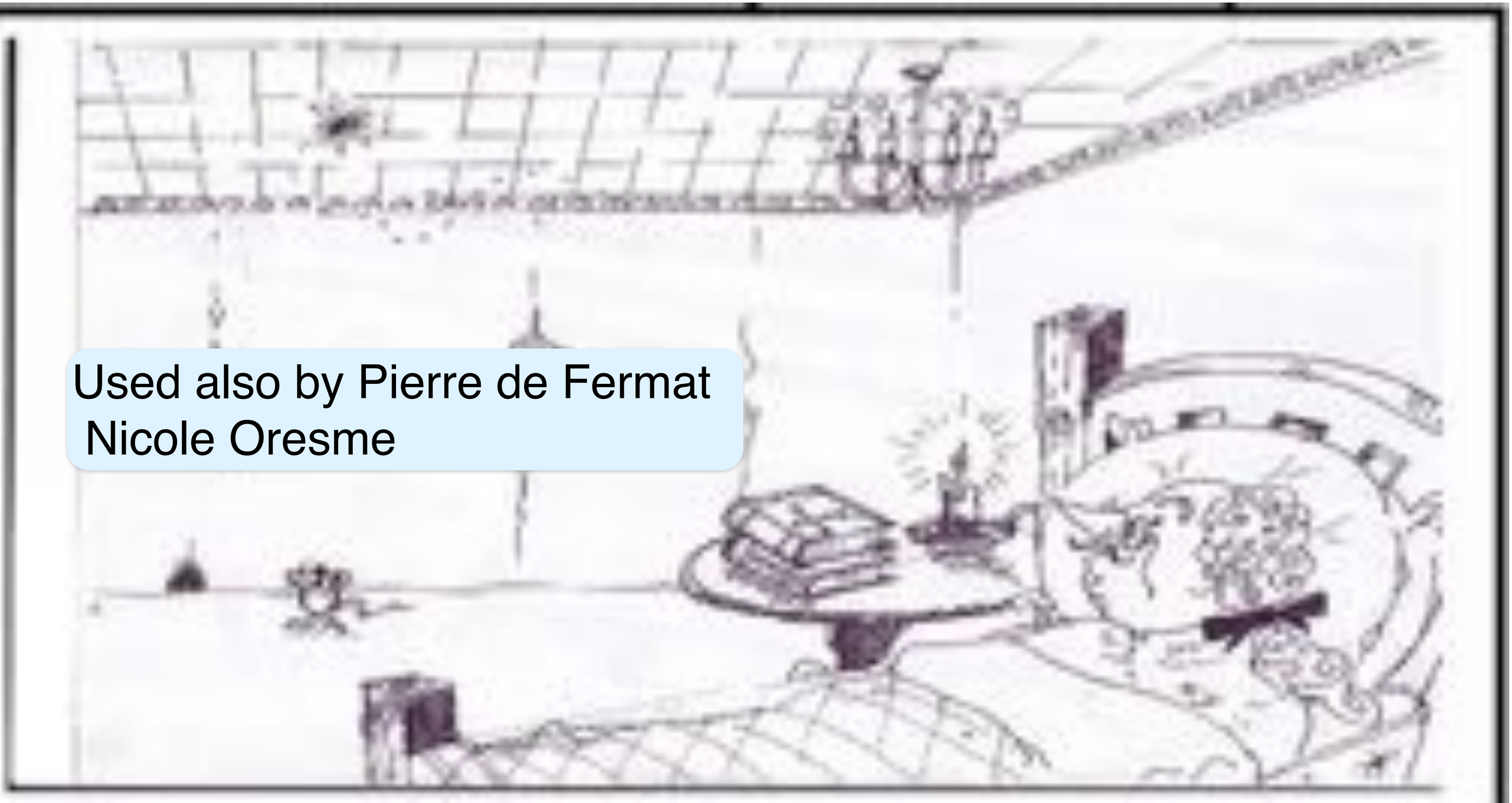
Divide each difficulty into as many parts as necessary to solve it

