
Final Report

Realistic Ocean Simulation using Fourier Transform

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BSc Computer Science**

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COMP3931 Individual Project

The candidate confirms that the following have been submitted.

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Final Report	PDF file	Uploaded to Minerva (DD/MM/YY)
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Summary

Other Solutions:

- Sum of sines
- Gerstner waves
- Particle based ocean simulation

Problems:

- Computationally expensive to calculate huge amounts of sins.
- Gerstner waves are not physically accurate.
- Both are not easily modifiable.
- Does not look good with stormy weather.
- Tiling is noticeable.
- Lacks water detail.

Problems I intend to solve:

- Simulate water based on empirical data for a more realistic look.
- Have an easy modifiable ocean system.
- Have an ocean that can simulate stormy weather.
- Reduce tiling.
- Have a more detailed ocean.

Main achievements:

- Calculate on GPU.
- Implement JONSWAP spectrum.
- Implement FFT algorithm.
- Simulate foam.
- Shade the ocean.

Acknowledgements

<The page should contain any acknowledgements to those who have assisted with your work. Where you have worked as part of a team, you should, where appropriate, reference to any contribution made by other to the project.>

Note that it is not acceptable to solicit assistance on ‘proof reading’ which is defined as the “the systematic checking and identification of errors in spelling, punctuation, grammar and sentence construction, formatting and layout in the test”; see

https://www.leeds.ac.uk/secretariat/documents/proof_reading_policy.pdf

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Chapter 1

Introduction and Background Research

1.1 Introduction

Ocean surface simulation 1.1 is a field of computer graphics that aims to create realistic representations of the ocean surface. It is an important area of research as it has a wide range of applications, including video games, movies. In video games it used for interactivity and visual appeal, while in movies it is used for visual appeal.

There are many techniques that have been developed to simulate ocean surfaces, from the simple and fast algorithms as sum of sines or Gerstner waves to more complex techniques such as particle simulations and Fast Fourier Transform (FFT) Ocean 1.1.

The primary objective of this project is to simulate a realistic ocean surface, one that does not exhibit any tiling or repeating patterns, and that can accurately represent stormy weather conditions. Therefore, using sum of sines or Gerstner waves is not sufficient.

This is achieved by leveraging the power of the Inverse Fourier Transform (IFFT), a mathematical technique that transforms data from the frequency domain back to the time (or spatial) domain. In the context of this project, it allows us to transform the frequency data of the ocean waves into a spatial representation, i.e., the height map of the ocean surface. This is desirable as it allows us to simulate realistic ocean surfaces that are not only visually appealing, but also physically accurate as we are using real-world data to generate frequencies.



Figure 1.1: FFT Ocean Simulation

1.2 Fourier Transform

The Fourier Transform is a mathematical method that enables us to convert our data from the time domain to the frequency domain, and vice versa. To simplify, imagine we have a smoothie that's too sour. With the Fourier Transform, we could deconstruct our smoothie (time domain) into its ingredients (frequency domain), remove the sour component, and then reconstruct it back. It was invented by Joseph Fourier[1] in 1822, although at his time it didn't have much practical use, however, nowadays it is used in many fields, including signal processing, image processing, and in our case ocean simulation.

To convert our data from the time domain to the frequency domain, we use the Fourier Transform:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx \quad (1.1)$$

, where $f(x)$ is the function in the time domain, $\tilde{f}(\omega)$ is the function in the frequency domain, and ω is a frequency. To convert our data from the frequency domain back to the time domain, we use the Inverse Fourier Transform:

$$f(x) = \int_{-\infty}^{\infty} \tilde{f}(\omega)e^{i\omega x}d\omega \quad (1.2)$$

As we are going to work with discrete data, we are going to use the Discrete Fourier Transform (DFT):

$$x_n = \sum_{k=0}^{N-1} \tilde{x}_k e^{-i2\pi kn/N} \quad (1.3)$$

and the Inverse Discrete Fourier Transform (IDFT):

$$\tilde{x}_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{i2\pi kn/N} \quad (1.4)$$

1.3 Fourier Transform Ocean

A significant challenge associated with the utilization of Gerstner waves, as will be elaborated upon in the related works section, is the necessity to generate multiple waves for each vertex in order to simulate an ocean. This process can be computationally intensive, particularly considering that an oceanic simulation may be constructed of thousands of vertices. In response to this computational demand, J. Tessendorf [2] proposed a method for generating ocean waves using the Fourier Transform.

The main idea is to generate a height map of the ocean surface in the frequency domain, and then convert it back to the time domain using the IFT. Since the IFT produces a periodic height map, we can tile it to create an infinite ocean surface. This means that we can generate one texture and apply it to the entire water body.

The advantage of this method becomes apparent when considering a 512x512 texture, which combines 262,144 distinct waves. In contrast, Gerstner waves experience noticeable

performance loss after 65 waves.

To produce the height map, we first need a function that approximates the frequencies. We call this function a spectrum. J. Tessendorf uses a modified Phillips spectrum that generates Fourier amplitudes in the frequency domain:

$$\tilde{h}_0(\mathbf{k}) = \frac{1}{\sqrt{2}}(\xi_r + \xi_i)\sqrt{P_h(\mathbf{k})} \quad (1.5)$$

where ξ_r and ξ_i is complex number made from real and imaginary part that was drawn from a gaussian random number generator, with mean 0 and standard deviation 1.

We then combine $\tilde{h}_0(\mathbf{k})$ with its conjugate $\tilde{h}_0^*(-\mathbf{k})$, to "produce waves towards and against the wave direction when propagating"[3]:

$$\tilde{h}(\mathbf{k}, t) = \tilde{h}_0(\mathbf{k})e^{i\omega(k)t} + \tilde{h}_0^*(-\mathbf{k})e^{-i\omega(k)t} \quad (1.6)$$

By performing the Inverse Discrete Fourier Transform (IDFT):

$$h(\mathbf{x}) = \sum_{\mathbf{k}} \tilde{h}(\mathbf{k}, t)e^{i\mathbf{k} \cdot \mathbf{x}} \quad (1.7)$$

we obtain the height map, where $\mathbf{x} = (x, y)$ is the position in the texture.



Figure 1.2: Height Map using J. Tessendorf's Spectrum

1.4 Ocean Spectrums

1.4.1 Jerry Tessendorf's Ocean Spectrum

The main thing that defines how ocean will look is the spectrum. Inside Jerry Tessendorf's [2] paper, he proposed modified Phillips spectrum 2.3:

$$P_h(\mathbf{k}) = A \frac{e^{-1/(kL)^2}}{k^4} |\hat{\mathbf{k}} \cdot \mathbf{w}|^6 \quad (1.8)$$

where A is numeric constant, $L = V^2/g$ is the largest possible wave arising from wind speed V and gravity g . The $|\hat{\mathbf{k}} \cdot \mathbf{w}|^6$ is used to suppress waves that are moving perpendicular to the wind direction. However, J. Tessendorf admits that this spectrum has "poor convergence properties at high values of the wavenumber $|\mathbf{k}|$ and suggests to suppress waves that are $l \leq L$, where l is some small constant and multiply the Phillips spectrum by $e^{-k^2 l^2}$.

