Graph Edge Colorings & the Uniqueness Spectrum

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Brief Background

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- There's many famous results such as the Four Color Theorem, Brooks' Theorem, Vizing's Theorem, Ramsey Theory, and many others.
- This is a very active field of research both in applied and pure mathematics.

Basic definitions

Definition

A graph $G = (V, E)$ is an ordered pair where V is a set containing elements called vertices and E is a set of paired vertices which are called edges. For edge $e = \{v_1, v_2\}$ we call v_1 and v_2 the endpoints.

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For a given graph G we denote the number of vertices as $|G|$ and the number of edges as $\|G\|$.

Graph

- We can draw a graph by representing the vertices as circles and the edges as lines which connect them.

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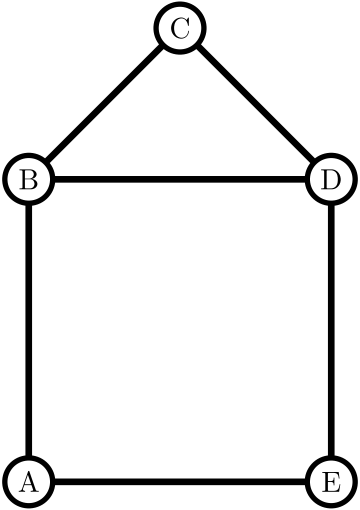

Coloring Definitions

Definition

A k**-edge-coloring** of a graph $G = (V, E)$ is a function $c : E \to [k]$. We call c **proper** if no pair of incident edges is colored the same. The **chromatic index of** G, written $\chi'(G)$, is the minimum $k \in \mathbb{N}$ for which a proper k-edge-coloring of G exists.

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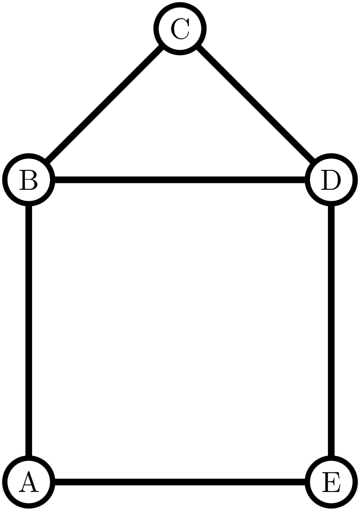
- We can think of edge coloring as assigning a color to an edge.
- We call this coloring proper if whenever two edges share an endpoint they are colored differently.
- We can then think of the chromatic index as the minimum number of colors you need to properly edge color a graph.

Let's Edge Color Something!



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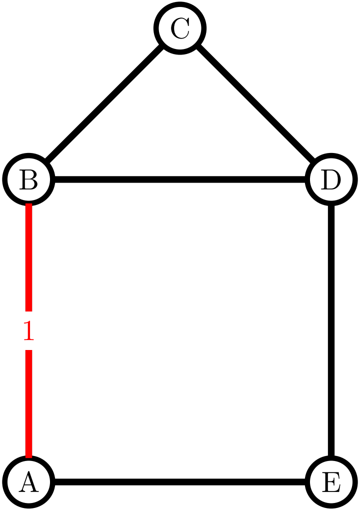

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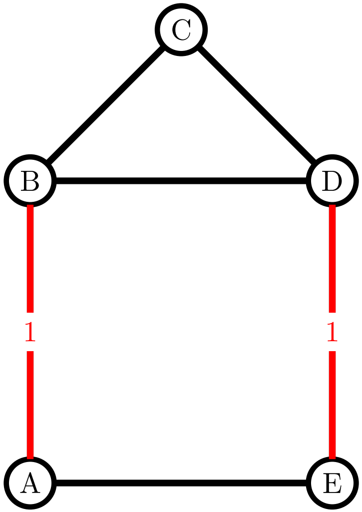