Each problem I solved became a rule I later used to solve other problems

A good mind is not enough, the main thing is to use it well

To improve the mind, learn less and contemplate more

Cartesian Coordinates



Used also by Pierre de Fermat
Nicole Oresme

Tuples and Ordered Pairs

Set Order and repetition do not matter $\{a, b, c\} = \{b, c, a\}$

Tuple Both order and repetition matter $(a, b, c) \neq (b, c, a)$

$(a, a, a) \neq (a)$

n-tuple Tuple with n elements $(a_1, a_2, \ldots, a_n)$

2-tuple Ordered pair $(3, 7)$

Also real interval

Tell from context

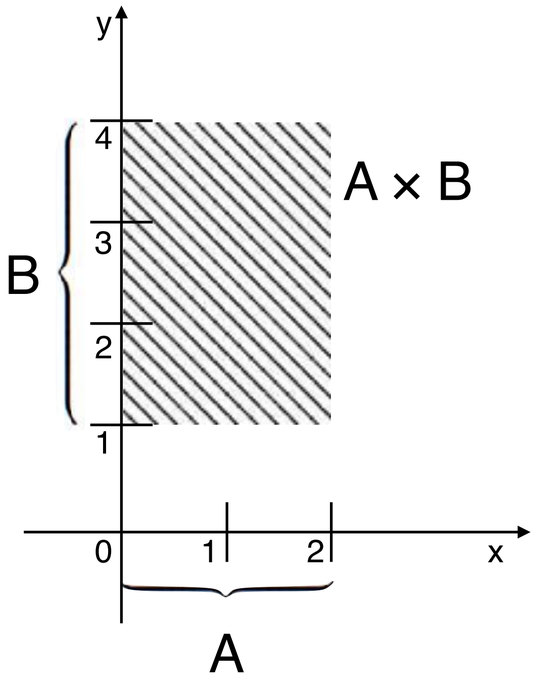
Cartesian Products

The cartesian product of A and B is the set $A \times B$ of ordered pairs (a, b) where $a \in A$ and $b \in B$

$A \times B = \{(a,b): a \in A,\ b \in B\}$

$A \times A$ denoted A^2

$\mathbb{R}^2 = \{ (x,y): x,y \in \mathbb{R} \}$ Cartesian Plane



$A, B \subseteq \mathbb{R}$ $A \times B \subseteq \mathbb{R}^2$ Rectangle

$A = [0, 2]$ $B = [1, 4]$

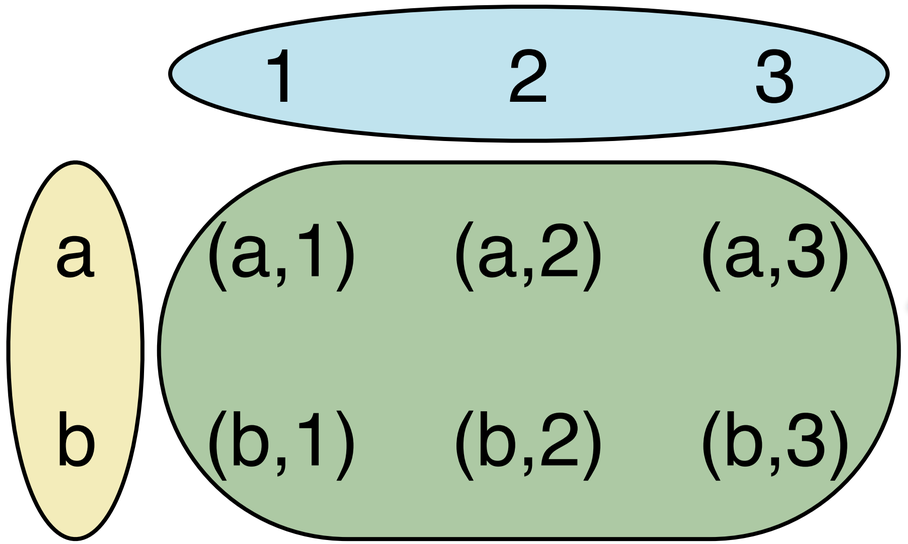
$A \times B = \{ (x, y) : x \in [0, 2], y \in [1, 4] \}$

