DERIVADA DE FUNÇÕES BÁSICAS

FUNÇÕES TRIGONOMÉTRICAS

a) $f(x) = 3\,sen(\underbrace{7x}_{u}) = 3\,sen(u)$

$f'(x) = 3\cos(u) \cdot u' = 3\cos(7x) \cdot 7 = 21\cos(7x)$

b) $y = \cos(5x^3 + x) = \cos(u)$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = -sen(u) \cdot u' = -sen(5x^3+x) \cdot (15x^2+1) =$
$= -(15x^2+1)\,sen(5x^3+x)$

c) $f(x) = sen(3x) \cdot \cos(3x)$

$f'(x) = [sen(3x)]' \cos(3x) + sen(3x) \cdot [\cos(3x)]' =$
$= 3\cos(3x)\cos(3x) + sen(3x) \cdot [-3\,sen(3x)] =$
$= 3\cos^2(3x) - 3\,sen^2(3x)$

d) $y = \dfrac{sen(x) + \cos(x)}{sen(x) - \cos(x)}$

$\dfrac{dy}{dx} = \dfrac{[sen(x)+\cos(x)]'\,[sen(x)-\cos(x)] - [sen(x)+\cos(x)] \cdot [sen(x)-\cos(x)]'}{[sen(x)-\cos(x)]^2} =$

$= \dfrac{[\cos(x) - sen(x)][sen(x)-\cos(x)] - [sen(x)+\cos(x)][\cos(x)+sen(x)]}{[sen(x)-\cos(x)]^2} =$

$= \dfrac{\cancel{\cos(x)sen(x)} - \cos^2(x) - sen^2(x) + \cancel{sen(x)\cos(x)} - \cancel{sen(x)\cos(x)} - sen^2(x) - \cos^2(x) - \cancel{\cos(x)sen(x)}}{[sen(x)-\cos(x)]^2}$

$= \dfrac{-2\cos^2(x) - 2\,sen^2(x)}{[sen(x)-\cos(x)]^2} = \dfrac{-2\,[\cos^2(x)+sen^2(x)]}{[sen(x)-\cos(x)]^2} =$

$= \dfrac{-2}{[sen(x)-\cos(x)]^2}$

e) $y = \frac{sen(2x)}{x^5}$ $\frac{dy}{dx} = \frac{[sen(2x)]' x^5 - sen(2x).[x^5]'}{(x^5)^2}$

$\frac{dy}{dx} = \frac{2cos(2x)\,x^5 - 5x^4 sen(2x)}{x^{10}} = \frac{2x cos(2x) - 5 sen(2x)}{x^6}$

f) $y = [sen(5x)]^4$ $\frac{dy}{dx} = 4sen^3(5x).cos(5x).5$

$\frac{dy}{dx} = 20\, sen^3(5x)\, cos(5x)$

g) $y = sen(\sqrt{x}) + \sqrt{sen x} = sen(x^{1/2}) + [sen(x)]^{1/2}$

$\frac{dy}{dx} = cos(\sqrt{x}).\frac{1}{2}x^{-1/2} + \frac{1}{2}[sen(x)]^{-1/2}.\, cos(x)$

$\frac{dy}{dx} = \frac{cos(\sqrt{x})}{2\sqrt{x}} + \frac{cos(x)}{2\sqrt{sen(x)}}$

h) $y = [1 - 2cos(x)]^{5/2}$

$\frac{dy}{dx} = \frac{5}{2}[1-2cos(x)]^{3/2}.\,(0 - 2(-sen(x))) =$

$= \frac{5}{\cancel{2}}[1-2cos(x)]^{3/2}.\,\cancel{2}sen(x) = 5 sen(x)\sqrt{(1-2cos x)^3}$

i) $y = sen(tg(4x^3))$

$\frac{dy}{dx} = cos(tg(4x^3)).\, sec^2(4x^3).\,12x^2$

j) $y = \left(\frac{x}{tg(x)}\right)^3$ $\frac{dy}{dx} = 3\left(\frac{x}{tg(x)}\right)^2 \cdot \left[\frac{tg(x) - x \cdot sec^2(x)}{tg^2(x)}\right]$

