

(Probability and Statistics)

Module - I :— Basic probability, independence, Discrete random variables, independent random variables, the multinomial distribution, Poisson approximation to the binomial distribution, infinite sequences of Bernoulli trials, sums of independent random variables; Expectation of Discrete random variables, Moments, variance of a sum, Correlation Coefficient, Chebyshev's Inequality.

* Random experiment :— An experiment which can be repeated any no. of times under similar conditions & the outcome of any particular experiment can't be predicted from the past experience is known as a random experiment.

e.g. :— Tossing a coin, Throwing a die, taking out a card from a pack etc.

* Sample space :— The set of all possible outcomes of a random experiment is called the sample space for that random experiment and is denoted by S .
e.g. if a fair die is thrown,
then $S = \{1, 2, 3, 4, 5, 6\}$

* Sample point :— The elements of the sample space are called sample points.

Event :— Any subset of the sample space S is called an event. Thus an event consisting of a single point of S is called sample point.

Mutually Exclusive Events:- Two or more events are said to be mutually exclusive or disjoint if the occurrence of any one of them prevents the occurrence of all others.

Classical approach:-

Definition of probability:- If an event can occur in n different ways out of a total no. of m possible ways, all of which are equally likely, then the probability of the event is n/m .

e.g. Tossing of a coin, $P(H) = \frac{1}{2}$

Limitations:- Applicable only in finite and discrete cases. e.g. height of a human being usually lies in the range $[0\text{ft}, 7\text{ft}]$. Hence if we want to find out the probability that height of a person is 5.5ft (or lies in the range 5ft to 6ft) the sample sp. is infinite. So this definition is not applicable.

2) Frequency approach:- If after n repetitions of an experiment, where n is very large, an event is observed to occur in h of these, then the prob. of the event is h/n .

Limitations:- The term very large is vague. According to this def if $n=1000$ is large and heads comes up 536 times in tossing a coin, then

then, $P(H) = 0.536$, which is wrong.

Q) what is the prob. that height of a person is 5.5 ft?

(Ans. to such qns. Can't be given directly with above defns. For that we need to introduce discrete and continuous random variables also the axioms of probability)

→ So, prob. of an event may be 0 but it can still happen.

The Atomic Defn. of Probability:-

Let S be a sample space. To each event E , a real no. $P(E)$ is associated, called the prob. of E , satisfying the following axioms:

1) $0 \leq P(E) \leq 1$

2) $P(S) = 1$, (prob. of the entire sample space is 1)

3) For any set of mutually exclusive events E_1, E_2, E_3, \dots
($E_i \cap E_j = \emptyset$ when $i \neq j$), $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

Q. P.T. $P(\{5\}) = \frac{1}{6}$ in throwing of a dice.

Some Properties of Probability:-

1) Prob. of the impossible event is zero, i.e. $P(\emptyset) = 0$
(e.g. getting 7 in tossing a dice)

2) If A^c is the complement of A , then $P(A^c) = 1 - P(A)$

3) If A & B are any 2 events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4) If $B \subseteq A$, then $P(B) \leq P(A)$