School of Computing
FACULTY OF ENGINEERING AND
PHYSICAL SCIENCES

Final Report

Realistic Ocean Simulation using Fourier Transform

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COMP3931 Individual Project

The candidate confirms that the following have been submitted.

Items	Format	Recipient(s) and Date
Final Report	PDF file	Uploaded to Minerva (DD/MM/YY)
<Example> Scanned participant consent forms	PDF file / file archive	Uploaded to Minerva (DD/MM/YY)
<Example> Link to online code repository	URL	Sent to supervisor and assessor (DD/MM/YY)
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Summary

Other Solutions:

- Sum of sines
- Gerstner waves
- Particle based ocean simulation

Problems:

- Computaiontally expensive to calculate huge amouts of sins.
- Gerstner waves are not physically accurate.
- Both are not easily modifiable.
- Does not look good with stormy weather.
- Tiling is noticable.
- Lacks water detail.

Problems I intend to solve:

- Simulate water based on emipirical data for a more realistic look.
- Have an easy modifiable ocean system.
- Have an ocean that can simulate stormy weather.
- Reduce tiling.
- Have a more detailed ocean.

Main achivements:

- Calculate on GPU.
- Implement JONSWAP spectrum.
- Implement FFT algorithm.
- Simulate foam.
- Shade the ocean.

Acknowledgements

<The page should contain any acknowledgements to those who have assisted with your work. Where you have worked as part of a team, you should, where appropriate, reference to any contribution made by other to the project.>

Note that it is not acceptable to solicit assistance on 'proof reading' which is defined as the "the systematic checking and identification of errors in spelling, punctuation, grammar and sentence construction, formatting and layout in the test"; see
`https://www.leeds.ac.uk/secretariat/documents/proof_reading_policy.pdf`

Contents

Chapter 1

Introduction and Background Research

1.1 Introduction

Ocean surface simulation 1.1 is a field of computer graphics that aims to create realistic representations of the ocean surface. It is an important area of research as it has a wide range of applications, including video games, movies. In video games it used for inteactivity and visual apeal, while in movies it is used for visual apeal.

There are many techniques that have been developed to simulate ocean surfaces, from the simple and fast algorithms as sum of sines or gersner waves to more complex techniques such as particle simulations and Fast Fourier Transform (FFT) Ocean 1.1.

The primary objective of this project is to simulate a realistic ocean surface, one that does not exhibit any tiling or repeating patterns, and that can accurately represent stormy weather conditions.

[Missing Logic Linking]

This is achieved by leveraging the power of the Inverse Fourier Transform (IFFT), a mathematical technique that transforms data from the frequency domain back to the time (or spatial) domain. In the context of this project, it allows us to transform the frequency data of the ocean waves into a spatial representation, i.e., the height map of the ocean surface. This is desireble as it allows us to simulate realistic ocean surfaces that are not only visually appealing, but also physically accurate as we are using real-world data to generate frequencies.

Figure 1.1: FFT Ocean Simulation

1.2 Fourier Transform

The Fourier Transform is a mathematical method that enables us to convert our data from the time domain to the frequency domain, and vice versa. To simplify, imagine we have a smoothie that's too sour. With the Fourier Transform, we could deconstruct our smoothie (time domain) into its ingredients (frequency domain), remove the sour component, and then reconstruct it back. It was invented by Joseph Fourier[1] in 1822, altough at his time it didn't have much practical use, however, nowdays it is used in many fields, including signal processing, image processing, and in our case ocean simulation.

To convert our data from the time domain to the frequency domain, we use the Fourier Transform:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx \tag{1.1}$$

, where $f(x)$ is the function in the time domain, $\tilde{f}(\omega)$ is the function in the frequency domain, and ω is a frequency. To convert our data from the frequency domain back to the time domain, we use the Inverse Fourier Transform:

$$f(x) = \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega x} d\omega \tag{1.2}$$

As, we going to work with discrete data, we are going to use the Discrete Fourier Transform (DFT):

$$x_n = \sum_{k=0}^{N-1} \tilde{x}_k e^{-i2\pi kn/N} \tag{1.3}$$

and the Inverse Discrete Fourier Transform (IDFT):

$$\tilde{x}_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{i2\pi kn/N} \tag{1.4}$$

1.3 Fourier Transform Ocean

A significant challenge associated with the utilization of Gerstner waves, as will be elaborated upon in the related works section, is the necessity to generate multiple waves for each vertex in order to simulate an ocean. This process can be computationally intensive, particularly considering that an oceanic simulation may be constructed of thousands vertices. In response to this computational demand, J. Tessendorf [2] proposed a method for generating ocean waves using the Fourier Transform.

The main idea is to generate a height map of the ocean surface in the frequency domain, and then convert it back to the time domain using the IFT. Since the IFT produces a periodic height map, we can tile it to create an infinite ocean surface. This means that we can generate one texture and apply it to the entire water body.

The advantage of this method becomes apparent when considering a 512x512 texture, which combines 262,144 distinct waves. In contrast, Gerstner waves experience noticeable performance

loss after 65 waves.

To produce the height map, we first need a function that approximates the frequencies. We call this function a spectrum. J. Tessendorf uses a modified Phillips spectrum that generates Fourier amplitudes in the frequency domain:

$$\tilde{h}_0(\mathbf{k}) = \frac{1}{\sqrt{2}}(\xi_r + \xi_i)\sqrt{P_h(\mathbf{k})} \tag{1.5}$$

where ξ_r and ξ_i is complex number made from real and imaginary part that was drawn from a gaussian random number generator, with mean 0 and stardart deviation 1.

We then combine $\tilde{h}_0(\mathbf{k})$ with its conjugate $\tilde{h}_0^*(-\mathbf{k})$, to "produce waves towards and against the wave direction when propagating"[3]:

$$\tilde{h}(\mathbf{k}, t) = \tilde{h}_0(\mathbf{k})e^{i\omega(k)t} + \tilde{h}_0^*(-\mathbf{k})e^{-i\omega(k)t} \tag{1.6}$$

By performing the Inverse Discrete Fourier Transform (IDFT):

$$h(\mathbf{x}) = \sum_{\mathbf{k}} \tilde{h}(\mathbf{k}, t)e^{i\mathbf{k}\cdot\mathbf{x}} \tag{1.7}$$

we obtain the height map, where $\mathbf{x} = (x, y)$ is the position in the texture.

Figure 1.2: Height Map using J. Tessendorf's Spectrum

1.4 Ocean Spectrums

1.4.1 Jerry Tessendorf's Ocean Spectrum

The main thing that defines how ocean will looks is the spectrum. Inside Jerry Tessendorf's [2] paper, he proposed modified Phillips spectrum 2.3:

$$P_h(\mathbf{k}) = A\frac{e^{-1/(kL)^2}}{k^4}|\hat{\mathbf{k}} \cdot \mathbf{w}|^6 \tag{1.8}$$

where A is numeric constant, $L = V^2/g$ is the largest possible wave arising from wind speed V and gravity g. The $|\hat{\mathbf{k}} \cdot \mathbf{w}|^6$ is used to surpress waves that are moving perpendicular to the wind direction. However, J. Tessendorf admits that this spectrum has "poor convergence properties at high values of the wavenumber $|\mathbf{k}|$ and suggests to surpress waves that are $l \leq L$, where l is some small constant and multiply the Phillips spectrum by $e^{-k^2 l^2}$.

1.4.2 JONSWAP Spectrum

Regarding all the issues that we experiance with "Phillips Spectrum", Horvath Christopher [3] proposed to use JONSWAP spectrum. JONSWAP stands for "Joint North Sea Wave Project" and was developed by Hasselmann [4, et al. in 1973]. This spectrum is improved version of Pierson-Moskowitz Spectrum [5] when Horvath noticed that the wave spectrum is never fully developed. Therefore he added extra peak enchancement factor γ^r.
The JONSWAP spectrum is defined as follows:

$$\begin{aligned} S_{\text{JONSWAP}}(\omega) &= \frac{\alpha g^2}{\omega^5} \exp\left(-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right)\gamma^r \\ r &= \exp\left(-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right) \\ \alpha &= 0.076\left(\frac{U_{10}^2}{Fg}\right)^{0.22} \\ \omega_p &= 22\left(\frac{g^2}{U_{10}F}\right)^{1/3} \\ \gamma &= 3.3 \\ \sigma &= \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases} \end{aligned} \tag{1.9}$$

where F is the fetch (distance to lee shore), U_{10} is the wind speed at 10 meters above the sea level and ω is angular frequency.

1.4.3 The Texel MARSEN ARSLOE (TMA) Spectrum

The biggest problem with with JONSWAP spectrum is that it only works with deep waters. Therefore, Horvath Christopher [3] proposed to use TMA correction for deep water spectrum. TMA was developed by Steven A. Hughes [6], based on the observations made by Kitaigordskii [7]. this is known as Kitaigordskii depth Attenuation function and it takes form:

$$\begin{aligned} \Phi(\omega_h) &= \left[\frac{(k(\omega,k))^{-3}\frac{\partial k(\omega,h)}{\partial\omega}}{(k(\omega,\infty))^{-3}\frac{\partial k(\omega,\infty)}{\partial\omega}}\right] \\ \omega_h &= \omega\sqrt{h/g} \end{aligned} \tag{1.10}$$

where h is the depth of the water. At first glance this function seems to be difficult, however looking at 1.3 we can see that this can be easilly approximated.

