Random variable :- A real valued function defined on the Sample space is called a random variable.

Ex: Suppose a coin is tossed twice. Then the Sample Sp. $S = \{HH, HT, TH, TT\}$

Let. X denote the no. of heads appearing. Then X is a random variable.

$\therefore X(H.H) = 2,\ X(H.T) = 1,\ X(TH) = 1,\ X(TT) = 0$

$(\because X : S \to \mathbb{R})$

• **Discrete random variable**:- A random variable that takes on a finite or countably infinite no. of values is called a discrete random variable. If it assumeses uncountable no of values, then it is called non-discrete.

Let X be a discrete random variable which assumes the values $X_1, X_2, X_3, \ldots$ with probabilities given by

$$P(X = X_K) = f(X_K) \text{ for } K = 1, 2, \ldots$$

and $P(X = x) = f(x) = 0$ for other values of x (i.e, when $X \neq X_K$)

then the fn $f(x)$ is called a prob. mass fn (PMF).

Note:- A function $f(x)$ is a PMF if i) $f(x) \geq 0\ \forall x$ &

ii) $\sum_{x} f(x) = 1$

Q) A box contains 2 black & 3 white balls. Balls are drawn successively without replacement. If the random variable X is the no. of draws until the last black ball is obtained, find the PMF of X.

Soln:- Let B denote the black ball & W denote the white ball.

Then the sample space $S = \{BB, BWB, WBB, BWWB, WBWB, WWBB, BWWWB, WWBWB, WWWBB, WBWWB\}$.

$\therefore$ PMF is $P(X=2) = f(2) = \frac{1}{10}$, $P(X=3) = f(3) = \frac{2}{10}$,

$P(X=4) = \frac{3}{10} = f(4)$, $P(X=5) = f(5) = \frac{4}{10}$

Q) Suppose 3 fair coins are tossed. Let X denote the no. of heads appearing. Find the PMF of X.

Soln:- $P(X=0) = P(TTT) = \frac{1}{8}$

$P(X=1) = P(HTT, THT, TTH) = \frac{3}{8}$

$P(X=2) = P(HHT, HTH, THH) = \frac{3}{8}$

$P(X=3) = P(HHH) = \frac{1}{8}$

$$\therefore \text{PMF is } f(x) = \begin{cases} \frac{1}{8}, & x = 0, 3 \\ 3/8, & x = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

Q) The PMF of a (discrete) random variable X is given by $P(i) = \frac{c\lambda^2}{i!}$, $i = 0, 1, 2,$ where λ is a +ve constant (fixed). Find a) the value of c, b) $P(X=0)$ c) $P(X>2)$

Solⁿ: a) $\sum_{i=0}^{\infty} P(i) = 1 \Rightarrow c \sum_{i=0}^{\infty} \frac{\lambda^2}{i!} = 1 \Rightarrow ce^{\lambda} = 1$

$\Rightarrow c = e^{-\lambda}$

b) $P(X=0) = P(i=0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda}$

c) $P(X>2) = 1 - P(X \leq 2) = 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2 e^{-\lambda}}{2}$

*Distribution function for Random Variables:–

The Cummulative distribution function (or briefly the distⁿ fⁿ) for a random variable X is defined by

$F(x) = P(X \leq x)$.

Note:– For a discrete random variable X, is given

$F(x) = P(X \leq x) = \sum_{u \leq x} f(u)$

Q) If PMF of a rand. varb. X is given by

$f(1) = \frac{1}{4}$, $f(2) = \frac{1}{2}$, $f(3) = \frac{1}{8}$, $f(4) = \frac{1}{8}$,

find its distⁿ fⁿ

Solⁿ: $F(x) = P(X \leq x) = \begin{cases} 0, & x < 1 \\ 1/4, & 1 \leq x < 2 \\ 3/4, & 2 \leq x < 3 \\ 7/8, & 3 \leq x < 4 \\ 1, & 4 \leq 2 \end{cases}$

Properties of the dist^n f^n :-

i) $F(x)$ is non-decreasing, $(F(x) \leq F(y)$ if $x \leq y)$

ii) $\lim_{x \to -\infty} F(x) = 0$ & $\lim_{x \to \infty} F(x) = 1$

iii) $F(x)$ is cont^s ~~function~~ from the right (i.e, $\lim_{h \to 0^+} F(x+h) = F(x)$)

Continuous Random variables :- A non-discrete random variable X is said to be cont^s if its distribution f^n may be represented by $F(x) = P(X \leq x)$

$= \int_{-\infty}^{x} f(u)\,dx \quad (-\infty < x < \infty)$, where the f^n $f(x)$ is called the probability Density function (PDF) of X and has the properties

i) $f(x) \geq 0$ & (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Q) ~~What is the prob. that a cont. rand. var. X~~

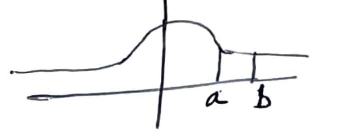
~~takes an~~

Q) If X is ~~the~~ a cont^s random variable with PDF What is the probability that x lies between a & b $(a < b)$?

Sol^n:- $F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$ & $F(b) = P(X \leq b) = \int_{-\infty}^{b} f(x)\,dx$

$\therefore P(a \leq X \leq b) = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx = \int_{a}^{b} f(x)\,dx$

$$\boxed{= F(b) - F(a)}$$



Q) What is the prob. that a Cont^s random varb. X takes an any one particular value ?

Ans⇒ The prob. is zero, for $P(a \leq X \leq a) = P(X=a)$

$$= \int_a^a f(x)\,dx = 0$$

Q) Find the Const. C s.t. the function $f(x) = \begin{cases} Cx^2; & 0<x<3 \\ 0, & \text{otherwise} \end{cases}$ is a PDF of a Cont^s random varb. X. Also find $P(1<X<2)$.

Ans⇒ $$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow C\int_0^3 x^2 dx = 1 \Rightarrow C = \frac{1}{9}$$

Further, $$P(1<X<2) = \frac{1}{9}\int_1^2 x^2 dx = \frac{7}{27}.$$

Q) Find the distribution function of the previous rand. var. whose PDF is given by $f(x) = \begin{cases} \frac{1}{9}x^2, & 0<x<3 \\ 0, & \text{otherwise} \end{cases}$

Also find $P(1<X<2)$ using the dist^n f^n

Ans⇒ The dist^n f^n is –

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

If $x \leq 0$, then $F(x) = 0$

If $0<x<3$, then $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x).\,dx$

$$= \int_0^x f(x)\,dx = \int_0^x \frac{1}{9}x^2 dx$$

If $x \geq 3$, then $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx = \int_0^3 \frac{1}{9}x^2 dx = 1$

$$\therefore F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^3}{27}, & 0<x<3 \\ 1, & x \geq 3 \end{cases}$$

Q) How to obtain density fn from the distribution fn?

Ans⇒ $F(X) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \Rightarrow \frac{d(F(X))}{dx} = f(x)$

Q) The distribution fn of a Conts random varb. X is:-

$$F(x) = \begin{cases} 1-e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find, (a) the density fn b) $P(X>2)$ c) $P(-3<X\leq$

Soln:- a) $f(x) = \frac{d}{dx}F(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

b) $P(X>2) = \int_{2}^{\infty} 2e^{-2x}\,dx = e^{-4}$

or

$P(X>2) = 1 - P(X\leq 2) = 1 - F(2) = 1-($

$= e^{-4}$

c) $P(-3<X\leq 4) = F(4) - F(-3)$

$= 1 - e^{-8} - 0$

$= 1 - e^{-8}$