$\frac{dy}{dx} = \frac{3x^2 (tg(x) - x\, sec^2(x))}{tg^4(x)}$

k) $y = sen\left(\frac{\sqrt{x^2+1}}{x^3}\right)$

$\underbrace{}_{u}$

$\frac{dy}{dx} = cos\left(\frac{\sqrt{x^2+1}}{x^3}\right) \cdot \left(\frac{\sqrt{x^2+1}}{x^3}\right)'$

$\left(\frac{(x^2+1)^{1/2}}{x^3}\right)' = \frac{\left[(x^2+1)^{1/2}\right]' x^3 - \sqrt{x^2+1} \cdot (x^3)'}{x^6} =$

$= \frac{\frac{1}{2\sqrt{x^2+1}} \cdot 2x \cdot x^3 - \sqrt{x^2+1} \cdot 3x^2}{x^6} = \frac{\frac{x^4}{\sqrt{x^2+1}} - 3x^2\sqrt{x^2+1}}{x^6} =$

$= \frac{\frac{x^4 - 3x^2(x^2+1)}{\sqrt{x^2+1}}}{x^6} = \frac{x^4 - 3x^4 - 3x^2}{x^6\sqrt{x^2+1}} =$

$= \frac{-2x^4 - 3x^2}{x^6\sqrt{x^2+1}} = \frac{-2x^2 - 3}{x^4\sqrt{x^2+1}}$

$\frac{dy}{dx} = cos\left(\frac{\sqrt{x^2+1}}{x^3}\right) \cdot \left(\frac{-2x^2-3}{x^4\sqrt{x^2+1}}\right)$

l) $y = \sqrt{1 + sen^2(3x)} = (1 + sen^2(3x))^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(1+sen^2(3x))^{-1/2} \cdot [1+sen^2(3x)]' =$$

$$= \frac{1}{2\sqrt{1+sen^2(3x)}} \cdot (0 + 2sen(3x) \cdot cos(3x) \cdot 3)$$

$$= \frac{6\,sen(3x)\,cos(3x)}{2\sqrt{1+sen^2(3x)}} = \frac{3\,sen(3x)\,cos(3x)}{\sqrt{1+sen^2(x)}}$$

DERIVADA DE FUNÇÕES DO TIPO LOGARÍTMICA

a) $y = \log_3 (\underbrace{x^3+1}_{u})$

$$\frac{dy}{dx} = \frac{1}{u\ln(3)} \cdot u' = \frac{1}{(x^3+1)\ln(3)} \cdot 3x^2 = \frac{3x^2}{(x^3+1)\ln(3)}.$$

b) $y = \log\left(\overset{u}{\frac{2x}{1+x}}\right)$

$$\frac{dy}{dx} = \frac{1}{u\ln(10)} \cdot u' = \frac{1}{\left(\frac{2x}{1+x}\right)\ln(10)} \cdot \left(\frac{2(1+x)-2x\cdot 1}{(1+x)^2}\right)$$

$$= \frac{1}{\frac{2x\ln(10)}{1+x}} \cdot \frac{2+\cancel{2x}-\cancel{2x}}{(1+x)^2} = \frac{\cancel{(1+x)}}{\cancel{2}x\ln(10)} \cdot \frac{\cancel{2}}{(1+x)^{\cancel{2}}}$$

$$\frac{dy}{dx} = \frac{1}{(x+x^2)\ln(10)}$$

c) $y = \ln(2x+3)$ $\qquad \dfrac{dy}{dx} = \dfrac{2}{2x+3}$

d) $y = \ln(x^2+2x+3)$ $\qquad \dfrac{dy}{dx} = \dfrac{2x+2}{x^2+2x+3}$

e) $y = \ln(x^4)$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{x^4} \cdot 4x^3 = \dfrac{4}{x}$