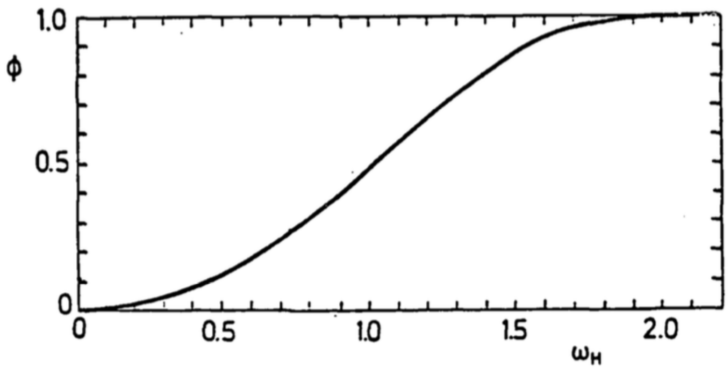

Figure 1.3: $\Phi(\omega, h)$ as a function of ω_h [6]

Approximation given by Thomson and Vincent [8] is:

$$\Phi(\omega, h) \approx \begin{cases} \frac{1}{2}\omega_h^2 & \text{if } \omega_h \leq 1 \\ 1 - \frac{1}{2}(2 - \omega_h)^2 & \text{if } \omega_h > 1 \end{cases} \tag{1.11}$$

With $\Phi(\omega, h)$ we can now correct the JONSWAP spectrum for shallow waters:

$$S_{\text{TMA}}(\omega, h) = S_{\text{JONSWAP}}(\omega)\Phi(\omega, h) \tag{1.12}$$

Donelan-Banner Directional Spreading

Currentlly we can't use the TMA sprectrum for our project as it has few problems. Firstlly, this spectrum is non-directional, so we need to add directionality to it. Secondly, this spectrum accepts ω as input, but as we following J. Tessendorf's [2] paper, we need to use $\mathbf{k}$ as input. To solve this problem Horvath Christopher [3] proposes to use Donelan-Banner Directional Spreading [9]:

$$D(\omega, \theta) = \frac{\beta_s}{2\tanh(\beta_s \pi)}\text{sech}(\beta_s \theta)^2 \tag{1.13}$$

where,

$$\beta_s = \begin{cases} 2.61(\omega/\omega_p)^{1.3} & \text{for } 0.56 < \omega/\omega_p < 0.95 \\ 2.28(\omega/\omega_p)^{-1.3} & \text{for } 0.95 \leq \omega/\omega_p < 1.6 \\ 10^{\epsilon} & \text{for } \omega/\omega_p \geq 1.6 \end{cases}$$

$$\epsilon = -0.4 + 0.8393 \cdot e^{-0.567 \ln(\omega/\omega_p)^2}$$

$$\theta = \arctan(k.y/k.x) - \theta_{\text{wind}}$$

In our project we will use $\omega/\omega_p < 0.95$ for first case as it produces more smooth results. Now we can combine the TMA spectrum with the Donelan-Banner Directional Spreading:

$$D_{\text{TMA}}(\omega, \theta) = S_{\text{TMA}}(\omega) \cdot D(\omega, \theta) \tag{1.14}$$

TMA transformation

Lastlly, to make this spectrum usable we need to transform it from ω to $\mathbf{k}$ and according to Horvath Christopher [3] we can transform it like this:

$$S_{\mathrm{TMA}}(\mathbf{k}) = 2S_{\mathrm{TMA}}(\omega, h) \cdot \frac{d\omega}{dk}/k \cdot \Delta k_x \cdot \Delta k_y \tag{1.15}$$

where in our case,

$$\frac{d\omega}{dk} = \frac{g}{2\sqrt{g * \mathbf{k}}}$$

1.5 Cooley-Tukey Fast Fourier Transform (FFT)

When simulating fourier transform ocean majority times is spent on converting from frequency domain to time domain, in other words performing Inverse Fourier Transform. Earlier mentioned IDFT 1.4 has time complexity of $O(n^2)$, and it's clearlly that this algorithm is not sufficient to simulate heiger resolution ocean surfaces in real time. In 1805 Gauss Carl Friedrich [10] invented FFT algorithm $O(n \log n)$ and in 1965 Cooley James W. and Tukey John W. [11] reinvented it. The FFT algorithm is based on that there is redundancy in the computation of the DFT, and that we can exploit it to reduce the time complexity, it is important to mention that the FFT algorithm works only when the number of samples is a power of 2.

The basic idea of the FFT algorithm is as follows:

1. Split the input data into two sets, one containing the even samples and the other containing the odd samples and repeat this proccess until a set has 2 samples (bit-reverse sort order).
2. Generate twiddle factor $W_N = e^{-i2\pi k/N}$ and rise it to the power of

   $$k = i * N/2^{\mathrm{stage}+1} \bmod N \tag{1.16}$$

   where i is the index of the sample, stage is the current stage and N is the number of samples.
3. Perform butterfly operations 1.4, where x and y are complex numbers and W is the twiddle factor

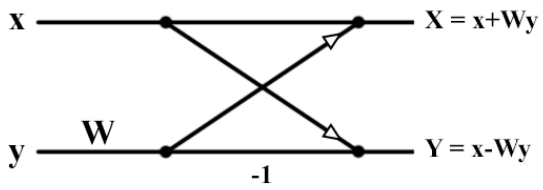

Figure 1.4: Butterfly Diagram

4. The butterfly operations are performed in stages 1.5. For example, if we have 8 samples, we will have log(8) = 3 stages. The first stage is for pairs of points (2-point DFTs), the second stage is for groups of four points (4-point DFTs), and so on.

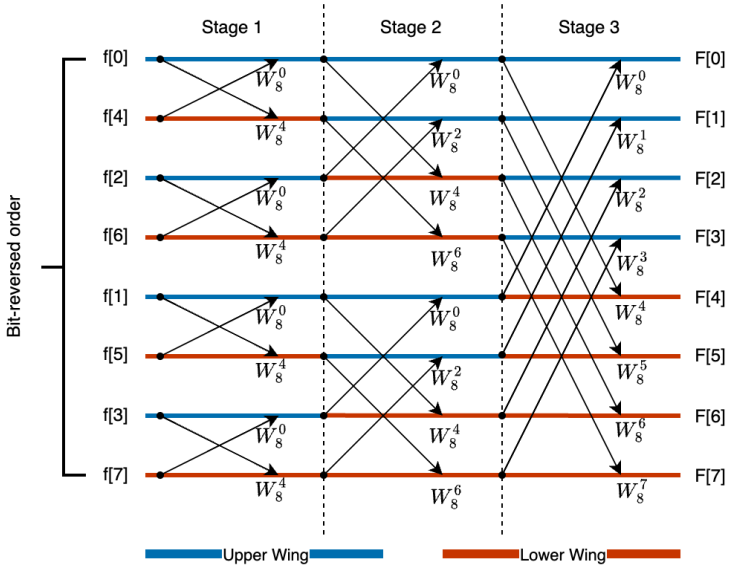

Figure 1.5: 8-point FFT Butterfly Diagram

1.6 Ocean Shading

1.6.1 Phong Shading

The Phong shading model, one of the simplest yet effective methods for approximating realistic shading, was proposed by [12, Phong Bui Tuong, 1975]. This model comprises three components: ambient shading, diffuse shading, and specular shading, as depicted in Figure 1.6. The ambient component is a constant value representing the light that is scattered about the entire scene. The diffuse component, on the other hand, can be calculated using the following equation:

$$\text{Diffuse} = \max(\text{Normal} \cdot \text{LightDirection}, 0.0); \tag{1.17}$$

Here, the Normal is a normalized vector perpendicular to the surface that points away from the surface. The specular component can be calculated as follows:

$$\text{Specular} = max(\text{ViewDir} \cdot \text{ReflectionDir}, 0.0)^{\text{Shininess}} \tag{1.18}$$

In this equation, the ViewDir is a normalized vector pointing towards the camera, and the ReflectionDir is a normalized reflection vector with respect to the Normal, as illustrated in Figure 1.7.