1.4.2 JONSWAP Spectrum

Regarding all the issues that we experience with "Phillips Spectrum", Horvath Christopher [3] proposed to use JONSWAP spectrum. JONSWAP stands for "Joint North Sea Wave Project" and was developed by Hasselmann [4, et al. in 1973]. This spectrum is improved version of Pierson-Moskowitz Spectrum [5] when Horvath noticed that the wave spectrum is never fully developed. Therefore he added extra peak enhancement factor γ^r .

The JONSWAP spectrum is defined as follows:

$$\begin{aligned}
S_{\text{JONSWAP}}(\omega) &= \frac{\alpha g^2}{\omega^5} \exp\left(-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right) \gamma^r \\
r &= \exp\left(-\frac{(\omega - \omega_p)^2}{2\sigma^2\omega_p^2}\right) \\
\alpha &= 0.076 \left(\frac{U_{10}^2}{Fg}\right)^{0.22} \\
\omega_p &= 22 \left(\frac{g^2}{U_{10}F}\right)^{1/3} \\
\gamma &= 3.3 \\
\sigma &= \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases}
\end{aligned} \tag{1.9}$$

where F is the fetch (distance to lee shore), U_{10} is the wind speed at 10 meters above the sea level and ω is angular frequency.

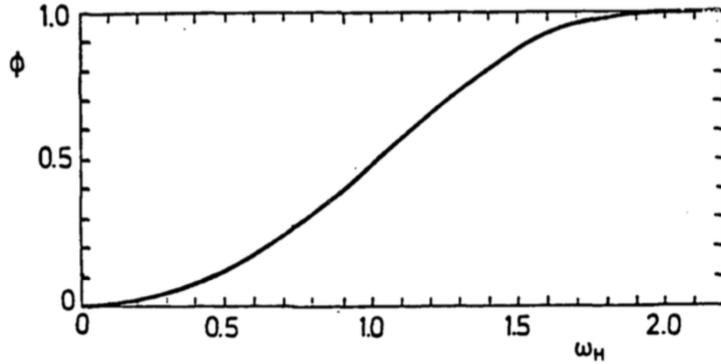
1.4.3 The Texel MARSEN ARSLOE (TMA) Spectrum

The biggest problem with JONSWAP spectrum is that it only works with deep waters. Therefore, Horvath Christopher [3] proposed to use TMA correction for deep water spectrum. TMA was developed by Steven A. Hughes [6], based on the observations made by Kitaigordskii [7]. this is known as Kitaigordskii depth Attenuation function and it takes form:

$$\Phi(\omega_h) = \left[\frac{(k(\omega, k))^{-3} \frac{\partial k(\omega, h)}{\partial \omega}}{(k(\omega, \infty))^{-3} \frac{\partial k(\omega, \infty)}{\partial \omega}} \right] \tag{1.10}$$

$$\omega_h = \omega \sqrt{h/g}$$

where h is the depth of the water. At first glance this function seems to be difficult, however looking at 1.3 we can see that this can be easily approximated.

Figure 1.3: $\Phi(\omega, h)$ as a function of ω_h [6]

Approximation given by Thomson and Vincent [8] is:

$$\Phi(\omega, h) \approx \begin{cases} \frac{1}{2}\omega_h^2 & \text{if } \omega_h \leq 1 \\ 1 - \frac{1}{2}(2 - \omega_h)^2 & \text{if } \omega_h > 1 \end{cases} \quad (1.11)$$

With $\Phi(\omega, h)$ we can now correct the JONSWAP spectrum for shallow waters:

$$S_{\text{TMA}}(\omega, h) = S_{\text{JONSWAP}}(\omega)\Phi(\omega, h) \quad (1.12)$$

Donelan-Banner Directional Spreading

Currently we can't use the TMA spectrum for our project as it has few problems. Firstly, this spectrum is non-directional, so we need to add directionality to it. Secondly, this spectrum accepts ω as input, but as we following J. Tessendorf's [2] paper, we need to use \mathbf{k} as input. To solve this problem Horvath Christopher [3] proposes to use Donelan-Banner Directional Spreading [9]:

$$D(\omega, \theta) = \frac{\beta_s}{2 \tanh(\beta_s \pi)} \operatorname{sech}(\beta_s \theta)^2 \quad (1.13)$$

where,

$$\beta_s = \begin{cases} 2.61(\omega/\omega_p)^{1.3} & \text{for } 0.56 < \omega/\omega_p < 0.95 \\ 2.28(\omega/\omega_p)^{-1.3} & \text{for } 0.95 \leq \omega/\omega_p < 1.6 \\ 10^\epsilon & \text{for } \omega/\omega_p \geq 1.6 \end{cases}$$

$$\epsilon = -0.4 + 0.8393 \cdot e^{-0.567 \ln(\omega/\omega_p)^2}$$

$$\theta = \arctan(k.y/k.x) - \theta_{\text{wind}}$$

In our project we will use $\omega/\omega_p < 0.95$ for first case as it produces more smooth results. Now we can combine the TMA spectrum with the Donelan-Banner Directional Spreading:

$$D_{\text{TMA}}(\omega, \theta) = S_{\text{TMA}}(\omega) \cdot D(\omega, \theta) \quad (1.14)$$

TMA transformation

Lastlly, to make this spectrum usable we need to transform it from ω to \mathbf{k} and according to Horvath Christopher [3] we can transform it like this:

$$S_{\text{TMA}}(\mathbf{k}) = 2S_{\text{TMA}}(\omega, h) \cdot \frac{d\omega}{dk} / k \cdot \Delta k_x \cdot \Delta k_y \quad (1.15)$$

where in our case,

$$\frac{d\omega}{dk} = \frac{g}{2\sqrt{g * \mathbf{k}}}$$

1.5 Cooley-Tukey Fast Fourier Transform (FFT)

When simulating fourier transform ocean majority times is spent on converting from frequency domain to time domain, in other words performing Inverse Fourier Transform. Earlier mentioned IDFT 1.4 has time complexity of $O(n^2)$, and it's clearly that this algorithm is not sufficient to simulate heiger resolution ocean surfaces in real time. In 1805 Gauss Carl Friedrich [10] invented FFT algorithm $O(n \log n)$ and in 1965 Cooley James W. and Tukey John W. [11] reinvented it. The FFT algorithm is based on that there is redundancy in the computation of the DFT, and that we can exploit it to reduce the time complexity, it is important to mention that the FFT algorithm works only when the number of samples is a power of 2.