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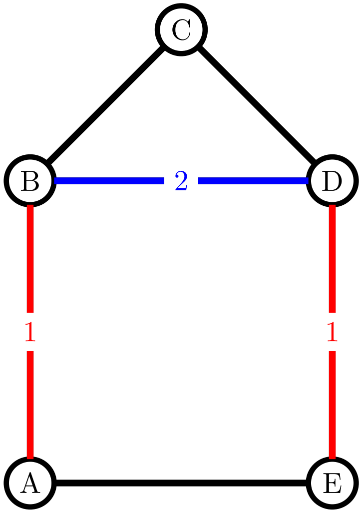

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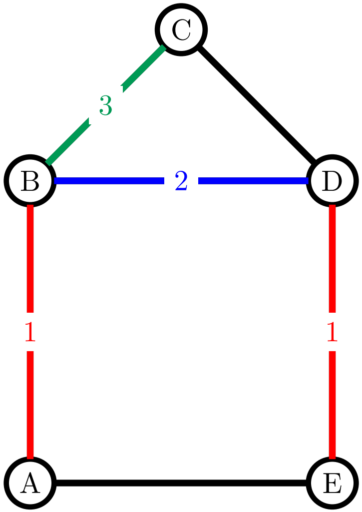

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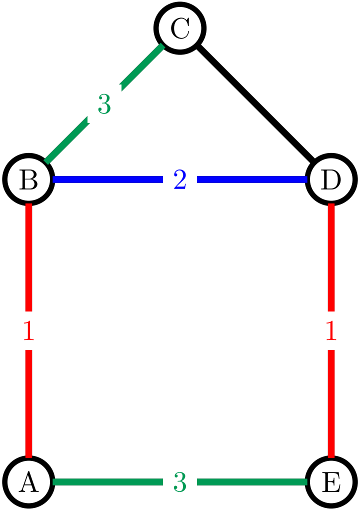

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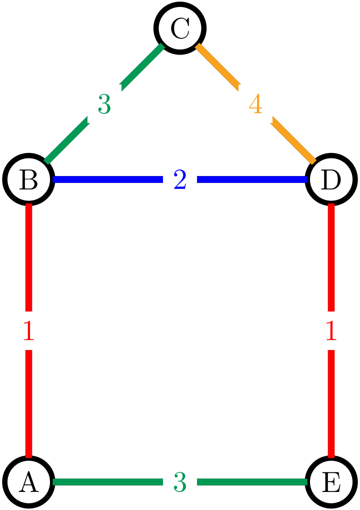

K-Uniqueness

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Given a proper k-edge-coloring c of a graph G, we say that a coloring of G is **k-unique** if there are exactly k edges of G whose colors are not repeated elsewhere in the coloring. That is, whenever $\left|\left\{k \in [k] : \left|c^{-1}(k)\right| = 1\right\}\right| = k$.

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- A coloring is k-unique when exactly k edges have colors which appear nowhere else.

More Coloring!

More Coloring!

This is 0-unique

More Coloring!

More Coloring!

This is 1-unique

More Coloring!

$\text{Spec} = \{0, 1\}$

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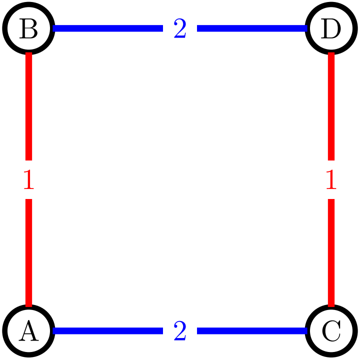

Spec $= \{0\}$

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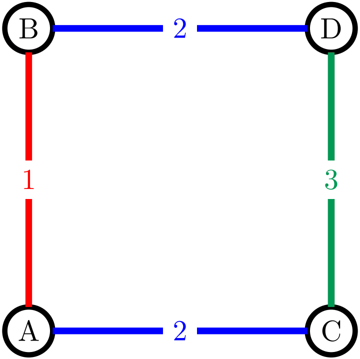

