Solution of Problem no 3:

Q3.

Cutoff frequency = 1000Hz

$\omega_c = \omega_p = 2\pi f_c = 2\times\pi\times 1000 = 2000\pi$ rad/sec

$\alpha_s = 10dB$; $\omega_s = 2\pi\times 350 = 700\pi$ rad/sec.

$$T = \frac{1}{\text{Sampling frequency}} = \frac{1}{5000Hz} = 2\times10^{-4}\text{sec}.$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times 2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.2\pi) = 7265 \text{ rad/sec}.$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{700\pi\times 2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.07\pi) = 2235 \text{ rad/sec}$$

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\frac{7265}{2235}}$$

$$= \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.93 \simeq 1$$

$\therefore \boxed{N = 1}$ (order = 1)

$\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

$\Omega_c = \Omega_p = 7265$ rad/sec

$$s = \frac{\Omega_c}{s}$$

i.e $s \rightarrow \frac{7265}{s}$

TF of HPF

$$H(s) = \frac{1}{s+1}\Big|_{s=\frac{7265}{s}} = \frac{s}{s+7265}$$

Using Bilinear Transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Bigg|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= 10000\left(\frac{1-z^{-1}}{1+z^{+1}}\right)$$

$$10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265$$

$$= \frac{10^4(1-z^{-1})}{10^4(1-z^{-1})+7265(1+z^{-1})}$$

$$= \frac{10^4(1-z^{-1})}{10000-10000z^{-1}+7265+7265z^{-1}}$$

$$= \frac{10000(1-z^{-1})}{17265-2735z^{-1}}$$

$$= \frac{10^4(1-z^{-1})}{17265(1-0.1582^{-1})} = \frac{0.5792(1-z^{-1})}{(1-0.1584z^{-1})}$$

$$H(z) = \frac{0.5792-0.5792z^{-1}}{1-0.1584z^{-1}}$$

$$y(n) - 0.1584y(n-1) = 0.5792x(n) - 0.5792x(n-1)$$

$$y(n) = 0.5792x(n) - 0.5792x(n-1) + 0.1584y(n-1)$$