

Forward = backward \rightarrow Standard or stationary wave

$$c = \frac{x}{t}$$

$$x = ct \quad \text{--- (1)}$$

wave travel from left to right

$$Q_1 = A \sin 2\pi \left(\frac{x}{\lambda} - vt \right) \quad \text{--- (2)}$$

$$Q_2 = A \sin 2\pi \left(\frac{x}{\lambda} + vt \right) \quad \text{--- (3)}$$

$$Q \propto Q_1 + Q_2$$

$$Q = Q_1 + Q_2$$

$$= A \sin 2\pi \left(\frac{x}{\lambda} - vt \right) + A \sin 2\pi \left(\frac{x}{\lambda} + vt \right)$$

$$= 2A \sin 2\pi \left(\frac{x}{\lambda} \right) \cos 2\pi vt \quad \text{--- (4)}$$

Schrodinger eqⁿ :-

microsystem = e^-, p^+, n \rightarrow these contents $\begin{cases} PE \\ KE \end{cases}$
 $e = m \rightarrow x$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - V) \psi = 0$$

$$\frac{d^2\psi}{dx^2} = -\frac{1}{c^2} \frac{d^2\psi}{dt^2} \quad \text{--- (1)}$$

$$\phi(x, t) = 2A \sin 2\pi \left(\frac{x}{\lambda} \right) \cos 2\pi \nu t \quad \text{--- (i)}$$

$$= \psi(x) \cdot f(t)$$

$$= \psi(x) \cdot \cos 2\pi \nu t \quad \text{--- (ii)}$$

$$\frac{d^2 \phi}{dx^2} = \frac{d^2 \psi}{dx^2} \times \cos 2\pi \nu t \quad \text{--- (iii)}$$

$$\frac{d\phi}{dt} = -2\pi \nu \sin 2\pi \nu t \times \psi(x)$$

$$\begin{aligned} \frac{d^2 \phi}{dt^2} &= -2\pi \nu \times 2\pi \nu \times \cos 2\pi \nu t \times \psi(x) \\ &= -4\pi^2 \nu^2 \cos 2\pi \nu t \times \psi(x) \quad \text{--- (iv)} \end{aligned}$$

Substituting (iii) and (iv) in (i)

$$\frac{d^2 \psi}{dx^2} \cos 2\pi \nu t = \frac{1}{c^2} \times (-4\pi^2 \nu^2 \cos 2\pi \nu t \times \psi(x))$$

we know,

$$\therefore \frac{d^2 \psi}{dx^2} = \frac{1}{\lambda^2 \nu^2} (-4\pi^2 \nu^2 \psi(x))$$

$$\Rightarrow \frac{d^2 \psi}{dx^2} = -\frac{4\pi^2}{\lambda^2} \psi(x)$$

From de-Broglie

$$\lambda = \frac{h}{m\nu}$$

$$= \frac{-4\pi^2}{\left(\frac{h^2}{m^2 \nu^2} \right)} \psi(x)$$

$$= \frac{-4\pi^2 m^2 \nu^2}{h^2} \psi(x) \quad \text{--- (v)}$$

Total Energy

$$E = KE + PE$$

$$E = \frac{1}{2}mv^2 + V$$

$$\frac{1}{2}mv^2 = E - V$$

$$mv^2 = 2(E - V)m \quad \text{--- (vii)}$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{-4\pi^2 2(E - V)m}{h^2} = \frac{-8\pi^2 m(E - V)\psi}{h^2}$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} (E - V)\psi = 0 \quad \text{--- 1D}$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{8\pi^2 m}{h^2} (E - V)\psi = 0 \quad \text{--- 2D}$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{y^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2} (E - V)\psi = 0 \quad \text{--- 3D}$$

Probability distribution of wave function

$$P \propto |\psi|^2 dx$$

$$P \propto |\psi|^2 dx dy$$

$$P \propto |\psi|^2 dx dy dz$$

$$\underline{ds = \text{volume element}}$$



Normalize:

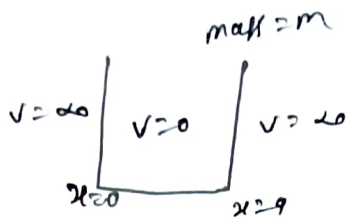
$$\int_{-\infty}^{\infty} \psi_i^* \psi_j ds = 1 \quad i=j$$

orthogonal

$$\int_{-\infty}^{\infty} \psi_i^* \psi_j ds = 0 \quad i \neq j$$

1-D kinetic energy :

Particle in a 1D box



outside the box :

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - \infty) \psi = 0 \quad \text{--- (i)}$$

$$E = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (0 - \infty) \psi = 0$$

$$\frac{d^2\psi}{dx^2} = \frac{8\pi^2m}{h^2} \infty \psi = 0 \quad \text{--- (ii)}$$

$$\frac{1}{\infty} \frac{d^2\psi}{dx^2} = \frac{8\pi^2m}{h^2} \infty \psi = 0$$

inside the box :

$$x=0, x=a, V=0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E \psi) = 0 \quad \text{--- (iii)}$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{--- (iv)}$$

$$\psi = A \sin kx + B \cos kx$$

$$\text{At } x=0, \psi=0$$

$$0 = A \sin k \cdot 0 + B \cos k \cdot 0$$

$$\therefore B=0$$

$$\text{①} \Rightarrow \psi = A \sin kx \quad \text{--- (v)}$$

$$\text{At } x=a, \psi=0$$

$$0 = A \sin ka$$

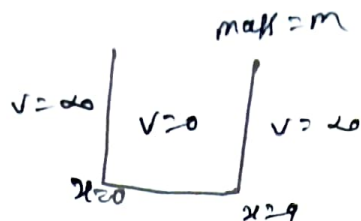
$$A=0, \sin ka=0$$

↓

Not accurate

1-D kinetic Energy :

Particle in a 1D box



outside the box :

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - \infty) \psi = 0 \quad \text{--- (i)}$$

$$E = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} (0 - \infty) \psi = 0$$

$$\frac{d^2\psi}{dx^2} = \frac{8\pi^2m}{h^2} \infty \psi = 0 \quad \text{--- (ii)}$$

$$\frac{1}{\infty} \frac{d^2\psi}{dx^2} = \frac{8\pi^2m}{h^2} \infty \psi = 0$$

inside the box :

$$x = 0, \quad x = a, \quad V = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} E \psi = 0 \quad \text{--- (iii)}$$

$$\frac{d^2\psi}{dx^2} + k^2 \psi = 0 \quad \text{--- (iv)}$$

$$\psi = A \sin kx + B \cos kx$$

$$\text{At } x = 0, \quad \psi = 0$$

$$0 = A \sin k \cdot 0 + B \cos k \cdot 0$$

$$\therefore B = 0$$

$$\text{(v)} \Rightarrow \psi = A \sin kx \quad \text{--- (vi)}$$

$$\text{At } x = a, \quad \psi = 0$$

$$0 = A \sin ka$$

$$A = 0, \quad \sin ka = 0$$

↓

Not accurate

$$3. \sin ka = 0 = \sin n\pi = ka = n\pi$$

$$k = \frac{n\pi}{a} \quad \text{--- (vii)}$$

$$k^2 = \frac{n^2\pi^2}{a^2}$$

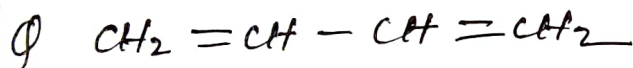
$$ka = n\pi$$

$$\frac{8\pi^2m}{n^2} E = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2h^2}{8ma^2}$$

$$n = 1, 2, 3, \dots$$

(n ≠ 0)



$$2 \times 0.154 \text{ nm} + 2 \times 0.154 \text{ nm}$$

$$= 0.57 \text{ nm}$$

$$E_1 = \frac{n^2h^2}{8ma^2} = \frac{1^2h^2}{8 \times 9.1 \times 10^{-31} \times (0.57)^2}$$

$$E_2 = \frac{n^2h^2}{8ma^2} = \frac{2^2h^2}{8 \times 9.1 \times 10^{-31} \times (0.57)^2}$$

$$E_3 = \frac{n^2h^2}{8ma^2} = \frac{3^2h^2}{8 \times 9.1 \times 10^{-31} \times (0.57)^2}$$