Conditional probability :— The Conditional prob. of an event B, assuming that the event A has occured is denoted $P(B/A)$ & defined as -

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) \neq 0}$$

Example:— A fair dice is tossed. The prob. of getting '1' given that an odd no. has been obtained is 1/3.

As per formula, $P(B/A) = \dfrac{1/6}{3/6} = \dfrac{1}{3}$

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{1\}$

Independent events :— Two events A & B are said to be independent if occurrence of one does not depend or affect the occurance of other.

Thus, if A & B are independent,

then $P(B/A) = P(B)$

i.e, $\dfrac{P(A \cap B)}{P(A)} = P(B) \Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$

8) A box contains tags marked 1, 2, ..., n. Two tags are chosen at random without replacement. Find the prob. that the no. on the tags are consecutive integers.

Sol^n:- Must be a pair from following:- (1,2), (2,3), ..., $(n-1, n) \rightarrow (n-1)$ pairs

No. of ways of choosing any pair from the above $= {}^{n-1}C_1 = n-1$

Total no. of ways of choosing 2 tags from n tags $= {}^nC_2 = \frac{n!}{2!(n-2)!}$

$\therefore$ Req. prob. $= \dfrac{n-1}{\frac{n(n-1)}{2}}$

$= \frac{2}{n}$

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8) In a shooting test, the prob. of hitting the target is $\frac{1}{2}$ for A, $2/3$ for B & $3/4$ for C. If all of them fire at the targets, find the prob. that

(i) none of them hits the target.

(ii) at least one of them hits the target.

(iii) all of them hits the target.

Sol^n:- Let A denote the event of A hitting the target and so on,

$P(A') = \frac{1}{2}, \quad P(B') = \frac{1}{3}, \quad P(C') = \frac{1}{4}$

i) P(none hits the target) $= P(A^c \cap B^c \cap C^c)$

$= P(A^c)\, P(B^c)\, P(C^c)$

$(\because$ independent$)$

$= \frac{1}{24}$

ii) P(at least one hits the target) $= 1 - P(\text{none hits})$

$= 1 - \frac{1}{24}$

$= \frac{23}{24}$

iii) P(all hit the target) $= P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= \frac{1}{4}$

<u>Theorem of Total probability</u> :— If an event A must result in one of the mutually exclusive events $A_1, A_2, \ldots, A_n$ then

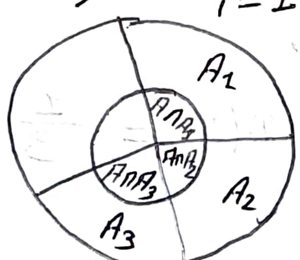
$$P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2) + \ldots + P(A_n)\,P(A/A_n)$$

<u>Pf.</u> The inner circle represents the event A, which can occur along with (or due to) $A_1, A_2, \ldots,$

$\therefore A \cap A_1,\ A \cap A_2, \ldots\ldots, A \cap A_n$ are mutually exclusive

$\therefore A = (A \cap A_1) \cup \ldots \cup (A \cap A_n)$

$$\therefore P(A) = \sum_{i=1}^{n} P(A \cap A_i) = \sum_{i=1}^{n} P(A_i)\,P(A/A_i)$$



Baye's theorem :— Let $A_1, A_2, \ldots, A_n$ be mutually exclusive events whose union is the sample space S, i.e., one of the events must occur. Then if A is any event, we have

$$P(A_k/A) = \frac{P(A_k)\,P(A/A_k)}{\sum_{j=1}^{n} P(A_j)\,P(A/A_j)}$$

Proof :- Since A results in one of the mutually exclusive events $A_1, A_2, \ldots, A_n$, by the theorem of total probability.,

$$P(A) = P(A_1)\,P(A/A_1) + \ldots + P(A_n)\,P(A/A_n)$$

$$\therefore P(A_k/A) = \frac{P(A_k \cap A)}{P(A)} = \frac{P(A_k)\,P(A/A_k)}{\sum_{j=1}^{n} P(A_j)\,P(A/A_j)}$$

Q) A bolt is manufactured by 3 machines A, B & C. A turns out twice as B, and B & C produce equal no. of items. 2% of bolts produced by A & B are defective and 4% of bolts produce by C are defective. All bolts are put together and one is chosen at random from this pile. What is the probability that it is defective ?