Discrete Sets

Similar simpler

$\{a,b\} \times \{1,2,3\} = \{ (x, y): \ x \in \{a,b\}, y \in \{1,2,3\} \}$

$= \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3) \}$



1st coordinate vertically,
2nd horizontally

Identities

$A \times \varnothing = \varnothing \times A = \varnothing$

$A \times (B \cup C) = A \times B \cup A \times C$

$A \times (B \cap C) = A \times B \cap A \times C$

$A \times (B - C) = A \times B - A \times C$

B

		1	2	3
A	a	a1	a2	a3
	b	b1	b2	b3

A x B

C

		2	3	4
A	a	a2	a3	a4
	b	b2	b3	b4

A x C

B ∪ C

		1	2	3	4
A	a	a1	a2	a3	a4
	b	b1	b2	b3	b4

A x (B ∪C)

Tables

Tables are Cartesian products

Column

attribute

name, gender, GPA

Row

record

student

Adam	M	3.5
Eve	F	3.7
John	M	3.4
Lisa	F	3.2
Mary	F	3.9

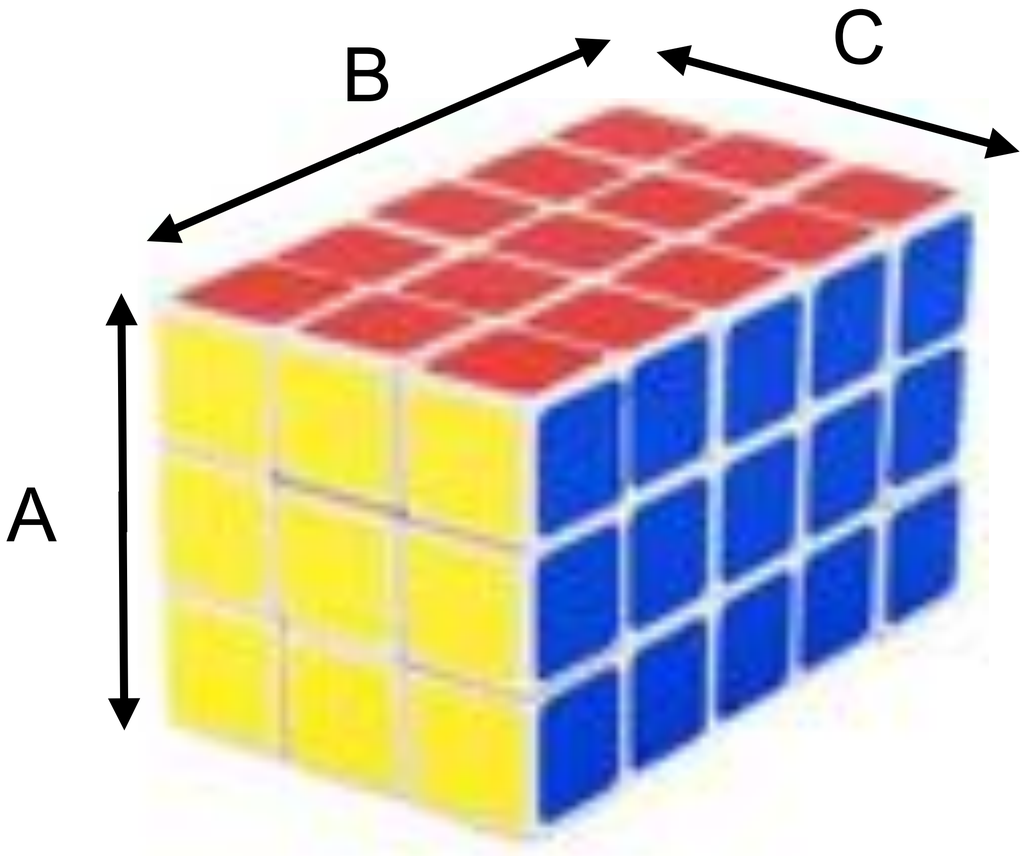
Cell

entry

Cartesian Product of 3 Sets

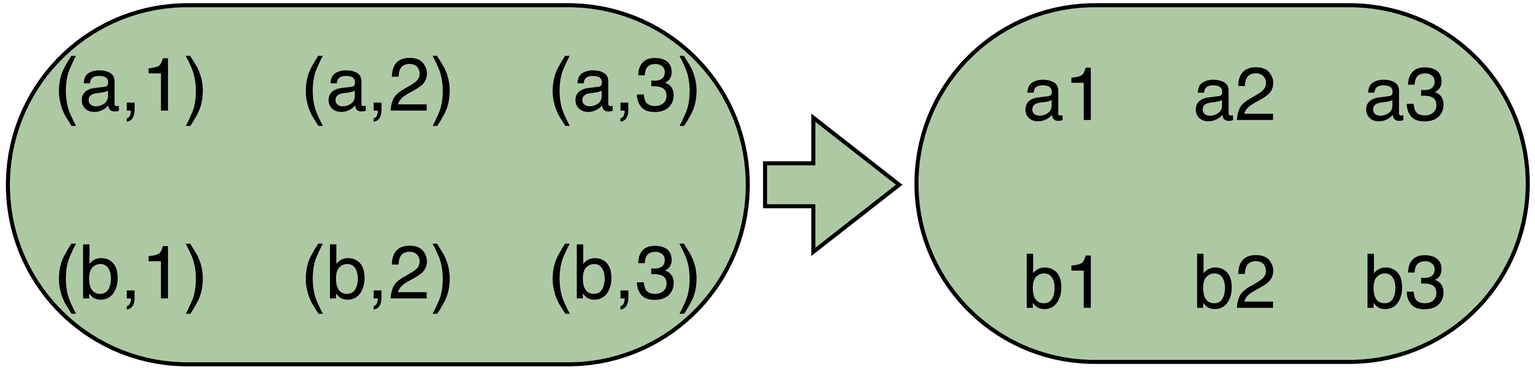
A x B — 2-dimensional rectangle

A x B x C — 3-dimensional “cuboid”



Sequences

Tuples, just without ( ) and sometimes without ,



$\{0,1\}^2 = \{ xy: x, y \in \{0,1\} \} = \{ 00, 01, 10, 11 \}$

$\{0,1\}^3 = \{ 000, 001, 010, 011, 100, 101, \ldots, 111 \}$

Cartesian Products

Cartesian Product

Use **product** function in **itertools** library

```
from itertools import product
Faces = set({‘J’,‘Q’,‘K’})
Suits = {‘♦’,’♠’}
for pair in product(Faces,Suits):
  print(pair)
(‘J’, ‘♦’)
(‘J’, ‘♠’)
(‘Q’, ‘♦’)
(‘Q’, ‘♠’)
(‘K’, ‘♦’)
(‘K’, ‘♠’)
```

Cartesian Products

Tuples $(a_1,\ldots,a_n)$