Figure 1.6: Phong Shading
Credits: learnopengl.com

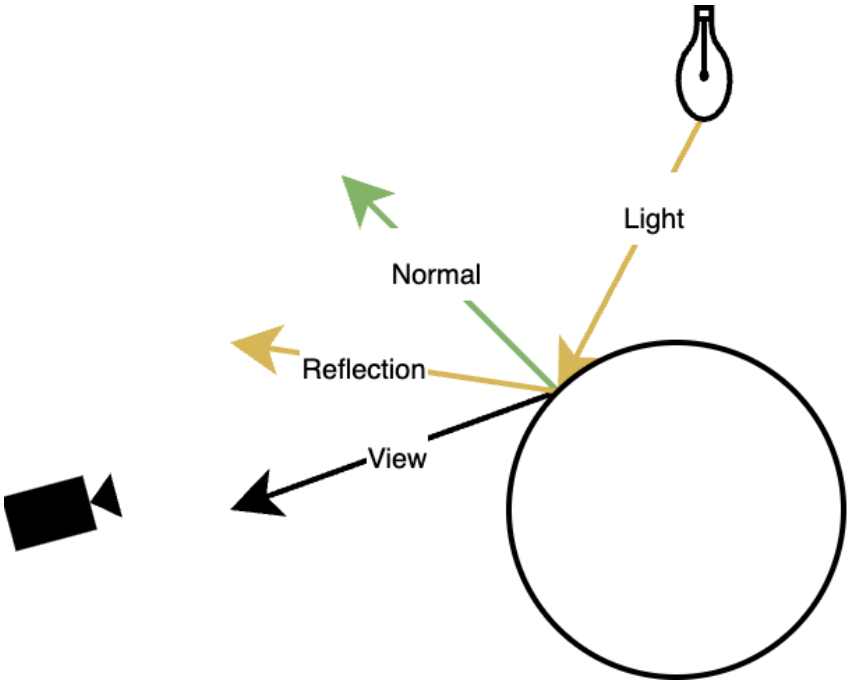

Figure 1.7: Phong Vectors

1.7 Related Work

1.7.1 Gerstner Waves

Gerstner waves provide a simple model for generating somewhat realistic ocean waves. This model is straightforward to implement and is predominantly used in video games. Even major titles, such as "Pokemon Legends: Arceus", utilize something similar to Gerstner waves to simulate oceanic environments. The concept of Gerstner waves was first introduced by F.J. Gerstner [13] in 1802. The initial known implementation of Gerstner waves in computer graphics was carried out by Fournier and Reeves [14] in 1986. The model is an attempt to approximate the motion of ocean waves, based on the principle that each point in the ocean undergoes a circular motion. If a point is represented as $\mathbf{x_0} = (x_0, z_0)$, the height and horizontal displacements can be calculated as follows:

$$\mathbf{x} = \mathbf{x_0} - (\mathbf{k}/k)A\sin(\mathbf{k} \cdot \mathbf{x_0} - \omega t) \tag{1.19}$$

$$y = A\cos(\mathbf{k} \cdot \mathbf{x_0} - \omega t) \tag{1.20}$$

$$\omega = \sqrt{gk} \tag{1.21}$$

Here, A is the amplitude, ω is dispertion relationship, and $\mathbf{k}$ is a wave vector that points horizontally in the direction of wave travel and is proportional to the wavelength λ of the wave.

$$k = 2\pi/\lambda \tag{1.22}$$

As presented, the Gerstner waves model is limited to a single wave. To simulate more complex ocean surfaces, multiple waves need to be summed together. This is achieved by summing the

waves as follows:

$$\mathbf{x} = \mathbf{x_0} - \sum_{i=1}^{N} (\mathbf{k_i}/k_i) A_i \sin(\mathbf{k_i} \cdot \mathbf{x_0} - \omega_i t) \tag{1.23}$$

Here, N is the number of waves, A_i is the set of amplitudes, ω_i is the set of frequencies, and $\mathbf{k}_i$ is the set of wave vectors. The primary challenge with this approach is that simulating realistic ocean surfaces requires summing a large number of waves together, which can be computationally expensive. Furthermore, Gerstner waves offer limited artistic control, has visible tiling 1.8, and underperforms in simulating stormy weather conditions.

Figure 1.8: Water Tiling in Pokemon Legends: Arceus

1.7.2 Particle Simulation and Machine Learning

Particle-based methodologies are frequently employed for the simulation of water in a manner that is both realistic and visually compelling. However, these methods are associated with a high computational cost.

Recent advancements in Graphics Processing Unit (GPU) technology, in conjunction with research such as that conducted by Libo Huang [15], are incrementally making particle simulations more viable for ocean simulation. Despite this progress, these techniques demand a high-performance GPU and are not yet amenable to widespread real-time applications.

Moreover, there have been considerable advancements in the application of machine learning to accelerate fluid simulations, as demonstrated by the work of Dmitrii Kochkov [16]. However, this is out of this projects scope and won't be disccused.

1.7.3 Physically Based Rendering (PBR)

Earlier mentioned Phong Shading model, does not provide a highly realistic approximation of lighting. To achieve a realistic representation of ocean shading, it is necessary to incorporate a multitude of custom parameters.

Physically Based Rendering (PBR) attempts to address this issue. However, it is important to note that PBR is more of a conceptual framework than a set of rigid rules [17, Joe Wilson, 2017]. The PBR rendering model adheres to three primary principles: energy conservation, microfacet

support, and the Fresnel effect.

[ELABORATE]

For the specific application of PBR in ocean shading, one can refer to “Wakes Explosions and Lighting: Interactive Water Simulation in Atlas” by Mark Mihelich and Tim Tcheblokov [18], which provides an in-depth discussion on the subject. For a more general understanding of PBR, “Physically Based Shading in Theory and Practice” by Stephen Hill and Stephen McAuley [19] offers comprehensive coverage of the topic from 2012 to 2020.

Chapter 2

Methods

2.1 Library Choice

In the context of our ocean generation project, we evaluated six potential technologies, which can be categorized into three groups: older APIs, modern APIs, and game engines.

The older APIs category includes OpenGL, an ideal API for learning and cross-platform use. It's relatively easy to use, but it's becoming outdated.

The second category, modern APIs, includes Metal, Vulkan, and DirectX. These APIs are more complex to use, but they provide more control over the GPU and lower CPU usage. However, Metal and DirectX are limited to Apple and Microsoft platforms, respectively. In the case of Vulkan, it requires significantly more code to accomplish the same task.

The final category is game engines, specifically Unity and Unreal. These engines are excellent for computer graphics as they offer a wealth of functionality, most importantly, useful UI tools that make development easier. However, they are massive tools with many custom needs, and you need experience to use them correctly.

All these options are viable as they all have the capability to fulfill our project requirements. However, after careful consideration, we chose Unity. The decision was influenced by Unity's comprehensive suite of tools, including UI design, GPU programming, and a built-in build system. These features significantly streamline the development process, making Unity an optimal choice for our project's needs.

2.2 Utilization of Unity Tools

In our project, we strategically employed Unity's toolset. We used C# for startup computations, setting the initial conditions and parameters for our project including communication between CPU and GPU.

For the intensive mathematical computations inside compute shaders and vertex and fragment shaders, we utilized the High-Level Shading Language (HLSL) for GPU programming. This approach allowed us to leverage the computational power of the GPU, enhancing the performance and visual fidelity of our project.

Additionally, Unity's built-in User Interface (UI) system and build system were used for creating interactive UIs and streamlining the compilation process, respectively. These tools collectively contributed to the effective and efficient development of our project.

2.3 Version Controll

In the development of our project, we adopted GitHub as our version control system. This platform facilitated the systematic management of different versions of our project files. By organizing our project into branches, we were able to work on individual features without affecting the main codebase. This approach not only enhanced the efficiency of our development process but also ensured the integrity and stability of our project.

2.4 Algorithm Overview

The algorithm can be split into 3 main parts as shown in figure 2.1

1. Spectrum generation (Only calculated on parameter value change)
2. Frequency generation
3. Convertion from frequency to time domain using IFFT

As shown in figure 2.1 majority of calculations are done on GPU using HLSL executed inside Unity. As FFT algorith requires input size to be 2^n we going to use 512x512 textures as this provides good performance and high enought details. For simplicity in this project we assume that the texture size is NxN, where N is the size of a texture and N is power of 2.