The basic idea of the FFT algorithm is as follows:

1. Split the input data into two sets, one containing the even samples and the other containing the odd samples and repeat this proccess until a set has 2 samples (bit-reverse sort order).
2. Generate twiddle factor $W_N = e^{-i2\pi k/N}$ and rise it to the power of

$$k = i * N/2^{\text{stage}+1} \bmod N \quad (1.16)$$

where i is the index of the sample, stage is the current stage and N is the number of samples.

3. Perform butterfly operations 1.4, where x and y are complex numbers and W is the twiddle factor

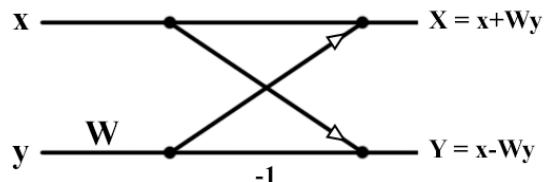


Figure 1.4: Butterfly Diagram

4. The butterfly operations are performed in stages 1.5. For example, if we have 8 samples, we will have $\log(8) = 3$ stages. The first stage is for pairs of points (2-point DFTs), the second stage is for groups of four points (4-point DFTs), and so on.

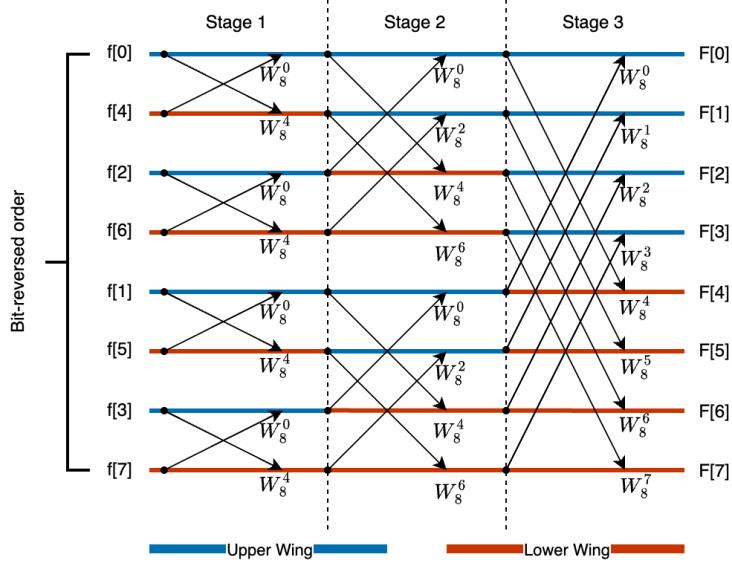


Figure 1.5: 8-point FFT Butterfly Diagram

1.6 Related Work

1.6.1 Gerstner Waves

Gerstner waves provide a simple model for generating somewhat realistic ocean waves. This model is straightforward to implement and is predominantly used in video games. Even major titles, such as “Pokemon Legends: Arceus”, utilize something similar to Gerstner waves to simulate oceanic environments. The concept of Gerstner waves was first introduced by F.J. Gerstner [12] in 1802. The initial known implementation of Gerstner waves in computer graphics was carried out by Fournier and Reeves [13] in 1986. The model is an attempt to approximate the motion of ocean waves, based on the principle that each point in the ocean undergoes a circular motion. If a point is represented as $\mathbf{x}_0 = (x_0, z_0)$, the height and horizontal displacements can be calculated as follows:

$$\mathbf{x} = \mathbf{x}_0 - (\mathbf{k}/k)A \sin(\mathbf{k} \cdot \mathbf{x}_0 - \omega t) \quad (1.17)$$

$$y = A \cos(\mathbf{k} \cdot \mathbf{x}_0 - \omega t) \quad (1.18)$$

$$\omega = \sqrt{gk} \quad (1.19)$$

Here, A is the amplitude, ω is dispersion relationship, and \mathbf{k} is a wave vector that points horizontally in the direction of wave travel and is proportional to the wavelength λ of the wave.

$$k = 2\pi/\lambda \quad (1.20)$$

As presented, the Gerstner waves model is limited to a single wave. To simulate more complex ocean surfaces, multiple waves need to be summed together. This is achieved by summing the waves as follows:

$$\mathbf{x} = \mathbf{x}_0 - \sum_{i=1}^N (\mathbf{k}_i/k_i) A_i \sin(\mathbf{k}_i \cdot \mathbf{x}_0 - \omega_i t) \quad (1.21)$$

Here, N is the number of waves, A_i is the set of amplitudes, ω_i is the set of frequencies, and \mathbf{k}_i is the set of wave vectors. The primary challenge with this approach is that simulating realistic ocean surfaces requires summing a large number of waves together, which can be computationally expensive. Furthermore, Gerstner waves offer limited artistic control, has visible tiling 1.6, and underperforms in simulating stormy weather conditions.



Figure 1.6: Water Tiling in Pokemon Legends: Arceus

1.6.2 Particle Simulation and Machine Learning

Particle-based methodologies are frequently employed for the simulation of water in a manner that is both realistic and visually compelling. However, these methods are associated with a high computational cost.

Recent advancements in Graphics Processing Unit (GPU) technology, in conjunction with research such as that conducted by Libo Huang [14], are incrementally making particle simulations more viable for ocean simulation. Despite this progress, these techniques demand a high-performance GPU and are not yet amenable to widespread real-time applications.

Moreover, there have been considerable advancements in the application of machine learning to accelerate fluid simulations, as demonstrated by the work of Dmitrii Kochkov [15]. However, this is out of this projects scope and won't be discussed.

Chapter 2

Methods

2.1 Algorithm Overview

The algorithm can be split into 3 main parts as shown in figure 2.1

1. Spectrum generation (Only calculated on parameter value change)
2. Frequency generation
3. Conversion from frequency to time domain using IFFT

As shown in figure 2.1 majority of calculations are done on GPU using HLSL executed inside Unity. As FFT algorithm requires input size to be 2^n we going to use 512x512 textures as this provides good performance and high enough details. For simplicity in this project we assume that the texture size is $N \times N$, where N is the size of a texture and N is power of 2.