Spec $= \{0, 2\}$

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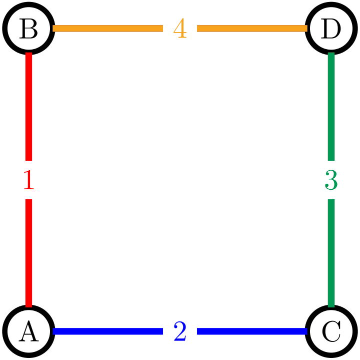

$\text{Spec} = \{0, 2, 4\}$

Important observations

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If we recolor an edge from a color used exactly twice into a new color, our uniqueness goes up by two.

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If we recolor an edge from a color used three (or more) times into a new color, our uniqueness goes up by one.

Uniqueness Spectrum

Definition (Bushaw-Bednar 2022)

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- Why is $\|G\| - 1$ missing?
- What kinds of graphs are full spectrum? What kinds are not?

Guaranteed Values

Lemma

For any graph G, no $\|G\| - 1$-unique coloring can exist.

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- Just color every edge something different!

Before Going Further

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The degree of a vertex v is the number of edges which use v as an endpoint.

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Theorem (Vizing 1964)

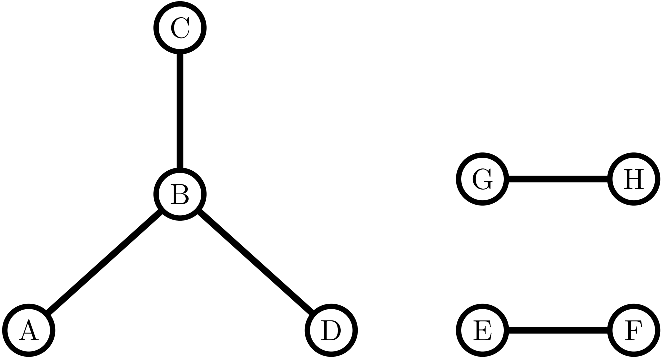
For any graph G the minimum number of colors ($\chi'(G)$) we need to edge color it is either $\Delta(G)$ or $\Delta(G)+1$

Graphs Which Aren't Full Spectrum

Lemma

If G is a graph with $2\Delta(G) > \|G\|$, then G is not full spectrum. Specifically, the graph can have no 0-unique coloring.

Graphs Which Aren't Full Spectrum



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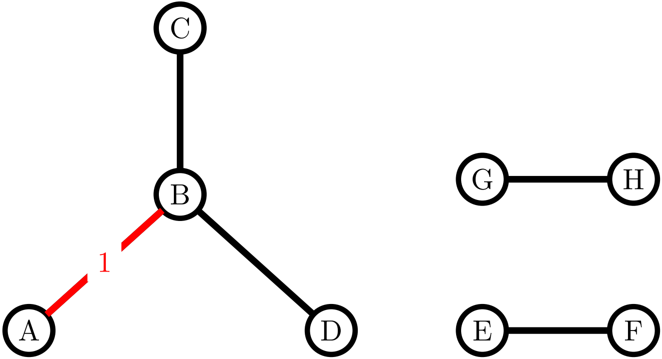

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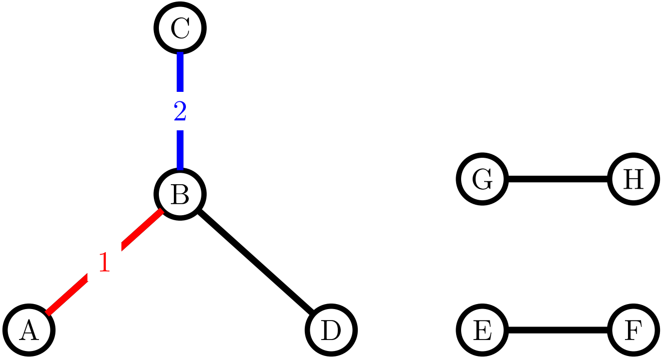