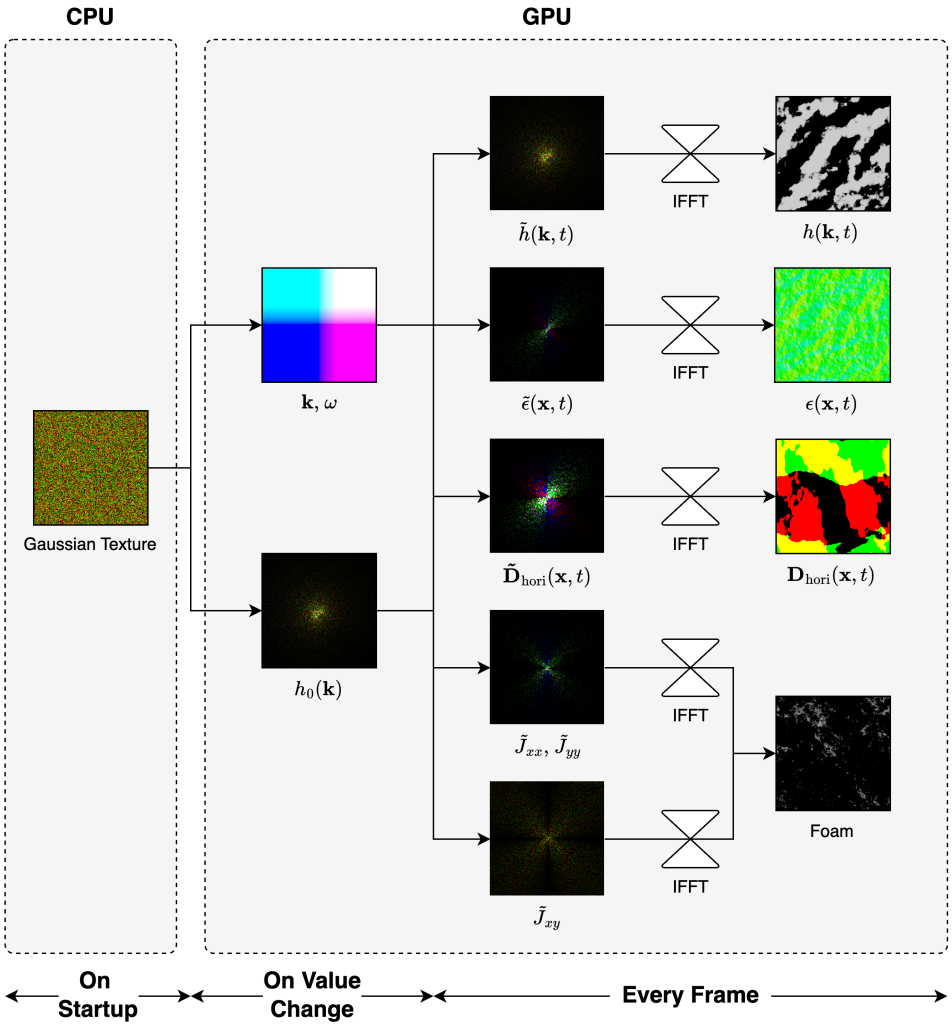

Figure 2.1: Ocean Algorithm

2.5 IFFT

2.5.1 Butterfly Texture 2.2

Firstlly we only intrested in IFFT algorithm and we will not use FFT. To perform IFFT we going to produce so called butterfly texture by [20, Flügge Fynn-Jorin]. This texture has width of $(log_2(n)$ and height of n and is precomputed and stored inside the GPU memory once, unless the data size changes. This is 4 chanel texture (W_r, W_i, y_t, y_b), where W_r and W_i are real and imaginary parts of the twiddle factor, y_t and y_b are top and bottom butterfly indices as shown in 1.4.

In butterfly texture coordinate x represents a stage 1.5, while each y represents a butterfly operation between two data points y_t and y_b.

In the first stage we assign y_t and y_b in bit reversed order, on the other stages:

- If the butterfly wing is top half

$$\begin{aligned} y_t &= y_{\text{current}} \\ y_b &= y_{\text{current}} + 2^{\text{stage}} \end{aligned} \tag{2.1}$$

- If the butterfly wing is bottom half

$$\begin{aligned} y_t &= y_{\text{current}} - 2^{\text{stage}} \\ y_b &= y_{\text{current}} \end{aligned} \tag{2.2}$$

We can determine if the wing is upper or lower half:

$$\text{wing} = y_{\text{current}} \bmod 2^{(\text{stage}+1)} \tag{2.3}$$

Lastlly we need to calculate twiddle factors:

$$\begin{aligned} k &= (y_{\text{current}} \cdot n/2^{\text{stage}+1}) \bmod n \\ W &= \exp(-2\pi i k/n) \end{aligned} \tag{2.4}$$

2.5.2 Performing IFFT

Currentlly our data is stored in 2D texture. While this algorithm only accepts 1D data. Therefore, we need to perform 1D IFFT on each row "horizontally" and then on each column "vertically" 2.2. By following pseudocode from [20] we can perform IFFT in 2D:

```
float4 butterflyData = ButterflyTexture[float2(Stage, id.x)];
const float2 twiddle = butterflyData.xy;
// fetch top butterfly input sample
topSignal = PingPong0[float2(butterflyData.z, id.y)].xy;
// fetch bottom butterfly input sample
bottomSignal = PingPong0[float2(butterflyData.w, id.y)].xy;
// perform butterfly operation
h = topSignal + ComplexMult(twiddle, bottomSignal);
```

Listing 2.1: Horizontal Butterfly Operation

Notice that we are using ping-pong buffers to store intermediate results, as we need to perform multiple IFFT passes, in total $log_2(n)$ passes. After we perform IFFT on each row, we need to perform for each column:

```
float4 butterflyData = ButterflyTexture[float2(Stage, id.y)];
const float2 twiddle = butterflyData.xy;
// fetch top butterfly input sample
topSignal = PingPong0[float2(id.x, butterflyData.z)].xy;
// fetch bottom butterfly input sample
bottomSignal = PingPong0[float2(id.x, butterflyData.w)].xy;
// perform butterfly operation
h = topSignal + ComplexMult(twiddle, bottomSignal);
```

Listing 2.2: Vertical Butterfly Operation

Permutation

Lastlly, our data needs to be permuted as you will see later our data is offseted:

$$[\text{freq}(-N/2),\ ...,\ \text{freq}(-1),\ \text{freq}(0),\ \text{freq}(1),\ ...,\ \text{freq}(N/2-1)] \tag{2.5}$$

While, our IFFT algorithm expected data to be in the following order:

$$[\text{freq}(0),\ \text{freq}(1),\ ...,\ \text{freq}(N-1)] \tag{2.6}$$

This causes our data to flip sign in grid like pattern therefore we need to permute our data:

```
float perms[] = {-1, 1};
uint index = int((id.x + id.y) % 2);
float perm = perms[index];
float h = perm * PingPong1[id.xy].x;

PingPong0[id.xy] = float4(h, h, h, 1);
```

Listing 2.3: Data Permutation [20]

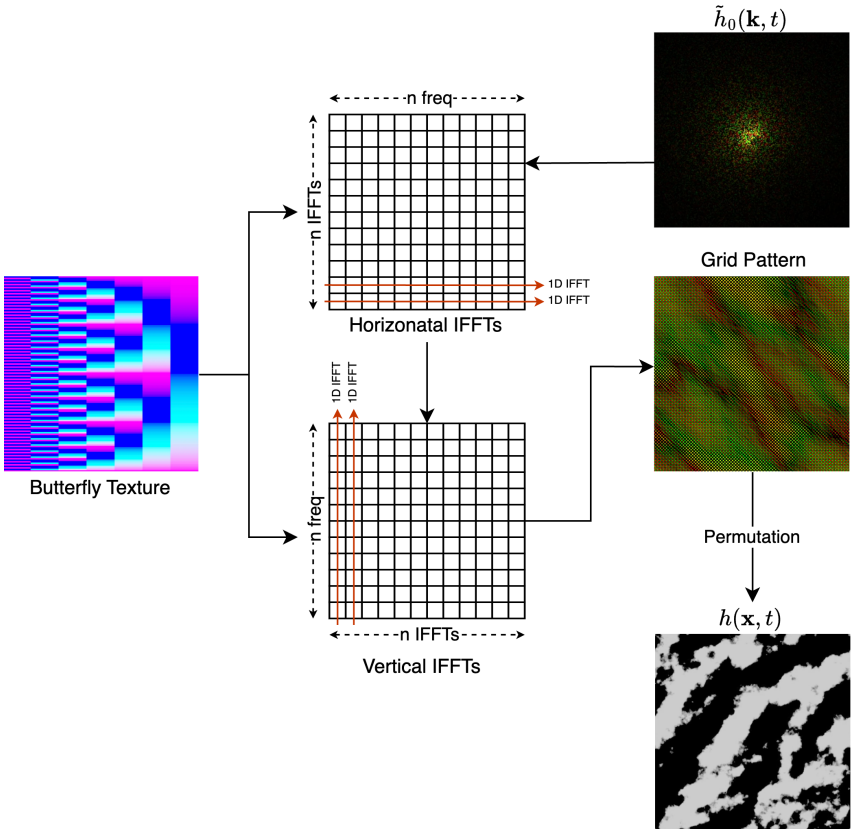

Figure 2.2: IFFT Algorithm

2.6 Ocean Geometry

2.6.1 Spectrum Generation

Symbol	Meaning
$S(\omega)$	Non directional wave spectrum
$S(\omega, \theta)$	Directional wave spectrum
$S(\mathbf{k})$	Directional wave spectrum
$\mathbf{k}$	wave vector
k	Magnetude of wave vector
l	Length scale of the ocean
ω	dispertion relationship (angular frequency)
U_{10}	Wind speed at 10m above the sea level
F	Fetch (Disntance from lee shore)
θ_{wind}	Wind angle
λ	Choppy factor

Table 2.1: Deffinition Table

Spectrums are responsable for whole look of the ocean. It's importatnt to pick the one that is based on real world data to make our ocean look realistic. Initialy, Tessendorf spectrum 1.8 was implemented as it was straight forward to implement. However, it didn't produce satisfying results, as waves didn't seem to transfer energy in convincing way, waves didn't seem to follow wave direction and it was hard to have artistic control over the ocean.