f) $y = \ln(\sqrt{x}) + \sqrt{\ln(x)} = \ln(x^{1/2}) + (\ln(x))^{1/2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{\ln(x)}} \cdot \frac{1}{x} = \frac{1}{2x} + \frac{1}{2x\sqrt{\ln(x)}}$$

$$\frac{dy}{dx} = \frac{\sqrt{\ln(x)} + 1}{2x\sqrt{\ln(x)}}$$

g) $y = \ln[(2-3x)^5]$

$$\frac{dy}{dx} = \frac{1}{(2-3x)^{\cancel{5}}} \cdot 5\cancel{(2-3x)^4} \cdot (-3) = \frac{-15}{2-3x}$$

h) $y = \ln((6x+7)^{1/3})$

$$\frac{dy}{dx} = \frac{1}{\sqrt[3]{6x+7}} \cdot \frac{1}{3\sqrt[3]{(6x+7)^2}} \cdot 6 = \frac{2}{6x+7}$$

i) $y = (\ln(x))^3$ $\qquad \dfrac{dy}{dx} = 3[\ln(x)]^2 \cdot \dfrac{1}{x}$

j) $y = \ln(x^2\sqrt{x^2+1})$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{x^2\sqrt{x^2+1}} \cdot \left(2x\sqrt{x^2+1} + x^2 \dfrac{1}{2\sqrt{x^2+1}} \cdot 2x\right)$

$$\frac{dy}{dx} = \frac{2x\cancel{\sqrt{x^2+1}}}{x^{\cancel{2}}\cancel{\sqrt{x^2+1}}} + \frac{\cancel{2}x^{\cancel{3}}}{\cancel{2}x^{\cancel{2}} \cdot (x^2+1)} = \frac{2}{x} + \frac{x}{x^2+1}$$

k) $y = \ln(\underbrace{\ln(x)}_{u})$

$$\frac{dy}{dx} = \frac{1}{\ln(x)} \cdot \frac{1}{x} = \frac{1}{x\ln(x)}$$

l) $y = \frac{[\ln(x)]^2}{1+x^2}$

$$\frac{dy}{dx} = \frac{2\ln(x) \cdot \frac{1}{x}(1+x^2) - [\ln(x)]^2 \cdot 2x}{(1+x^2)^2} \Rightarrow$$

$$\frac{dy}{dx} = \frac{\frac{2(1+x^2)\ln(x)}{x} - 2x[\ln(x)]^2}{(1+x^2)^2} = \frac{\frac{2(1+x^2)\ln(x) - 2x^2[\ln(x)]^2}{x}}{(1+x^2)^2} =$$

$$= \boxed{\frac{2.(1+x^2)\ln(x) - 2x^2[\ln(x)]^2}{(1+x^2)^2}}$$

m) $y = \ln(x) . \log(x) - \ln(a) \log_a x$

$$\frac{dy}{dx} = \frac{1}{x}\log(x) + \ln(x) . \frac{1}{x\ln(10)} - \left[\cancel{\ln(a)} . \frac{1}{x\cancel{\ln(a)}}\right]$$

$$\frac{dy}{dx} = \frac{\log(x)}{x} + \frac{\ln(x)}{x\ln(10)} - \frac{1}{x} = \frac{1}{x}\left[\log x + \ln(x) - 1\right]$$

n) $y = \log(\operatorname{sen}(x))$ $\quad \frac{dy}{dx} = \frac{1}{\operatorname{sen}(x)\ln(10)} . \cos(x) = \boxed{\frac{\operatorname{cotg}(x)}{\ln(10)}}$

o) $y = \ln[\cos(x^2)] \Rightarrow \frac{dy}{dx} = \frac{1}{\cos(x^2)} . -\operatorname{sen}(x^2) . 2x$

$$\frac{dy}{dx} = -2x\, tg(x^2)$$

p) $y = \sqrt{\ln(x)+1} + \ln(\sqrt{x}+1)$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\ln(x)+1}} \cdot \frac{1}{x} + \frac{1}{\sqrt{x}+1} \cdot \frac{1}{2\sqrt{x}} =$$