Soln:— Let A denote the event that the item has been prod. by A & so on.
D denote the event that the item is defective.

$P(A) = 1/2$, $P(B) = P(C) = 1/4$

$\therefore P(D/A) = P($ the item is defective given that has produced it$)$

$= \frac{2}{100} = P(D/B) \qquad P(D/C) = \frac{4}{100}$

By the theom. of total prob.

$$P(D) = P(A)\,P'(D/A) + P(B)\,P(D/B) + P(C)\,P(\ldots$$

$$= \frac{1}{2} \times \frac{2}{100} + \frac{1}{4} \times \frac{2}{200} + \frac{1}{4} \times \frac{4}{100}$$

$$= \frac{1}{40}$$

Eg) In a Coin-tossing experiment, if the coin shows head 1 Dice is thrown & the result is recorded. But if the coin shows tails, 2 dice are thrown and their sum is recorded. What is the prob. that the recorded no will be 2 ?

Sol^n:- when a single dice is thrown, $P(2) = \frac{1}{6}$

when 2 dice are thrown, their sum will be 2, only if each die shows 1.

$\therefore P($getting 2 as sum with 2 dice$) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

$(\because$ independent$)$

By the theom of total prob.

$$P(2) = P(H) \times P(2/H) + P(T)\,P(2/T)$$

$$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{36}$$

$$= 7/72$$

8) A bag contains 5 balls and it not known how many of them are white. Two balls are drawn at random from the bag & they are formed to be white. What is the prob. that all the balls in the beg are white?

Solⁿ

Solⁿ— Since 2 white balls are drawn, the bag must have 2, 3, 4 or 5 white balls.

Let B_1 denote the event of the bag containing 2 white balls and similarly $B_2, B_3, B_4, \dots$

Let A denote the event of drawing 2 white balls.

$$P(A/B_1) = \frac{2C_2}{5C_2} = \frac{1}{10}, \quad P(A/B_2) = \frac{3C_2}{5C_2}$$

$$P(A/B_3) = \frac{4C_2}{5C_2} = \frac{3}{5}, \quad P(A/B_4) = \frac{5C_2}{5C_2} = 1$$

Since the no. of white balls in the bag are not known, B_i S are equally likely,

$$\therefore P(B_1) = P(B_2) = P(B_3) = P(B_4) = \frac{1}{4}$$

By Bayes' therm, $$P(B_4/A) = \frac{P(B_4) \times P(A/B_4)}{\sum_{j=1}^{4} P(B_j) \times P(A/B_j)}$$

$$= \frac{1/4 \times 1}{\frac{1}{4} \times \left(\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1 \right)}$$

$$= \frac{1}{2} \text{ or } 10/23$$

Q) There are 3 true coins & 1 false coin with head on both sides. A coin is chosen at random & tossed 4 times. If head occurs all the 4 times, what is the prob. that the false coin has been chosen.

Solⁿ:
$$P(T) = P(\text{Coin is true}) = 3/4$$
$$P(F) = P(\text{false coin}) = 1/4$$

Let A be the event of getting all heads in 4 tosses

Then, $P(A/T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$$= \frac{1}{16}$$

$$P(A/F) = 1$$

By Baye's theorem, $P(F/A) = \dfrac{P(F)\,P(A/F)}{P(F)\,P(A/F) + P(T)\,P(A/T)}$

$$= \frac{1/4 \times 1}{\frac{1}{4} \times 1 + \frac{3}{4} \times \frac{1}{16}}$$

$$= \frac{16}{19}$$