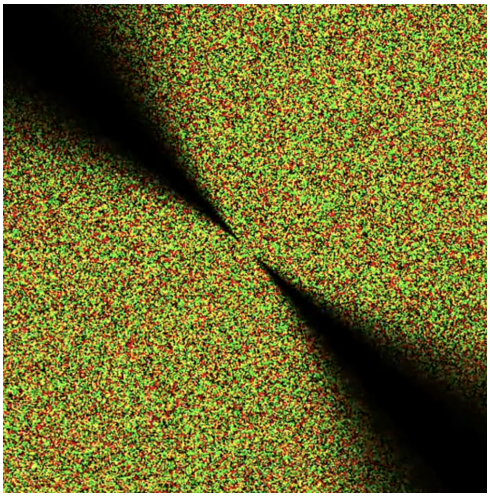

Figure 2.3: Jerry Tessendorf's Spectrum

Therefore, TMA spectrum 1.15 was implemented.

$$S_{\text{TMA}}(\mathbf{k}) = 2S_{\text{TMA}}(\omega, h) \cdot \frac{d\omega}{dk}/k \cdot \Delta k_x \cdot \Delta k_y$$

where $\mathbf{k} = (k_x, k_y)$, $k_x = 2 * \pi(x_x - n/2)/l$, $k_y = 2 * \pi(x_y - n/2)/l$, l is the the length scale of the ocean, and $\mathbf{x} = (x_x, x_y)$ is current position in the texture.

This spectrum resulted in more relistic looking ocean with intuitive controls: fetch, wind speed, wind angle, depth. Moreover, it didn't require any efort to make the ocean look relistic and it just worked out of the box.

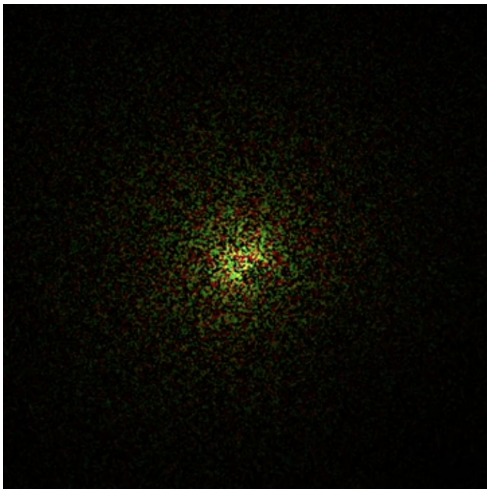

Figure 2.4: TMA spectrum, Frequency Domain
(100x for better visibility)

2.6.2 Height Map Generation

To generate height map, firstlly we need to have fourier amplitudes as shown in 1.5, where P_h is TMA Spectrum $S_{TMA}(\mathbf{k})$. For the next step we need to add fourier amplitude and it's complex conjugate to produce "produce waves towards and against the wave direction when propagating"[3]. In our luck we don't need to recalculate complex conjugate as fourier series are symetric, so we can just mirror the amplitudes:

$$\tilde{h}_0^* = T_{h_0}(x^*, y^*) \tag{2.7}$$

where $T_{h_0}(x, y)$ is fourier amplitude in precomputed texture at $x^* = (n - x) \bmod n$, $y^* = (n - y) \bmod n$, (x, y) is current position in the texture.

By having combined amplitudes 1.6 we can perform IFFT as shown in 2.2 to produce height map.

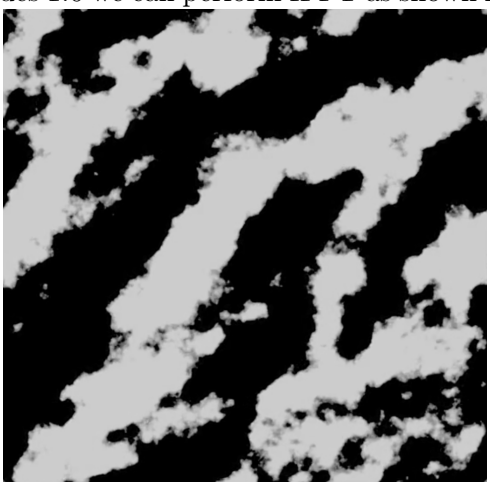

Figure 2.5: Height Map using S_{TMA}

2.6.3 Normal Map Generation

Latter we will need extra information about the ocean to calculate ocean shading. Therefore, we need to calculate normal map. Normal map is perpendicular direction to the surface of the ocean. To calculate normal map we need to calculate gradient, which is derivative of the height map. According to [21] the derivative is:

$$\epsilon(\mathbf{x}, t) = i\mathbf{k}\tilde{h}(\mathbf{k}, t) \tag{2.8}$$

Then we need to perform IFFT 2.2 to get normal map in the time domain.

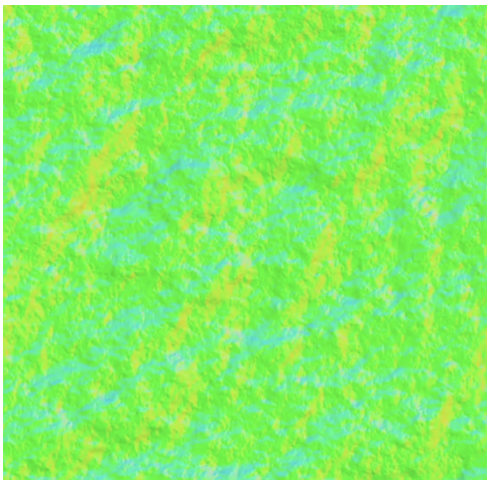

Figure 2.6: Normal Map using S_{TMA}

2.6.4 Choppy Waves

Currentlly, when rendering ocean it looks too smooth 2.7 and lacks choppiness.

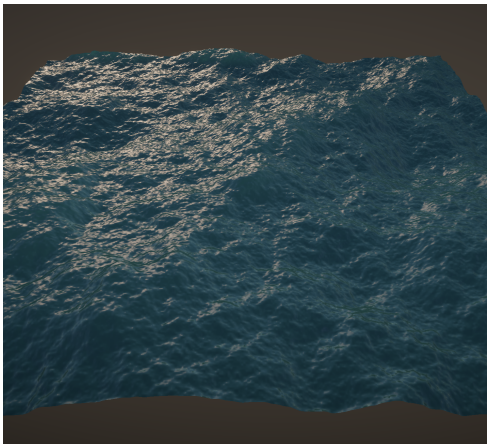

Figure 2.7: Ocean without Choppy Waves

To make waves choppy we need to introduce horizotal displacement. This will not only make waves more chopy but also make energy transfer between waves more relistic. Following formula from [21, J. Tessendorf] we can calculate horizontal displacement:

$$\mathbf{D}_{\text{hori}}(\mathbf{x}, t) = -i\frac{\mathbf{k}}{k}h(\tilde{\mathbf{k}}, t) \tag{2.9}$$

Using this horizotal displacement we can displace our verticies:

$$\mathbf{x} + \lambda \mathbf{D}_{\text{hori}}(\mathbf{x}, t) \tag{2.10}$$

, where λ is "choppy factor". Once again we need to perform IFFT 2.2 to get horizontal displacement in the time domain.

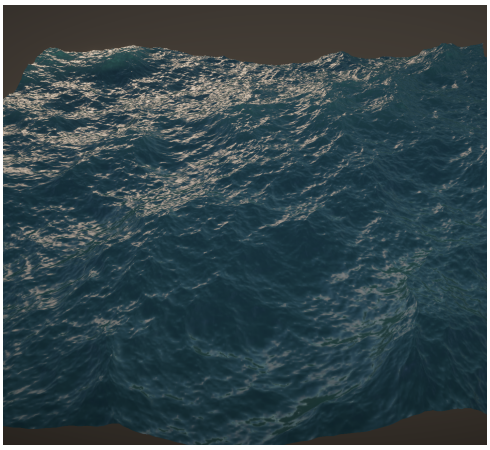

Figure 2.8: Ocean with Choppy Waves

2.7 Ocean Shading

Symbol	Parameter
L_a	Ambient Light
$L_s s$	Subsurface Scatter Light
L_s	Specular Light
L_r	Enviroment Reflection
N	Normal
D_s	Sun Direction
D_v	View Direction
C_a	Ambient Light Color
C_l	Light Color
C_b	Air Bubble Color
C_{ws}	Water Scattering Color
H	Ocean Height
F	Fresnel Effect
ρ_a	Air Bubble Density
k_a	Ambient Light Intensity
k_{ss_1}	Subsurface Scattering Intensity
k_{ss_2}	Subsurface Scattering Intensity

Table 2.2: Lighting Parameters

2.7.1 Lighting Model

Output

[MENTION PHON SHADING DISCCUSED IN BACKGROUND RESERCH]