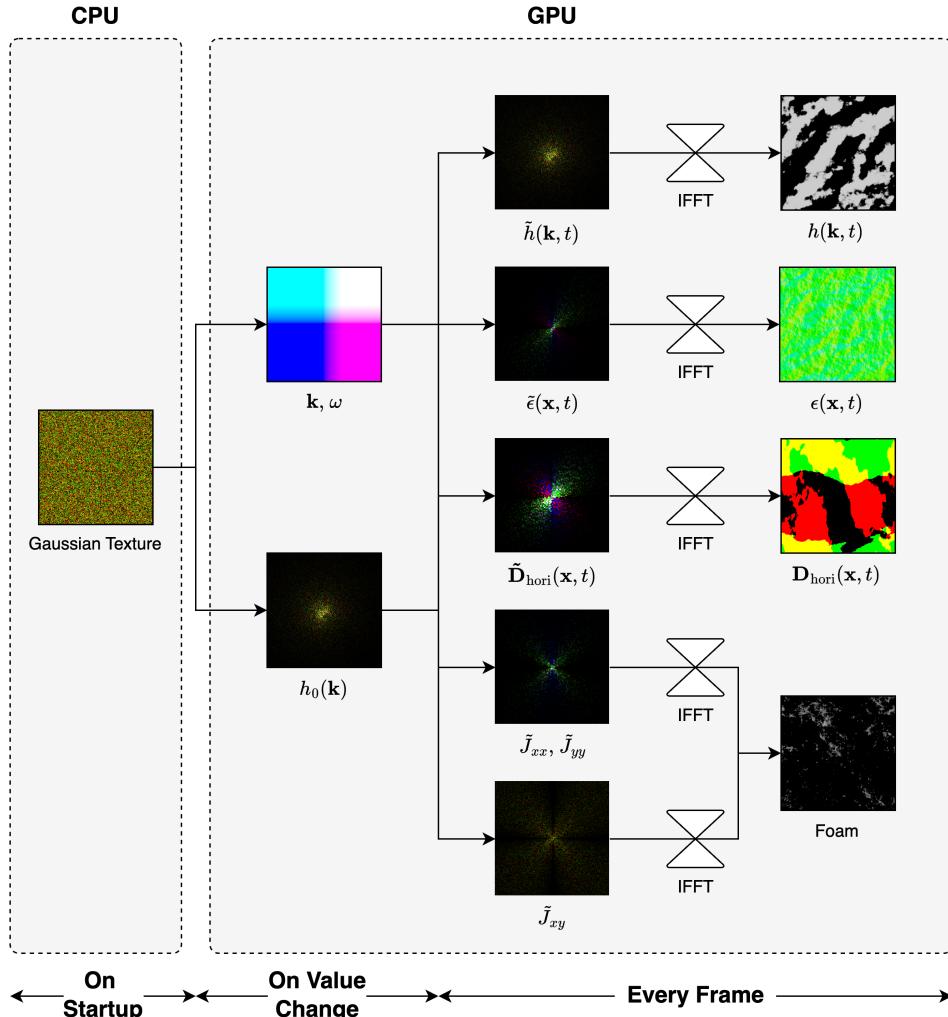


Figure 2.1: Ocean Algorithm

2.2 IFFT

2.2.1 Butterfly Texture 2.2

Firstly we only interested in IFFT algorithm and we will not use FFT. To perform IFFT we going to produce so called butterfly texture by [16, Flügge Flynn-Jorin]. This texture has width of $(\log_2(n))$ and height of n and is precomputed and stored inside the GPU memory once, unless the data size changes. This is 4 channel texture (W_r, W_i, y_t, y_b) , where W_r and W_i are real and imaginary parts of the twiddle factor, y_t and y_b are top and bottom butterfly indices as shown in 1.4.

In butterfly texture coordinate x represents a stage 1.5, while each y represents a butterfly operation between two data points y_t and y_b .

In the first stage we assign y_t and y_b in bit reversed order, on the other stages:

- If the butterfly wing is top half

$$\begin{aligned} y_t &= y_{\text{current}} \\ y_b &= y_{\text{current}} + 2^{\text{stage}} \end{aligned} \tag{2.1}$$

- If the butterfly wing is bottom half

$$\begin{aligned} y_t &= y_{\text{current}} - 2^{\text{stage}} \\ y_b &= y_{\text{current}} \end{aligned} \tag{2.2}$$

We can determine if the wing is upper or lower half:

$$\text{wing} = y_{\text{current}} \bmod 2^{(\text{stage}+1)} \tag{2.3}$$

Lastlly we need to calculate twiddle factors:

$$\begin{aligned} k &= (y_{\text{current}} \cdot n / 2^{\text{stage}+1}) \bmod n \\ W &= \exp(-2\pi i k / n) \end{aligned} \tag{2.4}$$

2.2.2 Performing IFFT

Currently our data is stored in 2D texture. While this algorithm only accepts 1D data. Therefore, we need to perform 1D IFFT on each row "horizontally" and then on each column "vertically" 2.2. By following pseudocode from [16] we can perform IFFT in 2D:

```

float4 butterflyData = ButterflyTexture[ float2( Stage , id.x ) ];
const float2 twiddle = butterflyData.xy;
// fetch top butterfly input sample
topSignal = PingPong0[ float2( butterflyData.z , id.y ) ].xy;
// fetch bottom butterfly input sample
bottomSignal = PingPong0[ float2( butterflyData.w , id.y ) ].xy;

```

```
// perform butterfly operation
h = topSignal + ComplexMult(twiddle, bottomSignal);
```

Listing 2.1: Horizontal Butterfly Operation

Notice that we are using ping-pong buffers to store intermediate results, as we need to perform multiple IFFT passes, in total $\log_2(n)$ passes. After we perform IFFT on each row, we need to perform for each column:

```
float4 butterflyData = ButterflyTexture[float2(Stage, id.y)];
const float2 twiddle = butterflyData.xy;
// fetch top butterfly input sample
topSignal = PingPong0[float2(id.x, butterflyData.z)].xy;
// fetch bottom butterfly input sample
bottomSignal = PingPong0[float2(id.x, butterflyData.w)].xy;
// perform butterfly operation
h = topSignal + ComplexMult(twiddle, bottomSignal);
```

Listing 2.2: Vertical Butterfly Operation

Permutation

Lastly, our data needs to be permuted as you will see later our data is offseted:

$$[\text{freq}(-N/2), \dots, \text{freq}(-1), \text{freq}(0), \text{freq}(1), \dots, \text{freq}(N/2 - 1)] \quad (2.5)$$

While, our IFFT algorithm expected data to be in the following order:

$$[\text{freq}(0), \text{freq}(1), \dots, \text{freq}(N - 1)] \quad (2.6)$$

This causes our data to flip sign in grid like pattern therefore we need to permute our data:

```
float perms[] = {-1, 1};
uint index = int((id.x + id.y) % 2);
float perm = perms[index];
float h = perm * PingPong1[id.xy].x;

PingPong0[id.xy] = float4(h, h, h, 1);
```