$$= \frac{1}{2x\sqrt{\ln(x)+1}} + \frac{1}{2x+2\sqrt{x}}$$

q) $y = \ln\left(\sqrt{\frac{1+x}{1-x}}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \left(\frac{1\cdot(1-x) - (1+x)\cdot(-1)}{(1-x)^2}\right) =$$

$$= \frac{1}{2\left(\frac{1+x}{1-x}\right)} \cdot \frac{(1-\cancel{x}+1+\cancel{x})}{(1-x)^2} = \frac{\cancel{2}}{\cancel{2}\left(\frac{1+x}{\cancel{1-x}}\right)\cdot(1-x)^{\cancel{2}}}$$

$$\frac{dy}{dx} = \frac{1}{(1+x)(1-x)} = \frac{1}{1-x^2}$$

r) $y = sen(\underbrace{\ln(x)}_{u})$

$$\frac{dy}{dx} = \cos(\ln(x)) \cdot \frac{1}{x} = \frac{\cos(\ln(x))}{x}$$

DERIVADA DE FUNÇÕES DO TIPO EXPONENCIAL.

a) $y = 2^{5x}$ u

$\frac{dy}{dx} = 2^{u}\ln(2).\,u' = 2^{5x}\ln(2).\,5$

b) $y = 4^{(-2x)}$

$\frac{dy}{dx} = 4^{(-2x)}.\ln(4).\,(-2) = -2\ln(4).\,4^{(-2x)}$

c) $y = 2^{(7x^2)}$ $\frac{dy}{dx} = 2^{(7x^2)}.\ln(2).\,14x$

d) $y = (x^3+3).\,2^{-7x}$

$\frac{dy}{dx} = 3x^2.\,2^{-7x} + (x^3+3)\,2^{-7x}.\ln(2).(-7)$

$\frac{dy}{dx} = 2^{-7x}.\,[3x^2 - 7(x^3+3)\ln(2)]$

e) $y = 2^{tg(x^2)}$

$\frac{dy}{dx} = 2^{tg(x^2)}.\ln(2).\,sec^2(x^2).\,2x$

f) $y = 7^{\sqrt{x^2+9}}$

$\frac{dy}{dx} = 7^{\sqrt{x^2+9}}.\ln(7).\frac{1}{\cancel{2}\sqrt{x^2+9}}.\cancel{2}x = \frac{\ln(7)\,x.\,7^{\sqrt{x^2+9}}}{\sqrt{x^2+9}}$

g) $y = \pi^x \qquad \dfrac{dy}{dx} = \pi^x \ln(\pi)$

h) $y = e^{2x} \ln(x)$

$$\frac{dy}{dx} = e^{2x} \cdot 2 \ln(x) + e^{2x} \cdot \frac{1}{x} = e^{2x}\left(2\ln(x) + \frac{1}{x}\right)$$

i) $y = e^{x\ln(x)}$

$$\frac{dy}{dx} = e^{x\ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot 1/x\right) = e^{x\ln(x)} \cdot (\ln(x) + 1)$$

j) $y = e^{\left(\frac{x}{\sqrt{4+x^2}}\right)}$

$$\frac{dy}{dx} = e^{\left(x/\sqrt{4+x^2}\right)} \cdot \left(\frac{1 \cdot \sqrt{4+x^2} - x \cdot \frac{1}{\cancel{2}\sqrt{4+x^2}} \cdot \cancel{2}x}{4+x^2}\right) =$$

$$= e^{\frac{x}{\sqrt{4+x^2}}}\left(\frac{\sqrt{4+x^2} - \frac{x^2}{\sqrt{4+x^2}}}{4+x^2}\right) = \left(\frac{\frac{4+\cancel{x^2}-\cancel{x^2}}{\sqrt{4+x^2}}}{4+x^2}\right) \cdot e^{\frac{x}{\sqrt{4+x^2}}}$$