Our lighting model, as depicted in Equation 2.11, comprises four integral components: ambient light, subsurface scattering, specular light, and environmental reflection. The culmination of these components results in the final output, as illustrated in Figure 2.9

$$L = L_a + L_{ss} + L_s + L_r \tag{2.11}$$

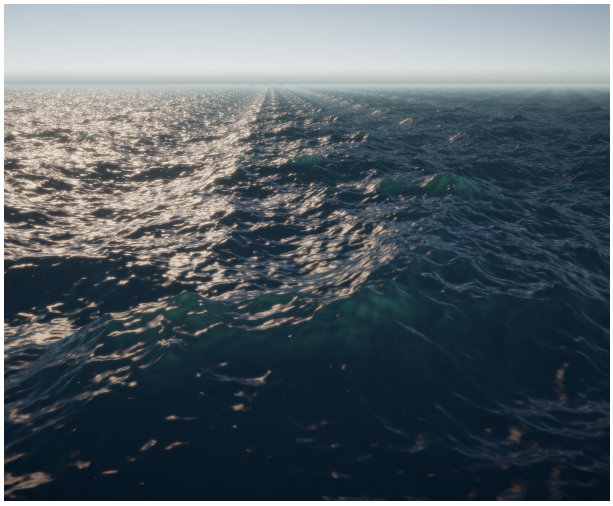

Figure 2.9: Finall Shader Output

Ambient Light

Ambient light, a constant illumination that does not depend on the direction of the light source, is the result of light scattering in the environment. For oceanic simulations, we utilize an approximation formula for ambient light derived from the GDC conference [18]:

$$L_a = k_a N C_a C_l + \rho_a C_b C_l \tag{2.12}$$

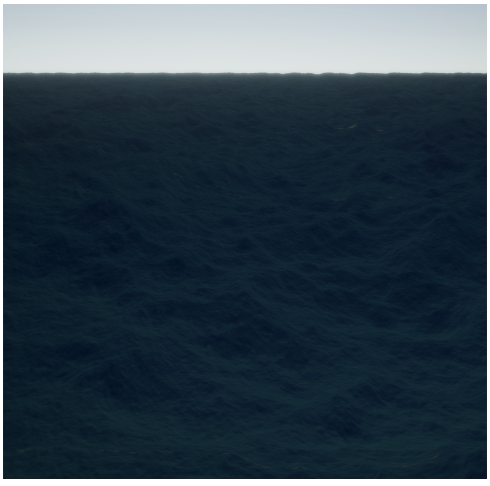

Figure 2.10: Ambient Light

Fresnel Effect

The Fresnel effect, a phenomenon where light is more reflective at grazing angles as shown in figure 2.11, is crucial for calculations involving specular and reflection:

$$F = (1 - \max(D_v \cdot N, 0.15))^5 \tag{2.13}$$

Figure 2.11: Fresnel Effect

Subsurface Scattering

[SAY AS EARLIER MENTIONED]

Subsurface scattering describes the interaction of light when it penetrates an object, in this case, water. While ray tracing would typically be required for realistic results, we can use an approximation from the GDC conference [18] due to the majority of our light being trapped in the ocean, with only light at wave peaks able to escape.

$$\begin{aligned} L_{ss} &= (L_{ss_1} + L_{ss_2}) C_{ws} C_l \\ L_{ss_1} &= k_{ss_1} \max(0, H)([D_s, -D_v])^4 (0.5 - 0.5(D_s \cdot N))^3 \\ L_{ss_2} &= k_{ss_2}([D_V, N])^2 \end{aligned} \tag{2.14}$$

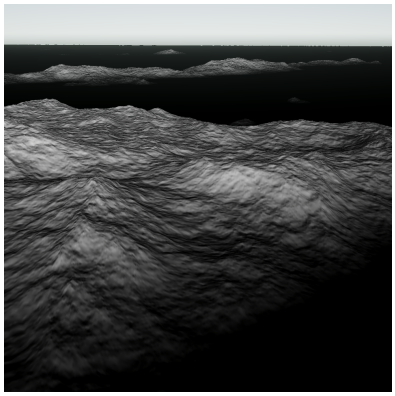

Figure 2.12: L_{ss_1}

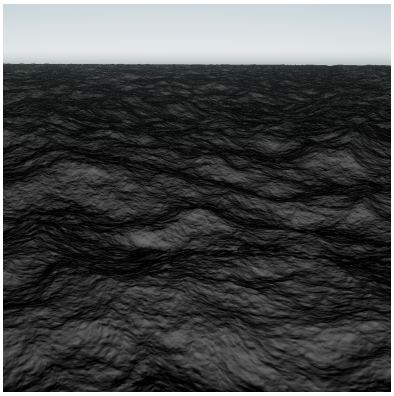

Figure 2.13: L_{ss_2}

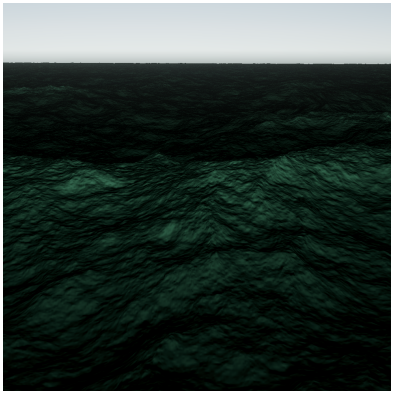

Figure 2.14: Subsurface Scattering

Specular Reflection

Specular reflection, the reflection of light from a light source on an object, is a characteristic of shiny objects like metals, plastic, or in our case, water. We employ a simple Phong scattering technique, as shown in Figure 2.15, and the formula takes the following form:

$$\begin{aligned} L_s &= C_l S C_s F \\ S &= [D_v, D_r]^{\text{Shininess}} \\ D_r &= \text{reflect}(\text{LightPos}, N) \end{aligned} \tag{2.15}$$

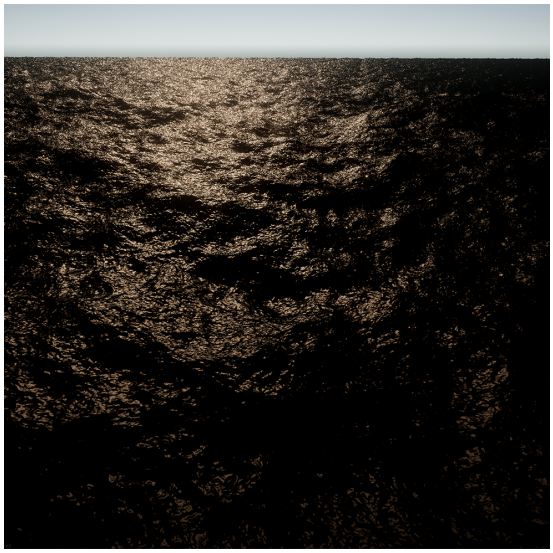

Figure 2.15: Specular Reflection

Enviroment Reflection

In our project, we only consider the reflection of the skybox on the water surface. As per Figure 2.16, we illustrate how skybox reflection operates on a calm ocean surface to better visualize environmental reflection.

$$\begin{aligned} L_r &= F k_r C_{\text{sky}} \\ C_{\text{sky}} &= \text{Sky}[\text{reflect} D_i, N] \end{aligned} \tag{2.16}$$

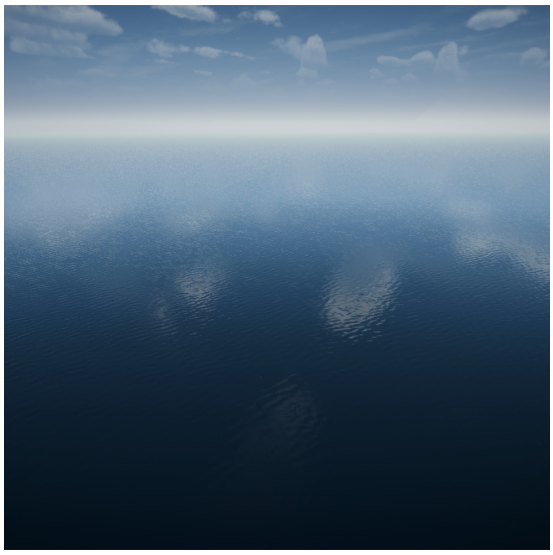

Figure 2.16: Skybox reflection on calm ocean

2.7.2 Foam

Foam Generation

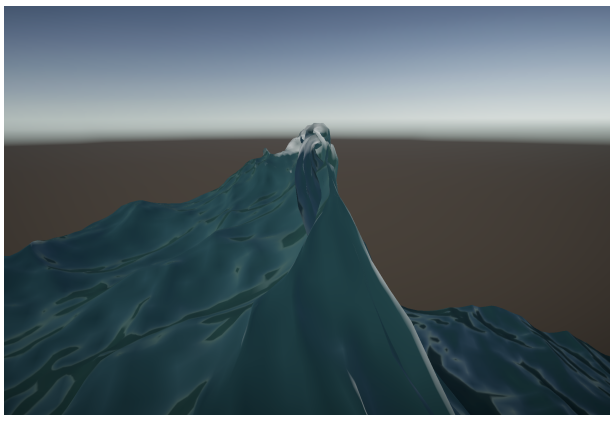

Figure 2.17: Wave Curl At Wave Peek

Ocean foam is a salient and realistic feature of the ocean, contributing to its distinct and authentic appearance. The primary occurrence of foam is observed during wave crashes, a phenomenon facilitated by horizontal displacement. This can be visually discerned at the peaks of waves where the water curls up, as illustrated in Figure 2.17. In essence, our horizontal transformation undergoes an inversion.

As proposed by Tessendorf [2], the rendering of foam can be achieved by calculating the determinant of the Jacobian matrix for horizontal displacement, which helps identify these inversions. In our ocean simulation, the Jacobian matrix provides insights into the changes over the x and z axes. When determinant of the Jacobian matrix is bellow zero the "wave crash" happens:

$$\mathrm{Det}(\mathbf{x}) = J_{xx} + J_{yy} - J_{xy}J_{yx} \tag{2.17}$$

where,

$$J(\mathbf{x}) = \begin{bmatrix} 1 + \lambda \frac{\partial D_x(\mathbf{x})}{\partial x} & 1 + \lambda \frac{\partial D_x(\mathbf{x})}{\partial y} \\ 1 + \lambda \frac{\partial D_y(\mathbf{x})}{\partial x} & 1 + \lambda \frac{\partial D_y(\mathbf{x})}{\partial y} \end{bmatrix} \tag{2.18}$$

in this case $J_{xy} = J_{yx}$.