Listing 2.3: Data Permutation [16]

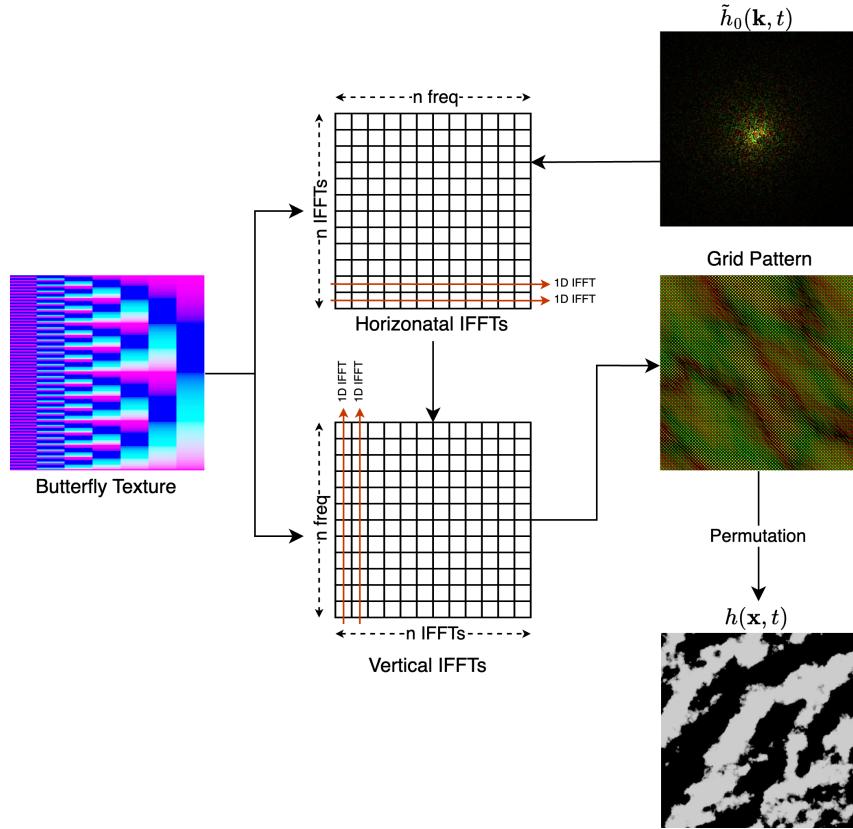


Figure 2.2: IFFT Algorithm

2.3 Ocean Geometry

2.3.1 Spectrum Generation

Symbol	Meaning
$S(\omega)$	Non directional wave spectrum
$S(\omega, \theta)$	Directional wave spectrum
$S(\mathbf{k})$	Directional wave spectrum
\mathbf{k}	wave vector
k	Magnitude of wave vector
l	Length scale of the ocean
ω	dispertion relationship (angular frequency)
U_{10}	Wind speed at 10m above the sea level
F	Fetch (Disntance from lee shore)
θ_{wind}	Wind angle
λ	Choppy factor

Table 2.1: Deffinition Table

Spectrums are responsable for whole look of the ocean. It's importatnt to pick the one that is based on real world data to make our ocean look realistic. Initialy, Tessendorf spectrum 1.8 was implemented as it was straight forward to implement. However, it didn't produce satisfying

results, as waves didn't seem to transfer energy in convincing way, waves didn't seem to follow wave direction and it was hard to have artistic control over the ocean.

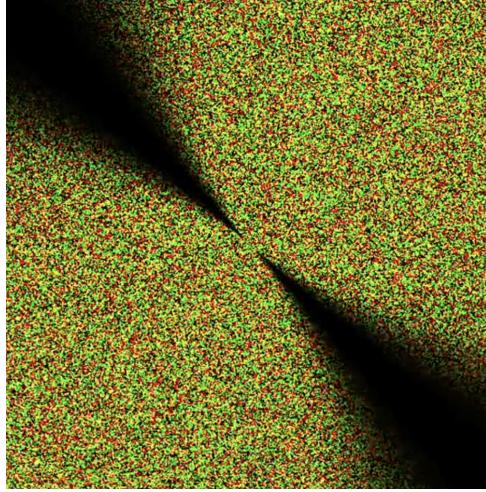


Figure 2.3: Jerry Tessendorf's Spectrum

Therefore, TMA spectrum 1.15 was implemented.

$$S_{\text{TMA}}(\mathbf{k}) = 2S_{\text{TMA}}(\omega, h) \cdot \frac{d\omega}{dk} / k \cdot \Delta k_x \cdot \Delta k_y$$

where $\mathbf{k} = (k_x, k_y)$, $k_x = 2 * \pi(x_x - n/2)/l$, $k_y = 2 * \pi(x_y - n/2)/l$, l is the length scale of the ocean, and $\mathbf{x} = (x_x, x_y)$ is current position in the texture.

This spectrum resulted in more realistic looking ocean with intuitive controls: fetch, wind speed, wind angle, depth. Moreover, it didn't require any effort to make the ocean look realistic and it just worked out of the box.

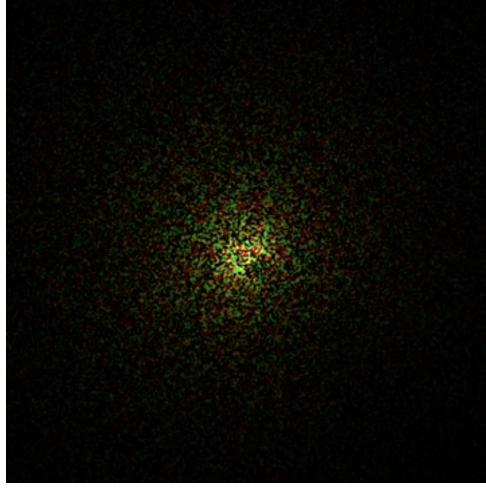


Figure 2.4: TMA spectrum, Frequency Domain
(100x for better visibility)

2.3.2 Height Map Generation

To generate height map, firstly we need to have fourier amplitudes as shown in 1.5, where P_h is TMA Spectrum $S_{\text{TMA}}(\mathbf{k})$. For the next step we need to add fourier amplitude and its complex conjugate to produce "produce waves towards and against the wave direction when

propagating"[3]. In our luck we don't need to recalculate complex conjugate as fourier series are symmetric, so we can just mirror the amplitudes:

$$\tilde{h}_0^* = T_{h_0}(x^*, y^*) \quad (2.7)$$

where $T_{h_0}(x, y)$ is fourier amplitude in precomputed texture at $x^* = (n - x) \bmod n$, $y^* = (n - y) \bmod n$, (x, y) is current position in the texture.