$$\frac{dy}{dx} = \frac{4e^{\frac{x}{\sqrt{4+x^2}}}}{\sqrt{4+x^2}\,(4+x^2)}$$

k) $y = x^{\pi} \cdot \pi^x \qquad \dfrac{dy}{dx} = \pi x^{\pi-1} \cdot \pi^x + x^{\pi} \cdot \pi^x \cdot \ln(\pi) =$

$$= x^{\pi-1}\pi^{x+1} + x^{\pi}\pi^x \ln(\pi)$$

l) $y = 3e^{\sqrt{x}}$

$$\frac{dy}{dx} = 3e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{3e^{\sqrt{x}}}{2\sqrt{x}}.$$

m) $y = \dfrac{e^x - 1}{e^x + 1}$

$$\frac{dy}{dx} = \frac{e^x.(e^x+1) - (e^x-1).e^x}{(e^x+1)^2} = \frac{\cancel{e^{2x}} + e^x - \cancel{e^{2x}} + e^x}{(e^x+1)^2}$$

$$\frac{dy}{dx} = \frac{2e^x}{(e^x+1)^2}$$

n) $y = \sqrt{xe^x + x} = (xe^x + x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{xe^x+x}} (xe^x+x)' = \frac{e^x + xe^x + 1}{2\sqrt{xe^x+x}}$$

o) $y = e^{[(sen(x))^2]}$

$$\frac{dy}{dx} = e^{sen^2(x)} \cdot 2sen(x) \cdot cos(x)$$

p) $y = e^x \ln(sen(x))$

$$\frac{dy}{dx} = e^x \ln(sen(x)) + e^x \cdot \frac{1}{sen(x)} \cdot cos(x) =$$

$$= e^x \left[\ln(sen(x)) + cotg(x) \right]$$

q) $y = \ln\left(\frac{e^{4x-1}}{e^{4x+1}}\right)$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{e^{4x-1}}{e^{4x+1}}\right)} \cdot \left(\frac{e^{4x-1} \cdot 4 \cdot e^{4x+1} - e^{4x-1} \cdot e^{4x+1} \cdot 4}{(e^{4x+1})^2}\right)$$

$$\frac{dy}{dx} = \frac{\cancel{e^{4x+1}}}{e^{4x-1}} \cdot \left(\frac{4e^{4x-1+4x+1} - 4e^{4x-1+4x+1}}{\cancel{e^{4x+1}} \cdot e^{4x+1}}\right) \Rightarrow$$

$$\frac{dy}{dx} = \frac{4e^{8x} - 4e^{8x}}{e^{4x-1} \cdot e^{4x+1}} = 0$$

r) $y = \ln\left(\frac{e^x}{1+e^x}\right)$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{e^x}{1+e^x}\right)} \cdot \left[\frac{e^x \cdot (1+e^x) - e^x \cdot e^x}{[1+e^x]^2}\right] =$$

$$= \frac{\cancel{1+e^x}}{e^x} \left[\frac{e^x + \cancel{e^{2x}} - \cancel{e^{2x}}}{[1+e^x]^{\cancel{2}}}\right] = \frac{1}{\cancel{e^x}} \cdot \frac{\cancel{e^x}}{1+e^x} = \frac{1}{1+e^x}$$

s) $y = e^{\log_4 x}$

$$\frac{dy}{dx} = e^{\log_4 x} \cdot \frac{1}{x\ln(4)}$$

t) $y = \log\left(\frac{3^x}{1+5^x}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{3^x}{1+5^x}\right) \cdot \ln(10)} \cdot \left(\frac{3^x\ln(3)(1+5^x) - 3^x \cdot 5^x \ln(5)}{(1+5^x)^2}\right)$

$$= \frac{\cancel{(1+5^x)}}{\cancel{3^x}\ln(10)} \cdot \left(\frac{\cancel{3^x}\ln(3)(1+5^x) - \cancel{3^x}5^x\ln(5)}{(1+5^x)^{\cancel{2}}}\right) = \frac{\ln(3)(1+5^x) - 5^x\ln(5)}{\ln(10)(1+5^x)}$$