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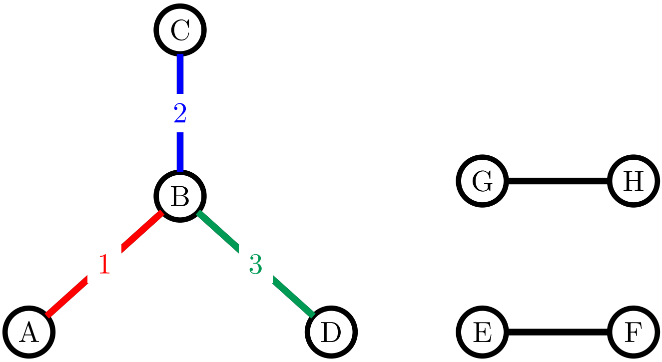

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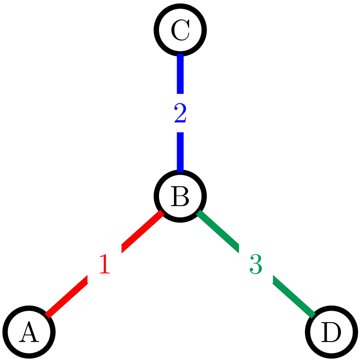

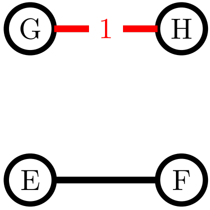

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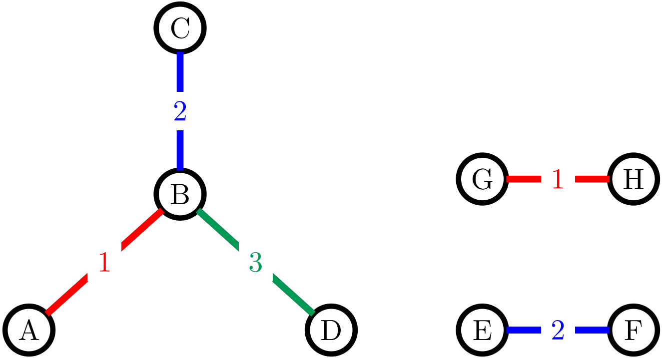

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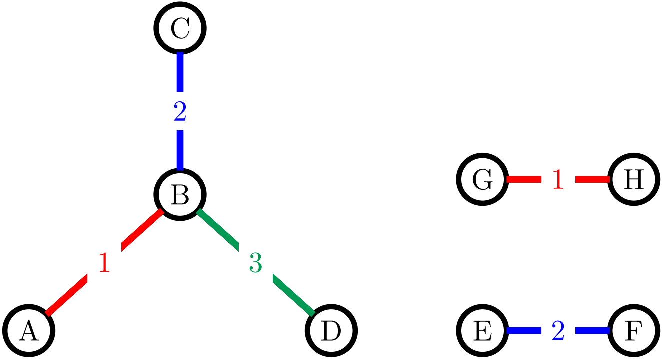

Not enough edges!

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- This is strictly weaker by Vizing's Theorem.

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Lemma (Swap Rule)

Given color classes C_i and C_j where $|C_i| = 1$ and $|C_j| \geq 3$, you can recolor an edge from the color of C_j to the color of C_i

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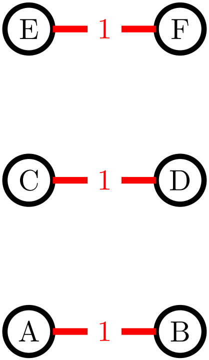
- This reduces the k-uniqueness!