By having combined amplitudes 1.6 we can perform IFFT as shown in 2.2 to produce height map.



Figure 2.5: Height Map using S_{TMA}

2.3.3 Normal Map Generation

Latter we will need extra information about the ocean to calculate ocean shading. Therefore, we need to calculate normal map. Normal map is perpendicular direction to the surface of the ocean. To calculate normal map we need to calculate gradient, which is derivative of the height map. According to [17] the derivative is:

$$\epsilon(\mathbf{x}, t) = i\mathbf{k}\tilde{h}(\mathbf{k}, t) \quad (2.8)$$

Then we need to perform IFFT 2.2 to get normal map in the time domain.

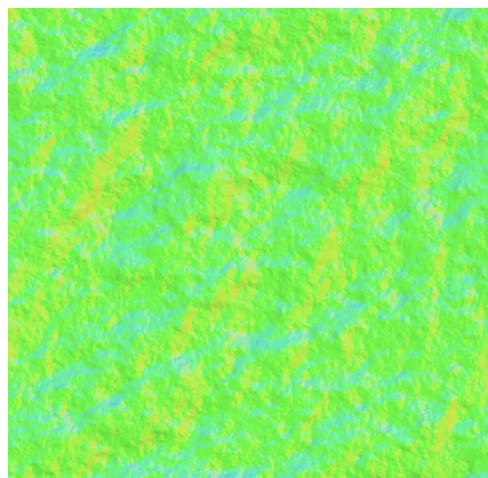


Figure 2.6: Normal Map using S_{TMA}

2.3.4 Choppy Waves

Currently, when rendering ocean it looks too smooth 2.7 and lacks choppiness.

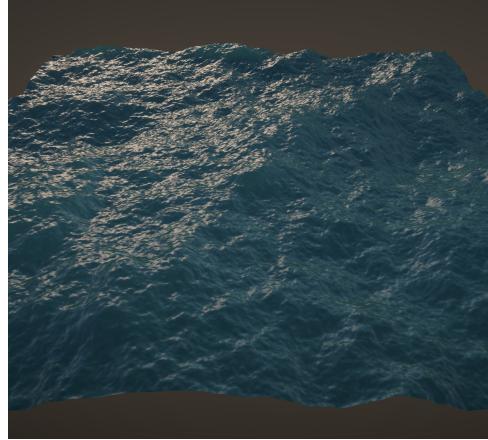


Figure 2.7: Ocean without Choppy Waves

To make waves choppy we need to introduce horizontal displacement. This will not only make waves more choppy but also make energy transfer between waves more realistic. Following formula from [17, J. Tessendorf] we can calculate horizontal displacement:

$$\mathbf{D}_{\text{hori}}(\mathbf{x}, t) = -i \frac{\mathbf{k}}{k} h(\tilde{\mathbf{k}}, t) \quad (2.9)$$

Using this horizontal displacement we can displace our vertices:

$$\mathbf{x} + \lambda \mathbf{D}_{\text{hori}}(\mathbf{x}, t) \quad (2.10)$$

, where λ is "choppy factor". Once again we need to perform IFFT 2.2 to get horizontal displacement in the time domain.

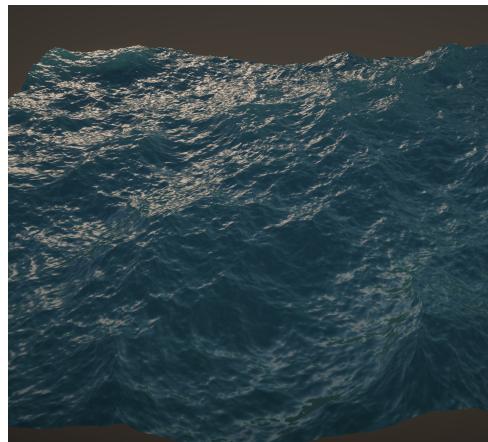


Figure 2.8: Ocean with Choppy Waves

2.4 Foam

2.4.1 Foam Generation

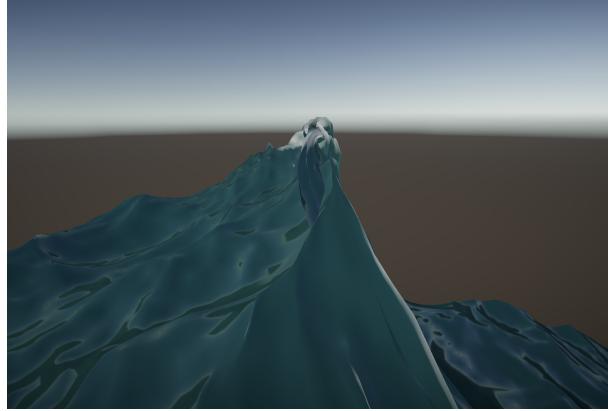


Figure 2.9: Wave Curl At Wave Peek

Ocean foam is a salient and realistic feature of the ocean, contributing to its distinct and authentic appearance. The primary occurrence of foam is observed during wave crashes, a phenomenon facilitated by horizontal displacement. This can be visually discerned at the peaks of waves where the water curls up, as illustrated in Figure 2.9. In essence, our horizontal transformation undergoes an inversion.

As proposed by Tessendorf [2], the rendering of foam can be achieved by calculating the determinant of the Jacobian matrix for horizontal displacement, which helps identify these inversions. In our ocean simulation, the Jacobian matrix provides insights into the changes over the x and z axes. When determinant of the Jacobian matrix is below zero the "wave crash" happens:

$$\text{Det}(\mathbf{x}) = J_{xx} + J_{yy} - J_{xy}J_{yx} \quad (2.11)$$

where,

$$J(\mathbf{x}) = \begin{bmatrix} 1 + \lambda \frac{\partial D_x(\mathbf{x})}{\partial x} & 1 + \lambda \frac{\partial D_x(\mathbf{x})}{\partial y} \\ 1 + \lambda \frac{\partial D_y(\mathbf{x})}{\partial x} & 1 + \lambda \frac{\partial D_y(\mathbf{x})}{\partial y} \end{bmatrix} \quad (2.12)$$

in this case $J_{xy} = J_{yx}$.