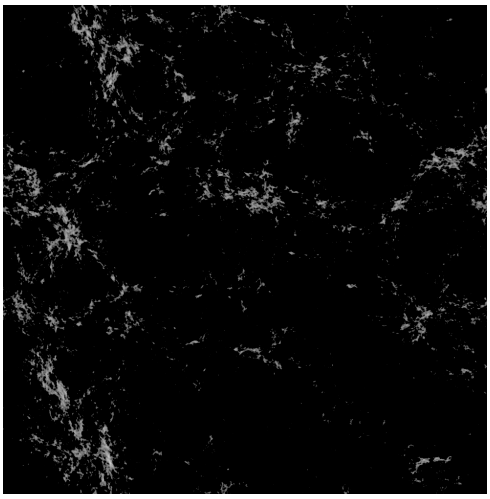

Figure 2.18: Foam Texture

Foam Accumilation

Curretlly the foam apears and diasapears quicly, however in real life the foam accumilates and disapears over time. We can introduce foam accumilation by compering previous and current foam values:

```
float accumulation =
LastFoamValue - FoamDecay * DeltaTime / max(currentFoam, 0.5);
float foam = max(accumulation, currentFoam);
```

Listing 2.4: Foam Accumilation

This results in pleasing foam accumilation:

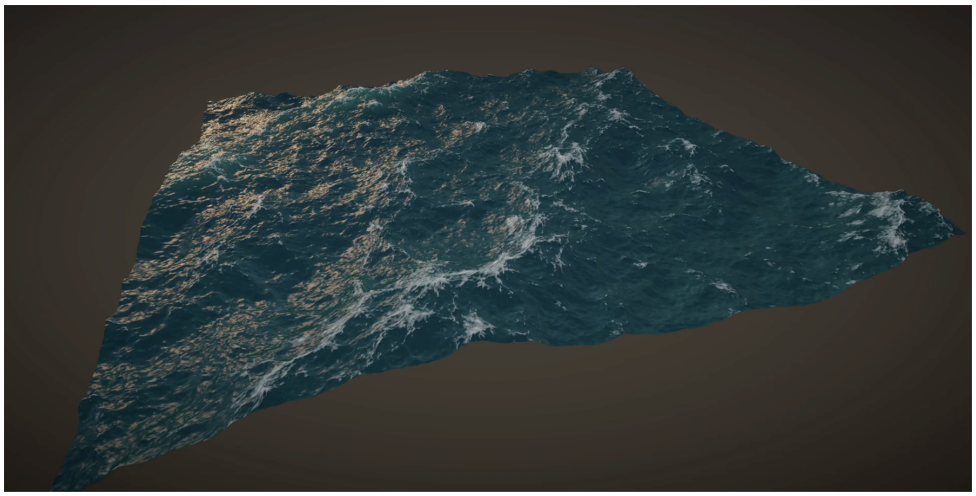

Figure 2.19: Ocean With Foam

2.8 Multiple Cascades

At this stage our ocean with texture 512x512 simulates 262,144 distinc waves however the tilling is still noticible 2.20.

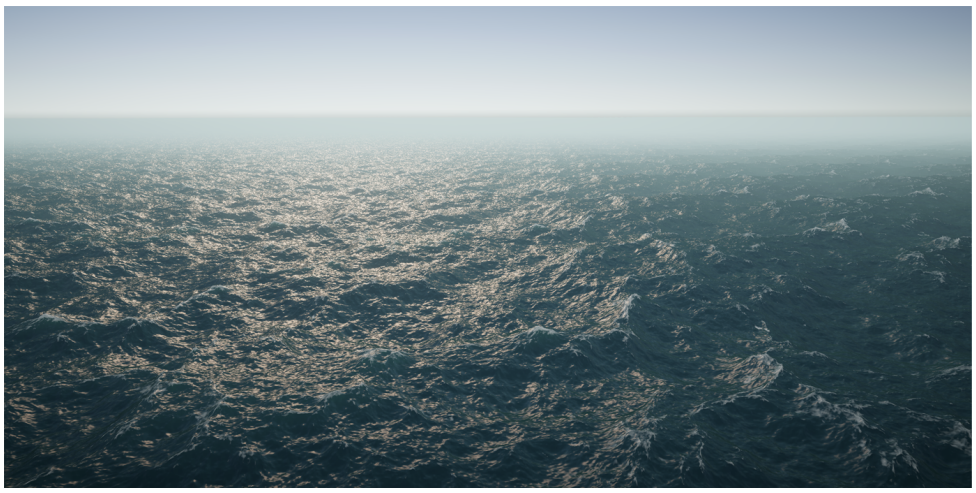

Figure 2.20: Ocean with Tilling, using 512x512 texture

To counter this we could increase our simulation texture however even FFT becomes expensive really fast. Another approuch is to simulate multiple cascades for diffrent wave lengths k 2.21 and diffrent l length scales. We can split our simulation into 3 parts, big waves, medium waves and small waves.

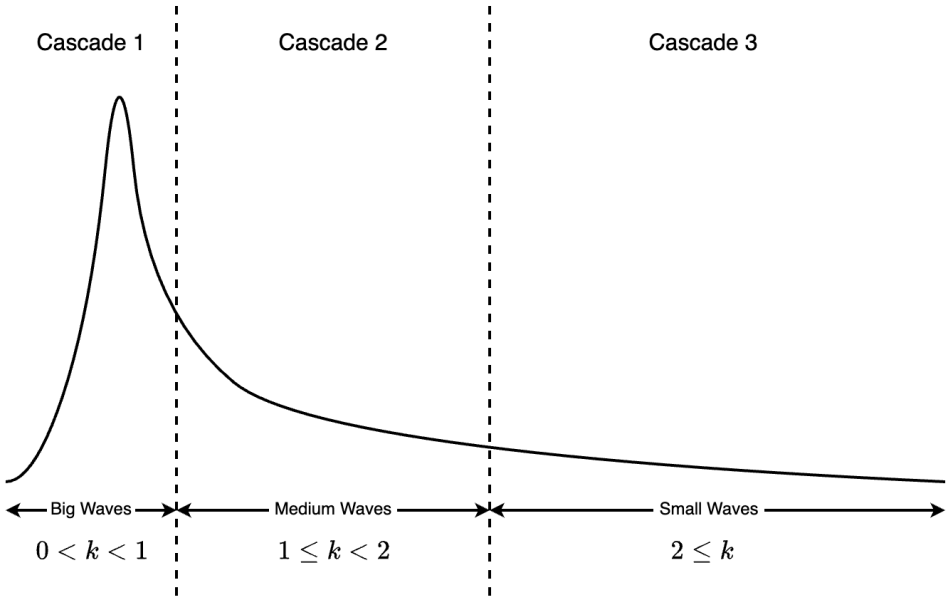

Figure 2.21: Cascades

When it comes to chosing l we need to follow these steps:

1. We chose bigger l for bigger waves as we are more likly to notice tilling with big waves
2. We chose smaller l for smaller waves as we are less likly to notice tilling with small waves

This results ocean that has less tilling and more detail 2.22.

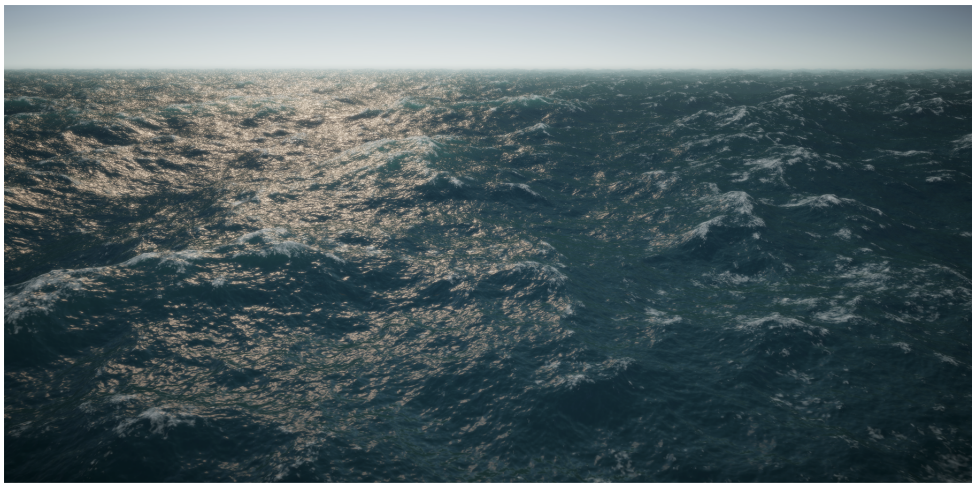

Figure 2.22: Ocean with Cascades, using 3x(512x512) textures

One of the main pros of having multiple cascades is reduction in performance cost, as we can have similar results of 512x512 with 3x(256x256) xtextures as shown in figure 2.23.