Swap Rule Continued



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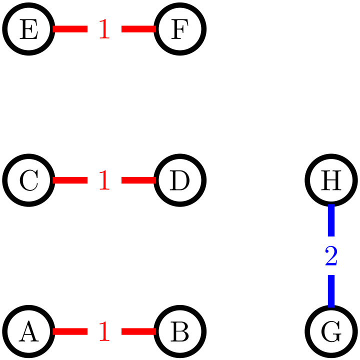

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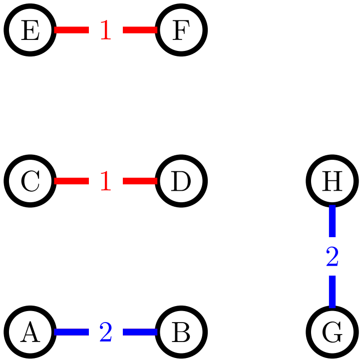

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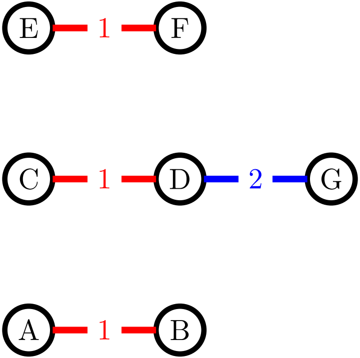

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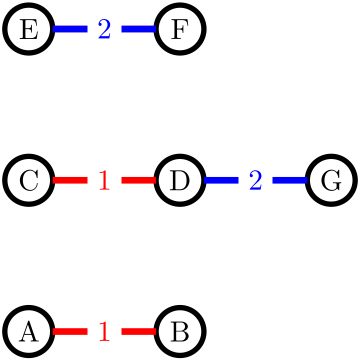

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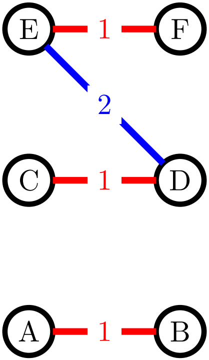

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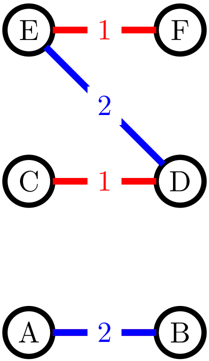

Chain Theorem

Theorem (Chain Theorem)

Given graph G with a k-unique coloring, if $k \leq \|G\| - 4$, then G also has a $(k+2)$-unique coloring

Chain Theorem Proof

Question

What causes the k-uniqueness to increase by 2?

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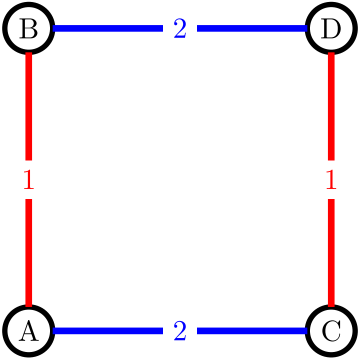
- Two colors get used exactly three times each

Chain Theorem Proof

- Some color gets used exactly twice



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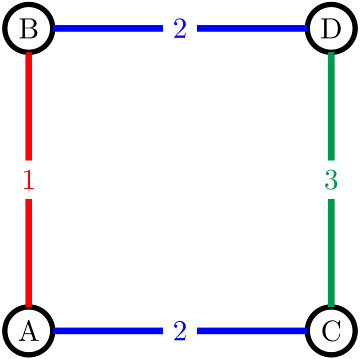

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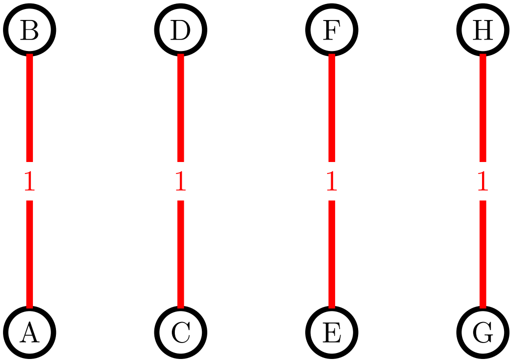
- Some color gets used four or more times