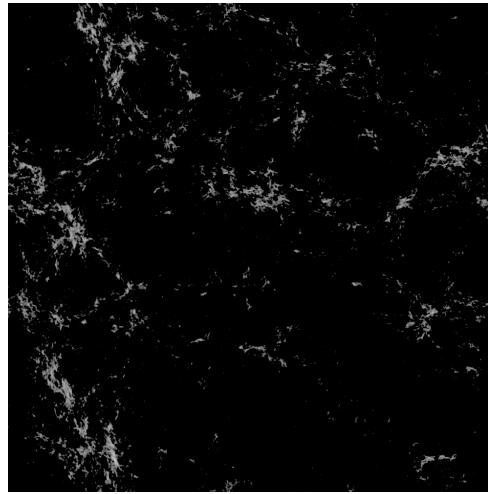


Figure 2.10: Foam Texture

2.4.2 Foam Accumulation

Curreltly the foam apears and diasapears quickly, however in real life the foam accumilates and disappears over time. We can introduce foam accumulation by compering previous and current foam values:

```
float accumulation =
LastFoamValue - FoamDecay * DeltaTime / max(currentFoam, 0.5);
float foam = max(accumulation, currentFoam);
```

Listing 2.4: Foam Accumilation

This results in pleasing foam accumilation:

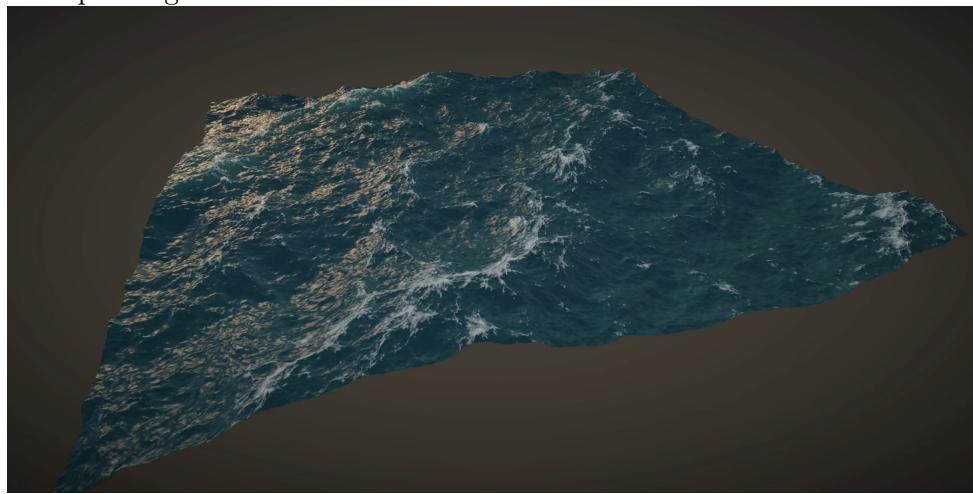


Figure 2.11: Ocean With Foam

2.5 Multiple Cascades

At this stage our ocean with texture 512x512 simulates 262,144 distinc waves however the tilling is still noticeable 2.12.



Figure 2.12: Ocean with Tilling, using 512x512 texture

To counter this we could increase our simulation texture however even FFT becomes expensive really fast. Another approach is to simulate multiple cascades for different wave lengths k 2.13 and different l length scales. We can split our simulation into 3 parts, big waves, medium waves and small waves.

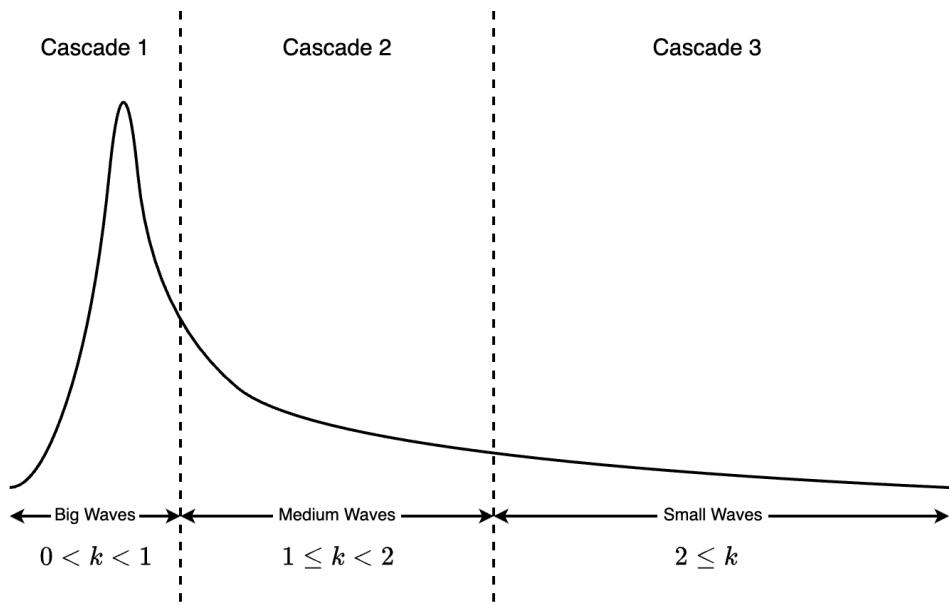


Figure 2.13: Cascades

When it comes to choosing l we need to follow these steps:

1. We chose bigger l for bigger waves as we are more likely to notice tilling with big waves
2. We chose smaller l for smaller waves as we are less likely to notice tilling with small waves

This results in an ocean that has less tilling and more detail 2.14.



Figure 2.14: Ocean with Cascades, using 3x(512x512) textures

One of the main pros of having multiple cascades is reduction in performance cost, as we can have similar results of 512x512 with 3x(256x256) xtextures as shown in figure 2.15.

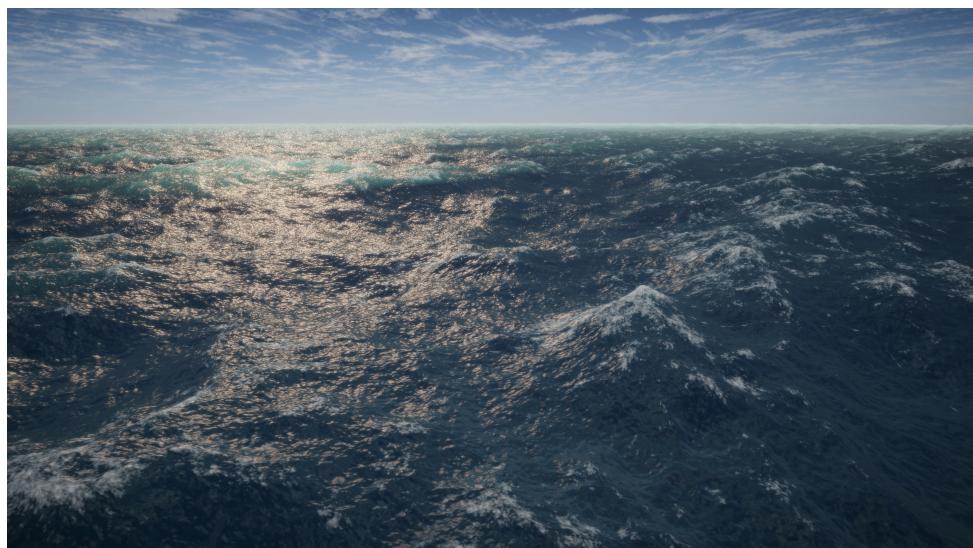


Figure 2.15: Ocean with Cascades, using 3x(256x256) textures

Chapter 3

Results

<Results, evaluation (including user evaluation) etc. should be described in one or more chapters. See the ‘Results and Discussion’ criterion in the mark scheme for the sorts of material that may be included here.>