Ordered pairs (a,b)

Sequences

Python `product` function in `itertools` library

A Perplexing Paradox

Bertrand Russell

Mathematician, philosopher, author

1950 literature Nobel Laureate

Democracy: fools have a right to vote.
Dictatorship: fools have a right to rule.

Bertrand Russell, 1872-1970

Most people would rather die than think.
In fact, most do.

Men are born ignorant, not stupid.
They are made stupid by education.

Sets in Sets

Sets can be elements $\{ \{0\}, \{1,2\} \}$

Every set is a subset of itself $\{0\} \subseteq \{0\}$

Can a set belong to (be an element of) itself? $S \in S$?

Typically, sets do not belong to themselves $\{0\} \notin \{0\}$ $\varnothing \notin \varnothing$

$0 \in \{0\}$

But some sets do belong to themselves!

NT = { anything that is not }

= { , 0, {1,2}, … , NT }

$NT \in NT$

Some sets ∈ themselves (NT), others don't ({0})

Russell's Paradox

Define a set that cannot exist

R = { sets that don't belong to themselves } = { S: S $\notin$ S }

$\{0\} \notin \{0\}$, hence $\{0\} \in R$

NT $\in$ NT, hence NT $\notin$ R

Must have $R \in R$ $\rightarrow$ $R \notin R$

or

$R \notin R$ $\rightarrow$ $R \in R$

Both lead to contradiction!

If R existed then both $R \in R$ and $R \notin R$ would hold

R defined but cannot exist!

What Happened?

Recursive definitions

$\{ S : S \in S \}$

$\{ S : S \notin S \}$

Will avoid

Not needed for exam

Variations

Who shaves the barber?

Woody Allen – Annie Hall