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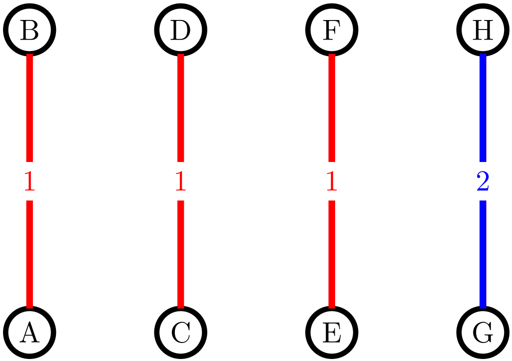

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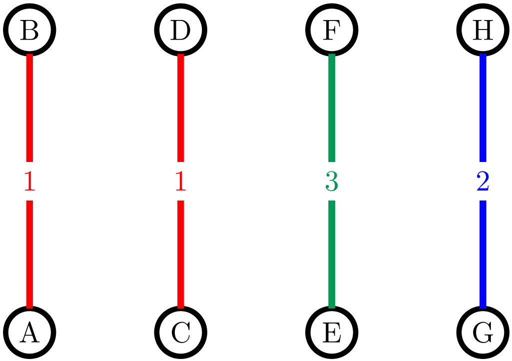

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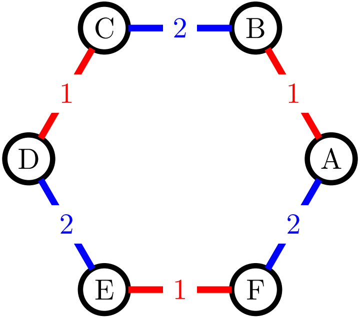

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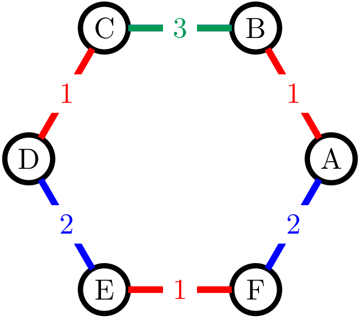

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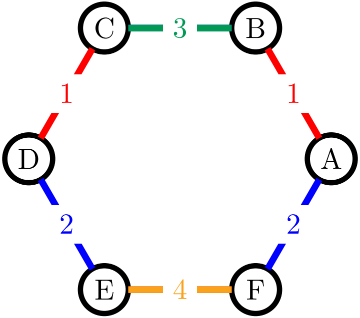

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- Consider a k-unique coloring where $k \leq \|G\| - 4$
- This means that there are at least 4 edges which use repeated colors

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  - Every color gets used exactly three times
- So we must also be $(k+2)$-unique!

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- There's an important observation.
- If a graph G has a 0-unique and 1-unique coloring it must have $2, 3, 4, \ldots, \|G\| - 3, \|G\| - 2$ in the spectrum.

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Given graph G with a k-unique coloring, if $k \leq \|G\| - 4$, then G also has a $(k+2)$-unique coloring

- There's an important observation.
- If a graph G has a 0-unique and 1-unique coloring it must have $2, 3, 4, \ldots, \|G\| - 3, \|G\| - 2$ in the spectrum.
- But we know that $\|G\| - 1$ is never in the spectrum and that $\|G\|$ is always in the spectrum!

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Corollary

If graph G is 0-unique and 1-unique, then it is full spectrum.

Graphs which are full spectrum

Lemma

If G is a graph such that $2\chi'(G) < \|G\|$, then G is full spectrum.

Graphs which are full spectrum

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- By the Chain Theorem we just need to show that G is 0 and 1 unique.

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- Furthermore, we still only use $\chi'(G)$ many colors.

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- Furthermore, we still only use $\chi'(G)$ many colors.
- Therefore, it is still true that there is color which is used three or more times
- We can repeat this process until all colors are used at least twice.
- Such a coloring is 0-unique!