3.1 Performance

GPU	Texture Size	Time
M1	256x256	4.97ms
M1	512x512	40.27ms
M1	1024x1024	x-code didn't show performance

[FINISH THIS WHEN RESULTS RETRIEVED FROM UNIVERSITY]

3.2 Comparison

3.2.1 Spectrums

The main difference comes to what kind of spectrum you use for FFT based oceans. The proposed "Phillips" spectrum 3.1 by [2, J. Tessendorf] has issues with energy transformation and following the wind direction. The proposed TMA spectrum 3.2 handles energy transformation way more realistic and does not have any issues following the wind direction. This is mostly because that TMA is based on empirical data. You can see clear difference between "Phillips" spectrum and TMA spectrum:

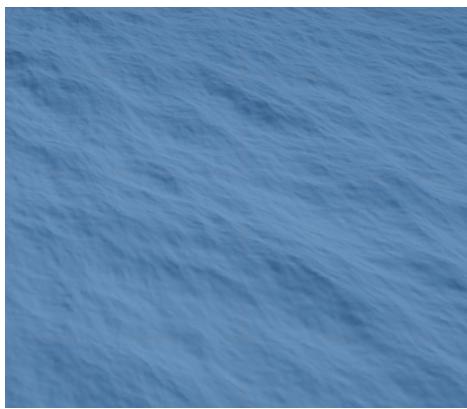


Figure 3.1: Height Map with P_h

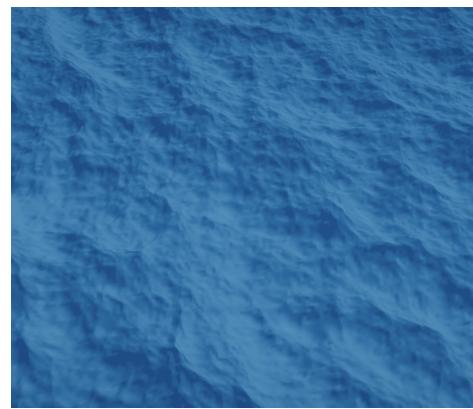


Figure 3.2: Height Map using TMA Spectrum

3.2.2 Real world oceans

When it comes to calm ocean, the FFT based ocean with TMA spectrum performs extremely well and holds the desired shape as shown in figures 3.3 and 3.4.

Spectrum	Size	l_1	l_2	l_3	λ	U_{10}	Fetch	Depth
256x256	TMA	256	128	70	0.9	1 m/s	1000 km	500 m

Table 3.1: Calm Ocean Parameters

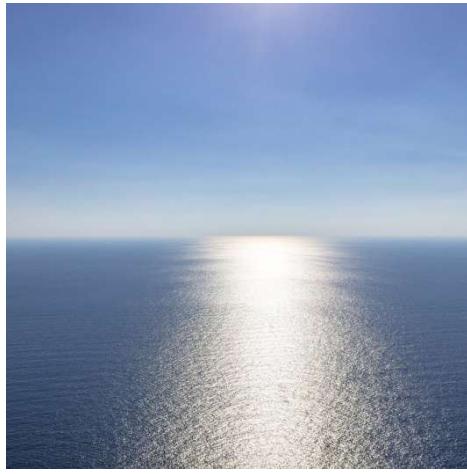


Figure 3.3: Calm Atlantic Ocean
Credits: CC0 Public Domain

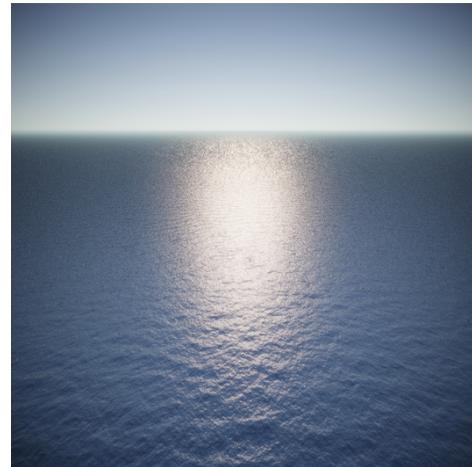


Figure 3.4: FFT Calm Ocean

When it comes to stormy ocean where huge waves is expected the general shape of an ocean is still realistic and can hadle the big waves without any trouble, and because of diffrent cascades the tilling is bearlly noticeable as shown in the following figures 3.5 and 3.6. where each l is the

Spectrum	Size	l_1	l_2	l_3	λ	U_{10}	Fetch	Depth
256x256	TMA	700	256	70	0.9	21 m/s	1000 km	500 m

Table 3.2: Stormy Ocean Parameters

wave length scale of each cascade.



Figure 3.5: Calm Atlantic Ocean
Credits: CC0 Public Domain



Figure 3.6: Height Map using
TMA Spectrum

Chapter 4

Discussion

<Everything that comes under the ‘Results and Discussion’ criterion in the mark scheme that has not been addressed in an earlier chapter should be included in this final chapter. The following section headings are suggestions only.>

4.1 Conclusions

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

4.2 Ideas for future work

This technique produces realistic oceans in a cost-efficient way suitable for real-time rendering. However, it does not support interactive waves. For that, J. Tessendorf proposed a method called “iWaves” [17] and a later upgraded version “eWaves” [18].

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Appendix A

Self-appraisal

<This appendix should contain everything covered by the 'self-appraisal' criterion in the mark scheme. Although there is no length limit for this section, 2—4 pages will normally be sufficient. The format of this section is not prescribed, but you may like to organise your discussion into the following sections and subsections.>

A.1 Critical self-evaluation

A.2 Personal reflection and lessons learned

A.3 Legal, social, ethical and professional issues

<Refer to each of these issues in turn. If one or more is not relevant to your project, you should still explain *why* you think it was not relevant.>

A.3.1 Legal issues

A.3.2 Social issues

A.3.3 Ethical issues

A.3.4 Professional issues

Appendix B

External Material

<This appendix should provide a brief record of materials used in the solution that are not the student's own work. Such materials might be pieces of codes made available from a research group/company or from the internet, datasets prepared by external users or any preliminary materials/drafts/notes provided by a supervisor. It should be clear what was used as ready-made components and what was developed as part of the project. This appendix should be included even if no external materials were used, in which case a statement to that effect is all that is required.>