Linear Colisions

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Abstract

In this paper, we analyze the collision of two objects experiencing minimal dissipative forces. We begin by doing a detailed study of the dissipative force present in our system, which is then followed up by a series of collisions under different initial conditions, different elasticity conditions, and finally under different masses, and calculate relevant coefficients and relations for each of them. In doing this, we are able to analyze how well momentum and kinetic energy are conserved.

1   Introduction

1.1   Objectives

The main objective of this paper is to study the degree of conservation of linear momentum and kinetic energy in one-dimensional elastic and inelastic collisions. To do this, we will pay special attention to the variations in total momentum and kinetic energy before and after the collisions.

1.2   Theoretical Introduction

In general, collisions can be quite a complicated subject to treat and analyze. Much of that is owed to the fact that we live in three dimensions, that bodies are extended and that dissipative forces are not monogenic[1].

As such, we will heavily restrict our problem: Instead of analyzing collisions in general, we will constrain the movement of our bodies to one dimension, drastically reducing our degrees of freedom. Furthermore, we will also use an experimental setup that minimizes the effects of friction. To achieve both, we will use so-called air track, quite common for experiments about rectilinear movement[2]

To begin our analysis of collisions, we will assume that linear momentum is conserved, or, equivalently, that there are no external forces[3]:

$$\boldsymbol{p_i} = \boldsymbol{p_f}$$

However, since we are working in a one-dimensional picture, we can talk about only the (signed) magnitude of momentum without loss of generality. Thus, to avoid getting lost in sub-indices and vectors, we will say that $\boldsymbol{p_i} = p_i \boldsymbol{e_x}$ and $\boldsymbol{p_f} = p_f \boldsymbol{e_x}$. From now on, we will only talk about p_i and p_f, not $\boldsymbol{p_i}$ and $\boldsymbol{p_f}$. We will follow the same convention for the velocities. As such;

$$p_i = m_1 v_{1i} + m_2 v2i = m_1 v_{1f} + m_2 v2f = p_f \tag{1}$$

Notation out of the way, we begin to talk about the kinetic energy of our system. A priori, it has no reason to be conserved: Work is done to (minimally) deform our solid. Therefore, we will write that:

$$T_f = T_i - \Delta Q \tag{2}$$

Where ΔQ represents all the polygenic work done in the system.

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[1]Polygenic forces are those that have origin in more than one source, such as a frictional forces. A general discussion of polygenic and monogenic forces is present in Cornelius Lanczos. *The variational principles of mechanics.* Dover Publications, 2005, p. 30

[2]A very basic treatment of the air-track can be found in Daniel Kleppner and Robert J. Kolenkow. *An introduction to mechanics.* Cambridge Univ. Press, 2014, p. 54

[3]This should sound strange, since nowhere have we claimed to have removed air resistance from the picture. As we will see, linear momentum *will not* be conserved, but only by an amount small enough to make this approximation valid for most situations.

2 Experimental Procedure

2.1 Measurements

3 Results

4 Final Discussion

References

Kleppner, Daniel and Robert J. Kolenkow. *An introduction to mechanics.* Cambridge Univ. Press, 2014.

Lanczos, Cornelius. *The variational principles of mechanics.* Dover Publications, 2005.