Graphs Which are Full Spectrum

Lemma

If G is a graph such that $2\chi'(G) < \|G\|$, then G is 1-unique.

Graphs Which are Full Spectrum

Lemma

If G is a graph such that $2\chi'(G) < \|G\|$, then G is 1-unique.

- We can create a 0-unique coloring of G using $\chi'(G)$ many colors.

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If G is a graph such that $2\chi'(G) < \|G\|$, then G is 1-unique.

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Lemma

If G is a graph such that $2\chi'(G) < \|G\|$, then G is 1-unique.

- We can create a 0-unique coloring of G using $\chi'(G)$ many colors.
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- Pick any edge with color c and recolor it into a color used nowhere else.

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If G is a graph such that $2\chi'(G) < \|G\|$, then G is 1-unique.

- We can create a 0-unique coloring of G using $\chi'(G)$ many colors.
- Such a coloring has some color c used 3 or more times by the pigeonhole principle.
- Pick any edge with color c and recolor it into a color used nowhere else.
- Such a coloring would be 1-unique!

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- We have that a graph is full spectrum if $2\chi'(G) < \|G\|$.

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- We have that a graph is full spectrum if $2\chi'(G) < \|G\|$.
- We have that a graph is not full spectrum if $2\chi'(G) > \|G\|$.
- What about if $2\chi'(G) = \|G\|$?

The Middle Case

Definition

A matching is a edges sharing no endpoints.

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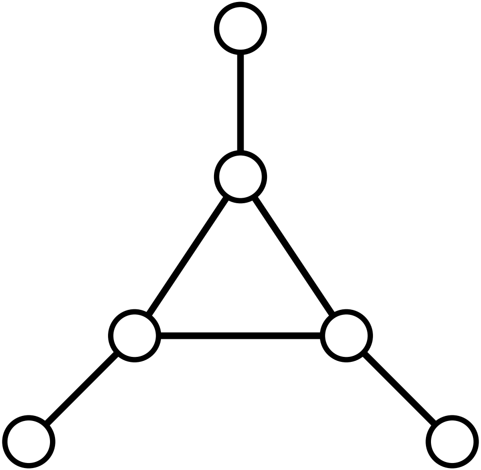
Lemma

If G has $2\chi'(G) = \|G\|$ and $\alpha'(G) \geq 3$ and G is not N_6, then G is full spectrum.

The Net Graph — N_6



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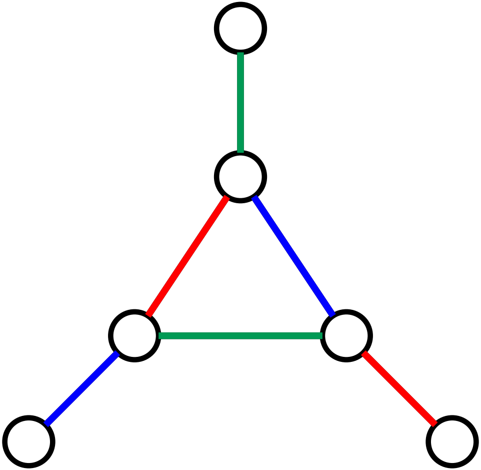

Final Result

Theorem

A graph G is full spectrum if and only if we have $2\chi'(G) \leq \|G\|$, $\alpha'(G) \geq 3$, and G is not N_6.

Common Graphs which are full spectrum

Question

What classes of graphs have full spectrum?

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What classes of graphs have full spectrum?

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What classes of graphs have full spectrum?

- Paths with 5 or more edges.
- Cycles with 6 or more edges.

Common Graphs which are full spectrum

Question

What classes of graphs have full spectrum?

- Paths with 5 or more edges.
- Cycles with 6 or more edges.
- Any class at all that satisfies the final result!

Looking Forward

- Hypergraphs!
- What about instead of just unique colors we look for color classes of other sizes?

Conclusion

Thank you everyone for listening!
Any questions?