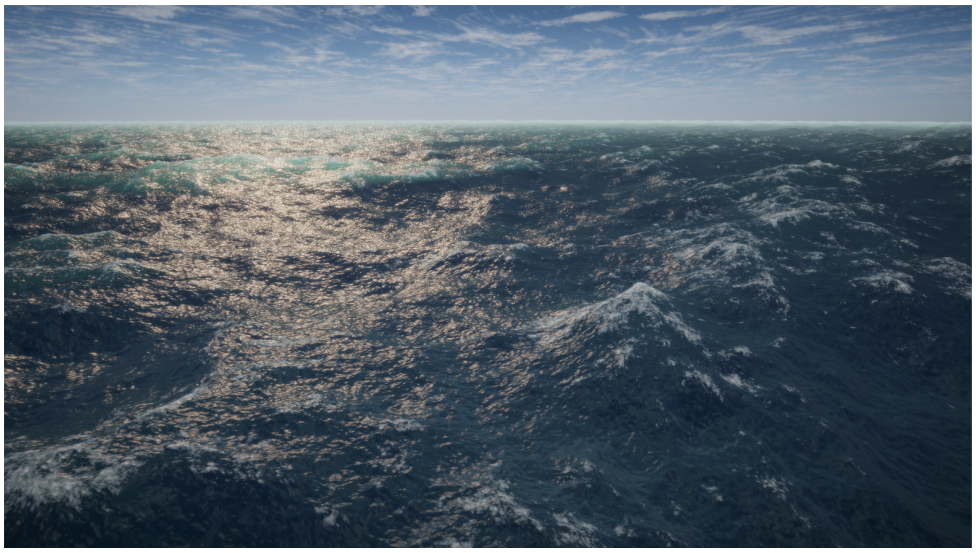

Figure 2.23: Ocean with Cascades, using 3x(256x256) textures

Chapter 3

Results

<Results, evaluation (including user evaluation) *etc.* should be described in one or more chapters. See the 'Results and Discussion' criterion in the mark scheme for the sorts of material that may be included here.>

3.1 Performance

GPU	Texture Size	Time
M1	256x256	8.56ms
M1	512x512	43.50ms
M1	1024x1024	128.25ms
Nvidia GeForce RTX 4070	256x256	1.65ms
Nvidia GeForce RTX 4070	512x512	3.29ms
Nvidia GeForce RTX 4070	1024x1024	10.70ms

[FINISH THIS WHEN RESULTS RETRIEVED FROM UNIVERSITY]

3.2 Comparison

3.2.1 Spectrums

The main diffrence comes to what kind of spectrum you use for FFT based oceans. The proposed "Phillips" spectrum 3.1 by [2, J. Tessendorf] has issues with energy transformation and following the wind direction. The proposed TMA spectrum 3.2 handles energy transformation way more relistic and does not have any issues following the wind direction. This is mostly because that TMA is based on empirical data. You can see clear diffrence between "Phillips" spectrum and TMA spectrum:

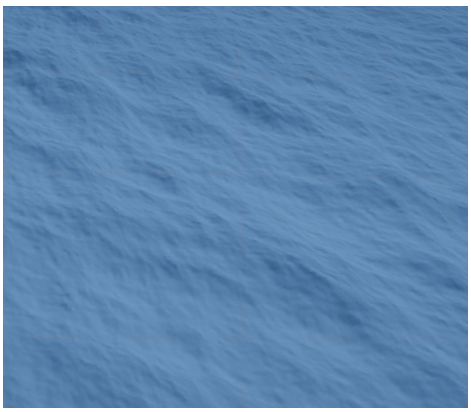

Figure 3.1: Height Map with P_h

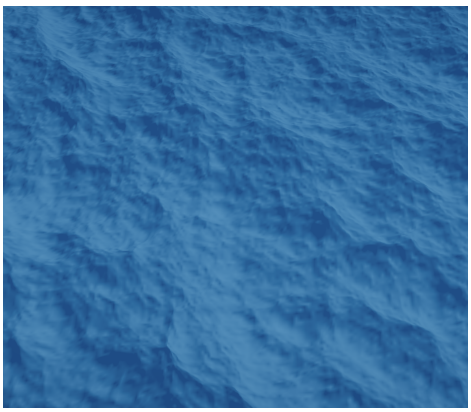

Figure 3.2: Height Map using TMA Spectrum

3.2.2 Real world oceans

When it comes to calm ocean, the FFT based ocean with TMA spectrum performs extremelly well and holds the desired shape as shown in figures 3.3 and 3.4.

Spectrum	**Size**	$\mathbf{l_1}$	$\mathbf{l_2}$	$\mathbf{l_3}$	λ	$\mathbf{U_{10}}$	**Fetch**	**Depth**
256x256	TMA	256	100	10	0.9	0.02 m/s	1000 km	500 m

Table 3.1: Calm Ocean Parameters

Figure 3.3: Calm Atlantic Ocean
Credits: CC0 Public Domain

Figure 3.4: FFT Calm Ocean

When it comes to stormy ocean where huge waves is expected the general shape of an ocean is still relistic and can hadle the big waves without any trouble, and because of diffrent cascades the tilling is bearlly noticible as shown in the following figures 3.5 and 3.6. where each l is the

Spectrum	**Size**	$\mathbf{l_1}$	$\mathbf{l_2}$	$\mathbf{l_3}$	λ	$\mathbf{U_{10}}$	**Fetch**	**Depth**
256x256	TMA	700	256	70	0.9	21 m/s	1000 km	500 m

Table 3.2: Stormy Ocean Parameters

wave length scale of each cascade.

Figure 3.5: Calm Atlantic Ocean
Credits: CC0 Public Domain

Figure 3.6: Height Map using
TMA Spectrum

3.3 Diffrent Outputs

3.4 Known Problems

Chapter 4

Discussion

4.1 Conclusions

The Fast Fourier Transform (FFT) has been instrumental in ocean rendering, providing a remarkably efficient and persuasive approximation. This efficiency is primarily attributable to the use of the JONSWAP spectrum, which is grounded in empirical data. The substantial enhancements in speed are largely credited to the FFT algorithm, without which the use of the Discrete Fourier Transform (DFT) algorithm would render real-time graphics unfeasible.

In our research, we found that the key to creating a realistic ocean lies in the selection of an empirically based spectrum, for which we employed the TMA spectrum. However, despite its efficiency, FFT-based ocean rendering is not yet sufficient to produce an ocean without tiling. To overcome this, we rendered multiple cascades for different wavelengths and superimposed them, effectively rendering the tiling inconspicuous. This approach not only removes tiling, but also enhances the quality of our ocean and reduces the computation expense as we can render multiple smaller textures.

One of the most significant challenges with FFT-based ocean rendering is its limited interactivity with surroundings. As suggested by Jerry Tessendorf in 2001[tessendorf2001], a hybrid approach could be a potential solution to this problem.

For ocean shading, we utilized a modified version of Phong shading, which yielded satisfactory results. However, to further enhance the realism of the ocean, it would be prudent to consider transitioning to Physically-Based Rendering (PBR). This transition could potentially provide a more convincing representation of the ocean.

4.2 Ideas for future work

In the pursuit of further enhancing the realism and interactivity of our ocean rendering, several advancements are proposed for future exploration.

Firstly, the current model lacks the representation of foam spray, a characteristic feature observed during significant wave crashes. To address this, we propose the integration of a GPU-based particle system, which could potentially simulate the foam spray effect, thereby adding to the visual complexity and realism of the ocean surface.

Secondly, the incorporation of buoyancy is another aspect worth considering. This would allow objects to interact with the ocean in a more realistic manner, adhering to the principles of fluid dynamics.

However, for more complex interactions, such as the ocean's reaction with the shoreline by

reducing wave amplitude, or with ships by creating wakes, a hybrid approach is recommended. This approach would combine the strengths of different methods to achieve a more comprehensive and realistic simulation.

Lastly, to further enhance the visual fidelity of the ocean, we suggest transitioning from our customized Phong shading to Physically-Based Rendering (PBR). This transition could potentially provide a more convincing and physically accurate representation of the ocean surface, thereby contributing to the overall realism of the scene.

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Appendix A

Self-appraisal

<This appendix should contain everything covered by the 'self-appraisal' criterion in the mark scheme. Although there is no length limit for this section, 2—4 pages will normally be sufficient. The format of this section is not prescribed, but you may like to organise your discussion into the following sections and subsections.>

A.1 Critical self-evaluation

A.2 Personal reflection and lessons learned

A.3 Legal, social, ethical and professional issues

<Refer to each of these issues in turn. If one or more is not relevant to your project, you should still explain *why* you think it was not relevant.>

A.3.1 Legal issues

A.3.2 Social issues

A.3.3 Ethical issues

A.3.4 Professional issues

Appendix B

External Material

<This appendix should provide a brief record of materials used in the solution that are not the student's own work. Such materials might be pieces of codes made available from a research group/company or from the internet, datasets prepared by external users or any preliminary materials/drafts/notes provided by a supervisor. It should be clear what was used as ready-made components and what was developed as part of the project. This appendix should be included even if no external materials were used, in which case a statement to that effect is all that is required.>